CSCI 567 - Home Work - 5

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1.1)

Computing $\alpha_t(j)$ recursively, till t=6 to compute $P(X_{1:6} = O_{1:6}; \theta)$

$\alpha_t(j) = P(x_t|Z_t = s_j) \sum_i a_{ij}\alpha_{t-1}(i)$

$= b_{ik} \sum_i a_{ij}\alpha_{t-1}(i)$

t=1:

$\alpha_1(1) = \pi_1 P(x_1 = A|Z_1 = s_1) = \pi_1 b_{1A} = 0.7 * 0.4 = 0.28$

$\alpha_1(2) = \pi_2 P(x_1 = A|Z_1 = s_2) = \pi_2 b_{2A} = 0.3 * 0.2 = 0.06$

t=2:

$\alpha_2(1) = b_{1G}[a_{11}\alpha_1(1) + a_{21}\alpha_1(2)] = 0.4(0.8 * 0.28 + 0.4 * 0.06) = 0.0992$

$\alpha_2(2) = b_{2G}[a_{12}\alpha_1(1) + a_{22}\alpha_1(2)] = 0.2(0.2 * 0.28 + 0.6 * 0.06) = 0.0184$

t=3:

$\alpha_3(1) = b_{1C}[a_{11}\alpha_2(1) + a_{21}\alpha_2(2)] = 0.1(0.8 * 0.0992 + 0.4 * 0.0184) = 8.672 * 10^{-3}$

$\alpha_3(2) = b_{2C}[a_{12}\alpha_2(1) + a_{22}\alpha_2(2)] = 0.3(0.2 * 0.0992 + 0.6 * 0.0184) = 9.264 * 10^{-3}$

t=4:

$\alpha_4(1) = b_{1G}[a_{11}\alpha_3(1) + a_{21}\alpha_3(2)] = 0.4(0.8 * 8.672 * 10^{-3} + 0.4 * 9.264 * 10^{-3}) = 4.25728 * 10^{-3}$

$\alpha_4(2) = b_{2G}[a_{12}\alpha_3(1) + a_{22}\alpha_3(2)] = 0.2(0.2 * 8.672 * 10^{-3} + 0.6 * 9.264 * 10^{-3}) = 1.45856 * 10^{-3}$

t=5:

$\alpha_5(1) = b_{1T}[a_{11}\alpha_4(1) + a_{21}\alpha_4(2)] = 0.1(0.8 * 4.25728 + 0.4 * 1.45856) * 10^{-3} = 3.989248 * 10^{-4}$

$\alpha_5(2) = b_{2T}[a_{12}\alpha_4(1) + a_{22}\alpha_4(2)] = 0.3(0.2 * 4.25728 + 0.6 * 1.45856) * 10^{-3} = 5.179776 * 10^{-4}$

t=6:

$\alpha_6(1) = b_{1A}[a_{11}\alpha_5(1) + a_{21}\alpha_5(2)] = 0.4(0.8 * 3.989248 + 0.4 * 5.179776) * 10^{-4}$

$= 2.10532352 * 10^{-4}$

$\alpha_6(2) = b_{2A}[a_{12}\alpha_5(1) + a_{22}\alpha_5(2)] = 0.2(0.2 * 3.989248 + 0.6 * 5.179776) * 10^{-4} = 7.8114304 * 10^{-5}$

$P(X_{1:6} = O_{1:6}; \theta) = \alpha_6(1) + \alpha_6(2) = 2.88646656 * 10^{-4}$

1.2)

$z^*_{1:6} = [z^*_1 z^*_2 z^*_3 z^*_4 z^*_5 z^*_6] = argMax_{z_{1:6}} P(Z_{1:6} = z_{1:6}|X_{1:6} = O_{1:6}; \theta) = argMax_j \delta_T(j)$

$\delta_t(j) = max_i \delta_{t-1}(i) a_{ij} P(x_t = k|Z_t = s_j)$

$= max_i \delta_{t-1}(i) a_{ij} b_{jk}$

t=1:

$\delta_1(1) = P(x_1 =' A'|Z_1 = s_1).\pi_1 = b_{1A}.\pi_1 = 0.4 * 0.7 = 0.28$

$\delta_1(2) = P(x_1 =' A'|Z_1 = s_2).\pi_2 = b_{2A}.\pi_2 = 0.2 * 0.3 = 0.06$

t=2:

$$\delta_2(1) = max\{\delta_1(1)a_{11}b_{1G}, \delta_1(2)a_{21}b_{1G}\} = max\{0.28 * 0.8 * 0.4, 0.06 * 0.4 * 0.4\}$$
$$= max\{0.0896, 9.6 * 10^{-3}\} = 0.0896$$
$$\delta_2(2) = max\{\delta_1(1)a_{12}b_{2G}, \delta_1(2)a_{22}b_{2G}\} = max\{0.28 * 0.2 * 0.2, 0.06 * 0.6 * 0.2\}$$
$$= max\{0.0112, 7.2 * 10^{-3}\} = 0.0112$$

t=1	t=2
1	1
1	2

t=3:

$$\delta_3(1) = max\{\delta_2(1)a_{11}b_{1C}, \delta_2(2)a_{21}b_{1C}\} = max\{0.0896 * 0.8 * 0.1, 0.0112 * 0.4 * 0.1\}$$
$$= max\{7.168 * 10^{-3}, 4.48 * 10^{-4}\} = 7.168 * 10^{-3}$$
$$\delta_3(2) = max\{\delta_2(1)a_{12}b_{2C}, \delta_2(2)a_{22}b_{2C}\} = max\{0.0896 * 0.2 * 0.3, 0.0112 * 0.6 * 0.3\}$$
$$= max\{5.376 * 10^{-3}, 2.016 * 10^{-3}\} = 5.376 * 10^{-3}$$

t=2	t=3
1	1
1	2

t=4:

$$\delta_4(1) = max\{\delta_3(1)a_{11}b_{1G}, \delta_3(2)a_{21}b_{1G}\} = max\{7.168 * 10^{-3} * 0.8 * 0.4, 5.376 * 10^{-3} * 0.4 * 0.4\}$$
$$= max\{2.29376 * 10^{-3}, 8.6016 * 10^{-4}\} = 2.29376 * 10^{-3}$$
$$\delta_4(2) = max\{\delta_3(1)a_{12}b_{2G}, \delta_3(2)a_{22}b_{2G}\} = max\{7.168 * 10^{-3} * 0.2 * 0.2, 5.376 * 10^{-3} * 0.6 * 0.2\}$$
$$= max\{2.8672 * 10^{-4}, 6.4512 * 10^{-4}\} = 6.4512 * 10^{-4}$$

t=3	t=4
1	1
2	2

t=5:

$$\delta_5(1) = max\{\delta_4(1)a_{11}b_{1T}, \delta_4(2)a_{21}b_{1T}\} = max\{2.29376*10^{-3}*0.8*0.1, 6.4512*10^{-4}*0.4*0.1\}$$
$$= max\{1.835008 * 10^{-4}, 2.58048 * 10^{-5}\} = 1.835008 * 10^{-4}$$
$$\delta_5(2) = max\{\delta_4(1)a_{12}b_{2T}, \delta_4(2)a_{22}b_{2T}\} = max\{2.29376*10^{-3}*0.2*0.3, 6.4512*10^{-4}*0.6*0.1\}$$
$$= max\{1.376256 * 10^{-4}, 1.61216 * 10^{-4}\} = 1.61216 * 10^{-4}$$

t=4	t=5
1	1
2	2

t=6:

$$\delta_6(1) = max\{\delta_5(1)a_{11}b_{1A}, \delta_5(2)a_{21}b_{1A}\}$$
$$= max\{1.835008 * 10^{-4} * 0.8 * 0.4, 1.376256 * 10^{-4} * 0.4 * 0.4\}$$
$$= max\{5.8720256 * 10^{-5}, 2.2020096 * 10^{-5}\} = 5.8720256 * 10^{-5}$$
$$\delta_6(2) = max\{\delta_5(1)a_{12}b_{2A}, \delta_5(2)a_{22}b_{2A}\}$$

$= max\{1.835008 * 10^{-4} * 0.2 * 0.2, 1.376256 * 10^{-4} * 0.6 * 0.2\}$
$= max\{7.340032 * 10^{-6}, 1.6515072 * 10^{-5}\} = 1.6515072 * 10^{-5}$

t=5	t=6
1	1
2	2

for t=6, path is s1 (as $\delta_6(1) > \delta_6(2)$) , by traversing backwards using tables,
$z^*_{1:6} = [z^*_1 z^*_2 z^*_3 z^*_4 z^*_5 z^*_6] = [s_1, s_1, s_1, s_1, s_1, s_1]$

1.3) By Bayes theorem,
$x^* = argMax_x = P(X_7 = x|X_{1:6} = O_{1:6}; \theta) = \frac{P(X_{1:7};\theta)}{P(X_{1:6};\theta)}$
$= argMax_x P(X_7 = x, X_{1:6} = O_{1:6}; \theta)$
Omitted denominator as it is same $\forall x \in A, C, T, G$.
Calculating $P(X_{1:7})$ for different x, i.e., calculating α_7 values as in Q (1.1) and adding extra step for t=7.

$X_7 = A$:
$\alpha_7(1) = b_{1A}[a_{11}\alpha_6(1) + a_{21}\alpha_6(2)] =$
$0.4(0.8 * 2.10532352 * 10^{-4} + 0.4 * 7.8114304 * 10^{-5}) = 7.986864128 * 10^{-5}$
$\alpha_7(2) = b_{2A}[a_{12}\alpha_6(1) + a_{22}\alpha_6(2)] =$
$0.2(0.2 * 2.10532352 * 10^{-4} + 0.6 * 7.8114304 * 10^{-5}) = 1.779501056 * 10^{-5}$
$P(X_7 =' A', X_{1:6} = O_{1:6}; \theta) = \alpha_7(1) + \alpha_7(2) = 9.766365184 * 10^{-5}$

$X_7 = C$:
$\alpha_7(1) = b_{1C}[a_{11}\alpha_6(1) + a_{21}\alpha_6(2)] =$
$0.1(0.8 * 2.10532352 * 10^{-4} + 0.4 * 7.8114304 * 10^{-5}) = 1.996716032 * 10^{-5}$
$\alpha_7(2) = b_{2C}[a_{12}\alpha_6(1) + a_{22}\alpha_6(2)] =$
$0.3(0.2 * 2.10532352 * 10^{-4} + 0.6 * 7.8114304 * 10^{-5}) = 2.669251584 * 10^{-5}$
$P(X_7 =' C', X_{1:6} = O_{1:6}; \theta) = \alpha_7(1) + \alpha_7(2) = 4.665967616 * 10^{-5}$

$X_7 = G$:
$\alpha_7(1) = b_{1G}[a_{11}\alpha_6(1) + a_{21}\alpha_6(2)] =$
$0.4(0.8 * 2.10532352 * 10^{-4} + 0.4 * 7.8114304 * 10^{-5}) = 7.986864128 * 10^{-5}$
$\alpha_7(2) = b_{2G}[a_{12}\alpha_6(1) + a_{22}\alpha_6(2)] =$
$0.2(0.2 * 2.10532352 * 10^{-4} + 0.6 * 7.8114304 * 10^{-5}) = 1.779501056 * 10^{-5}$
$P(X_7 =' G', X_{1:6} = O_{1:6}; \theta) = \alpha_7(1) + \alpha_7(2) = 9.766365184 * 10^{-5}$

$X_7 = T$:
$\alpha_7(1) = b_{1T}[a_{11}\alpha_6(1) + a_{21}\alpha_6(2)] =$
$0.1(0.8 * 2.10532352 * 10^{-4} + 0.4 * 7.8114304 * 10^{-5}) = 1.996716032 * 10^{-5}$
$\alpha_7(2) = b_{2T}[a_{12}\alpha_6(1) + a_{22}\alpha_6(2)] =$
$0.3(0.2 * 2.10532352 * 10^{-4} + 0.6 * 7.8114304 * 10^{-5}) = 2.669251584 * 10^{-5}$
$P(X_7 =' T', X_{1:6} = O_{1:6}; \theta) = \alpha_7(1) + \alpha_7(2) = 4.665967616 * 10^{-5}$

x^* = A or G as both of them have higher probability than other two. And both of them have equal probability of getting observed at t=7.