7)

Given:
- $\bar{X}_1 = \mu_1 = 10$; $\bar{X}_1 \sim N(10, 6)$
- $\bar{X}_0 = \mu_0 = 0$; $\bar{X}_0 \sim N(0, 6)$
- $\hat{\sigma}^2 = 36$
- $\hat{\sigma} = 6$
- $\pi_1 = 0.8$
- $\pi_0 = 0.2$

Find: $p(\pi_1 | X=4) = \dfrac{p(X=4|\pi_1) \cdot \pi_1}{p(X=4)}$

$p(X=4)$: LOTP (*)

(*) $p(X=4) = p(X=4|\pi_1) \cdot \pi_1 + p(X=4|\pi_0) \cdot \pi_2$

$p(X=4|\pi_1) \sim N(\mu_1, \hat{\sigma})$; $p(X=4|\pi_0) \sim N(\mu_0, \hat{\sigma})$

see R.