Discrete Mathematics and Algorithms (CSE 611)

Assignment Set 3

Total Marks: 150

Deadline: October 04, 2013 (Friday), 4:00 pm after class

Each question carries 10 marks.

1. If $F(x)$ denotes the distribution function of a random variable X, then show that
(i) $P(a < X < b) = F(b-0) - F(a)$, and
(ii) $P(a \le X \le b) = F(b) - F(a-0)$.

2. Show that a function $f(x)$ given by

$$f(x) \quad = \quad \begin{cases} x & 0 < x < 1 \\ k - x & 1 < x < 2 \\ 0 & \text{elsewhere} \end{cases}$$

is a probability density function for a suitable value of the constant k. Calculate the probability that the random variable lies between $\frac{3}{2}$ and $\frac{7}{6}$.

3. The mean of a continuous random variable X is defined by $E(X) = \int_{-\infty}^{\infty} x\, f(x)\, dx$, where $f(x)$ is the probability density function of X. If $\mu = E(X)$, we define the variation of X as $Var(X) = E[(X-\mu)^2] = E(X^2) - \mu^2$. Suppose X follows a gamma distribution with parameter $l (> 0)$. Compute $E(X)$ and $Var(X)$.

4. The probability density function of a random variable X is $A\ sech(x)$, where $sech(x)$ is the hyperbolic sec function and it is given by $sech(x) = \frac{1}{cosh(x)}$ and $cosh(x) = \frac{e^x + e^{-x}}{2}$. Find the value of the constant A. Show that $P(X < 1) = \frac{2}{\pi} \tan^{-1}(e)$ and $P(|X| \ge 1) = 2 - \frac{4}{\pi} \tan^{-1}(e)$.

5. Prove that
(i) $P[(A \cap B)|A] = P(B|A)$.
(ii) $P[(B \cup C)|A] = P(B|A) + P(C|A) - P[(B \cap C)|A]$.

6. Interest for money deposited in a savings account is paid at a rate of 0.5 percent per month, with interest compounded monthly. Suppose $\$50$ is deposited into a savings account each month for a period of five years. What is the total amount in the account four years after the first deposit? Twenty years after the first deposit?

7. Let

$$\begin{aligned} a_r &= \begin{cases} 1, r = 0 \\ 3, r = 1 \\ 2, r = 2 \\ 0, r \geq 3 \end{cases} \\ c_r &= 5^r \text{ for all } r \end{aligned}$$

Given that $c = a * b$, that is c is the convolution of numeric functions a and b. Show that $b_r = \frac{25}{42}5^r - \frac{1}{6}(-1)^r + \frac{4}{7}(-1)^r 2^r, r \geq 0$.

8. Using the generating function, show that solution of the following recurrence relation

$$a_k - 7a_{k-1} + 10a_{k-2} = 3^k$$

with initial conditions $a_0 = 0$ and $a_1 = 1$, is

$$a_k = \frac{8}{3}2^k - \frac{9}{2}3^k + \frac{11}{6}5^r$$

9. Consider an air traffic-control system in which the desired altitude of an aircraft, a_r, is computed by a computer every second and is compared with the actual altitude of the aircraft, b_{r-1}, determined by a tracking radar 1 second earlier. Depending on whether a_r is larger or smaller than b_{r-1}, the altitude of the aircraft will be changed accordingly. Specifically, the change in altitude at the r-th second, $b_r - b_{r-1}$, is proportional to the difference $a_r - b_{r-1}$. That is,

$$b_r - b_{r-1} = K(a_r - b_{r-1})$$

where K is a proportional constant.
(a) Determine b_r, given that $a_r = 1000(\frac{3}{2})^2, K = 3,$ and $b_0 = 0$.
(b) Determine b_r, given that

$$a_r = \begin{cases} 1000(\frac{3}{2})^r, 0 \leq r \leq 9 \\ 1000(\frac{3}{2})^{10}, r \geq 10 \end{cases}$$

$K = 3$, and $b_0 = 0$.

10. Consider the multiplication of bacteria in a controlled environment. Let a_r denote the number of bacteria there are on the r-th day. We define the rate of growth on the r-th day to be $a_r - 5.a_{r-1}$. It is known that the rate of growth at r-th day is three times the growth of the $(r-1)$-th day. Determine a_r, given that $a_0 = 1$.

11. Let a and b be constants, and m be a positive integer. Then prove that the solution of the recurrence relation

$$T(n) = \begin{cases} b, & n \leq 2 \\ mT(n/2) + an^2, & n > 2 \end{cases}$$

is $T(n) = O(n^{\log_2 m})$.

12. If k is a nonnegative constant, then prove that the recurrence

$$T(n) = \begin{cases} k, & n = 1 \\ 3T(n/2) + kn, & n > 1 \end{cases}$$

has the following solution (for n is a power of 2):
$T(n) = 3kn^{log_2 3} - 2kn.$

13. The sets A and B have m and n elements (respectively) from a linear order. These sets are not necessarily sorted. Also assume that $m \leq n$. Show how to compute $A \cup B$ and $A \cap B$ in $O(n \log m)$ time.

14. Consider the Strassen's matrix multiplication using the divide-and-conquer algorithm, when the order n of the matrices is not power of 2. In this case, explain the Strassen's matrix multiplication for $C = AB$, where $A = \begin{pmatrix} 2 & -1 & 6 \\ 0 & 1 & 8 \\ 1 & 2 & -3 \end{pmatrix}$, and $B = \begin{pmatrix} 0 & -1 & 5 \\ 1 & 1 & 3 \\ -1 & 1 & 0 \end{pmatrix}$.

15. Applying the dynamic programming approach, determine an LCS of the sequences $\langle 1, 0, 0, 1, 0, 1, 0, 1 \rangle$ and $\langle 0, 1, 0, 1, 1, 0, 1, 1, 0 \rangle$. Show, in details, the c and b tables.

Submission Instructions

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submitted by

Name: XYZ, Roll No: abc