lot of memory

import array

define array

not pointer based = less memory

only int, float, char

tuple, list, array

hold addresses

point to actual object

Array-Based Sequences

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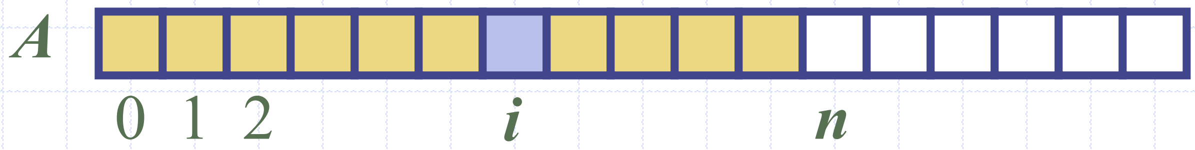
Python Sequence Classes

- Python has built-in types, **list**, **tuple**, and **str**.
- Each of these **sequence** types supports indexing to access an individual element of a sequence, using a syntax such as A[i]
- Each of these types uses an **array** to represent the sequence.
  - An array is a set of memory locations that can be addressed using consecutive indices, which, in Python, start with index 0.



List & tuple

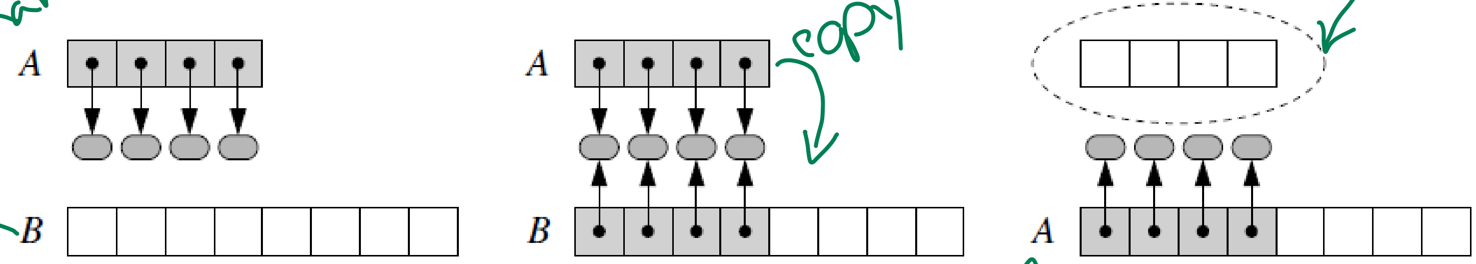
Dynamic Array

- A Python list can grow.
- Python list class uses an efficient dynamic array implementation

delete to save space

create more memory than needed

copy



Rename

(a) (b) (c)

(a) create new array *B* (b) store elements of *A* in *B* (c) reassign reference *A* to the new array.

Running times of `append` operations on dynamic arrays /list

↳ list.append(value)

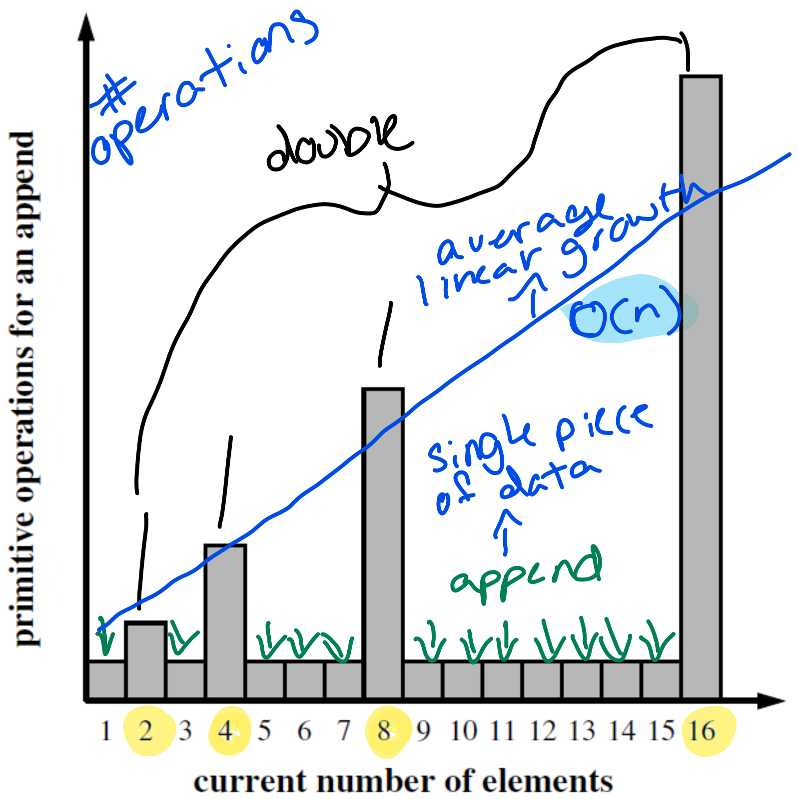
- After every n append operations, the list capacity doubles.
  - This is efficient.
- The total time to perform a series of n append operations is **O(n)**.

size = 1

size = size · 2 # 2

size = size · 2 # 4

size = size · 2 # 8



geometric progression

↳ O(n) ↳ multiplying

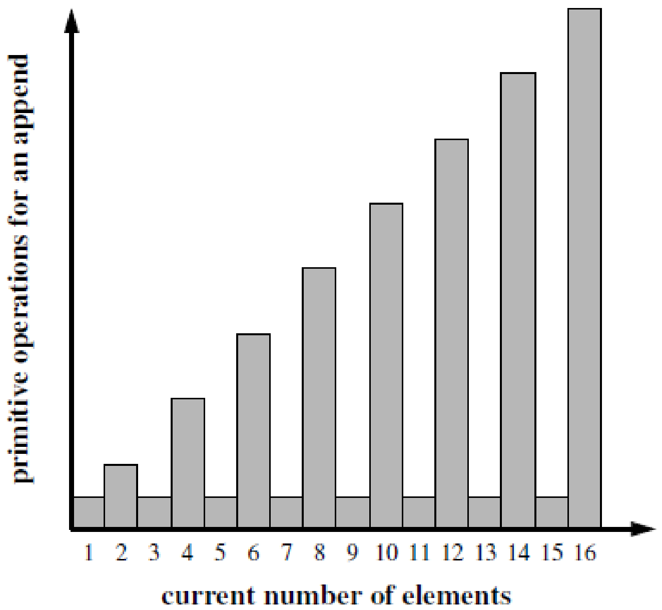
Beware of Arithmetic Progression

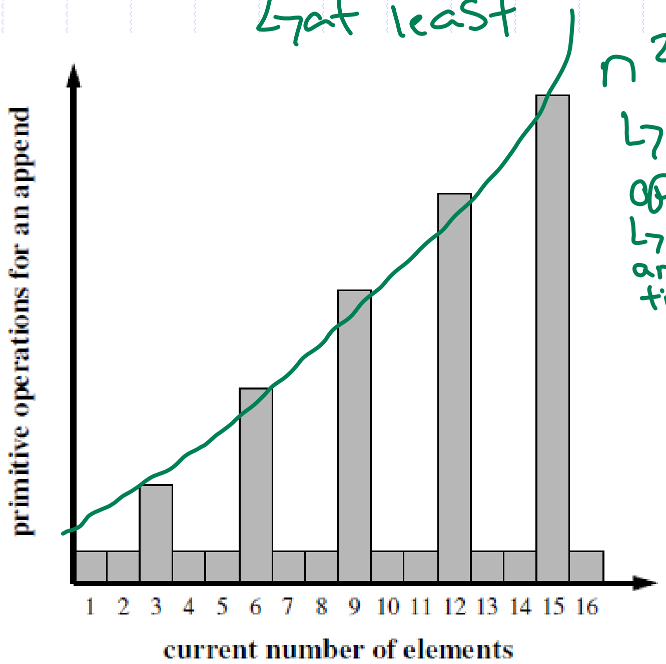
adding # each time

- Arithmetically increasing the array size by adding 2 or 3 may seem to save memory space.
- Run time will be in the order of $\Omega(n^2)$

at least

adding cells as needed





n^2
more opperations
copy whole array each time

Non-mutating behaviors
↳ worst case complexity of O(n)

Efficiency of `list` and `tuple`

↳ not dynamic
↳ cannot expand

- *Non-mutating* behaviors of tuples and lists

1 2 4 2
data.count(2)
↳ comparing each

Operation	Running Time
len(data)	$O(1)$ → creates list & saves size
reading value ← data[j]	$O(1)$
data.count(value)	$O(n)$ # times value in seq. ↳ repeats
data.index(value)	$O(k+1)$ → 1st
value **in** data	$O(k+1)$
data1 == data2 (similarly !=, <, <=, >, >=)	$O(k+1)$
slicing data[j:k]	$O(k-j+1)$
data1 + data2	$O(n_1+n_2)$
c * data	$O(cn)$

`data`, `data1`, and `data2` designate instances of the list or tuple class, and n, n_1, and n_2 their respective lengths.

Mutating Behaviors of `list` class

can mutate, dynamic
↳ no tuples

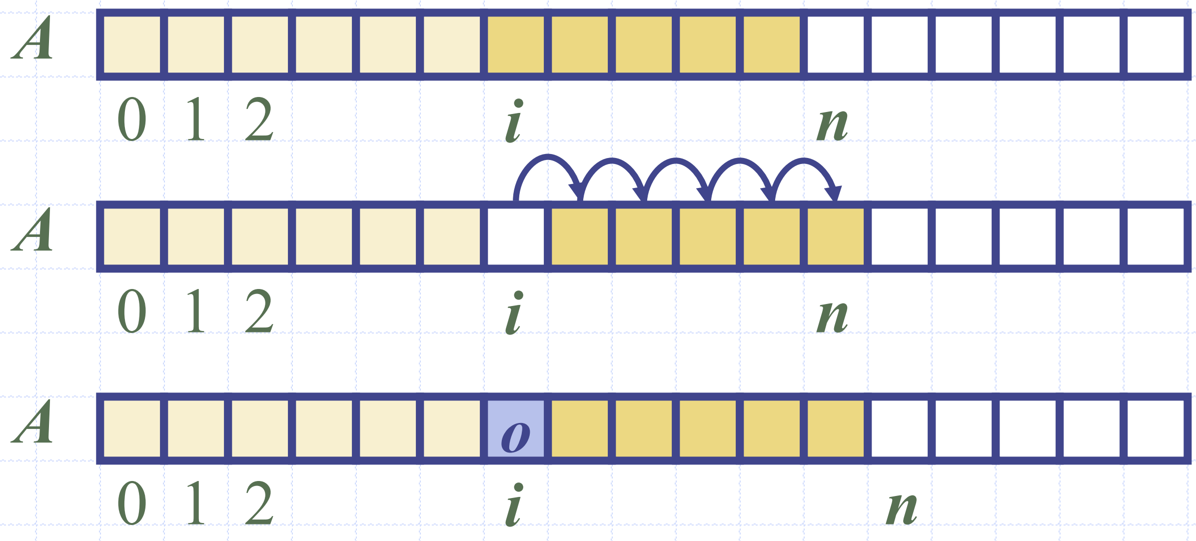
Operation	Running Time
data[j] = val	$O(1)$
data.append(value)	$O(1)^*$
data.insert(k, value)	$O(n-k+1)^*$
data.pop()	$O(1)^*$
data.pop(k) **del** data[k]	$O(n-k)^*$
data.remove(value)	$O(n)^*$
data1.extend(data2) data1 += data2	$O(n_2)^*$
data.reverse()	$O(n)$
data.sort()	$O(n \log n)$

*amortized → best case

add → algorithmic

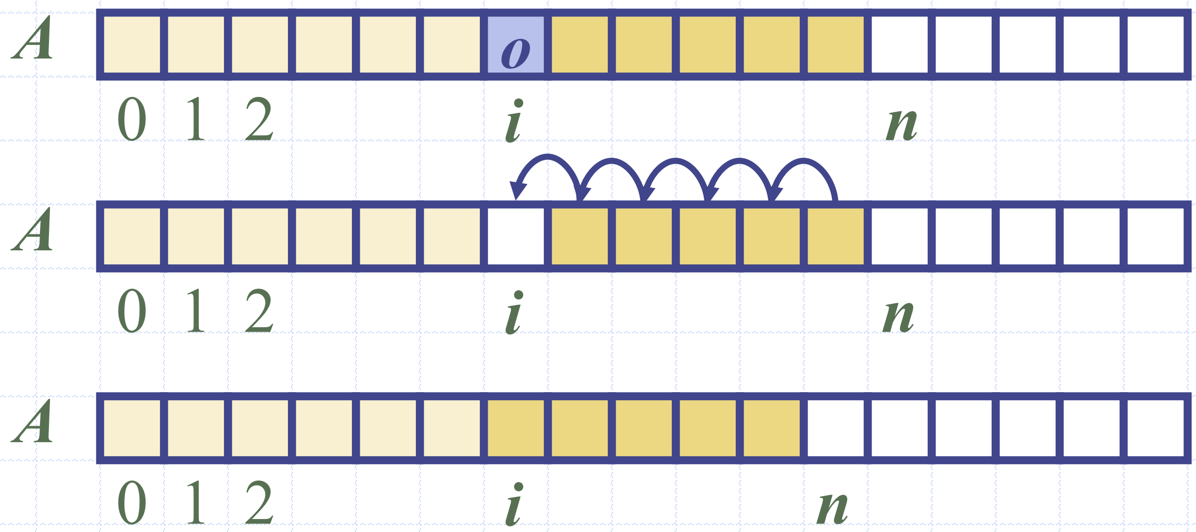
Insertion: `data.insert(i,o)`

- In an operation $add(i, o)$, we need to make room for the new element by shifting forward the $n - i$ elements $A[i], \ldots, A[n - 1]$
- In the worst case ($i = 0$), this takes $O(n)$ time



Element Removal: `data.pop(i)`

- In an operation *remove*(i), we need to fill the hole left by the removed element by shifting backward the $n - i - 1$ elements $A[i + 1], \ldots, A[n - 1]$
- In the worst case ($i = 0$), this takes $O(n)$ time



Extending a List

- Python provides a method named `extend` that is used to add all elements of one list to the end of a second list.

```
for element in other:
    data.append(element)
```

equivalent to :

```
data.extend(other)
```

data + = other

Preferred

↳ don't use append in loop -> call every time

↳ write code eliminating need to call many functions

[1, 4, 9, 16]

Constructing New `list`

- List comprehension:

```
squares = [ k*k for k in range(1, n+1) ]
```

- Equivalent to :

```
squares = [ ] empty
for k in range(1, n+1):
    squares.append(k*k)
```

add each

Faster, more efficient

- [0] * n : list of size n

Very efficient

Performance Conclusion

- In an array based implementation of a dynamic list:
  - The space used by the data structure is $O(n)$
  - Indexing the element at i takes $O(1)$ time
  - *add* and *remove* run in $O(n)$ time in worst case
- In an *add* operation, when the array is full, instead of throwing an exception, we can replace the array with a larger one…

Growable Array-based Array List

- In an add(o) operation (without an index), we could always add at the end
- When the array is full, we replace the array with a larger one
- How large should the new array be?
  - Incremental strategy: increase the size by a constant c
  - Doubling strategy: double the size

Algorithm $add(o)$
  if $t = S.length - 1$ **then**
    $A \leftarrow$ **new array of size …**
    for $i \leftarrow 0$ **to** $n-1$ **do**
      $A[i] \leftarrow S[i]$
    $S \leftarrow A$
  $n \leftarrow n + 1$
  $S[n-1] \leftarrow o$

Comparison of the Strategies

- We compare the incremental strategy and the doubling strategy by analyzing the total time $T(n)$ needed to perform a series of n add(o) operations
- We assume that we start with an empty stack represented by an array of size 1
- We call amortized time of an add operation the average time taken by an add over the series of operations, i.e., $T(n)/n$

Incremental Strategy Analysis

- We replace the array $k = n/c$ times
- The total time $T(n)$ of a series of n add operations is proportional to

$$n + c + 2c + 3c + 4c + \ldots + kc =$$
$$n + c(1 + 2 + 3 + \ldots + k) =$$
$$n + ck(k + 1)/2$$

- Since c is a constant, $T(n)$ is $O(n + k^2)$, i.e., $O(n^2)$
- The amortized time of an add operation is $O(n)$

Doubling Strategy Analysis

- We replace the array $k = \log_2 n$ times
- The total time $T(n)$ of a series of n add operations is proportional to

$$n + 1 + 2 + 4 + 8 + \ldots + 2^k =$$
$$n + 2^{k+1} - 1 =$$
$$3n - 1$$

- $T(n)$ is $O(n)$
- The amortized time of an add operation is $O(1)$

geometric series