PHYS 2211 Exam 1
Spring 2016

Name(print) ~Test Key~ **Section #** ______

Greco (K, M) and Schatz(N)			
Day	12-3pm	3-6pm	6-9pm
Monday	N07 M07	K02 K01	
Tuesday	M01 N01	M02 N02	M03 N03
Wednesday	K05 K03	K07 K04	M08 K06
Thursday	M04 N04	M05 N05	M06 N06

Instructions

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization must be clear.
- You must show all work, including correct vector notation.
- **Correct answers without adequate explanation will be counted wrong.**
- Incorrect work or explanations mixed in with correct work will be counted wrong. Cross out anything you do not want us to grade
- Make explanations correct but brief. You do not need to write a lot of prose.
- Include diagrams!
- **Show what goes into a calculation, not just the final number, e.g.:** $\frac{a \cdot b}{c \cdot d} = \frac{(8\times10^{-3})(5\times10^{6})}{(2\times10^{-5})(4\times10^{4})} = 5 \times 10^4$
- Give standard SI units with your numeric results.

Unless specifically asked to derive a result, you may start from the formulas given on the formula sheet, including equations corresponding to the fundamental concepts. If a formula you need is not given, you must derive it.

If you cannot do some portion of a problem, invent a symbol for the quantity you can not calculate (explain that you are doing this), and use it to do the rest of the problem.

Honor Pledge

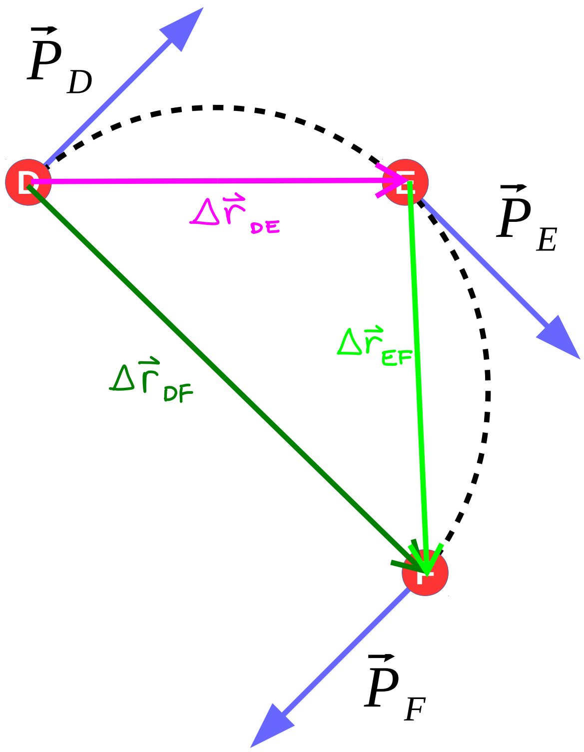
"In accordance with the Georgia Tech Honor Code, I have neither given nor received unauthorized aid on this test."

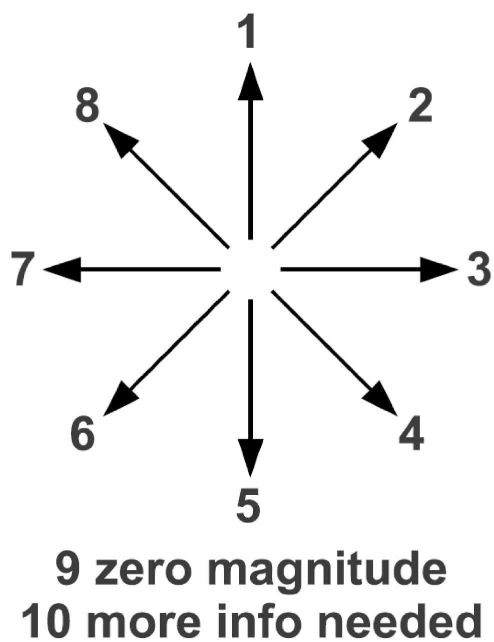
______

Sign your name on the line above

Problem 1 Grader: ______________________ Score (25pts): ____________

An object moves from location D to location F on a trajectory (dotted line) in the direction indicated; arrows representing the momentum at locations D, E, and F.

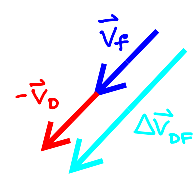




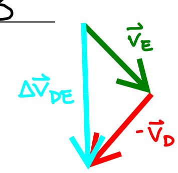
1pt each

(a 15pts) Using the numbered direction arrows shown, indicate (by number) which direction arrow best represents the direction of the quantities listed below. If the quantity has zero magnitude or cannot be determined, indicate using the corresponding number listed below.

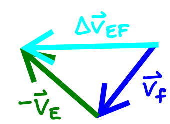
- The position vector at location D __10__ Need origin to determine position
- The change in position (the displacement) between location D and location F __4__
- The change in velocity between location D and location F __6__ $\Delta\vec{V}_{DF} = \vec{V}_F - \vec{V}_D$
- The change in momentum between location D and location F __6__ $\vec{p} = m\vec{v} \longrightarrow \Delta\vec{p} = m\Delta\vec{v}$
- The average net force between location D and location F __6__ $\vec{F}_{net} = \frac{\Delta\vec{p}}{\Delta t}$



- The position vector at location E 10
- The change in position (the displacement) between location D and location E 3
- The change in velocity between location D and location E 5 $\Delta\vec{v}_{DE} = \vec{v}_E - \vec{v}_D$



- The change in momentum between location D and location E 5 $\Delta\vec{p}_{DE} = m\Delta\vec{v}_{DE}$
- The average net force between location D and location E 5 $\vec{F}_{net} = \frac{\Delta\vec{p}}{\Delta t}$
- The position vector at location F 10
- The change in position (the displacement) between location E and location F 5
- The change in velocity between location E and location F 7 $\Delta\vec{v}_{EF} = \vec{v}_F - \vec{v}_E$



- The change in momentum between location E and location F 7
- The average net force between location E and location F 7

2 pts each (c ~~5pts~~ 10 pts) Write "T" next to each true statement below, and write "F" for every false statement.

- The displacement vector for an object can be in a different direction than its average velocity (during the same time interval). F $\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t}$
- An object's momentum is always in the same direction as the acceleration on that object. F $\vec{F} = m\vec{a} = \frac{m\Delta\vec{v}}{\Delta t}$ (at low $|\vec{v}|$)
- The change in an object's momentum can be in a different direction than the net force on the object. F $\vec{F}_{net} = \frac{\Delta\vec{p}}{\Delta t}$
- An object's momentum and its instantaneous velocity are always in the same direction. T $\vec{p} = m\vec{v}$
- If the net force on an object is constant, then the rate of change of its momentum is constant. T $\vec{F}_{net} = \frac{\Delta\vec{p}}{\Delta t}$

Problem 2 Grader: ____________________ Score (25pts): __________

Dr. Greco duct tapes a canister of compressed air to a skateboard and releases it from rest. The exhaust of the air results in a constant net force on the skateboard $\vec{F}_{net} =< 10, 0, 0 >$ N that last for 10 s. After the canister is empty, the skateboard continues to coast down the hall for and additional 30 s. During this time, the net force on the skateboard is zero. Calculate the total displacement of the skateboard during the complete 40 s interval. The skateboard and canister have a combined, constant, mass of 20 kg.

Consider 2 segments of motion: constant net force and zero net force

Segment 1: constant force

$\vec{F}_0 = \langle 10, 0, 0 \rangle N$

$\vec{p}_1 = \cancel{\vec{p}_0}^{\,0} + \vec{F}_0 \Delta t = \langle 10,0,0 \rangle N \cdot (10s) = \langle 100, 0, 0 \rangle$ kg·m/s

Use arithmetic average b/c $\vec{F}$ constant $\vec{v}_{avg} = \frac{\vec{v}_0 + \vec{v}_1}{2} = \frac{\vec{v}_1}{2} = \frac{\vec{p}_1}{2m} = \frac{\langle 100, 0, 0 \rangle \text{ kg·m/s}}{2 \cdot 20\text{kg}} = \langle 2.5, 0, 0 \rangle$ m/s

$\Delta \vec{r}_{10} = \vec{v}_{avg1} \Delta t = (\langle 2.5, 0, 0 \rangle \text{ m/s})(10s) = \langle 25, 0, 0 \rangle$ m

Segment 2: zero net force

$\vec{F}_2 = \langle 0, 0, 0 \rangle$

$\vec{p}_2 = \vec{p}_1 + \cancel{\vec{F}_2 \Delta t}^{\,0} = \langle 100, 0, 0 \rangle$ kg·m/s

Since $\vec{p}_1 = \vec{p}_2$, $\vec{v}_{avg,2} = \vec{v}_1 = \vec{v}_2$

$\Delta \vec{r}_{21} = \vec{v}_{avg,2} \Delta t = \frac{\langle 100,0,0 \rangle \text{kg·m/s}}{20\text{kg}} (30s) = \langle 150, 0, 0 \rangle$ m

Total displacement

$\Delta \vec{r} = \Delta \vec{r}_{10} + \Delta \vec{r}_{21}$

$= \langle 25, 0, 0 \rangle m + \langle 150, 0, 0 \rangle m$

$= \langle 175, 0, 0 \rangle m$

$\boxed{\Delta \vec{r} = \langle 175, 0, 0 \rangle m}$

-1	clerical	(-1)
-4	minor	(-5)
-7.5	major	(-10)
-20	BTN	(-20)

* Note: these values were calculated using a previous rubric. The numbers in purple are the equivalent penalties from the rubric on T-Square

Problem 3 Grader: ______________ Score (25pts): ________

A soccer ball of mass 0.30 kg is rolling with velocity $< 0, 0, 2.0 >$ m/s, when you kick it. Your kick delivers an impulse of $< 0, 3.4, 0 >$ Ns.

All

(a 5pts) What is the ball's momentum immediately after the kick? Express your answer as a vector with the proper units.

By definition $\Delta\vec{p} = \vec{F}_{net}\Delta t$ Recall: $\vec{F}_{net}\Delta t$ is impulse

$$\vec{p}_f = \vec{p}_i + \vec{F}_{net}\Delta t$$
$$= m\vec{v}_i + \vec{F}_{net}\Delta t$$
$$= (0.3\text{ kg})\langle 0, 0, 2.0\rangle \text{m/s} + \langle 0, 3.4, 0\rangle \text{Ns}$$
$$= \langle 0, 3.4, 0.6\rangle \text{ kg}\cdot\text{m/s}$$

$$\boxed{\vec{p}_f = \langle 0, 3.4, 0.6\rangle \text{ kg}\cdot\text{m/s}} \qquad \vec{v}_0 = \frac{\vec{p}_f}{m} = \frac{\langle 0, 3.4, 0.6\rangle \text{ kg}\cdot\text{m/s}}{0.3\text{ kg}} = \langle 0, 11.333, 2.0\rangle \text{m/s}$$

Note: "f" here is final from the kick → will be initial in next part

Immediately after the kick, the ball starts from an initial position $\vec{r} = < 0, -4, 6 >$ m and rolls with a net force (due to the air and the grass) with magnitude of 0.40 N and pointing in the direction opposite to the ball's momentum. Using a time step of $\Delta t = 0.5$ s, calculate step by step (iteratively) the following quantities:

└means $\vec{F}$ is in the direction $-\hat{p}$

(b 10pts) the position and velocity of the block at $t = 0.5$ s. Express your answer as a vector with the proper units.

Calculate net force

$$\hat{p} = \frac{\vec{p}}{|\vec{p}|} = \frac{\langle 0, 3.4, 0.6\rangle \text{ kg}\cdot\text{m/s}}{\sqrt{(0)^2 + (3.4\text{ kg}\cdot\text{m/s})^2 + (0.6\text{kg}\cdot\text{m/s})^2}} = \langle 0, 0.9848, 0.1738\rangle$$

$$\vec{F}_1 = -|\vec{F}|\hat{p} = -(0.4\text{ N})\langle 0, 0.9848, 0.1738\rangle = \langle 0, -0.39392, -0.06952\rangle \text{N}$$

Update momentum

$$\vec{p}_1 = \vec{p}_0 + \vec{F}_1\Delta t = \langle 0, 3.4, 0.6\rangle \text{kg}\cdot\text{m/s} + \langle 0, -0.39392, -0.06952\rangle \text{N}(0.5\text{s}) = \langle 0, 3.203, 0.565\rangle \text{ kg}\cdot\text{m/s}$$

$$\vec{v}_1 = \frac{\vec{p}_1}{m} = \frac{\langle 0, 3.203, 0.565\rangle \text{ kg}\cdot\text{m/s}}{0.3\text{ kg}} = \langle 0, 10.677, 1.883\rangle \text{ m/s}$$

Calculate average velocity — constant force * Original key didn't use this - would likely be treated as a minor error

$$\vec{v}_{avg,1} = \frac{\vec{v}_0 + \vec{v}_1}{2} = \frac{\langle 0, 11.333, 2.0\rangle \text{m/s} + \langle 0, 10.677, 1.883\rangle \text{m/s}}{2} = \langle 0, 11.005, 1.942\rangle \text{m/s}$$

Update position

$$\vec{r}_1 = \vec{r}_0 + \vec{v}_{avg,1}\Delta t = \langle 0, -4, 6\rangle \text{m} + (\langle 0, 11.005, 1.942\rangle \text{m/s})(0.5\text{s})$$
$$= \langle 0, 1.503, 6.971\rangle \text{m}$$

$$\boxed{\begin{array}{l}\vec{v}_1 = \langle 0, 10.677, 1.883\rangle \text{m/s} \\ \vec{r}_1 = \langle 0, 1.503, 6.971\rangle \text{m}\end{array}}$$

* Watch for POE *

-0.5	clerical	(-1)
-1.5	minor	(-2)
-3.0	major	(-4)
-8.0	BTN	(-8)

Same note as p4 (problem 2)

(c 10pts) the position and velocity of the block at $t = 1.0$ s. Express your answer as a vector with the proper units.

Calculate net force (Note: could have just said $\vec{F}$ constant)

$$\hat{p}_1 = \frac{\vec{p}_1}{|\vec{p}_1|} = \frac{\langle 0, 3.203, 0.565\rangle \, kg\cdot m/s}{\sqrt{(0)^2 + (3.203\,kg\cdot m/s)^2 + (0.565\,kg\cdot m/s)^2}} = \langle 0, 0.9848, 0.1738\rangle$$

$$\vec{F}_2 = -|\vec{F}|\hat{p}_1 = \langle 0, -0.39392, -0.06952\rangle \, N$$

Update momentum

$$\vec{p}_2 = \vec{p}_1 + \vec{F}_2 \Delta t = \langle 0, 3.203, 0.565\rangle \, kg\cdot m/s + (\langle 0, -0.39392, -0.06952\rangle N)(0.5\,s)$$

$$= \langle 0, 3.006, 0.530\rangle \, kg\cdot m/s$$

$$\vec{v}_2 = \frac{\vec{p}_2}{m} = \frac{\langle 0, 3.006, 0.530\rangle \, kg\cdot m/s}{0.3\,kg} = \langle 0, 10.02, 1.767\rangle \, m/s$$

Calculate average velocity

$$\vec{v}_{avg,2} = \frac{\vec{v}_1 + \vec{v}_2}{2} = \frac{\langle 0, 10.677, 1.883\rangle m/s + \langle 0, 10.02, 1.767\rangle m/s}{2} = \langle 0, 10.349, 1.825\rangle \, m/s$$

Update position

$$\vec{r}_2 = \vec{r}_1 + \vec{v}_{avg,2} \Delta t = \langle 0, 1.503, 6.971\rangle m + (\langle 0, 10.349, 1.825\rangle m/s)(0.5\,s)$$

$$= \langle 0, 6.678, 7.884\rangle \, m$$

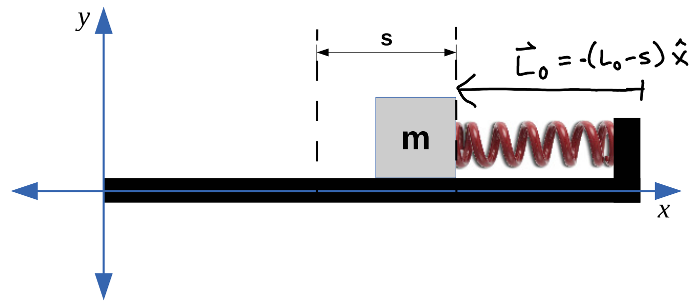
$$\boxed{\begin{array}{l} \vec{v}_2 = \langle 0, 10.02, 1.767\rangle \, m/s \\ \vec{r}_1 = \langle 0, 6.678, 7.884\rangle \, m \end{array}}$$

-0.5 clerical
-1.5 minor
-3 major
-8 BTN

* Watch for POE *

`Problem 4` Grader: ______________________ Score (25pts): __________

A spring with relaxed length L_0 is attached to a horizontal (i.e. flat) table in the physics lab as indicated in the diagram. You attach a mass m to the spring and compress it an amount s.



All

(a 5pts) Determine the direction of the spring force on the mass (your answer should be a vector).

Definition of spring force

$$\vec{F}_s = -k_s s \hat{L}$$

$\hat{L}$ points from fixed end of spring to free end of spring: $\hat{L} = \langle -1,0,0\rangle = -\hat{x}$

Spring compressed, so s is negative

Direction of spring force is

$$\hat{F}_s = \frac{-k_s s \hat{L}}{k_s |s|} = \frac{+\cancel{k_s}(+\cancel{|s|})\langle -1,0,0\rangle}{\cancel{k_s}\cancel{|s|}} = \langle -1,0,0\rangle$$

$$\boxed{\hat{F}_s = \langle -1,0,0\rangle}$$

(b 10pts) Determine the velocity of the block a short time Δt after being released from rest.

$\llcorner \vec{p}_0 = \vec{0}$

Calculate force:

$$\vec{F}_s = -k_s s \hat{L} = -k_s(-s)(-\hat{x}) = -k_s s \hat{x}$$

Update momentum:

$$\vec{p}_1 = \cancel{\vec{p}_0}^{\,0} + \vec{F}_{net}\Delta t = (-k_s s \hat{x})(\Delta t) = -k_s s \Delta t \hat{x}$$

Calculate velocity

$$\vec{V}_1 = -\frac{k_s s \Delta t}{m}\hat{x}$$

$$\boxed{\vec{V}_1 = -\frac{k_s s \Delta t}{m}\hat{x}}$$

Watch for POE

−0.5	clerical
−1.5	minor
−3.0	major
−8.0	BTW

*Same comment as page 5

(c 10pts) Determine the net force on the block a short time Δt after being released from rest.

First, need to update position $\rightarrow$ $\vec{F}_s$ not constant, so approximate $\vec{v}_{avg} \approx \vec{v}_f = \frac{\vec{p}_f}{m}$

$$\vec{L}_0 = -(L_0 - s)\hat{x}$$

$$\vec{L}_1 = \vec{L}_0 + \vec{v}_{avg}\Delta t \approx \vec{L}_0 + \vec{v}_1 \Delta t$$

$$= -(L_0 - s)\hat{x} - \left(\frac{k_s s \Delta t}{m}\hat{x}\right)\Delta t$$

$$= \left[-L_0 + s\left(1 - \frac{k_s(\Delta t)^2}{m}\right)\right]\hat{x}$$

$$= -\left[L_0 - s\left(1 - \frac{k_s(\Delta t)^2}{m}\right)\right]\hat{x}$$

Note: I calculated $\vec{L}$ with respect to the fixed end of the spring

Update net force

Since we know $L_0 > s$ and that $\frac{k_s s(\Delta t)^2}{m}$ is positive, $|\vec{L}| = L_0 - s\left(1 - \frac{k_s(\Delta t)^2}{m}\right)$

Calculate stretch $s = |\vec{L}| - L_0$

$$= \cancel{L_0} - s\left(1 - \frac{k_s(\Delta t)^2}{m}\right) - \cancel{L_0}$$

$$= -s\left(1 - \frac{k_s(\Delta t)^2}{m}\right)$$

Calculate force $\vec{F}_s = -k_s\left[-s\left(1 - \frac{k_s(\Delta t)^2}{m}\right)\right](-\hat{x})$

$$= -k_s s\left(1 - \frac{k_s(\Delta t)^2}{m}\right)\hat{x}$$

$$\boxed{\vec{F}_s = -k_s s\left(1 - \frac{k_s(\Delta t)^2}{m}\right)\hat{x}}$$

Watch for POE

-0.5	clerical
-1.5	minor
-3.0	major
-8.0	BTN

*Same comment as pg 5

This page is for extra work, if needed.

Things you must have memorized

The Momentum Principle Definition of Momentum	The Energy Principle Definition of Velocity	The Angular Momentum Principle Definition of Angular Momentum
Definitions of angular velocity, particle energy, kinetic energy, and work		

Other potentially useful relationships and quantities

$$\gamma \equiv \frac{1}{\sqrt{1-\left(\frac{|\vec{v}|}{c}\right)^2}} \qquad E^2-(pc)^2=\left(mc^2\right)^2$$

$$\frac{d\vec{p}}{dt}=\frac{d|\vec{p}|}{dt}\hat{p}+|\vec{p}|\frac{d\hat{p}}{dt} \qquad \vec{F}_{\parallel}=\frac{d|\vec{p}|}{dt}\hat{p} \text{ and } \vec{F}_{\perp}=|\vec{p}|\frac{d\hat{p}}{dt}=|\vec{p}|\frac{|\vec{v}|}{R}\hat{n}$$

$$\vec{F}_{grav}=-G\frac{m_1m_2}{|\vec{r}|^2}\hat{r} \qquad U_{grav}=-G\frac{m_1m_2}{|\vec{r}|}$$

$$\left|\vec{F}_{grav}\right|\approx mg \text{ near Earth's surface} \qquad \Delta U_{grav}\approx mg\Delta y \text{ near Earth's surface}$$

$$\vec{F}_{elec}=\frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{|\vec{r}|^2}\hat{r} \qquad U_{elec}=\frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{|\vec{r}|}$$

$$\left|\vec{F}_{spring}\right|=k_ss \qquad U_{spring}=\frac{1}{2}k_ss^2$$

$$U_i\approx\frac{1}{2}k_{si}s^2-E_M \qquad \Delta E_{thermal}=mC\Delta T$$

$$\vec{r}_{cm}=\frac{m_1\vec{r}_1+m_2\vec{r}_2+\ldots}{m_1+m_2+\ldots} \qquad I=m_1r_{1\perp}^2+m_2r_{2\perp}^2+\ldots$$

$$K_{tot}=K_{trans}+K_{rel} \qquad K_{rel}=K_{rot}+K_{vib}$$

$$K_{rot}=\frac{L_{rot}^2}{2I} \qquad K_{rot}==\frac{1}{2}I\omega^2$$

$$\vec{L}_A=\vec{L}_{trans,A}+\vec{L}_{rot} \qquad \vec{L}_{rot}=I\vec{\omega}$$

$$\omega=\sqrt{\frac{k_s}{m}} \qquad v=d\sqrt{\frac{k_{si}}{m_a}}$$

$$Y=\frac{F/A}{\Delta L/L}\text{ (macro)} \qquad Y=\frac{k_{si}}{d}\text{ (micro)}$$

$$\Omega=\frac{(q+N-1)!}{q!\,(N-1)!} \qquad S\equiv k\ln\Omega$$

$$\frac{1}{T}\equiv\frac{\partial S}{\partial E} \qquad \Delta S=\frac{Q}{T}\text{ (small } Q\text{)}$$

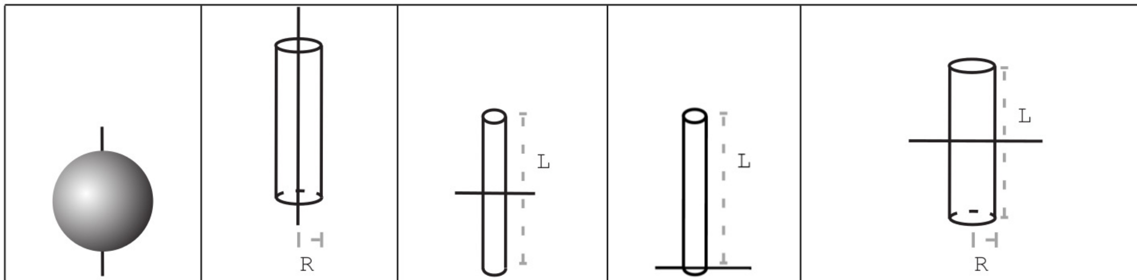
$$\text{prob}(E)\propto\Omega\,(E)\,e^{-\frac{E}{kT}} \qquad E_N=-\frac{13.6\text{eV}}{N^2}\text{ where } N=1,2,3\ldots$$

$$E_N=N\hbar\omega_0+E_0 \text{ where } N=0,1,2\ldots \text{ and } \omega_0=\sqrt{\frac{k_{si}}{m_a}} \text{ (Quantized oscillator energy levels)}$$

Moment of intertia for rotation about indicated axis

The cross product

$$\vec{A} \times \vec{B} = \langle A_y B_z - A_z B_y, A_z B_x - A_x B_z, A_x B_y - A_y B_x \rangle$$



$I = \frac{2}{5}MR^2$	$I = \frac{1}{2}MR^2$	$I = \frac{1}{12}ML^2$	$I = \frac{1}{3}ML^2$	$I = \frac{1}{12}ML^2 + \frac{1}{4}MR^2$

Constant	Symbol	Approximate Value
Speed of light	c	3×10^8 m/s
Gravitational constant	G	6.7×10^{-11} N $\cdot$ m^2/kg^2
Approx. grav field near Earth's surface	g	9.8 N/kg
Electron mass	m_e	9×10^{-31} kg
Proton mass	m_p	1.7×10^{-27} kg
Neutron mass	m_n	1.7×10^{-27} kg
Electric constant	$\frac{1}{4\pi\epsilon_0}$	9×10^9 N $\cdot$ m^2/C^2
Proton charge	e	1.6×10^{-19} C
Electron volt	1 eV	1.6×10^{-19} J
Avogadro's number	N_A	6.02×10^{23} atoms/mol
Plank's constant	h	6.6×10^{-34} joule $\cdot$ second
hbar $= \frac{h}{2\pi}$	$\hbar$	1.05×10^{-34} joule $\cdot$ second
specific heat capacity of water	C	4.2 J/g/K
Boltzmann constant	k	1.38×10^{-23} J/K

milli	m	1×10^{-3}	kilo	K	1×10^{3}
micro	μ	1×10^{-6}	mega	M	1×10^{6}
nano	n	1×10^{-9}	giga	G	1×10^{9}
pico	p	1×10^{-12}	tera	T	1×10^{12}