1.) P(no student answers twice)

$$\frac{\frac{15!}{7!}}{15^8} = \frac{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8}{15^8}$$

$$= \frac{259459200}{2562890625} = 0.1012 = 10.12\%$$

2.) Digits (unique)

5	4	7	6	5
ODD	ODD	RAND	RAND	EVEN
1	3	2	4	2
3	5	4	6	4
5	7	6	8	6
7	9	8	5	8
9		5	7	0
		7	9	
		9		

$$\frac{\text{unique combos}}{\text{total combos}} = \frac{5 \cdot 4 \cdot 7 \cdot 6 \cdot 5}{10^5}$$

$$= \frac{4200}{100000} = 0.042$$

3.) A = at least 2 dice show ≥ 4
B = all 3 dice show same value

A_1 = only 2 dice show ≥ 4

A_2 = all 3 dice show ≥ 4

Binomial distribution : $P(K) = \binom{n}{k} p^k (1-p)^{n-k}$

$$P(A) = P(A_1) + P(A_2) = \binom{3}{2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^1 + \binom{3}{3}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^0$$

$$= 3\left(\frac{1}{8}\right) + \frac{1}{8} = \frac{4}{8} = \frac{1}{2}$$

$$P(B) = \frac{6}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$$

$$P(A \wedge B) = \frac{3}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{72}$$

$$P(A) \cdot P(B) = \frac{1}{2} \cdot \frac{1}{36} = \frac{1}{72}$$

$P(A) \cdot P(B) = P(A \wedge B)$ independent

1.) Geometric distribution

$$P(\text{flush}) = \underset{\text{suit}}{\binom{4}{1}} \cdot \underset{\text{card}}{\binom{13}{5}} = \frac{4!}{3!} \cdot \frac{13!}{5!\,8!}$$

$$= 4 \cdot 1287 = 5148 \text{ hands}$$

$$P(\text{hands}) = \binom{52}{5} = \frac{52!}{5!\,47!} = 2598960 \text{ hands}$$

$$P = \frac{5148}{2598960} = 0.00198$$

$$E[x] = \frac{1}{P} = \frac{1}{0.00198} = 505 \text{ hands}$$

2.) W = team wins S = star played S' = didn't play

$P(S) = 0.75$ $P(S') = 0.25$

$$P(S|W) = \frac{P(W|S) \cdot P(S)}{P(W|S) \cdot P(S) + P(W|S') \cdot P(S')}$$

Binomial distribution : $\binom{n}{k} p^k (1-p)^{n-k}$

$$P(W|S) = \binom{5}{4}(0.70)^4(0.30)^1 = 0.36015$$

$$P(W|S') = \binom{5}{4}(0.5)^4(0.5)^1 = 0.15625$$

$$P(S|W) = \frac{0.36015 \times 0.75}{0.36015 \times 0.75 + 0.25 \times 0.15625}$$

$$= \frac{0.2701125}{0.309175} = 0.87$$

87% chance superstar played