

1.1 Sol: In the training set, when computing $(X^T X)^{-1}$, the matrix should not have non-zero rows in echelon form. i.e., if $N \geq D + 1$ and training set is linearly independent of each other we get unique solution.

1.2 Sol:

$$RSS(\omega) = \sum_n [y_n - (b + \sum_d \omega_d x_{nd})]^2$$

derivative w.r.t to b and equating to 0

$$\partial RSS(\omega) / \partial b = 2 \sum_n [y_n - (b + \sum_d \omega_d x_{nd})](-1) = 0$$

$$\sum_n y_n - Nb - \sum_n \sum_d \omega_d x_{nd} = 0$$

$$Nb = \sum_n y_n - \sum_n \sum_d \omega_d x_{nd}$$

$$\text{but, } \frac{1}{N} \sum_n x_{nd} = 0$$

$$b^* = \frac{\sum_n y_n}{N}$$

1.3 Sol:

As we do not have access to feature x, implies $x_n = 0$, implies $\omega = 0$

$$\partial \varepsilon(b) / \partial b = - \sum_n \{y_n [1 - \sigma(b)] - (1 - y_n) \sigma(b)\} = 0$$

$$\sigma(b) = \frac{1}{N} \sum_n y_n \Rightarrow e^{-b} = \frac{\sum_n y_n}{N} - 1, \text{ assuming } \sum_n 1 = N$$

$$b^* = \log\left(\frac{\sum_n y_n}{N - \sum_n y_n}\right)$$

$$\text{probability that } y=1 \text{ is } p(y=1) = \frac{\sum_n y_n}{N}$$