Practical 4

Question 1

Let $X_1, .., X_n$ denote n independent random variables, each of which has expectation μ and variance σ^2. The sample mean is defined as

$$\bar{X}_n = \frac{\sum_{i=1}^{n} X_i}{n}$$

Find

(a) $E(\bar{X}_n)$

(b) $Var(\bar{X}_n)$

Question 2

Assume that systolic blood pressure for 5 year old boys is normally distributed with a mean of 94 mm Hg and a standard deviation of 11 mm Hg.

What is the probability of a 5 year old boy having a systolic blood pressure:

(a) less than 70 mm Hg?

(b) higher than 100 mm Hg?

(c) between 80 and 100 mm Hg?

(d) What systolic blood pressure is necessary to put a 5 year old boy in the top 1% of the distribution?

(e) What systolic blood pressure is necessary to put a 5 year old boy in the bottom 10% of the distribution?

Question 3

A patient suffers epileptic fits at the rate of λ per day, according to a Poisson distribution. Let X be a random variable denoting the number of fits experienced up to time t, and let T be a random variable that is the time to the first fit.

(a) What distribution does X follow?

(b) Express the event $(T \leq t)$ in terms of the random variable X.

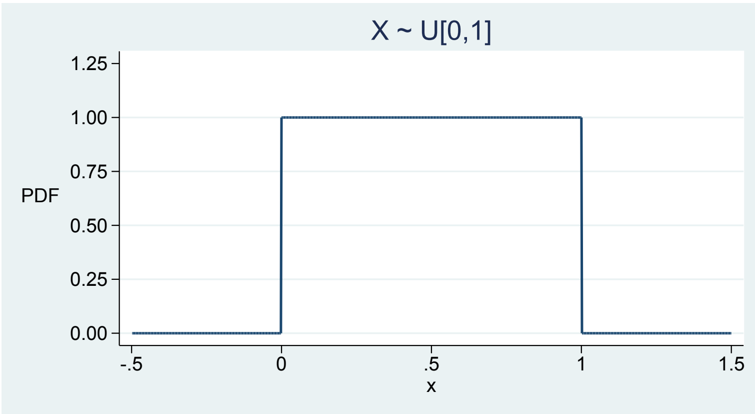
(c) Using your answer to (b), calculate $F(t) = Pr(T \leq t)$, the cumulative distribution function of T.

(d) Determine the probability density function of T, using the fact that this is the derivative of the CDF, i.e.

$$f(t) = \frac{d}{dt}\{F(t)\}$$

Additional: Question 4

As described in the notes, the continuous uniform distribution has a constant probability density function. The $U(0,1)$ uniform distribution has positive density equal to 1 between 0 and 1 (figure below).



(a) Explain why the probability density function for the uniform distribution makes it clear that a density function is not the same thing as a discrete random variable's probability distribution function.

(b) Let X follow the uniform distribution on $(0,1)$. Show from first principles that $E(X) = 1/2$ and $Var(X) = 1/12$.

(c) If Y is distributed uniformly between a and b, then by using your results and the relationship between Y and X, or otherwise, derive the expectation and variance of Y.

Additional: Question 5

Suppose that a continuous random variable Y has a probability density function of the form:

$$f(y) = ky^2(1-y),\ \ 0 < y < 1$$

(a) Find the constant, k, that makes this a valid probability density function.

(b) What is the general form of the cumulative distribution function? i.e. find $F(y) = P(Y \leq y)$.

(c) Hence find $P(Y < 0.2)$ and $P(Y > 0.6)$.