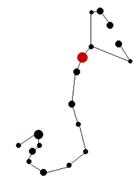
PHYS 2211 Test 2
Fall 2015



Name(print) ~~ Test Key ~~ **Section #** ____________

Fenton (C), Gumbart (M), Schatz(N)			
Day	12-3pm	3-6pm	6-9pm
Monday	C02 N02	C01 M01	
Tuesday	M03 N01	M06 C03	C04 N03
Wednesday	C05 N05	M02 N06	
Thursday	M04 C06	M05 N04	

Instructions

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization must be clear.
- You must show all work, including correct vector notation.
- **Correct answers without adequate explanation will be counted wrong.**
- Incorrect work or explanations mixed in with correct work will be counted wrong. Cross out anything you do not want us to grade
- Make explanations correct but brief. You do not need to write a lot of prose.
- Include diagrams!
- **Show what goes into a calculation, not just the final number, e.g.:** $\frac{a \cdot b}{c \cdot d} = \frac{(8\times10^{-3})(5\times10^{6})}{(2\times10^{-5})(4\times10^{4})} = 5\times10^{4}$
- Give standard SI units with your numeric results.

Unless specifically asked to derive a result, you may start from the formulas given on the formula sheet, including equations corresponding to the fundamental concepts. If a formula you need is not given, you must derive it.

If you cannot do some portion of a problem, invent a symbol for the quantity you can not calculate (explain that you are doing this), and use it to do the rest of the problem.

Honor Pledge

"In accordance with the Georgia Tech Honor Code, I have neither given nor received unauthorized aid on this test."

Antares

Sign your name on the line above

Please do not write on this page

Problem	Score	Grader
Problem 1 (25 pts)		
Problem 2 (25 pts)		
Problem 3 (25 pts)		
Problem 4 (25 pts)		

Problem 1 (25 Points)

Below is an incomplete program to predict the motion of a white particle. The net force on the white particle is exerted by a red particle; the force acts along the line joining the two particles and is attractive. The magnitude of the force is given by $k|\vec{r}|^7$, where k is a positive constant and $\vec{r}$ is the position vector pointing from the red particle to the white particle.

Write the necessary statements to complete the code to predict the white particle's motion. Assume the white particle starts from rest and the red particle is motionless.

```
GlowScript 1.1 VPython

# Objects
redParticle = sphere(pos=vector(5,4,0), radius=0.25, color=color.red)
whiteParticle = sphere(pos=vector(-3,-2,0), radius=0.25, color=color.white)
# Constants
k = 0.3
redParticle.m = 10
whiteParticle.m = 5e-3

#(a 5pts) Add the initializations required for the while loop below
```

```
whiteParticle.p = whiteParticle.m * vector(0,0,0)
redParticle.p = redParticle.m * vector(0,0,0)
```

5pts for momentum of white Particle

```
t = 0
deltat = 5e-6
while t < 1:
    #(b 20pts) update the force, momentum, and position for the white particle
```

Alternate but equivalent methods are OK too.

```
    r = whiteParticle.pos - redParticle.pos
    rmag = mag(r)
    rhat = norm(r)
```

5 pts

```
    Fmag = K * rmag**7
    Fnet = - Fmag * rhat
```

5 pts

5 pts

```
    whiteParticle.p = whiteParticle.p + Fnet * deltat
```

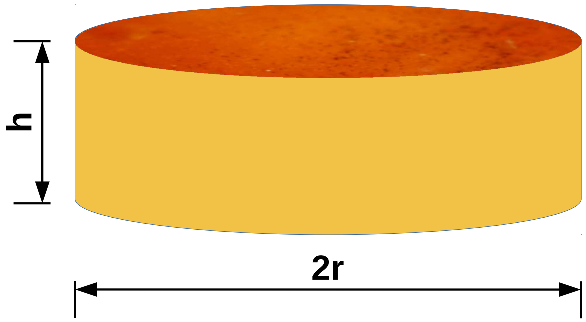
```
    whiteParticle.pos = whiteParticle.pos + (whiteParticle.p/whiteParticle.m) * deltat
```

5 pts

```
    t = t + deltat
```

Problem 2 (25 Points)

You are making flan for Prof. Fenton's birthday and want to add fruit to the top but are worried about crushing and ruining your dessert. The flan is a cylinder with a radius of $r = 0.13$ m, a height of $h = 0.10$ m, and a mass of 6.86 kg.



(a 5pts) Calculate the density of flan.

$$\rho = \frac{\text{Mass}}{\text{volume}} = \frac{6.86\ \text{kg}}{\pi(0.13\text{m})^2(0.10\text{m})} = \boxed{1292\ \text{kg}/\text{m}^3}$$

$$\boxed{\boxed{\text{−1pt for units or sign errors}}}$$

(b 5pts) When you slap the top of the flan, it oscillates up and down with a frequency of 2 Hz (recall $\omega = 2\pi f$). Determine the macroscopic stiffness k_s of the flan. Hint: think of the flan as one big spring with a mass of 6.86 kg attached to the end.

$$\omega = \sqrt{k/m} \Rightarrow \omega^2 = k/m \Rightarrow k = \omega^2 m$$

$$\Rightarrow k = (2\pi f)^2 m = (2\pi)^2(2\ \text{Hz})^2(6.86\ \text{kg}) = 1083\ \text{kg}/\text{s}^2$$

$$\Rightarrow \boxed{k = 1083\ \text{N}/\text{m}}$$

$$\boxed{\boxed{\text{−1pt for units or sign errors}}}$$

(c 5pts) A fruit topping is placed on the flan which compresses by 0.01 m. Calculate the mass of the fruit topping.

$$F_{net} = ks - mg = 0 \Rightarrow ks = mg \Rightarrow m = \frac{ks}{g}$$

$$\Rightarrow m = \frac{(1083\ N/\cancel{m})(0.01\cancel{m})}{9.8\ m/s^2} = 1.11\ N/m/s^2$$

$$\Rightarrow \boxed{m = 1.11\ kg}$$

−1pt for units or sign errors

* Watch for POE *

(d 5pts) Calculate the Young's modulus for flan.

$$Y = \frac{F/A}{\Delta L/L_0} = \frac{F}{A}\frac{L_0}{\Delta L} = \frac{(1.11\ kg)(9.8\ m/s^2)}{\pi(0.13m)^2}\cdot\frac{0.10\cancel{m}}{0.01\cancel{m}} = 2048.86\ \frac{kg\cdot m/s^2}{m^2}$$

$$\Rightarrow \boxed{Y = 2048.86\ N/m^2}$$

−1pt for units or sign errors

* Watch for POE *

(e 5pts) Prof. Schatz takes a slice that is one quarter of the flan (same height but 1/4 the area). How much does his flan compress if he piles half of the fruit, from the whole flan, on top of his single piece?

$$\left.\begin{matrix} A' = A/4 \\ F' = F/2 \end{matrix}\right\} \Rightarrow Y = \frac{F'}{A'}\frac{L_0}{\Delta L'} = \frac{F/2}{A/4}\frac{L_0}{\Delta L'} = \frac{\cancel{4}^{2}F}{\cancel{2}A}\frac{L_0}{\Delta L'} = \frac{2FL_0}{A\,\Delta L'}$$

$$\Rightarrow \frac{AY}{2FL_0} = \frac{1}{\Delta L'} \Rightarrow \Delta L' = \frac{2FL_0}{AY} = \frac{2FL_0}{A}\frac{1}{Y} = \frac{2\cancel{F}\cancel{L_0}}{\cancel{A}}\frac{\cancel{A}\Delta L}{\cancel{F}\cancel{L_0}} = 2\Delta L$$

$$\Rightarrow \Delta L' = 2\Delta L = 2(0.01m) = \boxed{0.02\ m}$$

−1pt for units or sign errors

* Watch for POE *

(extra credit 5pts) Before piling fruit onto his piece of flan, Prof Schatz's slaps the top of his flan and watches it oscillate up and down. Calculate the frequency of this oscillation in Hertz.

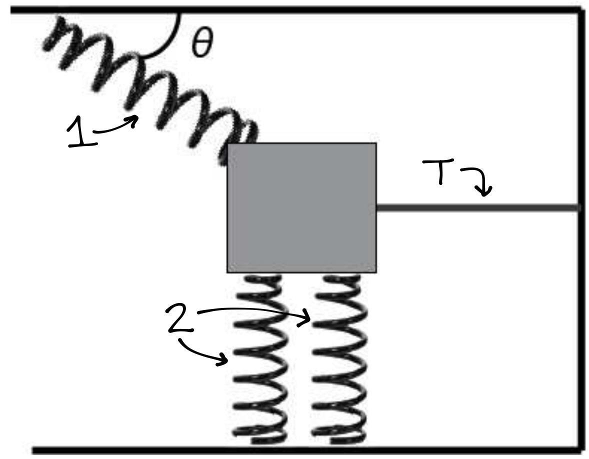
$$\left.\begin{matrix} \checkmark\ m' = m/4 \\ \checkmark\ A' = A/4 \Rightarrow K' = K/4 \\ \text{(b/c springs in parallel)} \end{matrix}\right\} \Rightarrow \omega' = \sqrt{K'/m'} = \sqrt{\frac{K/\cancel{4}}{m/\cancel{4}}} = \sqrt{K/m} = \omega$$

$$\Rightarrow \boxed{\omega' = \omega = 2\ Hz}$$

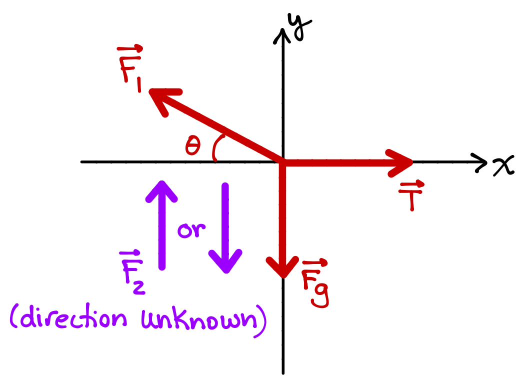
−1pt for units or sign errors

Problem 3 (25 Points)

A block of mass m is suspended motionless by a string and three identical springs as shown in the figure. Each spring has stiffness k and relaxed length L_0. The spring from above forms an angle θ with the horizontal, the string has an unknown tension and the Earth's gravity points down. Recall that the force for an ideal spring is given by $\vec{F} = -k(|\vec{L}| - L_0)\hat{L}$ where $\vec{L}$ points from the attachment point to the block.



(a 10pts) Determine the tension T in the string if the stretched length of the upper spring is $3L_0$. It will help if you start by identifying the forces acting on the system and sketch a corresponding force diagram.



x-components:

$$\vec{F}_{net,x} = \vec{T}_x - \vec{F}_{1x} = 0$$

$$T - F_1 \cos\theta = 0$$

$$T = F_1 \cos\theta$$

$$\Rightarrow T = F_1 \cos\theta = K(L - L_0)\cos\theta = K(3L_0 - L_0)\cos\theta = K(2L_0)\cos\theta$$

$$\Rightarrow \boxed{T = 2KL_0 \cos\theta}$$

−0.5 clerical
−1.5 minor
−3.0 major
−8.0 BTN

All (b 5pts) If you treat the two lower springs as a single spring, what is the effective spring constant k_{eff}?

springs are in parallel, so:

$$K_{eff} = K + K = \boxed{2K}$$

(c 10pts) Calculate their length in the current state ~~and indicate if they are extended or compressed~~. **(Pay careful attention to directions!)**

y-components:

$$\vec{F}_{net,y} = \vec{F}_{1y} + \vec{F}_2 - \vec{F}_g = 0 \Rightarrow \vec{F}_2 = (F_g - F_1 \sin\theta)\,\hat{y} = (mg - 2KL_0 \sin\theta)\,\hat{y}$$

Also: $\vec{F}_2 = -K_{eff}(L - L_0)\,\hat{L} = -K_{eff}(L - L_0)\langle 0,1,0\rangle = \langle 0, -K_{eff}(L-L_0), 0\rangle$

$$\vec{F}_2 = -K_{eff}(L - L_0)\,\hat{y} = -2K(L - L_0)\,\hat{y}$$

−0.5 clerical
−1.5 minor
−3.0 major
−8.0 BTN

* Watch for POE *

Combining:

$$mg - 2KL_0 \sin\theta = -2K(L - L_0)$$

$$mg = -2K(L - L_0) + 2KL_0 \sin\theta =$$

$$= -2KL + 2KL_0 + 2KL_0 \sin\theta = -2KL + 2KL_0(1 + \sin\theta)$$

$$mg - 2KL_0(1 + \sin\theta) = -2KL$$

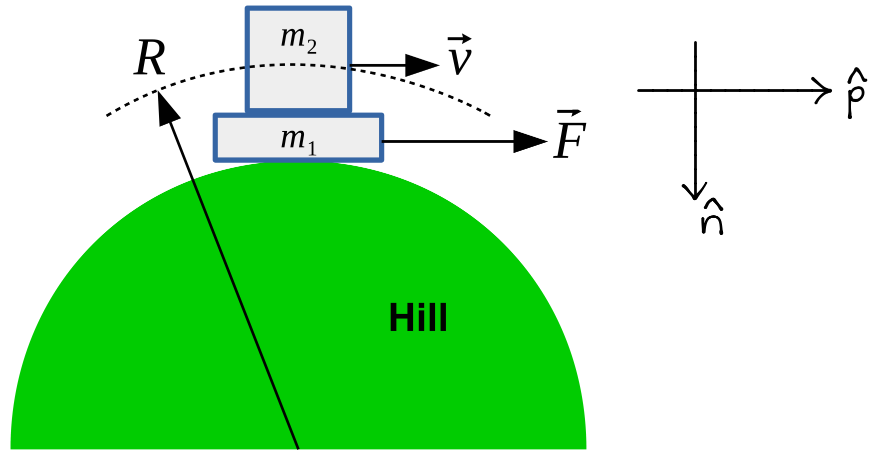
$$2KL = 2KL_0(1 + \sin\theta) - mg$$

$$\Rightarrow L = \frac{2KL_0(1 + \sin\theta) - mg}{2K} =$$

$$= \boxed{L_0(1 + \sin\theta) - \frac{mg}{2K}}$$

Problem 4 (25 Points)

Two blocks of mass m_1 and m_2 are pulled over a hill by an external force $\vec{F}$ as shown in the diagram. The coefficient of friction between the two blocks is μ and the center of the top block moves through a radius of curvature R. At the top of the hill, the top block slips (slides on the bottom block). At that instant, with respect to the hill, the top block is moving to the right with speed v.



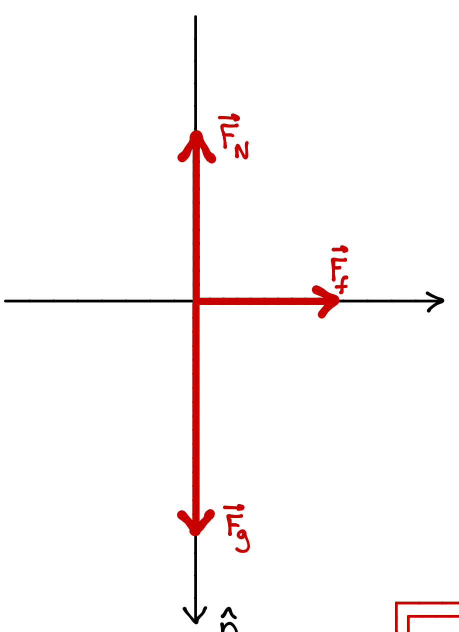
(a 4pts) Determine the magnitude and direction of $(\frac{d\vec{p}}{dt})_\perp$ for the top block at the instant the block is at the top of the hill.

$$\left(\frac{d\vec{p}}{dt}\right)_\perp = \frac{mv^2}{R}\hat{n}$$

magnitude $\Rightarrow \dfrac{m_2 v^2}{R}$ → 2pts

direction $\Rightarrow \hat{n}$ (down) → 2 pts

(b 5pts) Identify the forces acting on the top block and sketch a corresponding force diagram.



✓ $\vec{F}_g$ = gravity (by Earth)

✓ $\vec{F}_N$ = normal force (by bottom block)

✓ $\vec{F}_f$ = friction (by bottom block)

∗ Also allowed: $\vec{F}_c$ = contact force, equivalent to $\vec{F}_N + \vec{F}_f$. If done this way, then diagram should have only $\vec{F}_g$ (pointing down) and $\vec{F}_c$ (pointing up and to the right).

3pts for directions
2pts for magnitudes ($\vec{F}_g$ is biggest)

(c 10pts) Determine the magnitude and direction of the perpendicular component of the force the bottom block exerts on the top block. **Warning** $(\vec{F}_{net})_\perp \neq 0$**)!**

$$\vec{F}_{net\perp} = \vec{F}_{block\perp} + \vec{F}_{g\perp} = \frac{m_2 v^2}{R}\,\hat{n}$$

$$\vec{F}_{block\perp} + m_2 g\,\hat{n} = \frac{m_2 v^2}{R}\,\hat{n}$$

$$\vec{F}_{block\perp} = \left(\frac{m_2 v^2}{R} - m_2 g\right)\hat{n}$$

$$\boxed{\vec{F}_{block\perp} = m_2\left(\frac{v^2}{R} - g\right)\hat{n}}$$

−0.5 clerical
−1.5 minor
−3.0 major
−8.0 BTN

(d 6pts) Determine the magnitude and direction of the parallel component of the force the bottom block exerts on the top block.

$$\vec{F}_{net\parallel} = \vec{F}_{block\parallel} = \vec{F}_f = \mu\left|\vec{F}_N\right|(\hat{p}) = \boxed{\mu m_2\left(\frac{v^2}{R} - g\right)\hat{p}}$$

3 pts for magnitude
3 pts for direction

* Watch for POE *

This page is for extra work, if needed.

Things you must have memorized

The Momentum Principle Definition of Momentum	The Energy Principle Definition of Velocity	The Angular Momentum Principle Definition of Angular Momentum
Definitions of angular velocity, particle energy, kinetic energy, and work		

Other potentially useful relationships and quantities

$$\gamma \equiv \frac{1}{\sqrt{1-\left(\frac{|\vec{v}|}{c}\right)^2}} \qquad E^2-(pc)^2=\left(mc^2\right)^2$$

$$\frac{d\vec{p}}{dt}=\frac{d|\vec{p}|}{dt}\hat{p}+|\vec{p}|\frac{d\hat{p}}{dt} \qquad \vec{F}_{\parallel}=\frac{d|\vec{p}|}{dt}\hat{p} \text{ and } \vec{F}_{\perp}=|\vec{p}|\frac{d\hat{p}}{dt}=|\vec{p}|\frac{|\vec{v}|}{R}\hat{n}$$

$$\vec{F}_{grav}=-G\frac{m_1m_2}{|\vec{r}|^2}\hat{r} \qquad U_{grav}=-G\frac{m_1m_2}{|\vec{r}|}$$

$$\left|\vec{F}_{grav}\right| \approx mg \text{ near Earth's surface} \qquad \Delta U_{grav} \approx mg\Delta y \text{ near Earth's surface}$$

$$\vec{F}_{elec}=\frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{|\vec{r}|^2}\hat{r} \qquad U_{elec}=\frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{|\vec{r}|}$$

$$\left|\vec{F}_{spring}\right|=k_s s \qquad U_{spring}=\frac{1}{2}k_s s^2$$

$$U_i \approx \frac{1}{2}k_{si}s^2-E_M \qquad \Delta E_{thermal}=mC\Delta T$$

$$\vec{r}_{cm}=\frac{m_1\vec{r}_1+m_2\vec{r}_2+\ldots}{m_1+m_2+\ldots} \qquad I=m_1r_{1\perp}^2+m_2r_{2\perp}^2+\ldots$$

$$K_{tot}=K_{trans}+K_{rel} \qquad K_{rel}=K_{rot}+K_{vib}$$

$$K_{rot}=\frac{L_{rot}^2}{2I} \qquad K_{rot}==\frac{1}{2}I\omega^2$$

$$\vec{L}_A=\vec{L}_{trans,A}+\vec{L}_{rot} \qquad \vec{L}_{rot}=I\vec{\omega}$$

$$\omega=\sqrt{\frac{k_s}{m}} \qquad v=d\sqrt{\frac{k_{si}}{m_a}}$$

$$Y=\frac{F/A}{\Delta L/L} \text{ (macro)} \qquad Y=\frac{k_{si}}{d} \text{ (micro)}$$

$$\Omega=\frac{(q+N-1)!}{q!\,(N-1)!} \qquad S \equiv k\ln\Omega$$

$$\frac{1}{T} \equiv \frac{\partial S}{\partial E} \qquad \Delta S=\frac{Q}{T} \text{ (small } Q\text{)}$$

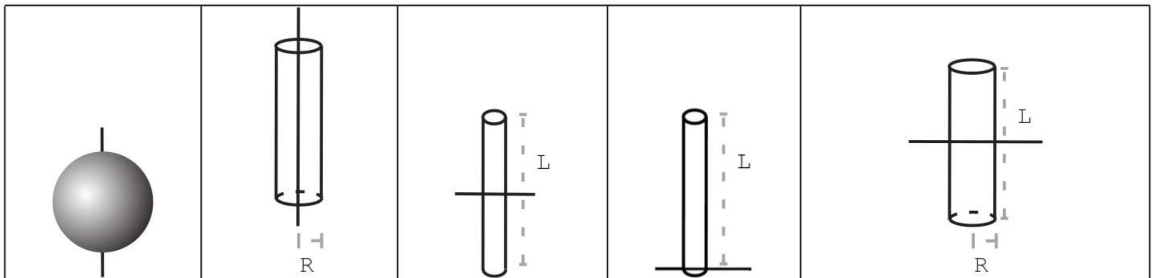
$$\text{prob}(E) \propto \Omega(E)\,e^{-\frac{E}{kT}} \qquad E_N=-\frac{13.6\text{eV}}{N^2} \text{ where } N=1,2,3\ldots$$

$$E_N=N\hbar\omega_0+E_0 \text{ where } N=0,1,2\ldots \text{ and } \omega_0=\sqrt{\frac{k_{si}}{m_a}} \text{ (Quantized oscillator energy levels)}$$

Moment of intertia for rotation about indicated axis

The cross product

$$\vec{A} \times \vec{B} = \langle A_yB_z - A_zB_y, A_zB_x - A_xB_z, A_xB_y - A_yB_x \rangle$$



$I = \frac{2}{5}MR^2$	$I = \frac{1}{2}MR^2$	$I = \frac{1}{12}ML^2$	$I = \frac{1}{3}ML^2$	$I = \frac{1}{12}ML^2 + \frac{1}{4}MR^2$

Constant	**Symbol**	**Approximate Value**
Speed of light	c	3×10^8 m/s
Gravitational constant	G	6.7×10^{-11} N $\cdot$ m^2/kg^2
Approx. grav field near Earth's surface	g	9.8 N/kg
Electron mass	m_e	9×10^{-31} kg
Proton mass	m_p	1.7×10^{-27} kg
Neutron mass	m_n	1.7×10^{-27} kg
Electric constant	$\frac{1}{4\pi\epsilon_0}$	9×10^9 N $\cdot$ m^2/C^2
Proton charge	e	1.6×10^{-19} C
Electron volt	1 eV	1.6×10^{-19} J
Avogadro's number	N_A	6.02×10^{23} atoms/mol
Plank's constant	h	6.6×10^{-34} joule $\cdot$ second
hbar $= \frac{h}{2\pi}$	$\hbar$	1.05×10^{-34} joule $\cdot$ second
specific heat capacity of water	C	4.2 J/g/K
Boltzmann constant	k	1.38×10^{-23} J/K

milli	m	1×10^{-3}	kilo	K	1×10^{3}
micro	μ	1×10^{-6}	mega	M	1×10^{6}
nano	n	1×10^{-9}	giga	G	1×10^{9}
pico	p	1×10^{-12}	tera	T	1×10^{12}