CHAPTER 6

Expected Rate of Return and Risk—Stock

Introduction

Although knowledge of present or past returns is important for security analysts, an investor is more concerned with what a security will yield in the future; that is, she wants to know the expected rate of return, as well as the chance that her expectations might be wrong. For example, after an in-depth analysis, an investor might think a stock would generate a 20 percent rate of return, but she also knows that under certain scenarios she might lose 10 percent or gain as much as 35 percent. Similarly, a bond investor who sees a bond trading to yield 10 percent also knows there is a chance of default in which he might lose 70 percent and also a chance that rates in the market might decrease, causing the bond to sell at a higher price and thus yielding a higher return than the current yield. Whether it is investments in stocks, bonds, investment fund shares, or commodities, investors must deal in a world of uncertainty. As a result, investors have to consider not only the expected rate of return, but also the risk that such a rate may not be attained.

In this chapter, we extend our analysis of security valuation and return presented in Chapter 3 to stock risk and expected return. We begin by first defining a risk-free security and the difference between nominal and real rates of return. We next briefly examine the nature of risk as it relates to stocks. With this background, we then look at a security's expected return and risk statistically by examining return distributions and characteristics—mean, variance, and correlation parameters. Finally, we conclude the chapter by examining the relations between expected return and risk for a stock.

Risk-Free Security, Nominal Rates, and Real Rates

Investment risk is the possibility that the realized return from holding a security will deviate from the expected. Most securities are subject to risk. However, some securities, like a U.S. T-bill, have a future return that is known in advance. For example, an investor who plans to hold a T-bill to its maturity knows with certainty that at that date she will receive the principal. Thus, the rate of return from the time of the investment to maturity is known. Treasury bonds and notes—the Treasury's coupon issues—are, in turn, considered default free with known coupon and principal payments. However, they technically are not risk-free securities since investors are subject to reinvestment risk when they reinvest their coupon income at unknown rates.

Denoted here as R_f , the rate of return on a security whose return is known in advance is defined as the *risk-free rate* (also called the sure rate or riskless rate). An investment in a risk-free security ensures the investor that she will receive a certain amount of cash. Such an investment, however, is not protected against inflation, which can lower the purchasing power of the investment money received. An investor who invested $100 in a one-year, risk-free security paying $105 at maturity, would earn a risk-free rate of

5 percent. If the average price of goods and services increased by 10 percent over the next year, the purchasing power of the $100 would have decreased (i.e., goods that cost $100 at the beginning of the year now cost $110). In this case, the investor would lose approximately 5 percent of purchasing power from the investment as a result of the 10 percent inflation.

Since inflation subjects an investment in a risk-free security to purchasing-power risk, analysts make a distinction between the nominal risk-free rate and the *real risk-free rate*. The nominal risk-free rate is the rate whose future cash is known in advance (i.e., R_f), whereas the real risk-free rate, RR_f, is the rate whose return reflects the investment return's purchasing power. The real rate can be found by discounting the future nominal risk-free return by the inflation rate. That is:

$$RR_f = \frac{1 + R_f}{1 + \text{Inflation rate}} - 1$$

Thus, with a nominal rate from the investment of 5 percent and an inflation rate of 10 percent, the real rate of return in our example is −4.545 percent:

$$RR_f = \frac{1.05}{1.10} - 1 = -0.04545$$

When inflation is expected to be high and the nominal rates on securities low, investors are better off increasing their consumption of goods and services and decreasing their savings and investments. In contrast, when deflation is expected, the real risk-free rate is higher than the nominal, and investors are better off increasing their saving and investment.

Stock Risk: Firm, Industry, and Market Risk

For investments in stocks, realized returns can deviate from expected returns when there are changes in expected dividends, growth rates in dividends, and factors that alter an investor's required returns. In Chapters 11 and 12, we will examine the fundamentals that determine a firm's income and balance sheet and the underlying factors that determine a firm's earnings, dividends, growth rates, and required return. We can, however, categorize the total risk of stock in terms of three general factors that influence a stock's return: factors related to the individual firm, the industry in which the firm competes, and the market in general.

Firm factors are unique to the company under consideration. They include the company's investment and financing decisions, marketing strategies, and competitive position in the industry. For example, an oil company acquiring a refinery and financing the acquisition by selling long-term bonds would be subjecting its equity investors to firm-related factors that would affect not only the expected return on the company's stock, but also the stock's risk. *Industry factors* affect all companies in the same industry. They include economic activities and regulations and laws that affect the industry. For example, the price of Inbev stock is influenced not only by firm factors such as the marketing strategies of the Samuel Adams Brewing Company, but also by industry factors such as consumers' relative taste for wine or other distilled spirits—a consideration that affects all beer companies. Finally, there are *market factors*, which by definition affect all stocks. They include macroeconomic factors, like the growth in the overall economy, the rate of inflation, the general level of interest rates, and the economic growth of emerging markets. For example, an economic recession would be a market factor since it tends to depress the earnings and returns of all companies.

The total risk of a stock can therefore be broken into firm risk, industry risk, and market risk. Total risk of a stock can also be divided into unsystematic risk and systematic risk. As we will examine later in this chapter, *unsystematic risk* is defined as the risk of a security that can be eliminated or reduced when that security is included in a diversified portfolio; it consists of risk due to firm and industry factors. *Systematic risk* is market risk, and in contrast to unsystematic risk, it cannot be eliminated by forming a portfolio of stocks. For example, the industry risk associated with investing in Inbev stock might by eliminated, or at least reduced, by investing in a wine stock, but forming a portfolio with both stocks would probably not eliminate the effect a recession would have on either stock's return. Thus, unlike firm and industry risk, market risk cannot be diversified away by forming a portfolio of stocks.

Statistical Measurements of Expected Return and Risk

In statistics, the term *random variable* describes a variable whose value is uncertain. For most securities, their future cash flows, prices, and rates of return are random variables. Signified with a tilde over the symbol (e.g., $\tilde{R}$), random variables are also called stochastic variables.

An important property of a random variable is its *probability distribution*. A probability distribution assigns probabilities to all possible values of the random variable. The distribution can be objective (as when using past frequencies or simply assuming the distribution takes a specific form) or subjective. Also, the distribution can be continuous, taking on all possible values over the range of the distribution, or discrete, where the distribution takes on only a few possible defined values. Exhibit 6.1 shows a probability distribution for next period's rate of return on stock ABC ($\tilde{r}$). This discrete distribution is defined by five possible rates (column 1) and their respective probabilities (column 2) and is shown graphically in Exhibit 6.1. The most common way to describe this distribution is in terms of its expected value, variance, and skewness. The expected value, or mean, is a measure of the central tendency of the distribution and the variance is a measure of the distribution's average squared deviation. The expected value and variance of a security's return distribution are good measures of that security's expected rate of return and risk, respectively.

(1) r_i	(2) p_i	(3) $p_i r_i$	(4) $[r_i - E(r)]$	(5) $[r_i - E(r)]^2$	(6) $p_i[r_i - E(r)]^2$	(7) $[r_i - E(r)]^3$	(8) $p_i[r_i - E(r)]^3$
4%	0.1	0.4	−2	4	0.4	−8	−0.8
5%	0.2	1.0	−1	1	0.2	−1	−0.2
6%	0.4	2.4	0	0	0.0	0	0.0
7%	0.2	1.4	1	1	0.2	1	0.2
8%	0.1	0.8	2	4	0.4	8	0.8
	1	$E(r) = 6\%$			$V(r) = 1.2$		$S_k(r) = 0$

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
r_i	p_i	$p_i r_i$	$[r_i - E(r)]$	$[r_i - E(r)]^2$	$p_i[r_i - E(r)]^2$	$[r_i - E(r)]^3$	$p_i[r_i - E(r)]^3$

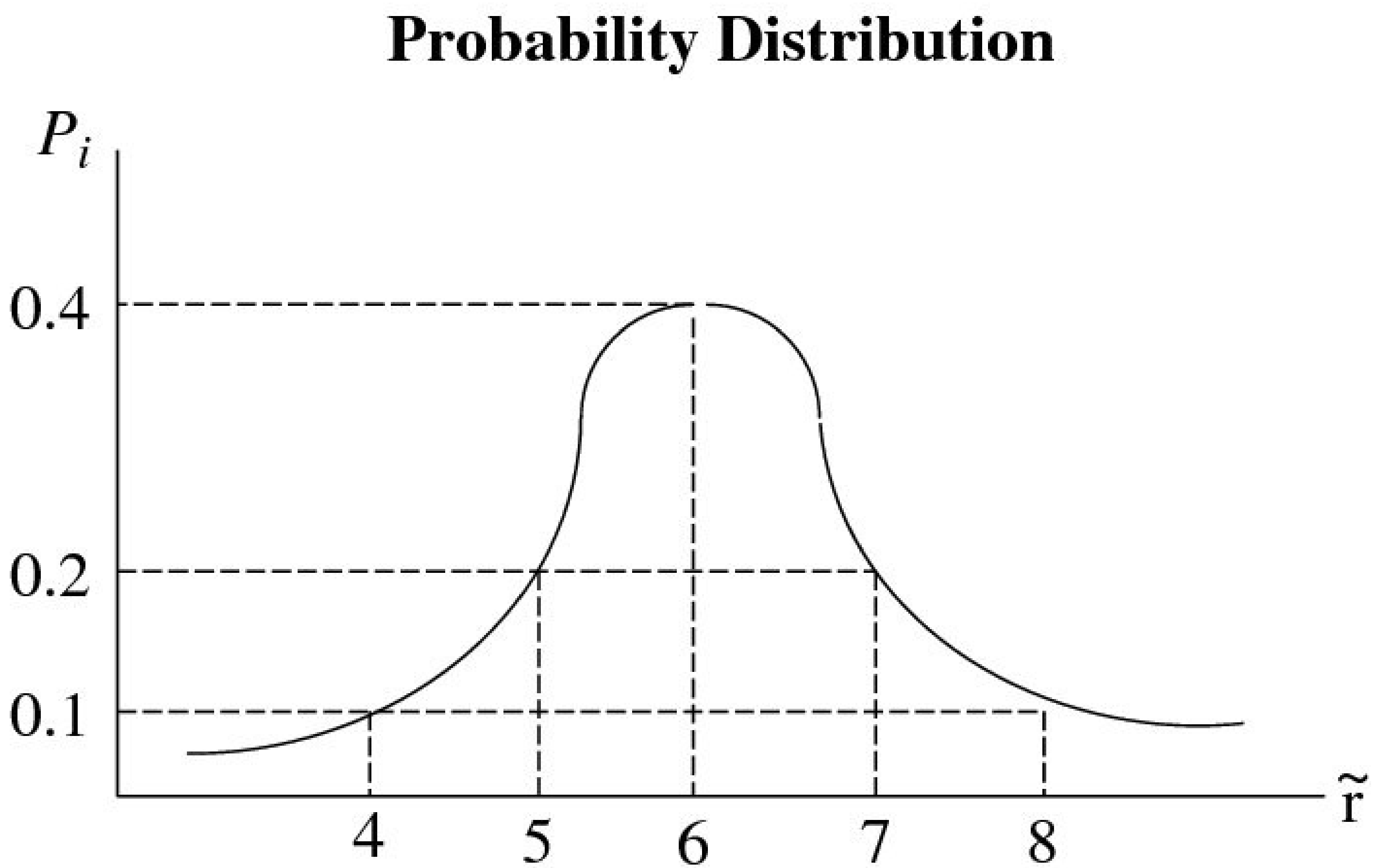


EXHIBIT 6.1 Probability Distribution for Stock ABC

Expected Value

The expected value of a random variable x, $E(x)$, or its mean, μ, is the weighted average of the possible values, with the weights being the probabilities assigned to each possible value (p_i). The expected value, along with the median and the mode, is a measure of the central tendency of the distribution. For example, the expected rate of return for stock ABC, $E(r)$, shown in Exhibit 6.1 is 6 percent:

$$E(\tilde{r}) = \sum_{i=1}^{T} p_i r_i = p_1 r_1 + p_2 r_2 + \cdots + p_T r_T$$
$$E(\tilde{r}) = (0.1)(4\%) + (0.2)(5\%) + (0.4)(6\%) + (0.2)(7\%) + (0.1)(8\%) = 6\%$$

Like any variable, a random variable can be a function of other variables. The rates of return on many stocks, for instance, are related to the rates of return on a market index (R^M), as measured by the proportional changes in the S&P 500 index or the Dow Jones Average. In such cases, the rate of return on a security can be expressed algebraically as:

$$\tilde{r} = \alpha + \beta \tilde{R}^M$$

where:

$$\alpha = \text{Interecept}$$
$$\beta = \text{Slope} = \frac{\Delta r}{\Delta R^M}$$

In this form, the expected value of r is equal to $E[\alpha + \beta R_M]$. The expected value of this equation, in turn, can be simplified by making use of the following expected value operator rules:

1. Expected value of a constant (α) is equal to the constant:
$$\text{EV Rule 1: } E(\alpha) = \alpha$$
2. Expected value of a constant times a random variable is equal to the constant times the expected value of the random variable:

$$\text{EV Rule 2: } E(\beta\tilde{R}^M) = \beta E(\tilde{R}^M)$$

3. Expected value of a sum is equal to the sum of the expected values:

$$\text{EV Rule 3: } E[X + Y] = E(X) + E(Y)$$

By applying these three rules to the equation $E[\alpha + \beta R_M]$, $E(r)$ can be expressed as

$$E(r) = E[\alpha + \beta\tilde{R}^M]$$
$$E(r) = E[\alpha] + E[\beta\tilde{R}^M] \text{ EV Rule 3}$$
$$E(r) = \alpha + \beta E(\tilde{R}^M) \text{ EV Rules 1 and 2}$$

Variance, Standard Deviation, and Coefficient of Variation

Variance

The variance of a random variable ($V(\tilde{r})$) is the expected value of the squared deviation from the mean:

$$V(r) = E[\tilde{r} - E(r)]^2$$

The variance is referred to as the second moment of the distribution. It is a measure of the distribution's dispersion, indicating the squared deviation most likely to occur. As an expected value, the variance can be measured by calculating the weighted average of each squared deviation, with the weights being each squared deviation's probability:

$$V(\tilde{r}) = \sum_{i=1}^{T} p_i[\tilde{r}_i - E(\tilde{r})]^2$$

$$V(\tilde{r}) = p_1[\tilde{r}_1 - E(\tilde{r})]^2 + p_2[\tilde{r}_2 - E(\tilde{r})]^2 + \cdots + p_T[\tilde{r}_T - E(\tilde{r})]^2$$

ABC stock described in Exhibit 6.1, in turn, has a variance of 1.2:

$$\begin{aligned} V(\tilde{r}) &= (0.1)[4\% - 6\%]^2 + (0.2)[5\% - 6\%]^2 + (0.4)[6\% - 6\%]^2 \\ &\quad + (0.2)[7\% - 6\%]^2 + (0.1)[8\% - 6\%]^2 \\ V(\tilde{r}) &= 1.2 \end{aligned}$$

Standard Deviation

The standard deviation, $\sigma(r)$, is the square root of the variance:

$$\sigma(\tilde{r}) = \sqrt{V(\tilde{r})}$$

The standard deviation provides a measure of dispersion that is on the same scale as the distribution's deviations. The standard deviation for ABC stock is 1.09544; this indicates that the distribution has an average deviation of plus or minus 1.0954451 (±1.09544).

Coefficient of Variation

As noted, the risk of a security is the uncertainty that the realized return will deviate from the expected return. By definition, the variance and standard deviation of a security's rate of return define a security's risk. That is, the greater a security's variance, the greater the realized return can deviate from the expected return, and thus the greater the security's risk.

Although the variance (or standard deviation) can be used to define and measure a security's risk relative to other securities, using just that parameter can be misleading if there is a large disparity in the expected rates of returns among the securities being evaluated. For example, suppose you are considering an investment in either Stock A or Stock B with the following expected returns and standard deviations:

Stock	A	B
Expected Return	0.05	0.12
Standard Deviation	0.04	0.07

Stock B with a standard deviation of 0.07 appears to be riskier than Stock A, whose standard deviation is only 0.04. However, Stock B's distribution dominates Stock A's distribution; that is, with an expected return of 0.12 and standard deviation of 0.07, Stock B has an average range between 0.05 and 0.19, whereas Stock A, with an expected return of 0.05 and a standard deviation of 0.04, has only an average range of between 0.01 and 0.09. In such cases, the *coefficient of variation* (*CV*) is a better measure of risk than the variance or standard deviation. The coefficient of variation expresses the standard deviation as a proportion of the expected value:

$$CV = \frac{\sigma(r)}{E(r)}$$

In this example, Stock A has a *CV* of 0.80, whereas B has a *CV* of only 0.583:

$$CV_A = \frac{\sigma(r)}{E(r)} = \frac{0.04}{0.05} = 0.80$$
$$CV_A = \frac{\sigma(r)}{E(r)} = \frac{0.07}{0.12} = 0.583$$

Thus, Stock B has less risk per unit of expected return than Stock A.

Skewness

Skewness measures the degree of symmetry of the distribution. A distribution that is symmetric about its mean is one in which the probability of $r = E(r) + x$ is equal to the probability of $r = E(r) - x$, for all values of x. Skewness, $S_k(r)$, is defined as the third moment of the distribution and can be measured by calculating the expected value of the cubic deviation:

$$S_k(\tilde{r}) = \sum_{i=1}^{T} p_i[\tilde{r}_i - E(\tilde{r})]^3$$
$$S_k(\tilde{r}) = p_1[\tilde{r}_1 - E(\tilde{r})]^3 + p_2[\tilde{r}_2 - E(\tilde{r})]^3 + \cdots + p_T[\tilde{r}_T - E(\tilde{r})]^3$$

The skewness of the distribution in Exhibit 6.1 is zero, indicating the distribution is symmetrical.

$$\begin{aligned} S_k(\tilde{r}) &= (0.1)[4\% - 6\%]^3 + (0.2)[5\% - 6\%]^3 + (0.4)[6\% - 6\%]^3 \\ &\quad + (0.2)[7\% - 6\%]^3 + (0.1)[8\% - 6\%]^3 \\ S_k(\tilde{r}) &= 0 \end{aligned}$$

In investments, security prices are characterized by increasing trends (e.g., a bull market) or decreasing trends (e.g., a bear market). These trends often tend to be characterized statistically by having a negative skewness and positive expected return when there is an increasing trend and positive skewness and negative expected return when there is a decreasing trend.

Normal Distribution

The above example dealt with a discrete probability distribution. Most securities have a large number of possible values at a future date and are better represented by a continuous distribution. The most well-known continuous distribution is the normal distribution. This is a symmetric, bell-shaped distribution extending from $-\infty$ to $+\infty$. The distribution is completely described by two parameters: its mean, μ, and standard deviation, σ. Thus, the only difference between one normal distribution and another is their respective means and standard deviations.

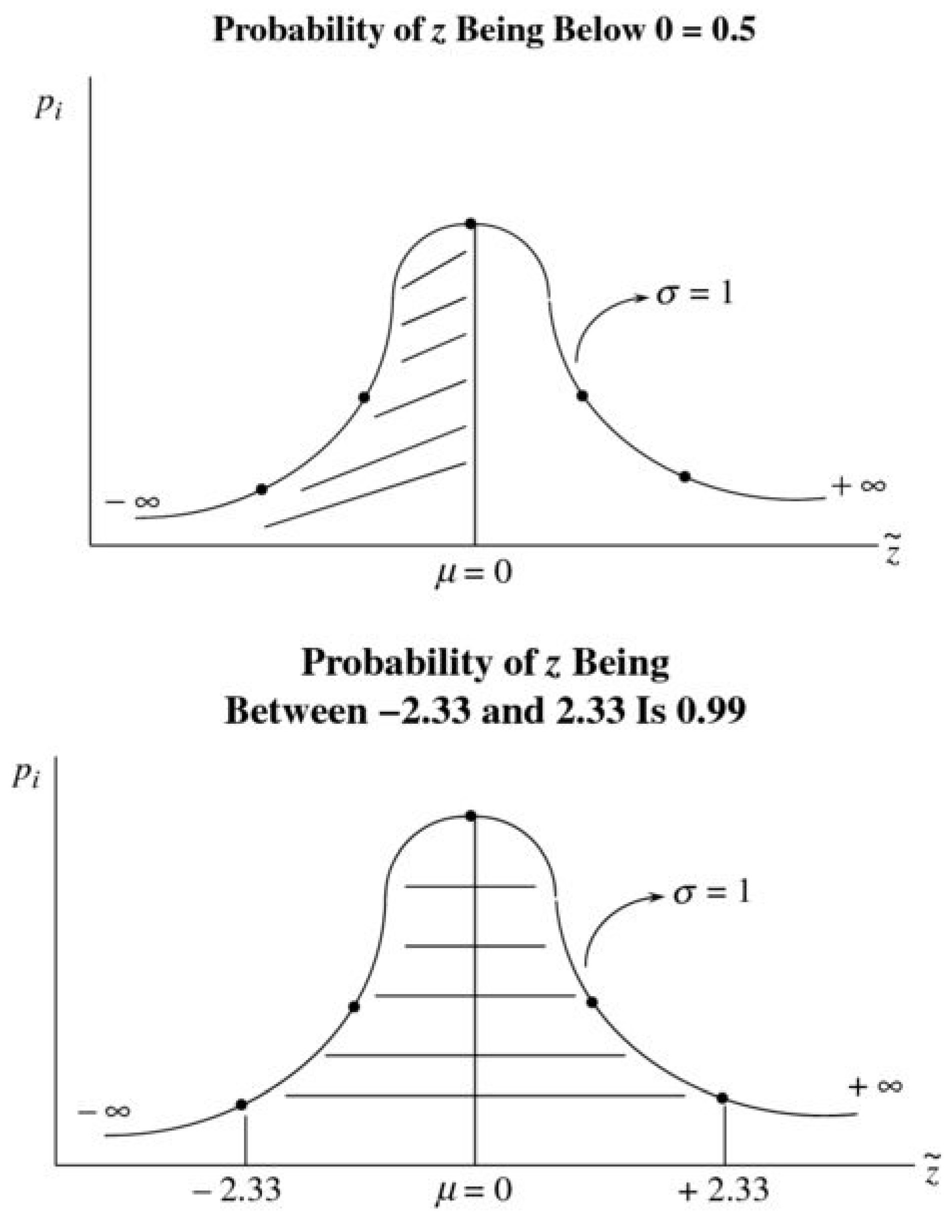


EXHIBIT 6.2 Standard Normal Distribution

A special type of normal distribution is the standard normal distribution. As shown in Exhibit 6.2, this distribution has a mean (or expected value) of zero, a standard deviation of one, and an area under the distribution equal to one. With the area under the normal distribution equal to one, the probability of an outcome occurring between any two points of a standard normal distribution equals the area between the two points. For example, the probability of an outcome being below the mean (i.e., being between 0 and $-\infty$) is 0.5, and the probability of it being above the mean (i.e., being between 0 and $+\infty$) is also 0.5 (see Exhibit 6.2). Similarly, the probability of an outcome being between one standard deviation below the mean and one standard deviation above is 0.68, whereas the probability of an outcome being between 2.33 standard deviations below the mean and 2.33 standard deviations above is 0.99 (see Exhibit 6.2). For the standard normal distribution, the number of standard deviations from the mean (e.g., 1 or 2.33) is referred to as the *z*-score. The area under a standard normal distribution corresponding to a given *z*-scores, N(*z*), can be obtained from a standard normal density table found in most statistics books; the area can also be found using the following formula:

$$\begin{aligned} \mathrm{n}(z) = 1 - 0.5[1 &+ 0.196854(|z|) + 0.115194(|z|)^2 \\ &+ 0.000344(|z|)^3 + 0.019527(|z|)^4]^{-4} \end{aligned}$$

where:

- $|z|$ = absolute value of z.

If z is negative, then the n(z) value obtained from the formula is subtracted from one; if z is positive, then the n(z) obtained from the formula is used:

$$\mathrm{N}(z) = 1 - \mathrm{n}(z), \text{ for } z < 0$$
$$\mathrm{N}(z) = \mathrm{n}(z), \text{ for } z > 0$$

Since other normal distributions differ only in terms of their means and standard deviations, the value of a normally distributed random variable x can be converted into its corresponding z-score by using the following formula:

$$\tilde{z} = \frac{\tilde{x} - \mu}{\sigma} \tag{6.1}$$

and the value of *x* corresponding to a given *z* value can be found using the following:

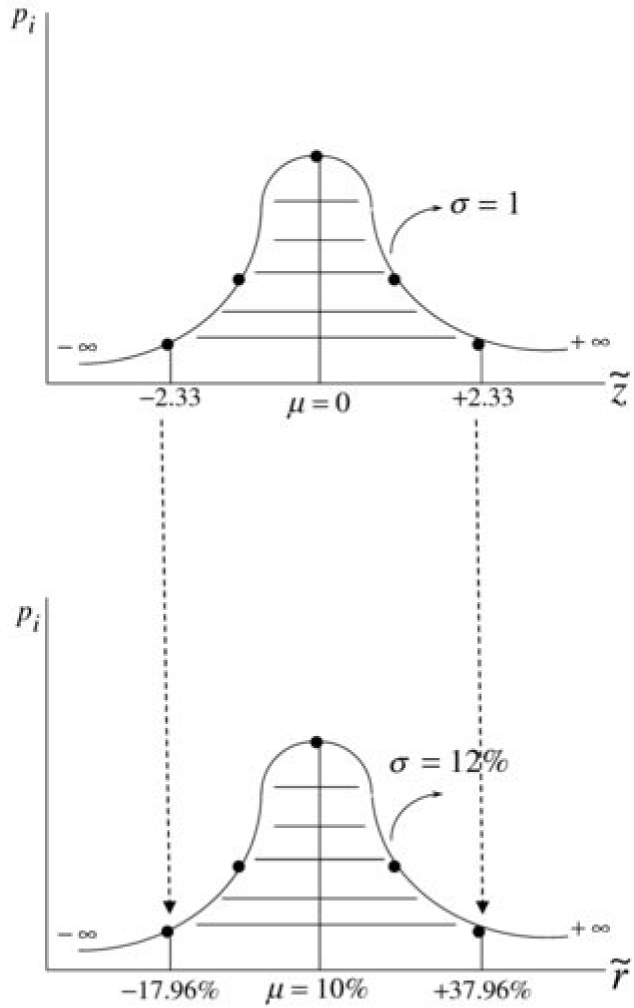
$$\tilde{x} = \mu + \tilde{z}\sigma \tag{6.2}$$

By specifying the *z*-score needed to achieve a certain probability, a confidence interval for a random variable *x* can be formed. For example, a 99 percent confidence interval is formed by setting *z* in Equation (6.2) equal to −2.33 and to 2.33:

$$\text{Probability}\,[\mu - 2.33\sigma < x < \mu + 2.33\sigma] = 0.99$$

If a security being analyzed has returns that are normally distributed, then given estimates of the security's expected return and standard deviation, a confidence interval for its returns can be formed. For example, a stock with an expected return of 10 percent and a standard deviation of 12 percent would have a 99 percent confidence interval of its return being between −17.96 percent and 37.96 percent (see Exhibit 6.3):

$$\text{Probability}\,[10\% - 2.33(12\%) < \tilde{r} < 10\% + 2.33(12\%)] = 0.99$$
$$\text{Probability}\,[-17.96 < \tilde{r} < 37.96] = 0.99$$



$$\text{Probability}\,[\mu - 2.33\,\sigma < x < \mu + 2.33\sigma] = 0.99$$
$$\text{Probability}\,[10\% - 2.33\,(12\%) < \tilde{r} < 10\% + 2.33\,(12\%) = 0.99$$
$$\text{Probability}\,[-17.96 < \tilde{r} < 37.96] = 0.99$$

EXHIBIT 6.3 Confidence Intervals

Correlation

Covariance

The covariance is a measure of the extent to which one random variable is above or below its mean at the same time or state that another random variable is above or below its mean. The covariance measures how two random variables move with each other. If two random variables, on average, are above their means at the same time and, on average, are below at the same time, then the random variables would be positively correlated with each other and would have a positive covariance (see Exhibit 6.4). In contrast, if one random variable, on average, is above its mean when another is below and vice versa, then the random variables would move inversely or negatively to each other and would have a negative covariance (see Exhibit 6.5).

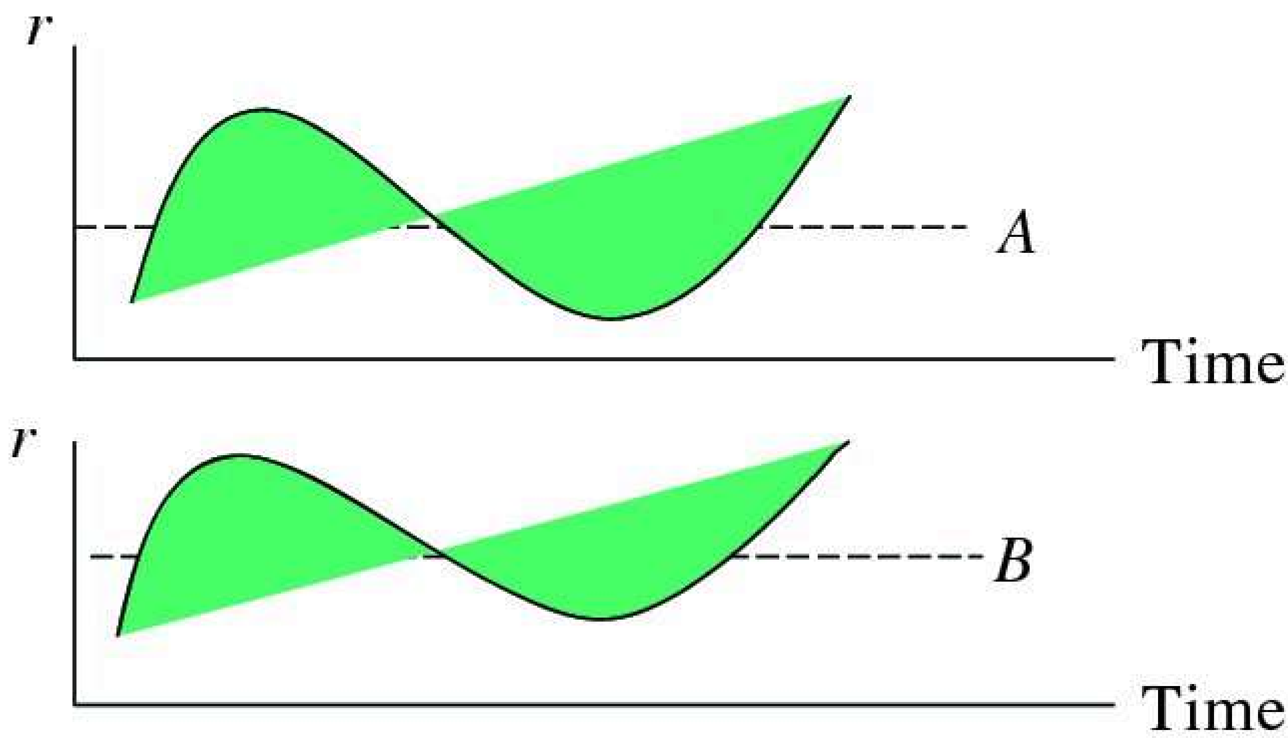


EXHIBIT 6.4 Positive Correlation

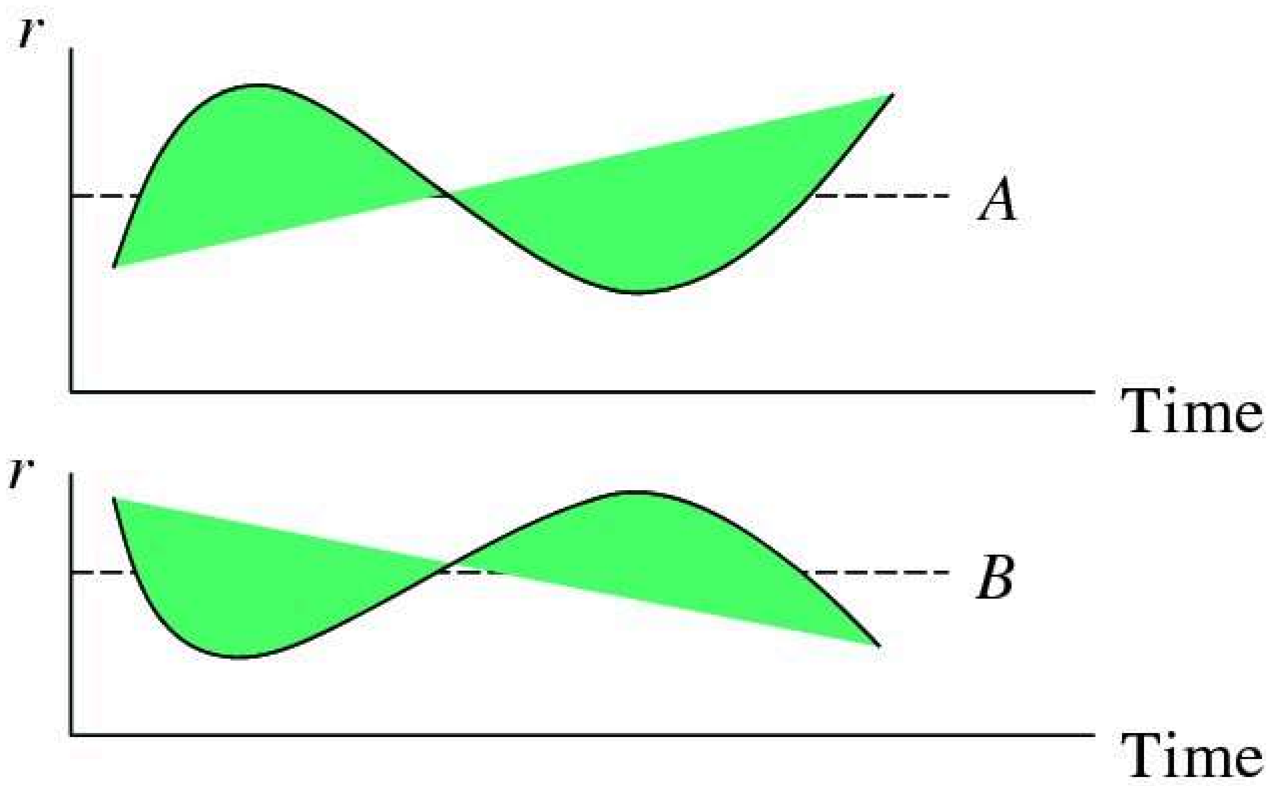


EXHIBIT 6.5 Negative Correlation

The covariance between two random variables, r_1 and r_2, is equal to the expected value of the product of the variables' deviations:

$$Cov(\tilde{r}_1\tilde{r}_2) = E[\tilde{r}_1 - E(\tilde{r}_1)][\tilde{r}_2 - E(\tilde{r}_2)]$$

$$Cov(\tilde{r}_1\tilde{r}_2) = \sum_{i=1}^{T} p_i[\tilde{r}_{1i} - E(\tilde{r}_1)][\tilde{r}_{2i} - E(\tilde{r}_2)]$$

In Exhibit 6.6, the possible rates of return for securities 1 and 2 are shown for three possible states (A, B, and C), along with the probabilities of occurrence of each state. As shown in the table, $E(r_1)$ = 18 percent, $V(r_1)$ = 36, $E(r_2)$ = 16 percent, and $V(r_2)$ = 16 (note the tilde $\tilde{r}$ over the symbol is dropped). In addition, the table also shows that in State A, security 1 yields a return below its mean, whereas security 2 yields a return above its mean. In State B, both yield rates of return equal to their mean, and in State C, security 1 yields a return above its mean whereas security 2 yields a return below. Securities 1 and 2 therefore are negatively correlated and, as shown in Exhibit 6.6, have a negative covariance of −24.

State	p_i	r_{1i}	r_{2i}	$p_i r_{1i}$	$p_i r_{2i}$	$p_i[r_{1i} - E(r_1)]^2$	$p_i[r_{2i} - E(r_2)]^2$	$p_i[r_{1i} - E(r_1)]\,[r_{2i} - E(r_2)]$
A	1/8	6%	24%	0.75	3.0	(1/8)(144)	(1/8)(64)	(1/8)(−12)(8) = −12
B	6/8	18%	16%	13.5	12.0	(6/8)(0)	(6/8)(0)	(6/8)(0)(0) = 0
C	1/8	30%	8%	3.75	1.0	(1/8)(144)	(1/8)(64)	(1/8)(12)(−8) = −12
				$E(r_1) = 18$	$E(r_2) = 16$	$V(r_1) = 36$ $\sigma(r_1) = 6$	$V(r_2) = 16$ $\sigma(r_2) = 4$	$\text{Cov}(r_1\, r_2) = -24$ $\rho_{12} = -1$

EXHIBIT 6.6 Negative Correlation Example

In contrast, Exhibit 6.7 shows a case of positive correlation. In this case, in State A both securities 1 and 2 have returns below their mean; in State B both have rates of return equal to their mean; in State C both have returns above their means. Securities 1 and 2 therefore are positively correlated and, as shown in the exhibit, the securities have a positive covariance of 24.

State	p_i	r_{1i}	r_{2i}	$p_i\, r_{1i}$	$p_i\, r_{2i}$	$p_i[r_{1i} - E(r_1)]^2$	$p_i[r_{2i} - E(r_2)]^2$	$p_i[r_{1i} - E(r_1)]\, [r_{2i} - E(r_2)]$
A	1/8	6%	8%	0.75	1.0	(1/8)(144)	(1/8)(64)	(1/8)(−12)(−8) =
B	6/8	18%	16%	13.5	12.0	(6/8)(0)	(6/8)(0)	12
C	1/8	30%	24%	3.75	3.0	(1/8)(144)	(1/8)(64)	(6/8)(0)(0) = 0
								(1/8)(12)(8) = 12
				$E(r_1)$ = 18	$E(r_2)$ = 16	$V(r_1)$ = 36 $\sigma(r_1)$ = 6	$V(r_2)$ = 16 $\sigma(r_2)$ = 4	$\mathrm{Cov}(r_1\, r_2)$ = 24 ρ_{12} = 1

EXHIBIT 6.7 Positive Correlation Example

Correlation Coefficient

The correlation coefficient between two random variables r_1 and $r_2(\rho_{12})$ is equal to the covariance between the variables divided by the product of each random variable's standard deviation:

$$\rho_{12} = \frac{Cov(r_1 r_2)}{\sigma(r_1)\rho(r_2)}$$

The correlation coefficient has the mathematical property that its value must be within the range of minus and plus one:

$$-1 \leq \rho_{12} \leq 1$$

If two random variables have a correlation coefficient equal to one, they are said to be perfectly positively correlated; if their coefficient is equal to a minus one, they are said to be perfectly negatively correlated; if their correlation coefficient is equal to zero, they are said to be zero correlated and statistically independent. That is:

$$\begin{aligned} &\text{If } \rho_{12} = -1 \Rightarrow \text{Perfect Negative Correlation} \\ &\text{If } \rho_{12} = 0 \Rightarrow \text{Uncorrelated} \\ &\text{If } \rho_{12} = 1 \Rightarrow \text{Perfect Positive Correlation} \end{aligned}$$

Correlation and Portfolio Risk

Correlation is important in measuring the risk of a portfolio of securities. If the returns of two stocks are perfectly negatively correlated, then a zero risk portfolio can be formed with the two stocks. Consider the two stocks described in Exhibit 6.6 that are perfectly negatively correlated with a covariance of −24. The negative correlation suggests that when stock 1's return (r_1) is above its mean, $E(r_1)$, stock 2's return (r_2) is below its mean, $E(r_2)$, and when r_2 is above $E(r_2)$, r_1 is below $E(r_1)$. If an investor were holding these securities in a portfolio with 40 percent of her investments in stock 1 and 60 percent in stock 2, her portfolio rate of return (R_p) would be 16.8 percent in each of the three possible states shown in Exhibit 6.6. For example, in State A, the return on stock 1 is 6 percent, and the return on stock 2 is 24 percent. With allocation of 40 percent in stock 1 and 60 percent in stock 2, the investor would earn only 2.4 percent [=(0.40)(6%)] from her investment in stock 1 but would earn a 14.4 percent rate of return [=(0.60)(24%)] from her investment in stock 2. Combined, the investor's portfolio return would be 16.8 percent [=(0.40)(6%) + (0.60)(24%)]. In State B, her return from stock 1would be 7.2 percent [=(0.40)(18%)] and her return from stock 2 would be 9.60 percent [=(0.60)(16%)], yielding again a portfolio return of 16.80 percent [=(0.40)(18%) + (0.60)(16%)]. Finally, in State C, the return from stock 1 would be

12 percent [=(0.40)(30%)] and the return from stock 2 would be 4.8 percent [=(0.60)(8%)], yielding again a portfolio return of 16.8 percent [=(0.40)(30%) + (0.60)(8%)]. In summary:

State	Stock 1	Stock 2	Proportional Investment in Stock 1	Proportional Investment in Stock 2	Portfolio Rate of Return
A	6%	24%	0.4	0.6	0.168
B	18%	16%	0.4	0.6	0.168
C	30%	8%	0.4	0.6	0.168

Thus, the investor would find for each of the three possible states that her portfolio rate of return is always 16.8 percent. Thus, since the investor can always attain a 16.8 percent rate of return, there is no portfolio risk. It is interesting that both stocks 1 and 2 are risky [$\sigma(r_1) = 6$] and [$\sigma(r_2) = 4$], but the portfolio formed with them has rate of return of 16.8 percent for all states and therefore a portfolio variance of zero. Again, this is due to the perfect negative correlation.

In contrast, the two stocks described in Exhibit 6.7 are perfectly positively correlated with a covariance of 24. Portfolios formed with these stocks will always be subject to risk. For example, a portfolio with allocations of 40 percent in stock 1 and 60 percent in stock 2, generates a portfolio return of 7.2 percent in State A, 16.8 percent in State B, and 26.4 percent in State C. The returns from different portfolio allocations would all be characterized by lower returns in State A, moderate returns in State B, and higher returns in State C.

Parameter Estimates: Historical Averages

In most cases, we do not know the probabilities associated with the possible values of the random variable and must therefore estimate the parameter characteristics. The simplest way to estimate the mean, variance, skewness, and covariance is to calculate the parameters' historical average values from a sample. The historical averages also provide analysts with important descriptive information about a security's past performance. For the rate of return on a security, the average rate of return per period can be calculated using the security past holding period yields, HPY_t (stock $HPY = [(P_t - P_{t-1}) + \text{dividend}]/P_{t-1}$) + coupon) over N historical periods:

$$\bar{r} = \frac{1}{N} \sum_{t=1}^{N} HPY_t$$

Similarly, the variance of a security can be estimated by averaging the security's squared deviations, and the covariance between two securities can be estimated by averaging the product of the securities' deviations. Note, in estimating the variance and covariance, averages usually are found by dividing by $N - 1$ instead of N in order to obtain better unbiased estimates:

$$\overline{V}(r) = \frac{1}{N-1} \sum_{t=1}^{N} (HPY_t - \bar{r})^2$$

$$\overline{Cov}(r_1 r_2) = \frac{1}{N-1} \sum_{t=1}^{N} (HPY_{1t} - \bar{r}_1)(HPY_{2t} - \bar{r}_2)$$

An example of estimating parameters is shown in Exhibit 6.8, in which the average HPY, variances, and covariance are computed for a stock and a hypothetical stock index (S_m).

Time	S	Dividend	HPY	(HPY − Av.)	$(HPY − Av.)^2$
1	100	0			
2	105	0	0.050000	0.032324	0.0010448
3	110	1	0.057143	0.039467	0.0015576
4	115	0	0.045455	0.027779	0.0007716
5	110	1	−0.034783	−0.052459	0.0027519
6	105	0	−0.045455	−0.063131	0.0039855
7	100	1	−0.038095	−0.055771	0.0031104
8	105	0	0.050000	0.032324	0.0010448
9	110	1	0.057143	0.039467	0.0015576
			0.141408		0.0158244
			Av. = 0.017676		Var = 0.0022606

Time	**S**	**Dividend**	**HPY**	**(HPY − Av.)**	**(HPY − Av.)^2**
					Stan. Dev. = 0.0475457
Time	S_m	R_m = HPY	(R_m − Av.)	(R_m − Av.)^2	(R_m − Av.)(HPY − Av.)
1	300				
2	315	0.050000	0.035583	0.001266	0.0011502
3	333	0.057143	0.042726	0.001825	0.0016863
4	346	0.039039	0.024622	0.000606	0.0006840
5	334	−0.034682	−0.049099	0.002411	0.0025757
6	319	−0.044910	−0.059327	0.003520	0.0037454
7	306	−0.040752	−0.055169	0.003044	0.0030769
8	320	0.045752	0.031335	0.000982	0.0010129
9	334	0.043750	0.029333	0.000860	0.0011577

Time	S	Dividend	HPY	(HPY − Av.)	$(\text{HPY} - \text{Av.})^2$
		0.115339		0.014514	0.0150888
		Av. = 0.014417		Var = 0.0020735	Cov = 0.002156

$$\hat{\alpha} = \bar{r} - \hat{\beta}\overline{R}_m$$
$$= 0.017676 - 1.04(0.014417) = 0.00268$$
$$\hat{\beta} = \frac{\hat{Cov}(r\ R_m)}{\hat{V}(R_m)} = \frac{0.002156}{0.002073} = 1.04$$
$$V(\varepsilon) = V(R) - \hat{\beta}^2 V(R_m)$$
$$= 0.0022606 - (1.04)^2 0.002073 = 0.0000184$$

$$E(r) = \alpha + \beta E(R_m)$$
$$= 0.00268 + 1.04E(R_m)$$
$$V(r) = \beta^2 V(R_m) + V(\varepsilon)$$
$$= (1.04)^2 V(R_m) + 0.0000184$$

EXHIBIT 6.8 Historical Averages and Regression Estimates

Geometric Mean and Arithmetic Mean

In computing historical averages, there are two ways to measure a security's average *HPY*: the arithmetic mean and the geometric mean. The arithmetic mean is simply the sum of the periodic *HPY*s divided by the number of periods (*N*):

$$\text{Arithmetic Mean} = \frac{1}{N}\sum_{t=1}^{N} HPY_t$$

The geometric mean is the Nth root of the products of one plus the *HPY*s for each of the *N* periods minus one:

$$\text{Geometric Mean} = \left[\prod_{t=1}^{N}(1 + HPY_1)\right]^{1/N} - 1$$

$$\text{Geometric Mean} = \left[(1 + HPY_1)(1 + HPY_2)\cdots(1 + HPY_N)\right]^{1/N} - 1$$

Different from the arithmetic mean, the geometric mean takes into account the compounding effect. For example, if a stock yields a 5 percent rate in one period, then the return for the second period would reflect the return earned in that period based on the compounded investment value at the beginning. As a result, the geometric mean is a better estimate of the rate at which the investment grew over the historical period.

To see the difference in the two average measures, consider the following three annual HPYs:

Year	0	1	2	3
XYZ Stock Price	**$100**	**$200**	**$100**	**$150**

	Period	Beginning Value	Ending Value	HPY
	1	$100	$200	1.00
	2	$200	$100	−0.50
	3	$100	$150	0.50

The *HPY*s represent the annual rates earned over a three-year period from investing in XYZ stock (assume no dividends). As shown, the *HPY*s of 100 percent, −50 percent, and 50 percent, reflect XYZ's value going from $100 to $200 in the first year, $200 to $100 in the second year, and $100 to $150 in the third year. The arithmetic mean of these three *HPY*s is 33.33 percent, whereas the geometric mean is 14.47 percent:

$$\text{Arithmetic Mean} = \frac{1}{N}\sum_{t=1}^{N} HPY_t$$

$$\text{Arithmetic Mean} = \frac{1.00 + (-0.50) + 0.50}{3} = 0.3333$$

$$\text{Geometric Mean} = \left[(1 + HPY_1)(1 + HPY_2) \cdots (1 + HPY_N)\right]^{1/N} - 1$$

$$\text{Geometric Mean} = [(1 + 1)(1 - 0.50)(1 + 0.50)]^{1/3} - 1 = 0.1447$$

Since the geometric mean takes into account the compounding of interest each period, it provides a measure of the rate at which the investment grows to equal its end-of-period value. The arithmetic mean, however, tends to overshoot the end-of-the period value. In this example, using the geometric mean of 14.47 percent, a $100 investment would grow annually at that rate of 14.47 percent to equal its correct value of $150 at the end of year three:

$$\$100(1.1447)^3 = \$150$$

In contrast, using the arithmetic rate of 33.33 percent as the average annual rate, the $100 investment would grow to equal a value of $237 at the end of three years:

$$\$100(1.3333)^3 = \$237$$

Similarly, looking at the *HPY*s for the first two years, the geometric mean is zero, correctly reflecting the fact that the investment of $100 is still worth $100 at end of year two; the arithmetic mean, on the other hand, is 25 percent even though there is no gain in value:

$$\text{Arithmetic Mean} = \frac{1.00 + (-0.50)}{2} = 0.25$$

$$\text{Geometric Mean} = [(1 + 1)(1 - 0.50)]^{1/2} - 1 = 0$$

Thus, unless the *HPY*s are the same in each period, the geometric mean will be less than the arithmetic mean and will correctly value the end-of-the period value, whereas the arithmetic mean will tend to overvalue the end-of-the period value.

Cumulative Returns

In addition to averages, analysts often look at how the past returns from an investment cumulate over time. Cumulative returns can be computed by either adding periodic *HPY*s starting from a base of zero or from a $1 investment or by calculating the compounded return each period. The cumulative *HPY* returns and compounded returns for the past *N* periods are:

$$\text{Cumulative return} = HPY_t + HPY_{t-1} + \cdots + HPY_{t-N}$$

$$\text{Compounded return} = \left[(1 + HPY_t)(1 + HPY_{t-1}) \cdots (1 + HPY_{t-N})\right] - 1$$

Cumulative *HPY* and compounded returns, in turn, allow an investor to evaluate the performance of the investment over time. Furthermore, by comparing the cumulative and compounded returns of different stocks or indexes, one can also evaluate the relative performance of an investment over time. Exhibit 6.9 shows graphically and in a table the cumulative *HPY*s and compounded returns starting from time period 0 to period 10. The cumulative and compounded returns are generated from the end-of-the period stock prices shown in Column 1 and the corresponding *HPY*s (assume no dividends). In examining the graphs, note that an investor who purchased the stock in time period 0 and sold it in period 6 would have realized a cumulative *HPY* of 20 percent and a compounded return of 20.06 percent. From period 6 to 7, however, the stock price declines by 20 percent, lowering the cumulative *HPY* to zero and the cumulative compounded return to −3.95 percent. Thus, if the investor had maintained the investment to year 7, she would have lost 3.95 percent of her initial investment. With the stock increasing in periods 8 through 10, the investor would have regained some of the loss with the cumulative *HPY* for the 10 periods being 20 percent and the compounded return being 16.48 percent.

Period	Price	HPY	Cumulative HPY	1 + Cumulative HPY	1 + Compound Return	Compounded Return
t	P_t	$(P_t/P_{t-1}) - 1$	$HPY_t + HPY_{t-1} + \cdots + HPY_{t-n}$		$[(1 + HPY_t)(1 + HPY_{t-1})\cdots(1 + HPY_{t-n})]$	$[(1 + HPY_t)(1 + HPY_{t-1})\cdots(1 + HPY_{t-n})] - 1$
0	$100.00	0.0000	0.0000	1.0000	1.0000	0.0000
1	$110.00	0.1000	0.1000	1.1000	1.1000	0.1000
2	$115.50	0.0500	0.1500	1.1500	1.1550	0.1550
3	$103.95	−0.1000	0.0500	1.0500	1.0395	0.0395
4	$114.35	0.1000	0.1500	1.1500	1.1435	0.1435
5	$114.35	0.0000	0.1500	1.1500	1.1435	0.1435
6	$120.06	0.0500	0.2000	1.2000	1.2006	0.2006
7	$96.05	−0.2000	0.0000	1.0000	0.9605	−0.0395

8	$105.65	0.1000	0.1000	1.1000	1.0565	0.0565
9	$110.94	0.0500	0.1500	1.1500	1.1094	0.1094
10	$116.48	0.0500	0.2000	1.2000	1.1648	0.1648

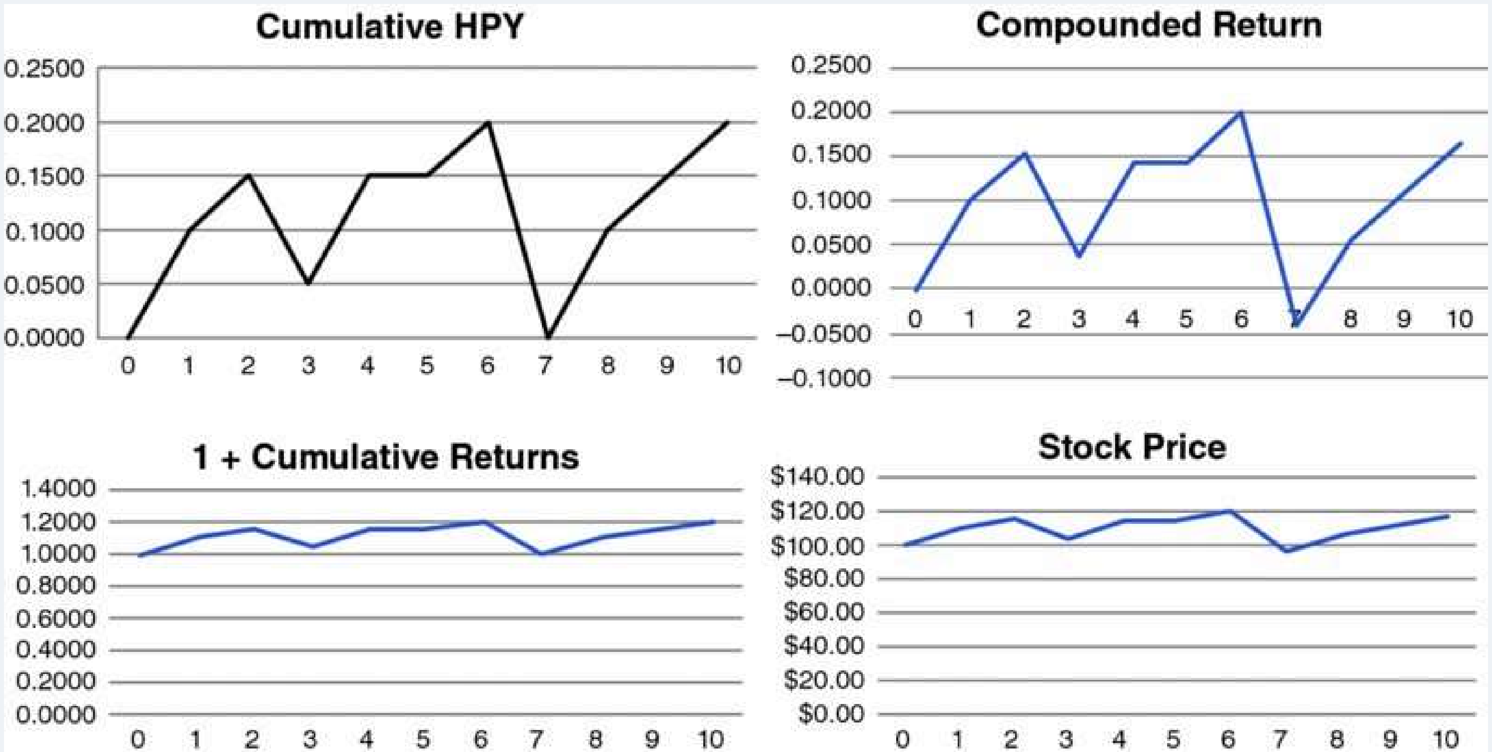


EXHIBIT 6.9 Cumulative and Compounded Returns

Note that the compounded return captures the actual rate at which the stock grows over time to equal its ending value and is therefore a more accurate measure of the investor's return over time than the cumulative *HPY*s.

BLOOMBERG STATISTICS

Statistical calculations of the prices and returns for stocks, bonds, indexes, and portfolios are common on many Bloomberg screens. Screens of note include Comparative Total Returns (COMP), Graph Volatility (GV), Historical Volatility Table (HVT), Company Risk (RSKV), CORR, and PC.

- **Total Return, COMP**: The COMP screen shows the cumulative holding period returns for a selected stock, bond, portfolio, or index.
- **Graph Volatility Screen**: The GV screen charts historical prices, volatility, and other volatility measures for up to four securities.
- **CORR:** The CORR screen can be used to create and save a number of correlation matrices for securities, indexes, currencies, interest rates, and commodities. The matrix also shows a variance-covariance matrix (Cov), correlation coefficient matrix (Correlation), beta, and other correlation and regression parameters. A correlation matrix allows you to compare a maximum of 500 peers. The number of peers that can appear depends on the type of matrix you create. A symmetric matrix displays a maximum of 10 rows and 10 columns (10x10). A nonsymmetric matrix displays a maximum of 500 rows and 10 columns (500x10). An additional column may be inserted as the first column for any security in the matrix to see its correlation to other instruments in the matrix.

BLOOMBERG EXCEL ADD-IN

There are a number of Bloomberg Excel Add-in templates that calculate returns and volatility, as well as screen securities by volatility and return. See the DAPI screen and click “Excel Template Library.” One useful template for calculating cumulative returns for different securities and for different periods is the XTOT Template. To access: DAPI <Enter>, click “Excel Template Library” and look for “Multiple Security Total Return Applications” in the “Equity” and “Other” category.

See Bloomberg Web Exhibit 6.1.

Parameter Estimates: Regression

Linear Regression

Historical averages provide analysts with important descriptive information about a security's past performance, but as noted, they are not usually good estimates of future values. Another approach to estimating expected returns and variances is to use regression analysis to estimate the behavioral relationship between the returns on a security and an identified explanatory variable such as the economy or the overall market. Given the estimated relation, the expected return, variability, and covariance can be estimated by independently projecting the future values of the explanatory variable.

Regression involves estimating the coefficients of an assumed algebraic equation. A *linear regression* model has only one explanatory variable, whereas a *multiple regression* model has more than one independent variable. As an example, consider a linear regression model relating the rate of return on a stock (dependent variable) to the market rate of return, R^M (independent variable), where R^M is measured by the proportional change in a stock index:

$$r_j = \alpha + \beta R_j^M + \varepsilon_j$$

where:

- α = intercept
- β = slope = $\Delta r / \Delta R^M$
- j = observation

- ϵ = error

In the above equation, ϵ_j is referred to as the error term or stochastic disturbance term. Thus, the model assumes that for each observation *j*, errors or deviations in the relationship between *r* and R^M can exist, causing *r* to deviate from the algebraic relation defined by $\alpha + \beta R^M$. Since, a priori, the errors are not known, the regression model needs to provide assumptions about ϵ. The standard assumptions are:

$$E(\varepsilon_j) = 0$$
$$V(\varepsilon_j) \text{ is constant}$$
$$Cov(\varepsilon, R^M) = 0$$

This regression model is depicted graphically in Exhibit 6.10. The regression line shown in the figure is referred to as the *characteristic line*. As shown in the figure, for each observation ($R^M{}_1$ and $R^M{}_2$), there are corresponding values for *r*, given α and β (r_1 and r_2), and there are possible errors, ϵ, that can cause *r* to be greater or less than the values determined by the intercept and slope. The assumption $E(\epsilon_j) = 0$, however, indicates that, on average, the errors cancel each other out, causing the $E(r_j)$ for observation $R^M{}_j$ to equal $E[\alpha + \beta R^M{}_j]$. The second assumption of a constant $V(\epsilon)$, in turn, implies that the distribution of errors is the same at each observation; the third assumption indicates that ϵ and $R^M{}_j$ are independent.

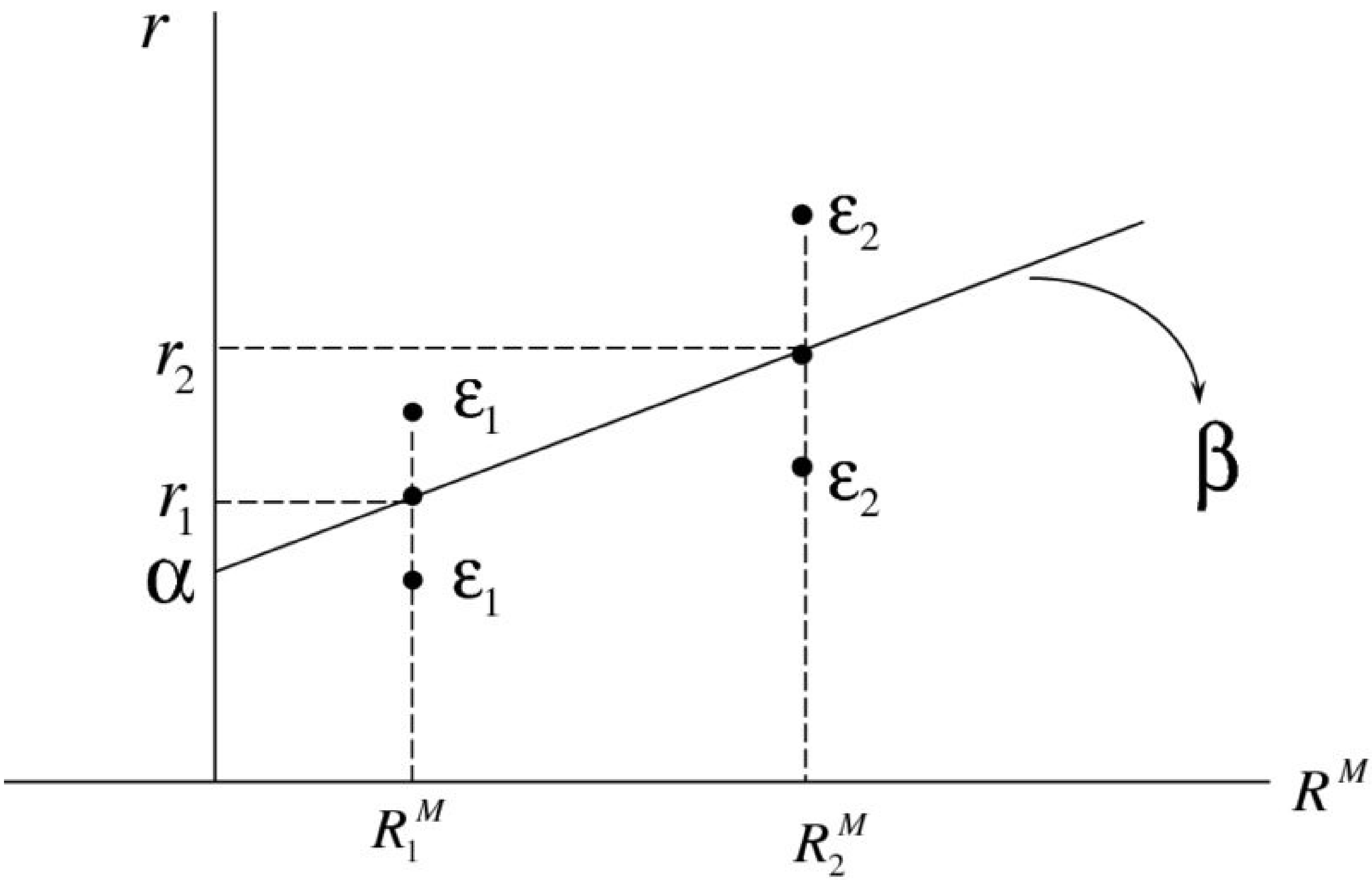


EXHIBIT 6.10 Characteristic Line

Using the above assumptions and the expected value operator rules, the expected value and variance can be defined in terms of the regression model as follows (see Exhibit 6.11 for the derivation of the equations):

$$E(r) = E[\alpha + \beta R^M + \varepsilon] \qquad (6.3)$$

$$E(r) = \alpha + \beta E(R^M) + E(\varepsilon)$$

$$E(r) = \alpha + \beta E(R^M)$$

$$V(r) = E[r - E(r)]^2 \qquad (6.4)$$

$$V(r) = \beta^2 V(R^M) + V(\varepsilon)$$

If two securities (1 and 2) both are related to R^M such as

$$r_1 = \alpha_1 + \beta_1 + \varepsilon_1$$

$$r_2 = \alpha_2 + \beta_2 + \varepsilon_2$$

(where the *j* subscript indicating observation is deleted) and ϵ_1 and ϵ_2 are independent ($Cov(\epsilon_1, \epsilon_2) = 0$), then the equation for the $Cov(r_1 r_2)$ is

$$Cov(r_1 r_2) = \beta_1 \beta_2 V(R_m) \qquad (6.5)$$

See Exhibit 6.11 for the derivations of Equation (6.4) and (6.5).

Variance

- Assume:

$$E(\varepsilon) = 0$$

$$Cov(R^M \varepsilon) = 0$$

$$Cov(R^M \varepsilon) = E[R^M - E(R^M)][\varepsilon - E(\varepsilon)]$$

$$V(\varepsilon) = E[\varepsilon - E(\varepsilon)]^2 = E[\varepsilon - 0] = E[\varepsilon^2]$$

$$V(r) = E[r - E(r)]^2$$

$$V(r) = E[\alpha + \beta R^M + \varepsilon - \alpha - \beta E(R^M)]^2$$

$$V(r) = E[\beta[R^M - E(R^M)] + \varepsilon]^2$$

$$V(r) = E[\beta^2[R^M - E(R^M)]^2 + \varepsilon^2 + 2\beta[R^M - E(R^M)]\varepsilon$$

$$V(r) = \beta^2 E[R^M - E(R^M)]^2 + E(\varepsilon^2) + 2\beta E[R^M - E(R^M)]\varepsilon$$

$$V(r) = \beta^2 V(R^M) + V(\varepsilon)$$

-

Covariance

- Given:

$$r_1 = \alpha_1 + \beta_1 R^M + \varepsilon_1$$

$$r_2 = \alpha_2 + \beta_2 R^M + \varepsilon_2$$

$$Cov(\varepsilon_1 \varepsilon_2) = E[\varepsilon_1 - E(\varepsilon_1)][\varepsilon_2 - E(\varepsilon_2)] = E[\varepsilon_1 - 0][\varepsilon_2 - 0] = E[\varepsilon_1 \varepsilon_2] = 0$$

Assume :

$$E(\varepsilon) = 0$$

$$Cov(\varepsilon_1 \varepsilon_2) = E[\varepsilon_1 \varepsilon_2] = 0$$

$$Cov(\varepsilon R^M) = 0$$

$$Cov(r_1 r_2) = E[r_1 - E(r_1)][r_2 - E(r_2)]$$

$$Cov(r_1 r_2) = E[\alpha_1 + \beta_1 R^M + \varepsilon_1 - \alpha_1 - \beta_1 E(R^M)][\alpha_2 + \beta_2 R^M + \varepsilon_2 - \alpha_2 - \beta_2 E(R^M)]$$

$$Cov(r_1 r_2) = E[\beta_1 \beta_2 [R^M - E(R^M)]^2 + \varepsilon_1 \varepsilon_2 + \beta_1 [R^M - E(R^M)]\varepsilon_2 + \beta_2 [R^M - E(R^M)]\varepsilon_1$$

$$Cov(r_1 r_2) = \beta_1 \beta_2 E[R^M - E(R^M)]^2 + E(\varepsilon_1 \varepsilon_2) + \beta_1 E[R^M - E(R^M)]\varepsilon_2 + \beta_2 E[R^M - E(R^M)]\varepsilon_1$$

$$Cov(r_1 r_2) = \beta_1 \beta_2 V(R^M) + Cov(\varepsilon_1 \varepsilon_2) + \beta_1 Cov(R^M \varepsilon_2) + \beta_2 Cov(R^M \varepsilon_1)$$

$$Cov(r_1 r_2) = \beta_1 \beta_2 V(R^M) + 0 + 0 + 0$$

$$Cov(r_1 r_2) = \beta_1 \beta_2 V(R^M)$$

-

EXHIBIT 6.11 Math Derivation of $V(R)$ and $Cov(R_1, R_2)$ in Terms of the Regression Model

Estimating the Regression Coefficients

The intercept and slope of a linear regression model can be estimated by the ordinary least squares (OLS) estimation procedure. This technique uses sample data for the dependent and independent variables (time series data or cross-sectional data) to find the estimates of α and β that minimize the sum of the squared errors. Graphically, OLS estimation can be described as finding the line (α and β) that cuts through a scatter diagram of data coordinates of the dependent and independent variables that minimizes the errors (or sum of squared errors). The estimating formulas for α and β that best cut through the scatter diagram in which the errors are minimized are:

$$\hat{\alpha} = \bar{r} - \hat{\beta}\overline{R}^{M} \tag{6.6}$$

$$\hat{\beta} = \frac{C\hat{o}v(r, R^{M})}{\hat{V}(R^{M})} \tag{6.7}$$

where:

- $C\hat{o}v(r, R_m), \hat{V}(R_m), \bar{r}, \text{and } \overline{R}_m$ = estimates (averages)
- ∧ = sign for estimate

An estimate of the $V(\epsilon)$, can be found using Equation (6.4) for $V(r)$. That is:

$$\hat{V}(\varepsilon) = \hat{V}(r) - \hat{\beta}^{2} V(\hat{R}^{M}) \tag{6.8}$$

where: $V(r)$ and $V(R^{M})$ can be estimated using the sample averages, and β can be estimated using the ordinary least squares estimating Equation (6.7).

Exhibit 6.8 shows how to estimate a regression model relating the rate of return on a stock to the market rate using quarterly data on the stock and the stock index. In the example in the exhibit, the estimated β is 1.04, α is 0.00268, and $V(\epsilon)$ is 0.0000184. Using these estimates, the expected return and risk of the stock in term of this regression model are:

$$E(r) = \hat{\alpha} + \hat{\beta}E(R^M)$$
$$E(r) = 0.00268 + 1.04E(R^M)$$
$$V(r) = \hat{\beta}^2 V(R^M) + \hat{V}(\varepsilon)$$
$$V(r) = (1.04)^2 \; V(R^M) + 0.0000184$$

Regression Qualifiers

The coefficients between any variable can be estimated with a regression model. Whether the relationship is good or not depends on the quality of the regression model. All regression models therefore need to be accompanied by information about the quality of the regression results. Two commonly used regression qualifiers are the coefficient of determination (R^2) and the t-statistic (or t-test).

The coefficient of variation measures how much of the variation in the dependent variable can be explained by the variation in the explanatory variables. The variance of a security's return defined in terms of the above regression model, Equation (6.4), measures how much variation in return can be attributed to the market, $\beta^2 V(R^M)$ and how much variation can be attributed to other factors, $V(\epsilon)$:

$$V(r) = \beta^2 V(R^M) + V(\varepsilon)$$

The variability can also be expressed as proportions of total variability by dividing Equation (6.4) by $V(r)$:

$$1 = \frac{\beta^2 V(R^M)}{V(r)} + \frac{V(\varepsilon)}{V(r)} \qquad (6.9)$$
$$1 = R^2 + \frac{V(\varepsilon)}{V(r)}$$

The first term on the right-hand side of Equation (6.9) measures the proportion of variation in r that can be attributed to the market, and the second term measures the proportion due to other factors.

Statistically, the first term on the right side of the equation is defined as the coefficient of variation, R^2:

$$R^2 = \frac{\beta^2 V(R^M)}{V(r)} \tag{6.10}$$

Note: Since both terms are positive, R^2 must be between 0 and 1:

$$0 \leq R^2 \leq 1$$

Thus, the higher the R^2, the more the variability is explained by the explanatory variables and the less by other factors, and therefore the better is the statistical quality of the regression model. If the R^2 coefficient is 0.9, for example, it indicates that 90 percent of the fluctuations in the security's return can be attributed to the market and 10 percent to other factors.

The other qualifier, a *t*-test or *t*-statistic, is equal to the ratio of the estimated slope coefficient (e.g., β) to standard deviation in the coefficient estimates, $\sigma_{\hat{\beta}}$. For our regression example, the *t*-statistic is

$$t = \frac{\hat{\beta}}{\sigma_{\hat{\beta}}}$$

Explanations of the *t*-distribution, *t*-statistics, and distributions of estimates can be found in most statistics texts. For our purposes, the *t*-statistic provides a test of the null hypothesis of the coefficient being equal to zero. A *t*-statistic greater than 2.5 in absolute value, indicates that one can reject the null hypothesis, H_0, that the coefficient is equal to zero (i.e., H_0: β = 0), whereas a relatively low *t*-statistic—that is, less than 2.5 in absolute value—indicates that one cannot reject the null hypothesis. Since the hypothesis of β = 0 implies there is no relation between the dependent and independent variable (i.e., between a security's return and the market return), a high *t*-value (one greater than 2.5 in absolute value) indicates the relation between the dependent, and independent variable is significant (at the 5 percent level).

(a)

(b)

EXHIBIT 6.12 Disney and GE Regressions

Regression Examples

Linear regressions for the Disney Corporation and General Electric are shown in Exhibit 6.12. Both regression analyses were done on Bloomberg's "Beta" screen. In each regression, the stock's daily percentage returns are regressed against the percentage changes in the S&P 500 (SPX) for the period from 9/8/2008 to 9/5/2013. The Beta screen shows the scatter diagram, regression estimates, and qualifiers. From this regression, Disney has a beta 1.106, alpha of 0.034, $\sigma(\epsilon)$ of 1.153 ($V(\epsilon) = 1.33$), t-statistic = $\beta/\sigma(\beta)$ = 54.494, and R^2 of 0.703:

$$E(r) = \hat{\alpha} + \hat{\beta}E(R^M)$$
$$E(r) = 0.034 + 1.106E(R^M)$$

$$V(r) = \hat{\beta}^2 V(R^M) + \hat{V}(\varepsilon)$$
$$V(r) = (1.106)^2 V(R^M) + 1.33$$

GE has a beta 1.172, alpha of −0.028, $\sigma(\epsilon)$ of 1.644 ($V(\epsilon) = 2.7027$), t-statistic = $\beta/\sigma(\beta)$ = 40.436, and R^2 of 0.566:

$$E(r) = \hat{\alpha} + \hat{\beta}E(R^M)$$
$$E(r) = -0.028 + 1.172E(R^M)$$

$$V(r) = \hat{\beta}^2 V(R^M) + \hat{V}(\varepsilon)$$
$$V(r) = (1.172)^2 \ V(R^M) + 2.7027$$

Given the betas of GE and Disney, the covariance formula for these stocks based on the estimated regression equation is

$$Cov(r_1 r_2) = \beta_1 \beta_2 V(R^M)$$
$$Cov(r_1 r_2) = (1.106)(1.172) V(R^M)$$

Regression analysis can be done using Excel spreadsheets and Excel regression commands. Exhibit 6.13 shows the regression of monthly returns of Disney stock against the S&P 500 for the period from 8/31/2010 to 8/30/2013. The data were downloaded from the Bloomberg Excel Add-in (described in the Bloomberg Exhibit Box). The exhibit shows three Excel commands for generating the regression parameters: "Data Analysis" Excel Add-in, Scatter diagrams, and Excel Regression Commands.

EXHIBIT 6.13 Statistical Analysis in Excel of Disney: Regression Commands

BLOOMBERG REGRESSION AND CORRELATION SCREENS

HRA: BLOOMBERG'S LINEAR REGRESSION SCREEN

The HRA screen is an analytical screen that estimates linear regression of a selected security (or the loaded security) to a change in the value of a second selected security or index. On the HRA screen, you can select (1) the dependent variable (e.g., IBM stock) and independent variable (S&P 500, SPX); (2) Data (e.g., last price or mid-price); (3) Period (e.g., daily, weekly, or monthly); regression time periods: two time periods for the regressions can be analyzed (e.g., two-year regression period and three-year period); (4) Regression analysis: regress values, differences in values, or percentage change in values.

The screen displays a scatter diagram of the data points (moving your curser to the diamond coordinate brings up the date and each security's return), the estimated regression parameters, alpha and beta, and qualifiers (R^2), standard error ($\sigma(\epsilon)$), and t-statistic $\beta/\sigma(\beta)$.

Other features: (1) Beta +/− (check box) calculates Beta+ when the independent variable is increasing, the Beta– when it is decreasing, and the difference or convexity. (2) The Non-Param check box converts the data to logs. (3) There is also a calculation of the security's adjusted beta (Adj Beta). The security's adjusted beta is derived from historical data but modified by moving the regression beta (raw beta) toward the market average.

BETA: BLOOMBERG'S LINEAR REGRESSION SCREEN

The Beta screen is similar to the HRA screen, estimating a linear regression of a selected security (or the loaded security) to a change in value of a second selected security or index. See Exhibit 6.12.

PC, PEER CORRELATION: STOCK AND PEER CORRELATION AND REGRESSION PARAMETERS

- The PC platform shows correlation and regression parameters of a selected stock with its peers and related indexes: S&P 500, Dow Jones, Sector Index, and other indexes.
- You can select different peers, indexes, portfolios, and securities from searches from the “Peer Source” dropdown, select different parameters (correlation, beta, covariance, or *t*-statistics) from the “Calculation” tab, different regression periods, and frequencies (e.g., daily, weekly, or monthly).
- From the red “Edit” tab, you can also change the securities (rows) and indexes (columns). Each security's information (e.g., price graphs) can also be accessed by clicking the cell associated with that security and index.
- The PC screen can be saved to the Correlation Matrix from the red “Save to CORR” tab, where it can be accessed for later study by bringing up the CORR screen (CORR <Enter>). See Exhibit 6.15.

BLOOMBERG ADD-IN FOR EXCEL

Data for conducting time-series analysis also can be done using the Bloomberg Add-in for Excel (see Chapter 2 for a description on the Bloomberg Excel Add-in). On the Bloomberg Add-in, click “Import Data” and “Real-Time/Historical” dropdowns and follow the “Data Wizard” steps. Linear regression and other statistical analysis can be done in Excel by using Excel Commands. See Exhibit 6.13 for an illustration.

See Bloomberg Web Exhibit 6.2.

Using β as a Measure of Market Risk

In the regression model $r = \alpha + \beta R^M + \epsilon$, the slope in that equation, β, measures the change in the stock's rate of return per change in the market rate of return:

$$\beta = \Delta r / \Delta R^M$$

β, in turn, is a measure of an individual security's response to the market. If r and R^M are linearly related, β also measures the proportional relation between r and R^M. Thus, a stock with $\beta > 1$ would have rates of return that changes more than the market changes. For example, a stock with $\beta = 1.5$ would have a rate of return that changes by 15 percent for a 10 percent change in the market. A stock with $\beta = 1$, in turn, would have a change in its rate that matches the changes is the market, a stock with a $\beta < 1$ would have a smaller change than the market, and one with a $\beta < 0$ would have rates that change inversely to the market:

- $\beta > 1$: Example $\beta = 2$: 10% ↑ in $R^M \Rightarrow$ 20% ↑ in r
    - 10% ↓ in $R^M \Rightarrow$ 20% ↓ in r
- $\beta = 1$: Example $\beta = 1$: 10% ↑ in $R^M \Rightarrow$ 10% ↑ in r
    - 10% ↓ in $R^M \Rightarrow$ 10% ↓ in r
- $\beta < 1$: Example $\beta = 0.5$: 10% ↑ in $R^M \Rightarrow$ 5% ↑ in r
    - 10% ↓ in $R^M \Rightarrow$ 5% ↓ in r
- $\beta = 0$: Example $\beta = 0.0$: 10% ↑ in $R^M \Rightarrow$ 0% ↑ in r
    - 10% ↓ in $R^M \Rightarrow$ 0% ↓ in r
- $\beta < 0$: Example $\beta = -0.5$: 10% ↑ in $R^M \Rightarrow$ 5% ↓ in r
    - 10% ↓ in $R^M \Rightarrow$ 5% ↑ in r

Systematic and Unsystematic Risk

As previously discussed, the total risk of a stock can be broken into firm risk, industry risk, and market risk. It can also be broken into unsystematic risk and systematic risk. Unsystematic risk is a stock's risk due to firm and industry factors, whereas systematic risk is market risk. Since β measures how a security's rate of return relates to the market, it can be used to measure the security's systematic risk. Thus, a stock with a $\beta > 1$ would have greater risk than the market and one with a $\beta < 1$ would have less.

The regression model also can be used to decompose the total risk of a security into systematic risk and unsystematic risk. Specifically, Equation (6.8) for the variance of a security's return measures how much variation in return can be attributed to the market, $\beta^2 V(R^M)$, and how much variation can be attributed to other (firm and industry) factors, $V(\epsilon)$. Thus, the first term measures a security's systematic risk and the second term measures its unsystematic risk:

Systematic and unsystematic risk can also be expressed as proportions of total risk by dividing Equation (6.8) by $V(r)$:

The first term on the right-hand side of the equation measures the proportion of variation in *r* that can be attributed to the market, and the second term measures the proportion due to unsystematic factors. As noted, the first term is defined as the coefficient of variation, R^2. If this coefficient is 0.9, for example, it implies that 90 percent of the fluctuations in the security's return can be attributed to the market, and 10 percent to other factors.

A Note on Adjusting Betas

A number of empirical studies have reported evidence that historically estimated betas for stocks are generally not good predictors of future betas. Marshall Blume found low R^2's on the regressions of randomly selected stocks (average R^2 of 0.36) and low correlations between stock betas in one period and the next (average correlation coefficient of 0.60). This study and others have pointed to the need to adjust the beta of a stock, especially if it is to be used to measure a stock's market risk.[1]

Observing a tendency for the forecast beta to move from the historical beta toward a portfolio beta, Oldrich Vasicek proposed adjusting betas using a Bayesian estimation technique. This technique calls for calculating the forecast beta as a weighted average of the historical beta and the average beta across a sample of stocks. In the Bayesian approach, the weights are estimated by using the square of the standard error of an estimate of beta and the variance of the betas over the sample of stocks. An adjusted beta is also given on the Bloomberg BETA screen (Adj Beta). The security's adjusted beta shown is derived using a Bayesian approach in which the historical beta is modified by moving the regression beta (raw beta) toward the market average. For example, the adjusted beta for Disney is 1.071 compared to its historical or raw beta of 1.106, and the adjusted beta for GE is 1.115 and its raw beta is 1.172 (see Exhibit 6.12).

Beta$^+$ and Beta$^-$ and Convexity

Some stocks are observed to change differently when the market is increasing than when it is decreasing. One way to statistically estimate whether this may or may not be the case is to divide the data in a regression analysis into increasing market rates and decreasing market rates and then run two regressions. The beta for the increasing rate data is referred to as Beta$^+$ and the beta for the decreasing rate data is referred to as Beta$^-$. If the Beta$^+$ does not equal the Beta$^-$, then there is an asymmetrical gain/loss relation. For example if Beta$^+$ were equal to 1.25 and Beta$^-$ were equal to 0.75, then for a 10 percent increase in the market, the stock would increase by 12.5 percent, and for a 10 percent decrease in the market, the stock would decrease only by 7.5 percent. If two stocks have the same betas, but one has in absolute value a Beta$^+$ greater than a Beta$^-$, whereas the other has equal Beta$^+$ and Beta$^-$, then the former would be more valuable; that is, it provides greater gains if the market increases and smaller losses if the market decreases. The asymmetrical gain/loss relation defined by Beta$^+$ and Beta$^-$ is referred to a stock's *convexity*, where convexity is measured as the difference between Beta$^+$ and Beta$^-$ divided by two.

The Beta$^+$, Beta$^-$ and convexity for Disney and GE calculated from the previous regression period are shown in Exhibit 6.14. As shown, Disney has a Beta$^+$ of 1.192, Beta$^-$ of 1.026, and convexity of 0.083. The stock is more responsive when the market increases than when it decreases, increasing 1.192 percent for a 1 percent increase in the market, while decreasing by 1.026 percent for a 1 percent decrease in the market. Thus, it has greater gain-to-loss relation or a positive convexity. In contrast, GE has a Beta$^+$ of

1.088, a Beta$^-$ of 1.250, and a convexity of −0.081. The stock is more responsive when the market decreases than when it increases, decreasing 1.250 percent for a 1 percent decrease in the market, while increasing by 1.088 percent for a 1 percent increase in the market. It has greater loss to gain relation, or a negative convexity.

EXHIBIT 6.14 B$^+$ and B$^-$

BETA Screen, Beta$^+$ and Beta$^-$ and Convexity. Bloomberg Beta Screen shows the stocks Beta$^+$ and Beta$^-$ and convexity. To access: Ticker <Equity>; Beta; click Beta +/– box; click “Show Detailed Statistics” in the Actions tab. Beta$^+$ and Beta$^-$ and convexity can also be accessed from the HRA screen.

Alpha

From the regression, the intercept term, alpha, can be used as a measure of the stock's return above or below its risk-adjusted return. That is, if the intercept is positive, it suggests the stock is generating a return in excess of the market risk premium. For example, a stock with a beta of one and an alpha of zero would increase by 10 percent when the market is up by 10 percent and decrease by 10 percent when the market is down by 10 percent. Its return matches the return in the market. If the stock had a beta of one and an alpha of 0.02 (or 2 percent), then it would increase by 12 percent when the market increases by 10 percent and decrease by only 8 percent when the market decreases by 10 percent. Its return is always 2 percent better than the market return. The alpha for the Disney stock shown in Exhibit 6.12 is a positive 0.034, whereas the alpha for GE is −0.028. It should be noted that with a portfolio it is possible to diversify away firm and industry risk. If this is the case, then a portfolio with a positive alpha would be providing its investors with an excess or abnormal return. Using the intercept as a portfolio performance measure was first introduced by Jensen and is often referred to as the Jensen index. This measure of excess return for evaluating portfolios is discussed in more detail in Chapter 7.

Exhibit 6.15 shows a comparison of the R^2 or systematic risk measures and alphas for GE and related companies in its industry taken from Bloomberg's PC screen. Based on its R^2, approximately 55 percent of GE's variability is explained by market factors. This is similar to the R^2's of many of its peers. GE, however, is one of only five in the peer group with a negative alpha.

EXHIBIT 6.15 Peer Comparison of Alpha and R^2

Multiple Regression Analysis

The return on many stocks are better explained (higher R^2) in terms of a multiple regression model in which there are more than one explanatory variable. In terms of explaining the relation between a security's return and multiple explanatory variables, a multiple regression model takes the following form:

where:

- α = intercept
- F_i = explanatory variable
- β_i = slope = $\Delta r/\Delta F_i$
- ϵ = error

When a number of stocks are explained by the same set of factors, the model is known as a factor model. A linear regression model is known as a *single-factor model*, and a multiple regression model is referred to as a *multifactor model*. Typically, a single-factor model is explained by the market return and is referred to as the market model. For multifactor models, the challenge is identifying the explanatory variables.

One of the early studies using specified factors was done by William Sharpe. In his study, he estimated the relationship between a stock's return and the market rate, the long-term bond rate, and the stock's dividend yield. He estimated this relationship for over 2,000 stocks using data covering 1931 through 1979. In comparing several regression models, Sharpe found that models with two or more coefficients had better regression results than the linear regression market model explained by just R^M. This finding provided some support for a multifactor model. A more recent study of multifactor models was done by Chen, Roll, and Ross. They found a relatively strong statistical relation between a stock's return and four factors measuring unanticipated changes in inflation, the term structure of rates (differences in long-term and short-term bond rates), industrial production, and risk premiums (differences in yields on low-quality and high-quality bonds). Thus, like the Sharpe study, the Chen, Roll, and Ross study provided evidence in support of the multifactor model as an explanation of what determines a stock's returns. These and other multifactor models are examined in more detail in Chapter 10.

MRA: BLOOMBERG'S MULTIPLE REGRESSION SCREEN

Multiple regression analysis can be done using the Bloomberg MRA screen. On the screen, one selects a set for inputting information or editing a previous set. The dependent and independent variables are inputted by their ticker and <Equity> or Index ticker <Index>. Economic information such as that found in ECOF or ECST has a ticker; to input into MRA, one types in its Ticker and presses the <Index> key. Once the variables are selected, the user can save the set by typing 1 and hitting <Enter> and can select the time period and frequency (daily, weekly, etc.) by hitting 2 <Enter>. The MRA output shows the coefficient estimates, *t*-tests, R^2, and F-Statistic.

MULTIPLE REGRESSION USING BLOOMBERG EXCEL ADD-IN AND DATA ANALYSIS EXCEL ADD-IN

Using Bloomberg data pulled from the Bloomberg's Excel Add-in or other data sources, multiple regressions can be done in Excel by using the “Data Analysis” Add-in.

See Bloomberg Web Exhibit 6.3.

Cross-Sectional Regression Analysis

The data used in the regression example (Exhibit 6.8) was time-series data. In estimating a security's expected return, risk, and other features, analysts often use cross-sectional data in which the rates of return on a number of securities are used. Cross-sectional regression analysis can be used to explain how the market prices securities or to identify market factors that are important to determining an investment's return. Some analysts use a cross-sectional regression model to estimate how the market prices a stock, *P*, relative to its earnings per share (*e*)—its price-to-earnings ratio, P/e. The cross-sectional models differ in terms of the explanatory variables used to explain P/e. Elton and Gruber, for example, regressed the P/e ratios of 150 stocks against their historical growth rates to estimate the following simple linear relation between the P/e of any stock *i* and its growth rate, g_i. For one given year, they estimated the following relation:

In an earlier study, Malkiel and Cragg regressed 150 stocks' P/e ratios against three variables: dividends-to-earnings ratio (d/e), historical growth rates, *g*, and betas. For the relation for one of the years in their study, they found:

The coefficients in the cross-sectional regression equations are estimates of the relative importance the market places on the explanatory variable to determine how much a stock's price trades relative to its earnings-per-share (eps). For example, the slope coefficient of the Elton-Gruber regression of 1.6 indicates that for each 1 percent of growth, the market prices a stock $1.60 more per its eps. Thus, a stock with a 5 percent growth rate the market would price at $11 per dollar of eps (P/e = 11) and a stock with a 6 percent growth rate the market would price at $12.60 per dollar of eps (P/e = 12.6). The cross-sectional regression model of Malkiel and Cragg shows not only the relative importance the market places on the growth rate in determining P/e, but also the positive impact dividend/earnings have on P/e and the negative impact a stock's beta has on P/e.

Regression analysis can be done using Excel spreadsheets and Excel regression commands. Exhibit 6.16 shows the regression of the P/e ratios of 80 stocks taken from the S&P 100 against their betas, growth rates, and dividend payout ratio. Different from the Malkiel and Craig study, the growth rates and payout ratios for each stock were the estimated value used in Bloomberg's DDM (described in Chapter 3), and for beta, the stocks' adjusted betas were used instead of historical betas. The data were downloaded from the Bloomberg Excel Add-in (described in the Bloomberg Exhibit Box). The exhibit shows the data and regression results from using the Excel Add-in: Data Analysis:

EXHIBIT 6.16 Cross-Sectional Regression Analysis Using Excel and Data Downloaded from Bloomberg Excel Add-In

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BLOOMBERG EXCEL ADD-IN: CROSS-SECTIONAL MULTIPLE REGRESSION

Data for conducting cross-sectional analysis can be pulled from Bloomberg's database using the Bloomberg Add-in for Excel and then pulling in stocks from an index or portfolio (see Chapter 2 for a description on Bloomberg Excel Add-in):

- On the Bloomberg Add-in, click “Real-Time/Current” from the “Import Data” and “Real-Time/Historical” dropdowns.
- On the Bloomberg Data Wizard Box, Step 1, click “Indexes” in the “From” dropdown and the name of index (e.g., S&P 100) from the “Indexes” dropdown, and then click “Add All.” This will bring up the stocks for the index. Once loaded, click “Next.”
- On the Bloomberg Data Wizard box, Step 2, search and then add stock returns (e.g., Price-to-earning (P/e), growth rates, DDM implied growth rates, DDM dividend payout ratio in growth stage, or Altman Z-Score).
- After loading variables, click “Next.”
- On the Bloomberg Data Wizard Box, Step 3, click “Finish” to export the data to Excel.

MULTIPLE-REGRESSION IN EXCEL

- Multiple-regression can be done in Excel by using the "Data Analysis" Add-in. For an example, see Exhibit 6.16.

Relationship between Return and Risk

In general, there is a direct relationship between the expected return and risk among securities available in the market. That is, the greater a security's risk, the greater its rate of return. This positive return-risk relationship is simply the result of rational investment decisions. For example, if there were two stocks available in the market that had the same expected returns but different risks, rational investors would obviously want only the lower risk security. This preference would increase the demand and price of that security (causing its rate to decrease) and lower the demand and price of the higher risk security (causing its rate of return to increase). After this market adjustment, the higher risk security's price would reflect a higher rate of return than the lower risk security.

Security Market Line

In the finance literature, the relationship between return and risk has been described by the *Capital Asset Pricing Model* (CAPM). The CAPM postulates that most investors hold portfolios and therefore are not exposed to unsystematic risk; that is, with a portfolio investors can diversify away firm and industry risk. As a result, a security's *equilibrium rate of return* is determined only by its systematic risk, which, in turn, can be measured by its beta. In terms of the CAPM, a stock with a $\beta > 1$ has more risk than the market, and therefore, in equilibrium it should be priced so that its equilibrium expected rate, $E(r)^*$, is greater than the expected market rate of return: $E(r)^* > E(R^M)$. In contrast, a security with a $\beta < 1$ should be priced so that, in equilibrium, its expected rate is less than the expected market rate: $E(r)^* < E(R^M)$. By the same reasoning, a security with a $\beta = 1$, would have the same risk as the market (if there is no unsystematic risk) and therefore should be priced such that its expected rate is equal to the expected market rate: $E(r)^* = E(R^M)$. Moreover, if systematic risk is the only determining factor, then a security with a $\beta = 0$ should be priced in equilibrium such that its rate of return is equal to the rate on a risk-free security: $E(r)^* = R_f$. Finally, if investors make decisions in a portfolio context, then a stock with a negative beta would be negatively correlated with the market. Its inclusion in a portfolio would serve to reduce the portfolio's risk more than a risk-free security, and as a result, it would be priced to yield a return less than the risk-free rate.

The relation between beta and a security equilibrium return is summarized in Exhibit 6.17. The direct relationship between an investment's equilibrium rate of return and β is also shown graphically in Exhibit 6.18. The line depicted in the figure is known as the *security market line*, SML. The equation of the SML is:

(6.11)

where:

- *i* = any investment (stock or portfolio) i
- *signifies equilibrium

EXHIBIT 6.17 Equilibrium Return and Beta Relation

EXHIBIT 6.18 SML

The SML represents a cross-sectional model; it shows the equilibrium rate for any security given that security's beta. (Note that the characteristic line shown in Exhibit 6.5 is a time-series model, showing the relationship between a security's return and the market.) The slope of the SML, $E(R^M) - R_f$, is the market risk premium, RP^M:

It measures the additional return over the risk-free rate investors require in order to invest in the market.[2]

Equation (6.11) is often used by analysts to determine a stock's required return, k, or to determine the required equity return in a capital budgeting problem. For example, if the risk-free rate were 6 percent and the market risk-premium were 4 percent, a stock with a β of 1.5 would have a required return of 12 percent using the SML equation:

If the stock were expected to generate dividends of $10 each year for three years and be worth $100 at the end of year three, then it would be valued at $95.20 given the required return of 12 percent:

The return-risk relationship depicted by the SML is not static. Changes in the SML occur as a result of either a change in the risk-free rate or a change in the market risk-premium. A change in the risk-free rate leads to a parallel shift in the SML: An increase in R_f causes an increase in the required returns for any β; a decrease in R_f causes a decrease in the required returns for any β. The risk-free rate, in turn, is influenced by macroeconomic factors such as changes in monetary policy, fiscal policy, international balance of payments, real economic growth, and inflation. Changes in the market risk premium, on the other hand, cause changes in the slope of the SML. Such changes occur as a result of changing investor attitudes toward risk. For example, if investors were to become more timid (perhaps because of information indicating an economic recession), they would require a greater risk premium to invest in stock. Under such conditions, investor actions would cause the market risk premium to increase and the SML to become steeper. The CAPM and other models for determining the equilibrium return on investments are examined in more detail in Chapter 9 and Chapter 10.

EQRP: BLOOMBERG'S EQUITY RISK PREMIUM SCREEN

The EQRP screen shows a stock's risk premium. The premium is equal to the forecasted market risk premium $E(R^M) - R_f$ times the stock's beta. The forecasted market risk premium is based on a projected market rate and risk-free rate. The risk-free rate is the 10-year Treasury for the country. Beta is the historical beta. The user can change the market risk-premium and beta. The screen also shows the historical premiums and betas.

See Bloomberg Web Exhibit 6.4.

Return-Risk Preferences

What return-risk combination an investor chooses depends on her risk-return preference. Risk-return preference of investors in general—the market's risk-return preference—has an impact on the risk premium. To see the relation between risk-return preferences and the risk premium, suppose there are only two securities available in the market: a risk-free security and a risky bond. Suppose the risk-free security is a zero-coupon bond promising to pay $1,000 at the end of one year and that it currently is trading for $909.09 to yield a one-year risk-free rate, R_f, of 10 percent:

Suppose the risky bond is also a one-year zero coupon bond with a principal of $1,000, but there is a chance it could default and pay nothing. In particular, suppose there were a 0.8 probability the bond would pay its principal of $1,000 and a 0.2 probability it would pay nothing. The expected dollar return from the risky bond is therefore $800:

Given the choice of two securities, suppose that the market were characterized by investors who are willing to pay $727.27 for the risky bond, in turn yielding them an expected rate of return of 10 percent:

By paying $727.27, investors would have a 0.8 probability of attaining a rate of return of 37.5 percent [($1,000/$727.27) – 1] and a 0.2 probability of losing their investment. In this case, investors would be willing to receive an expected return from the risky investment that is equal to the risk-free rate of 10 percent, and the risk premium, $E(R) - R_f$, would be equal to zero. In finance terminology, such a market is described as *risk neutral*. Thus, in a risk neutral market, the required return is equal to the risk-free rate and the risk premium is equal to zero.

Instead of paying $727.27, suppose investors like the chance of obtaining returns greater than 10 percent (even though there is a chance of losing their investment), and as a result they are willing to pay $750 for the risky bond. In this case, the expected return on the bond would be 6.67 percent, and the risk premium would be negative:

By definition, markets in which the risk premium is negative are called *risk loving*. Risk-loving markets can be described as ones in which investors enjoy the excitement of the gamble and are willing to pay for it by accepting an expected return from the risky investment that is less than the risk-free rate. Even though there are some investors who are risk loving, a risk-loving market is an aberration, with the exceptions being casinos, sports gambling markets, lotteries, and racetracks.

Whereas risk-loving and risk-neutral markets are rare, they do serve as a reference for defining the more normal behavior toward risk—*risk aversion*. In a risk-averse market, investors require compensation in the form of a positive risk premium to pay them for the risk they are assuming. Risk-averse investors view risk as a disutility, not a utility, as risk-loving investors do. In terms of our example, suppose most of the investors making up our market were risk averse and as a result were unwilling to pay $727.27 or more for the risky bond. In this case, if the price of the risky bond were $727.27 and the price of the risk-free were $909.09, there would be little demand for the risky bond and a high demand for the risk-free one. Holders of the risky bonds who wanted to sell would therefore have to lower their price, increasing the expected return. On the other hand, the high demand for the risk-free bond would tend to increase its price and lower its rate. For example, suppose the markets cleared when the price of the risky bond dropped to $701.75 to yield 14 percent, and the price of the risk-free bond increased to $917.43 to yield 9 percent:

In this case, the risk premium would be 5 percent and the market is defined as being risk averse.

In a risk-averse market, the positive risk premium required by an investor to hold the riskier bond is partly the result of uncertainty and partly due to liquidity. For example, if investors knew that the probability of default was in fact 0.8, then they would know that by buying a portfolio of such bonds (e.g., 100 bonds like B), 80 percent of the portfolio would pay $1,000 at the end of the year and 20 percent would pay nothing. Alternatively, if investors buy Bond Bs over time, then they would find that in eight out of 10 years, they would receive a $1,000 principal and two out of 10 years, they would receive nothing. Thus, if the price of Bond B were the risk-neutral price of $727.27, then investors' average portfolio return or their average return over time would be 10 percent. It would appear that provided the 0.8 probability is known, investors would be indifferent between a portfolio of B bonds and Bond A or a strategy of buying B bonds or A Bonds over time. However, to obtain the certain portfolio return of 10 percent would require that investors buy a portfolio of B bonds or buy such bonds over an extended period to realize the 10 percent rate. This would require more funds or time than simply buying the risk-free Bond A. As a result, investors would demand less of B because of these marketability or liquidity requirements. The liquidity concern would, in turn, push the price of the B Bond down and increase its yield above the risk-free rate.

Historically, security markets such as the stock and corporate bond markets have generated rates of return that on average have exceeded the rates on Treasury securities. This would suggest that such markets are risk averse. Since most markets are risk averse, a relevant question is the degree of risk aversion. The degree of risk aversion can be measured in terms of the size of the risk premium. The greater an investor's risk aversion, the greater the demand for risk-free securities and the lower the demand for risky ones, and thus the larger the risk premium.

Conclusion

Although securities can be described and evaluated in terms of their common characteristics, for most investors the most important security characteristics are the expected rate of return and risk. The risk of a security is the possibility that the realized return will deviate from the expected return. By definition, the variance, standard deviation, and coefficient of variation of a stock's rate of return define such risk. That is, the greater a security's variance, the greater the realized return can deviate from the expected return, and thus the greater the security's risk. In this chapter, we examined the expected return and risk of stock statistically by examining return distributions and characteristics—mean, variance, and correlation parameters. In the next chapters, we continue our analysis of return and risk by examining the return and risk of a portfolio of stocks.

Web Site Information

1. For financial information on stocks see:
    1. www.Finance.Yahoo.com
    2. http://www.hoovers.com
    3. http://www.bloomberg.com
    4. http://www.businessweek.com
    5. http://www.ici.org
    6. http://seekingalpha.com
    7. http://bigcharts.marketwatch.com
    8. http://www.morningstar.com

9. http://free.stocksmart.com
2. FINRA
   1. Go to http://www.finra.org/index.htm, “Sitemap,” “Market Data,” and “Equity & Options.”
   2. *Wall Street Journal.*
   3. Go to http://online.wsj.com/public/us, Market Data and U.S. Stocks.
3. Yahoo.com
   1. Go to http://finance.yahoo.com.
4. Stock Screener: Go to http://screener.finance.yahoo.com/newscreener.html, enter “name of issuer,” and for click “Search.”
5. Market Browser: Stocks and other security data.
6. Download program for Market Browser: http://www.marketbrowser.com

Notes

1. **Blume proposed adjusting betas by using a cross-sectional model in which the betas of a sample set of stocks estimated in one period (period 2) are regressed against those stocks' betas estimated from the preceding period (period 1). That is:**

In estimating this relationship for two five-year periods, Blume found:

Using Blume's estimated equation, a stock with a five-year historical beta of 2, would have a forecasted beta for the next five-year period of 1.697, and a stock with a beta of 0.5 would have a forecast beta of 0.682.

Observing a tendency for the forecasted beta to move from the historical beta toward a portfolio beta, Oldrich Vasicek proposed adjusting betas using a Bayesian estimation technique. This technique calls for calculating the forecast beta as a weighted average of the historical beta, β_{i1}, and the average beta across a sample of stocks, β_{ip}:

In the Bayesian approach, the weights are estimated by using the square of the standard error of an estimate of beta, , and the variance of the betas over the sample of stocks, $\sigma^2_{\beta p}$ such that:

Another approach is to simply use equal weights; that is, make w_i = 0.5 and w_p = 0.5.

The Blume approach and a Bayesian approach have been tested. Studies by Klemkosky and Martin and Elton, Gruber, and Urich, for example, found that both approaches led to more accurate forecasts of future betas than did the unadjusted historical betas. Elton, Gruber, and Urich also found that adjusted betas were better predictors of future correlation coefficients.

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2. Studies by Ibbotson and Associates have found that the market risk premium has varied from 4 percent to 7 percent over time, given different economic conditions.

Selected References

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Bloomberg Exercises

1. Select a stock of interest and examine its total returns for different periods and frequencies (daily, weekly, etc.) relative to the S&P 500 using the COMP screen: Stock Ticker <Equity> <Enter> and then click COMP.
2. Select a stock of interest and examine its price and volatility for different periods and frequencies using the GV screen. Stock Ticker <Equity> <Enter> and then click GV and select the following:
    1. Select: Price; Period: Daily; Select: Statistics check box to show Histogram window.
    2. Select: Historical Volatility; Level: 30 Days; Period: Daily; Select: Statistics to show Histogram window.

3. Select a stock of interest and compare its historical volatility for different periods (30-day, 60-day, etc.) using the HVT screen.
4. Study the historical correlation over different periods of the top 10 stocks by market cap of the Dow Jones Average, the S&P 100, or some other index using the CORR screen. CORR <Enter>; click the red "Create New" tab; click "Symmetric Matrix" box; click "Add from Source"; select "Equity Indexes (e.g., INDU, OEX, or SPX) under "Name" tab; Select "All," and click "Add."
5. Select a stock of interest and examine its historical regression with the S&P 500 over different periods and frequencies using the Beta screen: Stock Ticker <Equity> <Enter> and click BETA. In your examination, note the stock's regression parameters, qualifiers, and adjusted beta. Find the stocks $\beta+$, $\beta-$, and convexity by clicking Beta +/− checkbox.
6. Select a stock of interest and compare its beta, alpha, and systematic risk (R^2) with its peers using the PC screen.
7. Use Bloomberg's Excel Add-in to download price data for a stock of interest and the S&P 500. Using Excel, run a regression of the stock's rates of return against the S&P 500 rates.
   1. "Import Data"; "Real-Time/Historical"; "Historical End of Day."
   2. On the Bloomberg Data Wizard Box, Step 1, load Stock and SPX and click "Next."
   3. On the Bloomberg Data Wizard Box, Step 2, type price in the "Search Text" box and hit <Enter>. From the list, select Last Price and then click "Add" and "Next."
   4. On the Bloomberg Data Wizard Box, Step 3, select periodicity (e.g., monthly) and time period; click "Next."
   5. On the Bloomberg Data Wizard Box, Steps 4 and 5, accept "Set History Parameters" and pricing defaults and click "Next."
   6. On the Bloomberg Data Wizard Box, Step 6, Box, accept Excel layout options and click "Finish" to export the data to Excel.
   7. In Excel, calculate period rates (as proportional price changes) from your price data.
   8. Use Excel regression commands to run your regressions.

See Exhibit 6.13 for an example.

8. Use the MRA screen to run a multiple regression of the returns of a stock or index of interest. For your explanatory variables, consider the S&P 500 (SPX <Index>) and long-term Treasury yields (USGG30YR <Index>): MRA <Enter>; select a set for inputting information; on the set screen select dependent and independent variables; save the set by typing 1 and hitting <Enter> and select the time period and frequency (daily, weekly, etc.) by hitting 2 <Enter>. For an example, see Bloomberg box: "MRA: Bloomberg's Multiple Regression Screen."
9. In early studies of factors determining a stock's P/e ratio, Elton and Gruber and Malkiel and Craig found the growth rates in DPS, beta, and dividend payout ratios to be significant in explaining stock P/e ratios. Using Bloomberg's Excel Add-in for data and Excel regressions (found on Excel's "Data Analysis" Add-in) conduct a cross-section regression of stock P/e ratios against past or forecasted growth rates, betas, and dividend payout.
    1. On the Bloomberg Add-in, click "Real/Current" from the "Import Data" and "Real-Time/Historical" dropdowns.
    2. On the Bloomberg Data Wizard Box, Step 1, click "Indexes" in the "From" dropdown and the name of index (S&P 100) from the "Indexes" dropdown, and then click "Add All." This will bring up the stocks for the index. Once loaded, click "Next."
    3. On the Bloomberg Data Wizard Box, Step 2, search and then add, stock returns (e.g., price-to-earning (P/e), DDM implied growth rates, DDM dividend payout ratio in growth stage, and adjusted beta.
    4. After loading variables, click "Next."
    5. On the Bloomberg Data Wizard Box, Step 3, click "Finish" to export the data to Excel.

    To run you regression, use the Bloomberg Excel Add-in: Click "Data" and then "Data Analysis"; in Data Analysis Box: Click "Regression"; in the Regression Box: enter field for the return in "Input Y Range" box and enter field for explanatory variables in "Input X range" box; Click OK.

    See Exhibit 6.16 for an example.

10. The EQRP screen shows a stock's risk premium. The premium is equal to a forecasted market risk premium $E(R^M) - R_f$ times the stock's beta. The market risk premium on the EQRP screen is based on a forecasted market rate and a risk-free rate equal to the 10-year Treasury for the country, and the beta is the stock's historical beta. Using the screen, examine the risk premium for a stock of interest. Also examine the stock's historical premiums and betas.