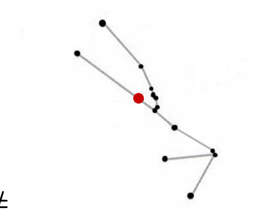
PHYS 2211 Test 3
Fall 2015



Name(print) ~~ Test Key ~~ **Section #** ______

Fenton (C), Gumbart (M), Schatz(N)			
Day	12-3pm	3-6pm	6-9pm
Monday	C02 N02	C01 M01	
Tuesday	M03 N01	M06 C03	C04 N03
Wednesday	C05 N05	M02 N06	
Thursday	M04 C06	M05 N04	

Instructions

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization must be clear.
- You must show all work, including correct vector notation.
- **Correct answers without adequate explanation will be counted wrong.**
- Incorrect work or explanations mixed in with correct work will be counted wrong. Cross out anything you do not want us to grade
- Make explanations correct but brief. You do not need to write a lot of prose.
- Include diagrams!
- **Show what goes into a calculation, not just the final number, e.g.:** $\frac{a \cdot b}{c \cdot d} = \frac{(8\times10^{-3})(5\times10^{6})}{(2\times10^{-5})(4\times10^{4})} = 5 \times 10^4$
- Give standard SI units with your numeric results.

Unless specifically asked to derive a result, you may start from the formulas given on the formula sheet, including equations corresponding to the fundamental concepts. If a formula you need is not given, you must derive it.

If you cannot do some portion of a problem, invent a symbol for the quantity you can not calculate (explain that you are doing this), and use it to do the rest of the problem.

Honor Pledge

"In accordance with the Georgia Tech Honor Code, I have neither given nor received unauthorized aid on this test."

Aldebaran

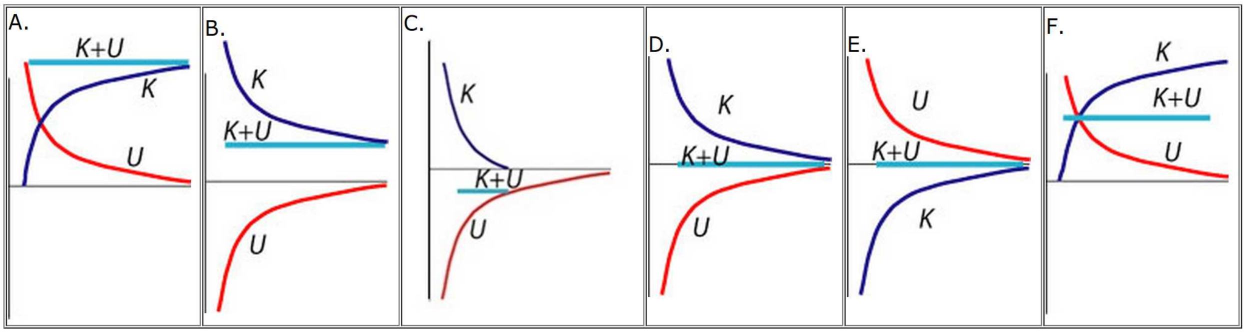
Sign your name on the line above

Please do not write on this page

Problem	Score	Grader
Problem 1 (25 pts)		
Problem 2 (25 pts)		
Problem 3 (25 pts)		
Problem 4 (25 pts)		

Problem 1 (25 Points)

The following diagrams are plots of energy as a function of separation distance r.



All (a 3pts) Which of the diagrams above corresponds to the system of the Sun and Halley's Comet, which orbits the Sun and passes near the Sun once every 76 years?

Circle one: **A** **B** **C** **D** **E** **F**

All (b 4pts) Which of the diagrams above corresponds to a system of a proton and an electron which start out infinitely far apart and are at rest (that is, their initial velocities are zero)?

Circle one: **A** **B** **C** **D** **E** **F**

All (c 4pts) Which of the diagrams above corresponds to a system of a proton and an electron which start out very far apart, with the proton moving toward the electron while the electron moves away from the proton (that is, their initial velocities are nonzero)?

Circle one: **A** **B** **C** **D** **E** **F**

All (d 4pts) An electron in a hydrogen atom has an initial velocity that is just enough for it to escape from the atom. (In other words, the hydrogen atom is ionized.) Which of the diagrams above corresponds to the system described (the hydrogen atom consisting of the ionized proton and electron)?

Circle one: **A** **B** **C** **D** **E** **F**

All (e 4pts) The Voyager 1 spacecraft is the only man-made object in interstellar space; it is very far from the Sun and moving away from the Sun. Which of the diagrams above corresponds to the Sun-Voyager 1 system?

Circle one: **A** **B** **C** **D** **E** **F**

All (f 3pts) Which of the diagrams above corresponds to a system of two electrons which are held at rest at some finite distance apart, moving away from each other after they are released (that is, their initial velocities are zero right after they are released)?

Circle one: **A** **B** **C** **D** **E** **F**

All (g 3pts) Which of the diagram(s) above **do NOT** correctly represent the sum of the kinetic and potential energy? (Choose all that apply)

Circle all that apply: **A** **B** **C** **D** **E** **F**

Problem 2 (25 Points)

A "free" neutron (that is, one outside of a nucleus) is unstable and decays into a proton (p^+), an electron (e^-), and an antineutrino ($\bar{\nu}_e$, uncharged). The mass of the neutron is 1.6749×10^{-27} kg, the mass of the proton is 1.6726×10^{-27} kg, the mass of the electron is 9×10^{-31} kg, and the mass of the antineutrino is much smaller than the electron mass, i.e., it can be assumed to be zero here.

(a 5pts) What is the total kinetic energy of all three products once they are far apart from each other? Assume the neutron is at rest before decaying and that the antineutrino can be ignored.

Initial: neutron alone $\rightarrow K_i = 0,\ U_i = 0,\ E_{rest,i} = m_n c^2$

Final: $p^+ + e^- + \bar{\nu}_e \longrightarrow K_f = ?,\ U_f = 0,\ E_{rest,f} = m_p c^2 + m_e c^2 + \cancel{m_\nu c^2} = (m_p + m_e) c^2$

$$\left.\begin{aligned} \Delta E &= \Delta K + \cancel{\Delta U} + \Delta E_{rest} = 0 \\ K_f - \cancel{K_i} &+ E_{rest,f} - E_{rest,i} = 0 \\ K_f + (m_p &+ m_e) c^2 - m_n c^2 = 0 \end{aligned}\right\} 3\text{ pts}$$

$$K_f + (m_p + m_e - m_n) c^2 = 0$$

$$K_f = (m_n - m_p - m_e) c^2$$

$$K_f = (1.6749e{-27} - 1.6726e{-27} - 9e{-31})(3e8)^2 = \boxed{1.26e{-13}\ \text{J}}$$

1pt 1pt

(b 5pts) Assume the distance between the electron and the proton is 1×10^{-15} m **immediately after the decay**. What is the potential energy? Note that the antineutrino is neutral and can again be ignored.

$$U = \underbrace{\frac{1}{4\pi\epsilon_0} \frac{Q_p Q_e}{d}}_{3\text{pts}} = \frac{(9e9)(1.6e{-19})(-1.6e{-19})}{1e{-15}} = \boxed{-2.304e{-13}\ \text{J}}$$

1pt 1pt

(c 10pts) What is the total kinetic energy **immediately after the decay**? Remember that antineutrino? Don't! Still ignore it here.

Initial: immediately after decay $\rightarrow K_i = ?$, $U_i =$ part b, $E_{rest,i} = (m_p + m_e)c^2$

Final: products far away $\rightarrow K_f =$ part a, $U_f = 0$, $E_{rest,f} = (m_p + m_e)c^2$

$$\Delta E = \Delta K + \Delta U + \cancel{\Delta E_{rest}} = 0$$

$$K_f - K_i + \cancel{U_f} - U_i = 0$$

$$K_f - K_i - U_i = 0$$

$$K_f - U_i = K_i$$

$$K_i = 1.26e\text{-}13 - (-2.304e\text{-}13) =$$

$$= \boxed{3.564e\text{-}13 \text{ J}}$$

-0.5 clerical
-1.5 minor
-3.0 major
-8.0 BTN

* Watch for POE *

(d 5pts) Up until this point, we've ignored the antineutrino, and it feels rather slighted. However, in a very small number of decays (4 in 1 million!) the electron does not escape the proton and becomes bound. As shown in class, the binding potential energy of a hydrogen atom ($p^+ + e^-$) is $U_{\text{electric}} = -2.1790 \times 10^{-18}$ J. For these extremely rare events, what is the kinetic energy of the antineutrino? Assume the resulting hydrogen atom product is stationary, i.e., $K_H = 0$.

Initial: neutron alone $\rightarrow K_i = 0$, $U_i = 0$, $E_{rest,i} = m_n c^2$

Final: $H + \bar{\nu}_e \longrightarrow K_f = \cancel{K_H} + K_\nu = K_\nu$, $U_f = U_{electric}$, $E_{rest,f} = m_H c^2 = (m_p + m_e)c^2$

$$\left.\begin{aligned} &\Delta E = \Delta K + \Delta U + \Delta E_{rest} = 0 \\ &K_f - \cancel{K_i} + U_f - \cancel{U_i} + E_{rest,f} - E_{rest,i} = 0 \\ &K_\nu + U_{electric} + (m_p + m_e)c^2 - m_n c^2 = 0 \\ &K_\nu = -U_{electric} + m_n c^2 - (m_p + m_e)c^2 = \end{aligned}\right\} \text{3pts}$$

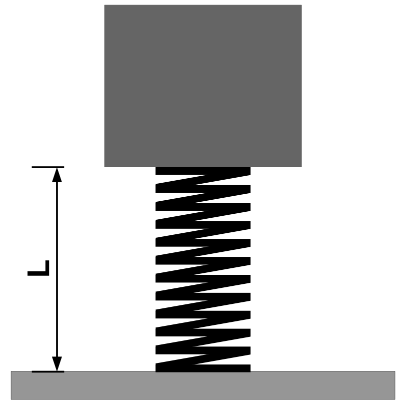
$$= -U_{electric} + (m_n - m_p - m_e)c^2 =$$

$$= -(-2.1790e\text{-}18) + (1.6749e\text{-}27 - 1.6726e\text{-}27 - 9e\text{-}31)(3e8)^2 =$$

$$= \boxed{1.26002e\text{-}13 \text{ J}}$$

1pt 1pt

Problem 3 (25 Points)



(a 5pts) A spring with stiffness k_s and relaxed length L stands vertically on a table. You release the mass M from rest just barely touching the top of the spring. The mass oscillates up and down. Determine the maximum compression of the spring. Hint: use the energy principle.

Initial:
release the mass
✓ $K_i = 0$
✓ $U_{gi} = mgL$
✓ $U_{si} = 0$

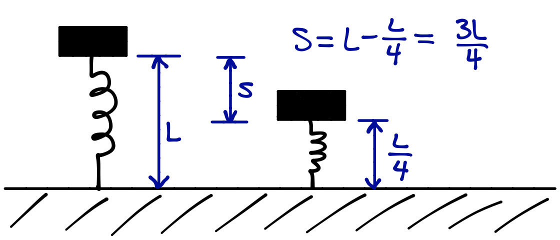
Final:
max compression
✓ $K_f = 0$
✓ $U_{gf} = mg(L - s_{max})$
✓ $U_{sf} = \frac{1}{2} k s_{max}^2$

3 pts
$$\Delta E = \cancel{\Delta K} + \Delta U_g + \Delta U_s = 0$$
$$U_{gf} - U_{gi} + U_{sf} - \cancel{U_{si}} = 0$$
$$mg(L - s_{max}) - mgL + \frac{1}{2} k s_{max}^2 = 0$$
$$mg(\cancel{L} - s_{max} - \cancel{L}) + \frac{1}{2} k s_{max}^2 = 0$$
$$-mg s_{max} + \frac{1}{2} k s_{max}^2 = 0$$
$$\frac{1}{2} k s_{max}^2 = mg s_{max}$$
$$\frac{1}{2} k s_{max} = mg$$
$$\boxed{s_{max} = \frac{2mg}{k}}$$
→ 2pts

(b 10pts) A spring with stiffness k_s and relaxed length L stands vertically on a table. You hold the mass, just barely touching the top of the spring, and very slowly let the mass down onto the spring until the spring has a length $L/4$. You hold the mass motionless at this location. Calculate how much work your hand performed.

Initial:
hold the mass @ top of spring
$K_i = 0$, $U_{gi} = mgL$, $U_{si} = 0$

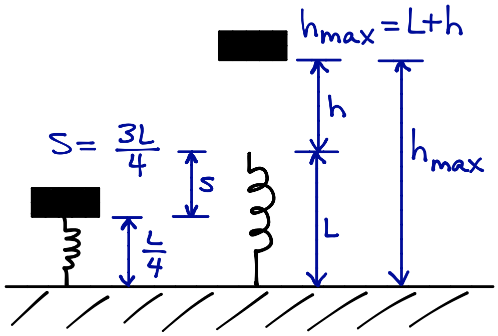
Final:
hold the mass with spring compressed
$K_f = 0$, $U_{gf} = mg(L/4)$, $U_{sf} = \frac{1}{2} k s^2$



$$\Delta E = \cancel{\Delta K} + \Delta U_g + \Delta U_s = W_{hand}$$
$$U_{gf} - U_{gi} + U_{sf} - \cancel{U_{si}} = W_{hand}$$
$$mg(L/4) - mgL + \frac{1}{2} k s^2 = W_{hand}$$
$$mg\left(\frac{L}{4} - L\right) + \frac{1}{2} k \left(\frac{3L}{4}\right)^2 = W_{hand}$$
$$-mg\left(\frac{3L}{4}\right) + \frac{1}{2} k \left(\frac{3L}{4}\right)^2 = W_{hand}$$
$$\boxed{W_{hand} = \frac{3L}{4}\left(\frac{3kL}{8} - mg\right)}$$

−0.5 clerical
−1.5 minor
−3.0 major
−8.0 BTN

(c 10pts) After compressing the spring, so that it's length is $L/4$ (as in part b), you release the mass from rest. The mass starts moving upward, losing contact with the spring (they are not attached to each other) and continues upward before turning around and falling back towards the table. Determine the maximum height reached by the block as measured from the table. When the block is not in contact with the spring, the spring has length L.



Initial: spring compressed

$K_i = 0,\ U_{gi} = mg(L/4),\ U_{si} = \frac{1}{2}ks^2$

Final: flying mass @ max height

$K_f = 0,\ U_{gf} = mgh_{max},\ U_{sf} = 0$

$$\Delta E = \cancel{\Delta K} + \Delta U_g + \Delta U_s = 0$$

$$U_{gf} - U_{gi} + \cancel{U_{sf}} - U_{si} = 0$$

$$mgh_{max} - mg(L/4) - \frac{1}{2}ks^2 = 0$$

$$mg\left(h_{max} - \frac{L}{4}\right) - \frac{1}{2}k\left(\frac{3L}{4}\right)^2 = 0$$

$$mg\left(h_{max} - \frac{L}{4}\right) = \frac{1}{2}k\left(\frac{3L}{4}\right)^2$$

$$h_{max} - \frac{L}{4} = \left(\frac{3L}{4}\right)^2\frac{k}{2mg}$$

$$h_{max} = \frac{L}{4} + \left(\frac{3L}{4}\right)^2\frac{k}{2mg} = \frac{L}{4} + \frac{3kL^2}{32mg} = \boxed{\frac{L}{4}\left(1 + \frac{3kL}{8mg}\right)}$$

-0.5 clerical
-1.5 minor
-3.0 major
-8.0 BTN

Problem 4 (25 Points)

(a 5pts) One Halloween morning, Prof. Fenton places a dessert flan with mass 6.86 kg and temperature 10° C into an oven. When the flan comes out of the oven it is at a temperature of 82° C. Determine the change in thermal energy for the flan. The specific heat C for flan is 4.18 J/(g C).

$$\Delta E_{th} = \overbrace{mC\,\Delta T}^{3\text{ pts}} = \left(\frac{6.86\,\cancel{\text{kg}}}{} \left| \frac{1000\,\cancel{g}}{1\,\cancel{\text{kg}}}\right.\right)\left(4.18\,\frac{\text{J}}{\cancel{g}\,\cancel{°C}}\right)\left(82\,\cancel{°C} - 10\,\cancel{°C}\right) =$$

$$= \boxed{2.065\text{e}6\ \text{J}}$$

1 pt 1 pt

(b 5pts) While in the oven, the macroscopic work W on the flan was zero. Taking the flan as your system, calculate the thermal transfer of energy Q between the system and the surroundings?

$$\left.\begin{aligned} \Delta E &= \cancel{W} + Q \\ \Delta E &= \Delta E_{th} = Q \end{aligned}\right\} 3\text{ pts}$$

$$\boxed{Q = 2.065\text{e}6\ \text{J}}$$

1 pt 1 pt

(c 5pts) When the flan comes out of the oven, at 82° C, it is too hot to eat. Prof. Fenton patiently waits until the internal temperature of the flan has reduced to 37° C to take his first bite. During the long wait, no macroscopic work W was performed. Taking the flan as your system, calculate the thermal transfer of energy Q between the system and the surroundings?

$$3\text{pts}\left\{\begin{aligned} \Delta E &= \cancel{W} + Q \\ \Delta E &= \Delta E_{th} = Q \\ Q &= mC\,\Delta T = (6.86\,\cancel{\text{kg}})(1000\,\cancel{g}/\cancel{\text{kg}})(4.18\,\text{J}/\cancel{g}\,\cancel{°C})(37\,\cancel{°C} - 82\,\cancel{°C}) = \end{aligned}\right.$$

$$= \boxed{-1.29\text{e}6\ \text{J}}$$

1 pt 1 pt

(d 10pts) Prof. Schatz takes a 0.86 kg piece of flan at 37° and places a piece of chocolate on top. The chocolate has a mass of 0.028 kg, an initial temperature of 21° C, and a specific heat of 1.6 J/(g C). Assuming the flan+chocolate are a closed system, calculate the final equilibrium temperature for the system.

1 = flan
2 = chocolate

−0.5 clerical
−1.5 minor
−3.0 major
−8.0 BTN

$$\Delta E = \Delta E_1 + \Delta E_2 = 0$$

$$m_1 C_1 \Delta T_1 + m_2 C_2 \Delta T_2 = 0$$

$$m_1 C_1 (T_f - T_1) + m_2 C_2 (T_f - T_2) = 0$$

$$m_1 C_1 T_f - m_1 C_1 T_1 + m_2 C_2 T_f - m_2 C_2 T_2 = 0$$

$$m_1 C_1 T_f + m_2 C_2 T_f = m_1 C_1 T_1 + m_2 C_2 T_2$$

$$(m_1 C_1 + m_2 C_2) T_f = m_1 C_1 T_1 + m_2 C_2 T_2$$

$$T_f = \frac{m_1 C_1 T_1 + m_2 C_2 T_2}{m_1 C_1 + m_2 C_2}$$

$$T_f = \frac{(0.86\,\cancel{kg})(1000\,\cancel{g}/\cancel{kg})(4.18\,J/\cancel{g}\,\cancel{°C})(37\,°\cancel{C}) + (0.028\,\cancel{kg})(1000\,\cancel{g}/\cancel{kg})(1.6\,J/\cancel{g}\,\cancel{°C})(21\,°\cancel{C})}{(0.86\,\cancel{kg})(1000\,\cancel{g}/\cancel{kg})(4.18\,J/\cancel{g}\,°C) + (0.028\,\cancel{kg})(1000\,\cancel{g}/\cancel{kg})(1.6\,J/\cancel{g}\,°C)} =$$

$$= \boxed{36.8\,°C}$$

This page is for extra work, if needed.

Things you must have memorized

The Momentum Principle Definition of Momentum	The Energy Principle Definition of Velocity	The Angular Momentum Principle Definition of Angular Momentum
Definitions of angular velocity, particle energy, kinetic energy, and work		

Other potentially useful relationships and quantities

$$\gamma \equiv \frac{1}{\sqrt{1-\left(\frac{|\vec{v}|}{c}\right)^2}} \qquad E^2-(pc)^2=\left(mc^2\right)^2$$

$$\frac{d\vec{p}}{dt}=\frac{d|\vec{p}|}{dt}\hat{p}+|\vec{p}|\frac{d\hat{p}}{dt} \qquad \vec{F}_{\parallel}=\frac{d|\vec{p}|}{dt}\hat{p} \text{ and } \vec{F}_{\perp}=|\vec{p}|\frac{d\hat{p}}{dt}=|\vec{p}|\frac{|\vec{v}|}{R}\hat{n}$$

$$\vec{F}_{grav}=-G\frac{m_1m_2}{|\vec{r}|^2}\hat{r} \qquad U_{grav}=-G\frac{m_1m_2}{|\vec{r}|}$$

$$\left|\vec{F}_{grav}\right|\approx mg \text{ near Earth's surface} \qquad \Delta U_{grav}\approx mg\Delta y \text{ near Earth's surface}$$

$$\vec{F}_{elec}=\frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{|\vec{r}|^2}\hat{r} \qquad U_{elec}=\frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{|\vec{r}|}$$

$$\left|\vec{F}_{spring}\right|=k_ss \qquad U_{spring}=\frac{1}{2}k_ss^2$$

$$U_i\approx\frac{1}{2}k_{si}s^2-E_M \qquad \Delta E_{thermal}=mC\Delta T$$

$$\vec{r}_{cm}=\frac{m_1\vec{r}_1+m_2\vec{r}_2+\dots}{m_1+m_2+\dots} \qquad I=m_1r_{1\perp}^2+m_2r_{2\perp}^2+\dots$$

$$K_{tot}=K_{trans}+K_{rel} \qquad K_{rel}=K_{rot}+K_{vib}$$

$$K_{rot}=\frac{L_{rot}^2}{2I} \qquad K_{rot}==\frac{1}{2}I\omega^2$$

$$\vec{L}_A=\vec{L}_{trans,A}+\vec{L}_{rot} \qquad \vec{L}_{rot}=I\vec{\omega}$$

$$\omega=\sqrt{\frac{k_s}{m}} \qquad v=d\sqrt{\frac{k_{si}}{m_a}}$$

$$Y=\frac{F/A}{\Delta L/L}\text{ (macro)} \qquad Y=\frac{k_{si}}{d}\text{ (micro)}$$

$$\Omega=\frac{(q+N-1)!}{q!\,(N-1)!} \qquad S\equiv k\ln\Omega$$

$$\frac{1}{T}\equiv\frac{\partial S}{\partial E} \qquad \Delta S=\frac{Q}{T}\text{ (small } Q\text{)}$$

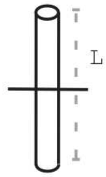
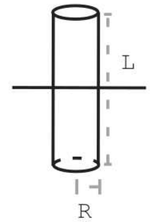
$$\text{prob}(E)\propto\Omega\,(E)\,e^{-\frac{E}{kT}} \qquad E_N=-\frac{13.6\text{eV}}{N^2}\text{ where } N=1,2,3\dots$$

$$E_N=N\hbar\omega_0+E_0 \text{ where } N=0,1,2\dots \text{ and } \omega_0=\sqrt{\frac{k_{si}}{m_a}} \text{ (Quantized oscillator energy levels)}$$

Moment of intertia for rotation about indicated axis

The cross product

$$\vec{A} \times \vec{B} = \langle A_y B_z - A_z B_y, A_z B_x - A_x B_z, A_x B_y - A_y B_x \rangle$$

				
$I = \frac{2}{5}MR^2$	$I = \frac{1}{2}MR^2$	$I = \frac{1}{12}ML^2$	$I = \frac{1}{3}ML^2$	$I = \frac{1}{12}ML^2 + \frac{1}{4}MR^2$

Constant	**Symbol**	**Approximate Value**
Speed of light	c	3×10^8 m/s
Gravitational constant	G	6.7×10^{-11} N $\cdot$ m^2/kg^2
Approx. grav field near Earth's surface	g	9.8 N/kg
Electron mass	m_e	9×10^{-31} kg
Proton mass	m_p	1.7×10^{-27} kg
Neutron mass	m_n	1.7×10^{-27} kg
Electric constant	$\frac{1}{4\pi\epsilon_0}$	9×10^9 N $\cdot$ m^2/C^2
Proton charge	e	1.6×10^{-19} C
Electron volt	1 eV	1.6×10^{-19} J
Avogadro's number	N_A	6.02×10^{23} atoms/mol
Plank's constant	h	6.6×10^{-34} joule $\cdot$ second
hbar $= \frac{h}{2\pi}$	$\hbar$	1.05×10^{-34} joule $\cdot$ second
specific heat capacity of water	C	4.2 J/g/K
Boltzmann constant	k	1.38×10^{-23} J/K

milli	m	1×10^{-3}	kilo	K	1×10^3
micro	μ	1×10^{-6}	mega	M	1×10^6
nano	n	1×10^{-9}	giga	G	1×10^9
pico	p	1×10^{-12}	tera	T	1×10^{12}