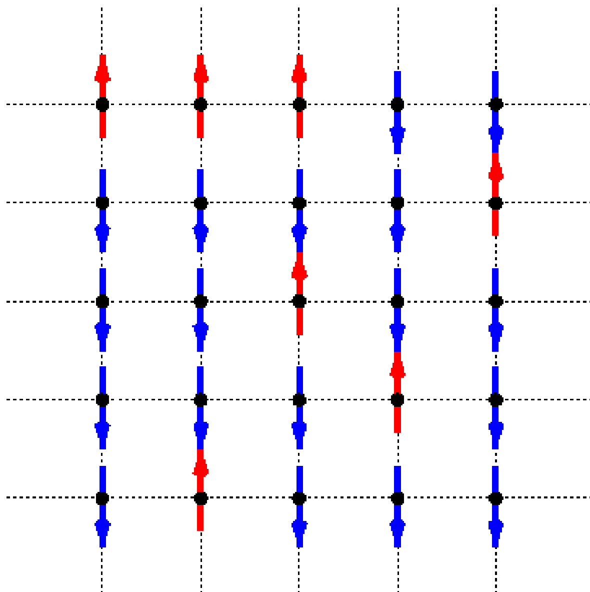
**Course Project for PHYS3151**
**Due Date: 2019.11.15 (group presentation)**

Please simulate a two dimensional Ising model using Monte Carlo method.

The two dimensional Ising model is shown below



There is a spin (S) on each site of the lattice. The value of the spin can take two values : +1 and -1. The energy of the whole lattice is defined as the sum of the interaction of the spins with their nearest neighbors.

$$E(C) = -J \sum_{<i,j>} S_i S_j$$

<i,j> denote the nearest neighboring sites. Usually we set J to be 1.

The magnetization is defined as

$$m = \frac{1}{N} \sum_i S_i$$

The procedure of Monte Carlo simulation is shown below. This is called Metropolis algorithm, please google it.

1. Initialize the lattice (randomly generate a configuration among $\{\pm 1, \pm 1, \pm 1, ..., \pm 1\}$ as the initial configuration). Set the size of the lattice to be L=4, 8, 16, 32.
2. Randomly choose a spin. Consider to flip the spin S to -S. Calculate the change of the energy of the system ( $\Delta E$ ) due to the flip.
    - If $\Delta E$ <0, accept the flip.
    - Else. Flip the spin with a probability of $e^{-\Delta E/T}$ .

3. Return to step 2 until convergence.

You should start sampling after the system is fully thermalized, that is, when the magnetization of the system doesn't change except for a small fluctuation. Typically we collect data of N steps and then average them to calculate the expectation values of the relevant physical quantities. The error of the quantity is determined by $error = \sqrt{var/N}$ , where "var" refers to the variance of the data. If A is a physical observable you want to measure, and $\{C_j\}$ is a set of configurations from which you evaluate the physical quantity. Then the expectation value can be calculated by

$$< A >= \frac{1}{N} \sum_{\{C_j\}} A(C_j)$$

After you successfully simulate the 2d Ising model, please draw <E>-T and <m>-T for different system sizes (L=4,8,16,32). You can also measure C and $\chi$ using the following relations

$$C = \frac{1}{N}\frac{d<E>}{dT} = \frac{1}{N}\frac{d}{dT}\frac{1}{Z}\sum_{\{C_j\}} E(C_j)e^{-E(C_j)/T} = \frac{1}{N}\frac{1}{T^2}(< E^2 > - < E >^2)$$

$$\chi = \frac{1}{N}\frac{1}{T}(< M^2 > - < M >^2)$$

C is the specific heat, $\chi$ is the susceptibility of the system, and M is just $\sum_i S_i$ .

Try to determine the phase transition point as precise as possible using the graphs you draw. Please also analyze some other properties which interest you, such as the critical exponent of m (which is 1/8).

Some useful references:

- http://physics.bu.edu/~py502/lectures5/mc.pdf
- https://en.wikipedia.org/wiki/Ising_model
- https://pdfs.semanticscholar.org/002a/23b6bc3f85f80dc74d075473b0f04746edbb.pdf
- https://www.phas.ubc.ca/~berciu/TEACHING/PHYS503/PROJECTS/05_dominic.pdf