

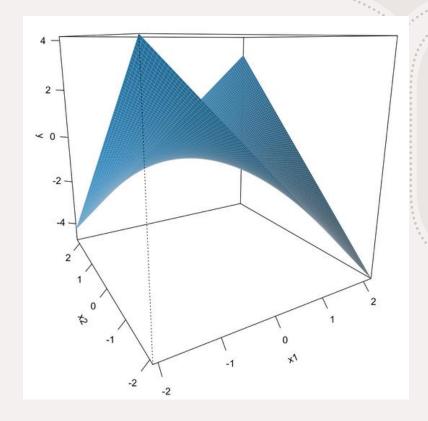
#### Interaction Effects

- An **interaction effect** occurs when the effect of a predictor variable on the outcome variables *varies depending on the level* of another predictor variable.
  - The second predictor variable is often called a moderator

• When we *don't* include an interaction term in a model with multiple continuous predictors, we are **assuming** that the relationship between  $X_1$  and Y does not vary depending on the level of  $X_2$  (and vice versa)

### Visualization of an Interaction Effect

- An interaction effect is represented by a curve in the plane that is being fit to the pattern in the data
- The sides of the plane do not have constant slope. The slope varies depending on the level of the other predictor(s) in the model.



## Example

A researcher is interested in the variables that predict people's ability to learn a new skill, like learning how to play a musical instrument. Previous research has found that the amount of practice people get at a new skill predicts better learning of that skill. They are also aware of new research, though, that suggests that sleep plays an important part in skill learning. The researcher wants to examine whether there is an interaction effect between practice and sleep. In other words, the researcher is interested in whether the relationship between practice and skill learning varies depending on the amount of sleep people get.

skill_learning	practice	sleep
		0
		0 0 0 0 0 0 0
		****************

# Example

- **skill\_learning** = an expert's rating on how well new players are able to play a song on the guitar from 1 (poorly) to 10 (very well)
- practice = number of hours spent practicing on the guitar
- sleep = number of hours of sleep people get on the nights after they practice

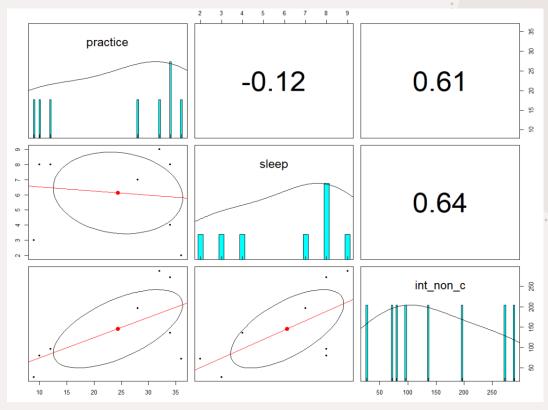
skill_learning	practice	sleep
10	34	8
3	34	4
1	12	8
2	9	3
3	36	2
8	28	7
10	32	9
2	10	8

# Centering Continuous Predictors

• It's now necessary to center the continuous predictors to reduce multicollinearity between them and the interaction effect

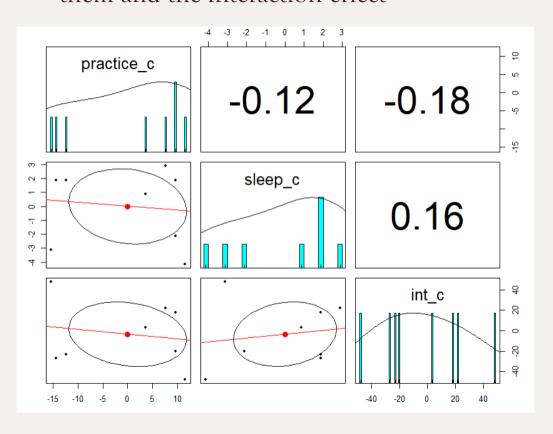
Uncentered Interaction Term

practice	sleep	prac*sleep
34	8	272
34	4	136
12	8	96
9	3	27
36	2	72
28	7	196
32	9	288
10	8	80



# Centering Continuous Predictors

• It's now necessary to center the continuous predictors to reduce multicollinearity between them and the interaction effect



#### Centered Interaction Term

practice_c	sleep_c	prac_c*sleep_c
9.625	1.875	18.05
9.625	-2.125	-20.45
-12.375	1.875	-23.20
-15.375	-3.125	48.05
11.625	-4.125	-47.95
3.625	0.875	3.17
7.625	2.875	21.92
-14.375	1.875	-26.95

### Interpreting Parameter Estimates

**Fit a model** predicting `skill\_learn` from `practice\_c`, `sleep\_c`, and the interaction between the two, `practice\_c\*sleep\_c`:

```
model <- lm(skill_learn ~ practice_c*sleep_c, data = data)</pre>
summary(model)
Coefficients:
                  Estimate Std. Error t value Pr(>|t|)
                              0.46329 10.962 0.000393 ***
(Intercept)
                   5.07856
practice_c
                   0.24344
                              0.04226
                                        5.760 0.004506 **
sleep_c
                              0.18574
                   0.70832
                                        3.814 0.018882 *
practice_c:sleep_c 0.05949
                              0.01595
                                        3.729 0.020312 *
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Full Model Estimate Equation:

```
skill_learn' = 5.08 + 0.24*practice_c + 0.71*sleep_c + 0.06*(practice_c*sleep_c)
```

# Rearranging the full model equation into "simple relationship" equations

• Full Model Estimate Equation:

- $b_0 = 5.08$
- $b_1 = 0.24$
- $b_2 = 0.71$
- $b_3 = 0.06$

• In order to accurately interpret the meaning of each parameter estimate in a model containing a continuous by continuous interaction term, we must rearrange the full model estimate equation into "simple" relationship equations that represent the relationship between  $X_1$  and Y across different levels of  $X_2$  (or vice versa)

#### "Simple" Relationship Equations

• Full Model Equation: skill\_learn' = b0 + b1\*practice\_c + b2\*sleep\_c + b3\*(practice\_c\*sleep\_c)

• "Simple" relationship between practice and skill learning at different levels of sleep:

• "Simple" relationship between sleep and skill learning at different levels of practice:

### "Simple" relationship between practice and skill learning

• Full equation: skill\_learn' = 5.08 + 0.24\*practice\_c + 0.71\*sleep\_c + 0.06\*(practice\_c\*sleep\_c)

• Simple relationship between practice and skill learning equation:

- $b_0 = 5.08$ , the intercept for the "simple" practice and skill-learning relationship when sleep\_c = 0 (at the mean of sleep)
- $b_2 = 0.71$ , the change in the intercept for "simple" practice and skill-learning relationship for every 1-unit increase in sleep\_c
- $b_1 = 0.24$ , the slope for the "simple" practice and skill-learning relationship when sleep\_c = 0 (at the mean of sleep)
- $b_3 = 0.06$ , the change in the slope for the "simple" practice and skill-learning relationship for every 1-unit increase in sleep\_c

### "Simple" relationship between sleep and skill learning

• Full equation: skill\_learn' = 5.08 + 0.24\*practice\_c + 0.71\*sleep\_c + 0.06\*(practice\_c\*sleep\_c)

• Simple relationship between practice and skill learning equation:

- $b_0 = 5.08$ , the intercept for the "simple" sleep and skill-learning relationship when practice\_c = 0 (at the mean of practice)
- $b_1 = 0.24$ , the change in the intercept for "simple" sleep and skill-learning relationship for every 1-unit increase in practice\_c
- $b_2 = 0.71$ , the slope for the "simple" sleep and skill-learning relationship when practice\_c = 0 (at the mean of practice)
- $b_3 = 0.06$ , the change in the slope for the "simple" sleep and skill-learning relationship for every 1-unit increase in practice\_c

# Significance Testing when Interactions are Included in the Model

```
model <- lm(skill_learn ~ practice_c*sleep_c, data = data)</pre>
summary(model)
Coefficients:
                   Estimate Std. Error t value Pr(>|t|)
(Intercept)
                    5.07856
practice_c
                    0.24344
                               0.04226
                               0.18574
sleep_c
                    0.70832
                                         3.814 0.018882 *
practice_c:sleep_c
                    0.05949
                               0.01595
                                         3.729 0.020312 *
                  '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 '
```

- Each row corresponds to a significance test for the parameter estimate on each row
- Q: What does the significance test for  $b_1 = 0.24$  correspond to?
  - A test of whether the relationship between practicing and skill learning is significantly different from zero when and only when sleep\_c is equal to zero
- Q: What does the significance test for  $b_2 = 0.71$  correspond to?
  - A test of whether the relationship between sleep and skill learning is significantly different from zero when and only when practice\_c is equal to zero

# Significance Testing when Interactions are Included in the Model

```
model <- lm(skill_learn ~ practice_c*sleep_c, data = data)</pre>
summary(model)
Coefficients:
                   Estimate Std. Error t value Pr(>|t|)
(Intercept)
                    5.07856
practice_c
                    0.24344
                               0.04226
                               0.18574
sleep_c
                    0.70832
                                          3.814 0.018882 *
practice_c:sleep_c
                    0.05949
                               0.01595
                                          3.729 0.020312 *
                  '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' '
```

- When interaction effects are included in the model:
  - The parameter estimates corresponding to the individual predictors in the model *do not correspond to* the "overall" relationship between each individual predictor and the outcome variable
  - Rather, the rows on which individual predictors are listed test the significance of the relationship between that predictor and the outcome variable at a particular value of the other predictor

# Significance Testing when Interactions are Included in the Model

```
model <- lm(skill_learn ~ practice_c*sleep_c, data = data)</pre>
summary(model)
Coefficients:
                   Estimate Std. Error t value Pr(>|t|)
(Intercept)
                    5.07856
practice_c
                    0.24344
                               0.04226
                                         5.760 0.004506 **
sleep_c
                    0.70832
                               0.18574
                                         3.814 0.018882 *
practice_c:sleep_c 0.05949
                               0.01595
                                         3.729 0.020312 *
                  '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ''
```

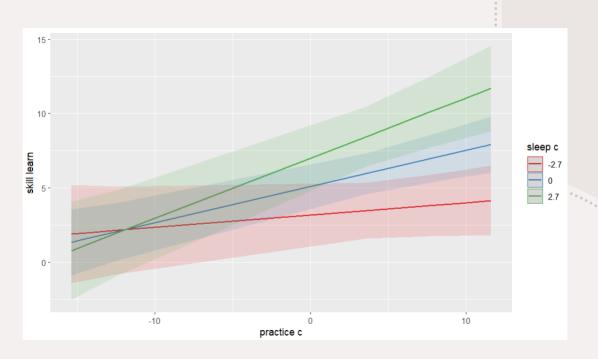
- Q: What does the significance test for  $b_3 = 0.06$  correspond to?
  - A test of whether the relationship between practice and skill learning significantly differs *depending* on the *level* of sleep (or vice versa)

# Simple Slopes Analysis

- Simple slopes analysis allows us to test the significance of the relationship between predictor 1 and the outcome variable at different levels of predictor 2
  - The most typical levels of predictor 2 that are chosen are ± 1SD and the mean
- For example, let's examine the relationship between practice and skill learning at the mean and ± 1SD on sleep

# Visualizing Simple Slopes

- Q: What appears to be the nature of the significant interaction effect?
  - The relationship between practicing and skill learning appears to be stronger at high (+1SD) levels of sleep compared to low (-1SD) levels of sleep



# Performing Simple Slopes Analysis

- Q: What is the nature and significance of the simple relationship between practice and skill learning at -1SD on sleep?
  - At -1SD on sleep, there is a slightly positive, but non-significant, relationship between practice and skill learning, b = 0.08, t(4) = 1.50, p = .207.
- Q: What is the nature and significance of the simple relationship between practice and skill learning at the mean of sleep?
  - At the mean of sleep, there is a significant, positive relationship between practice and skill learning, b = 0.24, t(4) = 5.76, p = .005.
- Q: What is the nature and significance of the simple relationship between practice and skill learning at +1SD on sleep?
  - At +1SD on sleep, there is a significant, positive relationship between practice and skill learning, b = 0.40, t(4) = 6.22, p = .003.

#### simple\_slopes(model)

```
practice_c sleep_c Test Estimate Std. Error t value df Pr(>|t|)
1 -11.879604 sstest 0.0016 0.2838 0.0057 4 0.995720
2 0 sstest 0.7083 0.1857 3.8136 4 0.018882
3 11.879604 sstest 1.4150 0.2456 5.7624 4 0.004500
4 sstest -2.695896 0.0831 0.0552 1.5037 4 0.207087
5 sstest 0 0.2434 0.0423 5.7604 4 0.004506
6 sstest 2.695896 0.4038 0.0650 6.2168 4 0.003408
```