Homework #4 Due 11/15/18

Salvatore Zerbo

**Problem 1**

```
% Load data
load ex4-2-2

% Setup tableau
T = totbl(A, b, p);

% Phase I
T = addcol(T, [0; 1; 0; 0], 'x0', 4);
T = addrow(T, [0 0 0 1 0], 'z0', 5);

% Special pivot
[maxviol, r] = max(b);
s = length(p) + 1;
T = ljx(T, r, s);

% Optimize phase I table and go to phase II
T = ljx(T, 2, 2);
T = delcol(T, 'x0');
T = delrow(T, 'z0');

% Optimize phase II table
T = ljx(T, 2, 1);
```

| | $x_2$ | $x_3$ | $x_5$ | 1 |
|---|---|---|---|---|
| $x_4 =$ | -2.0000 | 1.0000 | -1.0000 | 2.0000 |
| $x_1 =$ | -4.0000 | -1.0000 | 1.0000 | 1.0000 |
| $x_6 =$ | -6.0000 | 2.0000 | -2.0000 | 3.0000 |
| z = | 1.0000 | 1.0000 | 2.0000 | 2.0000 |

The minimum occurs at 2 with $x_1 = 1$, $x_2 = x_3 = 0$.

The dual is:

$$\begin{aligned} &max & -3u_1 + u_2 - 5u_3 \\ &subject\ to & -u_1 + u_2 - 2u_3 \leq 2 \\ && -6u_1 + 4u_2 - 14u_3 \leq 9 \\ && u_2 \leq 3 \\ && u_1, u_2, u_3 \geq 0 \end{aligned}$$

```
% Load data
load ex4-2-2

% Setup tableau
T = totbl(-A', -p, -b);
T = dualbl(T);

% Optimize dual
T = ljx(T, 1, 2);
```

| | | $u_4 =$ | $u_1 =$ | $u_6 =$ | w = |
|---|---|---|---|---|---|
| | | $x_1$ | $x_4$ | $x_3$ | 1 |
| $-u_5$ | $x_2 =$ | 1.0000 | -1.0000 | 2.0000 | 2.0000 |
| $-u_2$ | $x_5 =$ | 2.0000 | 4.0000 | 6.0000 | 1.0000 |
| $-u_3$ | $x_6 =$ | -1.0000 | 1.0000 | -2.0000 | 1.0000 |
| 1 | z = | 2.0000 | 1.0000 | 3.0000 | -2.0000 |

The maximum occurs at 2 with $u_2 = 2$ and $u_1 = u_3 = 0$.

**Problem 2**

```
% Load data
A = [3, 0; 2, 4; 2, 5];
b = [6; 10; 8];
p = [50; 80];

% Setup tableau
T = totbl(A, b, p);
```

```
T = dualbl(T);

% Optimize
T = ljx(T, 1, 1);
T = ljx(T, 3, 2);
T = ljx(T, 2, 2);
```

| | | $u_1 =$ | $u_2 =$ | w = |
|---|---|---|---|---|
| | | $x_3$ | $x_4$ | 1 |
| $-u_4$ | $x_1 =$ | 0.3333 | -0.0000 | 2.0000 |
| $-u_3$ | $x_5 =$ | -0.1667 | 1.2500 | 3.5000 |
| $-u_5$ | $x_2 =$ | -0.1667 | 0.2500 | 1.5000 |
| 1 | z = | 3.3333 | 20.0000 | 220.0000 |

The primal has a solution of $x_1 = 2$ and $x_2 = 1.5$ with minimum value of 220.
The dual has a solution of $u_1 = \frac{10}{3}$, $u_2 = 20$, and $u_3 = 0$ with a maximum value of 220.

## Problem 3

The primal form:

$$\begin{aligned} \mathit{min} \quad & -47x_1 + 13x_2 + 22x_3 \\ \mathit{subject\ to} \quad & -4x_1 + x_2 - 17x_3 \geq 2 \\ & -x_1 + x_2 + 39x_3 \geq 1 \\ & x_1, x_2 \geq 0 \end{aligned}$$

Formulating the dual:

$$\begin{aligned} \mathit{max} \quad & 2u_1 + u_2 \\ \mathit{subject\ to} \quad & -4u_1 - u_2 \leq -47 \\ & u_1 + u_2 \leq 13 \\ & -17u_1 + 39u_2 \leq 22 \\ & u_1, u_2 \geq 0 \end{aligned}$$

Consider the values $u_1 = 13$ and $u_2 = 0$. This satisfies the constraints of the dual:

$$\begin{aligned} -4 * 13 - 0 = -52 \leq -47 \\ 13 + 0 = 13 \leq 13 \\ -17 * 13 + 39 * 0 = -221 \leq 22 \\ 13 \geq 0 \\ 0 \geq 0 \end{aligned}$$

and has a value of $2 * 13 + 0 = 26$. By weak duality: $p^T\bar{x} \geq b^T u$, so a lower bound for the optimal value of the primal objective function is $\mathit{min} \geq 26$.

## Problem 4

### Part 1

```
% Load data
A = [1, -1; 1, 2];
b = [1, -3];
p = [1; -2];

% Setup tableau
T = totbl(A, b, p);

% Phase I
T = addcol(T, [1; 0; 0], 'x0', 3);
T = addrow(T, [0 0 1 0], 'z0', 4);

% Special pivot
[maxviol, r] = max(b);
s = length(p) + 1;
T = ljx(T, r, s);

% Optimize phase I and go to phase II
T = ljx(T, 1, 1);
T = delcol(T, 'x0');
T = delrow(T, 'z0');
```

| | $x_2$ | $x_3$ | 1 |
|---|---|---|---|
| $x_1 =$ | 1.0000 | 1.0000 | 1.0000 |
| $x_4 =$ | 3.0000 | 1.0000 | 4.0000 |
| z = | -1.0000 | 1.0000 | 1.0000 |

Since there is a negative value in the $x_2$ column, but the ratio test fails to identify a pivot row, the minimum must be unbounded. Letting $x_2 = \lambda$ and $x_3 = 0$, we get the equations:

$$x_1 = \lambda + 1 \qquad\qquad x_4 = 3\lambda + 4$$

Which yields the ray:

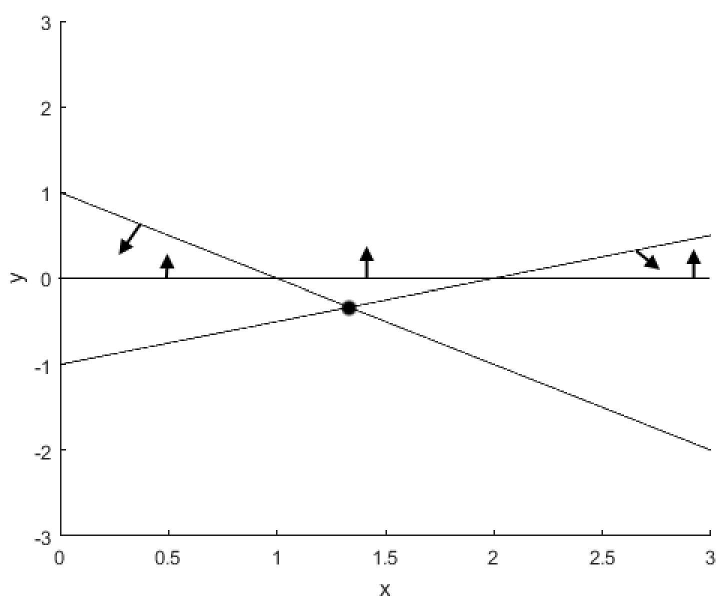
$$x(\lambda) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

**Part 2**

$$\begin{aligned} &max & u_1 - 3u_2 \\ &subject\ to & u_1 + u_2 \leq 1 \\ & & -u_1 + 2u_2 \leq -2 \\ & & u_1, u_2 \geq 0 \end{aligned}$$

**Part 3**

By the strong duality theorem, since the prime L.P. is unbounded, the dual L.P. must then be infeasible. This can be seen in the figure below where the only point that satisfies both equations is the intersection of the two lines; however, that point lies below the line y = 0, so it is also not feasible.




**Problem 5**

**Part 1**

$$\begin{aligned} &max & 5u_1 + 6u_2 \\ &subject\ to & u_1 + 2u_2 \leq 3 \\ & & 2u_1 + 2u_2 \leq 4 \\ & & 3u_1 + u_2 \leq 5 \\ & & u_1, u_2 \geq 0 \end{aligned}$$

```
% Load data
load ex4-6-4

% Setup dual tableau
```

```
T = totbl(-A', -p, -b);
T = dualbl(T);

% Optimize dual
T = ljx(T, 1, 2);
T = ljx(T, 2, 1);
```

| | | $u_2 =$ | $u_1 =$ | w = |
|---|---|---|---|---|
| | | $x_4$ | $x_3$ | 1 |
| $-u_5$ | $x_2 =$ | 0.5000 | -1.0000 | 1.0000 |
| $-u_4$ | $x_1 =$ | -1.0000 | 1.0000 | 1.0000 |
| $-u_3$ | $x_5 =$ | 2.5000 | -2.0000 | 1.0000 |
| 1 | z = | 2.0000 | 1.0000 | -11.0000 |

From this, the dual has the solution $u_1 = 1$ and $u_2 = 1$ with a maximum value of 11.

### Part 2

```
% Load data
load ex4-6-4

% Setup dual tableau
T = totbl(A, b, p);
T = dualbl(T);

% Optimize the dual
T = ljx(T, 1, 2);
T = ljx(T, 2, 1);
```

| | | $u_2 =$ | $u_1 =$ | $u_5 =$ | w = |
|---|---|---|---|---|---|
| | | $x_5$ | $x_4$ | $x_3$ | 1 |
| $-u_4$ | $x_2 =$ | -0.5000 | 1.0000 | -2.5000 | 2.0000 |
| $-u_3$ | $x_1 =$ | 1.0000 | -1.0000 | 2.0000 | 1.0000 |
| 1 | z = | 1.0000 | 1.0000 | 1.0000 | 11.0000 |

Taking the transpose and changing the signs yield:

$$-A^T = \begin{bmatrix} 0.5 & -1 & 2.5 \\ -1 & 1 & -2 \end{bmatrix}^T = \begin{bmatrix} 0.5 & -1 \\ -1 & 1 \\ 2.5 & -2 \end{bmatrix}$$

$$-b = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$-p = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}$$

$$w = -(11)$$

After transposing and changing signs, this yields the original table from part 1:

| | | $u_2 =$ | $u_1 =$ | w = |
|---|---|---|---|---|
| | | $x_4$ | $x_3$ | 1 |
| $-u_5$ | $x_2 =$ | 0.5000 | -1.0000 | 1.0000 |
| $-u_4$ | $x_1 =$ | -1.0000 | 1.0000 | 1.0000 |
| $-u_3$ | $x_5 =$ | 2.5000 | -2.0000 | 1.0000 |
| 1 | z = | 2.0000 | 1.0000 | -11.0000 |

## Problem 6

```
% Load data
load ex4-2-2

% Setup tableau
T = totbl(A, b, p);
T = dualbl(T);

% Optimize dual
T = ljx(T, 2, 1);
```

| | | $u_2 =$ | $u_5 =$ | $u_6 =$ | w = |
|---|---|---|---|---|---|
| | | $x_5$ | $x_2$ | $x_3$ | 1 |
| $-u_1$ | $x_4 =$ | -1.0000 | -2.0000 | 1.0000 | 2.0000 |
| $-u_4$ | $x_1 =$ | 1.0000 | -4.0000 | -1.0000 | 1.0000 |
| $-u_3$ | $x_6 =$ | -2.0000 | -6.0000 | 2.0000 | 3.0000 |
| 1 | z = | 2.0000 | 1.0000 | 1.0000 | 2.0000 |

The maximum occurs at 2 with $u_2 = 2$, $u_1 = u_3 = 0$. This is the same answer as question 1, part 2. Using the dual simplex method is much faster and requires less lines of code and is therefore better for this problem. Using the method in question 1 requires both phase I and II to obtain an optimal solution, while this method only requires phase II.