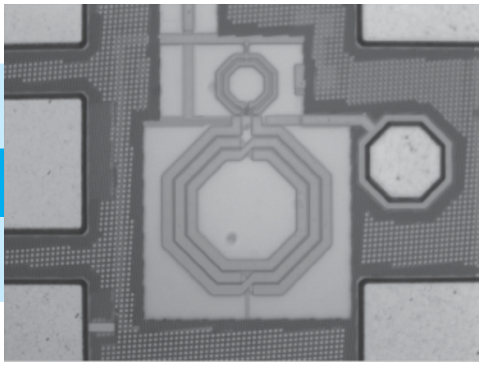


4

Physics of Bipolar Transistors

Qué onda con los Bell Laboratories?

The bipolar transistor was invented in 1945 by Shockley, Brattain, and Bardeen at Bell Laboratories, subsequently replacing vacuum tubes in electronic systems and paving the way for integrated circuits.

In this chapter, we analyze the structure and operation of bipolar transistors, preparing ourselves for the study of circuits employing such devices. Following the same thought process as in Chapter 2 for *pn* junctions, we aim to understand the physics of the transistor, derive equations that represent its I/V characteristics, and develop an equivalent model that can be used in circuit analysis and design. The outline below illustrates the sequence of concepts introduced in this chapter.

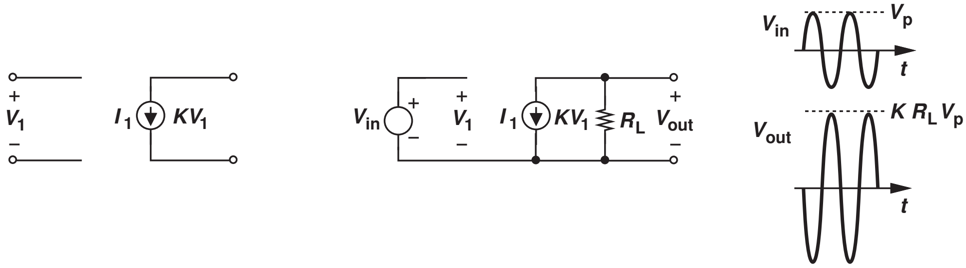
Voltage-Controlled Device as Amplifying Element → **Structure of Bipolar Transistor** → **Operation of Bipolar Transistor** → **Large-Signal Model** → **Small-Signal Model**

4.1 GENERAL CONSIDERATIONS

In its simplest form, the bipolar transistor can be viewed as a voltage-dependent current source. We first show how such a current source can form an amplifier and hence why bipolar devices are useful and interesting.

Consider the voltage-dependent current source depicted in Fig. 4.1(a), where I_1 is proportional to V_1: $I_1 = KV_1$. Note that K has a dimension of resistance^{-1}. For example, with $K = 0.001\ \Omega^{-1}$, an input voltage of 1 V yields an output current of 1 mA. Let us now construct the circuit shown in Fig. 4.1(b), where a voltage source V_{in} controls I_1 and the output current flows through a load resistor R_L, producing V_{out}. Our objective is to demonstrate that this circuit can operate as an amplifier, i.e., V_{out} is an amplified replica of V_{in}. Since $V_1 = V_{in}$ and $V_{out} = -R_L I_1$, we have

$$V_{out} = -KR_L V_{in}. \tag{4.1}$$



(a) (b)

Figure 4.1 (a) Voltage-dependent current source, (b) simple amplifier.

Interestingly, if $KR_L > 1$, then the circuit amplifies the input. The negative sign indicates that the output is an "inverted" replica of the input circuit [Fig. 4.1(b)]. The amplification factor or "voltage gain" of the circuit, A_V, is defined as

$$A_V = \frac{V_{out}}{V_{in}} \tag{4.2}$$

$$= -KR_L, \tag{4.3}$$

and depends on both the characteristics of the controlled current source and the load resistor. Note that K signifies how strongly V_1 controls I_1, thus directly affecting the gain.

Example 4.1 Consider the circuit shown in Fig. 4.2, where the voltage-controlled current source exhibits an "internal" resistance of r_{in}. Determine the voltage gain of the circuit.

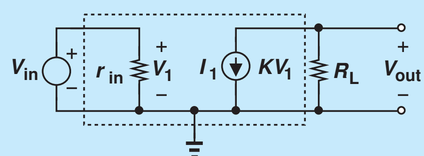


Figure 4.2 Voltage-dependent current source with an internal resistance r_{in}.

Solution Since V_1 is equal to V_{in} regardless of the value of r_{in}, the voltage gain remains unchanged. This point proves useful in our analyses later.

Exercise Repeat the above example if $r_{in} = \infty$.

Fuente de corriente controlada = Amplificador

The foregoing study reveals that a voltage-controlled current source can indeed provide signal amplification. Bipolar transistors are an example of such current sources and can ideally be modeled as shown in Fig. 4.3.Note that the device contains three terminals and its output current is an exponential function of V_1. We will see in Section 4.4.4 that under certain conditions, this model can be approximated by that in Fig. 4.1(a).

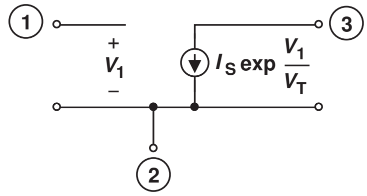


Figure 4.3 Exponential voltage-dependent current source.

As three-terminal devices, bipolar transistors make the analysis of circuits more difficult. Having dealt with two-terminal components such as resistors, capacitors, inductors, and diodes in elementary circuit analysis and the previous chapters of this book, we are accustomed to a one-to-one correspondence between the current through and the voltage across each device. With three-terminal elements, on the other hand, one may consider the current and voltage between every two terminals, arriving at a complex set of equations. Fortunately, as we develop our understanding of the transistor's operation, we discard some of these current and voltage combinations as irrelevant, thus obtaining a relatively simple model.

4.2 STRUCTURE OF BIPOLAR TRANSISTOR → Lo salteo

The bipolar transistor consists of three doped regions forming a sandwich. Shown in Fig. 4.4(a) is an example comprising of a *p* layer sandwiched between two *n* regions and called an "*npn*" transistor. The three terminals are called the "base," the "emitter," and the "collector." As explained later, the emitter "emits" charge carriers and the collector "collects" them while the base controls the number of carriers that make this journey. The circuit symbol for the *npn* transistor is shown in Fig. 4.4(b). We denote the terminal voltages by V_E, V_B, and V_C, and the voltage differences by V_{BE}, V_{CB}, and V_{CE}. The transistor is labeled Q_1 here.

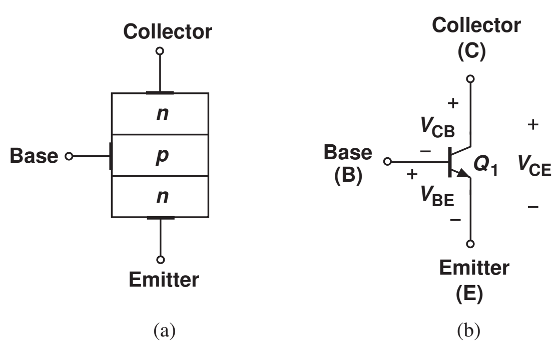


Figure 4.4 (a) Structure and (b) circuit symbol of bipolar transistor.

We readily note from Fig. 4.4(a) that the device contains two *pn* junction diodes: one between the base and the emitter and another between the base and the collector. For example, if the base is more positive than the emitter, $V_{BE} > 0$, then this junction is forward-biased. While this simple diagram may suggest that the device is symmetric with respect to the emitter and the collector, in reality, the dimensions and doping levels of these two regions are quite different. In other words, E and C cannot be interchanged. We will also see that proper operation requires a thin base region, e.g., about 100 Å in modern integrated bipolar transistors.

As mentioned in the previous section, the possible combinations of voltages and currents for a three-terminal device can prove overwhelming. For the device in Fig. 4.4(a), V_{BE}, V_{BC}, and V_{CE} can assume positive or negative values, leading to 2^3 possibilities for the terminal voltages of the transistor. Fortunately, only *one* of these eight combinations finds practical value and comes into our focus here.

Before continuing with the bipolar transistor, it is instructive to study an interesting effect in *pn* junctions. Consider the reverse-biased junction depicted in Fig. 4.5(a) and recall from Chapter 2 that the depletion region sustains a strong electric field. Now suppose an electron is somehow "injected" from outside into the right side of the depletion region. What happens to this electron? Serving as a minority carrier on the p side, the electron experiences the electric field and is rapidly swept away into the n side. The ability of a reverse-biased *pn* junction to efficiently "collect" externally-injected electrons proves essential to the operation of the bipolar transistor.

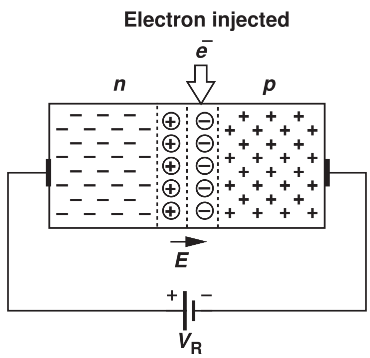


Figure 4.5 Injection of electrons into depletion region.

4.3 OPERATION OF BIPOLAR TRANSISTOR IN ACTIVE MODE

In this section, we analyze the operation of the transistor, aiming to prove that, under certain conditions, it indeed acts as a voltage-controlled current source. More specifically, we intend to show that (a) the current flow from the emitter to the collector can be viewed as a current source tied between these two terminals, and (b) this current is controlled by the voltage difference between the base and the emitter, V_{BE}.

We begin our study with the assumption that the base-emitter junction is forward-biased ($V_{BE} > 0$) and the base-collector junction is reverse-biased ($V_{BC} < 0$). Under these conditions, we say the device is biased in the "forward active region" or simply in the "active mode." For example, with the emitter connected to ground, the base voltage is set to about 0.8 V and the collector voltage to a *higher* value, e.g., 1 V [Fig. 4.6(a)]. The base-collector junction therefore experiences a reverse bias of 0.2 V.

Let us now consider the operation of the transistor in the active mode. We may be tempted to simplify the example of Fig. 4.6(a) to the equivalent circuit shown in Fig. 4.6(b). After all, it appears that the bipolar transistor simply consists of two diodes sharing their

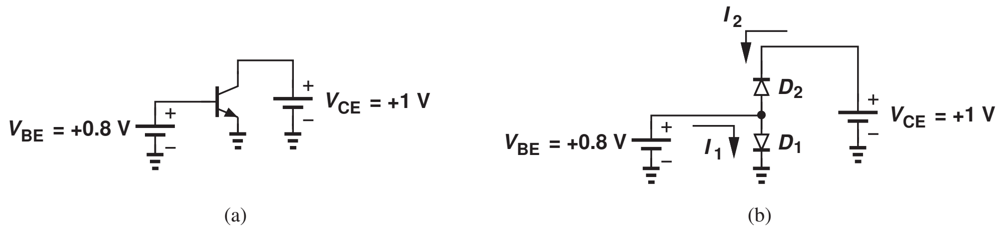


Figure 4.6 (a) Bipolar transistor with base and collector bias voltages, (b) simplistic view of bipolar transistor.

anodes at the base terminal. This view implies that D_1 carries a current and D_2 does not; i.e., we should anticipate current flow from the base to the emitter but no current through the collector terminal. Were this true, the transistor would not operate as a voltage-controlled current source and would prove of little value.

To understand why the transistor cannot be modeled as merely two back-to-back diodes, we must examine the flow of charge inside the device, bearing in mind that the base region is very thin. Since the base-emitter junction is forward-biased, electrons flow from the emitter to the base and holes from the base to the emitter. For proper transistor operation, the former current component must be much greater than the latter, requiring that the emitter doping level be much greater than that of the base (Chapter 2). Thus, we denote the emitter region with n^+, where the superscript emphasizes the high doping level. Figure 4.7(a) summarizes our observations thus far, indicating that the emitter

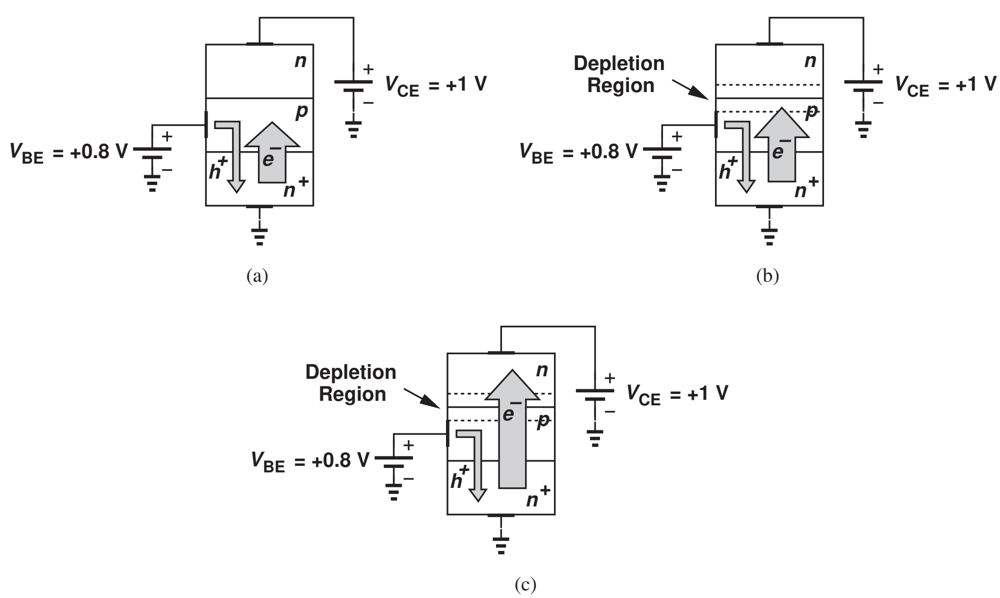


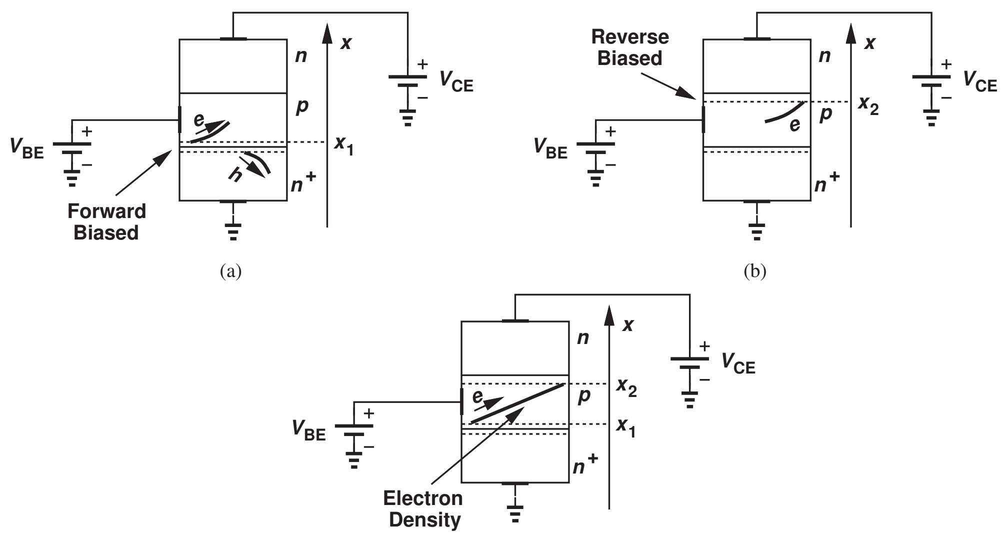
Figure 4.7 (a) Flow of electrons and holes through base-emitter junction, (b) electrons approaching collector junction, (c) electrons passing through collector junction.

injects a large number of electrons into the base while receiving a small number of holes from it.

What happens to electrons as they enter the base? Since the base region is thin, most of the electrons reach the edge of the collector-base depletion region, beginning to experience the built-in electric field. Consequently, as illustrated in Fig. 4.5, the electrons are swept into the collector region (as in Fig. 4.5) and absorbed by the positive battery terminal. Figures 4.7(b) and (c) illustrate this effect in "slow motion." We therefore observe that the reverse-biased collector-base junction carries a current because minority carriers are "injected" into its depletion region.

Let us summarize our thoughts. In the active mode, an *npn* bipolar transistor carries a large number of electrons from the emitter, through the base, to the collector while drawing a small current of holes through the base terminal. We must now answer several questions. First, how do electrons travel through the base: by drift or diffusion? Second, how does the resulting current depend on the terminal voltages? Third, how large is the base current?

Operating as a moderate conductor, the base region sustains but a small electric field, i.e., it allows most of the field to drop across the base-emitter depletion layer. Thus, as explained for *pn* junctions in Chapter 2, the drift current in the base is negligible,[1] leaving diffusion as the principal mechanism for the flow of electrons injected by the emitter. In fact, two observations directly lead to the necessity of diffusion: (1) redrawing the diagram of Fig. 2.29 for the emitter-base junction [Fig. 4.8(a)], we recognize that the density of electrons at $x = x_1$ is very high; (2) since any electron arriving at $x = x_2$ in Fig. 4.8(b) is swept away, the density of electrons falls to zero at this point. As a result,



(c)

Figure 4.8 (a) Hole and electron profiles at base-emitter junction, (b) zero electron density near collector, (c) electron profile in base.

[1]This assumption simplifies the analysis here but may not hold in the general case.

the electron density in the base assumes the profile depicted in Fig. 4.8(c), providing a gradient for the diffusion of electrons.

4.3.1 Collector Current

We now address the second question raised previously and compute the current flowing from the collector to the emitter.[2] As a forward-biased diode, the base-emitter junction exhibits a high concentration of electrons at $x = x_1$ in Fig. 4.8(c) given by Eq. (2.96):

$$\Delta n(x_1) = \frac{N_E}{\exp\dfrac{V_0}{V_T}}\left(\exp\frac{V_{BE}}{V_T} - 1\right) \tag{4.4}$$

$$= \frac{N_B}{n_i^2}\left(\exp\frac{V_{BE}}{V_T} - 1\right). \tag{4.5}$$

Here, N_E and N_B denote the doping levels in the emitter and the base, respectively, and we have utilized the relationship $\exp(V_0/V_T) = N_E N_B/n_i^2$. In this chapter, we assume $V_T = 26$ mV. Applying the law of diffusion [Eq. (2.42)], we determine the flow of electrons into the collector as

$$J_n = qD_n\frac{dn}{dx} \tag{4.6}$$

$$= qD_n \cdot \frac{0 - \Delta n(x_1)}{W_B}, \tag{4.7}$$

where W_B is the width of the base region. Multipling this quantity by the emitter cross section area, A_E, substituting for $\Delta n(x_1)$ from Eq. (4.5), and changing the sign to obtain the conventional current, we obtain

$$I_C = \frac{A_E q D_n n_i^2}{N_B W_B}\left(\exp\frac{V_{BE}}{V_T} - 1\right). \tag{4.8}$$

In analogy with the diode current equation and assuming $\exp(V_{BE}/V_T) \gg 1$, we write

$$I_C = I_S \exp\frac{V_{BE}}{V_T}, \tag{4.9}$$

where

Bernardo, pero si lo leo no voy a entender una gama

$$I_S = \frac{A_E q D_n n_i^2}{N_B W_B}. \tag{4.10}$$

Equation (4.9) implies that the bipolar transistor indeed operates as a voltage-controlled current source, proving a good candidate for amplification. We may alternatively say the transistor performs "voltage-to-current conversion."

[2]In an *npn* transistor, electrons go from the emitter to the collector. Thus, the conventional direction of the current is from the collector to the emitter.

Example 4.2

Determine the current I_X in Fig. 4.9(a) if Q_1 and Q_2 are identical and operate in the active mode and $V_1 = V_2$.

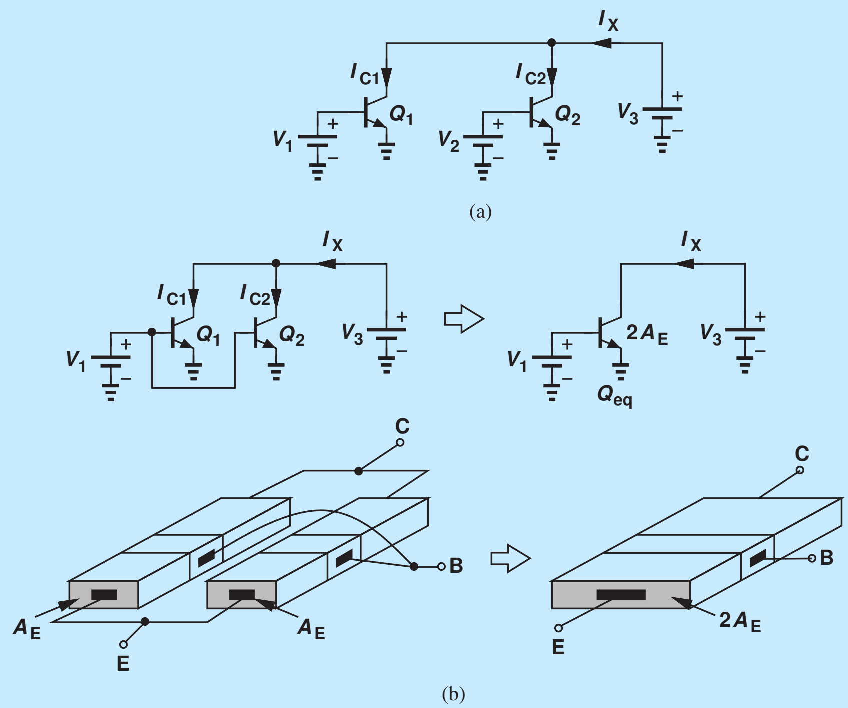


Figure 4.9 (a) Two identical transistors drawing current from V_C, (b) equivalence to a single transistor having twice the area.

Solution Since $I_X = I_{C1} + I_{C2}$, we have

$$I_X \approx 2\frac{A_E q D_n n_i^2}{N_B W_B} \exp\frac{V_1}{V_T}. \tag{4.11}$$

This result can also be viewed as the collector current of a *single* transistor having an emitter area of $2A_E$. In fact, redrawing the circuit as shown in Fig. 4.9(b) and noting that Q_1 and Q_2 experience identical voltages at their respective terminals, we say the two transistors are "in parallel," operating as a single transistor with twice the emitter area of each.

Exercise Repeat the above example if Q_1 has an emitter area of A_E and Q_2 an emitter area of $8A_E$.

Example 4.3 In the circuit of Fig. 4.9 (a), Q_1 and Q_2 are identical and operate in the active mode. Determine $V_1 - V_2$ such that $I_{C1} = 10I_{C2}$.

Solution From Eq. (4.9), we have

$$\frac{I_{C1}}{I_{C2}} = \frac{I_S \exp\frac{V_1}{V_T}}{I_S \exp\frac{V_2}{V_T}}, \tag{4.12}$$

and hence

$$\exp\frac{V_1 - V_2}{V_T} = 10. \tag{4.13}$$

That is,

$$V_1 - V_2 = V_T \ln 10 \tag{4.14}$$

$$\approx 60\,\text{mV at } T = 300\text{ K}. \tag{4.15}$$

Identical to Eq. (2.109), this result is, of course, expected because the exponential dependence of I_C upon V_{BE} indicates a behavior similar to that of diodes. We therefore consider the base-emitter voltage of the transistor relatively constant and approximately equal to 0.8 V for typical collector current levels.

Exercise Repeat the above example if Q_1 and Q_2 have different emitter areas, i.e., $A_{E1} = nA_{E2}$.

Example 4.4 Typical discrete bipolar transistors have a large area, e.g., 500 μm × 500 μm, whereas modern integrated devices may have an area as small as 0.5 μm × 0.2 μm. Assuming other device parameters are identical, determine the difference between the base-emitter voltage of two such transistors for equal collector currents.

Solution From Eq. (4.9), we have $V_{BE} = V_T \ln(I_C/I_S)$ and hence

$$V_{BEint} - V_{BEdis} = V_T \ln\frac{I_{S1}}{I_{S2}}, \tag{4.16}$$

where $V_{BEint} = V_T \ln(I_{C2}/I_{S2})$ and $V_{BEdis} = V_T \ln(I_{C1}/I_{S1})$ denote the base-emitter voltages of the integrated and discrete devices, respectively. Since $I_S \propto A_E$,

$$V_{BEint} - V_{BEdis} = V_T \ln\frac{A_{E2}}{A_{E1}}. \tag{4.17}$$

For this example, $A_{E2}/A_{E1} = 2.5 \times 10^6$, yielding

$$V_{BEint} - V_{BEdis} = 383\,\text{mV}. \tag{4.18}$$

In practice, however, $V_{BEint} - V_{BEdis}$ falls in the range of 100 to 150 mV because of differences in the base width and other parameters. The key point here is that $V_{BE} = 800$ mV is a reasonable approximation for integrated transistors and should be lowered to about 700 mV for discrete devices.

Exercise Repeat the above comparison for a very small integrated device with an emitter area of 0.15 μm × 0.15 μ.

Since many applications deal with *voltage* quantities, the collector current generated by a bipolar transistor typically flows through a resistor to produce a voltage.

Example 4.5 Determine the output voltage in Fig. 4.10 if $I_S = 5 \times 10^{-16}$ A.

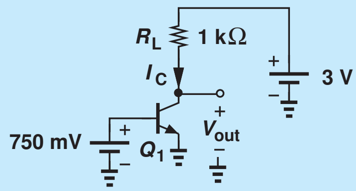


Figure 4.10 Simple stage with biasing.

Solution Using Eq. (4.9), we write $I_C = 1.69$ mA. This current flows through R_L, generating a voltage drop of $1\ \mathrm{k\Omega} \times 1.69\ \mathrm{mA} = 1.69$ V. Since $V_{CE} = 3\ \mathrm{V} - I_C R_L$, we obtain

$$V_{out} = 1.31\ \mathrm{V}. \tag{4.19}$$

Exercise What happens if the load resistor is halved?

Equation (4.9) reveals an interesting property of the bipolar transistor: the collector current does not depend on the collector voltage (so long as the device remains in the active mode). Thus, for a fixed base-emitter voltage, the device draws a constant current, acting as a current source [Fig. 4.11(a)]. Plotted in Fig. 4.11(b) is the current as a function of the collector-emitter voltage, exhibiting a constant value for $V_{CE} > V_1$.[3] Constant current sources find application in many electronic circuits and we will see numerous examples of their usage in this book. In Section 4.5, we study the behavior of the transistor for $V_{CE} < V_{BE}$.

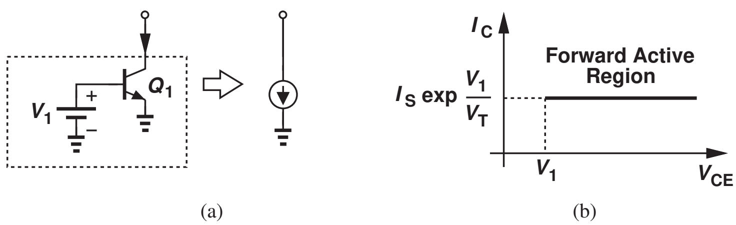


Figure 4.11 (a) Bipolar transistor as a current source, (b) I/V characteristic.

4.3.2 Base and Emitter Currents

Having determined the collector current, we now turn our attention to the base and emitter currents and their dependence on the terminal voltages. Since the bipolar transistor must satisfy Kirchoff's current law, calculation of the base current readily yields the emitter current as well.

[3]Recall that $V_{CE} > V_1$ is necessary to ensure the collector-base junction remains reverse biased.

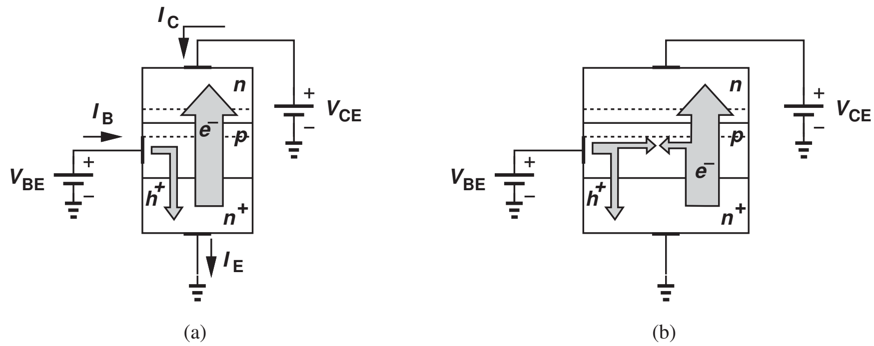


Figure 4.12 Base current resulting from holes (a) crossing to emitter and (b) recombining with electrons.

In the *npn* transistor of Fig. 4.12 (a), the base current, I_B, results from the flow of holes. Recall from Eq. (2.99) that the hole and electron currents in a forward-biased *pn* junction bear a *constant* ratio given by the doping levels and other parameters. Thus, the number of holes entering from the base to the emitter is a constant fraction of the number of electrons traveling from the emitter to the base. As an example, for every 200 electrons injected by the emitter, one hole must be supplied by the base.

In practice, the base current contains an additional component of holes. As the electrons injected by the emitter travel through the base, some may "recombine" with the holes [Fig. 4.12 (b)]; inessence, some electrons and holes are "wasted" as a result of recombination. For example, on the average, out of every 200 electrons injected by the emitter, one recombines with a hole.

In summary, the base current must supply holes for both reverse injection into the emitter and recombination with the electrons traveling toward the collector. We can therefore view I_B as a constant fraction of I_E or a constant fraction of I_C. It is common to write

$$I_C = \beta I_B, \tag{4.20}$$

where β is called the "current gain" of the transistor because it shows how much the base current is "amplified." Depending on the device structure, the β of *npn* transistors typically ranges from 50 to 200.

In order to determine the emitter current, we apply the KCL to the transistor with the current directions depicted in Fig. 4.12 (a):

$$I_E = I_C + I_B \tag{4.21}$$

$$= I_C\left(1 + \frac{1}{\beta}\right). \tag{4.22}$$

We can summarize our findings as follows:

$$I_C = I_S \exp\frac{V_{BE}}{V_T} \tag{4.23}$$

$$I_B = \frac{1}{\beta} I_S \exp\frac{V_{BE}}{V_T} \tag{4.24}$$

$$I_E = \frac{\beta+1}{\beta} I_S \exp\frac{V_{BE}}{V_T}. \tag{4.25}$$

It is sometimes useful to write $I_C = [\beta/(\beta+1)]I_E$ and denote $\beta/(\beta+1)$ by α. For $\beta = 100$, $\alpha = 0.99$, suggesting that $\alpha \approx 1$ and $I_C \approx I_E$ are reasonable approximations. In this book, we assume that the collector and emitter currents are approximately equal.

Example 4.6 A bipolar transistor having $I_S = 5 \times 10^{-16}$ A is biased in the forward active region with $V_{BE} = 750$ mV. If the current gain varies from 50 to 200 due to manufacturing variations, calculate the minimum and maximum terminal currents of the device.

Solution For a given V_{BE}, the collector current remains independent of β:

$$I_C = I_S \exp\frac{V_{BE}}{V_T} \tag{4.26}$$

$$= 1.685\ \text{mA}. \tag{4.27}$$

The base current varies from $I_C/200$ to $I_C/50$:

$$8.43\ \mu\text{A} < I_B < 33.7\ \mu\text{A}. \tag{4.28}$$

On the other hand, the emitter current experiences only a small variation because $(\beta + 1)/\beta$ is near unity for large β:

$$1.005 I_C < I_E < 1.02 I_C \tag{4.29}$$

$$1.693\ \text{mA} < I_E < 1.719\ \text{mA}. \tag{4.30}$$

Exercise Repeat the above example if the area of the transistor is doubled.

4.4 BIPOLAR TRANSISTOR MODELS AND CHARACTERISTICS

ARRANCO ACÁ

4.4.1 Large-Signal Model

With our understanding of the transistor operation in the forward active region and the derivation of Eqs. (4.23)–(4.25), we can now construct a model that proves useful in the analysis and design of circuits—in a manner similar to the developments in Chapter 2 for the *pn* junction.

Since the base-emitter junction is forward-biased in the active mode, we can place a diode between the base and emitter terminals. Moreover, since the current drawn from the collector and flowing into the emitter depends on only the base-emitter voltage, we add a voltage-controlled current source between the collector and the emitter, arriving at the model shown in Fig. 4.13. As illustrated in Fig. 4.11, this current remains independent of the collector-emitter voltage.

Did you know?

The first bipolar transistor introduced by Bell Labs in 1948 was implemented in germanium rather than silicon, had a base thickness of about 30 μm, and provided a maximum operation frequency (called the "transit frequency") of 10 MHz. By contrast, bipolar transistors realized in today's silicon integrated circuits have a base thickness less than 0.01 μm and a transit frequency of several hundred gigahertz.

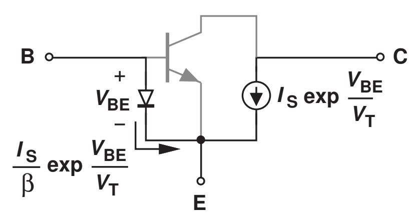


Figure 4.13 Large-signal model of bipolar transistor in active region.

But how do we ensure that the current flowing through the diode is equal to $1/\beta$ times the collector current? Equation (4.24) suggests that the base current is equal to that of a diode having a reverse saturation current of I_S/β. Thus, the base-emitter junction is modeled by a diode whose cross section area is $1/\beta$ times that of the actual emitter area.

¿El huevo o la gallina? ==> La tensión base-emisor

With the interdependencies of currents and voltages in a bipolar transistor, the reader may wonder about the cause and effect relationships. We view the chain of dependencies as $V_{BE} \to I_C \to I_B \to I_E$; i.e., the base-emitter voltage generates a collector current, which requires a proportional base current, and the sum of the two flows through the emitter.

Example 4.7

Consider the circuit shown in Fig. 4.14 (a), where $I_{S,Q1} = 5 \times 10^{-17}$ A and $V_{BE} = 800$ mV. Assume $\beta = 100$. (a) Determine the transistor terminal currents and voltages and verify that the device indeed operates in the active mode. (b) Determine the maximum value of R_C that permits operation in the active mode.

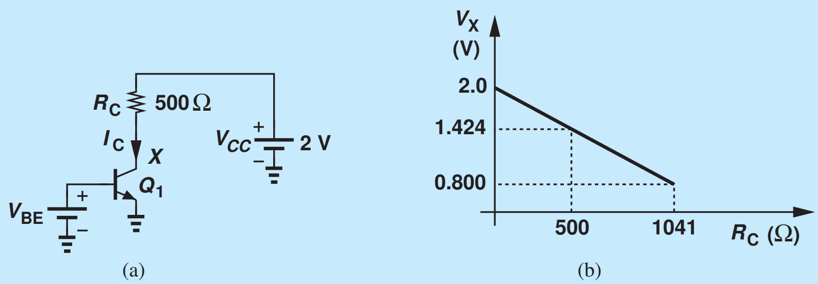


Figure 4.14 (a) Simple stage with biasing, (b) variation of collector voltage as a function of collector resistance.

Solution (a) Using Eq. (4.23)–(4.25), we have

$$I_C = 1.153 \text{ mA} \tag{4.31}$$

$$I_B = 11.53\ \mu\text{A} \tag{4.32}$$

$$I_E = 1.165 \text{ mA}. \tag{4.33}$$

The base and emitter voltages are equal to +800 mV and zero, respectively. We must now calculate the collector voltage, V_X. Writing a KVL from the 2-V power supply and across R_C and Q_1, we obtain

$$V_{CC} = R_C I_C + V_X. \tag{4.34}$$

That is,

$$V_X = 1.424 \text{ V}. \tag{4.35}$$

Since the collector voltage is more positive than the base voltage, this junction is reverse-biased and the transistor operates in the active mode.

(b) What happens to the circuit as R_C increases? Since the voltage drop across the resistor, $R_C I_C$, increases while V_{CC} is constant, the voltage at node X *drops*.

The device approaches the "edge" of the forward active region if the base-collector voltage falls to zero, i.e., as $V_X \to +800\,\text{mV}$. Rewriting Eq. (4.33) yields:

$$R_C = \frac{V_{CC} - V_X}{I_C}, \tag{4.36}$$

which, for $V_X = +800\,\text{mV}$, reduces to

$$R_C = 1041\,\Omega. \tag{4.37}$$

Figure 4.14(b) plots V_X as a function of R_C.

This example implies that there exists a maximum allowable value of the collector resistance, R_C, in the circuit of Fig. 4.14(a). As we will see in Chapter 5, this limits the voltage gain that the circuit can provide.

Exercise In the above example, what is the minimum allowable value of V_{CC} for transistor operation in the active mode? Assume $R_C = 500\,\Omega$.

The reader may wonder why the equivalent circuit of Fig. 4.13 is called the "large-signal model." After all, the above example apparently contains *no* signals! This terminology emphasizes that the model can be used for *arbitrarily* large voltage and current changes in the transistor (so long as the device operates in the active mode). For example, if the base-emitter voltage varies from 800 mV to 300 mV, and hence the collector current by many *orders of magnitude*,[4] the model still applies. This is in contrast to the small-signal model, studied in Section 4.4.4.

4.4.2 I/V Characteristics

The large-signal model naturally leads to the I/V characteristics of the transistor. With three terminal currents and voltages, we may envision plotting different currents as a function of the potential difference between every two terminals—an elaborate task. However, as explained below, only a few of such characteristics prove useful.

The first characteristic to study is, of course, the exponential relationship inherent in the device. Figure 4.15(a) plots I_C versus V_{BE} with the assumption that the collector voltage is constant and no lower than the base voltage. As shown in Fig. 4.11, I_C is independent of V_{CE}; thus, different values of V_{CE} do not alter the characteristic.

Next, we examine I_C for a given V_{BE} but with V_{CE} varying. Illustrated in Fig. 4.15(b), the characteristic is a horizontal line because I_C is constant if the device remains in the active mode ($V_{CE} > V_{BE}$). On the other hand, if different values are chosen for V_{BE}, the characteristic moves up or down.

The two plots of Fig. 4.15 constitute the principal characteristics of interest in most analysis and design tasks. Equations (4.24) and (4.25) suggest that the base and emitter currents follow the same behavior.

[4] A 500-mV change in V_{BE} leads to $500\,\text{mV}/60\,\text{mV} = 8.3$ decades of change in I_C.

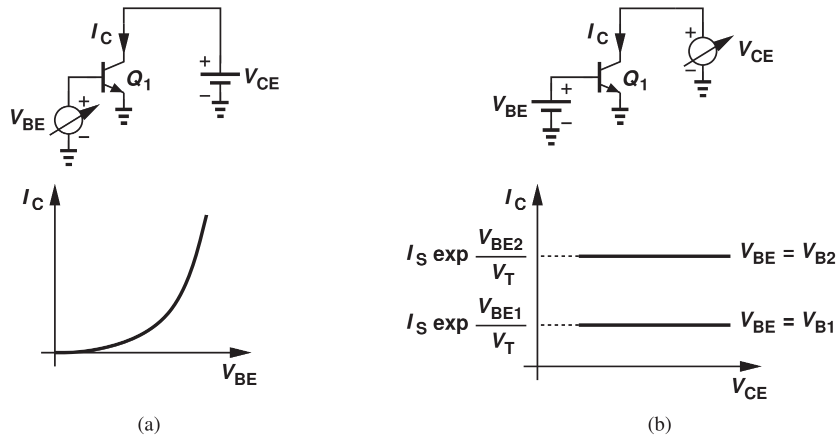


Figure 4.15 Collector current as a function of (a) base-emitter voltage and (b) collector-emitter voltage.

Example 4.8 For a bipolar transistor, $I_S = 5 \times 10^{-17}$ A and $\beta = 100$. Construct the I_C-V_{BE}, I_C-V_{CE}, I_B-V_{BE}, and I_B-V_{CE} characteristics.

Solution We determine a few points along the I_C-V_{BE} characteristics, e.g.,

$$V_{BE1} = 700\,\text{mV} \Rightarrow I_{C1} = 24.6\,\mu\text{A} \tag{4.38}$$

$$V_{BE2} = 750\,\text{mV} \Rightarrow I_{C2} = 169\,\mu\text{A} \tag{4.39}$$

$$V_{BE3} = 800\,\text{mV} \Rightarrow I_{C3} = 1.153\,\text{mA}. \tag{4.40}$$

The characteristic is depicted in Fig. 4.16 (a).

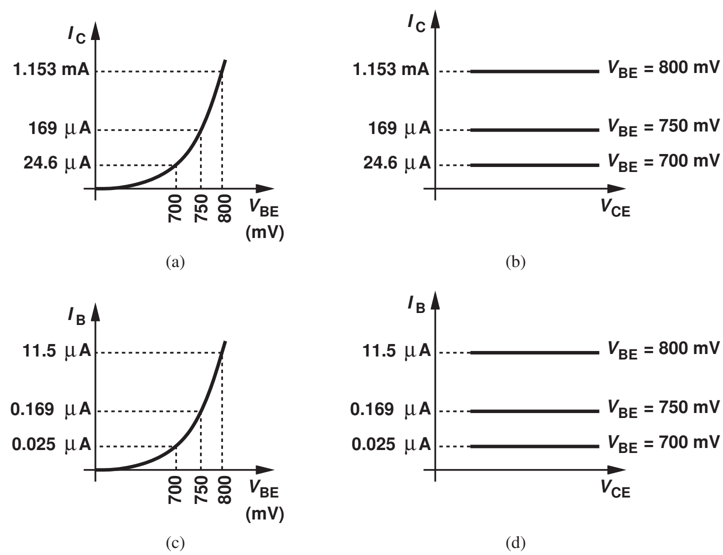


Figure 4.16 (a) Collector current as a function of V_{BE}, (b) collector current as a function of V_{CE}, (c) base current as a function of V_{BE}, (d) base current as a function of V_{CE}.

Using the values obtained above, we can also plot the I_C-V_{CE} characteristic as shown in Fig. 4.16(b), concluding that the transistor operates as a constant current source of, e.g., 169 μA if its base-emitter voltage is held at 750 mV. We also remark that, for equal increments in V_{BE}, I_C jumps by increasingly greater steps: 24.6 μA to 169 μA to 1.153 mA. We return to this property in Section 4.4.3.

For I_B characteristics, we simply divide the I_C values by 100 [Figs. 4.16(c) and (d)].

Exercise What change in V_{BE} doubles the base current?

The reader may wonder what exactly we learn from the I/V characteristics. After all, compared to Eqs. (4.23)–(4.25), the plots impart no additional information. However, as we will see throughout this book, the visualization of equations by means of such plots greatly enhances our understanding of the devices and the circuits employing them.

4.4.3 Concept of Transconductance

Our study thus far shows that the bipolar transistor acts as a voltage-dependent current source (when operating in the forward active region). An important question that arises here is, how is the *performance* of such a device quantified? In other words, what is the measure of the "goodness" of a voltage-dependent current source?

The example depicted in Fig. 4.1 suggests that the device becomes "stronger" as K increases because a given input voltage yields a larger output current. We must therefore concentrate on the voltage-to-current conversion property of the transistor, particularly as it relates to amplification of signals. More specifically, we ask, if a signal changes the base-emitter voltage of a transistor by a small amount (Fig. 4.17), how much *change* is produced in the collector current? Denoting the change in I_C by ΔI_C, we recognize that the "strength" of the device can be represented by $\Delta I_C/\Delta V_{BE}$. For example, if a base-emitter voltage change of 1 mV results in a ΔI_C of 0.1 mA in one transistor and 0.5 mA in another, we can view the latter as a better voltage-dependent current source or "voltage-to-current converter."

The ratio $\Delta I_C/\Delta V_{BE}$ approaches dI_C/dV_{BE} for very small changes and, in the limit, is called the "transconductance," g_m:*

$$g_m = \frac{dI_C}{dV_{BE}}. \tag{4.41}$$

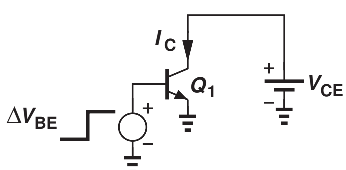


Figure 4.17 Test circuit for measurement of g_m.

*Note that V_{CE} is constant here.

Note that this definition applies to any device that approximates a voltage-dependent current source (e.g., another type of transistor described in Chapter 6). For a bipolar transistor, Eq. (4.9) gives

$$g_m = \frac{d}{dV_{BE}}\left(I_S \exp\frac{V_{BE}}{V_T}\right) \tag{4.42}$$

$$= \frac{1}{V_T} I_S \exp\frac{V_{BE}}{V_T} \tag{4.43}$$

$$= \frac{I_C}{V_T}. \tag{4.44}$$

The close resemblance between this result and the small-signal resistance of diodes [Eq. (3.58)] is no coincidence and will become clearer in the next chapter.

Equation (4.44) reveals that, as I_C increases, the transistor becomes a better amplifying device by producing larger collector current excursions in response to a given signal level applied between the base and the emitter. The transconductance may be expressed in Ω^{-1} or "siemens," S. For example, if $I_C = 1$ mA, then with $V_T = 26$ mV, we have

$$g_m = 0.0385\ \Omega^{-1} \tag{4.45}$$

$$= 0.0385\ \mathrm{S} \tag{4.46}$$

$$= 38.5\ \mathrm{mS}. \tag{4.47}$$

However, as we will see throughout this book, it is often helpful to view g_m as the inverse of a resistance; e.g., for $I_C = 1$ mA, we may write

$$g_m = \frac{1}{26\ \Omega}. \tag{4.48}$$

The concept of transconductance can be visualized with the aid of the transistor I/V characteristics. As shown in Fig. 4.18, $g_m = dI_C/dV_{BE}$ simply represents the slope of I_C-V_{BE} characteristic at a given collector current, I_{C0}, and the corresponding base-emitter voltage, V_{BE0}. In other words, if V_{BE} experiences a small perturbation $\pm\Delta V$ around V_{BE0}, then the collector current displays a change of $\pm g_m \Delta V$ around I_{C0}, where $g_m = I_{C0}/V_T$. Thus, the

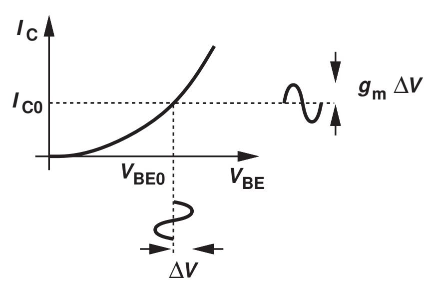


Figure 4.18 Illustration of transconductance.

value of I_{C0} must be chosen according to the required g_m and, ultimately, the required gain. We say the transistor is "biased" at a collector current of I_{C0}, meaning the device carries a bias (or "quiescent") current of I_{C0} in the absence of signals.[5]

Example 4.9

Consider the circuit shown in Fig. 4.19(a). What happens to the transconductance of Q_1 if the area of the device is increased by a factor of n?

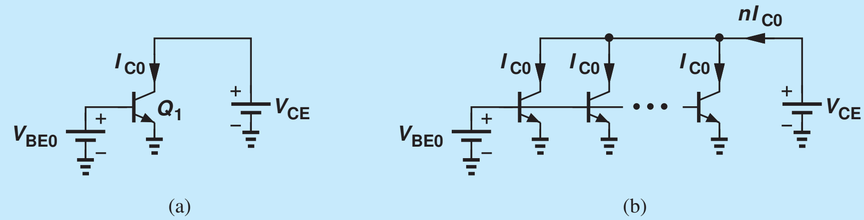


Figure 4.19 (a) One transistor and (b) n transistors providing transconductance.

Solution Since $I_S \propto A_E$, I_S is multiplied by the same factor. Thus, $I_C = I_S \exp(V_{BE}/V_T)$ also rises by a factor of n because V_{BE} is constant. As a result, the transconductance increases by a factor of n. From another perspective, if n identical transistors, each carrying a collector current of I_{C0}, are placed in parallel, then the composite device exhibits a transconductance equal to n times that of each [Fig. 4.19(b)]. On the other hand, if the total collector current remains unchanged, then so does the transconductance.

Exercise Repeat the above example if V_{BE0} is reduced by $V_T \ln n$.

It is also possible to study the transconductance in the context of the I_C-V_{CE} characteristics of the transistor with V_{BE} as a parameter. Illustrated in Fig. 4.20 for two different bias currents I_{C1} and I_{C2}, the plots reveal that a change of ΔV in V_{BE} results in a greater change in I_C for operation around I_{C2} than around I_{C1} because $g_{m2} > g_{m1}$.

The derivation of g_m in Eqs. (4.42)–(4.44) suggests that the transconductance is fundamentally a function of the collector current rather than the base current. For example, if I_C remains constant but β varies, then g_m does not change but I_B does. For this reason, the collector bias current plays a central role in the analysis and design, with the base current viewed as secondary, often undesirable effect.

As shown in Fig. 4.10,the current produced by the transistor may flow through a resistor to generate a proportional voltage. We exploit this concept in Chapter 5 to design amplifiers.

4.4.4 Small-Signal Model

Electronic circuits, e.g., amplifiers, may incorporate a large number of transistors, thus posing great difficulties in the analysis and design. Recall from Chapter 3 that diodes can

[5]Unless otherwise stated, we use the term "bias current" to refer to the collector bias current.

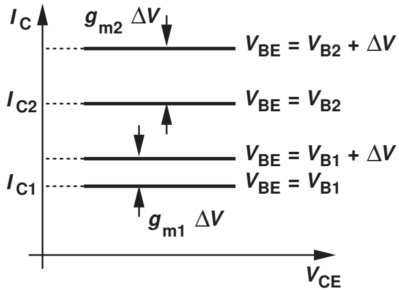


Figure 4.20 Transconductance for different collector bias currents.

be reduced to linear devices through the use of the small-signal model. A similar benefit accrues if a small-signal model can be developed for transistors.

The derivation of the small-signal model from the large-signal counterpart is relatively straightforward. We perturb the voltage difference between every two terminals (while the third terminal remains at a constant potential), determine the changes in the currents flowing through *all* terminals, and represent the results by proper circuit elements such as controlled current sources or resistors. Figure 4.21 depicts two conceptual examples where V_{BE} or V_{CE} is changed by ΔV and the changes in I_C, I_B, and I_E are examined.

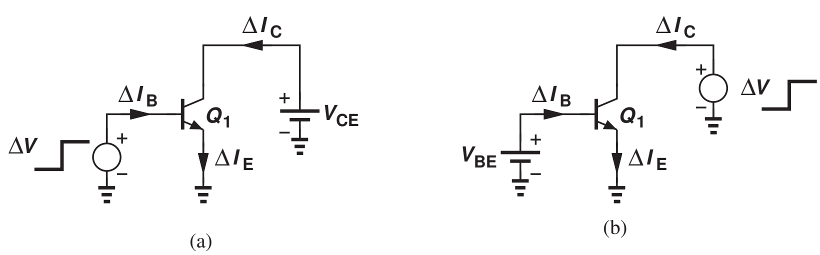


Figure 4.21 Excitation of bipolar transistor with small changes in (a) base-emitter and (b) collector-emitter voltage.

Let us begin with a change in V_{BE} while the collector voltage is constant (Fig. 4.22). We know from the definition of transconductance that

$$\Delta I_C = g_m\,\Delta V_{BE}, \tag{4.49}$$

concluding that a voltage-controlled current source must be connected between the collector and the emitter with a value equal to $g_m\,\Delta V$. For simplicity, we denote ΔV_{BE} by v_π and the change in the collector current by $g_m v_\pi$.

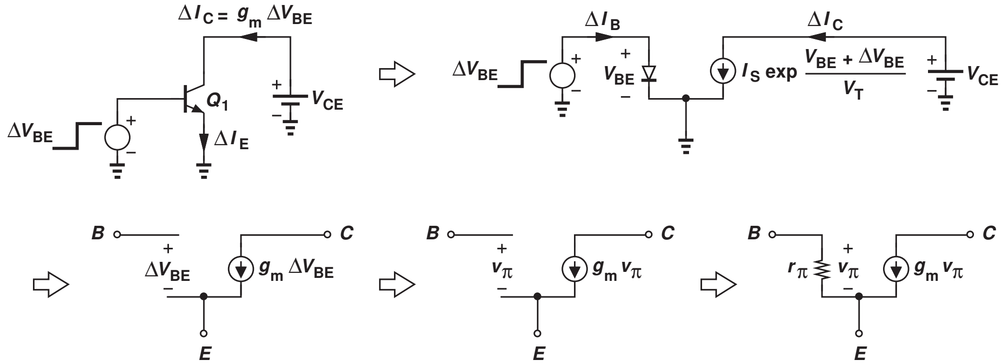


Figure 4.22 Development of small-signal model.

The change in V_{BE} creates another change as well:

$$\Delta I_B = \frac{\Delta I_C}{\beta} \tag{4.50}$$

$$= \frac{g_m}{\beta}\Delta V_{BE}. \tag{4.51}$$

That is, if the base-emitter voltage changes by ΔV_{BE}, the current flowing between these two terminals changes by $(g_m/\beta)\Delta V_{BE}$. Since the voltage and current correspond to the same two terminals, they can be related by Ohm's Law, i.e., by a resistor placed between the base and emitter having a value:

$$r_\pi = \frac{\Delta V_{BE}}{\Delta I_B} \tag{4.52}$$

$$= \frac{\beta}{g_m}. \tag{4.53}$$

Thus, the forward-biased diode between the base and the emitter is modeled by a small-signal resistance equal to β/g_m. This result is expected because the diode carries a bias current equal to I_C/β and, from Eq. (3.58), exhibits a small-signal resistance of $V_T/(I_C/\beta) = \beta(V_T/I_C) = \beta/g_m$.

We now turn our attention to the collector and apply a voltage change with respect to the emitter (Fig. 4.23). As illustrated in Fig. 4.11, for a constant V_{BE}, the collector voltage has no effect on I_C or I_B because $I_C = I_S \exp(V_{BE}/V_T)$ and $I_B = I_C/\beta$. Since ΔV_{CE} leads to no change in any of the terminal currents, the model developed in Fig. 4.22 need not be altered.

How about a change in the collector-base voltage? As studied in Problem 4.18, such a change also results in a zero change in the terminal currents.

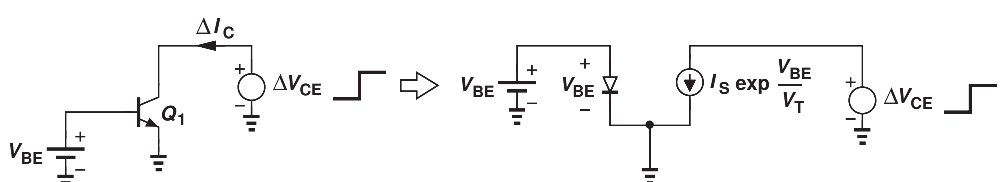


Figure 4.23 Response of bipolar transistor to small change in V_{CE}.

The simple small-signal model developed in Fig. 4.22 serves as a powerful, versatile tool in the analysis and design of bipolar circuits. We should remark that both parameters of the model, g_m and r_π, depend on the bias current of the device. With a high collector bias current, a greater g_m is obtained, but the impedance between the base and emitter falls to lower values. Studied in Chapter 5, this trade-off proves undesirable in some cases.

Example 4.10

Consider the circuit shown in Fig. 4.24(a), where v_1 represents the signal generated by a microphone, $I_S = 3 \times 10^{-16}$ A, $\beta = 100$, and Q_1 operates in the active mode. (a) If $v_1 = 0$, determine the small-signal parameters of Q_1. (b) If the microphone generates a 1-mV signal, how much change is observed in the collector and base currents?

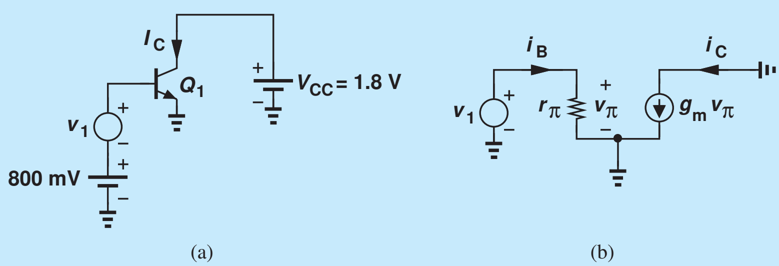


Figure 4.24 (a) Transistor with bias and small-signal excitation, (b) small-signal equivalent circuit.

Solution (a) Writing $I_C = I_S \exp(V_{BE}/V_T)$, we obtain a collector bias current of 6.92 mA for $V_{BE} = 800$ mV. Thus,

$$g_m = \frac{I_C}{V_T} \tag{4.54}$$

$$= \frac{1}{3.75\ \Omega}, \tag{4.55}$$

and

$$r_\pi = \frac{\beta}{g_m} \tag{4.56}$$

$$= 375\ \Omega. \tag{4.57}$$

(b) Drawing the small-signal equivalent of the circuit as shown in Fig. 4.24(b) and recognizing that $v_\pi = v_1$, we obtain the change in the collector current as:

$$\Delta I_C = g_m v_1 \tag{4.58}$$

$$= \frac{1\ \text{mV}}{3.75\ \Omega} \tag{4.59}$$

$$= 0.267\ \text{mA}. \tag{4.60}$$

The equivalent circuit also predicts the change in the base current as

$$\Delta I_B = \frac{v_1}{r_\pi} \tag{4.61}$$

$$= \frac{1\ \text{mV}}{375\ \Omega} \tag{4.62}$$

$$= 2.67\ \mu\text{A}, \tag{4.63}$$

which is, of course, equal to $\Delta I_C/\beta$.

Exercise Repeat the above example if I_S is halved.

The above example is not a useful circuit. The microphone signal produces a change in I_C, but the result flows through the 1.8-V battery. In other words, the circuit generates no output. On the other hand, if the collector current flows through a resistor, a useful output is provided.

Example 4.11 The circuit of Fig. 4.24 (a) is modified as shown in Fig. 4.25, where resistor R_C converts the collector current to a voltage. (a) Verify that the transistor operates in the active mode. (b) Determine the output signal level if the microphone produces a 1-mV signal.

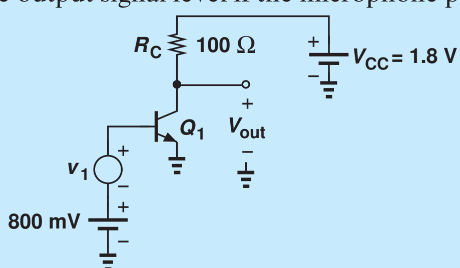


Figure 4.25 Simple stage with bias and small-signal excitation.

Solution (a) The collector bias current of 6.92 mA flows through R_C, leading to a potential drop of $I_C R_C = 692$ mV. The collector voltage, which is equal to V_{out}, is thus given by:

$$V_{out} = V_{CC} - R_C I_C \tag{4.64}$$

$$= 1.108\ \text{V}. \tag{4.65}$$

Since the collector voltage (with respect to ground) is more positive than the base voltage, the device operates in the active mode.

(b) As seen in the previous example, a 1-mV microphone signal leads to a 0.267-mA change in I_C. Upon flowing through R_C, this change yields a change of 0.267 mA × 100 Ω = 26.7 mV in V_{out}. The circuit therefore *amplifies* the input by a factor of 26.7.

Exercise What value of R_C results in a zero collector-base voltage?

Did you know?

The first revolution afforded by the transistor was the concept of *portable* radios. Up to the 1940s, radios incorporated vacuum tubes, which required very high supply voltages (e.g., 60 V) and hence a bulky and heavy radio design. The transistor, on the other hand, could operate with a few batteries. The portable "transistor radio" was thus introduced by Regency and Texas Instruments in 1954. Interestingly, a Japanese company called Tsushin Kogyo had also been working on a transistor radio around that time and was eager to enter the American market. Since the company's name was difficult to pronounce for westerners, they picked the Latin word "sonus" for sound and called themselves Sony.

The foregoing example demonstrates the amplification capability of the transistor. We will study and quantify the behavior of this and other amplifier topologies in the next chapter.

Small-Signal Model of Supply Voltage We have seen that the use of the small-signal model of diodes and transistors can simplify the analysis considerably. In such an analysis, other components in the circuit must also be represented by a small-signal model. In particular, we must determine how the supply voltage, V_{CC}, behaves with respect to small changes in the currents and voltages of the circuit.

The key principle here is that the supply voltage (ideally) remains *constant* even though various voltages and currents within the circuit may change with time. Since the supply does not change and since the small-signal model of the circuit entails only changes in the quantities, we observe that V_{CC} must be replaced with a *zero* voltage to signify the zero change. Thus, we simply "ground" the supply voltage in small-signal analysis. Similarly, any other constant voltage in the circuit is replaced with a ground connection. To emphasize that such grounding holds for only signals, we sometimes say a node is an "ac ground."

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Y si tuviera una fuente de corriente, abro el circuito (no cortocircuito), porque el cambio en al corriente es nulo

4.4.5 Early Effect

Our treatment of the bipolar transistor has thus far concentrated on the fundamental principles, ignoring second-order effects in the device and their representation in the large-signal and small-signal models. However, some circuits require attention to such effects if meaningful results are to be obtained. The following example illustrates this point.

Example 4.12 Considering the circuit of Example 4.11, suppose we raise R_C to 200 Ω and V_{CC} to 3.6 V. Verify that the device operates in the active mode and compute the voltage gain.

Solution The voltage drop across R_C now increases to $6.92\text{ mA} \times 200\ \Omega = 1.384\text{ V}$, leading to a collector voltage of $3.6\text{ V} - 1.384\text{ V} = 2.216\text{ V}$ and guaranteeing operation in the active mode. Note that if V_{CC} is not doubled, then $V_{out} = 1.8\text{ V} - 1.384\text{ V} = 0.416\text{ V}$ and the transistor is not in the forward active region.

Recall from part (b) of the above example that the change in the output voltage is equal to the change in the collector current multiplied by R_C. Since R_C is doubled, the voltage gain must also double, reaching a value of 53.4. This result is also obtained with the aid of the small-signal model. Illustrated in Fig. 4.26, the

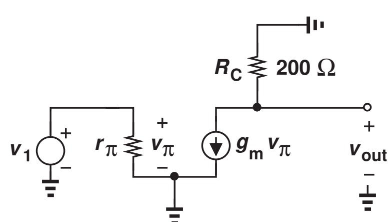

Figure 4.26 Small-signal equivalent circuit of the stage shown in Fig. 4.25.

equivalent circuit yields $v_{out} = -g_m v_\pi R_C = -g_m v_1 R_C$ and hence $v_{out}/v_1 = -g_m R_C$. With $g_m = (3.75\ \Omega)^{-1}$ and $R_C = 200\ \Omega$, we have $v_{out}/v_1 = -53.4$.

Exercise What happens if $R_C = 250\ \Omega$?

This example points to an important trend: if R_C increases, so does the voltage gain of the circuit. Does this mean that, if $R_C \to \infty$, then the gain also grows indefinitely? Does another mechanism in the circuit, perhaps in the transistor, limit the maximum gain that can be achieved? Indeed, the "Early effect" translates to a nonideality in the device that can limit the gain of amplifiers.

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To understand this effect, we return to the internal operation of the transistor and reexamine the claim shown in Fig. 4.11that "the collector current does not depend on the collector voltage." Consider the device shown in Fig. 4.27(a), where the collector voltage is somewhat higher than the base voltage and the reverse bias across the junction creates a certain depletion region width. Now suppose V_{CE} is raised to V_{CE2} [Fig. 4.27(b)], thus increasing the reverse bias and widening the depletion region in the collector and base areas. Since the base charge profile must still fall to zero at the edge of depletion region, x_2', the *slope* of the profile increases. Equivalently, the effective base width, W_B, in Eq. (4.8) decreases, thereby increasing the collector current. Discovered by Early, this phenomenon poses interesting problems in amplifier design (Chapter 5).

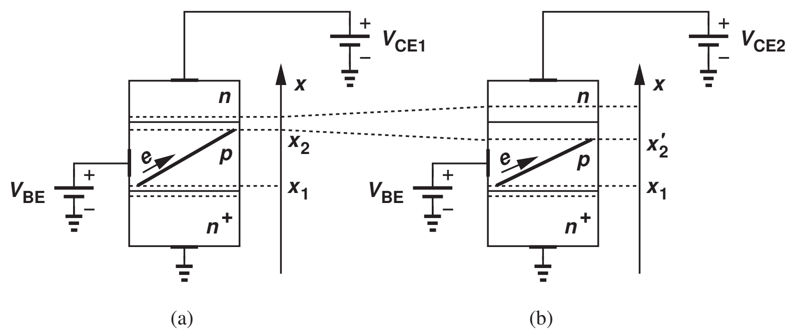

Figure 4.27 (a) Bipolar device with base and collector bias voltages, (b) effect of higher collector voltage.

How is the Early effect represented in the transistor model? We must first modify Eq. (4.9) to include this effect. It can be proved that the rise in the collector current with V_{CE} can be approximately expressed by a multiplicative factor:

$$I_C = \frac{A_E q D_n n_i^2}{N_E W_B}\left(\exp\frac{V_{BE}}{V_T} - 1\right)\left(1 + \frac{V_{CE}}{V_A}\right), \tag{4.66}$$

$$\approx \left(I_S \exp\frac{V_{BE}}{V_T}\right)\left(1 + \frac{V_{CE}}{V_A}\right). \tag{4.67}$$

where W_B is assumed constant and the second factor, $1 + V_{CE}/V_A$, models the Early effect. The quantity V_A is called the "Early voltage."

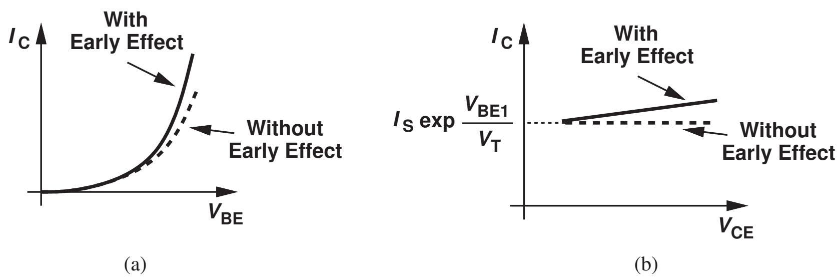

Figure 4.28 Collector current as a function of (a) V_{BE} and (b) V_{CE} with and without Early effect.

It is instructive to examine the I/V characteristics of Fig. 4.15 in the presence of the Early effect. For a constant V_{CE}, the dependence of I_C upon V_{BE} remains exponential but with a somewhat greater slope [Fig. 4.28(a)]. On the other hand, for a constant V_{BE}, the $I_C - V_{CE}$ characteristic displays a nonzero slope [Fig. 4.28(b)]. In fact, differentiation of Eq. (4.67) with respect to V_{CE} yields

$$\frac{\delta I_C}{\delta V_{CE}} = I_S\left(\exp\frac{V_{BE}}{V_T}\right)\left(\frac{1}{V_A}\right) \tag{4.68}$$

$$\approx \frac{I_C}{V_A}, \tag{4.69}$$

where it is assumed $V_{CE} \ll V_A$ and hence $I_C \approx I_S \exp(V_{BE}/V_T)$. This is a reasonable approximation in most cases.

The variation of I_C with V_{CE} in Fig. 4.28(b) reveals that the transistor in fact does not operate as an *ideal* current source, requiring modification of the perspective shown in Fig. 4.11(a). The transistor can still be viewed as a two-terminal device but with a current that varies to some extent with V_{CE} (Fig. 4.29).

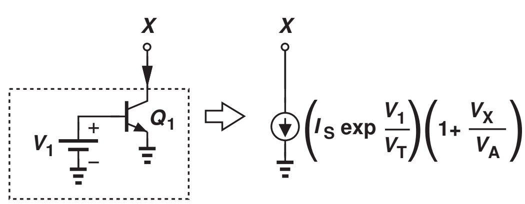

Figure 4.29 Realistic model of bipolar transistor as a current source.

Example 4.13 A bipolar transistor carries a collector current of 1 mA with $V_{CE} = 2$ V. Determine the required base-emitter voltage if $V_A = \infty$ or $V_A = 20$ V. Assume $I_S = 2 \times 10^{-16}$ A.

Solution With $V_A = \infty$, we have from Eq. (4.67)

$$V_{BE} = V_T \ln \frac{I_C}{I_S} \tag{4.70}$$

$$= 760.3\,\text{mV}. \tag{4.71}$$

If $V_A = 20$ V, we rewrite Eq. (4.67) as

$$V_{BE} = V_T \ln \left(\frac{I_C}{I_S} \frac{1}{1 + \dfrac{V_{CE}}{V_A}} \right) \tag{4.72}$$

$$= 757.8\,\text{mV}. \tag{4.73}$$

In fact, for $V_{CE} \ll V_A$, we have $(1 + V_{CE}/V_A)^{-1} \approx 1 - V_{CE}/V_A$

$$V_{BE} \approx V_T \ln \frac{I_C}{I_S} + V_T \ln \left(1 - \frac{V_{CE}}{V_A}\right) \tag{4.74}$$

$$\approx V_T \ln \frac{I_C}{I_S} - V_T \frac{V_{CE}}{V_A}, \tag{4.75}$$

where it is assumed $\ln(1 - \epsilon) \approx -\epsilon$ for $\epsilon \ll 1$.

Exercise Repeat the above example if two such transistors are placed in parallel.

Large-Signal and Small-Signal Models The presence of Early effect alters the transistor models developed in Sections 4.4.1 and 4.4.4. The large-signal model of Fig. 4.13 must now be modified to that in Fig. 4.30, where

$$I_C = \left(I_S \exp \frac{V_{BE}}{V_T}\right)\left(1 + \frac{V_{CE}}{V_A}\right) \tag{4.76}$$

$$I_B = \frac{1}{\beta}\left(I_S \exp \frac{V_{BE}}{V_T}\right) \tag{4.77}$$

$$I_E = I_C + I_B. \tag{4.78}$$

Note that I_B is independent of V_{CE} and still given by the base-emitter voltage.

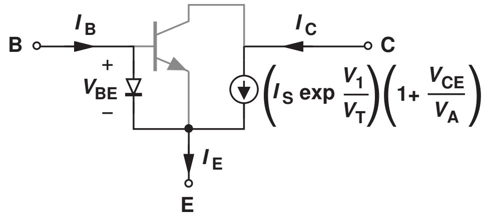

Figure 4.30 Large-signal model of bipolar transistor including Early effect.

For the small-signal model, we note that the controlled current source remains unchanged and g_m is expressed as

$$g_m = \frac{dI_C}{dV_{BE}} \tag{4.79}$$

$$= \frac{1}{V_T}\left(I_S \exp \frac{V_{BE}}{V_T}\right)\left(1 + \frac{V_{CE}}{V_A}\right) \tag{4.80}$$

$$= \frac{I_C}{V_T}. \tag{4.81}$$

Similarly,

$$r_\pi = \frac{\beta}{g_m} \tag{4.82}$$

$$= \beta \frac{V_T}{I_C}. \tag{4.83}$$

Considering that the collector current does vary with V_{CE}, let us now apply a voltage change at the collector and measure the resulting current change [Fig. 4.31(a)]:

$$I_C + \Delta I_C = \left(I_S \exp \frac{V_{BE}}{V_T}\right)\left(1 + \frac{V_{CE} + \Delta V_{CE}}{V_A}\right). \tag{4.84}$$

It follows that

$$\Delta I_C = \left(I_S \exp \frac{V_{BE}}{V_T}\right)\frac{\Delta V_{CE}}{V_A}, \tag{4.85}$$

which is consistent with Eq. (4.69). Since the voltage and current change correspond to the same two terminals, they satisfy Ohm's Law, yielding an equivalent resistor:

$$r_O = \frac{\Delta V_{CE}}{\Delta I_C} \tag{4.86}$$

$$= \frac{V_A}{I_S \exp \frac{V_{BE}}{V_T}} \tag{4.87}$$

$$\approx \frac{V_A}{I_C}. \tag{4.88}$$

Depicted in Fig. 4.31(b), the small-signal model contains only one extra element, r_O, to represent the Early effect. Called the "output resistance," r_O plays a critical role in high-gain amplifiers (Chapter 5). Note that both r_π and r_O are inversely proportionally to the bias current, I_C.

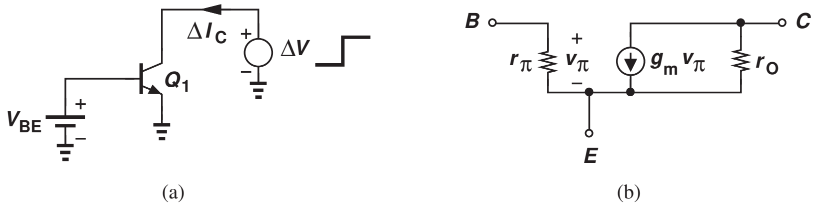

Figure 4.31 (a) Small change in V_{CE} and (b) small-signal model including Early effect.

Example 4.14 A transistor is biased at a collector current of 1 mA. Determine the small-signal model if $\beta = 100$ and $V_A = 15$ V.

Solution We have

$$g_m = \frac{I_C}{V_T} \tag{4.89}$$

$$= \frac{1}{26\ \Omega}, \tag{4.90}$$

and

$$r_\pi = \frac{\beta}{g_m} \tag{4.91}$$

$$= 2600\ \Omega. \tag{4.92}$$

Also,

$$r_O = \frac{V_A}{I_C} \tag{4.93}$$

$$= 15\ \text{k}\Omega. \tag{4.94}$$

Exercise What early voltage is required if the output resistance must reach 25 kΩ?

In the next chapter, we return to Example 4.12 and determine the gain of the amplifier in the presence of the Early effect. We will conclude that the gain is eventually limited by the transistor output resistance, r_O. Figure 4.32 summarizes the concepts studied in this section.

An important notion that has emerged from our study of the transistor is the concept of biasing. We must create proper dc voltages and currents at the device terminals to accomplish two goals: (1) guarantee operation in the active mode ($V_{BE} > 0$, $V_{CE} \geq 0$); e.g., the load resistance tied to the collector faces an upper limit for a given supply voltage (Example 4.7); (2) establish a collector current that yields the required values for the small-signal parameters g_m, r_O, and r_π. The analysis of amplifiers in the next chapter exercises these ideas extensively.

Finally, we should remark that the small-signal model of Fig. 4.31(b) does not reflect the high-frequency limitations of the transistor. For example, the base-emitter and base-collector junctions exhibit a depletion-region capacitance that impacts the speed. These properties are studied in Chapter 11.

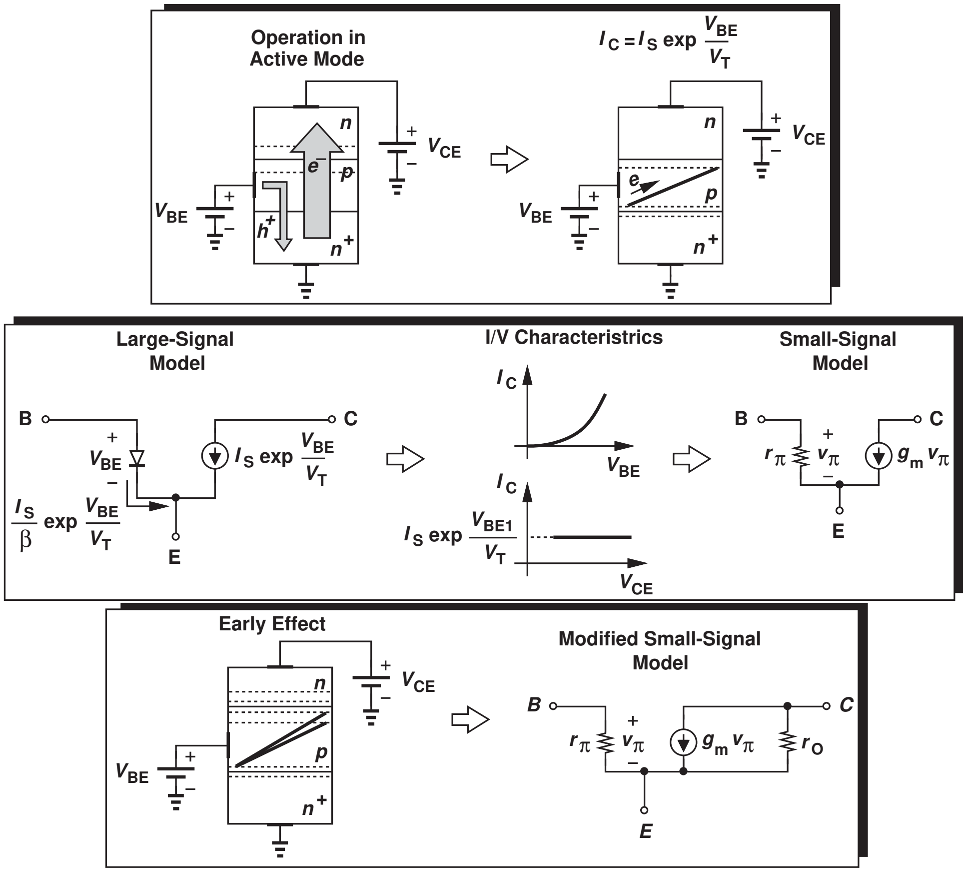

Figure 4.32 Summary of concepts studied thus far.

4.5 OPERATION OF BIPOLAR TRANSISTOR IN SATURATION MODE

As mentioned in the previous section, it is desirable to operate bipolar devices in the forward active region, where they act as voltage-controlled current sources. In this section, we study the behavior of the device outside this region and the resulting difficulties.

Let us set V_{BE} to a typical value, e.g., 750 mV, and vary the collector voltage from a high level to a low level [Fig. 4.33(a)]. As V_{CE} approaches V_{BE}, and V_{BC} goes from a negative value toward zero, the base-collector junction experiences less reverse bias. For $V_{CE} = V_{BE}$, the junction sustains a zero voltage difference, but its depletion region still absorbs most of the electrons injected by the emitter into the base. But what happens if $V_{CE} < V_{BE}$, i.e., $V_{BC} > 0$ and the B-C junction is forward biased? We say the transistor enters the "saturation region." Suppose $V_{CE} = 550\,\text{mV}$ and hence $V_{BC} = +200\,\text{mV}$. We know from Chapter 2 that a typical diode sustaining 200 mV of forward bias carries an extremely small current.[1] Thus, even in this case the

[1] About nine orders of magnitude less than one sustaining 750 mV: (750 mV − 200 mV)/(60 mV/dec) ≈ 9.2.

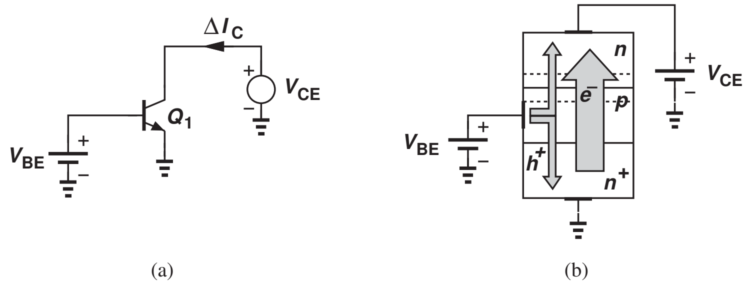

Figure 4.33 (a) Bipolar transistor with forward-biased base-collector junction, (b) flow of holes to collector.

transistor continues to operate as in the active mode, and we say the device is in "soft saturation."

If the collector voltage drops further, the B-C junction experiences greater forward bias, carrying a significant current [Fig. 4.33(b)]. Consequently, a large number of holes must be supplied to the base terminal—as if β is reduced. In other words, heavy saturation leads to a sharp rise in the base current and hence a rapid fall in β.

Example 4.15 A bipolar transistor is biased with $V_{BE} = 750$ mV and has a nominal β of 100. How much B-C forward bias can the device tolerate if β must not degrade by more than 10%? For simplicity, assume base-collector and base-emitter junctions have identical structures and doping levels.

Solution If the base-collector junction is forward-biased so much that it carries a current equal to one-tenth of the nominal base current, I_B, then the β degrades by 10%. Since $I_B = I_C/100$, the B-C junction must carry no more than $I_C/1000$. We therefore ask, what B-C voltage results in a current of $I_C/1000$ if $V_{BE} = 750$ mV gives a collector current of I_C? Assuming identical B-E and B-C junctions, we have

$$V_{BE} - V_{BC} = V_T \ln\frac{I_C}{I_S} - V_T \ln\frac{I_C/1000}{I_S} \tag{4.95}$$

$$= V_T \ln 1000 \tag{4.96}$$

$$\approx 180\ \text{mV}. \tag{4.97}$$

That is, $V_{BC} = 570$ mV.

Exercise Repeat the above example if $V_{BE} = 800$ mV.

It is instructive to study the transistor large-signal model and I-V characteristics in the saturation region. We construct the model as shown in Fig. 4.34(a), including the base-collector diode. Note that the net collector current *decreases* as the device enters

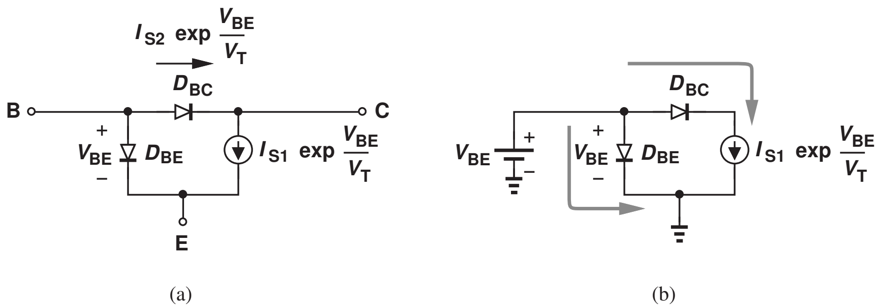

Figure 4.34 (a) Model of bipolar transistor including saturation effects, (b) case of open collector terminal.

saturation because part of the controlled current $I_{S1} \exp(V_{BE}/V_T)$ is provided by the B-C diode and need not flow from the collector terminal. In fact, as illustrated in Fig. 4.34(b), if the collector is left open, then D_{BC} is forward-biased so much that its current becomes equal to the controlled current.

The above observations lead to the I_C-V_{CE} characteristics depicted in Fig. 4.35, where I_C begins to fall for V_{CE} less than V_1, about a few hundred millivolts. The term "saturation" is used because increasing the base current in this region of operation leads to little change in the collector current.

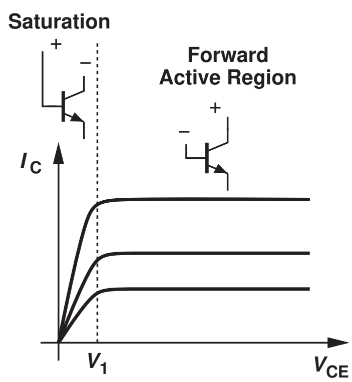

Figure 4.35 Transistor I/V characteristics in different regions of operation.

In addition to a drop in β, the *speed* of bipolar transistors also degrades in saturation (Chapter 11). Thus, electronic circuits rarely allow operation of bipolar devices in this mode. As a rule of thumb, we permit soft saturation with $V_{BC} < 400\,\text{mV}$ because the current in the B-C junction is negligible, provided that various tolerances in the component values do not drive the device into deep saturation.

It is important to recognize that the transistor simply draws a current from any component tied to its collector, e.g., a resistor. Thus, it is the external component that defines the collector voltage and hence the region of operation.

Example 4.16 For the circuit of Fig. 4.36, determine the relationship between R_C and V_{CC} that guarantees operation in soft saturation or active region.

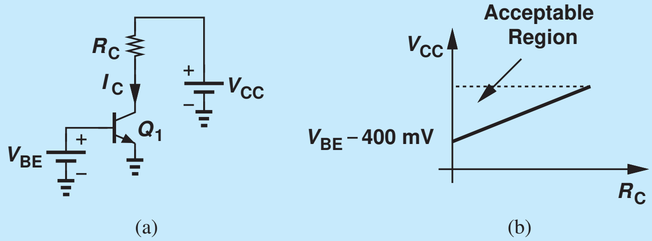

Figure 4.36 (a) Simple stage, (b) acceptable range of V_{CC} and R_C.

Solution In soft saturation, the collector current is still equal to $I_S \exp(V_{BE}/V_T)$. The collector voltage must not fall below the base voltage by more than 400 mV:

$$V_{CC} - R_C I_C \geq V_{BE} - 400\,\text{mV}. \tag{4.98}$$

Thus,

$$V_{CC} \geq I_C R_C + (V_{BE} - 400\,\text{mV}). \tag{4.99}$$

For a given value of R_C, V_{CC} must be sufficiently large so that $V_{CC} - I_C R_C$ still maintains a reasonable collector voltage.

Exercise Determine the maximum tolerable value of R_C.

In the deep saturation region, the collector-emitter voltage approaches a constant value called $V_{CE,sat}$ (about 200 mV). Under this condition, the transistor bears no resemblance to a controlled current source and can be modeled as shown in Fig. 4.37.(The battery tied between C and E indicates that V_{CE} is relatively constant in deep saturation.)

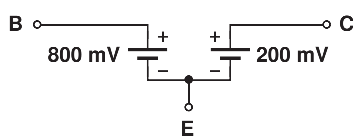

Figure 4.37 Transistor model in deep saturation.

4.6 THE *PNP* TRANSISTOR

We have thus far studied the structure and properties of the *npn* transistor, i.e., with the emitter and collector made of *n*-type materials and the base made of a *p*-type material. We may naturally wonder if the dopant polarities can be inverted in the three regions, forming a "*pnp*" device. More importantly, we may wonder why such a device would be useful.

4.6.1 Structure and Operation

SARASA

Figure 4.38(a) shows the structure of a *pnp* transistor, emphasizing that the emitter is heavily doped. As with the *npn* counterpart, operation in the active region requires forward-biasing the base-emitter junction and reverse-biasing the collector junction. Thus, $V_{BE} < 0$ and $V_{BC} > 0$. Under this condition, majority carriers in the emitter (holes) are injected into the base and swept away into the collector. Also, a linear profile of holes is formed in the base region to allow diffusion. A small number of base majority carriers (electrons) are injected into the emitter or recombined with the holes in the base region, thus creating the base current. Figure 4.38(b) illustrates the flow of the carriers. All of the operation principles and equations described for *npn* transistors apply to *pnp* devices as well.

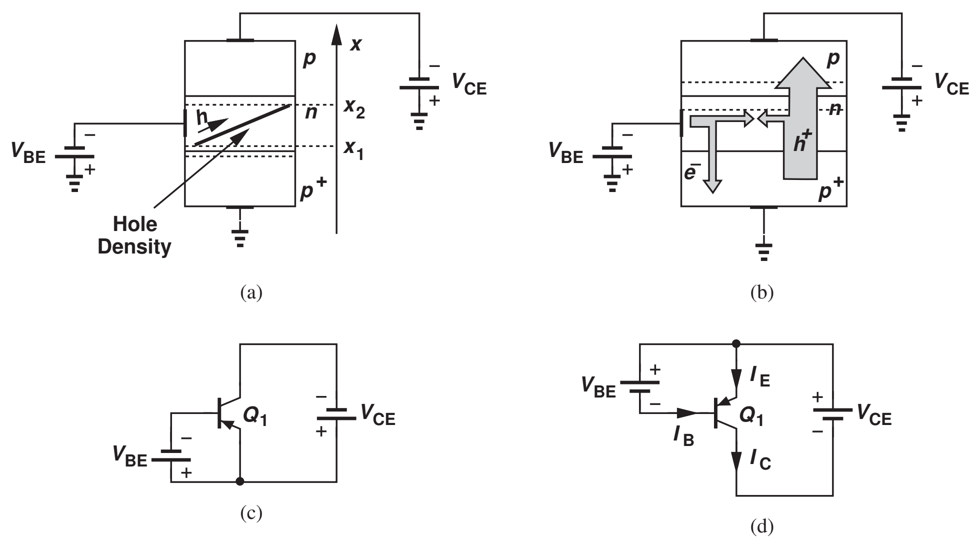

Figure 4.38 (a) Structure of *pnp* transistor, (b) current flow in *pnp* transistor, (c) proper biasing, (d) more intuitive view of (c).

Figure 4.38(c) depicts the symbol of the *pnp* transistor along with constant voltage sources that bias the device in the active region. In contrast to the biasing of the *npn* transistor in Fig. 4.6, here the base and collector voltages are *lower* than the emitter voltage. Following our convention of placing more positive nodes on the top of the page, we redraw the circuit as in Fig. 4.38(d) to emphasize $V_{EB} > 0$ and $V_{BC} > 0$ and to illustrate the actual direction of current flow into each terminal.

4.6.2 Large-Signal Model

The current and voltage polarities in *npn* and *pnp* transistors can be confusing. We address this issue by making the following observations. (1) The (conventional) current always flows from a positive supply (i.e., top of the page) toward a lower potential (i.e., bottom of the page). Figure 4.39(a) shows two branches employing *npn* and *pnp* transistors, illustrating that the (conventional) current flows from collector to emitter in *npn* devices and from emitter to collector in *pnp* counterparts. Since the base current must be included in the emitter current, we note that I_{B1} and I_{C1} add up to I_{E1}, whereas I_{E2} "loses" I_{B2} before emerging as I_{C2}. (2) The distinction between active and saturation regions is based on the

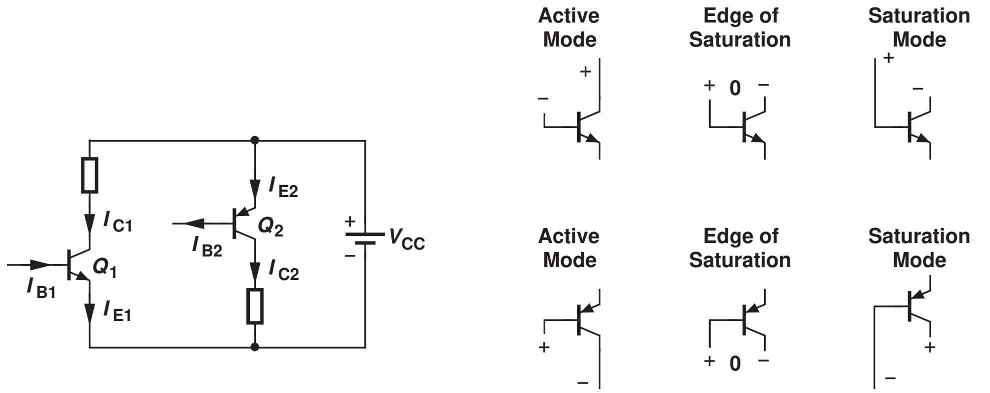

Figure 4.39 (a) Voltage and current polarities in *npn* and *pnp* transistors, (b) illustration of active and saturation regions.

B-C junction bias. The different cases are summarized in Fig. 4.39(b), where the relative position of the base and collector nodes signifies their potential difference. We note that an *npn* transistor is in the active mode if the collector (voltage) is *not* lower than the base (voltage). For the *pnp* device, on the other hand, the collector must not be *higher* than the base. (3) The *npn* current equations (4.23)–(4.25) must be modified as follows for the *pnp* device:

$$I_C = I_S \exp \frac{V_{EB}}{V_T} \tag{4.100}$$

$$I_B = \frac{I_S}{\beta} \exp \frac{V_{EB}}{V_T} \tag{4.101}$$

$$I_E = \frac{\beta + 1}{\beta} I_S \exp \frac{V_{EB}}{V_T}, \tag{4.102}$$

where the current directions are defined in Fig. 4.40. The only difference between the *npn* and *pnp* equations relates to the base-emitter voltage that appears in the

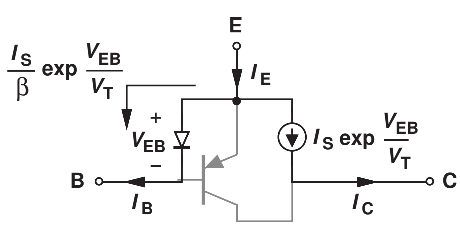

Figure 4.40 Large-signal model of *pnp* transistor.

exponent, an expected result because $V_{BE} < 0$ for *pnp* devices and must be changed to V_{EB} to create a large exponential term. Also, the Early effect can be included as

$$I_C = \left(I_S \exp \frac{V_{EB}}{V_T}\right)\left(1 + \frac{V_{EC}}{V_A}\right). \tag{4.103}$$

Example 4.17

In the circuit shown in Fig. 4.41, determine the terminal currents of Q_1 and verify operation in the forward active region. Assume $I_S = 2 \times 10^{-16}$ A and $\beta = 50$, but $V_A = \infty$.

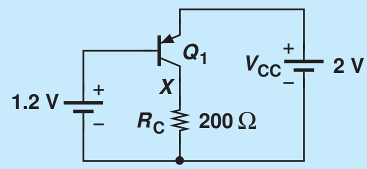

Figure 4.41 Simple stage using a *pnp* transistor.

Solution We have $V_{EB} = 2\text{ V} - 1.2\text{ V} = 0.8\text{ V}$ and hence

$$I_C = I_S \exp \frac{V_{EB}}{V_T} \tag{4.104}$$

$$= 4.61\text{ mA}. \tag{4.105}$$

It follows that

$$I_B = 92.2\ \mu\text{A} \tag{4.106}$$

$$I_E = 4.70\text{ mA}. \tag{4.107}$$

We must now compute the collector voltage and hence the bias across the B-C junction. Since R_C carries I_C,

$$V_X = R_C I_C \tag{4.108}$$

$$= 0.922\text{ V}, \tag{4.109}$$

which is *lower* than the base voltage. Invoking the illustration in Fig. 4.39(b), we conclude that Q_1 operates in the active mode and the use of equations (4.100)–(4.102) is justified.

Exercise What is the maximum value of R_C if the transistor must remain in soft saturation?

We should mention that some books assume all of the transistor terminal currents flow into the device, thus requiring that the right-hand side of Eqs. (4.100) and (4.101) be multiplied by a negative sign. We nonetheless continue with our notation as it reflects the actual direction of currents and proves more efficient in the analysis of circuits containing many *npn* and *pnp* transistors.

Example 4.18 In the circuit of Fig. 4.42, V_{in} represents a signal generated by a microphone. Determine V_{out} for $V_{in} = 0$ and $V_{in} = +5$ mV if $I_S = 1.5 \times 10^{-16}$ A.

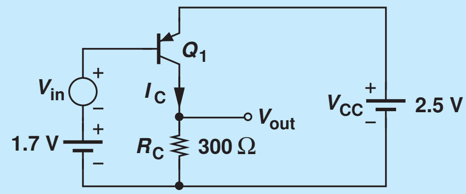

Figure 4.42 *PNP* stage with bias and small-signal voltages.

Solution For $V_{in} = 0$, $V_{EB} = +800$ mV and we have

$$I_C|_{V_{in}=0} = I_S \exp\frac{V_{EB}}{V_T} \tag{4.110}$$

$$= 3.46\,\text{mA}, \tag{4.111}$$

and hence

$$V_{out} = 1.038\,\text{V}. \tag{4.112}$$

If V_{in} increases to $+5$ mV, $V_{EB} = +795$ mV and

$$I_C|_{V_{in}=+5\,\text{mV}} = 2.85\,\text{mA}, \tag{4.113}$$

yielding

$$V_{out} = 0.856\,\text{V}. \tag{4.114}$$

Note that as the base voltage *rises*, the collector voltage *falls*, a behavior similar to that of the *npn* counterparts in Fig. 4.25. Since a 5-mV change in V_{in} gives a 182-mV change in V_{out}, the voltage gain is equal to 36.4. These results are more readily obtained through the use of the small-signal model.

Exercise Determine V_{out} if $V_{in} = -5$ mV.

4.6.3 Small-Signal Model

Since the small-signal model represents *changes* in the voltages and currents, we expect *npn* and *pnp* transistors to have similar models. Depicted in Fig. 4.43(a), the small-signal model of the *pnp* transistor is indeed *identical* to that of the *npn* device. Following the convention in Fig. 4.38(d), we sometimes draw the model as shown in Fig. 4.43(b).

The reader may notice that the terminal currents in the small-signal model bear an opposite direction with respect to those in the large-signal model of Fig. 4.40. This is not an inconsistency and is studied in Problem 4.50.

Did you know?

Some of the early low-cost radios used only two germanium (rather than silicon) pnp transistors to form two amplifier stages. (Silicon transistors became manufacturable later.) However, these radios had poor performance and could receive only one or two stations. For this reason, many manufacturers would proudly print the number of the transistors inside a radio on its front panel along with the brand name, e.g., "Admiral—Eight Transistors."

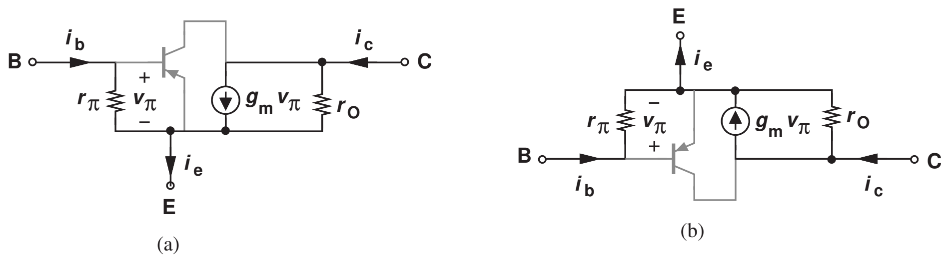

Figure 4.43 (a) Small-signal model of *pnp* transistor, (b) more intuitive view of (a).

The small-signal model of *pnp* transistors may cause confusion, especially if drawn as in Fig. 4.43(b). In analogy with *npn* transistors, one may automatically assume that the "top" terminal is the collector and hence the model in Fig. 4.43(b) is not identical to that in Fig. 4.31(b). We caution the reader about this confusion. A few examples prove helpful here.

Example 4.19

If the collector and base of a bipolar transistor are tied together, a two-terminal device results. Determine the small-signal impedance of the devices shown in Fig. 4.44(a). Assume $V_A = \infty$.

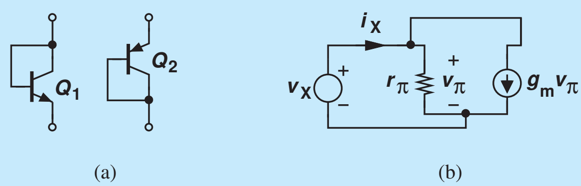

Figure 4.44

Solution We replace the bipolar transistor Q_1 with its small-signal model and apply a small-signal voltage across the device [Fig. 4.44(b)]. Noting that r_{pi} carries a current equal to v_X/r_π, we write a KCL at the input node:

$$\frac{v_X}{r_\pi} + g_m v_\pi = i_X. \tag{4.115}$$

Since $g_m r_\pi = \beta \gg 1$, we have

$$\frac{v_X}{i_X} = \frac{1}{g_m + r_\pi^{-1}} \tag{4.116}$$

$$\approx \frac{1}{g_m} \tag{4.117}$$

$$= \frac{V_T}{I_C}. \tag{4.118}$$

Interestingly, with a bias current of I_C, the device exhibits an impedance similar to that of a diode carrying the same bias current. We call this structure a "diode-connected transistor." The same results apply to the *pnp* configuration in Fig. 4.44(a).

Exercise What is the impedance of a diode-connected device operating at a current of 1 mA?

Example 4.20 Draw the small-signal equivalent circuits for the topologies shown in Figs. 4.45(a)–(c) and compare the results.

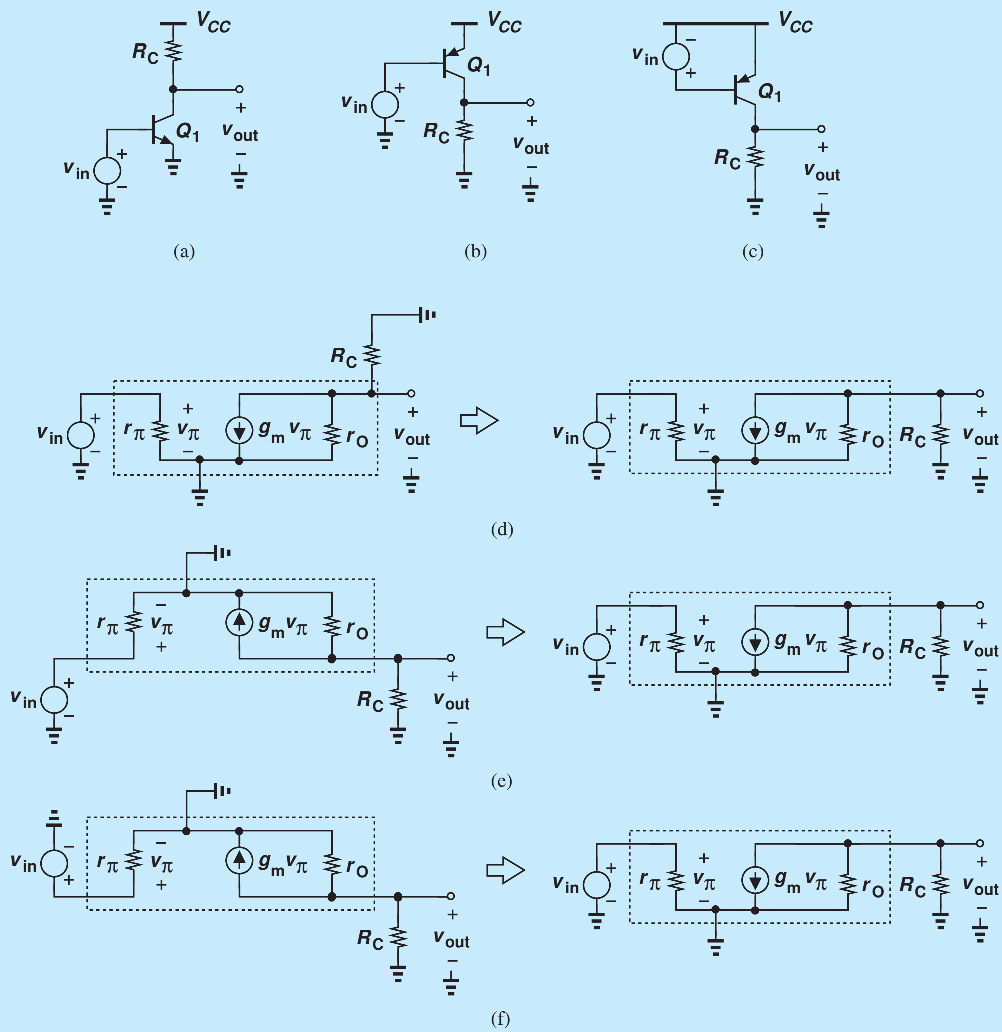

Figure 4.45 (a) Simple stage using an *npn* transistor, (b) simple stage using a *pnp* transistor, (c) another *pnp* stage, (d) small-signal equivalent of (a), (e) small-signal equivalent of (b), (f) small-signal equivalent of (f).

Solution As illustrated in Figs. 4.45(d)–(f), we replace each transistor with its small-signal model and ground the supply voltage. It is seen that all three topologies reduce to the same equivalent circuit because V_{CC} is grounded in the small-signal representation.

Exercise Repeat the preceding example if a resistor is placed between the collector and base of each transistor.

Example 4.21

Draw the small-signal equivalent circuit for the amplifier shown in Fig. 4.46(a).

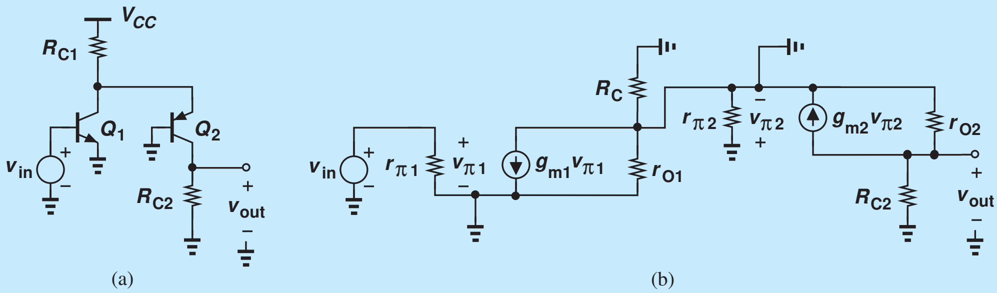

Figure 4.46 (a) Stage using *npn* and *pnp* devices, (b) small-signal equivalent of (a).

Solution Figure 4.46(b) depicts the equivalent circuit. Note that r_{O1}, R_{C1}, and $r_{\pi 2}$ appear in parallel. Such observations simplify the analysis (Chapter 5).

Exercise Show that the circuit depicted in Fig. 4.47 has the same small-signal model as the above amplifier.

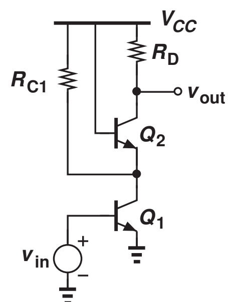

Figure 4.47 Stage using two *npn* devices.

4.7 CHAPTER SUMMARY

- A voltage-dependent current source can form an amplifier along with a load resistor. Bipolar transistors are electronic devices that can operate as voltage-dependent current sources.
- The bipolar transistor consists of two *pn* junctions and three terminals: base, emitter, and collector. The carriers flow from the emitter to the collector and are controlled by the base. the electrons?
- For proper operation, the base-emitter junction is forward-biased and the base-collector junction reverse-biased (forward active region). Carriers injected by the emitter into the base approach the edge of collector depletion region and are swept away by the high electric field.

- The base terminal must provide a small flow of carriers, some of which go to the emitter and some others recombine in the base region. The ratio of collector current and base current is denoted by β.
- In the forward active region, the bipolar transistor exhibits an exponential relationship between its collector current and base-emitter voltage.
- In the forward active region, a bipolar transistor behaves as a constant current source.
- The large-signal model of the bipolar transistor consists of an exponential voltage-dependent current source tied between the collector and emitter, and a diode (accounting for the base current) tied between the base and emitter.
- The transconductance of a bipolar transistor is given by $g_m = I_C/V_T$ and remains independent of the device dimensions.
- The small-signal model of bipolar transistors consists of a linear voltage-dependent current source, a resistance tied between the base and emitter, and an output resistance.
- If the base-collector junction is forward-biased, the bipolar transistor enters saturation and its performance degrades.
- The small-signal models of *npn* and *pnp* transistors are identical.

PROBLEMS

In the following problems, unless otherwise stated, assume the bipolar transistors operate in the active mode.

Section 4.1 General Considerations

4.1. Suppose the voltage-dependent current source of Fig. 4.1(a) is constructed with $K = 20\,\text{mA/V}$. What value of load resistance in Fig. 4.1(b) is necessary to achieve a voltage gain of 15?

4.2. A resistance of R_S is placed in series with the input voltage source in Fig. 4.2. Determine V_{out}/V_{in}.

4.3. Repeat Problem 4.2 but assuming that r_{in} and K are related: $r_{in} = a/x$ and $K = bx$. Plot the voltage gain as a function of x.

Section 4.3 Operation of Bipolar Transistor in Active Mode

4.4. Due to a manufacturing error, the base width of a bipolar transistor has increased by a factor of two. How does the collector current change?

4.5. In the circuit of Fig. 4.48, $I_{S1} = I_{S2} = 3 \times 10^{-16}$ A.

(a) Calculate V_B such that $I_X = 1$ mA.
(b) With the value of V_B found in (a), choose I_{S3} such that $I_Y = 2.5$ mA.

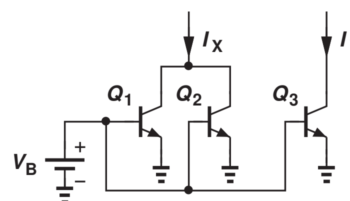

Figure 4.48

4.6. In the circuit of Fig. 4.49, it is observed that the collector currents of Q_1 and Q_2 are equal if $V_{BE1} - V_{BE2} = 20$ mV. Determine the ratio of transistor cross section areas if the other device parameters are identical.

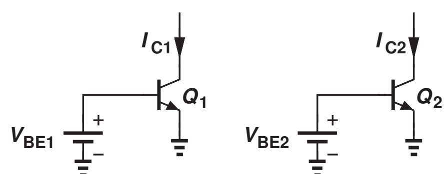

Figure 4.49

4.7. Consider the circuit shown in Fig. 4.50.
(a) If $I_{S1} = 2I_{S2} = 5 \times 10^{-16}$ A, determine V_B such that $I_X = 1.2$ mA.
(b) What value of R_C places the transistors at the edge of the active mode?

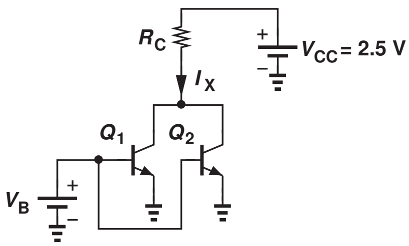

Figure 4.50

4.8. Repeat Problem 4.7 if V_{CC} is lowered to 1.5 V.

4.9. Calculate V_X in Fig. 4.51 if $I_S = 6 \times 10^{-16}$ A.

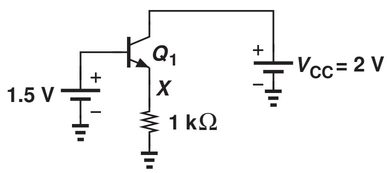

Figure 4.51

4.10. In the circuit of Fig. 4.52, determine the maximum value of V_{CC} that places Q_1 at the edge of saturation. Assume $I_S = 3 \times 10^{-16}$ A.

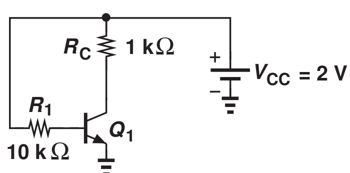

Figure 4.52

4.11. Consider the circuit shown in Fig. 4.53. Calculate the value of V_B that places Q_1 at the edge of the active region. Assume $I_S = 5 \times 10^{-16}$ A.

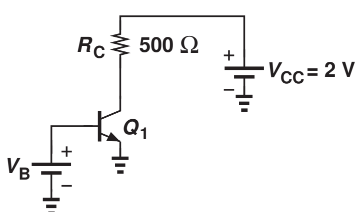

Figure 4.53

4.12. An integrated circuit requires two current sources: $I_1 = 1$ mA and $I_2 = 1.5$ mA. Assuming that only integer multiples of a unit bipolar transistor having $I_S = 3 \times 10^{-16}$ A can be placed in parallel, and only a single voltage source, V_B, is available (Fig. 4.54), construct the required circuit with minimum number of unit transistors.

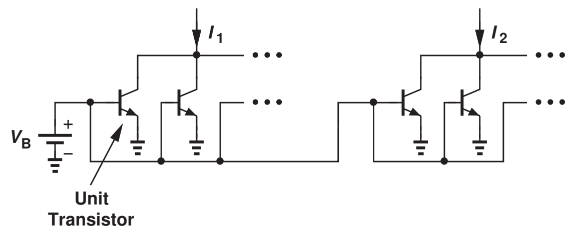

Figure 4.54

4.13. Repeat Problem 4.12 for three current sources $I_1 = 0.2$ mA, $I_2 = 0.3$ mA, and $I_3 = 0.45$ mA.

4.14. Consider the circuit shown in Fig. 4.55, assuming $\beta = 100$ and $I_S = 7 \times 10^{-16}$ A. If $R_1 = 10\,\text{k}\Omega$, determine V_B such that $I_C = 1$ mA.

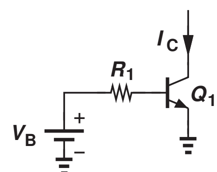

Figure 4.55

4.15. In the circuit of Fig. 4.55, $V_B = 800$ mV and $R_B = 10\,\text{k}\Omega$. Calculate the collector current.

4.16. In the circuit depicted in Fig. 4.56, $I_{S1} = 2I_{S2} = 4 \times 10^{-16}$ A. If $\beta_1 = \beta_2 = 100$ and $R_1 = 5\,\text{k}\Omega$, compute V_B such that $I_X = 1$ mA.

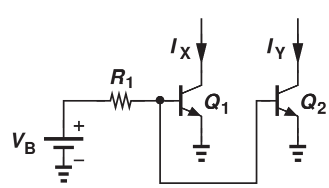

Figure 4.56

4.17. In the circuit of Fig. 4.56, $I_{S1} = 3 \times 10^{-16}$ A, $I_{S2} = 5 \times 10^{-16}$ A, $\beta_1 = \beta_2 = 100, R_1 = 5\,\text{k}\Omega$, and $V_B = 800$ mV. Calculate I_X and I_Y.

4.18. The base-emitter junction of a transistor is driven by a constant voltage. Suppose a voltage source is applied between the base and collector. If the device operates in the forward active region, prove that a change in base-collector voltage results in no change in the collector and base currents. (Neglect the Early effect.)

Section 4.4 Bipolar Transistor Models and Characteristics

4.19. Most applications require that the transconductance of a transistor remain relatively constant as the signal level varies. Of course, since the signal changes the collector current, $g_m = I_C/V_T$, does vary. Nonetheless, proper design ensures negligible variation, e.g., $\pm 10\%$. If a bipolar device is biased at $I_C = 1$ mA, what is the largest change in V_{BE} that guarantees only $\pm 10\%$ variation in g_m?

4.20. A transistor with $I_S = 6 \times 10^{-16}$ A must provide a transconductance of $1/(13\,\Omega)$. What base-emitter voltage is required?

4.21. Determine the operating point and the small-signal model of Q_1 for each of the circuits shown in Fig. 4.57. Assume $I_S = 8 \times 10^{-16}$ A, $\beta = 100$, and $V_A = \infty$.

4.22. Determine the operating point and the small-signal model of Q_1 for each of the circuits shown in Fig. 4.58. Assume $I_S = 8 \times 10^{-16}$ A, $\beta = 100$, and $V_A = \infty$.

4.23. A fictitious bipolar transistor exhibits an I_C-V_{BE} characteristic given by

$$I_C = I_S \exp \frac{V_{BE}}{nV_T}, \tag{4.119}$$

where n is a constant coefficient. Construct the small-signal model of the device if I_C is still equal to βI_B.

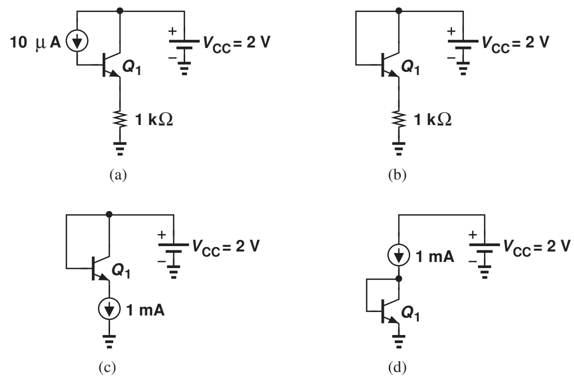

Figure 4.58

***4.24.** A fictitious bipolar transistor exhibits the following relationship between its base and collector currents:

$$I_C = aI_B^2, \tag{4.120}$$

where a is a constant coefficient. Construct the small-signal model of the device if I_C is still equal to $I_S \exp(V_{BE}/V_T)$.

4.25. The collector voltage of a bipolar transistor varies from 1 V to 3 V while the base-emitter voltage remains constant. What Early voltage is necessary to ensure that the collector current changes by less than 5%?

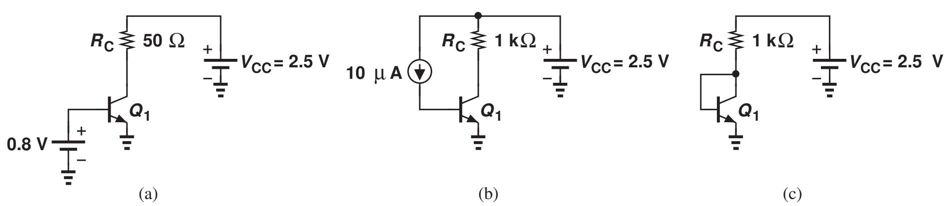

Figure 4.57

*__4.26.__ In the circuit of Fig. 4.59, $I_S = 5 \times 10^{-17}$ A. Determine V_X for (a) $V_A = \infty$, and (b) $V_A = 5$ V.

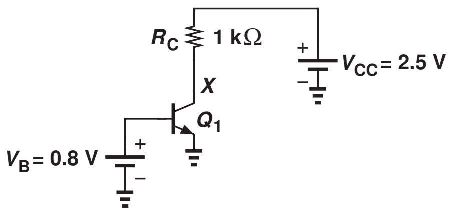

Figure 4.59

4.27. In the circuit of Fig. 4.60, V_{CC} changes from 2.5 to 3 V. Assuming $I_S = 1 \times 10^{-17}$ A and $V_A = 5$ V, determine the change in the collector current of Q_1.

4.28. In Problem 4.27, we wish to decrease V_B to compensate for the change in I_C. Determine the new value of V_B.

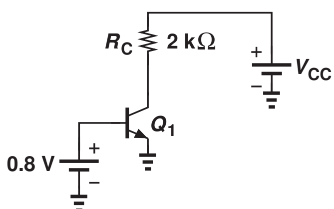

Figure 4.60

4.29. In the circuit of Fig. 4.61, n identical transistors are placed in parallel. If $I_S = 5 \times 10^{-16}$ A and $V_A = 8$ V for each device, construct the small-signal model of the equivalent transistor.

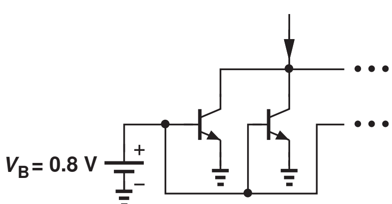

Figure 4.61

4.30. A bipolar current source is designed for an output current of 2 mA. What value of V_A guarantees an output resistance greater than 10 kΩ?

*__4.31.__ Consider the circuit shown in Fig. 4.62, where I_1 is a 1-mA ideal current source and $I_S = 3 \times 10^{-17}$ A.

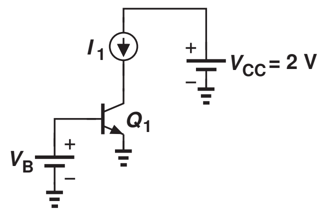

Figure 4.62

(a) Assuming $V_A = \infty$, determine V_B such that $I_C = 1$ mA.

(b) If $V_A = 5$ V, determine V_B such that $I_C = 1$ mA for a collector-emitter voltage of 1.5 V.

4.32. Consider the circuit shown in Fig. 4.63, where $I_S = 6 \times 10^{-16}$ A and $V_A = \infty$.

(a) Determine V_B such that Q_1 operates at the edge of the active region.

(b) If we allow soft saturation, e.g., a collector-base forward bias of 200 mV, by how much can V_B increase?

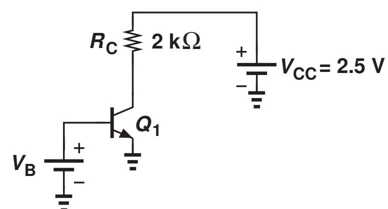

Figure 4.63

4.33. For the circuit depicted in Fig. 4.64, calculate the maximum value of V_{CC} that produces a collector-base forward bias of 200 mV. Assume $I_S = 7 \times 10^{-16}$ A and $V_A = \infty$.

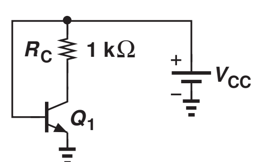

Figure 4.64

4.34. Consider the circuit shown in Fig. 4.65, where $I_S = 5 \times 10^{-16}$ A and $V_A = \infty$. If V_B is chosen to forward-bias the base-collector junction by 200 mV, what is the collector current?

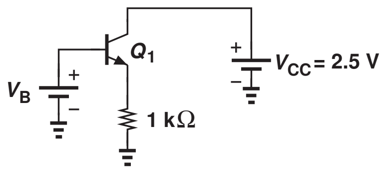

Figure 4.65

***4.35.** Assume $I_S = 2 \times 10^{-17}$ A, $V_A = \infty$, and $\beta = 100$ in Fig. 4.66. What is the maximum value of R_C if the collector-base must experience a forward bias of less than 200 mV?

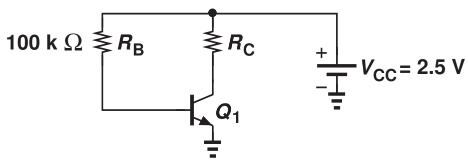

Figure 4.66

4.36. In the circuit of Fig. 4.67, $\beta = 100$ and $V_A = \infty$. Calculate the value of I_S such that the base-collector junction is forward-biased by 200 mV.

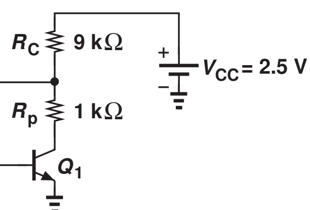

Figure 4.67

Section 4.6 *pnp* Transistor

4.37. If $I_{S1} = 3I_{S2} = 6 \times 10^{-16}$ A, calculate I_X in Fig. 4.68.

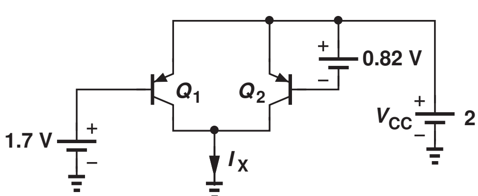

Figure 4.68

4.38. In the circuit of Fig. 4.69, it is observed that $I_C = 3$ mA. If $\beta = 100$, calculate I_S.

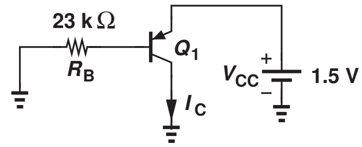

Figure 4.69

4.39. Determine the collector current of Q_1 in Fig. 4.70 if $I_S = 2 \times 10^{-17}$ A and $\beta = 100$.

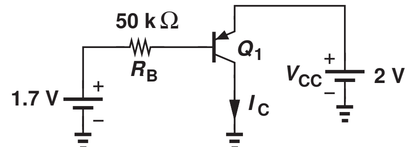

Figure 4.70

4.40. Determine the value of I_S in Fig. 4.71 such that Q_1 operates at the edge of the active mode.

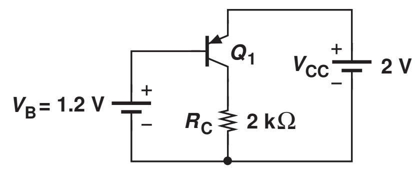

Figure 4.71

4.41. What is the value of β that places Q_1 at the edge of the active mode in Fig. 4.72? Assume $I_S = 8 \times 10^{-16}$ A.

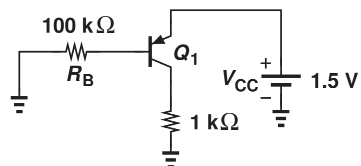

Figure 4.72

4.42. Calculate the collector current of Q_1 in Fig. 4.73 if $I_S = 3 \times 10^{-17}$ A.

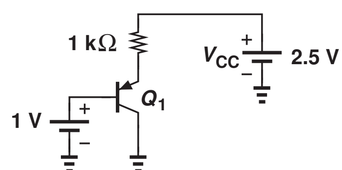

Figure 4.73

***4.43.** Determine the operating point and the small-signal model of Q_1 for each of the circuits shown in Fig. 4.74. Assume $I_S = 3 \times 10^{-17}$ A, $\beta = 100$, and $V_A = \infty$.

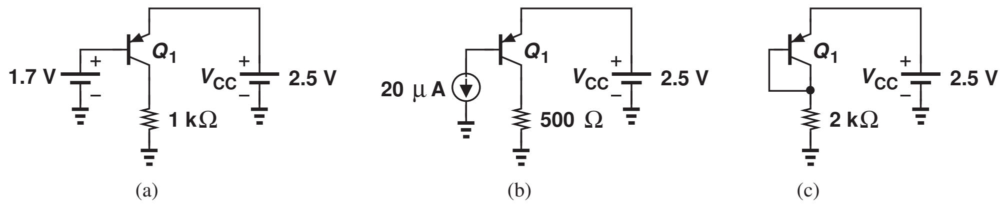

Figure 4.74

4.44. In the circuit of Fig. 4.75, $I_S = 5 \times 10^{-17}$ A. Calculate V_X for (a) $V_A = \infty$, and (b) $V_A = 6$ V.

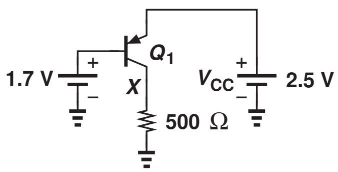

Figure 4.75

***4.45.** Determine the operating point and the small-signal model of Q_1 for each of the circuits shown in Fig. 4.76. Assume $I_S = 3 \times 10^{-17}$ A, $\beta = 100$, and $V_A = \infty$.

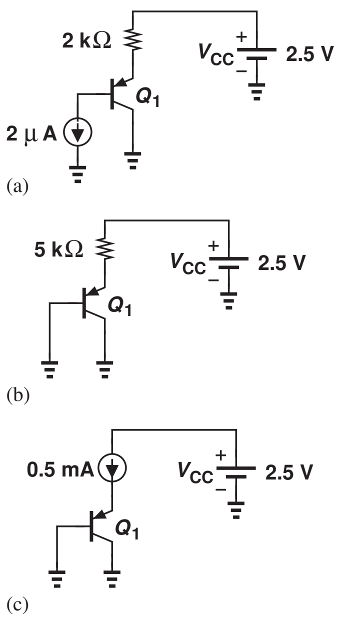

Figure 4.76

4.46. A *pnp* current source must provide an output current of 2 mA with an output resistance of 60 kΩ. What is the required Early voltage?

4.47. Repeat Problem 4.46 for a current of 1 mA and compare the results.

***4.48.** Suppose $V_A = 5$ V in the circuit of Fig. 4.77.

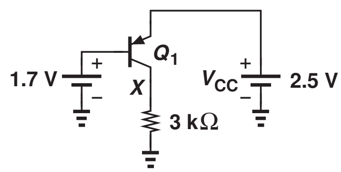

Figure 4.77

(a) What value of I_S places Q_1 at the edge of the active mode?

(b) How does the result in (a) change if $V_A = \infty$?

***4.49.** Consider the circuit depicted in Fig. 4.78, where $I_S = 6 \times 10^{-16}$ A, $V_A = 5$ V, and $I_1 = 2$ mA.

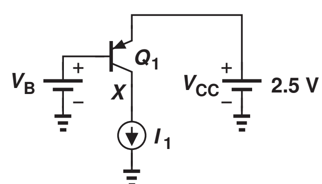

Figure 4.78

(a) What value of V_B yields $V_X = 1$ V?

(b) If V_B changes from the value found in (a) by 0.1 mV, what is the change in V_X?

(c) Construct the small-signal model of the transistor.

***4.50.** The terminal currents in the small-signal model of Fig. 4.43 do not seem to agree with those in the large-signal model of Fig. 4.40. Explain why this is not an inconsistency.

4.51. In the circuit of Fig. 4.79, $\beta = 100$ and $V_A = \infty$.

(a) Determine I_S such that Q_1 experiences a collector-base forward bias of 200 mV.

(b) Calculate the transconductance of the transistor.

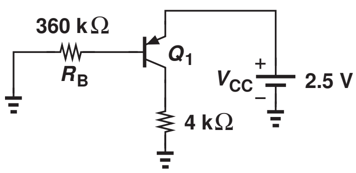

Figure 4.79

****4.52.** Determine the region of operation of Q_1 in each of the circuits shown in Fig. 4.80. Assume $I_S = 5 \times 10^{-16}$ A, $\beta = 100$, $V_A = \infty$.

****4.53.** Consider the circuit shown in Fig. 4.81, where $I_{S1} = 3I_{S2} = 5 \times 10^{-16}$ A, $\beta_1 = 100$, $\beta_2 = 50$, $V_A = \infty$, and $R_C = 500\ \Omega$.

(a) We wish to forward-bias the collector-base junction of Q_2 by no more than 200 mV. What is the maximum allowable value of V_{in}?

(b) With the value found in (a), calculate the small-signal parameters of Q_1 and Q_2 and construct the equivalent circuit.

****4.54.** Repeat Problem 4.53 for the circuit depicted in Fig. 4.82 but for part (a), determine the minimum allowable value of V_{in}. Verify that Q_1 operates in the active mode.

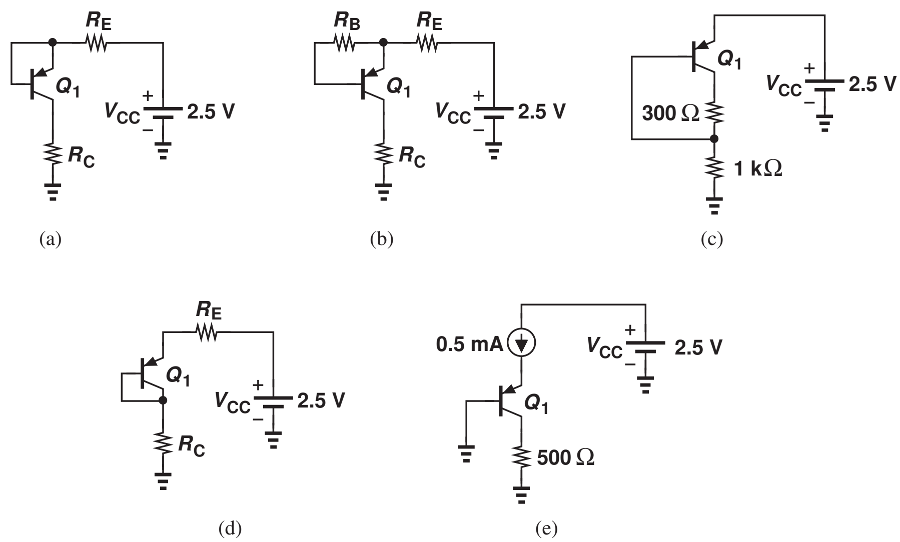

Figure 4.80

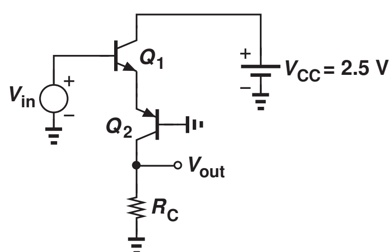

Figure 4.81

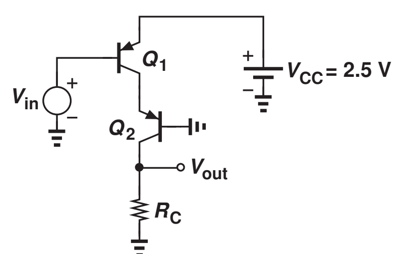

Figure 4.82

**4.55. Repeat Problem 4.53 for the circuit illustrated in Fig. 4.83.

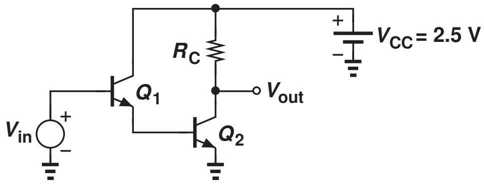

Figure 4.83

**4.56. In the circuit of Fig. 4.84, $I_{S1} = 2I_{S2} = 6 \times 10^{-17}$ A, $\beta_1 = 80$ and $\beta_2 = 100$.

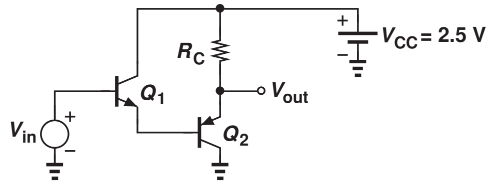

Figure 4.84

(a) What value of V_{in} yields a collector current of 2 mA for Q_2?

(b) With the value found in (a), calculate the small-signal parameters of Q_1 and Q_2 and construct the equivalent circuit.

SPICE PROBLEMS

In the following problems, assume $I_{S,npn} = 5 \times 10^{-16}$ A, $\beta_{npn} = 100$, $V_{A,npn} = 5$ V, $I_{S,pnp} = 8 \times 10^{-16}$ A, $\beta_{pnp} = 50$, $V_{A,pnp} = 3.5$ V.

4.57. Plot the input/output characteristic of the circuit shown in Fig. 4.85 for $0 < V_{in} < 2.5$ V. What value of V_{in} places the transistor at the edge of saturation?

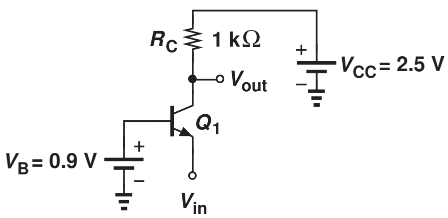

Figure 4.85

4.58. Repeat Problem 4.57 for the stage depicted in Fig. 4.86. At what value of V_{in} does Q_1 carry a collector current of 1 mA?

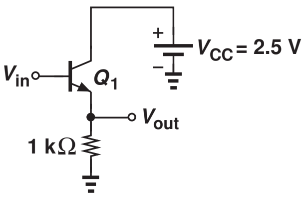

Figure 4.86

4.59. Plot I_{C1} and I_{C2} as a function of V_{in} for the circuits shown in Fig. 4.87 for$0 < V_{in} < 1.8$ V. Explain the dramatic difference between the two.

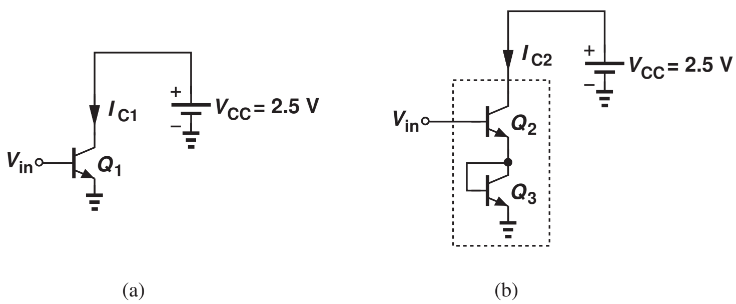

Figure 4.87

4.60. Plot the input/output characteristic of the circuit illustrated in Fig. 4.88 for $0 < V_{in} < 2$ V. What value of V_{in} yields a transconductance of $(50\ \Omega)^{-1}$ for Q_1?

4.61. Plot the input/output characteristic of the stage shown in Fig. 4.89 for $0 < V_{in} < 2.5$ V. At what value of V_{in} do Q_1 and Q_2 carry equal collector currents? Can you explain this result intuitively?

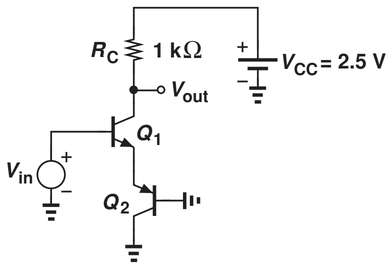

Figure 4.88

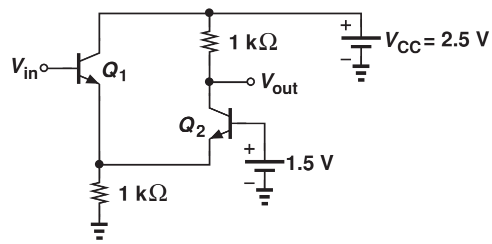

Figure 4.89