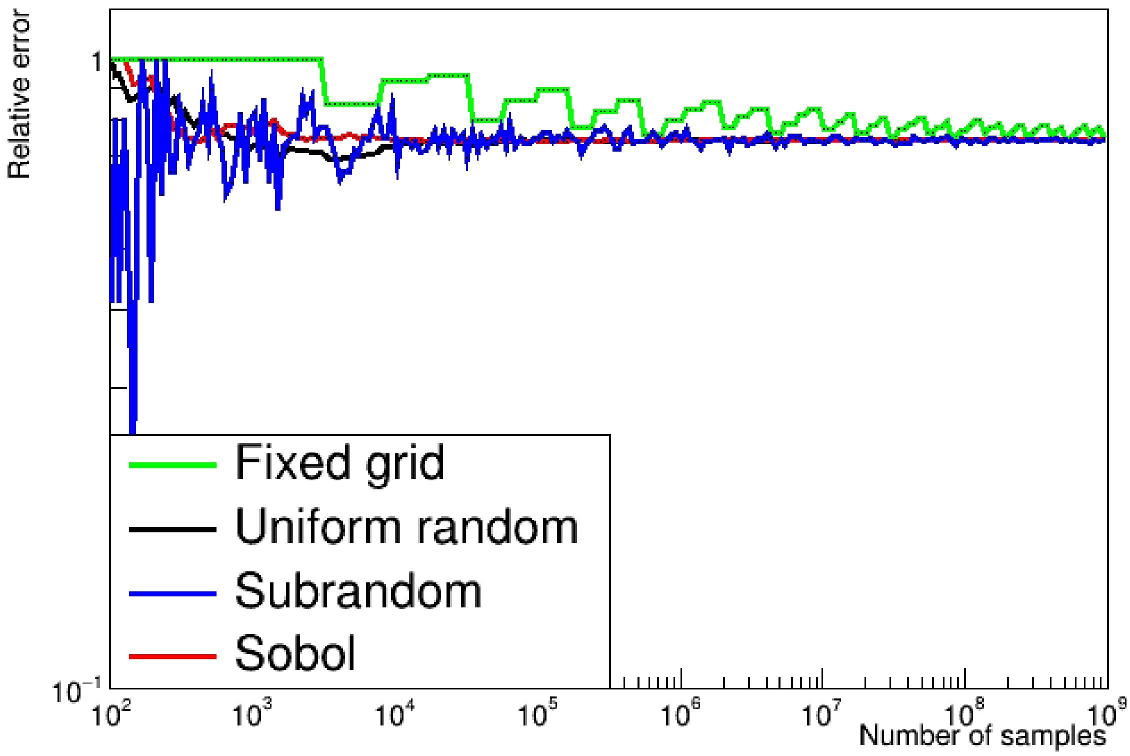
MC Integration – Sorawich Maichum – sm9cq



This is my plot shows the relative error of each method of integration.

This shows that the integration using Monte Carlo method is better than simply integrate by summing (Fixed grid).

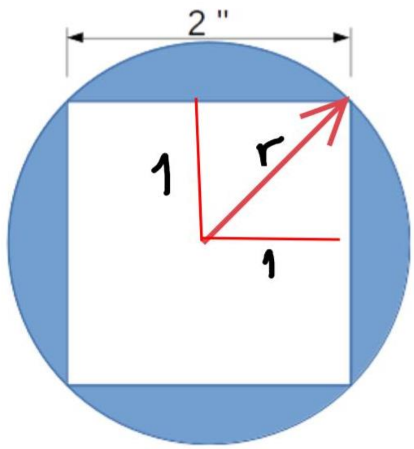
The relative error might be not good. They still not lower than 0.1.

In my opinion, it is not about the method of integration, but it is about using a wrong analytic solution for this problem.

Because the graphs show that they are converging to one value(which might be the good value for the analytic solution).

I have an idea and I will discuss next page.

Start with the picture for the problem.



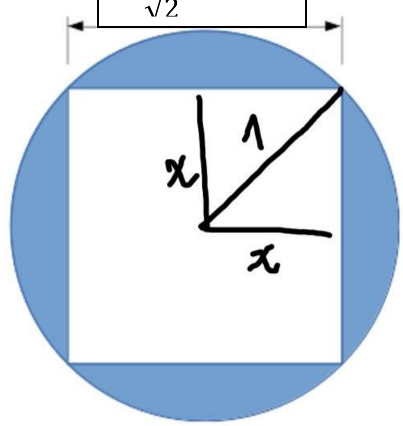
$$r^2 = 1^2 + 1^2$$

$$r = \sqrt{2}$$

I can find that the radius corresponding to this picture is r =√2.

Afterward, transform to unit circle which is easier to find its volume/area.

$$\boxed{\frac{2}{\sqrt{2}} = \sqrt{2}}$$



$$1^2 = 2x^2$$

$$x = \frac{1}{\sqrt{2}}$$

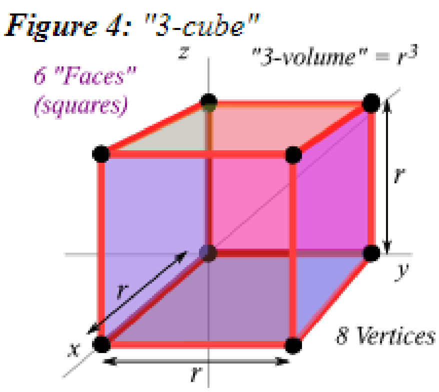
Then, we get the side of square is also x=√2.

But what is the problem?

The problem happens at 5D.

For this problem the volume of 5D-cube is $(2x)^5$ ref: http://www.physicsinsights.org/hypercubes_1.html

Or



$$
\begin{aligned}
\textit{n-volume}(\text{n-cube}) &= r^n \\
\textit{vertices}(\text{n-cube}) &= 2^n \\
\textit{edges}(\text{n-cube}) &= (n-1)\cdot 2^{n-1} \\
\textit{hyperfaces}(\text{n-cube}) &= 2\cdot n
\end{aligned}
$$

And for the n-ball, especially 5d-ball. The volume of a unit 5d-ball is $\frac{8\pi^2}{15} \approx 5.26$ ref: https://en.wikipedia.org/wiki/Volume_of_an_n-ball

Dimension	Volume of a ball of radius R	Radius of a ball of volume V
0	1	(all 0-balls have volume 1)
1	$2R$	$\frac{V}{2} = 0.5 \times V$
2	$\pi R^2 \approx 3.142 \times R^2$	$\frac{V^{1/2}}{\sqrt{\pi}} \approx 0.564 \times V^{\frac{1}{2}}$
3	$\frac{4\pi}{3} R^3 \approx 4.189 \times R^3$	$\left(\frac{3V}{4\pi}\right)^{1/3} \approx 0.620 \times V^{1/3}$
4	$\frac{\pi^2}{2} R^4 \approx 4.935 \times R^4$	$\frac{(2V)^{1/4}}{\sqrt{\pi}} \approx 0.671 \times V^{1/4}$
5	$\frac{8\pi^2}{15} R^5 \approx 5.264 \times R^5$	$\left(\frac{15V}{8\pi^2}\right)^{1/5} \approx 0.717 \times V^{1/5}$
6	$\frac{\pi^3}{6} R^6 \approx 5.168 \times R^6$	$\frac{(6V)^{1/6}}{\sqrt{\pi}} \approx 0.761 \times V^{1/6}$
7	$\frac{16\pi^3}{105} R^7 \approx 4.725 \times R^7$	$\left(\frac{105V}{16\pi^3}\right)^{1/7} \approx 0.801 \times V^{1/7}$

Then, the shaded volume is $V_{5d-ball} - V_{5-cube} = \frac{8\pi^2}{15} - \sqrt{2}^5 = 5.26 - 16\sqrt{2}$

$= -17.37$

Which is negative.

So I think. We cannot implement the square inside circle from 2D to 5D in this problem.

And it is also hard to get the analytic solution for this problem also(This is only my discussion).