# Nonuniformity two-point linear correction errors in infrared focal plane arrays

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**Abstract.** The error and hence the residual nonuniformity in an IR focal plane array, after two-point linear nonuniformity correction, which results from measurement errors and nonlinearity, is calculated, yielding different results than have been obtained in the past.  [S0091-3286(98)03704-0]

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## 1 Introduction

The signal transfer function (STF), i.e., the electrical signal from an IR focal plane array (FPA) as a function of the incident radiant power falling on it, differs from pixel to pixel. If this nonuniformity is not corrected to a certain level in the displayed output, it will degrade image quality meaningfully.[1] There are two different ways to remedy this problem. The first way is to calibrate each pixel in the FPA by the signal obtained when the FPA is facing a reference target(s) held at one or more fixed temperatures.[2,3] The second method is by realization of an algorithm that is based on the statistics of the signal levels from all FPA pixels during random (natural or initiated) motion of the FPA relative to the object.[4]

In this paper, we deal with the two-point, linear nonuniformity correction (NUC) method and calculate the residual nonuniformity after the NUC due to measurement errors and nonlinearity of the STF. These two error sources have been discussed erroneously elsewhere.[2]

Applying the two-point, linear, NUC to the FPA output under ideal conditions implies that the offset and gain of all the pixels are equal to predetermined values. The signal of a pixel $ij$, in the FPA in the linear approximation is given by

$$S_{ij}=R_{ij}P_s-O_{ij}, \tag{1}$$

where $R_{ij}$ and $O_{ij}$ are the gain (responsivity) and offset of the pixel, and $P_s$ is the radiant power incident on the pixel. After NUC, the signal of any pixel in the FPA is given by

$$S_{ij,o}=\frac{K(S_{ij,I}-L_{ij})}{H_{ij}-L_{ij}}+O, \tag{2}$$

where $H_{ij}$ and $L_{ij}$ are the electrical signals of the pixel when it senses the hot and cold targets, respectively; $S_{ij,I}$ is the signal at the input to the NUC; and $K$ and $O$ are arbitrary constants that are equal for all pixels that determine the gain and offset of the corrected output.

## 2 Measurement and Target Temperature Nonuniformity Error

The errors in $H_{ij}$, $L_{ij}$, and $S_{ij,I}$ affect the accuracy of the NUC, and hence, the residual (after correction) nonuniformity. Applying the standard linear approximation to Eq. (2) yields for the error after NUC

$$\delta S_{ij,o,\text{meas}}=\frac{K}{H_{ij}-L_{ij}}[Y_{ij}^2\delta H_{ij}^2+(Y_{ij}^2-2Y_{ij}+1)\delta L_{ij}^2 + \delta S_{ijI}^2]^{1/2}, \tag{3}$$

where $Y$ is a dimensionless parameter given by

$$Y_{ij}=\frac{S_{ij,I}-L_{ij}}{H_{ij}-L_{ij}}, \tag{4}$$

$\delta S_{ij,I}$ is the temporal noise of the incoming signal, and $\delta H_{ij}$ and $\delta L_{ij}$ are the errors in $H_{ij}$ and $L_{ij}$; note these errors might have two sources. The first source is the temporal noise of the pixel, which can be reduced by averaging the signal received from the targets, and the second is a spatial noise that appears if not all of the pixels see the same target temperature and emissivity. This can happen when the targets are located at a secondary image plane of the system. The error in $H_{ij}$ (and $L_{ij}$) is therefore given by

$$\delta H_{ij}=(\delta H_{ij,t}^2+\delta H_{ij,s}^2)^{1/2}, \tag{5}$$

where the subscripts $t$ and $s$ indicate temporal and spatial noise, respectively. In the middle of the range ($Y_{ij}=0.5$), the error is minimum and it equals

$$(\delta S_{ij,o,\text{meas}})_{\min}=\frac{K}{H_{ij}-L_{ij}}(0.5\delta H_{ij}^2+\delta S_{ij,I}^2)^{1/2}, \tag{6}$$

where it was assumed that $\delta H_{ij}=\delta L_{ij}$. In the case where the image is uniformly distributed between the hot and cold targets, i.e., $Y_{ij}$ is uniformly distributed between 0 and 1, we get that the root mean square (rms) (relative to $Y_{ij}$) error is

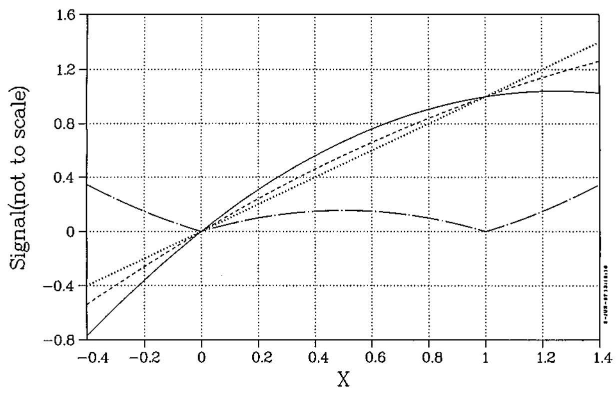



**Fig. 1** Residual nonuniformity caused by nonlinear STF; two-point linear corrected STFs of 2 pixels (the solid and dashed lines), the linear predicted STF (dotted line), and the residual nonuniformity W-shaped curve (dashed-dotted line).

$$(\delta S_{ij,o,\mathrm{meas}})_{\mathrm{rms}}=\frac{K}{H_{ij}-L_{ij}}\left(\frac{2}{3}\ \delta H_{ij}^2+\delta S_{ij,I}^2\right)^{1/2}. \tag{7}$$

If we assume, as was done in Ref. 2, that $\delta H_{ij}=\delta L_{ij}=\delta S_{ij,I}=\delta$, we get

$$\delta S_{ij,o,\mathrm{meas}}=\frac{\sqrt{2}K\delta}{H_{ij}-L_{ij}}\ (Y_{ij}^2-Y_{ij}+1)^{1/2}. \tag{8}$$

Milton et al.[2] calculated the same error for the worst-case condition and got a wrong result because $\delta L$, which appears twice in their expression for the error (Eq. 16 in Ref. 2), has a different sign each time, although it is a result of a single measurement and hence must be either positive or negative. This mistake resulted in an overestimation of the measurement error.

## 3 Residual Nonuniformity as Results from Nonlinear STF

The effect of nonlinear STF on the residual nonuniformity after two-point linear NUC is described in Fig. 1. For the sake of simplicity, we ignore the nonrelevant offsets in the STFs and define a dimensionless input parameter

$$X=\frac{P_S-P_L}{P_H-P_L}, \tag{9}$$

where $P_L$ and $P_H$ are the incident radiant powers on the FPA from the cold and hot targets. The two curved lines are the corrected STF of 2 pixels after NUC. The straight line is the predicted (after NUC) linear STF. The residual nonuniformity $\delta S_{o,\mathrm{NL}}$ defined as the standard deviation of the signals obtained from all the pixels when they face the same object temperature is depicted in the figure by the W-shaped curve, which stems from the fact that the signal of any pixel is given by its corrected STF [Eq. (11)], but this signal is transformed to a value at the display by the predicted STF.

We assume that the signal before NUC is of the form

$$S_{ij,I}=a_{ij}P_S-b_{ij}P_S^2, \tag{10}$$

and after two-point linear NUC, the corrected signal is

$$S_{ij,o}=\frac{K(S_{ij,I}-L_{ij})}{H_{ij}-L_{ij}}, \tag{11}$$

where

$$L_{ij}=a_{ij}P_L-b_{ij}P_L^2, \tag{12}$$

$$H_{ij}=a_{ij}P_H-b_{ij}P_H^2. \tag{13}$$

The error in $S_{ij,o}$ due to variations in $a_{ij}$ and $b_{ij}$ in the FPA is

$$\delta S_{o,\mathrm{NL}}=\left\{\left[\left(\frac{\delta S_{ij,o}}{\partial a_{ij}}\right)_{\overline{a_{ij}},\overline{b_{ij}}}\delta a_{ij}\right]^2+\left[\left(\frac{\partial S_{ij,o}}{\partial b_{ij}}\right)_{\overline{a_{ij}},\overline{b_{ij}}}\delta b_{ij}\right]^2\right\}^{1/2}$$
$$=\frac{K(P_H-P_L)X(1-X)(\overline{b_{ij}^2}\delta a_{ij}^2+\overline{a_{ij}^2}\delta b_{ij}^2)^{1/2}}{[\overline{a_{ij}}-b_{ij}(P_H+P_L)]^2}, \tag{14}$$

where $a_{ij}$ and $b_{ij}$ were taken as two independent random variables, and $\overline{a_{ij}}$ and $\overline{b_{ij}}$ are their average; $\delta a_{ij}$ and $\delta b_{ij}$ are the standard deviations of the values of $a_{ij}$ and $b_{ij}$ of all the pixels in the FPA. If we choose $K$ to keep the average output equal to the average input, i.e.,

$$K=\overline{a}_{ij}P_H-\overline{b}_{ij}P_H^2-(\overline{a}_{ij}P_L-\overline{b}_{ij}P_L^2), \tag{15}$$

and define

$$\delta S_{ij,I}=\overline{a}_{ij}N-\overline{b}_{ij}N^2, \tag{16}$$

where $N$ is the (temporal) noise equivalent power (NEP) of the detector

$$\delta a_{ij}=\overline{a}_{ij}\sigma_{a_{ij}}, \tag{17}$$

$$\delta b_{ij}=\frac{\overline{a}_{ij}\sigma_{b_{ij}}}{P_H}, \tag{18}$$

Eq. (14) can be written as

$$\delta S_{o,\mathrm{NL}}=\frac{\delta S_{ij,I}(P_H-P_L)X(1-X)[(P_H-P_L)-(\overline{b_{ij}}/\overline{a_{ij}})(P_H-P_L)][(\overline{b_{ij}^2}/\overline{a_{ij}^2})\sigma_{a_{ij}}^2+(1/P_H^2)\sigma_{b_{ij}}^2]^{1/2}}{[N-(b_{ij}/\overline{a_{ij}})N^2][1-(b_{ij}/\overline{a_{ij}})(P_H+P_L)]^2}. \tag{19}$$

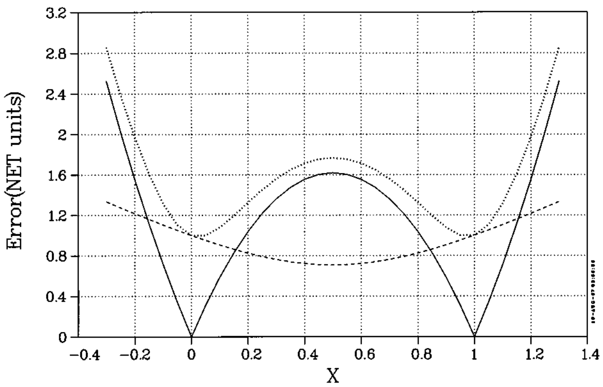



**Fig. 2** Residual nonuniformity of the numerical example error due to variation in the nonlinearity (solid line), measurement and target nonuniformity error (dashed line), and the total error (dotted line).

This equation gives the residual nonuniformity in terms of the temporal noise of the pixel $\delta_{S_{ij,I}}$ the relative (to $a_{ij}$) nonlinearity $P_H(b_{ij}/a_{ij})$ and the relative variances of this nonlinearity $\sigma_{a_{ij}}$ and $\sigma_{b_{ij}}$. Milton et al.[2] calculated the same error as a function of the fluctuations in the approximated linear STF of each pixel and of the fluctuations in the deviation from this line [$dR_i$ and $dH_i$ in Eq. (21) of Ref. 2] and treated them as independent random variables. This is not correct because these variables are correlated through the nonlinear (square in incident radiant power) term as defined in Eq. (19) in Ref. 2.

At $X=0.5$, the error is maximum. Under the current condition $\overline{b_{ij}}/\overline{a_{ij}} \ll 1$, it is

$$(\delta S_{o,\mathrm{NL}})_{\max}=\frac{\delta S_{ij,I}(P_H-P_L)^2}{4P_H N}\,\sigma_{b_{ij}}. \tag{20}$$

If we assume further that the incident radiant power is uniformly distributed between that of the cold and hot targets, the rms (over $X$) error is

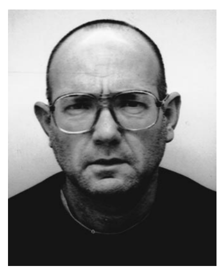
$$(\delta_{o,\mathrm{NL}})_{\mathrm{rms}}=\frac{4}{\sqrt{30}}\,(\delta_{o,\mathrm{NL}})_{\max}. \tag{21}$$

## 4 Numerical Example

The FPA operates in the 7.8- to 10.2-$\mu$m range. The required object temperature range is 10 to 40°C. In this case, it is worthwhile to fix the cold and hot target temperatures at 15 and 36°C because the error due to nonlinearity at object emittance of 0.2 out of the emittance span between targets $(=1)$ on both sides is approximately equal to the maximum error in the midrange. We assume also the following values for the other parameters: the average nonlinearity $\overline{b_{ij}P_H/a_{ij}}$ is 0.1 and $\sigma_{a_{ij}}$ and $\sigma_{b_{ij}}$ are 0.1 and 0.03, respectively. The noise equivalent temperature (NET) is 0.1°C (at 25°C) and $\delta H_{ij}=\delta L_{ij}$ is 0.5 NET. Figure 2 describes the measurement and target error [Eq. (3) without $\delta S_{ij,I}$], the nonlinearity error [Eq. (19)], and the total error for this example.

## References

1. J. M. Mooney, F. D. Shepherd, W. S. Ewing, J. E. Murguia, and J. E. Silverman, ''Responsivity nonuniformity limited performance of infrared staring cameras,'' *Opt. Eng.* **28**, 1151–1161 (1989).
2. A. F. Milton, F. R. Barone, and M. R. Kruer, ''Influence of nonuniformity on infrared focal plane arrays performance,'' *Opt. Eng.* **24**, 855–862 (1985).
3. D. L. Perry and E. L. Dereniak, ''Linear theory of nonuniformity correction in infrared staring sensors,'' *Opt. Eng.* **32**, 1854–1859 (1993).
4. D. A. Scribner, K. A. Sarkady, J. T. Caulfield, M. R. Kruer, G. Katz, and C. J. Gridley, ''Non-uniformity correction for staring IR focal plane arrays using scene-based techniques,'' *Proc. SPIE* **1308**, 224–233 (1990).



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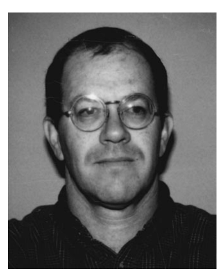

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