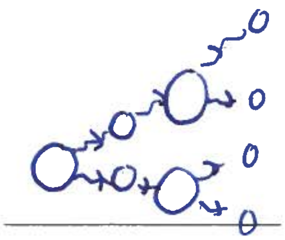
PHYS 2211 Exam 3
Spring 2016

Name(print) Enrico Fermi Section # 

Greco (K, M) and Schatz(N)			
Day	12-3pm	3-6pm	6-9pm
Monday	N07 M07	K02 K01	
Tuesday	M01 N01	M02 N02	M03 N03
Wednesday	K05 K03	K07 K04	M08 K06
Thursday	M04 N04	M05 N05	M06 N06

Instructions

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization must be clear.
- You must show all work, including correct vector notation.
- **Correct answers without adequate explanation will be counted wrong.**
- Incorrect work or explanations mixed in with correct work will be counted wrong. Cross out anything you do not want us to grade
- Make explanations correct but brief. You do not need to write a lot of prose.
- Include diagrams!
- **Show what goes into a calculation, not just the final number, e.g.:** $\frac{a \cdot b}{c \cdot d} = \frac{(8\times10^{-3})(5\times10^{6})}{(2\times10^{-5})(4\times10^{4})} = 5 \times 10^{4}$
- Give standard SI units with your numeric results.

Unless specifically asked to derive a result, you may start from the formulas given on the formula sheet, including equations corresponding to the fundamental concepts. If a formula you need is not given, you must derive it.

If you cannot do some portion of a problem, invent a symbol for the quantity you can not calculate (explain that you are doing this), and use it to do the rest of the problem.

Honor Pledge

"In accordance with the Georgia Tech Honor Code, I have neither given nor received unauthorized aid on this test."

Sign your name on the line above

Problem 1 Grader: ______________________ Score: ______/25 ______/25

An electron with a speed of $0.95c$ is emitted by a supernova, where c is the speed of light.

(a 5pts) What is the total (particle) energy of the electron?

$$E = \gamma mc^2 = \frac{mc^2}{\left(1-\left(\frac{v}{c}\right)^2\right)^{1/2}}$$

$$E = \frac{mc^2}{\left(1-\left(\frac{0.95c}{c}\right)^2\right)^{1/2}} = \frac{(9\times10^{-31}\,kg)(3\times10^{8}\,m/s)^2}{\left(1-(0.95)^2\right)^{1/2}}$$

$$\boxed{E = 2.6 \times 10^{-13}\ J}$$

All or -1 for units

(b 5pts) What is the kinetic energy of the electron?

$E = K + mc^2$ then $K = E - mc^2$

$E = 2.6\times10^{-13}$ then $K = 2.6\times10^{-13}\,J - (9\times10^{-31}\,kg)(3e8\,m/s)$

$$\boxed{K = 1.8 \times 10^{-13}\ J}$$

All or -1 for units

check for POE

(c 8pts) A force of $\langle 2\times10^{-8}, -4\times10^{-8}, 3\times10^{-8}\rangle$ N acts on the electron as it moves over a displacement of $\langle -0.03, 0.05, 0.07\rangle$ m. What is the work done on the electron by this force?

$$W = \int \vec{F}\cdot d\vec{r} = \vec{F}\cdot\Delta\vec{r} \text{ for constant forces}$$

$$\vec{F}\cdot\Delta\vec{r} = \sum_{i=1}^{3} F_i \Delta r_i = (2e\text{-}8)(-0.03) + (-4e\text{-}8)(0.05) + (3e\text{-}8)(0.07)$$

[CI: -0.5
Minor: -1.0
Major: -2.5
BTN: -6]

$$W = (-6\times10^{-10}\text{ J}) + (-2\times10^{-9}\text{ J}) + (2.1\times10^{-9}\text{ J})$$

$$\boxed{W = -5\times10^{-10}\text{ J}}$$

(d 7pts) Determine the new kinetic energy of the electron.

$$\Delta E = W$$

$$\Delta E_{rest} + \Delta K = W$$

$\Delta E_{rest} = 0$ then $K_f - K_i = W$

$$K_f = K_i + W$$

$$= 1.8\times10^{-13}\text{ J} - 5\times10^{-10}\text{ J}$$

$$\boxed{K_f = -4.99\times10^{-10}\text{ J}}$$

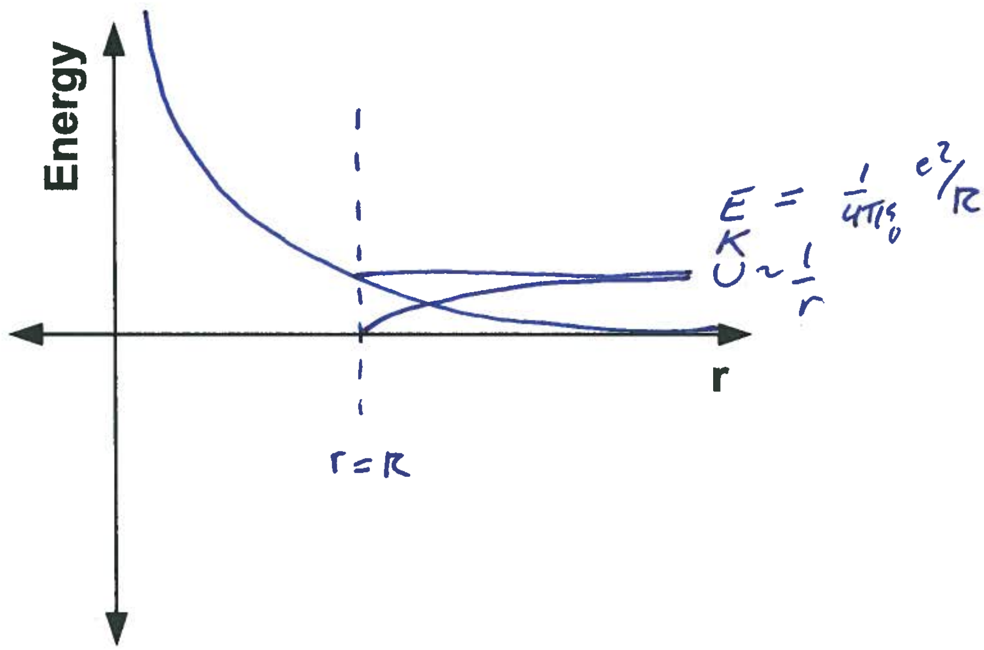
[-0.5
-1.0
-2.5
-6.0]

* negative K implies $\vec{F}$ could not cause $\Delta\vec{r}$ and another force must be present.

Problem 2 Grader: ______________________ Score: ______/25 ______/25 ______/25

For each system given below, sketch and label the graphs of kinetic (K), potential (U) and total energy (E=K+U).

(a 8pts) Two electrons start at rest (that is, their initial velocities zero) some finite distance R apart.



$E = K + U$

$U_{ele} = \frac{1}{4\pi\epsilon_0}\frac{(-e)(-e)}{r} = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r}$ (3pts)

$E = 0 + \frac{1}{4\pi\epsilon_0}\frac{e^2}{R}$ (2pts)

$K = \frac{1}{4\pi\epsilon_0}\frac{e^2}{R} - \frac{1}{4\pi\epsilon_0}\frac{e^2}{r}$ (3pts)

→ Qualitative plots only, but $E = U$ @ $r = R$

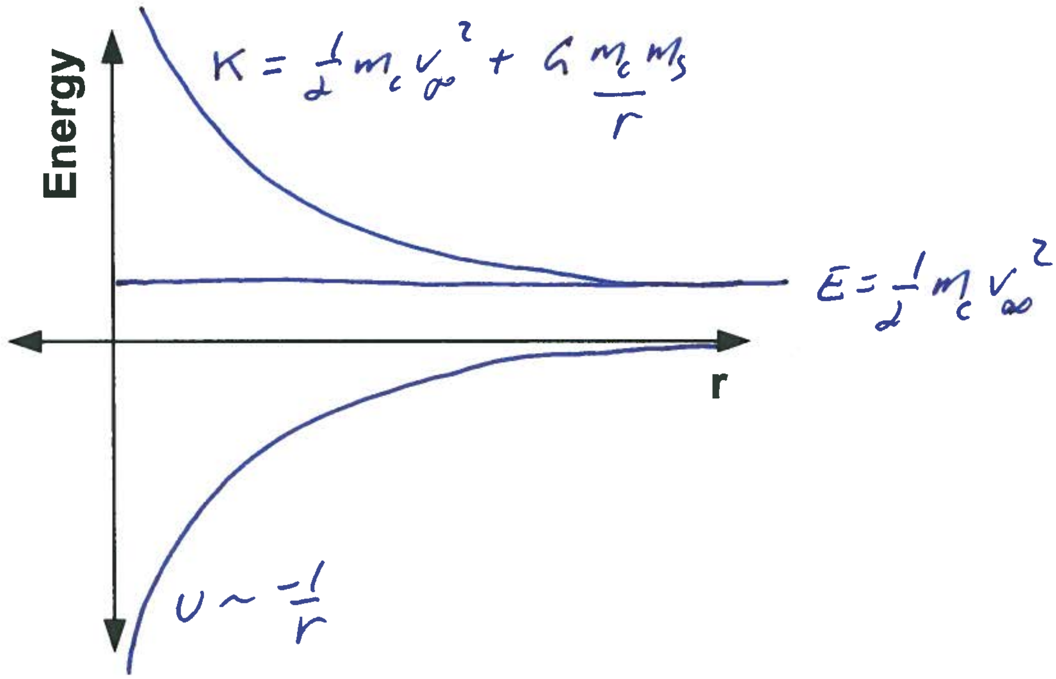
AND $U \to 0$, $K \to E$ for $r \to \infty$

- partial credit TA discuss

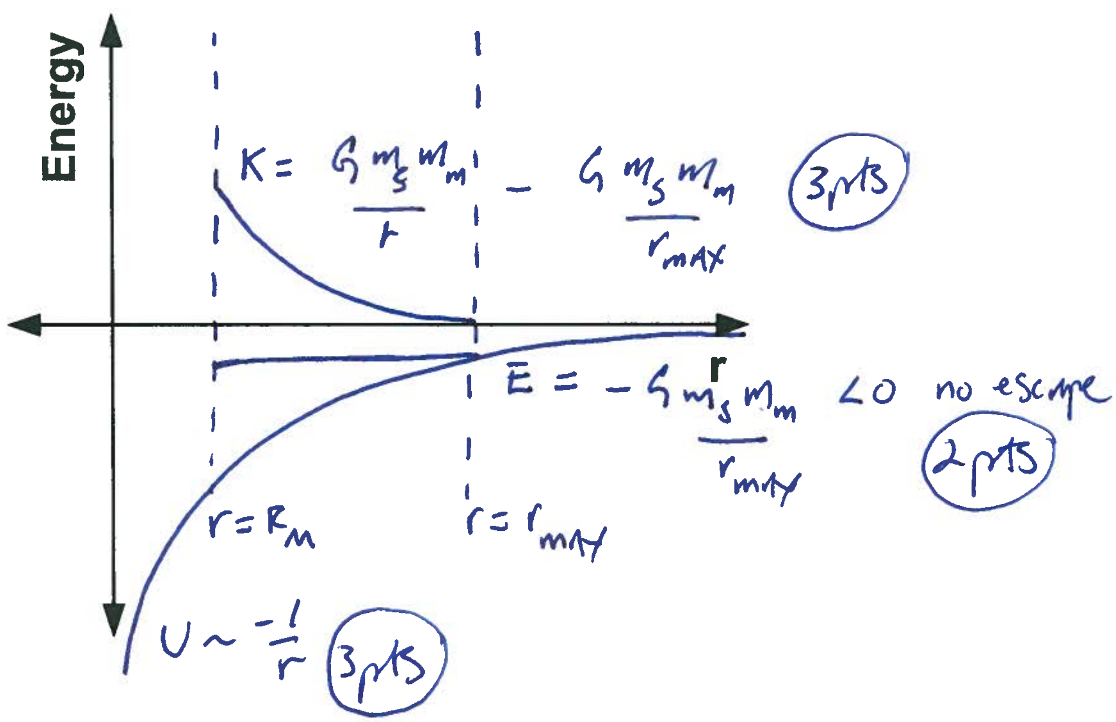
(b 9pts) A comet moves away from the Sun, never to return. Very far away from the Sun, the comet has a nonzero speed.

3pts each

partial credit follows same as in (a)



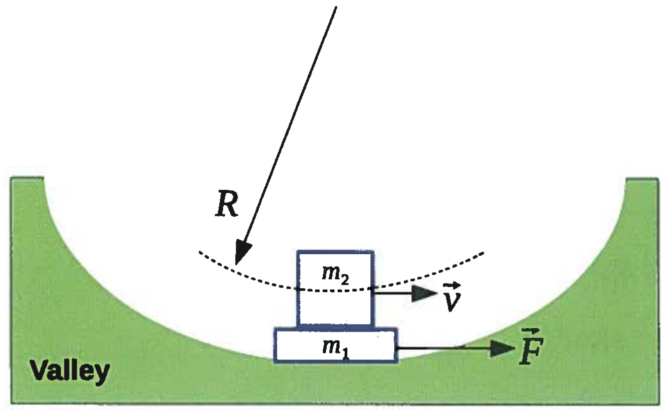
(c 8pts) A spacecraft leaves Mars with an initial velocity that is below the escape speed for Mars.



→ partial credit as in (a)

Problem 3 Grader: ____________________ Score: ______/25

Two blocks with mass m_1 and m_2 are pulled through a valley by an external force $\vec{F}$. The center of the top block moves through a radius of curvature R. When the blocks are at the bottom of the valley (as shown in the figure), friction is able to prevent the top block from sliding with respect to the bottom block. There is negligible friction between the bottom block and the valley floor. At the bottom of the valley, both blocks are moving together to the right with speed v.



(a 2pts) Determine the magnitude and direction of $(\frac{d\vec{p}}{dt})_{\parallel}$ for a system that includes both blocks at the instant when both blocks are at the bottom of the valley.

$$\left(\frac{d\vec p}{dt}\right)_{\parallel} = \vec F_{net_{\parallel}} = \vec F_{\parallel} + \vec F_{\mathcal{E}_{\parallel}} + \vec F_{valley_{\parallel}} = |\vec F|\hat p$$

All

(b 8pts) Determine the magnitude and direction of $(\frac{d\vec{p}}{dt})_{\parallel}$ for the top block at the instant when both blocks are at the bottom of the valley. (Hint: Choose your system to be the top block only, use the result from part (a) and note that $(\frac{d\vec{v}}{dt})_{\parallel}$ is the same for all choices of system.)

$$\vec a_{\parallel} = \frac{|\vec F|}{(m_1+m_2)}\hat p$$ As determined in part (a)

$$\left(\frac{d\vec p_{m_2}}{dt}\right)_{\parallel} = \left(\vec F_{net_{m_2}}\right)_{\parallel} = m_2 \vec a_{\parallel}$$

$$\boxed{\left(\frac{d\vec p}{dt}m_2\right)_{\parallel} = \left(\frac{m_2}{m_1+m_2}\right)|\vec F|\hat p}$$

$$\begin{bmatrix} -0.5 \\ -1.0 \\ -2.5 \\ -6.0 \end{bmatrix}$$

(c 5pts) Determine the magnitude and direction of $(\frac{dp}{dt})_{\perp}$ for the top block at the instant when both blocks are at the bottom of the valley.

$$\left(\frac{d\vec p}{dt}\right)_{\perp} = \frac{|\vec p||\vec v|}{R_{Kiss}}\hat n = \frac{m_2|\vec v|^2}{R}\hat n$$

All

* note $\hat n = \hat y$ ok

(d 2pts) Find the magnitude $|\vec{F}_{net}|$ on the top block at the instant when both blocks are at the bottom of the valley.

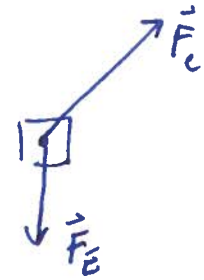
$$\vec{F}_{net} = \frac{d\vec{p}_{m_2}}{dt} = \left(\frac{d\vec{p}_{m_2}}{dt}\right)_{\parallel} + \left(\frac{d\vec{p}_{m_2}}{dt}\right)_{\perp}$$

$$= \left(\frac{m_2}{m_1+m_2}\right)|\vec{F}|\hat{p} + \frac{m_2|\vec{v}|^2}{R}\hat{n}$$

$$|\vec{F}_{net}| = \left[\left(\frac{m_2}{m_1+m_2}\right)^2|\vec{F}|^2 + \frac{m_2^2|\vec{v}|^4}{R^2}\right]^{1/2}$$

<u>All</u>

(e 4pts) At the bottom of the valley, determine the magnitude and direction of the perpendicular component of the force the bottom block exerts on the top block.



$$\vec{F}_{net,m_2} = \vec{F}_{earth} + \vec{F}_{contact\ m_1}$$ (2pts)

$$\frac{m_2|\vec{v}|^2}{R}\hat{n} = -m_2 g\hat{n} + (\vec{F}_{c,m_1})_{\perp}$$

$$(\vec{F}_{contact,m_2})_{\perp} = \left(\frac{m_2|\vec{v}|^2}{R} + m_2 g\right)\hat{n}$$ (2pts)

(f 4pts) Determine the magnitude and direction of the contact force from the top block on the bottom block.

→ Contact forces are equal & opposite (3rd Law) (2pts)

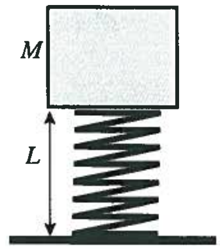
$$\vec{F}_{contact\ m_1} = \left(\frac{m_2|\vec{v}|^2}{R} + m_2 g\right)\hat{n} + \left(\frac{m_2}{m_1+m_2}\right)|\vec{F}|\hat{p}$$

<u>then</u>

$$\vec{F}_{contact\ m_2} = -\left(\frac{m_2|\vec{v}|^2}{R} + m_2 g\right)\hat{n} - \left(\frac{m_2}{m_1+m_2}\right)|\vec{F}|\hat{p}$$ (2pts)

Problem 4 Grader: ______________________ Score: ______/25 ______/25 ______/25 ______/25

A spring with stiffness k and relaxed length L_0 stands vertically on a table. A mass M sits on the spring in static equilibrium. The quantities k, L_0 and M are known; the compressed length L, shown in the figure, is unknown.



(a 5pts) Determine the compressed length of the spring L in terms of known quantities and constants.

$$\frac{d\vec{p}}{dt} = 0 = \vec{F}_{net} \Rightarrow \vec{F}_{net} = \vec{F}_S + \vec{F}_E = 0$$

2pts

$$-Ks\hat{L} + -mg\hat{y} = 0$$

2pts

$$-K(L - L_0) = Mg$$

$$\boxed{(KL_0 - Mg)/K = L}$$

1pt

(b 10pts) Using your hand, you compress the spring so that the spring now has a length of $L/2$ and you hold the spring motionless at this position. Calculate the work done by your hand. Briefly explain in words why the sign you obtained for this work is reasonable.

$\Delta E = W$ system; Block + spring + earth
surr; hand

$$\cancel{\Delta K} + \Delta U_{spring} + \Delta U_{grav} = W_{hand}$$

0

$$\frac{1}{2}K(s_f^2 - s_i^2) + mg\,\Delta y = W_{hand}$$

$$\frac{1}{2}K\left[(L/2 - L_0)^2 - (L - L_0)^2\right] + mg(L/2 - L) = W_{hand}$$

$$\rightarrow L - L_0 = -\frac{mg}{K} \quad \& \quad L/2 - L_0 = -\frac{1}{2}\left(L_0 + \frac{mg}{K}\right)$$

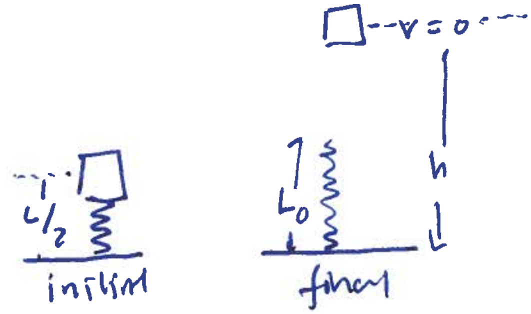
$$\begin{bmatrix} -0.5 \\ -1.5 \\ -3.0 \\ -8.0 \end{bmatrix}$$

see last page

$$W_{hand} \overset{?}{=} \frac{KL^2}{8} = \frac{K}{8}\left(L_0 - \frac{mg}{K}\right)^2 > 0$$

* F_{hand} is down & $\Delta\vec{r}$ is down so $W > 0$

(c 10pts) You let go of the block and watch it shoot straight up into the air. Find the maximum height reached by the block in terms of known quantities and constants.



System: Block + spring + earth

Surroundings: The Nothing

$$\Delta E = W$$

$$\Delta E = 0$$

$$\cancel{\Delta K} + \Delta U_{spring} + \Delta U_{grav} = 0$$

0

$$\rightarrow \quad (\cancel{U_{spring,f}} - U_{spring,i}) + (U_{grav,f} - U_{grav,i}) = 0$$

0

$$-\frac{1}{2} k s_i^2 + mg\,\Delta y = 0$$

$$\begin{bmatrix} -0.5 \\ -1.5 \\ -3.0 \\ -8.0 \end{bmatrix}$$

$$\rightarrow \quad -\frac{1}{2} k \left(L/2 - L_0\right)^2 + mg\left(h - L/2\right) = 0$$

$$mg\left(h - L/2\right) = \frac{1}{2} k \left(L/2 - L_0\right)^2$$

$$\boxed{h = \frac{L}{2} + \frac{1}{2}\frac{k}{mg}\left(L/2 - L_0\right)^2}$$

$$h = \frac{\left(L_0 - \frac{mg}{k}\right)}{2} + \frac{1}{2}\frac{k}{mg}\left(-\frac{1}{2}\left(L_0 + \frac{mg}{k}\right)\right)^2$$

$$h = \frac{L_0}{2} - \frac{mg}{2k} + \frac{1}{8}\frac{k}{mg}\left(L_0 + \frac{mg}{k}\right)^2$$

This page is for extra work, if needed.

$$\frac{1}{2}K\left[(L/2-L_0)^2-(L-L_0)^2\right]+mg(L/2-L)=W_{hand}$$

$$\frac{1}{2}K\left[\frac{L^2}{4}-LL_0+\cancel{L_0^2}-L^2+2LL_0-\cancel{L_0^2}\right]-mg\frac{L}{2}=W_{hand}$$

$$\frac{1}{2}K\left[-\frac{3L^2}{4}+LL_0\right]-mgL/2=W_{hand}$$

$$\frac{L}{2}\left(KL_0-\frac{3L}{4}K\right)-mg\frac{L}{2}=W_{hand}$$

$$\frac{L}{2}\left(KL_0-\frac{3L}{4}K-mg\right)=W_{hand}$$

<u>But</u> $L_0-\frac{mg}{K}=L \Rightarrow L_0K=LK+mg$

$$\frac{L}{2}\left(LK+\cancel{mg}-\frac{3L}{4}K-\cancel{mg}\right)=W_{hand}$$

$$\frac{L^2}{2}\left(K-\frac{3}{4}K\right)=W_{hand}$$

$$\frac{KL^2}{2}\left(1-\frac{3}{4}\right)=W_{hand}$$

$$\boxed{\frac{KL^2}{8}=W_{hand}}$$

Things you must have memorized

The Momentum Principle Definition of Momentum	The Energy Principle Definition of Velocity	The Angular Momentum Principle Definition of Angular Momentum
Definitions of angular velocity, particle energy, kinetic energy, and work		

Other potentially useful relationships and quantities

$$\gamma \equiv \frac{1}{\sqrt{1-\left(\frac{|\vec{v}|}{c}\right)^2}}$$

$$E^2 - (pc)^2 = \left(mc^2\right)^2$$

$$\frac{d\vec{p}}{dt} = \frac{d|\vec{p}|}{dt}\hat{p} + |\vec{p}|\frac{d\hat{p}}{dt}$$

$$\vec{F}_{\parallel} = \frac{d|\vec{p}|}{dt}\hat{p} \text{ and } \vec{F}_{\perp} = |\vec{p}|\frac{d\hat{p}}{dt} = |\vec{p}|\frac{|\vec{v}|}{R}\hat{n}$$

$$\vec{F}_{grav} = -G\frac{m_1 m_2}{|\vec{r}|^2}\hat{r}$$

$$U_{grav} = -G\frac{m_1 m_2}{|\vec{r}|}$$

$$\left|\vec{F}_{grav}\right| \approx mg \text{ near Earth's surface}$$

$$\Delta U_{grav} \approx mg\Delta y \text{ near Earth's surface}$$

$$\vec{F}_{elec} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}|^2}\hat{r}$$

$$U_{elec} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}|}$$

$$\left|\vec{F}_{spring}\right| = k_s s$$

$$U_{spring} = \frac{1}{2}k_s s^2$$

$$U_i \approx \frac{1}{2}k_{si}s^2 - E_M$$

$$\Delta E_{thermal} = mC\Delta T$$

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots}{m_1 + m_2 + \ldots}$$

$$I = m_1 r_{1\perp}^2 + m_2 r_{2\perp}^2 + \ldots$$

$$K_{tot} = K_{trans} + K_{rel}$$

$$K_{rel} = K_{rot} + K_{vib}$$

$$K_{rot} = \frac{L_{rot}^2}{2I}$$

$$K_{rot} == \frac{1}{2}I\omega^2$$

$$\vec{L}_A = \vec{L}_{trans,A} + \vec{L}_{rot}$$

$$\vec{L}_{rot} = I\vec{\omega}$$

$$\omega = \sqrt{\frac{k_s}{m}}$$

$$v = d\sqrt{\frac{k_{si}}{m_a}}$$

$$Y = \frac{F/A}{\Delta L/L} \text{ (macro)}$$

$$Y = \frac{k_{si}}{d} \text{ (micro)}$$

$$\Omega = \frac{(q+N-1)!}{q!\,(N-1)!}$$

$$S \equiv k\ln\Omega$$

$$\frac{1}{T} \equiv \frac{\partial S}{\partial E}$$

$$\Delta S = \frac{Q}{T} \text{ (small } Q\text{)}$$

$$\text{prob}(E) \propto \Omega(E)\, e^{-\frac{E}{kT}}$$

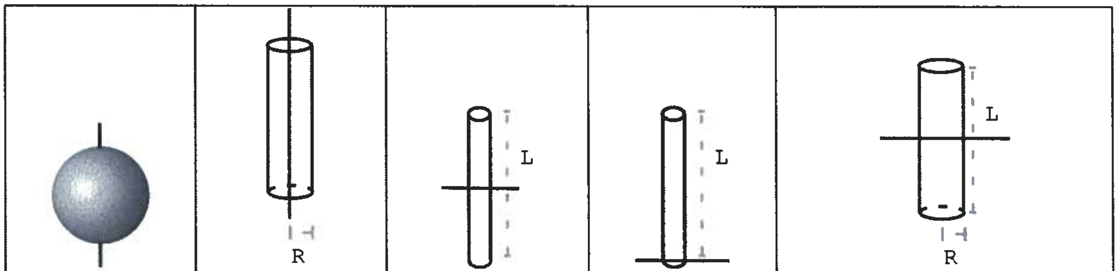
$$E_N = -\frac{13.6\text{eV}}{N^2} \text{ where } N = 1, 2, 3\ldots$$

$$E_N = N\hbar\omega_0 + E_0 \text{ where } N = 0, 1, 2\ldots \text{ and } \omega_0 = \sqrt{\frac{k_{si}}{m_a}} \text{ (Quantized oscillator energy levels)}$$

Moment of intertia for rotation about indicated axis

The cross product

$$\vec{A} \times \vec{B} = \langle A_y B_z - A_z B_y, A_z B_x - A_x B_z, A_x B_y - A_y B_x \rangle$$



$I = \frac{2}{5}MR^2$	$I = \frac{1}{2}MR^2$	$I = \frac{1}{12}ML^2$	$I = \frac{1}{3}ML^2$	$I = \frac{1}{12}ML^2 + \frac{1}{4}MR^2$

Constant	**Symbol**	**Approximate Value**
Speed of light	c	3×10^8 m/s
Gravitational constant	G	6.7×10^{-11} N $\cdot$ m^2/kg^2
Approx. grav field near Earth's surface	g	9.8 N/kg
Electron mass	m_e	9×10^{-31} kg
Proton mass	m_p	1.7×10^{-27} kg
Neutron mass	m_n	1.7×10^{-27} kg
Electric constant	$\frac{1}{4\pi\epsilon_0}$	9×10^9 N $\cdot$ m^2/C^2
Proton charge	e	1.6×10^{-19} C
Electron volt	1 eV	1.6×10^{-19} J
Avogadro's number	N_A	6.02×10^{23} atoms/mol
Plank's constant	h	6.6×10^{-34} joule $\cdot$ second
hbar $= \frac{h}{2\pi}$	$\hbar$	1.05×10^{-34} joule $\cdot$ second
specific heat capacity of water	C	4.2 J/g/K
Boltzmann constant	k	1.38×10^{-23} J/K

milli	m	1×10^{-3}	kilo	K	1×10^3
micro	μ	1×10^{-6}	mega	M	1×10^6
nano	n	1×10^{-9}	giga	G	1×10^9
pico	p	1×10^{-12}	tera	T	1×10^{12}