SM-2302 Software for Mathematicians

Introduction & Getting Started

Drs. Haziq Jamil & Huda Ramli
Mathematical Sciences, Faculty of Science, UBD
`https://sm2302.github.io`

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Overview

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- Getting started
- Module contents
- Purpose of mathematical software

Getting started

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- Lecturer information


```
Dr. Haziq Jamil
Assistant Professor in Statistics
Room M1.09
haziq.jamil@ubd.edu.bn
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Dr. Huda Ramli
Lecturer in Applied Mathematics
Room M1.04
huda.ramli@ubd.edu.bn
```


- **IMPORTANT: Read the syllabus.**
- Weekly contact hours
  - Lectures: 2 hours in computer lab on Thursdays 7:50 AM
  - Tutorials: 2 hours practice lab on Saturdays 2:10 PM
- Be aware of schedule and important deadlines.
- Check Canvas regularly for announcements and course materials.

Module description

Mathematical software is what bridges higher mathematics to real world applications. On completing this module, students should be able to use `MATLAB` and `R` to effectively implement mathematical solutions to real world problems. They should also be able to produce publication-quality mathematical documents using LaTeX. This module provides the computing skills required for an applied mathematics final year project.

Contents

1. Learning `MATLAB` and `R` languages for mathematical applications.
2. `MATLAB` specific outomes: Basic operations, programming, numerical techniques and root finding.
3. `R` specific outcomes: Logic and types, data frames and matrices, data wrangling, and visualisations.
4. Preparation of report-style documents using LaTeX.
5. Version control and social coding using Git and GitHub.

Assessment

Take note that this module is assessed wholly (100%) by coursework.

Formative assessment

- Lab-based tutorials

Summative assessment

- **[20%]** 4 $\times$ online quizzes
- **[20%]** 2 $\times$ mini individual assignments
- **[30%]** 2 $\times$ mini group assignments
- **[30%]** 1 $\times$ project assignment with written report

See the syllabus pdf file for further details.

Purpose of mathematical software

> *Software is essential for modelling, analysing and calculating numeric, symbolic, or geometric data.*

Generally speaking, mathematical software is very focused:

1. **Software calculator**: Performs simple mathematical operations.
2. **Computer algebra systems**: Designed to solve classical algebra equations and problems in human readable notation.
3. **Statistics**: Statistical analysis of data.
4. **Optimisation**: Selecting a best solution from a set of alternatives.
5. **Numerical analysis**: Numerical approximations for the problems of mathematical analysis.
6. etc.

Remark

While mathematical software produces useful solutions, they very often do not explain why the solutions are what they are.

How many primes are there?

Theorem 1 (Euclid's Theorem)

There are infinitely many primes.

- A prime number $p \in \mathbb{N}$ is divisible only by itself and 1.
- We might attempt to brute force the answer by writing a software loop.
- Can we prove this theorem by software?

```
INPUT n
  i := 2
  count := 0
  WHILE i <= n
    rem := n % i
    IF rem not equal to 0
      i := i + 1
      count := count + 1
    END IF
  END WHILE
OUTPUT count
```

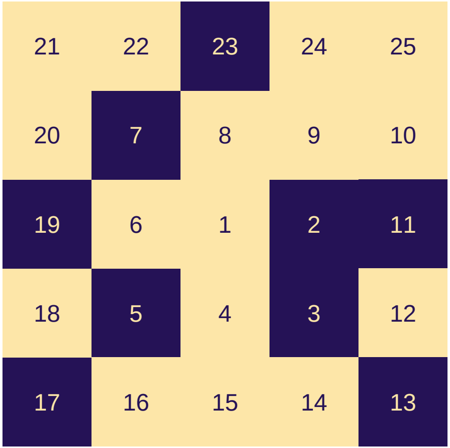
Software affords us insight

Let $\pi(x)$ be the prime counting function defined to be the number of primes less than or equal to x, for any $x \in \mathbb{R}$. Can we intuit a good approximation of $\pi(x)$?

A different (but related) question: How far apart are the prime numbers?

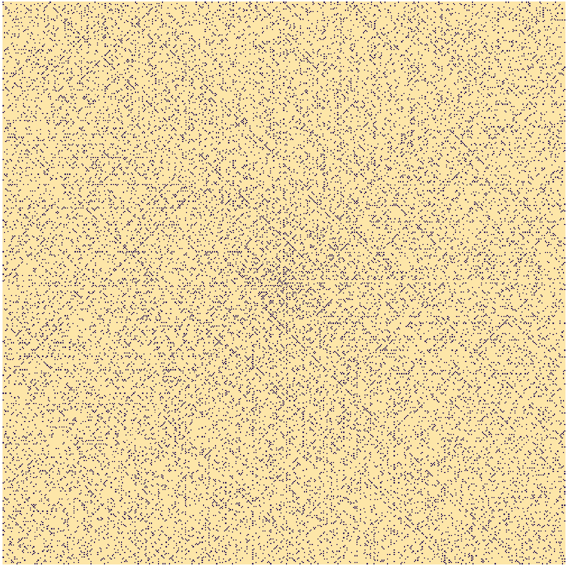
Define the *density* of primes as $\pi(x)/x$. This gives an idea of the distribution of primes up to x. It would be interesting to map this out.

Source code from
`https://github.com/johnistan/ulam-spirals-R`



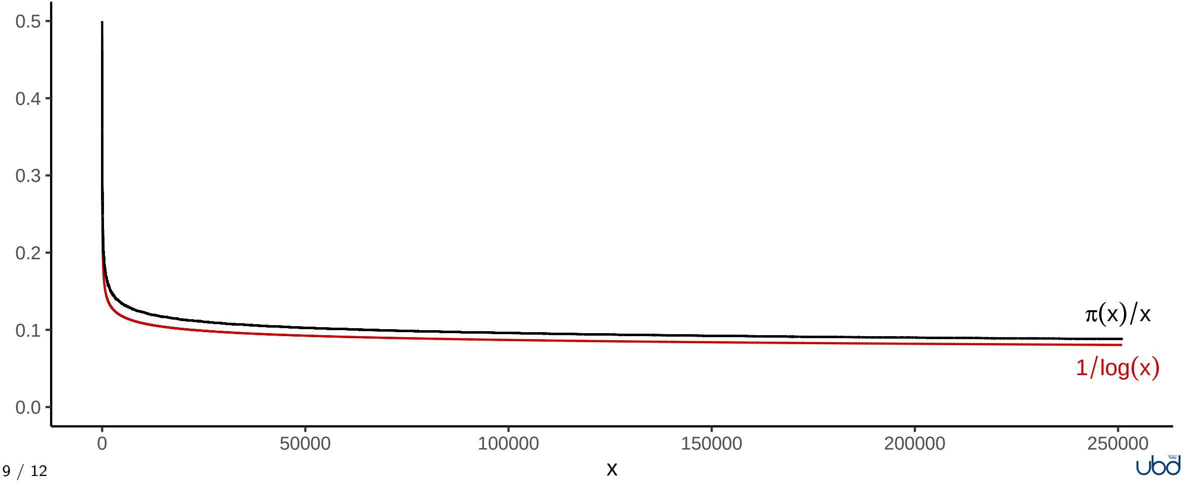
Ulam's spiral

- Prominent diagonal, horizontal and vertical lines containing large number of primes.
- Not unsurprising, as these correspond to certain prime-generating polynomials such as $x^2 - x + 41$ (Euler's).
- Nonetheless, connected to many unsolved areas of mathematics!
  - Riemann Hypothesis
  - Goldbach's conjecture
  - Twin prime conjecture
  - Legendre's conjecture



Does the density converge?

As $x \to \infty$, the prime density $\pi(x)/x$ diminishes at a slow rate. Reminiscent of an inverse logarithmic decrease!



The prime number theorem

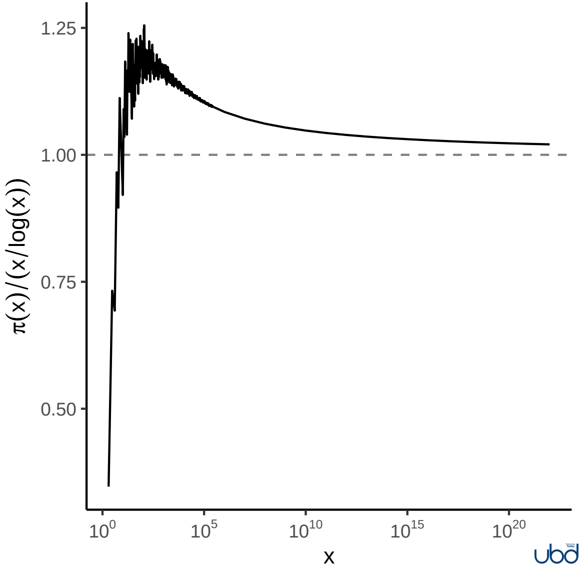
- The *asymptotic* law of distribution of prime numbers states that

$$\lim_{x\to\infty} \frac{\pi(x)/x}{1/\log(x)} = \frac{\pi(x)}{x/\log(x)} = 1$$

- From this, we have

$$\pi(x) \sim \frac{x}{\log(x)}$$

- We now have an approximation for the prime counting function, which improves as x increases. In particular, $\lim_{x\to\infty} x/\log(x) = \infty$.



Work flow

$$
\begin{tikzcd}
\text{Mathematics} \arrow[r, bend left, "\textit{ideation}"] & \texttt{Software} \arrow[l, bend left, "\textit{results}"]
\end{tikzcd}
$$

Use software as a tool to...

- Explore and visualise ideas
- Confirm ideas numerically
- Communicate results

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Getting started

Software

1. MATLAB
2. RStudio
3. Git, github.com and GitHub Desktop
4. Overleaf.com