PHYS 2211 Exam 1
Spring 2016

Name(print)____ **Section #** __________________

Greco (K, M) and Schatz(N)			
Day	12-3pm	3-6pm	6-9pm
Monday	N07 M07	K02 K01	
Tuesday	M01 N01	M02 N02	M03 N03
Wednesday	K05 K03	K07 K04	M08 K06
Thursday	M04 N04	M05 N05	M06 N06

Instructions

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization must be clear.
- You must show all work, including correct vector notation.
- **Correct answers without adequate explanation will be counted wrong.**
- Incorrect work or explanations mixed in with correct work will be counted wrong. Cross out anything you do not want us to grade
- Make explanations correct but brief. You do not need to write a lot of prose.
- Include diagrams!
- **Show what goes into a calculation, not just the final number, e.g.:** $\frac{a \cdot b}{c \cdot d} = \frac{(8\times10^{-3})(5\times10^{6})}{(2\times10^{-5})(4\times10^{4})} = 5 \times 10^4$
- Give standard SI units with your numeric results.

Unless specifically asked to derive a result, you may start from the formulas given on the formula sheet, including equations corresponding to the fundamental concepts. If a formula you need is not given, you must derive it.

If you cannot do some portion of a problem, invent a symbol for the quantity you can not calculate (explain that you are doing this), and use it to do the rest of the problem.

Honor Pledge

"In accordance with the Georgia Tech Honor Code, I have neither given nor received unauthorized aid on this test."

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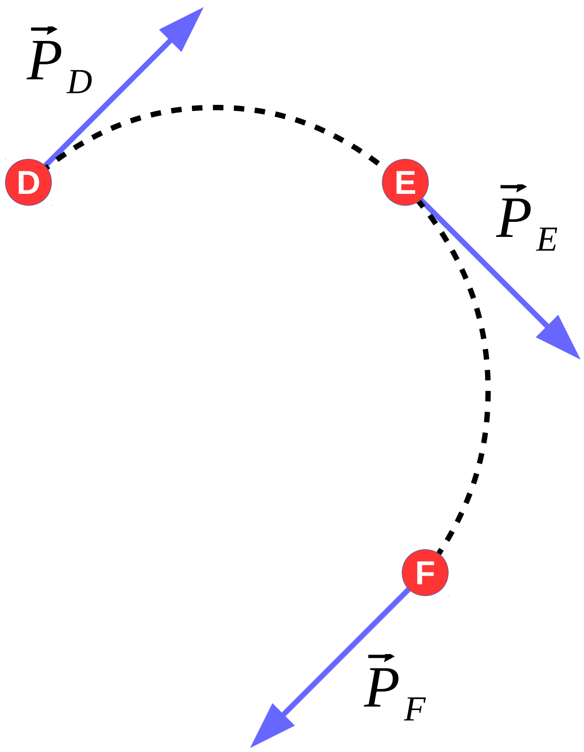
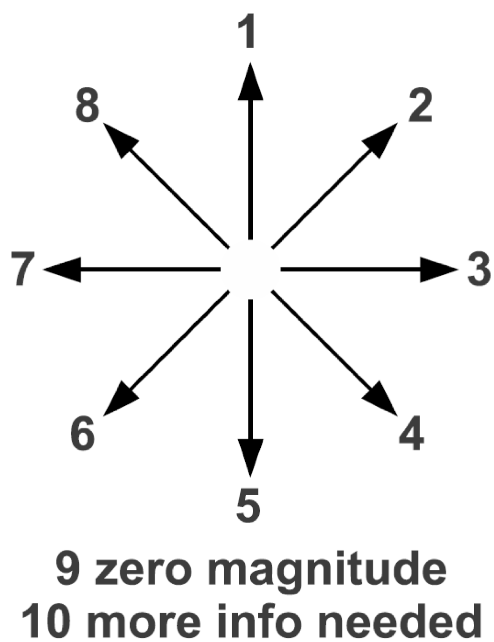
Sign your name on the line above

`Problem 1` Grader: ______________________ Score (25pts): ___________

An object moves from location D to location F on a trajectory (dotted line) in the direction indicated; arrows representing the momentum at locations D, E, and F.





(a 15pts) Using the numbered direction arrows shown, indicate (by number) which direction arrow best represents the direction of the quantities listed below. If the quantity has zero magnitude or cannot be determined, indicate using the corresponding number listed below. $\boxed{\text{1 pt each}}$

- The position vector at location D ___**10**___
- The change in position (the displacement) between location D and location F ___**4**___
- The change in velocity between location D and location F ___**6**___
- The change in momentum between location D and location F ___**6**___
- The average net force between location D and location F ___**6**___

- The position vector at location E **10**

- The change in position (the displacement) between location D and location E **3**

- The change in velocity between location D and location E **5**

- The change in momentum between location D and location E **5**

- The average net force between location D and location E **5**

- The position vector at location F **10**

- The change in position (the displacement) between location E and location F **5**

- The change in velocity between location E and location F **7**

- The change in momentum between location E and location F **7**

- The average net force between location E and location F **7**

(c 10pts) Write "T" next to each true statement below, and write "F" for every false statement. 2 pts each

- The displacement vector for an object can be in a different direction than its average velocity (during the same time interval). **F**

- An object's momentum is always in the same direction as the acceleration on that object. **F**

- The change in an object's momentum can be in a different direction than the net force on the object. **F**

- An object's momentum and its instantaneous velocity are always in the same direction. **T**

- If the net force on an object is constant, then the rate of change of its momentum is constant. **T**

`Problem 2` Grader: ______________________ Score (25pts): __________

Dr. Greco duct tapes a canister of compressed air to a skateboard and releases it from rest. The exhaust of the air results in a constant net force on the skateboard $\vec{F}_{\text{net}} = \langle 10, 0, 0\rangle\,\text{N}$ that last for 10 s. After the canister is empty, the skateboard continues to coast down the hall for and additional 30 s. During this time, the net force on the skateboard is zero. Calculate the total displacement of the skateboard during the complete 40 s interval. The skateboard and canister have a combined, constant, mass of 20 kg.

$\boxed{\{-1.0, -4.0, -7.5, -20\}}$

Solution:

Let us break this problem into two parts. We shall now consider the first part, where there is a non-zero force acting on the skateboard. Using the momentum principle, we see that the change in momentum is related to the force as

$$\Delta\vec{p} = \vec{F}_{\text{net}}\Delta t, \tag{1}$$

which we can rearrange to

$$\vec{p}_{10} = \cancel{\vec{p}_0} + \vec{F}_{\text{net}}\Delta t, \tag{2}$$

where $\vec{p}_i$ is zero and $\Delta t = 10\,\text{s}$. The force is constant, and so we must use the arithmetic average,

$$\vec{v}_{\text{avg},10} = \frac{\vec{v}_f + \vec{v}_i}{2} = \frac{\vec{p}_{10} + \cancel{\vec{p}_0}}{2m} = \frac{\vec{p}_{10}}{2m} \tag{3}$$

Using the iterative prediction method,

$$\Delta\vec{r}_{10} = \vec{v}_{\text{avg},10}\Delta t = \frac{\vec{p}_{10}}{2m}\Delta t \tag{4}$$

Let us now consider the second part of the problem, where the skateboard is coasting with no force acting on it. By the Newton's first law, we know that the skateboard will move at a constant velocity because no force is acting on it. Mathematically, we see this as

$$\vec{p}_{40} = \vec{p}_{10} + \cancel{\vec{F}_{\text{net},2}}\Delta T = \vec{p}_{10}, \tag{5}$$

where $\vec{F}_{\text{net},2} = \vec{0}\,\text{N}$ and $\Delta T = 30\,\text{s}$. We can thus find the final position of the skateboard as

$$\Delta\vec{r}_{40} = \Delta\vec{r}_{10} + \frac{\vec{p}_{10}}{m}\Delta T = \langle 25, 0, 0\rangle + \frac{1}{20}\langle 100, 0, 0\rangle(30) = \langle 175, 0, 0\rangle\,\text{m}. \tag{6}$$

Alternatively, we can calculate the change of position between the 10 s and 40 s,

$$\Delta\vec{r}_{30} = \frac{\vec{p}_{10}}{m}\Delta T = \vec{F}_{\frac{1}{m}}\text{net}\Delta t\Delta T, \tag{7}$$

and add this to the total change in position,

$$\Delta\vec{r}_{40} = \Delta\vec{r}_{10} + \Delta\vec{r}_{30} \tag{8a}$$

$$= \frac{\vec{p}_{10}}{2m}\Delta t + \frac{1}{m}\vec{F}_{\text{net}}\Delta t\Delta T \tag{8b}$$

$$= \frac{1}{2m}\vec{F}_{\text{net}}\Delta t^2 + \frac{1}{m}\vec{F}_{\text{net}}\Delta t\Delta T \tag{8c}$$

$$= \frac{1}{m}\vec{F}_{\text{net}}\Delta t\left(\frac{1}{2}\Delta t + \Delta T\right) \tag{8d}$$

$$= \langle 175, 0, 0\rangle\,\text{m} \tag{8e}$$

`Problem 3` Grader: ______________________ Score (25pts): ____________

A soccer ball of mass 0.30 kg is rolling with velocity $\langle 0, 0, 2.0\rangle\,\mathrm{m\,s^{-1}}$, when you kick it. Your kick delivers an impulse of $\langle 0, 3.4, 0\rangle\,\mathrm{N\,s}$.

(a 5pts) What is the ball's momentum immediately after the kick? Express your answer as a vector with the proper units. $\boxed{\text{All; -1 for cleric}}$

Solution:
Recall that

$$\Delta\vec{p} = \vec{F}\Delta t. \tag{9}$$

Rewriting this and substituting $\vec{p}_i = m\vec{v}$, we find

$$\vec{p} = m\vec{v} + \vec{F}\Delta t = 0.30\,\langle 0, 0, 2.0\rangle + \langle 0, 3.4, 0\rangle = \langle 0, 3.4, 0.6\rangle\,\mathrm{kg\,m\,s^{-1}}. \tag{10}$$

Immediately after the kick, the ball starts from an initial position $\vec{r} = \langle 0, -4, 6\rangle\,\mathrm{m}$ and rolls with a net force (due to the air and the grass) with magnitude of $0.40\,\mathrm{N}$ and pointing in the direction opposite to the ball's momentum. Using a time step of $\Delta t = 0.5\,\mathrm{s}$, calculate step by step (iteratively) the following quantities:

(b 10pts) the position and velocity of the block at $t = 0.5\,\mathrm{s}$. Express your answer as a vector with the proper units. $\boxed{\{-0.5, -1.5, -3.0, -8.0\}}$

Solution:
Let us denote the magnitude of the drag force as $F = 40\,\mathrm{N}$. Then, from the problem statement,

$$\vec{F}_{\text{net}} = -F\hat{p}, \tag{11}$$

where $F_{\text{net}} = 0.40\,\mathrm{N}$ and $\hat{p}$ is the unit vector of the momentum vector calculated above,

$$\hat{p} = \frac{\vec{p}}{p} = \frac{\langle 0, 3.4, 0.6\rangle}{\sqrt{0^2 + 3.4^2 + 0.6^2}} = \langle 0, 0.985, 0.174\rangle\,. \tag{12}$$

Note: Rationalizing the $\vec{p}$ and performing the same operations will yield similar results and may be easier to read; some may have done so. If this is the case, they would instead obtain

$$\hat{p} = \frac{1}{\sqrt{298}}\,\langle 0, 17, 3\rangle\,. \tag{13}$$

We are now in position to calculate the position, using the iterative step method and approximating $\vec{v}_f \approx \frac{\vec{p}_f}{m}$,

$$\vec{p}_f = \vec{p} + \vec{F}_{\text{net}}\Delta t = \langle 0, 3.4, 0.6\rangle - (0.40)\,\langle 0, 0.985, 0.174\rangle\,(0.5) = \langle 0, 3.20, 0.565\rangle\,\mathrm{kg\,m\,s^{-1}}, \tag{14a}$$

$$\vec{v}_f = \frac{\vec{p}_f}{m} = \langle 0, 10.7, 1.88\rangle\,\mathrm{m\,s^{-1}}, \tag{14b}$$

$$\vec{r}_f = \vec{r} + \frac{\vec{p}_f}{m}\Delta t = \vec{r} + \frac{\vec{p}\Delta t}{mp}\,(p + F\Delta t) = \langle 0, -4, 6\rangle + \frac{1}{0.30}\,\langle 0, 3.20, 0.565\rangle\,(0.5) = \langle 0, 1.34, 6.94\rangle\,\mathrm{m} \tag{14c}$$

(c 10pts) the position and velocity of the block at $t = 1.0\,\mathrm{s}$. Express your answer as a vector with the proper units. $\boxed{\{-0.5, -1.5, -3.0, -8.0\}}$

Solution:
The only trick here is to not change $\Delta t = 1.0\,\mathrm{s}$ as it should still be treated as $\Delta t = 0.5\,\mathrm{s}$ and to recalculate the net force, momentum, and finally the position:

$$\vec{F}_{\mathrm{net},2} = -F\hat{p}_f == -F\frac{\vec{p}_f}{p_f} = -\langle 0, 0.394, 0.0695\rangle\,\mathrm{N}, \tag{15a}$$

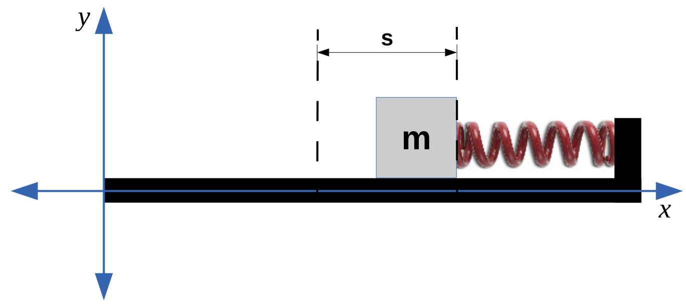
$$\vec{p}_{f,2} = \vec{p}_f + \vec{F}_{\mathrm{net},2}\Delta t = \vec{p}_f + \frac{F\vec{p}_f}{p_f}\Delta t = \vec{p}_f\left(1 + \frac{F}{p_f}\Delta t\right) = \langle 0, 3.01, 0.530\rangle\,\mathrm{kg\,m\,s^{-1}}, \tag{15b}$$

$$\vec{r}_{f,2} = \frac{\vec{p}_{f,2}}{m} = \langle 0, 10.0, 1.77\rangle\,\mathrm{m\,s^{-1}}, \tag{15c}$$

$$\vec{r}_{f,2} = \vec{r}_f + \frac{\vec{p}_{f,2}}{m}\Delta t = \vec{r}_f + \frac{\vec{p}_f\Delta t}{m}\left(1 + \frac{F}{p_f}\right) = \langle 0, 6.35, 7.83\rangle\,\mathrm{m} \tag{15d}$$

`Problem 4` Grader: ____________________ Score (25pts): __________

A spring with relaxed length L_0 is attached to a horizontal (i.e. flat) table in the physics lab as indicated in the diagram. You attach a mass m to the spring and compress it an amount s.



(a 5pts) Determine the direction of the spring force on the mass (your answer should be a vector). $\boxed{\text{All}}$

Solution:
Let us define the initial length of the spring $\vec{L}_i$, which lies along the negative x axis as shown in the figure, as

$$\vec{L} = -(L_0 - s)\,\hat{x}, \qquad |s| < L_0. \tag{16}$$

We note that if $s = L_0$, the spring would be completely compressed ($L_i = 0\,\mathrm{m}$), which is unreasonable for what we are considering. Using Hooke's law, we see that the force of the spring, denoted as $\vec{F}_s$, is given by

$$\vec{F}_s = -k\,(L_i - L_0)\,\hat{L} = -ks\hat{L} \tag{17}$$

where we assume a spring stiffness k_s and the compression s is given by

$$s = L_i - L_0 < 0. \tag{18}$$

We find then that the direction of the spring force is

$$\hat{F}_s = -\hat{x} = \langle -1, 0, 0 \rangle\,. \tag{19}$$

(b 10pts) Determine the velocity of the block a short time Δt after being released from rest.

Solution:
From the problem problem statement, we know the length L of the spring is $L_0 - s$. From Hooke's law, we know that the force is given as

$$\vec{F}_{\text{net}} = -k_s \left(L_i - L_0\right) \left(-\hat{x}\right) = -k_s s\, \hat{x} \tag{20}$$

Because the force lies solely in the x direction, we do not need to consider the force due to gravity. We can now use the momentum principle,

$$\Delta \vec{p} = \vec{p}_f - \cancel{\vec{p}_i} = \vec{F}_{\text{net}} \Delta t = -k_s s \Delta t\, \hat{x} \tag{21}$$

Using the approximation that $\vec{v}_f \approx m^{-1} \vec{p}_f$, we thus obtain the velocity as

$$\vec{v}_f = -\frac{1}{m} k_s s \Delta t\, \hat{x}. \tag{22}$$

(c 10pts) Determine the net force on the block a short time Δt after being released from rest. $\boxed{\{-0.5, -1.5, -3.0, -8.0\}}$

Solution:
This is not the force calculated above. First, we need to calculate the change in position,

$$\Delta \vec{L} = \vec{L}_f - \vec{L}_i = \Delta \vec{v}_{\text{avg}} \Delta t \tag{23a}$$

$$\implies \vec{L}_f = \vec{L}_i + \left(\vec{v}_f - \cancel{\vec{v}_i}\right) \Delta t, \tag{23b}$$

where $\vec{L}_i = -\left(L_0 - s\right)\hat{x}$ and $\vec{v}_f = -\frac{1}{m} k_s s \Delta t$,

$$\therefore \vec{L}_f = \left[-L_0 + s - \frac{1}{m} k_s s \left(\Delta t\right)^2\right] \hat{x}, \tag{23c}$$

which we can now use to calculate the force $\vec{F}$, which by Hooke's law is

$$\vec{F} = -k_s \left(L_f - L_0\right) \left(-\hat{x}\right) \tag{24a}$$

$$= k_s \left[\left(L_0 - |s| - \frac{1}{m} k_s s \Delta t^2\right) - L0\right] \hat{x} \tag{24b}$$

$$= -k_s s \left(-1 + \frac{1}{m} k_s \Delta t^2\right) \hat{x}. \tag{24c}$$

This page is for extra work, if needed.

Things you must have memorized

The Momentum Principle Definition of Momentum	The Energy Principle Definition of Velocity	The Angular Momentum Principle Definition of Angular Momentum
Definitions of angular velocity, particle energy, kinetic energy, and work		

Other potentially useful relationships and quantities

$$\gamma \equiv \frac{1}{\sqrt{1-\left(\frac{|\vec{v}|}{c}\right)^2}} \qquad E^2-(pc)^2=\left(mc^2\right)^2$$

$$\frac{d\vec{p}}{dt}=\frac{d|\vec{p}|}{dt}\hat{p}+|\vec{p}|\frac{d\hat{p}}{dt} \qquad \vec{F}_{\parallel}=\frac{d|\vec{p}|}{dt}\hat{p} \text{ and } \vec{F}_{\perp}=|\vec{p}|\frac{d\hat{p}}{dt}=|\vec{p}|\frac{|\vec{v}|}{R}\hat{n}$$

$$\vec{F}_{grav}=-G\frac{m_1m_2}{|\vec{r}|^2}\hat{r} \qquad U_{grav}=-G\frac{m_1m_2}{|\vec{r}|}$$

$$\left|\vec{F}_{grav}\right|\approx mg \text{ near Earth's surface} \qquad \Delta U_{grav}\approx mg\Delta y \text{ near Earth's surface}$$

$$\vec{F}_{elec}=\frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{|\vec{r}|^2}\hat{r} \qquad U_{elec}=\frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{|\vec{r}|}$$

$$\left|\vec{F}_{spring}\right|=k_s s \qquad U_{spring}=\frac{1}{2}k_s s^2$$

$$U_i\approx\frac{1}{2}k_{si}s^2-E_M \qquad \Delta E_{thermal}=mC\Delta T$$

$$\vec{r}_{cm}=\frac{m_1\vec{r}_1+m_2\vec{r}_2+\dots}{m_1+m_2+\dots} \qquad I=m_1r_{1\perp}^2+m_2r_{2\perp}^2+\dots$$

$$K_{tot}=K_{trans}+K_{rel} \qquad K_{rel}=K_{rot}+K_{vib}$$

$$K_{rot}=\frac{L_{rot}^2}{2I} \qquad K_{rot}==\frac{1}{2}I\omega^2$$

$$\vec{L}_A=\vec{L}_{trans,A}+\vec{L}_{rot} \qquad \vec{L}_{rot}=I\vec{\omega}$$

$$\omega=\sqrt{\frac{k_s}{m}} \qquad v=d\sqrt{\frac{k_{si}}{m_a}}$$

$$Y=\frac{F/A}{\Delta L/L}\ \text{(macro)} \qquad Y=\frac{k_{si}}{d}\ \text{(micro)}$$

$$\Omega=\frac{(q+N-1)!}{q!\,(N-1)!} \qquad S\equiv k\ln\Omega$$

$$\frac{1}{T}\equiv\frac{\partial S}{\partial E} \qquad \Delta S=\frac{Q}{T}\ \text{(small } Q\text{)}$$

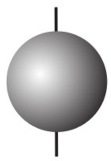
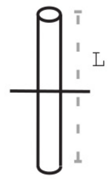
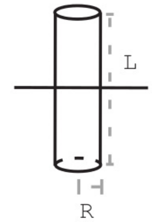
$$\text{prob}(E)\propto\Omega(E)\,e^{-\frac{E}{kT}} \qquad E_N=-\frac{13.6\text{eV}}{N^2} \text{ where } N=1,2,3\dots$$

$$E_N=N\hbar\omega_0+E_0 \text{ where } N=0,1,2\dots \text{ and } \omega_0=\sqrt{\frac{k_{si}}{m_a}} \text{ (Quantized oscillator energy levels)}$$

Moment of intertia for rotation about indicated axis

The cross product

$$\vec{A} \times \vec{B} = \langle A_y B_z - A_z B_y, A_z B_x - A_x B_z, A_x B_y - A_y B_x \rangle$$

				
$I = \frac{2}{5}MR^2$	$I = \frac{1}{2}MR^2$	$I = \frac{1}{12}ML^2$	$I = \frac{1}{3}ML^2$	$I = \frac{1}{12}ML^2 + \frac{1}{4}MR^2$

Constant	**Symbol**	**Approximate Value**
Speed of light	c	3×10^8 m/s
Gravitational constant	G	6.7×10^{-11} N $\cdot$ m^2/kg^2
Approx. grav field near Earth's surface	g	9.8 N/kg
Electron mass	m_e	9×10^{-31} kg
Proton mass	m_p	1.7×10^{-27} kg
Neutron mass	m_n	1.7×10^{-27} kg
Electric constant	$\frac{1}{4\pi\epsilon_0}$	9×10^9 N $\cdot$ m^2/C^2
Proton charge	e	1.6×10^{-19} C
Electron volt	1 eV	1.6×10^{-19} J
Avogadro's number	N_A	6.02×10^{23} atoms/mol
Plank's constant	h	6.6×10^{-34} joule $\cdot$ second
hbar $= \frac{h}{2\pi}$	$\hbar$	1.05×10^{-34} joule $\cdot$ second
specific heat capacity of water	C	4.2 J/g/K
Boltzmann constant	k	1.38×10^{-23} J/K

milli	m	1×10^{-3}	kilo	K	1×10^3
micro	μ	1×10^{-6}	mega	M	1×10^6
nano	n	1×10^{-9}	giga	G	1×10^9
pico	p	1×10^{-12}	tera	T	1×10^{12}