Welcome To The Course

Is Investing Monkey-Business?

Source: Eric Isselee, Getty Images

Who am I?

- Professor of Finance

Vrije Universiteit Brussel

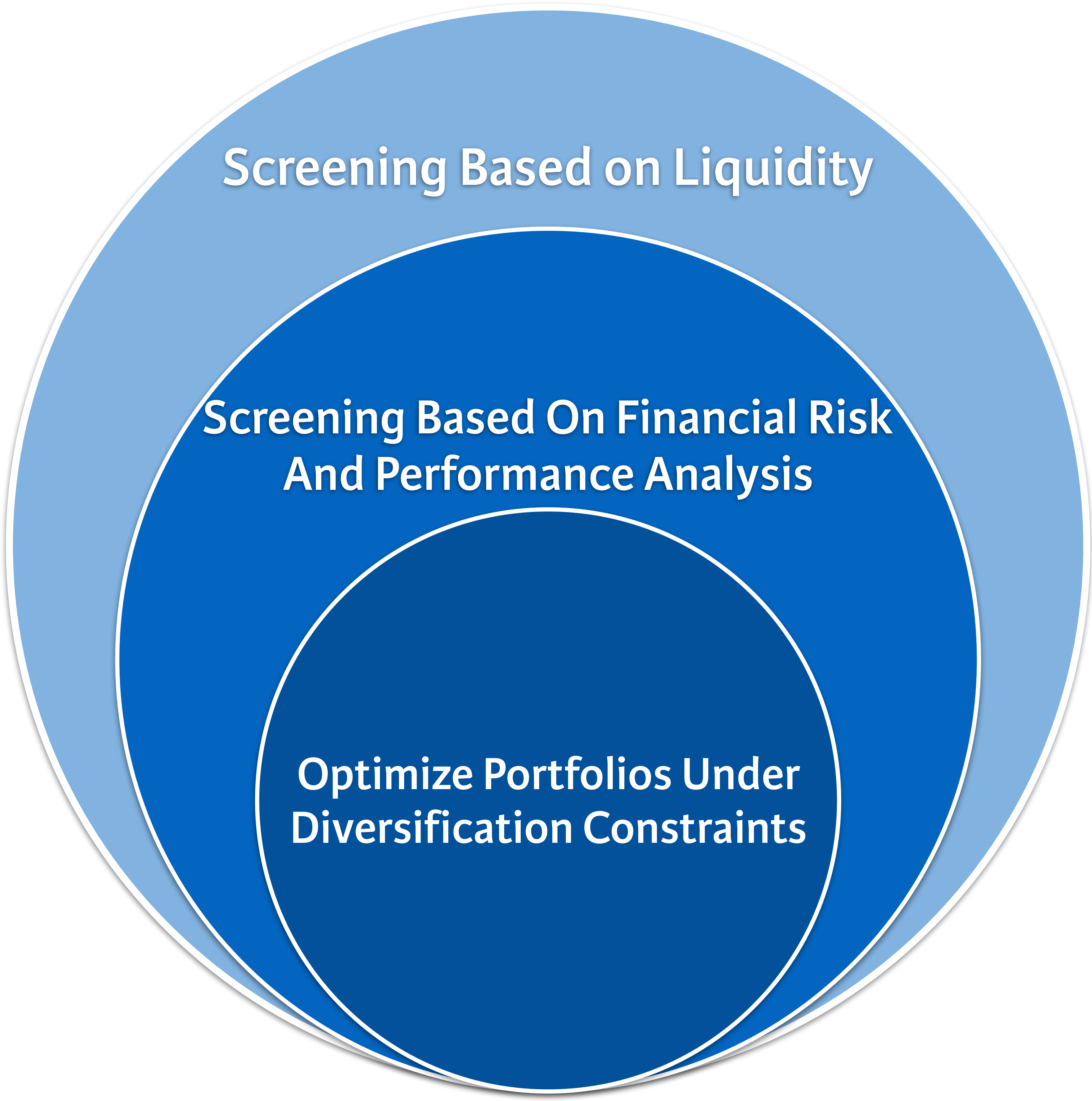
VU VRIJE UNIVERSITEIT AMSTERDAM

Who am I?

- Advisor to investment companies about risk optimized investment: Winning by losing less.

Source: Finvex

Diversify To Avoid Losses



Simple Tricks

Simple Tricks

- To avoid large losses:
  - Carefully select diversified portfolios
  - Use backtesting and online performance monitoring

Course Overview

Chapter 1: *Portfolio Weights & Returns*

↓

Chapter 2: *Portfolio Performance Evaluation*

↓

Chapter 3: *Drivers of Performance*

↓

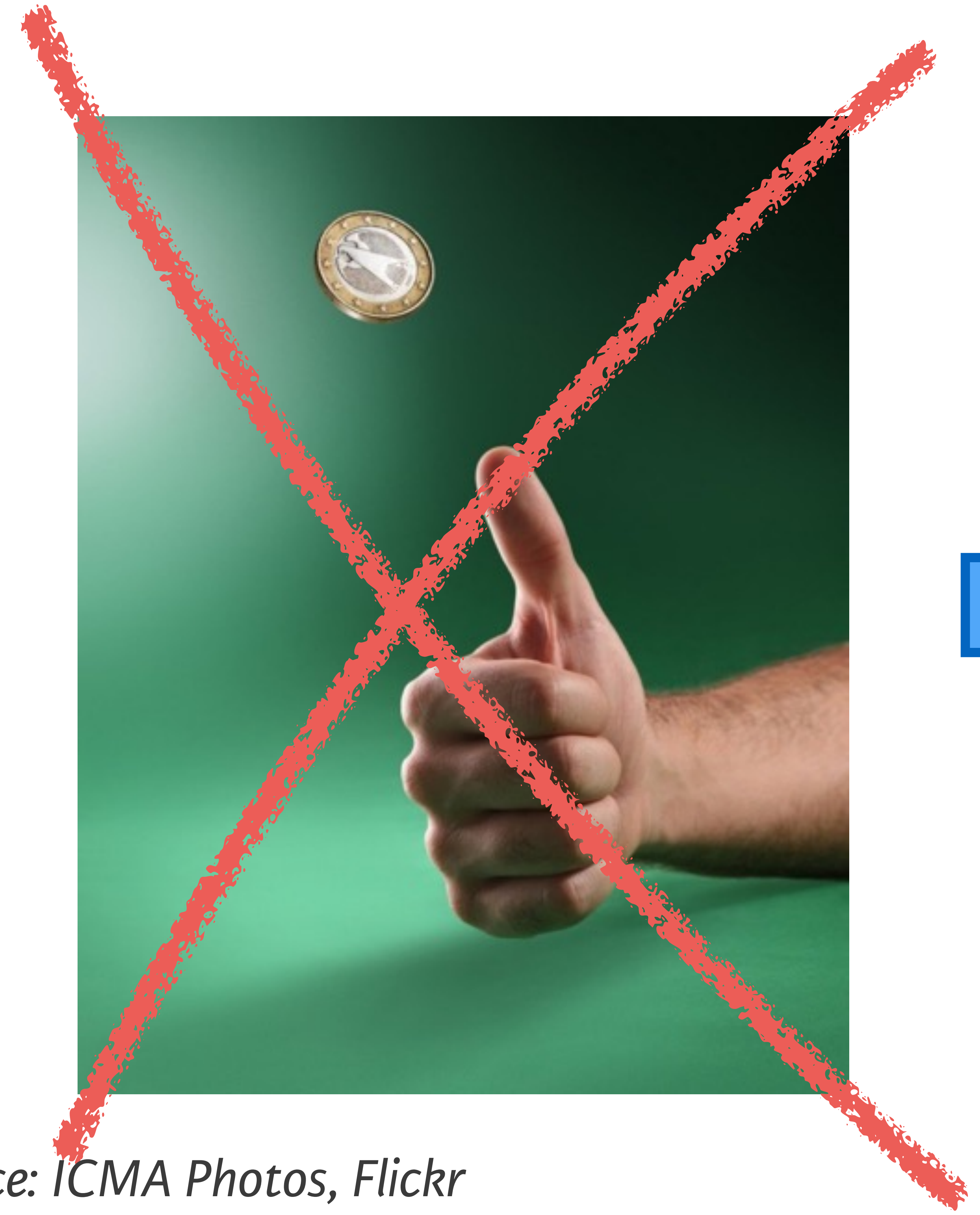
Chapter 4: *Portfolio Optimization*

Let’s practice!

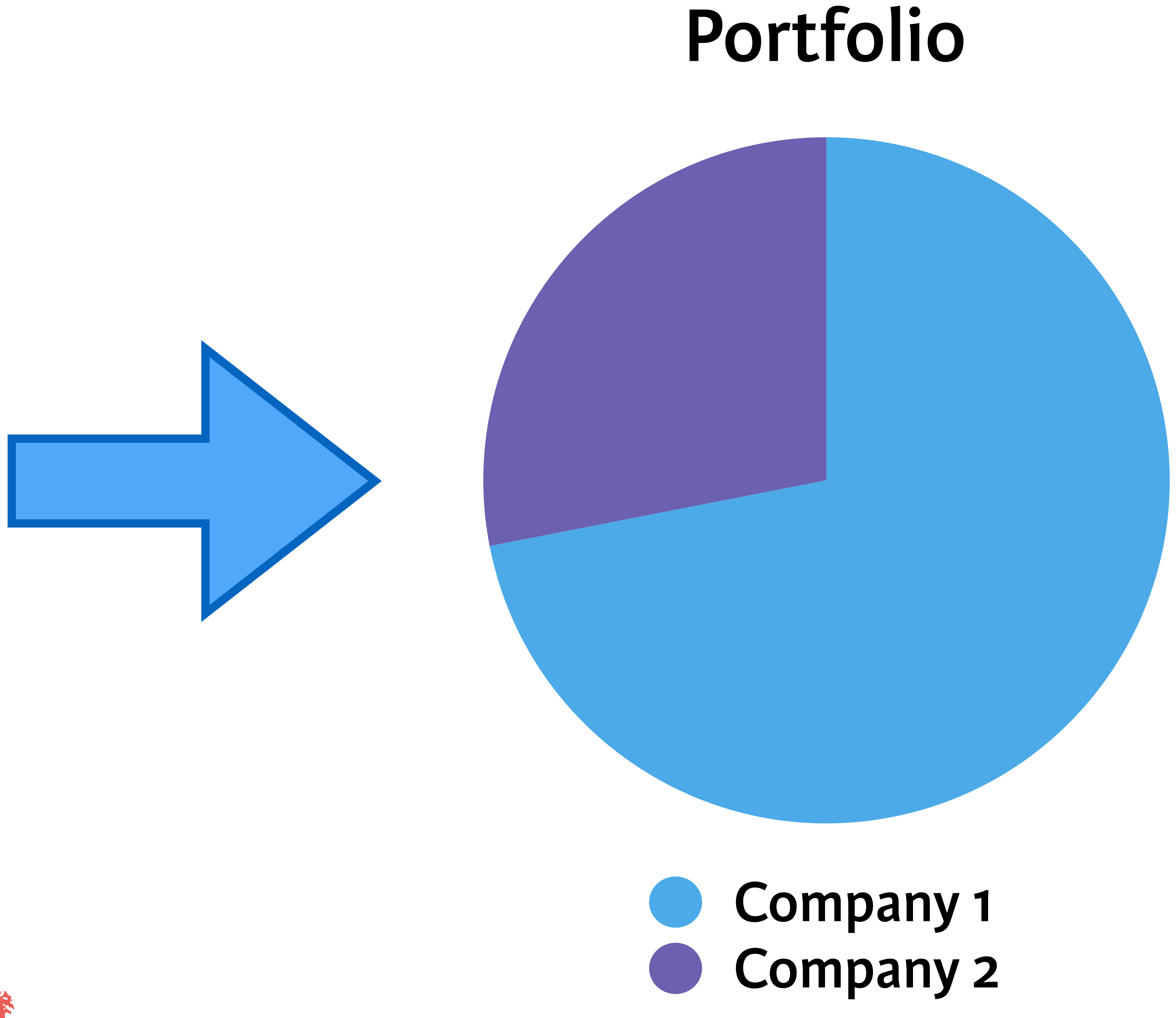
The Portfolio Weights

Investment Decision Choices

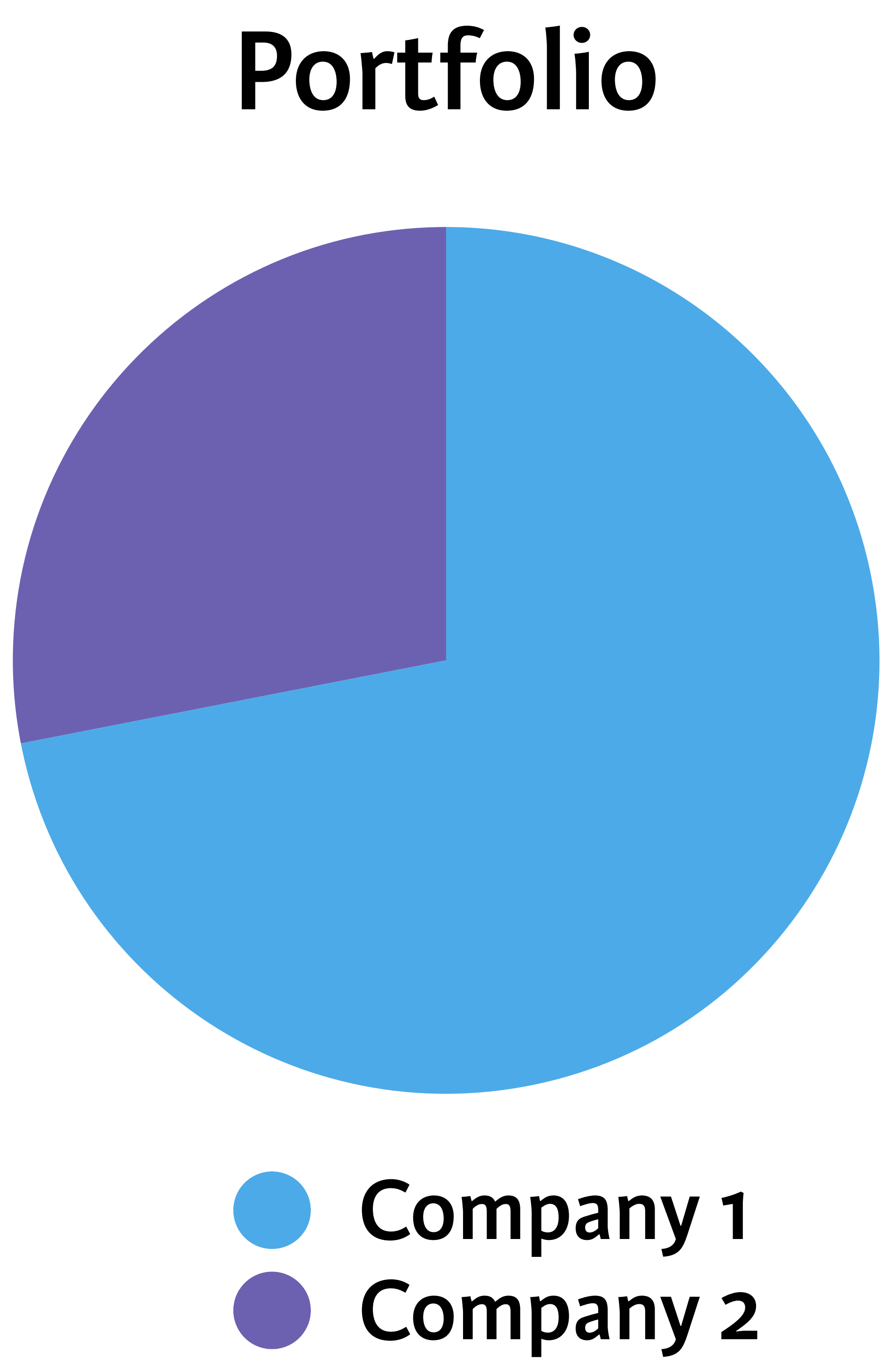
- There are two similar companies:
  - Do you invest in either of them based on a coin toss?



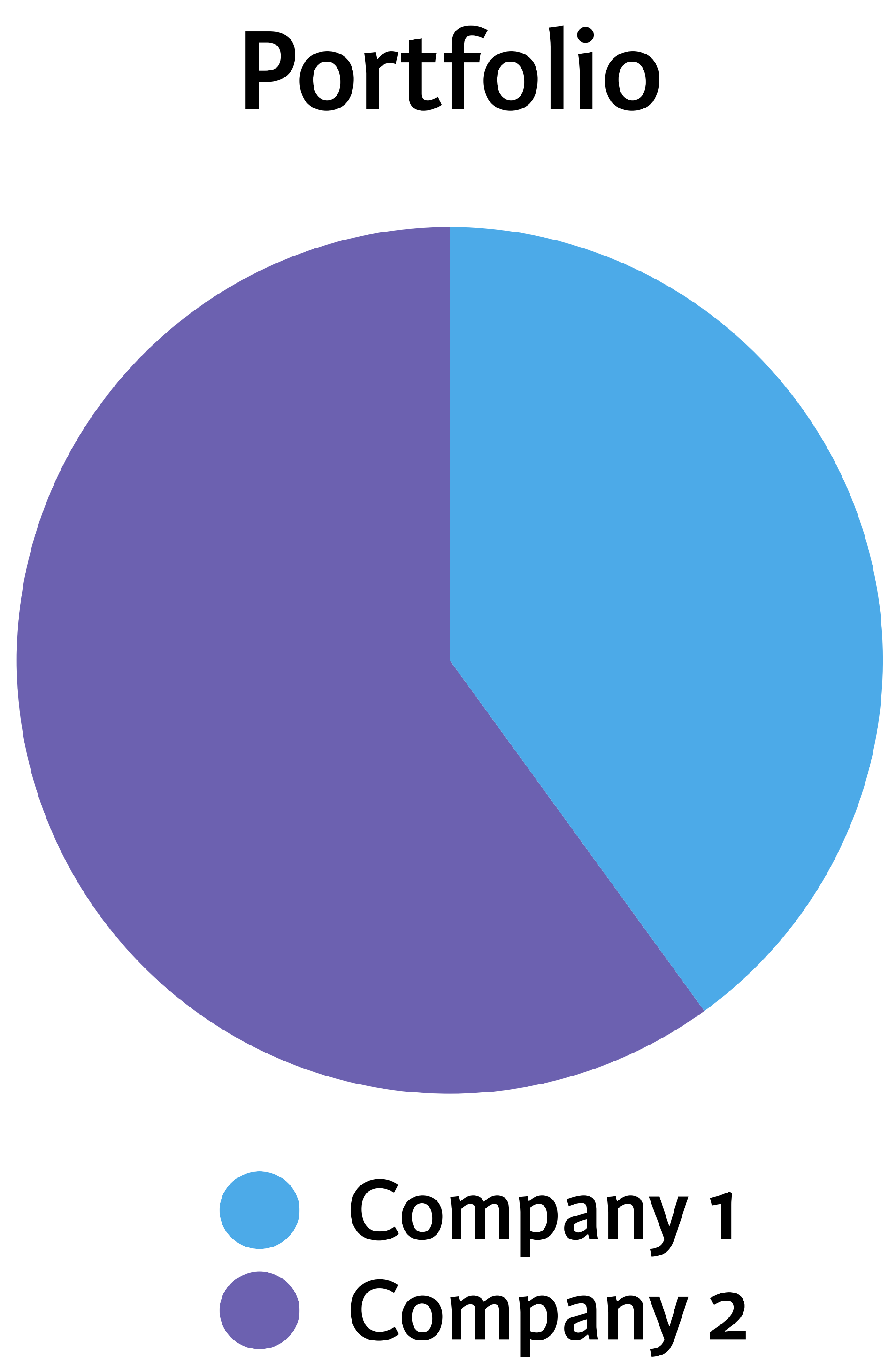
Source: ICMA Photos, Flickr



Investment Decision Choices



or



or ... ?

compute portfolio weights

Asset Weighting

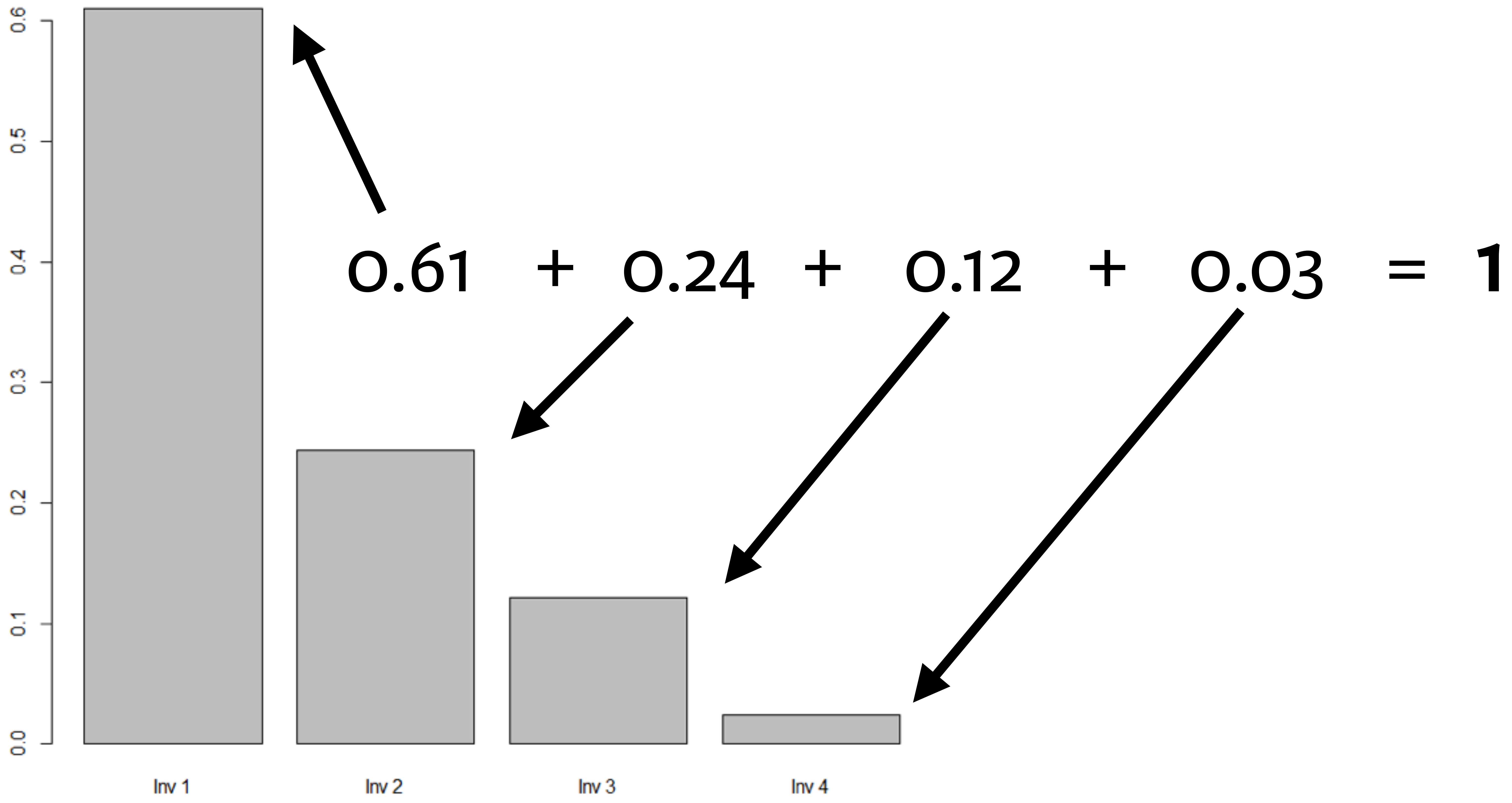
Investment	Value Invested	Weight
1	V_1	$w_1 = \frac{V_1}{V_1 + \ldots + V_N}$
2	V_2	$w_2 = \frac{V_2}{V_1 + \ldots + V_N}$
$\vdots$	$\vdots$	$\vdots$
N	V_N	$w_N = \frac{V_N}{V_1 + \ldots + V_N}$

Calculating Weight

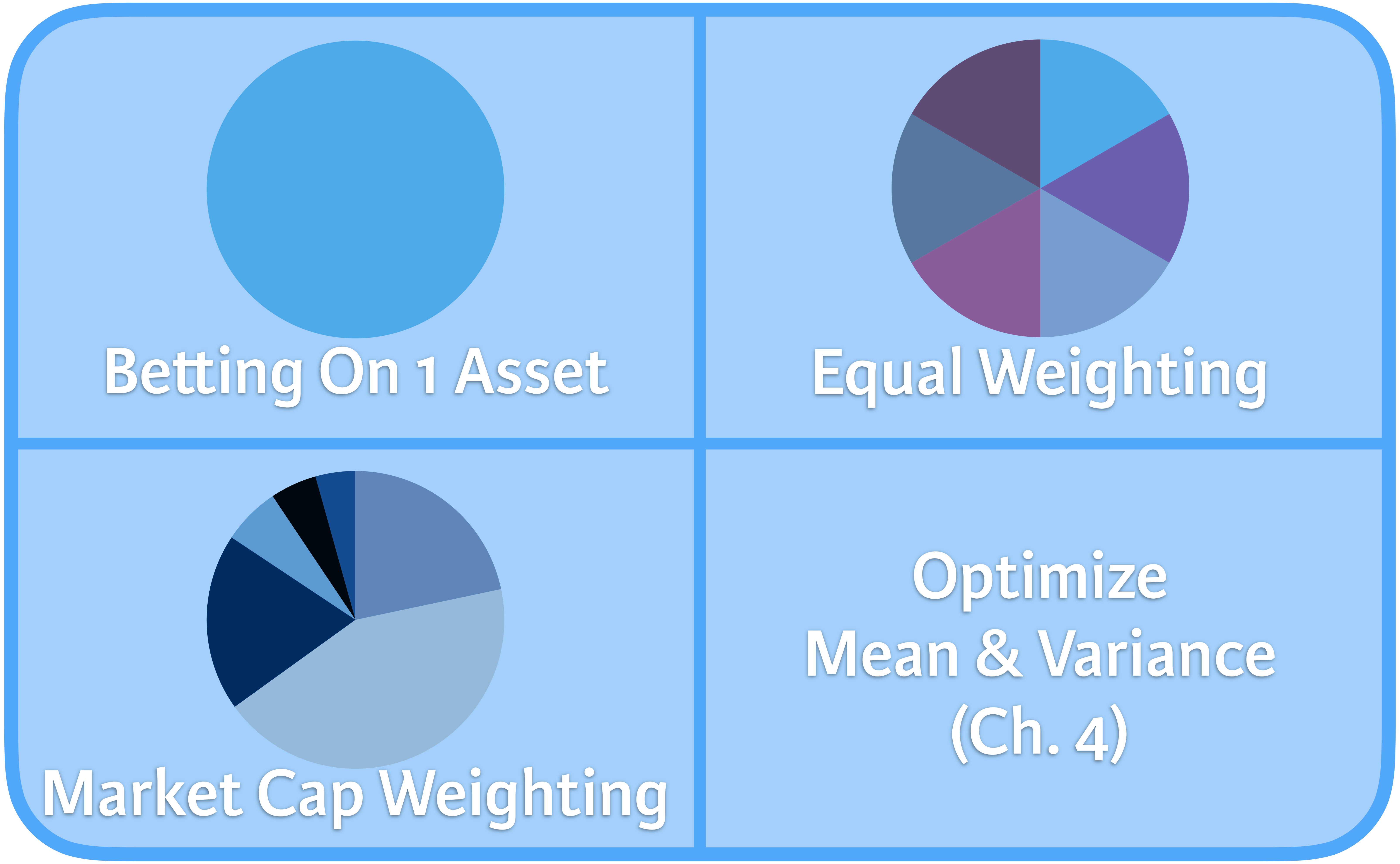
```
values <- c(500000, 200000, 100000, 20000)
names(values) <- c(“Inv 1”, “Inv 2”, “Inv 3”, “Inv 4”)
weights <- values/sum(values)

barplot(weights)
```

Calculating Weight



Allocation Strategies



Source: http://www.falibo.com/vocabulary/idiom-dont-put-all-your-eggs-in-one/

Let's practice!

The Portfolio Return

Portfolio Returns: Relative Value

- Weights reveal active investment bets
- Returns are the relative changes in value:

$$\frac{final\ value\ -initial\ value}{initial\ value}$$

Initial Value	100
Final Value	120

$$\frac{120-100}{100}=20\%$$

Three Steps

Asset $_1$	...	Asset$_N$
InValue.Asset$_1$	...	InValue.Asset$_N$
FinValue.Asset$_1$	...	FinValue.Asset$_N$

InValue.Portfolio = InValue.Asset$_1$ + ... + InValue.Asset$_N$
FinValue.Portfolio = FinValue.Asset$_1$ + ... + FinValue.Asset$_N$

$$Portfolio\ Return = \frac{FinValue.Portfolio - InValue.Portfolio}{InValue.Portfolio}$$

Example: Two Assets

$Asset_1$	$Asset_2$
$InValue.Asset_1 = \$200$	$InValue.Asset_2 = \$300$
$FinValue.Asset_1 = \$180$	$FinValue.Asset_2 = \$330$

$InValue.Portfolio = \$200 + \$300 = \$500$
$FinValue.Portfolio = \$180 + \$330 = \$510$

$$Portfolio\ Return = \frac{FinValue.Portfolio - InValue.Portfolio}{InValue.Portfolio} = \frac{510-500}{500} = 2\%$$

Portfolio Returns: Weighted Average Return

$$Portfolio\ Return = w_1 R_1 + w_2 R_2 + \ldots + w_n R_n$$

Where:

$$w_i = \frac{InValue.Asset_i}{\sum_{j=1}^{N} InValue.Asset_j}$$

$$R_i = \frac{FinValue.Asset_i - InValue.Asset_i}{InValue.Asset_i}$$

Three Steps

Asset $_1$	...	Asset$_N$
InValue.Asset$_1$	...	InValue.Asset$_N$
FinValue.Asset$_1$	...	FinValue.Asset$_N$

Asset $_1$	Asset$_N$
$w_1 = \frac{InValue.Asset_1}{InValue.Portfolio}$	$w_n = \frac{InValue.Asset_n}{InValue.Portfolio}$
$R_1 = \frac{FinValue.\ Asset_1 - InValue.\ Asset_1}{InValue.\ Asset_1}$	$R_n = \frac{FinValue.\ Asset_n - InValue.\ Asset_n}{InValue.\ Asset_n}$

$$Portfolio\ Return = w_1 R_1 + w_2 R_2 + \ldots + w_n R_n$$

Example: Two Assets

Asset $_1$	Asset $_2$
InValue.Asset $_1$ = \$200	InValue.Asset $_2$ = \$300
FinValue.Asset $_1$ = \$180	FinValue.Asset $_2$ = \$300

Asset $_1$	Asset $_2$
$w_1 = \frac{200}{500} = 40\%$	$w_2 = \frac{300}{500} = 60\%$
$R_1 = \frac{180-200}{200} = -10\%$	$R_2 = \frac{330-300}{300} = 10\%$

$$Portfolio\ Return = 0.4^*(-10\%) + 0.6^*(10\%) = 2\%$$

Let's practice!

PerformanceAnalytics

The Practitioner’s Challenge

- In practice, time series of portfolio returns
- Longer history ⟶ more info on portfolio
- Good package = **PerformanceAnalytics**

The Creators

- PerformanceAnalytics is the go-to package for portfolio return analysis in R

Peter Carl

Brian Peterson

Calculating Returns

Calculating Returns

- **Return.calculate**: to compute the asset returns
- **Return.portfolio**: to compute the portfolio return
- Return.calculate(prices)

  xts-object

- Dates structure: **YYYY-MM-DD**

Calculating Returns

Return.calculate

In: Prices **Out: Returns**

```
> returns <- Return.calculate(prices)
> returns <- returns[(-1),]
```

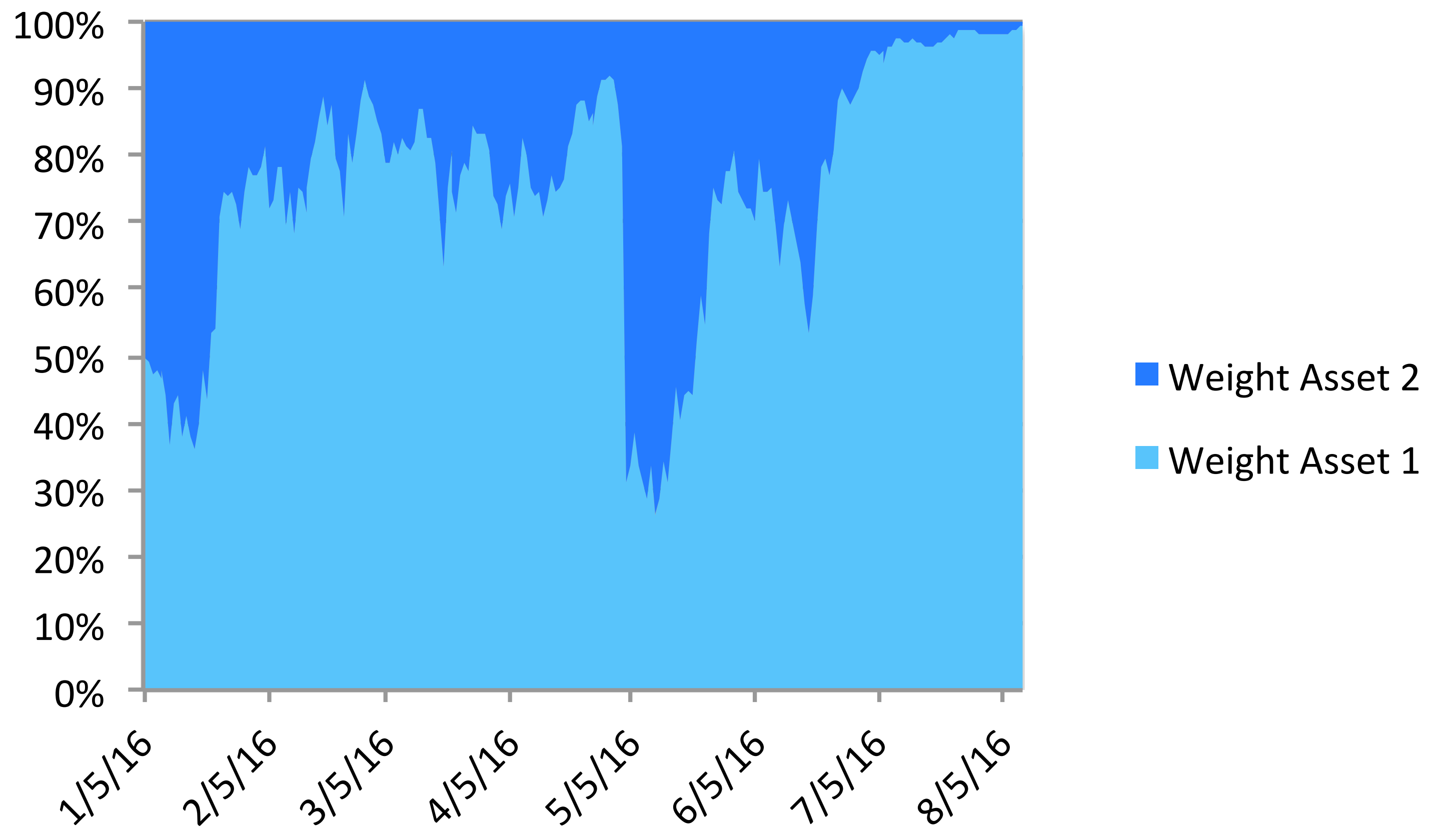
```
> head(prices)
>                AAPL     MSFT
> 2006-01-03 9.829465  21.07395
> 2006-01-04 9.858394  21.17603
> 2006-01-05 9.780810  21.19173
> 2006-01-06 10.033286 21.12891
> 2006-01-09 10.000411 21.08966
> 2006-01-10 10.632916 21.19958
```

```
> head(returns)
>                    AAPL          MSFT
> 2006-01-03         NA            NA
> 2006-01-04  0.002943090  0.0048434670
> 2006-01-05 -0.007869842  0.0007415934
> 2006-01-06  0.025813404 -0.0029640809
> 2006-01-09 -0.003276594 -0.0018579752
> 2006-01-10  0.063247901  0.0052121756
```

Dynamics of Portfolio Weights

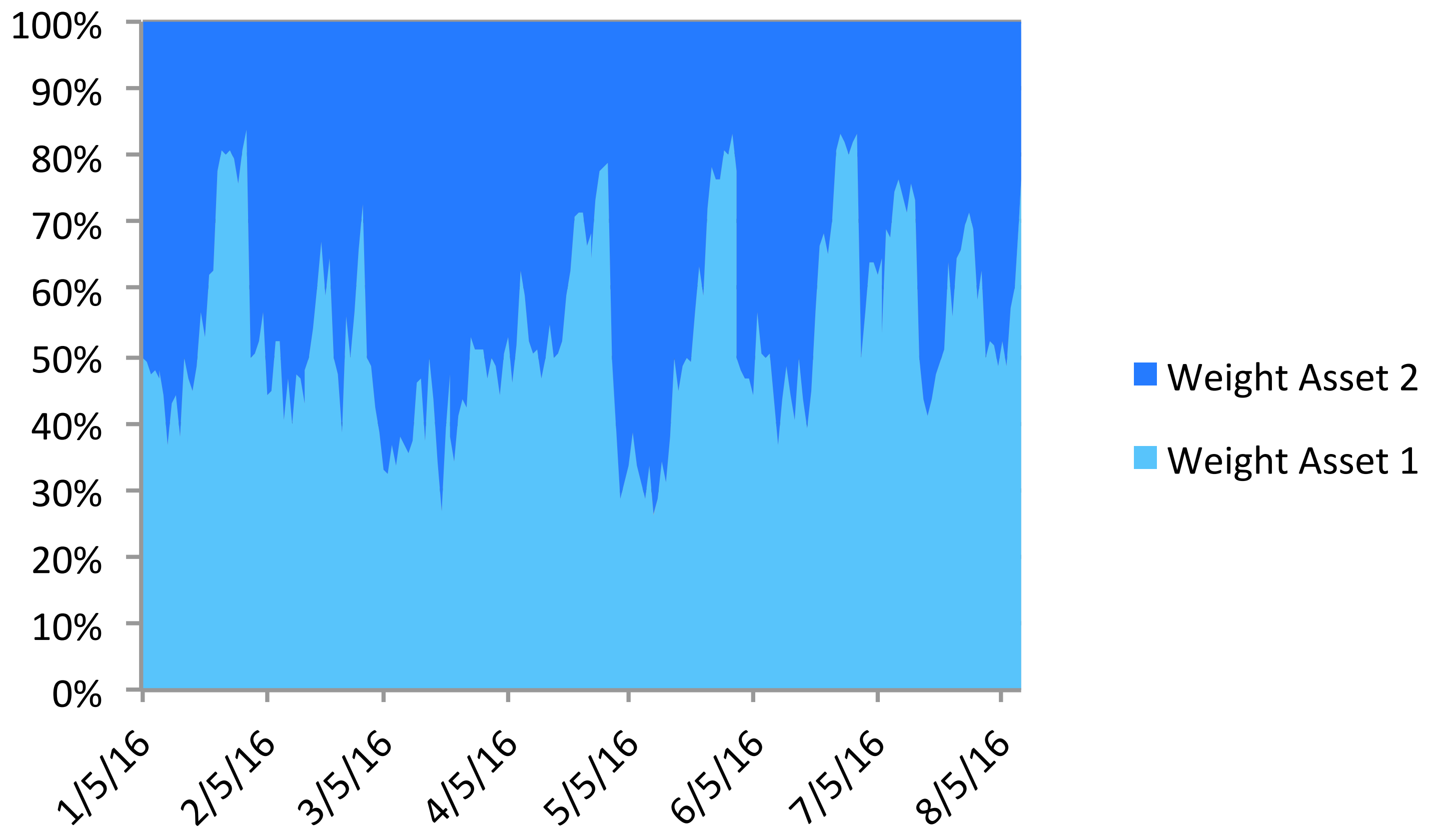
Set Initial Weights & Do Not Intervene

Example: Initial 50/50 weight



Actively Change Portfolio Weights

Example: 50/50 Weight With Rebalance



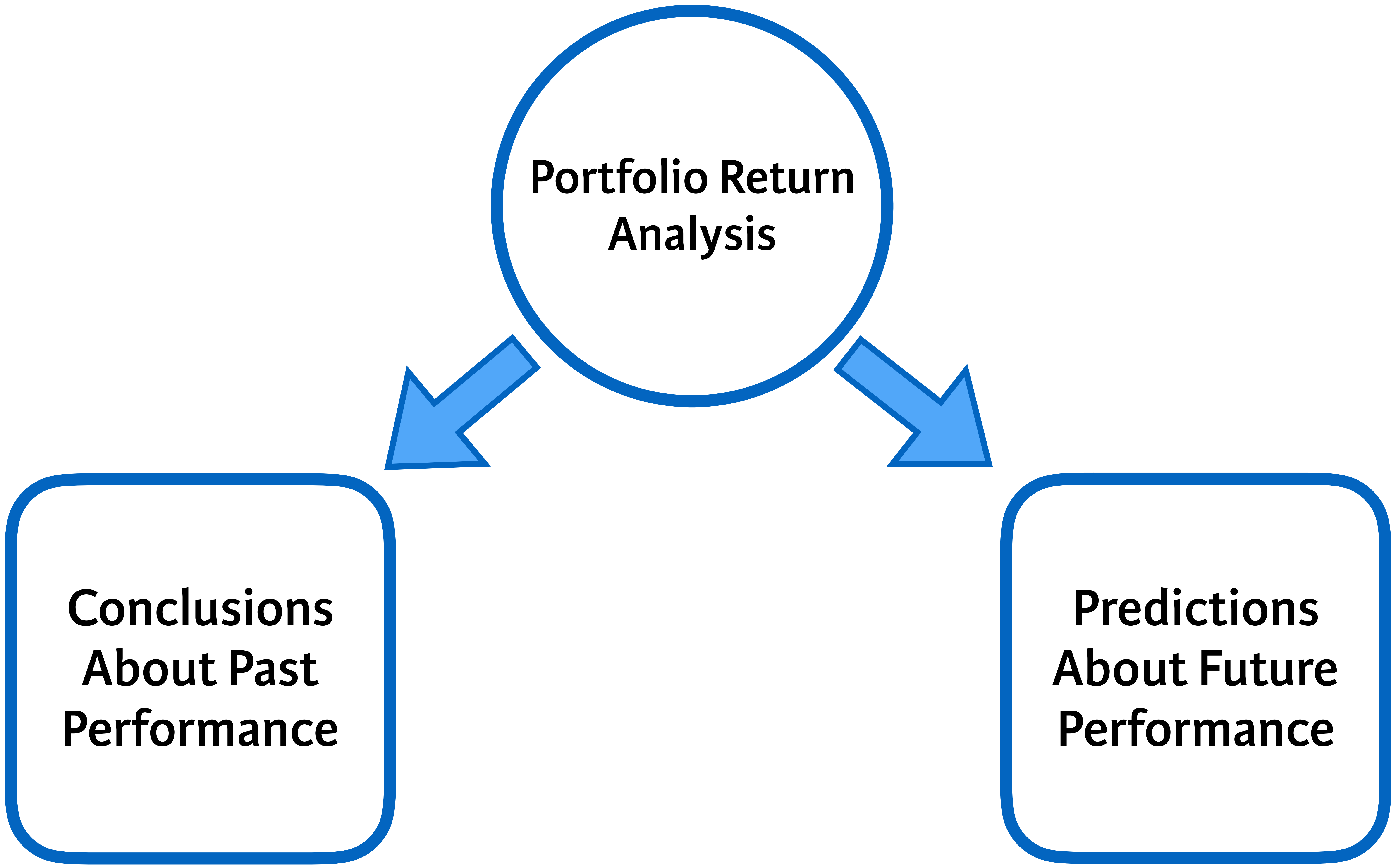
Portfolio Returns

```
> Return.portfolio <- function(R, weights = NULL,
      rebalance_on = c(NA, “years”, “quarters”, “months”, “weeks”, “days”))
```

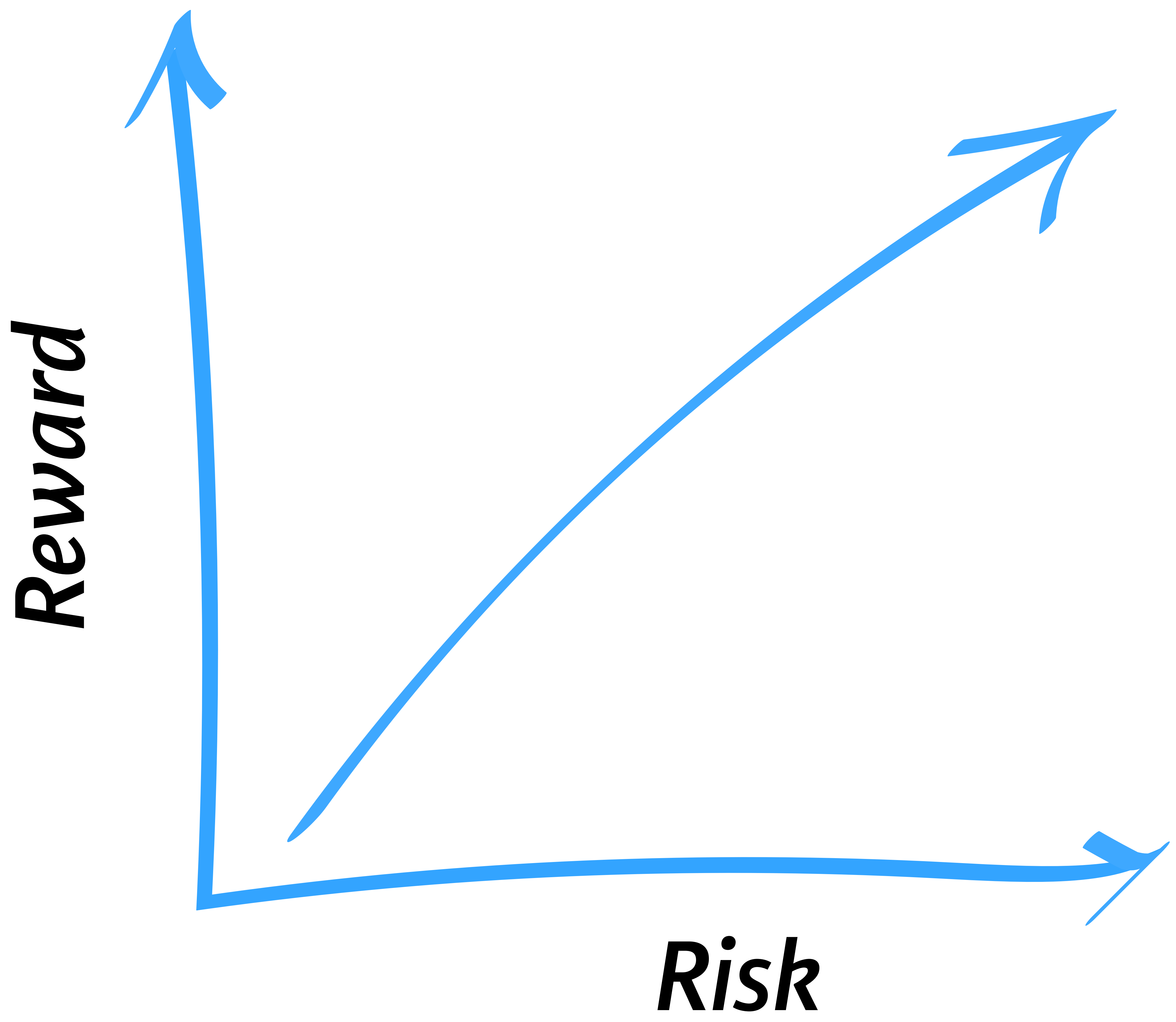
Let’s practice!

Dimensions of Portfolio Performance

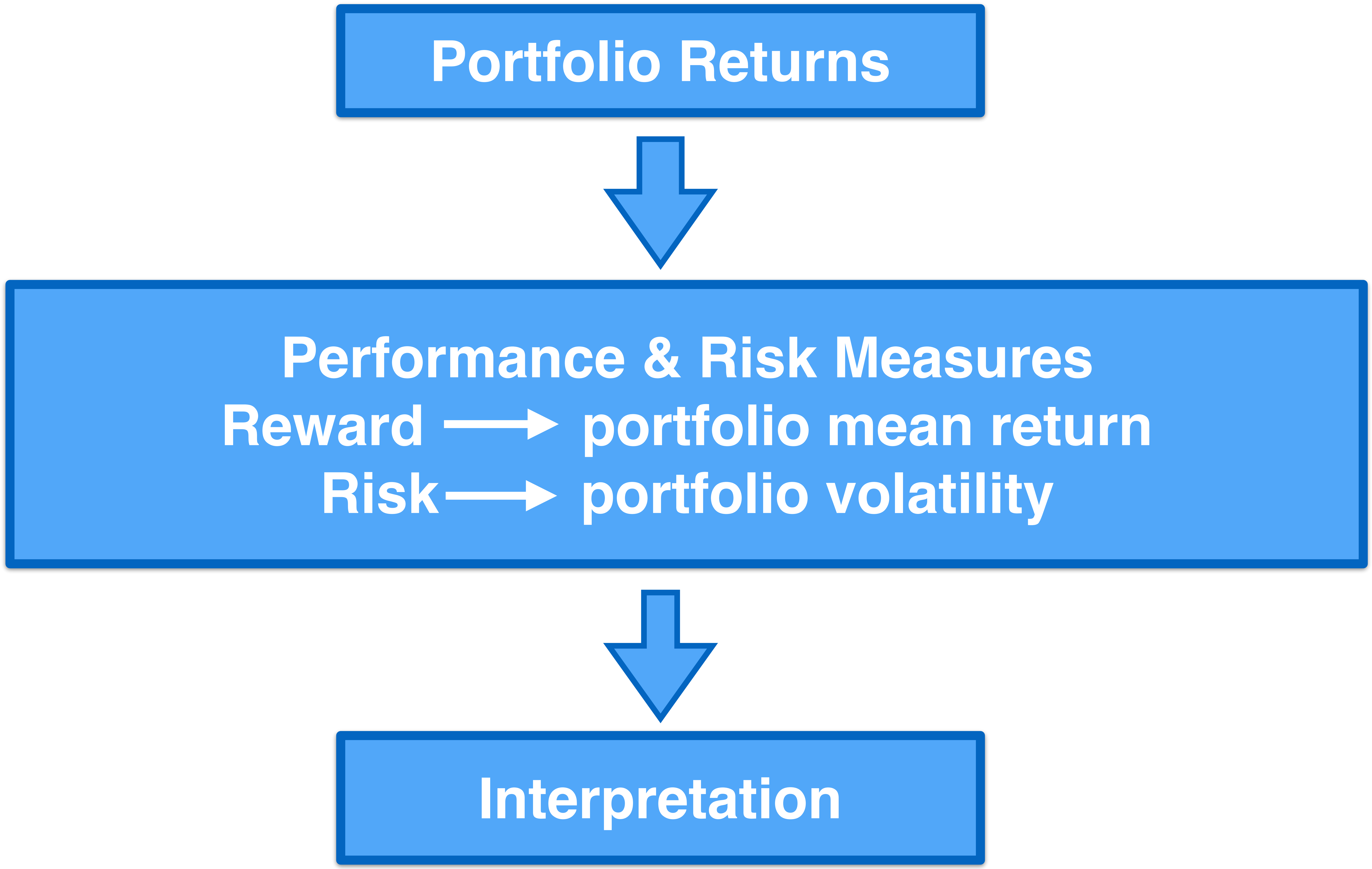
Interpretation of Portfolio Returns



Risk vs. Reward



Need For Performance Measure



Arithmetic Mean Return

- Assume a sample of *T* portfolio return observations:

$$R_1, R_2, \ldots, R_T$$

- Reward Measurement: Arithmetic mean return is given:

$$\hat{\mu} = \frac{R_1 + R_2 + \ldots + R_T}{T}$$

- It shows how large the portfolio return is on average

Risk: Portfolio Volatility

- De-meaned return

$$R_i - \hat{\mu}$$

- Variance of the portfolio

$$\hat{\sigma}^2 = \frac{(R_1 - \hat{\mu})^2 + (R_2 - \hat{\mu})^2 + \cdots + (R_T - \hat{\mu})^2}{T - 1}$$

- Portfolio Volatility:

$$\hat{\sigma} = \sqrt{\hat{\sigma}^2}$$

No Linear Compensation In Return

- Mismatch between average return and effective return

final value=
*initial value * (1 +0.5)*(1-0.5)= 0.75 * initial value*

↓

Average Return = (0.5 - 0.5) / 2 = 0

Geometric Mean Return

- Formula for *Geometric Mean* for a sample of **T** portfolio return observations $R_1, R_2, ..., R_T$:

$$\text{Geometric mean} = [(1+R_1)*(1+R_2)*...(1+R_T)]^{\frac{1}{T}} - 1$$

- Example: +50% & -50% return

$$\begin{aligned} \text{Geometric mean} &= [(1+0.50)*(1-0.50)]^{\frac{1}{2}} - 1 \\ &= 0.75^{\frac{1}{2}} - 1 \\ &= -13.4\% \end{aligned}$$

Application to the S&P 500

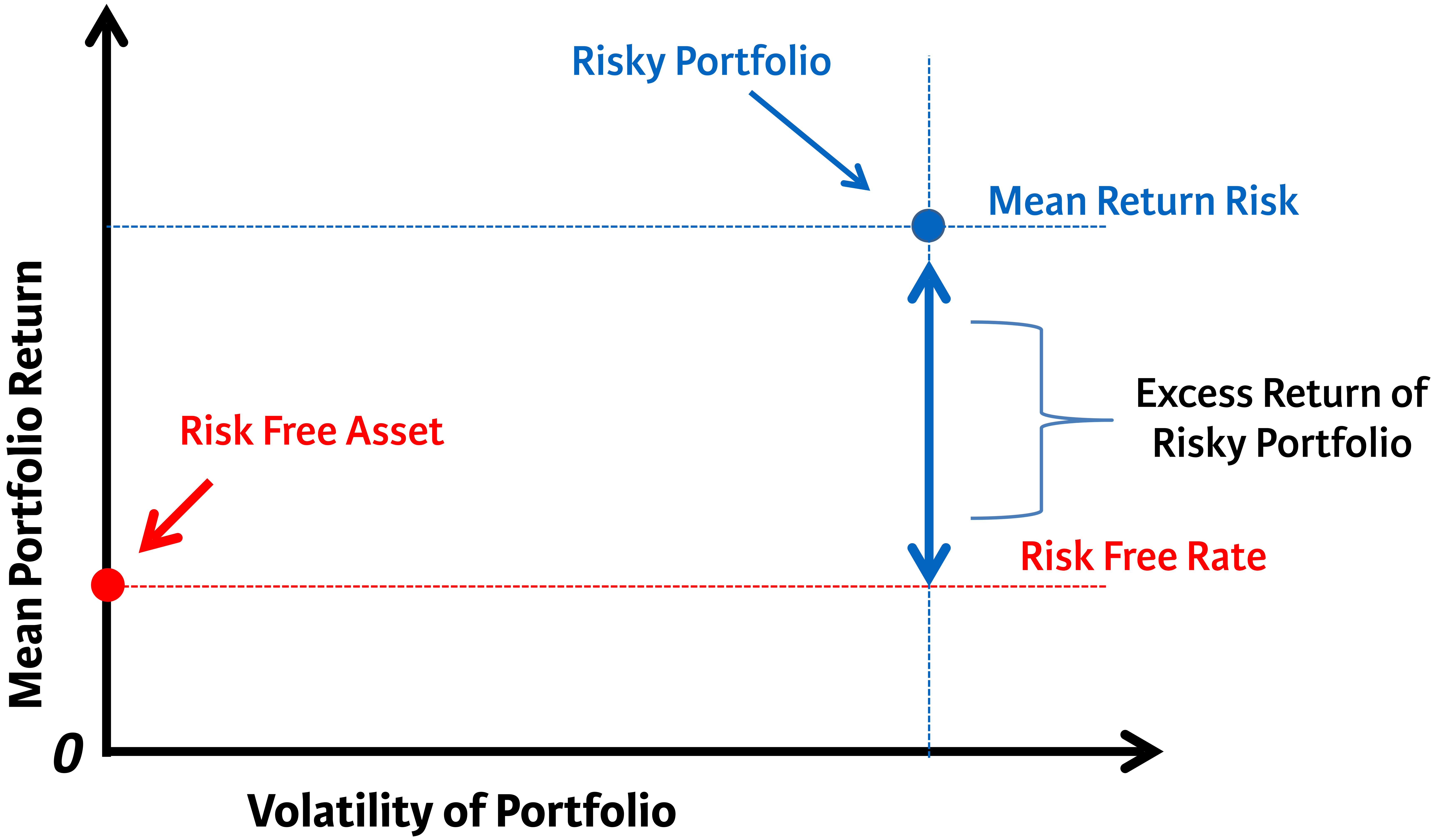
Let's practice!

The (Annualized) Sharpe Ratio

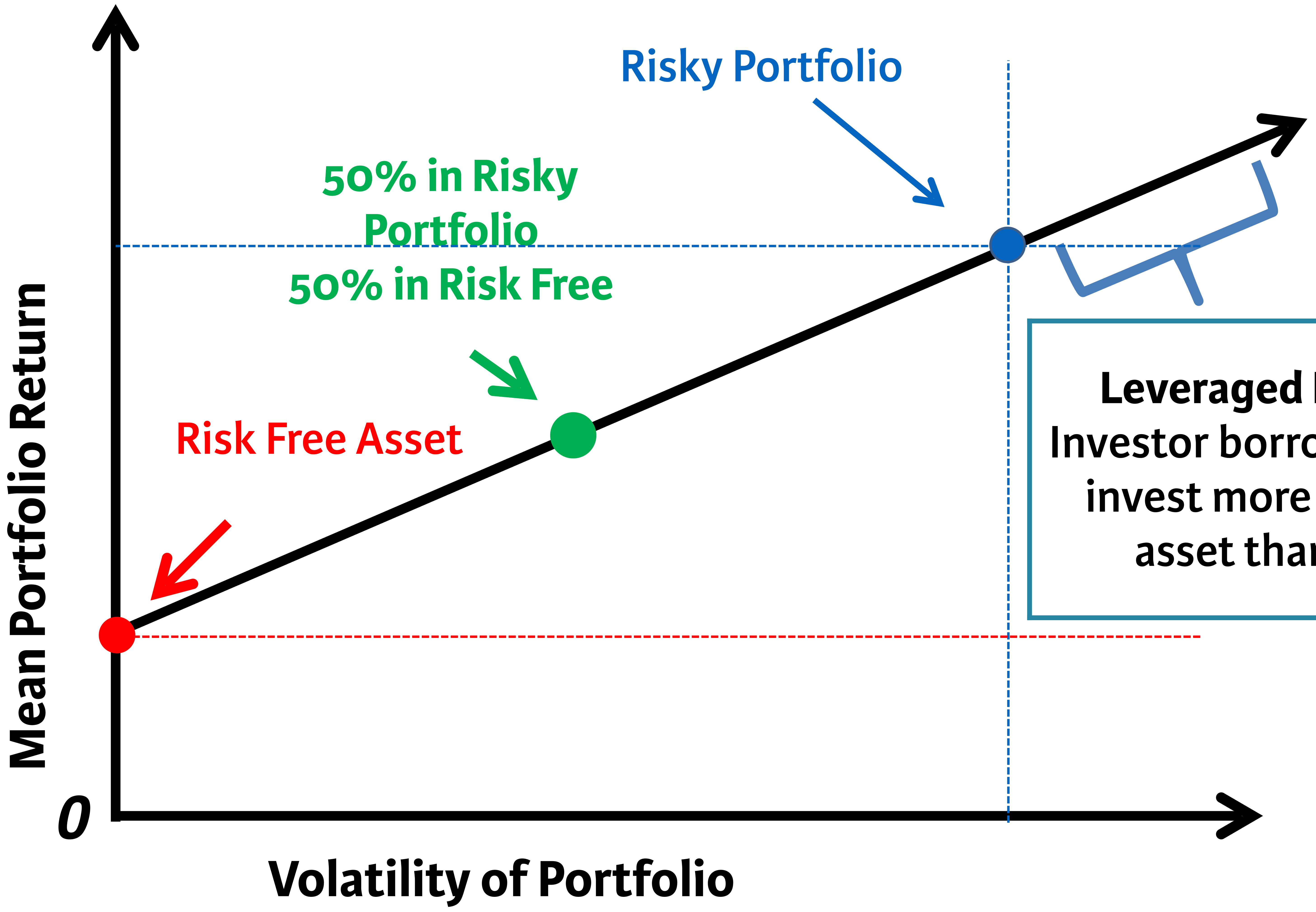
Benchmarking Performance

Risky Portfolio **E.g:** portfolio invested in stocks, bonds, real estate, and commodities	Risk Free Asset **E.g:** US Treasury Bill
Reward: measured by mean portfolio return	**Reward**: measured by risk free rate
Risk: measured by volatility of the portfolio returns	**Risk**: No risk, volatility = 0 return = risk free rate

Risk-Return Trade-Off

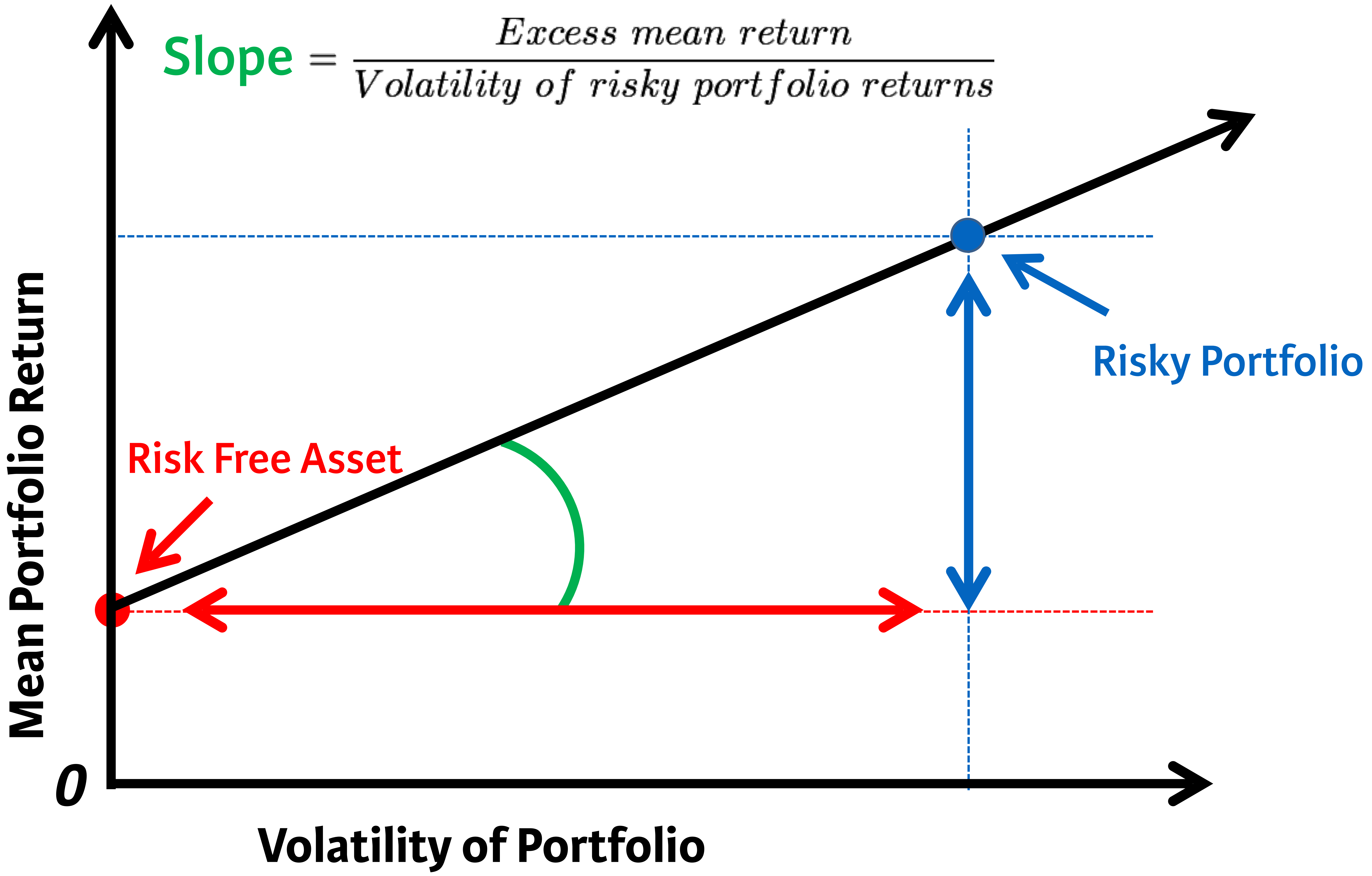


Capital Allocation Line



Leveraged Portfolios: Investor borrows capital to invest more in the risky asset than she has

The Sharpe Ratio



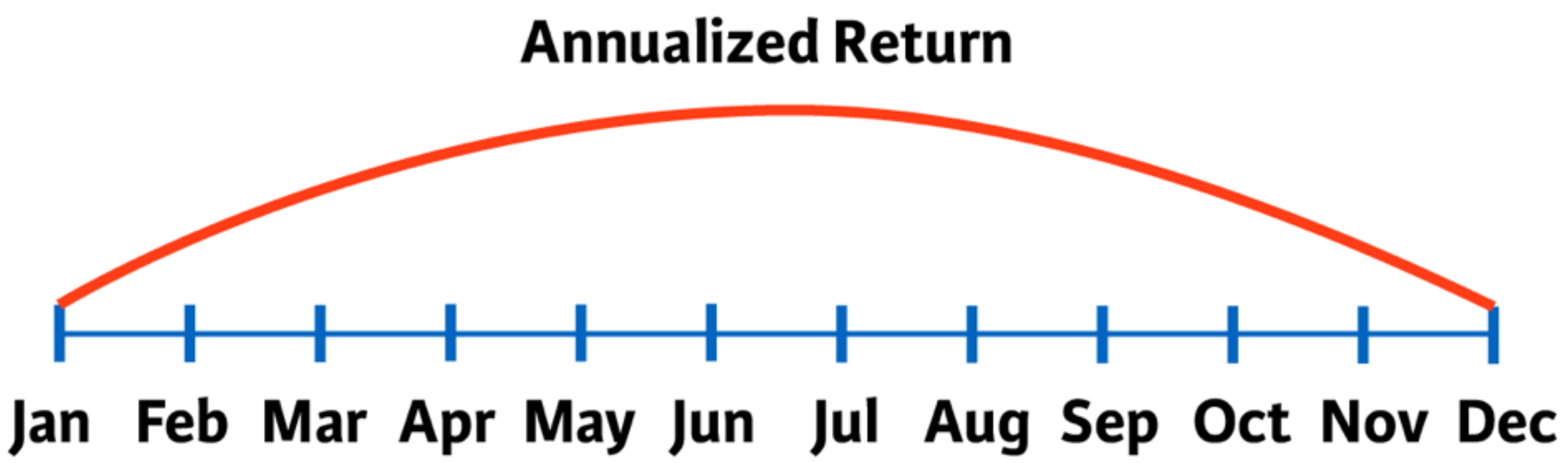
Performance Statistics In Action

```
> library(PerformanceAnalytics)
> sample_returns <- c( -0.02, 0.00, 0.00, 0.06, 0.02, 0.03, -0.01, 0.04)

> (mean(sample_returns) - 0.004)/StdDev(sample_returns)
```

returns	-0.02, 0 , 0 , 0.06, 0.02, 0.03, -0.01, 0.04
arithmetic mean	0.015
geometric mean	0.01468148
volatility	0.02725541
sharpe ratio	0.4035897

Annualize Monthly Performance



Arithmetic mean: monthly mean * 12

Geometric mean, when **R_i** are monthly returns:

$$[(1+R_1)*(1+R_2)*\ldots*(1+R_T)]^{\frac{12}{T}}-1$$

Volatility: monthly volatility * sqrt(12)

Performance Statistics In Action

```
> library(PerformanceAnalytics)
> sample_returns <- c( -0.02, 0.00, 0.00, 0.06, 0.02, 0.03, -0.01, 0.04)

> Return.annualized(sample_returns, scale = 12) /
Std.Dev.annualized(sample_returns, scale = 12)
```

	monthly	FACTOR	annualized
arithmetic mean	0.015	12	0.18
geometric mean	0.01468148	$[0.98 * \ldots * 1.04]^{\frac{12}{8}} - 1$	0.1911235
volatility	0.02725541	sqrt(12)	0.0944155
sharpe ratio	0.4035897	sqrt(12)	1.398076

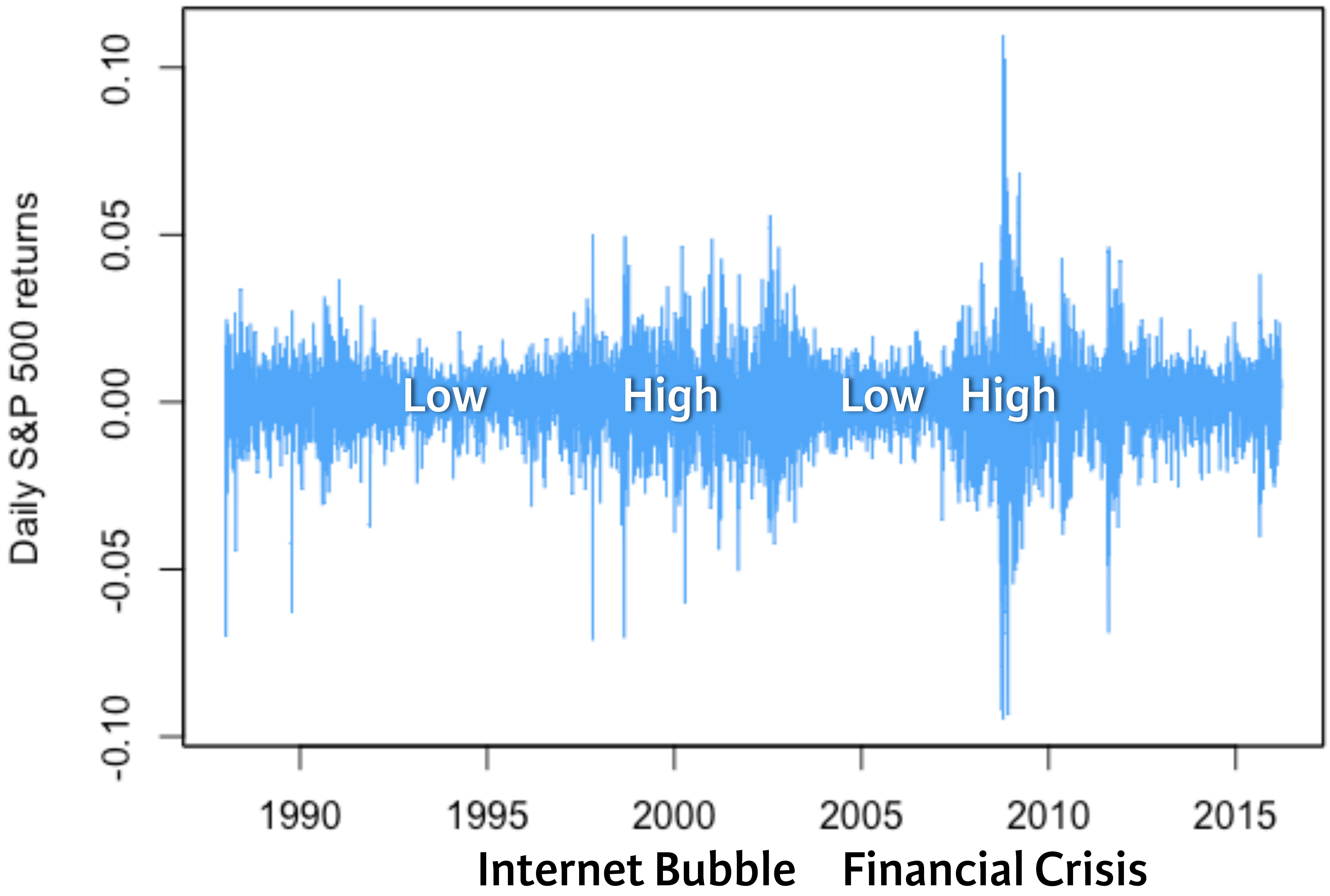
Let's practice!

Time-Variation In Portfolio Performance

Bulls & Bears

- Business cycle, news, and swings in the market psychology affect the market

Clusters of High & Low Volatility

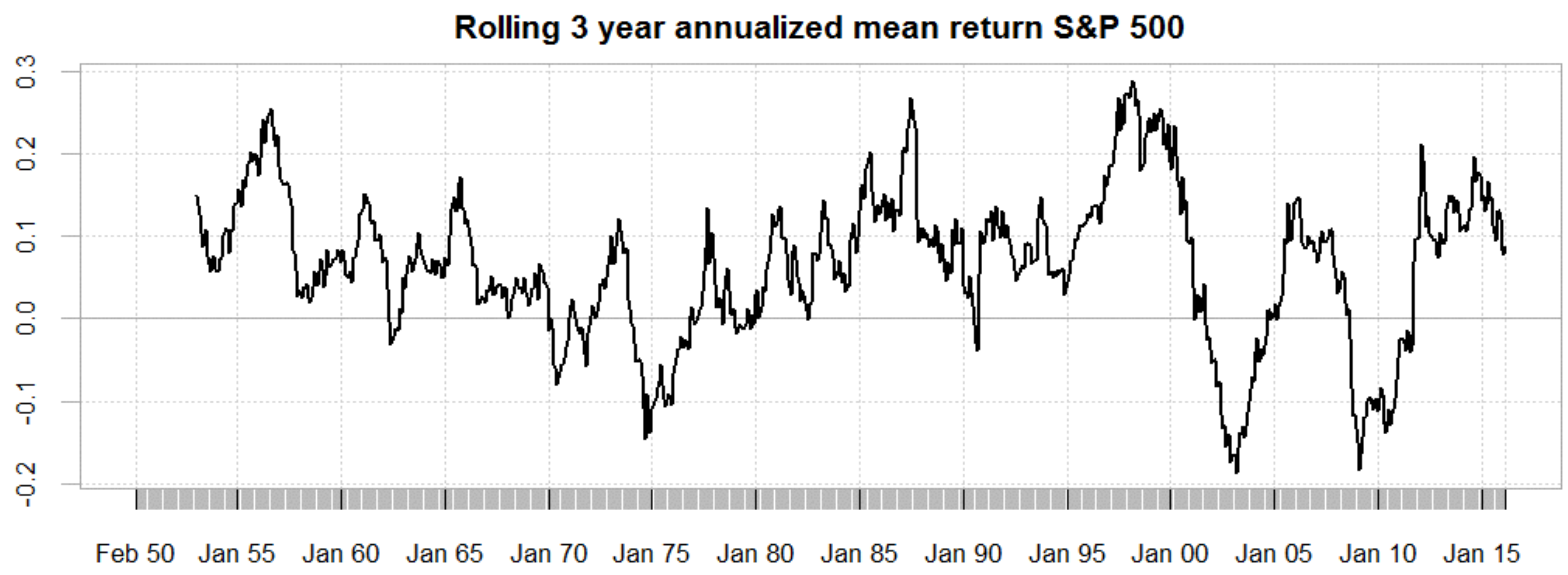


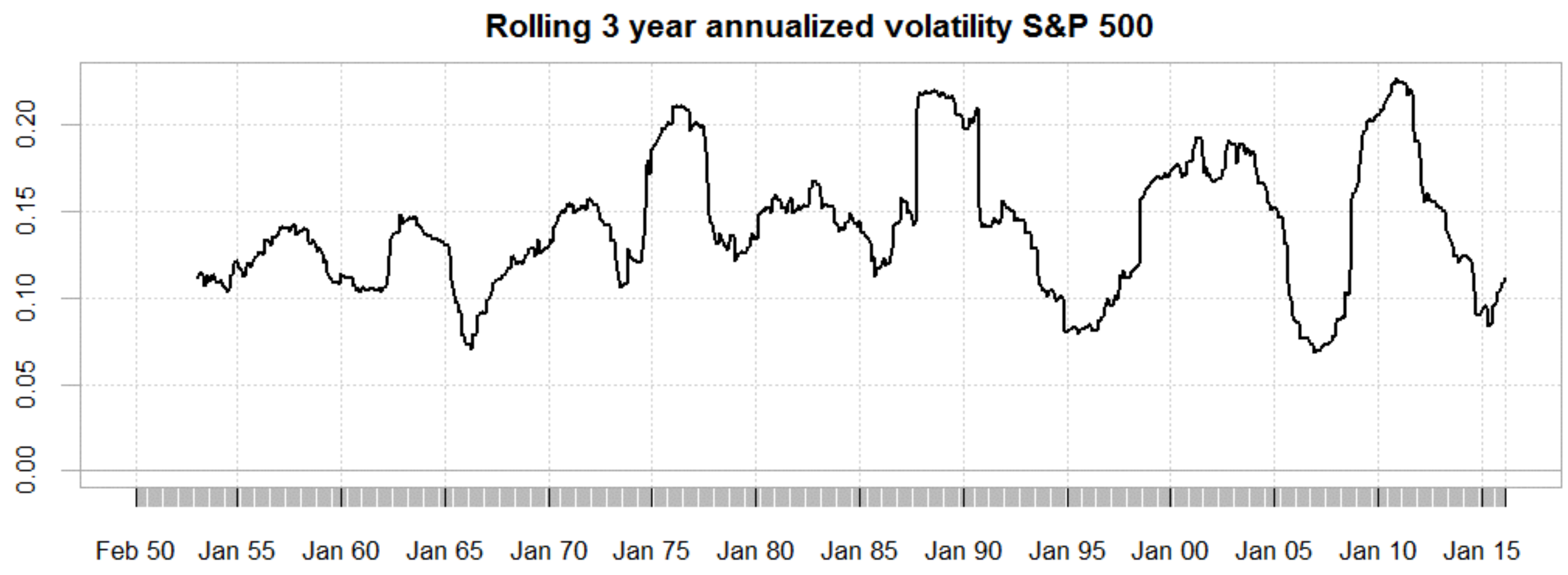
Rolling Estimation Samples

- Rolling samples of K observations:
    - Discard the most distant and include the most recent

R_{t-k+1}	R_{t-k+2}	R_{t-k+3}	…	R_t	R_{t+1}	R_{t+2}	R_{t+3}

Rolling Performance Calculation





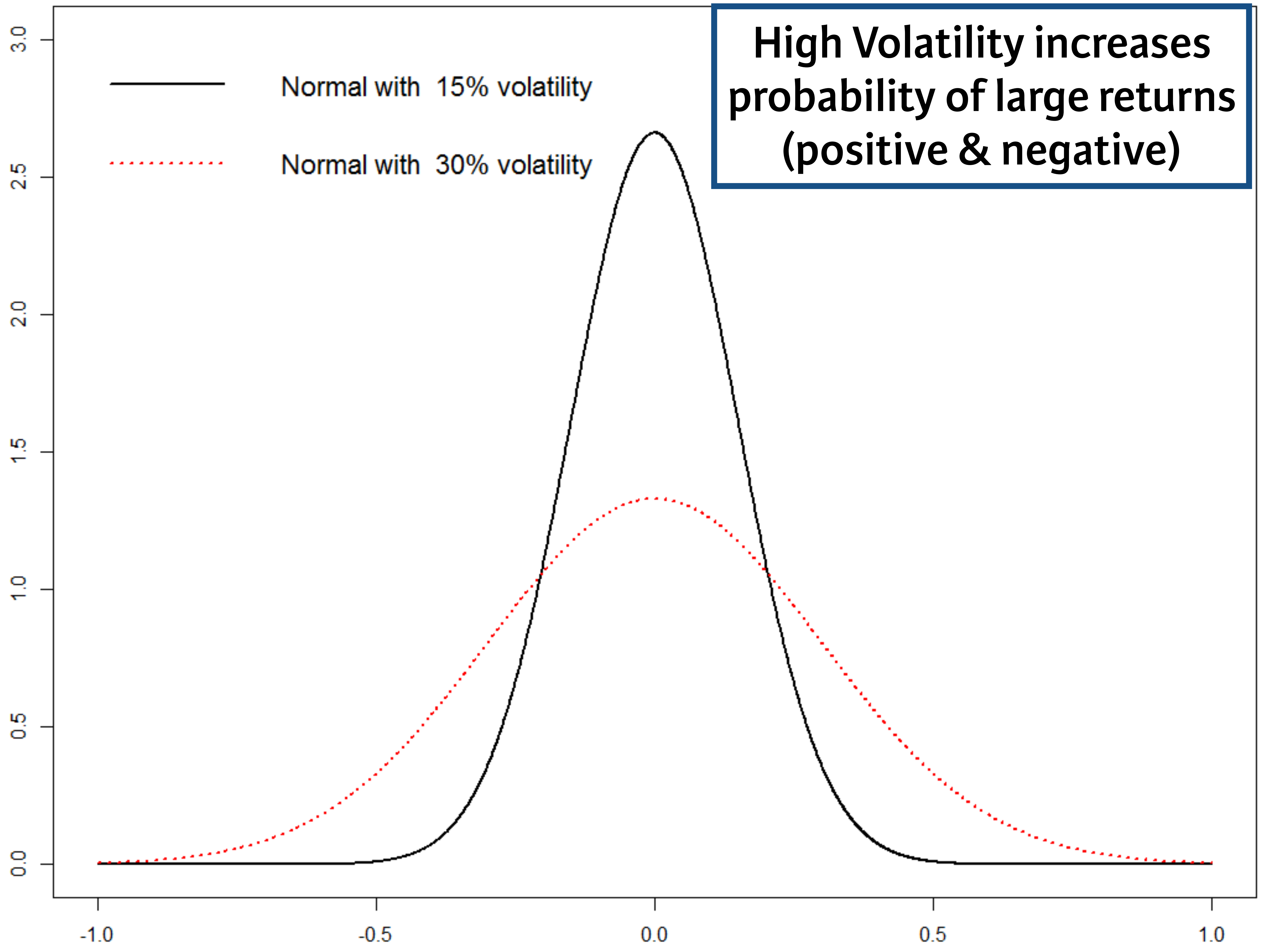
Choosing Window Length

- Need to balance noise (long samples) with recency (shorter samples)
- Longer sub-periods smooth highs and lows
- Shorter sub-periods provide more information on recent observations

Let's practice!

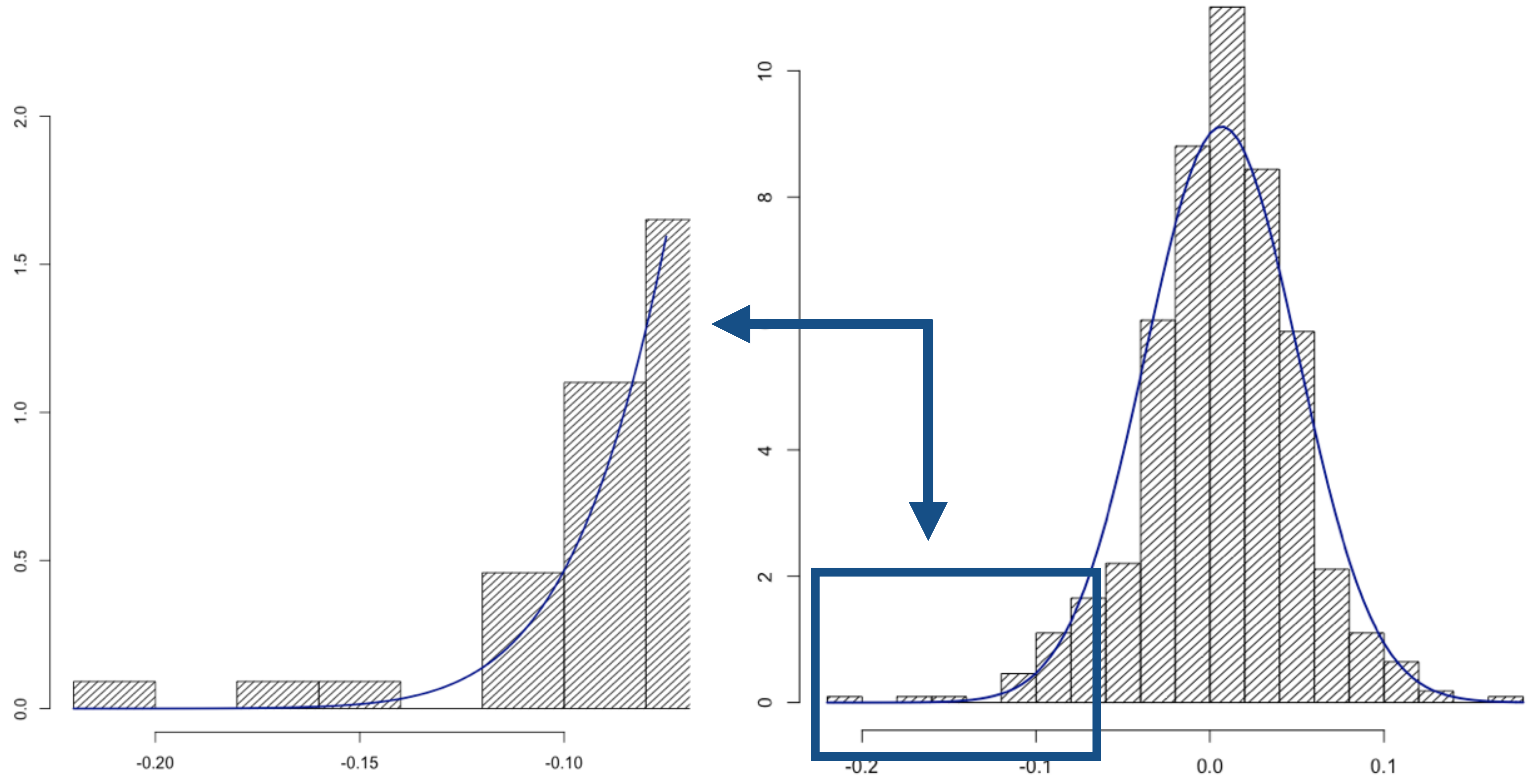
Non-Normality of the Return Distribution

Volatility Describes “Normal” Risk



High Volatility increases probability of large returns (positive & negative)

Non-Normality of Return



Portfolio Return Semi-Deviation

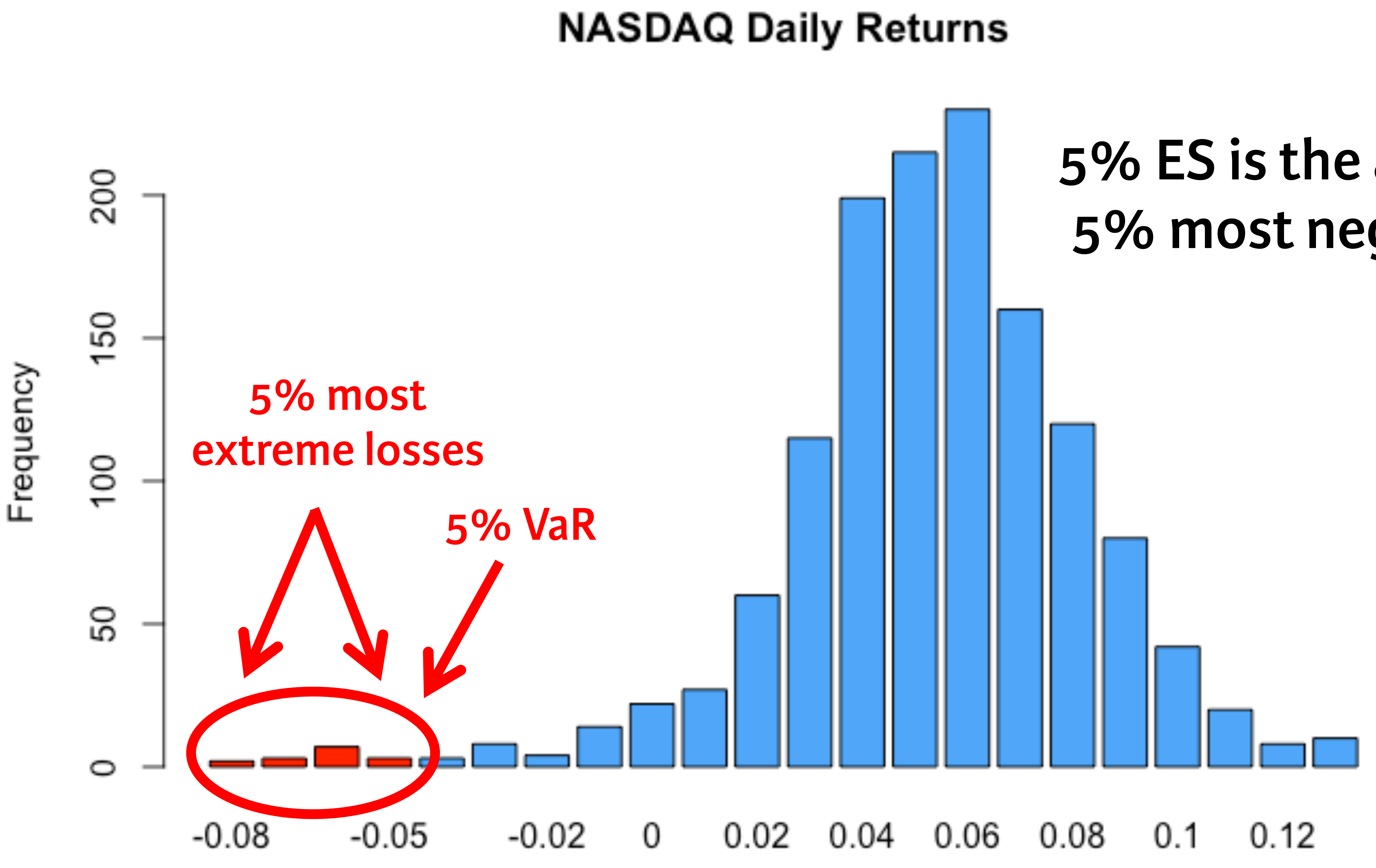
- **Standard Deviation** of Portfolio Returns:
  - Take the *full sample* of returns

$$SD = \sqrt{\frac{(R_1 - \mu)^2 + (R_2 - \mu)^2 + ... + (R_T - \mu)^2}{T - 1}}$$

- **Semi-Deviation** of Portfolio Returns:
  - Take the *subset* of returns below the mean

$$SemiDev = \sqrt{\frac{(Z_1 - \mu)^2 + (Z_2 - \mu)^2 + ... + (Z_n - \mu)^2}{n}}$$

Value-at-Risk & Expected Shortfall



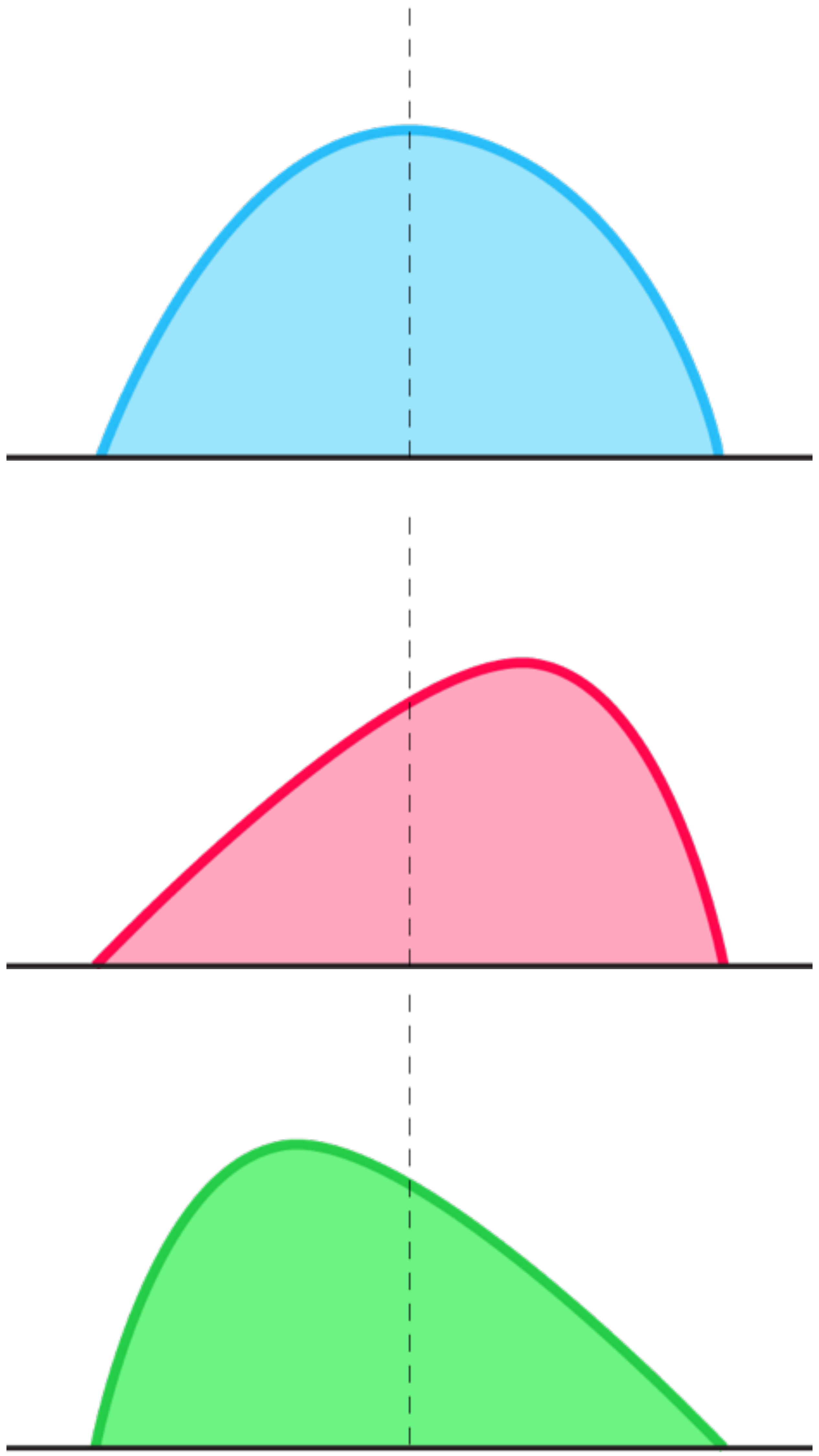
5% ES is the average of the 5% most negative returns

Shape of the Distribution

- Is it symmetric?
    - Check the skewness
- Are the tails fatter than those of the normal distribution?
    - Check the excess kurtosis

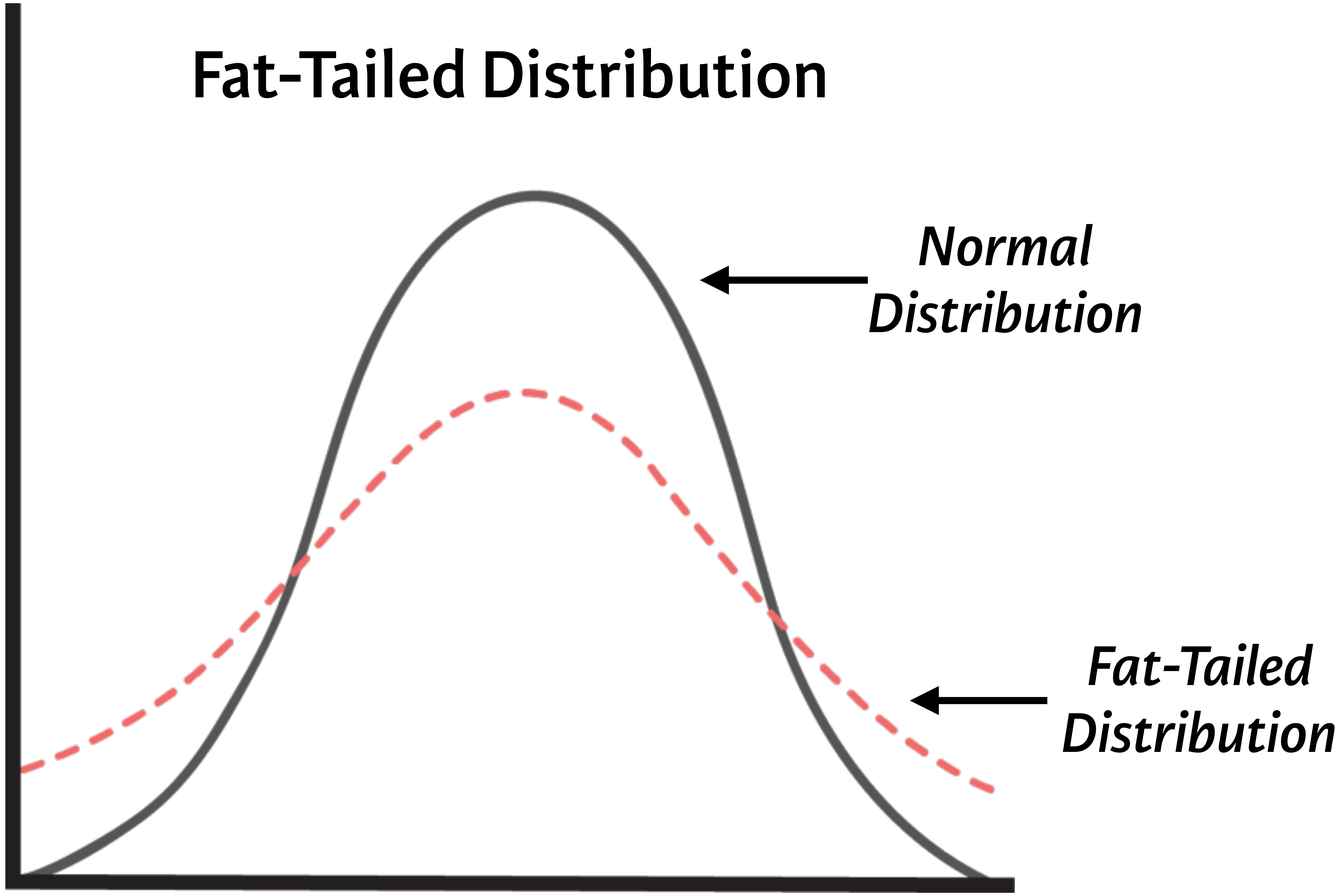
Skewness

- Zero Skewness
  - Distribution is symmetric
- Negative Skewness
  - Large negative returns occur more often than large positive returns
- Positive Skewness
  - Large positive returns occur more often than large negative returns



Kurtosis

- The distribution is fat-tailed when the excess kurtosis > 0



Let's practice!

Drivers in the Case of Two Assets

Future Returns Are Random In Nature

Optimizing Portfolio requires expectations:

- about average portfolio return (mean)
- about how far off it may be (variance)

Why?

Portfolio Return Is A Random Variable

Past Performance to Predictions

	Mean Portfolio Return
Computed on a sample of T Historical Returns	$\hat{\mu} = \frac{R_1 + R_2 + \ldots + R_T}{T}$
When the return is a random variable	$\mu = E[R]$

	Portfolio Return Variance
Computed on a sample of T Historical Returns	$\hat{\sigma}^2 = \frac{(R_1 - \hat{\mu})^2 + (R_2 - \hat{\mu})^2 + \ldots + (R_T - \hat{\mu})^2}{T - 1}$
When the return is a random variable	$\sigma^2 = E[(R - \mu)^2]$

Drivers of Mean & Variance

- Assume two assets:

Asset 1	Asset 2
Weight: w_1	Weight: w_2
Return: R_1	Return: R_2

- **Portfolio Return** $P = w_1 * R_1 + w_2 * R_2$
- Thus: $\boxed{E[P] = w_1 * E[R_1] + w_2 * E[R_2]}$

Portfolio Return Variance

Again, for a portfolio with 2 assets

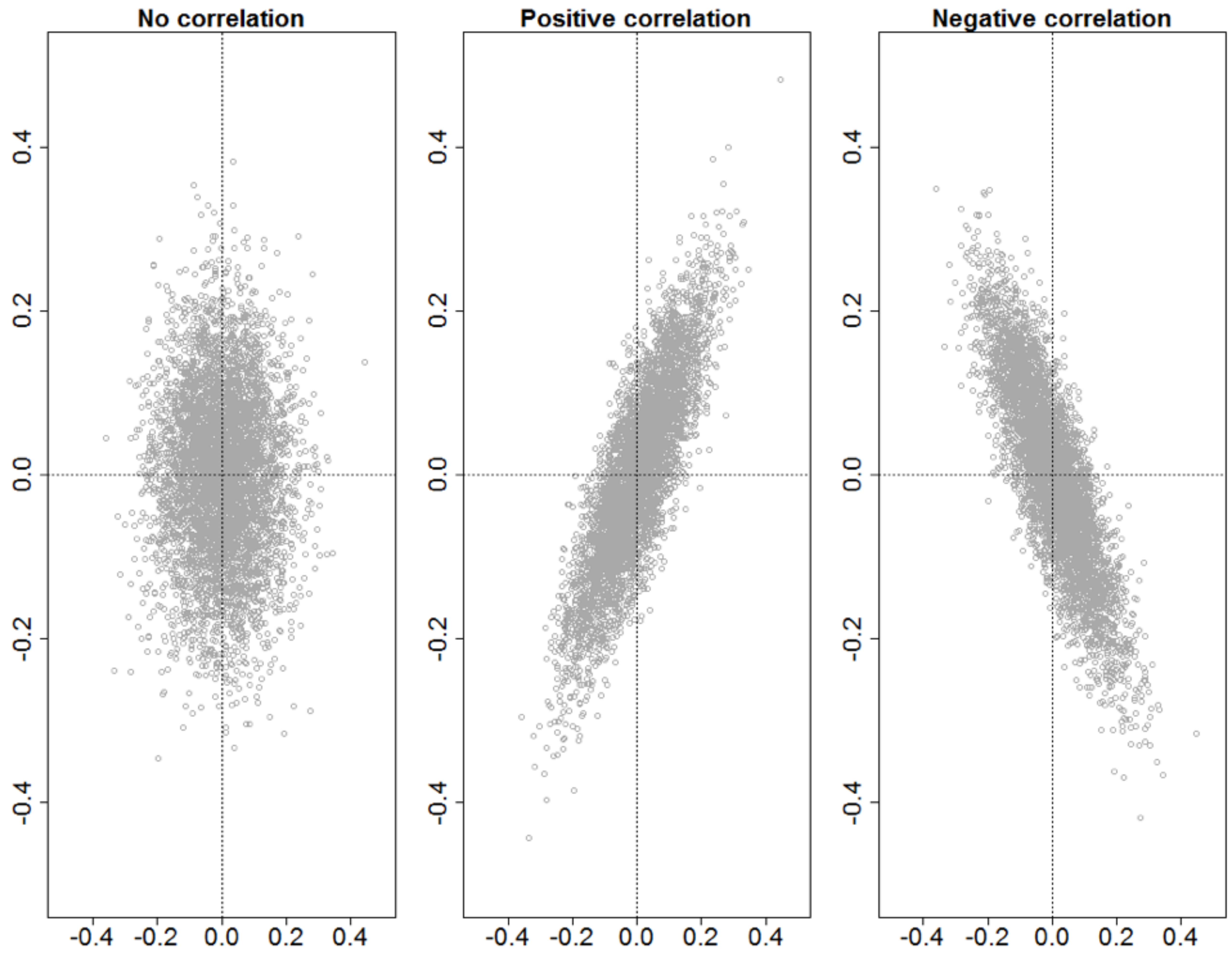
Variance of Portfolio Return

$$
\begin{aligned}
var(P) = \boxed{E[(P - E[P])^2]} = & \; w_1^2 * var(R_1) \\
& + w_2 * var(R_2) \\
& + 2 * w_1 * w_2 * \boxed{cov(R_1, R_2)}
\end{aligned}
$$

Covariance between return 1 and 2

$$
\begin{aligned}
Cov(R_1, R_2) &= \boxed{E[(R_1 - E[R_1])(R_2 - E(R_2))]} \\
&= StdDev(R_1) * StdDev(R_2) * corr(R_1, R_2)
\end{aligned}
$$

Correlations



Take Away Formulas

- E[Portfolio Return] = $E(P) = w_1 * E[R_1] + w_2 * E[R_2]$
- var(Portfolio Return) = $var(P) = w_1^2 * var(R_1)$

$$+w_2^2 * var(R_1)$$
$$+2 * w_1 * w_2 * cov(R_1, R_2)$$

Let's practice!

Using Matrix Notation

Variables at Stake for N Assets

- w: the N x 1 column-matrix of portfolio weights

$$w = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_N \end{bmatrix}$$

- R: the N x 1 column-matrix of asset returns

$$R = \begin{bmatrix} R_1 \\ R_2 \\ \vdots \\ R_N \end{bmatrix}$$

- μ: the N x 1 column-matrix of expected returns

$$\mu = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_N \end{bmatrix}$$

Variables at Stake for N Assets

- Σ: The N x N covariance matrix of the N asset returns:

$$\sigma_i^2 \qquad \sigma_{ij}$$

$$\Sigma = \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1N} \\ \sigma_{21} & \sigma_2^2 & & \sigma_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{N1} & \sigma_{N2} & \cdots & \sigma_N^2 \end{bmatrix}$$

Covariance: Outside Diagonal
Variance: On Diagonal

Generalizing from 2 to N Assets

Portfolio Return

$$w_1 * R_1 + w_2 * R_2 \Rightarrow w_1 * R_1 + \ldots + w_N * R_N$$

Portfolio Expected Return

$$w_1 * \mu_1 + w_2 * \mu_2 \Rightarrow w_1 * \mu_1 + \ldots + w_N * \mu_N$$

Portfolio Variance

$$\begin{gathered} w_1^2 * var(R_1) + w_2^2 * var(R_2) \\ +2 * w_1 * w_2 * cov(R_1, R_2) \end{gathered} \Rightarrow \begin{aligned} & w_1^2 + var(R_1) + \ldots + w_N^2 * var(R_N) \\ & +2 * w_1 * w_2 * cov(R_1, R_2) + \ldots \\ & +2 * w_{N-1} * w_N * cov(R_{N-1}, R_N) \end{aligned}$$

Matrices Simplify the Notation

- Avoid large number of terms by using matrix notation
- We have 4 matrices:
  - weights (w), returns (R), expected returns (μ), and covariance matrix (Σ)

$$w = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_N \end{bmatrix} \qquad w' = \begin{bmatrix} w_1 & w_2 & \cdots & w_N \end{bmatrix}$$

Simplifying the Notation

Portfolio Return

$$\boxed{w_1 * R_1 + \ldots + w_N * R_N} \Rightarrow \boxed{w' R}$$

Portfolio Expected Return

$$\boxed{w_1 * \mu_1 + \ldots + w_N * \mu_N} \Rightarrow \boxed{w' \mu}$$

Portfolio Variance

$$\boxed{\begin{array}{l} w_1^2 + var(R_1) + \ldots + w_N^2 * var(R_N) \\ +2 * w_1 * w_2 * cov(R_1, R_2) + \ldots \\ +2 * w_{N-1} * w_N * cov(R_{N-1}, R_N) \end{array}} \Rightarrow \boxed{w' \Sigma w}$$

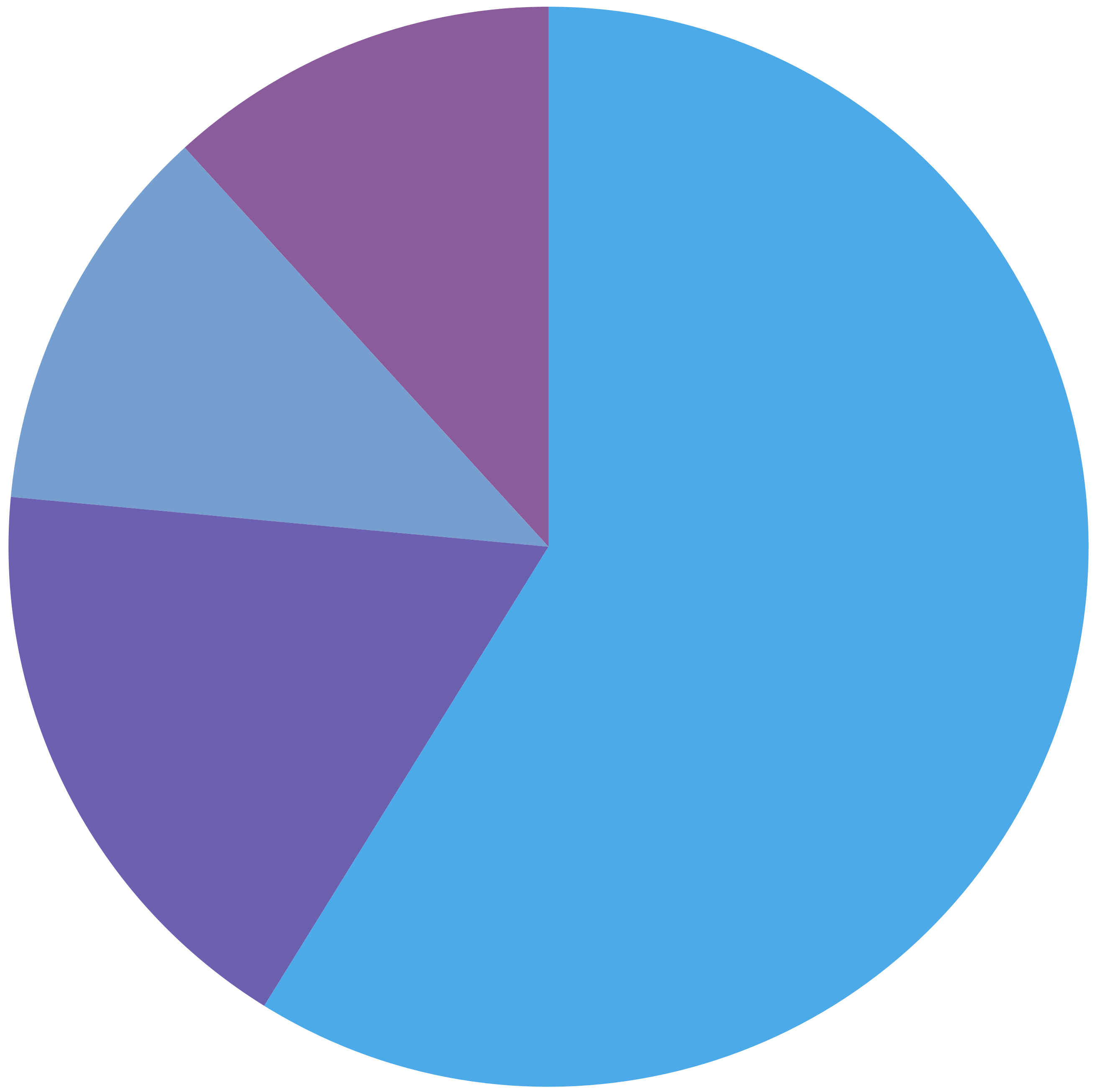
Let's practice!

INTRODUCTION TO PORTFOLIO ANALYSIS

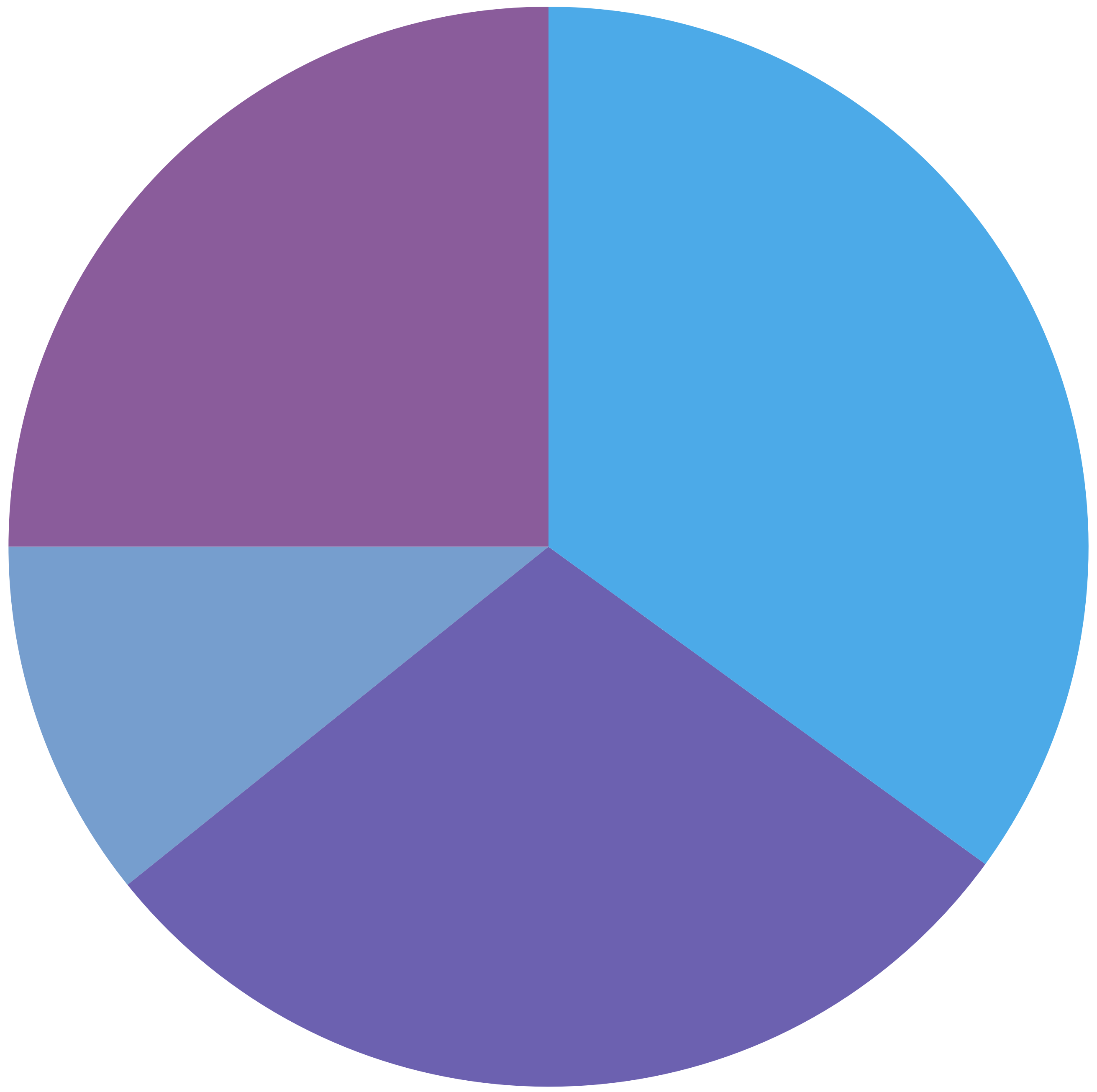
Portfolio Risk Budget

Who Did It?

Capital Allocation Budget



Portfolio Volatility Risk



Asset 1 Asset 2 Asset 3 Asset 4

Portfolio Volatility In Risk Contribution

- Portfolio Volatility = $\sum_{i=1}^{N} RC_i$
    - Where: $RC_i = \frac{w_i(\sum w)_i}{\sqrt{w' \sum w}}$
- risk contribution of asset *i* depends on
    1. the complete matrix of weights w
    2. the full covariance matrix $\sum$

Percent Risk Contribution

$$\boxed{\%RC_i = \frac{RC_i}{\text{portfolio volatility}}}$$

where $\sum_{i=1}^{N} \%RC_i = 1$

Relatively more risky assets: $\%RC_i > w_i$

Relatively less risky assets: $\%RC_i < w_i$

Let's practice!

Modern Portfolio Theory of Harry Markowitz

Portfolio Weights Are Optimal...

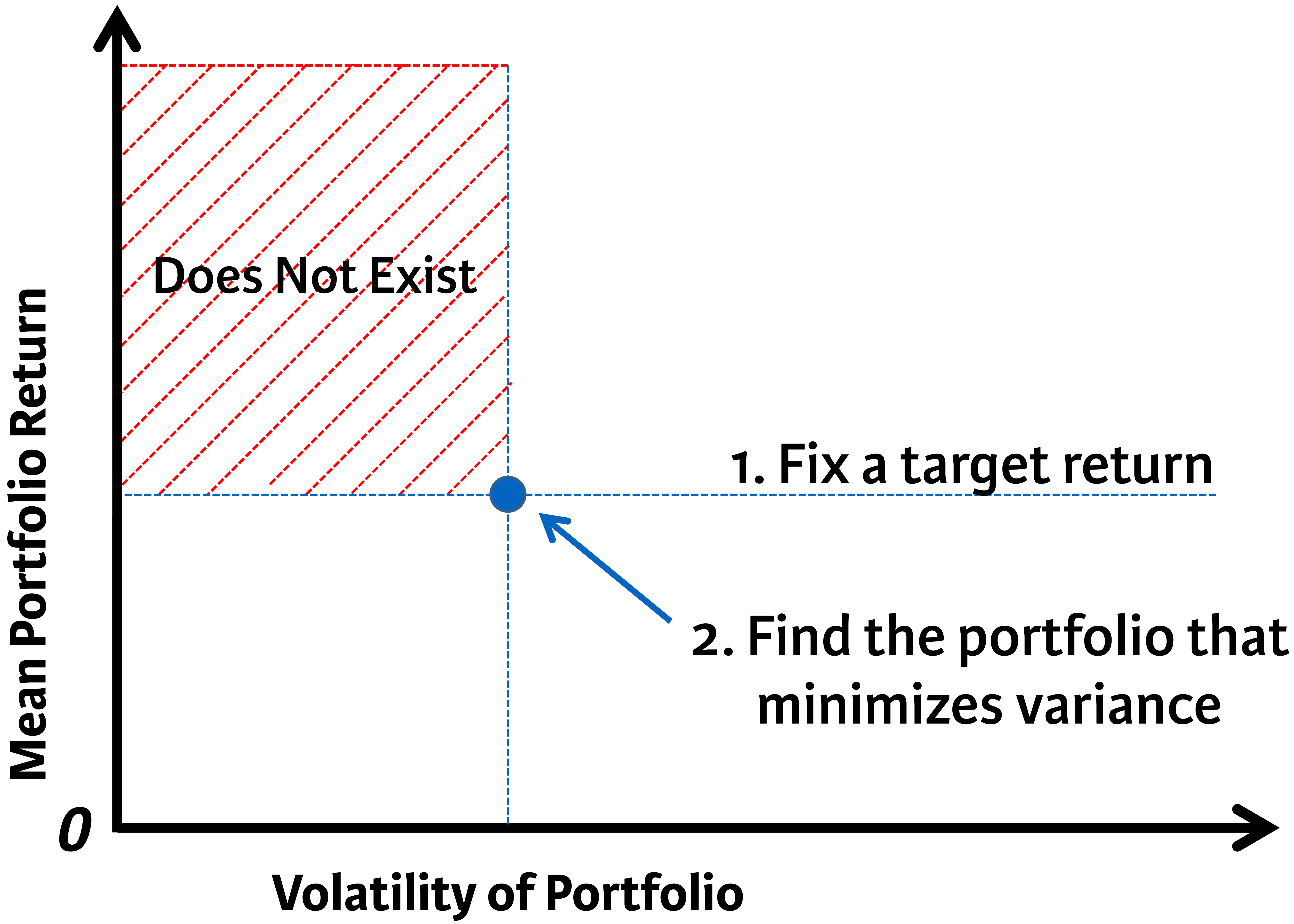
... when they optimize an objective function while satisfying the constraints

Possible Objectives	Possible Constraints
Maximize expected return	Only positive weights
Minimize the variance	Weights sum to 1 (all capital needs to be invested)
Maximize the Sharpe Ratio	Portfolio expected return equals a target value

Harry Markowitz

- Nobel Prize Winner
- Recommends finding optimal portfolios by minimizing portfolio variance
    - Constraint: Expected return should be equal to a pre-specified target return

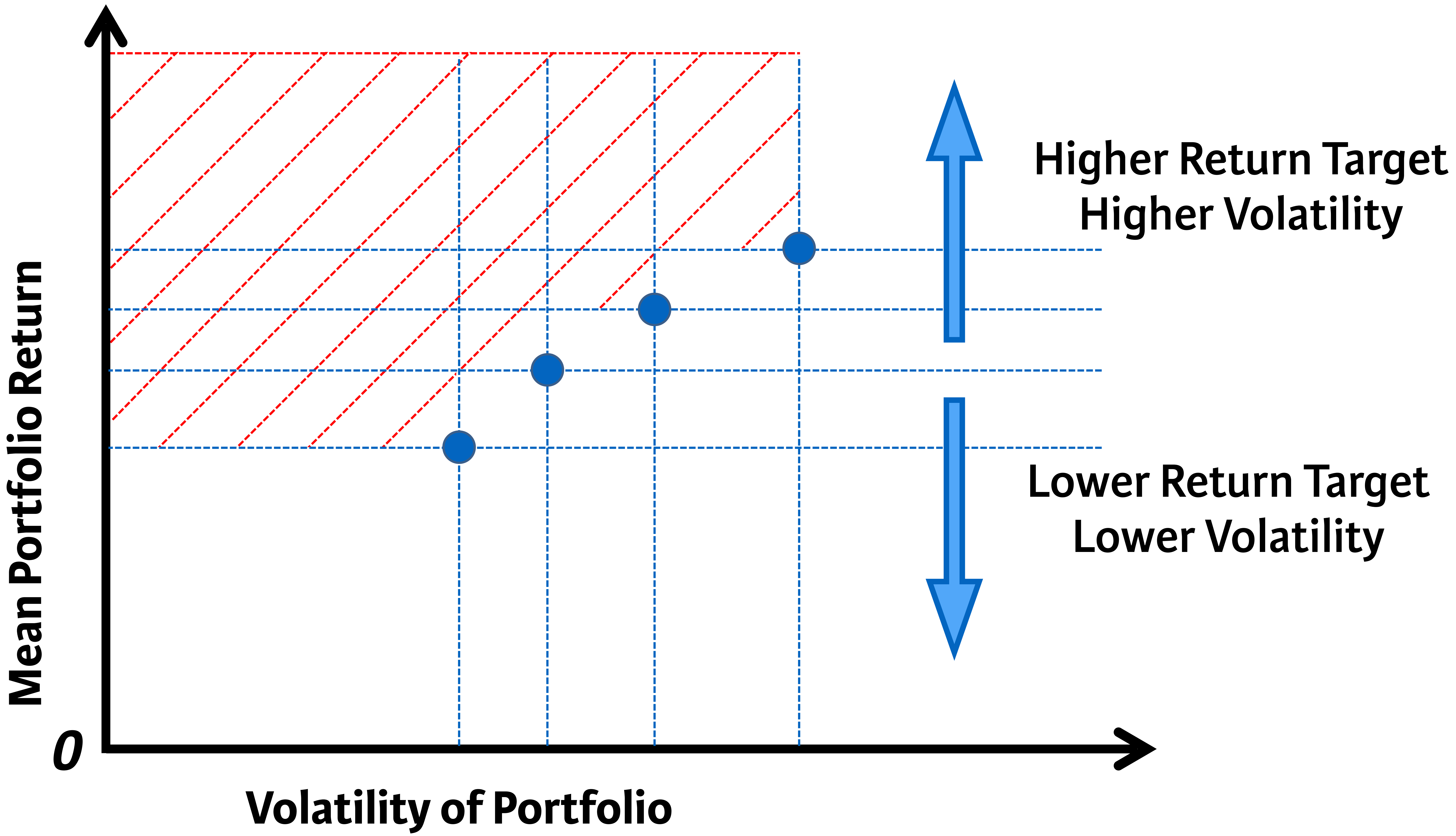
The H. Markowitz Approach



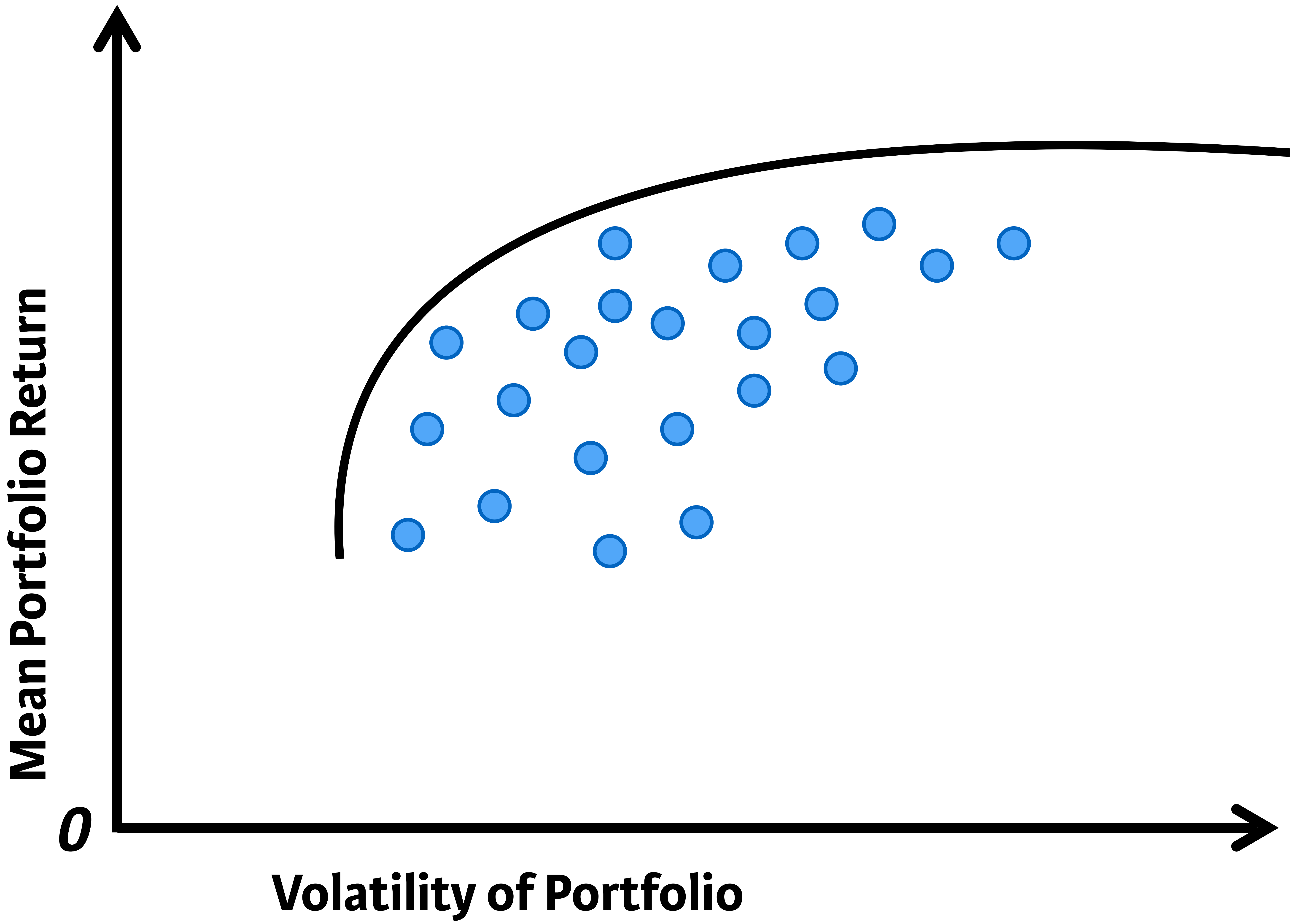
Let’s practice!

The Efficient Frontier

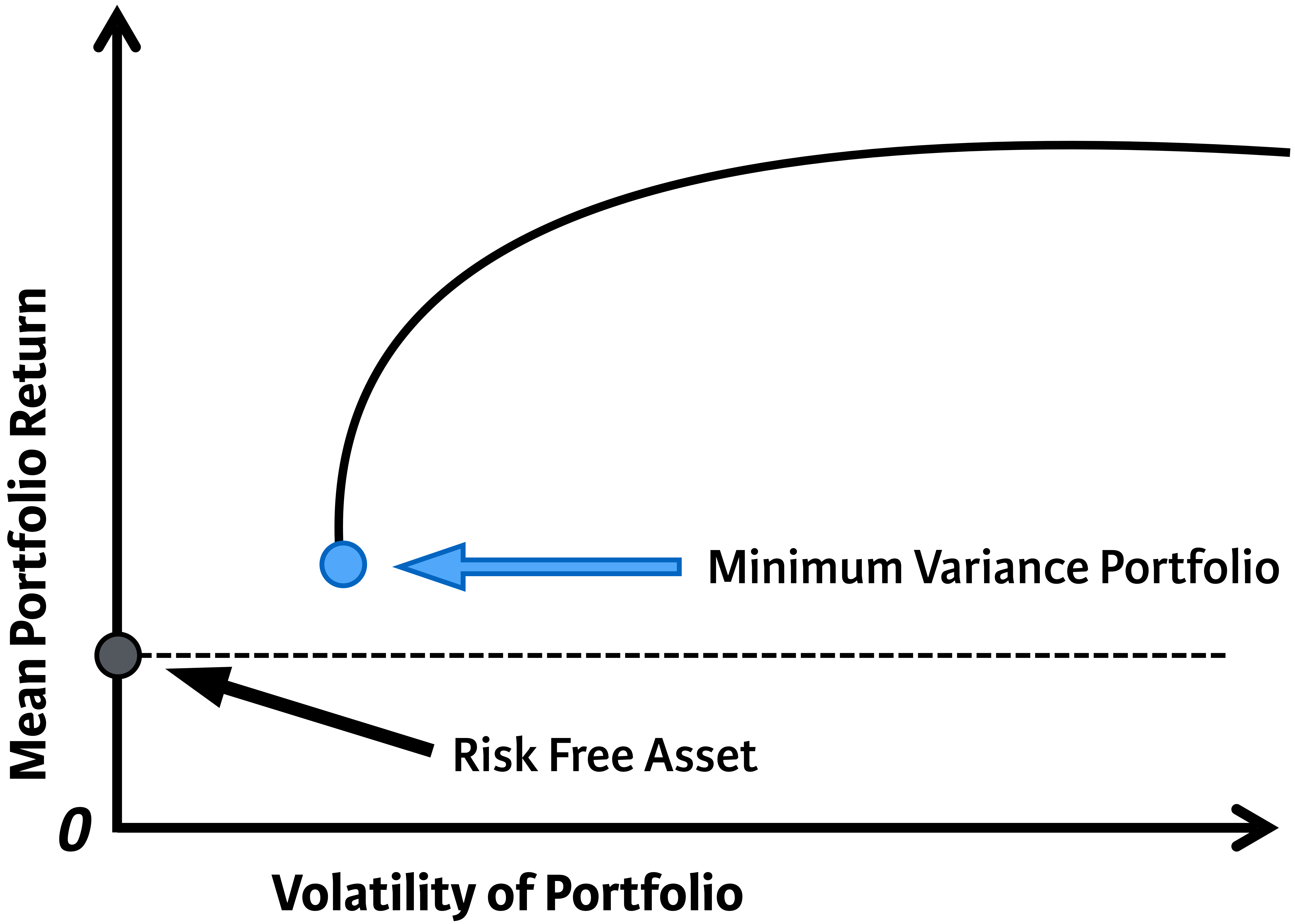
Changing Target Return



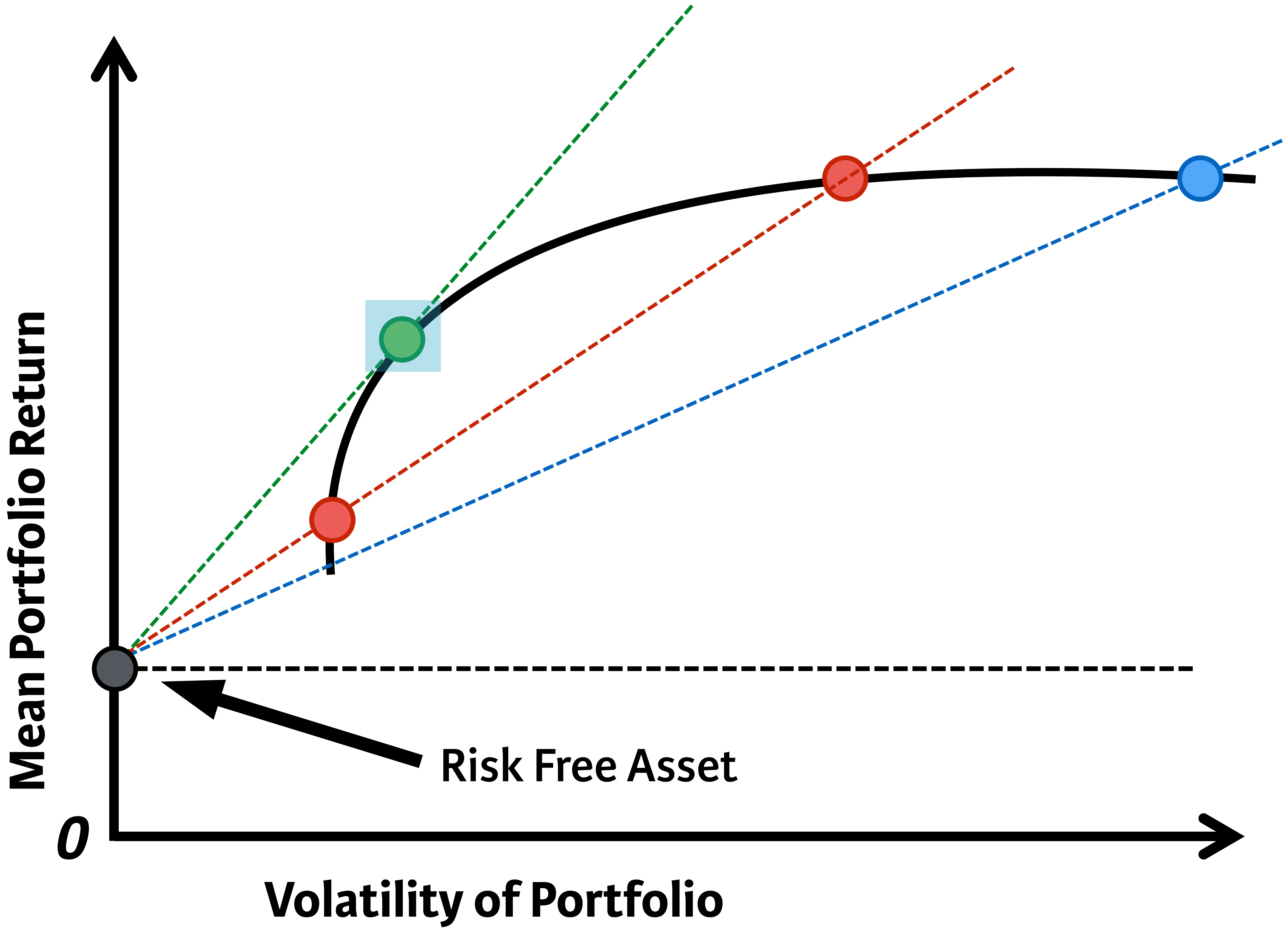
The Efficient Frontier



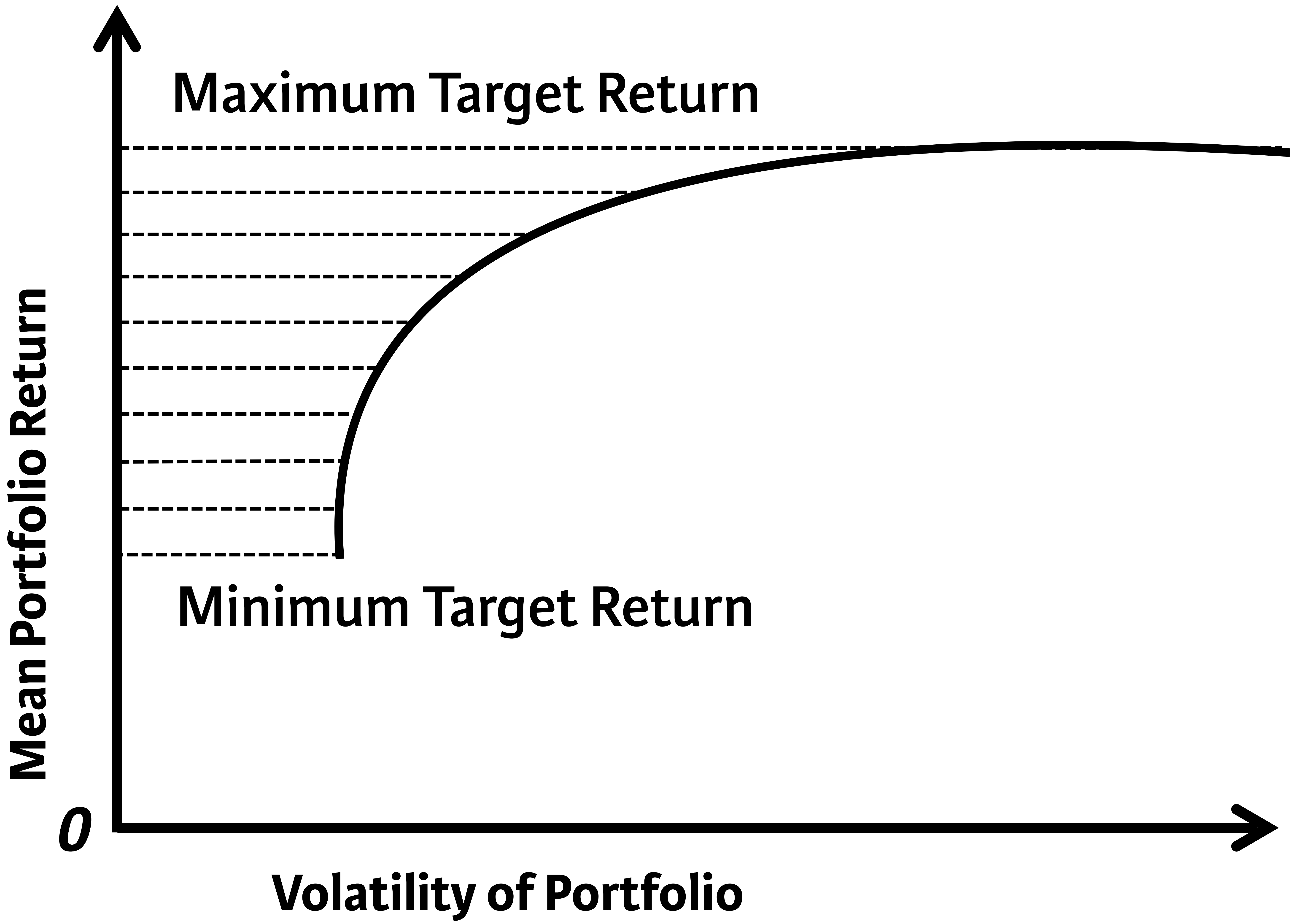
Minimum Variance Portfolio



Maximum Sharpe Ratio Portfolio



Time For Practice



Let’s practice!

In-Sample vs. Out-of-Sample

Bad News: Estimation Error

- Limitation to data-driven portfolio allocation:

Use in Practice
Estimated mean $\hat{\mu}$
Estimated variance $\hat{\sigma}^2$

Use In Theory
True (unknown) mean μ
True (unknown) variance σ^2

Optimized weights based on estimated mean & variance: $\hat{w}$

True optimal portfolio: w

Good News: Opportunities

- Do not ignore estimation error
- Use split-sample analysis to do a realistic evaluation of portfolio performance

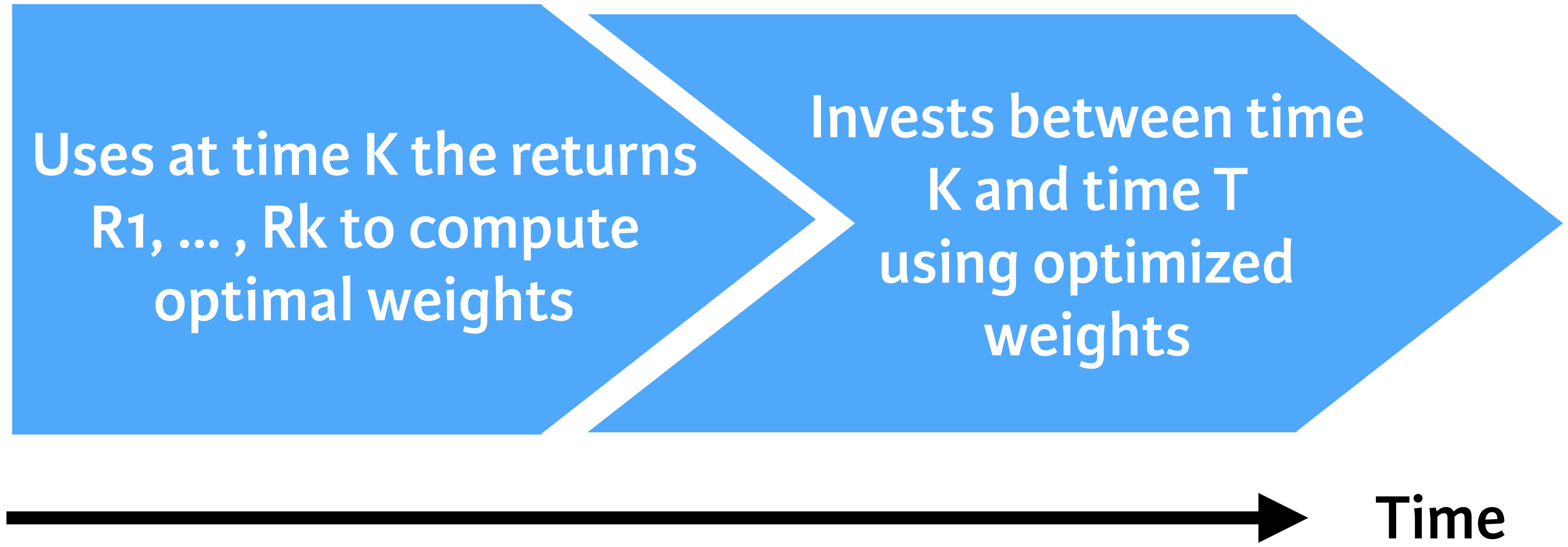
R_1	R_2	...	R_K	R_{K+1}	R_{K+2}	...	R_T

Estimation sample used to find the optimal weights

Out-of-Sample evaluation to give a realistic view on portfolio performance

No Look-Ahead Bias In Optimized Weights

- Split-sample design matches with the investor who:



- Function `window()` to do split-sample analysis in R

Let’s practice!