Section 1.1.3

1 We have an edge for every distinct pairs of vertices, so $\binom{n}{2} = \frac{n(n-1)}{2}$.

2 We prove the contrapositive. Suppose $r_1 \neq r_2$, $|X| = r_1$ and $|Y| = r_2$. Then for $x \in X$, $deg(x) = r2$ and for $y \in Y$ $deg(y) = r1$, but $r2 \neq r1$, so K_{r_1,r_2} is not regular.

3 No, no matter which 4 vertices you choose, 2 of them will be in the same subset, and would have no edges between them.

4 No. Having all 4 vertices in the same set will induce a disconnected graph, 3 in X and 1 in Y would result in a edges between the single vertex in a partition set to the other three, and 2 in X and 2 in Y induces a graph with a cycle.

6

$$\text{order} = \sum_{i=1}^{k} r_i$$

We use $\sum deg(v) = 2|E|$ to solve for size. For $v \in A_i$, $deg(v) = \sum_{j=1}^{k} r_j - r_i$. All vertices of A_i have the same degree, thus $r_i(\sum_{j=1}^{k} r_j - r_i)$ is the sum of degrees of vertices in A_i, thus

$$\text{size} = \sum_{i=1}^{k} [r_j(\sum_{j=1}^{k} r_j - r_i)]$$

7 For part c), we have $order = m$, $size = \sum_{i=1}^{n} \frac{(r_i-1)r_i}{2}$.

8 Suppose G and H are isomorphic. There exists a bijection f from $V(G)$ to $V(H)$ such that $xy \in E(G) \Leftrightarrow f(x)f(y) \in E(H)$.

$$xy \in E(\overline{G}) \Leftrightarrow xy \notin E(G) \Leftrightarrow f(x)f(y) \notin E(H) \Leftrightarrow f(x)f(y) \in E(\overline{H})$$

. Thus the complements are isomorphic.