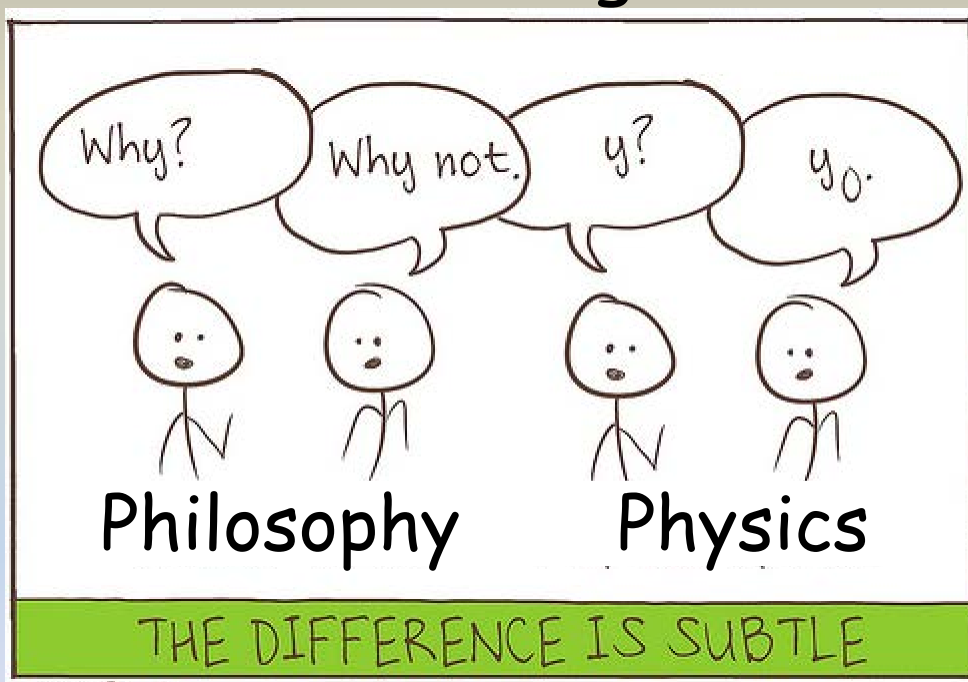
Physics 2211: *Matter and Interactions*

Class III (interactions and momentum)

Todays objective:

Using the Momentum Principle with NON-Constant Forces

Having Fun With Programs!!



- **The momentum principle**

$$\Delta \vec{p} = \vec{F}_{net} \Delta t = \text{Impulse}$$

$$\vec{p}_f = \vec{p}_i + \vec{F}_{net} \Delta t$$

$$X_f = X_o + V_{ox}\, t + F_x t^2/2m$$

$$Y_f = Y_o + V_{oy}\, t + F_y t^2/2m$$

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Howey Room C203
Office Hours: Weds 7-9 am or by appt.

Clicker: *A ball is initially on the ground, and you kick it with initial velocity < 3,7,0> m/s. At this speed air resistance is negligible. Assume the usual coordinate system. Which components of the ball's momentum will change in the* ***next*** *half second?*

(1) P_x

(2) P_y

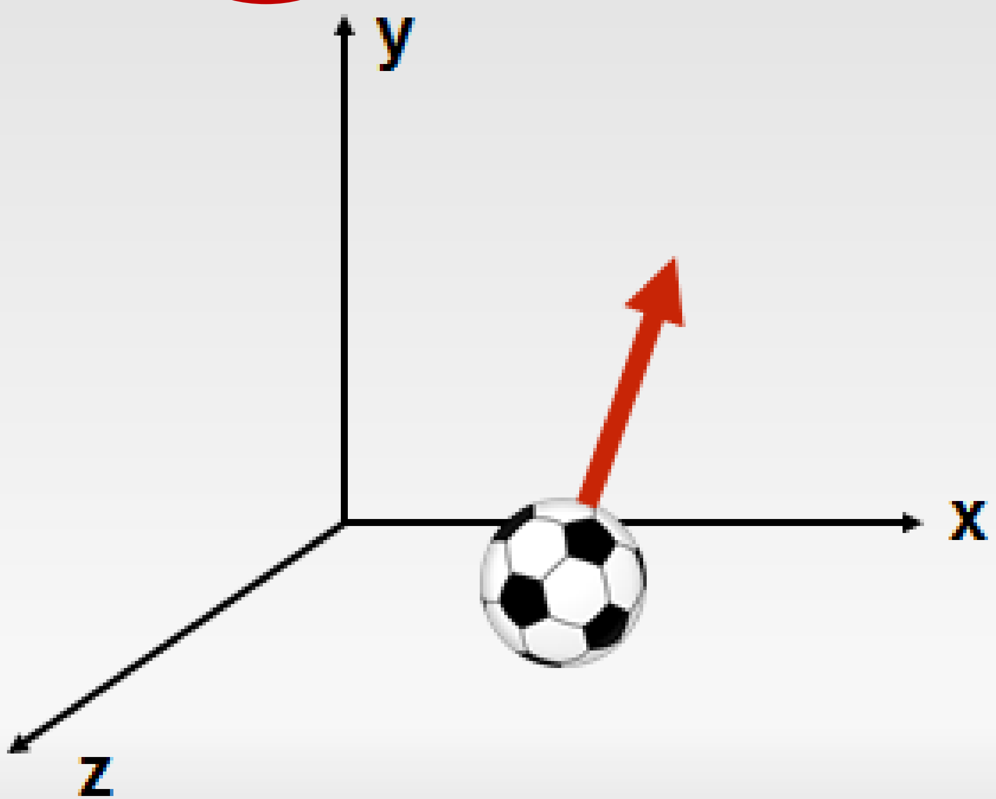
(3) P_z

(4) P_x & P_y

(5) P_y & P_z

(6) P_z & P_x

(7) P_x, P_y, & P_z



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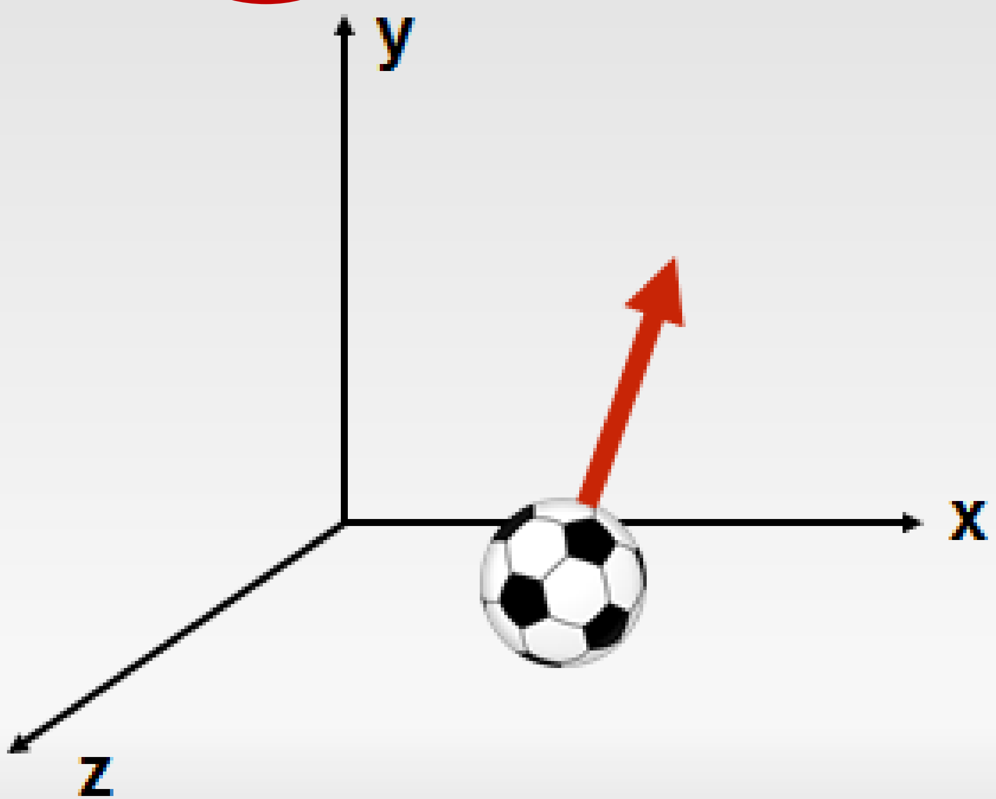
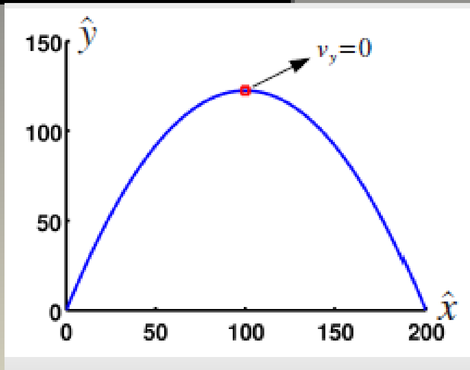
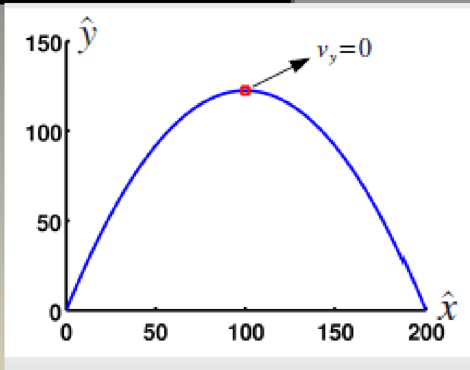


Projectile motion:



Example: A 1 kg ball is kicked from location <9,0,-5>m (on the ground) giving it an initial velocity of <-10,13,-5> m/s. (F_g=mg; g= 9.8m/s^2)

1. What is the velocity of the ball 0.6s after being kicked?
2. What is the location of the ball at that time?
3. At what time the ball will reach it's maximum height?
4. What is the maximum height?
5. At what time the ball will hit the ground?
6. What is the location of the ball when it hits the ground?
7. What is the direction of Delta P at the highest point?



Projectile motion: Constant Force (F=-mg)

Example: A 1 kg ball is kicked from location <9,0,-5>m (on the ground) giving it an initial velocity of <-10,13,-5> m/s. (F_g=mg; g= 9.8m/s^2)

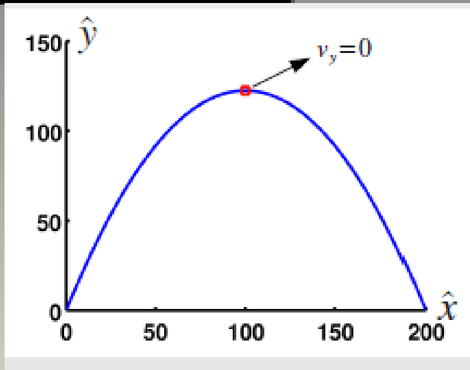
1. What is the velocity of the ball 0.6s after being kicked?

Since γ =1, $\vec{Vf} = \vec{Vi} + \vec{F}\Delta t/m$

$\vec{Vf}$ = <-10,13,-5> m/s+ < 0, -9.8m_{ball}, 0> *0.6Ns/m_{ball}kg

$\vec{Vf}$ = <-10, 7.12, -5>m/s

Projectile motion:



Example: A 1 kg ball is kicked from location <9,0,-5>m (on the ground) giving it an initial velocity of <-10,13,-5> m/s. (F_g=mg; g= 9.8m/s^2)

2. What is the location of the ball at that time?

$$\overline{r_f} = \overline{r_i} + \overline{V_{avg}}\,\Delta t$$

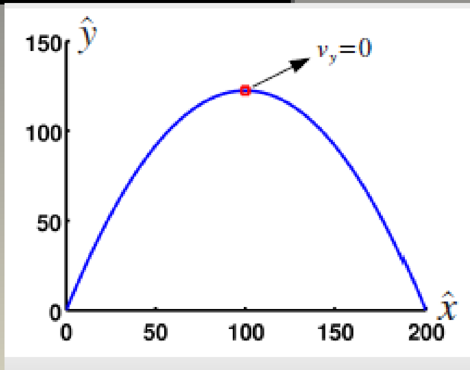
Note, we can use V_{avg} because:

F_{net}= constant ➔ V changes linearly!

So $V_{avg} = (V_i+V_f)/2$

$$\overline{r_f} = \langle 9,0,-5\rangle + (\langle -10,13,-5\rangle + \langle -10, 7.2, -5\rangle)*0.6/2$$

$$\overline{r_f} = \langle 3,6.036,-8\rangle$$



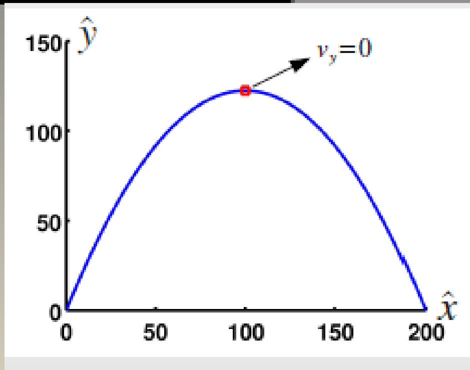
Projectile motion:

Example: A 1 kg ball is kicked from location <9,0,-5>m (on the ground) giving it an initial velocity of <-10,13,-5> m/s. (F_g=mg; g= 9.8m/s^2)

3. At what time the ball will reach it's maximum height?

$\Delta\vec{P} = \vec{F}_{net}\,\Delta t$ at the maximum point V_{fy} =0 (Note $V_{fx} \neq 0$)

$t_f = V_{iy}\, m_{ball}/F_{net\_y}$ = 13 m/s / 9.8m/s^2 = 1.326 s



Projectile motion:

Example: A 1 kg ball is kicked from location <9,0,-5>m (on the ground) giving it an initial velocity of <-10,13,-5> m/s. (F_g=mg; g= 9.8m/s^2)

4. What is the maximum height?

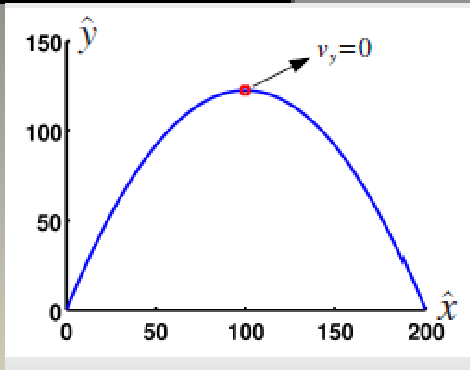
$$r_{fy} = r_{iy} + V_{avg\_y}\Delta t$$

Note, we can use V_{avg} because:
F_{net}= constant ➔ V changes linearly!

$$r_{fy} = 0 + (13+0)\text{m/s}*1.326\text{s}/2 = 8.64\text{m}$$

Also could use: $X_f = X_o + V_{oy}\,t + F_y t^2/2m$

Projectile motion:

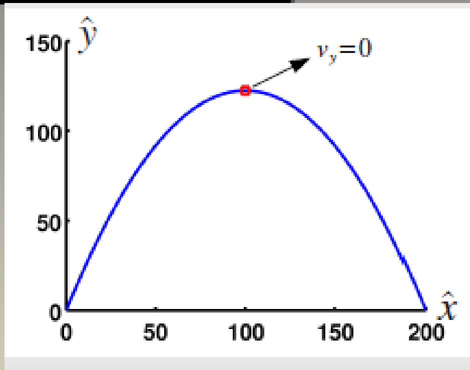


Example: A 1kg ball is kicked from location <9,0,-5>m (on the ground) giving it an initial velocity of <-10,13,-5> m/s. (F_g=mg; g= 9.8m/s^2)

5. At what time the ball will hit the ground?

$t = t_{up}+t_{down}$ (since force is constant, both are the same)

t= 2*1.326s = 2.652s



Projectile motion:

Example: A 1 kg ball is kicked from location <9,0,-5>m (on the ground) giving it an initial velocity of <-10,13,-5> m/s. (F_g=mg; g= 9.8m/s^2)

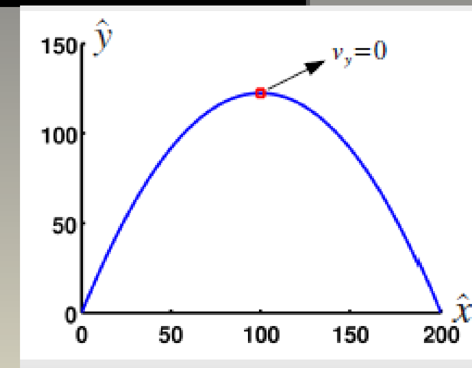
6. What is the location of the ball when it hits the ground?

What is V_f? And so what is V_{avg}?

$$V_{avg} = < -10, 0, -5 >$$

$$r_f = r_0 + Vavg * \Delta t_{total}$$

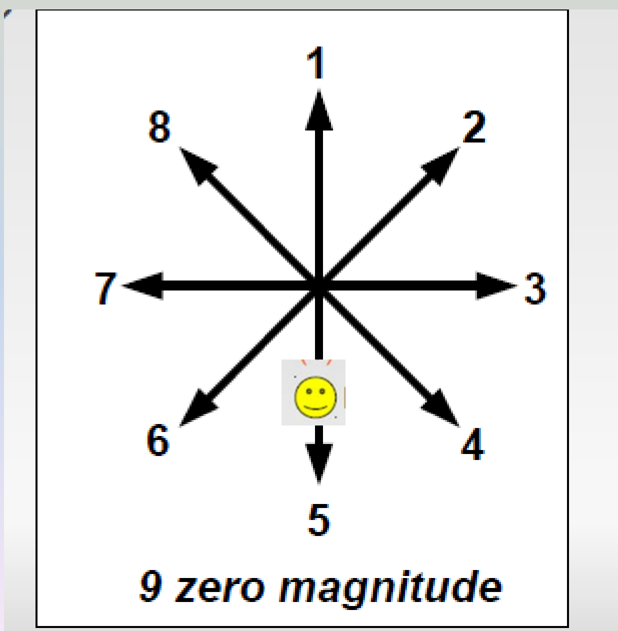
Physics 2211: *Matter and Interactions*

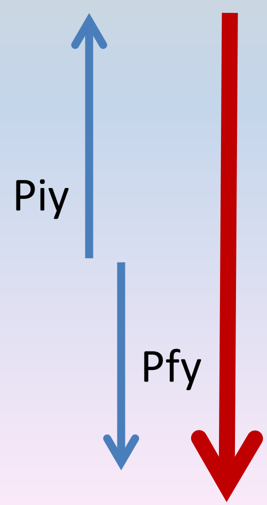


Projectile motion:

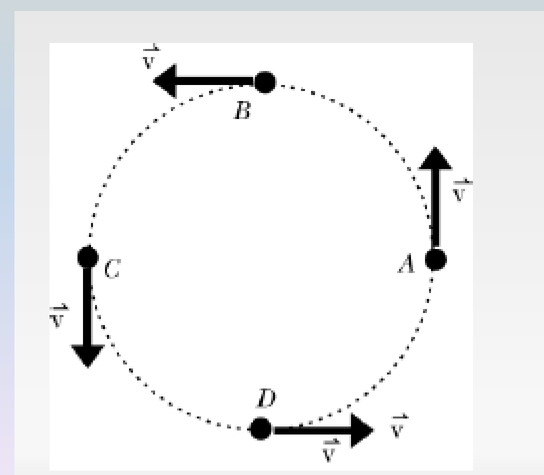
Example: A 1 kg ball is kicked from location <9,0,-5>m (on the ground) giving it an initial velocity of <-10,13,-5> m/s. (F_g=mg; g= 9.8m/s^2)

7. What is the direction of Delta Py at the highest point?

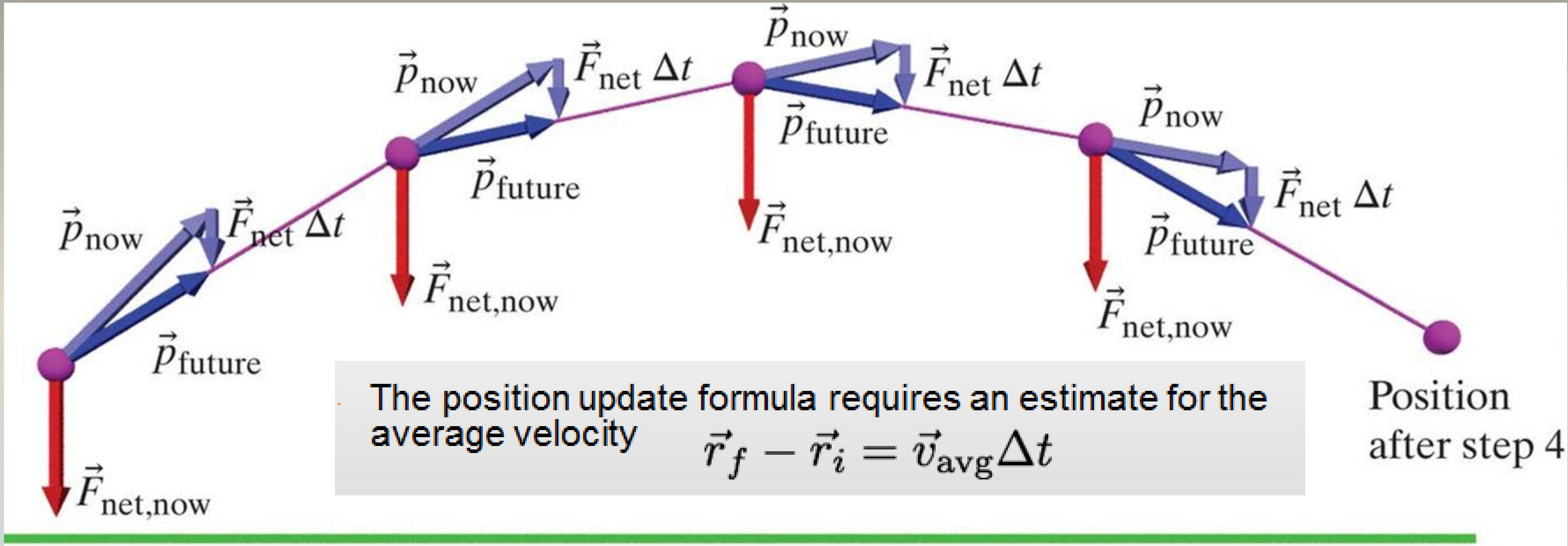




Remember the merry go round example?



projectile motion in a computer



- The position update formula requires an estimate for the average velocity $\vec{r}_f - \vec{r}_i = \vec{v}_{\rm avg}\Delta t$

Iterative solution

- Calculate the net force $\vec{F}_{\rm net,now}$ acting on the system
- Update momentum: $\vec{p}_{\rm future} = \vec{p}_{\rm now} + \vec{F}_{\rm net,now}\Delta t$
- Update position: $\vec{r}_{\rm future} = \vec{r}_{\rm now} + \vec{v}_{\rm avg}\Delta t$

repeat!

Springs vs Gravity

A simple non-constant force

- Hooke's Law (1678): k_s *is the spring stiffness, s is the stretch of the spring, L is the length of spring when stretched or compressed and* L_0 *is the length of the relaxed spring*

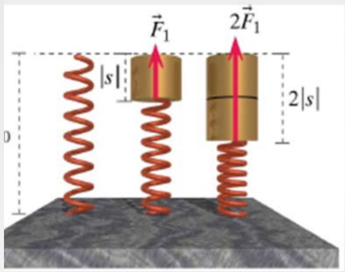
$$\left|\vec{F}_{spring}\right| = k_s |s| \qquad s = L - L_0$$

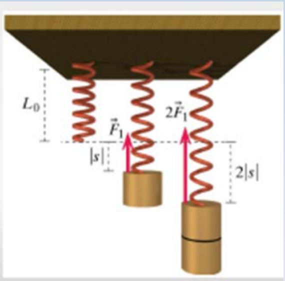
- The force acts in a direction to restore the spring to its relaxed length.

$$\left|\vec{F}_{spring}\right| = k_s |s| \qquad s = L - L_0$$

$$\vec{F}_{spring} = -k_s (L - L_0)\hat{L}$$

$$\vec{F}_{spring} = -k_s (\vec{L} - \vec{L}_0)$$





- **SRS:** *A spring is 12 cm (0.12 m) long when relaxed. Its stiffness is 30 N/m. You push on the spring, compressing it so its length is now 10 cm (0.10 m). What is the magnitude of the force the spring now exerts on your hand?*

1) 0.6 N

(2) 3 N

(3) 3.6 N

(4) 30 N

$$\left|\vec{F}_{spring}\right| = k_s |s| \quad s = L - L_0$$

$$\vec{F}_{spring} = -k_s (L - L_0) \hat{L}$$

$$\vec{F}_{spring} = -k_s (\vec{L} - \vec{L}_0)$$

Physics 2211: *Matter and Interactions*

A 0.02 kg mass oscillates up and down at the end of a spring whose stiffness is 1.5 N/m and whose relaxed length is 0.3 m. At a particular moment the length of the spring is 0.42 m. What is the force exerted on the mass by the spring at this instant?

1) < 0, –0.42, 0 > N
2) < 0, –1.96, 0 > N
3) < 0, –0.18, 0> N
4) < 0, –0.63, 0> N

5) < 0, 1.96, 0 > N
6) < 0, 0.18, 0> N
7) < 0, 0.63, 0> N
8) < 0, 0.42, 0 > N

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4) $\langle 0, -0.63, 0 \rangle$ N
5) $\langle 0, 1.96, 0 \rangle$ N
6) $\langle 0, 0.18, 0 \rangle$ N
7) $\langle 0, 0.63, 0 \rangle$ N
8) $\langle 0, 0.42, 0 \rangle$ N

How do we know if it is moving down or moving up?

Non Constant Forces (Force changes in time)

- **Example: (Step 1)** *Calculate iteratively (three steps), the position of a block attached to a compressed spring after 0.3 seconds. The relaxed length of the spring is 20 cm, the spring stiffness is 8 N/m, the initial length is 10 cm and the mass of the block is 0.06 kg.*

In applying the Momentum Principle iteratively to predict the motion of a block on a spring, what quantities must be recalculated for each time step?

A: the stretch of the spring B: the force by the spring on the block C: the force by the Earth on the block	D: the momentum of the block E: the new position of the block

1) A, B, C, D, and E
2) A, B, D, and E only
3) D and E only
4) A, B, and C only
5) some other combination of quantities

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- Update position: $\vec{r}_{\text{future}} = \vec{r}_{\text{now}} + \vec{v}_{\text{avg}}\Delta t$

repeat!

Non Constant Forces (Force changes in time)

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  - System: Block, Surroundings: Spring + Earth
    Time interval = 0.1 s

$$\vec{p}_f = \vec{p}_i + \vec{F}_{net} \Delta t$$

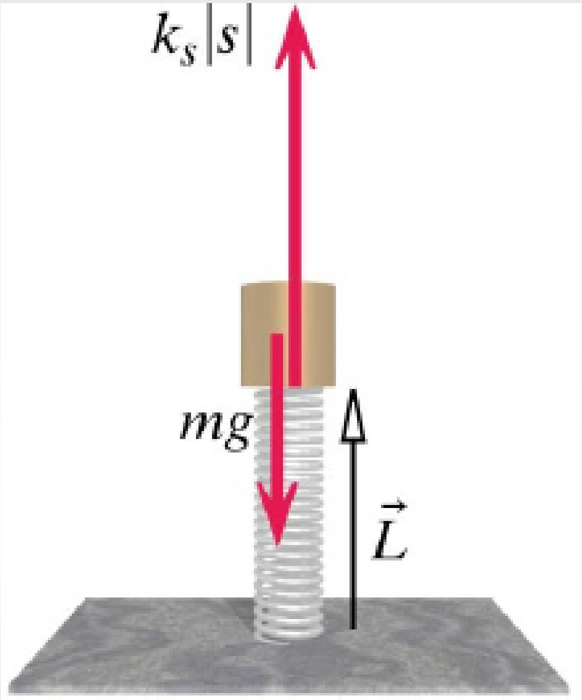
$$\vec{p}_f = (-8\mathrm{N}/m(0.1-0.2) - 0.06*9.8)(0.1\mathrm{s})$$

$$\vec{p}_f = 0.0212\,\hat{y}\; kg \cdot m/s$$

$$\vec{r}_f = \vec{r}_i + \vec{v}_{avg} \Delta t$$

$$\vec{r}_f = 0.1\,\hat{y} + \frac{0.0212}{0.06}(0.1)\,\hat{y} = 0.135\,\hat{y}\; m$$

Note:
We will use V_f for V_{avg}!



Non Constant Forces (Force changes in time)

- **Example: (Step 2)** *Now the final position & momentum from step 1 become the initial position & momentum for step 2.*
  - Update the force and repeat the iteration

$$\vec{F}_{net} = (-8\mathrm{N}/m(0.135-0.2) - 0.06*9.8)\,\hat{y} = -0.0707\,\hat{y}$$

The force changed direction

$$\vec{p}_f = 0.0212\,\hat{y} + (-0.0707)(0.1\mathrm{s})\,\hat{y}$$

$$\vec{p}_f = 0.0141\,\hat{y}\ kg \cdot m/s$$

$$\vec{r}_f = 0.135\,\hat{y} + \frac{0.0141}{0.06}(0.1)\,\hat{y}$$

$$\vec{r}_f = 0.159\,\hat{y}\ m$$

Note:
We used V_f for V_{avg}!

Non Constant Forces (Force changes in time)

- **Example: (Step 3)** *Now the final position & momentum from step 2 become the initial position & momentum for step 3.*
  - Update the force and repeat the iteration

$$\vec{F}_{net} = (-8\text{N}/m(0.159-0.2) - 0.06*9.8)\,\hat{y} = -0.2597\,\hat{y}$$

$$\vec{p}_f = 0.0141\,\hat{y} + (-0.2597)(0.1\text{s})\,\hat{y} = -0.118\,\hat{y}\ kg\cdot m/s$$

The momentum changed direction

$$\vec{r}_f = 0.159\,\hat{y} + \frac{-0.118}{0.06}(0.1)\,\hat{y}$$

$$\vec{r}_f = 0.139\,\hat{y}\ m$$

Note:
We used V_f for V_{avg}!

The mass is moving down now

Non Constant Forces (Force changes in time)

Which of the following would make the biggest improvement in the prediction of the motion of the block?

1) Use more significant figures
2) Use a shorter time step and take more steps
3) Use a larger time step and take fewer steps
4) There is nothing we can do to improve this prediction

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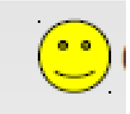
2) Use a shorter time step and take more steps 
3) Use a larger time step and take fewer steps
4) There is nothing we can do to improve this prediction

What values to use for dt? In an iterative integration?