# **Problem Set #2**

## **1. Question 1**

Consider a directed graph with distinct and nonnegative edge lengths and a source vertex s. Fix a destination vertex t, and assume that the graph contains at least one s-t path. Which of the following statements are true? [Check all that apply.]

The shortest (i.e., minimum-length) s-t path might have as many as n−1 edges, where n is the number of vertices.

The shortest s-t path must include the minimum-length edge of G.

There is a shortest s-t path with no repeated vertices (i.e., a "simple" or "loopless" such path).

The shortest s-t path must exclude the maximum-length edge of G.

## **2. Question 2**

Consider a directed graph G with a source vertex s, a destination t, and nonnegative edge lengths. Under what conditions is the shortest s-t path guaranteed to be unique?

When all edge lengths are distinct powers of 2.

When all edge lengths are distinct positive integers.

None of the other options are correct.

When all edges lengths are distinct positive integers and the graph G contains no directed cycles.

## **3. Question 3**

Consider a directed graph G=(V,E) and a source vertex s with the following properties: edges that leave the source vertex s have arbitrary (possibly negative) lengths; all other edge lengths are nonnegative; and there are no edges from any other vertex to the source s. Does Dijkstra's shortest-path algorithm correctly compute shortest-path distances (from s) in this graph?

Always

Maybe, maybe not (depends on the graph)

Only if we add the assumption that G contains no directed cycles with negative total weight.

Never

## **4. Question 4**

Consider a directed graph G and a source vertex s. Suppose G has some negative edge lengths but no negative cycles, meaning G does not have a directed cycle in which the sum of the edge lengths is negative. Suppose you run Dijkstra's algorithm on G (with source s). Which of the following statements are true? [Check all that apply.]

It's impossible to run Dijkstra's algorithm on a graph with negative edge lengths.

Dijkstra's algorithm always terminates, but in some cases the paths it computes will not be the shortest paths from s to all other vertices.

Dijkstra's algorithm might loop forever.

Dijkstra's algorithm always terminates, and in some cases the paths it computes will be the correct shortest paths from s to all other vertices.

## **5. Question 5**

Consider a directed graph G and a source vertex s. Suppose G contains a negative cycle (a directed cycle in which the sum of the edge lengths is negative) and also a path from s to this cycle. Suppose you run Dijkstra's algorithm on G (with source s). Which of the following statements are true? [Check all that apply.]

Dijkstra's algorithm always terminates, but in some cases the paths it computes will not be the shortest paths from s to all other vertices.

Dijkstra's algorithm always terminates, and in some cases the paths it computes will be the correct shortest paths from s to all other vertices.

Dijkstra's algorithm might loop forever.

It's impossible to run Dijkstra's algorithm on a graph with a negative cycle.