Homework #3 — Due 11/25/18

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Problem 1

```
% Load in data
load ex3-3-2;

% Create tableau
T = totbl(A, b, p);

% b has no positive elements - go to phase 2 immediately
% Exchange elements
T = ljx(T, 1, 3);
T = ljx(T, 2, 2);

% Next would be column 4, but all positive values in this column
% so stop here
```

	x_1	x_6	x_5	x_4	1
$x_3 =$	0.6667	-0.3333	-0.3333	1.0000	3.0000
$x_2 =$	1.3333	-0.6667	0.3333	3.0000	2.0000
$x_7 =$	0.3333	-0.6667	0.3333	1.0000	5.0000
z	-4.3333	2.6667	0.6667	-6.0000	-16.0000

Setting $x_1 = \lambda$ and $x_4 = x_5 = x_6 = 0$, we obtain the equations:

$$x_3 = \frac{2}{3}\lambda + 3 \qquad x_2 = \frac{4}{3}\lambda + 2 \qquad x_7 = \frac{1}{3}\lambda + 5$$

Which yields a more general equation for x_1, x_2, x_3, x_4, :

$$x(\lambda) = \begin{bmatrix} 0 \\ 2 \\ 3 \\ 0 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ \frac{4}{3} \\ \frac{2}{3} \\ 0 \end{bmatrix}$$

Letting $\lambda = -415$ yields:

$$x = u - 415\lambda = \begin{bmatrix} -415.0000 \\ -551.3333 \\ -273.6667 \\ 0 \end{bmatrix}$$

Problem 2

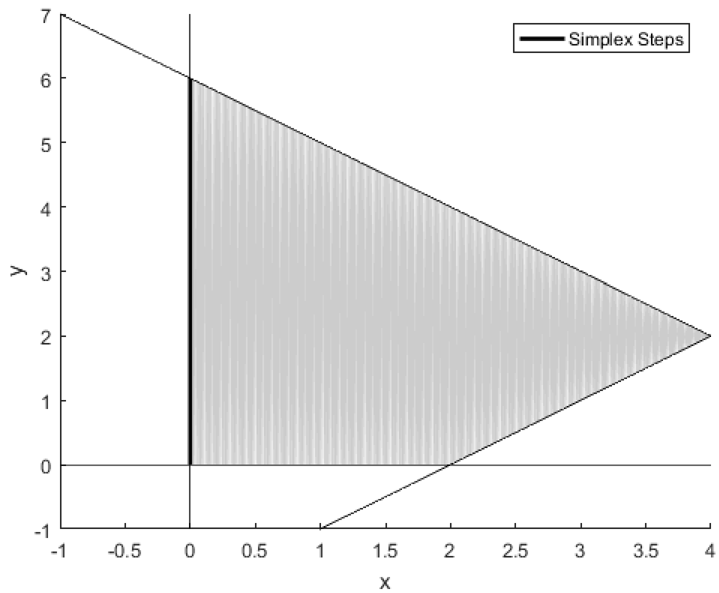
Part 1

```
% Load in data
load ex3-3-5-1

%Create Tableau
T = totbl(A, b, p);

% b has no positive elements - go to phase 2 immediately
% Exchange elements
T = ljx(T, 2, 2);

% Only useful column left is column 1, but z is not negative
% so we stop here
```

	x_1	x_4	1
$x_3 =$	-2.0000	-1.0000	8.0000
$x_2 =$	-1.0000	-1.0000	6.0000
z	2.0000	1.0000	-6.0000

There are no negative z-values, so we must stop. This gives optimal values of $x_1 = 0$ and $x_2 = 6$.

Part 2



Part 3

```
% Load in data
load ex3-3-5-3

%Create Tableau
T = totbl(A, b, p);

% b has no positive elements - go to phase 2 immediately
% Exchange elements
T = ljx(T, 1, 1); % First vertex
T = ljx(T, 2, 2); % Second vertex
```

We obtain vertex solutions at $x_1 = 2$, $x_2 = 0$ and $x_1 = 4$, $x_2 = 2$. Setting $x_3 = 0$ and $x_4 = \lambda$ yields the solution:

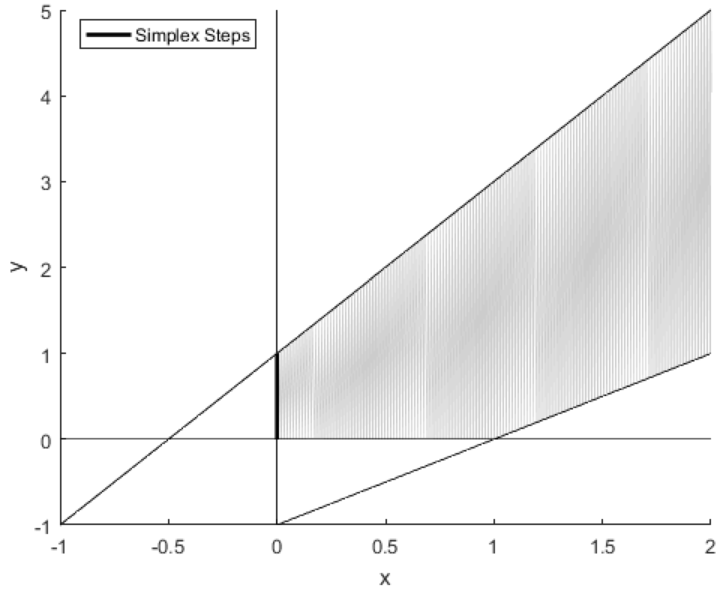
$$\{x_=(\frac{-1}{2}\lambda + 4, \frac{-1}{2}\lambda + 2) \mid 0 \leq \lambda \leq 4, \}$$

Part 4

```
% Load in data
load ex3-3-5-4

%Create Tableau
T = totbl(A, b, p);

% b has no positive elements - go to phase 2 immediately
% Exchange elements
T = ljx(T, 2, 1);
```

	x_4	x_2	1
$x_3 =$	-2.0000	1.0000	3.0000
$x_1 =$	-1.0000	1.0000	1.0000
z	1.0000	-2.0000	-1.0000

Since there is a negative value in the z row, but there no negative values in the x_2 column, the region is unbounded, as seen in the figure below. Since the objective function decrease as x_1 and x_2 increase, this means that the minimum of the function is $-\infty$.



Problem 3

	x_1	x_5	x_3	x_4	1
$x_2 =$	1.0000	-1.0000	-1.0000	-0.5000	2.0000
$x_6 =$	4.7500	-2.2500	-2.7500	3.1250	8.5000
$x_7 =$	0.0000	-1.0000	-1.0000	-2.5000	5.0000
z =	0.0000	3.0000	1.0000	6.5000	-6.0000

The feasible region is bounded since there are no more negative values left in the z row. Since the only value left is x_2, it is impossible for there to be more than one value that is a solution to the system, meaning the solution set is not unbounded.

Problem 4

Part 1

```
% Load data
A = [-1, -1; 2, 2];
b = [-2; 10];
p = [-3, 1];

% Create tableau
T = totbl(A, b, p);

% Phase 1 procedures
neg = [0; 1; 0];
T = addcol(T, neg, 'x0', 3);
T = addrow(T, [0 0 1 0], 'z0', 4);

% Special pivot
[maxviol, r] = max(b);
s = length(p) + 1;
T = ljx(T, r, s);

% Continue as usual
T = ljx(T, 1, 1);
```

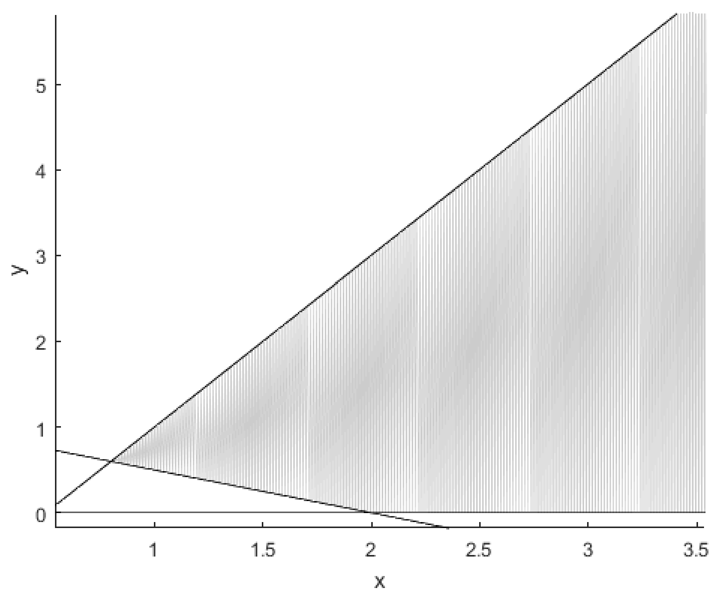
	x_3	x_2	x_4	1
$x_1 =$	-1.0000	-1.0000	0.0000	2.0000
$x_0 =$	2.0000	0.0000	1.0000	6.0000
z =	3.0000	4.0000	0.0000	-6.0000
$z_0 =$	2.0000	0.0000	1.0000	6.0000

We are unable to continue any further due to the z_0 row not containing any negative elements. Also note that x_0 has not been moved back to the top. Therefore the initial system must be infeasible.

Part 2

```
% Load data
A = [2, -1; 1, 2];
b = [1; 2];
p = [-1; 1];

% Create tableau
T = totbl(A, b, p);

% Phase 1 procedures
neg = [1; 1; 0];
T = addcol(T, neg, 'x0', 3);
T = addrow(T, [0 0 1 0], 'z0', 4);

% Special pivot
[maxviol, r] = max(b);
s = length(p) + 1;
T = ljx(T, r, s);

% Continue as usual
T = ljx(T, 2, 1);

% Remove phase 1 stuff
T = delrow(T, 'z0');
T = delcol(T, 'x0');
```

	x_2	x_4	1
$x_3 =$	-5.0000	2.0000	3.0000
$x_1 =$	-2.0000	1.0000	2.0000
z =	3.0000	-1.0000	-2.0000



There is a negative element in the z row, but we are unable to continue due to the x_4 column not containing any negative values. This means that the system is unbounded as seen in the figure above.

Problem 5

```
% Load data
load ex3-4-4

% Create tableau
T = totbl(A, b, p);

% Phase 1 procedures
neg = [1; 1; 0];
T = addcol(T, neg, 'x0', 5);
T = addrow(T, [0 0 0 0 1 0], 'z0', 4);

% Special pivot
[maxviol, r] = max(b);
s = length(p) + 1;
T = ljx(T, r, s);

% Continue as usual
T = ljx(T, 1, 3);

% Remove phase 1 stuff
T = delrow(T, 'z0');
T = delcol(T, 'x0');

% Continue exchanging
T = ljx(T, 2, 2);
T = ljx(T, 1, 3);
```

	x_1	x_6	x_3	x_5	1
$x_4 =$	-0.2000	0.4000	-0.2000	-0.2000	0.2000
$x_2 =$	-0.4000	-0.2000	-1.4000	0.6000	2.4000
z =	0.0000	1.0000	1.0000	1.0000	8.0000

A solution for the system is found at $x_1 = x_3 = 0$, $x_2 = 2.4$, and $x_4 = 0.2$ with a minimum value of $z = 8$.