Linear Regression with Multiple Categorical Predictors

aka, Factorial ANOVA

Multiple Categorical Predictors

- Previously, we've discussed linear models that included a single categorical predictor with 2 or more categorical predictors

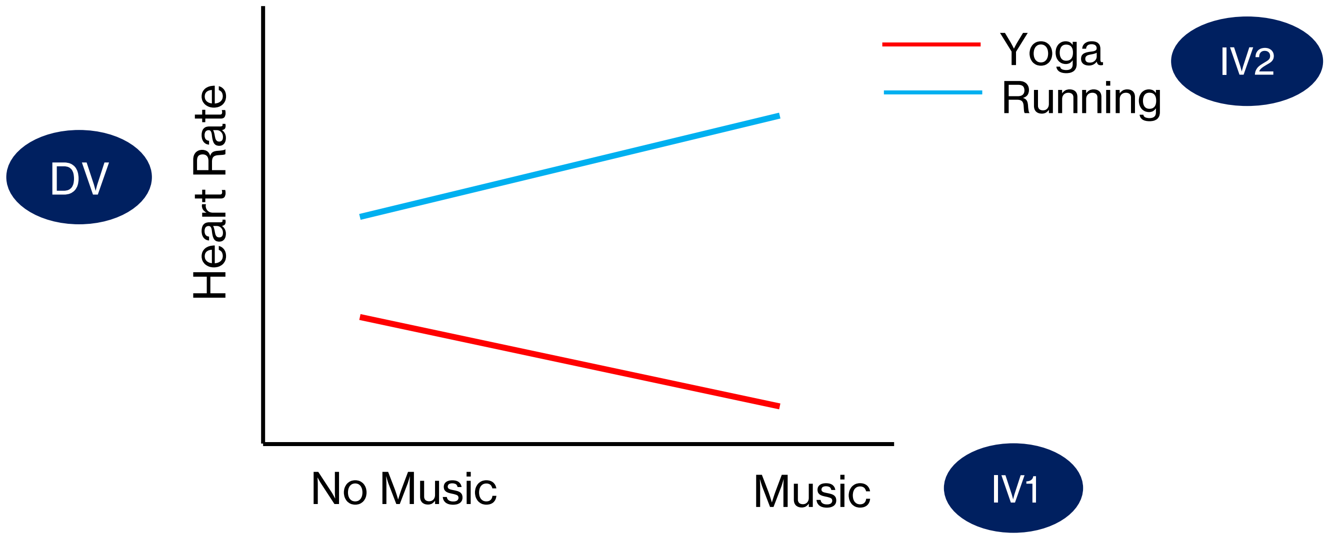
- Next, we'll introduce research scenarios where there are **multiple categorical predictors** (i.e., multiple independent variables)

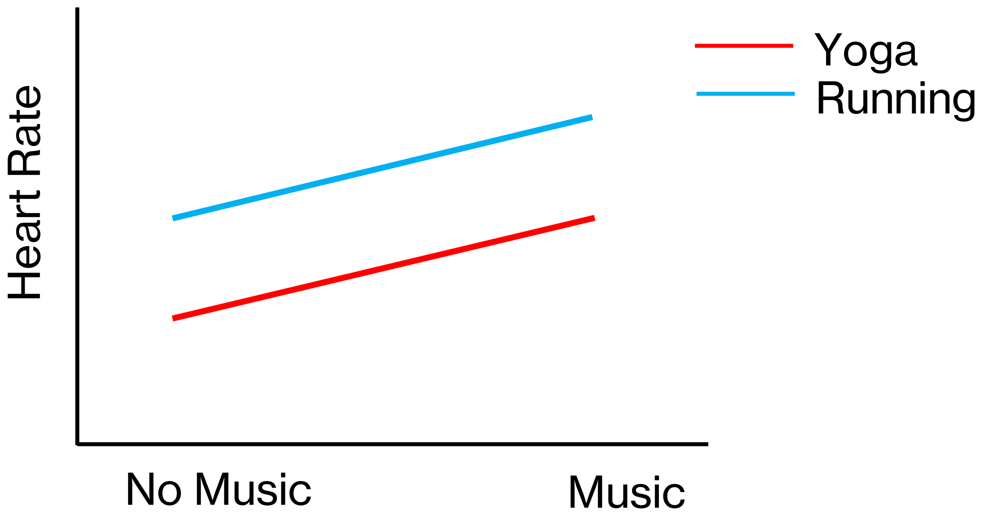
- The benefits of including multiple categorical predictors are:
  - Can examine the effect of each IV individually on the DV (called **main effects**)
  - Can examine whether there is an **interaction effect** between the categorical predictors

Interaction Effects

- An **interaction effect** is when the effect of one of the independent variables on the dependent variable *varies depending on the level of the second independent variable*.
  - Example: Does the effect of listening to music on heart rate *depend on* the type of activity one is performing?

- The best way to understand an interaction effect is by graphing it.
  - One of the IVs goes on the x-axis, and its effect across each level of the second IV is represented using different colors, or different patterns, of lines





Interpreting Interaction Effects

- Example: Say you're interested in evaluating how the time of day and in what environment a student studies affects his or her studying efficiency. You want to see if people study better in the morning or in the evening and under what environmental conditions: a quiet environment or in an environment where there is a television playing. Studying efficiency scores are computed for each student. Higher numbers indicate greater studying efficiency. The table of means is presented below.

1. Is there evidence of a main effect of time of day?
2. Is there evidence of a main effect of type of environment?
3. Is there evidence of an interaction effect between time of day and type of environment?

	Quiet	TV
Morning	8	9
Evening	4	5

Performing a Linear Regression Analysis with Multiple Categorical Predictors

- Example: A researcher is interested in whether caffeine has an effect on students' test scores. Additionally, the researcher is interested in whether the effect of caffeine on student test scores varies depending on whether the student is extraverted or introverted. The raw data is shown in the table below.
  - IV1: Caffeine dosage
    - Small dose (0 mg)
    - Large dose (4 mg)
  - IV2: Personality
    - Extraverts
    - Introverts
  - DV: Scores on a practice GRE exam

	Small dose (0 mg)	**Large dose (4 mg)**
Extravert	3	9
	8	9
	3	13
	3	6
	3	8
Introvert	2	0
	5	0
	9	0
	7	5
	7	0

Table of Means

- The data for a model with two categorical predictors is often represented in a table of means:

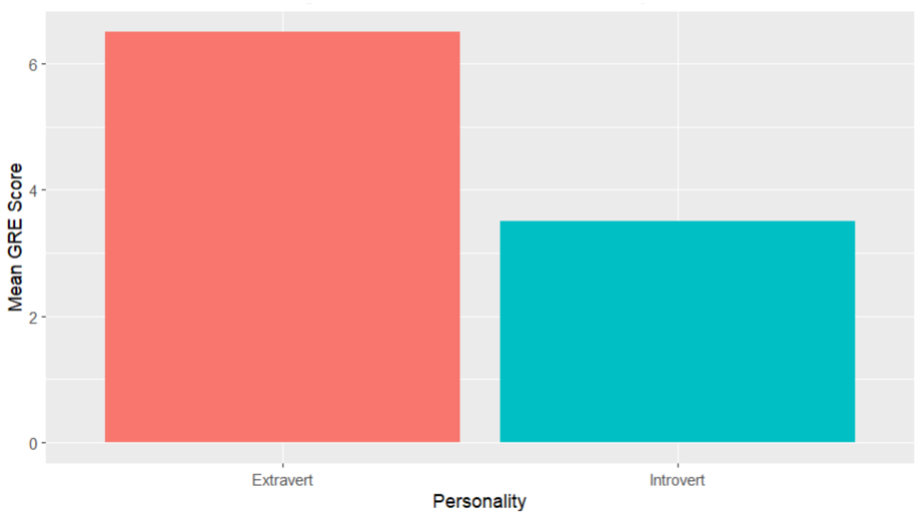
	Small dose (0 mg)	Large dose (4 mg)	Row Means
Extravert	4	9	6.50
Introvert	6	1	3.50
Column Means	5.00	5.00	

- Marginal Row Means: Capture the **main effect of personality**
- Marginal Column Means: Capture the **main effect of caffeine dosage**
- Cell means: Capture the **interaction effect between personality and caffeine dosage**

Visualizations

- Visualizations like **bar graphs** and **line graphs** can be used to easily illustrate whether there is evidence of potential main effects or an interaction effect.

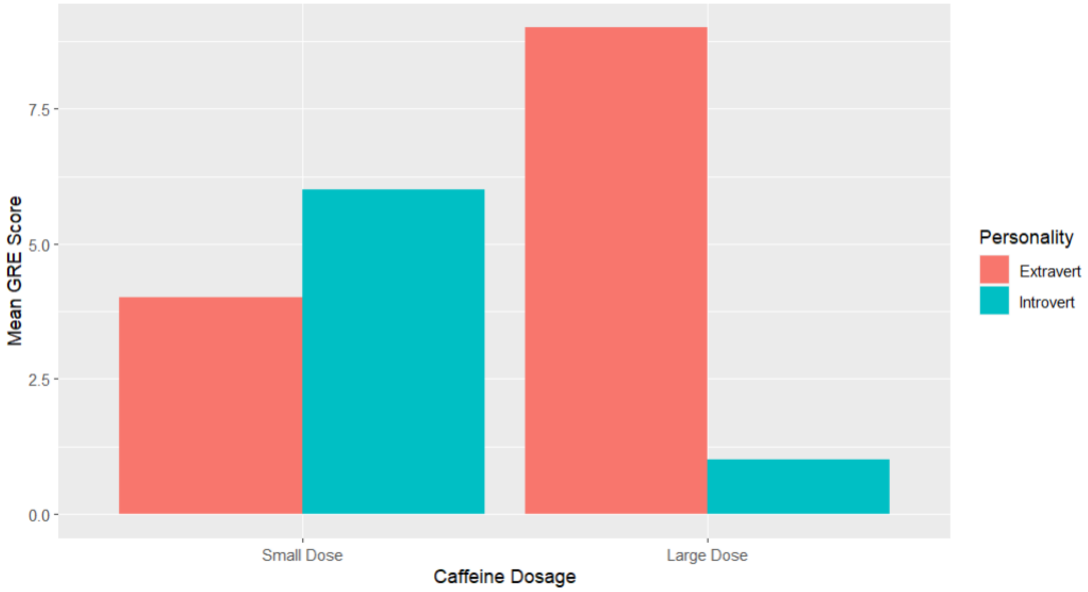
Main Effect of Personality



Main Effect of Caffeine

Visualizations

- To visualize the interaction effect for this example, we'll put caffeine dosage (small or large) on the x-axis, and represent its effect separately for extraverts and introverts using different colors of bars
  - If the change in the bars' heights is different depending on the level of the second IV, there is evidence of a potential interaction effect



Contrast Coding

- To contrast code a model with multiple categorical predictors, we need to consider how many *total combinations of conditions* there are
  - Number of contrast codes needed: $m - 1$
    - m is the total number of combinations of conditions

- For our example, each IV has two levels, resulting in 4 total conditions
    - Extraverts/ Small dose
    - Extraverts / Large dose
    - Introverts / Small dose
    - Introverts / Large dose

- Number of contrast codes needed: 4-1 = 3

	Small dose (0 mg)	**Large dose (4 mg)**	**Row Means**
Extravert	4	9	6.50
Introvert	6	1	3.50
Column Means	5.00	5.00	

Contrast Coding

To construct a set of contrast codes for multiple categorical predictors:

1. Contrast code each of the categorical predictors individually
   - Personality: Extravert = +1/2, Introvert = -1/2
   - Caffeine: Large dose = +1/2, Small dose = -1/2

2. Multiply their contrast codes to construct the contrast codes for the interaction effect
   - To represent the contrast codes, we need to construct a table with columns corresponding to all four combinations of conditions.

Contrast Coding

	Extravert / Small	Extravert / Large	Introvert / Small	Introvert / Large
CC1 (Main effect of personality)	1/2	1/2	-1/2	-1/2
CC2 (Main effect of caffeine)	-1/2	1/2	-1/2	1/2
CC3 (Interaction effect)	-1/4	1/4	1/4	-1/4

- There are **three effects** we can examine:
    - The main effect of personality (CC1)
    - The main effect of caffeine (CC2)
    - The interaction effect (CC3)

Model Comparison: Main Effect of IV1

- First, let's construct the model comparison that tests whether there is a significant **main effect of personality**

- Model A: $Y_i = \beta_0 + \beta_1 CC1_i + \beta_2 CC2_i + \beta_3 CC3_i + \varepsilon_i$
- Model C: $Y_i = \beta_0 + \beta_1 CC2_i + \beta_2 CC3_i + \varepsilon_i$

- H0: $\beta_1 = 0$
- H1: $\beta_1 \neq 0$

	Extravert / Small	**Extravert / Large**	**Introvert / Small**	**Introvert / Large**
CC1	1/2	1/2	-1/2	-1/2
CC2	-1/2	1/2	-1/2	1/2
CC3	-1/4	1/4	1/4	-1/4

Model A's Parameter Estimates

- **Model A: $Y_i = \beta_0 + \beta_1 CC1_i + \beta_2 CC2_i + \beta_3 CC3_i + \varepsilon_i$**

- What will the parameter estimates for Model A be equal to?
  - **b_0:** The mean of the group means = 5.00
  - **b_1:** The main effect of personality: $M_{Extravert} - M_{Introvert} = 3.00$
  - **b_2:** The main effect of caffeine: $M_{LargeDose} - M_{SmallDose} = 0.00$
  - **b_3:** The interaction effect, i.e., the difference in the effect of caffeine on GRE scores between extraverts and introverts
    - Effect of caffeine on extraverts: 9-4 = 5.00
    - Effect of caffeine on introverts: 1-6 = -5.00
    - b_3 = Difference between the effect on extraverts vs introverts: 5 – (-5) = 10.00

- **Estimate of Model A:** $Y_i = 5 + 3*CC1 + 0*CC2 + 10*CC3$

	Extravert / Small	**Extravert / Large**	**Introvert / Small**	**Introvert / Large**
CC1	1/2	1/2	-1/2	-1/2
CC2	-1/2	1/2	-1/2	1/2
CC3	-1/4	1/4	1/4	-1/4

	Small dose (0 mg)	**Large dose (4 mg)**	**Row Means**
Extravert	4	9	6.50
Introvert	6	1	3.50
Column Means	5.00	5.00	

Calculating SSE(A)

	Extravert / Small	Extravert / Large	Introvert / Small	Introvert / Large
CC1	1/2	1/2	-1/2	-1/2
CC2	-1/2	1/2	-1/2	1/2
CC3	-1/4	1/4	1/4	-1/4

- **Estimate of Model A: $Y_i = 5 + 3*CC1 + 0*CC2 + 10*CC3$**

Personality	Caffeine	Y_i	Y'	$(Y_i – Y')$	$(Y_i – Y')^2$
Extravert	Small	3	5 + 3*(1/2) + 0*(-1/2) + 10*(-1/4) = 4	-1	1
Extravert	Small	8	4	4	16
Extravert	Small	3	4	-1	1
Extravert	Small	3	4	-1	1
Extravert	Small	3	4	-1	1
Introvert	Small	2	5 + 3*(-1/2) + 0*(-1/2) + 10*(1/4) = 6	-4	16
Introvert	Small	5	6	-1	1
Introvert	Small	9	6	3	9
Introvert	Small	7	6	1	1
Introvert	Small	7	6	1	1
Extravert	Large	9	5 + 3*(1/2) + 0*(1/2) + 10*(1/4) = 9	0	0
Extravert	Large	9	9	0	0
Extravert	Large	13	9	4	16
Extravert	Large	6	9	-3	9
Extravert	Large	8	9	-1	1
Introvert	Large	0	5 + 3*(-1/2) + 0*(1/2) + 10*(-1/4) = 1	-1	1
Introvert	Large	0	1	-1	1
Introvert	Large	0	1	-1	1
Introvert	Large	5	1	4	16
Introvert	Large	0	1	-1	1

SSE(A)
= 94

Model C's Parameter Estimates

- **Model C: $Y_i = \beta_0 + \beta_1 CC2_i + \beta_2 CC3_i + \varepsilon_i$**

- What will the parameter estimates for Model C be equal to?
  - **b_0:** 5.00
  - **b_1:** 0.00
  - **b_2:** 10.00

- **Estimate of Model C:** $Y_i = 5 + 0*CC2 + 10*CC3$

	Extravert / Small	**Extravert / Large**	**Introvert / Small**	**Introvert / Large**
CC1	1/2	1/2	-1/2	-1/2
CC2	-1/2	1/2	-1/2	1/2
CC3	-1/4	1/4	1/4	-1/4

	Small dose (0 mg)	**Large dose (4 mg)**	**Row Means**
Extravert	4	9	6.50
Introvert	6	1	3.50
Column Means	5.00	5.00	

Calculating SSE(C)

	Extravert / Small	Extravert / Large	Introvert / Small	Introvert / Large
CC1	1/2	1/2	-1/2	-1/2
CC2	-1/2	1/2	-1/2	1/2
CC3	-1/4	1/4	1/4	-1/4

- Estimate of Model C: $Y_i = 5 + 0*CC2 + 10*CC3$

Personality	Caffeine	Y_i	Y’	$(Y_i – Y’)$	$(Y_i – Y’)^2$
Extravert	Small	3	5 + 0*(-1/2) + 10*(-1/4) = 2.5	0.5	0.25
Extravert	Small	8	2.5	5.5	30.25
Extravert	Small	3	2.5	0.5	0.25
Extravert	Small	3	2.5	0.5	0.25
Extravert	Small	3	2.5	0.5	0.25
Introvert	Small	2	5 + 0*(-1/2) + 10*(1/4) = 7.5	-5.5	30.25
Introvert	Small	5	7.5	-2.5	6.25
Introvert	Small	9	7.5	1.5	2.25
Introvert	Small	7	7.5	-0.5	0.25
Introvert	Small	7	7.5	-0.5	0.25
Extravert	Large	9	5 + 0*(1/2) + 10*(1/4) = 7.5	1.5	2.25
Extravert	Large	9	7.5	1.5	2.25
Extravert	Large	13	7.5	5.5	30.25
Extravert	Large	6	7.5	-1.5	2.25
Extravert	Large	8	7.5	0.5	0.25
Introvert	Large	0	5 + 0*(1/2) + 10*(-1/4) = 2.5	-2.5	6.25
Introvert	Large	0	2.5	-2.5	6.25
Introvert	Large	0	2.5	-2.5	6.25
Introvert	Large	5	2.5	2.5	6.25
Introvert	Large	0	2.5	-2.5	6.25

SSE(C)
= 139

Main Effect of IV1: Comparing Model A and Model C

- **Model A:** $Y_i = \beta_0 + \beta_1 CC1_i + \beta_2 CC2_i + \beta_3 CC3_i + \varepsilon_i$
  - **SSE(A) = 94**
- **Model C:** $Y_i = \beta_0 + \beta_1 CC2_i + \beta_2 CC3_i + \varepsilon_i$
  - **SSE(C) = 139**

- By how much did the inclusion of personality in the model reduce unaccounted for error?
  - **SSR = 139 – 94 = 45**

- As a percentage?
  - **PRE = 45/139 = .3237 * 100 = 32%**
  - The main effect of personality accounted for approximately 32% more of the variability in participants' GRE scores compared to a model that did not include the main effect of personality.

Main Effect of IV1: Comparing Model A and Model C

- **Calculating the *F*-statistic**
  - *F* represents, as a ratio, the variance explained by Model A versus the variance left unexplained by Model A

- $F = \frac{SSR/df_{Reduced}}{SSE(A)/df_{ModelA}} = \frac{45/(4-3)}{94/(20-4)} = 7.66$

- Use this *F*-statistic to determine whether Model A is ***significantly*** better than Model C
  - Using R to obtain the corresponding *p*-value of our *F*-statistic
  - Or find an *F*-critical value for 1 & 16 degrees of freedom

ANOVA Summary Table

	SS	*df*	*MS*	*F*	*p*
Personality	45	1	45/1 = 45	45/5.875 = 7.66	Use R to obtain
Caffeine					
Personality* Caffeine					
Model A	94	16	94/16 = 5.875		

Model Comparison: Main Effect of IV2

- Second, let's construct the model comparison that tests whether there is a significant **main effect of caffeine**

	Extravert / Small	Extravert / Large	Introvert / Small	Introvert / Large
CC1	1/2	1/2	-1/2	-1/2
CC2	-1/2	1/2	-1/2	1/2
CC3	-1/4	1/4	1/4	-1/4

- Model A: $Y_i = \beta_0 + \beta_1 CC1_i + \beta_2 CC2_i + \beta_3 CC3_i + \varepsilon_i$
- Model C: $Y_i = \beta_0 + \beta_1 CC1_i + \beta_2 CC3_i + \varepsilon_i$

- H0: $\beta_2 = 0$
- H1: $\beta_2 \neq 0$

- Model A is the same as the one we used to test the main effect of IV1.

Model C's Parameter Estimates

- **Model C: $Y_i = \beta_0 + \beta_1 CC1_i + \beta_2 CC3_i + \varepsilon_i$**

- What will the parameter estimates for Model C be equal to?
  - **b_0:** 5.00
  - **b_1:** 3.00
  - **b_2:** 10.00

- **Estimate of Model C:** $Y_i = 5 + 3{*}CC1 + 10{*}CC3$

	Extravert / Small	**Extravert / Large**	**Introvert / Small**	**Introvert / Large**
CC1	1/2	1/2	-1/2	-1/2
CC2	-1/2	1/2	-1/2	1/2
CC3	-1/4	1/4	1/4	-1/4

	Small dose (0 mg)	**Large dose (4 mg)**	**Row Means**
Extravert	4	9	6.50
Introvert	6	1	3.50
Column Means	5.00	5.00	

Calculating SSE(C)

	Extravert / Small	Extravert / Large	Introvert / Small	Introvert / Large
CC1	1/2	1/2	-1/2	-1/2
CC2	-1/2	1/2	-1/2	1/2
CC3	-1/4	1/4	1/4	-1/4

- **Estimate of Model C: $Y_i = 5 + 3*CC1 + 10*CC3$**

Personality	Caffeine	Y_i	Y'	$(Y_i – Y')$	$(Y_i – Y')^2$
Extravert	Small	3	5 + 3*(1/2) + 10*(-1/4) = 4	-1	1
Extravert	Small	8	4	4	16
Extravert	Small	3	4	-1	1
Extravert	Small	3	4	-1	1
Extravert	Small	3	4	-1	1
Introvert	Small	2	5 + 3*(-1/2) + 10*(1/4) = 6	-4	16
Introvert	Small	5	6	-1	1
Introvert	Small	9	6	3	9
Introvert	Small	7	6	1	1
Introvert	Small	7	6	1	1
Extravert	Large	9	5 + 3*(1/2) + 10*(1/4) = 9	0	0
Extravert	Large	9	9	0	0
Extravert	Large	13	9	4	16
Extravert	Large	6	9	-3	9
Extravert	Large	8	9	-1	1
Introvert	Large	0	5 + 3*(-1/2) + 10*(-1/4) = 1	-1	1
Introvert	Large	0	1	-1	1
Introvert	Large	0	1	-1	1
Introvert	Large	5	1	4	16
Introvert	Large	0	1	-1	1

SSE(C)
= 94

Main Effect of IV2: Comparing Model A and Model C

- **Model A: $Y_i = \beta_0 + \beta_1 CC1_i + \beta_2 CC2_i + \beta_3 CC3_i + \varepsilon_i$**
  - **SSE(A) = 94**
- **Model C: $Y_i = \beta_0 + \beta_1 CC1_i + \beta_2 CC3_i + \varepsilon_i$**
  - **SSE(C) = 94**

- By how much did the inclusion of caffeine in the model reduce unaccounted for error?
  - **SSR = 94 – 94 = 0**

- As a percentage?
  - **PRE = 0/94 = 0.00**
  - The main effect of caffeine accounted for approximately 0% more of the variability in participants' GRE scores compared to a model that did not include the main effect of caffeine.

Main Effect of IV2:
Comparing Model A and Model C

- **Calculating the *F*-statistic**

- $F = \dfrac{SSR/df_{Reduced}}{SSE(A)/df_{ModelA}} = \dfrac{0/(4-3)}{94/(20-4)} = 0.00$

- Use this *F*-statistic to determine whether Model A is ***significantly*** better than Model C
  - Using R to obtain the corresponding *p*-value of our *F*-statistic
  - Or find an *F*-critical value for 1 & 16 degrees of freedom

ANOVA Summary Table

	SS	*df*	*MS*	*F*	*p*
Personality	45	1	45/1 = 45	45/5.875 = 7.66	Use R to obtain
Caffeine	0	1	0/1 = 0	0/5.875 = 0	Use R to obtain
Personality* Caffeine					
Model A	94	16	94/16 = 5.875		

Model Comparison: Interaction Effect

- Third, let's construct the model comparison we would use to test **whether there is a significant interaction effect** between personality and caffeine dosage

- Model A: $Y_i = \beta_0 + \beta_1 CC1_i + \beta_2 CC2_i + \beta_3 CC3_i + \varepsilon_i$
- Model C: $Y_i = \beta_0 + \beta_1 CC1_i + \beta_2 CC2_i + \varepsilon_i$

- H0: $\beta_3 = 0$
- H1: $\beta_3 \neq 0$

	Extravert / Small	**Extravert / Large**	**Introvert / Small**	**Introvert / Large**
CC1	1/2	1/2	-1/2	-1/2
CC2	-1/2	1/2	-1/2	1/2
CC3	-1/4	1/4	1/4	-1/4

Model C's Parameter Estimates

- **Model C: $Y_i = \beta_0 + \beta_1 CC1_i + \beta_2 CC2_i + \varepsilon_i$**

- What will the parameter estimates for Model C be equal to?
  - b_0: 5
  - b_1: 3
  - b_2: 0

	Small dose (0 mg)	Large dose (4 mg)	Row Means
Extravert	4	9	6.50
Introvert	6	1	3.50
Column Means	5.00	5.00	

Calculating SSE(C)

- **Estimate of Model C: $Y_i = 5 + 3*CC1 + 0*CC2$**

	Extravert / Small	Extravert / Large	Introvert / Small	Introvert / Large
CC1	1/2	1/2	-1/2	-1/2
CC2	-1/2	1/2	-1/2	1/2
CC3	-1/4	1/4	1/4	-1/4

Personality	Caffeine	Y_i	Y'	$(Y_i – Y')$	$(Y_i – Y')^2$
Extravert	Small	3	5 + 3*(1/2) + 0*(-1/2) = 6.5	-3.5	12.25
Extravert	Small	8	6.5	1.5	2.25
Extravert	Small	3	6.5	-3.5	12.25
Extravert	Small	3	6.5	-3.5	12.25
Extravert	Small	3	6.5	-3.5	12.25
Introvert	Small	2	5 + 3*(-1/2) + 0*(-1/2) = 3.5	5.5	30.25
Introvert	Small	5	3.5	5.5	30.25
Introvert	Small	9	3.5	9.5	90.25
Introvert	Small	7	3.5	2.5	6.25
Introvert	Small	7	3.5	4.5	20.25
Extravert	Large	9	5 + 3*(1/2) + 0*(1/2) = 6.5	-4.5	20.25
Extravert	Large	9	6.5	-1.5	2.25
Extravert	Large	13	6.5	2.5	6.25
Extravert	Large	6	6.5	0.5	0.25
Extravert	Large	8	6.5	0.5	0.25
Introvert	Large	0	5 + 3*(-1/2) + 0*(1/2) = 3.5	-3.5	12.25
Introvert	Large	0	3.5	-3.5	12.25
Introvert	Large	0	3.5	-3.5	12.25
Introvert	Large	5	3.5	1.5	2.25
Introvert	Large	0	3.5	-3.5	12.25

SSE(C)
= 219

Interaction Effect: Comparing Model A and Model C

- **Model A: $Y_i = \beta_0 + \beta_1 CC1_i + \beta_2 CC2_i + \beta_3 CC3_i + \varepsilon_i$**
  - **SSE(A) = 94**
- **Model C: $Y_i = \beta_0 + \beta_1 CC1_i + \beta_2 CC2_i + \varepsilon_i$**
  - **SSE(C) = 219**

- By how much did the inclusion of the interaction effect reduce unaccounted for error?
  - **SSR = 219 – 94 = 125**

- As a percentage?
  - **PRE = 125/219 = .57077 * 100 = 57%**
  - The interaction term accounted for approximately 57% for variability in GRE scores than a model that did not include the interaction term.

Interaction Effect: Comparing Model A and Model C

- **Calculating the *F*-statistic**

- $F = \dfrac{{}^{SSR}/_{df_{Reduced}}}{{}^{SSE(A)}/_{df_{ModelA}}} = \dfrac{{}^{125}/_{(4-3)}}{{}^{94}/_{(20-4)}} = 21.28$

- Use this *F*-statistic to determine whether Model A is ***significantly*** better than Model C
  - Using R to obtain the corresponding *p*-value of our *F*-statistic
  - Or find an *F*-critical value for 1 & 16 degrees of freedom

ANOVA Summary Table

	SS	*df*	*MS*	*F*	*p*
Personality	45	1	45/1 = 45	45/5.875 = 7.66	Use R to obtain
Caffeine	0	1	0/1 = 0	0/5.875 = 0	Use R to obtain
Personality* Caffeine	125	1	125/1 = 125	125/5.875 = 21.28	Use R to obtain
Model A	94	16	94/16 = 5.875		

Performing the Analysis in R

- Set up the data so that each IV is its own column and the DV is its own column
  - The categorical IVs should be factors, and
  - The DV should be integer or numeric

Personality	Caffeine	GRE
Extravert	Small Dose	3
Extravert	Small Dose	8
Extravert	Small Dose	3
Extravert	Small Dose	3
Extravert	Small Dose	3
Extravert	Large Dose	9
Extravert	Large Dose	9
Extravert	Large Dose	13
Extravert	Large Dose	6
Extravert	Large Dose	8
Introvert	Small Dose	2
Introvert	Small Dose	5
Introvert	Small Dose	9
Introvert	Small Dose	7
Introvert	Small Dose	7

Performing the Analysis in R

- Assign contrast codes to the two categorical predictors
  - Personality: Extravert = +1/2, Introvert = -1/2
  - Caffeine: Small dose = -1/2, Large dose = +1/2

```
cc1 <- c(1/2, -1/2)
cc2 <- c(-1/2, 1/2)

contrasts(data$Personality) <- cc1
contrasts(data$Caffeine) <- cc2
```

- Fit the model using lm()
  - When both categorical predictors are between-subjects factors
  - Using the syntax: lm(DV ~ IV1*IV2, data = data)
    - IV1*IV2 represents the interaction effect
    - This syntax automatically also includes the main effects of each IV
    - The above model is equivalent to: lm(DV ~ IV1 + IV2 + IV1*IV2, data = data)

```
model <- lm(GRE ~ Personality*Caffeine, data = data)
```

Performing the Analysis in R

- Interpreting the output:
  - anova(model)

- Each row corresponds to a Model A-Model C comparison for that effect:
  - The main effect of personality was significant, $F(1, 16) = 7.66$, $p = .014$
  - The main effect of caffeine was non-significant, $F(1, 16) = 0.00$, $p = 1.000$.
  - The interaction effect was significant, $F(1, 16) = 21.28$, $p < .001$

```
> anova(model)
Analysis of Variance Table

Response: GRE
                     Df Sum Sq Mean Sq F value    Pr(>F)    
Personality           1     45  45.000  7.6596 0.0137281 *  
Caffeine              1      0   0.000  0.0000 1.0000000    
Personality:Caffeine  1    125 125.000 21.2766 0.0002882 ***
Residuals            16     94   5.875                      
---
```

Performing the Analysis in R

- Interpreting the output:
  - To get effect sizes, use the `etaSquared()` function

```
> etaSquared(model)
                         eta.sq eta.sq.part
Personality                0.17        0.32
Caffeine                   0.00        0.00
Personality:Caffeine       0.47        0.57
```

```
> SS_Total <- sum((data$GRE - mean(data$GRE))^2)
> SS_Total
[1] 264
```

- Eta-squared
  - Synonymous with R^2
  - Uses SS_{Total} in the denominator
  - Interpretation: The proportion of the total variability in the DV that is explained by an effect
    - Personality: 45/264 = 0.17 * 100 = 17%
    - Caffeine: 0/264 = 0.00 * 100 = 0%
    - Personality*Caffeine = 125/264 = 0.47 * 100 = 47%

Performing the Analysis in R

- Interpreting the output:
  - To get effect sizes, use the `etaSquared()` function

```
> etaSquared(model)
                     eta.sq eta.sq.part
Personality            0.17        0.32
Caffeine               0.00        0.00
Personality:Caffeine   0.47        0.57
```

- Partial eta-squared
  - Synonymous with *PRE*
  - Uses SSE(C) in the denominator
  - Interpretation: The proportion of the variability in the DV that is explained by an effect that was left unexplained by the other predictors in the model
    - Personality: 45/139 = 0.32 * 100 = 32%
    - Caffeine: 0/94 = 0.00 * 100 = 0%
    - Personality*Caffeine = 125/219 = 0.57 * 100 = 57%

Performing the Analysis in R

- Interpreting the output:
  - summary(model)
  - Provides the parameter estimates

```
Coefficients:
                                   Estimate            Std. Error t value    Pr(>|t|)    
(Intercept)            5.000000000000000000  0.541987084716969991    9.23 0.000000083 ***
Personality1           3.000000000000000000  1.083974169433940204    2.77     0.01373 *  
Caffeine1              0.000000000000000124  1.083974169433940204    0.00     1.00000    
Personality1:Caffeine1 10.000000000000000000 2.167948338867880409    4.61     0.00029 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.4 on 16 degrees of freedom
Multiple R-squared:  0.644,	Adjusted R-squared:  0.577 
F-statistic: 9.65 on 3 and 16 DF,  p-value: 0.000713
```

- Estimate of Model A:
  $Y_i = 5.00 + 3.00*CC1 + 0.00*CC2 + 10.00*CC3$

- Each row corresponds to a test of the significance of each parameter estimate.
  - b_0: The mean of the group means (M = 5.00) was significantly different from zero, $t(16) = 9.23$, $p < .001$
  - b_1: The main effect of personality ($M_{Extravert} - M_{Introvert} = 3.00$) was significant, $t(16) = 2.77$, $p = .014$
  - b_2: The main effect of caffeine ($M_{LargeDose} - M_{SmallDose} = 0.00$) was non-significant, $t(16) = 0.00$, $p = 1.000$
  - b_3: The interaction effect between personality and caffeine dosage was significant, $t(16) = 4.61$, $p < .001$

Performing the Analysis in R

- The bottom of the summary output corresponds to a test of the significance of the **overall model**
  - Personality, caffeine, and the interaction between the two altogether

```
Coefficients:
                              Estimate            Std. Error t value    Pr(>|t|)
(Intercept)          5.000000000000000000  0.541987084716969991    9.23 0.000000083 ***
Personality1         3.000000000000000000  1.083974169433940204    2.77     0.01373 *
Caffeine1            0.000000000000000124  1.083974169433940204    0.00     1.00000
Personality1:Caffeine1 10.000000000000000000  2.167948338867880409    4.61     0.00029 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 2.4 on 16 degrees of freedom
Multiple R-squared:  0.644,     Adjusted R-squared:  0.577
F-statistic: 9.65 on 3 and 16 DF,  p-value: 0.000713
```

- The overall model accounted for a significant amount of variability in GRE scores, $F(3, 16) = 9.65$, $p < .001$. The model accounted for approximately 64% of the variability in GRE scores ($R^2 = 64.40$).

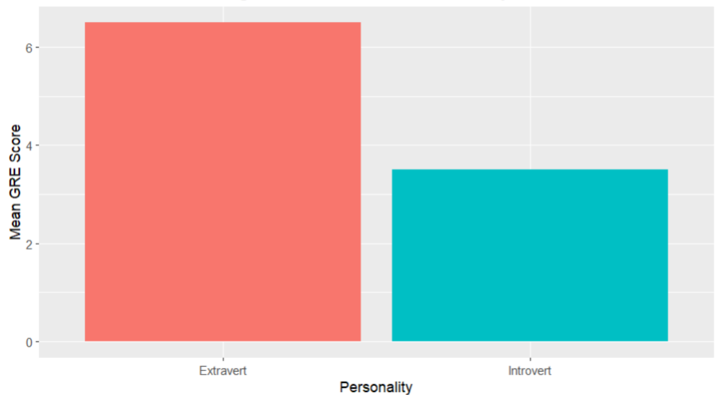
Interpreting Main Effects

- The main effects of personality and caffeine are very straightforward to interpret
  - Examine the marginal means, or
  - Examine a graph of the main effects

	Small dose (0 mg)	Large dose (4 mg)	Row Means
Extravert	4	9	6.50
Introvert	6	1	3.50
Column Means	5.00	5.00	

Main effect of personality:

- Overall, extraverts (*M* = 6.50) had significantly higher GRE scores compared to introverts (*M* = 3.50), $F(1, 16) = 7.66$, $p = .014$.



Interpreting Main Effects

- The main effects of personality and caffeine are very straightforward to interpret
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	Small dose (0 mg)	Large dose (4 mg)	Row Means
Extravert	4	9	6.50
Introvert	6	1	3.50
Column Means	5.00	5.00	

Main effect of caffeine:

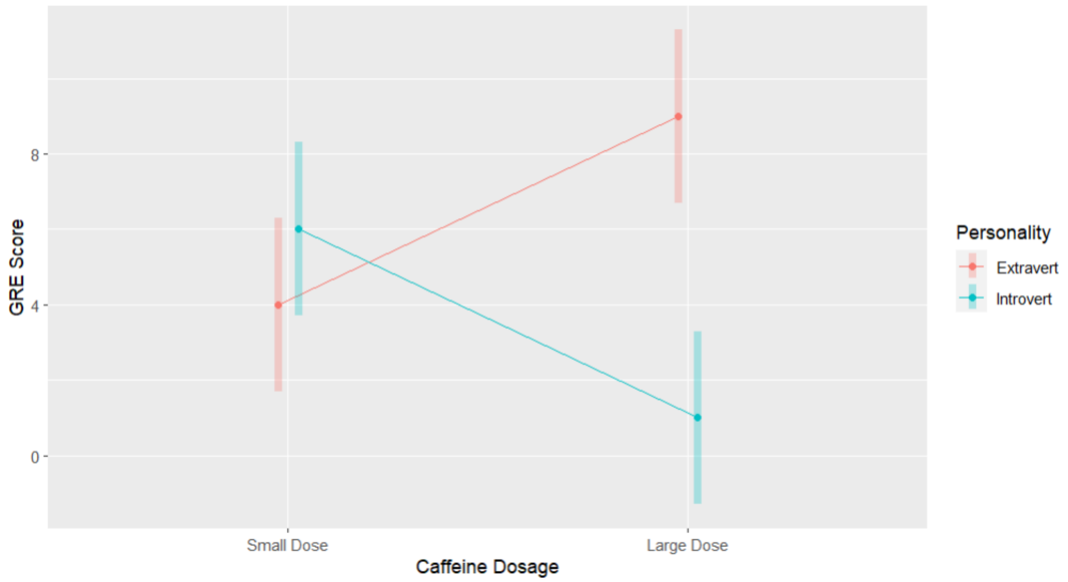
- Overall, people who consumed a large dose of caffeine (*M* = 5.00) scored non-significantly differently on the GRE compared to people who took a small dose (*M* = 5.00), *F*(1, 16) = 0.00, *p* = 1.000.

Interpreting Interaction Effects

- To interpret interaction effects, you can either:
  - Examine the pattern of the cell means, or
  - Examine a plot of the interaction effect

- How would you describe the nature of the significant interaction between personality and caffeine dosage?
  - The interaction effect between personality and caffeine dosage was significant, $F(1, 16) = 21.28$, $p < .001$. For extraverts, consuming a large dose of caffeine improved their GRE scores. However, it had the opposite effect on introverts. For introverts, consuming a large dose of caffeine worsened their performance on the GRE.

	Small dose (0 mg)	Large dose (4 mg)	Row Means
Extravert	4	9	6.50
Introvert	6	1	3.50
Column Means	5.00	5.00	



Simple Effects Analysis

- When an interaction effect is significant, we can perform **simple effects analyses** to further unpack the nature of the interaction effect.

- **Simple effects** are used to examine the effect of IV1 on the DV separately at each level of IV2.
  - In this case, let's examine the effect of personality (introverts vs extraverts) on GRE scores separately for each level of caffeine (small vs large dose).

Simple Effects Analysis

- Recall the full model we obtained from the summary() output:
  - GRE_i = 5.00 + 3.00*Personality + 0.00*Caffeine + 10.00*(PersonalityXCaffeine)

- We can express the "simple" relationship between personality & GRE scores at specific values of caffeine by rearranging the full model:
  - GRE_i = (5.00 + 0*Caffeine) + (3.00 + 10*Caffeine)*Personality
    - The intercept for the "simple" relationship between Personality & GRE scores changes by 0 units for every 1-unit change in caffeine dosage
    - The slope for the "simple" relationship between Personality & GRE changes by 10 units for every 1-unit change in caffeine dosage

	Small dose (0 mg)	Large dose (4 mg)	Row Means
Extravert	4	9	6.50
Introvert	6	1	3.50
Column Means	5.00	5.00	$M_{Grand} = 5$

Simple Effects Analysis

- **The "simple" relationship between Personality & GRE Scores:**
  - GRE_i = (5.00 + 0*Caffeine) + (3.00 + 10*Caffeine)*Personality

- When participants had a small dosage of caffeine (Caffeine = -1/2), the relationship between personality and GRE scores is:
  - GRE_i = (5 + 0*(-1/2)) + (3 + 10(-1/2))*Personality
  - **GRE_i = 5 - 2*Personality**
    - Extraverts: GRE_i = 5 – 2*(+1/2) = 4
    - Introverts: GRE_i = 5 – 2*(-1/2) = 6

- When participants had a large dosage of caffeine (Caffeine = +1/2), the relationship between personality and GRE scores is:
  - GRE_i = (5 + 0*(+1/2)) + (3 + 10(+1/2))*Personality
  - **GRE_i = 5 + 8*Personality**
    - Extraverts: GRE_i = 5 + 8*(+1/2) = 9
    - Introverts: GRE_i = 5 + 8*(-1/2) = 1

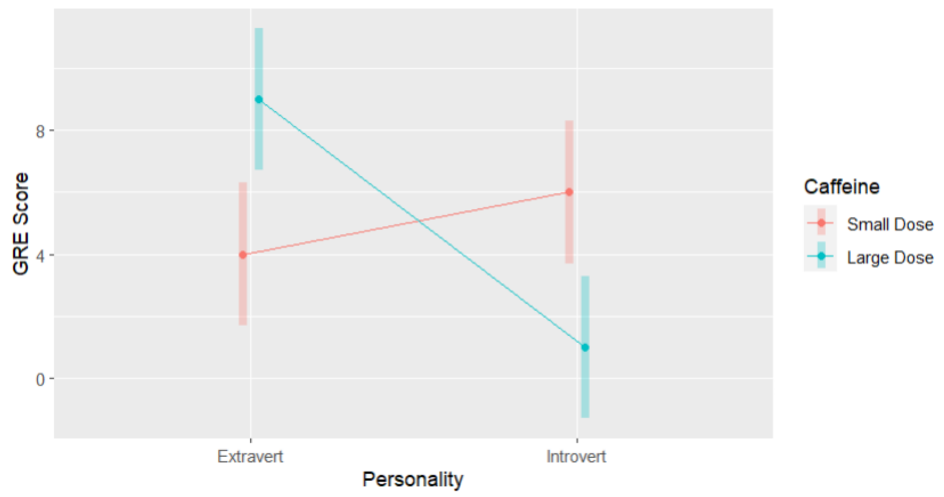
Simple Effects Analysis

- Using simple effects analyses, we can ask **at which levels of caffeine dosage does personality have a significant effect**?

- When participants had a small does of caffeine, was there a significant effect of personality on GRE scores?
  - No, when participants had a small dose of a caffeine, there was no significant difference in average GRE scores between extraverts (*M* = 4.00) and introverts (*M* = 6.00), *t*(16) = 1.30, *p* = .210.

- When participants had a large dose of caffeine, was there a significant effect of personality on GRE scores?
  - Yes, when participants had a large dose of caffeine, extraverts (*M* = 9.00) had significantly higher GRE scores than introverts (*M* = 1.00), *t*(16) = -5.20, *p* < .001.

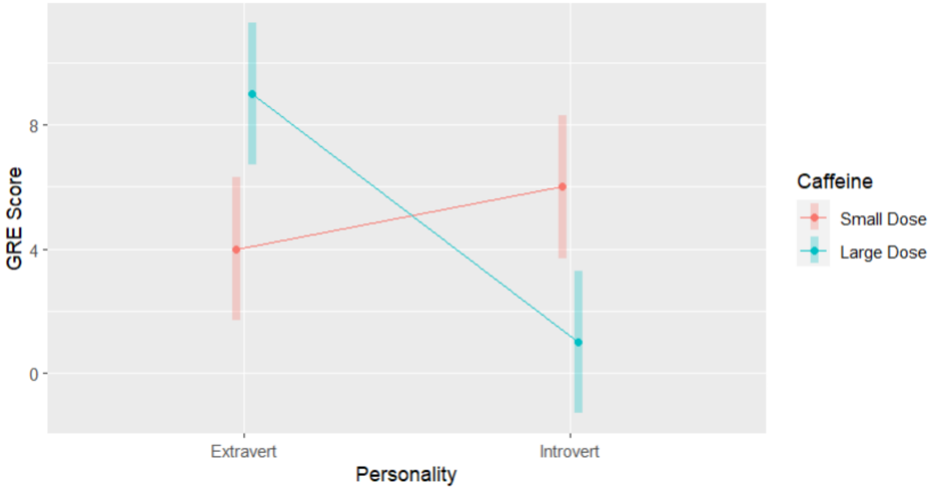
	Small dose (0 mg)	Large dose (4 mg)	Row Means
Extravert	4	9	6.50
Introvert	6	1	3.50
Column Means	5.00	5.00	




```
Caffeine = Small Dose:
 Personality_consec    estimate   SE df t.ratio p.value
 Introvert - Extravert        2 1.53 16   1.300  0.2100

Caffeine = Large Dose:
 Personality_consec    estimate   SE df t.ratio p.value
 Introvert - Extravert       -8 1.53 16  -5.200  <.0001
```

Simple Effects Analysis



- Using simple effects analyses, we could also ask whether effect of personality is *significantly different* depending on whether people had a small, or large, dose of caffeine:
  - The effect of personality at small dose of caffeine: -2
  - The effect of personality at large dose of caffeine: 8
  - The difference in their effects: -2 – 8 = -10
    - You'll learn the syntax for this simple effects analysis in lab!

```
Personality_consec     Caffeine_consec          estimate   SE df t.ratio p.value
Introvert - Extravert Large Dose - Small Dose        -10 2.17 16  -4.600  0.0003
```

- Yes, the effect of personality on GRE scores was significantly stronger when participants drank a large dose of caffeine (M_{Diff} = 8.00) than when participants drank a small dose of caffeine (M_{Diff} = -2.00), $t(16) = -4.60$, $p < .001$.