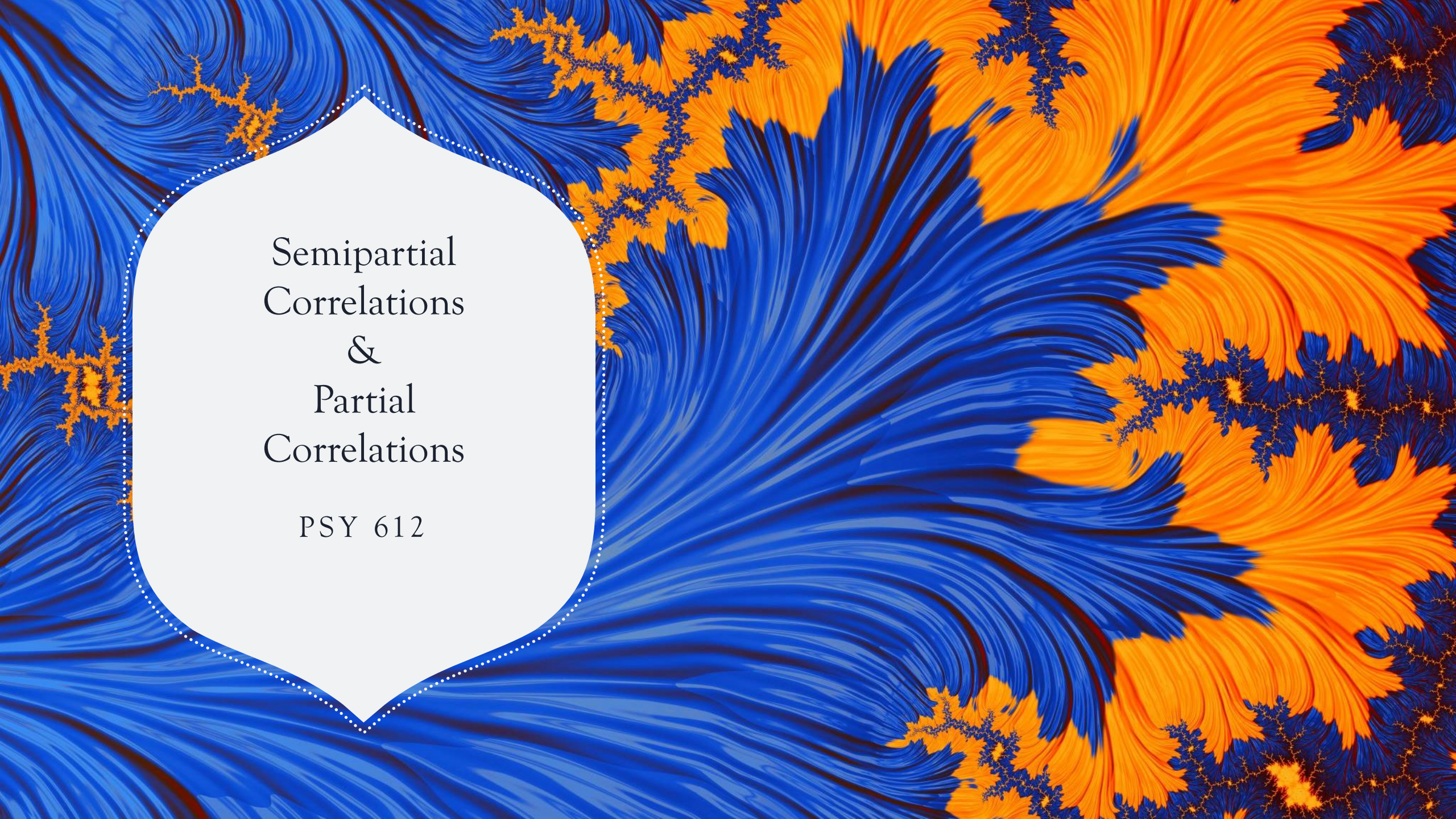

Semipartial
Correlations
&
Partial
Correlations
PSY 612

Types of Correlations

- **Zero-order correlation**: the correlation between a predictor variable, X, and an outcome variable, Y, ignoring their relationship with any other variables

- **Semipartial correlation**: the correlation between an outcome variable, Y, with the part of a predictor variable, X_1, that is unrelated to a second predictor variable, X_2

- **Partial correlation**: the correlation between the part of an outcome variable, Y, and the part of a predictor variable, X_1, that are both unrelated to a second predictor variable, X_2

Example

A researcher is interested in the theory that people who plan their goals are more likely to achieve them. To test this theory, the researcher wants to examine the relationship between **goal achievement** (Y) and **planfulness** (X1)**.**

However, **grit**, or one's perseverance and passion for pursuing long-term goals, has also been found to relate to people's goal achievement.

The current researcher wants to examine the **unique** relationship between **goal achievement** and **planfulness** when ***controlling for grit*** to demonstrate that planfulness has a relationship with goal achievement **over and above** the relationship that grit has with goal achievement.

Goal achievement Y	**Planfulness X1**	**Grit X2**

Example

Goal achievement was measured using the number of days during winter term that students went to the rec center among those who set a new years goal to exercise regularly.

Planfulness was measured on a scale from 1 (strongly disagree) to 5 (strongly agree) using items like, "Developing a clear plan when I have a goal is important to me."

Grit was measured on a scale from 1 (not at all like me) to 5 (very much like me) using items like, "I finish whatever I begin."

Goal achievement Y	Planfulness X1	Grit X2
48	5	4
21	4	2
15	3	3
6	1	1
10	2	2

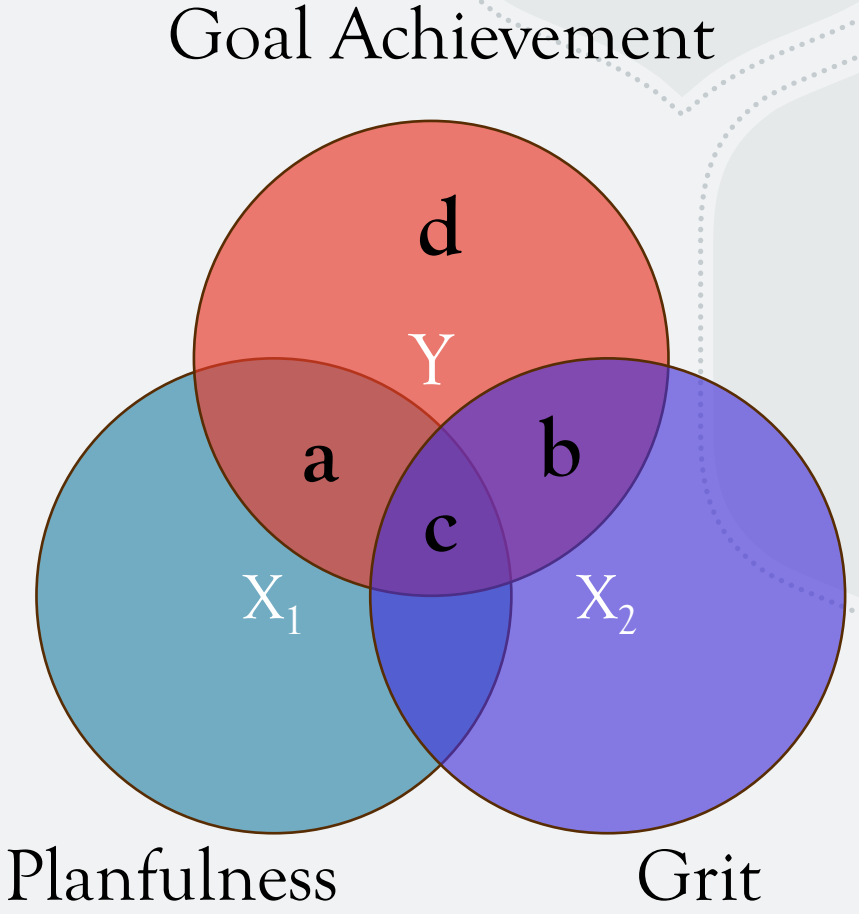
Example

The zero-order correlations between these three variables are shown in the correlation matrix below:

	Goal achievement	**Planfulness**	**Grit**
Goal achievement	1	-	-
Planfulness	0.9033	1	-
Grit	0.8571	0.8321	1

Explaining Variance in Y

- The venn diagram represents the variance in goal achievement (Y) that is related to planfulness (X1) and grit (X2)
- Notice that, because planfulness and grit are correlated with each other, there is **overlap** in the variance they explain in goal achievement
- Some of the variance explained by planfulness is **redundant** with the variance explained by grit in goal achievement scores



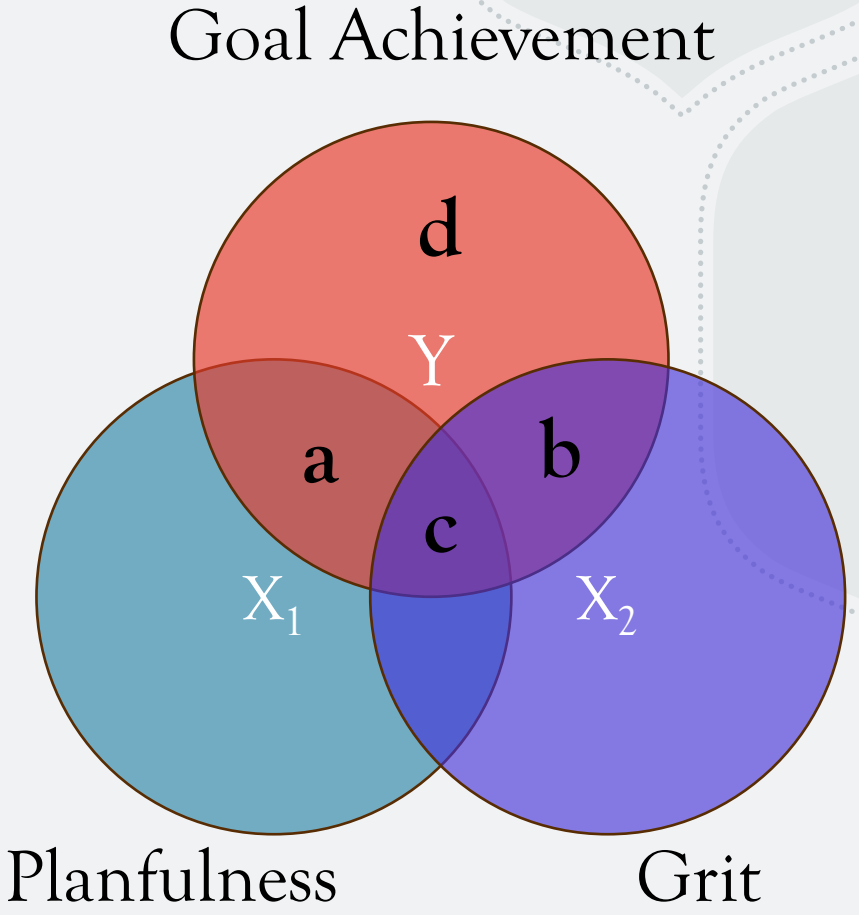
Explaining Variance in Y

a = the variance in goal achievement that is **uniquely** explained by its relationship with planfulness

b = the variance in goal achievement that is **uniquely** explained by its relationship with grit

c = the variance in goal achievement that is **redundantly explained by *both*** planfulness and grit

a + b + c + d = the total variance in goal achievement scores ($SS_{Total} = SS_Y$)



Semipartial Correlation

Semipartial correlation: the correlation between an outcome variable, Y, with the part of a predictor variable, X_1, that is unrelated to a second predictor variable, X_2

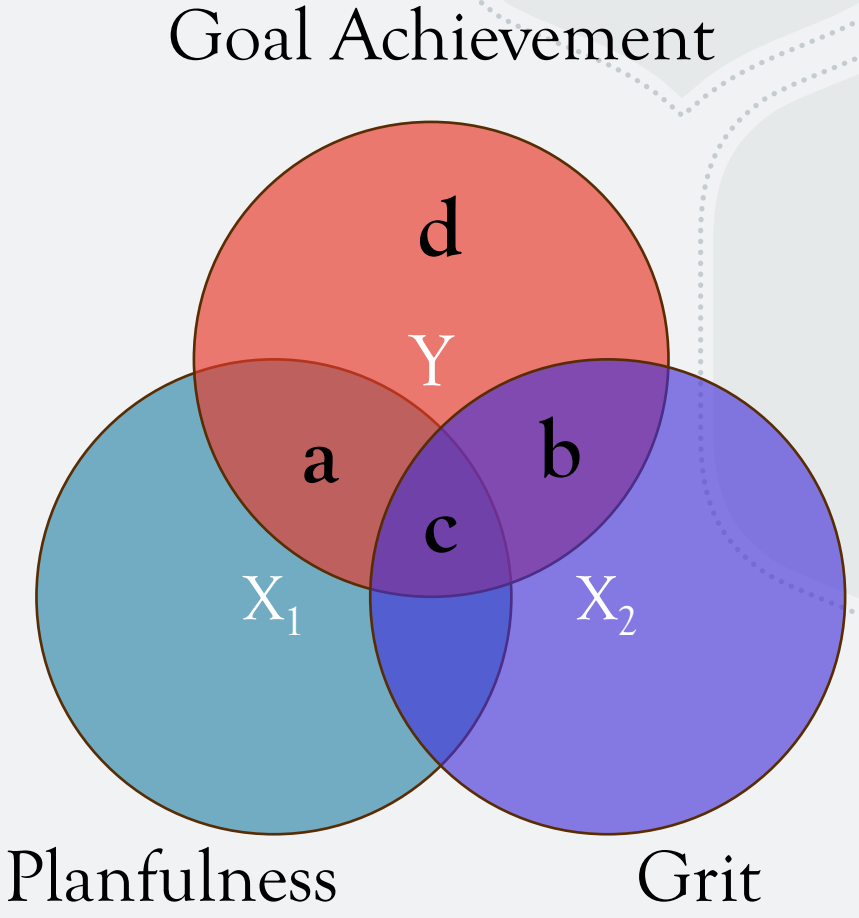
The formula for the semipartial correlation between Y and X1 is:

$$sr_1 = \frac{r_{Y1} - r_{Y2}r_{12}}{\sqrt{1 - r^2{}_{12}}}$$

r_{Y1} = the zero-order correlation between Y and X1

r_{Y2} = the zero-order correlation between Y and X2

r_{12} = the zero-order correlation between X1 and X2

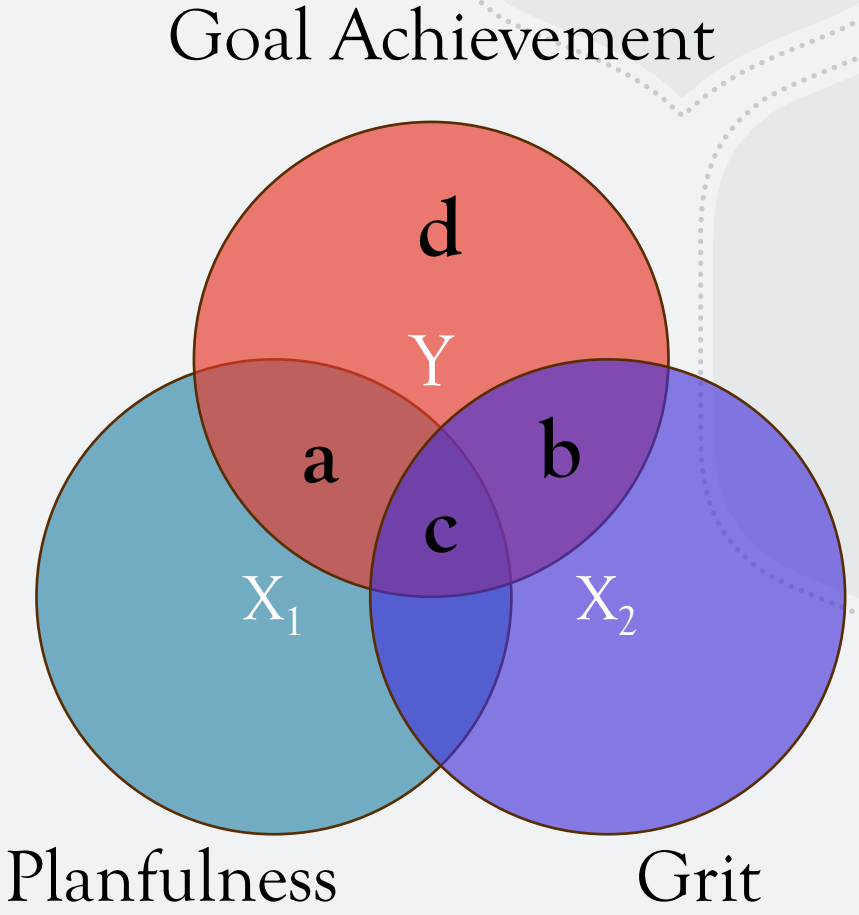


Semipartial Correlation

- When examining the unique relationship between planfulness and goal achievement using a **semipartial correlation**, the variance explained is equal to:

$$sr_1{}^2 = \frac{a}{a + b + c + d}$$

- The **semipartial correlation** measures the relationship between the part of X1 that is unrelated to X2 and <u>all</u> of Y



Semipartial Correlation

For our example (by hand):

	Goal Ach (Y)	Planfulness (X1)	Grit (X2)
Goal Ach (Y)	1	-	-
Planfulness (X1)	0.9033	1	-
Grit (X2)	0.8571	0.8321	1

$$sr_1 = \frac{0.9033 - (0.8571 * 0.8321)}{\sqrt{1 - 0.8321^2}}$$

$$sr_1 = \frac{0.1901}{\sqrt{0.3076}} = 0.34$$

$sr_1^2 = (0.34^2) = .1156$, or $\approx 12\%$

For our example (in R):

```
spcor.test(data$goal_ach, # Y
           data$planfulness, # X1
           data$grit) # X2
```

```
   estimate   p.value statistic n gp  Method
1 0.3428421 0.6571579 0.5161332 5  1 pearson
```

```
> (0.3428)^2
[1] 0.1175118
```

Planfulness explains approximately 12% of the variance in goal achievement over and above the variance explained by grit.

Partial Correlation

Partial correlation: the correlation between the part of an outcome variable, Y, and the part of a predictor variable, X_1, that are both unrelated to a second predictor variable, X_2

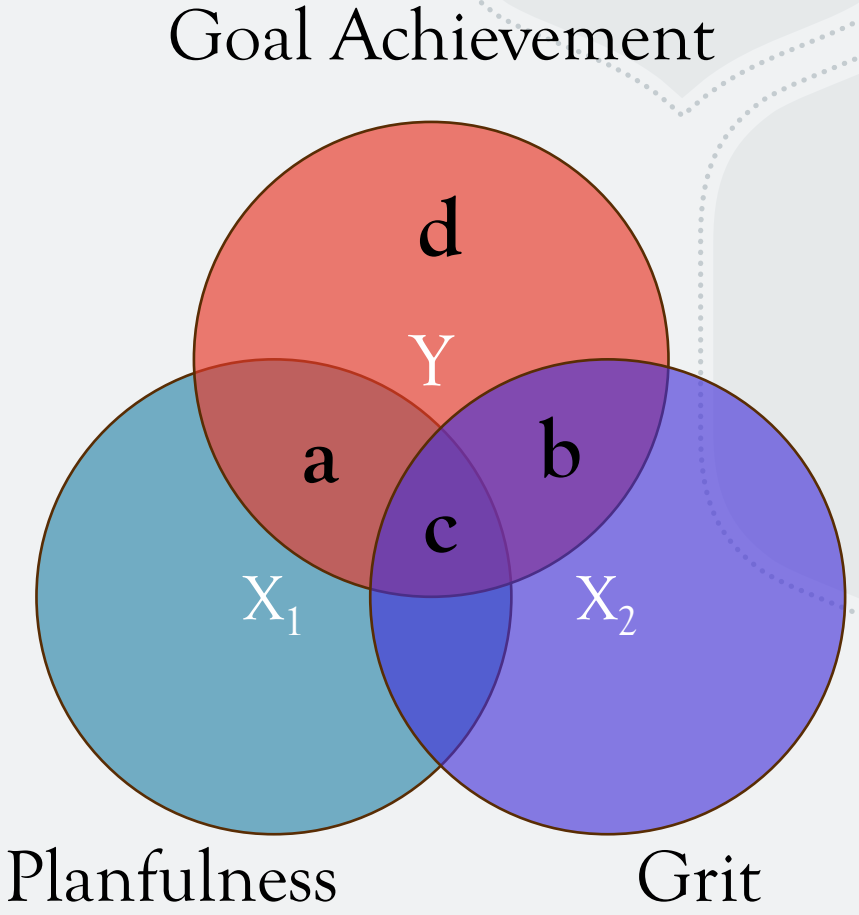
The formula for the partial correlation between Y and X1 is:

$$pr_1 = \frac{r_{Y1} - r_{Y2}r_{12}}{\sqrt{1 - r^2_{Y2}}\sqrt{1 - r^2_{12}}}$$

r_{Y1} = the zero-order correlation between Y and X1

r_{Y2} = the zero-order correlation between Y and X2

r_{12} = the zero-order correlation between X1 and X2

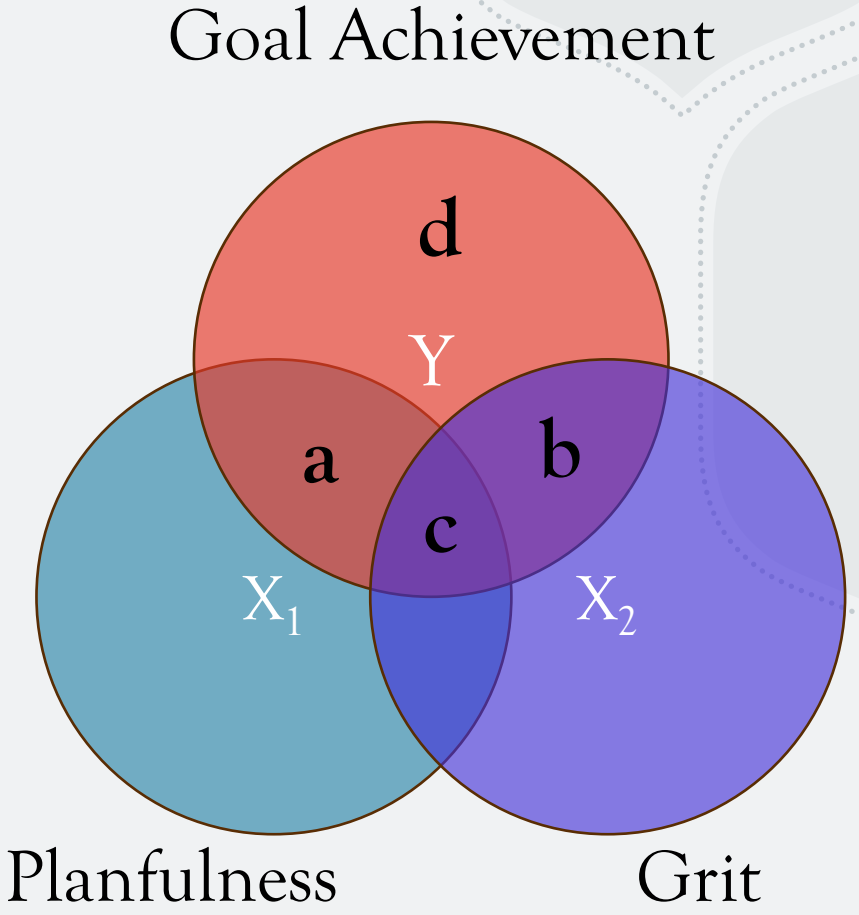


Partial Correlation

- When examining the unique relationship between planfulness and goal achievement using a **partial correlation**, the variance explained is equal to:

$$pr_1{}^2 = \frac{a}{a + d}$$

- The **partial correlation** measures the relationship between the part of X1 that is unrelated to X2 and the part of Y that is unrelated to X2



Partial Correlation

For our example (by hand):

	Goal Ach (Y)	Planfulness (X1)	Grit (X2)
Goal Ach (Y)	1	-	-
Planfulness (X1)	0.9033	1	-
Grit (X2)	0.8571	0.8321	1

$$pr_1 = \frac{.9033 - (.8571 * .8321)}{\sqrt{1 - .8571^2}\sqrt{1 - .8321^2}}$$

$$pr_1 = \frac{.1901}{\sqrt{.2654}\sqrt{.3076}} = 0.67$$

$pr_1^2 = (0.67^2) = .4489$, or $\approx 45\%$

For our example (in R):

```
pcor.test(data$goal_ach, # Y
          data$planfulness, # X1
          data$grit) # X2
```

```
  estimate p.value statistic n gp  Method
1  0.66553 0.33447  1.261037 5  1 pearson
```

```
> (0.6655)^2
[1] 0.4428902
```

Planfulness explains approximately 45% of the variance in goal achievement after partialling out the relationship that both goal achievement and planfulness have with grit.

Semipartial Correlations

- Semipartial correlations are **most similar conceptually** to regression coefficients (i.e., the parameter estimates) in a linear model with multiple continuous predictors.

Semipartial correlation between X1 and Y

$$sr_1 = \frac{r_{Y1} - r_{Y2}r_{12}}{\sqrt{1 - r^2_{12}}}$$

Standardized regression coefficient for X1 predicting Y

$$b^*_1 = \frac{r_{Y1} - r_{Y2}r_{12}}{1 - r^2_{12}}$$

- The only difference is the square root in the denominator, which bounds the semipartial correlation to between ± 1, whereas the standardized regression coefficient can be larger than 1.

Semipartial Correlations

- A semipartial correlation is used to demonstrate that a predictor has a relationship with an outcome variable **over and above** the relationship that other predictor variables have with the outcome variable
- This is how we will interpret the meaning of the **parameter estimates** (aka, regression coefficients) in a model with **multiple continuous predictors**

Semipartial correlation between X1 and Y

$$sr_1 = \frac{r_{Y1} - r_{Y2}r_{12}}{\sqrt{1 - r^2_{12}}}$$

Standardized regression coefficient for X1 predicting Y

$$b^*_1 = \frac{r_{Y1} - r_{Y2}r_{12}}{1 - r^2_{12}}$$

Question: What is the standardized regression coefficient equal to if there is **no correlation (i.e., no redundancy)** between predictors X1 and X2?

Partial Correlations

- Partial correlations are used to rule out **third variable** arguments by removing X2's relationship from *both* X1 and Y

- Demonstrates whether X1 and Y are still related even after completely removing the relationship between each and a third variable