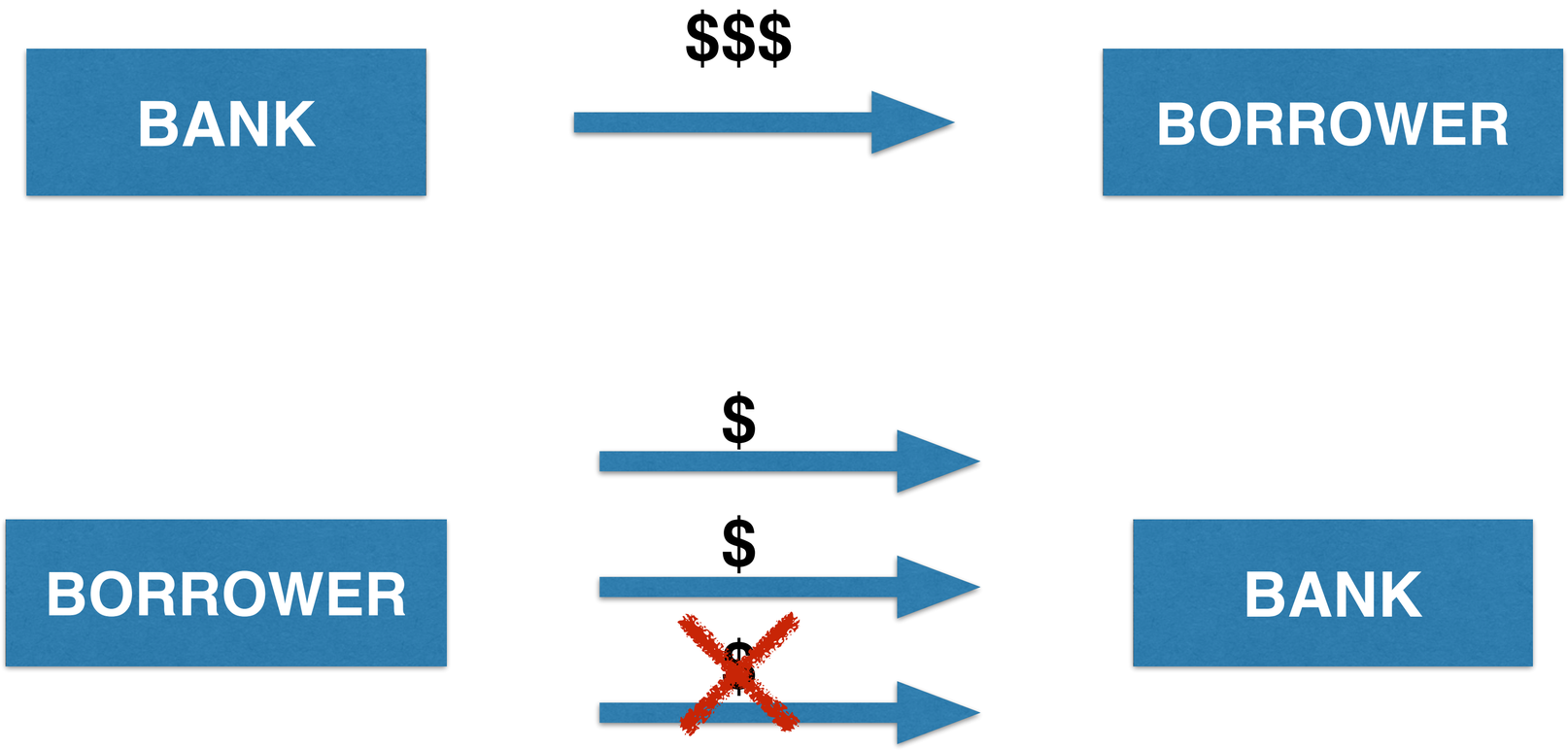
CREDIT RISK MODELING IN R

Introduction and data structure

What is loan default?



Components of expected loss (EL)

- Probability of default (PD)
- Exposure at default (EAD)
- Loss given default (LGD)

$$EL = PD \times EAD \times LGD$$

Information used by banks

- Application information:
  - income
  - marital status
  - ...
- Behavioral information
  - current account balance
  - payment arrears in account history
  - ...

The data

```
> head(loan_data, 10)
   loan_status loan_amnt int_rate grade emp_length home_ownership annual_inc age
1            0      5000    10.65     B          10           RENT      24000  33
2            0      2400       NA     C          25           RENT      12252  31
3            0     10000    13.49     C          13           RENT      49200  24
4            0      5000       NA     A           3           RENT      36000  39
5            0      3000       NA     E           9           RENT      48000  24
6            0     12000    12.69     B          11            OWN      75000  28
7            1      9000    13.49     C           0           RENT      30000  22
8            0      3000     9.91     B           3           RENT      15000  22
9            1     10000    10.65     B           3           RENT     100000  28
10           0      1000    16.29     D           0           RENT      28000  22
```

CrossTable

```
> library(gmodels)
> CrossTable(loan_data$home_ownership)

   Cell Contents
|-------------------------|
|                       N |
|         N / Table Total |
|-------------------------|


Total Observations in Table:  29092


           |  MORTGAGE |     OTHER |       OWN |      RENT |
           |-----------|-----------|-----------|-----------|
           |     12002 |        97 |      2301 |     14692 |
           |     0.413 |     0.003 |     0.079 |     0.505 |
           |-----------|-----------|-----------|-----------|
```

CrossTable

```
> CrossTable(loan_data$home_ownership, loan_data$loan_status, prop.r = TRUE,
prop.c = FALSE, prop.t = FALSE, prop.chisq = FALSE)

                           | loan_data$loan_status
loan_data$home_ownership   |         0 |         1 | Row Total |
---------------------------|-----------|-----------|-----------|
                  MORTGAGE |     10821 |      1181 |     12002 |
                           |     0.902 |     0.098 |     0.413 |
---------------------------|-----------|-----------|-----------|
                     OTHER |        80 |        17 |        97 |
                           |     0.825 |     0.175 |     0.003 |
---------------------------|-----------|-----------|-----------|
                       OWN |      2049 |       252 |      2301 |
                           |     0.890 |     0.110 |     0.079 |
---------------------------|-----------|-----------|-----------|
                      RENT |     12915 |      1777 |     14692 |
                           |     0.879 |     0.121 |     0.505 |
---------------------------|-----------|-----------|-----------|
              Column Total |     25865 |      3227 |     29092 |
---------------------------|-----------|-----------|-----------|
```

CREDIT RISK MODELING IN R

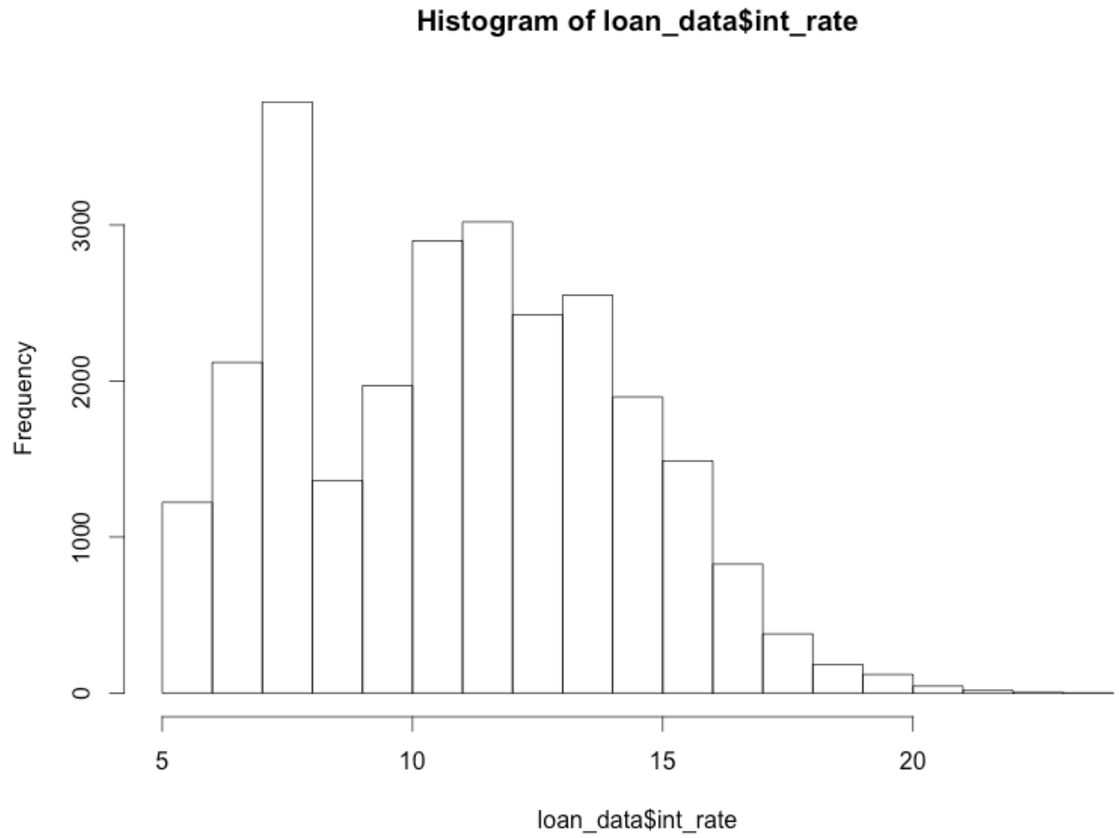
Let’s practice!

CREDIT RISK MODELING IN R

Histograms and outliers

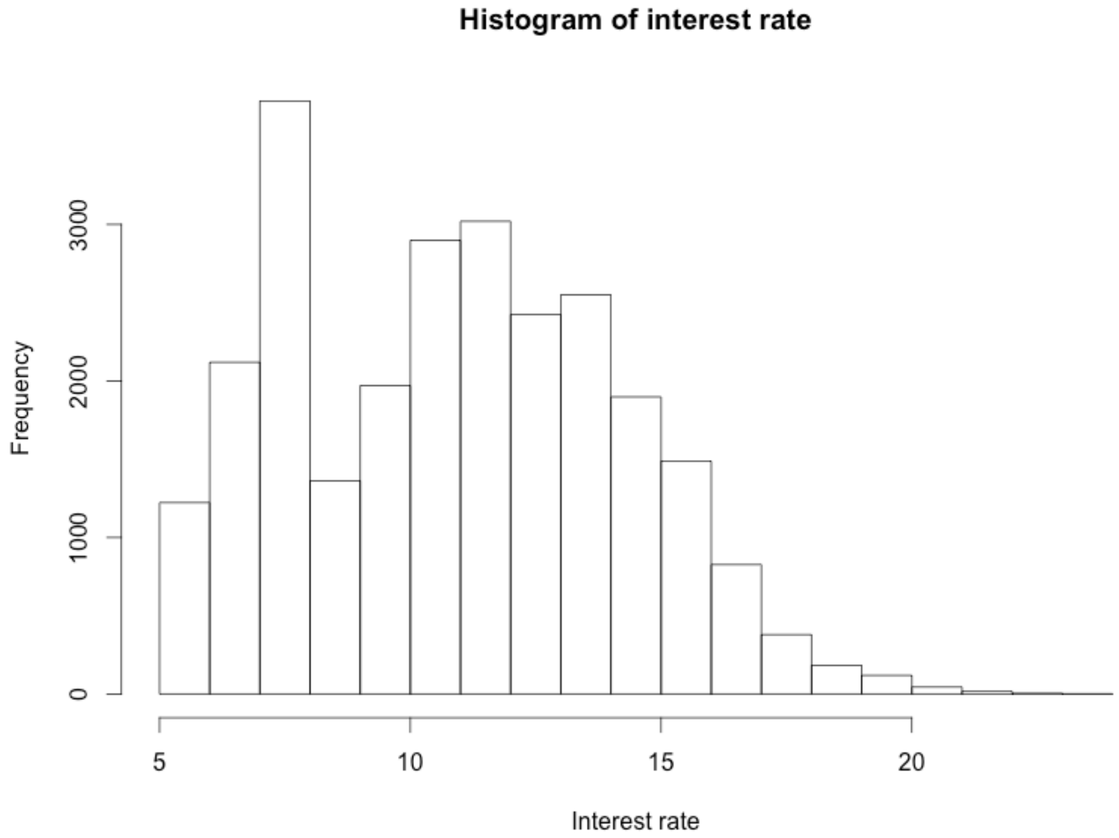
Using function hist()

```
> hist(loan_data$int_rate)
```



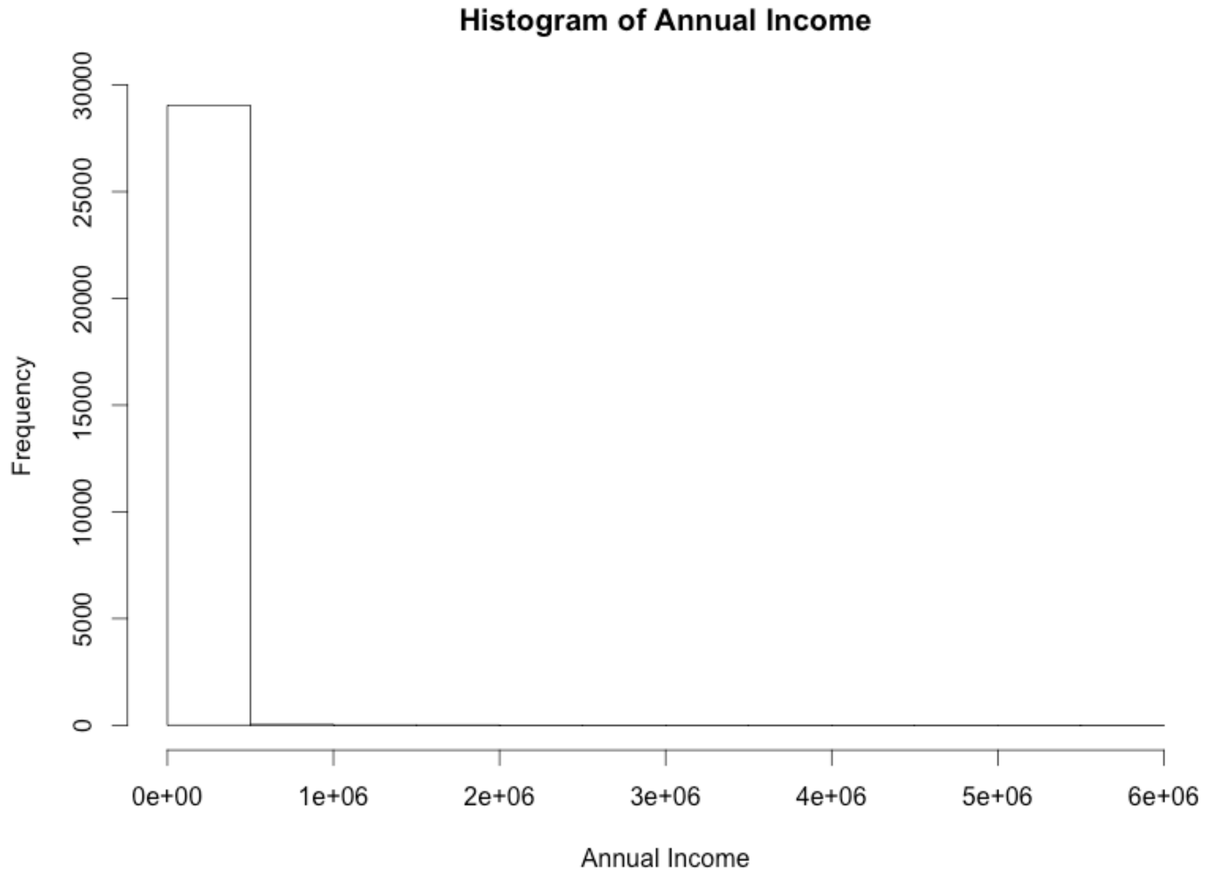
Using function hist()

```
> hist(loan_data$int_rate, main = "Histogram of interest rate", xlab = "Interest rate")
```



Using function hist() on annual_inc

```
hist(loan_data$annual_inc, xlab= "Annual Income”, main= "Histogram of Annual Income")
```

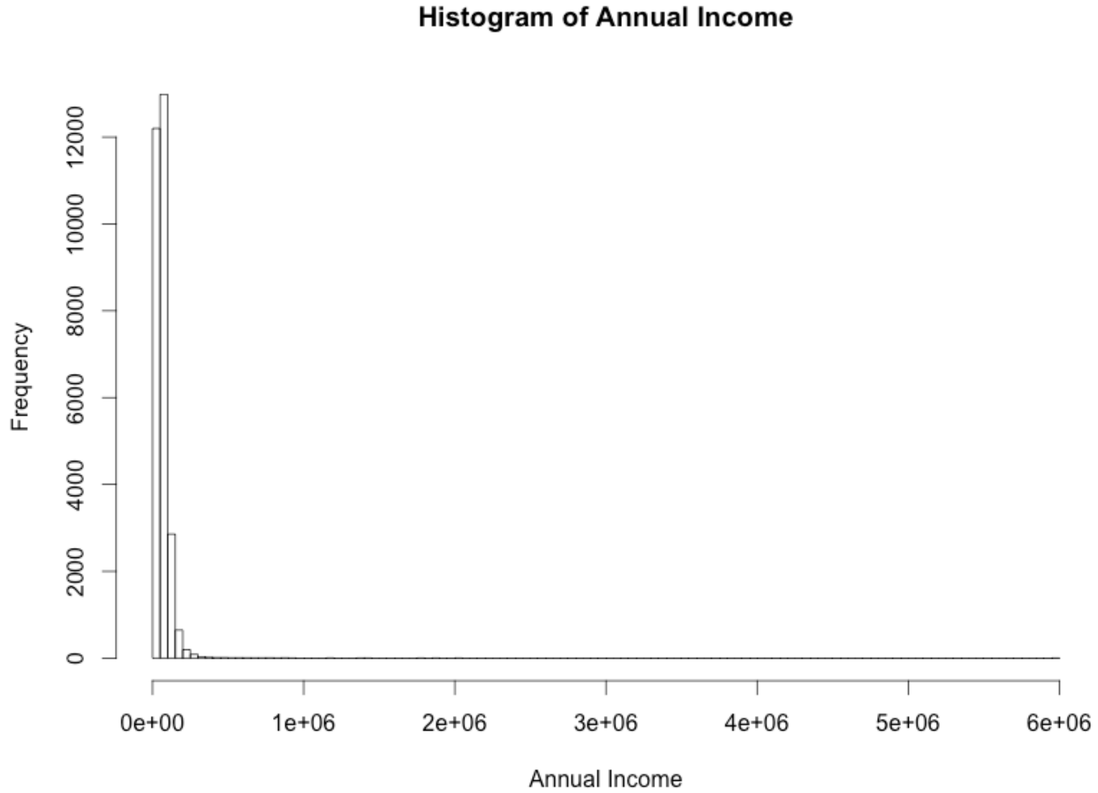


Using function hist() on annual_inc

```
> hist_income <- hist(loan_data$annual_inc, xlab = "Annual Income”, main =
"Histogram of Annual Income”)

> hist_income$breaks
 [1]       0  500000 1000000 1500000 2000000 2500000 3000000 3500000 4000000
4500000 5000000 5500000 6000000
```

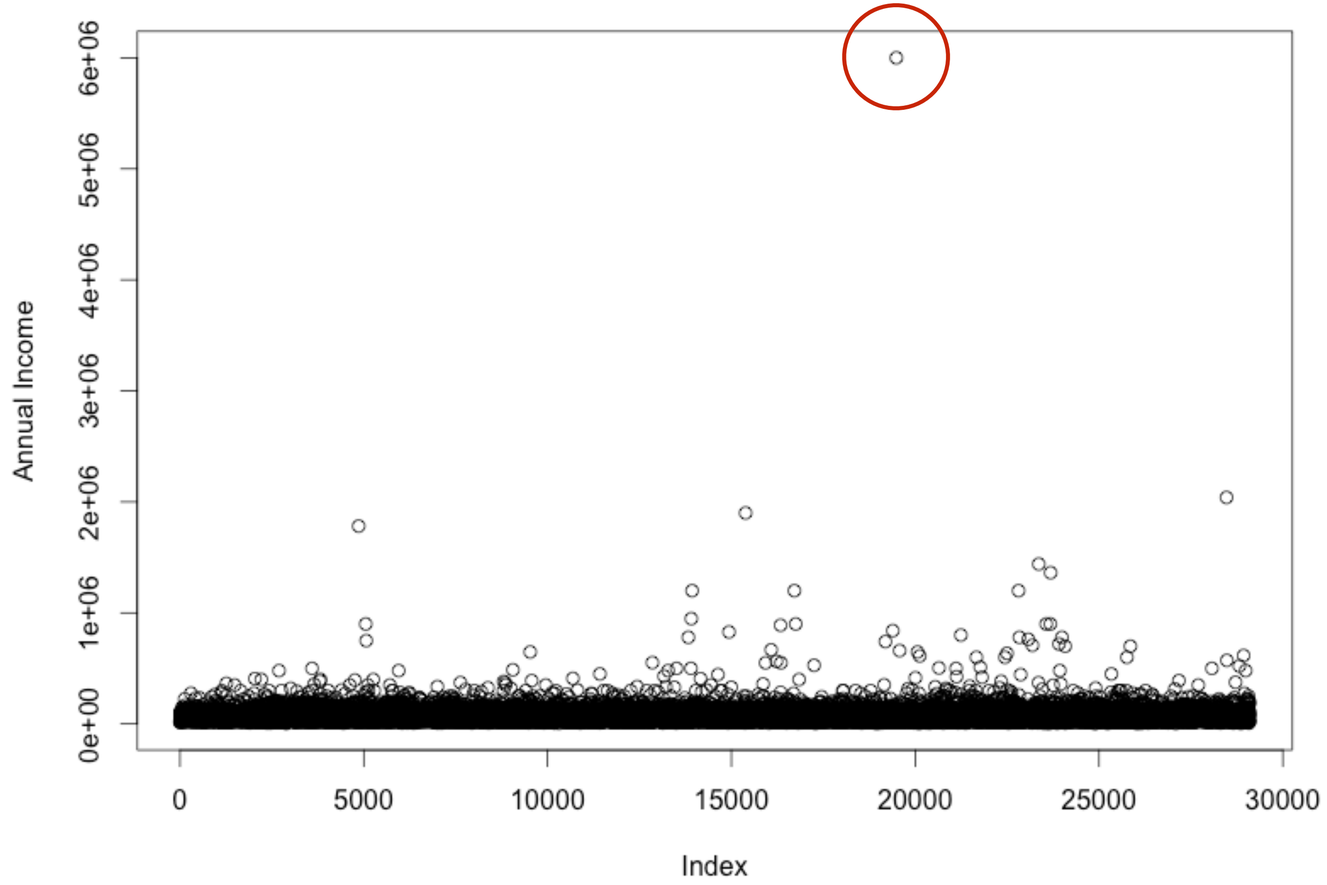
The breaks-argument

```
> n_breaks <- sqrt(nrow(loan_data)) # = 170.5638

> hist_income_n <- hist(loan_data$annual_inc, breaks= n_breaks, xlab = "Annual
Income”, main = "Histogram of Annual Income")
```



annual_inc

```
plot(loan_data$annual_inc, ylab = "Annual Income")
```



Outliers

- When is a value an outlier?
  - expert judgement
  - rule of thumb: Q1 - 1.5 * IQR
    Q3 + 1.5 * IQR
  - mostly: combination of both

Expert judgement - rule of thumb

“Annual salaries > $ 3 million are outliers”

```
> index_outlier_expert <- which(loan_data$annual_inc > 3000000)

> loan_data_expert <- loan_data[-index_outlier_expert, ]
```

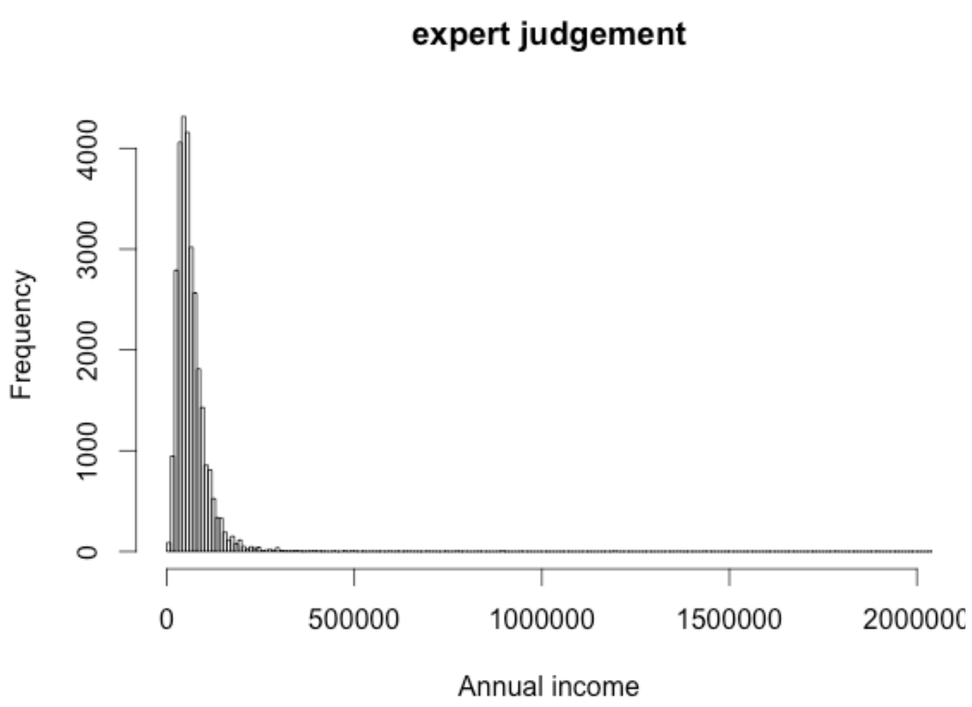
Use of a rule of thumb: outlier if bigger than Q3 + 1.5 * IQR

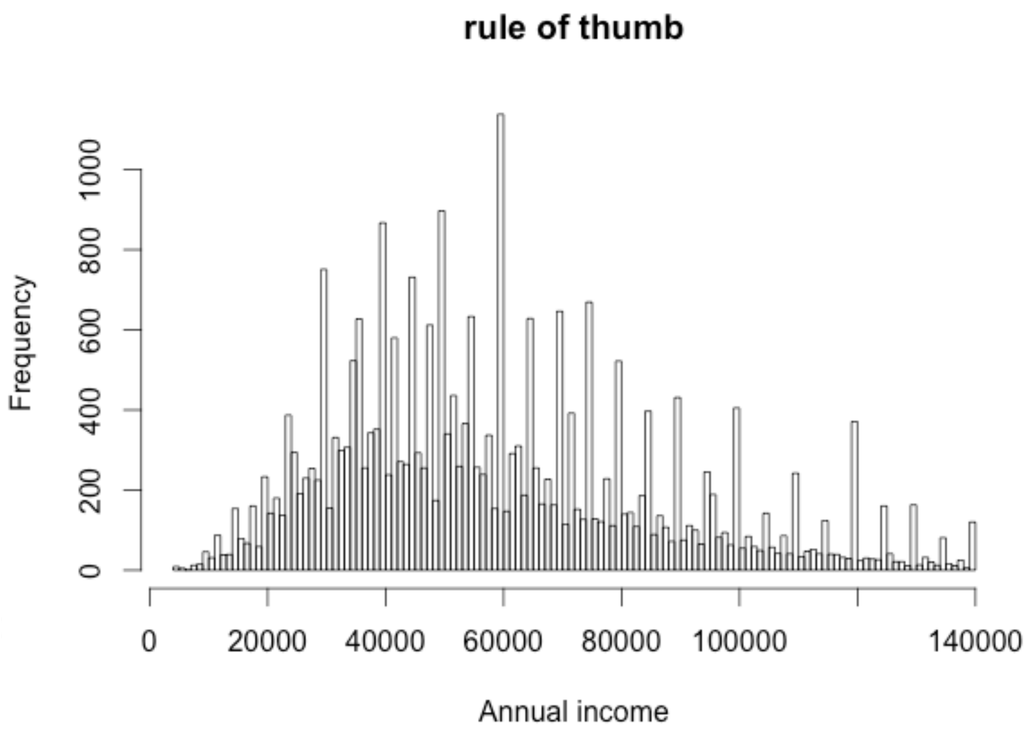
```
outlier_cutoff <- quantile(loan_data$annual_inc, 0.75) + 1.5 * IQR(loan_data$annual_inc)

index_outlier_ROT <- which(loan_data$annual_inc > outlier_cutoff)

loan_data_ROT <- loan_data[-index_outlier_ROT, ]
```

histograms

```
hist(loan_data_expert$annual_inc,
sqrt(nrow(loan_data_expert)), xlab =
"Annual income expert judgement”)
```

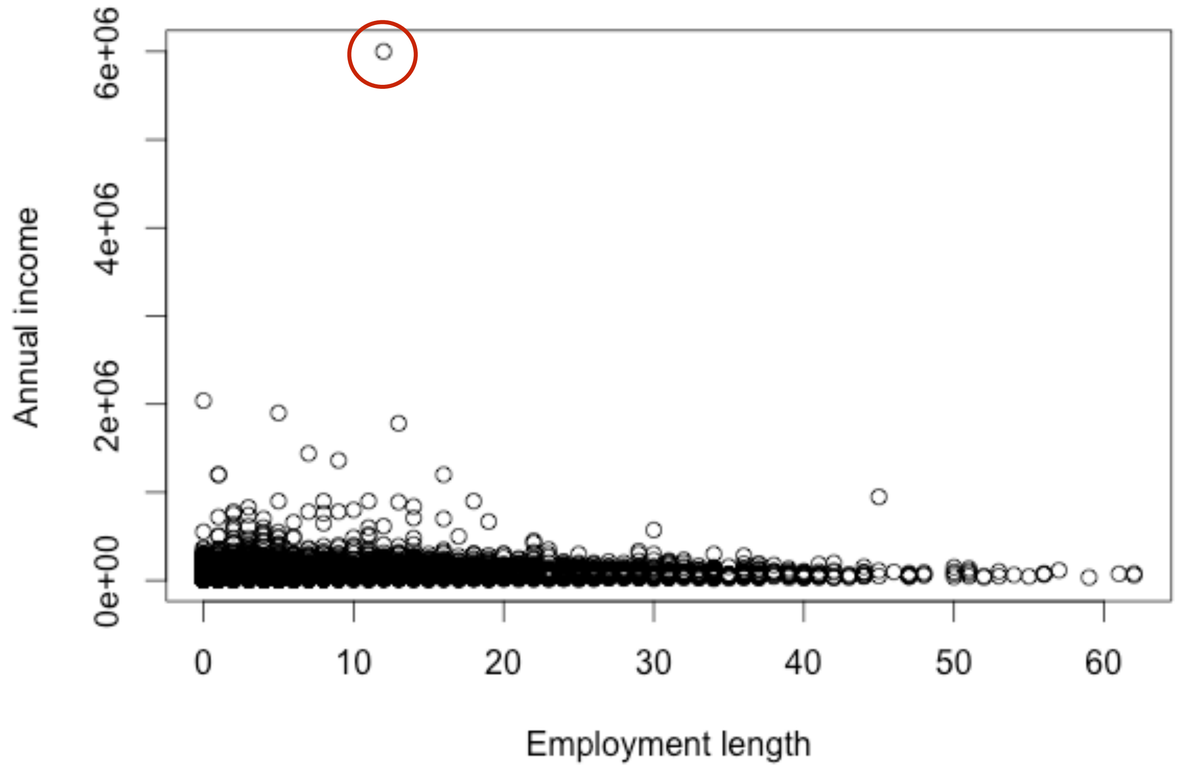
```
hist(loan_data_ROT$annual_inc,
sqrt(nrow(loan_data_ROT)), xlab =
"Annual income rule of thumb")
```





bivariate plot

```
plot(loan_data$emp_length, loan_data$annual_inc, xlab= "Employment length”,
ylab= "Annual income")
```



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Let’s practice!

CREDIT RISK MODELING IN R

Missing data and coarse classification

Outlier deleted

loan_status	loan_amnt	int_rate	grade	emp_length	home_ownership	annual_inc	age
0	5000	12.73	C	12	MORTGAGE	6000000	144

Missing inputs

	loan_status	loan_amnt	int_rate	grade	emp_length	home_ownership	annual_inc	age
…	…	…	…	…	…	…	…	…
125	0	6000	14.27	C	14	MORTGAGE	94800	23
126	1	2500	7.51	A	NA	OWN	12000	21
127	0	13500	9.91	B	2	MORTGAGE	36000	30
128	0	25000	12.42	B	2	RENT	225000	30
129	0	10000	NA	C	2	RENT	45900	65
130	0	2500	13.49	C	4	RENT	27200	26
…	…	…	…	…	…	…	…	…
2108	0	8000	7.90	A	8	RENT	64000	24
2109	0	12000	8.90	A	0	RENT	38400	26
2110	0	4000	NA	A	7	RENT	48000	30
2111	0	7000	9.91	B	20	MORTGAGE	130000	30
2112	0	7600	6.03	A	41	MORTGAGE	70920	28
2113	0	10000	11.71	B	5	RENT	48132	22
2114	0	8000	6.62	A	17	OWN	42000	24
2115	0	4475	NA	B	NA	OWN	15000	23
2116	0	5750	8.90	A	3	RENT	17000	21
2117	0	4900	6.03	A	12	MORTGAGE	77000	27
…	…	…	…	…	…	…	…	…

Missing inputs

```
> summary(loan_data$emp_length)

   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.    NA's
  0.000   2.000   4.000   6.145   8.000  62.000     809
```

Missing inputs: strategies

- Delete row/column
- Replace
- Keep

Delete rows

```
index_NA <- which(is.na(loan_data$emp_length)
loan_data_no_NA <- loan_data[-c(index_NA), ]
```

```
     loan_status loan_amnt  int_rate  grade  emp_length home_ownership annual_inc age
  …          …         …         …      …          …              …          …    …
125          0      6000      14.27      C         14       MORTGAGE      94800   23
126          1      2500       7.51      A         NA            OWN      12000   21
127          0     13500       9.91      B          2       MORTGAGE      36000   30
128          0     25000      12.42      B          2           RENT     225000   30
129          0     10000         NA      C          2           RENT      45900   65
130          0      2500      13.49      C          4           RENT      27200   26
  …          …         …         …      …          …              …          …    …
2112         0      7600       6.03      A         41       MORTGAGE      70920   28
2113         0     10000      11.71      B          5           RENT      48132   22
2114         0      8000       6.62      A         17            OWN      42000   24
2115         0      4475         NA      B         NA            OWN      15000   23
2116         0      5750       8.90      A          3           RENT      17000   21
  …          …         …         …      …          …              …          …    …
```

Delete column

```
loan_data_delete_employ <- loan_data
loan_data_delete_employ$emp_length <- NULL
```

```
     loan_status loan_amnt  int_rate  grade  emp_length home_ownership annual_inc age
  …          …         …         …      …          …              …          …    …
125          0      6000      14.27      C         14       MORTGAGE      94800   23
126          1      2500       7.51      A         NA            OWN      12000   21
127          0     13500       9.91      B          2       MORTGAGE      36000   30
128          0     25000      12.42      B          2           RENT     225000   30
129          0     10000         NA      C          2           RENT      45900   65
130          0      2500      13.49      C          4           RENT      27200   26
  …          …         …         …      …          …              …          …    …
2112         0      7600       6.03      A         11       MORTGAGE      70920   28
2113         0     10000      11.71      B          5           RENT      48132   22
2114         0      8000       6.62      A         17            OWN      42000   24
2115         0      4475         NA      B         NA            OWN      15000   23
2116         0      5750       8.90      A          3           RENT      17000   21
  …          …         …         …      …          …              …          …    …
```

Replace: median imputation

```
index_NA <- which(is.na(loan_data$emp_length)
loan_data_replace <- loan_data
loan_data_replace$emp_length[index_NA] <- median(loan_data$emp_length, na.rm = TRUE)
```

```
     loan_status loan_amnt  int_rate  grade  emp_length home_ownership annual_inc age
  …          …         …         …      …          …              …          …    …
125          0      6000      14.27      C         14       MORTGAGE      94800   23
126          1      2500       7.51      A         NA            OWN      12000   21
127          0     13500       9.91      B          2       MORTGAGE      36000   30
128          0     25000      12.42      B          2           RENT     225000   30
129          0     10000         NA      C          2           RENT      45900   65
130          0      2500      13.49      C          4           RENT      27200   26
  …          …         …         …      …          …              …          …    …
2112         0      7600       6.03      A         41       MORTGAGE      70920   28
2113         0     10000      11.71      B          5           RENT      48132   22
2114         0      8000       6.62      A         17            OWN      42000   24
2115         0      4475         NA      B         NA            OWN      15000   23
2116         0      5750       8.90      A          3           RENT      17000   21
  …          …         …         …      …          …              …          …    …
```

Replace: median imputation

```
index_NA <- which(is.na(loan_data$emp_length)
loan_data_replace <- loan_data
loan_data_replace$emp_length[index_NA] <- median(loan_data$emp_length, na.rm = TRUE)
```

```
     loan_status loan_amnt  int_rate  grade  emp_length home_ownership annual_inc age
  …          …         …         …      …          …              …          …    …
125          0      6000      14.27      C         14       MORTGAGE      94800   23
126          1      2500       7.51      A          4            OWN      12000   21
127          0     13500       9.91      B          2       MORTGAGE      36000   30
128          0     25000      12.42      B          2           RENT     225000   30
129          0     10000         NA      C          2           RENT      45900   65
130          0      2500      13.49      C          4           RENT      27200   26
  …          …         …         …      …          …              …          …    …
2112         0      7600       6.03      A         41       MORTGAGE      70920   28
2113         0     10000      11.71      B          5           RENT      48132   22
2114         0      8000       6.62      A         17            OWN      42000   24
2115         0      4475         NA      B          4            OWN      15000   23
2116         0      5750       8.90      A          3           RENT      17000   21
  …          …         …         …      …          …              …          …    …
```

Keep

- Keep NA
- Problem: will cause row deletions for many models
- Solution: coarse classification, put variable in “bins”
  - new variable emp_cat
  - range: 0-62 years —> make bins of +/- 15 years
  - categories: “0-15”, “15-30”, “30-45”, “45+”, “missing”

Keep: coarse classification

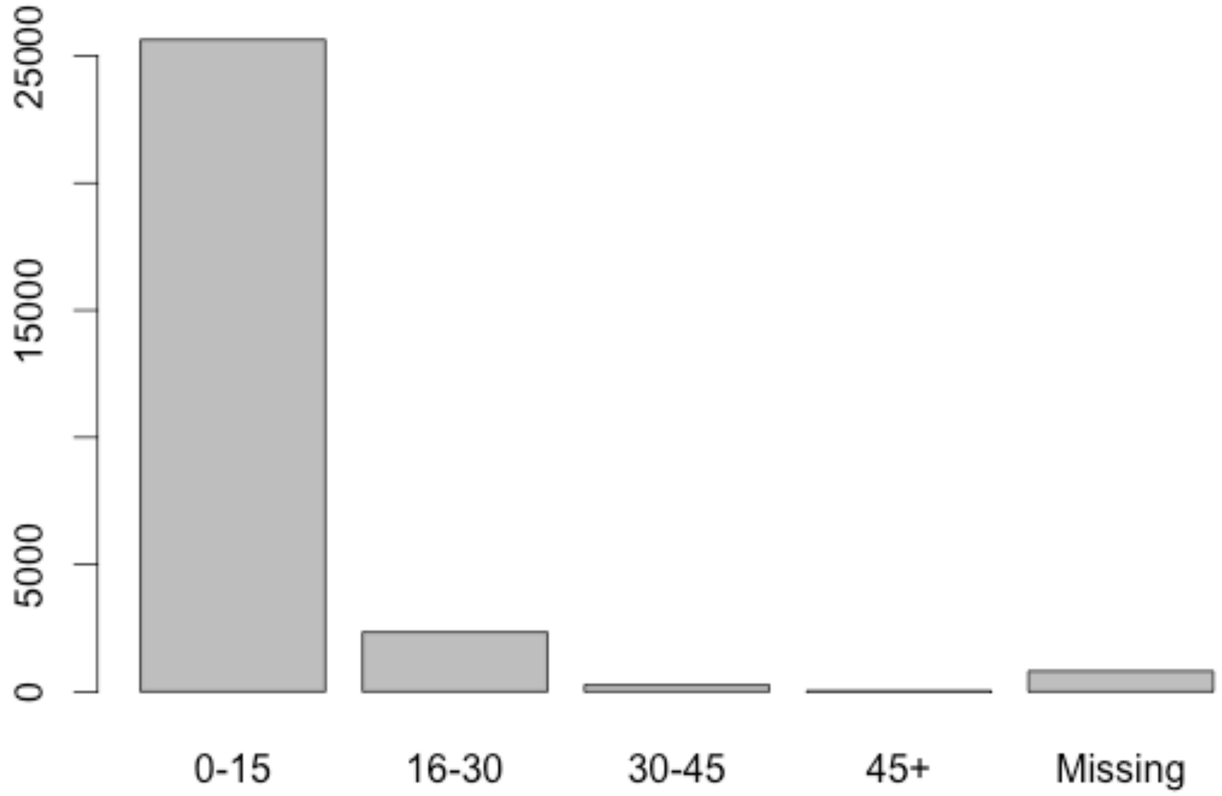
	loan_status	loan_amnt	int_rate	grade	emp_length	home_ownership	annual_inc	age
…	…	…	…	…	…	…	…	…
125	0	6000	14.27	C	14	MORTGAGE	94800	23
126	1	2500	7.51	A	NA	OWN	12000	21
127	0	13500	9.91	B	2	MORTGAGE	36000	30
128	0	25000	12.42	B	2	RENT	225000	30
129	0	10000	NA	C	2	RENT	45900	65
130	0	2500	13.49	C	4	RENT	27200	26
…	…	…	…	…	…	…	…	…
2112	0	7600	6.03	A	41	MORTGAGE	70920	28
2113	0	10000	11.71	B	5	RENT	48132	22
2114	0	8000	6.62	A	17	OWN	42000	24
2115	0	4475	NA	B	NA	OWN	15000	23
2116	0	5750	8.90	A	3	RENT	17000	21
…	…	…	…	…	…	…	…	…

Keep: coarse classification

	loan_status	loan_amnt	int_rate	grade	emp_cat	home_ownership	annual_inc	age
…	…	…	…	…	…	…	…	…
125	0	6000	14.27	C	0-15	MORTGAGE	94800	23
126	1	2500	7.51	A	Missing	OWN	12000	21
127	0	13500	9.91	B	0-15	MORTGAGE	36000	30
128	0	25000	12.42	B	0-15	RENT	225000	30
129	0	10000	NA	C	0-15	RENT	45900	65
130	0	2500	13.49	C	0-15	RENT	27200	26
…	…	…	…	…	…	…	…	…
2112	0	7600	6.03	A	30-45	MORTGAGE	70920	28
2113	0	10000	11.71	B	0-15	RENT	48132	22
2114	0	8000	6.62	A	15-30	OWN	42000	24
2115	0	4475	NA	B	Missing	OWN	15000	23
2116	0	5750	8.90	A	0-15	RENT	17000	21
…	…	…	…	…	…	…	…	…

Bin frequencies

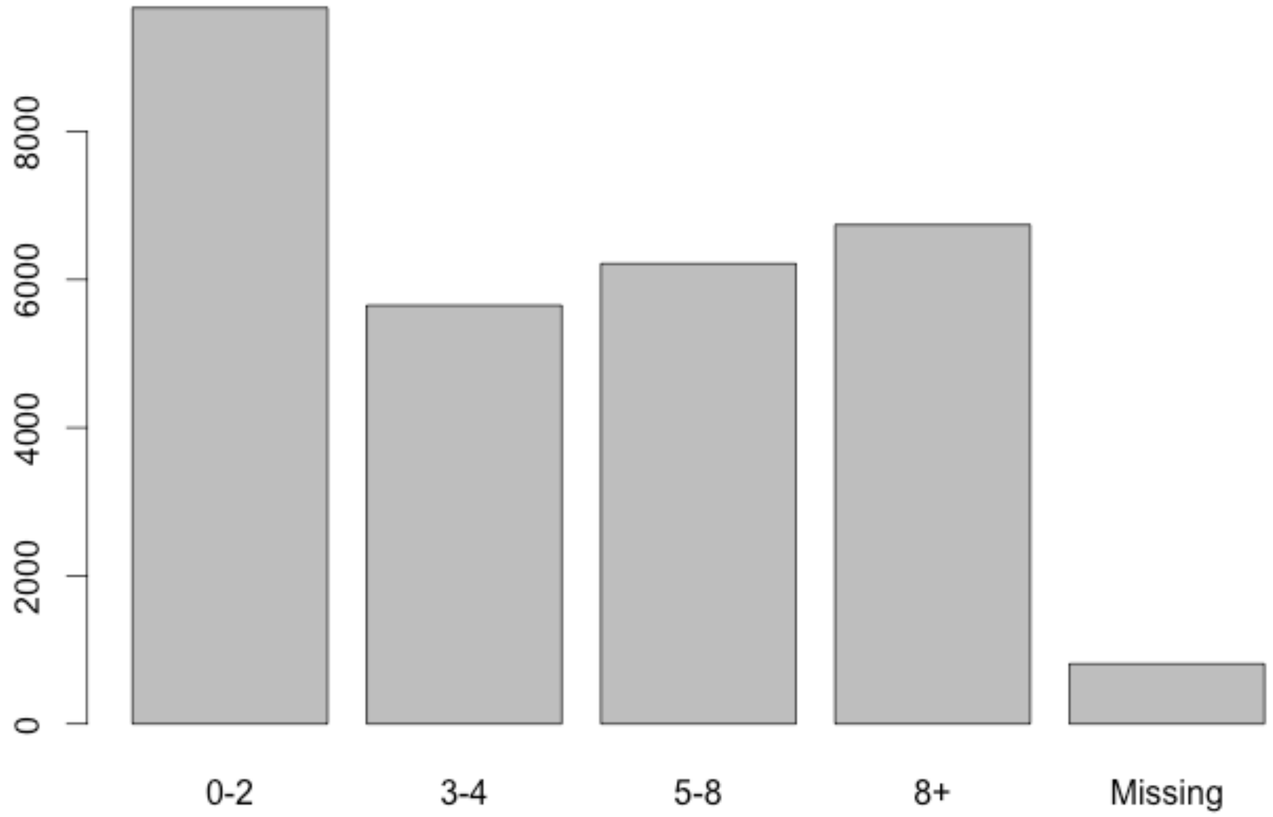
```
plot(loan_data$emp_cat)
```




```
    emp_cat
       …
     0-15
Missing
     0-15
     0-15
     0-15
     0-15
       …
    30-45
     0-15
    15-30
Missing
     0-15
       …
```

Bin frequencies

```
plot(loan_data$emp_cat)
```




```
emp_cat
      …
     8+
Missing
    0-2
    0-2
    0-2
    3-4
      …
     8+
    5-8
     8+
Missing
    3-4
      …
```

Final remarks

	CONTINUOUS	CATEGORICAL
DELETE	Delete rows (observations with NAs) Delete column (entire variable)	Delete rows (observations with NAs) Delete column (entire variable)
REPLACE	replace using median	replace using most frequent category
KEEP	keep as NA (not always possible) keep using coarse classification	NA category

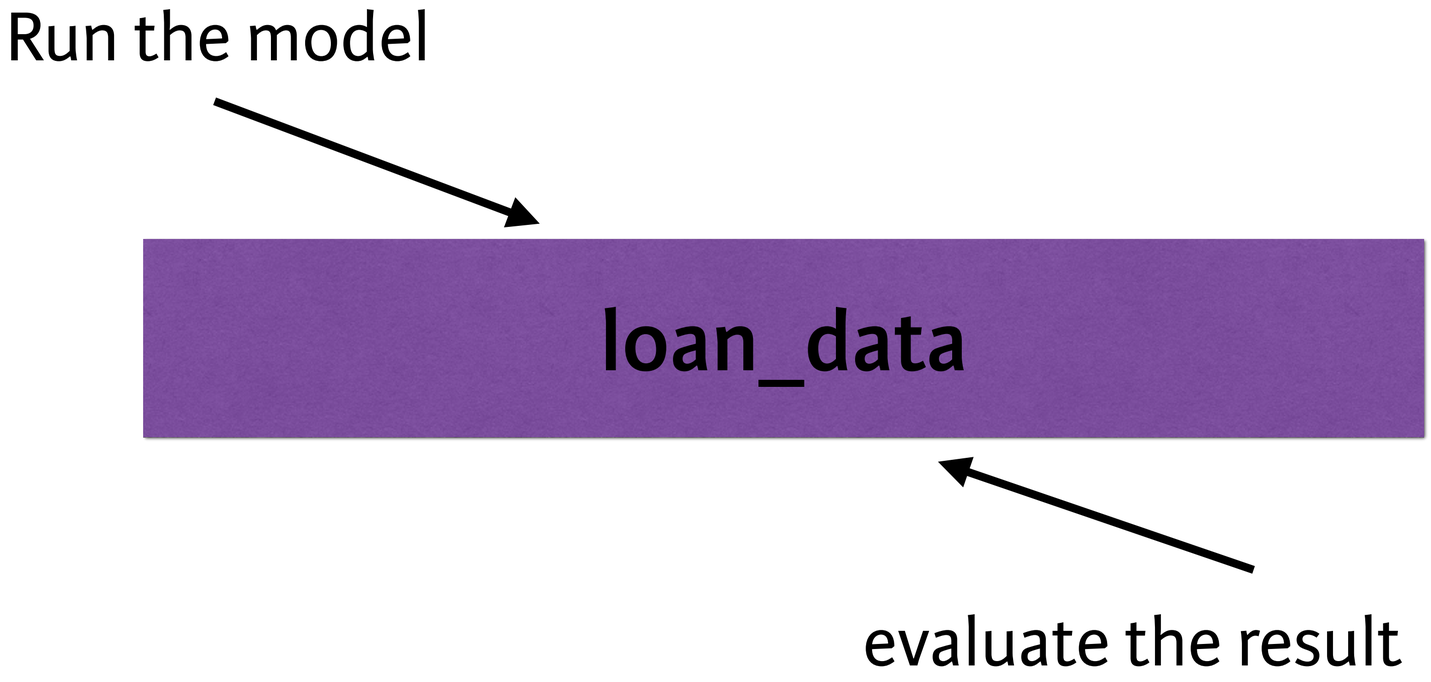
CREDIT RISK MODELING IN R

Let’s practice!

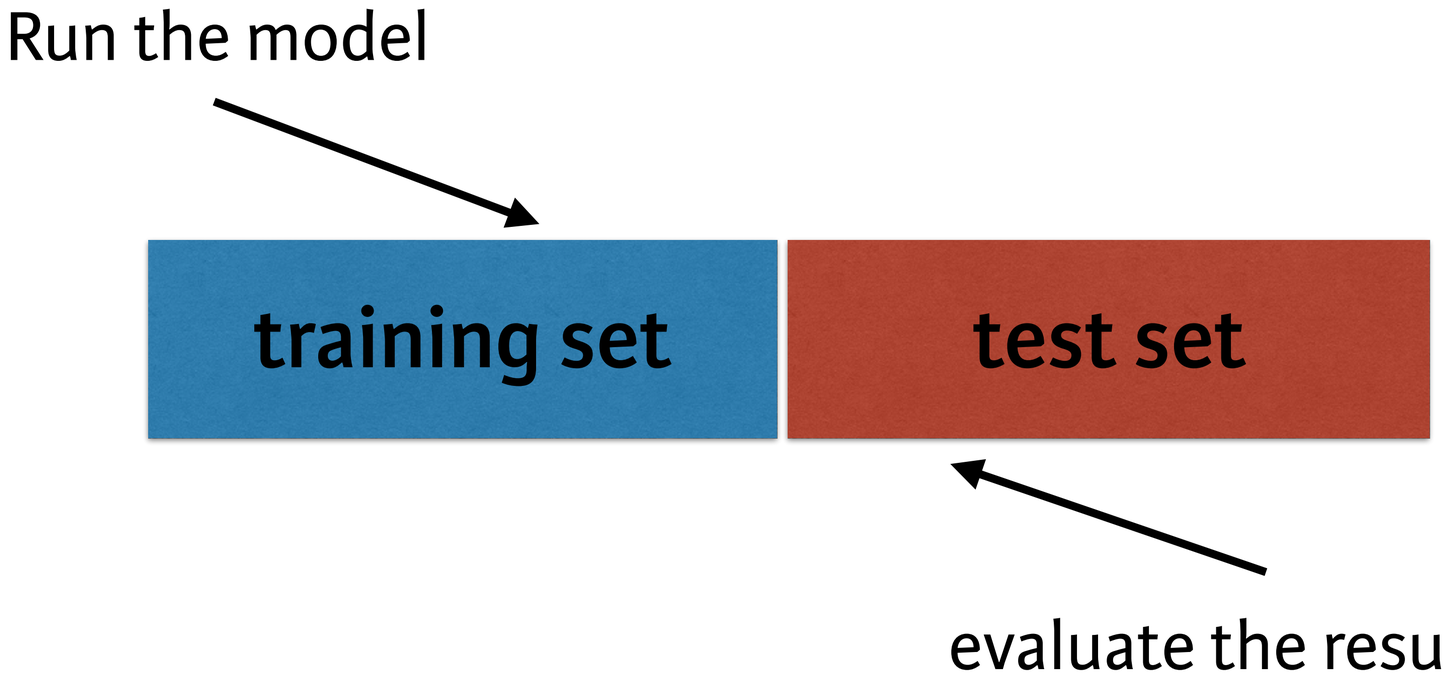
CREDIT RISK MODELING IN R

Data splitting and confusion matrices

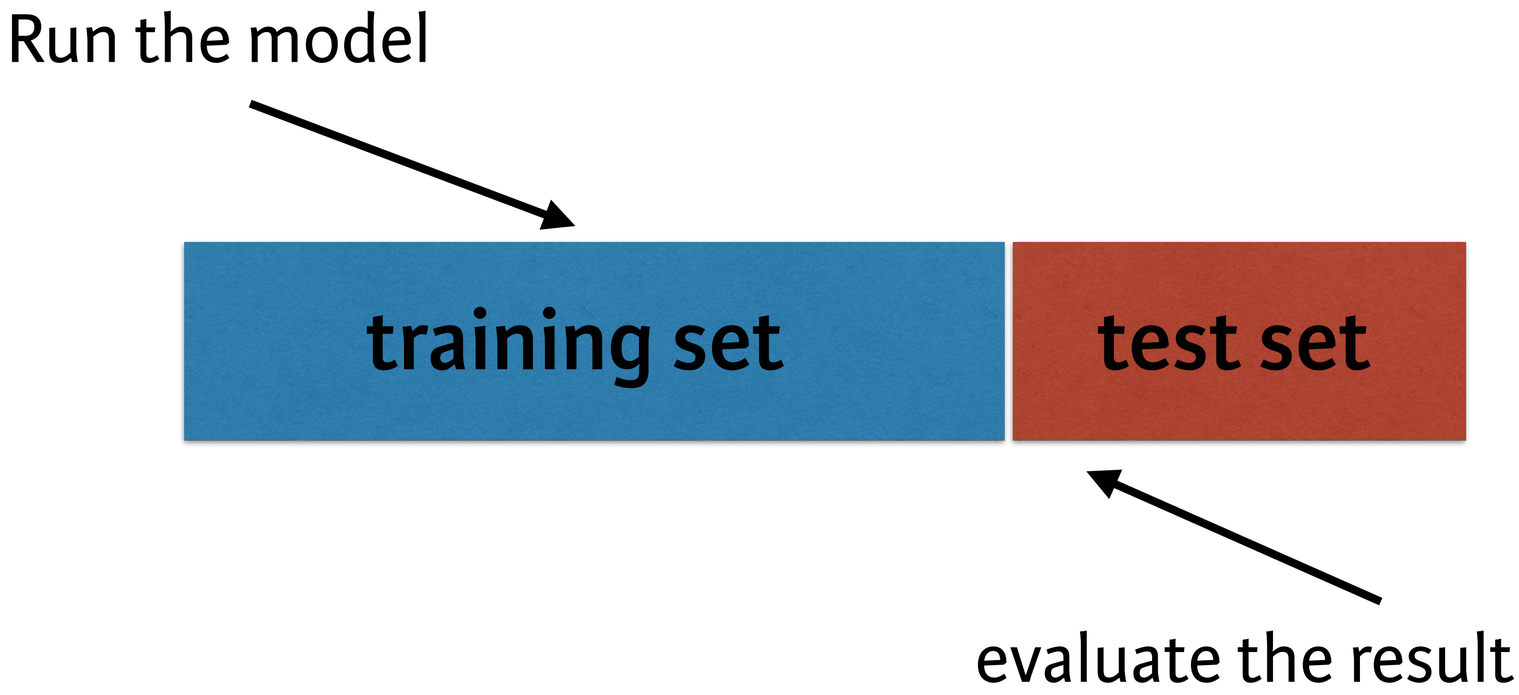
Start analysis



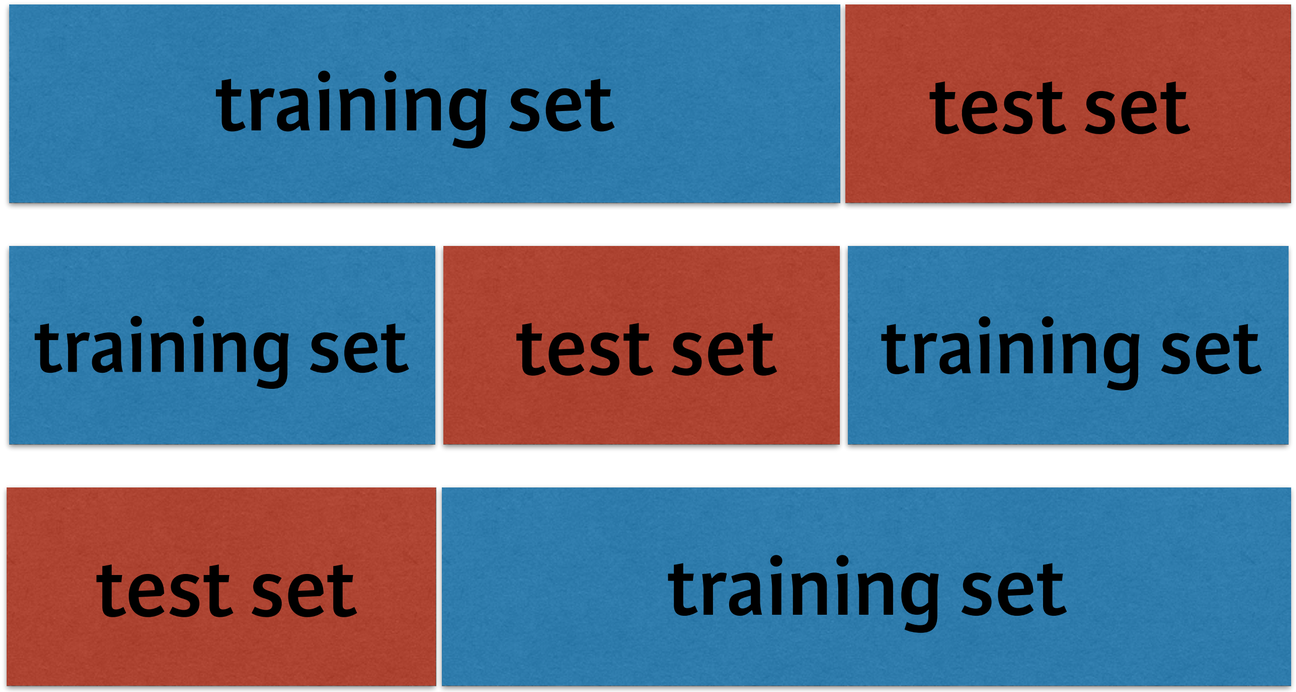
training and test set



training and test set



cross-validation



evaluate a model

	test_set$loan_status	model_prediction
	…	…
[8066,]	1	1
[8067,]	0	0
[8068,]	0	0
[8069,]	0	0
[8070,]	0	0
[8071,]	0	1
[8072,]	1	0
[8073,]	1	1
[8074,]	0	0
[8075,]	0	0
[8076,]	0	0
[8077,]	1	1
[8078,]	0	0
[8079,]	0	1
	…	…

model prediction

actual loan status

	no default (0)	default (1)
no default (0)	8	2
default (1)	1	3

evaluate a model

	test_set$loan_status	model_prediction
	…	…
[8066,]	1	1
[8067,]	0	0
[8068,]	0	0
[8069,]	0	0
[8070,]	0	0
[8071,]	0	1
[8072,]	1	0
[8073,]	1	1
[8074,]	0	0
[8075,]	0	0
[8076,]	0	0
[8077,]	1	1
[8078,]	0	0
[8079,]	0	1
	…	…

model prediction

actual loan status

	no default (0)	default (1)
no default (0)	**TN**	**FP**
default (1)	**FN**	**TP**

some measures...

- Accuracy = (8 + 3) / 14 = 78.57%
- Sensitivity = 3 / (1 + 3) = 75 %
- Specificity = 8/ (8 + 2)= 80%

model prediction

actual loan status

	no default (0)	default (1)
no default (0)	8	2
default (1)	1	3

CREDIT RISK MODELING IN R

Let’s practice!

CREDIT RISK MODELING IN R

Logistic regression: introduction

Final data structure

```
> str(training_set)

'data.frame': 19394 obs. of  8 variables:
 $ loan_status   : Factor w/ 2 levels "0","1": 1 1 1 1 1 1 1 1 1 1 ...
 $ loan_amnt     : int  25000 16000 8500 9800 3600 6600 3000 7500 6000 22750 ...
 $ grade         : Factor w/ 7 levels "A","B","C","D",..: 2 4 1 2 1 1 1 2 1 1 ...
 $ home_ownership: Factor w/ 4 levels "MORTGAGE","OTHER",..: 4 4 1 1 1 3 4 3 4 1 ...
 $ annual_inc    : num  91000 45000 110000 102000 40000 ...
 $ age           : int  34 25 29 24 59 35 24 24 26 25 ...
 $ emp_cat       : Factor w/ 5 levels "0-15","15-30",..: 1 1 1 1 1 2 1 1 1 1 ...
 $ ir_cat        : Factor w/ 5 levels "0-8","11-13.5",..: 2 3 1 4 1 1 1 4 1 1 ...
```

What is logistic regression?

A regression model with output between 0 and 1

$$P(\text{loan\_status} = 1 \mid x_1, \ldots, x_m) = \frac{1}{1 + e^{-(\beta_0 + \beta_1 x_1 + \ldots + \beta_m x_m)}}$$

$x_1, \ldots, x_m$ →

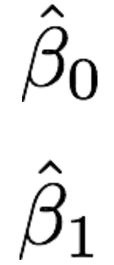
```
loan_amnt    grade    age    annual_inc
home_ownership    emp_cat    ir_cat
```

$\beta_0, \ldots, \beta_m$ → Parameters to be estimated

$\beta_0 + \beta_1 x_1 + \ldots + \beta_m x_m$ → Linear predictor

Fitting a logistic model in R

```
> log_model <- glm(loan_status ~ age , family= "binomial", data = training_set)
> log_model

Call:  glm(formula = loan_status ~ age, family = "binomial", data = training_set)

Coefficients:
(Intercept)          age
  -1.793566    -0.009726

Degrees of Freedom: 19393 Total (i.e. Null);  19392 Residual
Null Deviance:      13680
Residual Deviance: 13670  AIC: 13670
```



$$P(\text{loan\_status} = 1 \mid \text{age}) = \frac{1}{1 + e^{-(\hat{\beta}_0 + \hat{\beta}_1 age)}}$$

Probabilities of default

$$P(\text{loan\_status} = 1 \mid x_1, \ldots, x_m) = \frac{1}{1 + e^{-(\beta_0 + \beta_1 x_1 + \ldots + \beta_m x_m)}} = \frac{e^{\beta_0 + \beta_1 x_1 + \ldots + \beta_m x_m}}{1 + e^{\beta_0 + \beta_1 x_1 + \ldots + \beta_m x_m}}$$

$$P(\text{loan\_status} = 0 \mid x_1, \ldots, x_m) = 1 - \frac{e^{\beta_0 + \beta_1 x_1 + \ldots + \beta_m x_m}}{1 + e^{\beta_0 + \beta_1 x_1 + \ldots + \beta_m x_m}} = \frac{1}{1 + e^{\beta_0 + \beta_1 x_1 + \ldots + \beta_m x_m}}$$

$$\frac{P(\text{loan\_status} = 1 \mid x_1, \ldots, x_m)}{P(\text{loan\_status} = 0 \mid x_1, \ldots, x_m)} = e^{\beta_0 + \beta_1 x_1 + \ldots + \beta_m x_m}$$ → odds in favor of loan_status=1

Interpretation of coefficient

If variable x_j goes up by 1 → The odds are multiplied by e^{β_j}

$\beta_j < 0$ → $e^{\beta_j} < 1$ → The odds decrease as x_j increases

$\beta_j > 0$ → $e^{\beta_j} > 1$ → The odds increase as x_j increases

Applied to our model

If variable age goes up by 1 → The odds are multiplied by $e^{-0.009726}$

→ The odds are multiplied by 0.991

CREDIT RISK MODELING IN R

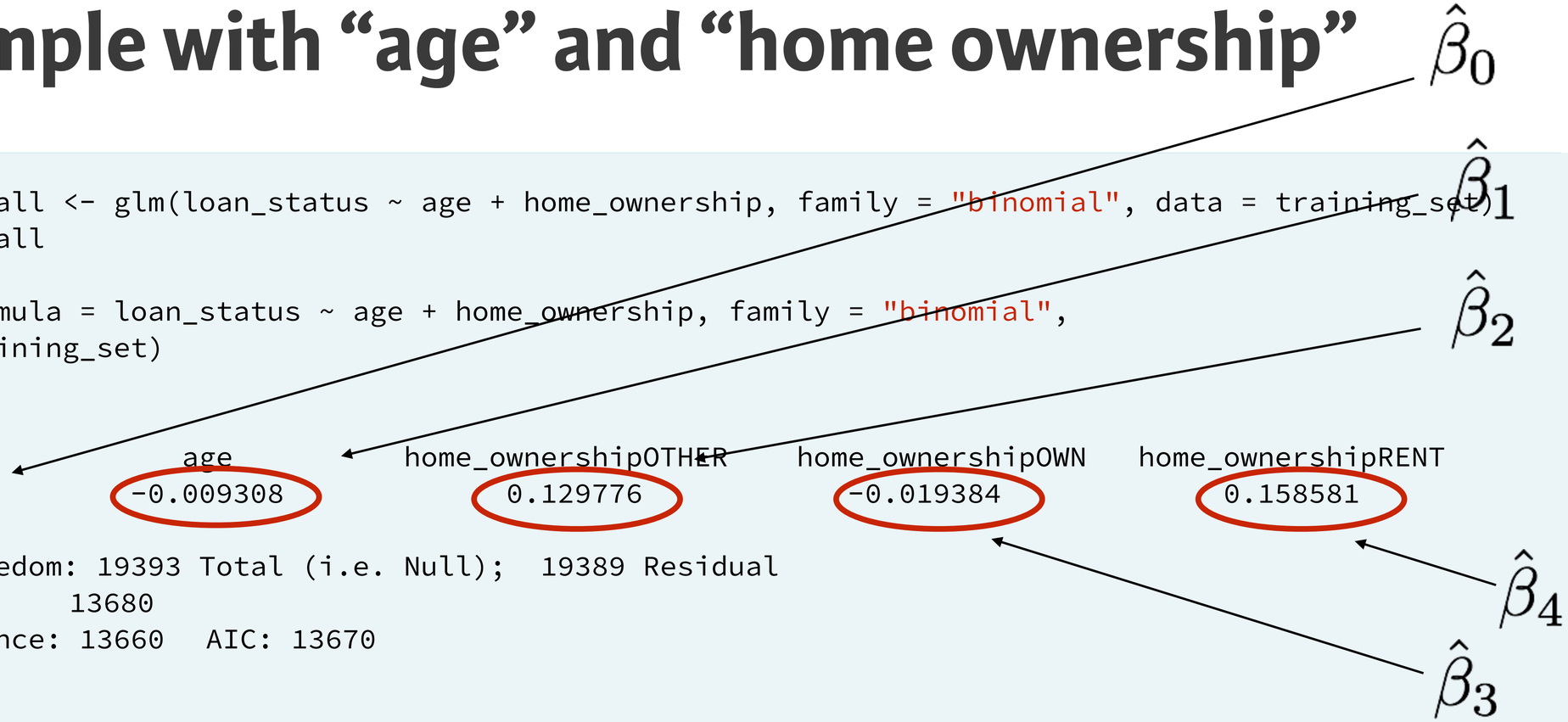
Let’s practice!

CREDIT RISK MODELING IN R

Logistic regression: predicting the probability of default

An example with “age” and “home ownership”

```
> log_model_small <- glm(loan_status ~ age + home_ownership, family = "binomial", data = training_set)
> log_model_small

Call:  glm(formula = loan_status ~ age + home_ownership, family = "binomial",
    data = training_set)

Coefficients:
 (Intercept)           age  home_ownershipOTHER  home_ownershipOWN  home_ownershipRENT
   -1.886396     -0.009308             0.129776          -0.019384            0.158581

Degrees of Freedom: 19393 Total (i.e. Null);  19389 Residual
Null Deviance:     13680
Residual Deviance: 13660   AIC: 13670
```



$$P(\text{loan\_status} = 1 \mid \text{age}, \text{home\_ownership}) = \frac{1}{1 + e^{-(\hat{\beta}_0 + \hat{\beta}_1 \text{age} + \hat{\beta}_2 \text{OTHER} + \hat{\beta}_3 \text{OWN} + \hat{\beta}_4 \text{RENT})}}$$

Test set example

$$P(\text{loan\_status} = 1 \mid \text{age} = 33, \text{home\_ownership} = \text{RENT})$$

$$= \frac{1}{1 + e^{-(\hat{\beta}_0 + \hat{\beta}_1 * 33 + \hat{\beta}_2 * 0 + \hat{\beta}_3 * 0 + \hat{\beta}_4 * 1)}}$$

$$= \frac{1}{1 + e^{-(-1.886396 + (-0.009308) * 33 + (0.158581) * 1)}}$$

$$= 0.115579$$

Making predictions in R

```
> test_case <- as.data.frame(test_set[1,])

> test_case
  loan_status loan_amnt  grade home_ownership annual_inc  age  emp_cat   ir_cat
1           0      5000      B          RENT      24000   33    0-15     8-11

> predict(log_model_small, newdata = test_case)
       1
-2.03499
```

$$\hat{\beta}_0 + \hat{\beta}_1 \text{age} + \hat{\beta}_2 \text{OTHER} + \hat{\beta}_3 \text{OWN} + \hat{\beta}_4 \text{RENT}$$

```
> predict(log_model_small, newdata = test_case, type = "response")
        1
0.1155779
```

CREDIT RISK MODELING IN R

Let’s practice!

CREDIT RISK MODELING IN R

Evaluating the logistic regression model result

Recap: model evaluation

```
          test_set$loan_status    model_prediction
                             …                   …
[8066,]                      1                   1
[8067,]                      0                   0
[8068,]                      0                   0
[8069,]                      0                   0
[8070,]                      0                   0
[8071,]                      0                   1
[8072,]                      1                   0
[8073,]                      1                   1
[8074,]                      0                   0
[8075,]                      0                   0
[8076,]                      0                   0
[8077,]                      1                   1
[8078,]                      0                   0
[8079,]                      0                   1
                             …                   …
```

model prediction

actual loan status

	no default (0)	default (1)
no default (0)	**8**	**2**
default (1)	**1**	**3**

In reality...

	test_set$loan_status	model_prediction
	…	…
[8066,]	1	0.09881492
[8067,]	0	0.09497852
[8068,]	0	0.21071984
[8069,]	0	0.04252119
[8070,]	0	0.21110838
[8071,]	0	0.08668856
[8072,]	1	0.11319341
[8073,]	1	0.16662207
[8074,]	0	0.15299176
[8075,]	0	0.08558058
[8076,]	0	0.08280463
[8077,]	1	0.11271048
[8078,]	0	0.08987446
[8079,]	0	0.08561631
	…	…

model prediction

actual loan status

	no default (0)	default (1)
no default (0)	?	?
default (1)	?	?

In reality...

	test_set$loan_status	model_prediction
	…	…
[8066,]	1	0.09881492
[8067,]	0	0.09497852
[8068,]	0	0.21071984
[8069,]	0	0.04252119
[8070,]	0	0.21110838
[8071,]	0	0.08668856
[8072,]	1	0.11319341
[8073,]	1	0.16662207
[8074,]	0	0.15299176
[8075,]	0	0.08558058
[8076,]	0	0.08280463
[8077,]	1	0.11271048
[8078,]	0	0.08987446
[8079,]	0	0.08561631
	…	…

Cutoff
or
treshold value

between 0 and 1

Cutoff = 0.5

	test_set$loan_status	model_prediction
	…	…
[8066,]	1	0
[8067,]	0	0
[8068,]	0	0
[8069,]	0	0
[8070,]	0	0
[8071,]	0	0
[8072,]	1	0
[8073,]	1	0
[8074,]	0	0
[8075,]	0	0
[8076,]	0	0
[8077,]	1	0
[8078,]	0	0
[8079,]	0	0
	…	…

model prediction

actual loan status

	no default (0)	default (1)
no default (0)	**10**	**0**
default (1)	**4**	**0**

Accuracy = 10/(10+4+0+0) = 71.4%

Sensitivity = 0/(4+0) = 0%

Cutoff = 0.1

	test_set$loan_status	model_prediction
	…	…
[8066,]	1	0
[8067,]	0	0
[8068,]	0	1
[8069,]	0	0
[8070,]	0	1
[8071,]	0	0
[8072,]	1	1
[8073,]	1	1
[8074,]	0	1
[8075,]	0	0
[8076,]	0	0
[8077,]	1	1
[8078,]	0	0
[8079,]	0	0
	…	…

model prediction

actual loan status	no default (0)	default (1)
no default (0)	**7**	**3**
default (1)	**1**	**3**

Accuracy = 10/(10+4+0+0) = 71.4%

Sensitivity = 3/(3+1) = 75%

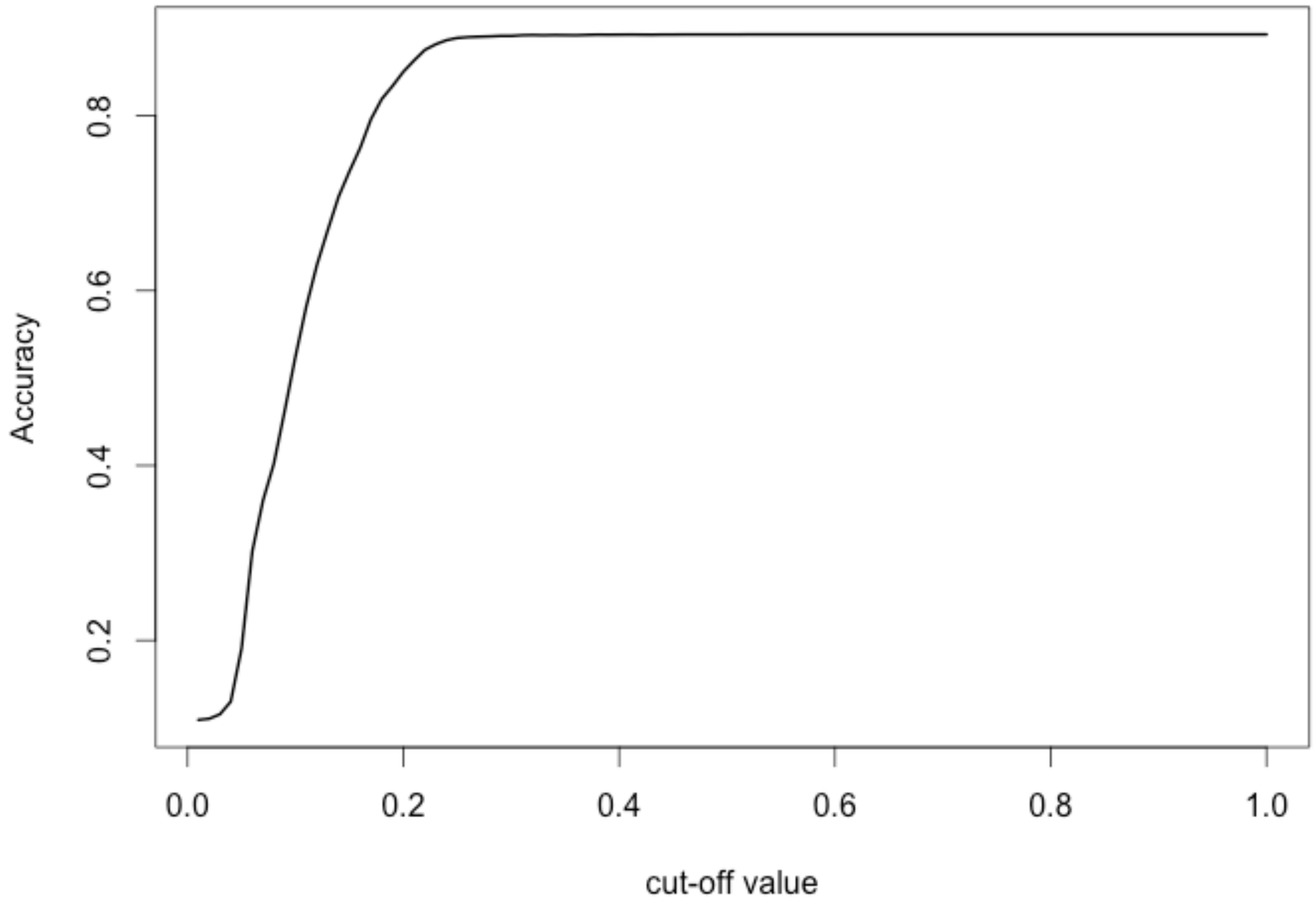
CREDIT RISK MODELING IN R

Let’s practice!

CREDIT RISK MODELING IN R

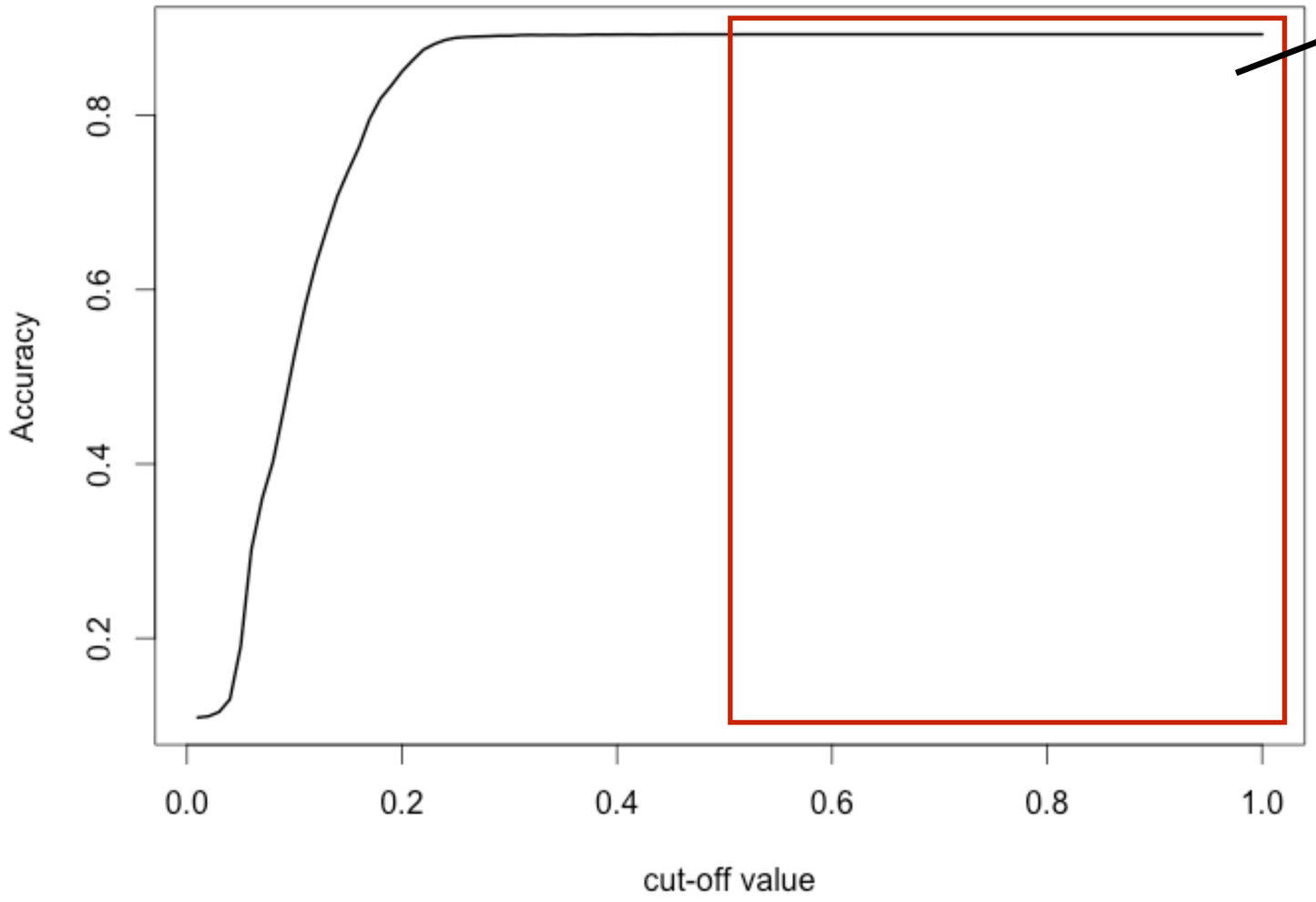
wrap-up and remarks

best cut-off for accuracy?



$$\text{Accuracy} = \frac{TP + TN}{TP + FP + TN + FN}$$

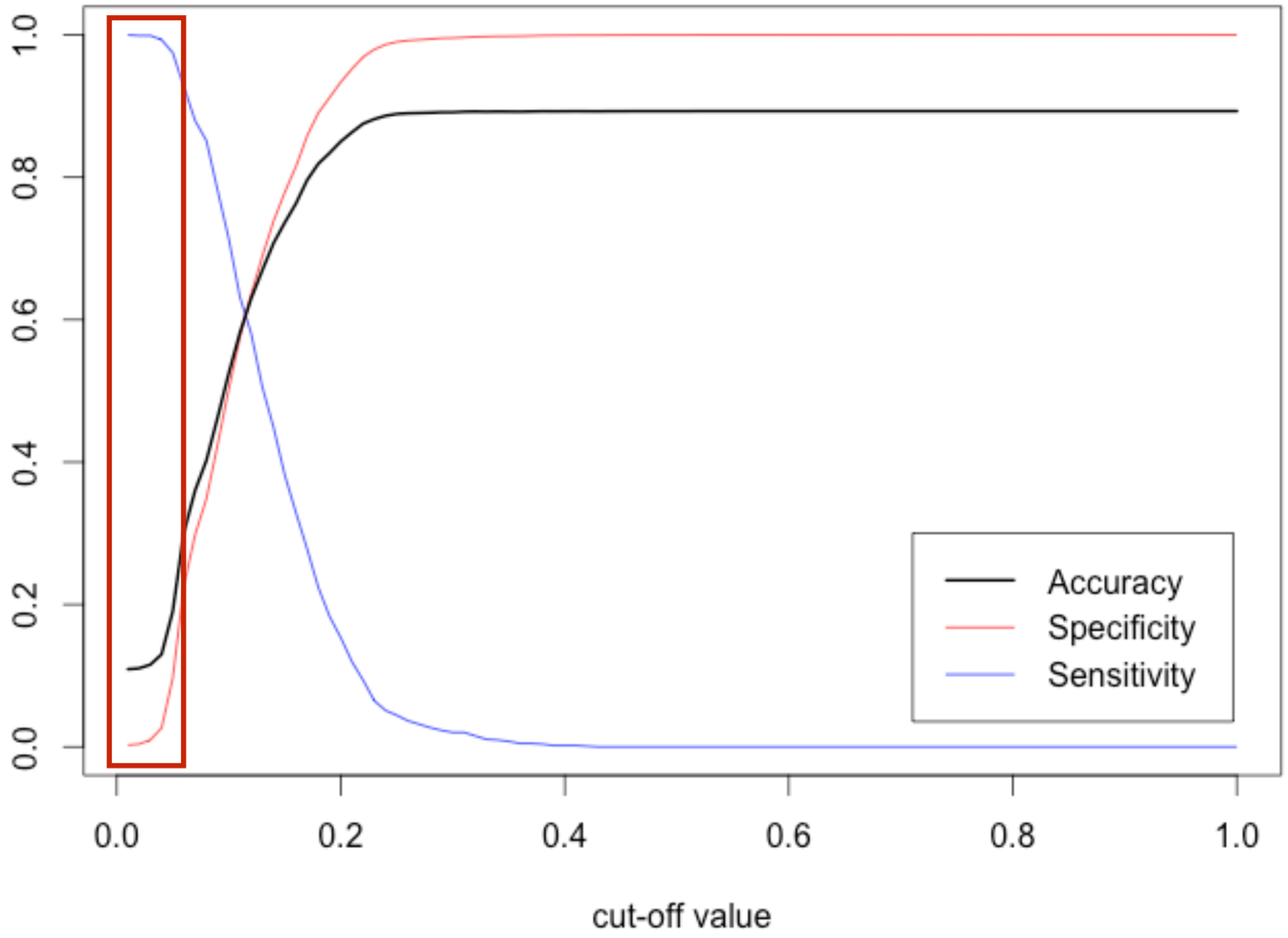
best cut-off for accuracy?



Accuracy = 89.31 %

ACTUAL defaults in test set=
10.69 % = (100 - 89.31) %

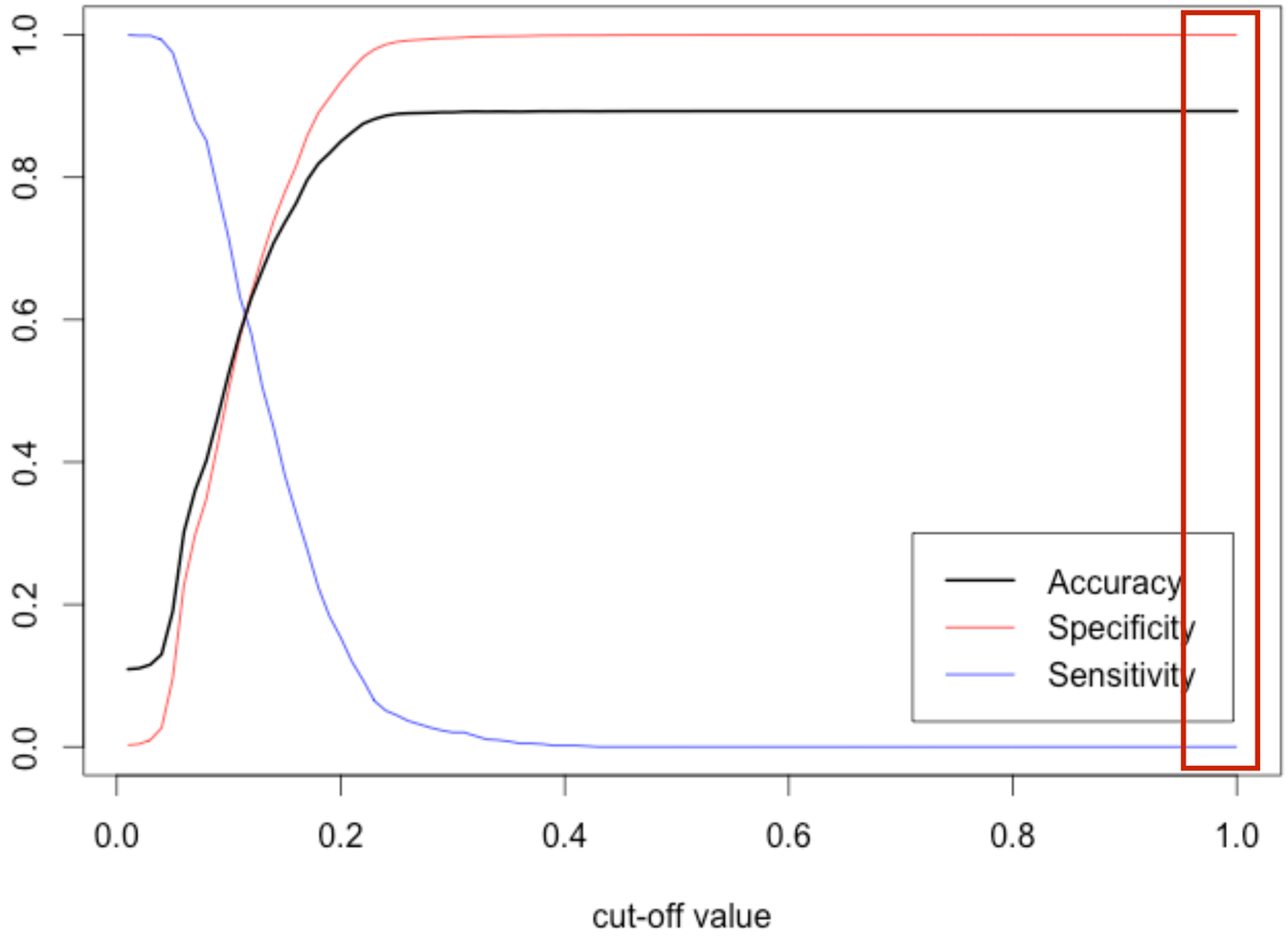
What about sensitivity or specificity?



Sensitivity = 1037 / (1037 +0) = 100%

Specificity = 0 / (0 + 864) = 0%

What about sensitivity or specificity?



Sensitivity = 0 / (0 + 1037) = 0%

Specificity = 8640 / (8640 + 0) = 100%

About logistic regression...

```
log_model_full <- glm(loan_status ~ ., family = "binomial", data = training_set)
```

is the same as

```
log_model_full <- glm(loan_status ~ ., family = binomial(link = logit),
data = training_set)
```

recall

$$P(\text{loan\_status} = 1 \mid x_1, \ldots, x_m) = \frac{1}{1 + e^{-(\beta_0 + \beta_1 x_1 + \ldots + \beta_m x_m)}}$$

Other logistic regression models

```
log_model_full <- glm(loan_status ~ ., family = binomial(link = probit),
data = training_set)
```

```
log_model_full <- glm(loan_status ~ ., family = binomial(link = cloglog),
data = training_set)
```

$\beta_j < 0$ → The probability of default decreases as x_j increases

$\beta_j > 0$ → The probability of default decreases as x_j increases

BUT

$$P(\text{loan\_status} = 1 \mid x_1, \ldots, x_m) = \frac{1}{1 + e^{-(\beta_0 + \beta_1 x_1 + \ldots + \beta_m x_m)}}$$

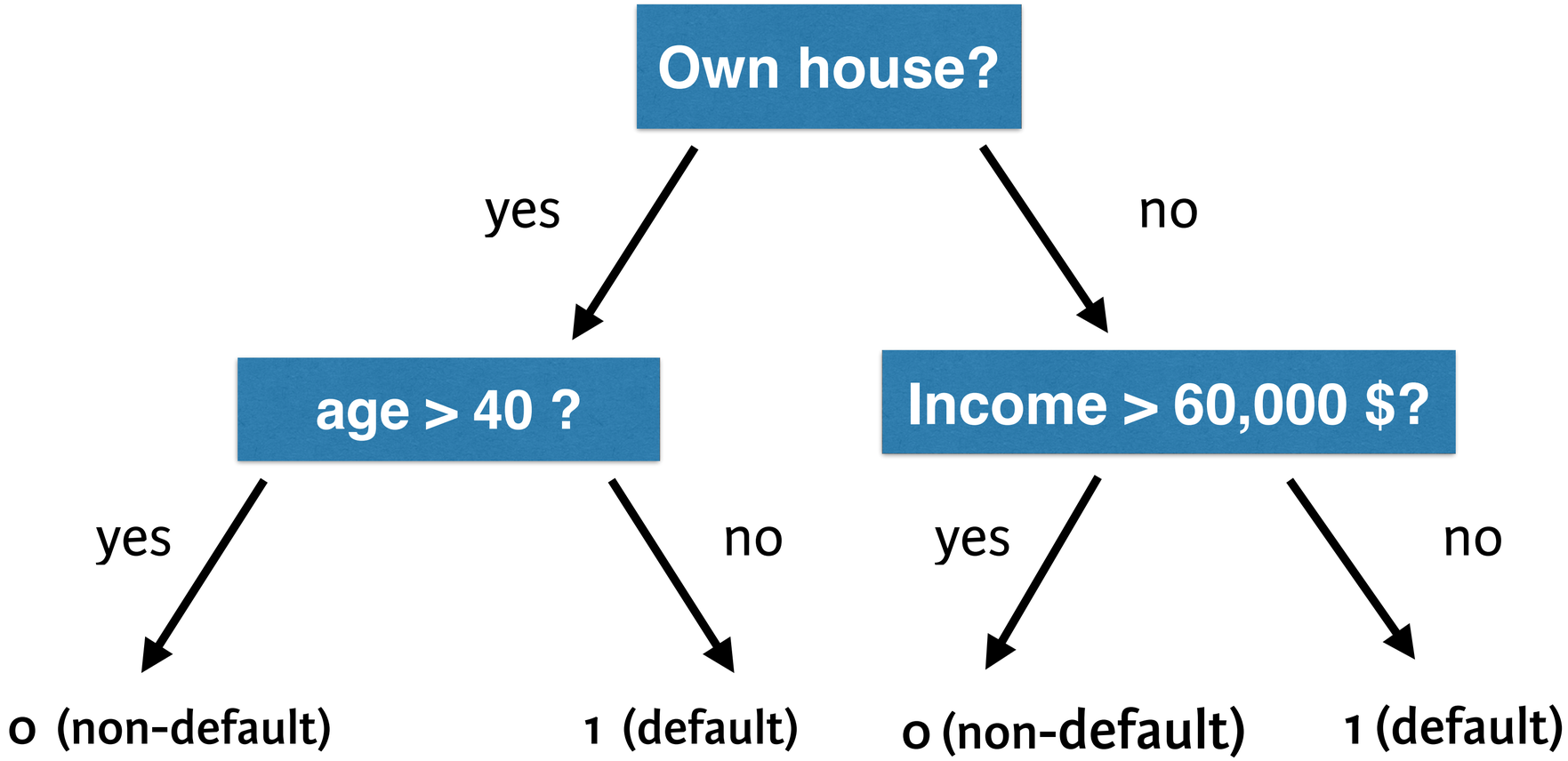
CREDIT RISK MODELING IN R

Let’s practice!

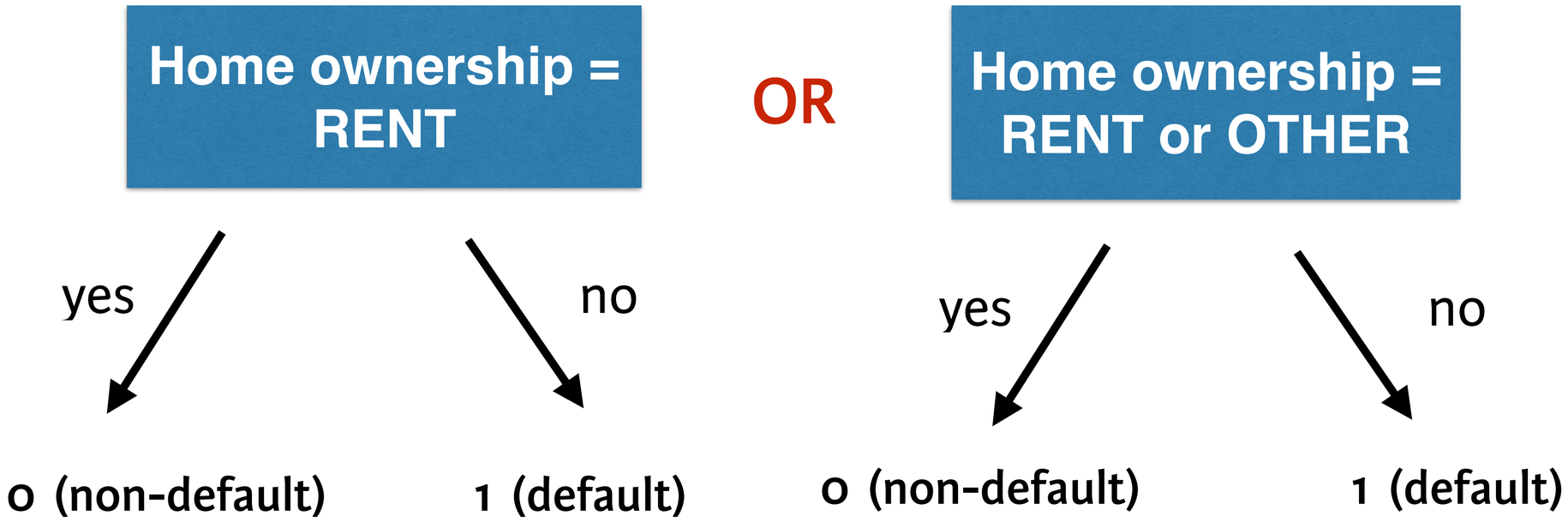
CREDIT RISK MODELING IN R

What is a decision tree?

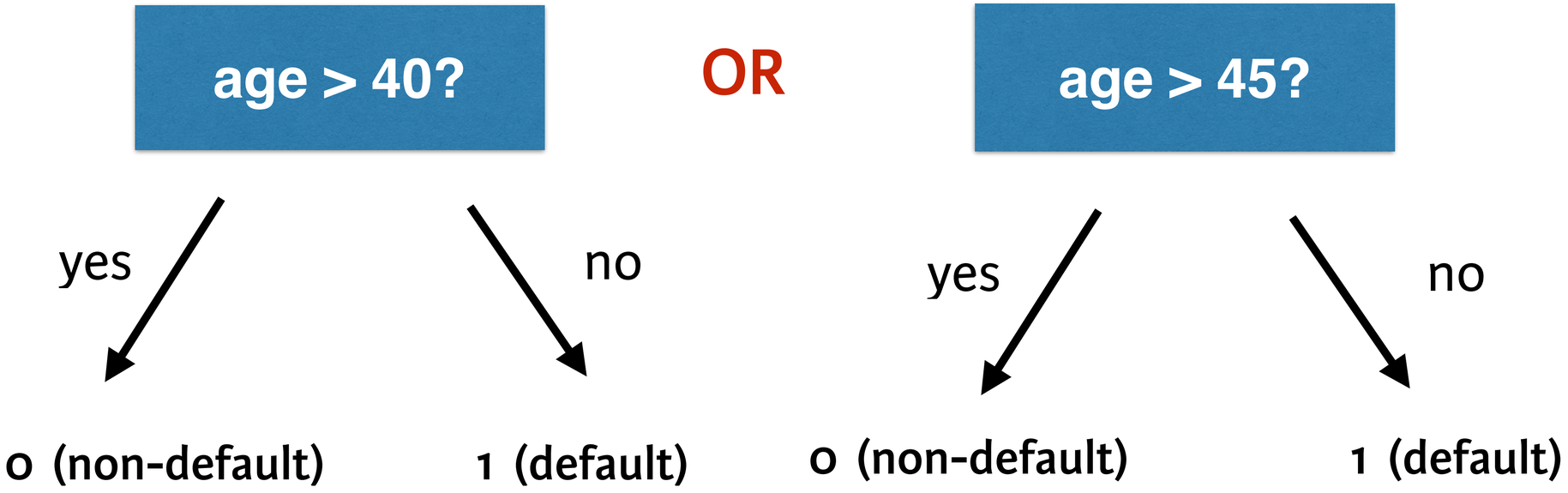
Decision tree example



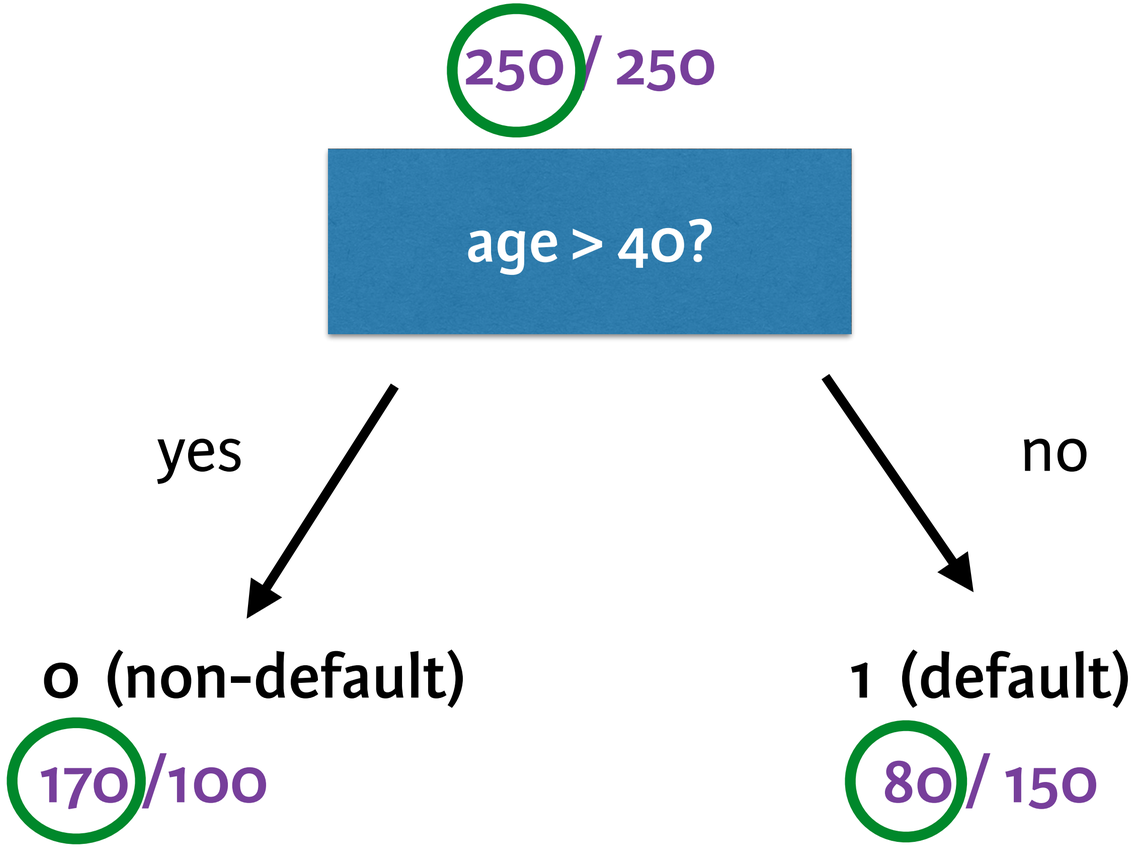
How to make splitting decision?



How to make splitting decision?

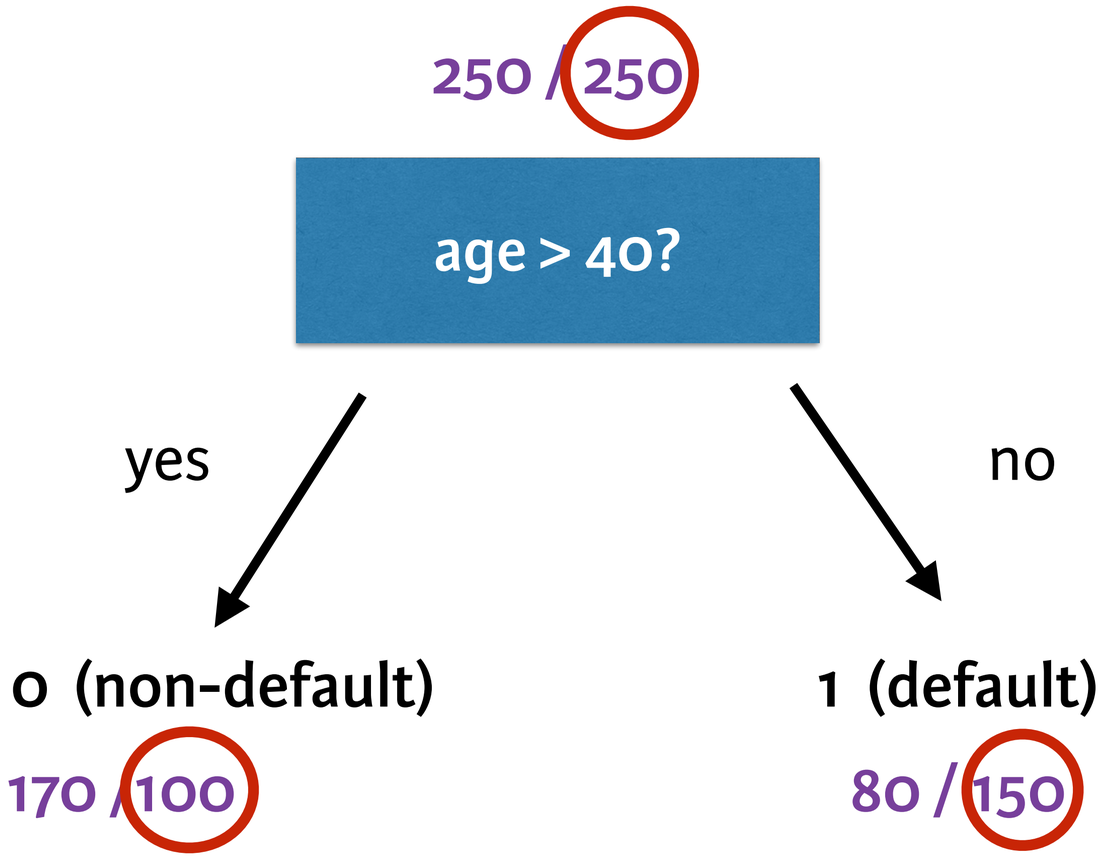


Example



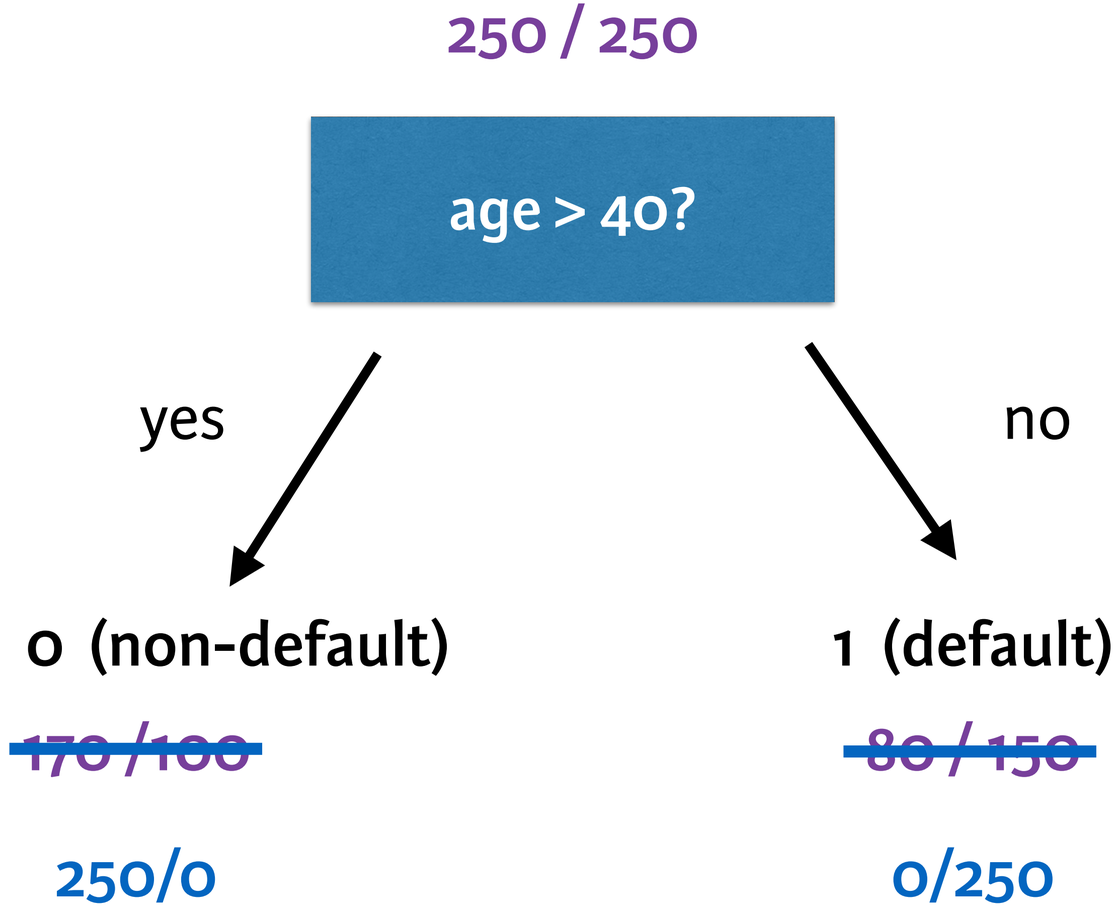
Actual **non-defaults** in this node using this split

Example



Actual **defaults** in this node using this split

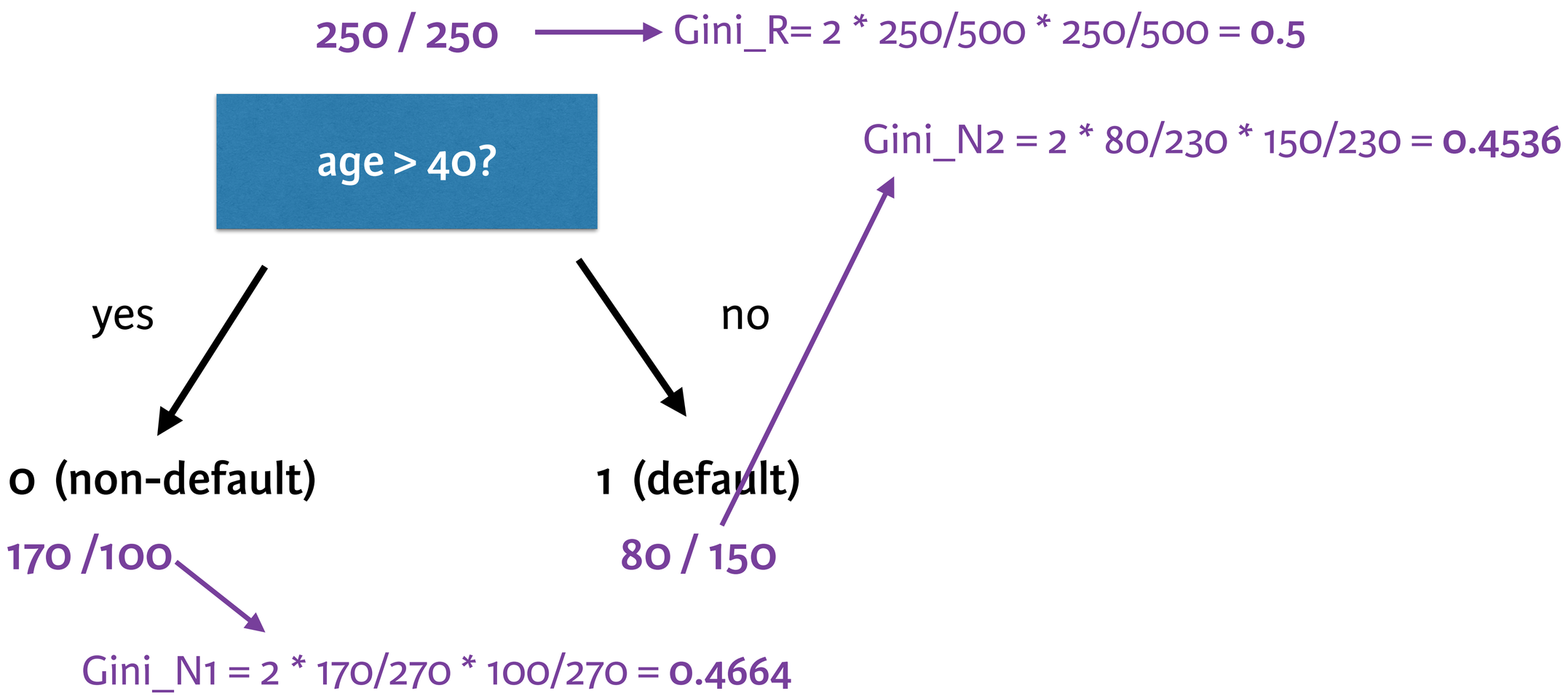
Example



= IDEAL SCENARIO

Example

Gini = 2*prop(default)*prop(non-default)



Example

Gain = **Gini_R** - prop(cases in N1) * **Gini_N1**

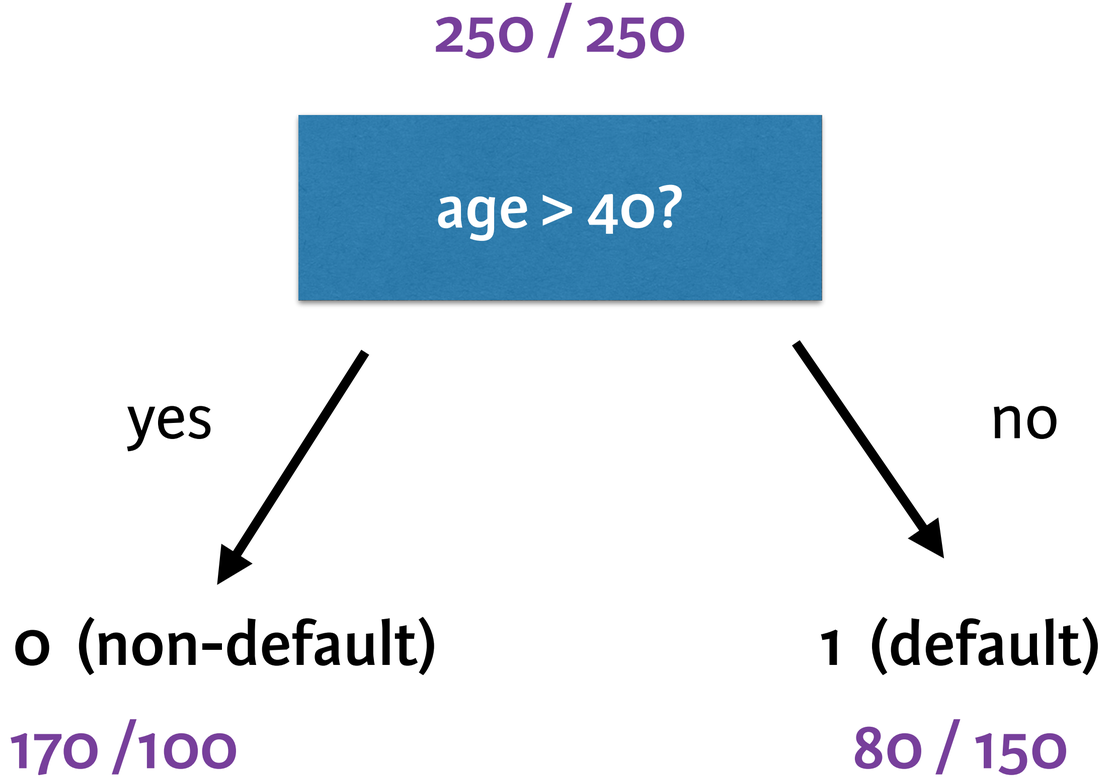
- prop(cases in N2) * **Gini_N1**

MAXIMIZE GAIN

= 0.5 - 270/500 * 0.4664

- 230/500*0.4536

= 0.039488



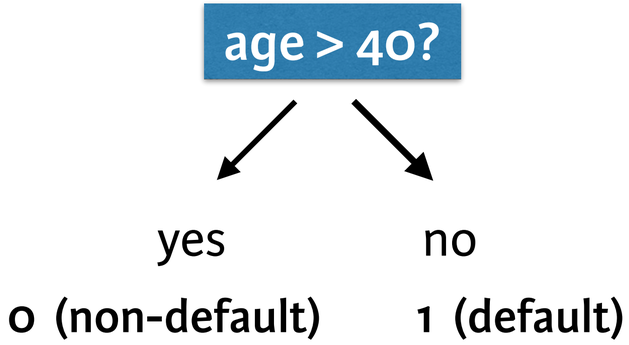
CREDIT RISK MODELING IN R

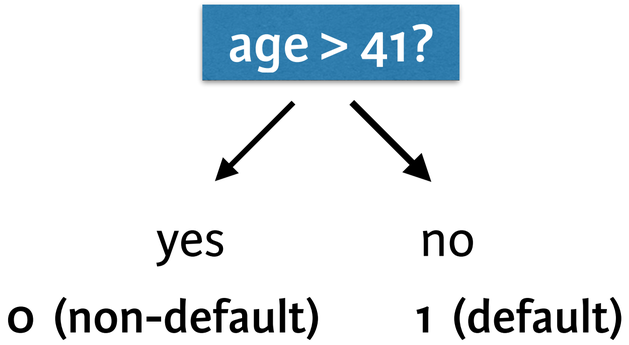
Let’s practice!

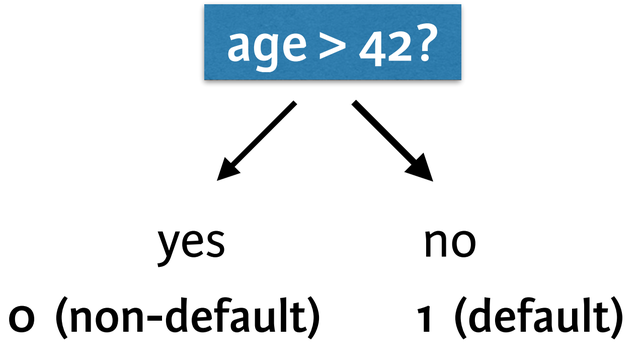
CREDIT RISK MODELING IN R

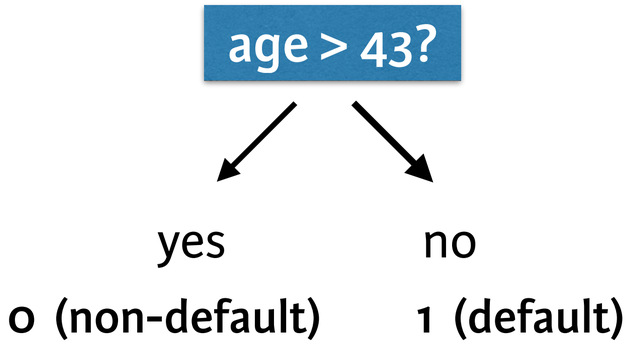
Building decision trees using the rpart()-package

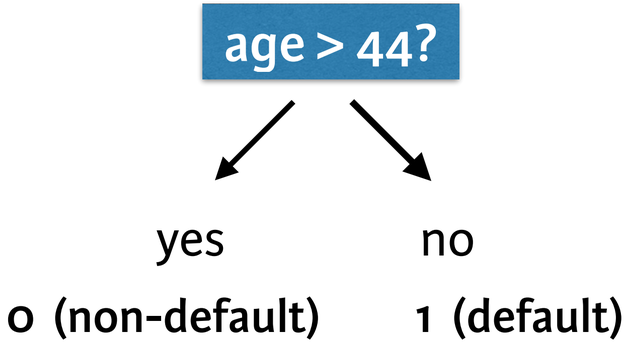
Imagine...


age > 40?
yes
no
0 (non-default)
1 (default)


age > 41?
yes
no
0 (non-default)
1 (default)


age > 42?
yes
no
0 (non-default)
1 (default)


age > 43?
yes
no
0 (non-default)
1 (default)


age > 44?
yes
no
0 (non-default)
1 (default)

. . .

rpart() package! But...

- hard building nice decision tree for credit risk data
- main reason: unbalanced data

```
> fit_default <- rpart(loan_status ~ ., method = “class",
 data = training_set)

> plot(fit_default)
Error in plot.rpart(fit_default) : fit is not a tree, just a root
```

Three techniques to overcome unbalance

- Undersampling or oversampling
  - Accuracy issue will disappear
  - Only training set
- Changing the prior probabilities
- Including a loss matrix

Validate model to see what is best!

CREDIT RISK MODELING IN R

Let’s practice!

CREDIT RISK MODELING IN R

Pruning the decision tree

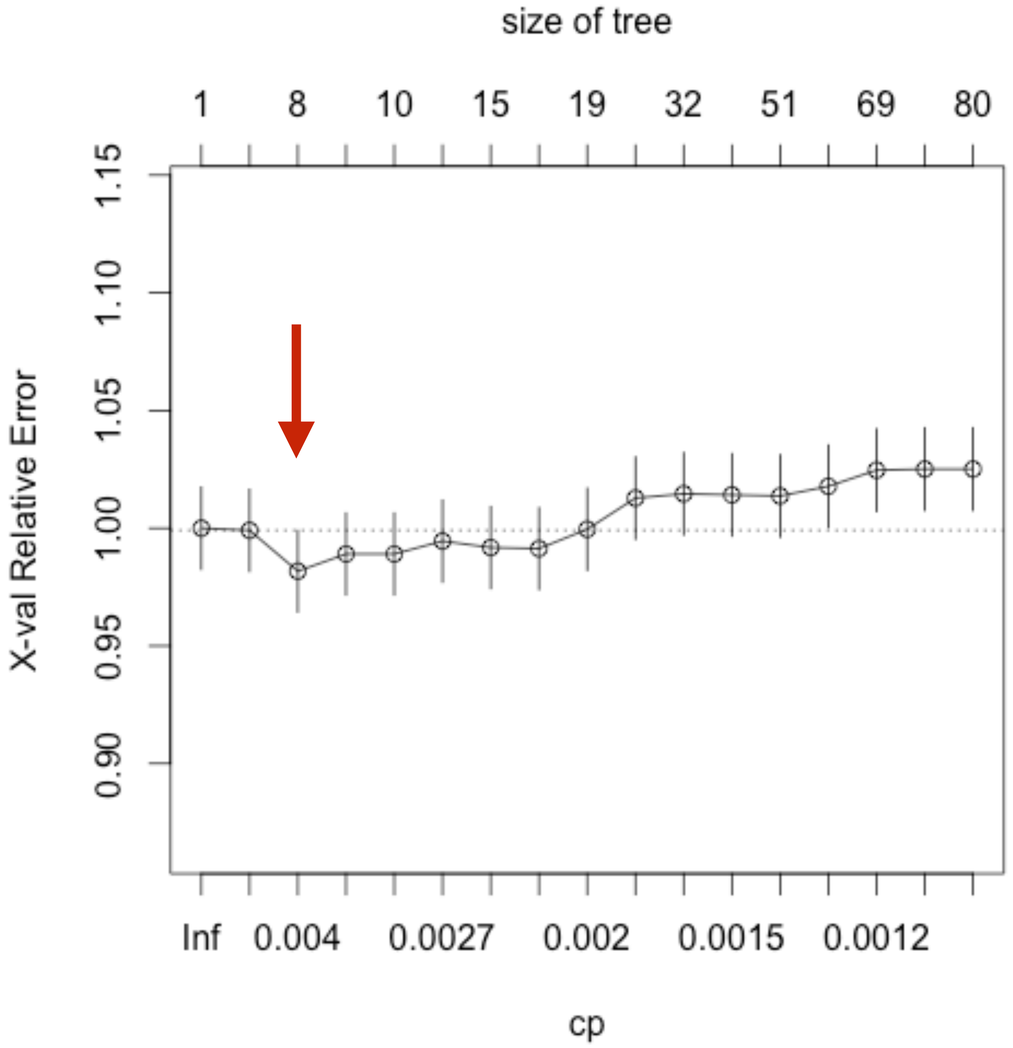
Problems with large decision trees

- Too complex: not clear anymore
- **Overfitting when applying to test set**
- Solution: use printcp(), plotcp() for pruning purposes

Printcp and tree_undersample

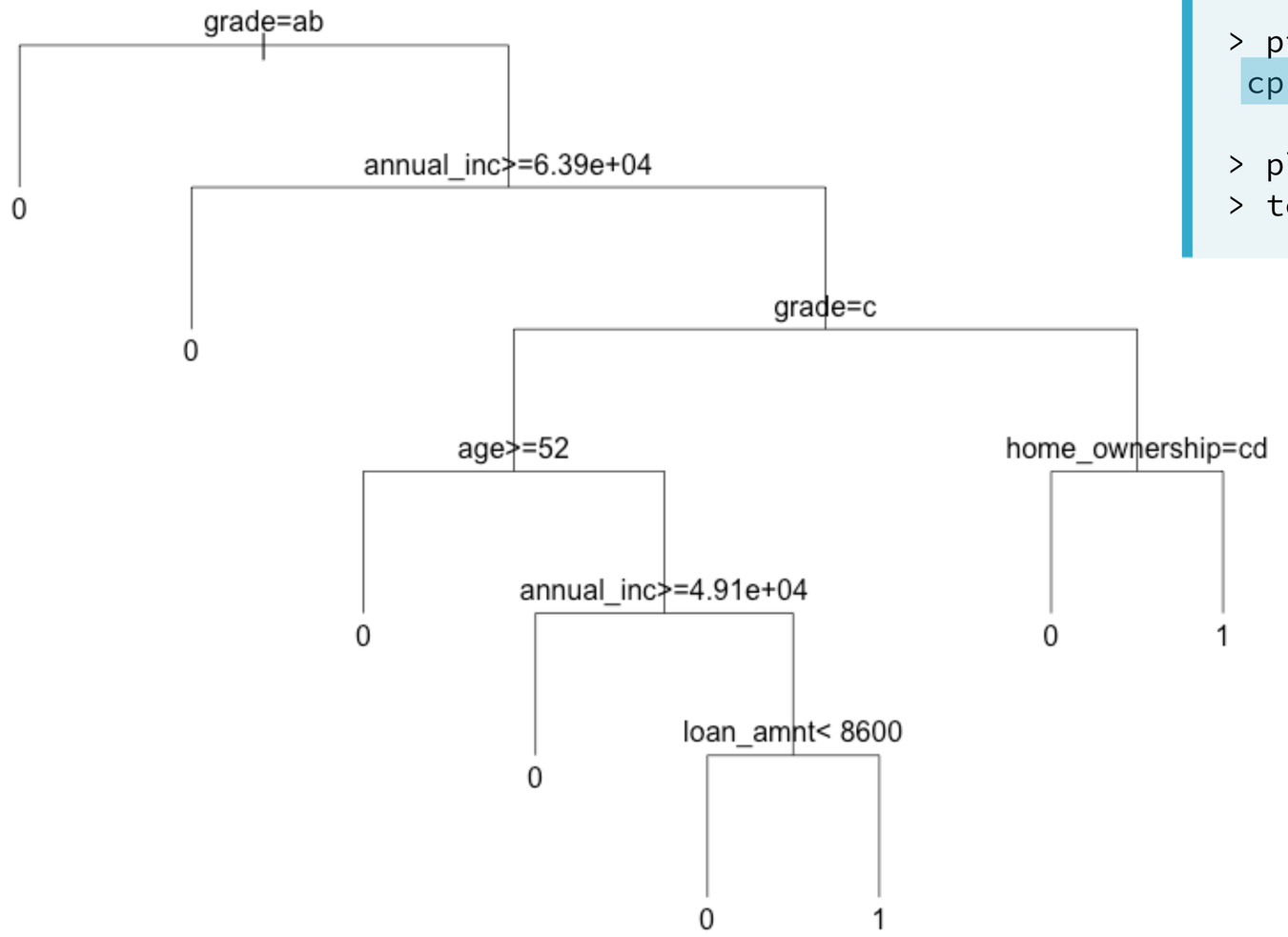
```
> printcp(tree_undersample)

Classification tree:
rpart(formula = loan_status ~ ., data = undersampled_training_set, method = "class",
 control = rpart.control(cp = 0.001))

Variables actually used in tree construction:
[1] age    annual_inc     emp_cat     grade    home_ownership   ir_cat     loan_amnt

Root node error: 2190/6570 = 0.33333

n= 6570

         CP    nsplit  rel error    xerror       xstd
1  0.0059361       0    1.00000   1.00000   0.017447
2  0.0044140       4    0.97443   0.99909   0.017443
3  0.0036530       7    0.96119   0.98174   0.017366
4  0.0031963       8    0.95753   0.98904   0.017399
                      …
16 0.0010654      76    0.84247   1.02511   0.017554
17 0.0010000      79    0.83927   1.02511   0.017554
```

Plotcp and tree_undersample



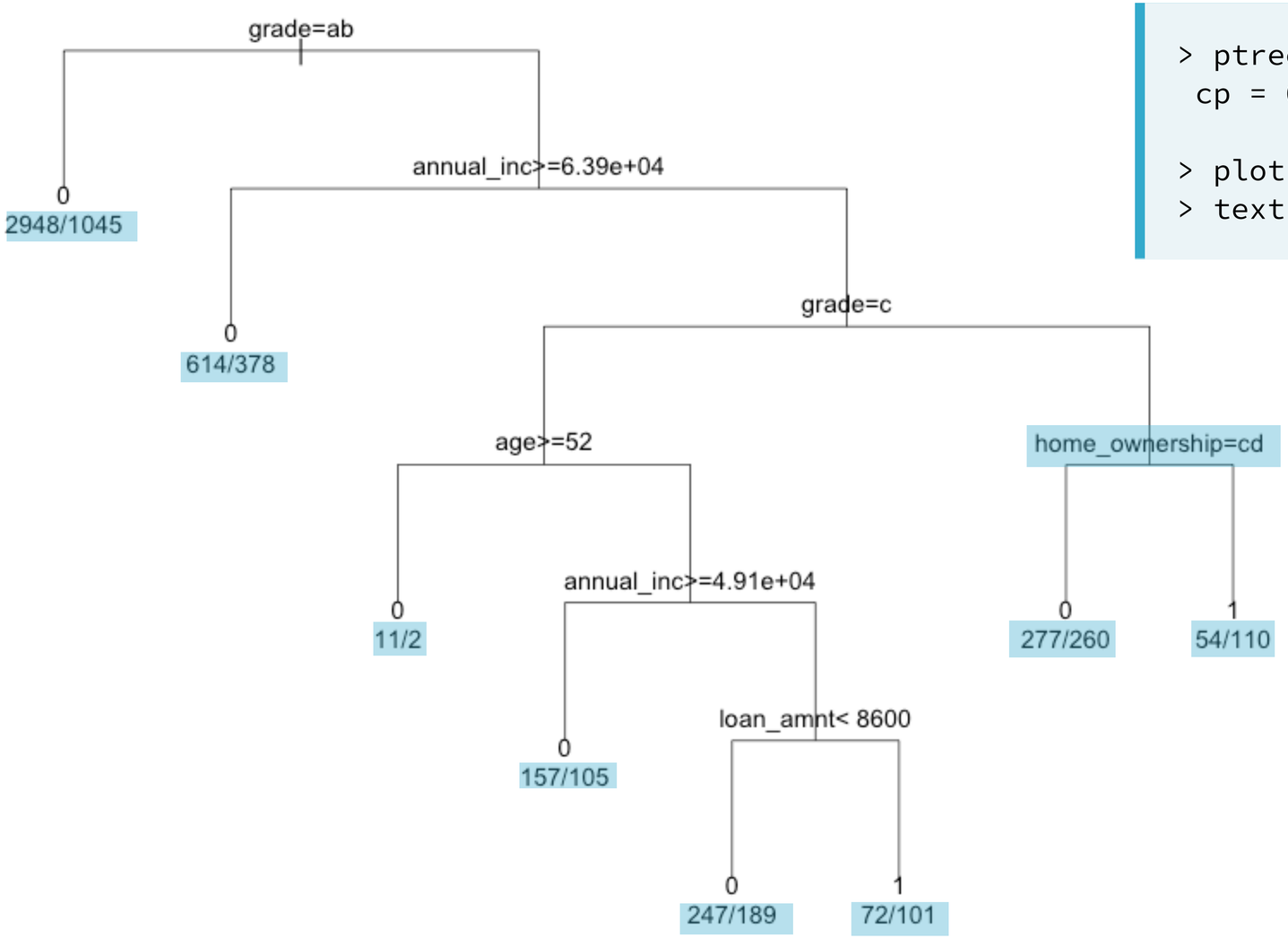
cp= 0.003653

plot the pruned tree



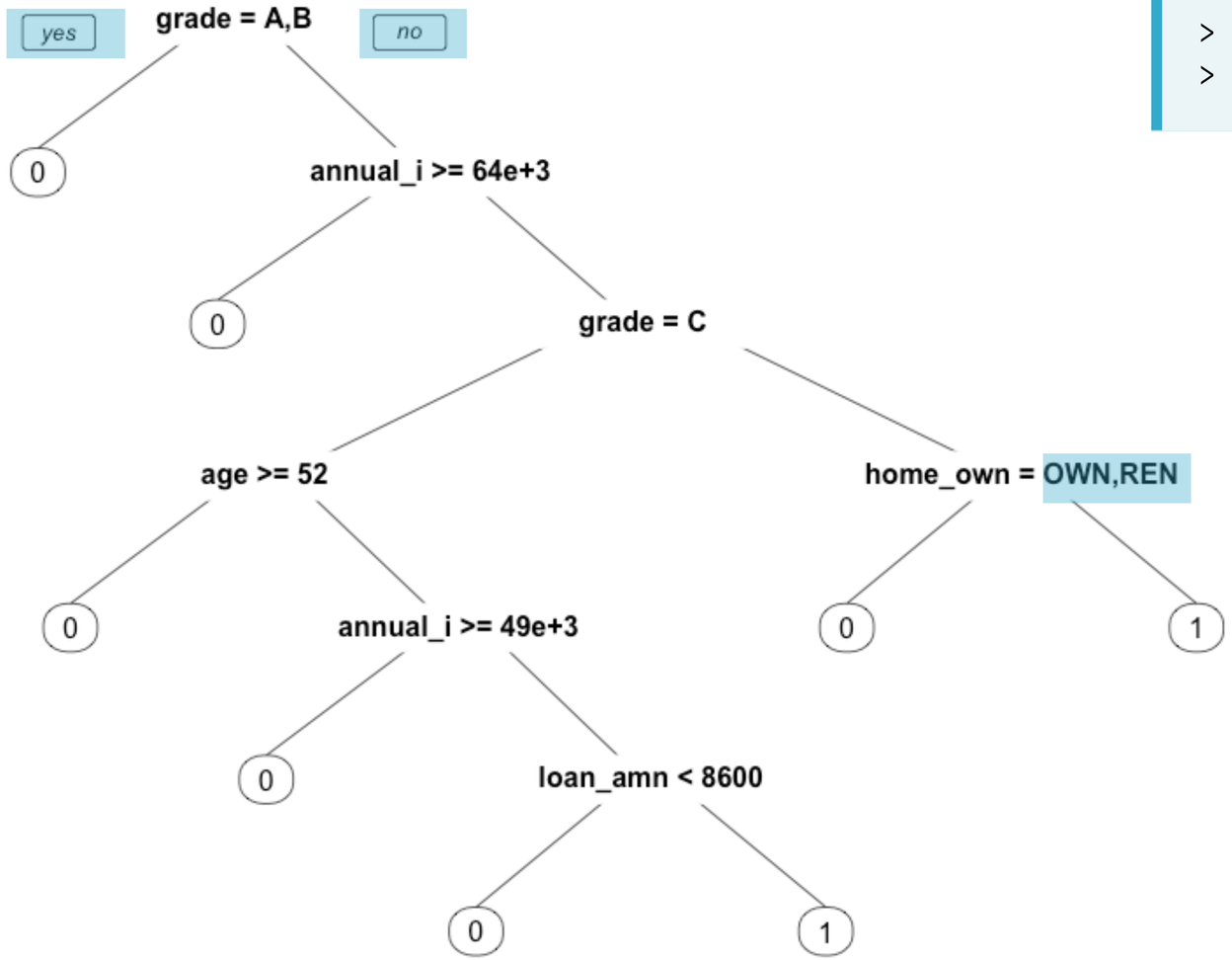

```
> ptree_undersample=prune(tree_undersample,
 cp = 0.003653)

> plot(ptree_undersample, uniform=TRUE)
> text(ptree_undersample)
```

plot the pruned tree



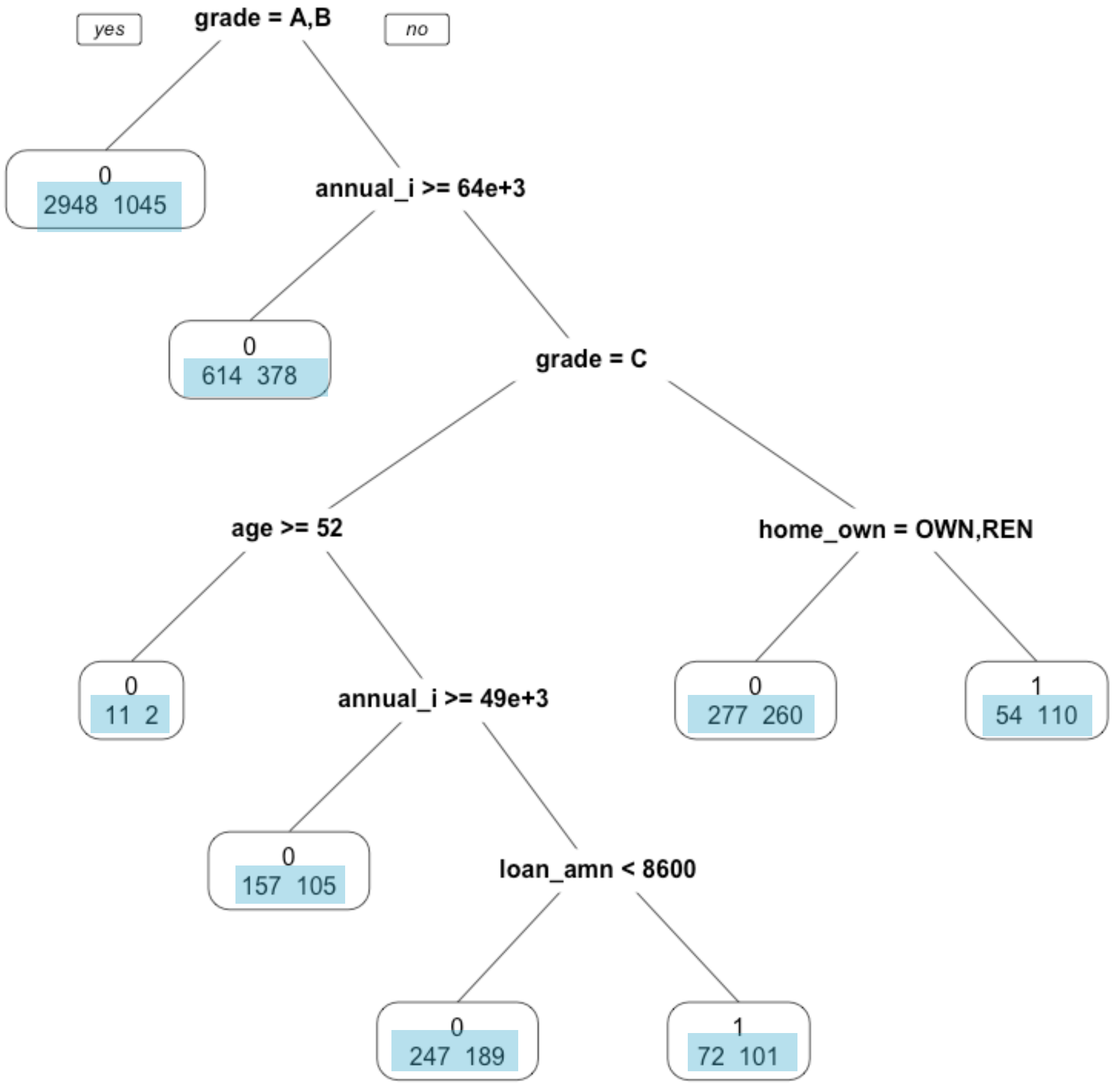

```
> ptree_undersample=prune(tree_undersample,
 cp = 0.003653)

> plot(ptree_undersample, uniform=TRUE)
> text(ptree_undersample, use.n=TRUE)
```

prp() in the rpart.plot-package




```
> library(rpart.plot)
> prp(ptree_undersample)
```

prp() in the part.plot-package




```
> library(rpart.plot)
> prp(ptree_undersample, extra = 1)
```

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Other tree options and the construction of confusion matrices.

Other interesting rpart()-arguments

...in rpart()

- `weights:` include case weights

...in the control argument of rpart (rpart.control)

- `minsplit:` minimum number of observations for split attempt
- `minbucket:` minimum number of observations in leaf node

Making predictions using the decision tree

```
> pred_undersample_class = predict(ptree_undersample, newdata = test_set,
type ="class")

1     2     3    …   29073 29079 29084 29090 29091
0     0     0    …     1     0     0     0     0
```

OR

```
> pred_undersample = predict(ptree_undersample, newdata = test_set)

           0         1
1     0.7382920 0.2617080
2     0.5665138 0.4334862
3     0.5992366 0.4007634
          …         …
29073 0.4161850 0.5838150
29079 0.6189516 0.3810484
29084 0.7382920 0.2617080
29090 0.7382920 0.2617080
29091 0.7382920 0.2617080
```

Constructing a confusion matrix

```
> table(test_set$loan_status, pred_undersample_class)

   pred_undersample_class
       0    1
  0 8314  346
  1  964   73
```

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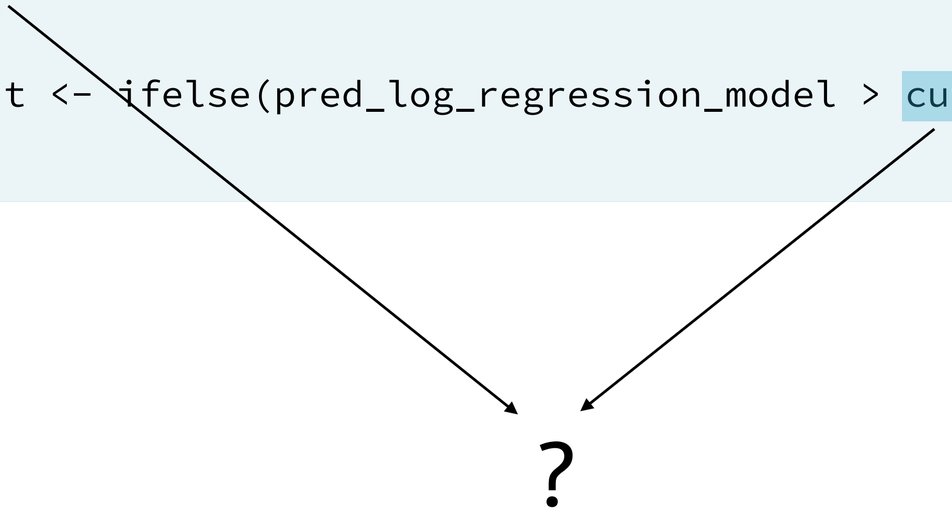
Finding the right cut-off: the strategy curve

Constructing a confusion matrix

```
> predict(log_reg_model, newdata = test_set, type = "response")

 1           2           3           4           5           …
0.08825517 0.3502768 0.28632298 0.1657199 0.11264550    …



> predict(class_tree, new data = test_set)
           0         1
1 0.7873134 0.2126866
2 0.6250000 0.3750000
3 0.6250000 0.3750000
4 0.7873134 0.2126866
5 0.5756867 0.4243133
```

Cut-off?

```
> pred_log_regression_model <- predict(log_reg_model, newdata = test_set,
type = “response")

> cutoff <- 0.14

> class_pred_logit <- ifelse(pred_log_regression_model > cutoff, 1, 0)
```



A certain strategy...

```
> log_model_full <- glm(loan_status ~ ., family = "binomial", data = training_set)
> predictions_all_full<- predict(log_reg_model, newdata = test_set, type = “response")

> cutoff <- quantile(predictions_all_full, 0.8)
  cutoff
      80%
0.1600124

> pred_full_20 <- ifelse(predictions_all_full > cutoff, 1, 0)
```

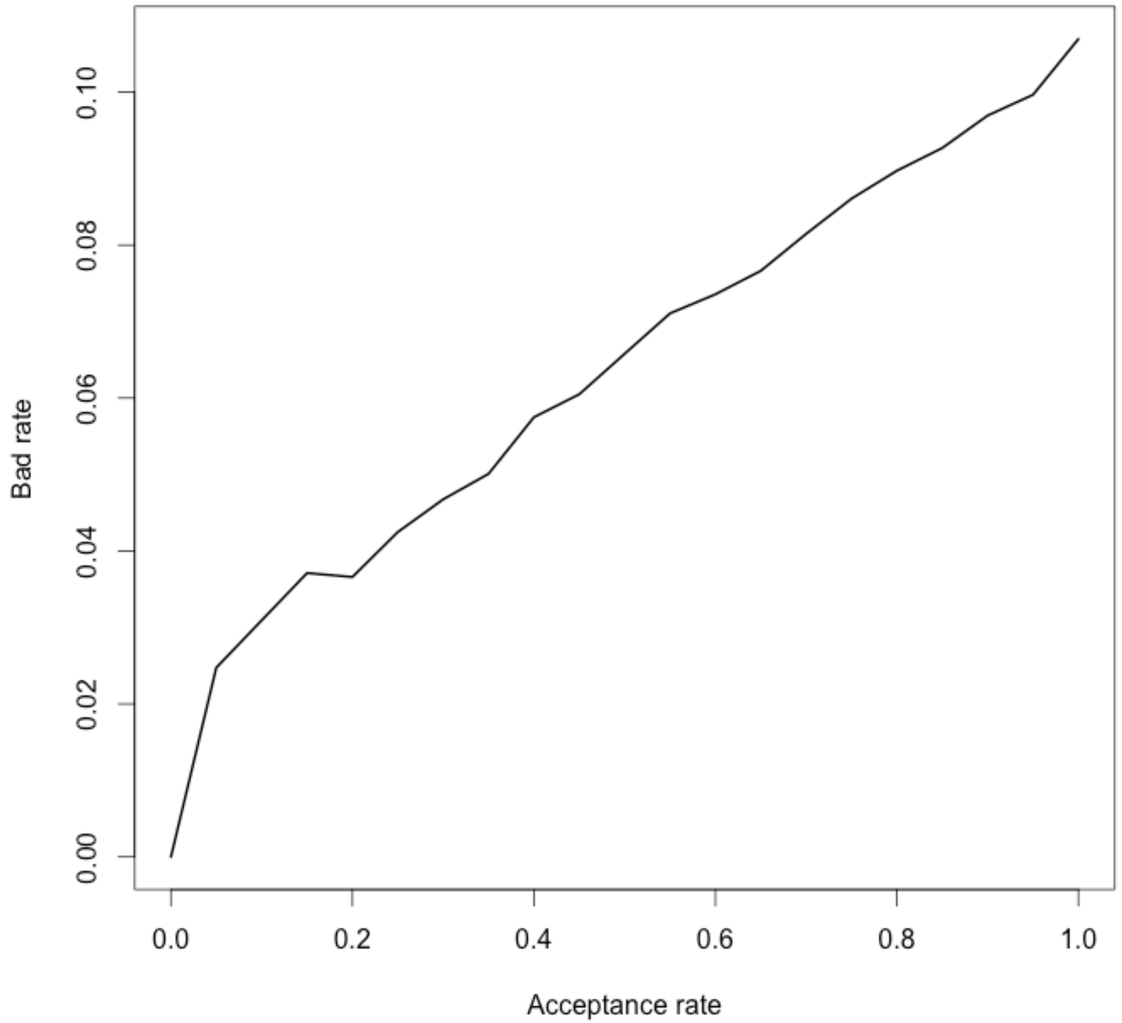
A certain strategy (continued)

```
> true_and_predval <- cbind(test_set$loan_status, pred_full_20)
  true_and_predval

     test_set$loan_status    pred_full_20
1                    0                   0
2                    0                   0
3                    0                   1
4                    0                   0
5                    0                   1
…                    …                   …


> accepted_loans <- pred_and_trueval[pred_full_20 == 0,1]

> bad_rate <- sum(accepted_loans)/length(accepted_loans)
> bad_rate
[1] 0.08972541
```

The strategy table

```
       accept_rate  cutoff  bad_rate
 [1,]        1.00  0.5142    0.1069
 [2,]        0.95  0.2122    0.0997
 [3,]        0.90  0.1890    0.0969
 [4,]        0.85  0.1714    0.0927
 [5,]        0.80  0.1600    0.0897
 [6,]        0.75  0.1471    0.0861
 [7,]        0.70  0.1362    0.0815
 [8,]        0.65  0.1268    0.0766
  …             …       …         …
[16,]        0.25  0.0644    0.0425
[17,]        0.20  0.0590    0.0366
[18,]        0.15  0.0551    0.0371
[19,]        0.10  0.0512    0.0309
[20,]        0.05  0.0453    0.0247
[21,]        0.00  0.0000    0.0000
```

The strategy curve



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The ROC-curve

Until now

- strategy table/curve : still make assumption
- what is “overall” best model?

Confusion matrix

model prediction

actual loan status

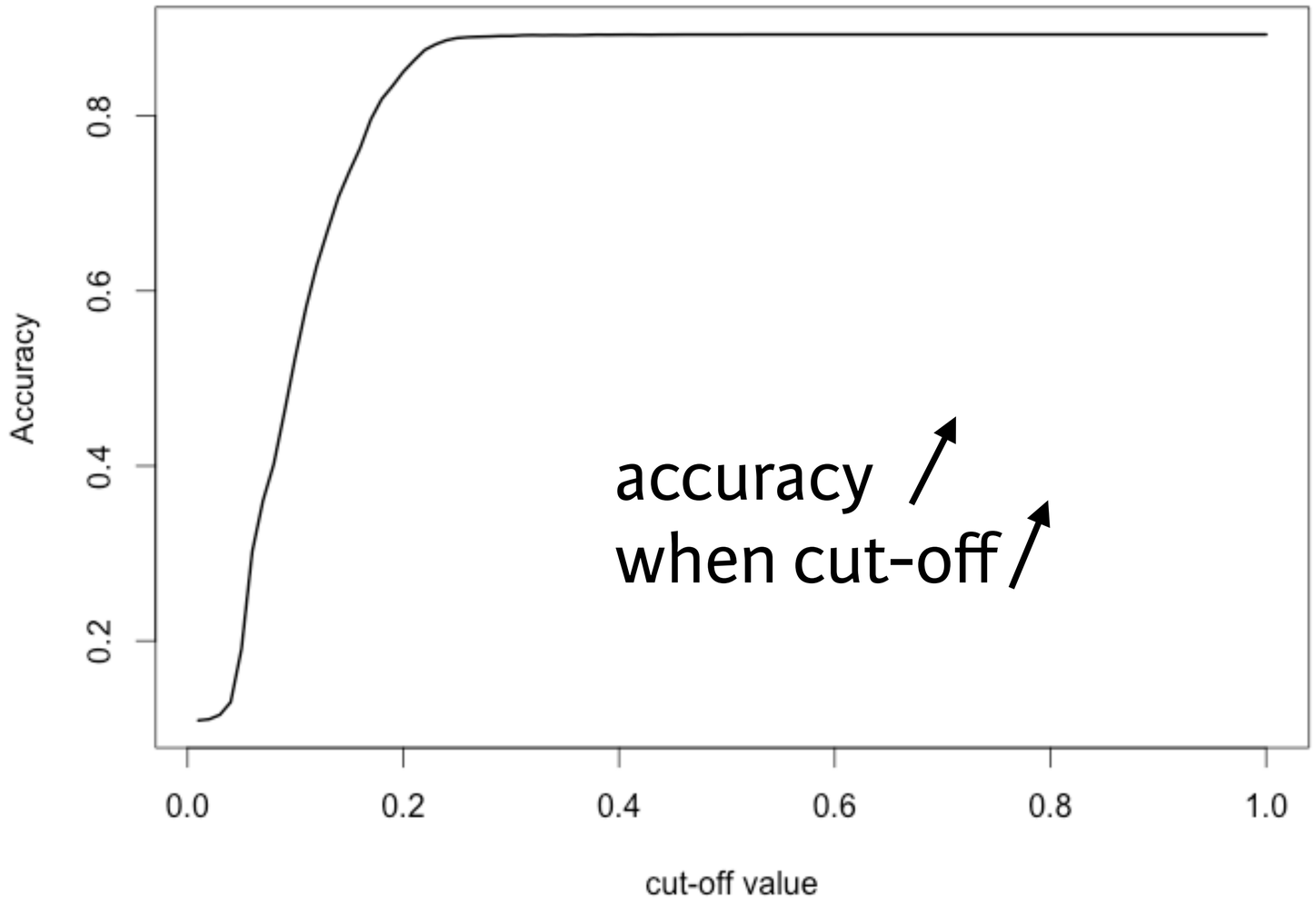
	no default (0)	default (1)
no default (0)	**TN**	**FP**
default (1)	**FN**	**TP**

$$\text{Accuracy} = \frac{TP + TN}{TP + FP + TN + FN}$$

$$\text{Sensitivity} = \frac{TP}{TP + FN}$$

$$\text{Specificity} = \frac{TN}{TN + FP}$$

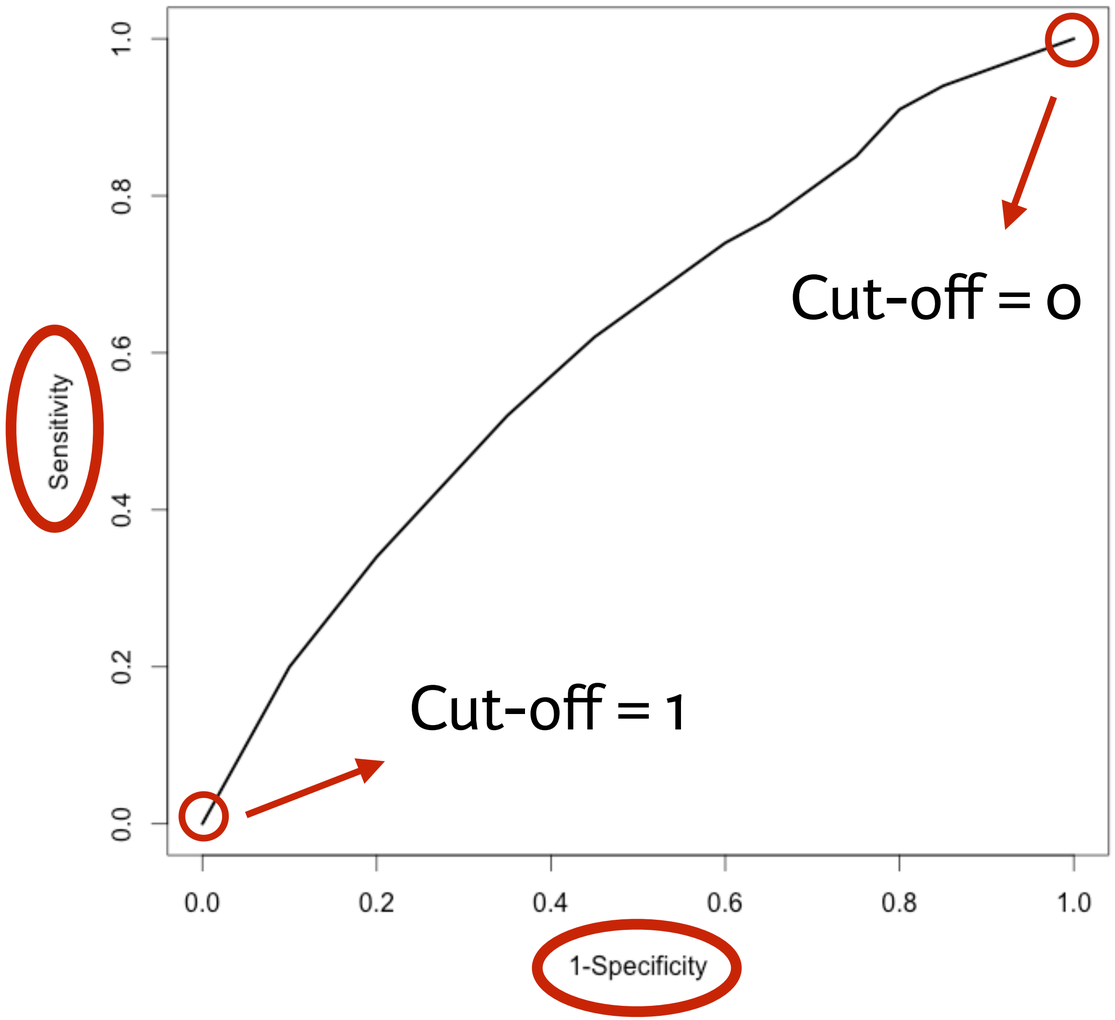
Accuracy?



$$\text{Sensitivity} = \frac{TP}{TP + FN}$$

$$\text{Specificity} = \frac{TN}{TN + FP}$$

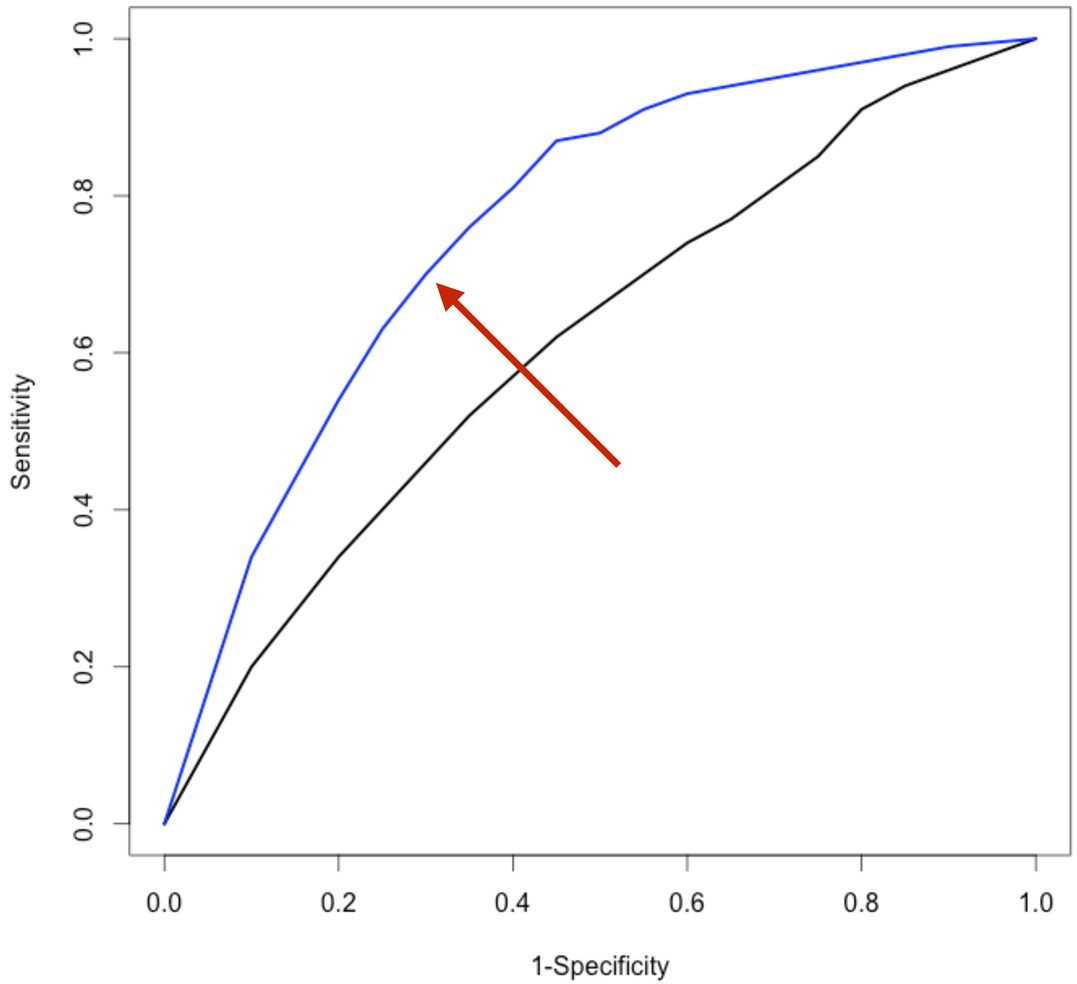
The ROC-curve



$$\text{Sensitivity} = \frac{TP}{TP + FN}$$

$$\text{Specificity} = \frac{TN}{TN + FP}$$

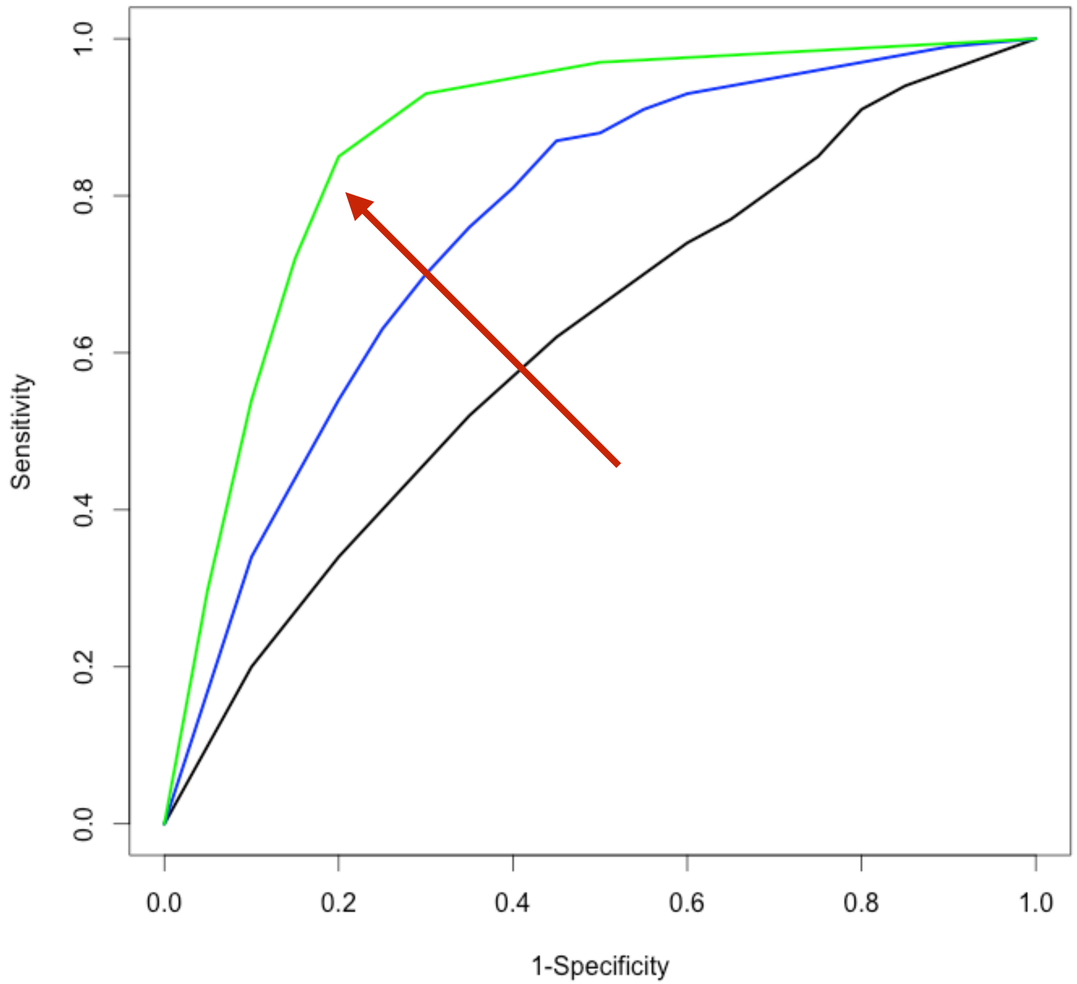
The ROC-curve



$$\text{Sensitivity} = \frac{TP}{TP + FN}$$

$$\text{Specificity} = \frac{TN}{TN + FP}$$

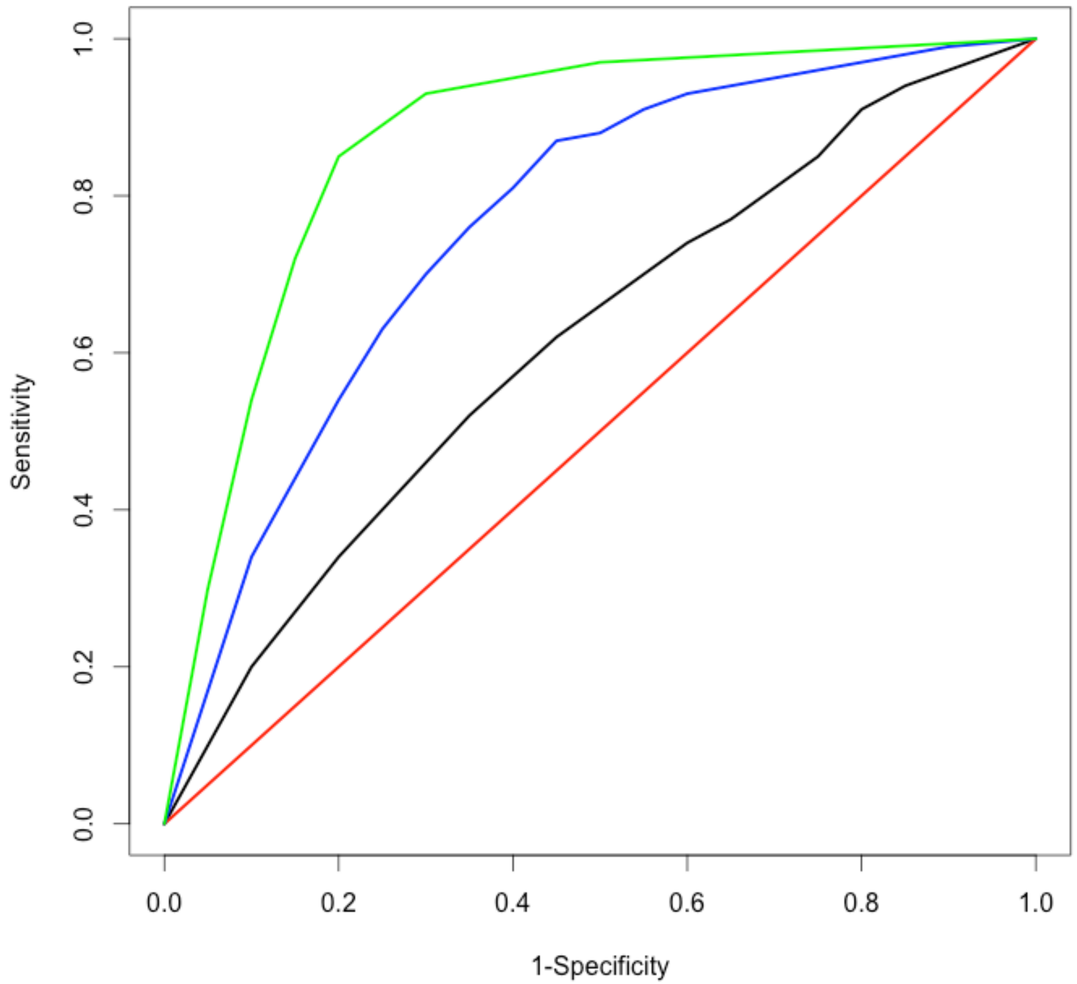
The ROC-curve



$$\text{Sensitivity} = \frac{TP}{TP + FN}$$

$$\text{Specificity} = \frac{TN}{TN + FP}$$

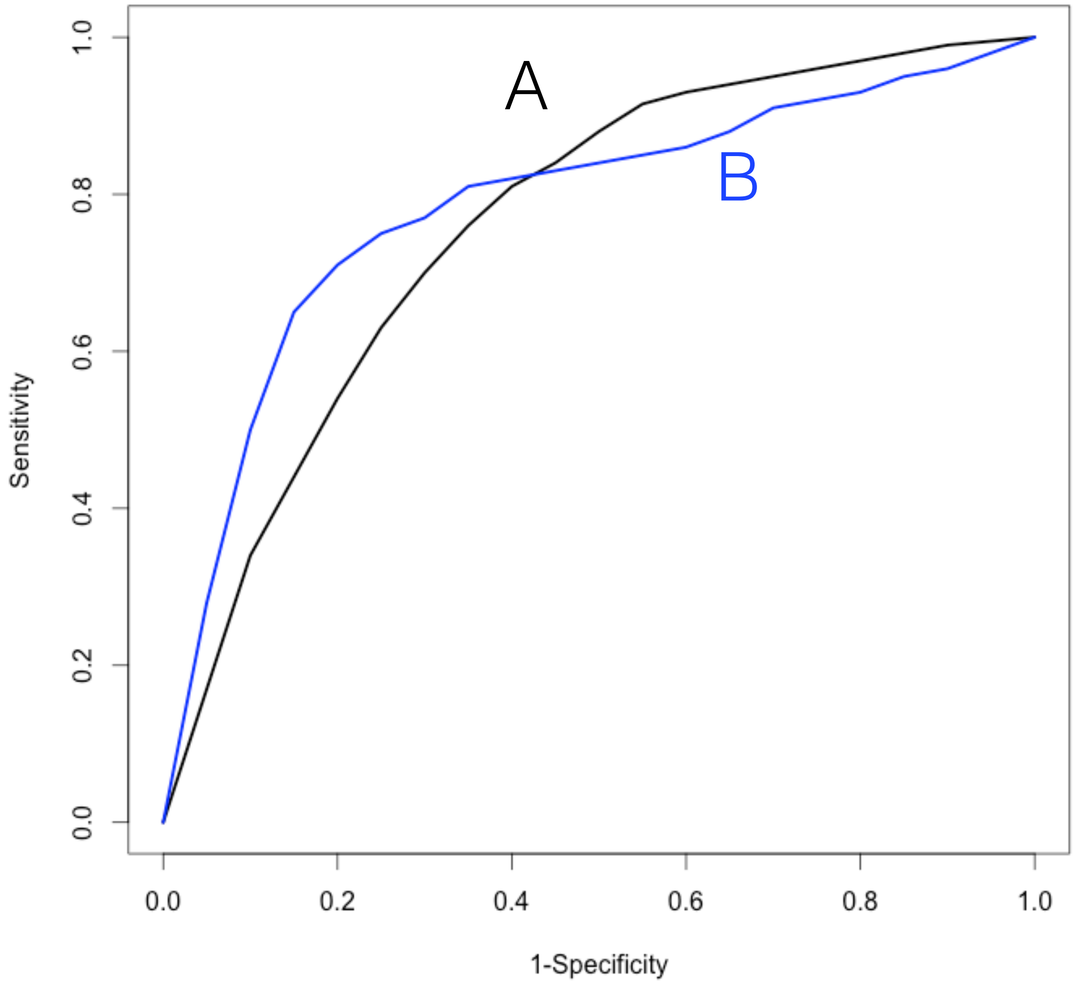
The ROC-curve



$$\text{Sensitivity} = \frac{TP}{TP + FN}$$

$$\text{Specificity} = \frac{TN}{TN + FP}$$

Which one is better?



AUC ROC-curve A = 0.75

AUC ROC-curve B = 0.78

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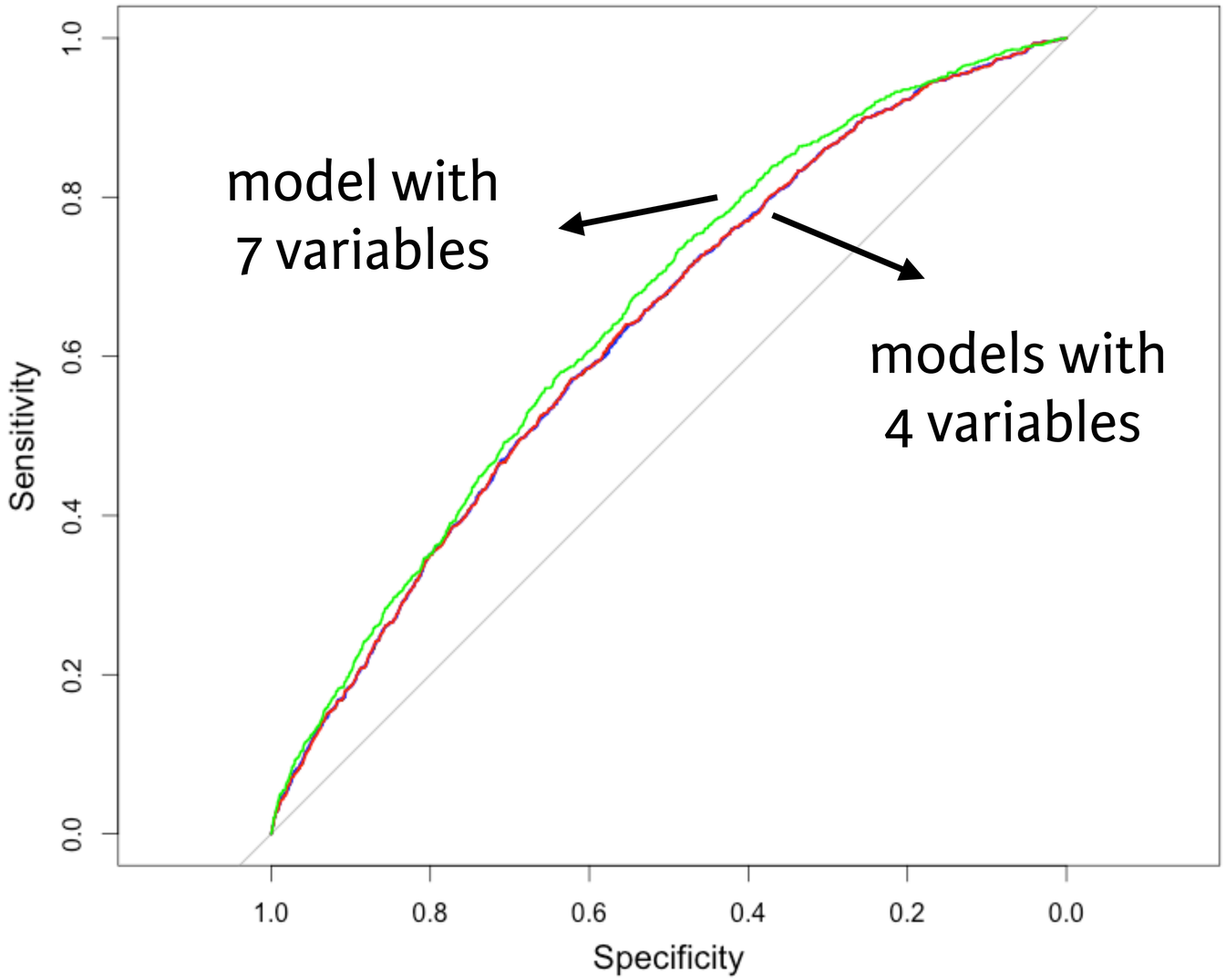
Input selection based on the AUC

ROC curves for 4 logistic regression models



AUC-based pruning

1) Start with a model including all variables (in our case, 7) and compute AUC

```
> log_model_full <- glm(loan_status ~ loan_amnt + grade + home_ownership +
annual_inc + age + emp_cat + ir_cat, family = "binomial", data = training_set)

> predictions_model_full <- predict(log_model_full, newdata = test_set, type =
“response")

> AUC_model_full <- auc(test_set$loan_status, predictions_model_full)
Area under the curve: 0.6512
```

AUC-based pruning

2) Build 7 new models, where each time one of the variables is removed, and make PD-predictions using the test set

```
log_1_remove_amnt <- glm(loan_status ~ grade + home_ownership + annual_inc + age
+ emp_cat + ir_cat, family = "binomial", data = training_set)

log_1_remove_grade <- glm(loan_status ~ loan_amnt + home_ownership + annual_inc +
age + emp_cat + ir_cat, family = "binomial", data = training_set)

log_1_remove_home <- glm(loan_status ~ loan_amnt + grade + annual_inc + age +
emp_cat + ir_cat, family = "binomial", data = training_set)

…

pred_1_remove_amnt <- predict(log_1_remove_amnt, newdata = test_set, type =
"response")
pred_1_remove_grade <- predict(log_1_remove_grade, newdata = test_set, type =
"response")
pred_1_remove_home <- predict(log_1_remove_home, newdata = test_set, type =
"response")
…
```

AUC-based pruning

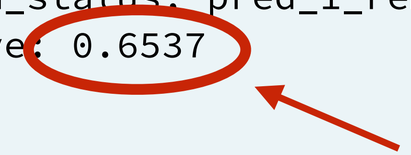
3) Keep the model that led to the best AUC (AUC full model: 0.6512)

```
> auc(test_set$loan_status, pred_1_remove_amnt)
Area under the curve: 0.6514

> auc(test_set$loan_status, pred_1_remove_grade)
Area under the curve: 0.6438

> auc(test_set$loan_status, pred_1_remove_home)
Area under the curve: 0.6537

…
```



Remove variable “home_ownership”

4) Repeat until AUC decreases (significantly)

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Course wrap-up

Other methods

- Discriminant analysis
- Random forest
- Neural networks
- Support vector machines

But... very classification-focused

- Timing aspect is neglected
- New popular method: survival analysis
    - PD's that change over time
    - time-varying covariates can be included

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The end!