

Condicionad: $P(A|B) = \frac{P(A \cap B)}{P(B)}$

A, B indep $\rightarrow P(A|B) = P(A)$; $P(A \cap B) = P(A)P(B)$ Generalizaci3n de $\forall x: f_x(x) = \sum_{y \in \mathcal{X}} P_x(y) \delta(x-y)$

Bayes: $P(A|B) = \frac{P(B|A)P(A)}{P(B)}$

$P(B) = \sum_{i=1}^n P(B \cap A_i) = \sum_{i=1}^n P(B|A_i)P(A_i)$

$f_{x_n}(x) = \frac{\partial}{\partial x_n} F_{x_n}(x) \leftarrow F_{x_n}(x) = \int_{-\infty}^x f_{x_n}(x) dx_n$

$F_{x|A}(x) = \frac{P(x \leq x, A)}{P(A)}$ Sea $y = \sum x_i$ suma de VA indep
 $f_y(y) = f_{x_1} * f_{x_2} * \dots * f_{x_n}$

Transformaci3n biyectiva y suave
 $\vec{z} = g(\vec{x}) : \mathbb{R}^n \rightarrow \mathbb{R}^n$
 $f_{\vec{z}}(\vec{z}) = \frac{f_{\vec{x}}(\vec{x})}{|\det(J(g))|}$ Evolu3n o lo inverso
 $\vec{g}^{-1}(\vec{z}) = \vec{x}$

m3dulo del determinante del Jacobiano

f marginal: $f_{x_i}(x) = \int f_{\vec{x}}(\vec{x}) dx_{2,3,\dots}$
 n3ven las que toc3s

$Var(x) = \sigma_x^2 = E[(x - \mu_x)^2]$ $f_{x|y}(x|y) = \frac{f_{x,y}(x,y)}{f_y(y)}$
 $\sigma_x \rightarrow$ desvia est3ndar

$E[g(x)] = \int_{-\infty}^{\infty} g(x) f_x(x) dx$ $E[ax+b] = aE[x] + b$
 Media $\rightarrow E[x]$

A, B indep $f_{A,B}(a,b) = f_A \cdot f_B$ $F_{A,B}(a,b) = F_A \cdot F_B$

Distribuci3n	Notaci3n	$p_X(x)$	Soporte	Par3metros	$E[X]$	$var(X)$
Bernoulli	Ber(p)	$p^x(1-p)^{1-x}$	$\{0, 1\}$	$p \in (0, 1)$	p	$p(1-p)$
Binomial	$\mathcal{B}(n, p)$	$\binom{n}{x} p^x (1-p)^{n-x}$	$[0, n]$	$p \in (0, 1), n \in \mathbb{N}$	np	$np(1-p)$
Geom3trica	$\mathcal{G}(p)$	$(1-p)^{x-1} p$	\mathbb{N}	$p \in (0, 1)$	$1/p$	$(1-p)/p^2$
Pascal	Pas(k, p)	$\binom{x-1}{k-1} (1-p)^{x-k} p^k$	\mathbb{Z}_k	$p \in (0, 1), k \in \mathbb{N}$	k/p	$k(1-p)/p^2$
Poisson	Poi(μ)	$(\mu^x e^{-\mu})/x!$	\mathbb{Z}_0	$\mu > 0$	μ	μ
Hipergeom3trica	$\mathcal{H}(N, d, n)$	$\frac{\binom{d}{x} \binom{N-d}{n-x}}{\binom{N}{n}}$	$[m, M]^{\dagger}$	$d \leq N, n \leq N \in \mathbb{N}$	$\frac{nd}{N}$	$\frac{nd(N-d)(N-n)}{N^2(N-1)}$

Distribuci3n	Notaci3n	$f_X(x)$	Soporte	Par3metros	$E[X]$	$var(X)$
Uniforme	$\mathcal{U}[a, b]$	$1/(b-a)$	$[a, b]$	$a < b$	$(a+b)/2$	$(b-a)^2/12$
Exponencial	$\mathcal{E}(\lambda)$	$\lambda e^{-\lambda x}$	$[0, +\infty)$	$\lambda > 0$	$1/\lambda$	$1/\lambda^2$
Gamma	$\Gamma(\nu, \lambda)$	$\frac{\lambda^\nu}{\Gamma(\nu)} x^{\nu-1} e^{-\lambda x}$	$[0, +\infty)$	$\nu > 0, \lambda > 0$	ν/λ	ν/λ^2
Normal	$\mathcal{N}(\mu, \sigma^2)$	$\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$	\mathbb{R}	$\mu \in \mathbb{R}, \sigma^2 > 0$	μ	σ^2
Chi cuadrado	χ_k^2	$\frac{1}{2^{\frac{k}{2}} \Gamma(\frac{k}{2})} x^{\frac{k}{2}-1} e^{-\frac{x}{2}}$	$[0, +\infty)$	$k \in \mathbb{N}$	k	2k
t-Student	t_ν	$\frac{\Gamma(\frac{\nu+1}{2})}{\sqrt{\nu\pi} \Gamma(\frac{\nu}{2})} \left(1 + \frac{t^2}{\nu}\right)^{-\frac{\nu+1}{2}}$	\mathbb{R}	$\nu > 0$	0	$\frac{\nu}{\nu-2}^*$
Weibull	Wei(c, α)	$\frac{c}{\alpha} \left(\frac{x}{\alpha}\right)^{c-1} e^{-\left(\frac{x}{\alpha}\right)^c}$	$[0, +\infty)$	$c > 0, \alpha > 0$	$\alpha \Gamma(1 + \frac{1}{c})$	$\alpha^2 [\Gamma(1 + \frac{2}{c}) - \Gamma^2(1 + \frac{1}{c})]$
Rayleigh	Ray(σ)	$\frac{x}{\sigma^2} e^{-x^2/(2\sigma^2)}$	$[0, +\infty)$	$\sigma > 0$	$\sigma \sqrt{\pi/2}$	$\frac{4-\pi}{2} \sigma^2$
Pareto	Par(m, α)	$\frac{\alpha m^\alpha}{x^{\alpha+1}}$	$[m, +\infty)$	$m > 0, \alpha > 0$	$\frac{\alpha m}{\alpha-1}^{\dagger}$	$\frac{m^2 \alpha}{(\alpha-1)^2 (\alpha-2)}^{\ddagger}$
Beta	$\beta(a, b)$	$\frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} x^{a-1} (1-x)^{b-1}$	$(0, 1)$	$a > 0, b > 0$	$a/(a+b)$	$\frac{ab}{(a+b)^2 (a+b+1)}$
Cauchy	Cau(x_0, γ)	$\frac{1}{\pi \gamma} \left[\frac{\gamma^2}{(x-x_0)^2 + \gamma^2} \right]$	\mathbb{R}	$x_0 \in \mathbb{R}, \gamma > 0$	no existe	no existe

Señal	<i>h</i>	<i>ROC</i>
$\delta(t)$	1	\mathbb{C}
$u(t)$	$\frac{1}{s}$	$\{s \in \mathbb{C} : \text{Re}\{s\} > 0\}$
$-u(-t)$	$\frac{1}{s}$	$\{s \in \mathbb{C} : \text{Re}\{s\} < 0\}$
$\frac{t^{n-1}}{(n-1)!}u(t)$	$\frac{1}{s^n}$	$\{s \in \mathbb{C} : \text{Re}\{s\} < 0\}$
$e^{-\alpha t}u(t)$	$\frac{1}{s+\alpha}$	$\{s \in \mathbb{C} : \text{Re}\{s\} > -\text{Re}\{\alpha\}\}$
$-e^{-\alpha t}u(-t)$	$\frac{1}{s+\alpha}$	$\{s \in \mathbb{C} : \text{Re}\{s\} < -\text{Re}\{\alpha\}\}$
$\frac{t^{n-1}}{(n-1)!}e^{-\alpha t}u(t)$	$\frac{1}{(s+\alpha)^n}$	$\{s \in \mathbb{C} : \text{Re}\{s\} > -\text{Re}\{\alpha\}\}$
$-\frac{t^{n-1}}{(n-1)!}e^{-\alpha t}u(-t)$	$\frac{1}{(s+\alpha)^n}$	$\{s \in \mathbb{C} : \text{Re}\{s\} < -\text{Re}\{\alpha\}\}$
$\sin(\omega_0 t)u(t)$	$\frac{\omega_0}{s^2+\omega_0^2}$	$\{s \in \mathbb{C} : \text{Re}\{s\} > 0\}$
$\cos(\omega_0 t)u(t)$	$\frac{s}{s^2+\omega_0^2}$	$\{s \in \mathbb{C} : \text{Re}\{s\} > 0\}$

Desplazamiento Laplace
$x(t-t_0) \xleftrightarrow{\mathcal{L}} e^{-st_0}X(s), \text{ ROC}\{e^{-st_0}X(s)\} = \text{ROC}\{X(s)\}$
$x(t)e^{s_0t} \xleftrightarrow{\mathcal{L}} X(s-s_0), \text{ ROC}\{X(s-s_0)\} = \text{ROC}\{X(s)\} + \text{Re}\{s_0\}$
$\mathcal{F}\left[\frac{dx(t)}{dt}\right] = j\omega X(j\omega) \quad \mathcal{F}\left[\int_{-\infty}^t x(\tau)d\tau\right] = \frac{X(j\omega)}{j\omega}$
$\int_{-\infty}^t x(\tau)d\tau \xleftrightarrow{\mathcal{L}} \frac{X(s)}{s}, \text{ ROC}\left\{\frac{X(s)}{s}\right\} \supseteq \text{ROC}\{X(s)\} \cap \{s \in \mathbb{C} : \text{Re}\{s\} > 0\}$
$\frac{dx(t)}{dt} \xleftrightarrow{\mathcal{L}} sX(s), \text{ ROC}\{sX(s)\} \supseteq \text{ROC}\{X(s)\}$

$$e^{-\alpha t} \sin(\omega_0 t) \xrightarrow{\mathcal{L}} \frac{\omega_0}{(s+\alpha)^2+\omega_0^2}$$

$$e^{-\alpha t} \cos(\omega_0 t) \xrightarrow{\mathcal{L}} \frac{s+\alpha}{(s+\alpha)^2+\omega_0^2}$$

Serie geométrica:
$\sum_{n=0}^{\infty} \alpha^n = \frac{1}{1-\alpha}, \quad \alpha \in \mathbb{C}, \quad \alpha < 1$
Suma geométrica parcial:
$\sum_{n=N_1}^{N_2-1} \alpha^n = \frac{\alpha^{N_1} - \alpha^{N_2}}{1-\alpha}, \quad \alpha \in \mathbb{C}, \quad N_1, N_2 \in \mathbb{N}$

- Teoremas del valor inicial y final: Sea $x(t)$ una señal que vale cero para $t < 0$ y que no tiene impulsos de ningún orden en el origen. Entonces valen los siguientes resultados:

$$x(0+) = \lim_{s \rightarrow \infty} sX(s)$$

$$\lim_{t \rightarrow \infty} x(t) = \lim_{s \rightarrow 0} sX(s)$$

Si $\lim_{t \rightarrow \infty} x(t)$ existe:

Convolución

$$x(t) * y(t) = \int_{-\infty}^{\infty} x(\tau)y(t-\tau)d\tau$$

Lineal: $\mathcal{T}[\alpha x(t) + \beta y(t)] = \alpha \mathcal{T}[x(t)] + \beta \mathcal{T}[y(t)]$

Ti: $\mathcal{T}[x(t-t_0)] = \mathcal{S}(t-t_0)$ Memoria $\rightarrow h(t) = \mathcal{K}\delta(t)$

Ec dif: $\sum_{k=0}^N a_k \frac{d^k y(t)}{dt^k} = \sum_{k=0}^M b_k \frac{d^k x(t)}{dt^k}$

Nyquist $f_{\text{sample}} > 2 f_{\text{max}}$

$$\mathcal{F}\{x[n-n_0]\} = e^{-j\Omega n_0} X(\Omega)$$

$$\mathcal{F}\{x(t-t_0)\} = e^{-j\omega t_0} X(\omega)$$

$$\sum_{p=0}^N y[n-p]\alpha_p = \sum_{q=0}^M x[n-q]\beta_q$$

$$x[n] * y[n] = \sum_{k=-\infty}^{\infty} x[k]y[n-k]$$

Relación entre transformadas

$$X(z) \xrightarrow{z=e^{j\Omega}} X(\Omega) \xrightarrow{\text{DFT}} X[k]$$

Bound $\rightarrow h(t)=0 \quad \forall t < 0$

Estable $\rightarrow \text{ROC} \circ \int |h(t)| < \infty$

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Cont $X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt, \omega \in \mathbb{R}$

dis $X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n}, \Omega \in [-\pi, \pi)$

Periódica en 2π

Cont $x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}d\omega, t \in \mathbb{R}$

dis $x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\Omega})e^{j\Omega n}d\Omega, n \in \mathbb{Z}$

$$H(z) = \frac{\sum_{q=0}^M z^{-q} \beta_q}{\sum_{p=0}^N z^{-p} \alpha_p} = \frac{\beta_0 + \beta_1 \bar{z}^{-1} + \dots + \beta_n \bar{z}^{-n}}{\alpha_1 + \alpha_2 \bar{z}^{-1} + \dots + \alpha_n \bar{z}^{-n}}$$

zeros

poles

Señal	Transformada Z	ROC
$\delta[n]$	1	\mathbb{C}
$u[n]$	$\frac{1}{1-z^{-1}}$	$\{z \in \mathbb{C} : z > 1\}$
$-u[-n-1]$	$\frac{1}{1-z^{-1}}$	$\{z \in \mathbb{C} : z < 1\}$
$\alpha^n u[n]$	$\frac{1}{1-\alpha z^{-1}}$	$\{z \in \mathbb{C} : z > \alpha \}$
$-\alpha^n u[-n-1]$	$\frac{1}{1-\alpha z^{-1}}$	$\{z \in \mathbb{C} : z < \alpha \}$
$n\alpha^n u[n]$	$\frac{\alpha z^{-1}}{(1-\alpha z^{-1})^2}$	$\{z \in \mathbb{C} : z > \alpha \}$
$-n\alpha^n u[-n-1]$	$\frac{\alpha z^{-1}}{(1-\alpha z^{-1})^2}$	$\{z \in \mathbb{C} : z < \alpha \}$
$r^n \cos(\omega_0 n)u[n], \quad r > 0$	$\frac{1-r \cos(\omega_0)z^{-1}}{1-2r \cos(\omega_0)z^{-1}+r^2z^{-2}}$	$\{z \in \mathbb{C} : z > r\}$
$r^n \sin(\omega_0 n)u[n], \quad r > 0$	$\frac{r \sin(\omega_0)z^{-1}}{1-2r \cos(\omega_0)z^{-1}+r^2z^{-2}}$	$\{z \in \mathbb{C} : z > r\}$
$x[n-n_0] \xleftrightarrow{\mathcal{Z}} z^{-n_0}X(z) \qquad nx[n] \xleftrightarrow{\mathcal{Z}} -z \frac{dX(z)}{dz}$		

Pivotes - * es parámetro desconocido

Dist.	Parám. θ	Pivote $Q(\underline{X}^{(n)}, \theta)$	Distribución de Q
$\mathcal{N}(\mu, \sigma^2)$	μ	$\frac{\bar{X}-\mu}{\sqrt{\sigma^2/n}}$	$\mathcal{N}(0, 1)$
$\mathcal{N}(\mu, \sigma^{2*})$	μ	$\frac{\bar{X}-\mu}{\sqrt{S^2/n}}$	t_{n-1}
$\mathcal{N}(\mu, \sigma^2)$	σ^2	$\frac{n\widehat{\sigma_{mv}^2}}{\sigma^2}$	χ_n^2
$\mathcal{N}(\mu^*, \sigma^2)$	σ^2	$\frac{(n-1)S^2}{\sigma^2}$	χ_{n-1}^2
$\mathcal{E}(\lambda)$	λ	$2n\bar{X}\lambda$	χ_{2n}^2
$\mathcal{E}(\lambda)$	λ	$2nX_{min}\lambda$	χ_2^2
$\text{Poi}(\mu)$	μ	$\frac{\bar{X}-\mu}{\sqrt{\mu/n}}$	$\mathcal{N}(0, 1)^\dagger$
$\text{Poi}(\mu)$	μ	$\frac{\bar{X}-\mu}{\sqrt{\bar{X}/n}}$	$\mathcal{N}(0, 1)^\ddagger$
$\text{Ber}(p)$	p	$\frac{\bar{X}-p}{\sqrt{p(1-p)/n}}$	$\mathcal{N}(0, 1)^\dagger$
$\text{Ber}(p)$	p	$\frac{\bar{X}-p}{\sqrt{\bar{X}(1-\bar{X})/n}}$	$\mathcal{N}(0, 1)^\ddagger$
$\mathcal{U}(0, \theta)$	θ	$\frac{X_{max}}{\theta}$	$q_\alpha = \alpha^n$
$\mathcal{N}(\mu_i, \sigma_i^2)$	Δ	$\frac{\bar{X}-\bar{Y}-\Delta}{\sqrt{\frac{\sigma_X^2}{m} + \frac{\sigma_Y^2}{n}}}$	$\mathcal{N}(0, 1)$
$\mathcal{N}(\mu_i, \sigma_{=}^{2*})$	Δ	$\frac{\bar{X}-\bar{Y}-\Delta}{\sqrt{S_P^2\left(\frac{1}{m} + \frac{1}{n}\right)}}$	t_{m+n-2}
$\mathcal{N}(\mu_i^*, \sigma_i^2)$	R	$\frac{1}{R} \frac{S_X^2}{S_Y^2}$	$F_{m-1, n-1}$
$\text{Ber}(p_i)$	Δ	$\frac{\bar{X}-\bar{Y}-\Delta}{\sqrt{\frac{\bar{X}(1-\bar{X})}{m} + \frac{\bar{Y}(1-\bar{Y})}{n}}}$	$\mathcal{N}(0, 1)^\ddagger$

Señal	Transformada de Fourier	Serie de Fourier
$\delta(t)$	1	
$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0)$	$a_1 = 1$ $a_k = 0 \quad \forall k \neq 1$
$\cos(\omega_0 t)$	$\pi [\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$	$a_1 = a_{-1} = \frac{1}{2}$ $a_k = 0 \quad \forall k \neq 1$
$\sin(\omega_0 t)$	$\frac{\pi}{j} [\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$	$a_1 = -a_{-1} = \frac{1}{2j}$ $a_k = 0 \quad \forall k \neq 1$
1	$2\pi\delta(\omega)$	$a_0 = 1$ $a_k = 0 \quad \forall k \neq 0$
$\sum_{n=-\infty}^{\infty} \delta(t - nT)$	$\frac{2\pi}{T} \sum_{n=-\infty}^{\infty} \delta(\omega - \frac{2\pi n}{T})$	$a_k = \frac{1}{T}, \quad \forall k$
$\sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$	$2\pi \sum_{k=-\infty}^{\infty} a_k \delta(\omega - k\omega_0)$	a_k
$u(t + T_1) - u(t - T_1)$	$\frac{2 \sin(\omega T_1)}{\omega}$	
$x(t) = u(t + T_1) - u(t - T_1)$ $x(t + T) = x(t)$	$\sum_{k=-\infty}^{\infty} \frac{2 \sin(k\omega_0 T_1)}{k} \delta(\omega - k\omega_0)$	$\frac{\sin(k\omega_0 T_1)}{k\pi}$
$\frac{\sin(Wt)}{\pi t}$	$u(\omega + W) - u(\omega - W)$	
$u(t)$	$\frac{1}{j\omega} + \pi\delta(\omega)$	
$e^{-\alpha t} u(t), \text{Re}(\alpha) > 0$	$\frac{1}{\alpha + j\omega}$	
$\frac{t^{n-1}}{(n-1)!} e^{-\alpha t} u(t), \text{Re}(\alpha) > 0$	$\frac{1}{(\alpha + j\omega)^n}$	
$\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{t^2}{2\sigma^2}}$	$e^{-\frac{\omega^2 \sigma^2}{2}}$	

Señal	Transformada de Fourier	Coeficientes de la serie de Fourier
$\delta[n]$	1	
$e^{j\omega_0 n}$	$2\pi \sum_{l=-\infty}^{\infty} \delta(\omega - \omega_0 - 2\pi l)$	$\text{Si } w_0 = \frac{2\pi m}{N}$ $a_k = \begin{cases} 1 & k = m, m \pm N, m \pm 2N, \dots \\ 0 & \text{en otro caso} \end{cases}$
$\cos(\omega_0 n)$	$\pi \sum_{l=-\infty}^{\infty} [\delta(\omega - \omega_0 - 2\pi l) + \delta(\omega + \omega_0 - 2\pi l)]$	$\text{Si } w_0 = \frac{2\pi m}{N}$ $a_k = \begin{cases} \frac{1}{2} & k = \pm m, \pm m \pm N, \pm m \pm 2N, \dots \\ 0 & \text{en otro caso} \end{cases}$
$\sin(\omega_0 n)$	$\frac{\pi}{j} \sum_{l=-\infty}^{\infty} [\delta(\omega - \omega_0 - 2\pi l) - \delta(\omega + \omega_0 - 2\pi l)]$	$\text{Si } w_0 = \frac{2\pi m}{N}$ $a_k = \begin{cases} \frac{1}{2j} & k = m, m \pm N, m \pm 2N, \dots \\ -\frac{1}{2j} & k = -m, -m \pm N, -m \pm 2N, \dots \\ 0 & \text{en otro caso} \end{cases}$
1	$2\pi \sum_{l=-\infty}^{\infty} \delta(\omega - 2\pi l)$	$a_k = \begin{cases} 1 & k = 0, \pm N, \pm 2N, \dots \\ 0 & \text{en otro caso} \end{cases}$
$\sum_{k=-\infty}^{\infty} \delta[n - kN]$	$\frac{2\pi}{N} \sum_{k=-\infty}^{\infty} \delta\left(\omega - \frac{2\pi k}{N}\right)$	$a_k = \frac{1}{N} \quad \forall k$
$\sum_{k=\langle N \rangle} a_k e^{j\frac{2\pi k}{N} n}$	$2\pi \sum_{k=-\infty}^{\infty} a_k \delta(\omega - \frac{2\pi k}{N})$	a_k
$u[n] - u[n - N]$	$e^{-j\omega \frac{(N-1)}{2}} \frac{\sin(\omega N/2)}{\sin(\omega/2)}$ periódica con periodo 2π	
$x[n] = u[n + N_1] - u[n - N_1 - 1]$ $x[n] = x[n + N]$	$2\pi \sum_{k=-\infty}^{\infty} \delta\left(\omega - \frac{2\pi k}{N}\right)$	$a_k = \frac{\sin\left(\frac{2\pi k}{N} \left(N_1 + \frac{1}{2}\right)\right)}{N \sin(\frac{2\pi k}{2N})} \quad k \neq 0, \pm N, \pm 2N, \dots$ $a_k = \frac{2N_1 + 1}{N} \quad k = 0, \pm N, \pm 2N, \dots$
$\frac{\sin(Wn)}{\pi n}, \quad 0 < W < \pi$	$u(\omega + W) - u(\omega - W)$ periódica con periodo 2π	
$u[n]$	$\frac{1}{1 - e^{-j\omega}} + \pi\delta(\omega)$ periódica con periodo 2π	
$\alpha^n u[n], \alpha < 1$	$\frac{1}{1 - \alpha e^{-j\omega}}$ periódica con periodo 2π	
$\frac{(n+r-1)!}{n!(r-1)!} \alpha^n u[n], \alpha < 1$	$\frac{1}{(1 - \alpha e^{-j\omega})^r}$ periódica con periodo 2π	