Discrete Mathematics and Algorithms (CSE 611)

Assignment Set 4

Total Marks: 150

Deadline: November 16, 2013 (Saturday), 4:00 pm in office (B3-307)

Questions 1-9 carry each 10 marks and questions 10-13 carry each 15 marks.

1. Let $[A, \cdot]$ be an algebraic system (structure) such that for all a, b, c in A, (i) $a \cdot a = a$ and (ii) $(a \cdot b) \cdot (c \cdot d) = (a \cdot c) \cdot (b \cdot d)$. Show that $a \cdot (b \cdot c) = (a \cdot b) \cdot (a \cdot c)$.

2. Let $[A, \cdot]$ be a semigroup. Furthermore, let there be an element $a \in A$ such that for every $x \in A$ there exist $u, v \in A$ satisfying the relation $a \cdot u = v \cdot a = x$. Show that there is an identity element in A.

3. Show that if z is a left zero of a semigroup S, then so are all its left multiples xz $(x \in S)$.

4. Exhibit a semigroup with left identity and no right identity. Examine whether the structure is a semigroup.

5. Prove carefully and rigorously, in full detail, that every subgroup of a cyclic group is cyclic.

6. Prove that a cyclic group is necessarily abelian. But the converse is not true.

7. Prove that the group $[G, \circ]$ is abelian if and only if $(a \circ b)^{-1} = b^{-1} \circ a^{-1}, \forall a, b \in G$.

8. If H be a subgroup of a group G and $h \in H$, then $hH = Hh = H$.

9. Prove that every monoid $[M, *, e]$ is isomorphic to a submonoid of $[M^M, \circ, i_M]$, where e is the identity element in M and i_M the identity function of M.

10. For a fixed k define $kCOLOR = \{\langle G \rangle |$ the undirected graph G is k-colorable $\}$. Prove that:
    (i) $2COLOR$ is in P.
    (ii) $3COLOR$ is NP-complete, in general, $kCOLOR$ is NP-complete.
    [Note: A coloring of a graph $G = (V, E)$ is a function $f : V \to \{1, 2, \ldots, k\}$ defined for all $i \in V$. If $(u, v) \in E$, then $f(u) \neq f(v)$.]

11. Let DOUBLE-SAT = $\{\langle\phi\rangle|\phi$ has at least two satisfying assignments $\}$. Show that DOUBLE-SAT is NP-complete.

12. A vertex-cover of an undirected graph G is the subset of $V(G)$ where every edge of G touches one of those vertices. Define the following formal problem:
VERTEX-COVER = $\{\langle G, k\rangle|G$ is an undirected graph having a vertex-cover of size k, $k > 0$ $\}$.
Prove that VERTEX-COVER is NP-complete.

13. Let IS-HAM-CYCLE denote the computational problem that, given an undirected graph G, decides whether G contains just those edges necessary to form a Hamiltonian cycle in G (no more, no less). Prove or disprove: IS-HAM-CYCLE is NP-Complete.

Submission Instructions

Copying in assignments leads to award ZERO marks in assignment marks. Also, the source from which you have copied, that source student will be treated under the same rule.

Please submit the assignment in hard copy stating the following at the top:

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submitted by
Name: XYZ, Roll No: abc