CHAPTER 3
Stock and Bond Valuation and Return

Introduction

In Chapter 1, we discussed how securities can be evaluated in terms of eight common characteristics: value, rate of return, risk, life, divisibility, marketability, liquidity, and taxability. For most investors, the most important of these characteristics are value and return. In this chapter, we examine the valuation of bonds and stocks and how to measure the rates of return from investing in such securities. In Chapter 11, we will examine more closely the value, return, and risk relations for stocks when we begin our analysis of equity securities. Understanding how securities are valued and their rates determined, however, is fundamental to being able to evaluate and select these securities.

Rate of Return

Holding Period Yield

The rate of return an investor earns from holding a security is equal to the total dollar return received from the security per period of time (e.g., year) expressed as a proportion of the price paid for the security. The total dollar return includes income payments (coupon interests or dividends), interest earned from reinvesting the income during the period, and capital gains or losses realized when the security is sold or matures. For example, an investor who purchased XYZ stock for S_0 = \$100, then received \$10 in dividends (*D*) two years later when he sold the stock for S_T = \$110, would realize a rate of return for this two-year period of 20 percent:

$$\text{Rate of Return} = \frac{D + S_T - S_0}{S_0} = \frac{\$10 + \$110 - \$100}{\$100} = 0.20$$

Similarly, a bond investor who bought a Treasury bond on April 20 for $P^B{}_0$ = \$95,000 and then sold it for $P^B{}_T$ = \$96,000 on October 20 just after receiving a coupon of C = \$4,000, would earn a rate of return for this six-month period of 5.263 percent:

$$\text{Rate of Return} = \frac{C + P^B_T - P^B_0}{P^B_0} = \frac{\$4{,}000 + \$96{,}000 - \$95{,}000}{\$95{,}000} = 0.05263$$

Both the bond and stock rates of return are measured as *holding period yields*, *HPY*. The *HPY* is the rate earned from holding the security for one period (e.g., one year or six months). The *HPY* can alternatively be expressed in terms of the security's *holding period return*, *HPR*, minus one, where the *HPR* is the ratio of the ending-period value (e.g., $D + S_T$) to the beginning period value (S_0). That is:

$$HPY = \frac{\text{Ending Value}}{\text{Beginning Value}} - 1$$

$$HPY = HPR - 1$$

$$HPY = \frac{D + S_T}{S_0} - 1; \; HPY = \frac{C + P^B_T}{P^B_0} - 1$$

Annualized HPY

To evaluate alternative investments with different holding periods, investment analysts often annualize the rate of return. The simplest way to annualize a return is to multiply the periodic rate of return by the number of periods of that length in a year. Thus, to annualize the *HPY* on the bond investment, we would multiply the six-month *HPY* of 0.05263 by 2 to obtain 0.10526; to annualize the stock's rate of return, we would multiple the two-year *HPY* of 0.20 by 1/2 to get 0.10. This method for annualizing rates of return, however, does not take into account the interest that could be earned from reinvesting the cash flows. That is, a $1 investment in the bond would yield $1.0526 after six months, which could be reinvested. If it is reinvested for six months at the same six-month rate of 5.26 percent, then the dollar investment would be worth $1.108 after one year. Thus, the *effective annual rate* (i.e., the rate that takes into account the reinvestment of interest or the compounding of interest) is 10.8 percent. The *effective annualized HPY* (HPY^A) can be calculated using the following formula:

$$HPY^A = HPR^{1/M} - 1$$

where: *M* = number of years the investment is held.

Thus, the effective annualized *HPY* for the bond investment would be 10.8 percent, and the effective HPY^A for the stock investment would be 9.544 percent:

$$HPY^A = HPR^{1/M} - 1$$

$$HPY^A = \left[\frac{\$4{,}000 + \$96{,}000}{\$95{,}000}\right]^{1/0.5} - 1 = 0.108$$

$$HPY^A = \left[\frac{\$10 + \$110}{\$100}\right]^{1/2} - 1 = 0.0954$$

The two-year *HPY* on the stock of 20 percent reflects annual compounding. That is, $1 after one year would be worth $1.0954, which reinvested for the next year would equal $1.20:

$$\$1.00(1.0954)(1.0954) = (1.0954)2 = \$1.20$$

Required Rates of Return and Value for a Single-Period Cash Flow

The price of a security and its rate of return are related. When an investor knows the price of the security and its cash flow, she can determine the rate of return. Alternatively, when the investor knows her *required rate of return* and the security's cash flow, she can determine the value of the security or the price she is willing to pay. For example, an investor who requires an annual 10 percent rate of return in order to invest in a one-year AAA bond paying a single cash flow (coupon interest) of $10 and a principal of

\$100 at maturity would value the bond at \$100. This price can be found by expressing the equation for the *HPY* in terms of its price:

$$\text{Rate of Return} = R = \frac{C + P_T^B}{P_0^B} - 1$$

$$P_0^B(1 + R) = C + P_T^B$$

$$P_0^B = \frac{C + P_0^B}{(1 + R)}$$

$$P_0^B = \frac{\$10 + \$100}{1.10} = \$100$$

If the bond were priced in the market below \$100, the investor would consider it underpriced, yielding a rate of return that exceeds her required rate of 10 percent; if the bond were priced above \$100, she would consider the bond to be overpriced, yielding a rate of return less than 10 percent.

Future and Present Values

Future Value

The above value and return relations can be described in terms of the present values and future values of investments and future receipts. More formally, the future value of any amount invested today is

$$P_N = P_0(1 + R)^N$$

where:

- N = number of periods of the investment
- P_N = future value of investment N periods from present (future value, FV)
- P_0 = initial investment value (present value, PV)
- R = rate per period (periodic rate)
- $(1 + R)^N$ = future value of \$1 invested today for N periods at a compound rate of R

In terms of the preceding bond example, the future value (FV) of the bond investment of $100 at 10 percent is $110 (coupon and principal):

$$P_1 = P_0(1 + R)^1$$
$$P_1 = \$100(1.10) = \$110$$

An investment fund that invested $1,000,000 in a security that paid 10 percent per year for three years would in turn have $1,331,000 at the end of three years:

$$P_N = P_0(1 + R)^N$$
$$P_3 = \$1{,}000{,}000\,(1.10)^3 = \$1{,}331{,}000$$

If the interest is paid more than once a year, then the rate of return and the number of periods must be adjusted. Specifically, let

$$n = \text{the number of times interest is paid per year}$$
$$M = \text{number of years of the investment}$$
$$\text{Period Rate} = R = \frac{\text{Annual Rate}}{n}$$
$$N = \text{number of periods of the investment} = (n)(M)$$

If an investment fund invested $1,000,000 in a three-year security that paid annual interest at 10 percent for three years with the interest paid semiannually, then the investment would be worth $1,340,095.64 after three years:

$$n = 2$$
$$M = 3 \text{ years}$$
$$\text{Period Rate} = R = \frac{\text{Annual Rate}}{n} = \frac{0.10}{2} = 0.05$$
$$N = \text{number of periods of the investment} = (n)(M) = (2)(3) = 6$$
$$P_N = P_0(1 + R)^N$$
$$P_6 = \$1{,}000{,}000\,(1.05)^6 = \$1{,}340{,}095.64$$

Note that with semiannual interest payments, there are more opportunities for reinvesting the interest received. As a result, the future value of the investment is greater with interest paid semiannually than annually.

Future Value of an Annuity

An annuity is a periodic investment or receipt. For example, an investment of $1 million each year for three years would be an example of an investment annuity, and a security paying $50 every six months for 10 years would be an example of a receipt annuity. The future value of an annuity (*A*) is equal to the sum of the future values of each investment at the investment horizon:

$$P_N = A(1+R)^{N-1} + A(1+R)^{N-2} + A(1+R)^{N-3} + \cdot\cdot\cdot + A(1+R)^{N-N}$$

$$P_N = \sum_{t=1}^{N} A(1+R)^{N-t}$$

As an example, suppose an investment fund owned $50,000,000 of bonds maturing in three years that promised to pay 10 percent per year and $50,000,000 at the end of three years. If the fund reinvested the annual interest of $5,000,000 at a rate of 10 percent, then at the end of three years the sum of the annual interest payments would be worth $16,550,000:

Year	**0**	**1**	**2**	**3**	**3**	**Values**
A		$5,000,000			$5,000,000(1.10)^2	$6,050,000
A			$5,000,000		$5,000,000(1.10)^1	$5,500,000
A				$5,000,000	$5,000,000(1.10)^0	$5,000,000
					Horizon Value = P_N	$16,550,000

$$P_N = \sum_{t=1}^{N} A(1+R)^{N-t}$$

$$P_3 = \sum_{t=1}^{3} \$5{,}000{,}000\,(1+R)^{3-t}$$

$$P_N = \$5{,}000,000(1.10)^{3-1} + \$5{,}000,000(1.10)^{3-2} + \$5{,}000{,}000(1.10)^{3-3}$$

$$P_N = \$16{,}550{,}000$$

At the end of three years, the fund would have $500 million in principal, $15 million in interest, and $1,550,000 (= $16,550,000 - $15,000,000) in interest earned from reinvesting the interest.

The equation for the future value of an annuity is equal to the annuity times the future value of $1 invested each period for *N* periods:

$$P_N = \sum_{t=1}^{N} A(1+R)^{N-t}$$

$$P_N = A\sum_{t=1}^{N} (1+R)^{N-t}$$

The future value of $1 invested each period for *N* periods is defined as the future value interest factor of an annuity, $FVIF_a$. The formula for determining $FVIF_a$ is:

$$FVIF_a = \sum_{t=1}^{N} (1+R)^{N-t} = \left[\frac{(1+R)^N - 1}{R}\right]$$

Substituting the formula for the $FVIF_a$ into the equation for P_N, the future value of annuity can alternatively be expressed as:

$$P_N = A \sum_{t=1}^{N} (1 + R)^{N-t}$$

$$P_N = A \left[\frac{(1 + R)^N - 1}{R} \right]$$

In terms of our example:

$$P_3 = \$5{,}000{,}000 \sum_{t=1}^{3} (1.10)^{3-t}$$

$$P_3 = \$5{,}000{,}000 \left[\frac{(1.10)^3 - 1}{0.10} \right]$$

$$P_3 = \$16{,}550{,}000$$

Note: If the bond investment fund received interest semiannually, then the fund would receive $2,500,000 every six months. If the semiannual reinvestment rate were 5 percent, then the sum of the future values of the interest payments would be $17,004,782:

Year	0.0	0.5	1.0	1.5	2.0	2.5	3.0	3	V
N		**1**	**2**	**3**	**4**	**5**	**6**	**6**	
A		$2,500,000						$2,500,000(1.05)^5	$3
A			$2,500,000					$2,500,000(1.05)^4	$3
A				$2,500,000				$2,500,000(1.05)^3	$2
A					$2,500,000			$2,500,000(1.05)^2	$2
A						$2,500,000		$2,500,000(1.05)^1	$2
A							$2,500,000	$2,500,000(1.05)^0	$2
								Horizon Value = P_N	$1

$$P_N = A \sum_{t=1}^{N} (1 + R)^{N-t}$$

$$P_6 = \$2{,}500{,}000 \sum_{t=1}^{6} (1.05)^{6-t}$$

$$P_6 = \$2{,}500{,}000 \left[\frac{(1.05)^6 - 1}{0.05} \right]$$

$$P_6 = \$17{,}004{,}782$$

Present Value

The present value is the amount that must be invested today to realize a specific future value. The present value of one future receipt is:

$$P_0 = \frac{P_N}{(1 + R)^N}$$

Thus, \$1,331,000 received three years from now would be worth \$1 million given a rate of return of 10 percent and annual compounding:

$$P_0 = \frac{P_N}{(1 + R)^N}$$
$$P_0 = \frac{\$1{,}331{,}000}{(1.10)^3} = \$1{,}000{,}000$$

The amount of \$1,340,095.64 received three years from now (M = 3), would be worth \$1 million given a 5 percent semiannual rate and two compoundings per year (n = 2 and $N = nM$ = (2)(3) = 6):

$$P_0 = \frac{P_N}{(1 + R)^N}$$
$$P_0 = \frac{\$1{,}340{,}095.64}{(1.05)^6} = \$1{,}000{,}000$$

The method of computing the present value is referred to as *discounting*, and the interest rate used to discount is referred to as the *discount rate.*

Present Value of an Annuity

When a fixed dollar annuity is received each period, the series is also called an annuity. If the first payment is received one period from the present, the annuity is referred to as an *ordinary annuity;* if the first payment is immediate, then the annuity is called an *annuity due.* The present value of an ordinary annuity is the sum of the present values of each annuity received:

$$P_0 = \frac{A}{(1+R)^1} + \frac{A}{(1+R)^2} + \frac{A}{(1+R)^3} + \cdots + \frac{A}{(1+R)^N}$$

$$P_0 = \sum_{t=1}^{N} \frac{A}{(1+R)^t}$$

$$P_0 = A \sum_{t=1}^{N} \frac{1}{(1+R)^t}$$

$\sum_{t=1}^{N} \frac{1}{(1+R)^t}$ is the present value of $1 received each period for *N* periods. It is referred to as the present value interest factor of an annuity, $PVIF_a$. $PVIF_a$ is equal to:

$$PVIF_a = \sum_{t=1}^{N} \frac{1}{(1+R)^t} = \left[\frac{1-(1/(1+R)^N}{R}\right]$$

Thus, an investor who received $100 at the end of each year for three years would have an investment currently worth $248.69 given a discount rate of 10 percent:

Year	0	0	1	2	3
A	$90.91	$100/(1.10)^1	$100.00		
A	$82.64	$100/(1.10)^2		$100.00	
A	$75.13	$100/(1.10)^3			$100.00
	$248.69	Present Value = P_0			

If the investment paid $50 every six months and the appropriate six-month rate were 5 percent, then the present value of the $50 annuity would be $253.78:

Year		0	0	0.5	1.0	1.5	2.0	2.5	3.0
	Number of Semiannual Periods from Present	**0**	**0**	**1**	**2**	**3**	**4**	**5**	**6**
	A	\$47.62	$\$50/(1.05)^1$	\$50					
	A	\$45.35	$\$50/(1.05)^2$		\$50				
	A	\$43.19	$\$50/(1.05)^3$			\$50			
	A	\$41.14	$\$50/(1.05)^4$				\$50		
	A	\$39.18	$\$50/(1.05)^5$					\$50	
	A	\$37.31	$\$50/(1.05)^6$						\$50
		\$253.78	Present Value = P_0						

Bond Valuation

Like the value of any asset, the value of a bond is equal to the sum of the present values of its future cash flow:

(3.1)

where

- V^B_0 = the value or price of the bond
- CF_t = the bond's expected cash flow in period t, including both coupon income and repayment of principal
- R = the discount rate
- N = the term to maturity on the bond

The discount rate is the required rate, that is, the rate investors require to buy the bond. This rate is typically estimated by determining the rate on a security with comparable features: same risk, liquidity, taxability, and maturity.

Many bonds pay a fixed coupon interest each period, with the principal repaid at maturity. The coupon payment, C, is typically quoted in terms of the bond's coupon rate, C^R. The coupon rate is the contractual rate the issuer agrees to pay on the bond. This rate is often expressed as a proportion of the bond's face value (or par) and is usually stated on an annual basis. Thus, a bond with a face value (F) of $1,000 and a 10 percent coupon rate would pay an annual coupon of $100 each year for the life of the bond: $C = C^R F$ = (0.10)($1,000) = $100. The value of a bond paying a fixed coupon interest each year (annual coupon payment) and the principal at maturity (M), in turn, would be

(3.2)

The fixed coupon payments are an annuity, and their value can be found by computing the present value of an annuity. The value of the principal payment, in turn, can be found by simply computing the present value of the principal:

Thus, if investors require a 10 percent annual rate of return on a 10-year, corporate bond paying a coupon equal to 9 percent of par each year and a principal of $1,000 at maturity (*N* = *M* = 10 years), then they would price the bond at $938.55:

Bond Price Relations

In the above example, the value of the bond is not equal to its par value. This can be explained by the fact that the discount rate and coupon rate are different. Specifically, for investors in the above case to obtain the 10 percent rate per year from a bond promising to pay an annual rate of C^R = 9 percent of par, they would have to buy the bond at a value, or price, below par: The bond would have to be purchased at a discount from its par, $V^B_0 < F$. In contrast, if the coupon rate is equal to the discount rate (i.e., R = 9 percent), then the bond's value would be equal to its par value, $V^B_0 = F$. In this case, investors would be willing to pay $1,000 for this bond, with each investor receiving $90 each year in coupons. Finally, if the required rate is lower than the coupon rate, then investors would be willing to pay a premium over par for the bond, $V^B_0 > F$. This might occur if bonds with comparable features were trading at rates below 9 percent. In this case, investors would be willing to pay a price above $1,000 for a bond with a coupon rate of 9 percent.

In addition to the above relations, the relation between the coupon rate and required rate also explains how the bond's value changes over time. If the required rate is constant over time, and if the coupon rate is equal to it (i.e., the bond is priced at par), then the value of the bond will always be equal to its face value throughout the life of the bond. This is illustrated in Exhibit 3.1 by the horizontal line that shows that the value of the 9 percent coupon bond is always equal to the par value. Here investors would pay

$1,000 regardless of the terms to maturity. On the other hand, if the required rate is constant over time and the coupon rate is less (i.e., the bond is priced at a discount), then the value of the bond will increase as it approaches maturity; if the required rate is constant and the coupon rate is greater (i.e., the bond is priced at a premium), then the value of the bond will decrease as it approaches maturity. These relationships are also illustrated in Exhibit 3.1.

	Discount Bond	Par Bond	Premium Bond
	Price of Bond	Price of Bond	Price of Bond
Year	Selling to Yield 11%	Selling to Yield 9%	Selling to Yield 7%
10	$882.22	$1,000.00	$1,140.47
9	$889.26	$1,000.00	$1,130.30
8	$897.08	$1,000.00	$1,119.43
7	$905.76	$1,000.00	$1,107.49
6	$915.39	$1,000.00	$1,095.33
5	$926.08	$1,000.00	$1,082.00
4	$937.95	$1,000.00	$1,067.74
3	$951.13	$1,000.00	$1,052.49
2	$965.75	$1,000.00	$1,036.16
1	$981.98	$1,000.00	$1,018.69
0	$1,000.00	$1,000.00	$1,000.00

EXHIBIT 3.1 The Value Over Time of an Original 10-Year, 9 Percent Annual Coupon Bond Selling at Par, Discount, and Premium

Another bond relationship to note is the inverse relation between the price of the bond and its rate of return. That is, given known coupon and principal payments, the only way an investor can obtain a higher rate of return on a bond is for its price (value) to be lower. In contrast, the only way for a bond to yield a lower rate is for its price to be higher. Thus, an inverse relationship exists between the price of a bond and its rate of return.

Finally, it should be noted that a bond's price sensitivity to interest rate changes and will be greater the longer its maturity and the lower its coupon rate. Specifically, the greater the bond's term to maturity, the greater its price sensitivity to a given change in interest rates. This relationship can be seen by comparing the price sensitivity to interest rate changes of the 10-year, 9 percent coupon bond in our above example with a 1-year, 9 percent coupon bond. If the required rate is 10 percent for both bonds, then the 10-year bond would trade at $938.55, whereas the 1-year bond would trade at $990.91 ($1,090/1.10). If the interest rate decreases to 9 percent for each bond (a 10 percent change in rates), both bonds would increase in price to $1,000. For the 10-year bond, the percentage increase in price would be 6.55 percent [($1,000 – $938.55)/$938.55], whereas the percentage increase for the 1-year bond would be only 0.9 percent. Thus, the 10-year bond's price is more sensitive to the interest rate change than the 1-year bond. Similarly, a lower coupon bond's price is more responsive to a given interest rate change than the price of the higher coupon bond.

Valuing Bonds with Different Cash Flows and Compounding Frequencies

Equation (3.2) defines the value of a bond that pays coupons on an annual basis and a principal at maturity. Bonds, of course, differ in the frequency in which they pay coupons each year, and many bonds have maturities less than one year. Also, when investors buy bonds, they often do so at noncoupon dates. Equation (3.2), therefore, needs to be adjusted to take these practical factors into account.

Semiannual Coupon Payments

Many bonds pay coupon interest semiannually. When bonds make semiannual payments, three adjustments to Equation (3.2) are necessary: (1) The number of periods is doubled; (2) the annual coupon rate is halved; (3) the annual discount rate is halved. Thus, if our illustrative 10-year, 9 percent coupon bond trading at a quoted annual rate of 10 percent paid interest semiannually instead of annually, it would be worth \$937.69:

Note that the rule for valuing semiannual bonds is easily extended to valuing bonds paying interest even more frequently. For example, to determine the value of a bond paying interest four times a year, we would quadruple the periods and quarter the annual coupon payment and discount rate. In general, if we let n be equal to the number of payments per year (i.e., the compoundings per year), M be equal to the maturity in years, R^A be the discount rate quoted on an annual basis (simple annual rate), and R be equal to the periodic rate, then we can express the general formula for valuing a bond as follows:

(3.3)

where:

- C^A = annual coupon = $(C^R)(F)$
- n = number of payments per year
- Periodic coupon = annual coupon/n
- M = term to maturity in years
- N = number of periods to maturity = $(n)(M)$
- Required periodic rate = R = annual rate/n = R^A/n

Valuing Zero-Coupon Bonds with Maturities Less than One Year

Some bonds do not make any periodic coupon payments. Instead, the investor realizes interest as the difference between the maturity value and the purchase price. These bonds are called *zero-coupon bonds* (also called *zeros* and *pure discount bonds*, PDB). The value of a zero-coupon bond is

For example, a zero-coupon bond maturing in 10 years and paying a maturing value of \$1,000 would be valued at \$385.54 if the required rate is 10 percent and annual compound is assumed:

If the convention is to double the number of years and half the annual discount rate, then the bond would be valued at \$376.89 to yield a semiannual rate of 5 percent, simple annual rate of 10 percent, and effective annual rate of 10.25 percent (= $1.05)^2 - 1$):

Many *zero-coupon bonds* have maturities less than a year. In valuing such bonds, the convention is to discount by using an annual rate and to express the bond's maturity as a proportion of a year. Thus, on March 1 a zero-coupon bond promising to pay \$100 on September 1 (184 days) and trading at an annual discount rate of 8 percent would be worth \$96.19:

Valuing Bonds at Noncoupon Dates

The above equations for pricing a bond are for valuing at dates in which the coupons are to be paid in exactly one period. However, most bonds purchased are not bought on coupon dates, but rather at dates between coupon dates. An investor who purchases a bond between coupon payments must compensate the seller for the coupon interest earned from the time of the last coupon payment to the settlement date of the bond. This amount is known as *accrued interest* (*AI*). (An exception to this rule would occur when a bond is in default. Such a bond is said to be quoted flat, that is, without accrued interest.) The formula for determining accrued interest is:

For U.S. Treasury coupon securities, the convention is to use the actual number of days since the last coupon date and the actual number of days between coupon payments: an actual/actual ratio. For example, consider a T-note whose last coupon payment was on March 1 and whose next coupon is six months later on September 1. Suppose the note is purchased with a settlement date of July 20. The actual number of days in the coupon period (sometime referred to as the basis) is 184 days, and the actual number of days between coupons is 43:

- July 20 to July 31 = 11 days.
- August = 31 days.
- September 1 = 1 day.
- Total = 43 days.

For corporate, agency, and municipal bonds, the practice is to use 30-day months and a 360-day year: 30/360 ratio for which each month is assumed to have 30 days and each year is assumed to have 360 days. If the preceding T-note were a corporate credit with a 30/360 day count convention, then the number of days in the coupon period would be 180 and the days between coupons would be 41:

- Remainder of July = 10 days.
- August = 30 days.
- September 1 = 1 day.
- Total = 41 days.

In trading bonds on a noncoupon date, the amount the buyer pays to the seller is the agreed-upon price plus the accrued interest. This amount is often called the *full-price* or *dirty price.* The price of a bond without accrued interest is called the *clean price:*

The full price of the bond can be found by doing the following:

1. Move to the next coupon date and determine the value of the bond at that date based on the future coupons.
2. Add the coupon at the next coupon date to the value of the bond.
3. Discount the bond value plus coupon back to the current date.

Thus, the 8 percent corporate bond maturing in 2019, purchased with a settlement date of July 20, 2013, at a required yield of 10 percent would have value of 91.693586 per $100 face at the next coupon date. Discounting the sum of that value and the $4 coupon back to the settlement date (0.227778 years) yields the full-bond price of $94.636:

- Value of the bond at next coupon date in 41days or 41/180 = 0.227778 year.
- Value of the bond at the next coupon date plus coupon paid at that date.

- Current value of the bond—full price:

The full or dirty price of $94.636 includes the portion of the coupon interest the buyer will receive but the seller has earned. Even though the price the buyer pays the seller is the full price, in the United States the convention is to quote a bond's clean price. In this example, given that there are 41 days to the next coupon and 180 days in the coupon period, the number of days from the last coupon is 139 (= 180 - 41). The accrued interest (*AI*) per $100 par is $3.0222, and the clean price or flat price is $91.547 (full price minus the *AI*):

Exhibit 3.2 shows Bloomberg Description and Yield Analysis (YAS) screens for a Procter & Gamble bond that pays an annual coupon of 2.3 percent, principal of $1,000, and matures on 2/6/2022. On 7/5/2013, the bond was trading at 96.335 per $100 face value (clean price). As shown on the bond's YAS screen, the bond has a settlement date of 7/10/2013, 154 days of accrued interest on that date of $9.84 (for $1,000 face), and an invoice price of $973.19. Finally, the bond yields a rate of return of 2.7834 percent; that is, 2.7834 percent is the discount rate that equates the price of the bond to the present value of its semiannual coupon payments and principal payment at 2/6/2022.

EXHIBIT 3.2 P&G Bond, Bloomberg Screens: DES and YAS

Price Quotes, Fractions, and Basis Points

Whereas many corporate bonds pay principals of $1,000, this is not the case for many non-corporate bonds and other fixed income securities. As a result, many traders quote bond prices as a percentage of their par value. For example, if a bond is selling at par, it would be quoted at 100 (100 percent of par); thus, a bond with a face value of $10,000 and quoted at 80 1/8 would be selling at (0.80125)($10,000) = $8,012.50. When a bond's price is quoted as a percentage of its par, the quote is usually expressed in points and fractions of a point, with each point equal to $1. Thus, a quote of 97 points means that the bond is selling for $97 for each $100 of par. The fractions of points differ among bonds. Fractions are either in thirds, eighths, quarters, halves, or 64ths. On a $100 basis, a 1/2 point is $0.50 and a 1/32 point is $0.03125. A price quote of 97 4/32 (97 – 4) is 97.125 for a bond with a 100 face value. It should also be noted that when the yield on a bond or other security changes over a short period, such as a day, the yield and subsequent price changes are usually quite small. As a result, fractions on yields are often quoted in terms of basis points (bp). A bp is equal to 1/100 of a percentage point, or 100 bps = 1 percent. Thus, 6.5 percent may be quoted as 6 percent plus 50 bps or 650 bps, and an increase in yield from 6.5 percent to 6.55 percent would represent an increase of 5 bps.

Exhibit 3.3 shows the Bloomberg FIT screen showing the prices and yields of U.S. Treasury bonds, Treasury notes, and Treasury bills that were recently issued and actively traded as of 2/15/2013. As shown on the screen, there is an approximate 10-year T-note paying a coupon of 2 percent. The Bloomberg description screen (DES) for the Treasury provides more details; specifically, the bond was issued on 2/15/13, matures on 2/15/23, pays a 2 percent coupon semiannually, its average quoted bid and ask prices from dealers (ALLQ screen) are 94-11 and 94-12. The Bloomberg yield analysis (YAS) screen shows that for an investment period from the settlement date of 7/8/13 to the 2/15/23 maturity, the yield or total return on the bond (with semiannual compounding) is 2.675 percent given a clean price of

94-10 (94.3125 per $100 face); the full price is $951.03, and the accrued interest is 7.90 per $1,000 face.

EXHIBIT 3.3 Bloomberg Price Quotes and Descriptions of U.S. Treasury Securities: Bloomberg FIT, YAS, Cash Flows (CSHF), Composite Quotes (ALLQ)

The Yield to Maturity and Other Rates of Return Measures for Bonds

The financial markets serve as conduits through which funds are distributed from borrowers to lenders. The allocation of funds is determined by the relative rates paid on bonds, loans, and other financial securities, with the differences in rates among claims being determined by risk, maturity, and other factors that serve to differentiate the claims. There are a number of different measures of the rates of return on bonds and loans. Some measures, for example, determine annual rates based on cash flows received over 365 days, whereas others use 360 days; some measures determine rates that include the compounding of cash flows, whereas some do not; and some measures include capital gains and losses, whereas others exclude price changes. In this section, we examine some of the measures of rates of return, including the most common measure—the yield to maturity—and in subsequent sections we look at three other important rate measures for bonds—the spot rate, the total return, and the geometric mean.

Common Measures of Rates of Return

When the term “rate of return” is used, it can mean a number of different rates, including the interest rate, coupon rate, current yield, or discount yield. The term *interest rate* is sometimes referred to the price a borrower pays a lender for a loan. Unlike other prices, this price of credit is expressed as the ratio of the cost or fee for borrowing and the amount borrowed. This price is typically expressed as an annual percentage of the loan (even if the loan is for less than one year). Today, financial economists often refer to the yield to maturity on a bond as the interest rate. In this book, the term interest rate will mean yield to maturity.

Another measure of rate of return is a bond's *coupon rate*. As noted in the last section, the coupon rate, C^R, is the contractual rate the issuer agrees to pay each period. It is usually expressed as a proportion of the annual coupon payment to the bond's face value:

Unless the bond is purchased at par, the coupon rate is not a good measure of the bond's rate of return because it fails to take into account the price paid for the bond.

In examining corporate bond quotes, the *current yield* on a bond is often provided. This rate is computed as the ratio of the bond's annual coupon to its current price. This measure provides a quick estimate of a bond's rate of return, but in many cases not an accurate one because it does not capture price changes. The current yield is a good approximation to the bond's yield, if the bond's price is selling at or near its face value or if it has a long maturity. That is, we noted earlier that if a bond is selling at par, its coupon rate is equal to the discount rate. In this case, the current yield is equal to the bond's yield to maturity. Thus, the closer the bond's price is to its face value, the closer the current yield is to the bond's yield to maturity. As for maturity, note that a coupon bond with no maturity or repayment of principal, known as a *perpetuity* or *consul*, pays a fixed amount of coupons forever. As shown in Exhibit 3.4, the value of such a bond is:

If the bond is priced in the market to equal V^B_0, then the rate on the bond would be equal to the current yield: $R = C/V^B_0$. Thus, when a coupon bond has a long-term maturity (e.g., 20 years), then it is similar to a perpetuity, making its current yield a good approximation of its rate of return.

EXHIBIT 3.4 Value of Perpetuity

Finally, the *discount yield* is the bond's return expressed as a proportion of its face value. For example, a one-year zero-coupon bond costing $900 and paying a par value of $1,000 yields $100 in interest and a discount yield of 10 percent:

The discount yield used to be the rate frequently quoted by financial institutions on their loans (because the discount rate is lower than a rate quoted on the borrowed amount). The difficulty with this rate measure is that it does not capture the conceptual notion of the rate of return being the rate at which the investment grows. In this example, the $900 bond investment grew at a rate of over 11 percent, not 10 percent:

Because of tradition, the rates on Treasury bills are quoted by dealers in terms of the bills' discount yield (R_D). Whereas Treasury bills have maturities less than one year, the discount yields are quoted on an annualized basis. Dealers quoting the annualized rates use a day count convention of actual days to maturity but with a 360-day year:

Given the dealer's discount yield, the bid or ask price can be obtained by solving the yield equation for the bond's price, P_0. Doing this yields:

Yield to Maturity

The most widely used measure of a bond's rate of return is the *yield to maturity* (YTM). As noted previously, the YTM, or simply the yield, is the rate that equates the purchase price of the bond, P^B_0, with the present value of its future cash flows. Mathematically, the YTM (y) is found by solving the following equation for y (YTM):

(3.4)

The YTM is analogous to the internal rate of return used in capital budgeting. It is a measure of the rate at which the investment grows. From our first example, if the 10-year, 9 percent annual coupon bond were actually trading in the market for $938.55, then the YTM on the bond would be 10 percent. The 2.3 percent P&G bond with a maturity of 2/16/2022 shown in Exhibit 3.2, in turn, is priced at 96.335 (per $100 face value). At the 7/10/13 settlement date, the cost of the bond (per $1,000 face value) is $973.19 (equal to the clean price of $963.35 plus 9.84 of 154 days of accrued interest). Based on this cost and using semiannual compounding the annualized yield of the P&G bond is 2.7824 percent. Unlike the current yield, the YTM incorporates all of the bonds cash flows (CFs). It also assumes the bond is held to maturity and that of all CFs from the bond are reinvested to maturity at the calculated YTM.

Estimating YTM: Average Rate to Maturity

If the cash flows on the bond (coupons and principal) are not equal, then Equation (3.4) cannot be solved directly for the YTM. Alternatively, one must use an iterative (trial and error) procedure: substituting different y values into Equation (3.4), until that y is found that equates the present value of the bond's cash flows to the market price. An estimate of the YTM, however, can be found using the bond's *average rate to maturity* (ARTM; also referred to as the *yield approximation formula*). This measure determines the rate as the average return per year as a proportion of the average price of the bond per year. For a coupon bond with a principal paid at maturity, the average return per year on the bond is its annual coupon plus its average annual capital gain. For a bond with an M-year maturity, its average gain is calculated as the total capital gain realized at maturity divided by the number of years to maturity: $(F - P^B{}_0)/M$. The average price of the bond is computed as the average of two known prices, the current price and the price at maturity (F): $(F + P^B{}_0)/2$. Thus, the ARTM is:

(3.5)

The ARTM for the 10-year, 9 percent annual coupon bond trading at $938.55 is 0.0992:

Bond Equivalent Yields

The YTM calculated above represents the yield for the period (in the above example this was an annual rate, given annual coupons). If a bond's *CF*s were semiannual, then solving Equation (3.4) for y would yield a six-month rate; if the *CF*s were monthly, then solving (3.4) for y would yield a monthly rate. To obtain a *simple annualized rate* (with no compounding), y^A, one needs to multiply the periodic rate, y, by the number of periods in the year. Thus, if a 10-year bond paying $45 every six months and $1,000 at maturity were selling for $937.69, its six-month yield would be 0.05 and its simple annualized rate, y^A, would be 10 percent:

In this example, the simple annualized rate is obtained by determining the periodic rate on a bond paying coupons semiannually and then multiplying by two. Because Treasury bonds and many corporate bonds pay coupons semiannually, the rate obtained by multiplying the semiannual periodic rate by two is called the *bond-equivalent yield.* Bonds with different payment frequencies often have their rates expressed in terms of their bond-equivalent yields so that their rates can be compared to each other on a common basis. This bond-equivalent yield, though, does not take into account the reinvestment of the bond's cash flows during the year. Therefore, it underestimates the actual rate of return earned. Thus, an investor earning 5 percent semiannually would have $1.05 after six months from a $1 investment that she can reinvest for the next six months. If she reinvests at 5 percent, then her annual rate would be 10.25 percent (= (1.05)(1.05) − 1 = $(1.05)^2$ – 1), not 10 percent. As noted earlier, the 10.25 percent annual rate, which takes into account compounding, is known as the effective rate.

Yield to Call, Yield to Put, and Yield to Worst

Yield to Call

Many bonds have a call feature that allows the issuer to buy back the bond at a specific price known as the call price (*CP*). Given a bond with a call option, the *yield to call* (YTC) is the rate obtained by assuming the bond is called on the call date. Like the YTM, the YTC is found by solving for the rate that equates the present value of the *CF*s to the market price:

where:

- CP = call price
- N_{CD} = number of periods to the call date

Thus, a 10-year, 9 percent coupon bond callable in 5 years at a call price of $1,100, paying interest semiannually and trading at $937.69, would have a YTM of 10 percent and an annualized YTC of 12.2115 percent:

Yield to Put

An issue can be putable, allowing the bondholder the right to sell the bond back to the issuer at a specified price, known as the put price (*PP*). As with callable bonds, putable bonds can have a constant put price or a put schedule. When a bond is putable, the convention is to calculate the yield to put (YTP). Like the YTM and YTC, the YTP is found by solving for the rate that equates the present value of the *CF*s to the market price:

where:

- PP = put price
- N_{PD} = number of periods to the put date

A 10-year, 9 percent coupon bond, first putable in five years at a put price of $950, paying interest semiannually and trading at $937.69, would have an annualized YTP of 9.807741 percent:

Yield to Worst

Many investors calculate the YTC for all possible call dates and the YTP for all possible put dates, as well as the YTM. They then select the lowest of the yields as their yield return measure. The lowest yield is sometimes referred to as the *yield to worst*. Exhibit 3.5 shows the Bloomberg call schedule from the

Bloomberg's YAS screen (Call tab) for a 3.5 percent Dow Chemical bond with a maturity of 12/15/16. The Dow bond has a YTM of 3.766 percent, yield to the first call of 6.5866 percent, and a yield to worst of 3.766 percent.

EXHIBIT 3.5 Call Schedule and Yield-to-Call Calculations, Bloomberg YAS Screen (Call Tab)

Bond Portfolio Yields

The yield for a portfolio of bonds is found by solving the rate that will make the present value of the portfolio's cash flow equal to the market value of the portfolio. For example, a portfolio consisting of a two-year, 5 percent annual coupon bond priced at par (100) and a three-year, 10 percent annual coupon bond priced at 107.87 to yield 7 percent (YTM) would generate a three-year cash flow of $15, $115, and $110 and would have a portfolio market value of $207.87. The rate that equates this portfolio's cash flow to its portfolio value is 6.2 percent:

Note that this yield is not the weighted average of the YTMs of the bonds comprising the portfolio. In this example, the weighted average (R_P) is 6.04 percent:

Rate on Zero-Coupon Bond

Whereas no algebraic solution for the YTM exists when a bond pays coupons and principal that are not equal, a solution does exist in the case of a zero-coupon bond or pure discount bonds (PDBs; we will use both expressions) in which there is only one cash flow (F). That is

(3.6)

where M = maturity in years. Thus, a zero-coupon bond with a par value of $1,000, a maturity of three years, and trading for $800 would have an annualized YTM of 7.72 percent:

If the convention is to assume semiannual compounding, then the semiannual YTM would be 3.798 percent and the simple annual rate or bond-equivalent yield would be 7.55782 percent:

Similarly, a pure discount bond paying $100 at the end of 182 days and trading at $96 would yield an annual rate of 8.53 percent (using a 365-day year):

Total Return

Equation (3.6) provides the formula for finding the YTM for a zero-discount bond. A useful extension of

Equation (3.6) is the *total return* (*TR*), also called the *realized return* and *average realized return* (ARR). The total return is the yield obtained by assuming the cash flows are reinvested to the investor's horizon at an assumed reinvestment rate and at the horizon the bond is sold at an assumed rate given the horizon is not maturity or pays its principal if the horizon is maturity. The *TR* is computed by first determining the investor's horizon, HD; next, finding the HD value, defined as the total funds the investor would have at HD; and third, solving for the *TR* using a formula for the zero-coupon bond [Equation (3.6)].

To illustrate, suppose an investor buys a four-year, 10 percent coupon bond, paying coupons annually, and selling at its par value of $1,000. Assume the investor needs cash at the end of year three (HD = 3), is certain he can reinvest the coupons during the period in securities yielding 10 percent, and expects to sell the bond at his HD at a rate of 10 percent. To determine the investor's *TR*, we first need to find the HD value. This value is equal to the price the investor obtains from selling the bond at HD and the value of the coupons at the HD. In this case, the investor, at his HD, will be able to sell a one-year bond paying a $100 coupon and a $1,000 par at maturity for $1,000, given an assumed discount rate of 10 percent:

Also at the HD, the $100 coupon paid at the end of the first year will be worth $121, given the assumption it can be reinvested at 10 percent for two years and there is annual compounding, $\$100(1.10)^2 =$ $121, and the $100 received at the end of year two will, in turn, be worth $110 in cash at the HD, $100(1.10) = $110. Finally, at the HD the investor would receive his third coupon of $100. Combined, the investor would have $1,331 in cash at the HD: HD value = $1,331. The horizon value of $1,330 consists of a bond valued at $1,000, coupons of $300, and interest earned from reinvesting coupons of $31 (HD coupon value – total coupon received = $331 – $300 – $31; see Exhibit 3.6). Note that if the rates at which coupons can be reinvested (reinvestment rates) are the same (as assumed in this example), then the coupon value at the horizon would be equal to the periodic coupon times the future value of an annuity of (FVIF_a):

EXHIBIT 3.6 Total Realized Return

The reinvestment income or interest earned from reinvesting coupons (interest on interest), in turn, is equal to the coupon value at HD minus the total coupons received, $(N)(C)$:

Given the HD value of $1,331, the *TR* is found in the same way as the YTM for a zero-coupon bond. In this case, a $1,000 investment in a bond returning $1,331 at the end of three years yields a total return of 10 percent:

(3.7)

Note that the total return is the rate that makes the initial investment grow to equal the horizon value. That is, $1,000 grows at an annual rate of 10 percent to equal the horizon value of $1,331 at the end of year three:

The total return can be applied to any period length. For example, if the four-year bond purchased by the investor made semiannual payments and the six-month yield were at 5 percent (a simple annual yield of 10 percent and an effective annual yield of 10.25 percent [= $(1.05)^2$ –1)], then the investor's coupon value, reinvestment income, price at HD, and HD value at his HD would respectively be $340.10, $40.10, $1,000, and $1,340.10 (see Exhibit 3.7):

Maturity = 4-years, annual coupon rate = 10%,
interest paid semiannually, par = $1,000, reinvestment rate = 5% semiannually, purchase price = $1,000, horizon = 3 years, bond expected to
sell at the HD at a 5% semiannual rate.

Horizon Value = $1,340, Semiannual Total Return = 5%, Simple Total Return = 10%, and Effective Total Rate = 10.25%:								
Year	0.0	0.5	1.0	1.5	2.0	2.5	3.0		Values
		$50.00						$50(1.05)^5$	$63.81
			$50.00					$50(1.05)^4$	$60.78
				$50.00				$50(1.05)^3$	$57.88
					$50.00			$50(1.05)^2$	$55.13
						$50.00		$50(1.05)^1$	$52.50

	$50.00	$50	$50.00
	$1,000.00	$50/1.05+ ($1,050/(1.05)^2	$1,000.00
		Horizon Value	$1,340.10

EXHIBIT 3.7 Total Realized Return with Semiannual Cash Flows

The investor's semiannual total return would be 5 percent, the simple annual rate would be 10 percent, and the effective annual rate would be 10.25 percent:

In this example, the semiannual *TR* of 5 percent is the same rate at which the bond was purchased; that is, a 10 percent coupon bond, paying interest twice a year, and selling at par, yields a semiannual YTM of 5 percent and bond-equivalent yield of 10 percent. In this case, obtaining a total return equal to the initial YTM should not be surprising because the coupons are assumed to be reinvested at the same semiannual rate as the initial YTM (5 percent) and the bond is also assumed to be sold at that rate as well (recall, the YTM measure assumes that all coupons are reinvested at the calculated YTM).

It should be noted that the yield on the Procter & Gamble bond shown on the YAS screen in Exhibit 3.2 is obtained by using a total return calculation and assuming the coupons are reinvested at the YTM.

Market Risk and Horizon Analysis

If the coupons in our above total return examples were expected to be reinvested at different rates or the bonds sold at a different YTM, then a total return equal to the initial YTM would not have been realized. For example, if the rates on all maturities were to increase from 10 percent to 12 percent (simple annual rate), just after the four-year, 10 percent bond was purchased at par, then the total semiannual total return would decrease to 4.8735 percent and the simple annual rate would be 9.7471 percent:

In this case, the rate increase has augmented the reinvestment income by $0.67 from to $48.10 to $48.77, but it has also lowered the horizon price by $18.33 from $1,000 to $981.67. As a result, the price decrease has reduced the total return more than the interest-on-interest increase has increased the realized return. Thus, the increase in rates from 10 percent to 12 percent lowers the total return from 10 percent to 9.7471 percent.

Exhibit 3.8 shows the Bloomberg Total Return Analysis (TRA) screen for the P&G bond paying a coupon of 2.3 percent and maturing on 2/6/22 with a settlement date of 7/10/13 and a selected one-year horizon. The screen shows the calculated price of the bond at the horizon at the yield reflecting the different yield shifts ranging from plus 75 basis points to minus 75 basis points and the total return based on the price, coupon, and interest on interest at the horizon.

EXHIBIT 3.8 Total Return, P&G Bond

The possibility of the actual return on a bond deviating from the expected return because of a change in interest rates is known as *market risk.* As illustrated in the above total return example, a change in interest rates has two effects on a bond's return. First, interest rate changes affect the price of a bond; this is referred to as *price risk.* If the investor's horizon is different from the bond's maturity date, then the investor will be uncertain about the price he will receive from selling the bond (if HD < *M*), or the price he will have to pay for a new bond (if HD > *M*). Also, as we noted earlier in discussing the properties of bonds, the price of a bond is inversely related to interest rates and is more price responsive to a change in interest rates if it has a longer term to maturity and its coupon rates are less. Thus, if interest rates change, the price effect on the total return will be negative (i.e., lower rates increase the horizon price and therefore the total return), with the effect being greater for bonds with greater terms to maturity and lower coupon rates. Secondly, interest rate changes affect the return the investor expects from reinvesting the coupon—*reinvestment risk.* If an investor buys a coupon bond, he automatically is subject to market risk. Thus, if interest rates change, the interest-on-interest effect on the total return will be direct (i.e., greater rates increase the reinvestment return and therefore the total return), with the effect being greater for bonds with greater coupon rates.

One way to evaluate market risk for a bond is to estimate the bond's total returns given different interest rate scenarios. Such analysis is known at *horizon analysis.* Moreover, by conducting horizon analysis on one or more bonds or bond portfolios, an investor or portfolio manager can project the performance of the bonds or portfolios and can compare different bonds or bond portfolios based on a planned investment horizon and expectations concerning the market. Exhibit 3.9 shows the total returns for a three-year horizon for four bonds with different coupons and maturities under three interest rate scenarios: yields stay at 7.5 percent, yields decrease to 5 percent, and yields increase to 10 percent. For each scenario, it is assumed the reinvestment rate and the rate for determining the horizon price of the bond are equal to the scenario yield. In terms of market risk, Bond A has the smallest deviations in total returns, with the lowest rate being 7.23 percent and the highest being 7.77 percent. Bond A's total return also decreases when rates decrease and increases when rates increase, suggesting Bond A's interest-on-interest effect dominates its price effect. In contrast, longer-term Bond D has the greatest market risk, with the range in total returns being 2.01 percent to 13.8 percent, and with its total return increasing when rates decrease and decreasing when rates increase, implying its price effect dominates its interest-on-interest effect. Ultimately, which bond an investor should select depends on her expectations about futures interest rates and the degree of market risk she wants to assume. Horizon analysis, though, is a useful tool for analyzing market risk and facilitating bond investment decisions.

Total returns for a 3-year horizon for four bonds under three interest rate scenarios: yields stay at 7.5%,
yields decrease to 5%, and yields increase to 10%. For each scenario it is assumed the reinvestment rate and the rate for determining the horizon price of the bond are equal to the scenario yield.

				Total Return	Total Return	Total Return
Bond	Annual Coupon Rate	Maturity	Price at 7.5%	5.00%	7.50%	10.00%
A	10.00%	3 yrs	106.61	7.23%	7.50%	7.77%
B	7.50%	5 yrs	100.00	8.59%	7.50%	6.48%
C	10.00%	10 yrs	117.37	10.85%	7.50%	4.47%
D	5.00%	15 yrs	77.71	13.80%	7.50%	2.01%
	Horizon = 3 years					
	Semi-annual payments					
	Yield same on all matuirites					

EXHIBIT 3.9 Horizon Analysis

BLOOMBERG BOND YIELD AND RETURN ANALYSIS SCREENS

Bloomberg Bond Yield Analysis Screen: YAS calculates the yield on the bond given the bond's prices using different market conventions: Current yield = coupon / price; street convention = yield based on the convention the bond is calculated on in the market; corporate bond equivalent, and after-tax yield (see Exhibits 3.2 and 3.3).

Bond Payment Schedule: The **CSHF** screen shows the bond's cash-flow schedule. On the CSHF screen, you can select either the cash flow or its present value.

FIHZ Screen: The BLOOMBERG FIHZ Screen calculates the total return on a selected bond. You can select the horizon period, the discount yield at the HD, and the reinvestment rate.

COMP Screen: Historical Comparative Return Screen compares a bond's historical total returns with a comparable bond index and other selected bonds.

Bloomberg Total Return Screen: The **TRA** screen calculates total returns given different interest rate cases. You can select different horizons, reinvestment rates (semiannual rate, S/A Reinv), and yield shifts (YLD SHFT). See Exhibit 3.8.

FISA: FISA compares and analyzes the profit/loss and percentage return of a selected bond under different scenarios based on changes in specific yields, dates, reinvestment rates, or specified yield curve shifts.

See Bloomberg Web Exhibit 3.1.

Spot Rates and Equilibrium Prices

The rate on a zero-coupon bond is called the *spot rate*. We previously examined how bonds are valued by discounting their cash flows at a common discount rate. Given different spot rates on similar bonds with different maturities, the correct approach to valuing a bond, however, is to price it by discounting each of the bond's *CF*s by the appropriate spot rates for that period (S_t). Theoretically, if the market does not price a bond with spot rates, arbitrageurs would be able to realize a free lunch by buying the bond and stripping it into a number of zero-coupon bonds or by buying strip bonds and bundling them into a coupon bond to sell. Thus, in the absence of arbitrage, the *equilibrium price* of a bond is determined by discounting each of its *CF*s by its appropriate spot rates.

To illustrate this relationship, suppose there are three risk-free zero-coupon bonds, each with principals of $100 and trading at annualized spot rates of S_1 = 7 percent, S_2 = 8 percent, and S_3 = 9 percent, respectively. If we discount the *CF* of a three-year, 8 percent coupon bond, paying an $8 coupon annually and a principal of $100 at maturity at these spot rates, its equilibrium price, P^*_0, would be $97.73:

Suppose this coupon bond is trading in the market at a price (P^B_0) of $95.03 to yield 10 percent:

At the price of $95.03, an arbitrageur could buy the bond, then strip it into three risk-free zero-coupon bonds: a one-year zero paying $8 at maturity, a two-year zero paying $8 at maturity, and a three-year zero bond paying $108 at maturity. If the arbitrageur could sell the bonds at their appropriate spot rates, she would be able to realize an initial cash flow (CF_0) from the sale of 97.73 and a risk-free profit of $2.70 (see Exhibit 3.10). Given this risk-free opportunity, this arbitrageur, as well as others, would exploit this strategy of buying and stripping the bond until the price of the coupon bond was bid up to equal its equilibrium price of $97.73.

EXHIBIT 3.10 Equilibrium Bond Price: Arbitrage for Underpriced Bond

However, if the 8 percent coupon bond were trading above its equilibrium price of $97.73, then arbitrageurs could profit by reversing the above strategy. For example, if the coupon bond were trading at $100, then arbitrageurs would be able to go into the market and buy proportions (assuming perfect divisibility) of the three pure discount bonds (8 percent of Bond 1, 8 percent of Bond 2, and 108 percent of Bond 3) at a cost of $97.73, and bundle them into one three-year, 8 percent coupon bond to be sold at $100. As shown in Exhibit 3.11, this strategy would result in a risk-free cash flow of $2.27.

EXHIBIT 3.11 Equilibrium Bond Price Arbitrage for Overpriced Bond

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BLOOMBERG SCREENS FOR STRIPS AND STRIP ANALYSIS

CSHF Screen: On the CSHF screen for a selected bond, one determines the present value of each payment using spot rates selected from a yield curve. See the CSHF screen for the Treasury security in Exhibit 3.3.

Strip Analysis Screen, SP: On the SP screen, one can analyze stripping a selected bond of its interest and principal payments.

Treasury Strips: U.S. Treasury Strip securities can be found from FIT, Strips Tab, and using the SECF screen.

See Bloomberg Web Exhibit 3.2.

Geometric Mean

Another useful measure of the return on a bond is its *geometric mean.* Conceptually, the geometric mean can be viewed as an average of current and future rates. To see this, consider one of our previous examples in which we computed a YTM of 7.72 percent for a zero-discount bond selling for $800 and paying $1,000 at the end of year three. The rate of 7.72 percent represents the annual rate at which $800 must grow to be worth $1,000 at the end of three years assuming annual compounding. If we do not restrict ourselves to the same rate in each year, then there are other ways $800 could grow to equal $1,000 at the end of three years. For example, suppose one-year bonds are currently trading at a 10 percent rate, a one-year bond purchased one year from the present is expected to yield 8 percent (R_{Mt} = R_{11} = 8 percent), and a one-year bond to be purchased two years from the present is expected to be 5.219 percent

($R_{Mt} = R_{12}$ = 5.219 percent). With these rates, $800 would grow to $1,000 at the end of year 3. Specifically, $800 after the first year would be $880 = $800(1.10), after the second, $950.40 = $800(1.10)(1.08), and after the third, $1,000 = $800(1.10)(1.08)(1.05219). Thus, an investment of $800 that yielded $1,000 at the end of three years could be thought of as an investment that yielded 10 percent the first year, 8 percent the second, and 5.219 percent the third. Moreover, 7.72 percent can be viewed not only as the annual rate at which $800 grows to equal $1,000, but also as the average of three rates: one-year rates today ($R_{Mt} = R_{10}$), one-year rates available one year from the present ($R_{Mt} = R_{11}$), and one-year rates available two years from the present ($R_{Mt} = R_{12}$):

(3.8)

Mathematically, the expression for the average rate on an M-year bond in terms of today's and future one-year rates (and assuming annual compounding) can be found by solving Equation (3.8) for YTM_M:

(3.9)

Equation (3.9) defines the rate of return on an M-year bond in terms of expected future rates. A more practical rate than an expected rate, however, is the implied forward rate.

Implied Forward Rate

An implied forward rate, f_{Mt}, is a future rate of return implied by the present interest rate structure. This rate can be attained by going long and short in current bonds. To see this, suppose the rate on a one-year, zero-coupon bond is 10 percent (i.e., spot rate is S_1 = 10 percent) and the rate on a similar two-year zero is S_2 = 9 percent. Knowing these current rates, we could solve for f_{11} in the equation below to determine the implied forward rate. That is:

With one-year and two-year zeros presently trading at 10 percent and 9 percent, respectively, the rate implied on 1-year bonds to be bought one year from the present is 8 percent. This 8 percent rate, however, is simply an algebraic result. This rate actually can be attained by implementing the following locking-in strategy:

1. Execute a short sale by borrowing the one-year bond and selling it at its market price of $909.09 = $1,000/1.10 (or borrow $909.09 at 10 percent).
2. With two-year bonds trading at $841.68 = $1,000/(1.09)^2$, buy $909.09/ $841.68 = 1.08 issues of the two-year bond.
3. At the end of the first year, cover the short sale by paying the holder of the one-year bond his principal of $1,000 (or repay loan).
4. At the end of the second year, receive the principal on the maturing two-year bond issue of (1.08) ($1,000) = $1,080.

With this locking-in strategy the investor does not make an investment until the end of the first year when he covers the short sale; in the present, the investor simply initiates the strategy. Thus, the investment of $1,000 is made at the end of the first year. In turn, the return on the investment is the principal payment of $1,080 on the 1.08 holdings of the two-year bonds that comes one year after the investment is made. Moreover, the rate of return on this 1-year investment is 8 percent [($1,080 -- $1,000)/ $1,000]. Hence, by using a locking-in strategy, an 8 percent rate of return on a one-year investment to be made one year in the future is attained, with the rate being the same rate obtained by solving algebraically for f_{11}.

Given the concept of implied forward rates, the geometric mean now can be formally defined as the geometric average of the current one-year spot rate and the implied forward rates:

(3.10)

Two points regarding the geometric mean should be noted. First, the geometric mean is not limited to one-year rates. That is, just as 7.72 percent can be thought of as an average of three one-year rates of 10 percent, 8 percent, and 5.219 percent, the implied rate on a two-year bond purchased at the end of one year, $f_{Mt} = f_{21}$, can be thought of as the average of one-year implied rates purchased one and two years, respectively, from now. Second, note that for bonds with maturities of less than one year, the same general formula for the geometric mean applies. For example, the annualized YTM on a zero-coupon maturing in 182 days (YTM_{182}) is equal to the geometric average of a current 91-day bond's annualized rate (YTM_{91}) and the annualized implied forward rate on a 91-day investment made 91 days from the present, $f_{91,91}$:[1]

BLOOMBERG FORWARD RATE CURVE MATRIX: FWCM

The FWCM screen can be used to find projected forward rates based on current interest rate.

Stock Valuation and Return

Stock Returns

An investor who has purchased common stock can expect to earn a possible return from the stock's periodic dividends, capital gains (or losses) when the stock is sold, and interest earned from reinvesting dividends. As noted, the stock's rate of return or yield is that rate that equates the present value of the stock's cash flow to its market price. For example, an investor planning to buy a stock currently priced in the market at $100 ($P_0$) and expected to pay dividends (d_t) of $10 in each of the next three years, and to sell for $100 at end of the third year (P_3), would expect to earn a rate of return of 10 percent; that is, the rate that equates the present value of the stock's expected cash flows to the market price of $100 is 10 percent:

Like the total return on a bond, the *total return on a stock* is obtained by assuming the expected cash flows (dividends) are reinvested to the investor's horizon at an assumed reinvestment rate and at the horizon the stock is valued at a forecasted expected price. In this case, the investor's total return would be 10 percent if she could reinvest the dividends at the discount rate of 10 percent and the stock was worth $100 at the investor's three-year horizon.

Year	**1**	**2**	**3**	**CF**	**Values**
	$10			$10(1.10)^2$	$12.10
		$10		$10(1.10)	$11.00
			$10	$10	$10.00
			$P^S_3 = \$100$	$100	$100
$P^S_0 = \$100$				Horizon Value	$133.10

Total return:

BLOOMBERG TOTAL RETURN ANALYSIS FOR STOCK: TRA

The TRA screen for a selected stock can be used to analyze the historical cumulative total returns for a selected stock.

Stock Valuation

Alternatively, if an investor knows the rate she requires on a stock, then she can determine the value of the stock (V^S_0) by discounting the stock's expected cash flows by her required return. Thus, if our above investor required a rate of return of 10 percent on the stock based on her assessment of the stock's risk, then she would value the stock at $100 and would consider the stock to be correctly priced if it were trading for $100 in the market:

If she required a rate of return of 15 percent, however, then she would value the stock at $88.58 and would consider it to be overpriced at $100 and not a good investment:

On the other hand, if the investor required a return of only 7.5 percent, then she would value the stock at $106.50 and consider it to be underpriced at $100 and therefore a good investment:

Note, that like bonds, there is an inverse relationship between the value of stock and the required return.

In addition to determining the required return, the valuation of stocks also requires estimating dividends that can be expected to change over time given changes in the company's earnings and investment prospects. In valuing stocks, many analysts try to determine the stock's *growth rate* in dividends over time, g_t. Given the estimated growth rate, expected dividends can then be related to previous dividends to determine the stock's value. For example, suppose instead of a constant annual dividend of $10 over the next three years, the investor in the above example estimates that the projected future revenue of the company will result in an annual growth rate of 3 percent in dividends for the next three years. Given this growth rate, in year 1 the investor would expect a dividend of $10.30 [= $10(1+.03)]; in year two, $10.61 [= $10.30(1.03) = $10(1.03)^2]; and in year 3, $10.93 [= $10.61(1.03) = $10(1.03)^3]. Suppose the investor also projects that the company would be worth $109.30 at the end of year three and estimates her required rate of return to be 10 percent. Given these estimates, the investor would value the stock at $108.46:

Based on her cash flow analysis, the investor would consider the stock to be underpriced if it were trading below $108.46 in the market and overpriced if it were trading above $108.46.

Note, if the investor can reinvest her expected dividends at the discount rate of 10 percent to her three-year horizon, then her total return would be equal to the required return of 10 percent:

Year	0	1	2	3	CF	Values
		\$10(1.03)			$\$10(1.03)(1.10)^2$	\$12.46
			$\$10(1.03)^2$		$\$10(1.03)^2$ (1.10)	\$11.67
				$\$10(1.03)^3$	$\$10(1.03)^3$	\$10.93
				$P^S{}_3$ = \$109.30	\$109.30	\$109.30
	$P^S{}_0$ = \$108.46				Horizon Value	\$144.36

Total return:

Fundamental Stock Valuation

The above example suggests that determining the value of a stock requires estimating the stock's growth rate in dividends, g, required return, k_e, and future price, $V^S{}_t$. Since the future price of a stock depends on the remaining future dividends, this valuation approach can be simplified by valuing the stock solely in terms of its future dividends. Thus, the stock value of a company expected to last N periods can be expressed as:

(3.11)

where: k_e = required rate ($k_e = R$).

This stock valuation expression can be simplified further by relating dividends in different periods to the growth rate in dividends: $d_t = d_{t-1}(1 + g_t)$, and by assuming the growth rate is constant over a certain number of periods. These assumptions allow one to relate dividends in one period all the way back to the current period. For example, if the growth rate in dividends is constant over the first T periods, as in our previous example (T = 3 years), then the dividend in period T can be related to the dividend in the current period, d_0:

If we assume dividends will grow at a constant rate, g, over the entire life of the stock of N years, then the value of the stock would be:

(3.12)

Valuing a stock solely in term of its dividends and growth rate in dividends (and not in terms of its expected future prices), requires that one estimate the life of the firm—N. Most companies have an indefinite life, with many expected to last many years. It is interesting that, as N gets large, Equation (3.12) for the value of the stock approaches a maximum value equal to $d_1/(k_e - g)$ at a discrete N value, such as 30 to 50 years (see Exhibit 3.4 for a proof). The N value where a variable approaches its optimum (maximum or minimum) is known as the asymptote or critical number and the relationship is described as an asymptotic relation. Thus, for a company expected to have a constant growth rate in dividends for the next 40 to 50 years—a profile that describes many large, blue-chip companies—the value of its stock would be:

Thus, if the investor in our above example expected the stock to grow at an annual rate of 3 percent for a long time and required a return of $k_e = 0.10$, she would value the stock at $147.14:

Exhibit 3.12 shows the relation between value of the stock and its life, N. The graph highlights the asymptotic relation in which the stock value increases at a decreasing rate, with the asymptote occurring at N^* equal to 60, where the maximum stock price is approaching $147: $d_1/(k_e - g)$ = $10(1.03)/(0.10 - 0.03) = $147. Note that the value of the stock would still be $147.14 even if the investor planned to sell the stock at a future horizon date, provided that the expected selling price at the investor's horizon is equal to the present value of the stock's future dividends. For example, in term of our previous example, if the investor's horizon were three years, then dividends in year four would be $11.25 [= $10(1.03)^4] and the value of the stock at year three would be $160.79. Discounting this value and the dividends back to the present yields a current value of the stock of $147.14:

EXHIBIT 3.12 Stock Values for Different *N*

In practice, estimating a stock's future dividend growth is very challenging, requiring knowledge of the company's operations, the external factors that affect the company, and the company's investment opportunities. The two most important factors influencing the equity value of a company, however, are its potential earnings from current operations and its potential investments. Exhibit 3.13 shows Bloomberg dividend per share (DVD) and earnings per share (ERN) screens for P&G stock. In Chapter 11, we examine how a company's operations and investment decisions determine its earnings, dividends, and dividend growth rates.

EXHIBIT 3.13 Bloomberg Dividends and Earnings for P&G: Bloomberg DVD and ERN Screens, 7/5/13

Two-Stage and Three-Stage Growth Models

The assumption that a company can maintain a constant growth rate for a long time fits the profile of many well-established companies. However, companies in relatively new industries that may be in their early stages of development or companies in industries going through fundamental changes may experience different stages of growth. Valuing the stocks of these firms requires assuming different growth rates. The two most common models are the two-stage growth model and the three-stage growth model.

The two-stage growth model assumes that dividends will grow at an extraordinary growth rate of g_1 for N years and then grow at a steady-state rate of g_2 thereafter. The value of the stock given these assumptions is

where:

Thus, if a company's current dividends per share were d_0 = \$10, and as a result of its current and potential investment opportunities investors expected dividends to grow at a rate g_1 = 5 percent for three years and thereafter at g_2 = 3 percent, then they would value the stock to be \$155.34:

where:

The three-stage growth model assumes a period with an extraordinary growth rate of g_1, a transitional period in which the growth rate moves from the extraordinary rate to the steady-state rate, and then a steady-state period in which the stock dividend grows at the steady-state rate. During the transitional period, it is generally assumed that the growth rate in dividends will change at a constant rate. In terms of the above example, suppose investors expect the dividends to initially grow at an annual rate of 6 percent for three years, then steadily decline with growth rates of 5 percent, 4 percent, and 3 percent in years 4, 5, and 6, respectively, and then starting in year 6 to grow at a constant steady-state rate of 3 percent (see Exhibit 3.14). Given these assumptions, the investor would value the stock at $163.31:

EXHIBIT 3.14 Three-Stage Growth

Determining which growth-rate model an analyst should use to estimate the value of a stock depends on the type of the company being evaluated. As noted, companies in emerging industries such as new technology, for example, are typically characterized by multistage growth periods. For such companies, there is often an initial period of extraordinary growth in which the companies in the new industry are expanding their manufacturing and marketing base to meet the immense domestic and possibly world demand for their products. The length of this initial period can vary depending on capital requirements, the ease of entry of new firms into the industry, the emergence of subsequent technology, and the potential demand. The initial growth stage for General Motors, Ford, and Chrysler arguably lasted as long as 50 years (from 1920 to 1970), whereas the initial growth stage for companies in the consumer product industry may have been only 10 years. In contrast, companies in more mature industries, such as metals, manufacturing, food processing, or mining, may be better characterized by a constant growth rate model in which their dividends and earnings are expected to grow at a steady-state rate. However, it is important to remember that such companies can be influenced by external factors, both good and bad, which can transform them into an emerging industry again. For example, many analysts considered the banking industry to be mature and stable in the 1960s, and then to become again a growth industry as a result of the liberalization of banking laws and the emergence of new technology. Similarly, the fall of Communism in the 1980s or the rise of emerging economies like China and India in 2000 led to new markets for the products of many companies in mature industries.

The Bloomberg Three-Stage Growth Model—DDM

Growth rate stages can also be classified based on cross-sectional trends. The Bloomberg DDM screen, for example, determines the values of a loaded stock using a three-stage growth model. Based on the stock's growth rate, the program defaults to one of four growth stage scenarios: explosive growth, high growth, average growth, and slow/mature growth. This classification is based on the normalized distribution of the forecasted growth rate for all equities.

Explosive growth firms are at the high end of the distribution, with growth rates significantly above the normalized mean or median, whereas low/mature growth firms are those at the low end of the distribution. The Bloomberg DDM model initially sets the length of the growth stage to three years for explosive growth, five years for high growth, seven years for average growth, and nine years for slow growth. The growth rate defaults to the mean secular growth rate.

In the third stage—steady state or mature stage—the DDM model assumes the growth rate is equal to the retention rate (100 percent minus the dividend-payout rate) times the stock's required rate. The growth rates in the transition period are based on assuming that the growth rate decreases annually from the rate in the growth stage period to the rate in the mature stage. In the Bloomberg DDM model, EPS for

FY1, FY2, and FY3 are based on consensus earnings projections. The EPS for the remaining years in the initial growth period reflect the long-term growth rate assumption: $EPS_{FY4} = EPS_{FY3}(1 + \text{growth rate})$, $EPS_{FY5} = EPS_{FY4}(1 + \text{growth rate})$ and so on.

Given EPS growth, dividends per share (DPS) for FY2 and FY3 and the growth years' periods are based on the current dividend-payout ratio (DPS/EPS). This ratio is multiplied by EPS in FY2 times the payout ratio to obtain the DPS in FY2. Similarly, DPS in FY3 is obtained by multiplying EPS in FY3 by the dividend-payout ratio [= $(DPS/EPS)(EPS_{FY3})$], and so on. The DDM model defaults to a dividend-payout ratio of 45 percent starting in the first year of the mature stage. For the transition period, the model assumes that the payout ratio moves to 45 percent (e.g., if payout ratio in the initial growth period is 20 percent, and the transition period is five years, then the payout ratio would increase by annual increments of 5 percent to reach 45 percent).

Finally, the DDM model defaults to a discount rate equal to the risk-free rate of the 10-year Treasury bond plus a risk premium (also known as the market's required rate of return) as the discount rate. Required returns on stock are examined in Chapter 9. The boxes on the DDM screen allow one to change the default assumptions (see Bloomberg exhibit box: "Bloomberg DDM Screen").

DDM Example

Exhibit 3.15 shows the DDM screen for Disney on 8/29/2013. The intrinsic value of the stock of approximately $44 is determined by discounting the flow of estimated DPS by a required rate based on the following estimates and assumptions:

1. As shown in the Bloomberg slide in Exhibit 3.15, the default model EPS is $3.37 for FY1, $3.91 for FY2, and $4.56 for FY3. These estimates are based on Bloomberg's consensus estimates drawn from the ANR screen.
2. For Disney, the length of the initial growth stage is seven years, with the annual growth rate estimated to be 9.94 percent for that period (see Bloomberg slide box). The length of the transitional stage is 10 years.
3. The mature growth period starts in year 20 with an assumed dividend payout ratio of 45 percent and with a growth rate of 5.636 percent. The mature growth rate is equal to the discount rate of 10.26

percent times the retention ratio: (1 − 0.45).

4. The transition period starts in year 11, with the growth rate decreasing by increments of 0.43 percent per year (= 9.94 percent − 5.636 percent/10) from the 9.94 percent to 5.636 percent. Column 3 in the table in Exhibit 3.15 shows the growth rates for each year (starting in year 4) and its corresponding EPS ($EPS_t = EPS_{t-1}(1 + g_t)$].
5. The dividend-payout ratio for the first 10 years (FY1 to FY3 plus the seven year growth year) is 0.217 (Column 5) and is equal to the current dividend-payout ratio (= DPS_{FY1}/EPS_{FY1} = $0.733/$3.374).
6. Starting in year 11 (the first year of the transition period), the payout ratios start to increase by annual increments of 2.33 percent [= (45 percent − 21.7 percent)/10] to reach the model's assume payout rate 45 percent at year 20, the last year of the transition period.
7. From year 20 on, the dividend payout stays at 45 percent.
8. The dividends per share each year are equal to the EPS times the payout ratio. They are shown for each year in Column 6 of the exhibit table.
9. The value of Disney's equity in year 20 is determined in the DDM by using the constant growth model ($V = d/(k_e - g)$). As shown in the table, dividends in year 21 are estimated to be 8.7164. With the discount rate at 10.26 percent and the steady-state growth rate at 5.636, the value of the stock at that year is $188.51 [= 8.7164/(0.1026 − 0.05636)].
10. Finally, the intrinsic value of approximately $44 is calculated by discounting each DPS and the terminal value by the discount rate of 10.26 percent (Column 7). The difference in the Bloomberg screen value shown at $44.035 and the Table value of $44.19 is due to rounding differences.

(b)

EXHIBIT 3.15 Valuation of Disney Using Bloomberg's DDM Model DDM Screen for Disney, 8/29/2013

In addition to determining the intrinsic value of the stock, the DDM also calculates the rate of return—internal rate of return (IRR)—by solving for the discount rate that equates the present value of the projected dividends to the current market price. The market price of Disney at the time of the analysis was $61.40. The IRR based on that price is 9.183 percent (see bottom of Exhibit 3.15).

BLOOMBERG DDM SCREEN

Bloomberg's DDM model estimates the intrinsic value of a selected equity using a three-stage growth model. The screen also can be used to calculate the IRR, expected return, and implied growth rate based on a series of assumptions. Estimates for EPS for FY1–FY3, dividend payout ratio, growth rates, length of years for stage 1 growth and transition years, and discount rates appear in the amber boxes. You can change any of the assumptions.

Bloomberg defaults:

1. Discount Rate: The model uses the risk-free rate of the 10-year Treasury bond plus a risk premium (also known as the market's required rate of return) as the discount rate. Required returns on stock are examined in Chapter 9. You can select you own risk-free rate and risk premium.
2. EPS for FY1, FY2, and FY3 are pulled from Bloomberg analysts' estimates. You can change these values based on your own estimates.
3. Growth Stages: DDM defaults to a three-stage dividend discount model consisting of a growth, transition, and mature or steady-state stage.
4. The long-term growth rate is Bloomberg's consensus estimate.
5. The dividend payout ratio for the first three years and growth years defaults to the current payout ratio.
6. The dividend payout rate for the mature stage defaults to 45 percent.
7. Growth rate in the mature stage is equal to the required rate times retention ratio [= (Discount Rate) (1 – dividend payout ratio)].
8. Growth rate in the transitional period decreases annually from the rate in the growth stage period to the rate in the mature stage.
9. The payout ratios in the transitional period change by annual increments to reach the payout rate at the mature stage (set at 45 percent).

   Note: You can modify both the payout rate and the growth rate in the mature stage, but the growth rate cannot exceed the market's required rate of return.

   See Bloomberg Web Exhibit 3.3.

Valuation of Preferred Stock

As noted in Chapter 1, preferred stock can be thought of as a limited ownership share. It provides its owners with only limited income potential in the form of a stipulated dividend (preferred dividend) that is usually expressed as a percentage of a stipulated par value. Preferred stock also gives its holders fewer voting privileges and less control over the business than common stock does. To make preferred stock more attractive, companies frequently sell preferred stock with special rights. Among the most common of these special rights is the priority over common stockholders over earnings and assets upon dissolution of the company and the right to cumulative dividends: If preferred dividends are not paid, then all past dividends must be paid before any common dividends are paid. Many preferred stocks are sold by financial institutions as a trust preferred and by utilities.

With a dividend payment that is fixed and with an indefinite life or long-term maturity, preferred stock takes the form of a bond perpetuity. As noted, the value of a perpetuity is equal to the coupon divided by the discount rate (= C/R). Similarly, the value of a preferred stock with a fixed dividend and long-term maturity or indefinite life is equal to the dividend divided by the required rate, k_e^{PS}, (the rate preferred investor's require for investing in the preferred stock). The rate of return on a preferred stock, in turn, is equal to its dividend divided by its market prices, P^{PS}:

Conclusion

For most investors the most important characteristics of an asset are its value and its rate of return. In this chapter, we have examined how the value and rate of return for bonds and stocks are measured. The measurement of security value and return, however, is based on future cash flows—coupons, dividends, and future prices—that are uncertain. Uncertainty, in turn, means that investors have to deal with risk—the possibility that their realized return will deviate from their expected return. In Chapters 6 and 7, we will examine in more detail the value, return, and other stock characteristics such as risk that are needed to evaluate and select stocks and construct stock portfolio.

Web Site Information

Bond Information

- **Investinginbonds.com**
  - Go to http://investinginbonds.com.
  - **FINRA**
  - Go to http://www.finra.org/index.htm, Sitemap, Market Data, and Bonds.
- ***Wall Street Journal***
  - Go to http://online.wsj.com/public/us, Market Data, Rates.
- **Yahoo.com**
  - Go to http://finance.yahoo.com/bonds, click "Advanced Bond Screener."
- **Finance Calculator—FICALC**

  The online FICALC calculator computes a bond's price given its yield or its yield given its price, as well as other information, such as cash flows and total returns. The FICALC calculator is designed so it can either be used as a stand-alone fixed income calculator, or integrated into a Web site that has information on fixed income securities:

  http://www.ficalc.com/calc.tips

Notes

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1. One of the practical uses of the geometric mean is in comparing investments in bonds with different maturities. For example, if the present interest rate structure for zero-coupon bonds were such that two-year bonds were providing an average annual rate of 9 percent and one-year bonds were at 10 percent, then the implied forward rate on a one-year bond, one year from now, would be 8 percent. With these rates, an investor could equate an investment in the two-year bond at 9 percent as being equivalent to an investment in a one-year bond today at 10 percent and a one-year investment to be made one year later yielding 8 percent (possibly through a locking-in strategy). Accordingly, if the investor knew with certainty that one-year bonds at the end of one year would be trading at 9 percent (6 percent)—a rate higher (lower) than the implied forward rate—then he would prefer an investment in the series of one-year bonds (two-year bond) over the two-year bond (series of one-year bonds). That is, by investing in a

one-year bond today and a one-year bond one year from now, the investor would obtain 10 percent and 9 percent, respectively, for an average annual rate on the two-year investment of 9.5 percent.