

MSc Medical Statistics

Term 1 Foundations of Medical Statistics (Probability)

Assignment

To be handed out on Monday 10th October 2022.

The deadline to submit your assignment is 1pm on Monday 24th October 2022. You must submit your assignment electronically via the submission point on the Foundations of Medical Statistics page on Moodle.

Assignments do not need to be word-processed, but hand-written assignments must be legible. The assignment contains 4 questions. There is no page limit, but it should be possible to fully answer all questions in 5-6 pages.

You must show your calculations/derivations to obtain marks.

Question 1: [20%]

In a randomised trial in patients with coronary heart disease, patients are randomised to receive a new treatment with probability 0.5 or to receive an existing treatment with probability 0.5. Patients receiving the new treatment have a 0.9 probability of surviving the next year, whereas those receiving the existing treatment only have a 0.7 probability of surviving the next year.

- (a) Draw a probability tree that represents this trial.
- (b) Given that a randomly selected patient survives the next year, what is the probability that they received the new treatment?

Question 2: [25%]

The number of patients arriving at a hospital's accident and emergency unit during the peak period on a Saturday night and who require immediate attention has been shown to follow a Poisson distribution with parameter $\mu = 3$. The unit can handle up to four such patients during this period. If more than four patients arrive in this period, the patients in excess of four must be sent to another hospital. The following questions refer to the peak period on Saturdays.

Note: For parts (a) and (b), simply looking the answer up in the Neave tables will not obtain full marks — please provide the calculations required to obtain the answer.

- (a) On a given Saturday, what is the probability of having to send patients to another hospital?
- (b) By how much must present facilities be increased to permit handling of all patients arriving on at least 95% of Saturdays?
- (c) What is the expected number of patients arriving each Saturday?
- (d) What is the most probable number of patients arriving each Saturday?
- (e) Give the main assumptions we make when we model the above situation using a Poisson distribution.

Question 3: [25%]

In a small clinical trial, 10 patients with chronic pain were randomly assigned to receive either drug A or drug B for one month. After completing the month, each patient had a two-week "wash-out" period during which no drug was taken. After the wash-out period, each patient then received the alternate drug for one month.

At the end of the study, each patient was asked to state which drug they preferred, in terms of which drug gave greater relief from their pain. Each patient had to choose one drug. Eight patients said that they preferred drug A.

- (a) If drug A and drug B were truly equal in their effects in reducing pain, what would be the probability that a randomly selected patient expressed a preference for drug A over drug B?
- (b) How many ways are there of obtaining 10 patients expressing a preference for drug A? Repeat for 9 patients, 8 patients,..., 1, 0 patients. Present your results in a table.
- (c) What is the probability of eight patients expressing a preference for drug A, if drugs A and B were truly equal in their effects? What is the probability of eight or more patients expressing a preference for drug A under the same condition?
- (d) Clearly describe how the probabilities in part (c) can be obtained only from your table in part (b).
- (e) *Based only on what you have learnt in the 5 probability sessions* and your answers above, what conclusion might one draw regarding the effectiveness of drug A compared to drug B?
- (f) State the assumptions you have made in carrying out your calculations. Discuss their appropriateness in the context of the study design described above.

Question 4: [30%]

When testing subjects for HIV in surveillance studies WHO protocol states that subjects whose test result is negative on the first test should be classified as 'HIV negative'. Subjects whose test result is positive on a first test should be retested twice and classified as 'HIV positive' if one or both of these retests are positive. If both of the retests are negative then the subject is classified as 'HIV negative'.

Let the sensitivity of a single test be α and the specificity be β .

- (a) Write down, in terms of α and β , the sensitivity and specificity of the test procedure carried out under the WHO protocol.
- (b) Use these expressions to compare the effects of using the WHO protocol as opposed to using a single test.
- (c) Discuss some of the practical advantages and disadvantages of applying the WHO protocol in comparison with using the single test.
- (d) State the main assumption that you have made in carrying out your calculations. Comment on whether this assumption seems reasonable in this situation.

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