Study 1: Observe trajectory vs step time

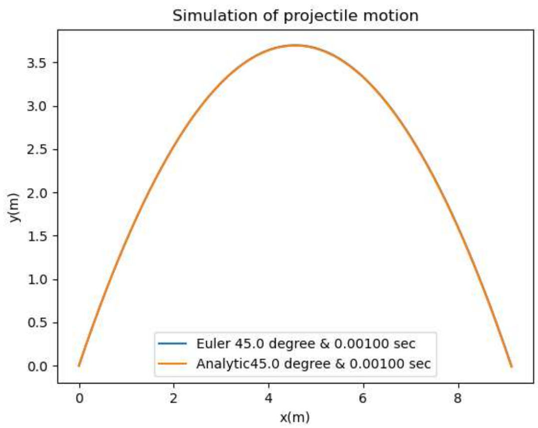

Figure 1: dt = 0.001 sec

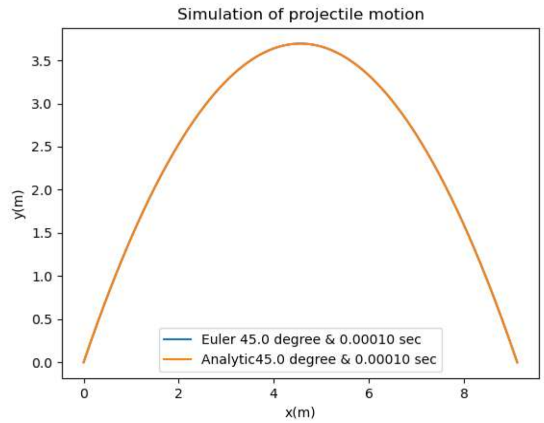

Figure2: dt=0.0001 sec

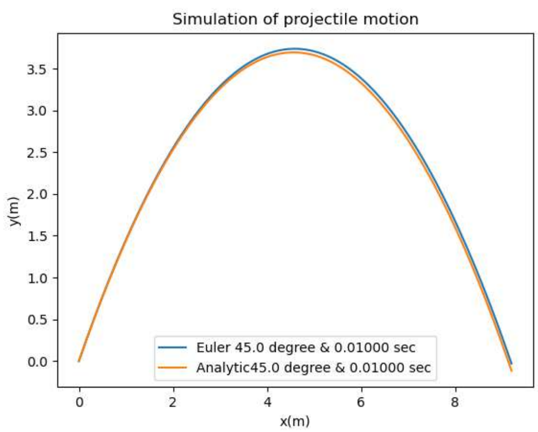

Figure 3 : default variables dt=0.01 sec

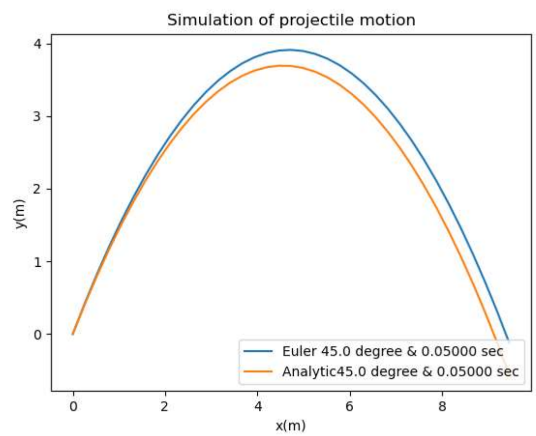

Figure 4 : dt=0.05 sec

From figures above, I will start with the figure 4 which uses the most rough time step, 0.05 sec. This figure shows the biggest deviation from the analytical path. Afterward, if I decrease the time step results the smaller deviation with the analytical trajectory.

I can conclude that the step of time has an effect on the accuracy of a numerical Eueler's method solving ODEs. If the dt is very small the error of each point compared to the true trajectory will be decreased.

For how I decide small or big? I have to compare with the initial value, namely initial velocity(in the 2nd study).

Study 2: Observe trajectory vs initial velocity

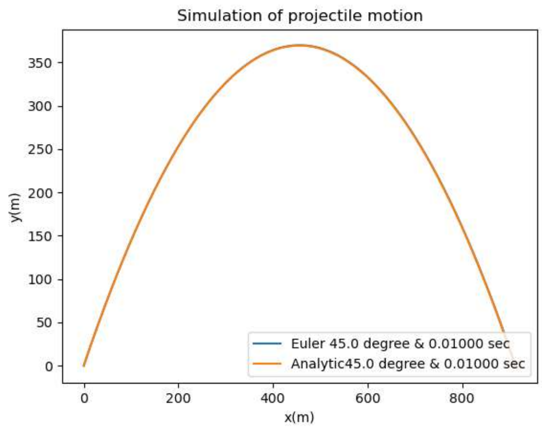

Figure 1: vinit=100 m/s

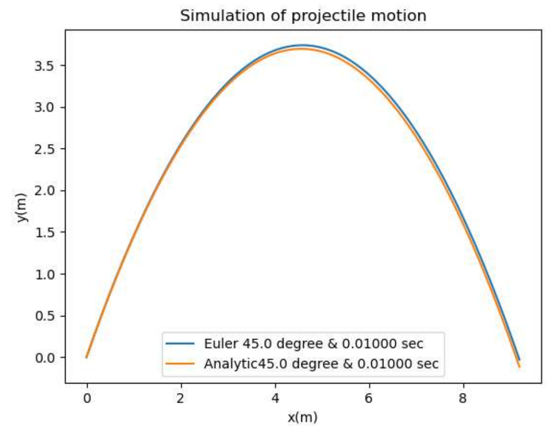

Figure2: default variables vinit=10 m/s

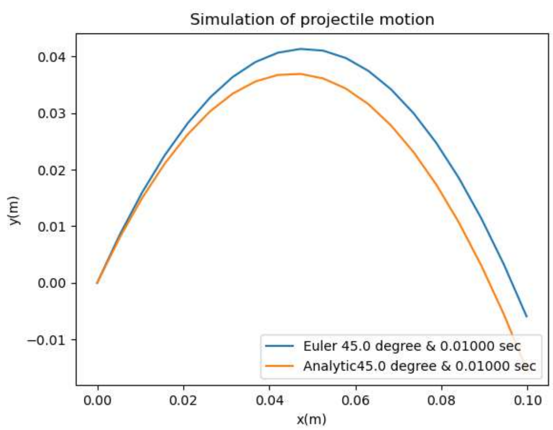

Figure 3 : vinit=1 m/s

As I mentioned about the initial velocity of a particle in the previous part, these plots show the difference between numerical trajectory and analytical trajectory. I use the same time step as 0.01 sec.

Let figure the Eueler's method

y=y0+f'dt

I can conclude that the error of the numerical method depends on the time step and the first order derivative which is used in the code.
Thus, f'/dt must be very big to get more accurate results.

Study 3: Observe Energy vs time step

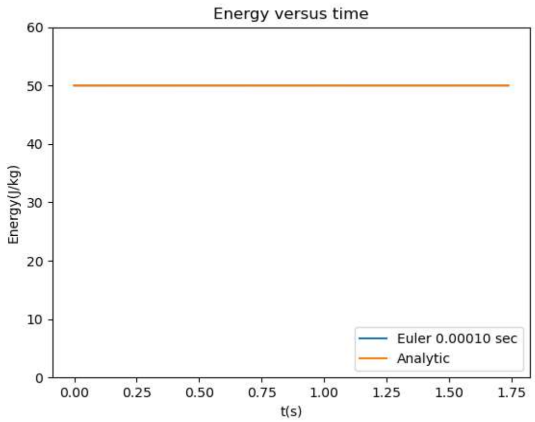

Figure 1: dt = 0.0001 sec

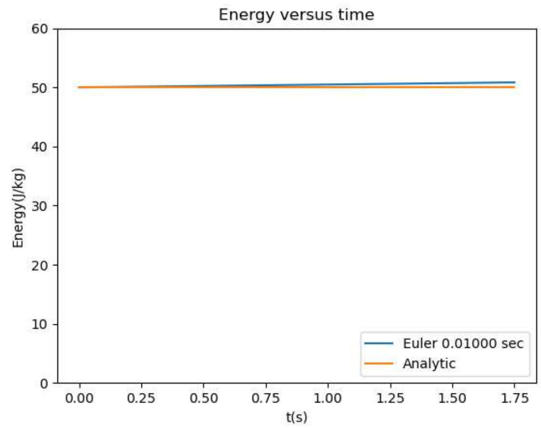

Figure2: dt=0.01 sec

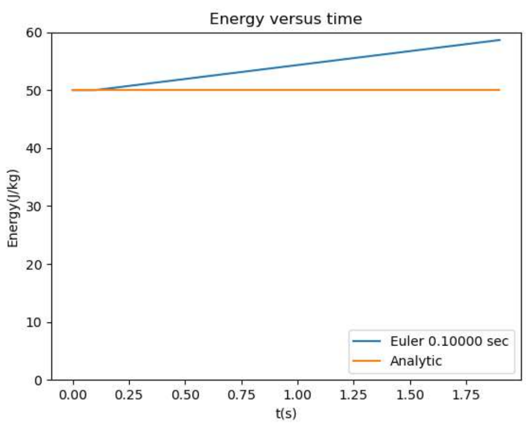

Figure3: dt=0.1 sec

For this section, these plots show the difference between numerical energy and analytical trajectory. I use the same initial velocity as 10 m/s.

Let figure the Eueler's method again with velocity(1st derivative) as well,

$$y=y0+f'dt$$

And $$f'=f'0+f''dt$$

On the 2nd line, the numerical velocity is approximated by previous velocity and step time. I can observe from very rough dt (Figure3). The Euler energy and analytic energy are diverged. Consequently, decrease of a step time result the convergence of numerical energy and analytical energy.

Same as before, if I use a very small step of time, the result would be nearly analytical energy as we know that it is conserved by a conservative of energy.