

A ball of mass $m = 2 \text{ kg}$ is located at $\langle 24, -12, 6 \rangle \text{ m}$ at time $t = 14.9 \text{ s}$.
 At this time, it is traveling with a velocity $\langle -1, 4, 3 \rangle \text{ m/s}$.
 It experiences a net force $\vec{F}_{\text{net}} = \langle 2, 1.6, -1 \rangle \text{ N}$. What is the new position at $t = 15.0 \text{ s}$?

Solution: Update momentum:

$$\begin{aligned}\vec{p}_f &= \vec{p}_i + \vec{F}_{\text{net}} \Delta t = m\vec{v}_i + \vec{F}_{\text{net}}(t_f - t_i) \\ &= (2 \text{ kg})\langle -1, 4, 3 \rangle \text{ m/s} + \langle 2, 1.6, -1 \rangle \text{ N} (15.0 - 14.9) \text{ s} \\ &= \langle -1.8, 8.16, 5.9 \rangle \text{ kg}\cdot\text{m/s}\end{aligned}$$

-0%
(perfect)

Net force constant:
 use mathematical
 definition of \vec{v}_{avg}

$$\begin{aligned}\vec{v}_{\text{avg}} &= \frac{1}{2}(\vec{v}_i + \vec{v}_f) = \frac{1}{2}\left(\vec{v}_i + \frac{\vec{p}_f}{m}\right) \\ &= \frac{1}{2}\left(\langle -1, 4, 3 \rangle \text{ m/s} + \frac{1}{2 \text{ kg}} \langle -1.8, 8.16, 5.9 \rangle \text{ kg}\cdot\text{m/s}\right) \\ &= \langle -0.95, 4.04, 2.975 \rangle \text{ m/s}\end{aligned}$$

Update position:

$$\begin{aligned}\vec{r}_f &= \vec{r}_i + \vec{v}_{\text{avg}} \Delta t = \vec{r}_i + \vec{v}_{\text{avg}}(t_f - t_i) \\ &= \langle 24, -12, 6 \rangle \text{ m} + \langle -0.95, 4.04, 2.975 \rangle \text{ m/s} (15.0 - 14.9) \text{ s} \\ &= \langle 23.905, -11.596, 6.298 \rangle \text{ m}\end{aligned}$$

$$\vec{r}_f = \langle 23.91, -11.60, 6.30 \rangle \text{ m}$$

Analytic with error: Update momentum:

$$\begin{aligned}\vec{p}_f &= \vec{p}_i + \vec{F}_{\text{net}} \Delta t = m\vec{v}_i + \vec{F}_{\text{net}} \Delta t \\ &= 2 \cdot \langle -1, 4, 3 \rangle + \langle 2, 1.6, -1 \rangle (15.0 - 14.8) \\ &= \langle -1.6, 8.32, 5.8 \rangle\end{aligned}$$

-30%

Update position

$$\begin{aligned}\vec{r}_f &= \vec{r}_i + \frac{\vec{p}_f}{m} \Delta t \\ &= \langle 24, -12, 6 \rangle + \frac{\langle -1.6, 8.32, 5.8 \rangle}{2} (15.0 - 14.8) \\ &= \langle 23.84, -11.168, 6.58 \rangle \text{ m}\end{aligned}$$

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 Didn't average velocity

$$\vec{r}_f = \langle 23.84, -11.17, 6.58 \rangle \text{ m}$$

Numeric with error:

$$\begin{aligned}\vec{p}_f &= \langle -1, 4, 3 \rangle + \langle 2, 1.6, -1 \rangle (0.2) \\ &= \langle -0.6, 4.32, 2.8 \rangle\end{aligned}$$

-80%

$$\begin{aligned}\vec{r}_f &= \langle 24, -12, 6 \rangle + \frac{\langle -0.6, 4.32, 2.8 \rangle}{2} 0.2 \\ &= \langle 23.9, -11.5, 6.3 \rangle\end{aligned}$$

$$\vec{r}_f = \langle 23.9, -11.5, 6.3 \rangle$$

No units in final answer
 Errors: Incorrect calculation of Δt (clerical)
 Didn't multiply initial velocity by mass to get momentum (minor)
 Didn't average velocity (minor)