

3. (a) 

```
int i=2;
while(i<n){
    i=i*i;
}
```

$i = 2, 4, 16, 256$   
 $\swarrow \quad \swarrow \quad \swarrow$   
 $\times 2 \quad \times 4 \quad \times 16$   
 $\swarrow \quad \swarrow$   
 $\times 2 \quad \times 4$   
 $\swarrow$   
 $\times 2$

$$i = 2^{2^x}$$

$$2^{2^x} < n$$

$$\log(2^{2^x}) < \log(n) = 2^x \log(2) < \log(n)$$

$$x \log(2) < \log(\log n)$$

$$\Theta(\log(\log(n)))$$

(b) 

```
for(int i=1; i<=n; i++) {
    if((i % (int)sqrt(n)) == 0) {
        for(int k=0; k<=pow(i,3); k++) {
            O(1)
        }
    }
}
```

$$\sum_{i=1}^n (\Theta(1) + O(\sum_{k=0}^{i^3-1} \Theta(1)))$$

$$= \Theta(n) + \sum_{i=1}^{\sqrt{n}} \sum_{k=0}^{i^3-1} \Theta(1)$$

$$= \Theta(n) + \sum_{i=1}^{\sqrt{n}} \Theta(i^2) = \Theta(n) + \Theta(\sqrt{n}^3)$$

$$= \Theta(n) + \Theta(n^2) = \Theta(n^2)$$

(c) 

```
for(int i=1; i<=n; i++) { n times
    for(int k=1; k<=n; k++) { n times
        if(A[k] == i) { worst: i times best: 0
            for(int m=1; m<=n; m=m*m) {
```

$m = 1, 2, 4, 8, 16, 32, 64$   
 $\swarrow \quad \swarrow \quad \swarrow \quad \swarrow \quad \swarrow \quad \swarrow$   
 $\times 1 \quad \times 2 \quad \times 4 \quad \times 8 \quad \times 16 \quad \times 32$

$$\sum_{i=1}^n \left( \Theta(1) + \sum_{k=1}^n \left( \Theta(1) + O\left( \sum_{m=1}^{\log(n)+1} \Theta(1) \right) \right) \right)$$

$$\sum_{i=1}^n \Theta(1) + \sum_{i=1}^n \sum_{k=1}^n \Theta(1) + \sum_{i=1}^n \sum_{k=1}^n \sum_{m=1}^{\log(n)+1} \Theta(1)$$

$$\sum_{i=1}^n \Theta(n) + \sum_{i=1}^n \sum_{k=1}^n \Theta(\log(n)+1)$$

$$\sum_{i=1}^n \Theta(n \cdot (\log(n)+1)) = \Theta(n) + \Theta(n^2) + \Theta(n^2 \log(n) + n^2)$$

$$\Theta(n^2 \log(n))$$

```

d) int *a = new int [10];
    int size = 10;
    for (int i = 0; i < n; i++) { O(n)
        if (i == size) {
            int newSize = 3 * size / 2;
            int *b = new int [newSize]
            for (int j = 0; j < size; j++) b[j] = a[j];
            delete [] a;
            a = b;
            size = newSize;
        }
        a[i] = i * i;
    }
}

```

depends on size

$\Theta(n)$