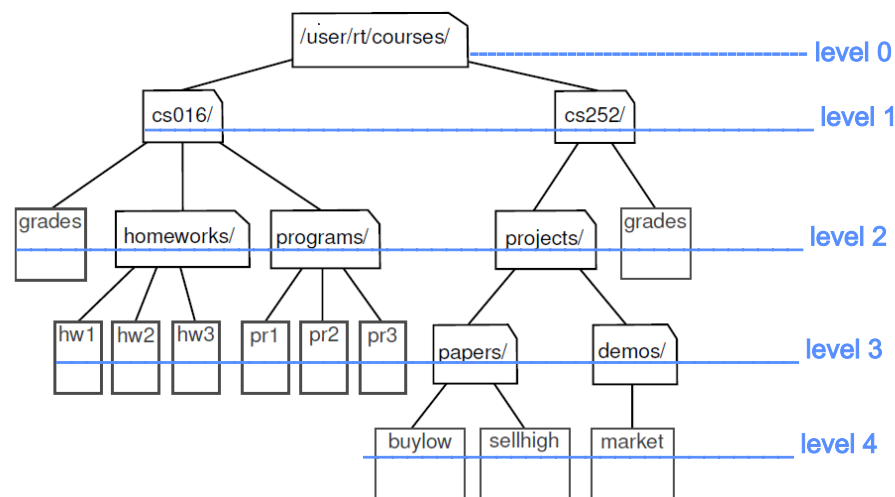


1- The following questions refer to the tree below:

- Which node is the root? `/user/rt/courses/`
- What are the internal nodes? `cs016/`, `homeworks/`, `programs/`, `cs252/`, `projects/`, `papers/`, `demos/`
- How many descendants does node `cs016/` have? 9
- How many ancestors does node `cs016/` have? 1
- What are the siblings of node `homeworks/`? `grades` & `programs/`
- Which nodes are in the subtree rooted at node `projects/`? `papers/` & `demos/`
- What is the depth of node `papers/`? 3
- What is the height of the tree? 4
- Indicate all the levels of this tree with lines.



2- Answer the following questions with justifications:

- What is the minimum number of external nodes for a proper binary tree with height  $h$ ? Justify your answer.
- What is the maximum number of external nodes for a proper binary tree with height  $h$ ? Justify your answer.
- Let  $T$  be a proper binary tree with height  $h$  and  $n$  nodes. Show that

$$\log(n+1) - 1 \leq h \leq (n-1)/2$$

- For which values of  $n$  and  $h$  can the above lower and upper bounds on  $h$  become an equality?

#2

a) The minimum number of external nodes would be one, if the tree only consisted of a single node, it would be considered external.

b) The maximum number of external nodes would equal the internal nodes +1. If you divide the internal nodes into one pile, and the external nodes into another pile, for a proper binary tree, the external node pile will always have one more than the internal node pile.

c)  $\log(n+1) - 1 \leq h \leq (n-1)/2$

Above I have proven that the number of external nodes for a proper binary tree is  $2^h$ . H can vary between these two intervals.

The bottom interval,  $\log(n+1) - 1$ :

$2^{(h+1)} - 1 = n$ , max # of nodes from the lecture

- rearranged to be  $\Rightarrow n + 1 = 2^{h+1}$

- log of both sides  $\Rightarrow \log(n+1) = h + 1$

- rearranged to be  $\Rightarrow h = \log(n+1) - 1$

The upper interval,  $(n-1)/2$ , is the total number of nodes for the proper binary tree.

d) When the binary tree is proper or perfect, meaning each parent node has two descendants, they become equalities, as stated in the lecture.

$\log(3+1)-1 \leq h \leq (3-1)/2$

$2-1 \leq h \leq 2/2$

$1 \leq h \leq 1$

3



3- Draw the binary tree representation of the following arithmetic expression:

$$(((5+2) * (2-1))/((2+9)+((7-2)-1)) * 8)$$

4- Draw a binary tree T that simultaneously satisfies the following:

- Each internal node of T stores a single character.
- A preorder traversal of T yields EXAMFUN.
- An inorder traversal of T yields MAFXUEN.

