Section 1.2.2

1 We have an edge for every distinct pairs of vertices, so $\binom{n}{2} = \frac{n(n-1)}{2}$.

2 We prove the contrapositive. Suppose $r_1 \neq r_2$, $|X| = r_1$ and $|Y| = r_2$. Then for $x \in X$, $deg(x) = r2$ and for $y \in Y$ $deg(y) = r1$, but $r2 \neq r1$, so K_{r_1,r_2} is not regular.

3 No, no matter which 4 vertices you choose, 2 of them will be in the same subset, and would have no edges between them.

4 a) $[A^3]_{j,j}$ equals the number of length 3 walks from v_j to itself. Length 3 closed walks form a triangle that contains v_j, but the walk v_j, v_x, v_y, v_j and v_j, v_y, v_x, v_j are both counted. These two walks form the same triangle, so we must divide the entry by two.

b) a) implies that $\frac{1}{2}\mathrm{Tr}(A^3)$ equals the number of triangles that contains v_1 or v_2 or ... or v_n. However every triangle consists of three vertices, so we are counting every triangle 3 times. Thus the number of unique triangles is $\frac{1}{6}\mathrm{Tr}(A^3)$.

5 This entry is 0. To show this, we prove that no odd length walk exists from 1 to any vertex labeled by an odd number. This is certainly true for walk of length 1. Suppose it is true for $2n-1$, and to show a contradiction suppose it is not true for $2n+1$. Without loss of generality, suppose that 5 is the vertex that is reached. The only length 2 walks from 5 are from 5 to 3, 5 to 7 or 5 to 5, thus the walk must consist of one of these, and a length $2n-1$ walk. However no $2n+1-2 = 2n-1$ length walk exists from 1 to 3, 5 or 7 by hypothesis.

6 a) Let i be the row with all positive entries in S_r. From 1. of Theorem 1.9, we know $ecc(v_i) = r$. Since for $a < r$, no row had all positive entries, none of the eccentricity for other vertices in G are less than r, so r is the smallest eccentricity, the radius of G.

b) Let i be the row with all positive entries in S_m, but not in S_{m-1}. From 1. of Theorem 1.9, $ecc(v_i) = m$, and since S_{m-1} contained zeros, all other eccentricities are $\leq m$, thus $diam(G) = m$.

8 We prove the contrapositive. Suppose G was not complete. $\exists v_i, v_j \in V(G)$ such that $uv \notin E(G)$, meaning $A_{i,j} = 0$. If there exists a v_iv_j path, then $D_{i,j} > 0$. Otherwise, $D_{i,j} = \inf$. In either case, $D_{i,j} \neq 0$, thus $A \neq D$.

Algorithm 1 Finding center of graph

7 S = [0], n by n zero matrix
k = 0
while everyrow of S has a 0 **do**
    S += A^k
    k += 1
return k