

CSCI 567 - HOME WORK - 2

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1.1)

$$\partial l / \partial u = (\partial l / \partial a) \cdot W^{(2)} \cdot H(u)$$

$$\partial l / \partial a = -\frac{y^T}{Z} \cdot Z'(a); \text{ where each element of (kxk) matrix } Z'(a) \text{ is } \mathbf{Z}'(a) = \frac{e^{a_m}}{\sum_k e^{a_k}} [I(m == n) - \frac{e^{a_n}}{\sum_k e^{a_k}}]$$

$$\partial l / \partial W^{(1)} = (\partial l / \partial u) \cdot x^T$$

$$\partial l / \partial b^{(1)} = \partial l / \partial u$$

$$\partial l / \partial W^{(2)} = (\partial l / \partial a) \cdot h$$

1.2)

When $W^{(1)}, W^{(2)}, b^{(1)}$ is initialized to zero vectors/matrices, loss derivatives w.r.t these also become zero., so at every step of gradient descent ($v = \alpha V - \eta g$ and $\omega = \omega + V$), will not change from initial values.

1.3)

$$u = W^{(1)} \cdot x + b^{(1)} \Rightarrow a = W^{(2)} \cdot (W^{(1)} \cdot x + b^{(1)}) + b^{(2)}$$

comparing this to $a = Ux + v$

$$U = W^{(2)} \cdot W^{(1)}$$

$$v = [W^{(2)} \cdot b^{(1)}] + b^{(2)}$$

2.1)

$$J(\omega) = \min_{\omega} \sum_n l(\omega^T \phi(x_n), y_n) + (\lambda/2) \|\omega\|_2^2$$

$$\partial J(\omega) / \partial \omega = \sum_n \partial l(s, y) / \partial s \cdot (\phi(x_n)) + \lambda \omega = 0$$

$$\lambda \omega^* = - \sum_n \partial l(s, y) / \partial s \cdot \phi(x_n)$$

$$\omega^* = \phi^T \alpha$$

$$\text{where } \phi^T = (\phi(x_1) \phi(x_2) \dots \phi(x_n)) \Rightarrow R^{M \times N}$$

$$\text{and } \alpha_n = \frac{-1}{\lambda} \cdot \frac{\partial l(s, y)}{\partial s}$$

$$\omega^* = \sum_n \alpha_n \cdot \phi(x_n)$$

which proves that optimal parameter is a linear combination of features.

2.2)

$$J(\omega) = \min_{\omega} \sum_n l(\omega^T \phi(x_n), y_n) + (\lambda/2) \|\omega\|_2^2$$

$$J(\omega) = l(\phi \phi^T \alpha, y) + \frac{\lambda}{2} \|\phi^2 \alpha\|_2^2$$

objective function in terms of α and α

$$J(\alpha) = l(K\alpha, y) + \frac{\lambda}{2} \alpha^T K \alpha$$