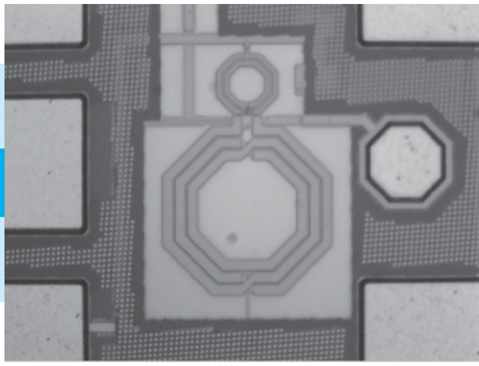


2

Basic Physics of Semiconductors

Microelectronic circuits are based on complex semiconductor structures that have been under active research for the past six decades. While this book deals with the analysis and design of *circuits*, we should emphasize at the outset that a good understanding of *devices* is essential to our work. The situation is similar to many other engineering problems, e.g., one cannot design a high-performance automobile without a detailed knowledge of the engine and its limitations.

Nonetheless, we do face a dilemma. Our treatment of device physics must contain enough depth to provide adequate understanding, but must also be sufficiently brief to allow quick entry into circuits. This chapter accomplishes this task.

Our ultimate objective in this chapter is to study a fundamentally important and versatile device called the "diode." However, just as we need to eat our broccoli before having dessert, we must develop a basic understanding of "semiconductor" materials and their current conduction mechanisms before attacking diodes.

In this chapter, we begin with the concept of semiconductors and study the movement of charge (i.e., the flow of current) in them. Next, we deal with the "*pn* junction," which also serves as diode, and formulate its behavior. Our ultimate goal is to represent the device by a circuit model (consisting of resistors, voltage or current sources, capacitors, etc.), so that a circuit using such a device can be analyzed easily. The outline is shown below.

Semiconductors
- Charge Carriers
- Doping
- Transport of Carriers

PN Junction
- Structure
- Reverse and Forward Bias Conditions
- I/V Characteristics
- Circuit Models

It is important to note that the task of developing accurate models proves critical for *all* microelectronic devices. The electronics industry continues to place greater demands on circuits, calling for aggressive designs that push semiconductor devices to their limits. Thus, a good understanding of the internal operation of devices is necessary.[1]

2.1 SEMICONDUCTOR MATERIALS AND THEIR PROPERTIES

Since this section introduces a multitude of concepts, it is useful to bear a general outline in mind:

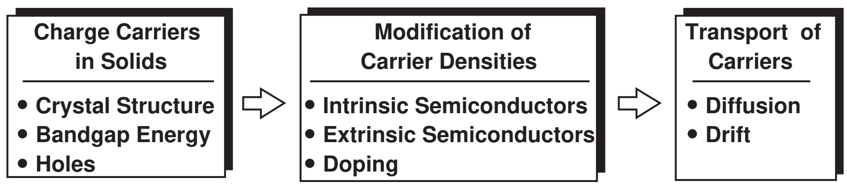


Figure 2.1 Outline of this section.

This outline represents a logical thought process: (a) we identify charge carriers in solids and formulate their role in current flow; (b) we examine means of modifying the density of charge carriers to create desired current flow properties; (c) we determine current flow mechanisms. These steps naturally lead to the computation of the current/voltage (I/V) characteristics of actual diodes in the next section.

2.1.1 Charge Carriers in Solids

Recall from basic chemistry that the electrons in an atom orbit the nucleus in different "shells." The atom's chemical activity is determined by the electrons in the outermost shell, called "valence" electrons, and how complete this shell is. For example, neon exhibits a complete outermost shell (with eight electrons) and hence no tendency for chemical reactions. On the other hand, sodium has only one valence electron, ready to relinquish it, and chloride has seven valence electrons, eager to receive one more. Both elements are therefore highly reactive.

The above principles suggest that atoms having approximately four valence electrons fall somewhere between inert gases and highly volatile elements, possibly displaying interesting chemical and physical properties. Shown in Fig. 2.2 is a section of the periodic table containing a number of elements with three to five valence electrons. As the most popular material in microelectronics, silicon merits a detailed analysis.[2]

Covalent Bonds A silicon atom residing in isolation contains four valence electrons [Fig. 2.3(a)], requiring another four to complete its outermost shell. If processed properly, the silicon material can form a "crystal" wherein each atom is surrounded by exactly four others [Fig. 2.3(b)]. As a result, each atom *shares* one valence electron with its neighbors, thereby completing its own shell and those of the neighbors. The "bond" thus formed between atoms is called a "covalent bond" to emphasize the sharing of valence electrons.

The uniform crystal depicted in Fig. 2.3(b) plays a crucial role in semiconductor devices. But, does it carry current in response to a voltage? At temperatures near absolute zero,

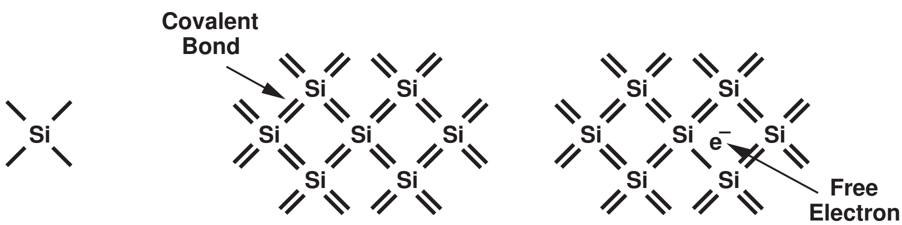
[1] As design managers often say, "If you do not push the devices and circuits to their limit but your competitor does, then you lose to your competitor."

[2] Silicon is obtained from sand after a great deal of processing.

	III	IV	V	
	Boron (B)	Carbon (C)		
• • •	Aluminum (Al)	Silicon (Si)	Phosphorus (P)	• • •
	Galium (Ga)	Germanium (Ge)	Arsenic (As)	
		• • •		

Figure 2.2 Section of the periodic table.

the valence electrons are confined to their respective covalent bonds, refusing to move freely. In other words, the silicon crystal behaves as an insulator for $T \to 0K$. However, at higher temperatures, electrons gain thermal energy, occasionally breaking away from the bonds and acting as free charge carriers [Fig. 2.3(c)] until they fall into another incomplete bond. We will hereafter use the term "electrons" to refer to free electrons.



(a) (b) (c)

Figure 2.3 (a) Silicon atom, (b) covalent bonds between atoms, (c) free electron released by thermal energy.

Holes When freed from a covalent bond, an electron leaves a "void" behind because the bond is now incomplete. Called a "hole," such a void can readily absorb a free electron if one becomes available. Thus, we say an "electron-hole pair" is generated when an electron is freed, and an "electron-hole recombination" occurs when an electron "falls" into a hole.

Why do we bother with the concept of the hole? After all, it is the free electron that actually moves in the crystal. To appreciate the usefulness of holes, consider the time evolution illustrated in Fig. 2.4. Suppose covalent bond number 1 contains a hole after

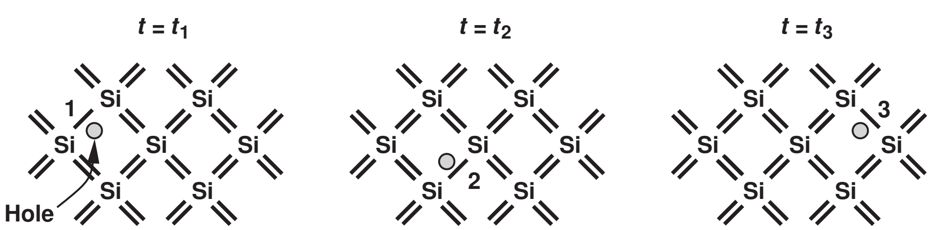


Figure 2.4 Movement of electron through crystal.

losing an electron some time before $t = t_1$. At $t = t_2$, an electron breaks away from bond number 2 and recombines with the hole in bond number 1. Similarly, at $t = t_3$, an electron leaves bond number 3 and falls into the hole in bond number 2. Looking at the three "snapshots," we can say one electron has traveled from right to left, or, alternatively, one hole has moved from left to right. This view of current flow by holes proves extremely useful in the analysis of semiconductor devices.

Bandgap Energy We must now answer two important questions. First, does *any* thermal energy create free electrons (and holes) in silicon? No, in fact, a minimum energy is required to dislodge an electron from a covalent bond. Called the "bandgap energy" and denoted by E_g, this minimum is a fundamental property of the material. For silicon, $E_g = 1.12$ eV.[3]

The second question relates to the conductivity of the material and is as follows. How *many* free electrons are created at a given temperature? From our observations thus far, we postulate that the number of electrons depends on both E_g and T: a greater E_g translates to fewer electrons, but a higher T yields more electrons. To simplify future derivations, we consider the *density* (or concentration) of electrons, i.e., the number of electrons per unit volume, n_i, and write for silicon:

$$n_i = 5.2 \times 10^{15} T^{3/2} \exp \frac{-E_g}{2kT} \text{ electrons/cm}^3 \tag{2.1}$$

where $k = 1.38 \times 10^{-23}$ J/K is called the Boltzmann constant. The derivation can be found in books on semiconductor physics, e.g., [1]. As expected, materials having a larger E_g exhibit a smaller n_i. Also, as $T \to 0$, so do $T^{3/2}$ and $\exp[-E_g/(2kT)]$, thereby bringing n_i toward zero.

The exponential dependence of n_i upon E_g reveals the effect of the bandgap energy on the conductivity of the material. Insulators display a high E_g; for example, $E_g = 2.5$ eV for diamond. Conductors, on the other hand, have a small bandgap. Finally, *semi*conductors exhibit a moderate E_g, typically ranging from 1 eV to 1.5 eV.

Example 2.1 Determine the density of electrons in silicon at $T = 300$ K (room temperature) and $T = 600$ K.

Solution Since $E_g = 1.12 \text{ eV} = 1.792 \times 10^{-19}$ J, we have

$$n_i(T = 300 \text{ K}) = 1.08 \times 10^{10} \text{ electrons/cm}^3 \tag{2.2}$$

$$n_i(T = 600 \text{ K}) = 1.54 \times 10^{15} \text{ electrons/cm}^3. \tag{2.3}$$

Since for each free electron, a hole is left behind, the density of holes is also given by (2.2) and (2.3).

Exercise Repeat the above exercise for a material having a bandgap of 1.5 eV.

The n_i values obtained in the above example may appear quite high, but, noting that silicon has 5×10^{22} atoms/cm^3, we recognize that only one in 5×10^{12} atoms benefit from a

[3]The unit eV (electron volt) represents the energy necessary to move one electron across a potential difference of 1 V. Note that 1 eV = 1.6×10^{-19} J.

free electron at room temperature. In other words, silicon still seems a very poor conductor. But, do not despair! We next introduce a means of making silicon more useful.

2.1.2 Modification of Carrier Densities

Intrinsic and Extrinsic Semiconductors
The "pure" type of silicon studied thus far is an example of "intrinsic semiconductors," suffering from a very high resistance. Fortunately, it is possible to modify the resistivity of silicon by replacing some of the atoms in the crystal with atoms of another material. In an intrinsic semiconductor, the electron density, $n(=n_i)$, is equal to the hole density, p. Thus,

$$np = n_i^2. \tag{2.4}$$

We return to this equation later.

Did you know?
The semiconductor industry manufactures microprocessors, memories, RF transceivers, imaging chips, and many other products, bringing in an annual revenue of 300 billion dollars. This means that, of the seven billion people in the world, each person spends an average of about \$40 on semiconductor chips every year. This starts when children buy their first video game device.

Recall from Fig. 2.2 that phosphorus (P) contains five valence electrons. What happens if some P atoms are introduced in a silicon crystal? As illustrated in Fig. 2.5, each P atom shares four electrons with the neighboring silicon atoms, leaving the fifth electron "unattached." This electron is free to move, serving as a charge carrier. Thus, if N phosphorus atoms are uniformly introduced in each cubic centimeter of a silicon crystal, then the density of free electrons rises by the same amount.

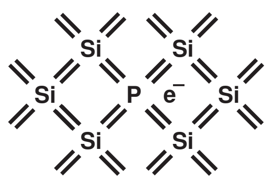


Figure 2.5 Loosely-attached electon with phosphorus doping.

The controlled addition of an "impurity" such as phosphorus to an intrinsic semiconductor is called "doping," and phosphorus itself a "dopant." Providing many more free electrons than in the intrinsic state, the doped silicon crystal is now called "extrinsic," more specifically, an "n-type" semiconductor to emphasize the abundance of free electrons.

As remarked earlier, the electron and hole densities in an intrinsic semiconductor are equal. But, how about these densities in a doped material? It can be proved that even in this case,

$$np = n_i^2, \tag{2.5}$$

where n and p respectively denote the electron and hole densities in the extrinsic semiconductor. The quantity n_i represents the densities in the intrinsic semiconductor (hence the subscript i) and is therefore independent of the doping level [e.g., Eq. (2.1) for silicon].

Example 2.2 The above result seems quite strange. How can np remain constant while we add more donor atoms and increase n?

Solution Equation (2.5) reveals that p must fall *below* its intrinsic level as more n-type dopants are added to the crystal. This occurs because many of the new electrons donated by the dopant "recombine" with the holes that were created in the intrinsic material.

Exercise Why can we not say that $n + p$ should remain constant?

Example 2.3 A piece of crystalline silicon is doped uniformly with phosphorus atoms. The doping density is 10^{16} atoms/cm^3. Determine the electron and hole densities in this material at the room temperature.

Solution The addition of 10^{16} P atoms introduces the same number of free electrons per cubic centimeter. Since this electron density exceeds that calculated in Example 2.1 by six orders of magnitude, we can assume

$$n = 10^{16} \text{ electrons/cm}^3. \tag{2.6}$$

It follows from (2.2) and (2.5) that

$$p = \frac{n_i^2}{n} \tag{2.7}$$

$$= 1.17 \times 10^4 \text{ holes/cm}^3. \tag{2.8}$$

Note that the hole density has dropped below the intrinsic level by six orders of magnitude. Thus, if a voltage is applied across this piece of silicon, the resulting current consists predominantly of electrons.

Exercise At what doping level does the hole density drop by three orders of magnitude?

This example justifies the reason for calling electrons the "majority carriers" and holes the "minority carriers" in an n-type semiconductor. We may naturally wonder if it is possible to construct a "p-type" semiconductor, thereby exchanging the roles of electrons and holes.

Indeed, if we can dope silicon with an atom that provides an *insufficient* number of electrons, then we may obtain many *incomplete* covalent bonds. For example, the table in Fig. 2.2 suggests that a boron (B) atom—with three valence electrons—can form only three complete covalent bonds in a silicon crystal (Fig. 2.6). As a result, the fourth bond contains a hole, ready to absorb a free electron. In other words, N boron atoms contribute N boron holes to the conduction of current in silicon. The structure in Fig. 2.6 therefore exemplifies a p-type semiconductor, providing holes as majority carriers. The boron atom is called an "acceptor" dopant.

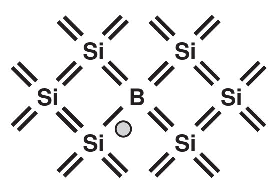


Figure 2.6 Available hole with boron doping.

Let us formulate our results thus far. If an intrinsic semiconductor is doped with a density of N_D ($\gg n_i$) donor atoms per cubic centimeter, then the mobile charge densities are given by

$$\text{Majority Carriers: } n \approx N_D \tag{2.9}$$

$$\text{Minority Carriers: } p \approx \frac{n_i^2}{N_D}. \tag{2.10}$$

Similarly, for a density of N_A ($\gg n_i$) acceptor atoms per cubic centimeter:

$$\text{Majority Carriers: } p \approx N_A \tag{2.11}$$

$$\text{Minority Carriers: } n \approx \frac{n_i^2}{N_A}. \tag{2.12}$$

Since typical doping densities fall in the range of 10^{15} to 10^{18} atoms/cm^3, the above expressions are quite accurate.

Example 2.4 Is it possible to use other elements of Fig. 2.2 as semiconductors and dopants?

Solution Yes, for example, some early diodes and transistors were based on germanium (Ge) rather than silicon. Also, arsenic (As) is another common dopant.

Exercise Can carbon be used for this purpose?

Figure 2.7 summarizes the concepts introduced in this section, illustrating the types of charge carriers and their densities in semiconductors.

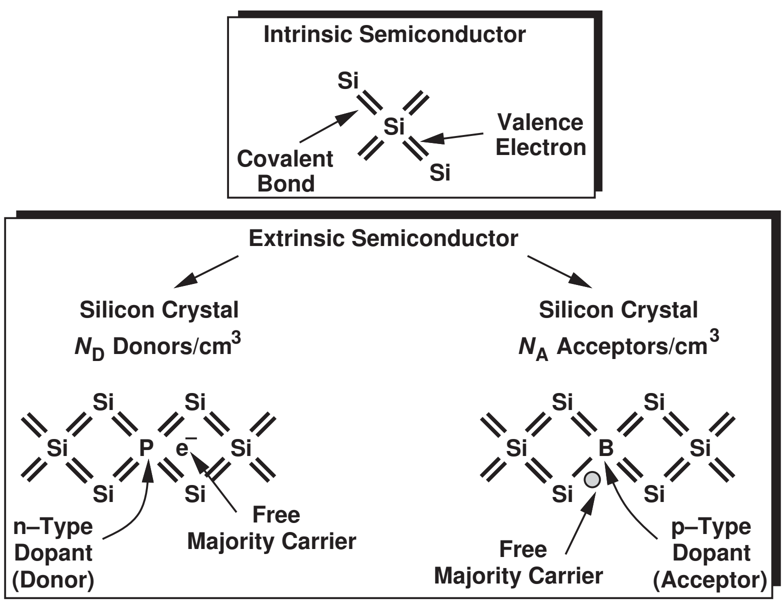


Figure 2.7 Summary of charge carriers in silicon.

2.1.3 Transport of Carriers

Having studied charge carriers and the concept of doping, we are ready to examine the *movement* of charge in semiconductors, i.e., the mechanisms leading to the flow of current.

Drift We know from basic physics and Ohm's law that a material can conduct current in response to a potential difference and hence an electric field.[4] The field accelerates the charge carriers in the material, forcing some to flow from one end to the other. Movement of charge carriers due to an electric field is called "drift."[5]

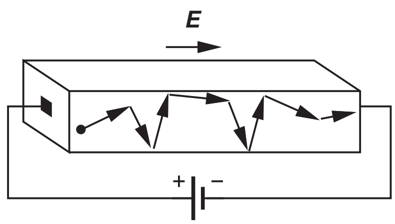


Figure 2.8 Drift in a semiconductor.

Semiconductors behave in a similar manner. As shown in Fig. 2.8, the charge carriers are accelerated by the field and accidentally collide with the atoms in the crystal, eventually reaching the other end and flowing into the battery. The acceleration due to the field and the collision with the crystal counteract, leading to a *constant* velocity for the carriers.[6] We expect the velocity, v, to be proportional to the electric field strength, E:

$$v \propto E, \tag{2.13}$$

and hence

$$v = \mu E, \tag{2.14}$$

where μ is called the "mobility" and usually expressed in $\text{cm}^2/(\text{V}\cdot\text{s})$. For example in silicon, the mobility of electrons, $\mu_n = 1350\,\text{cm}^2/(\text{V}\cdot\text{s})$, and that of holes, $\mu_p = 480\,\text{cm}^2/(\text{V}\cdot\text{s})$. Of course, since electrons move in a direction opposite to the electric field, we must express the velocity vector as

$$\vec{v_e} = -\mu_n \vec{E}. \tag{2.15}$$

For holes, on the other hand,

$$\vec{v_h} = \mu_p \vec{E}. \tag{2.16}$$

[4]Recall that the potential (voltage) difference, V, is equal to the negative integral of the electric field, E, with respect to distance: $V_{ab} = -\int_b^a E dx$.

[5]The convention for direction of current assumes flow of *positive* charge from a positive voltage to a negative voltage. Thus, if electrons flow from point A to point B, the current is considered to have a direction from B to A.

[6]This phenomenon is analogous to the "terminal velocity" that a sky diver with a parachute (hopefully, open) experiences.

Example 2.5 A uniform piece of n-type of silicon that is 1 μm long senses a voltage of 1 V. Determine the velocity of the electrons.

Solution Since the material is uniform, we have $E = V/L$, where L is the length. Thus, $E = 10{,}000$ V/cm and hence $v = \mu_n E = 1.35 \times 10^7$ cm/s. In other words, electrons take $(1\ \mu\text{m})/(1.35 \times 10^7\ \text{cm/s}) = 7.4$ ps to cross the 1-μm length.

Exercise What happens if the mobility is halved?

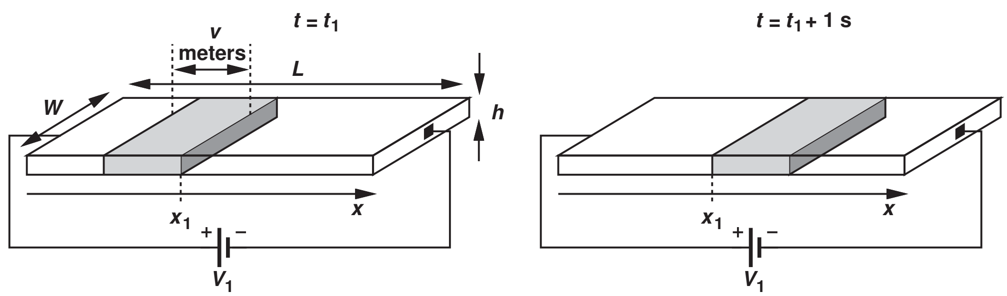


Figure 2.9 Current flow in terms of charge density.

With the velocity of carriers known, how is the current calculated? We first note that an electron carries a negative charge equal to $q = 1.6 \times 10^{-19}$ C. Equivalently, a hole carries a positive charge of the same value. Now suppose a voltage V_1 is applied across a uniform semiconductor bar having a free electron density of n (Fig. 2.9). Assuming the electrons move with a velocity of v m/s, considering a cross section of the bar at $x = x_1$ and taking two "snapshots" at $t = t_1$ and $t = t_1 + 1$ second, we note that the total charge in v meters passes the cross section in 1 second. In other words, the current is equal to the total charge enclosed in v meters of the bar's length. Since the bar has a width of W, we have:

$$I = -v \cdot W \cdot h \cdot n \cdot q, \tag{2.17}$$

where $v \cdot W \cdot h$ represents the volume, $n \cdot q$ denotes the charge density in coulombs, and the negative sign accounts for the fact that electrons carry negative charge.

Let us now reduce Eq. (2.13) to a more convenient form. Since for electrons, $v = -\mu_n E$, and since $W \cdot h$ is the cross section area of the bar, we write

$$J_n = \mu_n E \cdot n \cdot q, \tag{2.18}$$

where J_n denotes the "current density," i.e., the current passing through a *unit* cross section area, and is expressed in A/cm^2. We may loosely say, "the current is equal to the charge velocity times the charge density," with the understanding that "current" in fact refers to current density, and negative or positive signs are taken into account properly.

In the presence of both electrons and holes, Eq. (2.18) is modified to

$$J_{tot} = \mu_n E \cdot n \cdot q + \mu_p E \cdot p \cdot q \tag{2.19}$$

$$= q(\mu_n n + \mu_p p)E. \tag{2.20}$$

This equation gives the drift current density in response to an electric field E in a semiconductor having uniform electron and hole densities.

Example 2.6 In an experiment, it is desired to obtain equal electron and hole drift currents. How should the carrier densities be chosen?

Solution We must impose

$$\mu_n n = \mu_p p, \tag{2.21}$$

and hence

$$\frac{n}{p} = \frac{\mu_p}{\mu_n}. \tag{2.22}$$

We also recall that $np = n_i^2$. Thus,

$$p = \sqrt{\frac{\mu_n}{\mu_p}} n_i \tag{2.23}$$

$$n = \sqrt{\frac{\mu_p}{\mu_n}} n_i. \tag{2.24}$$

For example, in silicon, $\mu_n/\mu_p = 1350/480 = 2.81$, yielding

$$p = 1.68 n_i \tag{2.25}$$

$$n = 0.596 n_i. \tag{2.26}$$

Since p and n are of the same order as n_i, equal electron and hole drift currents can occur for only a very lightly doped material. This confirms our earlier notion of majority carriers in semiconductors having typical doping levels of 10^{15}–10^{18} atoms/cm^3.

Exercise How should the carrier densities be chosen so that the electron drift current is twice the hole drift current?

Velocity Saturation* We have thus far assumed that the mobility of carriers in semiconductors is *independent* of the electric field and the velocity rises linearly with E according to $v = \mu E$. In reality, if the electric field approaches sufficiently high levels, v no longer follows E linearly. This is because the carriers collide with the lattice so frequently and the time between the collisions is so short that they cannot accelerate much. As a result, v varies "sublinearly" at high electric fields, eventually reaching a saturated level, v_{sat} (Fig. 2.10). Called "velocity saturation," this effect manifests itself in some modern transistors, limiting the performance of circuits.

In order to represent velocity saturation, we must modify $v = \mu E$ accordingly. A simple approach is to view the slope, μ, as a field-dependent parameter. The expression

*This section can be skipped in a first reading.

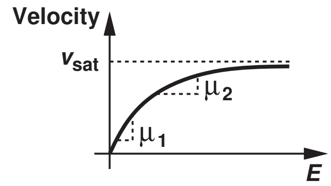


Figure 2.10 Velocity saturation.

for μ must therefore gradually fall toward zero as E rises, but approach a constant value for small E; i.e.,

$$\mu = \frac{\mu_0}{1 + bE}, \tag{2.27}$$

where μ_0 is the "low-field" mobility and b a proportionality factor. We may consider μ as the "effective" mobility at an electric field E. Thus,

$$v = \frac{\mu_0}{1 + bE} E. \tag{2.28}$$

Since for $E \to \infty$, $v \to v_{sat}$, we have

$$v_{sat} = \frac{\mu_0}{b}, \tag{2.29}$$

and hence $b = \mu_0 / v_{sat}$. In other words,

$$v = \frac{\mu_0}{1 + \frac{\mu_0 E}{v_{sat}}} E. \tag{2.30}$$

Example 2.7

A uniform piece of semiconductor 0.2 μm long sustains a voltage of 1 V. If the low-field mobility is equal to 1350 $cm^2/(V \cdot s)$ and the saturation velocity of the carriers 10^7 cm/s, determine the effective mobility. Also, calculate the maximum allowable voltage such that the effective mobility is only 10% lower than μ_0.

Solution

We have

$$E = \frac{V}{L} \tag{2.31}$$

$$= 50\,\mathrm{kV/cm}. \tag{2.32}$$

It follows that

$$\mu = \frac{\mu_0}{1 + \frac{\mu_0 E}{v_{sat}}} \tag{2.33}$$

$$= \frac{\mu_0}{7.75} \tag{2.34}$$

$$= 174\,\mathrm{cm^2/(V \cdot s)}. \tag{2.35}$$

If the mobility must remain within 10% of its low-field value, then

$$0.9\mu_0 = \frac{\mu_0}{1 + \dfrac{\mu_0 E}{v_{sat}}}, \tag{2.36}$$

and hence

$$E = \frac{1}{9}\frac{v_{sat}}{\mu_0} \tag{2.37}$$

$$= 823\ \text{V/cm}. \tag{2.38}$$

A device of length 0.2 μm experiences such a field if it sustains a voltage of $(823\ \text{V/cm}) \times (0.2 \times 10^{-4}\ \text{cm}) = 16.5\ \text{mV}$.

This example suggests that modern (submicron) devices incur substantial velocity saturation because they operate with voltages much greater than 16.5 mV.

Exercise At what voltage does the mobility fall by 20%?

Diffusion In addition to drift, another mechanism can lead to current flow. Suppose a drop of ink falls into a glass of water. Introducing a high local concentration of ink molecules, the drop begins to "diffuse," that is, the ink molecules tend to flow from a region of high concentration to regions of low concentration. This mechanism is called "diffusion."

A similar phenomenon occurs if charge carriers are "dropped" (injected) into a semiconductor so as to create a *nonuniform* density. Even in the absence of an electric field, the carriers move toward regions of low concentration, thereby carrying an electric current so long as the nonuniformity is sustained. Diffusion is therefore distinctly different from drift.

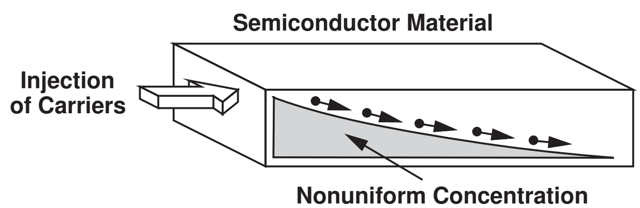


Figure 2.11 Diffusion in a semiconductor.

Figure 2.11 conceptually illustrates the process of diffusion. A source on the left continues to inject charge carriers into the semiconductor, a nonuniform charge profile is created along the x-axis, and the carriers continue to "roll down" the profile.

The reader may raise several questions at this point. What serves as the source of carriers in Fig. 2.11? Where do the charge carriers go after they roll down to the end of the profile at the far right? And, most importantly, why should we care?! Well, patience is a virtue and we will answer these questions in the next section.

Example 2.8 A source injects charge carriers into a semiconductor bar as shown in Fig. 2.12. Explain how the current flows.

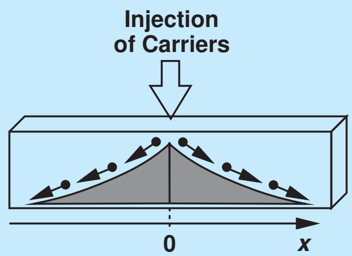


Figure 2.12 Injection of carriers into a semiconductor.

Solution In this case, two symmetric profiles may develop in both positive and negative directions along the x-axis, leading to current flow from the source toward the two ends of the bar.

Exercise Is KCL still satisfied at the point of injection?

Our qualitative study of diffusion suggests that the more nonuniform the concentration, the larger the current. More specifically, we can write:

$$I \propto \frac{dn}{dx}, \tag{2.39}$$

where n denotes the carrier concentration at a given point along the x-axis. We call dn/dx the concentration "gradient" with respect to x, assuming current flow only in the x direction. If each carrier has a charge equal to q, and the semiconductor has a cross section area of A, Eq. (2.39) can be written as

$$I \propto Aq\frac{dn}{dx}. \tag{2.40}$$

Thus,

$$I = AqD_n\frac{dn}{dx}, \tag{2.41}$$

where D_n is a proportionality factor called the "diffusion constant" and expressed in cm^2/s. For example, in intrinsic silicon, $D_n = 34$ cm^2/s (for electrons), and $D_p = 12$ cm^2/s (for holes).

As with the convention used for the drift current, we normalize the diffusion current to the cross section area, obtaining the current density as

$$J_n = qD_n\frac{dn}{dx}. \tag{2.42}$$

Similarly, a gradient in hole concentration yields:

$$J_p = -qD_p\frac{dp}{dx}. \tag{2.43}$$

With both electron and hole concentration gradients present, the total current density is given by

$$J_{tot} = q\left(D_n \frac{dn}{dx} - D_p \frac{dp}{dx}\right). \tag{2.44}$$

Example 2.9

Consider the scenario depicted in Fig. 2.11 again. Suppose the electron concentration is equal to N at $x = 0$ and falls linearly to zero at $x = L$ (Fig. 2.13). Determine the diffusion current.

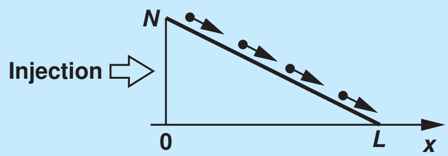


Figure 2.13 Current resulting from a linear diffusion profile.

Solution We have

$$J_n = qD_n \frac{dn}{dx} \tag{2.45}$$

$$= -qD_n \cdot \frac{N}{L}. \tag{2.46}$$

The current is constant along the x-axis; i.e., all of the electrons entering the material at $x = 0$ successfully reach the point at $x = L$. While obvious, this observation prepares us for the next example.

Exercise Repeat the above example for holes.

Example 2.10

Repeat the above example but assume an exponential gradient (Fig. 2.14):

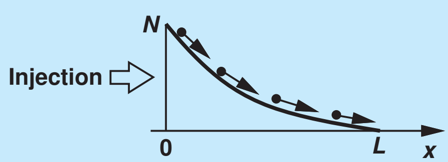


Figure 2.14 Current resulting from an exponential diffusion profile.

$$n(x) = N \exp \frac{-x}{L_d}, \tag{2.47}$$

where L_d is a constant.[7]

[7]The factor L_d is necessary to convert the exponent to a dimensionless quantity.

Solution We have

$$J_n = qD_n \frac{dn}{dx} \tag{2.48}$$

$$= \frac{-qD_n N}{L_d} \exp \frac{-x}{L_d}. \tag{2.49}$$

Interestingly, the current is *not* constant along the x-axis. That is, some electrons vanish while traveling from $x = 0$ to the right. What happens to these electrons? Does this example violate the law of conservation of charge? These are important questions and will be answered in the next section.

Exercise At what value of x does the current density drop to 1% of its maximum value?

Einstein Relation Our study of drift and diffusion has introduced a factor for each: μ_n (or μ_p) and D_n (or D_p), respectively. It can be proved that μ and D are related as:

$$\frac{D}{\mu} = \frac{kT}{q}. \tag{2.50}$$

Called the "Einstein Relation," this result is proved in semiconductor physics texts, e.g., [1]. Note that $kT/q \approx 26\,\text{mV}$ at $T = 300\,\text{K}$.

Figure 2.15 summarizes the charge transport mechanisms studied in this section.

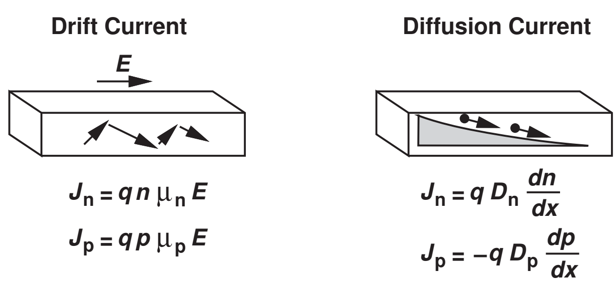


Figure 2.15 Summary of drift and diffusion mechanisms.

2.2 *pn* JUNCTION

We begin our study of semiconductor devices with the *pn* junction for three reasons. (1) The device finds application in many electronic systems, e.g., in adaptors that charge the batteries of cellphones. (2) The *pn* junction is among the simplest semiconductor devices, thus providing a good entry point into the study of the operation of such complex structures as transistors.

Did you know?

The *pn* junction was inadvertantly invented by Russel Ohl of Bell Laboratories in 1940. He melted silicon in quartz tubes to achieve a high purity. During the cooling process, the *p*-type and *n*-type impurities redistributed themselves, creating a *pn* junction. Ohl even observed that the *pn* junction produced a current when it was exposed to light. One wonders if Ohl ever predicted that this property would eventually lead to the invention of the digital camera.

(3) The *pn* junction also serves as part of transistors. We also use the term "diode" to refer to *pn* junctions.

We have thus far seen that doping produces free electrons or holes in a semiconductor, and an electric field or a concentration gradient leads to the movement of these charge carriers. An interesting situation arises if we introduce *n*-type and *p*-type dopants into two adjacent sections of a piece of semiconductor. Depicted in Fig. 2.16 and called a "*pn* junction," this structure plays a fundamental role in many semiconductor devices. The *p* and *n* sides are called the "anode" and the "cathode," respectively.

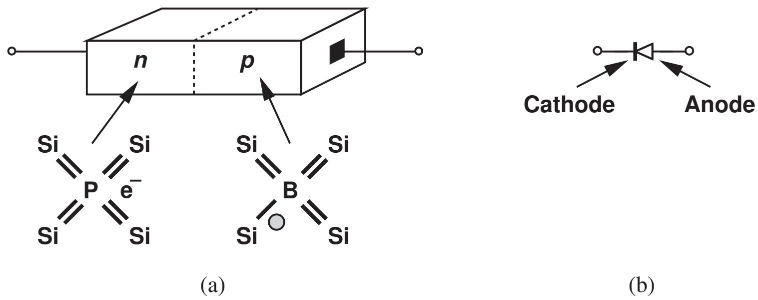


Figure 2.16 *pn* junction.

In this section, we study the properties and I/V characteristics of *pn* junctions. The following outline shows our thought process, indicating that our objective is to develop *circuit* models that can be used in analysis and design.

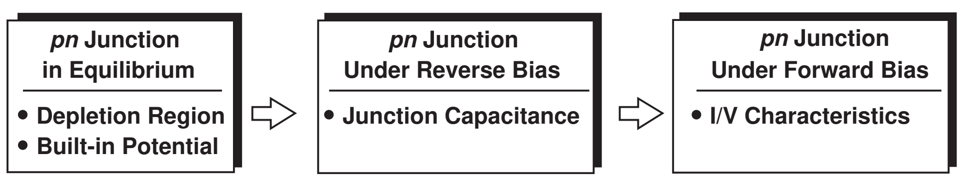


Figure 2.17 Outline of concepts to be studied.

2.2.1 *pn* Junction in Equilibrium

Let us first study the *pn* junction with no external connections, i.e., the terminals are open and no voltage is applied across the device. We say the junction is in "equilibrium." While seemingly of no practical value, this condition provides insights that prove useful in understanding the operation under nonequilibrium as well.

We begin by examining the interface between the *n* and *p* sections, recognizing that one side contains a large excess of holes and the other, a large excess of electrons. The sharp concentration gradient for both electrons and holes across the junction leads to two large diffusion currents: electrons flow from the *n* side to the *p* side, and holes flow in the opposite direction. Since we must deal with both electron and hole concentrations on each side of the junction, we introduce the notations shown in Fig. 2.18.

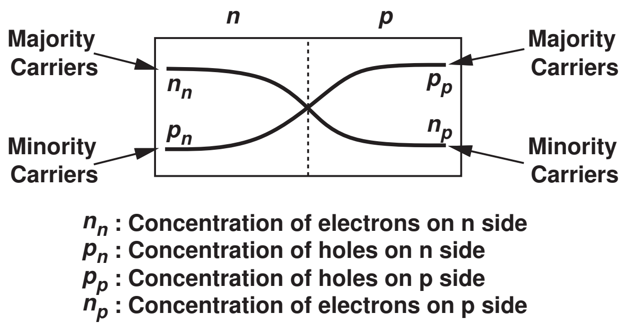


Figure 2.18

Example 2.11

A *pn* junction employs the following doping levels: $N_A = 10^{16}\ \text{cm}^{-3}$ and $N_D = 5 \times 10^{15}\ \text{cm}^{-3}$. Determine the hole and electron concentrations on the two sides.

Solution From Eqs. (2.11) and (2.12), we express the concentrations of holes and electrons on the *p* side respectively as:

$$p_p \approx N_A \tag{2.51}$$

$$= 10^{16}\ \text{cm}^{-3} \tag{2.52}$$

$$n_p \approx \frac{n_i^2}{N_A} \tag{2.53}$$

$$= \frac{(1.08 \times 10^{10}\ \text{cm}^{-3})^2}{10^{16}\ \text{cm}^{-3}} \tag{2.54}$$

$$\approx 1.1 \times 10^4\ \text{cm}^{-3}. \tag{2.55}$$

Similarly, the concentrations on the *n* side are given by

$$n_n \approx N_D \tag{2.56}$$

$$= 5 \times 10^{15}\ \text{cm}^{-3} \tag{2.57}$$

$$p_n \approx \frac{n_i^2}{N_D} \tag{2.58}$$

$$= \frac{(1.08 \times 10^{10}\ \text{cm}^{-3})^2}{5 \times 10^{15}\ \text{cm}^{-3}} \tag{2.59}$$

$$= 2.3 \times 10^4\ \text{cm}^{-3}. \tag{2.60}$$

Note that the majority carrier concentration on each side is many orders of magnitude higher than the minority carrier concentration on either side.

Exercise Repeat the above example if N_D drops by a factor of four.

The diffusion currents transport a great deal of charge from each side to the other, but they must eventually decay to zero. This is because if the terminals are left open (equilibrium condition), the device cannot carry a net current indefinitely.

We must now answer an important question: what stops the diffusion currents? We may postulate that the currents stop after enough free carriers have moved across the junction so as to equalize the concentrations on the two sides. However, another effect dominates the situation and stops the diffusion currents well before this point is reached.

To understand this effect, we recognize that for every electron that departs from the n side, a *positive ion* is left behind, i.e., the junction evolves with time as conceptually shown in Fig. 2.19. In this illustration, the junction is suddenly formed at $t = 0$, and the diffusion currents continue to expose more ions as time progresses. Consequently, the immediate vicinity of the junction is depleted of free carriers and hence called the "depletion region."

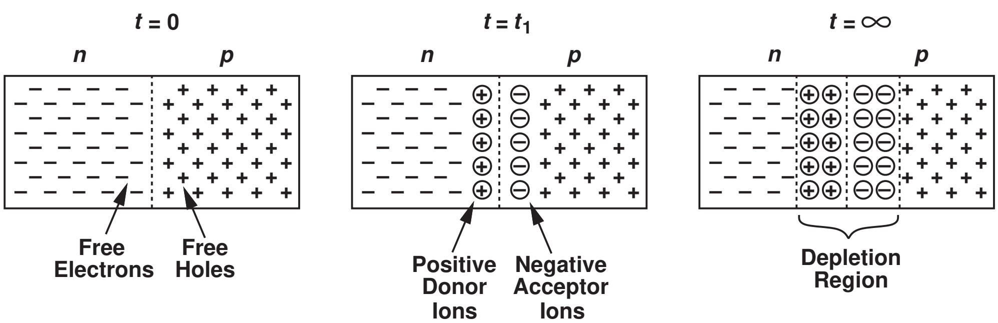


Figure 2.19 Evolution of charge concentrations in a *pn* junction.

Now recall from basic physics that a particle or object carrying a net (nonzero) charge creates an electric field around it. Thus, with the formation of the depletion region, an electric field emerges as shown in Fig. 2.20.[8] Interestingly, the field tends to force positive charge flow from left to right whereas the concentration gradients necessitate the flow of holes from right to left (and electrons from left to right). We therefore surmise that the junction reaches *equilibrium* once the electric field is strong enough to completely stop the diffusion currents. Alternatively, we can say, in equilibrium, the drift currents resulting from the electric field exactly cancel the diffusion currents due to the gradients.

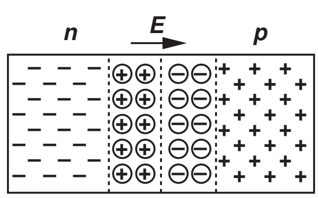


Figure 2.20 Electric field in a *pn* junction.

[8]The direction of the electric field is determined by placing a small positive test charge in the region and watching how it moves: away from positive charge and toward negative charge.

Example 2.12 In the junction shown in Fig. 2.21, the depletion region has a width of b on the n side and a on the p side. Sketch the electric field as a function of x.

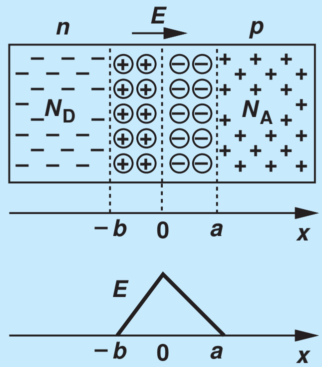


Figure 2.21 Electric field profile in a *pn* junction.

Solution Beginning at $x < -b$, we note that the absence of net charge yields $E = 0$. At $x > -b$, each positive donor ion contributes to the electric field, i.e., the magnitude of E rises as x approaches zero. As we pass $x = 0$, the negative acceptor atoms begin to contribute negatively to the field, i.e., E falls. At $x = a$, the negative and positive charge exactly cancel each other and $E = 0$.

Exercise Noting that potential voltage is negative integral of electric field with respect to distance, plot the potential as a function of x.

From our observation regarding the drift and diffusion currents under equilibrium, we may be tempted to write:

$$|I_{\text{drift},p} + I_{\text{drift},n}| = |I_{\text{diff},p} + I_{\text{diff},n}|, \tag{2.61}$$

where the subscripts p and n refer to holes and electrons, respectively, and each current term contains the proper polarity. This condition, however, allows an unrealistic phenomenon: if the number of the electrons flowing from the n side to the p side is equal to that of the holes going from the p side to the n side, then each side of this equation is zero while electrons continue to accumulate on the p side and holes on the n side. We must therefore impose the equilibrium condition on *each* carrier:

$$|I_{\text{drift},p}| = |I_{\text{diff},p}| \tag{2.62}$$

$$|I_{\text{drift},n}| = |I_{\text{diff},n}|. \tag{2.63}$$

Built-in Potential The existence of an electric field within the depletion region suggests that the junction may exhibit a "built-in potential." In fact, using (2.62) or (2.63), we can compute this potential. Since the electric field $E = -dV/dx$, and since (2.62) can be written as

$$q\mu_p pE = qD_p \frac{dp}{dx}, \tag{2.64}$$

we have

$$-\mu_p p \frac{dV}{dx} = D_p \frac{dp}{dx}. \tag{2.65}$$

Dividing both sides by p and taking the integral, we obtain

$$-\mu_p \int_{x_1}^{x_2} dV = D_p \int_{p_n}^{p_p} \frac{dp}{p}, \tag{2.66}$$

where p_n and p_p are the hole concentrations at x_1 and x_2, respectively (Fig. 2.22). Thus,

$$V(x_2) - V(x_1) = -\frac{D_p}{\mu_p} \ln \frac{p_p}{p_n}. \tag{2.67}$$

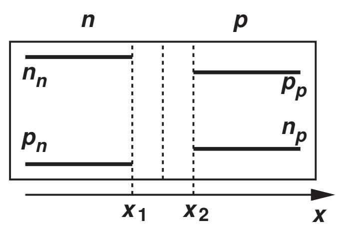


Figure 2.22 Carrier profiles in a *pn* junction.

The right side represents the voltage difference developed across the depletion region and will be denoted by V_0. Also, from Einstein's relation, Eq. (2.50), we can replace D_p/μ_p with kT/q:

$$|V_0| = \frac{kT}{q} \ln \frac{p_p}{p_n}. \tag{2.68}$$

Exercise Writing Eq. (2.64) for electron drift and diffusion currents, and carrying out the integration, derive an equation for V_0 in terms of n_n and n_p.

Finally, using (2.11) and (2.10) for p_p and p_n yields

$$V_0 = \frac{kT}{q} \ln \frac{N_A N_D}{n_i^2}. \tag{2.69}$$

Expressing the built-in potential in terms of junction parameters, this equation plays a central role in many semiconductor devices.

Example 2.13 A silicon *pn* junction employs $N_A = 2 \times 10^{16}\ \text{cm}^{-3}$ and $N_D = 4 \times 10^{16}\ \text{cm}^{-3}$. Determine the built-in potential at room temperature ($T = 300$ K).

Solution Recall from Example 2.1 that $n_i(T = 300\ \text{K}) = 1.08 \times 10^{10}\ \text{cm}^{-3}$. Thus,

$$V_0 \approx (26\ \text{mV}) \ln \frac{(2 \times 10^{16}) \times (4 \times 10^{16})}{(1.08 \times 10^{10})^2} \tag{2.70}$$

$$\approx 768\ \text{mV}. \tag{2.71}$$

Exercise By what factor should N_D be changed to lower V_0 by 20 mV?

Example 2.14 Equation (2.69) reveals that V_0 is a weak function of the doping levels. How much does V_0 change if N_A or N_D is increased by one order of magnitude?

Solution We can write

$$\Delta V_0 = V_T \ln \frac{10N_A \cdot N_D}{n_i^2} - V_T \ln \frac{N_A \cdot N_D}{n_i^2} \tag{2.72}$$

$$= V_T \ln 10 \tag{2.73}$$

$$\approx 60\,\text{mV (at } T = 300\,\text{K)}. \tag{2.74}$$

Exercise How much does V_0 change if N_A or N_D is increased by a factor of three?

An interesting question may arise at this point. The junction carries no net current (because its terminals remain open), but it sustains a voltage. How is that possible? We observe that the built-in potential is developed to *oppose* the flow of diffusion currents (and is, in fact, sometimes called the "potential barrier"). This phenomenon is in contrast to the behavior of a uniform conducting material, which exhibits no tendency for diffusion and hence no need to create a built-in voltage.

2.2.2 *pn* Junction Under Reverse Bias

Having analyzed the *pn* junction in equilibrium, we can now study its behavior under more interesting and useful conditions. Let us begin by applying an external voltage across the device as shown in Fig. 2.23, where the voltage source makes the *n* side more *positive* than the *p* side. We say the junction is under "reverse bias" to emphasize the connection of the positive voltage to the *n* terminal. Used as a noun or a verb, the term "bias" indicates operation under some "desirable" conditions. We will study the concept of biasing extensively in this and following chapters.

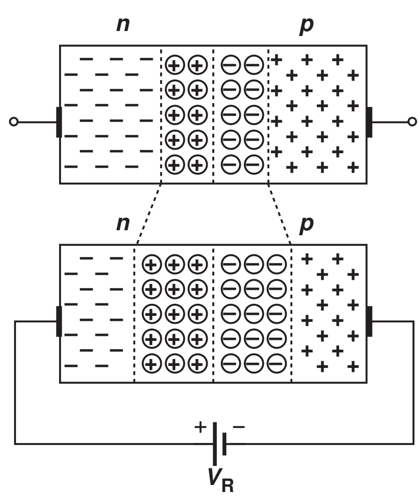


Figure 2.23 *pn* junction under reverse bias.

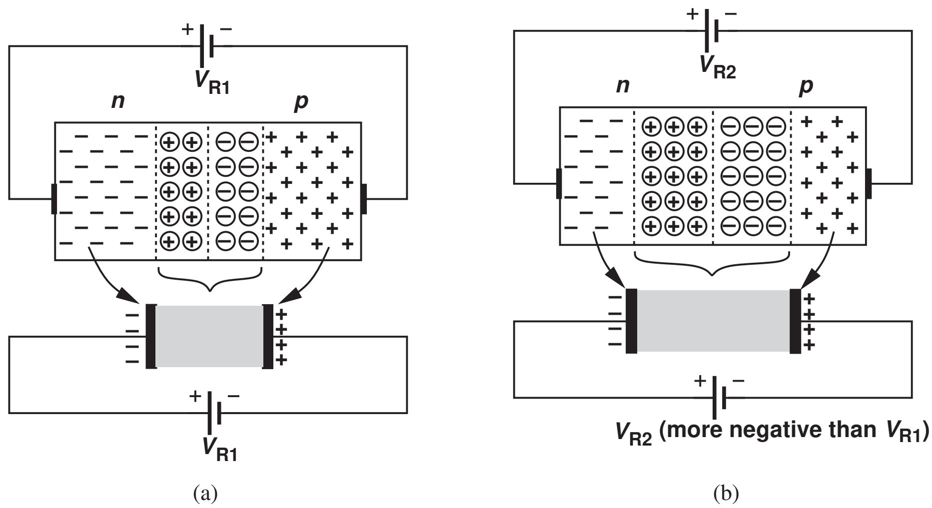


Figure 2.24 Reduction of junction capacitance with reverse bias.

We wish to reexamine the results obtained in equilibrium for the case of reverse bias. Let us first determine whether the external voltage *enhances* the built-in electric field or *opposes* it. Since under equilibrium, $\overrightarrow{E}$ is directed from the n side to the p side, V_R enhances the field. But, a higher electric field can be sustained only if a larger amount of fixed charge is provided, requiring that more acceptor and donor ions be exposed and, therefore, the depletion region be widened.

What happens to the diffusion and drift currents? Since the external voltage has strengthened the field, the barrier rises even higher than that in equilibrium, thus prohibiting the flow of current. In other words, the junction carries a negligible current under reverse bias.[9]

With no current conduction, a reverse-biased pn junction does not seem particularly useful. However, an important observation will prove otherwise. We note that in Fig. 2.23, as V_B increases, more positive charge appears on the n side and more negative charge on the p side. Thus, the device operates as a *capacitor* [Fig. 2.24(a)]. In essence, we can view the conductive n and p sections as the two plates of the capacitor. We also assume the charge in the depletion region equivalently resides on each plate.

The reader may still not find the device interesting. After all, since any two parallel plates can form a capacitor, the use of a pn junction for this purpose is not justified. But, reverse-biased pn junctions exhibit a unique property that becomes useful in circuit design. Returning to Fig. 2.23, we recognize that, as V_R increases, so does the width of the depletion region. That is, the conceptual diagram of Fig. 2.24(a) can be drawn as in Fig. 2.24(b) for increasing values of V_R, revealing that the capacitance of the structure *decreases* as the two plates move away from each other. The junction therefore displays a voltage-dependent capacitance.

It can be proved that the capacitance of the junction per unit area is equal to

$$C_j = \frac{C_{j0}}{\sqrt{1 - \dfrac{V_R}{V_0}}}, \tag{2.75}$$

[9] As explained in Section 2.2.3, the current is not exactly zero.

where C_{j0} denotes the capacitance corresponding to zero bias ($V_R = 0$) and V_0 is the built-in potential [Eq. (2.69)]. (This equation assumes V_R is negative for reverse bias.) The value of C_{j0} is in turn given by

$$C_{j0} = \sqrt{\frac{\epsilon_{si} q}{2} \frac{N_A N_D}{N_A + N_D} \frac{1}{V_0}}, \tag{2.76}$$

where ϵ_{si} represents the dielectric constant of silicon and is equal to $11.7 \times 8.85 \times 10^{-14}$ F/cm.[10] Plotted in Fig. 2.25, C_j indeed decreases as V_R increases.

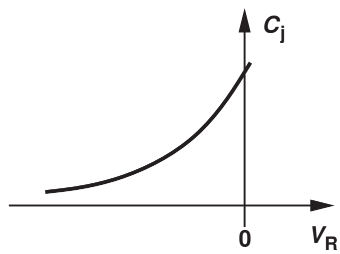


Figure 2.25 Junction capacitance under reverse bias.

Example 2.15 A *pn* junction is doped with $N_A = 2 \times 10^{16}$ cm^{-3} and $N_D = 9 \times 10^{15}$ cm^{-3}. Determine the capacitance of the device with (a) $V_R = 0$ and $V_R = 1$ V.

Solution We first obtain the built-in potential:

$$V_0 = V_T \ln \frac{N_A N_D}{n_i^2} \tag{2.77}$$

$$= 0.73 \text{ V}. \tag{2.78}$$

Thus, for $V_R = 0$ and $q = 1.6 \times 10^{-19}$ C, we have

$$C_{j0} = \sqrt{\frac{\epsilon_{si} q}{2} \frac{N_A N_D}{N_A + N_D} \cdot \frac{1}{V_0}} \tag{2.79}$$

$$= 2.65 \times 10^{-8} \text{ F/cm}^2. \tag{2.80}$$

In microelectronics, we deal with very small devices and may rewrite this result as

$$C_{j0} = 0.265 \text{ fF}/\mu\text{m}^2, \tag{2.81}$$

[10] The dielectric constant of materials is usually written in the form $\epsilon_r \epsilon_0$, where ϵ_r is the "relative" dielectric constant and a dimensionless factor (e.g., 11.7), and ϵ_0 the dielectric constant of vacuum (8.85×10^{-14} F/cm).

where 1 fF (femtofarad) $= 10^{-15}$ F. For $V_R = 1$ V,

$$C_j = \frac{C_{j0}}{\sqrt{1 + \frac{V_R}{V_0}}} \tag{2.82}$$

$$= 0.172\ \text{fF}/\mu\text{m}^2. \tag{2.83}$$

Exercise Repeat the above example if the donor concentration on the N side is doubled. Compare the results in the two cases.

The variation of the capacitance with the applied voltage makes the device a "nonlinear" capacitor because it does not satisfy $Q = CV$. Nonetheless, as demonstrated by the following example, a voltage-dependent capacitor leads to interesting circuit topologies.

Example 2.16 A cellphone incorporates a 2-GHz oscillator whose frequency is defined by the resonance frequency of an LC tank (Fig. 2.26). If the tank capacitance is realized as the pn junction of Example 2.15, calculate the change in the oscillation frequency while the reverse voltage goes from 0 to 2 V. Assume the circuit operates at 2 GHz at a reverse voltage of 0 V, and the junction area is 2000 μm^2.

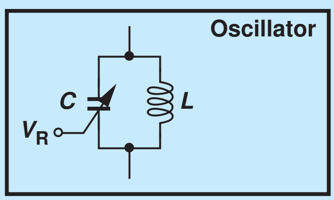


Figure 2.26 Variable capacitor used to tune an oscillator.

Solution Recall from basic circuit theory that the tank "resonates" if the impedances of the inductor and the capacitor are equal and opposite: $jL\omega_{res} = -(jC\omega_{res})^{-1}$. Thus, the resonance frequency is equal to

$$f_{res} = \frac{1}{2\pi}\frac{1}{\sqrt{LC}}. \tag{2.84}$$

At $V_R = 0, C_j = 0.265\ \text{fF}/\mu\text{m}^2$, yielding a total device capacitance of

$$C_{j,tot}(V_R = 0) = (0.265\ \text{fF}/\mu\text{m}^2) \times (2000\ \mu\text{m}^2) \tag{2.85}$$

$$= 530\ \text{fF}. \tag{2.86}$$

Setting f_{res} to 2 GHz, we obtain

$$L = 11.9\ \text{nH}. \tag{2.87}$$

If V_R goes to 2 V,

$$C_{j,tot}(V_R = 2\text{ V}) = \frac{C_{j0}}{\sqrt{1 + \frac{2}{0.73}}} \times 2000\ \mu\text{m}^2 \tag{2.88}$$

$$= 274\text{ fF}. \tag{2.89}$$

Using this value along with $L = 11.9$ nH in Eq. (2.84), we have

$$f_{res}(V_R = 2\text{ V}) = 2.79\text{ GHz}. \tag{2.90}$$

An oscillator whose frequency can be varied by an external voltage (V_R in this case) is called a "voltage-controlled oscillator" and used extensively in cellphones, microprocessors, personal computers, etc.

Exercise Some wireless systems operate at 5.2 GHz. Repeat the above example for this frequency, assuming the junction area is still 2000 μm^2 but the inductor value is scaled to reach 5.2 GHz.

Did you know?

Voltage-dependent capacitors are called "varactors" (or, in older literature, "varicaps"). The ability to "tune" a capacitor's value by a voltage proves essential in many systems. For example, your TV changes channels by changing the voltage applied to a varactor. By contrast, old TVs had a knob that mechanically switched different capacitors into the circuit. Imagine turning that knob by remote control!

In summary, a reverse-biased *pn* junction carries a negligible current but exhibits a voltage-dependent capacitance. Note that we have tacitly developed a circuit model for the device under this condition: a simple capacitance whose value is given by Eq. (2.75).

Another interesting application of reverse-biased diodes is in digital cameras (Chapter 1). If light of sufficient energy is applied to a *pn* junction, electrons are dislodged from their covalent bonds and hence electron-hole pairs are created. With a reverse bias, the electrons are attracted to the positive battery terminal and the holes to the negative battery terminal. As a result, a current flows through the diode that is proportional to the light intensity. We say the *pn* junction operates as a "photodiode."

2.2.3 *pn* Junction Under Forward Bias

Our objective in this section is to show that the *pn* junction carries a current if the *p* side is raised to a more *positive* voltage than the *n* side (Fig. 2.27). This condition is called "forward bias." We also wish to compute the resulting current in terms of the applied voltage and the junction parameters, ultimately arriving at a circuit model.

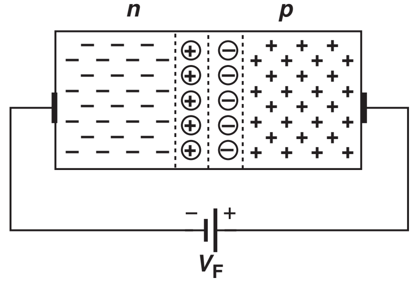


Figure 2.27 *pn* junction under forward bias.

From our study of the device in equilibrium and reverse bias, we note that the potential barrier developed in the depletion region determines the device's desire to conduct. In forward bias, the external voltage, V_F, tends to create a field directed from the p side toward the n side—opposite to the built-in field that was developed to stop the diffusion currents. We therefore surmise that V_F in fact *lowers* the potential barrier by weakening the field, thus allowing greater diffusion currents.

To derive the I/V characteristic in forward bias, we begin with Eq. (2.68) for the built-in voltage and rewrite it as

$$p_{n,e} = \frac{p_{p,e}}{\exp\dfrac{V_0}{V_T}}, \tag{2.91}$$

where the subscript e emphasizes equilibrium conditions [Fig. 2.28(a)] and $V_T = kT/q$ is called the "thermal voltage" ($\approx$26 mV at $T = 300$ K). In forward bias, the potential barrier is lowered by an amount equal to the applied voltage:

$$p_{n,f} = \frac{p_{p,f}}{\exp\dfrac{V_0 - V_F}{V_T}}. \tag{2.92}$$

where the subscript f denotes forward bias. Since the exponential denominator drops considerably, we expect $p_{n,f}$ to be much higher than $p_{n,e}$ (it can be proved that $p_{p,f} \approx p_{p,e} \approx N_A$). In other words, the *minority* carrier concentration on the p side rises rapidly with the forward bias voltage while the majority carrier concentration remains relatively constant. This statement applies to the n side as well.

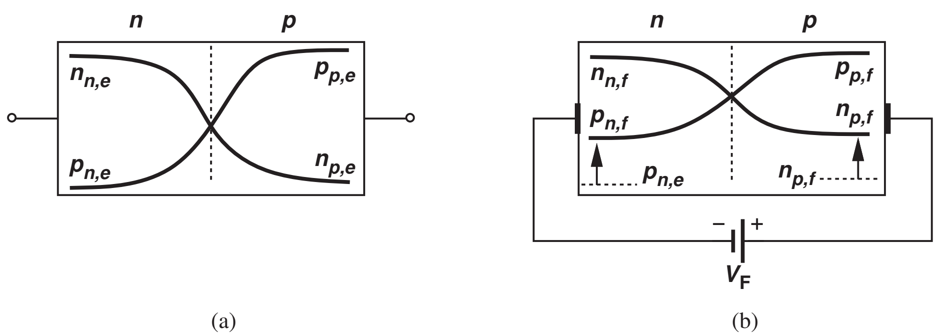


Figure 2.28 Carrier profiles (a) in equilibrium and (b) under forward bias.

Figure 2.28(b) illustrates the results of our analysis thus far. As the junction goes from equilibrium to forward bias, n_p and p_n increase dramatically, leading to a proportional change in the diffusion currents.[11] We can express the change in the hole concentration on the n side as:

$$\Delta p_n = p_{n,f} - p_{n,e} \tag{2.93}$$

$$= \frac{p_{p,f}}{\exp\dfrac{V_0 - V_F}{V_T}} - \frac{p_{p,e}}{\exp\dfrac{V_0}{V_T}} \tag{2.94}$$

$$\approx \frac{N_A}{\exp\dfrac{V_0}{V_T}}\left(\exp\frac{V_F}{V_T} - 1\right). \tag{2.95}$$

[11]The width of the depletion region actually decreases in forward bias, but we neglect this effect here.

Similarly, for the electron concentration on the p side:

$$\Delta n_p \approx \frac{N_D}{\exp\frac{V_0}{V_T}}\left(\exp\frac{V_F}{V_T}-1\right). \tag{2.96}$$

Note that Eq. (2.69) indicates that $\exp(V_0/V_T) = N_A N_D/n_i^2$.

The increase in the minority carrier concentration suggests that the diffusion currents must rise by a proportional amount above their equilibrium value, i.e.,

$$I_{tot} \propto \frac{N_A}{\exp\frac{V_0}{V_T}}\left(\exp\frac{V_F}{V_T}-1\right) + \frac{N_D}{\exp\frac{V_0}{V_T}}\left(\exp\frac{V_F}{V_T}-1\right). \tag{2.97}$$

Indeed, it can be proved that [1]

$$I_{tot} = I_S\left(\exp\frac{V_F}{V_T}-1\right), \tag{2.98}$$

where I_S is called the "reverse saturation current" and given by

$$I_S = Aqn_i^2\left(\frac{D_n}{N_A L_n}+\frac{D_p}{N_D L_p}\right). \tag{2.99}$$

In this equation, A is the cross section area of the device, and L_n and L_p are electron and hole "diffusion lengths," respectively. Diffusion lengths are typically in the range of tens of micrometers. Note that the first and second terms in the parentheses correspond to the flow of electrons and holes, respectively.

Example 2.17 Determine I_S for the junction of Example 2.13 at $T = 300\text{K}$ if $A = 100\ \mu\text{m}^2$, $L_n = 20\ \mu\text{m}$, and $L_p = 30\ \mu\text{m}$.

Solution Using $q = 1.6\times10^{-19}$ C, $n_i = 1.08\times10^{10}$ electrons/cm^3 [Eq. (2.2)], $D_n = 34\ \text{cm}^2/\text{s}$, and $D_p = 12\ \text{cm}^2/\text{s}$, we have

$$I_S = 1.77\times10^{-17}\ \text{A}. \tag{2.100}$$

Since I_S is extremely small, the exponential term in Eq. (2.98) must assume very large values so as to yield a useful amount (e.g., 1 mA) for I_{tot}.

Exercise What junction area is necessary to raise I_S to 10^{-15} A?

An interesting question that arises here is: are the minority carrier concentrations *constant* along the x-axis? Depicted in Fig. 2.29(a), such a scenario would suggest that electrons continue to flow from the n side to the p side, but exhibit no tendency to go beyond $x = x_2$ because of the lack of a gradient. A similar situation exists for holes, implying that the charge carriers do not flow deep into the p and n sides and hence no net current results! Thus, the minority carrier concentrations must vary as shown in Fig. 2.29(b) so that diffusion can occur.

This observation reminds us of Example 2.10 and the question raised in conjunction with it: if the minority carrier concentration falls with x, what happens to the carriers and how can the current remain constant along the x-axis? Interestingly, as the electrons enter

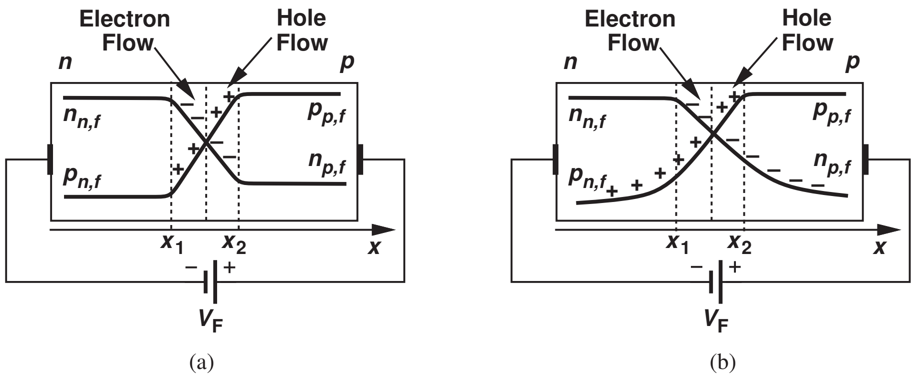


Figure 2.29 (a) Constant and (b) variable majority carrier profiles outside the depletion region.

the p side and roll down the gradient, they gradually *recombine* with the holes, which are abundant in this region. Similarly, the holes entering the n side recombine with the electrons. Thus, in the immediate vicinity of the depletion region, the current consists of mostly minority carriers, but towards the far contacts, it is primarily comprised of majority carriers (Fig. 2.30). At each point along the x-axis, the two components add up to I_{tot}.

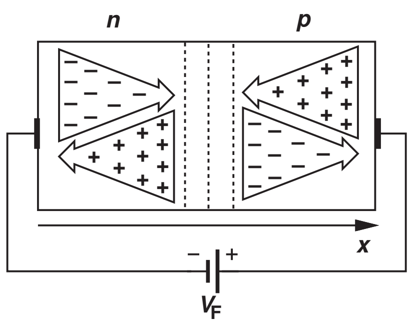


Figure 2.30 Minority and majority carrier currents.

2.2.4 I/V Characteristics

Let us summarize our thoughts thus far. In forward bias, the external voltage opposes the built-in potential, raising the diffusion currents substantially. In reverse bias, on the other hand, the applied voltage enhances the field, prohibiting current flow. We hereafter write the junction equation as:

$$I_D = I_S\left(\exp\frac{V_D}{V_T} - 1\right), \tag{2.101}$$

where I_D and V_D denote the diode current and voltage, respectively. As expected, $V_D = 0$ yields $I_D = 0$. (Why is this expected?) As V_D becomes positive and exceeds several V_T, the exponential term grows rapidly and $I_D \approx I_S \exp(V_D/V_T)$. We hereafter assume $\exp(V_D/V_T) \gg 1$ in the forward bias region.

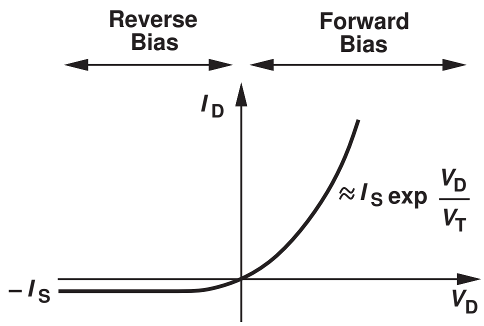


Figure 2.31 I/V characteristic of a *pn* junction.

It can be proved that Eq. (2.101) also holds in reverse bias, i.e., for negative V_D. If $V_D < 0$ and $|V_D|$ reaches several V_T, then $\exp(V_D/V_T) \ll 1$ and

$$I_D \approx -I_S. \tag{2.102}$$

Figure 2.31 plots the overall I/V characteristic of the junction, revealing why I_S is called the "reverse saturation current." Example 2.17 indicates that I_S is typically very small. We therefore view the current under reverse bias as "leakage." Note that I_S and hence the junction current are proportional to the device cross section area [Eq. (2.99)]. For example, two identical devices placed in parallel (Fig. 2.32) behave as a single junction with twice the I_S.

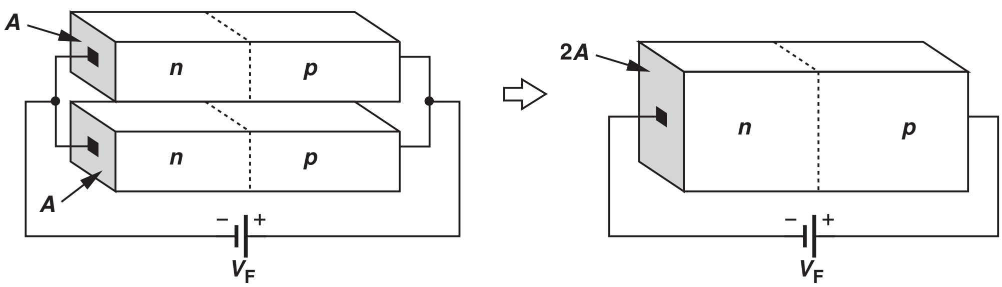


Figure 2.32 Equivalence of parallel devices to a larger device.

Example 2.18

Each junction in Fig. 2.32 employs the doping levels described in Example 2.13. Determine the forward bias current of the composite device for $V_D = 300$ mV and 800 mV at $T = 300$ K.

Solution From Example 2.17, $I_S = 1.77 \times 10^{-17}$ A for each junction. Thus, the total current is equal to

$$I_{D,tot}(V_D = 300\,\text{mV}) = 2I_S\left(\exp\frac{V_D}{V_T} - 1\right) \tag{2.103}$$

$$= 3.63\,\text{pA}. \tag{2.104}$$

Similarly, for $V_D = 800\,\text{mV}$:

$$I_{D,tot}(V_D = 800\,\text{mV}) = 82\,\mu\text{A}. \tag{2.105}$$

Exercise How many of these diodes must be placed in parallel to obtain a current of 100 μA with a voltage of 750 mV?

Example 2.19 A diode operates in the forward bias region with a typical current level [i.e., $I_D \approx I_S \exp(V_D/V_T)$]. Suppose we wish to increase the current by a factor of 10. How much change in V_D is required?

Solution Let us first express the diode voltage as a function of its current:

$$V_D = V_T \ln \frac{I_D}{I_S}. \tag{2.106}$$

We define $I_1 = 10I_D$ and seek the corresponding voltage, V_{D1}:

$$V_{D1} = V_T \ln \frac{10I_D}{I_S} \tag{2.107}$$

$$= V_T \ln \frac{I_D}{I_S} + V_T \ln 10 \tag{2.108}$$

$$= V_D + V_T \ln 10. \tag{2.109}$$

Thus, the diode voltage must rise by $V_T \ln 10 \approx 60\,\text{mV}$ (at $T = 300\,\text{K}$) to accommodate a tenfold increase in the current. We say the device exhibits a 60-mV/decade characteristic, meaning V_D changes by 60 mV for a decade (tenfold) change in I_D. More generally, an n-fold change in I_D translates to a change of $V_T \ln n$ in V_D.

Exercise By what factor does the current change if the voltages changes by 120 mV?

Example 2.20 The cross section area of a diode operating in the forward bias region is increased by a factor of 10. (a) Determine the change in I_D if V_D is maintained constant. (b) Determine the change in V_D if I_D is maintained constant. Assume $I_D \approx I_S \exp(V_D/V_T)$.

Solution (a) Since $I_S \propto A$, the new current is given by

$$I_{D1} = 10I_S \exp\frac{V_D}{V_T} \tag{2.110}$$

$$= 10I_D. \tag{2.111}$$

(b) From the above example,

$$V_{D1} = V_T \ln \frac{I_D}{10 I_S} \tag{2.112}$$

$$= V_T \ln \frac{I_D}{I_S} - V_T \ln 10. \tag{2.113}$$

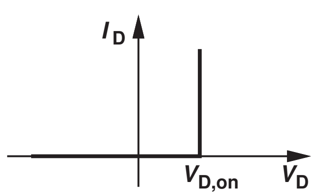
Thus, a tenfold increase in the device area lowers the voltage by 60 mV if I_D remains constant.

Exercise A diode in forward bias with $I_D \approx I_S \exp(V_D/V_T)$ undergoes two simultaneous changes: the current is raised by a factor of m and the area is increased by a factor of n. Determine the change in the device voltage.

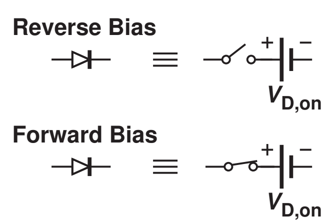
Constant-Voltage Model The exponential I/V characteristic of the diode results in nonlinear equations, making the analysis of circuits quite difficult. Fortunately, the above examples imply that the diode voltage is a relatively weak function of the device current and cross section area. With typical current levels and areas, V_D falls in the range of 700–800 mV. For this reason, we often approximate the forward bias voltage by a *constant* value of 800 mV (like an ideal battery), considering the device fully off if $V_D < 800$ mV. The resulting characteristic is illustrated in Fig. 2.33(a) with the turn-on voltage denoted by $V_{D,on}$. Note that the current goes to infinity as V_D tends to exceed $V_{D,on}$ because we assume the forward-biased diode operates as an ideal voltage source. Neglecting the leakage current in reverse bias, we derive the circuit model shown in Fig. 2.33(b). We say the junction operates as an open circuit if $V_D < V_{D,on}$ and as a constant voltage source if we attempt to increase V_D beyond $V_{D,on}$. While not essential, the voltage source placed in series with the switch in the off condition helps simplify the analysis of circuits: we can say that in the transition from off to on, only the switch turns on and the battery always resides in series with the switch.

A number of questions may cross the reader's mind at this point. First, why do we subject the diode to such a seemingly inaccurate approximation? Second, if we indeed intend to use this simple approximation, why did we study the physics of semiconductors and *pn* junctions in such detail?

The developments in this chapter are representative of our treatment of *all* semiconductor devices: we carefully analyze the structure and physics of the device to understand its operation; we construct a "physics-based" circuit model; and we seek to approximate



(a)



(b)

Figure 2.33 Constant-voltage diode model.

the resulting model, thus arriving at progressively simpler representations. Device models having different levels of complexity (and, inevitably, different levels of accuracy) prove essential to the analysis and design of circuits. Simple models allow a quick, intuitive understanding of the operation of a complex circuit, while more accurate models reveal the true performance.

Example 2.21

Consider the circuit of Fig. 2.34. Calculate I_X for $V_X = 3\,\text{V}$ and $V_X = 1\,\text{V}$ using (a) an exponential model with $I_S = 10^{-16}\,\text{A}$ and (b) a constant-voltage model with $V_{D,on} = 800\,\text{mV}$.

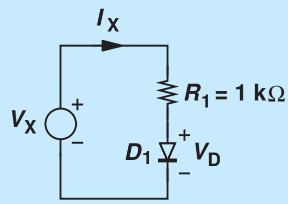


Figure 2.34 Simple circuit using a diode.

Solution (a) Noting that $I_D = I_X$, we have

$$V_X = I_X R_1 + V_D \tag{2.114}$$

$$V_D = V_T \ln \frac{I_X}{I_S}. \tag{2.115}$$

This equation must be solved by iteration: we guess a value for V_D, compute the corresponding I_X from $I_X R_1 = V_X - V_D$, determine the new value of V_D from $V_D = V_T \ln(I_X/I_S)$ and iterate. Let us guess $V_D = 750\,\text{mV}$ and hence

$$I_X = \frac{V_X - V_D}{R_1} \tag{2.116}$$

$$= \frac{3\,\text{V} - 0.75\,\text{V}}{1\,\text{k}\Omega} \tag{2.117}$$

$$= 2.25\,\text{mA}. \tag{2.118}$$

Thus,

$$V_D = V_T \ln \frac{I_X}{I_S} \tag{2.119}$$

$$= 799\,\text{mV}. \tag{2.120}$$

With this new value of V_D, we can obtain a more accurate value for I_X:

$$I_X = \frac{3\,\text{V} - 0.799\,\text{V}}{1\,\text{k}\Omega} \tag{2.121}$$

$$= 2.201\,\text{mA}. \tag{2.122}$$

We note that the value of I_X rapidly converges. Following the same procedure for $V_X = 1$ V, we have

$$I_X = \frac{1\text{ V} - 0.75\text{ V}}{1\text{ k}\Omega} \tag{2.123}$$

$$= 0.25\text{ mA}, \tag{2.124}$$

which yields $V_D = 0.742$ V and hence $I_X = 0.258$ mA. (b) A constant-voltage model readily gives

$$I_X = 2.2\text{ mA for } V_X = 3\text{ V} \tag{2.125}$$

$$I_X = 0.2\text{ mA for } V_X = 1\text{ V}. \tag{2.126}$$

The value of I_X incurs some error, but it is obtained with much less computational effort than that in part (a).

Exercise Repeat the above example if the cross section area of the diode is increased by a factor of 10.

2.3 REVERSE BREAKDOWN*

Recall from Fig. 2.31 that the *pn* junction carries only a small, relatively constant current in reverse bias. However, as the reverse voltage across the device increases, eventually "breakdown" occurs and a sudden, enormous current is observed. Figure 2.35 plots the device I/V characteristic, displaying this effect.

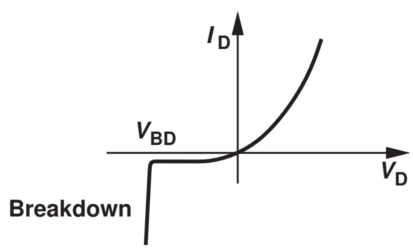


Figure 2.35 Reverse breakdown characteristic.

The breakdown resulting from a high voltage (and hence a high electric field) can occur in *any* material. A common example is lightning, in which case the electric field in the air reaches such a high level as to ionize the oxygen molecules, thus lowering the resistance of the air and creating a tremendous current.

The breakdown phenomenon in *pn* junctions occurs by one of two possible mechanisms: "Zener effect" and "avalanche effect."

*This section can be skipped in a first reading.

2.3.1 Zener Breakdown

The depletion region in a *pn* junction contains atoms that have lost an electron or a hole and, therefore, provide no loosely-connected carriers. However, a high electric field in this region may impart enough energy to the remaining covalent electrons to tear them from their bonds [Fig. 2.36(a)]. Once freed, the electrons are accelerated by the field and swept to the *n* side of the junction. This effect occurs at a field strength of about 10^6 V/cm (1 V/μm).

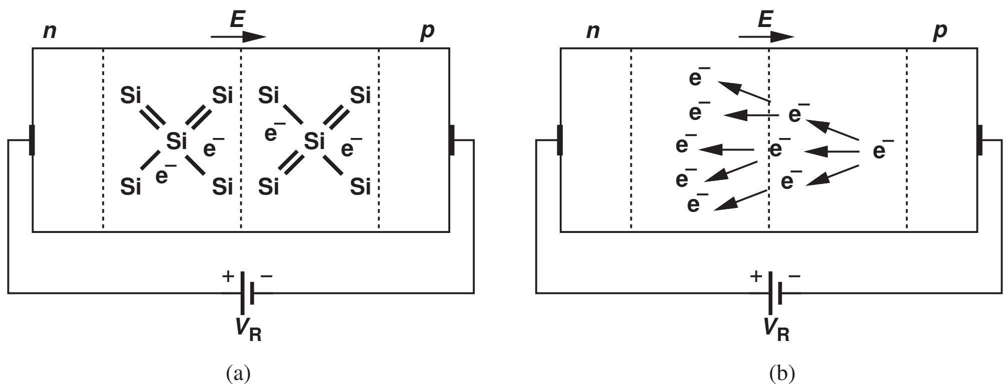


Figure 2.36 (a) Release of electrons due to high electric field, (b) avalanche effect.

In order to create such high fields with reasonable voltages, a *narrow* depletion region is required, which from Eq. (2.76) translates to high doping levels on both sides of the junction (why?). Called the "Zener effect," this type of breakdown appears for reverse bias voltages on the order of 3-8 V.

2.3.2 Avalanche Breakdown

Junctions with moderate or low doping levels ($<10^{15}$ cm^3) generally exhibit no Zener breakdown. But, as the reverse bias voltage across such devices increases, an avalanche effect takes place. Even though the leakage current is very small, each carrier entering the depletion region experiences a very high electric field and hence a large acceleration, thus gaining enough energy to break the electrons from their covalent bonds. Called "impact ionization," this phenomenon can lead to avalanche: each electron freed by the impact may itself speed up so much in the field as to collide with another atom with sufficient energy, thereby freeing one more covalent-bond electron. Now, these two electrons may again acquire energy and cause more ionizing collisions, rapidly raising the number of free carriers.

An interesting contrast between Zener and avalanche phenomena is that they display opposite temperature coefficients (TCs): V_{BD} has a negative TC for Zener effect and positive TC for avalanche effect. The two TCs cancel each other for $V_{BD} \approx 3.5$ V. For this reason, Zener diodes with 3.5-V rating find application in some voltage regulators.

The Zener and avalanche breakdown effects do not damage the diodes if the resulting current remains below a certain limit given by the doping levels and the geometry of the junction. Both the breakdown voltage and the maximum allowable reverse current are specified by diode manufacturers.

2.4 CHAPTER SUMMARY

- Silicon contains four atoms in its last orbital. It also contains a small number of free electrons at room temperature.
- When an electron is freed from a covalent bond, a "hole" is left behind.

- The bandgap energy is the minimum energy required to dislodge an electron from its covalent bond.
- To increase the number of free carriers, semiconductors are "doped" with certain impurities. For example, addition of phosphorus to silicon increases the number of free electrons because phosphorus contains five electrons in its last orbital.
- For doped or undoped semiconductors, $np = n_i^2$. For example, in an n-type material, $n \approx N_D$ and hence $p \approx n_i^2/N_D$.
- Charge carriers move in semiconductors via two mechanisms: drift and diffusion.
- The drift current density is proportional to the electric field and the mobility of the carriers and is given by $J_{tot} = q(\mu_n n + \mu_p p)E$.
- The diffusion current density is proportional to the gradient of the carrier concentration and given by $J_{tot} = q(D_n dn/dx - D_p dp/dx)$.
- A pn junction is a piece of semiconductor that receives n-type doping in one section and p-type doping in an adjacent section.
- The pn junction can be considered in three modes: equilibrium, reverse bias, and forward bias.
- Upon formation of the pn junction, sharp gradients of carrier densities across the junction result in a high current of electrons and holes. As the carriers cross, they leave ionized atoms behind, and a "depletion region" is formed. The electric field created in the depletion region eventually stops the current flow. This condition is called equilibrium.
- The electric field in the depletion results in a built-in potential across the region equal to $(kT/q) \ln (N_A N_D)/n_i^2$, typically in the range of 700 to 800 mV.
- Under reverse bias, the junction carries negligible current and operates as a capacitor. The capacitance itself is a function of the voltage applied across the device.
- Under forward bias, the junction carries a current that is an exponential function of the applied voltage: $I_S[\exp(V_F/V_T) - 1]$.
- Since the exponential model often makes the analysis of circuits difficult, a constant-voltage model may be used in some cases to estimate the circuit's response with less mathematical labor.
- Under a high reverse bias voltage, pn junctions break down, conducting a very high current. Depending on the structure and doping levels of the device, "Zener" or "avalanche" breakdown may occur.

PROBLEMS

Sec. 2.1 Semiconductor Materials and Their Properties

2.1. The intrinsic carrier concentration of germanium (GE) is expressed as

$$n_i = 1.66 \times 10^{15} T^{3/2} \exp\frac{-Eg}{2kT}\ \text{cm}^{-3}, \qquad (2.127)$$

where $Eg = 0.66$ eV.

(a) Calculate n_i at 300 K and 600 K and compare the results with those obtained in Example 2.1 for Si.

(b) Determine the electron and hole concentrations if Ge is doped with P at a density of 5×10^{16} cm^{-3}.

2.2. An n-type piece of silicon experiences an electric field equal to 0.1 V/μm.

(a) Calculate the velocity of electrons and holes in this material.

(b) What doping level is necessary to provide a current density of 1 mA/μm^2 under these conditions? Assume the hole current is negligible.

2.3. A n-type piece of silicon with a length of 0.1 μm and a cross section area of 0.05 μm $\times$ 0.05 μm sustains a voltage difference of 1 V.
(a) If the doping level is 10^{17} cm^{-3}, calculate the total current flowing through the device at $T = 300$ K.
(b) Repeat (a) for $T = 400$ K assuming for simplicity that mobility does not change with temperature. (This is not a good assumption.)

2.4. From the data in Problem 2.1, repeat Problem 2.3 for Ge. Assume $\mu_n = 3900\ \text{cm}^2/(\text{V}\cdot\text{s})$ and $\mu_p = 1900\ \text{cm}^2/(\text{V}\cdot\text{s})$.

2.5. Figure 2.37 shows a p-type bar of silicon that is subjected to electron injection from the left and hole injection from the right. Determine the total current flowing through the device if the cross section area is equal to 1 μm $\times$ 1 μm.

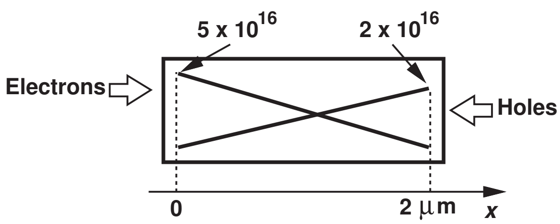


Figure 2.37

2.6. In Example 2.9, compute the total number of electrons "stored" in the material from $x = 0$ to $x = L$. Assume the cross section area of the bar is equal to a.

2.7. Repeat Problem 2.6 for Example 2.10 but for $x = 0$ to $x = \infty$. Compare the results for linear and exponential profiles.

***2.8.** Repeat Problem 2.7 if the electron and hole profiles are "sharp" exponentials, i.e., they fall to negligible values at $x = 2\ \mu$m and $x = 0$, respectively (Fig. 2.38).

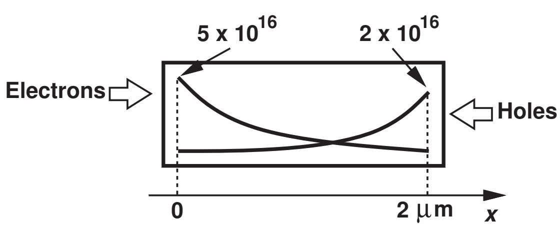


Figure 2.38

2.9. How do you explain the phenomenon of drift to a high school student?

Sec. 2.2 *pn* Junctions

2.10. A pn junction with $N_D = 3 \times 10^{16}$ cm^{-3} and $N_A = 2 \times 10^{15}$ cm^{-3} experiences a reverse bias voltage of 1.6 V.
(a) Determine the junction capacitance per unit area.
(b) By what factor should N_A be increased to double the junction capacitance?

2.11. Due to a manufacturing error, the p-side of a pn junction has not been doped. If $N_D = 3 \times 10^{16}$ cm^{-3}, calculate the built-in potential at $T = 300$ K.

2.12. A junction employs $N_D = 5 \times 10^{17}$ cm^{-3} and $N_A = 4 \times 10^{16}$ cm^{-3}.
(a) Determine the majority and minority carrier concentrations on both sides.
(b) Calculate the built-in potential at $T = 250$ K, 300 K, and 350 K. Explain the trend.

2.13. An oscillator application requires a variable capacitance with the characteristic shown in Fig. 2.39. Determine N_A and N_D.

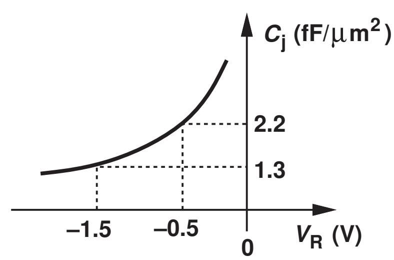


Figure 2.39

***2.14.** Two identical pn junctions are placed in series.
(a) Prove that this combination can be viewed as a single two-terminal device having an exponential characteristic.
(b) For a tenfold change in the current, how much voltage change does such a device require?

2.15. Figure 2.40 shows two diodes with reverse saturation currents of I_{S1} and I_{S2} placed in parallel.
(a) Prove that the parallel combination operates as an exponential device.

(a) If the total current is I_{tot}, determine the current carried by each diode.

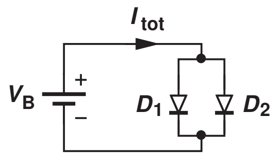


Figure 2.40

2.16. Consider a *pn* junction in forward bias.

(a) To obtain a current of 1 mA with a voltage of 750 mV, how should I_S be chosen?

(b) If the diode cross section area is now doubled, what voltage yields a current of 1 mA?

2.17. Figure 2.41 shows two diodes with reverse saturation currents of I_{S1} and I_{S2} placed in series. Calculate I_B, V_{D1}, and V_{D2} in terms of V_B, I_{S1}, and I_{S2}.

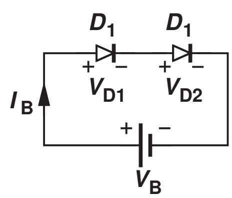


Figure 2.41

2.18. In the circuit of Problem 2.17, we wish to increase I_B by a factor of 10. What is the required change in V_B?

Sec. 2.2.4 I/V Characteristics

2.19. Consider the circuit shown in Fig. 2.42, where $I_S = 2 \times 10^{-15}$ A. Calculate V_{D1} and I_X for $V_X = 0.5\,\text{V}, 0.8\,\text{V}, 1\,\text{V}$, and 1.2 V. Note that V_{D1} changes little for $V_X \geq 0.8$ V.

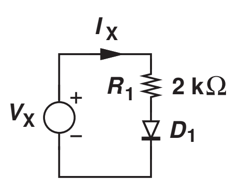


Figure 2.42

2.20. In the circuit of Fig. 2.42, the cross section area of D_1 is increased by a factor of 10. Determine V_{D1} and I_X for $V_X = 0.8$ V and 1.2 V. Compare the results with those obtained in Problem 2.19.

2.21. Suppose D_1 in Fig. 2.42 must sustain a voltage of 850 mV for $V_X = 2$ V. Calculate the required I_S.

2.22. For what value of V_X in Fig. 2.42, does R_1 sustain a voltage equal to $V_X/2$? Assume $I_S = 2 \times 10^{-16}$ A.

2.23. We have received the circuit shown in Fig. 2.43 and wish to determine R_1 and I_S. We note that $V_X = 1\,\text{V} \rightarrow I_X = 0.2\,\text{mA}$ and $V_X = 2\,\text{V} \rightarrow I_X = 0.5\,\text{mA}$. Calculate R_1 and I_S.

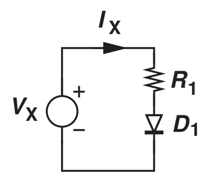


Figure 2.43

***2.24.** Figure 2.44 depicts a parallel resistor-diode combination. If $I_S = 3 \times 10^{-16}$ A, calculate V_{D1} for $I_X = 1\,\text{mA},\ 2\,\text{mA}$, and 4 mA.

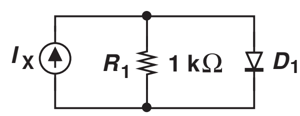


Figure 2.44

2.25. In the circuit of Fig. 2.44, we wish D_1 to carry a current of 0.5 mA for $I_X = 1.3$ mA. Determine the required I_S.

2.26. For what value of I_X in Fig. 2.44, does R_1 carry a current equal to $I_X/2$? Assume $I_S = 3 \times 10^{-16}$ A.

***2.27.** We have received the circuit shown in Fig. 2.45 and wish to determine R_1 and I_S. Measurements indicate that $I_X = 1\,\text{mA} \rightarrow V_X = 1.2\,\text{V}$ and $I_X = 2\,\text{mA} \rightarrow V_X = 1.8\,\text{V}$. Calculate R_1 and I_S.

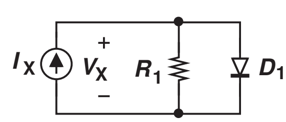


Figure 2.45

**2.28. In the circuit of Fig. 2.46, determine the value of R_1 such that this resistor carries 0.5 mA. Assume $I_S = 5 \times 10^{-16}$ A for each diode.

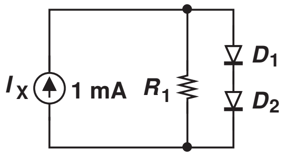


Figure 2.46

**2.29. The circuit illustrated in Fig. 2.47 employs two identical diodes with $I_S = 5 \times 10^{-16}$ A. Calculate the voltage across R_1 for $I_X = 2\,\text{mA}$.

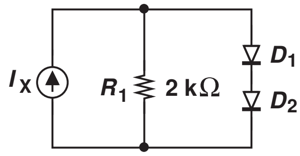


Figure 2.47

**2.30. Sketch V_X as a function of I_X for the circuit shown in Fig. 2.48. Assume (a) a constant-voltage model, (b) an exponential model.

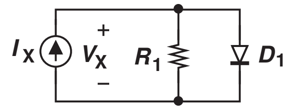


Figure 2.48

SPICE PROBLEMS

In the following problems, assume $I_S = 5 \times 10^{-16}$ A.

2.31. For the circuit shown in Fig. 2.49, plot V_{out} as a function of I_{in}. Assume I_{in} varies from 0 to 2 mA.

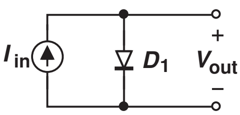


Figure 2.49

2.32. Repeat Problem 2.31 for the circuit depicted in Fig. 2.50, where $R_1 = 1\,\text{k}\Omega$. At what value of I_{in} are the currents flowing through D_1 and R_1 equal?

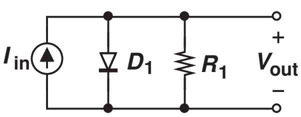


Figure 2.50

2.33. Using SPICE, determine the value of R_1 in Fig. 2.50 such that D_1 carries 1 mA if $I_{in} = 2\,\text{mA}$.

2.34. In the circuit of Fig. 2.51, $R_1 = 500\,\Omega$. Plot V_{out} as a function of V_{in} if V_{in} varies from -2 V to $+2$ V. At what value of V_{in} are the voltage drops across R_1 and D_1 equal?

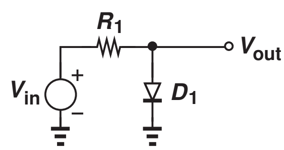


Figure 2.51

2.35. In the circuit of Fig. 2.51, use SPICE to select the value of R_1 such that $V_{out} < 0.7$ V for $V_{in} < 2$ V. We say the circuit "limits" the output.

REFERENCE

1. B. Streetman and S. Banerjee, *Solid-State Electronic Device*, fifth edition, Prentice-Hall, 1999.