Let 0<α<.5 be some constant (independent of the input array length n). Recall the Partition subroutine employed by the QuickSort algorithm, as explained in lecture. What is the probability that, with a randomly chosen pivot element, the Partition subroutine produces a split in which the size of the smaller of the two subarrays is ≥α times the size of the original array?

1−2∗α

α

1−α

2−2∗α

Now assume that you achieve the approximately balanced splits above in every recursive call --- that is, assume that whenever a recursive call is given an array of length k, then each of its two recursive calls is passed a subarray with length between αk and (1−α)k (where α is a fixed constant strictly between 0 and .5). How many recursive calls can occur before you hit the base case? Equivalently, which levels of the recursion tree can contain leaves? Express your answer as a range of possible numbers d, from the minimum to the maximum number of recursive calls that might be needed.

−log⁡(n)log⁡(α)≤d≤−log⁡(n)log⁡(1−α)

0≤d≤−log⁡(n)log⁡(α)

−log⁡(n)log⁡(1−α)≤d≤−log⁡(n)log⁡(α)

−log⁡(n)log⁡(1−2∗α)≤d≤−log⁡(n)log⁡(1−α)

Define the recursion depth of QuickSort to be the maximum number of successive recursive calls before it hits the base case --- equivalently, the number of the last level of the corresponding recursion tree. Note that the recursion depth is a random variable, which depends on which pivots get chosen. What is the minimum-possible and maximum-possible recursion depth of QuickSort, respectively?

Minimum: Θ(log⁡(n)) ; Maximum: Θ(n)

Minimum: Θ(log⁡(n)) ; Maximum: Θ(nlog⁡(n))

Minimum: Θ(1) ; Maximum: Θ(n)

Minimum: Θ(n) ; Maximum: Θ(n)

Consider a group of k people. Assume that each person's birthday is drawn uniformly at random from the 365 possibilities. (And ignore leap years.) What is the smallest value of k such that the expected number of pairs of distinct people with the same birthday is at least one?

[Hint: define an indicator random variable for each ordered pair of people. Use linearity of expectation.]

366

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27

28

23

Let X1,X2,X3 denote the outcomes of three rolls of a six-sided die. (I.e., each Xi is uniformly distributed among 1,2,3,4,5,6, and by assumption they are independent.) Let Y denote the product of X1 and X2 and Z the product of X2 and X3. Which of the following statements is correct?

Y and Z are not independent, and E[Y∗Z]≠E[Y]∗E[Z].

Y and Z are not independent, but E[Y∗Z]=E[Y]∗E[Z].

Y and Z are independent, and E[Y∗Z]=E[Y]∗E[Z].

Y and Z are independent, but E[Y∗Z]≠E[Y]∗E[Z].