2) Suppose we obtain a bootstrap sample from a set of n observations.

(a) what is the prob that the 1^{st} Bootstrap observation is not the j^{th} observation from the original sample?

- $P(j^{th}) = \frac{1}{n} \rightarrow P('j^{th}) = \boxed{1 - \frac{1}{n}}$

(b) What is the prob that the second bootstrap observation is not the j^{th} observation from the original sample?

- Given bootstrap samples use replacement, the case is the same: $= \boxed{1 - \frac{1}{n}}$

(c) Argue that the prob that j^{th} obs is not in bootstrap sample at all is $(1 - \frac{1}{n})^n$.

(i) Due to replacement, denominator does not Δ;
(ii) By compliment principle, prob of not $j = 1 - \frac{1}{n}$;
(iii) for consecutive draws w/ replacement, you take the product of probabilities, so for the entire sample n: $(1 - \frac{1}{n})^n$

(d) when $n=5$, what is the prob that the j^{th} observation is in the bootstrap sample

$p(\text{at least once}) = 1 - p(\text{none}) = 1-(1-\frac{1}{5})^5 = \boxed{0.6723}$

(e) " $n=100$ "

$= 1-(1-\frac{1}{100})^{100} \approx \boxed{0.6339}$

(f) " $n=10000$ "

$= 1-(1-{}^{1}/_{10000})^{10000} \approx \boxed{0.6321}$

(g) See R

8)

(a) $X \sim N(0,1)$; $\varepsilon \sim N(0,1)$; $n=100$

$Y = X - 2X^2 + \varepsilon$