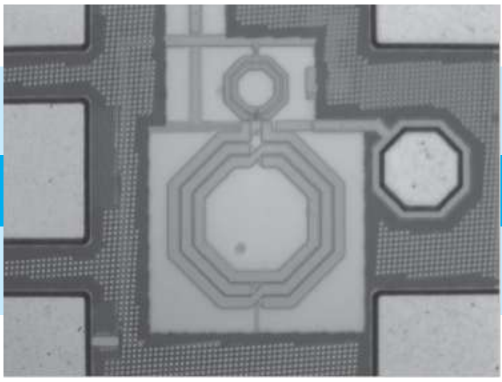


12

25/4 Voy a leer este capítulo x encima, ya debería estar leyendo multietapa.
Además hay un montón de cosas que no usamos

Feedback

Feedback is an integral part of our lives. Try touching your fingertips together with your eyes closed; you may not succeed the first time because you have broken a feedback loop that ordinarily "regulates" your motions. The regulatory role of feedback manifests itself in biological, mechanical, and electronic systems, allowing precise realization of "functions." For example, an amplifier targeting a precise gain of 2.00 is designed much more easily with feedback than without.

This chapter deals with the fundamentals of (negative) feedback and its application to electronic circuits. The outline is shown below.

General Considerations
- Elements of Feedback Systems
- Loop Gain
- Properties of Negative Feedback

Amplifiers and Sense/Return Methods
- Types of Amplifiers
- Amplifier Models
- Sense/Return Methods
- Polarity of Feedback

Analysis of Feedback Circuits
- Four Types of Feedback
- Effect of Finite I/O Impedances

Stability and Compensation
- Loop Instability
- Phase Margin
- Frequency Compensation

12.1 GENERAL CONSIDERATIONS

As soon as he reaches the age of 18, John eagerly obtains his driver's license, buys a used car, and begins to drive. Upon his parents' stern advice, John continues to observe the speed limit while noting that *every* other car on the highway drives faster. He then reasons that the speed limit is more of a "recommendation" and exceeding it by a small amount would not be harmful. Over the ensuing months, John gradually raises his speed so as to catch up with the rest of the drivers on the road, only to see flashing lights in his rear-view mirror one day. He pulls over to the shoulder of the road, listens to the sermon given by the police officer, receives a speeding ticket, and, dreading his parents' reaction, drives home—now strictly adhering to the speed limit.

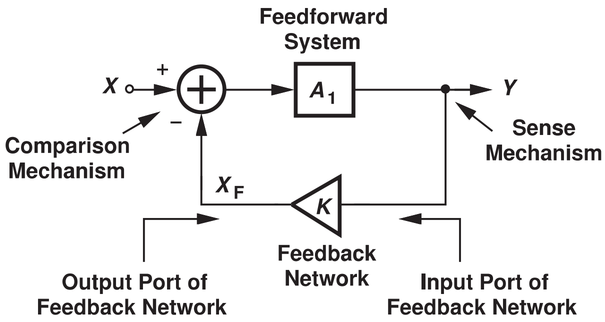


Figure 12.1 General feedback system.

John's story exemplifies the "regulatory" or "corrective" role of negative feedback. Without the police officer's involvement, John would probably continue to drive increasingly faster, eventually becoming a menace on the road.

Shown in Fig. 12.1, a negative feedback system consists of four essential components. (1) The "feedforward" system:[1] the main system, probably "wild" and poorly controlled. John, the gas pedal, and the car form the feedforward system, where the input is the amount of pressure that John applies to the gas pedal and the output is the speed of the car. (2) Output sense mechanism: a means of measuring the output. The police officer's radar serves this purpose here. (3) Feedback network: a network that generates a "feedback signal," X_F, from the sensed output. The police officer acts as the feedback network by reading the radar display, walking to John's car, and giving him a speeding ticket. The quantity $K = X_F/Y$ is called the "feedback factor." (4) Comparison or return mechanism: a means of subtracting the feedback signal from the input to obtain the "error," $E = X - X_F$. John makes this comparison himself, applying less pressure to the gas pedal—at least for a while.

The feedback in Fig. 12.1 is called "negative" because X_F is subtracted from X. Positive feedback, too, finds application in circuits such as oscillators and digital latches. If $K = 0$, i.e., no signal is fed back, then we obtain the "open-loop" system. If $K \neq 0$, we say the system operates in the "closed-loop" mode. As seen throughout this chapter, analysis of a feedback system requires expressing the closed-loop parameters in terms of the open-loop parameters. Note that the input port of the feedback network refers to that sensing the *output* of the forward system.

As our first step towards understanding the feedback system of Fig. 12.1, let us determine the closed-loop transfer function Y/X. Since $X_F = KY$, the error produced by the subtractor is equal to $X - KY$, which serves as the input of the forward system:

$$(X - KY)A_1 = Y. \tag{12.1}$$

corrección

That is, Lo vimos en clase

$$\frac{Y}{X} = \frac{A_1}{1 + KA_1}. \tag{12.2}$$

This equation plays a central role in our treatment of feedback, revealing that negative feedback reduces the gain from A_1 (for the open-loop system) to $A_1/(1 + KA_1)$.

[1]Also called the "forward" system.

Trade-off

The quantity $A_1/(1+KA_1)$ is called the "closed-loop gain." Why do we deliberately lower the gain of the circuit? As explained in Section 12.2, the benefits accruing from negative feedback very well justify this reduction of the gain.

Example 12.1 Analyze the noninverting amplifier of Fig. 12.2 from a feedback point of view.

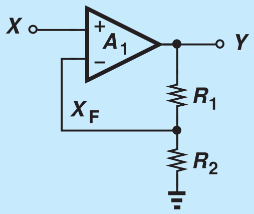


Figure 12.2

Solution The op amp A_1 performs two functions: subtraction of X and X_F and amplification. The network R_1 and R_2 also performs two functions: sensing the output voltage and providing a feedback factor of $K = R_2/(R_1 + R_2)$. Thus, Eq. (12.2) gives

$$\frac{Y}{X} = \frac{A_1}{1 + \dfrac{R_2}{R_1 + R_2} A_1}, \tag{12.3}$$

which is identical to the result obtained in Chapter 8.

Exercise Perform the above analysis if $R_2 = \infty$.

It is instructive to compute the error, E, produced by the subtractor. Since $E = X - X_F$ and $X_F = KA_1E$,

$$E = \frac{X}{1 + KA_1}, \tag{12.4}$$

suggesting that the difference between the feedback signal and the input diminishes as KA_1 increases. In other words, the feedback signal becomes a close "replica" of the input (Fig. 12.3). This observation leads to a great deal of insight into the operation of feedback systems.

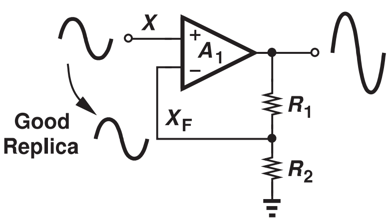


Figure 12.3 Feedback signal as a good replica of the input.

Example 12.2 Explain why in the circuit of Fig. 12.2, Y/X approaches $1 + R_1/R_2$ as $[R_2/\,(R_1 + R_2)]A_1$ becomes much greater than unity.

Solution If $KA_1 = [R_2/(R_1 + R_2)]A_1$ is large, X_F becomes almost identical to X, i.e., $X_F \approx X$. The voltage divider therefore requires that

$$Y\frac{R_2}{R_1 + R_2} \approx X \tag{12.5}$$

and hence

$$\frac{Y}{X} \approx 1 + \frac{R_1}{R_2}. \tag{12.6}$$

Of course, Eq. (12.3) yields the same result if $[R_2/(R_1 + R_2)]A_1 \gg 1$.

Exercise Repeat the above example if $R_2 = \infty$.

12.1.1 Loop Gain

In Fig. 12.1, the quantity KA_1, which is equal to product of the gain of the forward system and the feedback factor, determines many properties of the overall system. Called the "loop gain," KA_1 has an interesting interpretation. Let us set the input X to zero and "break" the loop at an arbitrary point, e.g., as depicted in Fig. 12.4(a).

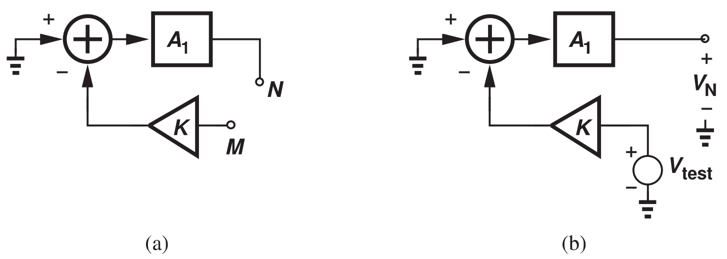


Figure 12.4 Computation of the loop gain by (a) breaking the loop and (b) applying a test signal.

The resulting topology can be viewed as a system with an input M and an output N. Now, as shown in Fig. 12.4(b), let us apply a test signal at M and follow it through the feedback network, the subtractor, and the forward system to obtain the signal at N.[2] The input of A_1 is equal to $-KV_{test}$, yielding

$$V_N = -KV_{test}A_1 \tag{12.7}$$

and hence

$$KA_1 = -\frac{V_N}{V_{test}}. \tag{12.8}$$

Meto una señal por la red de feedback. ¿qué sale?
==> Sale (-ganancia de lazo) * Señal

In other words, if a signal "goes around the loop," it experiences a gain equal to $-KA_1$; hence the term "loop gain." It is important not to confuse the closed-loop gain, $A_1/(1 + KA_1)$, with the loop gain, KA_1.

[2]We use voltage quantities in this example, but other quantities work as well.

Example 12.3 Compute the loop gain of the feedback system of Fig. 12.1 by breaking the loop at the input of A_1.

Solution Illustrated in Fig. 12.5 is the system with the test signal applied to the input of A_1. The output of the feedback network is equal to KA_1V_{test}, yielding

$$V_N = -KA_1V_{test} \tag{12.9}$$

and hence the same result as in Eq. (12.8).

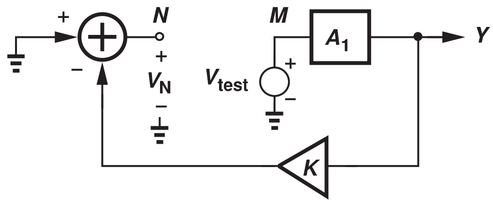


Figure 12.5

Exercise Compute the loop gain by breaking the loop at the input of the subtractor.

The reader may wonder if an ambiguity exists with respect to the *direction* of the signal flow in the loop gain test. For example, can we modify the topology of Fig. 12.4(b) as shown in Fig. 12.6? This would mean applying V_{test} to the *output* of A_1 and expecting to observe a signal at its *input* and eventually at N. While possibly yielding a finite value, such a test does not represent the actual behavior of the circuit. In the feedback system, the signal flows from the input of A_1 to its output and from the input of the feedback network to its output.

Did you know?

Negative feedback is a common phenomenon in nature. When you enter a bright room, your brain commands your pupils to become smaller. If you listen to very loud music for several hours, you feel hard of hearing afterwards because your ear has adjusted your hearing threshold in response to the volume. And if you try writing with your eyes closed, your handwriting will not be good because the feedback loop monitoring your pen strokes is broken.

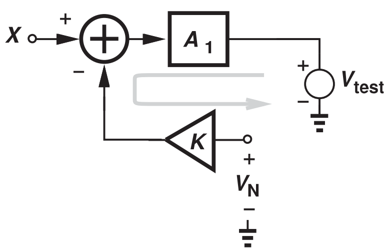


Figure 12.6 Incorrect method of applying test signal.

12.2 PROPERTIES OF NEGATIVE FEEDBACK

12.2.1 Gain Desensitization

Suppose A_1 in Fig. 12.1 is an amplifier whose gain is poorly controlled. For example, a CS stage provides a voltage gain of $g_m R_D$ while both g_m and R_D vary with process and temperature; the gain thus may vary by as much as ±20%. Also, suppose we require a voltage gain of 4.00.[3] How can we achieve such precision? Equation (12.2) points to a potential solution: if $KA_1 \gg 1$, we have

$$\frac{Y}{X} \approx \frac{1}{K}, \tag{12.10}$$

a quantity independent of A_1. From another perspective, Eq. (12.4) indicates that $KA_1 \gg 1$ leads to a small error, forcing X_F to be nearly equal to X and hence Y nearly equal to X/K. Thus, if K can be defined precisely, then A_1 impacts Y/X negligibly and a high precision in the gain is attained. The circuit of Fig. 12.2 exemplifies this concept very well. If $A_1 R_2/(R_1 + R_2) \gg 1$, then

Por qué hacer un amplificador entonces? Es precisa la red de fb?

$$\frac{Y}{X} \approx \frac{1}{K} \tag{12.11}$$

$$\approx 1 + \frac{R_1}{R_2}. \tag{12.12}$$

Why is R_1/R_2 more precisely defined than $g_m R_D$ is? If R_1 and R_2 are made of the same material and constructed identically, then the variation of their value with process and temperature does not affect their ratio. As an example, for a closed-loop gain of 4.00, we choose $R_1 = 3R_2$ and implement R_1 as the series combination of three "unit" resistors equal to R_2. Illustrated in Fig. 12.7, the idea is to ensure that R_1 and R_2 "track" each other; if R_2 increases by 20%, so does each unit in R_1 and hence the total value of R_1, still yielding a gain of $1 + 1.2R_1/(1.2R_2) = 4$.

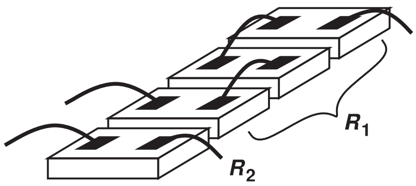


Figure 12.7 Construction of resistors for good matching.

Example 12.4

The circuit of Fig. 12.2 is designed for a nominal gain of 4. (a) Determine the actual gain if $A_1 = 1000$. (b) Determine the percentage change in the gain if A_1 drops to 500.

Solution

For a nominal gain of 4, Eq. (12.12) implies that $R_1/R_2 = 3$. (a) The actual gain is given by

$$\frac{Y}{X} = \frac{A_1}{1 + KA_1} \tag{12.13}$$

$$= 3.984. \tag{12.14}$$

[3] Some analog-to-digital converters (ADCs) require very precise voltage gains. For example, a 10-bit ADC may call for a gain of 2.000.

Note that the loop gain $KA_1 = 1000/4 = 250$. (b) If A_1 falls to 500, then

$$\frac{Y}{X} = 3.968. \tag{12.15}$$

Thus, the closed-loop gain changes by only $(3.984/3.968)/3.984 = 0.4\%$ if A_1 drops by factor of 2.

Exercise Determine the percentage change in the gain if A_1 falls to 200.

The above example reveals that the closed-loop gain of a feedback circuit becomes relatively independent of the open-loop gain so long as the loop gain, KA_1, remains sufficiently higher than unity. This property of negative feedback is called "gain desensitization."

We now see why we are willing to accept a reduction in the gain by a factor of $1 + KA_1$. We begin with an amplifier having a high, but poorly-controlled gain and apply negative feedback around it so as to obtain a better-defined, but inevitably lower gain. This concept was also extensively employed in the op amp circuits described in Chapter 8.

The gain desensitization property of negative feedback means that *any* factor that influences the open-loop gain has less effect on the closed-loop gain. Thus far, we have blamed only process and temperature variations, but many other phenomena change the gain as well.

- As the signal *frequency* rises, A_1 may fall, but $A_1/(1 + KA_1)$ remains relatively constant. We therefore expect that negative feedback *increases* the bandwidth (at the cost of gain).
- If the *load resistance* changes, A_1 may change; e.g., the gain of a CS stage depends on the load resistance. Negative feedback, on the other hand, makes the gain less sensitive to load variations.

X q' queremos al feedback

- The signal *amplitude* affects A_1 because the forward amplifier suffers from nonlinearity. For example, the large-signal analysis of differential pairs in Chapter 10 reveals that the small-signal gain falls at large input amplitudes. With negative feedback, however, the variation of the open-loop gain due to nonlinearity manifests itself to a lesser extent in the closed-loop characteristics. That is, negative feedback improves the linearity.

We now study these properties in greater detail.

¿¿

12.2.2 Bandwidth Extension

Let us consider a one-pole open-loop amplifier with a transfer function

$$A_1(s) = \frac{A_0}{1 + \dfrac{s}{\omega_0}}. \tag{12.16}$$

Here, A_0 denotes the low-frequency gain and ω_0 the -3 dB bandwidth. Noting from Eq. (12.2) that negative feedback lowers the low-frequency gain by a factor of $1 + KA_1$, we wish to determine the resulting bandwidth improvement. The closed-loop transfer function is obtained by substituting Eq. (12.16) for A_1 in Eq. (12.2):

$$\frac{Y}{X} = \frac{\dfrac{A_0}{1+\dfrac{s}{\omega_0}}}{1+K\dfrac{A_0}{1+\dfrac{s}{\omega_0}}}. \tag{12.17}$$

Multiplying the numerator and the denominator by $1 + s/\omega_0$ gives

$$\frac{Y}{X}(s) = \frac{A_0}{1+KA_0+\dfrac{s}{\omega_0}} \tag{12.18}$$

$$= \frac{\dfrac{A_0}{1+KA_0}}{1+\dfrac{s}{(1+KA_0)\omega_0}}. \tag{12.19}$$

In analogy with Eq. (12.16), we conclude that the closed-loop system now exhibits:

$$\text{Closed–Loop Gain} = \frac{A_0}{1+KA_0} \tag{12.20}$$

$$\text{Closed–Loop Bandwidth} = (1+KA_0)\omega_0. \tag{12.21}$$

In other words, the gain and bandwidth are scaled by the same factor but in opposite directions, displaying a *constant* product.

Example 12.5 Plot the closed-loop frequency response given by Eq. (12.19) for $K = 0$, 0.1, and 0.5. Assume $A_0 = 200$.

Solution For $K = 0$, the feedback vanishes and Y/X reduces to $A_1(s)$ as given by Eq. (12.16). For $K = 0.1$, we have $1 + KA_0 = 21$, noting that the gain decreases and the bandwidth increases by the same factor. Similarly, for $K = 0.5, 1 + KA_0 = 101$, yielding a proportional reduction in gain and increase in bandwidth. The results are plotted in Fig. 12.8.

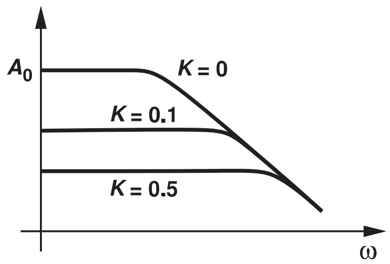


Figure 12.8

Exercise Repeat the above example for $K = 1$.

Example 12.6 Prove that the unity-gain bandwidth of the above system remains independent of K if $1 + KA_0 \gg 1$ and $K^2 \ll 1$.

Solution The magnitude of Eq. (12.19) is equal to

$$\left|\frac{Y}{X}(j\omega)\right| = \frac{\dfrac{A_0}{1+KA_0}}{\sqrt{1+\dfrac{\omega^2}{(1+KA_0)^2\omega_0^2}}}. \tag{12.22}$$

Equating this result to unity and squaring both sides, we write

$$\left(\frac{A_0}{1+KA_0}\right)^2 = 1 + \frac{\omega_u^2}{(1+KA_0)^2\omega_0^2}, \tag{12.23}$$

where ω_u denotes the unity-gain bandwidth. It follows that

$$\omega_u = \omega_0\sqrt{A_0^2 - (1+KA_0)^2} \tag{12.24}$$

$$\approx \omega_0\sqrt{A_0^2 - K^2A_0^2} \tag{12.25}$$

$$\approx \omega_0 A_0, \tag{12.26}$$

which is equal to the gain-bandwidth product of the open-loop system. Figure 12.9 depicts the results.

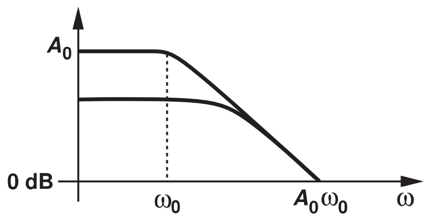


Figure 12.9

Exercise If $A_0 = 1000$, $\omega_0 = 2\pi \times (10\text{ MHz})$, and $K = 0.5$, calculate the unity-gain bandwidth from Eqs. (12.24) and (12.26) and compare the results.

12.2.3 Modification of I/O Impedances

As mentioned above, negative feedback makes the closed-loop gain less sensitive to the load resistance. This effect fundamentally arises from the modification of the *output impedance* as a result of feedback. Feedback modifies the *input* impedance as well. We will formulate these effects carefully in the following sections, but it is instructive to study an example at this point.

Example 12.7

Figure 12.10 depicts a transistor-level realization of the feedback circuit shown in Fig. 12.2. Assume $\lambda = 0$ and $R_1 + R_2 \gg R_D$ for simplicity. (a) Identify the four components of the feedback system. (b) Determine the open-loop and closed-loop voltage gain. (c) Determine the open-loop and closed-loop I/O impedances.

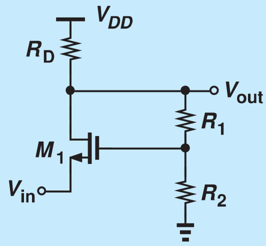


Figure 12.10

Solution

(a) In analogy with Fig. 12.10, we surmise that the forward system (the main amplifier) consists of M_1 and R_D, i.e., a common-gate stage. Resistors R_1 and R_2 serve as both the sense mechanism and the feedback network, returning a signal equal to $V_{out}R_2/(R_1 + R_2)$ to the subtractor. Transistor M_1 itself operates as the subtractor because the small-signal drain current is proportional to the *difference* between the gate and source voltages:

$$i_D = g_m(v_G - v_S). \tag{12.27}$$

(b) The forward system provides a voltage gain equal to

$$A_0 \approx g_m R_D \tag{12.28}$$

because $R_1 + R_2$ is large enough that its loading on R_D can be neglected. The closed-loop voltage gain is thus given by

$$\frac{v_{out}}{v_{in}} = \frac{A_0}{1 + KA_0} \tag{12.29}$$

$$= \frac{g_m R_D}{1 + \dfrac{R_2}{R_1 + R_2} g_m R_D}. \tag{12.30}$$

We should note that the overall gain of this stage can also be obtained by simply solving the circuit's equations—as if we know nothing about feedback. However, the use of feedback concepts both provides a great deal of insight and simplifies the task as circuits become more complex.

(c) The open-loop I/O impedances are those of the CG stage:

$$R_{in,open} = \frac{1}{g_m} \tag{12.31}$$

$$R_{out,open} = R_D. \tag{12.32}$$

At this point, we do not know how to obtain the closed-loop I/O impedances in terms of the open-loop parameters. We therefore simply solve the circuit. From Fig. 12.11(a), we recognize that R_D carries a current approximately equal to i_X because $R_1 + R_2$ is assumed large. The drain voltage of M_1 is thus given by $i_X R_D$, leading to a gate voltage equal to $+i_X R_D R_2/(R_1+R_2)$. Transistor M_1 generates a drain current proportional to v_{GS}:

$$i_D = g_m v_{GS} \tag{12.33}$$

$$= g_m\left(\frac{+i_X R_D R_2}{R_1+R_2} - v_X\right). \tag{12.34}$$

Since $i_D = -i_X$, Eq. (12.34) yields

$$\frac{v_X}{i_X} = \frac{1}{g_m}\left(1 + \frac{R_2}{R_1+R_2} g_m R_D\right). \tag{12.35}$$

That is, the input resistance *increases* from $1/g_m$ by a factor equal to $1 + g_m R_D R_2/(R_1+R_2)$, the same factor by which the gain decreases.

¿Miller?

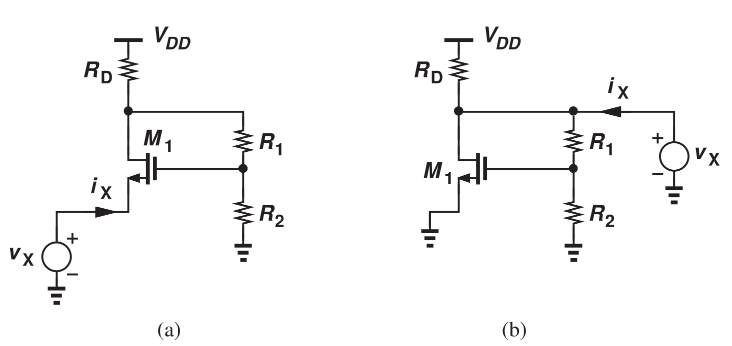


Figure 12.11

To determine the output resistance, we write from Fig. 12.11(b),

$$v_{GS} = \frac{R_2}{R_1+R_2} v_X, \tag{12.36}$$

and hence

$$i_D = g_m v_{GS} \tag{12.37}$$

$$= g_m \frac{R_2}{R_1+R_2} v_X. \tag{12.38}$$

Noting that, if $R_1 + R_2 \gg R_D$, then $i_X \approx i_D + v_X/R_D$, we obtain

$$i_X \approx g_m \frac{R_2}{R_1+R_2} v_X + \frac{v_X}{R_D}. \tag{12.39}$$

It follows that

$$\frac{v_X}{i_X} = \frac{R_D}{1 + \dfrac{R_2}{R_1+R_2} g_m R_D}. \tag{12.40}$$

The output resistance thus *decreases* by the "universal" factor $1 + g_m R_D R_2/\,(R_1+R_2)$.

The above computation of I/O impedances can be greatly simplified if feedback concepts are employed. As exemplified by Eqs. (12.35) and (12.40), the factor $1 + KA_0 = 1 + g_m R_D R_2/(R_1 + R_2)$ plays a central role here. Our treatment of feedback circuits in this chapter will provide the foundation for this point.

Exercise In some applications, the input and output impedances of an amplifier must both be equal to 50 Ω. What relationship guarantees that the input and output impedances of the above circuit are equal?

The reader may raise several questions at this point. Do the input impedance and the output impedance always scale down and up, respectively? Is the modification of I/O impedances by feedback *desirable*? We consider one example here to illustrate a point and defer more rigorous answers to subsequent sections.

Example 12.8 The common-gate stage of Fig. 12.10 must drive a load resistance $R_L = R_D/2$. How much does the gain change (a) without feedback, (b) with feedback?

Solution (a) Without feedback [Fig. 12.12(a)], the CG gain is equal to $g_m(R_D||R_L) = g_m R_D/3$. That is, the gain drops by factor of three.

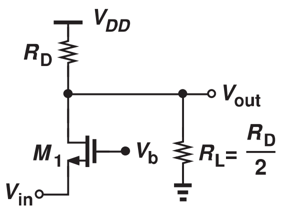


Figure 12.12

(b) With feedback, we use Eq. (12.30) but recognize that the open-loop gain has fallen to $g_m R_D/3$:

$$\frac{v_{out}}{v_{in}} = \frac{g_m R_D/3}{1 + \dfrac{R_2}{R_1 + R_2} g_m R_D/3} \tag{12.41}$$

$$= \frac{g_m R_D}{3 + \dfrac{R_2}{R_1 + R_2} g_m R_D}. \tag{12.42}$$

For example, if $g_m R_D R_2/(R_1 + R_2) = 10$, then this result differs from the "unloaded" gain expression in Eq. (12.30) by about 18%. Feedback therefore desensitizes the gain to load variations.

Exercise Repeat the above example for $R_L = R_D$.

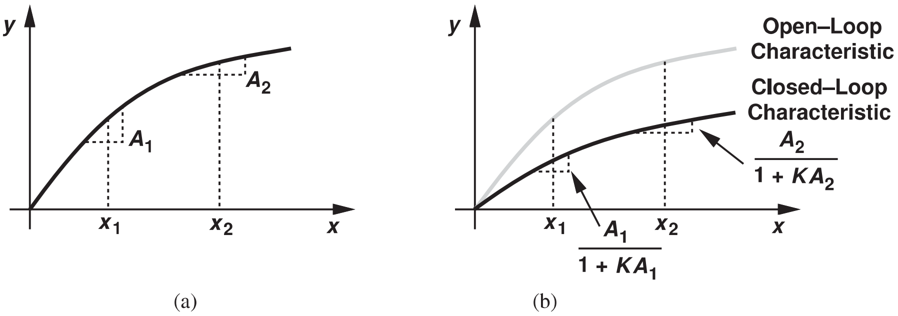


Figure 12.13 (a) Nonlinear open-loop characteristic of an amplifier, (b) improvement in linearity due to feedback.

12.2.4 Linearity Improvement

Consider a system having the input/output characteristic shown in Fig. 12.13(a). The nonlinearity observed here can also be viewed as the variation of the *slope* of the characteristic, i.e., the small-signal gain. For example, this system exhibits a gain of A_1 near $x = x_1$ and A_2 near $x = x_2$. If placed in a negative-feedback loop, the system provides a more uniform gain for different signal levels and, therefore, operates more linearly. In fact, as illustrated in Fig. 12.13(b) for the closed-loop system, we can write

$$\text{Gain at } x_1 = \frac{A_1}{1 + KA_1} \tag{12.43}$$

$$\approx \frac{1}{K}\left(1 - \frac{1}{KA_1}\right), \tag{12.44}$$

where it is assumed $KA_1 \gg 1$. Similarly,

$$\text{Gain at } x_2 = \frac{A_2}{1 + KA_2} \tag{12.45}$$

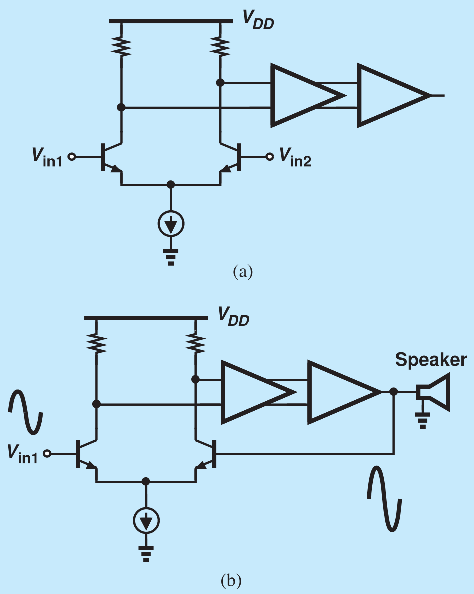
$$\approx \frac{1}{K}\left(1 - \frac{1}{KA_2}\right). \tag{12.46}$$

Thus, so long as KA_1 and KA_2 are large, the variation of the closed-loop gain with the signal level remains much less than that of the open-loop gain.

All of the above attributes of negative feedback can also be considered a result of the minimal error property illustrated in Fig. 12.3. For example, if at different signal levels, the forward amplifier's gain varies,

Did you know?

The large-signal behavior of circuits is generally nonlinear. For example, a low-quality audio amplifier driving a speaker suffers from nonlinearity as we increase the volume (i.e., the output signal swing); the result is the distinct distortion in the sound. A familiar type of nonlinearity is the type observed in differential pairs, e.g., the hyperbolic relationship obtained for bipolar implementations. In fact, the audio amplifier in your stereo system incorporates a differential pair at the input and other stages that can deliver a high output power [Fig. (a)]. To reduce distortion, the circuit is then placed in a negative feedback loop [Fig. (b)].



Amplifier driving a speaker (a) without, and (b) with feedback.

the feedback still ensures the feedback signal is a close replica of the input, and so is the output.

12.3 TYPES OF AMPLIFIERS

The amplifiers studied thus far in this book sense and produce voltages. While less intuitive, other types of amplifiers also exist, i.e., those that sense and/or produce currents. Figure 12.14 depicts the four possible combinations along with their input and output impedances in the ideal case. For example, a circuit sensing a current must display a *low* input impedance to resemble a current meter. Similarly, a circuit generating an output current must achieve a *high* output impedance to approximate a current source. The reader is encouraged to confirm the other cases as well. The distinction among the four types of amplifiers becomes important in the analysis of feedback circuits. Note that the "current-voltage" and "voltage-current" amplifiers of Figs. 12.14(b) and (c) are commonly known as "transimpedance" and "transconductance" amplifiers, respectively.

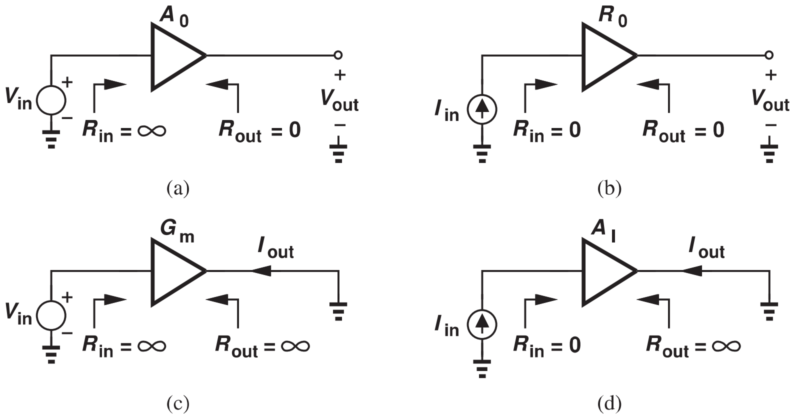


Figure 12.14 (a) Voltage, (b) transimpedance, (c) transconductance, and (d) current amplifiers.

12.3.1 Simple Amplifier Models

For our studies later in this chapter, it is beneficial to develop simple models for the four amplifier types. Depicted in Fig. 12.15 are the models for the ideal case. The voltage amplifier in Fig. 12.15(a) provides an *infinite* input impedance so that it can sense voltages as an *ideal* voltmeter, i.e., without loading the preceding stage. Also, the circuit exhibits a

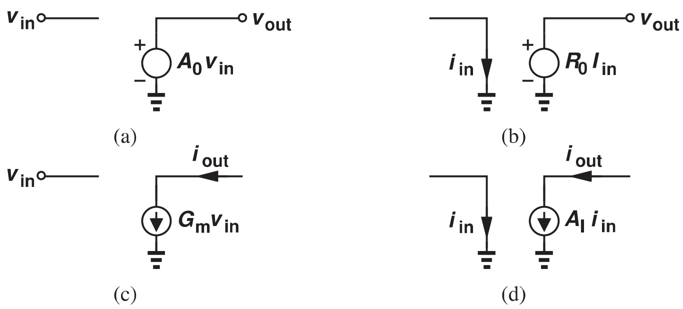


Figure 12.15 Ideal models for (a) voltage, (b) transimpedance, (c) transconductance, and (d) current amplifiers.

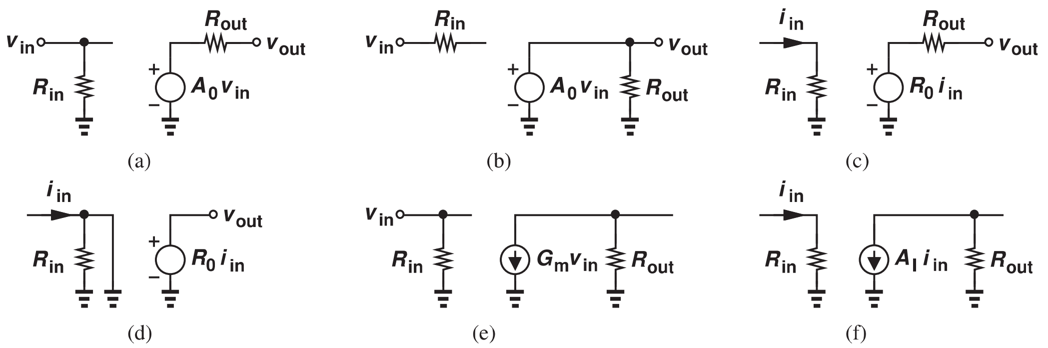


Figure 12.16 (a) Realistic model of voltage amplifier, (b) incorrect voltage amplifier model, (c) realistic model of transimpedance amplifier, (d) incorrect model of transimpedance amplifier, (e) realistic model of transconductance amplifier, (f) realistic model of current amplifier.

zero output impedance so as to serve as an ideal voltage source, i.e., deliver $v_{out} = A_0 v_{in}$ regardless of the load impedance.

The transimpedance amplifier in Fig. 12.15(b) has a *zero* input impedance so that it can measure currents as an ideal current meter. Similar to the voltage amplifier, the output impedance is also zero if the circuit operates as an ideal voltage source. Note that the "transimpedance gain" of this amplifier, $R_0 = v_{out}/i_{in}$, has a dimension of resistance. For example, a transimpedance gain of 2 kΩ means a 1-mA change in the input current leads to a 2-V change at the output.

The I/O impedances of the topologies in Figs. 12.15(c) and (d) follow similar observations. It is worth noting that the amplifier of Fig. 12.15(c) has a "transconductance gain," $G_m = i_{out}/v_{in}$, with a dimension of transconductance.

Nunca lo dijimos, pero nuestros los amplificadores con transistores sensan tensión y ponen corriente, son amplificadores de trans-conductrancia

In reality, the ideal models in Fig. 12.15 may not be accurate. In particular, the I/O impedances may not be negligibly large or small. Figure 12.16 shows more realistic models of the four amplifier types. Illustrated in Fig. 12.16(a), the voltage amplifier model contains an input resistance in *parallel* with the input port and an output resistance in *series* with the output port. These choices are unique and become clearer if we attempt other combinations. For example, if we envision the model as shown in Fig. 12.16(b), then the input and output impedances remain equal to infinity and zero, respectively, regardless of the values of R_{in} and R_{out}. (Why?) Thus, the topology of Fig. 12.16(a) serves as the only possible model representing finite I/O impedances.

Figure 12.16(c) depicts a nonideal transimpedance amplifier. Here, the input resistance appears in *series* with the input. Again, if we attempt a model such as that in Fig. 12.16(d), the input resistance is zero. The other two amplifier models in Figs. 12.16(e) and (f) follow similar concepts.

12.3.2 Examples of Amplifier Types

It is instructive to study examples of the above four types. Figure 12.17(a) shows a cascade of a CS stage and a source follower as a "voltage amplifier." The circuit indeed provides a high input impedance (similar to a voltmeter) and a low output impedance (similar to a voltage source). Figure 12.17(b) depicts a cascade of a CG stage and a source follower as a transimpedance amplifier. Such a circuit displays low input and output impedances to serve as a "current sensor" and a "voltage generator." Figure 12.17(c) illustrates a single

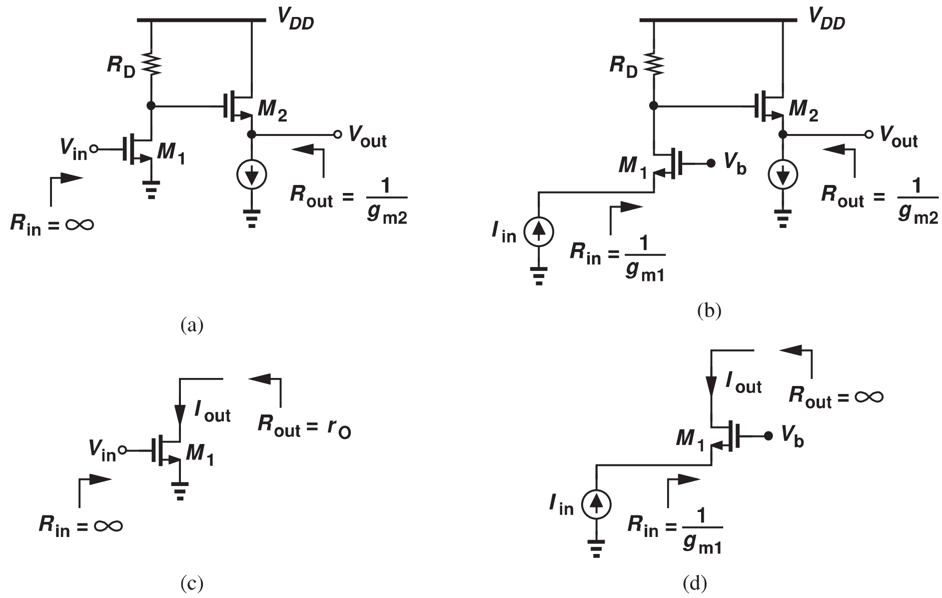


Figure 12.17 Examples of (a) voltage, (b) transimpedance, (c) transconductance, and (d) current amplifiers.

MOSFET as a transconductance amplifier. With high input and output impedances, the circuit efficiently senses voltages and generates currents. Finally, Fig. 12.17(d) shows a common-gate transistor as a current amplifier. Such a circuit must provide a low input impedance and a high output impedance.

Let us also determine the small-signal "gain" of each circuit in Fig. 12.17, assuming $\lambda = 0$ for simplicity. The voltage gain, A_0, of the cascade in Fig. 12.17(a) is equal to $-g_m R_D$ if $\lambda = 0$.[4] The gain of the circuit in Fig. 12.17(b) is defined as v_{out}/i_{in}, called the "transimpedance gain," and denoted by R_T. In this case, i_{in} flows through M_1 and R_D, generating a voltage equal to $i_{in}R_D$ at both the drain of M_1 and the source of M_2. That is, $v_{out} = i_{in}R_D$ and hence $R_T = R_D$.

For the circuit in Fig. 12.17(c), the gain is defined as i_{out}/v_{in}, called the "transconductance gain," and denoted by G_m. In this example, $G_m = g_m$. For the current amplifier in Fig. 12.17(d), the current gain, A_I, is equal to unity because the input current simply flows to the output.

Example 12.9

With a current gain of unity, the topology of Fig. 12.17(d) appears hardly better than a piece of wire. What is the advantage of this circuit?

Solution

The important property of this circuit lies in its input impedance. Suppose the current source serving as the input suffers from a large parasitic capacitance, C_p. If applied directly to a resistor R_D [Fig. 12.18(a)], the current would be wasted

[4] Recall from Chapter 7 that the gain of the source follower is equal to unity in this case.

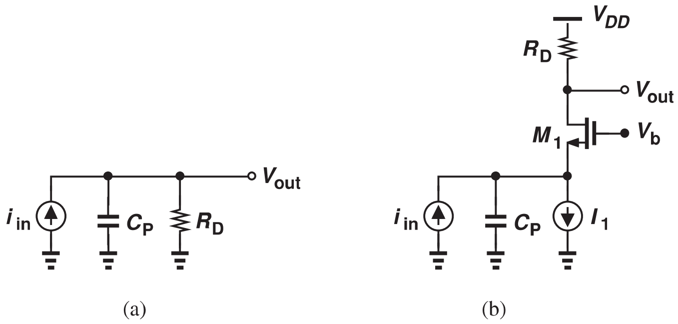


Figure 12.18

through C_p at high frequencies, exhibiting a -3 dB bandwidth of only $(R_D C_p)^{-1}$. On the other hand, the use of a CG stage [Fig. 12.18(b)] moves the input pole to g_m/C_p, a much higher frequency.

Exercise Determine the transfer function V_{out}/I_{in} for each of the above circuits.

12.4 SENSE AND RETURN TECHNIQUES

Recall from Section 12.1 that a feedback system includes means of sensing the output and "returning" the feedback signal to the input. In this section, we study such means so as to recognize them easily in a complex feedback circuit.

How do we measure the voltage across a port? We place a voltmeter in *parallel* with the port, and require that the voltmeter have a *high* input impedance so that it does not disturb the circuit [Fig. 12.19(a)]. By the same token, a feedback circuit sensing an output voltage must appear in parallel with the output and, ideally, exhibit an infinite impedance [Fig. 12.19(b)]. Shown in Fig. 12.19(c) is an example in which the resistive divider consisting of R_1 and R_2 senses the output voltage and generates the feedback signal, v_F. To approach the ideal case, $R_1 + R_2$ must be very large so that A_1 does not "feel" the effect of the resistive divider.

ESTO es un libro. Para vos Zola

How do we measure the current flowing through a wire? We *break* the wire and place a current meter in *series* with the wire [Fig. 12.20(a)]. The current meter in fact consists of a small resistor, so that it does not disturb the circuit, and a voltmeter that measures

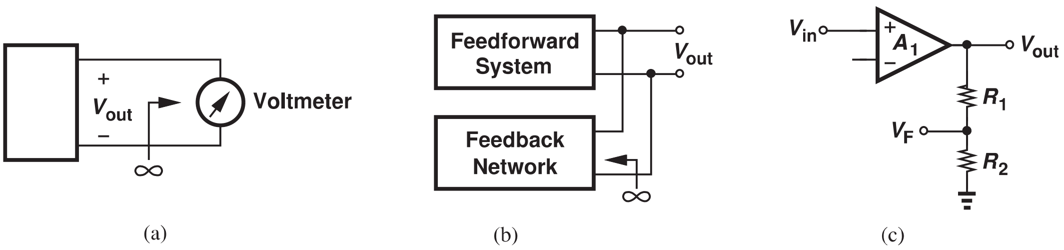
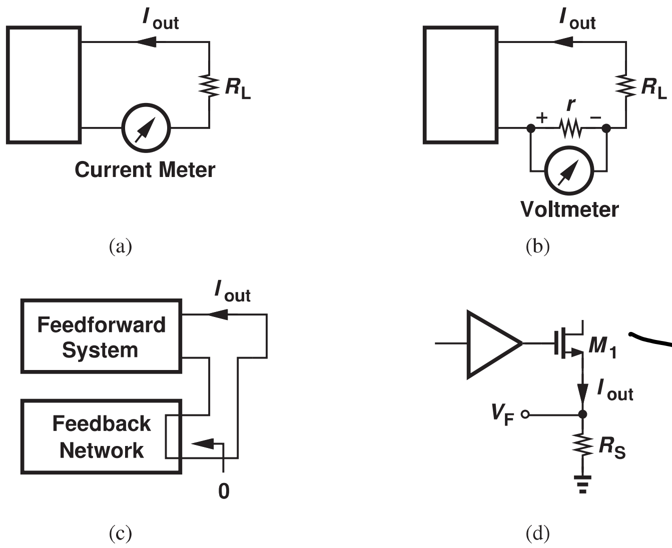


Figure 12.19 (a) Sensing a voltage by a voltmeter, (b) sensing the output voltage by the feedback network, (d) example of implementation.



SOURCE DEGENERADO!

MCST

Figure 12.20 (a) Sensing a current by a current meter, (b) actual realization of current meter, (c) sensing the output current by the feedback network, (d) example of implementation.

the voltage drop across the resistor [Fig. 12.20(b)]. Thus, a feedback circuit sensing an output current must appear in *series* with the output and, ideally, exhibit a zero impedance [Fig. 12.20(c)]. Depicted in Fig. 12.20(d) is an implementation of this concept. A resistor placed in series with the source of M_1 senses the output current, generating a proportional feedback voltage, V_F. Ideally, R_S is so small ($\ll 1/g_{m1}$) that the operation of M_1 remains unaffected.

To return a voltage or current to the input, we must employ a mechanism for adding or subtracting such quantities.[5] To add two voltage sources, we place them in *series* [Fig. 12.21(a)]. Thus, a feedback network returning a voltage must appear in series with the input signal [Fig. 12.21(b)], so that

$$v_e = v_{in} - v_F. \tag{12.47}$$

For example, as shown in Fig. 12.21(c), a differential pair can subtract the feedback voltage from the input. Alternatively, as mentioned in Example 12.7, a single transistor can operate as a voltage subtractor [Fig. 12.21(d)].

To add two current sources, we place them in *parallel* [Fig. 12.22(a)]. Thus, a feedback network returning a current must appear in parallel with the input signal, Fig. 12.22(b), so that

$$i_e = i_{in} - i_F. \tag{12.48}$$

For example, a transistor can return a current to the input [Fig. 12.22(c)]. So can a resistor if it is large enough to approximate a current source [Fig. 12.22(d)].

[5]Of course, only quantities having the same dimension can be added or subtracted. That is, a voltage cannot be added to a current.

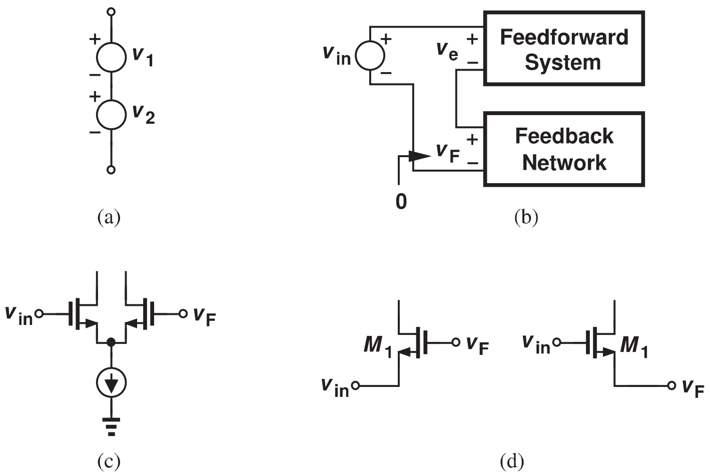


Figure 12.21 (a) Addition of two voltages, (b) addition of feedback and input voltages, (c) differential pair as a voltage subtractor, (d) single transistor as a voltage subtractor.

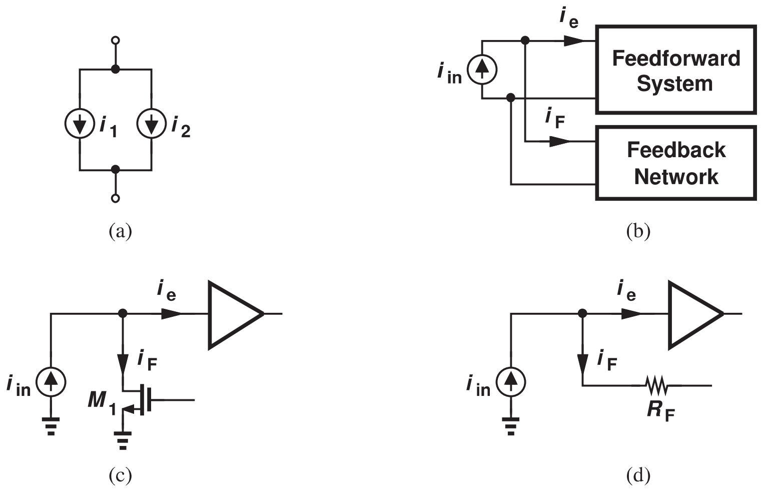


Figure 12.22 (a) Addition of two currents, (b) addition of feedback current and input current, (c) circuit realization, (d) another realization.

Example 12.10

Determine the types of sensed and returned signals in the circuit of Fig. 12.23.

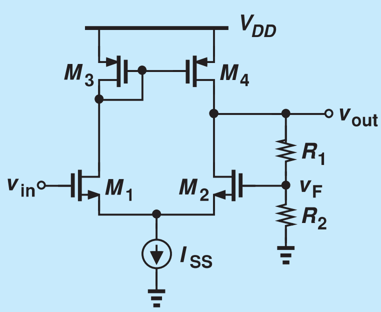


Figure 12.23

Solution This circuit is an implementation of the noninverting amplifier shown in Fig. 12.2. Here, the differential pair with the active load plays the role of an op amp. The resistive divider senses the output voltage and serves as the feedback network, producing $v_F = [R_2/(R_1 + R_2)]v_{out}$. Also, M_1 and M_2 operate as both part of the op amp (the forward system) and a voltage subtractor. The amplifier therefore combines the topologies in Figs. 12.19(c) and 12.21(c).

Exercise Repeat the above example if $R_2 = \infty$.

Example 12.11 Compute the feedback factor, K, for the circuit depicted in Fig. 12.24. Assume $\lambda = 0$.

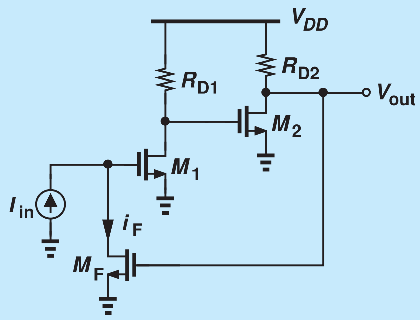


Figure 12.24

Solution Transistor M_F both senses the output voltage and returns a current to the input. The feedback factor is thus given by

$$K = \frac{i_F}{v_{out}} = g_{mF}, \tag{12.49}$$

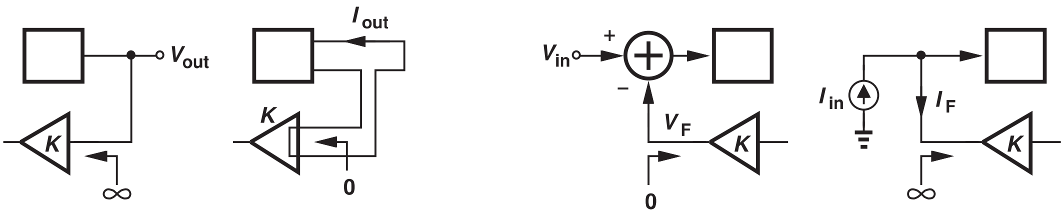
where g_{mF} denotes the transconductance of M_F.

Exercise Calculate the feedback factor if M_F is degenerated by a resistor of value R_S.

Let us summarize the properties of the "ideal" feedback network. As illustrated in Fig. 12.25(a), we expect such a network to exhibit an infinite input impedance if sensing a voltage and a zero input impedance if sensing a current. Moreover, the network must provide a zero output impedance if returning a voltage and an infinite output impedance if returning a current.

12.5 POLARITY OF FEEDBACK

While the block diagram of a feedback system, e.g., Fig. 12.1, readily reveals the polarity of feedback, an actual circuit implementation may not. The procedure of determining this polarity involves three steps: (a) assume the input signal goes up (or down); (b) follow the change through the forward amplifier and the feedback network; (c) determine whether the returned quantity *opposes* or *enhances* the original "effect" produced by the input change. A simpler procedure is as follows: (a) set the input to zero; (b) break the loop; (c) apply a test signal, V_{test}, travel around the loop, examine the returned signal, V_{ret}, and determine the polarity of $V_{\text{ret}}/V_{\text{test}}$.



(a) (b)

Figure 12.25 (a) Input impedance of ideal feedback networks for sensing voltage and current quantities, (b) output impedance of ideal feedback networks for producing voltage and current quantities.

Example 12.12 Determine the polarity of feedback in the circuit of Fig. 12.26.

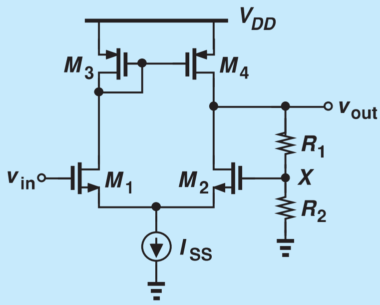


Figure 12.26

Solution If V_{in} goes up, I_{D1} tends to increase and I_{D2} tends to decrease. As a result, V_{out} and hence V_X tend to rise. The rise in V_X tends to increase I_{D2} and decrease I_{D1}, counteracting the effect of the change in V_{in}. The feedback is therefore negative. The reader is encouraged to apply the second procedure.

Exercise Suppose the top terminal of R_1 is tied to the drain of M_1 rather than the the drain of M_2. Determine the polarity of feedback.

Example 12.13 Determine the polarity of feedback in the circuit of Fig. 12.27.

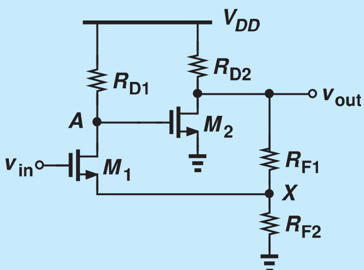


Figure 12.27

Solution If V_{in} goes up, I_{D1} tends to increase. Thus, V_A falls, V_{out} rises, and so does V_X. The rise in V_X tends to *reduce* I_{D1} (why?), thereby opposing the effect produced by V_{in}. The feedback is therefore negative.

Exercise Repeat the above example if M_2 is converted to a CG stage, i.e., its source is tied to node A and its gate to a bias voltage.

Example 12.14 Determine the polarity of feedback in the circuit of Fig. 12.28.

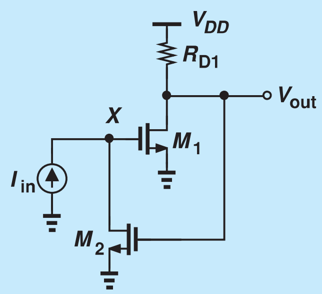


Figure 12.28

Solution If I_{in} goes up, V_X tends to rise (why?), thus raising I_{D1}. As a result, V_{out} falls and I_{D2} decreases, allowing V_X to *rise* (why?). Since the returned signal enhances the effect produced by I_{in}, the polarity of feedback is positive.

Exercise Repeat the above example if M_2 is a PMOS device (still operating as a CS stage). What happens if $R_D \to \infty$? Is this result expected?

Did you know?

While we focus on negative feedback in this book, positive feedback also finds its own applications. Suppose, for example, that a new ice cream store adopts a unique recipe and begins to attract many customers. If the store now adds one more scoop to each ice cream *for free*, then it will draw even more customers. That is, the store owner is providing a feedback signal that *enhances* the customer response (positive feedback). On the other hand, if the owner is greedy and *reduces* the size of the ice cream, then the store loses customers (negative feedback).

12.6 FEEDBACK TOPOLOGIES

Our study of different types of amplifiers in Section 12.3 and sense and return mechanisms in Section 12.4 suggests that four feedback topologies can be constructed. Each topology includes one of four types of amplifiers as its forward system. The feedback network must, of course, sense and return quantities compatible with those produced and sensed by the forward system, respectively. For example, a voltage amplifier requires that the feedback network sense and return voltages, whereas a transimpedance amplifier must employ a feedback network that senses a voltage and returns a current. In this section, we study each topology and compute the closed-loop characteristics such as gain and I/O impedances with the assumption that the feedback network is ideal (Fig. 12.25).

12.6.1 Voltage-Voltage Feedback

Illustrated in Fig. 12.29(a), this topology incorporates a voltage amplifier, requiring that the feedback network sense the output voltage and return a voltage to the subtractor. Recall from Section 12.4 that such a feedback network appears in *parallel* with the output and in *series* with the input,[6] ideally exhibiting an infinite input impedance and a zero output impedance.

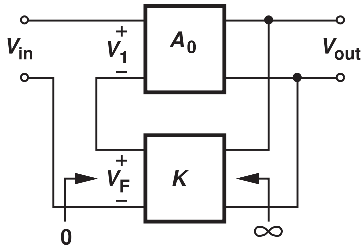


Figure 12.29 Voltage-voltage feedback.

We first calculate the closed-loop gain. Since

$$V_1 = V_{in} - V_F \tag{12.50}$$

$$V_{out} = A_0 V_1 \tag{12.51}$$

$$V_F = K V_{out}, \tag{12.52}$$

we have

$$V_{out} = A_0(V_{in} - K V_{out}), \tag{12.53}$$

1) GANANCIA

and hence

$$\frac{V_{out}}{V_{in}} = \frac{A_0}{1 + K A_0}, \tag{12.54}$$

an expected result.

Example 12.15

Determine the closed-loop gain of the circuit shown in Fig. 12.30, assuming $R_1 + R_2$ is very large.

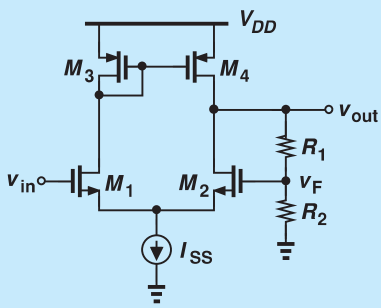


Figure 12.30

[6]For this reason, this type of feedback is also called the "series-shunt" topology, where the first term refers to the return mechanism at the input and the second term to the sense mechanism at the output.

Solution As evident from Examples 12.10 and 12.12, this topology indeed employs negative voltage-voltage feedback: the resistive network senses V_{out} with a high impedance (because $R_1 + R_2$ is very large), returning a voltage to the gate of M_2. As mentioned in Example 12.10, M_1 and M_2 serve as the input stage of the forward system and as a subtractor.

Noting that A_0 is the gain of the circuit consisting of M_1-M_4, we write from Chapter 10

$$A_0 = g_{mN}(r_{ON}||r_{OP}), \tag{12.55}$$

where the subscripts N and P refer to NMOS and PMOS devices, respectively.[7] With $K = R_2/(R_1 + R_2)$, we obtain

$$\frac{V_{out}}{V_{in}} = \frac{g_{mN}(r_{ON}||r_{OP})}{1 + \dfrac{R_2}{R_1 + R_2} g_{mN}(r_{ON}||r_{OP})}. \tag{12.56}$$

As expected, if the loop gain remains much greater than unity, then the closed-loop gain is approximately equal to $1/K = 1 + R_1/R_2$.

Exercise If $g_{mN} = 1/(100\,\Omega)$, $r_{ON} = 5\,\text{k}\Omega$, and $r_{OP} = 2\ \text{k}\Omega$, determine the required value of $R_2/(R_1 + R_2)$ for a closed loop gain of 4. Compare the result with the nominal value of $(R_2 + R_1)/R_2 = 4$.

2) R_{IN}

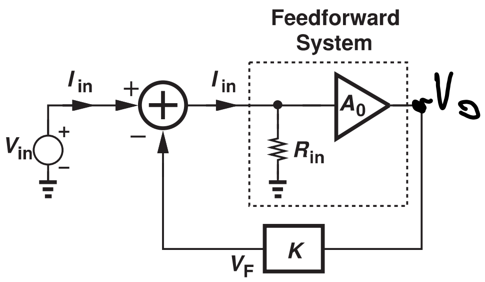


Figure 12.31 Calculation of input impedance.

In order to analyze the effect of feedback on the I/O impedances, we assume the forward system is a nonideal voltage amplifier (i.e., it exhibits finite I/O impedances) while the feedback network remains ideal. Depicted in Fig. 12.31 is the overall topology including a finite input resistance for the forward amplifier. Without feedback, of course, the entire input signal would appear across R_{in}, producing an input current of V_{in}/R_{in}.[8] With feedback, on the other hand, the voltage developed at the input of A_0 is equal to $V_{in} - V_F$ and also equal to $I_{in}R_{in}$. Thus,

$$I_{in}R_{in} = V_{in} - V_F \tag{12.57}$$

$V_f = kV_o = kA_oV_i = kA_oI_{in}R_{in}$

$$= V_{in} - (I_{in}R_{in})A_0K. \tag{12.58}$$

[7] We observe that $R_1 + R_2$ must be much greater than $r_{ON}||r_{OP}$ for this to hold. This serves as the definition of $R_1 + R_2$ being "very large."

[8] Note that V_{in} and R_{in} carry equal currents because the feedback network must appear in *series* with the input [Fig. 12.21(a)].

¿ $R_{oc} = r_o(1+g_m R_E)$?

It follows that

$$\frac{V_{in}}{I_{in}} = R_{in}(1 + KA_0). \tag{12.59}$$

Interestingly, negative feedback around a voltage amplifier *raises* the input impedance by the universal factor of one plus the loop gain. This impedance modification brings the circuit closer to an ideal voltage amplifier.

Example 12.16 Determine the input impedance of the stage shown in Fig. 12.32(a) if $R_1 + R_2$ is very large.

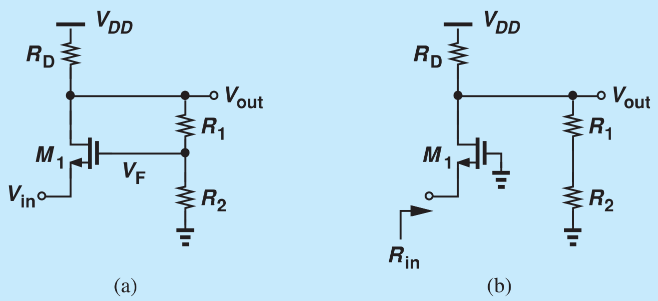


Figure 12.32

Solution We first open the loop to calculate R_{in} in Eq. (12.59). To open the loop, we break the gate of M_1 from the feedback signal and tie it to ground [Fig. 12.32(b)]:

$$R_{in} = \frac{1}{g_m}. \tag{12.60}$$

The closed-loop input impedance is therefore given by

$$\frac{V_{in}}{I_{in}} = \frac{1}{g_m}\left(1 + \frac{R_2}{R_1 + R_2} g_m R_D\right). \tag{12.61}$$

Exercise What happens if $R_2 \to \infty$? Is this result expected?

The effect of feedback on the output impedance can be studied with the aid of the diagram shown in Fig. 12.33, where the forward amplifier exhibits an output impedance of R_{out}. Expressing the error signal at the input of A_0 as $-V_F = -KV_X$, we write the output voltage of A_0 as $-KA_0V_X$ and hence

$$I_X = \frac{V_X - (-KA_0V_X)}{R_{out}}, \tag{12.62}$$

where the current drawn by the feedback network is neglected. Thus,

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$$\frac{V_X}{I_X} = \frac{R_{out}}{1 + KA_0}, \tag{12.63}$$

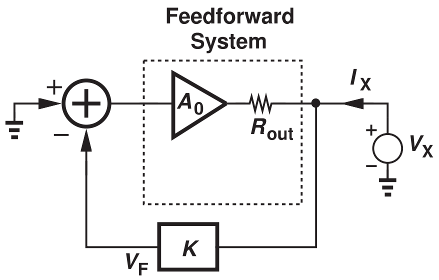


Figure 12.33 Calculation of output impedance.

revealing that negative feedback *lowers* the output impedance if the topology senses the output voltage. The circuit is now a better voltage amplifier—as predicted by our gain desensitization analysis in Section 12.2.

Example 12.17

Calculate the output impedance of the circuit shown in Fig. 12.34 if $R_1 + R_2$ is very large.

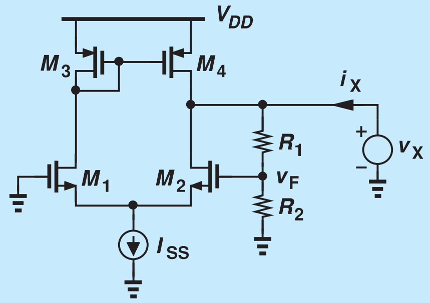


Figure 12.34

Solution

Recall from Example 12.15 that the open-loop output impedance is equal to $r_{ON}||r_{OP}$ and $KA_0 = [R_2/(R_1 + R_2)]g_{mN}(r_{ON}||r_{OP})$. Thus, the closed-loop output impedance, $R_{out,closed}$, is given by

$$R_{out,closed} = \frac{r_{ON}||r_{OP}}{1 + \dfrac{R_2}{R_1 + R_2} g_{mN}(r_{ON}||r_{OP})}. \tag{12.64}$$

If the loop gain is much greater than unity,

$$R_{out,closed} \approx \left(1 + \frac{R_1}{R_2}\right)\frac{1}{g_{mN}}, \tag{12.65}$$

a value independent of r_{ON} and r_{OP}. In other words, while the open-loop amplifier suffers from a *high* output impedance, the application of negative feedback lowers R_{out} to a multiple of $1/g_{mN}$.

Exercise What happens if $R_2 \to \infty$? Can you prove this result by direct analysis of the circuit?

In summary, voltage-voltage feedback lowers the gain and the output impedance by $1 + KA_0$ and raises the input impedance by the same factor.

12.6.2 Voltage-Current Feedback

Depicted in Fig. 12.35, this topology employs a transimpedance amplifier as the forward system, requiring that the feedback network sense the output voltage and return a current to the subtractor. In our terminology, the first term in "voltage-current feedback" refers to the quantity *sensed* at the output, and the second, to the quantity returned to the input. (This terminology is not standard.) Also, recall from Section 12.4 that such a feedback network must appear in parallel with the output and with the input,[9] ideally providing both an infinite input impedance and an infinite output impedance (why?). Note that the feedback factor in this case has a dimension of *conductance* because $K = I_F/V_{out}$.

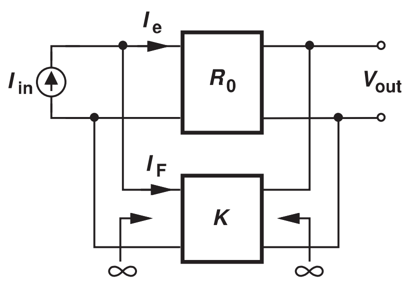
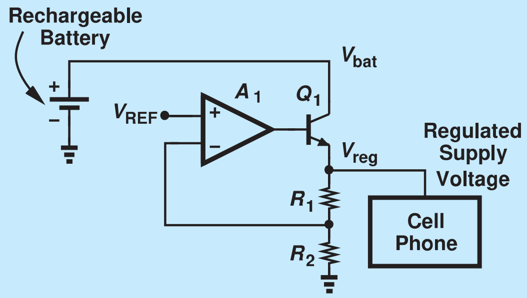


Figure 12.35 Voltage-current feedback.

Did you know?

Most electronic devices incorporate negative feedback in their "voltage regulators." Since the integrated circuits in a device such as a cell phone must operate with a tight supply voltage range, a regulator follows the battery so as to deliver a relatively constant voltage as the battery discharges and its voltage falls. The figure below shows an example, where Q_1, the resistor divider, and the op amp form a negative-feedback loop. To regulate the output voltage, V_{reg}, the loop divides it and compares the result with a reference voltage, V_{REF}. With a high loop gain, we must have $V_{reg}R_1/(R_1 + R_2) \approx V_{REF}$ and hence $V_{reg} \approx (1 + R_2/R_1)V_{REF}$. That is, the output is independent of V_{bat} and A_1. Of course, V_{REF} itself must remain constant. This voltage is provided by a Zener diode in discrete implementations or a "bandgap circuit" in integrated designs.



Voltage regulator using negative feedback.

We first compute the closed-loop gain, expecting to obtain a familiar result. Since $I_e = I_{in} - I_F$ and $V_{out} = I_eR_0$, we have

$$V_{out} = (I_{in} - I_F)R_0 \tag{12.66}$$

$$= (I_{in} - KV_{out})R_0, \tag{12.67}$$

and hence

$$\frac{V_{out}}{I_{in}} = \frac{R_0}{1 + KR_0}. \tag{12.68}$$

Example 12.18

For the circuit shown in Fig. 12.36(a), assume $\lambda = 0$ and R_F is very large and (a) prove that the feedback is negative; (b) calculate the open-loop gain; (c) calculate the closed-loop gain.

[9] For this reason, this type is also called "shunt-shunt" feedback.

Solution (a) If I_{in} increases, I_{D1} *decreases* and V_X rises. As a result, V_{out} falls, thereby reducing I_{RF}. Since the currents injected by I_{in} and R_F into the input node change in opposite directions, the feedback is negative.

(b) To calculate the open-loop gain, we consider the forward amplifier without the feedback network, exploiting the assumption that R_F is very large [Fig. 12.36(b)].

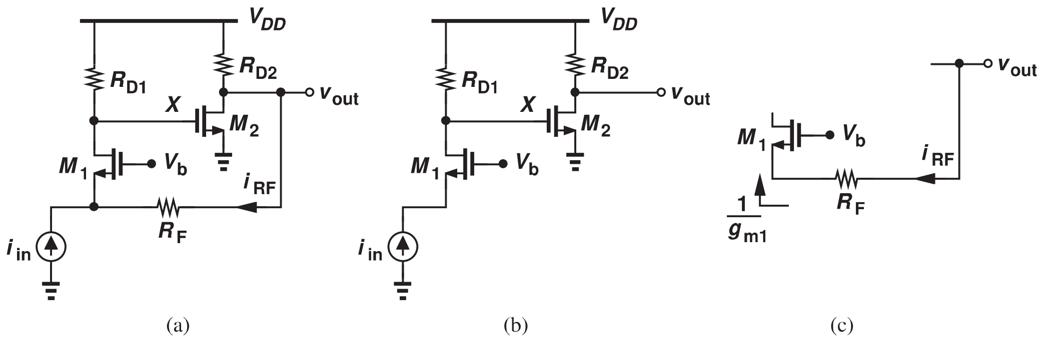


Figure 12.36

The transimpedance gain is given by the gain from I_{in} to V_X (i.e., R_{D1}) multiplied by that from V_X to V_{out} (i.e., $-g_{m2}R_{D2}$):

$$R_0 = R_{D1}(-g_{m2}R_{D2}). \tag{12.69}$$

Note that this result assumes $R_F \gg R_{D2}$ so that the gain of the second stage remains equal to $-g_{m2}R_{D2}$.

(c) To obtain the closed-loop gain, we first note that the current returned by R_F to the input is approximately equal to V_{out}/R_F if R_F is very large. To prove this, we consider a section of the circuit as in Fig. 12.36(c) and write

$$I_{RF} = \frac{V_{out}}{R_F + \dfrac{1}{g_{m1}}}. \tag{12.70}$$

Thus, if $R_F \gg 1/g_{m1}$, the returned current is approximately equal to V_{out}/R_F. (We say "R_F operates as a current source.") That is, $K = -1/R_F$, where the negative sign arises from the direction of the current *drawn* by R_F from the input node with respect to that in Fig. 12.35. Forming $1 + KR_0$, we express the closed-loop gain as

$$\left.\frac{V_{out}}{I_{in}}\right|_{closed} = \frac{-g_{m2}R_{D1}R_{D2}}{1 + \dfrac{g_{m2}R_{D1}R_{D2}}{R_F}}, \tag{12.71}$$

which reduces to $-R_F$ if $g_{m2}R_{D1}R_{D2} \gg R_F$.

It is interesting to note that the assumption that R_F is very large translates to two conditions in this example: $R_F \gg R_{D2}$ and $R_F \gg 1/g_{m1}$. The former arises from the output network calculations and the latter from the input network calculations. What happens if one or both of these assumptions are not valid? We deal with this (relatively common) situation in Section 12.7.

Exercise What is the closed-loop gain if $R_{D1} \to \infty$? How can this result be interpreted? (Hint: the infinite open-loop gain creates a virtual ground node at the source of M_1.)

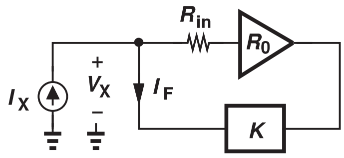


Figure 12.37 Calculation of input impedance.

We now proceed to determine the closed-loop I/O impedances. Modeling the forward system as an ideal transimpedance amplifier but with a finite input impedance R_{in} (Section 12.3), we construct the test circuit shown in Fig. 12.37. Since the current flowing through R_{in} is equal to V_X/R_{in} (why?), the forward amplifier produces an output voltage equal to $(V_X/R_{in})R_0$ and hence

$$I_F = K\frac{V_X}{R_{in}}R_0. \tag{12.72}$$

Writing a KCL at the input node thus yields

$$I_X - K\frac{V_X}{R_{in}}R_0 = \frac{V_X}{R_{in}} \tag{12.73}$$

and hence

$$\frac{V_X}{I_X} = \frac{R_{in}}{1+KR_0}. \tag{12.74}$$

That is, a feedback loop returning current to the input *lowers* the input impedance by a factor of one plus the loop gain, bringing the circuit closer to an ideal "current sensor."

Example 12.19 Determine the closed-loop input impedance of the circuit studied in Example 12.18.

Solution The open-loop amplifier shown in Fig. 12.36(b) exhibits an input impedance $R_{in} = 1/g_{m1}$ because R_F is assumed to be very large. With $1 + KR_0$ from the denominator of Eq. (12.71), we obtain

$$R_{in,closed} = \frac{1}{g_{m1}} \cdot \frac{1}{1 + \dfrac{g_{m2}R_{D1}R_{D2}}{R_F}}. \tag{12.75}$$

Exercise Explain what happens if $R_{D1} \to \infty$ and why.

From our study of voltage-voltage feedback in Section 12.6.1, we postulate that voltage-current feedback too lowers the output impedance because a feedback loop "regulating" the output voltage tends to stabilize it despite load impedance variations. Drawing the circuit as shown in Fig. 12.38, where the input current source is set to zero and R_{out} models the open-loop output resistance, we observe that the feedback network produces a current of $I_F = KV_X$. Upon flowing through the forward amplifier, this current translates

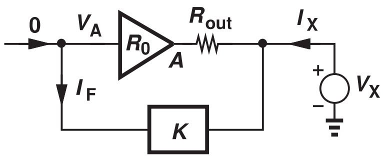


Figure 12.38 Calculation of output impedance.

to $V_A = -KV_XR_0$ and hence

$$I_X = \frac{V_X - V_A}{R_{out}} \tag{12.76}$$

$$= \frac{V_X + KV_XR_0}{R_{out}}, \tag{12.77}$$

where the current drawn by the feedback network is neglected. Thus,

$$\frac{V_X}{I_X} = \frac{R_{out}}{1 + KR_0}, \tag{12.78}$$

an expected result.

Example 12.20 Calculate the closed-loop output impedance of the circuit studied in Example 12.18.

Solution From the open-loop circuit in Fig. 12.36(b), we have $R_{out} \approx R_{D2}$ because R_F is assumed very large. Writing $1 + KR_0$ from the denominator of Eq. (12.71) gives

$$R_{out,closed} = \frac{R_{D2}}{1 + \dfrac{g_{m2}R_{D1}R_{D2}}{R_F}}. \tag{12.79}$$

Exercise Explain what happens if $R_{D1} \to \infty$ and why.

12.6.3 Current-Voltage Feedback

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Shown in Fig. 12.39(a), this topology incorporates a transconductance amplifier, requiring that the feedback network sense the output current and return a voltage to the subtractor. Again, in our terminology, the first term in "current-voltage feedback" refers to the

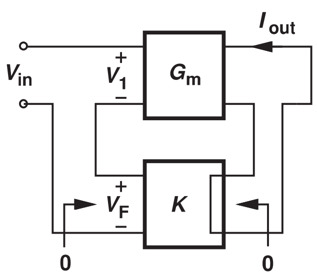

Figure 12.39 Current-voltage feedback.

quantity sensed at the output, and the second, to the quantity returned to the input. Recall from Section 12.4 that such a feedback network must appear in series with the output and with the input,[10] ideally exhibiting zero input and output impedances. Note that the feedback factor in this case has a dimension of *resistance* because $K = V_F/I_{out}$.

Let us first confirm that the closed-loop gain is equal to the open-loop gain divided by one plus the loop gain. Since the forward system produces a current equal to $I_{out} = G_m(V_{in} - V_F)$ and since $V_F = KI_{out}$, we have

$$I_{out} = G_m(V_{in} - KI_{out}) \tag{12.80}$$

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and hence

$$\frac{I_{out}}{V_{in}} = \frac{G_m}{1 + KG_m}. \tag{12.81}$$

Example 12.21

We wish to deliver a well-defined current to a laser diode as shown in Fig. 12.40(a),[11] but the transconductance of M_1 is poorly controlled. For this reason, we "monitor" the current by inserting a small resistor R_M in series, sensing the voltage across R_M, and returning the result to the input of an op amp [Fig. 12.40(b)]. Estimate I_{out} if the op amp provides a very high gain. Calculate the closed-loop gain for the implementation shown in Fig. 12.40(c).

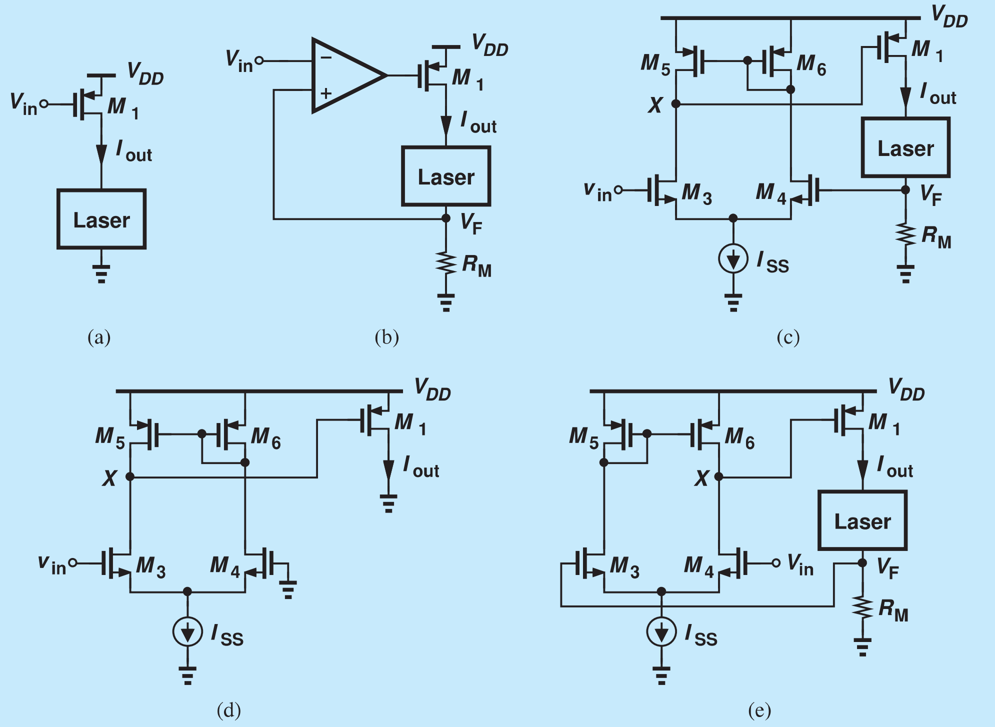

Figure 12.40

[10]For this reason, this type is also called "series-series" feedback.

[11]Laser diodes convert electrical signals to optical signals and are widely used in DVD players, long-distance communications, etc.

Solution If the gain of the op amp is very high, the difference between V_{in} and V_F is very small. Thus, R_M sustains a voltage equal to V_{in} and hence

$$I_{out} \approx \frac{V_{in}}{R_M}. \tag{12.82}$$

We now determine the open-loop gain of the transistor-level implementation in Fig. 12.40(c). The forward amplifier can be identified as shown in Fig. 12.40(d), where the gate of M_4 is grounded because the feedback signal (voltage) is set to zero. Since $I_{out} = -g_{m1}V_X$ (why?) and $V_X = -g_{m3}(r_{O3}||r_{O5})V_{in}$, we have

$$G_m = g_{m1}g_{m3}(r_{O3}||r_{O5}). \tag{12.83}$$

The feedback factor $K = V_F/I_{out} = R_M$. Thus,

$$\left.\frac{I_{out}}{V_{in}}\right|_{closed} = \frac{g_{m1}g_{m3}(r_{O3}||r_{O5})}{1 + g_{m1}g_{m3}(r_{O3}||r_{O5})R_M}. \tag{12.84}$$

Note that if the loop gain is much greater than unity, then

$$\left.\frac{I_{out}}{V_{in}}\right|_{closed} \approx \frac{1}{R_M}. \tag{12.85}$$

We must now answer two questions. First, why is the drain of M_1 *shorted* to ground in the open-loop test? The simple answer is that, if this drain is left open, then $I_{out} = 0$! But, more fundamentally, we can observe a duality between this case and that of voltage outputs, e.g., in Fig. 12.36. If driving no load, the output port of a voltage amplifier is left open. Similarly, if driving no load, the output port of a circuit delivering a current must be shorted to ground.

Second, why is the active-load amplifier in Fig. 12.40(c) drawn with the diode-connected device on the right? This is to ensure negative feedback. For example, if V_{in} goes up, V_X goes down (why?), M_1 provides a greater current, and the voltage drop across R_M rises, thereby steering a larger fraction of I_{SS} to M_4 and opposing the effect of the change in V_{in}. Alternatively, the circuit can be drawn as shown in Fig. 12.40(e).

Exercise Suppose V_{in} is a sinusoid with a peak amplitude of 100 mV. Plot V_F and the current through the laser as a function of time if $R_M = 10\ \Omega$ and $G_m = 1/(0.5\ \Omega)$. Is the voltage at the gate of M_1 necessarily a sinusoid?

From our analysis of other feedback topologies in Sections 12.6.1 and 12.6.2, we postulate that current-voltage feedback increases the input impedance by a factor of $1 + KG_m$. In fact, the test circuit shown in Fig. 12.41(a) is similar to that in Fig. 12.31—except that the forward system is denoted by G_m rather than A_0. Thus, Eq. (12.59) can be rewritten as

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$$\frac{V_{in}}{I_{in}} = R_{in}(1 + KG_m). \tag{12.86}$$

The output impedance is calculated using the test circuit of Fig. 12.41(b). Note that, in contrast to the cases in Figs. 12.33 and 12.38, the test voltage source is inserted in *series* with the output port of the forward amplifier and the input port of the feedback network. The voltage developed at port A is equal to $-KI_X$ and the current drawn by the G_m stage

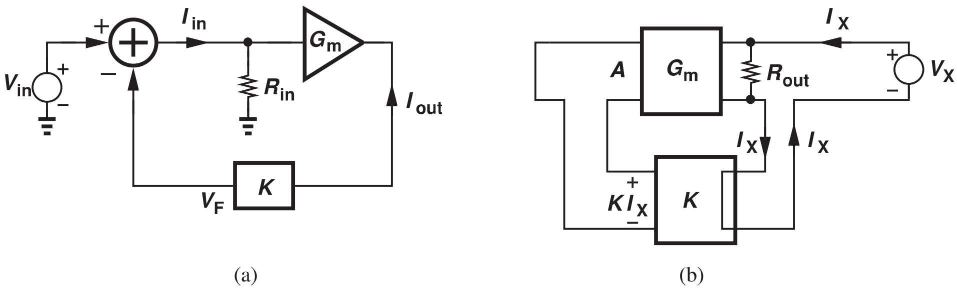

Figure 12.41 Calculation of (a) input and (b) output impedances.

equal to $-KG_mI_X$. Since the current flowing through R_{out} is given by V_X/R_{out}, a KCL at the output node yields

$$I_X = \frac{V_X}{R_{out}} - KG_mI_X \tag{12.87}$$

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and hence

$$\frac{V_X}{I_X} = R_{out}(1 + KG_m). \tag{12.88}$$

Interestingly, a negative feedback loop sensing the output current *raises* the output impedance, bringing the circuit closer to an ideal current generator. As in other cases studied thus far, this occurs because negative feedback tends to regulate the output quantity that it senses.

Example 12.22 An alternative approach to regulating the current delivered to a laser diode is shown in Fig. 12.42(a). As in the circuit of Fig. 12.40(b), the very small resistor R_M monitors the current, generating a proportional voltage and feeding it back to the subtracting device, M_1. Determine the closed-loop gain and I/O impedances of the circuit.

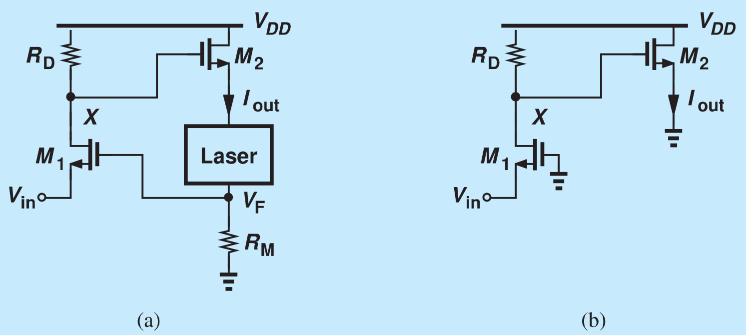


Figure 12.42

Solution Since R_M is very small, the open-loop circuit reduces to that shown in Fig. 12.42(b), where the gain can be expressed as

$$G_m = \frac{V_X}{V_{in}} \cdot \frac{I_{out}}{V_X} \tag{12.89}$$

$$= g_{m1}R_D \cdot g_{m2}. \tag{12.90}$$

The input impedance is equal to $1/g_{m1}$ and the output impedance equal to $1/g_{m2}$.[12] The feedback factor is equal to R_M, yielding

$$\left.\frac{I_{out}}{V_{in}}\right|_{closed} = \frac{g_{m1}g_{m2}R_D}{1 + g_{m1}g_{m2}R_D R_M}, \tag{12.91}$$

which reduces to $1/R_M$ if the loop gain is much greater than unity. The input impedance rises by a factor of $1 + G_m R_M$:

$$R_{in,closed} = \frac{1}{g_{m1}}(1 + g_{m1}g_{m2}R_D R_M), \tag{12.92}$$

and so does the output impedance (i.e., that seen by the laser):

$$R_{out,closed} = \frac{1}{g_{m2}}(1 + g_{m1}g_{m2}R_D R_M). \tag{12.93}$$

Exercise If an input impedance of 500 Ω and an output impedance of 5 kΩ are desired, determine the required values of g_{m1} and g_{m2}. Assume $R_D = 1$ kΩ and $R_M = 100$ Ω.

Example 12.23 A student attempts to calculate the output impedance of the current-voltage feedback topology with the aid of circuit depicted in Fig. 12.43. Explain why this topology is an incorrect representation of the actual circuit.

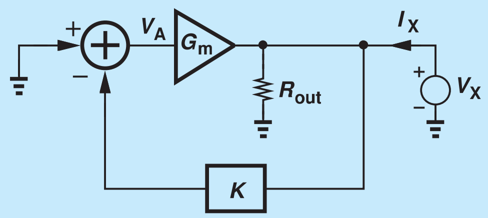


Figure 12.43

Solution If sensing the output current, the feedback network must remain in *series* with the output port of the forward amplifier, and so must the test voltage source. In other words, the output current of the forward system must be equal to both the input current of the feedback network and the current drawn by V_X [as in Fig. 12.41(b)]. In the arrangement of Fig. 12.43, however, these principles are violated because V_X is placed in parallel with the output.[13]

Exercise Apply the above (incorrect) test to the circuit of Fig. 12.42 and examine the results.

[12]To measure the output impedance, the test voltage source must be placed in series with the output wire.

[13]If the feedback network is ideal and hence has a zero input impedance, then V_X must supply an infinite current.

12.6.4 Current-Current Feedback

From the analysis of the first three feedback topologies, we predict that this type lowers the gain, raises the output impedance, and lowers the input impedance, all by a factor of one plus the loop gain.

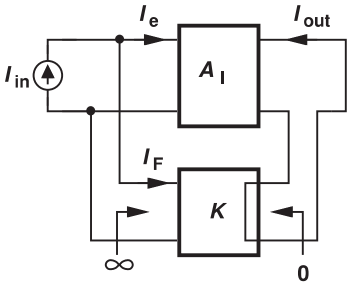


Figure 12.44 Current-current feedback.

As shown in Fig. 12.44, current-current feedback senses the output in series and returns the signal in parallel with the input. The forward system has a current gain of A_I and the feedback network a dimensionless gain of $K = I_F/I_{out}$. Given by $I_{in} - I_F$, the current entering the forward amplifier yields

$$I_{out} = A_I(I_{in} - I_F) \tag{12.94}$$

$$= A_I(I_{in} - KI_{out}) \tag{12.95}$$

and hence

$$\frac{I_{out}}{I_{in}} = \frac{A_I}{1 + KA_I}. \tag{12.96}$$

The input impedance of the circuit is calculated with the aid of the arrangement depicted in Fig. 12.45. As in the case of voltage-current feedback (Fig. 12.37), the input impedance of the forward amplifier is modeled by a series resistor, R_{in}. Since the current flowing through R_{in} is equal to V_X/R_{in}, we have $I_{out} = A_I V_X/R_{in}$ and hence $I_F = KA_I V_X/R_{in}$. A KCL at the input node therefore gives

$$I_X = \frac{V_X}{R_{in}} + I_F \tag{12.97}$$

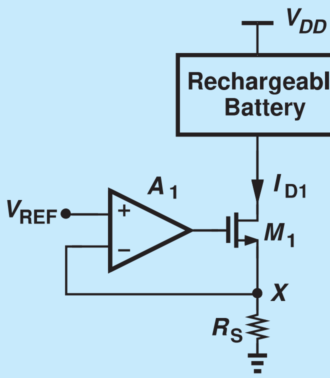
$$= \frac{V_X}{R_{in}} + KA_I\frac{V_X}{R_{in}}. \tag{12.98}$$

That is,

$$\frac{V_X}{I_X} = \frac{R_{in}}{1 + KA_I}. \tag{12.99}$$

Did you know?

While less intuitive than voltage regulation, current regulation is in fact quite common, namely, in battery chargers. A battery charged with a constant voltage may have a short lifetime. (Imagine what happens if two ideal voltage sources of unequal values are placed in parallel!) For this reason, we prefer to charge batteries with a constant current. The figure below shows an example where op amp A_1 forces V_X to be close to the reference voltage, V_{REF}. We say R_S and A_1 monitor and stabilize the source current of M_1. The drain current of M_1 drives the battery. Of course, as the battery charges, its voltage eventually exceeds the permitted value, causing damage. The charger must therefore incorporate additional circuitry for overcharge protection.



Current regulation in a battery charger.

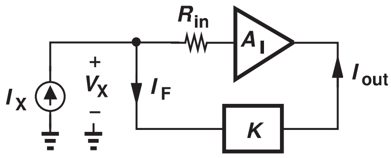


Figure 12.45 Calculation of input impedance.

For the output impedance, we utilize the test circuit shown in Fig. 12.46, where the input is left open and V_X is inserted in series with the output port. Since $I_F = KI_X$, the forward amplifier produces an output current equal to $-KA_I I_X$. Noting that R_{out} carries a current of V_X/R_{out} and writing a KCL at the output node, we have

$$I_X = \frac{V_X}{R_{out}} - KA_I I_X. \tag{12.100}$$

It follows that

$$\frac{V_X}{I_X} = R_{out}(1 + KA_I). \tag{12.101}$$

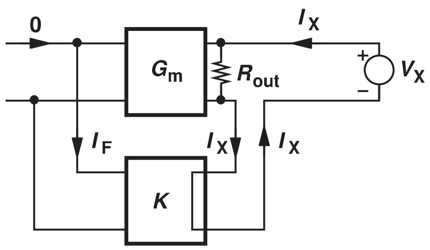


Figure 12.46 Calculation of output impedance.

Example 12.24

Consider the circuit shown in Fig. 12.47(a), where the output current delivered to a laser diode is regulated by negative feedback. Prove that the feedback is negative and compute the closed-loop gain and I/O impedances if R_M is very small and R_F very large.

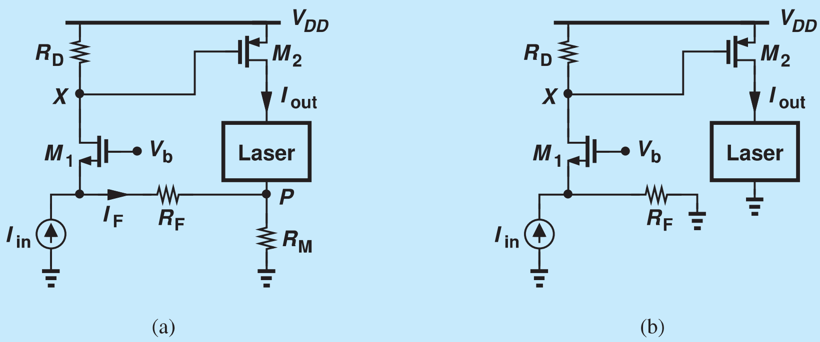


Figure 12.47

Solution Suppose I_{in} increases. Then, the source voltage of M_1 tends to rise, and so does its drain voltage (why?). As a result, the overdrive of M_2 decreases, I_{out} and hence V_P fall, and I_F *increases*, thereby lowering the source voltage of M_1. Since the feedback signal, I_F, opposes the effect produced by I_{in}, the feedback is negative.

We must now analyze the open-loop system. Since R_M is very small, we assume V_P remains near zero, arriving at the open-loop circuit depicted in Fig. 12.47(b). The assumption that R_F is very large ($\gg 1/g_{m1}$) indicates that almost all of I_{in} flows through M_1 and R_D, thus generating $V_X = I_{in}R_D$ and hence

$$I_{out} = -g_{m2}V_X \tag{12.102}$$

$$= -g_{m2}R_D I_{in}. \tag{12.103}$$

That is,

$$A_I = -g_{m2}R_D. \tag{12.104}$$

The input impedance is approximately equal to $1/g_{m1}$ and the output impedance is equal to r_{O2}.

To obtain the closed-loop parameters, we must compute the feedback factor, I_F/I_{out}. Recall from Example 12.18 that the current returned by R_F can be approximated as $-V_P/R_F$ if $R_F \gg 1/g_{m1}$. We also note that $V_P = I_{out}R_M$, concluding that

$$K = \frac{I_F}{I_{out}} \tag{12.105}$$

$$= \frac{-V_P}{R_F} \cdot \frac{1}{I_{out}} \tag{12.106}$$

$$= -\frac{R_M}{R_F}. \tag{12.107}$$

The closed-loop parameters are therefore given by:

$$A_{I,closed} = \frac{-g_{m2}R_D}{1 + g_{m2}R_D\dfrac{R_M}{R_F}} \tag{12.108}$$

$$R_{in,closed} = \frac{1}{g_{m1}} \cdot \frac{1}{1 + g_{m2}R_D\dfrac{R_M}{R_F}} \tag{12.109}$$

$$R_{out,closed} = r_{O2}\left(1 + g_{m2}R_D\frac{R_M}{R_F}\right). \tag{12.110}$$

Note that if $g_{m2}R_D R_M/R_F \gg 1$, then the closed-loop gain is simply given by $-R_F/R_M$.

Exercise Noting that $R_{out}|_{closed}$ is the impedance seen by the laser in the closed-loop circuit, construct a Norton equivalent for the entire circuit that drives the laser.

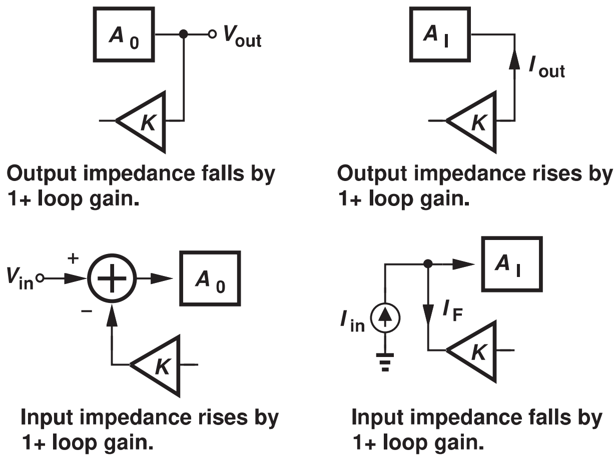


Figure 12.48 Effect of feedback on input and output impedances.

The effect of feedback on the input and output impedances of the forward amplifier is summarized in Fig. 12.48.

12.7 EFFECT OF NONIDEAL I/O IMPEDANCES

Our study of feedback topologies in Section 12.6 has been based on idealized models for the feedback network, always assuming that the I/O impedances of this network are very large or very small depending on the type of feedback. In practice, however, the finite I/O impedances of the feedback network may considerably alter the performance of the circuit, thereby necessitating analysis techniques to account for these effects. In such cases, we say the feedback network "loads" the forward amplifier and the "loading effects" must be determined.

Before delving into the analysis, it is instructive to understand the difficulty in the context of an example.

Example 12.25 Suppose in the circuit of Example 12.7, $R_1 + R_2$ is *not* much greater than R_D. How should we analyze the circuit?

Solution In Example 12.7, we constructed the open-loop circuit by simply neglecting the effect of $R_1 + R_2$. Here, on the other hand, $R_1 + R_2$ tends to reduce the open-loop gain because it appears in parallel with R_D. We therefore surmise that the open-loop circuit must be configured as shown in Fig. 12.49, with the open-loop gain given by

$$A_O = g_{m1}[R_D||(R_1 + R_2)], \tag{12.111}$$

and the output impedance

$$R_{out,open} = R_D||(R_1 + R_2). \tag{12.112}$$

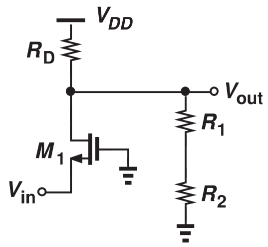


Figure 12.49

Other forward and feedback parameters are identical to those calculated in Example 12.7. Thus,

$$A_{v,closed} = \frac{g_{m1}[R_D||(R_1+R_2)]}{1+\dfrac{R_2}{R_1+R_2}g_{m1}[R_D||(R_1+R_2)]} \tag{12.113}$$

$$R_{in,closed} = \frac{1}{g_{m1}}\left\{1+\frac{R_2}{R_1+R_2}g_{m1}[R_D||(R_1+R_2)]\right\} \tag{12.114}$$

$$R_{out,closed} = \frac{R_D||(R_1+R_2)}{1+\dfrac{R_2}{R_1+R_2}g_{m1}[R_D||(R_1+R_2)]}. \tag{12.115}$$

Exercise Repeat the above example if R_D is replaced with an ideal current source.

The above example easily lends itself to intuitive inspection. But many other circuits do not. To gain more confidence in our analysis and deal with more complex circuits, we must develop a systematic approach.

12.7.1 Inclusion of I/O Effects

We present a methodology here that allows the analysis of the four feedback topologies even if the I/O impedances of the forward amplifier or the feedback network depart from their ideal values. The methodology is based on a formal proof that is somewhat beyond the scope of this book and can be found in [1].

Our methodology proceeds in six steps:

1. Identify the forward amplifier.
2. Identify the feedback network.
3. Break the feedback network according to the rules described below.
4. Calculate the open-loop parameters.
5. Determine the feedback factor according to the rules described below.
6. Calculate the closed-loop parameters.

Rules for Breaking the Feedback Network The third step is carried out by "duplicating" the feedback network at both the input and the output of the overall system.

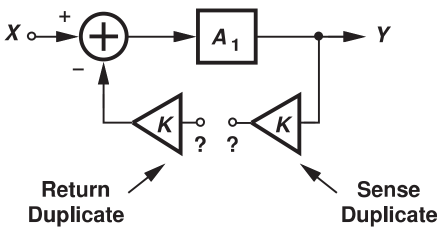


Figure 12.50 Method of breaking the feedback loop.

Illustrated in Fig. 12.50, the idea is to "load" both the input and the output of the forward amplifier by proper copies of the feedback network. The copy tied to the output is called the "sense duplicate" and that connected to the input, the "return duplicate." We must also decide what to do with the output port of the former and the input port of the latter, i.e., whether to short or open these ports. This is accomplished through the use of the "termination" rules depicted in Fig. 12.51. For example, for voltage-voltage feedback [Fig. 12.51(a)], the output port of the sense replica is left *open* while the input of the return duplicate is shorted. Similarly, for voltage-current feedback [Fig. 12.51(b)], both the output port of the sense duplicate and the input port of the return duplicate are shorted.

The formal proof of these concepts is given in [1] but it is helpful to remember these rules based on the following intuitive (but not quite rigorous) observations. In an ideal situation, a feedback network sensing an output *voltage* is driven by a zero impedance, namely, the output impedance of the forward amplifier. Thus, the input port of the *return* duplicate is *shorted*. Moreover, a feedback network returning a voltage to the input ideally sees an infinite impedance, namely, the input impedance of the forward amplifier. Thus, the output port of the *sense* duplicate is left open. Similar observations apply to the other three cases.

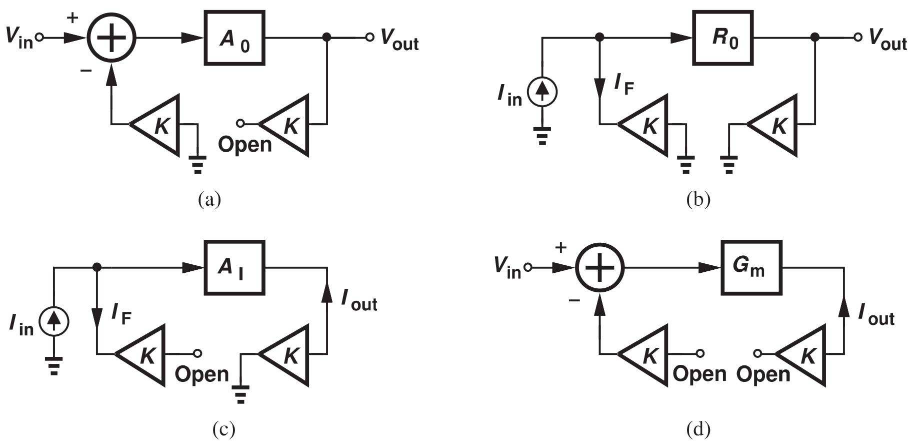


Figure 12.51 Proper termination of duplicates in (a) voltage-voltage, (b) voltage-current, (c) current-current, and (d) current-voltage feedback.

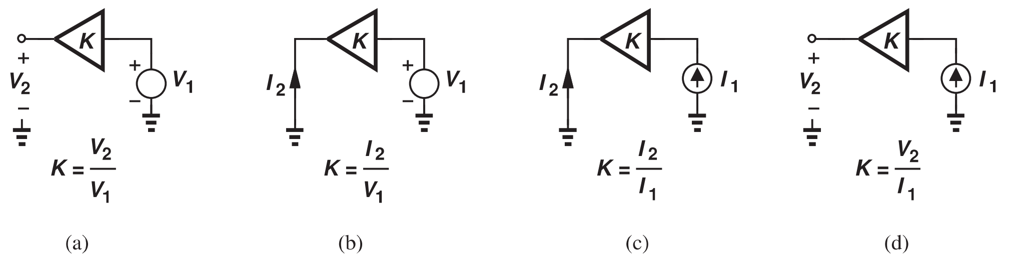


Figure 12.52 Calculation of feedback factor for (a) voltage-voltage, (b) voltage-current, (c) current-current, and (d) current-voltage feedback.

Calculation of Feedback Factor The fifth step entails the calculation of the feedback factor, a task requiring the rules illustrated in Fig. 12.52. Depending on the type of feedback, the output port of the feedback network is shorted or opened, and the ratio of the output current or voltage to the input is defined as the feedback factor. For example, in a voltage-voltage feedback topology, the output port of the feedback network is open [Fig. 12.52(a)] and $K = V_2/V_1$.

The proof of these rules is provided in [1], but an intuitive view can also be developed. First, the stimulus (voltage or current) applied to the *input* of the feedback network is of the same type as the quantity sensed at the *output* of the forward amplifier. Second, the output port of the feedback network is opened (shorted) if the returned quantity is a voltage (current)—just as in the case of the sense duplicates in Fig. 12.51. Of course, if the output port of the feedback network is left open, the quantity of interest is a voltage, V_2. Similarly, if the port is shorted, the quantity of interest is a current, I_2.

In order to reinforce the above principles, we reconsider the examples studied thus far in this chapter and determine the closed-loop parameters if I/O impedance effects are not negligible.

Example 12.26

Analyze the amplifier depicted in Fig. 12.53(a) if $R_1 + R_2$ is not much less than R_D.

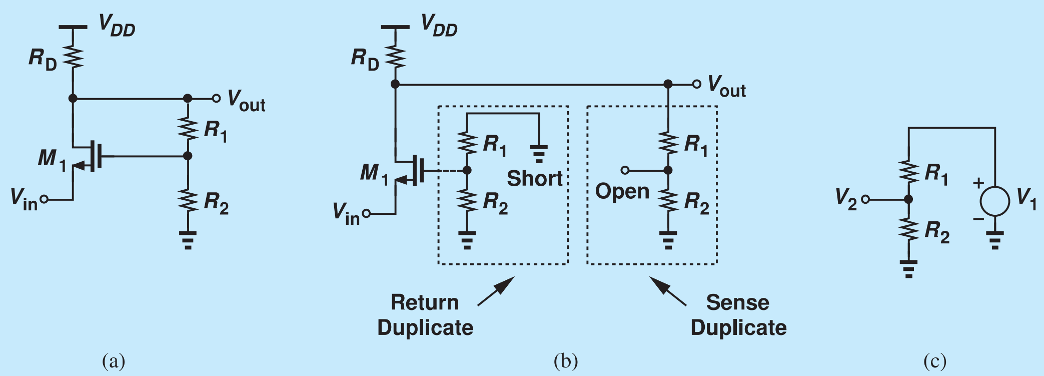


Figure 12.53

Solution We identify the forward system as M_1 and R_D, and the feedback network as R_1 and R_2. We construct the open-loop circuit according to Fig. 12.51(a), as shown in Fig. 12.53(b). Note that the feedback network appears twice. The sense duplicate output port is left open and the input port of the return duplicate is shorted. The open-loop parameters of this topology were computed in Example 12.25.

To determine the feedback factor, we follow the rule in Fig. 12.52(a) to form the circuit shown in Fig. 12.53(c), arriving at

$$K = \frac{V_2}{V_1} \tag{12.116}$$

$$= \frac{R_2}{R_1 + R_2}. \tag{12.117}$$

It follows that

$$KA_0 = \frac{R_2}{R_1 + R_2} g_{m1}[R_D||(R_1 + R_2)], \tag{12.118}$$

and hence

$$A_{v,closed} = \frac{g_{m1}[R_D||(R_1 + R_2)]}{1 + \dfrac{R_2}{R_1 + R_2} g_{m1}[R_D||(R_1 + R_2)]} \tag{12.119}$$

$$R_{in,closed} = \frac{1}{g_{m1}}\left\{1 + \frac{R_2}{R_1 + R_2} g_{m1}[R_D||(R_1 + R_2)]\right\} \tag{12.120}$$

$$R_{out,closed} = \frac{R_D||(R_1 + R_2)}{1 + \dfrac{R_2}{R_1 + R_2} g_{m1}[R_D||(R_1 + R_2)]}. \tag{12.121}$$

Obtained through our general methodology, these results agree with those found by inspection in Example 12.25.

Exercise Repeat the above analysis if R_D is replaced with an ideal current source.

Example 12.27 Analyze the circuit of Fig. 12.54(a) if $R_1 + R_2$ is not much greater than $r_{OP}||r_{ON}$.

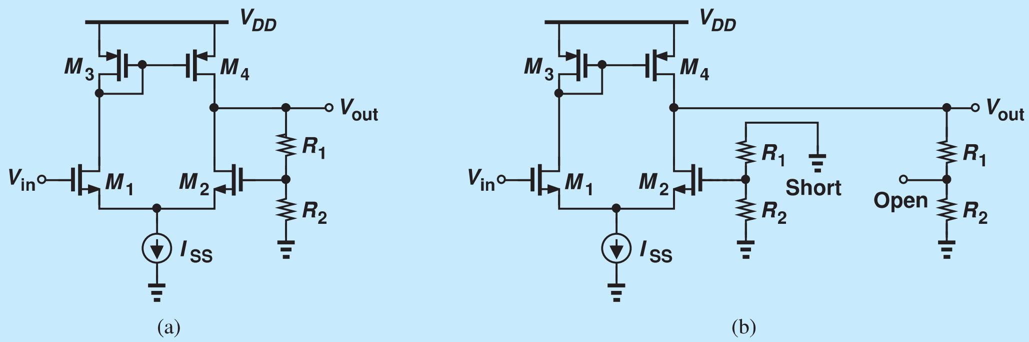


Figure 12.54

Solution Here, M_1-M_4 constitute the forward amplifier, and R_1 and R_2 the feedback network. The loop is broken in a manner similar to that in Example 12.26 because the type of feedback is the same [Fig. 12.54(b)]. The open-loop parameters are therefore given by

$$A_0 = g_{mN}[r_{ON}||r_{OP}||(R_1 + R_2)] \tag{12.122}$$

$$R_{in,open} = \infty \tag{12.123}$$

$$R_{out,open} = r_{ON}||r_{OP}||(R_1 + R_2). \tag{12.124}$$

The test circuit for calculation of the feedback factor is identical to that in Fig. 12.53(c), yielding

$$K = \frac{R_2}{R_1 + R_2}. \tag{12.125}$$

It follows that

$$\left.\frac{V_{out}}{V_{in}}\right|_{closed} = \frac{g_{mN}[r_{ON}||r_{OP}||(R_1 + R_2)]}{1 + \dfrac{R_2}{R_1 + R_2} g_{mN}[r_{ON}||r_{OP}||(R_1 + R_2)]} \tag{12.126}$$

$$R_{in,closed} = \infty \tag{12.127}$$

$$R_{out,closed} = \frac{r_{ON}||r_{OP}||(R_1 + R_2)}{1 + \dfrac{R_2}{R_1 + R_2} g_{mN}[r_{ON}||r_{OP}||(R_1 + R_2)]}. \tag{12.128}$$

Exercise Repeat the above example if a load resistor of R_L is tied between the output of the circuit and ground.

Example 12.28 Analyze the circuit of Fig. 12.55(a).

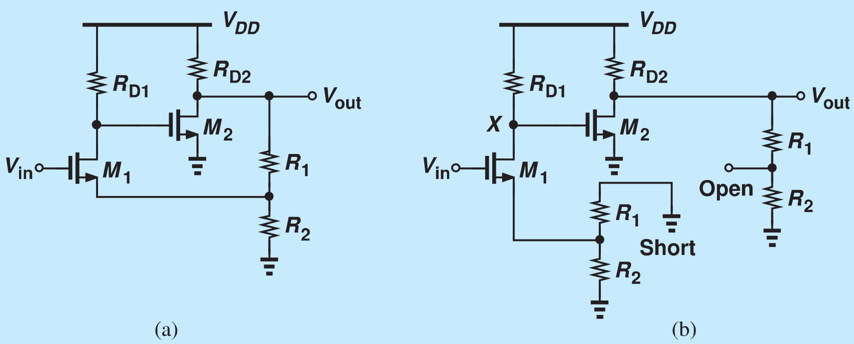


Figure 12.55

Solution We identify the forward system as M_1, R_{D1}, M_2, and R_{D2}. The feedback network consists of R_1 and R_2 and returns a voltage to the source of the subtracting transistor, M_1. In a manner similar to the above two examples, the open-loop circuit is constructed as shown in Fig. 12.55(b). Note that M_1 is now degenerated by $R_1||R_2$. Writing $A_0 = (V_X/V_{in})(V_{out}/V_X)$, we have

$$A_0 = \frac{-R_{D1}}{\frac{1}{g_m} + R_1||R_2} \cdot \{-g_{m2}[R_{D2}||(R_1 + R_2)]\} \tag{12.129}$$

$$R_{in,open} = \infty \tag{12.130}$$

$$R_{out,open} = R_{D2}||(R_1 + R_2). \tag{12.131}$$

As in the above example, the feedback factor is equal to $R_2/(R_1 + R_2)$, yielding

$$\left.\frac{V_{out}}{V_{in}}\right|_{closed} = \frac{A_0}{1 + \frac{R_2}{R_1 + R_2}A_0} \tag{12.132}$$

$$R_{in,closed} = \infty \tag{12.133}$$

$$R_{out,closed} = \frac{R_{D2}||(R_1 + R_2)}{1 + \frac{R_2}{R_1 + R_2}A_0}, \tag{12.134}$$

where A_0 is given by Eq. (12.129).

Exercise Repeat the above example if M_2 is degenerated by a resistor of value R_S.

Example 12.29 Analyze the circuit of Fig. 12.56(a), assuming that R_F is not very large.

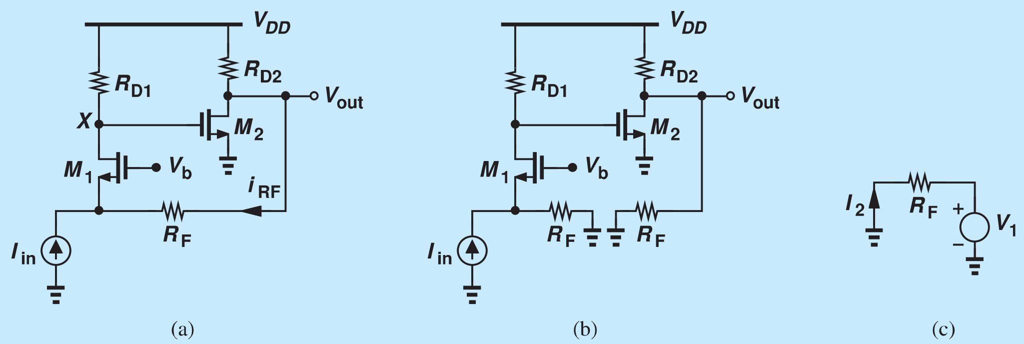


Figure 12.56

Solution As a voltage-current feedback topology, this circuit must be handled according to the rules in Figs. 12.51(b) and 12.52(b). The forward amplifier is formed by M_1, R_{D1}, M_2, and R_{D2}. The feedback network simply consists of R_F. The loop is opened as shown in Fig. 12.56(b), where, from Fig. 12.51(b), the output port of the sense duplicate is *shorted*. Since I_{in} splits between R_F and M_1, we have

$$V_X = I_{in}\frac{R_F R_{D1}}{R_F + \dfrac{1}{g_{m1}}}. \tag{12.135}$$

Noting that $R_0 = V_{out}/I_{in} = (V_X/I_{in})(V_{out}/V_X)$, we write

$$R_0 = \frac{R_F R_{D1}}{R_F + \dfrac{1}{g_{m1}}} \cdot [-g_{m2}(R_{D2}||R_F)]. \tag{12.136}$$

The open-loop input and output impedances are respectively given by

$$R_{in,open} = \frac{1}{g_{m1}}||R_F \tag{12.137}$$

$$R_{out,open} = R_{D2}||R_F. \tag{12.138}$$

To obtain the feedback factor, we follow the rule in Fig. 12.52(b) and construct the test circuit shown in Fig. 12.56(c), obtaining

$$K = \frac{I_2}{V_1} \tag{12.139}$$

$$= -\frac{1}{R_F}. \tag{12.140}$$

Note that both R_0 and K are negative here, yielding a positive loop gain and hence confirming that the feedback is negative. The closed-loop parameters are thus expressed as

$$\left.\frac{V_{out}}{I_{in}}\right|_{closed} = \frac{R_0}{1 - \dfrac{R_0}{R_F}} \tag{12.141}$$

$$R_{in,closed} = \frac{\dfrac{1}{g_{m1}}||R_F}{1 - \dfrac{R_0}{R_F}} \tag{12.142}$$

$$R_{out,closed} = \frac{R_{D2}||R_F}{1 - \dfrac{R_0}{R_F}}, \tag{12.143}$$

where R_0 is given by Eq. (12.136).

Exercise Repeat the above example if R_{D2} is replaced with an ideal current source.

Example 12.30

Analyze the circuit of Fig. 12.57(a), assuming R_M is not small, $r_{O1} < \infty$, and the laser diode has an impedance of R_L.

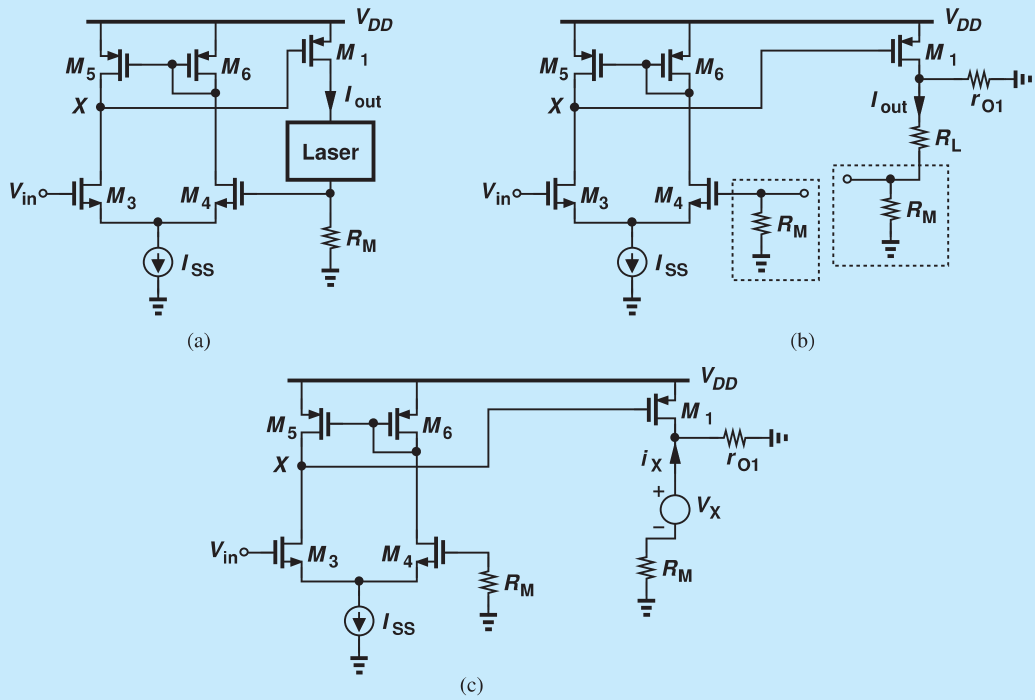


Figure 12.57

Solution This circuit employs current-voltage feedback and must be opened according to the rules shown in Figs. 12.51(d) and 12.52(d). The forward amplifier is formed by M_1 and M_3-M_6, and the feedback network consists of R_M. Depicted in Fig. 12.52(d), the open-loop circuit contains two instances of the feedback network, with the output port of the sense duplicate and the input port of the return duplicate left open. The open-loop gain $G_m = I_{out}/V_{in} = (V_X/V_{in})(I_{out}/V_X)$, and

$$\frac{V_X}{V_{in}} = -g_{m3}(r_{O3}||r_{O5}). \tag{12.144}$$

To calculate I_{out}/V_X, we note that the current produced by M_1 is divided between r_{O1} and $R_L + R_M$:

$$I_{out} = -\frac{r_{O1}}{r_{O1} + R_L + R_M} g_{m1} V_X, \tag{12.145}$$

where the negative sign arises because I_{out} flows *out* of the transistor. The open-loop gain is therefore equal to

$$G_m = \frac{g_{m3}(r_{O3}||r_{O5})g_{m1}r_{O1}}{r_{O1} + R_L + R_M}. \tag{12.146}$$

The output impedance is measured by replacing R_L with a test voltage source and measuring the small-signal current [Fig. 12.57(c)]. The top and bottom terminals of V_X respectively see an impedance of r_{O1} and R_M to ac ground; thus,

$$R_{in,open} = \frac{V_X}{I_X} \tag{12.147}$$

$$= r_{O1} + R_M. \tag{12.148}$$

The feedback factor is computed according to the rule in Fig. 12.52(d):

$$K = \frac{V_2}{I_1} \tag{12.149}$$

$$= R_M. \tag{12.150}$$

Forming KG_m, we express the closed-loop parameters as

$$\left.\frac{I_{out}}{V_{in}}\right|_{closed} = \frac{G_m}{1 + R_M G_m}, \tag{12.151}$$

$$R_{in,closed} = \infty \tag{12.152}$$

$$R_{out,closed} = (r_{O1} + R_M)(1 + R_M G_m), \tag{12.153}$$

where G_m is given by Eq. (12.146).

Exercise Construct the Norton equivalent of the entire circuit that drives the laser.

Example 12.31 Analyze the circuit of Fig. 12.58(a), assuming R_M is not small, and the laser diode exhibits an impedance of R_L.

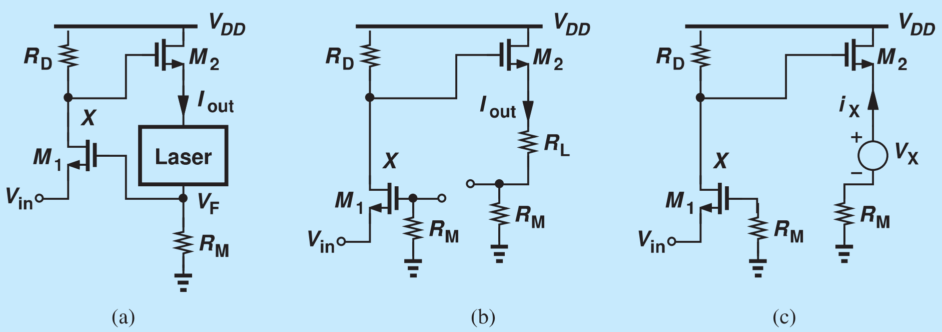


Figure 12.58

Solution The forward amplifier consisting of M_1, R_D, and M_2 senses a voltage and delivers a current to the load, and resistor R_M plays the role of the feedback network. In a manner similar to Example 12.30, we open the loop as shown in Fig. 12.58(b), where $G_m = I_{out}/V_{in} = (V_X/V_{in})(I_{out}/V_X)$. As a common-gate stage, M_1 and R_D yield

$V_X/V_{in} = g_{m1}R_D$. To determine I_{out}, we first view M_2 as a source follower and calculate the voltage gain V_A/V_X from Chapter 7:

$$\frac{V_A}{V_X} = \frac{R_L + R_M}{R_L + R_M + \dfrac{1}{g_{m2}}}. \tag{12.154}$$

Thus,

$$I_{out} = \frac{V_A}{R_L + R_M} \tag{12.155}$$

$$= \frac{V_X}{R_L + R_M + \dfrac{1}{g_{m2}}}, \tag{12.156}$$

yielding the open-loop gain as

$$G_m = \frac{g_{m1}R_D}{R_L + R_M + \dfrac{1}{g_{m2}}}. \tag{12.157}$$

The open-loop input impedance is equal to $1/g_{m1}$. For the open-loop output impedance, we replace R_L with a test voltage source [Fig. 12.58(c)], obtaining

$$\frac{V_X}{I_X} = \frac{1}{g_{m2}} + R_M. \tag{12.158}$$

The feedback factor remains identical to that in Example 12.30, leading to the following expressions for the closed-loop parameters:

$$\left.\frac{I_{out}}{V_{in}}\right|_{closed} = \frac{G_m}{1 + R_M G_m} \tag{12.159}$$

$$R_{in,closed} = \frac{1}{g_{m1}}(1 + R_M G_m) \tag{12.160}$$

$$R_{out,closed} = \left(\frac{1}{g_{m2}} + R_M\right)(1 + R_M G_m), \tag{12.161}$$

where G_m is given by Eq. (12.157).

Exercise Repeat the above example if a resistor of value of R_1 is tied between the source of M_2 and ground.

Example 12.32 Analyze the circuit of Fig. 12.59(a), assuming R_F is not large, R_M is not small, and the laser diode is modeled by a resistance R_L. Also, assume $r_{O2} < \infty$.

Solution As a current-current feedback topology, the amplifier must be analyzed according to the rules illustrated in Figs. 12.51(c) and 12.52(c). The forward system consists of M_1, R_D, and M_2, and the feedback network includes R_M and R_F. The loop is opened as depicted in Fig. 12.59(b), where the output port of the sense duplicate is shorted because the feedback network returns a current to the input. Given by $(V_X/I_{in})(I_{out}/V_X)$,

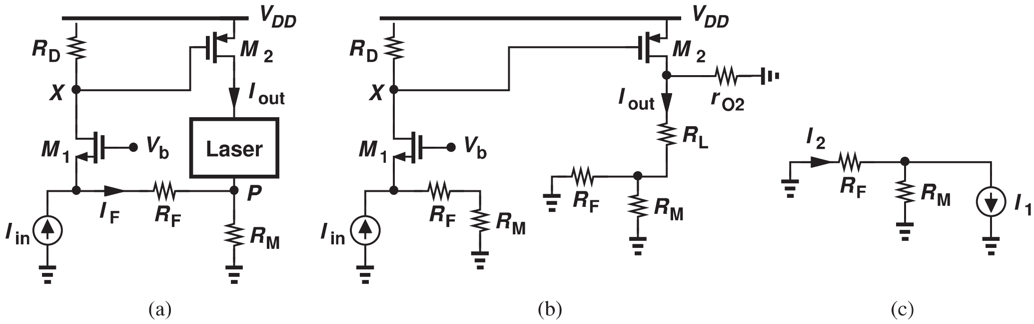


Figure 12.59

the open-loop gain is computed as

$$A_{I,open} = \frac{(R_F + R_M)R_D}{R_F + R_M + \dfrac{1}{g_{m1}}} \cdot \frac{-g_{m2}r_{O2}}{r_{O2} + R_L + R_M||R_F}, \tag{12.162}$$

where the two fractions account for the division of I_{in} between $R_F + R_M$ and M_1, and the division of I_{D2} between r_{O2} and $R_L + R_M||R_F$.

The open-loop I/O impedances are expressed as

$$R_{in,open} = \frac{1}{g_{m1}}||(R_F + R_M) \tag{12.163}$$

$$R_{out,open} = r_{O2} + R_F||R_M, \tag{12.164}$$

with the latter obtained in a manner similar to that depicted in Fig. 12.57(c).

To determine the feedback factor, we apply the rule of Fig. 12.52(c) as shown in Fig. 12.59(c), thereby obtaining

$$K = \frac{I_2}{I_1} \tag{12.165}$$

$$= -\frac{R_M}{R_M + R_F}. \tag{12.166}$$

The closed-loop parameters are thus given by

$$A_{I,closed} = \frac{A_{I,open}}{1 - \dfrac{R_M}{R_M + R_F}A_{I,open}} \tag{12.167}$$

$$R_{in,closed} = \frac{\dfrac{1}{g_{m1}}||(R_F + R_M)}{1 - \dfrac{R_M}{R_M + R_F}A_{I,open}} \tag{12.168}$$

$$R_{out,closed} = (r_{O2} + R_F||R_M)\left(1 - \frac{R_M}{R_M + R_F}A_{I,open}\right), \tag{12.169}$$

where $A_{I,open}$ is expressed by Eq. (12.162).

Exercise Construct the Norton equivalent of the entire circuit that drives the laser.

Example 12.33

Figure 12.60(a) depicts a circuit similar to that in Fig. 12.59(a), but the output of interest here is V_{out}. Analyze the amplifier and study the differences between the two.

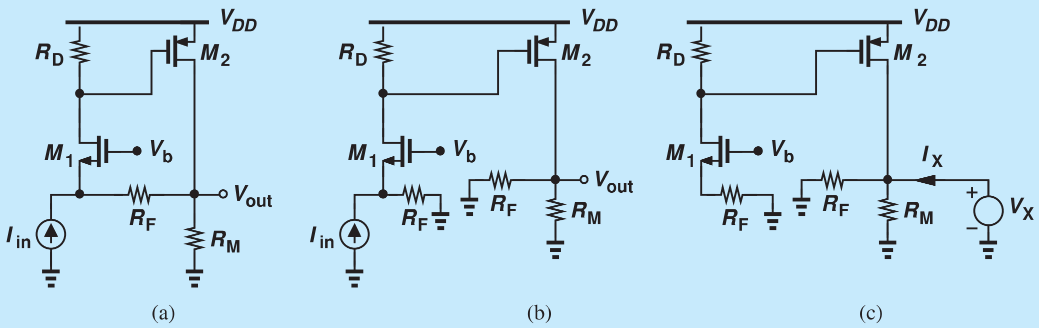


Figure 12.60

Solution The circuit incorporates voltage-current feedback. In contrast to the previous case, R_M now belongs to the forward amplifier rather than the feedback network. After all, M_2 would not be able to generate an output *voltage* without R_M. In fact, this circuit resembles the configuration of Fig. 12.56(a), except that the common-source stage employs a PMOS device.

Opening the loop with the aid of the rules in Fig. 12.51(b), we arrive at the topology in Fig. 12.60(b). Note that the return duplicate in this case (R_F) differs from that in Fig. 12.59(b) ($R_F + R_M$). The open-loop gain is then equal to

$$R_O = \frac{V_{out}}{I_{in}} \tag{12.170}$$

$$= \frac{R_F R_D}{R_F + \frac{1}{g_{m1}}}[-g_{m2}(R_F||R_M)], \tag{12.171}$$

and the open-loop input impedance is given by

$$R_{in,open} = \frac{1}{g_{m1}}||R_F. \tag{12.172}$$

The output impedance is computed as illustrated in Fig. 12.60(c):

$$R_{out,open} = \frac{V_X}{I_X} \tag{12.173}$$

$$= R_F||R_M. \tag{12.174}$$

If $r_{O2} < \infty$, then it simply appears in parallel with R_F and R_M in both Eqs. (12.171) and (12.174).

The feedback factor also differs from that in Example 12.32 and is determined with the aid of Fig. 12.52(b):

$$K = \frac{I_2}{V_1} \tag{12.175}$$

$$= -\frac{1}{R_F}, \tag{12.176}$$

yielding the following expressions for the closed-loop parameters:

$$\left.\frac{V_{out}}{I_{in}}\right|_{closed} = \frac{R_O}{1 - \dfrac{R_O}{R_F}} \tag{12.177}$$

$$R_{in,closed} = \frac{\dfrac{1}{g_{m1}}||R_F}{1 - \dfrac{R_O}{R_F}} \tag{12.178}$$

$$R_{out,closed} = \frac{R_F||R_M}{1 - \dfrac{R_O}{R_F}}. \tag{12.179}$$

In contrast to Example 12.32, the output impedance in this case decreases by $1 - R_O/R_F$ even though the circuit topology remains unchanged. This is because the output impedance is measured very differently in the two cases.

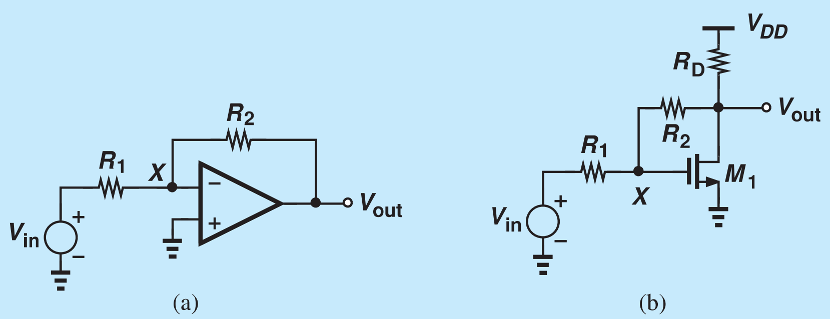
Exercise Repeat the above example if M_2 is degenerated by a resistor of value R_S.

12.8 STABILITY IN FEEDBACK SYSTEMS

Our studies in this chapter have thus far revealed many important benefits of negative feedback. Unfortunately, if designed poorly, negative-feedback amplifiers may behave “badly” or even oscillate. We say the system is marginally stable or simply unstable. In this section, we reexamine our understanding of feedback so as to define the meaning of “behaving badly” and determine the sources of instability.

Did you know?

A simple feedback circuit that, in fact, creates quite a bit of confusion is the op-amp-based noninverting amplifier [Fig. (a)] or a “poor man’s” realization thereof [Fig. (b)]. What type of feedback do we have here? The quantity sensed at the output is a voltage, but what is returned to the input? Since the input signal, V_{in}, is a voltage quantity, we may suspect that the feedback network (R_2) returns a voltage. However, for two voltages to add or subtract, they must be placed *in series*. The proper view here is to assume R_1 and R_2, respectively, convert V_{in} and V_{out} to current, and the resulting currents are summed at the virtual ground node, X. We study these commonly-used circuits in more advanced courses.



Inverting amplifier with feedback.

12.8.1 Review of Bode's Rules

In our review of Bode's rules in Chapter 11, we noted that the slope of the magnitude of a transfer function decreases (increases) by 20 dB/dec as the frequency passes a pole (zero). We now review Bode's rule for plotting the phase of the transfer function:

The phase of a transfer function begins to decrease (increase) at one-tenth of the pole (zero) frequency, incurs a change of $-45°$ ($+45°$) at the pole (zero) frequency, and experiences a total change of nearly $-90°$ ($+90°$) at ten times the pole (zero) frequency.[14]

Since the phase begins to change at one-tenth of a pole or zero frequency, even poles or zeros that seem far may affect it significantly—a point of contrast to the behavior of the magnitude.

Example 12.34 Figure 12.61(a) depicts the magnitude response of an amplifier. Using Bode's rule, plot the phase response.[15]

Solution Plotted in Fig. 12.61(b), the phase begins to fall at $0.1\omega_{p1}$, reaches $-45°$ at ω_{p1} and $-90°$ at $10\omega_{p1}$, begins to rise at $0.1\omega_z$, reaches $-45°$ at ω_z and approximately zero at $10\omega_z$, and finally begins to fall at $0.1\omega_{p2}$, reaching $-45°$ at ω_{p2} and $-90°$ at $10\omega_{p2}$. In this example, we have assumed that the pole and zero frequencies are so wide apart that $10\omega_{p1} < 0.1\omega_z$ and $10\omega_z < 0.1\omega_{p2}$. In practice, this may not hold, requiring more detailed calculations.

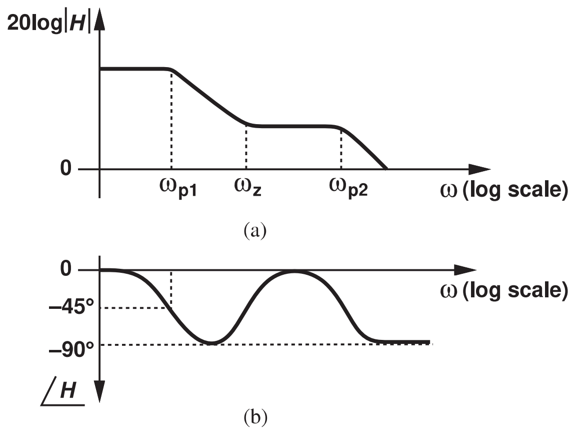


Figure 12.61

Exercise Repeat the above example if ω_{p1} falls between ω_z and ω_{p2}.

Amplifiers having multiple poles may become unstable if placed in a negative-feedback loop. The following example serves as our first step toward understanding this phenomenon.

[14] It is assumed that the poles and the zeros are located in the left half plane.

[15] In general, it may not be possible to construct the phase profile from the magnitude plot.

Example 12.35 Construct the magnitude and phase response of an amplifier having (a) one pole, (b) two poles, or (c) three poles.

Solution Figures 12.62(a)-(c) show the response for the three cases. The phase shift between the input and the output asymptotically approaches $-90°$, $-180°$, and $-270°$ in one-pole, two-pole, and three-pole systems, respectively. An important observation here is that the three-pole system introduces a phase shift of $-180°$ at a finite frequency ω_1, reversing the sign of an input sinusoid at this frequency [Fig. 12.62(d)]. For example, a 1-GHz sinusoid is shifted (delayed) by 0.5 ns.

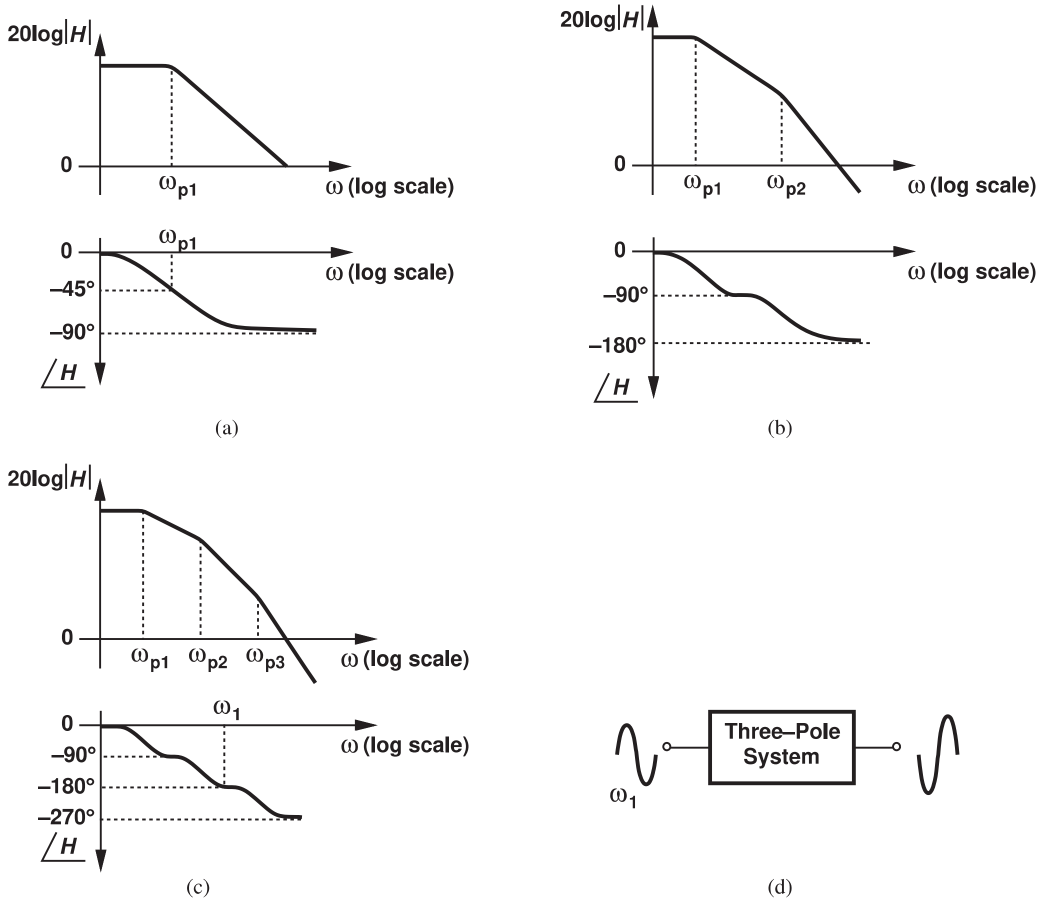


Figure 12.62

Exercise Repeat the above analysis for a three-pole system if $\omega_{p1} = \omega_{p2}$.

12.8.2 Problem of Instability

Suppose an amplifier having a transfer function $H(s)$ is placed in a negative feedback loop (Fig. 12.63). As with the cases studied in Section 12.1, we write the closed-loop transfer function as

$$\frac{Y}{X}(s) = \frac{H(s)}{1 + KH(s)}, \tag{12.180}$$

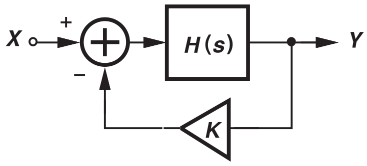


Figure 12.63 Negative feedback system.

where $KH(s)$ is sometimes called the "loop transmission" rather than the loop gain to emphasize its frequency dependence. Recall from Chapter 11 that for a sinusoidal input, $x(t) = A\cos\omega t$, we simply make the substitution $s = j\omega$ in the above transfer function. We assume the feedback factor, K, exhibits no frequency dependence. (For example, it is equal to the ratio of two resistors.)

An interesting question that arises here is, what happens if at a certain input frequency ω_1, the loop transmission, $KH(j\omega_1)$, becomes equal to -1? Then, the closed-loop system provides an infinite gain (even though the open-loop amplifier does not). To understand the consequences of infinite gain, we recognize that even an infinitesimally small input at ω_1 leads to a finite output component at this frequency. For example, the devices comprising the subtractor generate electronic "noise" containing all frequencies. A small noise component at ω_1 therefore experiences a very high gain and emerges as a large sinusoid at the output. We say the system oscillates at ω_1.[16]

It is also possible to understand the above oscillation phenomenon intuitively. Recall from Example 12.35 that a three-pole system introducing a phase shift of -180° reverses the sign of the input signal. Now, if $H(s)$ in Fig. 12.63 contains three poles such that $\angle H = -180^\circ$ at ω_1, then the feedback becomes *positive* at this frequency, thereby producing a feedback signal that *enhances* the input. Circulating around the loop, the signal may thus continue to grow in amplitude. In practice, the final amplitude remains bounded by the supply voltage or other "saturation" mechanisms in the circuit.

For analysis purposes, we express the condition $KH(j\omega_1) = -1$ in a different form. Viewing KH as a complex quantity, we recognize that this condition is equivalent to

$$|KH(j\omega_1| = 1 \tag{12.181}$$

$$\angle KH(j\omega_1) = -180^\circ, \tag{12.182}$$

the latter confirming our above intuitive perspective. In fact, Eq. (12.182) guarantees positive feedback (sufficient delay) and Eq. (12.181) ensures sufficient loop gain for the circulating signal to grow. Called "Barkhausen's criteria" for oscillation, Eqs. (12.181) and (12.182) prove extremely useful in the study of stability.

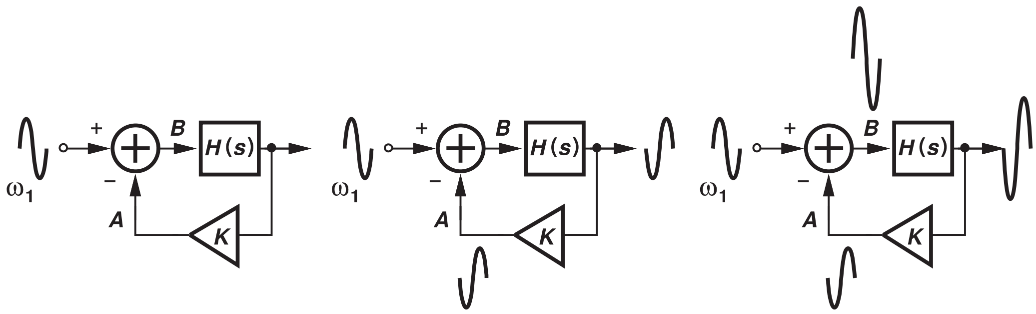
Example 12.36 Explain why a two-pole system cannot oscillate.[17]

Solution As evident from the Bode plots in Fig. 12.62(b), the phase shift produced by such a system reaches -180° only at $\omega = \infty$, where $|H| \to 0$. In other words, at no frequency are both Eqs. (12.181) and (12.182) satisfied.

Exercise What happens if one of the poles is at the origin?

[16] It can be proved that the circuit continues to produce a sinusoid at ω_1 even if the electronic noise of the devices ceases to exist. The term "oscillation" is thus justified.

[17] It is assumed that at least one of the poles is not at the origin.



(a) (b) (c)

Figure 12.64 Evolution of oscillatory system with time: (a) a component at ω_1 is sensed at input, (b) the component returns to subtractor with a 180° phase shift, (c) the subtractor enhances the signal at node B.

In summary, a negative feedback system may become unstable if the forward amplifier introduces a phase shift of -180° at a finite frequency, ω_1, *and* the loop transmission $|KH|$ is equal to unity at that frequency. These conditions become intuitive in the time domain as well. Suppose, as shown in Fig. 12.64(a), we apply a small sinusoid at ω_1 to the system and follow it around the loop as time progresses. The sinusoid incurs a sign reversal as it emerges at the output of the forward amplifier [Fig. 12.64(b)]. Assumed frequency-independent, the feedback factor simply scales the result by a factor of K, producing an inverted replica of the input at node A if $|KH(j\omega_1)| = 1$. This signal is now *subtracted* from the input, generating a sinusoid at node B with *twice* the amplitude [Fig. 12.64(c)]. The signal level thus continues to grow after each trip around the loop. On the other hand, if $|KH(j\omega_1)| < 1$, then the output cannot grow indefinitely.

Example 12.37

A three-pole feedback system exhibits the frequency response depicted in Fig. 12.65. Does this system oscillate?

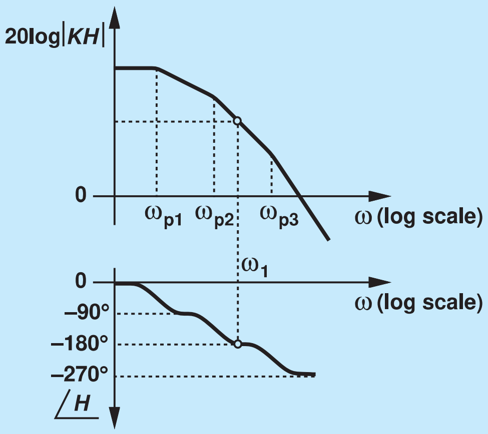


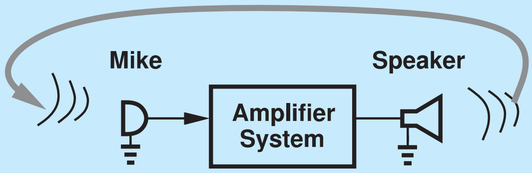
Figure 12.65

Solution Yes, it does. The loop gain at ω_1 is *greater* than unity, but we note from the analysis in Fig. 12.64 that indefinite signal growth still occurs, in fact more rapidly. After each trip around the loop, a sinusoid at ω_1 experiences a gain of $|KH| > 1$ and returns with opposite phase to the subtractor.

Exercise Suppose ω_{p1} is halved in value. Does the system still oscillate?

Did you know?

We have seen that a negative-feedback system can become unstable if the loop experiences a large phase shift. Since phase shift and delay are roughly equivalent, we can say excess delay causes instability. Two people walking past each other in a narrow hallway sometimes end up dancing to the left and to the right because of the slight delay they incur in correcting themselves. A microphone connected to a speaker system (as shown in the figure below) may whistle because the sound coming back from the speaker to the microphone through the air experiences delay. In this case, the feedback system consisting of the microphone, the amplifier, the speaker, and the return path through the air oscillates. You can change the frequency of oscillation (the pitch of the whistle) by adjusting the delay.



Unwanted feedback from speaker to microphone.

12.8.3 Stability Condition

Our foregoing investigation indicates that if $|KH(j\omega_1)| \geq 1$ and $\angle H(j\omega_1) = -180^\circ$, then the negative feedback system oscillates. Thus, to avoid instability, we must ensure that these two conditions do not occur at the same frequency.

TL2

Figure 12.66 depicts two scenarios wherein the two conditions do not coincide. Are both of these systems stable? In Fig. 12.66(a), the loop gain at ω_1 *exceeds* unity (0 dB), still leading to oscillation. In Fig. 12.66(b), on the other hand, the system cannot oscillate at ω_1 (due to insufficient phase shift) or ω_2 (because of inadequate loop gain).

The frequencies at which the loop gain falls to unity or the phase shift reaches -180° play such a critical role as to deserve specific names. The former is called the "gain crossover frequency" (ω_{GX}) and the latter, the "phase crossover frequency" (ω_{PX}). In Fig. 12.66(b), for example, $\omega_{GX} = \omega_1$ and $\omega_{PX} = \omega_2$. The key point emerging from the two above scenarios is that stability requires that

$$\omega_{GX} < \omega_{PX}. \tag{12.183}$$

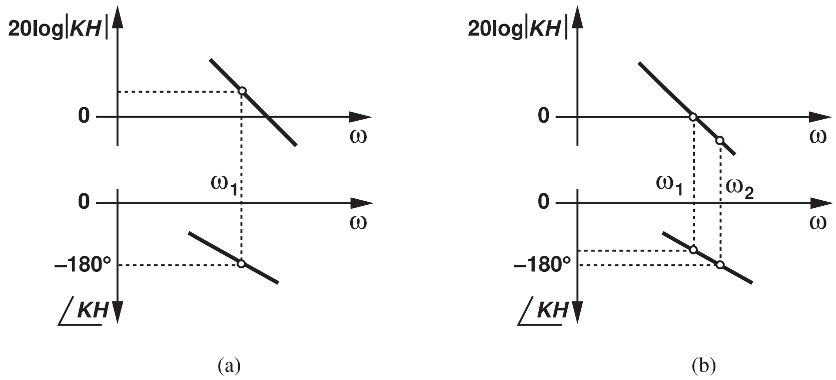


Figure 12.66 Systems with phase reaching -180° (a) before and (b) after the loop gain reaches unity.

In summary, to guarantee stability in negative-feedback systems, we must ensure that the loop gain falls to unity *before* the phase shift reaches −180° so that Barkhausen's criteria do not hold at the same frequency.

↳ Lo q·decla oscar

Example 12.38 We wish to apply negative feedback with $K = 1$ around the three-stage amplifier shown in Fig. 12.67(a). Neglecting other capacitances and assuming identical stages, plot the frequency response of the circuit and determine the condition for stability. Assume $\lambda = 0$.

Solution The circuit exhibits a low-frequency gain of $(g_m R_D)^3$ and three *coincident* poles given by $(R_D C_1)^{-1}$. Thus, as depicted in Fig. 12.67(b), $|H|$ begins to fall at a rate of 60 dB/dec at $\omega_p = (R_D C_1)^{-1}$. The phase begins to change at one-tenth of this frequency,[18] reaches −135° at ω_p, and approaches −270° at $10\omega_p$.

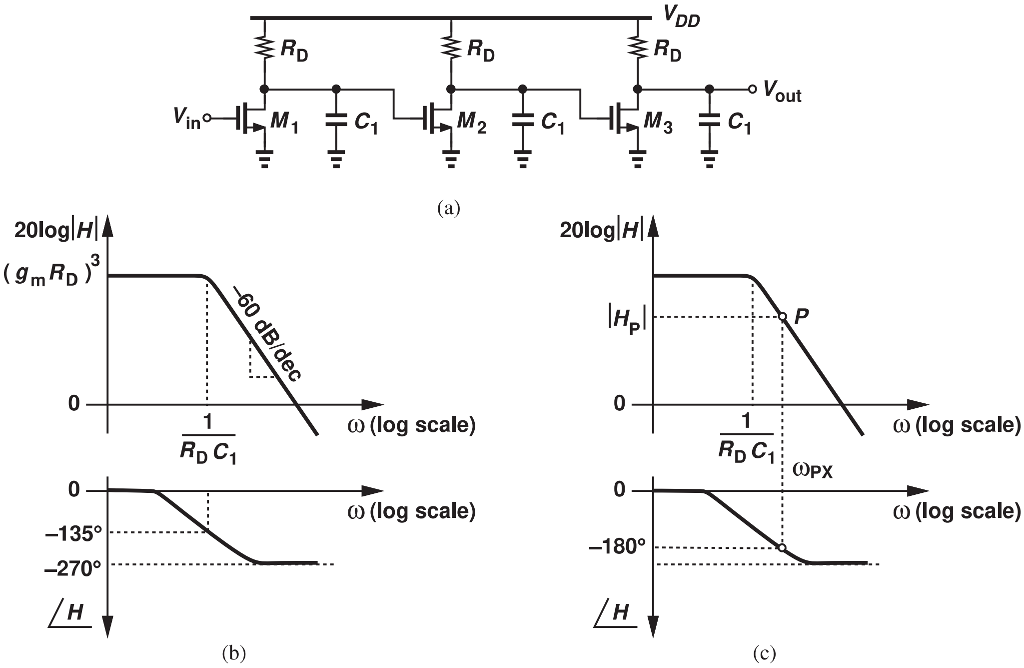


Figure 12.67

To guarantee that a unity-feedback system incorporating this amplifier remains stable, we must ensure that $|KH|(= |H|)$ falls below unity at the phase crossover frequency. Illustrated in Fig. 12.67(c), the procedure entails identifying ω_{PX} on the phase response, finding the corresponding point, P, on the gain response, and requiring that $|H_P| < 1$.

[18] Strictly speaking, we note that the three coincident poles affect the phase at frequencies even below $0.1\omega_p$.

We now repeat the procedure analytically. The amplifier transfer function is given by the product of those of the three stages:[19]

$$H(s) = \frac{(g_m R_D)^3}{\left(1 + \frac{s}{\omega_p}\right)^3}, \tag{12.184}$$

where $\omega_p = (R_D C_1)^{-1}$. The phase of the transfer function can be expressed as[20]

$$\angle H(j\omega) = -3\tan^{-1}\frac{\omega}{\omega_p}, \tag{12.185}$$

where the factor 3 accounts for the three coincident poles. The phase crossover occurs if $\tan^{-1}(\omega/\omega_p) = 60^\circ$ and hence

$$\omega_{PX} = \sqrt{3}\omega_p. \tag{12.186}$$

The magnitude must remain less than unity at this frequency:

$$\frac{(g_m R_D)^3}{\left[\sqrt{1 + \left(\frac{\omega_{PX}}{\omega_p}\right)^2}\right]^3} < 1. \tag{12.187}$$

It follows that

$$g_m R_D < 2. \tag{12.188}$$

If the low-frequency gain of each stage exceeds 2, then a feedback loop around this amplifier with $K = 1$ becomes unstable.

Exercise Repeat the above example if the last stage incorporates a load resistance equal to $2R_D$.

Example 12.39 A common-source stage is placed in a unity-gain feedback loop as shown in Fig. 12.68. Explain why this circuit does not oscillate.

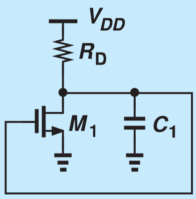


Figure 12.68

Solution Since the circuit contains only one pole, the phase shift cannot reach 180° at any frequency. The circuit is thus stable.

Exercise What happens if $R_D \to \infty$ and $\lambda \to 0$?

[19]For simplicity, we drop the negative sign in the gain of each stage here. The final result is still valid.

[20]Recall that the phase of a complex number $a + jb$ is given by $\tan^{-1}(b/a)$. Also, the phase of a product is equal to the sum of phases.

Example 12.40 Repeat Example 12.38 if the target value of K is $1/2$, i.e., the feedback is weaker.

Solution We plot $|KH| = 0.5|H|$ and $\angle KH = \angle H$ as shown in Fig. 12.69(a). Note the $|KH|$ plot is simply shifted down by 6 dB on a logarithmic scale. Starting from the phase crossover frequency, we determine the corresponding point, P, on $|KH|$ and require that $0.5|H_P| < 1$. Recognizing that Eqs. (12.185) and (12.186) still hold, we write

$$\frac{0.5(g_m R_D)^3}{\left[\sqrt{1+\left(\frac{\omega_{PX}}{\omega_p}\right)^2}\right]^3} < 1. \tag{12.189}$$

That is,

$$(g_m R_D)^3 < \frac{2^3}{0.5}. \tag{12.190}$$

Thus, the weaker feedback permits a greater open-loop gain.

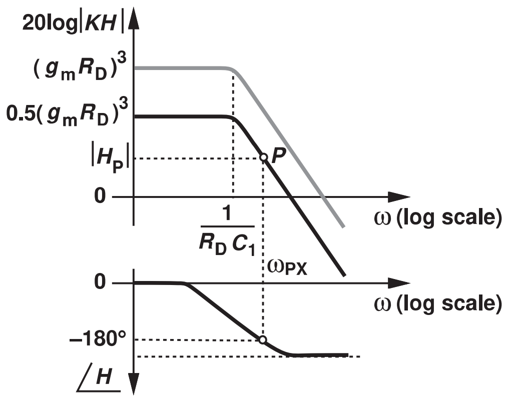


Figure 12.69

Exercise Repeat the above example if the third stage incorporates a load resistor of value $2R_D$.

12.8.4 Phase Margin

Our study of instability in negative-feedback systems reveals that ω_{GX} must remain below ω_{PX} to avoid oscillation. But by how much? We surmise that if $\omega_{GX} < \omega_{PX}$ but the difference between the two is small, then the feedback system displays an almost-oscillatory response. Shown in Fig. 12.70 are three cases illustrating this point. In Fig. 12.70(a), Barkhausen's criteria are met and the system produces oscillation in response to an input step. In Fig. 12.70(b), $\omega_{GX} < \omega_{PX}$, but the step response "rings" for a long time because the system is "marginally" stable and behaves "badly." We therefore postulate that a

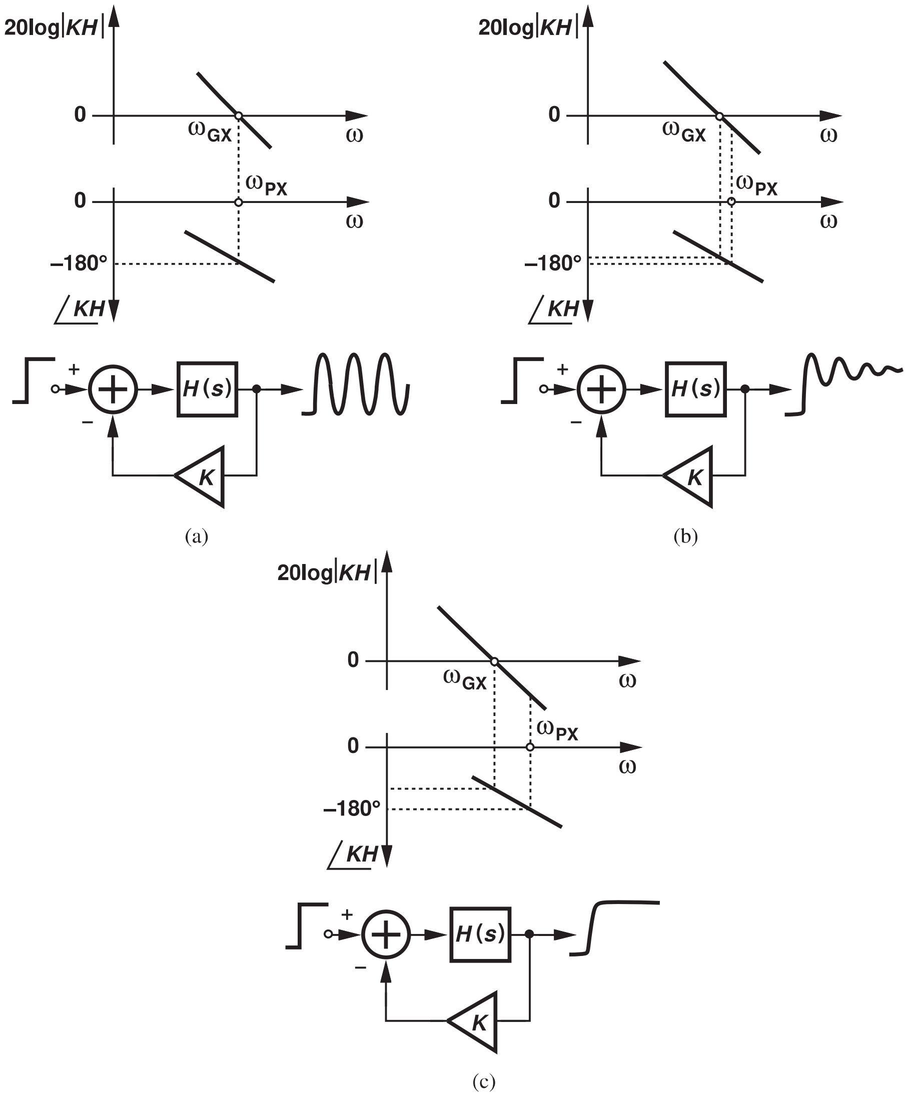


Figure 12.70 Systems with (a) coincident gain and phase crossovers, (b) gain crossover slightly below phase crossover, (c) gain crossover well below phase crossover.

well-behaved system is obtained only if a sufficient "margin" is allowed between ω_{GX} and ω_{PX} [Fig. 12.70(c)]. Note that in this case, $\angle KH$ at ω_{GX} remains significantly more positive than $-180°$.

A measure commonly used to quantify the stability of feedback systems is the "phase margin" (PM). As exemplified by the cases in Fig. 12.70, the more stable a system is, the greater is the *difference* between $\angle H(\omega_{GX})$ and $-180°$. Indeed, this difference is called the phase margin:

$$\text{Phase Margin} = \angle H(\omega_{GX}) + 180°. \tag{12.191}$$

Example 12.41

Figure 12.71 plots the frequency response of a multipole amplifier. The magnitude drops to unity at the second pole frequency. Determine the phase margin of a feedback system employing this amplifier with $K = 1$.

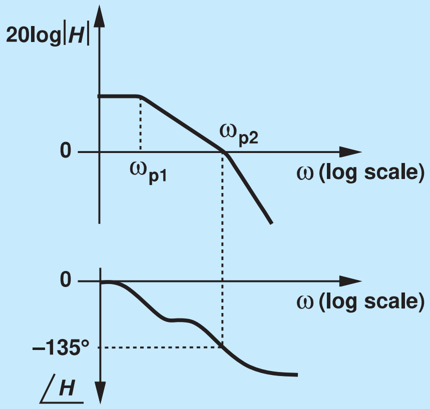


Figure 12.71

Solution The plot suggests that the phase reaches $-135°$ at the second pole frequency (i.e., the poles are far apart). Thus, the phase margin is equal to $45°$.

Exercise Is the phase margin greater or less than $45°$ if $K = 0.5$?

How much phase margin is necessary? For a well-behaved response, we typically require a phase margin of $60°$. Thus, the above example is not considered an acceptable design. In other words, the gain crossover must fall below the *second* pole.

12.8.5 Frequency Compensation

It is possible that after the design of an amplifier is completed, the phase margin proves inadequate. How is the circuit modified to improve the stability? For example, how do we make the three-stage amplifier of Example 12.38 stable if $K = 1$ and $g_m R_D > 2$? The solution is to make two of the poles unequal in magnitude. Called "frequency compensation," this task can be accomplished by shifting ω_{GX} toward the origin (without changing ω_{PX}). In other words, if $|KH|$ is forced to drop to unity at a lower frequency, then the phase margin increases [Fig. 12.72(a)].

How can ω_{GX} be shifted toward the origin? We recognize that if the *dominant* pole is translated to lower frequencies, so is ω_{GX}. Figure 12.72(b) illustrates an example where the first pole is shifted from ω_{p1} to ω'_{p1}, but other poles are constant. As a result, ω_{GX} decreases in magnitude.

What happens to the phase after compensation? As shown in Fig. 12.72(b), the low-frequency section of $\angle KH$ changes because ω_{p1} is moved to ω'_{P1}, but the critical section near ω_{GX} does not. Consequently, the phase margin increases.

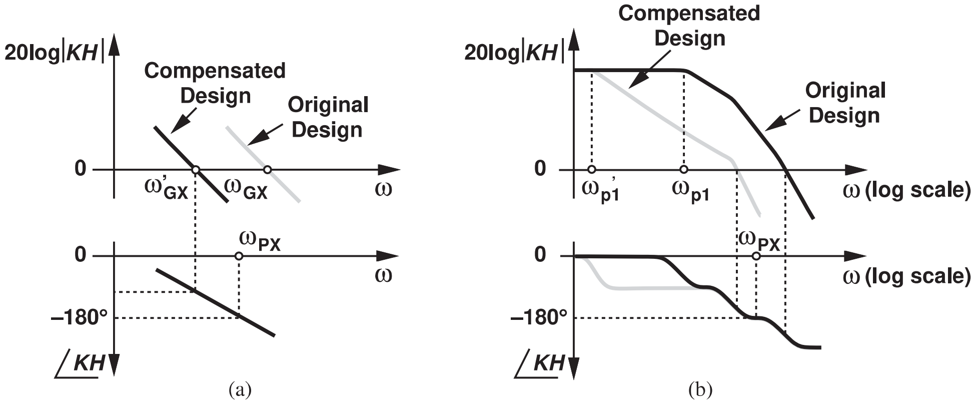


Figure 12.72 (a) Concept of frequency compensation, (b) effect on phase profile.

Example 12.42

The amplifier shown in Fig. 12.73(a) employs a cascode stage and a CS stage. Assuming that the pole at node B is dominant, sketch the frequency response and explain how the circuit can be "compensated."

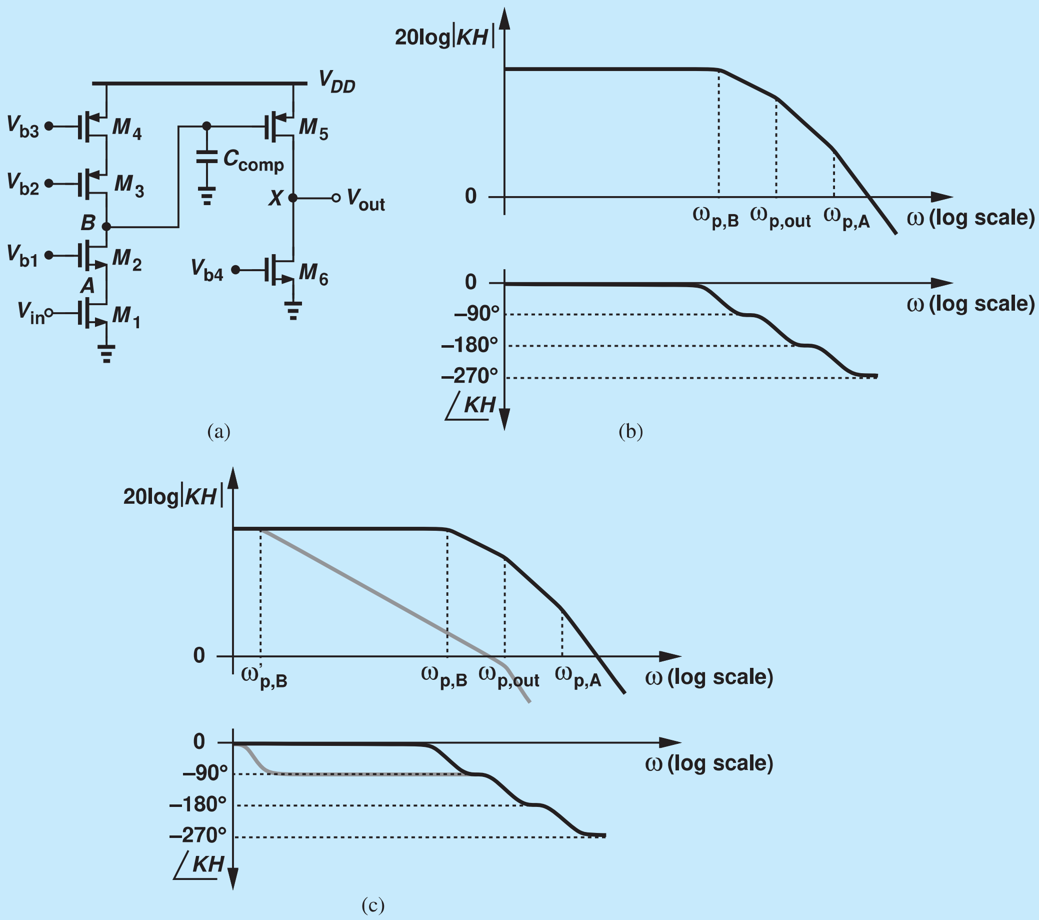


Figure 12.73

Solution Recall from Chapter 11 that the cascode stage exhibits one pole arising from node A and another from node B, with the latter falling much closer to the origin. We can express these poles respectively as

$$\omega_{p,A} \approx \frac{g_{m2}}{C_A} \tag{12.192}$$

$$\omega_{p,B} \approx \frac{1}{[(g_{m2}r_{O2}r_{O1})||(g_{m3}r_{O3}r_{O4})]C_B}, \tag{12.193}$$

where C_A and C_B denote the total capacitance seen at each node to ground. The third pole is associated with the output node:

$$\omega_{p,out} = \frac{1}{(r_{O5}||r_{O6})C_{out}}, \tag{12.194}$$

where C_{out} represents the total capacitance at the output node. We assume $\omega_{P,B} < \omega_{P,out} < \omega_{P,A}$. The frequency response of the amplifier is plotted in Fig. 12.73(b).

To compensate the circuit for use in a feedback system, we can add capacitance to node B so as to reduce $\omega_{p,B}$. If C_{comp} is sufficiently large, the gain crossover occurs well below the phase crossover, providing adequate phase margin [Fig. 12.73(c)]. An important observation here is that frequency compensation inevitably degrades the speed because the dominant pole is reduced in magnitude.

Exercise Repeat the above example if M_2 and M_3 are omitted, i.e., the first stage is a simple CS amplifier.

We now formalize the procedure for frequency compensation. Given the frequency response of an amplifier and the desired phase margin, we begin from the phase response and determine the frequency, ω_{PM}, at which $\angle H = -180^\circ + \text{PM}$ (Fig. 12.74). We then mark ω_{PM} on the magnitude response and draw a line having a slope of 20 dB/dec toward the vertical axis. The point at which this line intercepts the original magnitude response represents the new position of the dominant pole, ω'_{p1}. By design, the compensated amplifier now exhibits a gain crossover at ω_{PM}.

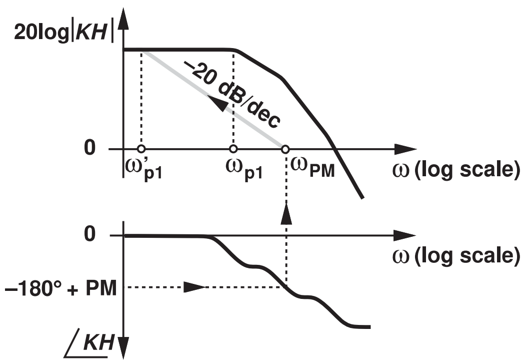


Figure 12.74 Systematic method for frequency compensation.

Example 12.43 A multipole amplifier exhibits the frequency response plotted in Fig. 12.75(a). Assuming the poles are far apart, compensate the amplifier for a phase margin of 45° with $K = 1$.

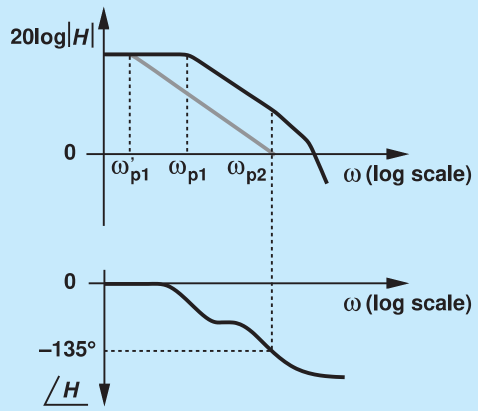


Figure 12.75

Solution Since the phase reaches $-135°$ at $\omega = \omega_{p2}$, in this example $\omega_{PM} = \omega_{p2}$. We thus draw a line with a slope of -20 dB/dec from ω_{p2} toward the vertical axis. The dominant pole must therefore be translated to ω'_{p1}. Since this phase margin is generally inadequate, in practice, $\omega_{PM} < \omega_{p2}$.

Exercise Repeat the above example for $K = 0.5$ and compare the results.

Did you know?

It is interesting to know how one capacitor changed the fate of a product forever. A general-purpose, innovative op amp (LM101) introduced by Robert Widlar at National Semiconductor in 1967 employed Miller compensation but with an *external* 30 pF capacitor. Evidently, the compensation capacitor was left off-chip to provide greater flexibility for the user. In 1968, David Fullagar at Fairchild Semiconductor introduced another op amp very similar to the LM101, except that the compensation capacitor was integrated on the chip. This was the 741 op amp, which quickly became popular and has served the electronics industry for nearly half a century.

The reader may wonder why the line originating at ω_{PM} must rise at a slope of 20 dB/dec (rather than 40 or 60 dB/dec) toward the vertical axis. Recall from Examples 12.41 and 12.43 that, for adequate phase margin, the gain crossover must occur *below* the second pole. Thus, the magnitude response of the *compensated* amplifier does not "see" the second pole as it approaches ω_{GX}; i.e., the magnitude response has a slope of only -20 dB/dec.

12.8.6 Miller Compensation

In Example 12.42, we noted that a capacitor can be tied from node B to ground to compensate the amplifier. The required value of this capacitor may be quite large, necessitating a large area on an integrated circuit. But recall from Miller's theorem in Chapter 11 that the apparent value of a capacitor increases if the device is connected between the input and output of an inverting amplifier. We also observe that the two-stage amplifier of Fig. 12.73(a) can employ Miller multiplication because the second stage provides some voltage gain. Illustrated in Fig. 12.76, the idea

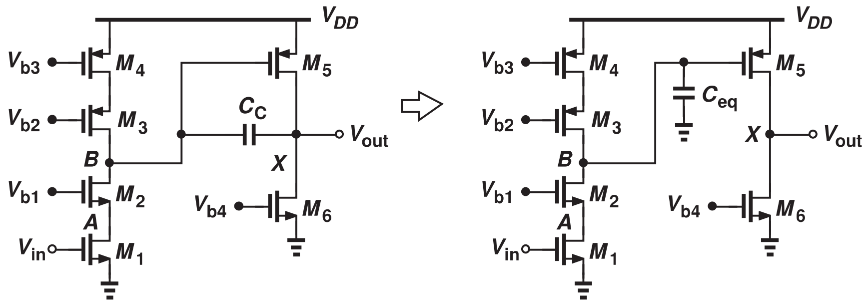


Figure 12.76 Example of Miller compensation.

is to introduce the compensation capacitor between the input and output of the second stage, thereby creating an equivalent *grounded* capacitance at B given by

$$C_{eq} = (1 - A_v)C_c \tag{12.195}$$

$$= [1 + g_{m5}(r_{O5}||r_{O6})]C_c. \tag{12.196}$$

Called "Miller compensation," this technique reduces the required value of C_c by a factor of $1 + g_{m5}(r_{O5}||r_{O6})$.

Miller compensation entails a number of interesting side effects. For example, it shifts not only the dominant pole but also the output pole. Such phenomena are studied in more advanced texts, e.g., [1].

12.9 CHAPTER SUMMARY

- Negative feedback can be used to regulate the behavior of systems that are otherwise "untamed" and poorly controlled.
- A negative feedback system consists of four components: forward system, output sense mechanism, feedback network, and input comparison mechanism.
- The loop gain of a feedback system can, in principle, be obtained by breaking the loop, injecting a test signal, and calculating the gain as the signal goes around the loop. The loop gain determines many properties of feedback systems, e.g., gain, frequency response, and I/O impedances.
- The loop gain and closed-loop gain should not be confused with each other. The latter refers to the overall gain from the main input to the main output while the feedback loop is closed.
- If the loop gain is much greater than unity, the closed-loop gain becomes approximately equal to the inverse of the feedback factor.
- Making the closed-loop gain relatively independent of the open-loop gain, negative feedback provides many useful properties: it reduces the sensitivity of the gain to component variations, load variations, frequency variations, and signal level variations.
- Amplifiers can generally be viewed as one of four types with voltage or current inputs and outputs. In the ideal case, the input impedance of a circuit is infinite if it senses a voltage or zero if it senses a current. Also, the output impedance of an ideal circuit is zero if it generates a voltage or infinite if it generates a current.

- Voltage quantities are sensed in parallel and current quantities in series. Voltage quantities are summed in series and current quantities in parallel.
- Depending on the type of the forward amplifier, four feedback topologies can be constructed. The closed-loop gain of each is equal to the open-loop gain divided by one plus the loop gain.
- A negative-feedback loop sensing and regulating the output voltage lowers the output impedance by a factor of one plus the loop gain, making the circuit a better voltage source.
- A negative-feedback loop sensing and regulating the output current raises the output impedance by a factor of one plus the loop gain, making the circuit a better current source.
- A negative-feedback loop returning a voltage to the input raises the input impedance by one plus the loop gain, making the circuit a better voltage sensor.
- A negative-feedback loop returning a current to the input lowers the input impedance by one plus the loop gain, making the circuit a better current sensor.
- If the feedback network departs from its ideal model, then it "loads" the forward amplifier characteristics. In this case, a methodical method must be followed that included the effect of finite I/O impedances.
- A high-frequency signal traveling through a forward amplifier experiences significant phase shift. With several poles, it is possible that the phase shift reaches 180°.
- A negative-feedback loop that introduces a large phase shift may become a positive-feedback loop at some frequency and begin to oscillate if the loop gain at that frequency is unity or higher.
- To avoid oscillation, the gain crossover frequency must fall below the phase crossover frequency.
- Phase margin is defined as 180° minus the phase of the loop transmission at the gain crossover frequency.
- To ensure a well-behaved time and frequeny response, a negative-feedback system must realize sufficient phase margin, e.g., 60°.
- If a feedback circuit suffers from insufficient phase margin, then it must be "frequency-compensated." The most common method is to lower the dominant pole so as to reduce the gain crossover frequency (without changing the phase profile). This typically requires adding a large capacitor from the dominant pole node to ground.
- To lower the dominant pole, one can exploit Miller multiplication of capacitors.

PROBLEMS

Sec. 12.1 General Considerations

12.1. Determine the transfer function, Y/X, for the systems shown in Fig. 12.77.

12.2. For the systems depicted in Fig. 12.77, compute the transfer function W/X.

12.3. For the systems depicted in Fig. 12.77, compute the transfer function E/X.

12.4. Calculate the loop gain of the circuits illustrated in Fig. 12.78. Assume the op amp exhibits an open-loop gain of A_1, but is otherwise ideal. Also, $\lambda = 0$.

12.5. Using the results obtained in Problem 12.4, compute the closed-loop gain of the circuits shown in Fig. 12.78.

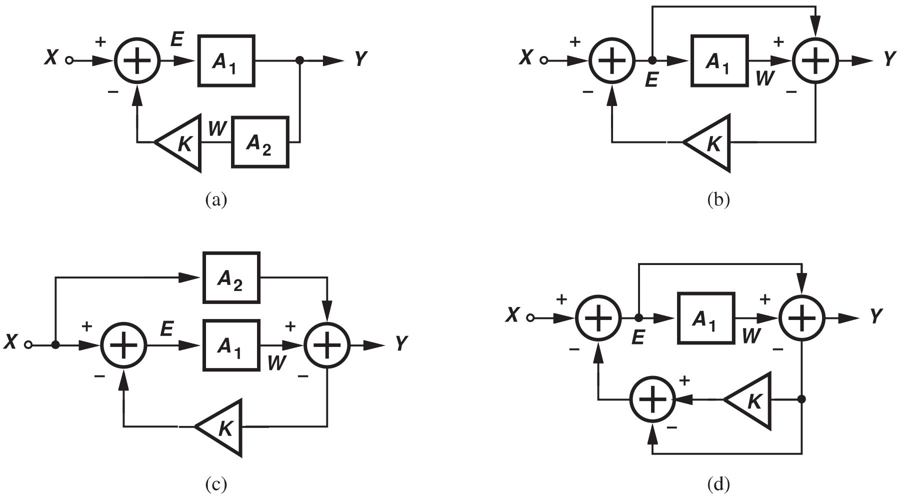


Figure 12.77

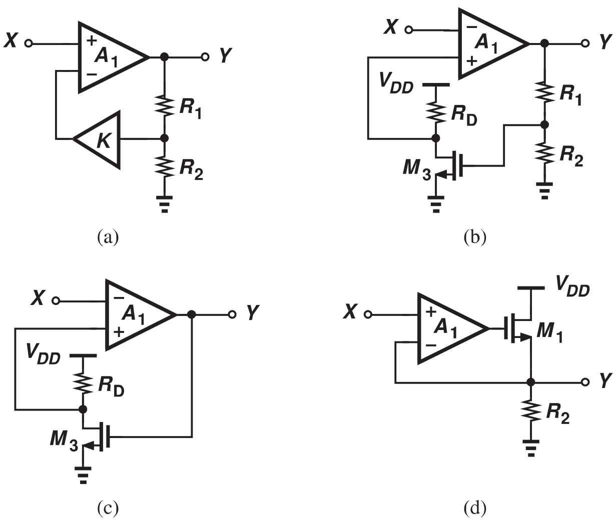


Figure 12.78

12.6. In the circuit of Fig. 12.3, the input is a sinusoid with a peak amplitude of 2 mV. If $A_1 = 500$ and $R_1/R_2 = 7$, determine the amplitude of the output waveform and the feedback waveform.

Sec. 12.2 Properties of Negative Feedback

12.7. Suppose the open-loop gain A_1 in Fig. 12.1 changes by 20%. Determine the minimum loop gain necessary to ensure the closed-loop gain changes by less than 1%.

12.8. In some applications, we may define a "-1 dB bandwidth" as the frequency at which the gain falls by 10%. Determine the -1 dB bandwidth of the open-loop and closed-loop first-order systems described by Eqs. (12.16) and (12.19). Can we say the -1 dB bandwidth increases by $1 + KA_0$ as a result of feedback?

12.9. Consider the feedback system shown in Fig. 12.79, where the common-source stage serves as the feedforward network. Assume $\mu_n C_{ox}$ may vary by $\pm 10\%$ and λ by $\pm 20\%$. What is the minimum loop gain necessary to guarantee that the closed-loop gain varies by less than $\pm 5\%$?

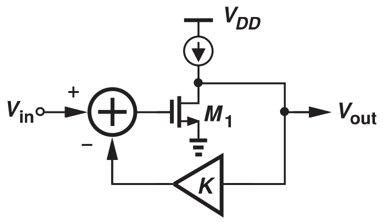


Figure 12.79

12.10. The circuit of Fig. 12.80 must achieve a closed-loop −3 dB bandwidth of B. Determine the required value of K. Neglect other capacitances and assume $\lambda > 0$.

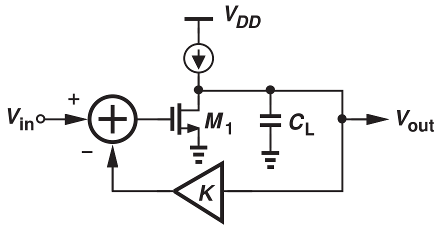


Figure 12.80

12.11. Repeat Example 12.7 for the circuit depicted in Fig. 12.81. Assume the impedance of C_1 and C_2 at the frequency of interest is much higher than R_D.

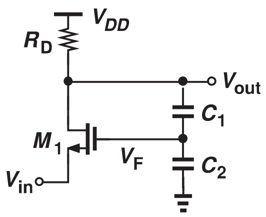


Figure 12.81

12.12. In Example 12.8, the closed-loop gain of the circuit must fall below its "unloaded" value by less than 10%. What is the lowest tolerable value of R_L?

12.13. In Fig. 12.13, $A_1 = 500$ and $A_2 = 420$. What value of K guarantees that the closed-loop gains at x_1 and x_2 differ by less than 5%? What closed-loop gain is achieved under this condition?

***12.14.** The characteristic in Fig. 12.13(a) is sometimes approximated as

$$y = \alpha_1 x - \alpha_3 x^3, \qquad (12.197)$$

where α_1 and α_3 are constant.

(a) Determine the small-signal gain $\partial y/\partial x$ at $x = 0$ and $x = \Delta x$.

(b) Determine the closed-loop gain at $x = 0$ and $x = \Delta x$ for a feedback factor of K.

12.15. Using the developments in Fig. 12.16, draw the amplifier model for each stage in Fig. 12.17.

12.16. Determine the amplifier model for the circuit depicted in Fig. 12.82. Assume $\lambda > 0$.

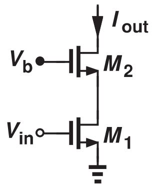


Figure 12.82

12.17. Repeat Problem 12.16 for the circuit in Example 12.7.

Sec. 12.4 Sense and Return Mechanisms

12.18. Identify the sense and return mechanisms in each amplifier depicted in Fig. 12.83.

12.19. Identify the sense and return mechanisms in each amplifier depicted in Fig. 12.84.

12.20. Identify the sense and return mechanisms in each amplifier depicted in Fig. 12.85.

***12.21.** Identify the sense and return mechanisms in each amplifier depicted in Fig. 12.86.

Sec. 12.5 Polarity of Feedback

***12.22.** Determine the polarity of feedback in each of the stages illustrated in Fig. 12.87.

12.23. Determine the polarity of feedback in the circuit of Example 12.11.

12.24. Determine the polarity of feedback in the circuits depicted in Figs. 12.83–12.86.

Sec. 12.6.1 Voltage-Voltage Feedback

12.25. Consider the feedback circuit shown in Fig. 12.88, where $R_1 + R_2 \gg R_D$. Compute the closed-loop gain and I/O impedances of the circuit. Assume $\lambda \neq 0$.

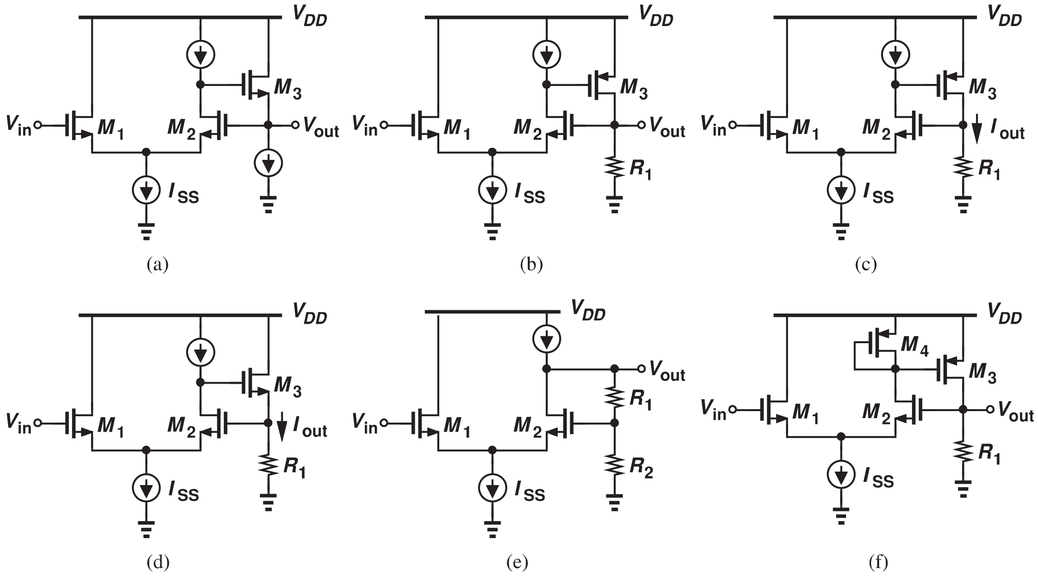


Figure 12.83

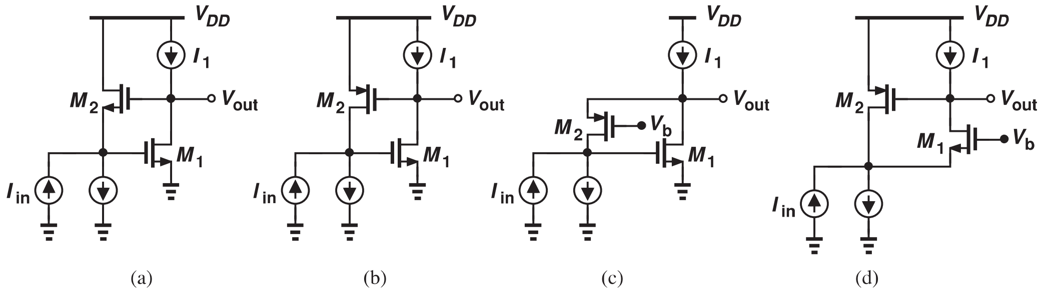


Figure 12.84

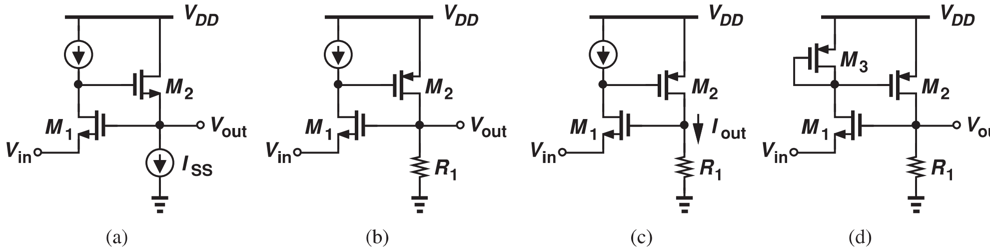


Figure 12.85

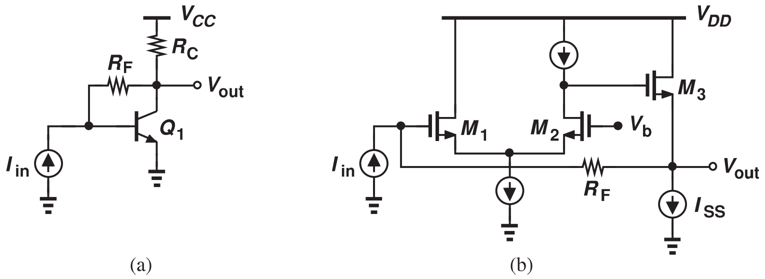


Figure 12.86

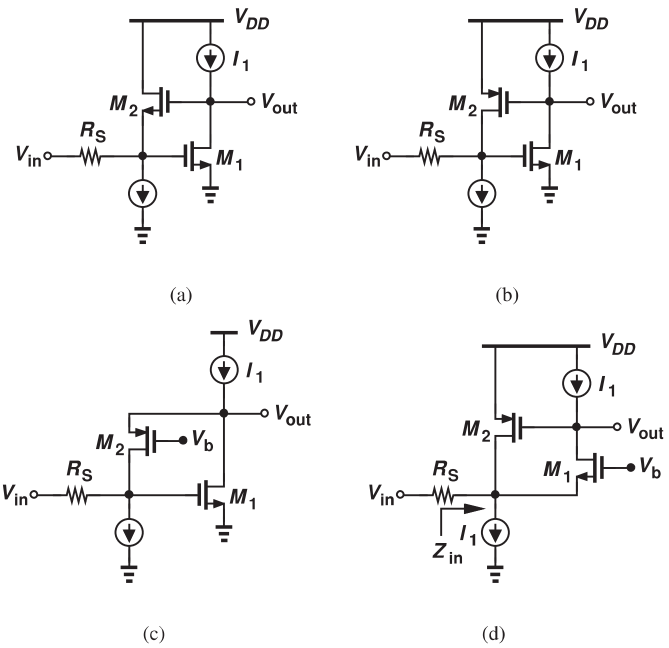


Figure 12.87

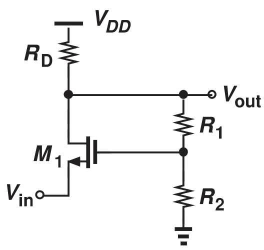


Figure 12.88

12.26. Repeat Problem 12.25 for the topology of Fig. 12.89. Assume C_1 and C_2 are very small and neglect other capacitances.

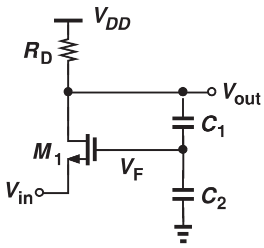


Figure 12.89

12.27. The amplifier shown in Fig. 12.90 provides a closed-loop gain close to unity but a very low output impedance. Assuming $\lambda > 0$, determine the closed-loop gain and output impedance and compare the results with those of a simple source follower.

***12.28.** An adventurous student replaces the NMOS source follower in Fig. 12.90 with a PMOS common-source stage (Fig. 12.91). Unfortunately, the amplifier does not operate well.

(a) Prove by inspection that the feedback is positive.

(b) Breaking the loop at the gate of M_2, determine the loop gain and prove that the feedback is positive.

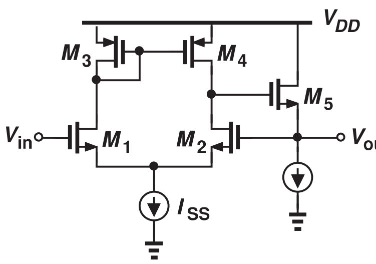


Figure 12.90

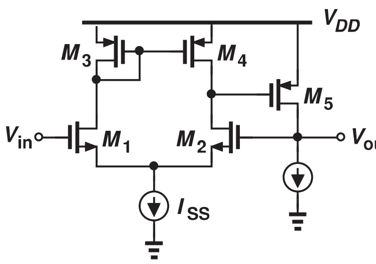


Figure 12.91

*12.29. Having discovered the polarity of feedback, the student in Problem 12.28 modifies the circuit as shown in Fig. 12.92. Determine the closed-loop gain and I/O impedances of the circuit and compare the results with those obtained in Problem 12.27.

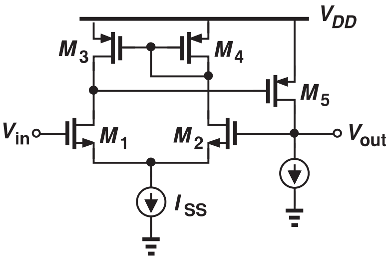


Figure 12.92

Sec. 12.6.2 Voltage-Current Feedback

12.30. Repeat Example 12.18 for the circuit illustrated in Fig. 12.93.

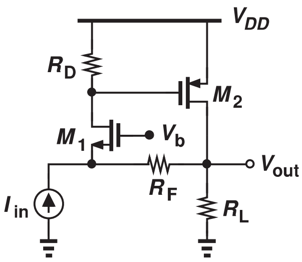


Figure 12.93

12.31. A student adventurously modifies the circuit of Example 12.18 to that shown in Fig. 12.94. Assume $\lambda = 0$.

(a) Prove by inspection that the feedback is positive.

(b) Assuming R_F is very large and breaking the loop at the gate of M_2, calculate the loop gain and prove that the feedback is positive.

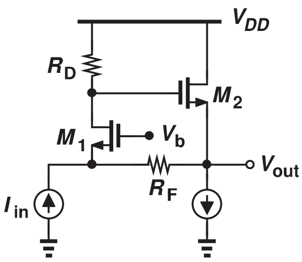


Figure 12.94

12.32. Determine the closed-loop I/O impedances of the circuit shown in Fig. 12.94.

**12.33. The amplifier depicted in Fig. 12.95 consists of a common-gate stage (M_1 and R_D) and a feedback network (R_1, R_2, and M_2). Assuming $R_1 + R_2$ is very large and $\lambda = 0$, compute the closed-loop gain and I/O impedances.

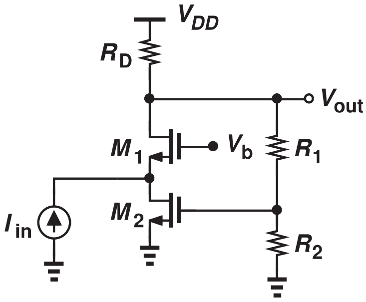


Figure 12.95

**12.34. Repeat Problem 12.33 for the circuit illustrated in Fig. 12.96. Assume C_1 and C_2 are very small and neglect other capacitances.

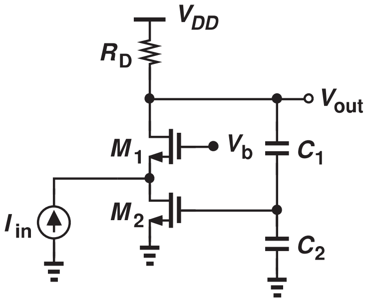


Figure 12.96

Sec. 12.6.3 Current-Voltage Feedback

12.35. A "laser diode" converts current to light (as in laser pointers). We wish to design a circuit that delivers a well-defined current to a laser diode. Shown in Fig. 12.97 is an example in which resistor R_M measures the current flowing through D_1 and amplifier A_1 subtracts the resulting voltage drop from V_{in}. Assume R_M is very small and $V_A = \infty$.

(a) Following the procedure used in Example 12.21, determine the open-loop gain.

(b) Calculate the loop gain and the closed-loop gain.

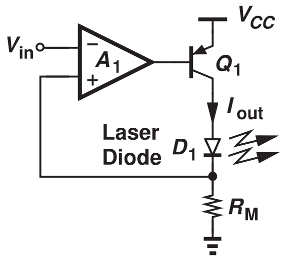


Figure 12.97

12.36. Following the procedure used in Example 12.22, compute the open-loop and closed-loop output impedances of the circuit depicted in Fig. 12.97.

*__12.37.__ A student mistakenly replaces the common-emitter *pnp* device in Fig. 12.97 with an *npn* emitter follower (Fig. 12.98). Repeat Problems 12.35 and 12.36 for this circuit and compare the results.

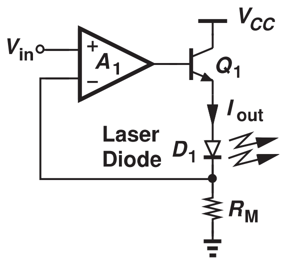


Figure 12.98

12.38. The amplifier A_1 in Fig. 12.98 can be realized as a common-base stage (Fig. 12.99). Repeat Problem 12.37 for this circuit. For simplicity, assume $\beta \to \infty$.

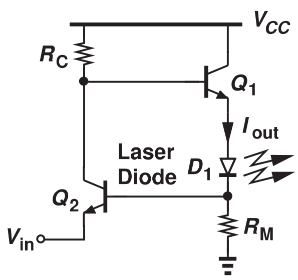


Figure 12.99

Sec. 12.6.4 Current-Current Feedback

12.39. A student has adventurously replaced the PMOS common-source stage in Fig. 12.47(a) with an NMOS source follower (Fig. 12.100).

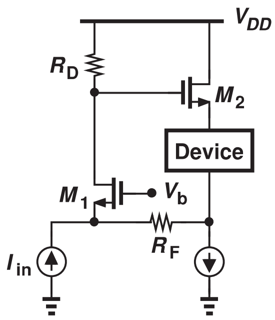


Figure 12.100

(a) Prove by inspection that the feedback is positive.

(b) Break the loop at the gate of M_2, determine the loop gain, and prove that the feedback is positive.

**__12.40.__ Consider the feedback circuit depicted in Fig. 12.101. Assume $V_A = \infty$.

(a) Suppose the output quantity of interest is the collector current of Q_2, I_{out}. Assuming R_M is very small and R_F is very large, determine the closed-loop gain and I/O impedances of the circuit.

(b) Now, suppose the output quantity of interest is V_{out}. Assuming R_F is very large, compute the closed-loop gain and I/O impedances of the circuit.

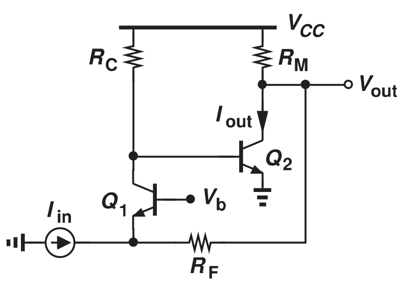


Figure 12.101

Sec. 12.7 Effect of Finite I/O Impedances

12.41. The common-gate stage shown in Fig. 12.102 employs an ideal current source as its load, requiring that the loading introduced by R_1 and R_2 be taken into account. Repeat Example 12.26 for this circuit.

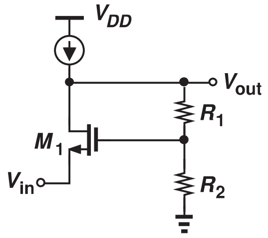


Figure 12.102

12.42. Figure 12.103 depicts the bipolar counterpart of the circuit studied in Example 12.26. Assuming $R_1 + R_2$ is not very large, $1 \ll \beta < \infty$ and $V_A = \infty$, determine the closed-loop gain and I/O impedances.

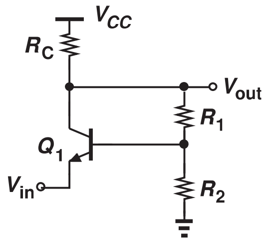


Figure 12.103

12.43. Repeat Problem 12.42 for the amplifier illustrated in Fig. 12.104.

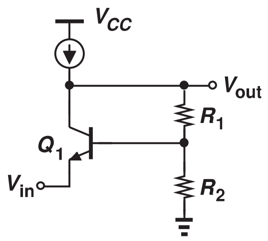


Figure 12.104

*__12.44.__ Repeat Example 12.28 for the circuit shown in Fig. 12.105. Assume $V_A = \infty$.

*__12.45.__ Repeat Example 12.28 for the circuit shown in Fig. 12.106. Assume $\lambda = 0$.

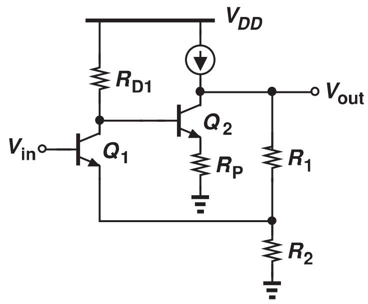


Figure 12.105

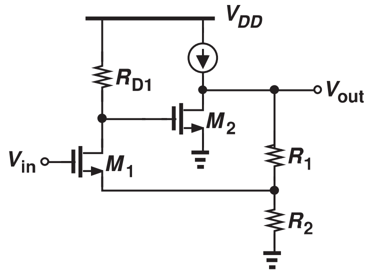


Figure 12.106

12.46. Assuming $V_A = \infty$, determine the closed-loop gain and I/O impedances of the amplifier depicted in Fig. 12.107. (For open-loop calculations, it is helpful to view Q_1 and Q_2 as a follower and a common-base stage, respectively.)

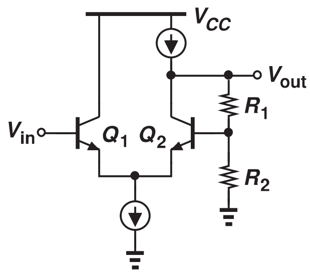


Figure 12.107

12.47. Repeat Example 12.29 for the bipolar transimpedance amplifier shown in Fig. 12.108. Assume $V_A = \infty$.

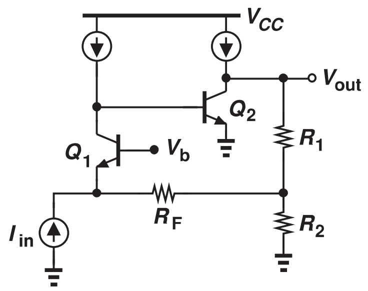


Figure 12.108

**12.48. Repeat Example 12.29 for the circuit illustrated in Fig. 12.109. Assume $\lambda > 0$.

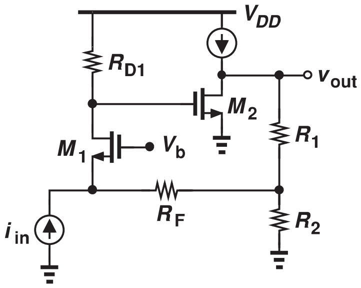


Figure 12.109

12.49. Figure 12.110 depicts a popular transimpedance amplifier topology. Repeat the analysis of Example 12.29 for this circuit. Assume $V_A < \infty$.

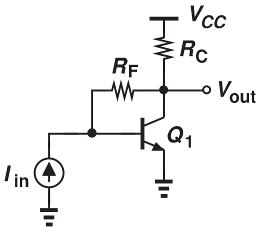


Figure 12.110

*12.50. The circuit of Fig. 12.110 can be improved by inserting an emitter follower at the output (Fig. 12.111). Assuming $V_A < \infty$, repeat Example 12.29 for this topology.

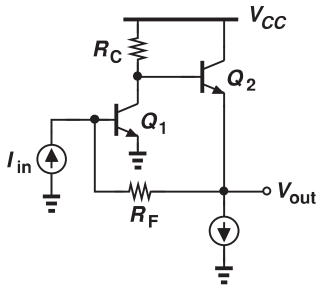


Figure 12.111

**12.51. Determine the closed-loop gain and I/O impedances of the circuits shown in Fig. 12.112, including the loading effects of each feedback network. Assume $\lambda = 0$.

12.52. The circuit of Fig. 12.97, repeated in Fig. 12.113, employs a value for R_M that is not very small. Assuming $V_A < \infty$ and the diode exhibits an impedance of R_L, repeat the analysis of Example 12.30 for this circuit.

12.53. Repeat Example 12.32 for the circuit shown in Fig. 12.114. Note that R_M is replaced with a current source but the analysis proceeds in a similar manner.

12.54. Repeat Problem 12.53 for the circuit illustrated in Fig. 12.115. Assume $V_A = \infty$.

12.55. Repeat Problem 12.53 for the topology depicted in Fig. 12.116. Assume $V_A = \infty$.

**12.56. Compute the closed-loop gain and I/O impedances of the stages illustrated in Fig. 12.117.

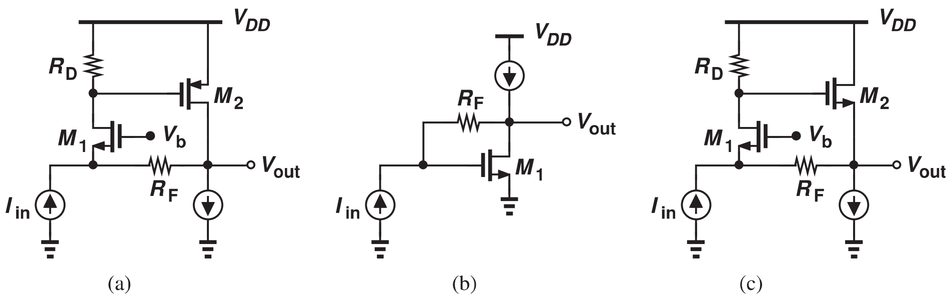


Figure 12.112

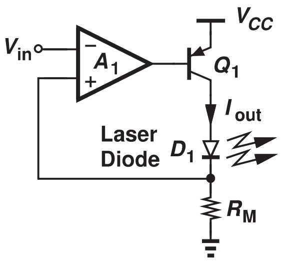


Figure 12.113

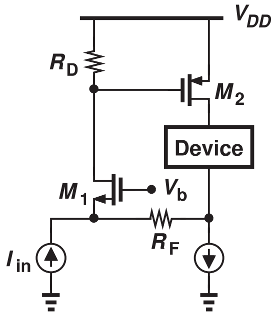


Figure 12.114

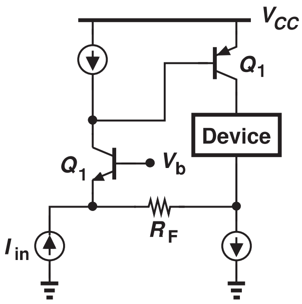


Figure 12.115

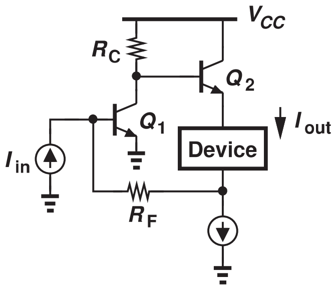


Figure 12.116

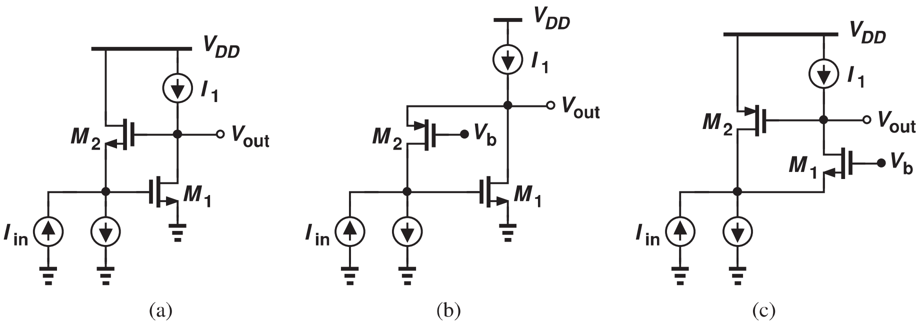


Figure 12.117

Sec. 12.8.1 Review of Bode's Rules

12.57. Construct the Bode plots for the magnitude and phase of the following systems:

(a) $\omega_{p1} = 2\pi \times (10\,\text{MHz})$, $\omega_{p2} = 2\pi \times (120\,\text{MHz})$, $\omega_z = 2\pi \times (1\,\text{GHz})$.

(b) $\omega_z = 2\pi \times (10\,\text{MHz})$, $\omega_{p1} = 2\pi \times (120\,\text{MHz})$, $\omega_{p2} = 2\pi \times (1\,\text{GHz})$.

(c) $\omega_z = 0$, $\omega_{p1} = 2\pi \times (10\,\text{MHz})$, $\omega_{p2} = 2\pi \times (120\,\text{MHz})$.

(d) $\omega_{p1} = 0$, $\omega_z = 2\pi \times (10\,\text{MHz})$, $\omega_{p2} = 2\pi \times (120\,\text{MHz})$.

12.58. In the Bode plots of Fig. 12.61, explain qualitatively what happens as ω_z comes closer to ω_{p1} or ω_{p2}.

****12.59.** Assuming $\lambda = 0$ and without using Miller's theorem, determine the transfer function of the circuit depicted in Fig. 12.118 and construct its Bode plots.

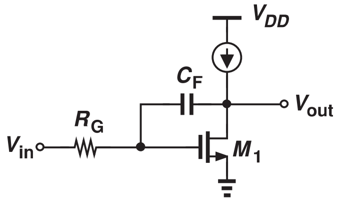


Figure 12.118

Sec. 12.8.2 Problem of Instability

12.60. In the system of Example 12.37, we gradually decrease the value of K without changing the position of the poles. Explain why decreasing K can make the system stable?

***12.61.** Unlike a one-pole system, the magnitude response of the circuit in Example 12.38 falls by more than 3 dB at the pole frequency. Determine $|H|$ at ω_p. Can we say $|H|$ falls 9 dB due to three coincident poles?

***12.62.** The three coincident poles in Example 12.38 do affect the phase even at $0.1\omega_p$. Calculate the phase of the transfer function at $\omega = 0.1\omega_p$.

12.63. Repeat Example 12.38 for $K = 0.1$.

12.64. Repeat Example 12.38 for four identical stages and compare the results.

12.65. Consider a one-pole circuit whose open-loop transfer function is given by

$$H(s) = \frac{A_0}{1 + \dfrac{s}{\omega_0}}. \tag{12.198}$$

Determine the phase margin of a feedback network using this circuit with $K = 1$.

12.66. Repeat Problem 12.65 for $K = 0.5$.

12.67. In each case illustrated in Fig. 12.70, what happens if K is reduced by a factor of 2?

12.68. Suppose the amplifier in Example 12.41 is described by

$$H(s) = \frac{A_0}{\left(1 + \dfrac{s}{\omega_{p1}}\right)\left(1 + \dfrac{s}{\omega_{p2}}\right)}, \tag{12.199}$$

where $\omega_{p2} \gg \omega_{p1}$. Compute the phase margin if the circuit is employed in a feedback system with $K = 0.5$.

12.69. Explain what happens to the characteristics illustrated in Fig. 12.72 if K drops by a factor of two. Assume ω_{p1} and ω'_{p1} remain constant.

****12.70.** Figure 12.119 depicts the amplifier of Example 12.38 with a compensation capacitor added to node X. Explain how the circuit can be compensated for a phase margin of 45°.

Design Problems

In the following problems, unless otherwise stated, assume $\mu_n C_{ox} = 2\mu_p C_{ox} = 100\ \mu\text{A/V}^2$ and $\lambda_n = 0.5\lambda_p = 0.1\ \text{V}^{-1}$.

12.71. Design the circuit of Example 12.15 for an open-loop gain of 50 and a nominal closed-loop gain of 4. Assume $I_{SS} = 0.5$ mA. Choose $R_1 + R_2 \approx 10(r_{O2}||r_{O4})$.

12.72. Design the circuit of Example 12.16 for an open-loop gain of 10, a closed-loop input impedance of 50 Ω, and a nominal closed-loop gain of 2. Calculate the closed-loop I/O impedances. Assume $R_1 + R_2 \approx 10R_D$.

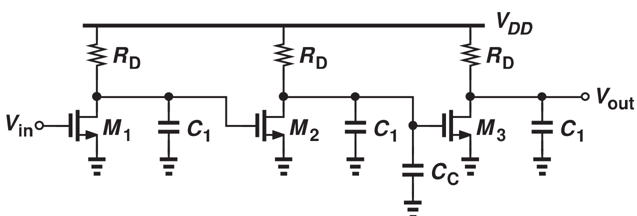


Figure 12.119

12.73. Design the transimpedance amplifier of Example 12.18 for an open-loop gain of 10 kΩ, a closed-loop gain of 1 kΩ, a closed-loop input impedance of 50 Ω, and a closed-loop output impedance of 200 Ω. Assume $R_{D1} = 1$ kΩ and R_F is very large.

12.74. Repeat Problem 12.73 for the circuit shown in Fig. 12.94.

12.75. We wish to design the transimpedance amplifier depicted in Fig. 12.101 for a closed-loop gain of 1 kΩ. Assume each transistor carries a collector bias current of 1 mA, $\beta = 100$, $V_A = \infty$, and R_F is very large.
(a) Determine the values of R_C and R_M for an open-loop gain of 20 kΩ and an open-loop output impedance of 500 Ω.
(b) Compute the required value of R_F.
(c) Calculate the closed-loop I/O impedances.

12.76. Design the circuit illustrated in Fig. 12.105 for an open-loop voltage gain of 20, an open-loop output impedance of 2 kΩ, and a closed-loop voltage gain of 4. Assume $\lambda = 0$. Is the solution unique? If not, how should the circuit parameters be chosen to minimize the power dissipation?

12.77. Design the circuit of Fig. 12.107 for a closed-loop gain of 2, a tail current of 1 mA, and minimum output impedance. Assume $\beta = 100$ and $V_A = \infty$.

12.78. Design the transimpedance amplifier of Fig. 12.111 for a closed-loop gain of 1 kΩ and an output impedance of 50 Ω. Assume each transistor is biased at a collector current of 1 mA and $V_A = \infty$.

SPICE PROBLEMS

In the following problems, use the MOS device models given in Appendix A. For bipolar transistors, assume $I_{S,npn} = 5 \times 10^{-16}$ A, $\beta_{npn} = 100$, $V_{A,npn} = 5$ V, $I_{S,pnp} = 8 \times 10^{-16}$ A, $\beta_{pnp} = 50$, $V_{A,pnp} = 3.5$ V. Also, SPICE models the effect of charge storage in the base by a parameter called $\tau_F = C_b/g_m$. Assume $\tau_F(tf) = 20$ ps.

12.79. Figure 12.120 shows a transimpedance amplifier often used in optical communications. Assume $R_F = 2$ kΩ.
(a) Select the value of R_C so that Q_1 carries a bias current of 1 mA.
(b) Estimate the loop gain.
(c) Determine the closed-loop gain and I/O impedances.
(d) Determine the change in the closed-loop gain if V_{CC} varies by $\pm 10\%$.

12.80. Figure 12.121 depicts another transimpedance amplifier, where the bias current of M_1 is defined by the mirror arrangement (M_2 and M_3). Assume $W/L = 20\ \mu\text{m}/0.18\ \mu\text{m}$ for M_1-M_3.
(a) What value of R_F yields a closed-loop gain of 1 kΩ?
(b) Determine the change in the closed-loop gain if V_{DD} varies by $\pm 10\%$.
(c) Suppose the circuit drives a load capacitance of 100 fF. Verify that the input impedance exhibits *inductive* behavior and explain why.

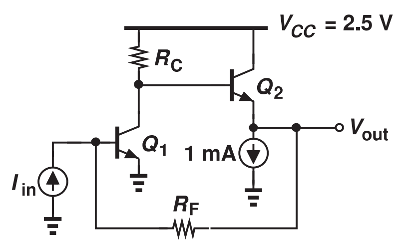


Figure 12.120

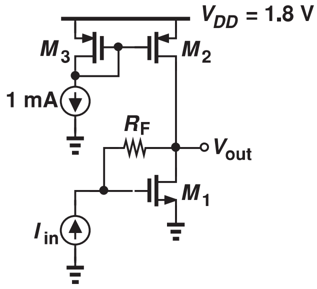


Figure 12.121

12.81. In the circuit shown in Fig. 12.122, $W/L = 20\,\mu\text{m}/0.18\,\mu\text{m}$ for M_1 and M_2.

(a) Determine the circuit's operating points for an input dc level of 0.9 V.

(b) Determine the closed-loop gain and I/O impedances.

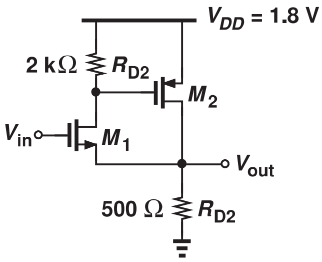


Figure 12.122

12.82. In the circuit of Fig. 12.123, the three stages provide a high gain, approximating an op amp. Assume $(W/L)_{1-6} = 10\,\mu\text{m}/0.18\,\mu\text{m}$.

(a) Explain why the circuit is potentially unstable.

(b) Determine the step response and explain the circuit's behavior.

(c) Place a capacitor between nodes X and Y and adjust its value to obtain a well-behaved step response.

(d) Determine the gain error of the circuit with respect to the nominal value of 10.

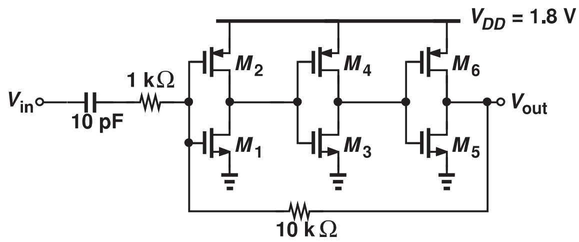


Figure 12.123

12.83. In the three-stage amplifier of Fig. 12.124, $(W/L)_{1-7} = 20\,\mu\text{m}/0.18\,\mu\text{m}$.

(a) Determine the phase margin.

(b) Place a capacitor between nodes X and Y so as to obtain a phase margin of 60°. What is the unity-gain bandwidth under this condition?

(c) Repeat (b) if the compensation capacitor is tied between X and ground and compare the results.

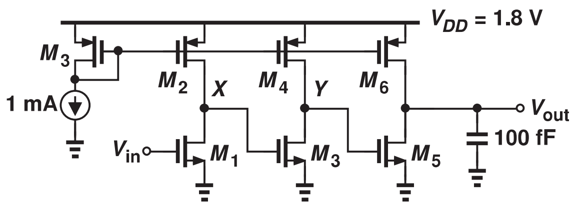


Figure 12.124

REFERENCE

1. B. Razavi, *Design of Analog CMOS Integrated Circuits*, McGraw-Hill, 2001.