CHAPTER 8

Portfolio Selection—Markowitz Model

Introduction

In his 1952 seminal article, Harry Markowitz stated that the objective of portfolio selection is to determine the allocation of securities in a portfolio such that it yields the maximum expected return given a specified risk or, alternatively, the minimum portfolio risk given a specified portfolio expected return. Given Equation (7.2) for the portfolio's expected return and Equation (7.11) for the portfolio variance, the Markowitz portfolio selection objective therefore involves determining the weights (w_i) for those equations that yield either the maximum $E(R_p)$ given a specified $V(R_p)$ or the minimum $V(R_p)$ given a specified $E(R_p)$. In this chapter, we examine the Markowitz portfolio selection model for determining optimum portfolio allocations. We begin our analysis by first examining the importance of correlation in determining the different return-risk combinations attainable by varying the security allocations of a two-stock portfolio. With this background, we next examine Markowitz portfolio selection and define an efficiency frontier in terms of that process. We conclude the chapter by extending the selection process to the single-index and multi-index models that were introduced in Chapter 6.

Two-Security Portfolio Return-Risk Relation

As we change the allocation of securities, we change both the portfolio's expected return and risk. How portfolio return and risk change in relation to each other depends on the correlations between the securities. This can be seen by examining the return-risk relationship of a two-security portfolio. Consider first a portfolio formed from two perfectly positively correlated stocks A and B with the following expected returns and variances:

Stock A	**Stock B**
$E(r_A) = 12\%$	$E(r_B) = 18\%$
$V(r_A) = 16\%$	$V(r_B) = 36\%$
$\sigma(r_A) = 4\%$	$\sigma(r_B) = 6\%$
Correlation:	$Cov(r_A\ r_B) = 24$
	$\rho_{AB} = +1$

The return-risk relation of portfolios formed with these two stocks can be seen by varying the allocations of investment funds between the two stocks. For example, if all investment funds are placed in A ($w_A = 1$, $w_B = 0$), the portfolio return and standard deviation would be equal to A's expected return and standard deviation of 12 percent and 4 percent, respectively. If half of the funds are invested in A and half in B, then the portfolio expected return would be 15 percent and the portfolio standard deviation would be 5 percent:

$$E(R_p) = (0.5)(12\%) + (0.5)(18\%) = 15\%$$
$$\sigma(R_p) = \sqrt{(0.5)^2(16) + (0.5)^2(36) + 2(0.5)(0.5)(24)} = 5\%$$

Finally, if all funds are place in B, then the portfolio return and risk would equal stock B's expected return and standard deviation of 18 percent and 6 percent, respectively. Exhibit 8.1 shows these and other portfolio return-risk combinations obtained by varying funds between A and B, and the figure in Exhibit 8.1 shows a graph of the relationship (note: the relation does not include any short positions; that is, all the weights are nonnegative).

Portfolio	w_A	w_B	$E(R_p)$	$\sigma\ (R_p)$
1	1	0	$E(R_A)$ = 12%	$\sigma\ (R_A)$ = 4
2	0.75	0.25	13.5%	4.5
3	0.5	0.5	15.0%	5.0
4	0.25	0.75	16.5%	5.5
5	0	1	$E(R_B)$ = 18	$\sigma\ (R_B)$ = 6

Portfolio	w_A	w_B	$E(R_p)$	$\sigma(R_p)$

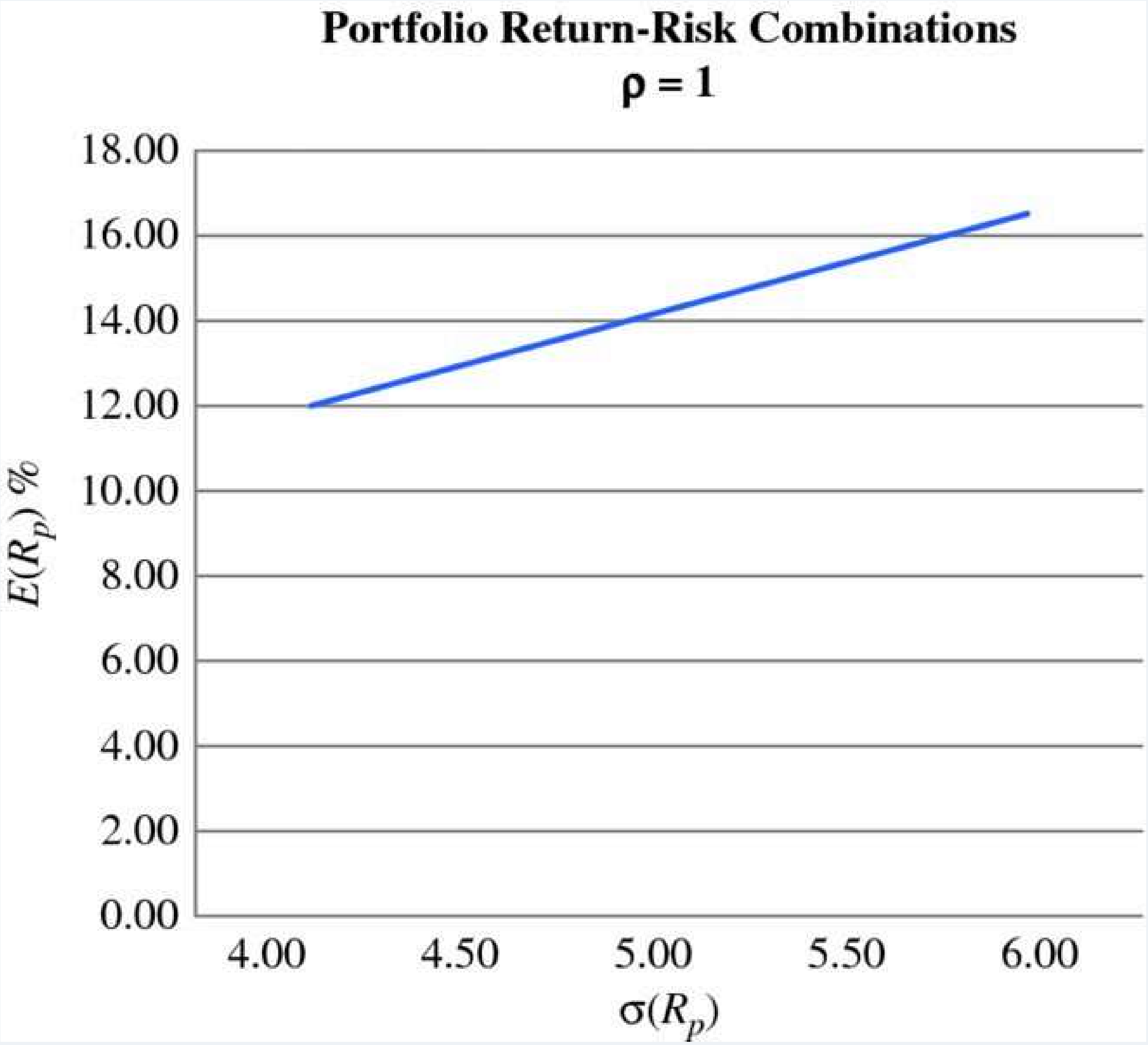


EXHIBIT 8.1 Portfolio Return and Risk with Perfect Positive Correlation

The figure in Exhibit 8.1 shows a positive linear relationship between portfolio return and risk. The linear relationship suggests that the portfolio's return and risk are simply linear combinations of the return and risk of the two securities and do not depend on the correlation between securities. That is, when the correlation coefficient is one, the portfolio variance (or standard deviation) depends only on the securities' variances (or standard deviations). That is:

$$V(R_p) = w_1^2 V(r_1) + w_2^2 V(r_2) + 2w_1 w_2 Cov(r_1 r_2)$$
$$V(R_p) = w_1^2 V(r_1) + w_2^2 V(r_2) + 2w_1 w_2 \rho_{12} \sigma(r_1)\sigma(r_2)$$
$$V(R_p) = w_1^2 \sigma(r_1)^2 + w_2^2 \sigma(r_2)^2 + 2w_1 w_2 (1)\sigma(r_1)\sigma(r_2)$$
$$V(R_p) = [w_1 \sigma(r_1) + w_2 \sigma(r_2)]^2$$
$$\sigma(R_p) = [w_1 \sigma(r_1) + w_2 \sigma(r_2)]$$

Intuitively, if two securities move in perfect unison with each other, there is no correlation benefit, and therefore the different portfolio return and risk combinations are linear combinations of the two securities' returns and risks.

Consider next the portfolio return-risk relation shown in Exhibit 8.2 where the returns of stocks A and B are assumed to be perfectly negatively correlated (−24 covariance). In this case, the portfolio return-risk relationship is characterized by two linear segments: a negatively sloped segment and a positively sloped segment. The negatively sloped segment extends from the 12 percent and 4 percent return-risk combination obtained by placing all funds in the low return-risk stock A to the 14.4 percent return and zero risk combination on the vertical axis obtained by investing 60 percent in A and 40 percent in B. The positively sloped segment extends from the vertical intercept to the 18 percent and 6 percent return-risk combination obtained by investing all funds in the high return-risk stock B. The positive-sloped portion of the figure includes all efficient portfolios and the negatively sloped portion consists of inefficient portfolios. *Efficient portfolios* are defined as those that yield the maximum return for a given risk, whereas inefficient portfolios are those that yield the minimum return for a given risk. Thus, at the 12 percent, 4 percent coordinate the return of 12 percent is the lowest return an investor can obtain for assuming a risk of 4. By changing the allocation from $w_A = 1$ and $w_B = 0$, to $w_A = 0.20$ and $w_B = 0.80$, the investor can move up to the positively sloped segment where for a risk of 4 percent the maximum return of 16.8 percent is obtained. Finally, note that the vertical intercept represents a zero risk portfolio. Whenever securities are perfectly negatively correlated, a graph of the portfolio's return-risk relation will always touch the vertical axis.[1]

Portfolio	w_A	w_B	$E(R_p)$	$\sigma (R_p)$
1	1	0	$E(R_A) = 12\%$	$\sigma (R_A) = 4$
2	0.8	0.2	13.2%	2
3	0.6	0.4	14.4%	0
4	0.4	0.6	15.6%	2
5	0.2	0.8	16.8%	4
6	0	1	$E(R_B) = 18\%$	$\sigma (R_B) = 6$

Portfolio	w_A	w_B	$E(R_p)$	$\sigma\ (R_p)$

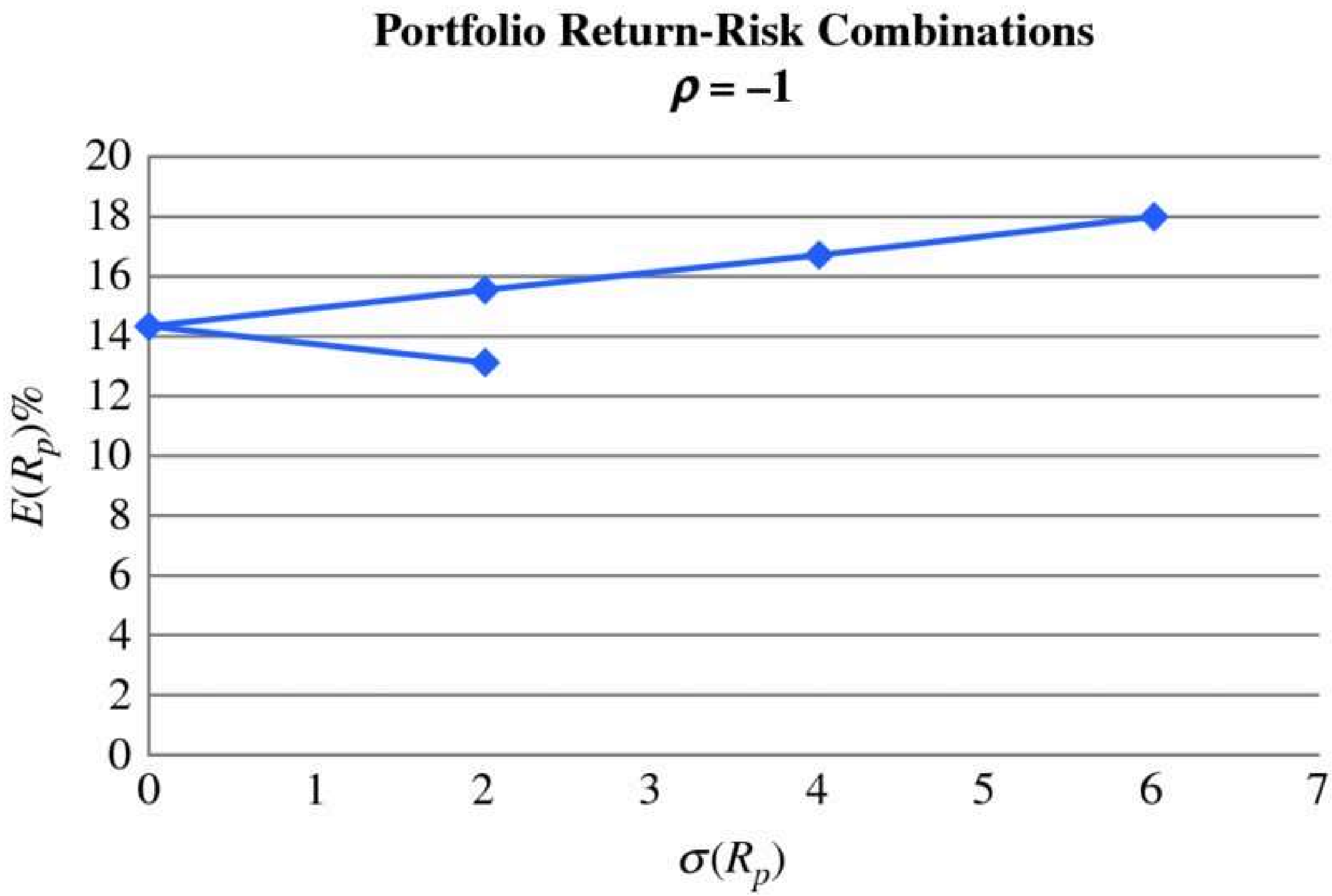


EXHIBIT 8.2 Portfolio Return and Risk with Perfect Negative Correlation

In Exhibit 8.3, the return-risk relationships for both correlation cases are plotted on the same graph. The two curves define the limits within which all portfolios of these two securities must lie for any intermediate correlation coefficient between $\rho_{AB} = -1$ and $\rho_{AB} = +1$. To fit into the triangle ACB, the curve depicting the return-risk relation for an intermediate correlation has to be convex. This convex relation is illustrated in Exhibit 8.4, where the portfolio return-risk combinations are shown for the case in which the correlation coefficient for stocks A and B is zero. The convex shape of the portfolio return-risk curve means that for equal increases in the portfolio return, the portfolio risk as measured by the standard deviation will increase at an increasing rate. For example, as the portfolio return increases by equal increments of 1.2 percent, the portfolio standard deviation goes from 3.39 percent to 3.93 (change of 0.54), from 3.93 to 4.86 (change of 0.93), and from 4.86 to 6 (change of 1.14).

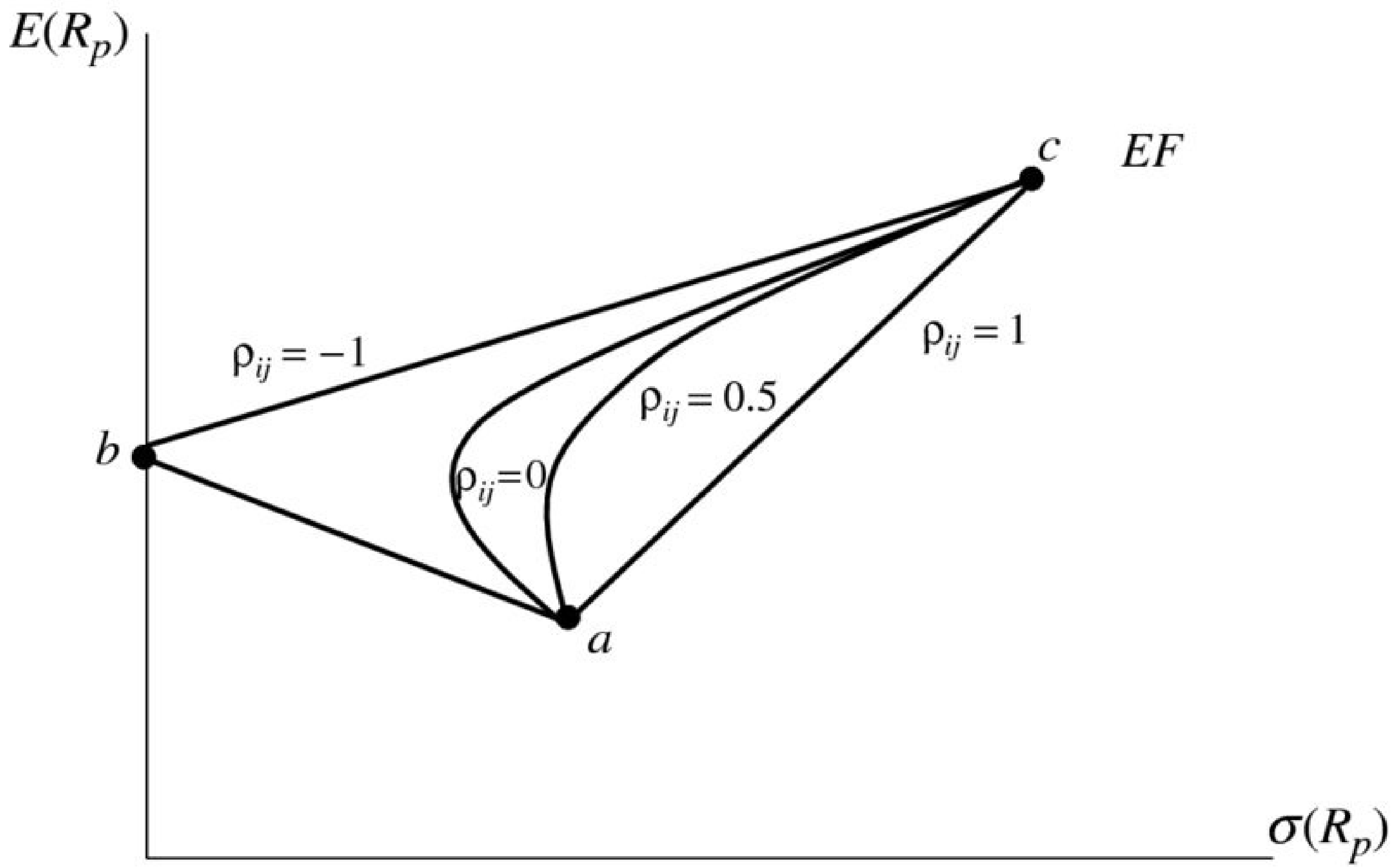


EXHIBIT 8.3 Portfolio Return and Risk with Different Correlations

Portfolio	w_A	w_B	$E(R_p)$	$\sigma(R_p)$
1	1	0	$E(R_A) = 12\%$	$\sigma(R_A) = 4$
2	0.8	0.2	13.2%	3.42
3	0.6	0.4	14.4%	3.39

Portfolio	w_A	w_B	$E(R_p)$	$\sigma (R_p)$
4	0.4	0.6	15.6%	3.93
5	0.2	0.8	16.8%	4.86
6	0	1	$E(R_B)$ = 18%	$\sigma (R_B)$ = 6

Portfolio	w_A	w_B	$E(R_p)$	$\sigma(R_p)$

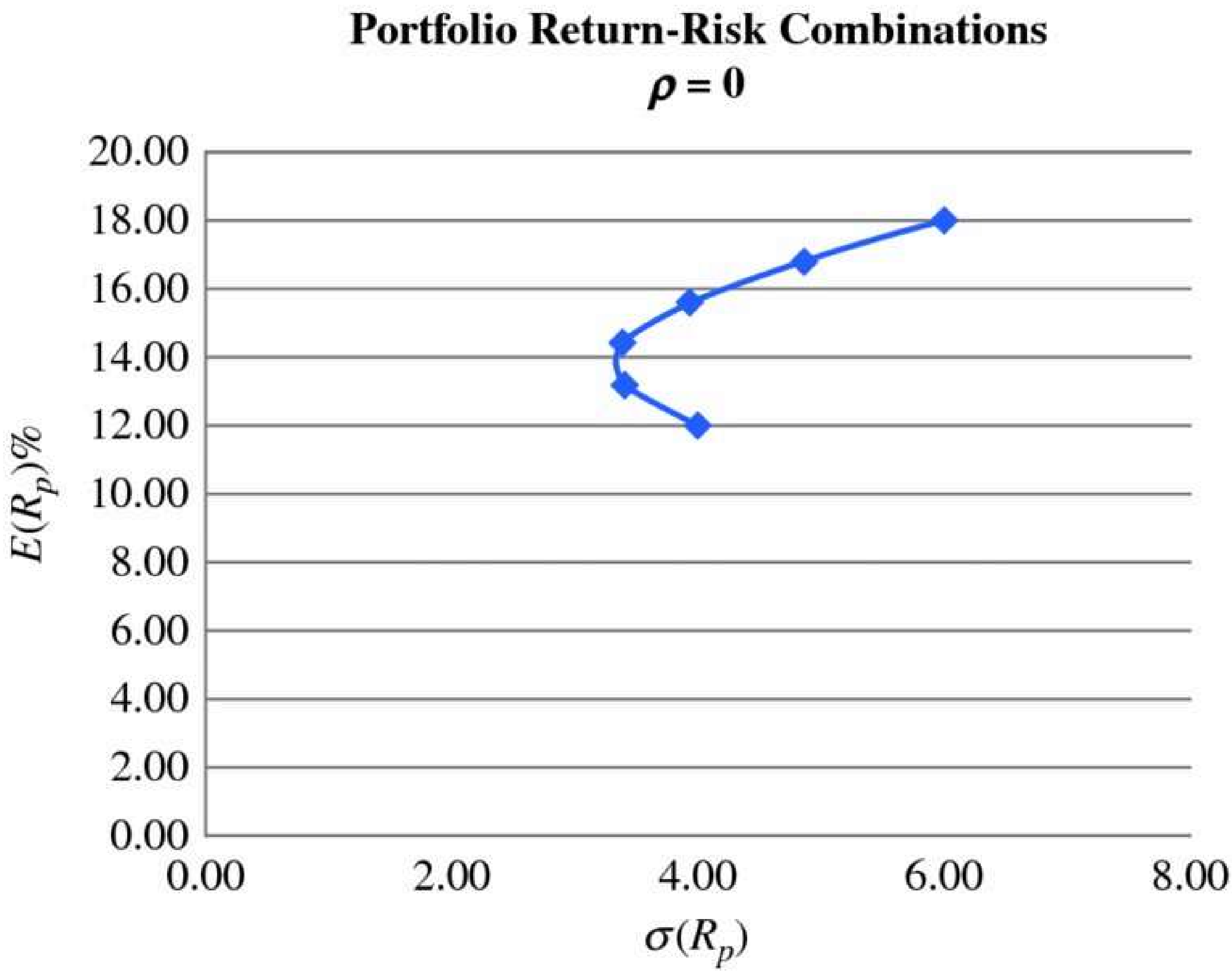


EXHIBIT 8.4 Portfolio Return and Risk with Zero Correlation

Intuitively, the convex return-risk relation implies that as you move up from the middle of the return-risk graph, you become more specialized in the high return-risk stock. As a result, the portfolio risk takes on progressively more and more of the risk of the high-risk security. In addition, as the portfolio becomes more specialized (and therefore less diversified) it loses the covariance effect. Combined, the increasing proportion allocated to the risky security and the loss of the covariance effect due to specialization causes the portfolio risk to increase at an increasing rate. Since the correlations among many securities are less than one, many portfolio return-risk relations are characterized by this convex relation.

In examining Exhibit 8.3, several additional points should be noted. First, the lower the correlation coefficient, the more dominant the portfolio's return-risk combinations. Thus, for a given risk, over the positively sloped portion of the curves portfolio returns are greater for lower correlation coefficients. Secondly, note that each of the return-risk curves with intermediate correlations has a vertical point (inflection point) where the slope of the curve is zero. This point represents the minimum variance portfolio. For a two-security portfolio, this portfolio can be found with calculus by taking the derivative of the portfolio variance equation with respect to one of the weights, setting the derivative equal to zero, and solving the resulting equation for the weight. Doing this, we obtain:

$$w_{\mathrm{A}} = \frac{\sigma(r_{\mathrm{B}})^2 - \sigma(r_{\mathrm{A}})\sigma(r_{\mathrm{B}})\rho_{\mathrm{AB}}}{\sigma(r_{\mathrm{B}})^2 + \sigma(r_{\mathrm{A}})^2 - 2\sigma(r_{\mathrm{A}})\sigma(r_{\mathrm{B}})\rho_{\mathrm{AB}}}$$

Finally, note that each of the return-risk combinations is unique; that is, each return-risk combination is associated with one allocation. This is because we have limited our analysis to two securities. If we expand our portfolio to more than two securities, then we will find that there are a number of allocations that can yield the same portfolio return and a number of allocations that can yield the same portfolio risk; that is, when the number of securities in a portfolio exceeds two, then the portfolio selection problem is one of determining the allocation that will yield an efficient portfolio—Markowitz portfolio selection.

Markowitz Portfolio Selection

Math Approach

There are several approaches that can be used to solve for the security allocations that satisfy the Markowitz portfolio selection objective. One of these is the mathematical approach and another is quadratic programming. The math approach uses differential calculus to find the allocation that will minimize the portfolio variance subject to the constraints that the weights sum to one and a specified portfolio return is attained, or the allocation that will maximize the portfolio return subject to the constraints that the weights sum to one and a specified portfolio variance is attained.

Thus, for a three-stock portfolio, the objective of portfolio variance minimization would be to solve for the w_1, w_2, and w_3 allocations that would yield the minimum portfolio (Min V_p), subject to the constraints that w_1, w_2, and w_3 sum to one and yield a specified portfolio return, E_p*:

Portfolio Inputs:

Given Estimates: $V_1, V_2, V_3, C_{12}, C_{13}, C_{23}$

Objective Function:

$$\text{Minimize } V_p = w_1^2 V_1 + w_2^2 V_2 + w_3^2 V_3 + 2w_1 w_2 C_{12} + 2w_1 w_3 C_{13} + 2w_2 w_3 C_{23}$$

where:

$$V_p = V(R_p)$$
$$V_i = V(r_i)$$
$$C_{ij} = Cov(r_i r_j)$$

First constraint:

$$w_1 + w_2 + w_3 = 1$$

Second constraint:

$$w_1 E_1 + w_2 E_2 + w_3 E_3 = E_p^*$$

where:

$$E_i = E(r_i)$$

The math approach for portfolio maximization given a specified portfolio variance is similar to the variance minimization approach. In this case, the objective function is the portfolio expected return, and the constraint is the portfolio variance. The portfolio return maximization approach is consistent with the portfolio variance minimization approach, yielding the same allocations as the portfolio variance approach.

This constrained optimization problem can be solved mathematically using the Lagrangian technique. The approach is presented in Appendix 8A (text Web site) along with an example. The approach requires using matrix algebra to solve a set of equations simultaneously. The Bloomberg Exhibit box, "Using the Bloomberg CORR Screen and Excel to Solve for Markowitz Efficient Portfolios," explains how to download a variance-covariance matrix for a portfolio from its CORR screen to Excel and then explains how to use Excel matrix multiplication commands to solve for a Markowitz efficient portfolio using the math approach. See the supplemental Appendix B (text Web site) on matrix algebra and for a listing of matrix algebra commands in Excel.

The math approach for solving for Markowitz efficient portfolios is capable of handling large portfolios. Its limitation is that the solutions do not necessarily exclude negative weights. Thus, it is possible to obtain an optimum portfolio that requires taking a short position in a poor security and using the proceeds to invest in other securities in the portfolio. Since most investors do not consider shorting poor securities, the mathematical approach may not be practical.

USING THE BLOOMBERG CORR SCREEN AND EXCEL TO SOLVE FOR MARKOWITZ EFFICIENT PORTFOLIOS

1. The CORR screen can be used to create and save a number of correlation matrices for securities, indexes, currencies, interest rates, and commodities. As noted in Chapter 6, the matrix also shows a variance-covariance matrix (Cov) for portfolios up to 10 stocks.
2. A portfolio created in PRTU can be imported into CORR. To import: (1) Click “Create New” tab; (2) Select dates and period for statistical analysis (e.g., Date Range: 8/5/2006 to 8/8/2013 and weekly periods); (3) in Matrix Securities Box, click “Symmetric Matrix box,” and “Add from Sources” tab, Select “Portfolio,” Name of Portfolio (e.g., Blue Rock), click “Select All,” and click “Update.”
3. On the CORR screen, you can obtain the variance-covariance matrix by selecting “Covariance” from the dropdown “Calculations” tab.
4. Matrices in CORR can be exported to Excel by clicking “Export to Excel” in the dropdown “Export” tab in the far right corner of the screen.
5. A *coefficient matrix*, **A**, formed from a variance-covariance matrix and its inverse, $\mathbf{A}^{-1}$, can be used to solve for the Markowitz efficient portfolio; these matrices can be created in Excel.

See Appendix 8A for an explanation of the math approach and Appendix B for a primer on matrix algebra and matrix Excel commands.

See Bloomberg Web Exhibit 8.1.

BLOOMBERG SCREENS FOR THE STOCK RETURNS OF A PORTFOLIO

PORT SCREEN, PERFORMANCE TAB

The PORT screen can used to find historical returns of the portfolio and the returns on the stock:

- For portfolio: Performance tab, “Total Return,” “Portfolio vs Index” (e.g., Dow (INDU)).
- For stocks: Performance tab, “Main View,” “By” dropdown: None, “Portfolio vs None.”
- Time = MTD, YTD, and Custom (for Custom, the select time period must be within the period history of the portfolio created in PRTU).
- Note: The Heat Map for the portfolio and stocks can be uploaded by clicking the “Chart” icon on the right.

MRR SCREEN

The MRR can be used to find average returns for stocks for different time periods. MRR <Enter>; select “Portfolio” from Source tab and your portfolio from the “Name” tab; select the time period for your analysis.

DDM SCREEN

The DDM estimates a stock's intrinsic value using a three-stage growth model. You can input different values for the lengths of each growth period, growth rates, and discount rates. The DDM also calculated an IRR that can be used as an estimate of expected rate. See Bloomberg exhibit box in Chapter 3.

See Bloomberg Web Exhibit 8.2.

Quadratic Programming Approach

An alternative to the mathematical approach is quadratic programming (QP). QP is an algorithm that iteratively solves for the security weights that yield the minimum portfolio variance subject to three constraints: The weights sum to one, they yield a specified portfolio expected return, and each weight is nonnegative. For a three-stock portfolio, the QP approach can be defined as follows:

Portfolio Inputs:

Given Estimates: V_1, V_2, V_3, C_{12}, C_{13}, C_{23}

Objective Function:

Minimize: $V_p = w^2{}_1V_1 + w^2{}_2V_2 + w^2{}_3V_3 + 2w_1w_2C_{12} + 2w_1w_3C_{13} + 2w_2w_3C_{23}$

First constraint:

$$w_1 + w_2 + w_3 = 1$$

Second constraint:

$$w_1E_1 + w_2E_2 + w_3E_3 = E_p^*$$

Third constraint of nonnegative weights:

$$w_1 \geq 0; \quad w_2 \geq 0; \quad w_3 \geq 0$$

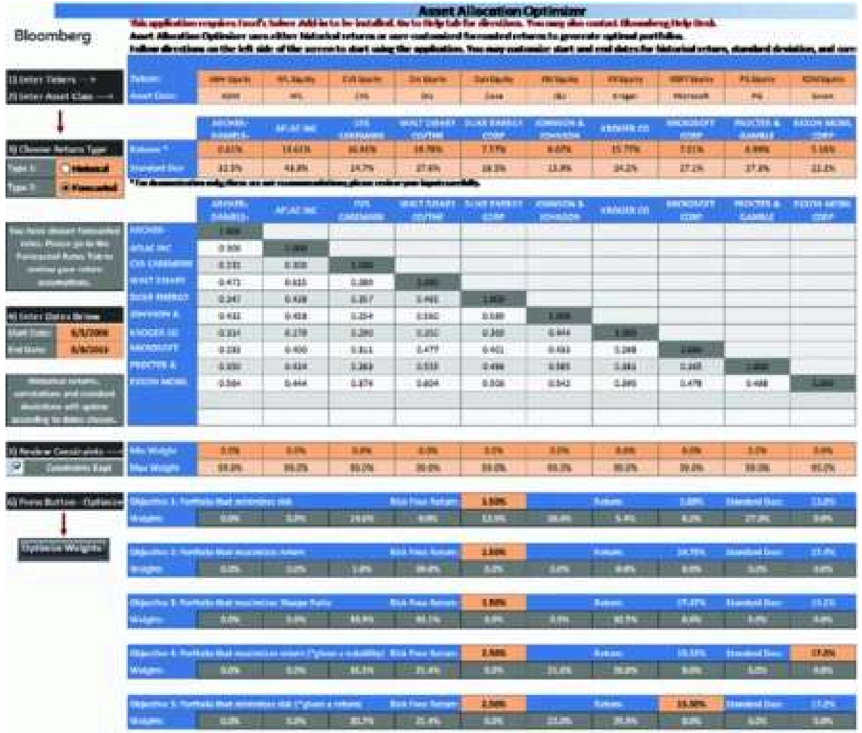
With the nonnegative weight constraints, quadratic programming provides a more practical approach to constructing portfolios that satisfy the Markowitz objective.

Excel Solver Approach: Bloomberg's Asset Allocation Optimizer Template

In Excel, efficient portfolios (maximum E_p* given V_p or minimum V_p* given E_p) with nonnegative weight constraints can be generated using the Excel Solver Add-In. Such programs yield results similar to QP-generated portfolios. A Bloomberg Excel program that uses Excel Solver for solving for such portfolios, "Asset Allocation Optimizer," can be downloaded from the Bloomberg Excel Template library found on the DAPI screen (DAPI <Enter>; Click Excel Template Library and Portfolios topic and then select "Asset Allocation Optimizer.") The template is shown in Exhibit 8.5. To use the program, the user:

1. Inputs the stock tickers (for stocks, ticker with the "Equity" moniker, for indexes, ticker with "Index" moniker, etc.).
2. Inputs average returns or expected returns.
3. Selects the time period for calculating the variance-covariance matrix (the template in Exhibit 8.5 shows the correlation matrix).
4. Selects minimum weight and maximum weight constraints for each stock; here the user can set the minimum weights to zero and the maximum weights at 99 percent (or another specified constraint).
5. Inputs the risk-free rate and values for the optimization programs: the portfolio standard deviation for portfolio maximization optimization and portfolio return for portfolio variance minimization.
6. Clicks "Optimize Weights" tab to run the program.

Exhibit 8.5 shows the optimum solutions for the 10 stocks making up the Blue Rock Fund. The variance-covariance matrix is calculated for the time period from 8/5/2006 to 8/8/2013 (weekly prices are used), the expected stock returns are based on averages over a more recent time period, the specified portfolio return is 15.5 percent, and the specified annualized portfolio standard deviation is 17 percent.[2] As shown at the bottom of the table in Exhibit 8.5, the optimum portfolio weights for the variance minimization are 30.7 percent in CVS, 21.4 percent in Disney, 22 percent in Johnson & Johnson, and 25.9 percent in Kroger.



(a)

Risk	Return	ADM	AFL	CVS	Dis	Duke	J&J	Kroger	Microsoft	PG	Exxon
13.8%	9.8%	0.0%	0.0%	14.6%	0.0%	13.9%	38.6%	5.4%	0.2%	27.3%	0.0%
13.9%	10.5%	0.0%	0.0%	18.3%	0.0%	10.2%	39.5%	9.2%	0.0%	22.8%	0.0%
14.1%	11.2%	0.0%	0.0%	22.2%	0.0%	6.4%	39.5%	13.1%	0.0%	18.8%	0.0%
14.4%	11.9%	0.0%	0.0%	24.4%	2.5%	4.2%	38.4%	15.3%	0.0%	15.1%	0.0%
14.8%	12.6%	0.0%	0.0%	25.6%	5.8%	2.1%	37.6%	17.5%	0.0%	11.4%	0.0%
15.3%	13.4%	0.0%	0.0%	27.4%	9.7%	0.0%	35.8%	19.5%	0.0%	7.7%	0.0%
15.8%	14.1%	0.0%	0.0%	28.3%	13.2%	0.0%	33.8%	21.8%	0.0%	2.9%	0.0%
16.3%	14.8%	0.0%	0.0%	29.7%	17.2%	0.0%	29.8%	23.2%	0.0%	0.0%	0.0%
17.0%	15.5%	0.0%	0.0%	30.7%	21.4%	0.0%	22.0%	25.9%	0.0%	0.0%	0.0%
17.7%	16.2%	0.0%	0.0%	31.9%	25.6%	0.0%	14.3%	28.2%	0.0%	0.0%	0.0%
18.5%	16.9%	0.0%	0.0%	33.1%	29.8%	0.0%	6.6%	30.4%	0.0%	0.0%	0.0%
19.3%	17.6%	0.0%	0.0%	33.4%	36.1%	0.0%	0.0%	30.4%	0.0%	0.0%	0.0%
20.9%	18.3%	0.0%	0.0%	28.3%	55.3%	0.0%	0.0%	16.4%	0.0%	0.0%	0.0%
23.5%	19.0%	0.0%	0.0%	23.2%	74.2%	0.0%	0.0%	2.6%	0.0%	0.0%	0.0%
27.4%	19.8%	0.0%	0.0%	1.0%	99.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%

(b)

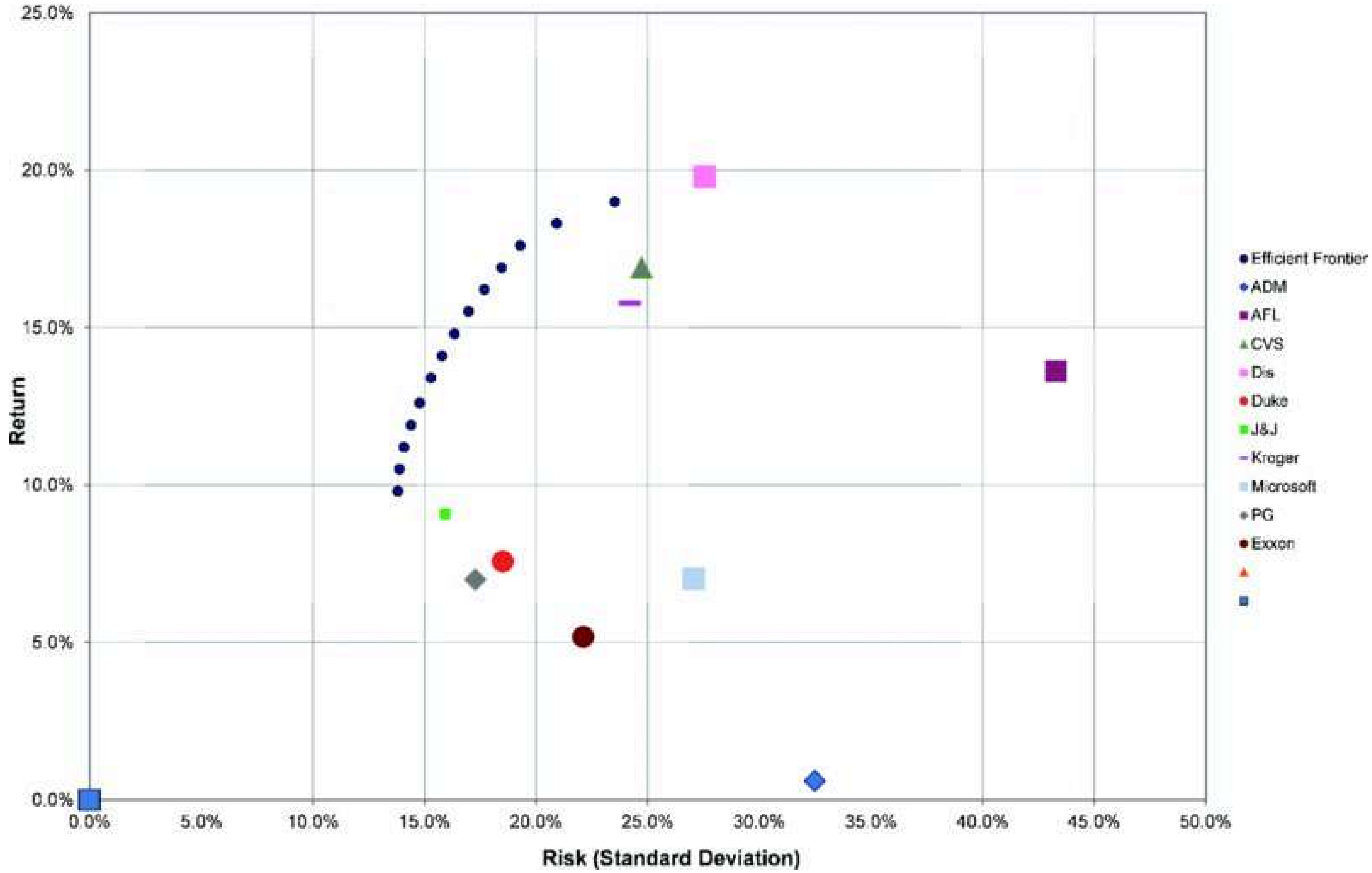


EXHIBIT 8.5 Markowitz Efficient Portfolios Generated Using the Asset Allocation Optimizer Excel Program

The ex-post total returns from the one-year period from 8/13/12 to 8/13/13 and for the three-year period from 8/13/10 to 8/13/13 for the Markowitz efficient portfolio and the S&P 500 are shown in Exhibit 8.6 (Bloomberg's PORT screen, Performance tab, and Total Return tab). As shown in the exhibit, the back testing results show the portfolio out-performs the market for both the one-year period (total return of 44.82 percent compared to 16.11 percent total return for S&P 500) and the three-year period (total return of 96.22 percent compared to 56.03 percent total return for S&P 500). Using the Bloomberg PORT screen (Performance tab and Statistics Summary tab) the Markowitz portfolio has a beta close to one based on year-to-date calculation, as well as a very large alpha, suggesting abnormal returns.

(a)

(b)

(c)

(d)

EXHIBIT 8.6 Ex-Post Performance, 8/13/2012–8/12/13 Markowitz Portfolio (Blue Rock Equity) and S&P 500

Elton, Gruber, and Padberg also have developed an algorithm based on the single-index model (examined later) for determining the best efficient portfolio. The technique they derive for generating efficient portfolios, in turn, can also be constrained so that the weights are nonnegative. The technique is presented in Appendix 8B (text Web site) and its application is presented later in this chapter.

Efficiency Frontier

The Markowitz portfolio selection objective can be restated as one of deriving an efficiency frontier, EF. An efficiency frontier is a graph showing the portfolio expected return, $E(R_p)$, and standard deviation, $\sigma(R_p)$, combinations that are Markowitz efficient; that is, satisfy the Markowitz objective of maximum $E(R_p)$ given a specified $V(R_p)$, or minimum $V(R_p)$ given a specified $E(R_p)$.

There are three steps involved in generating an efficiency frontier. The first step is to estimate the portfolio inputs: $E(r_i)$, $V(r_i)$, and $Cov(r_i\, r_j)$. These parameters can be estimated using either historical averages or a regression model. There are several regression models that can be used. The simplest is the single-index model in which each security's return is regressed against the market return. As discussed in the next section, the single-index model reduces the number of estimates needed to generate efficient portfolios, and it has been shown to lead to better estimates of the portfolio inputs than historical averages. Some practitioners also use a multi-index model in which each security's return is regressed against several explanatory variables.

Once the expected returns, variances, and covariances of the securities included in the portfolio are estimated, the next step is to generate Markowitz efficient portfolios. As noted above, this can be done using either the math approach, quadratic programming, or with Excel Solver. With any of these approaches, one would first specify a number of portfolio expected returns (or variances), then solve for the weights that would yield the minimum portfolio variance (or maximum portfolio return) for each return (or variance). Each portfolio return and variance would be either a Markowitz efficient or inefficient portfolio. The last step is to plot each portfolio's expected return and standard deviation (not portfolio variance) to generate the efficiency frontier.

The EF curve shown in Exhibit 8.7 depicts a typical efficiency frontier. The curve has features similar to the return-risk graph for a two-security portfolio previously discussed. First, like the two-security return-risk graph, EF is characterized by both a negatively sloped portion and positively sloped portion. The negatively sloped portion of EF represents the inefficient portfolios [minimum $E(R_p)$, given $\sigma(R_p)$], whereas the positively-sloped portion shows the efficient portfolios. Second, the efficiency frontier, like the portfolio return and risk curve for a two-security portfolio, is convex from below, except for cases in which the securities are perfectly positively or negatively correlated. As discussed with the two-security portfolio, the convexity of the efficiency frontier is explained by the increase specialization in the high-risk security and the loss of the covariance effect that occurs as one moves up the efficiency frontier. Finally, the efficiency frontier has a vertical segment (inflection point) that defines the minimum variance portfolio. The efficiency frontier and table for the portfolio in Exhibit 8.5 generated by using the Bloomberg “Asset Allocation Optimizer” is shown in Exhibit 8.5c.

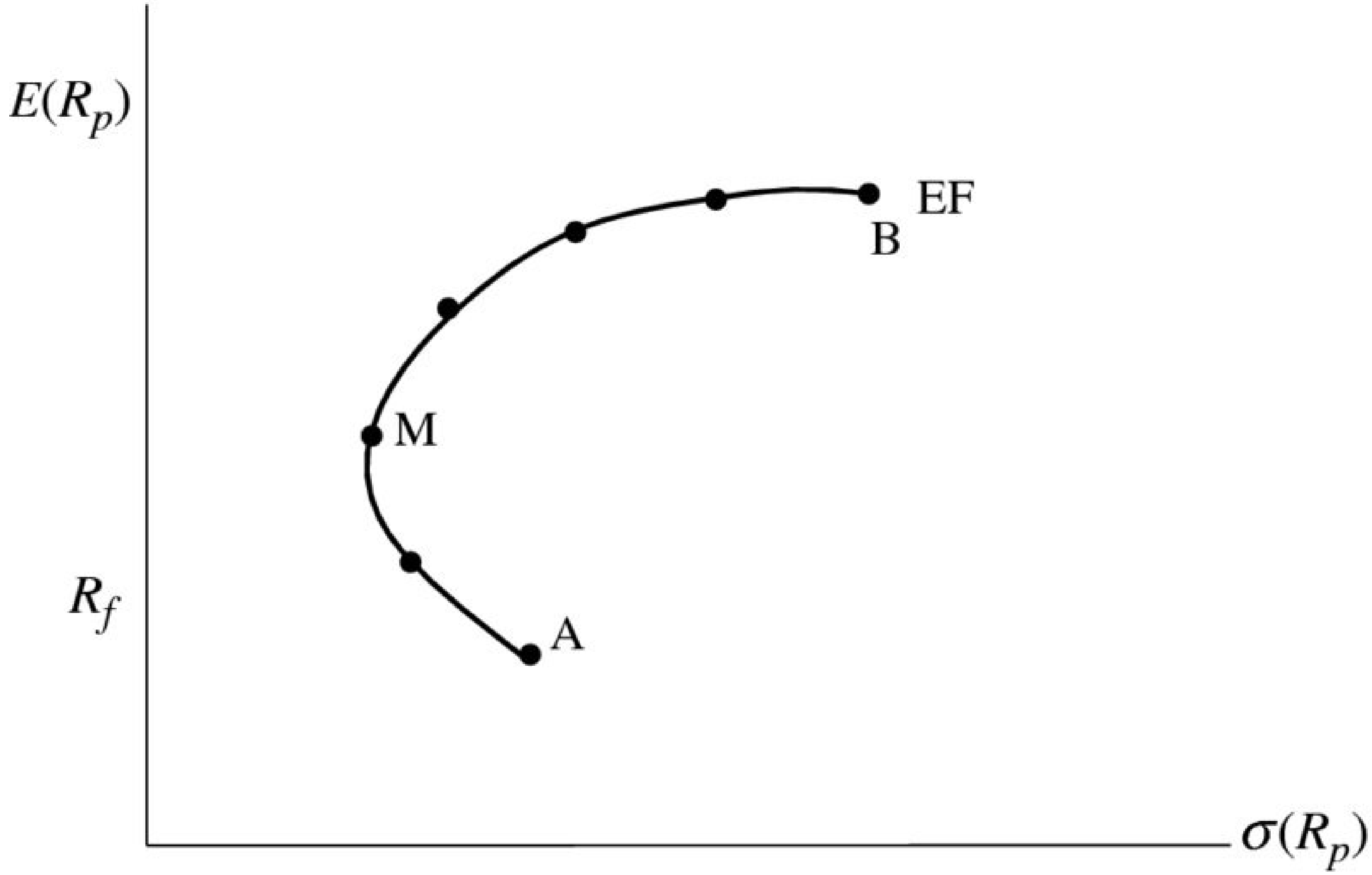


EXHIBIT 8.7 Efficiency Frontier

Best Efficient Portfolio

All portfolios along the positively sloped portion of the efficiency frontier are Markowitz efficient; that is, all have the maximum $E(R_p)$ given $V(R_p)$, or minimum $V(R_p)$ given $E(R_p)$. From this set of efficient portfolios, it is possible to determine the best portfolio by using the concept of portfolio ranking examined in Chapter 7. This can be seen in Exhibit 8.8, in which portfolio E represents the best portfolio. This is because the borrowing and lending line constructed with portfolio E has the steepest slope (largest λ). Thus, the return-risk combinations available with portfolio E and a risk-free security dominate the return-risk opportunities available from any other efficient portfolio and the risk-free security. If the efficiency frontier is convex (as we expect for many portfolios), then the best portfolio, such as E, can be determined at the point of tangency of the efficiency frontier and the borrowing and lending line. Furthermore, if the efficiency frontier is convex, then the best portfolio would be defined as one of the middle points on EF (not at a corner), implying that the best portfolio would be diversified.

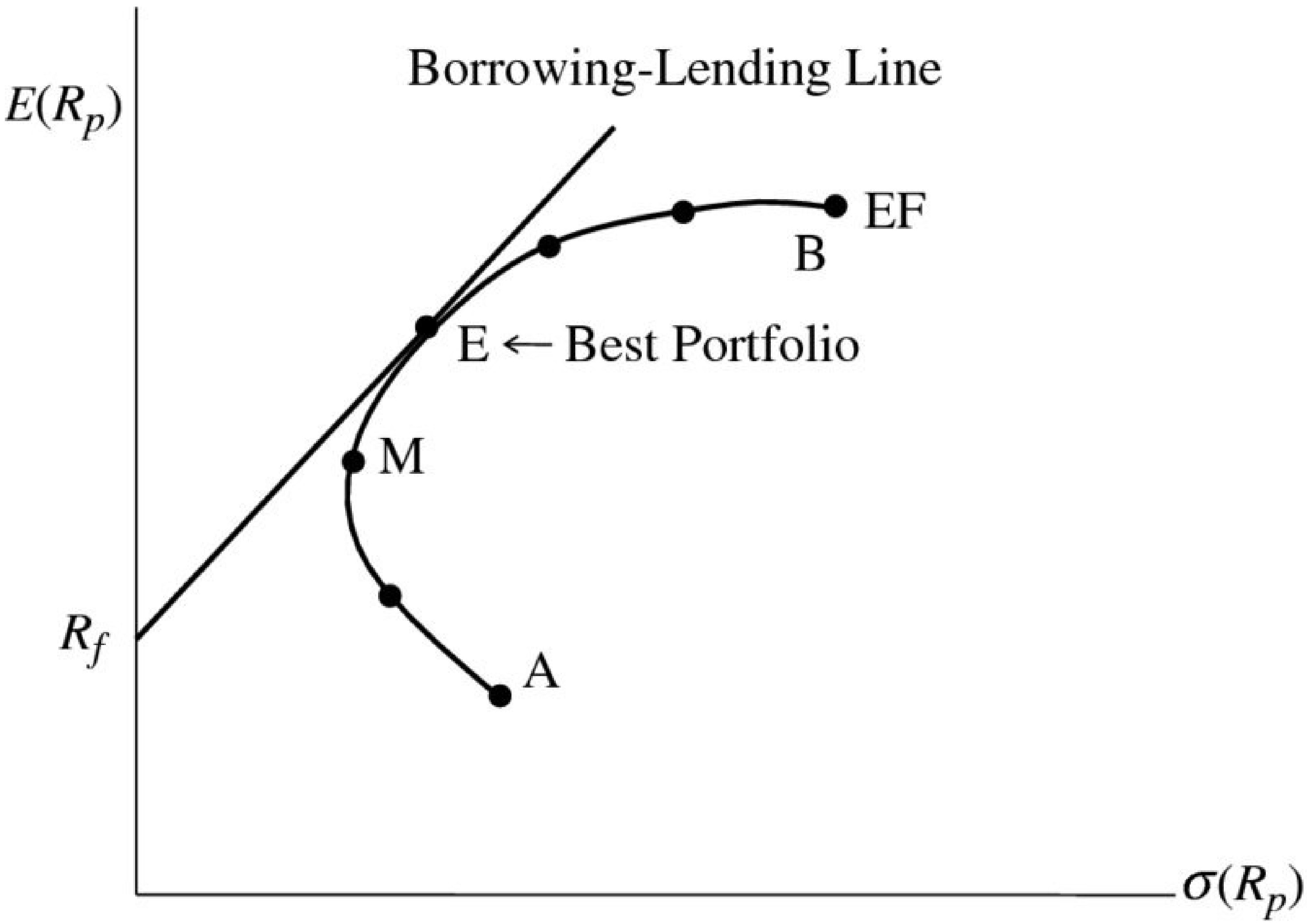


EXHIBIT 8.8 Best Efficiency Portfolio

Efficiency Frontiers for Stocks with Perfect Positive and Negative Correlations

It should be noted that if the securities in the portfolio are perfectly positively correlated, then the efficiency frontier is linear and the best portfolio will be one that is at one of the corners. Since the corner points of an efficiency frontier define a one-security portfolio (either the low return-risk security or the high return-risk security), the best portfolio therefore consists of only one security (see Exhibit 8.9). Thus, if the securities in the portfolio are perfectly positively correlated, then there is no benefit to diversification and the best portfolio consists of either the low return-risk security or the high return-risk security. This observation confirms a previously stated observation that if securities are moving in unison, there is no diversification benefit and a portfolio with a large number of perfectly positively correlated securities would be superfluous.

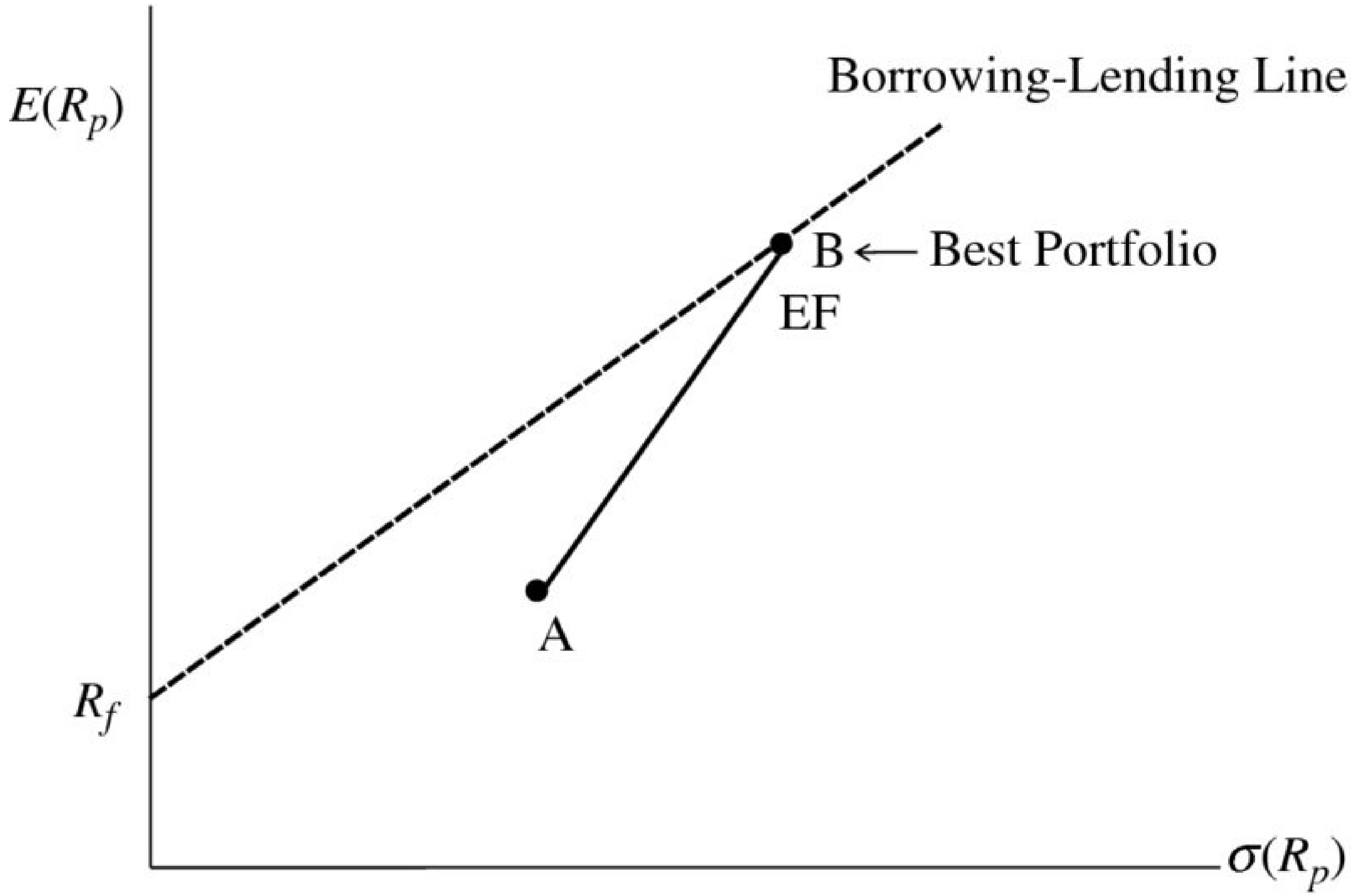


EXHIBIT 8.9 Efficiency Frontier with Perfect Positive Correlation

Finally, note that if the securities are perfectly negatively correlated, then the best portfolio (i.e., the one with the steepest borrowing-lending line) would be the zero risk portfolio obtained with limited diversification in two securities that are perfectly negatively correlated (see Exhibit 8.10). In this case, if the securities are in fact perfectly negatively correlated, or if two perfectly negatively correlated positions are formed (e.g., long stock position and a short stock position formed with stock option positions), then an arbitrage opportunity would exist if the risk-free rate were different than the portfolio rate associated with the zero risk portfolio. For example, if the risk-free rate were less than the zero-risk portfolio's rate, then an arbitrageur would borrow as much as she could to invest in the portfolio. By doing this, the arbitrageur would realize a free lunch: a future dollar return with no risk and no investment—an arbitrage.

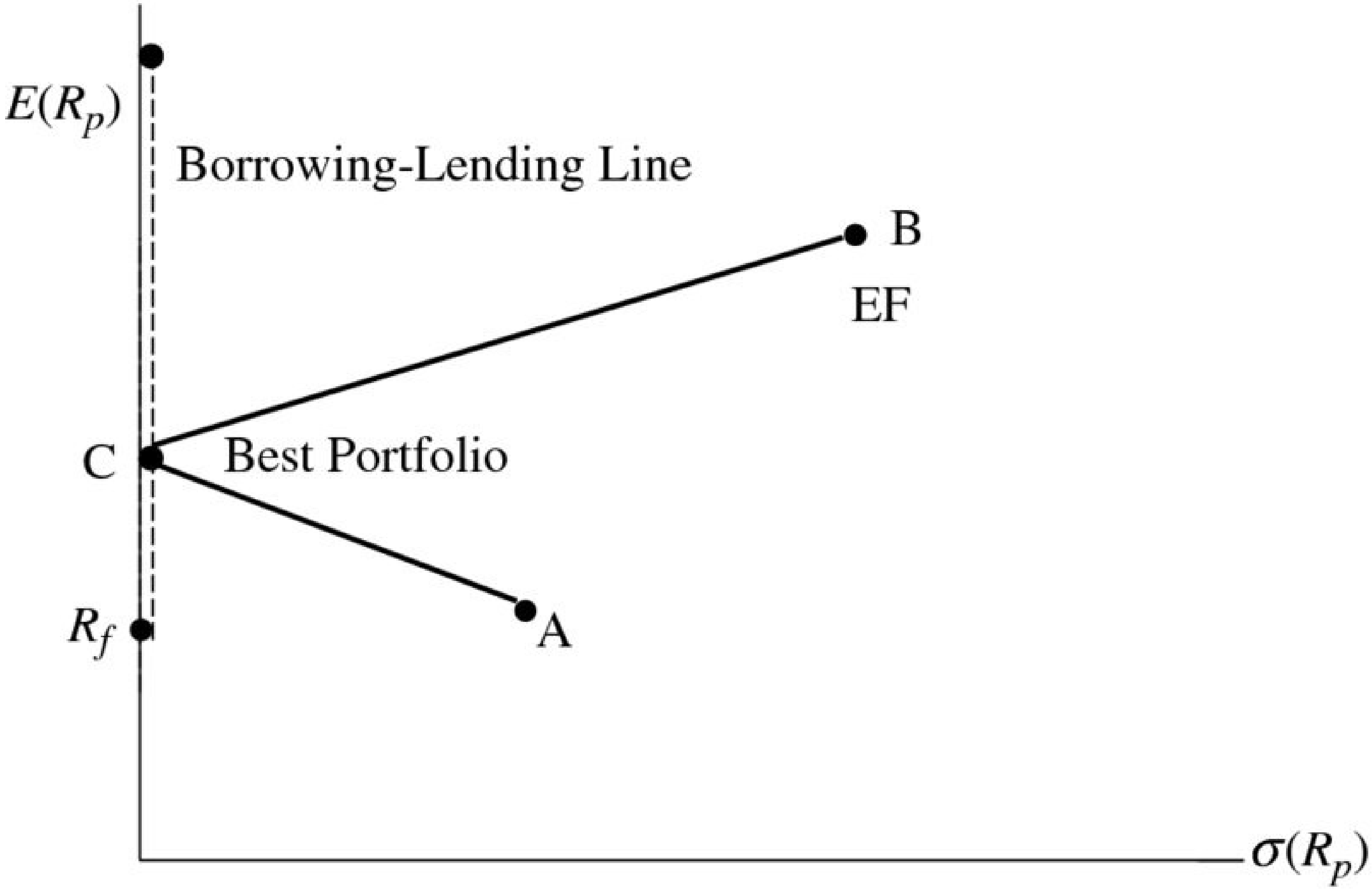


EXHIBIT 8.10 Efficiency Frontier with Perfect Negative Correlation

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BLOOMBERG'S ASSET ALLOCATION OPTIMIZER

To access Asset Allocation Optimizer Excel template go to the template library found on the DAPI screen (DAPI <Enter>) and click “Excel Template Library,” “Equity,” and “Portfolios.” The template is shown in Exhibit 8.5.

As described in the Template's “Help” sheet, the optimization program uses historical returns or user-customized forecasted returns to generate optimal portfolios. You can customize beginning and ending dates for the historical returns, standard deviation, and correlation matrix data. The spreadsheet uses Microsoft Excel's Solver Add-in to solve portfolio equations to find optimal portfolios. Instructions are provided in Help for uploading the Add-in if it is not already on your Excel spreadsheet. The program generates an optimal portfolio in the “Optimizer tab” and builds an efficient frontier in the “Efficient Frontier” tab. For an example of how to use the template, see Exhibit 8.5, “Markowitz Efficient Portfolios Generated Using the Asset Allocation Optimizer Excel Program.”

See Bloomberg Web Exhibit 8.3.

Single-Index Model

Markowitz Portfolio Theory was introduced over 60 years ago. Since then most of the academic research on Markowitz portfolio analysis has focused on how to implement the theory. One area that initially drew the interest of researchers was the simplification of the computational process. Recall that a portfolio with 100 stocks would require estimates of 4,950 covariances, 100 variances, and 100 expected returns. Two models that reduce the number of calculations needed to generate efficient portfolios are the *single-index model* and the *multi-index model*.

Developed by William Sharpe, the single-index model (also called the *diagonal model*) assumes that all securities in a portfolio are related just to the market return and that there is no correlation between the unsystematic risks of securities. Combined, these assumptions imply that comovements between securities in a portfolio are related to a single factor—the market return. As a result, in the single-index model one does not have to estimate the correlations between stocks in the portfolio; instead, one only has to estimate each security's relation to the common factor. In Chapter 6, we examined this model when we looked at estimating stock's expected return, variance, and covariance. Formally, the single-index model assumes that each security *i* in the portfolio being evaluated is only related to the market as described by the regression model:

$$r_i = \alpha_i + \beta_i R^M + \varepsilon_i \tag{8.1}$$

The model also assumes that the standard regression assumptions hold for each security in the portfolio and that there is no correlation between the error terms of the stocks; that is, the covariance between the errors of any two securities *j* and *k* is zero:

1. Each stock's ϵ is normally distributed.
2. $E(\epsilon_i) = 0$, for all securities.
3. Each stock's $V(\epsilon)$ is constant over all observations.
4. $Cov(R^M, \epsilon) = E(R^M \epsilon) = 0$, for all securities.
5. $Cov(\epsilon_j \epsilon_k) = 0$.

From these assumptions, the expected returns, variances, and covariances of the stocks in the portfolio are:

$$E(r_i) = \alpha_i + \beta_i E(R^M) \tag{8.2}$$

$$V(r_i) = \beta_i^2 V(R^M) + V(\varepsilon) \tag{8.3}$$

$$Cov(r_j r_k) = \beta_j \beta_k V(R^M) \tag{8.4}$$

As discussed in Chapter 6, Equation (8.3) shows that the variance of each stock in the portfolio depends on its sensitivity to the market as measured by β_i, the variability of the market, $V(R^M)$, and the stock's unsystematic risk as measured by $V(\epsilon_i)$. Equation (8.4) shows that the comovement of securities is related just to the movement of the market. This result follows directly from the assumption that ϵ_j is independent of ϵ_k. This, in turn, implies that when there is no correlation between industry and firm factors among securities, then the comovement of securities is explained only in terms of each security's relative movements to the market. By contrast, if we do assume that the errors are correlated, then each covariance term in the portfolio would include a $Cov(\epsilon_j\ \epsilon_k)$. Such a model that assumes each security in the portfolio is related only to the market but does not assume $Cov(\epsilon_j\ \epsilon_k) = 0$ is known as the *market model*.

Equations (8.2), (8.3), and (8.4) define the expected return, variance, and covariance for the single-index model. To estimate this model requires estimating an α_i, β_i, and $V(\epsilon_i)$ for each security and then estimating the expected return and variance of the market, $E(R^M)$ and $V(R^M)$. These parameters can be estimated either through a regression analysis using historical data or they can be provided independently. For an n-security portfolio, the number of parameters to estimate is $3n + 2$. In contrast, to estimate the portfolio inputs using historical averages would require $([n^2 - n]/2) + 2n$ estimates. Thus, for a 100-security portfolio, the single-index model would require 302 estimates, and for a 200-security portfolio, it would require 602 estimates. Using averages, on the other hand, would require estimating 5,150 parameters for the 100-security portfolio and 20,300 parameters for the 200-security portfolio. Thus, the single-index model greatly simplifies the computation for generating portfolio inputs.

BLOOMBERG REGRESSION SCREENS

HRA AND BETA: BLOOMBERG'S LINEAR REGRESSION SCREEN

The Bloomberg HRA and Beta screens show the linear regression of a loaded security and an index or other security. See the Bloomberg exhibit box in Chapter 6: “Bloomberg Regression and Correlation Screens.”

Portfolio Return and Risk in Terms of the Single-Index Model

In the single-index model, the portfolio expected return and portfolio variance can be expressed in forms similar to a stock's expected return and variance: Equations (8.2) and (8.3). The simplest way to derive the portfolio return and risk equations is to substitute Equation (8.1) into the expression for portfolio return. Doing this we obtain:

$$R_p = \sum_{i=1}^{n} w_i r_i$$
$$R_p = \sum_{i=1}^{n} w_i [\alpha_i + \beta_i R^M + \varepsilon_i]$$
$$R_p = \sum_{i=1}^{n} w_i \alpha_i + \left[\sum_{i=1}^{n} w_i \beta_i\right] R^M + \sum_{i=1}^{n} w_i \varepsilon_i$$
$$R_p = \alpha_p + \beta_p R^M + \varepsilon_p \tag{8.5}$$

where the portfolio coefficients α_p, β_p, and ϵ_p are equal to the weighted sum of the stock's parameters.

Equation (8.5) for the portfolio return has the same form as the regression equation for a stock. Thus, by analogy, the portfolio expected return and variance are

$$E(R_p) = \alpha_p + \beta_p E(R^M) \tag{8.6}$$

$$V(R_p) = \beta_p^2 V(R^M) + V(\varepsilon_p) \tag{8.7}$$

where:

$$V(\varepsilon_p) = \sum_{i=1}^{n} w_i^2 V(\varepsilon_i)$$

Equations (8.6) and (8.7) for the portfolio expected return and variance are similar in form to the regression equations for a stock's return and variance. In practice, analysts often regress a portfolio's rate return against the market. When a portfolio's return is regressed against the market, then the intercept and slope of the regression equation (α_p and β_p) can be interpreted as being the weighted α's and β's of the stocks making up the portfolio.

Like the variance equation for an individual security, Equation (8.7) decomposes the portfolio's risk into its systematic and unsystematic risk components. As with a security, the systematic risk of a portfolio is that risk that can be explained by market factors (those factors that affect all securities), whereas unsystematic risk is risk that can be explained by the industry and firm factors affecting each security that makes up the portfolio. In our Chapter 7 discussion of portfolio risk and size, we noted that unsystematic risk can be diversified away with a portfolio consisting of approximately 30 stocks. If the portfolio under consideration is of this size or more, then $V(\epsilon_p)$ would approach zero.[3] Thus for a portfolio consisting of approximately 30 stocks, the portfolio variance and standard deviation would be:[4]

$$V(R_p) = \beta_p^2 V(R^M)$$
$$\sigma(R_p) = \beta_p \sigma(R^M)$$

Example

Exhibit 8.11(a) shows the α_i, β_i, and $V(\epsilon_i)$ for the 10 stocks presented in our earlier example. The parameters values were pulled from Bloomberg's RV screen. The Bloomberg values for alpha, beta, and the standard deviation of the error term for the 10 stocks are based on the more recent 2012–2013 time period rather than the 2006–2013 time period used to calculate the average returns, variances, and covariances in our earlier example; the parameter values are also calculated from daily observations compared to weekly that were used in the earlier example. As a result, the expected returns, variances, and covariances for the 10 stocks differ in these examples.

$E(r_i) = \alpha_i + \beta_i E(R^M)$, $V(r_i) = \beta^2{}_i V(R^M) + V(\epsilon)$, $Cov(r_j r_k) = \beta_j \beta_k V(R^M)$

$E(R^M) = 16\%$, $V(R^M) = 20$, $R_f = 5\%$

(a) Portfolio Inputs

	Stock	Ticker	α_j	β_j	$E(r_j)$	$\beta^2 V(R^M)$	$V(\epsilon_j)$	$V(r_j)$	$\sigma(R_j)$	λ_T
1	Archer-Daniels-Midland	ADM	0.0717	1.146	18.41	26.28	5.79	32.07	5.66	11.70
2	Walt Disney	DIS	0.0418	1.121	17.98	25.14	3.71	28.85	5.37	11.58
3	Kroger	KR	0.1523	1.007	16.27	20.29	4.48	24.76	4.98	11.19
4	CVS Caremark	CVS	0.0516	1.005	16.14	20.21	3.41	23.62	4.86	11.08
5	Aflac Inc	AFL	0.0465	0.947	15.20	17.93	7.51	25.44	5.04	10.77
6	Exxon Mobil	XOM	−0.0503	0.826	13.17	13.66	1.68	15.34	3.92	9.89

7	Procter & Gamble	PG	0.0285	0.801	12.85	12.83	3.64	16.47	4.06	9.79
8	Microsoft	MSFT	−0.0408	0.800	12.76	12.80	6.79	19.59	4.43	9.70
9	Johnson & Johnson	JNJ	0.0742	0.696	11.21	9.70	1.73	11.43	3.38	8.93
10	Duke Energy	DUK	−0.0273	0.666	10.62	8.86	3.23	12.09	3.48	8.45
	Sum		0.3483	9.016	144.61		41.9651			
	Portfolio (w_i = 1/10)		α_p	β_p	$E(R_p)$	$\beta_p{}^2 V(R^M)$	$V(\epsilon_p)$	$V(R_p)$	$\sigma(R_p)$	
			0.03483	0.9016	14.46	16.26	0.41965	16.68	4.08	
	Market		0.00	1.00	16.00	1.00	0.00	20.00	4.47	
$\lambda_T = (E(r_j) - R_f)/\beta_j$										
$R_f = 5\%$										

(b) Variance-Covariance Matrix

		ADM	DIS	KR	CVS	AFL	XOM	PG	MSFT	JNJ	DUK
		1	2	3	4	5	6	7	8	9	10
ADM	1	32.07	25.70	23.09	23.05	21.71	18.95	18.37	18.34	15.96	15.26
DIS	2		28.85	22.58	22.54	21.24	18.53	17.96	17.94	15.61	14.92
KR	3			24.76	20.25	19.07	16.65	16.14	16.11	14.02	13.41
CVS	4				23.62	19.04	16.62	16.11	16.09	14.00	13.38
AFL	5					25.44	15.65	15.17	15.15	13.19	12.60
XOM	6						15.34	13.24	13.22	11.51	11.00
PG	7							16.47	12.82	11.16	10.66
MSFT	8								19.59	11.14	10.65
JNJ	9									11.43	9.27
DUK	10										12.09

EXHIBIT 8.11 Single-Index Model

The expected returns and variances for each stock are shown, respectively, in columns (6) and (9). These estimates were generated using Equations (8.2) and (8.3) and by assuming an expected market return, $E(R^M)$, of 16 percent and a market variance, $V(R^M)$ of 20. The stocks are ranked in the order of their Treynor index:

$$\lambda_T = [E(r) - R_f]/\beta.$$

The 45 covariances between the 10 stocks were calculated using Equation (8.4) and are shown in the variance-covariance matrix in Exhibit 8.11(b). For an equally allocated portfolio (w_i = 1/10) formed with these 10 stocks, the portfolio beta is 0.9016, the portfolio alpha is 0.0348, and the portfolio's unsystematic risk, $V(\epsilon_p)$, is 0.41965:

$$\sum_{i=1}^{n} w_i\alpha_i = (1/10)\sum_{i=1}^{n} \alpha_i = (1/10)(0.348) = 0.0348$$

$$\sum_{i=1}^{n} w_i\beta_i = (1/10)\sum_{i=1}^{n} \beta_i = (1/10)(9.016) = 0.9016$$

$$V(\varepsilon_p) = \sum_{i=1}^{n} w_i^2 V(\varepsilon_i) = (1/10)^2(41.965) = 0.41965$$

For an expected market return of $E(R^M)$ = 16 percent and market variance of $V(R^M)$ = 20 ($\sigma(R^M)$ = 4.47), the portfolio expected return is 14.46 percent, its variance is 16.68, and its standard deviation is 4.084.

$$E(R_p) = \alpha_p + \beta_p E(R^M)$$
$$E(R_p) = 0.0348 + 0.9016(16\%) = 14.46\%$$

$$V(R_p) = \beta_p^2 V(R^M) + V(\varepsilon_p)$$
$$V(R_p) = (0.9016)^2(20) + 0.419651 = 16.68$$
$$\sigma(R_p) = 4.084$$

PORTFOLIO REGRESSION/BLOOMBERG CIXB SCREEN AND CORR SCREEN

CIXB

The returns of a portfolio in EQS or PRTU can be evaluated using historical regression by putting the portfolio into a CIXB basket, creating historical data, and then treating the portfolio as an index.

Steps

1. CIXB <Enter>.
2. On the CIXB screen, name the ticker and the portfolio in the ".Ticker" and "Name" yellow box and hit <Enter> to update (.XSIF13 for ticker and XSIF 2013 for Name.
3. Click "Import" from the Actions dropdown tab.
4. On the "Import from Excel" box, click "Import from List" tab at bottom to bring up "Import from List" tab.
5. On "Import from List" tab: Select Portfolio (or EQS search or index) from the "Source" dropdown and the name of the portfolio (search, or index) from the "Name" dropdown, and then click the "Import" tab. These steps will import the portfolio's stocks, shares, and prices to the CIXB screen.
6. On CIXB screen, click the "Create" tab to bring up a time period box for selecting the time period for price and return data. After selecting the time period, hit "Save." This will activate a Bloomberg program for calculating the portfolio's daily historical returns.
7. The data will be sent to a report, RPT. To access this report, type "RPT" and hit<Enter>.

8. To access the menu screen for your portfolio: Ticker <Index> <Enter>. For regression, bring up HRA.

BLOOMBERG'S CORR SCREEN TO COMPUTE R^2, ALPHAS, AND BETAS FOR STOCKS IN A PORTFOLIO

To create a CORR screen for the stocks of a portfolio created in PRTU:

- Enter CORR <Enter>.
- Click "Create New" tab; select data time period.
- In the "Matrix Securities" box, unclick "Symmetric Matrix" button, import portfolio from "Add from Source" tab, and click update.
- In Column Securities Box, add stock index (e.g., S&P 500) by typing index ticker and index moniker (e.g., SPX <Index>), and then click the "Next" tab; name your CORR Screen.
- On your portfolio's CORR screen, select the data time period for analysis and then use the Calculation tab to find each stock's R^2, alphas, and betas.

See Bloomberg Web Exhibit 8.4.

Elton, Gruber, and Padberg

Technique for Determining the Best Efficient Portfolio

In a 1976 article, Elton, Gruber, and Padberg showed how the single-index model can be extended to determine the best efficient portfolio (tangency point of the borrowing-lending line with the efficiency frontier). The technique they derive for generating efficient portfolios, in turn, is much simpler than using the calculus minimization approach or quadratic programming. Moreover, the approach can be set up with a constraint that weights are nonnegative.

The Elton, Gruber, and Padberg (EGP) algorithm starts by ranking each stock in the portfolio by its Treynor index, $\lambda_{\beta j}$ (stock j's risk premium per level of systematic risk as measured by the stock's beta). Next calculating an index C_i for a set of portfolios starting first with a one-security portfolio (i = 1) consisting of the security with the highest rank, $\lambda_{\beta 1}$, then a two-security portfolio (i = 2) consisting of the first two securities with the highest ranks, $\lambda_{\beta 2}$, and so on, with the final C_i calculation consisting of a portfolio of all the securities. Columns 2 and 7 of Exhibit 8.12 show respectively the $\lambda_{\beta j}$ and the C_i formula and calculations for the portfolios formed with the 10 stocks in the example presented in Exhibit 8.11. Note that as you increase the size of the portfolio by adding the next higher ranked security to the portfolio, the C_i values increase until you get to portfolio i = 4. After that point, adding each successive higher ranked stock reduces the value of C_i. The highest C_i is defined as the *cutoff index* and is denoted as C^*. In this example, the cutoff index is C^* = 10.8860. Given the cutoff index, the next step is to select all securities with $\lambda_{\beta j} > C^*$ for inclusion in the portfolio. With C^* = 10.8860, there are four stocks in our example with $\lambda_{\beta j}$ values exceeding C^*. The final step is to determine the portfolio allocations of each of the selected securities, w_j. Each security's w_j is determined as a proportion of an index Z_j for the security to the sum of indexes for all securities in the portfolio. That is:

$$w_j = \frac{Z_j}{\sum_{j=1}^{n} Z_j}$$

where:

$$Z_j = \frac{\beta_j}{V(\varepsilon_j)} \left[\frac{E(r_j) - R_f}{\beta_j} - C^* \right]$$

	1	2	3	4	5	6	7
Stock	**Portfolio *i***	$\lambda_{\beta_j} = \frac{E(r_j) - R_f}{\beta_j}$	$\frac{(E(r_j) - R_f)\beta_j}{V(\varepsilon_j)}$	$\frac{\beta_j^2}{V(\varepsilon_j)}$	$\sum_{j=1}^{i} \frac{(E(r_j) - R_f)\beta_j}{V(\varepsilon_j)}$	$\sum_{j=1}^{i} \frac{\beta_j^2}{V(\varepsilon_j)}$	$C_i \frac{V(R^M) \sum_{j=1}^{i} \frac{[E(r_j) - R_f]\beta_j}{V(\varepsilon_j)}}{1 + V(R^M) \sum_{j=1}^{i} \frac{\beta_j^2}{V(\varepsilon_j)}}$
Archer-Daniels-Midland	1	11.70	2.6533	0.2268	2.6533	0.2268	9.5868
Walt Disney	2	11.58	3.9268	0.3392	6.5801	0.5659	10.6832
Kroger	3	11.19	2.5327	0.2264	9.1127	0.7923	10.8185
CVS Caremark	4	11.08	3.2845	0.2965	12.3972	1.0888	C* = 10.8860
Aflac Inc	5	10.77	1.2864	0.1195	13.6836	1.2083	10.8749
Exxon	6	9.89	4.0236	0.4069	17.7073	1.6152	10.6340

Mobil							
Procter & Gamble	7	9.79	1.7268	0.1763	19.4341	1.7915	10.5536
Microsoft	8	9.70	0.9150	0.0943	20.3491	1.8858	10.5119
Johnson & Johnson	9	8.93	2.4988	0.2800	22.8478	2.1658	10.3114
Duke Energy	10	8.45	1.1569	0.1370	24.0047	2.3027	10.2029

	1	2	3	4	5	6	7
	Stock *j*	$\frac{\beta_j}{V(\varepsilon_j)}$	$\lambda_{\beta_j} = \frac{E(r_j) - R_f}{\beta_j}$	C*	Max(Col(3)-Col(4), 0)	Z_j Col(2) X Col(5)	$w_j = Z_j / \sum Z_j$ Col(6)/0.4946
Archer-Daniels-Midland	1	0.1978	11.7005	10.8860	0.8146	0.1612	0.3258

Walt Disney	2	0.3025	11.5779	10.8860	0.6919	0.2093	0.4232
Kroger	3	0.2248	11.1865	10.8860	0.3006	0.0676	0.1366
CVS Caremark	4	0.2949	11.0777	10.8860	0.1917	0.0565	0.1143
Aflac Inc	5	0.1261	10.7691	10.8860	0.0000	0.0000	0.0000
Exxon Mobil	6	0.4923	9.8890	10.8860	0.0000	0.0000	0.0000
Procter & Gamble	7	0.2201	9.7940	10.8860	0.0000	0.0000	0.0000
Microsoft	8	0.1179	9.6993	10.8860	0.0000	0.0000	0.0000
Johnson & Johnson	9	0.4021	8.9253	10.8860	0.0000	0.0000	0.0000
Duke Energy	10	0.2058	8.4463	10.8860	0.0000	0.0000	0.0000

$$\sum Z_j = 0.4946$$

EXHIBIT 8.12 Elton, Gruber, and Padberg Technique for Determining Allocations

The second panel of Exhibit 8.12 shows the Z_i calculations for the four stocks included in the portfolio and their allocations. The Elton, Gruber, and Padberg technique is described in more detail in Appendix 8B. An Excel program, “Elton-Gruber Optimum Portfolio,” which calculates the optimum weights using the algorithm, can be accessed from the text's Web site.

The ex-post total returns from the one-year period from 8/8/12 to 8/8/13 for the Markowitz efficient portfolio using the EGP algorithm and the S&P 500 are shown in the figure in Exhibit 8.13 (Bloomberg's PORT screen, Performance tab, and Total Return tab). The back testing results show the portfolio outperforms the market for the one-year period (total return of 42.24 percent compared to 16.44 percent for S&P 500). A one-year regression of the portfolio against the S&P 500 (generated using the Bloomberg CIXB and HRA screens), shows the Markowitz portfolio has a beta close to one but a relatively large alpha of 0.059, indicating abnormal returns.

<HELP> for explanation.
Performance
Total Return
BLUE ROCK MARK EG 42.2436
SPX 16.4438
Performance Difference
Aug Sep Oct Nov Dec Jan Feb Mar Apr May Jun Jul Aug
2012
2013

(a)

<HELP> for explanation.
Attribution
Name
Tot Rtn
BLUE ROCK MARK EG 42.24
ARCHER-DANIELS-MIDLAND CO 48.32
CVS CAREMARK CORP 32.26
KROGER CO 78.04
WALT DISNEY CO/THE 32.21
Heat Map
CVS CAREMARK CORP (+32.26)
KROGER CO (+78.04)
BLUE ROCK MARK EG (+42.24)
WALT DISNEY CO/THE (+32.21)

(b)

(b)

(c)

EXHIBIT 8.13 Performance of Markowitz Efficient Portfolio Using the EGP Algorithm and the S&P 500

Note also that this efficient portfolio invested 32.58 percent in Archer-Daniels. In contrast, the portfolio shown in Exhibit 8.5 generated using the Asset Allocation template has a zero allocation to this stock. The difference can be explained in terms of the time periods. In the template example, the portfolio input averages were calculated over the seven-year period from 2006 to 2013. By contrast, in this case the averages were based on the more recent period from 2011 to 2013. For the longer period, Archer-Daniels has a relatively lower alpha of 2.7 percent (compared to 7.7 percent for the 2011–2013 period) and a beta of only 0.421. Several other stocks, such as Aflac, also have alphas and betas that differ significantly given the time period analyzed. These differences in the portfolios do highlight the problem of using historical averages to estimated portfolio inputs.

BLOOMBERG: FINDING ELTON, GRUBER, AND PADBERG PORTFOLIO INPUTS USING BETA, RV, AND PC SCREENS

The portfolio inputs for solving for the best efficient portfolio using the Elton, Gruber, and Padberg algorithm are each stocks' α_i, β_i, and $V(\epsilon_i)$. Estimates of these parameters can be generated from a number of Bloomberg screens: Stock beta, HRA, PC, and RV screens.

See Bloomberg Web Exhibit 8.5.

Bloomberg/Markowitz Excel Program

A Markowitz Excel Program that determines portfolio allocations using the Elton, Gruber, and Padberg technique for a portfolio imported from the Bloomberg PRTU screen can be downloaded from the text's Web site. Using the “Markowitz” Excel program, one can import the names of the stocks from a portfolio created in PRTU into the program (see Bloomberg Exhibit Box: “Markowitz Excel Program”). The user can then select a risk-free rate from a dropdown, an index (e.g., S&P 500 or Dow Jones), a regression time period, and a length of period (daily or weekly). The program then calculates α_i, β_i, and $V(\epsilon_i)$, and then each stock's $E(r_i)$ and $V(r_i)$, and $\lambda_{\beta j}$ based on the index's average market return and variability: $E(R^M) = \text{Av}R^M$ and $V(R^M) = \text{Av}V(R^M)$. The user can also elect to use either Bloomberg's adjusted beta or the regression beta (raw beta). Calculation Sheet 2 of the Excel program shows each stock's parameter values in the order of their $\lambda_{\beta j}$'s and the Elton, Gruber, and Padberg parameter calculations of C_i, and optimum weights. Exhibit 8.14 shows (1) the Bloomberg PRTU slide of the illustrative 10-stock portfolio; (2) the input page of the Excel program where the S&P 500, 10-year Treasury, and a weekly time period from 8/5/2006 to 8/8/2013 were selected for the regressions (note that this a different time period than the previous example in Exhibit 8.12 but similar to Exhibit 8.5 generated from the Asset Allocation Optimizer program); and (3) the Calculation Sheet 2, where the allocations for the portfolio are shown. The portfolio consists of the same stocks as the Optimizer program with high (but not identical) allocations to CVS, Disney, Kroger, and Johnson & Johnson. Exhibit 8.15 shows the ex-post performance of the portfolio (named Blue Mark Bloom) relative to the S&P 500 for the 8/8/2006–8/19/2013 period. The fund dramatically outperforms the market during this period with a total return for the period of 127.82 percent compared to a return of 78.01 for the S&P 500, with the most significant gains occurring in 2013.

(a)

Rf		10 yr treas
Rm	Index	SPX
	Start date	8/5/2006
	Ending date	8/8/2013
	Daily or weekly	W
Beta	Type	raw beta
	Relativev Index	SPX
	Start date	8/5/2006
	Ending date	8/8/2013
	Daily or weekly	W

Import Data Type	Portfolio
ID or Name	u5945505-128

(b)

Name	$E(r_i)$	β_i	$V(e_i)$	Rf	$V(R^m)$	λ_p	C_i	W_i
KROGER CO	15.77	0.56	8.70	2.58	2.84	23.73	2.1750	22.5%
CVS CAREMARK CORP	16.91	0.61	8.71	2.58	2.84	23.44	4.2959	24.3%
WALT DISNEY CO	19.78	1.08	6.08	2.58	2.84	15.99	8.3425	35.7%
JOHNSON & JOHNSON	9.07	0.50	2.89	2.58	2.84	12.92	8.8792	16.6%
DUKE ENERGY CORP	7.57	0.55	4.43	2.58	2.84	9.09	8.8967	0.5%
PROCTER & GAMBLE CO	6.99	0.49	3.87	2.58	2.84	9.04	8.9068	0.4%
AFLAC INC	13.61	1.51	17.61	2.58	2.84	7.33	8.7041	0.0%
MICROSOFT CORP	7.01	0.80	8.98	2.58	2.84	5.56	8.4977	0.0%
EXXON MOBIL CORP	5.15	0.78	4.45	2.58	2.84	3.30	7.9118	0.0%
ARCHER-DANIELS-MIDLAND CO	- 0.61	0.88	13.60	2.58	2.84	- 3.62	7.4022	0.0%

(c)

EXHIBIT 8.14 Bloomberg and Markowitz Excel Program Using Elton, Gruber, and Padberg Technique for Determining Allocation

(a)

(b)

EXHIBIT 8.15 Ex-Post Performance of Markowitz Fund and S&P 500, 8/8/2006–8/9/2013

BLOOMBERG: THE MARKOWITZ EXCEL PROGRAM

The Markowitz Excel Program can be downloaded from the text Web site. The program uses the Elton, Gruber, and Padberg Technique for determining the allocation for the best efficient portfolio. The user imports the stocks of a portfolio created in PRTU into the program by typing in the portfolio's ID number found on the top left corner of the PRTU screen. The user then selects a risk-free rate from a dropdown, an index (S&P 500 or Dow Jones), a regression time period, and a length of period (daily or weekly). The program then calculates α_i, β_i, and $V(\epsilon_i)$, and then each stock's $E(r_i)$, $V(r_i)$, and $\lambda_{\beta j}$ based on the average market return and variability. The user can also elect to use Bloomberg's adjusted beta or the regression beta (raw beta). Calculation Sheet 2 of the Excel program shows each stocks parameter values in the order of their $\lambda_{\beta j}$'s and the Elton, Gruber, and Padberg parameter calculations of C_i, C^*, and the optimum weights. See Exhibit 8.14.

Multi-Index Models

The single-index model assumes that all stocks in the portfolio are related to only one factor, with that factor typically being the market return. In a multi-index model, the number of factors affecting each security in the portfolio is extended to include more than one explanatory variable. Specifically, the model assumes that the return of each stock i in the portfolio being evaluated is related to the same set of factors, I_{ij}:

$$r_i = \alpha_i + \beta_{i1} I_1 + \beta_{i2} I_2 + \cdots + \beta_{in} I_n + \varepsilon_i \tag{8.8}$$

This model also assumes that the standard regression assumptions hold for each security (ϵ is normally distributed with $E(\epsilon) = 0$, $Cov(\epsilon_i\ I_j) = 0$, and $V(\epsilon)$ is constant over observations), and that there is no correlation in the error terms for securities ($Cov(\epsilon_j\ \epsilon_k) = 0$).

There are three potential problems in applying multi-index models to explain what determines a stock's return. The first is identifying those factors that are statistically significant in explaining the returns.

Multi-index models vary in terms of the factors used to explain returns. For example, there are *industry models* that explain stock returns in terms of the market and the average returns of the stock's industry; *pseudo industry models* in which the indexes are formed from stocks grouped into categories such as growth, cyclical, and stable; and *macroeconomic models* in which factors such as the market return, inflation, and bond returns explain each stock's return. As noted in Chapter 6, empirical research has provided evidence that provides some support for the construction of multi-index models based on macroeconomic factors that affect the value of stock as measured by the present value of the stock's future cash flows. These models are discussed further in Chapter 10 in conjunction with the testing of equilibrium models.

The second problem in applying multi-index models relates to their manageability and statistical accuracy. In a multi-index model, the number of computations needed to determine an optimal portfolio is greater than the computations needed for the single-index model but less than the requirements needed using historical averages.

The third problem relates to the possibility that the explanatory variables in a multiple regression model could be linearly related—a condition referred to as multicollinearity. When multicollinearity exists, one of the variables is redundant (i.e., it is simply a linear transformation of the other), and the regression qualifiers (*t*-test and *F*-test) are biased. As a result, the quality of the regression cannot be determined. Procedures for converting multi-index models into ones with uncorrelated indexes are presented in a number of statistics books.[5]

The latter two problems (number of computations and multicollinearity) can be eliminated (or minimized) by converting the multi-index model into another multi-index model in which the indexes, I^*, are uncorrelated.

MRA: BLOOMBERG'S MULTIPLE REGRESSION SCREEN

Multiple regression analysis can be done using the Bloomberg MRA screen. See the Bloomberg exhibit box in Chapter 6: "MRA: Bloomberg's Multiple Regression Screen."

Conclusion

Harry Markowitz introduced his model for determining portfolios in a 1952 article in the *Journal of Finance*. Even though the model is over 60 years old, it is still referred to as *modern portfolio theory*. Such persistence, in part, reflects the number of years it took to appreciate the importance and subtleties of his theory and in part the time it has taken the discipline to learn how to use the theory. Perhaps the most significant contribution of Markowtiz's work is that it introduced a statistical and mathematical methodology for approaching investment decisions. In 1989, Harry Markowitz, along with Merton Miller and William Sharpe, was awarded the Noble Prize in Economics for his contribution to Economics and Finance.

BLOOMBERG CASE: CONSTRUCTING AND BACK TESTING MARKOWITZ PORTFOLIOS IN BLOOMBERG

STEPS FOR CONSTRUCTING AND ANALYZING

- **Step 1**: Construct a portfolio in PRTU consisting of one share in each of the DJIA stocks: PRTU <Enter>; click “Create;” from Settings Screen, click “Import”; from “Actions” tab, select “Equity Index” from “Source” dropdown and INDU from “Name” dropdown.
- **Step 2**: Use the Markowitz Excel program to determine the best efficient portfolio using the Elton, Gruber, and Padberg algorithm box: “Markowitz Excel Program.”
- **Step 3**: Create your Markowitz portfolio in PRTU. In constructing the portfolio, use fixed weights:
  - PRTU <Enter>.
  - Click “Create” tab.
  - From Settings Screen click “Fixed Weight” in the “Position Type” dropdown.
  - Enter the Markowitz weights.
- **Step 4**: Create history for the Markowitz portfolio:
- **Step 5**: Analyze the portfolio's performance relative to an index (e.g., Dow) in PORT: PORT <Enter>, select “INDU” for comparison, click Performance tab, Total Return tab, and select time period for analysis.

See Bloomberg Web Exhibit 8.6.

OTHER BLOOMBERG PORTFOLIO SCREENS

PORT's Trade Simulation Tab allows you to select and edit hypothetical trading positions for your portfolio in order to assess the impact these moves may have on your portfolio.

PORT's VAR Tab: Value-at-Risk: VAR displays value at risk analytics for a portfolio. One can select from the tabs Absolute VAR/Relative VAR (relative to a benchmark) from the toolbar to display the desired data. The VAR calculation is based on the following methodologies: Historical 1 year, Historical 2 years, Historical 3 years, Monte Carlo, or Parametric. VAR is based on a given confidence interval (95 percent, 97.5 percent, or 99 percent), and a given time horizon (between 1 day and 1 quarter) and shows the range of possible values of the portfolio, industry, stocks, and index.

Notes

1. **For a two-security portfolio, the allocation that yields a zero risk portfolio can be found by setting $V(R_p)$ equal to zero and then solving for one of the weights. For this example, in which there is perfect negative correlation:**

$$V(R_p) = w_1^2\sigma(r_1)^2 + w_2^2\sigma(r_2)^2 + 2w_1w_2\rho_{12}\sigma(r_1)\sigma(r_2)$$
$$V(R_p) = w_1^2\sigma(r_1)^2 + w_2^2\sigma(r_2)^2 + 2w_1w_2(-1)\sigma(r_1)\sigma(r_2)$$
$$V(R_p) = [w_1\sigma(r_1) - w_2\sigma(r_2)]^2$$
$$0 = [w_1\sigma(r_1)^2 - w_2\sigma(r_2)^2]^2$$
$$w_1 = \frac{\sigma(r_1)}{\sigma(r_1) + \sigma(r_2)}$$
$$w_1 = \frac{4}{4+6} = 0.4$$

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2. The annualized variance is equal to the periodic variance (e.g., weekly) times the number of periods of that length (week) in a year (52). The annualized standard deviation is equal to the square root of the annualized variance.

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3. More formally, given

$$V(\varepsilon_p) = \sum_{i=1}^{n} w_i^2 V(\varepsilon_i)$$

and assuming an equal allocation strategy such that $w_i = 1/n$ and

$$V(\varepsilon_p) = \sum_{i=1}^{n} \left(\frac{1}{n}\right)^2 V(\varepsilon_i)$$

the larger the portfolio (i.e., the greater n), the smaller $V(\epsilon_p)$.

$$V(\varepsilon_p) = \sum_{i=1}^{n} \left(\frac{1}{n}\right)^2 V(\varepsilon_i) \to 0 \text{ as } n \to \infty$$

Thus for portfolio consisting of approximately 30 stocks, the portfolio variance and standard deviation simplify to:

$$V(R_p) = \beta_p^2 V(R^M)$$
$$\sigma(R_p) = \beta_p \sigma(R^M)$$

4. Note that if the portfolio consists of all securities in the market and the allocations reflect each security's proportional value to the market, then the portfolio represents the market portfolio and $E(R_p)$ will be equal to $E(R_M)$. To ensure that $E(R_p) = E(R^M)$, α_p must be equal to zero and β_p must be equal to one.

5. Elton and Gruber have derived a simplified technique for generating the best efficient portfolio using a multi-index model. See: Elton, Gruber, Brown, and Goetzmann (2003).

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Bloomberg Exercises

CREATING HISTORICAL DATA FOR PORTFOLIOS IN PRTU

Bring up the PRTU screen for your portfolio. On the screen, change the date in the amber “Date” box (e.g., Month/Day/2006). You may see the stocks disappear from the screen. Click the “Edit” tab (if needed) and then the “Actions” tab. From the “Actions” dropdown, select “Import” to bring up the import box. On the import box, select “Portfolio” from the “Source” dropdown, the name of your portfolio from the “Name” dropdown, change the date (e.g., back to the current period), hit the “Import” tab, and then click “Save.” You should now have a portfolio with historical data.

1. Using the some of the stocks comprising the Dow, generate an efficiency frontier using Bloomberg's Asset Allocation Optimizer Template. To download the program you may want to use DAPI: DAPI <Enter> and click “Excel Template Library,” “Equity,” “Portfolios,” and “Asset Allocation Optimizer.” For the DJIA stocks, select no more than 12 stocks, set each stock's minimum weight to zero and maximum weight to 99 percent, and use the average risk-free rate for the period.
2. Study the characteristics of some the stocks composing the DJIA or the stocks of the Markowitz portfolio you constructed in Exercise 1 in terms of their R^2's, alphas, betas, and $V(\epsilon)$'s using Bloomberg's PC screen (PC <Enter>). On the PC screen, examine the R^2's, alphas, betas, and $V(\epsilon)$'s over different time periods. Note: $V(\epsilon) = V(r)\,(1 - R^2)$.

   To create a PC screen for the DJIA (or your portfolio), use the PC screen for one of the stocks in the DJIA (portfolio) (e.g., IBM) and then type: IBM <Equity> PC <Enter>. On the PC screen, you can import the index (portfolio) from the “Peer Source” tab by clicking “Equity Index” (“Portfolio”) and then INDU (name of your portfolio) from the “Name Source” tab. You may want to use the edit screen to delete some of the indexes. Use the edit screen to also add an index (e.g., S&P 500) to the rows for stocks in order to have its variance. After saving your edited screen, you can set the time period for your analysis. Alphas and betas for each stock for the selected time period can be accessed from the "Calculation" tab and exported to Excel from the "Export" tab (upper right corner). Each stock's variance of the error, $V(\epsilon)$, can be calculated from the stock's R^2 and its variance using the stock variances (found by clicking "Covariance" and R^2 from the "Calculation" tab).

3. Stock rankings:
   1. Estimate the expected returns for each of the stocks you analyzed in Bloomberg Exercise 2 using the Bloomberg DDM model (see Bloomberg Exhibit in Chapter 3: "Bloomberg DDM Screen").
   2. Given your expected return estimates for each stock, estimate their betas and then determine the risk-free rate (e.g., current 10-year Treasury yield; see Bloomberg FIT screen). For beta, you may want to use the stock's adjusted beta (found on the stock's beta screen) or a beta from one of the PC screen calculations you made in Exercise 2.
   3. Given your stocks' expected returns and betas and the risk-free rate, rank the stocks using the Treynor index.

   Exercise 4 is based on material presented in Appendix 8A (text Web site).

4. Using Bloomberg's CORR screen, select 10 stocks to analyze from the DJIA: CORR <Enter>; click "Create New" tab; select data time periods (e.g., daily, last seven years); on the "Matrix Securities" box, click the "Add from Sources" tab and then click "Equity Indexes" from "Source" dropdown and INDU from the "Name" dropdown; add 10 of the DJIA stocks from the list appearing in the right box; click "Update."
   1. On the CORR screen for your 10 stocks, select a data time period for analysis and then select "Covariance" from the "Calculation" tab to bring up the variance-covariance matrix. Export the matrix to Excel (click "Export to Excel" from the "Export" tab in upper right corner).
   2. In Excel, create a coefficient matrix **A** from the variance-covariance matrix that you exported from Bloomberg. For your stock expected returns, use averages or formulate you own expected returns.
   3. Using the math approach for portfolio variance minimization, solve for the allocations for several efficient portfolios. Use Excel and the Excel matrix multiplication commands (see Appendix 8A

   (text Web site) and the Bloomberg Box: “Using the Bloomberg CORR Screen and Excel to Determine Markowitz Efficient Portfolios”).

   4. Comment on your results and the limitations of using the math approach to solve for Markowitz efficient portfolios.

5. Use EQS to find stocks making up the S&P 500 with a market cap greater than $30 billion and then import your stocks into PRTU to form an equally allocated portfolio.
   1. EQS <Enter>; select Standard and Poor's 500 from the Indexes tab; in the amber ribbon box, type Market Cap, and enter $30B; save your screen.
   2. PRTU <Enter>; On the Settings Screen, click “Actions” and “Import,” and then select “Equity Search (EQS)” from the Source dropdown and the Name of your search from the Source dropdown.
   3. Create history for your portfolio (see the box in the exercises, “Creating Historical Data for Portfolios in PRTU”).
      1. Examine the correlation of the stocks in your portfolio using the PC screen: Bring up the equity screen for one of the stocks in the portfolio (Stock Ticker <Equity> <Enter>); type in PC; on the PC screen, import your portfolio from the “Peer Source” yellow dropdown tab.
      2. Evaluate your portfolio using tabs on the PORT screen.
      3. Evaluate the performance of your portfolio using the “Performance” tab on the PORT screen.
      4. Examine the top movers and laggers in your portfolio for different periods using the MRR screen: MRR <Enter>; select “Portfolio” from the “Source” dropdown and find your portfolio from the “Name” dropdown.
      5. Examine your portfolio's regression relation with the S&P 500, its historical total return relative to the S&P 500, and its price graph by importing your portfolio to a basket using CIXB and then using the HRA, COMP, and GP screens. See Bloomberg Exhibit Box: Portfolio Regression, Bloomberg CIXB Screen.
6. Bloomberg's FMAP screen provides information on investment funds by category, areas, performance, and type.
   1. Using FMAP, examine several equity funds in different categories.

    2. Select a U.S. equity fund with a certain investment style (e.g., Value and Large Cap) and study it using the functions on the fund's menus screen: FMAP <Enter>; select “Objective” from “View by” dropdown and United States from “All Region” dropdown; click equity from the listing on the right-side of FMAP box, and then select type and the fund. To examine the fund go to the fund's menu screen: Fund Ticker <Equity> <Enter>. Screens on the fund's menu screen to consider: DES, historical fund analysis (HFA), fund holdings (MHD), price graph (GP), and comparative returns (COMP).
    3. Examine the fund you selected in 6b in more detail using the PORT screen.
7. Form a portfolio in PRTU consisting of some of the stocks of your selected fund in Exercise 6. For an easy way to load securities of the fund into PRTU, see the Bloomberg box in this exercise section: Dragging and Dropping Securities from MHD into PRTU. Your portfolio could consist of one share or 1,000 shares of each stock or you can determine an equal allocation for each. In forming your portfolio in PRTU, be sure to also create history (see Bloomberg Box in this exercise section: Creating Historical Data for Portfolios in PRTU).
    1. Using PORT, evaluate the historical performance of your portfolio relative to an index.
    2. Using the CORR screen, create a CORR screen by importing the portfolio: CORR <Enter>; click “Create New” tab; select data time period; in the “Matrix Securities” unclick “Symmetric Matrix” button, import portfolio from “Add from Source” tab and click update; in Column Securities Box, add stock index (e.g., S&P 500) by typing index ticker and index moniker (e.g., SPX <Index>) and then click the “Next” tab; name your CORR Screen.
    3. On your portfolio's CORR screen, evaluate the stocks' R^2, alphas, and betas.

DRAGGING AND DROPPING SECURITIES FROM MHD INTO PRTU

The Security holding of a fund appearing on the fund's MHD screen can be dragged and dropped into the PRTU setting screen. To do this: (1) open the MHD screen for a fund and the PRTU setting screen for inputting securities; (2) drag the green icon in the upper right corner of the MHD screen and drop it in the blank security input box in PRTU. You will then see the fund's securities in PRTU with each security holding consisting of one share.

8. The following list shows 20 equity investment funds with different equity styles.
   1. Study the policy statements for several of the funds found on the fund's description page: Ticker <Equity>; DES.
   2. Create a portfolio of the funds in PRTU and analyze it using the PORT screen.
   3. Create a basket of portfolio returns and make it an index using CIXB. As an index, use HRA to determine the portfolio's regression relation with the S&P 500.
   4. Examine each fund's alpha, beta, and R^2 using the CORR Screen: Enter CORR <Enter>; click “Create New” tab; select data time period; in the “Matrix Securities” unclick “Symmetric Matrix” button, import portfolio from “Add from Source” tab and click update; in Column Securities Box, add stock index (e.g., S&P 500) by typing index ticker and pressing the index moniker (e.g., SPX <Index>) and then click the “Next” tab; name your CORR Screen. On your portfolio's CORR screen select the data time period for analysis and then use the Calculation tab to find the stocks' R^2, alphas, and betas.

Ticker	Name
BTO US Equity	JOHN HANCOCK FINANCIAL OPPOR
CHN US Equity	CHINA FUND INC
CISGX US Equity	TOUCHSTONE SANDS CAP INC GRW
EISMX US Equity	EATON VANCE-ATLANTA SMID-I
FCPVX US Equity	FIDELITY SMALL CAP VALUE-I
FEN US Equity	FIRST TRUST ENERGY INCOME AN
FMFAX US Equity	FIDELITY ADVISOR MATERIAL-A
FRIFX US Equity	FIDELITY REAL ESTATE INCOME
FSGRX US Equity	FRANKLIN SMALL CAP GRW FD-A
HCPIX US Equity	PROFUNDS HEALTH CARE ULTR-IV
JCE US Equity	NUVEEN CORE EQUITY ALPHA FUN
JTIVX US Equity	JPMORGAN INTL VALUE SMA

Ticker	Name
MAPCX US Equity	BLACKROCK PACIFIC FUND-I
MERDX US Equity	MERIDIAN GROWTH FUND INC
MFVSX UQ Equity	MASSMUTUAL SEL FOC VALUE-S
OSMAX US Equity	OPPENHEIMER INTL SMALL CO-A
PEUGX US Equity	PUTNAM EUROPE EQUITY FUND-A
RYZAX US Equity	RYDEX LARGE CAP VALUE FUND-H
VINAX US Equity	VANGUARD INDUSTRIALS IDX-ADM
VUIAX US Equity	VANGUARD UTILITIES INDEX-ADM

9. Create your own fund of funds portfolio:
    1. Using the FMAP screen, select funds with different styles: FMAP <Enter>; View by = Objective; Region = United States; Analyze by = Last 3-Year Total Return.
    2. Create a portfolio of your funds in PRTU and analyze the portfolio using PORT.
    3. Create a basket of your portfolio returns and make it an index in CIXB. As an index, use HRA to determine its regression relation with the S&P 500.
    4. Examine each fund's alpha, beta, and R^2 using the CORR Screen: Enter CORR <Enter>; click “Create New” tab; select data time period; in the “Matrix Securities” unclick “Symmetric Matrix” button, import portfolio from “Add from Source” tab and click update; in Column Securities Box, add stock

index (e.g., S&P 500) by typing index ticker and pressing the index moniker (e.g., SPX <Index>), and then click the "Next" tab; name your CORR Screen. On your portfolio's CORR screen, select the data time period for analysis and then use the Calculation tab to find the stocks' R^2, alphas, and betas.

10. Examine a portfolio you formed in one of the exercises (or some other portfolio you have formed) in terms of the portfolio's regression relation with the S&P 500, its historical total return relative to the S&P 500, and its price graph. Import your portfolio to a basket and make it an index using CIXB; then use the HRA, COMP, and GP screens on the index menu created for your portfolio. Comment on the portfolio's systematic and unsystematic risk and its alpha.
    Create a CORR screen for your portfolio to evaluate each stock's relationship to the market (S&P 500): CORR <Enter>; click "Create New" tab; select data time period; in the "Matrix Securities" unclick "Symmetric Matrix" button, import portfolio from "Add from Source" tab and click update; in Column Securities Box, add S&P 500 index by typing index ticker and pressing the index moniker (SPX <Index>) and then click the "Next" tab; name your CORR Screen.
    On your portfolio's CORR screen, evaluate the stocks' R^2, alphas, and betas.