$$D(\alpha,\beta \mid X) = -2\sum \log f(X_i \mid \alpha,\beta)$$

$$\frac{\partial D}{\partial \alpha} = 0$$

$$= -2 \frac{\partial}{\partial \alpha}\left[\sum \log\left(\frac{\alpha\beta^{\alpha}}{x^{\alpha+1}}\right)\right]$$

$$= -2 \frac{\partial}{\partial \alpha}\left[\sum\left[\log(\alpha\beta^{\alpha}) - \log(x^{\alpha+1})\right]\right]$$

$$= -2 \frac{\partial}{\partial \alpha}\Big[\left(\log(\alpha\beta^{\alpha}) - \log(x_1^{\alpha+1})\right) + \left(\log(\alpha\beta^{\alpha}) - \log(x_2^{\alpha+1})\right) + \dots\dots\dots$$

$$\dots + \left(\log(\alpha\beta^{\alpha}) - \log(x_n^{\alpha+1})\right)\Big]$$

$$= -2 \frac{\partial}{\partial \alpha}\Big[\left(n\log(\alpha\beta^{\alpha})\right) - \Big[\log(x_1^{\alpha+1}) + \log(x_2^{\alpha+1})$$

$$\dots + \log(x_n^{\alpha+1})\Big]\Big]$$

$$= -2 \frac{\partial}{\partial \alpha}\left[n\log(\alpha\beta^{\alpha})\right] - \left(-2\frac{\partial}{\partial \alpha}\right)\Big[\log(x_1^{\alpha+1}) + \dots$$

$$\log(x_2^{\alpha+1}) + \dots + \log(x_n^{\alpha+1})\Big]$$

$$= \underbrace{-2n \frac{\partial}{\partial \alpha} \left[\log(\alpha \beta^{\alpha}) \right]}_{1} + \underbrace{2 \frac{\partial}{\partial \alpha} \left[\log(x_1^{\alpha+1}) + \dots \log(x_2^{\alpha+1}) + \dots + \log(x_n^{\alpha+1}) \right]}_{2}$$

$$1 = -2n \frac{\partial}{\partial x} \left[\log \alpha + \alpha \log \beta \right]$$

$$= -2n \left[\frac{1}{\alpha} + \log \beta \right]$$

$$2 = 2 \frac{\partial}{\partial \alpha} \left[\log(x_1^{\alpha} \cdot x_1) + \dots + \log(x_n^{\alpha} \cdot x_n) \right]$$

$$= 2 \frac{\partial}{\partial \alpha} \left[\log(x_1^{\alpha}) + \log(x_1) + \dots + \log(x_n^{\alpha}) + \log(x_n) \right]$$

$$= 2 \frac{\partial}{\partial \alpha} \left[\alpha \log x_1 + \log(x_1) + \dots + \alpha \log x_n + \log(x_n) \right]$$

$$= 2 \left[\log x_1 + \log x_2 + \dots + \log x_n \right]$$

$$\frac{\partial D}{\partial \alpha} = 0$$

$$= -2n\left[\frac{1}{\alpha} + \log\beta\right] + 2\left[\log x_1 + \log x_2 + \dots + \log x_n\right] = 0$$

$$= -2n\left[\frac{1}{\alpha} + \log\beta\right] = -2\left[\log x_1 + \log x_2 + \dots + \log x_n\right]$$

$$= n\left[\frac{1}{\alpha} + \log\beta\right] = \log x_1 + \log x_2 + \dots + \log x_n$$

$$= \frac{1}{\alpha} + \log\beta = \frac{\log x_1 + \log x_2 + \dots + \log x_n}{n}$$

$$= \frac{1}{\alpha} = \frac{\log x_1 + \log x_2 + \dots + \log x_n}{n} - \log\beta$$

$$= \frac{1}{\alpha} = \frac{\log x_1 + \log x_2 + \dots + \log x_n}{n} - \frac{\log\beta}{1} \begin{matrix} \times n \\ \times n \end{matrix}$$

$$= \frac{1}{\alpha} = \frac{\log x_1 + \log x_2 + \dots + \log x_n - \log\beta}{n}$$

$$= \hat{\alpha} = \frac{n}{\left[\log x_1 + \log x_2 + \dots + \log x_n\right] - \log\hat{\beta}} \quad //$$

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