Astro-286 - Week 6

1. **(proto)Planet-(proto)planet interactions (a true story)**

   (a) In a paper the authors consider (just as we did in class) the following energy equation:

   $$E = -\frac{M_\star M_p}{a} = -\frac{GM_\star M_p}{2r_1} - \frac{GM_\star M_p}{2r_2} - \Delta E \ . \tag{1}$$

   To estimate the energy they considered a sphere of influence r_i and then wrote:

   $$\frac{\Delta E}{E} \sim \frac{M_p^2/r_i}{M_\star M_p/a} \ , \tag{2}$$

   similarly as we did in class. However than they got that

   $$\frac{\Delta E}{E} \sim \left(\frac{M_p}{M_\star}\right)^{1/2} \ . \tag{3}$$

   (we got that $\Delta E/E \sim (M_p/M_\star)^{2/3}$). What did they do?
   Hint 1: Work backward what kind of equality needs to be done to get this relation
   Hint 2: This what was our sphere of influence and what perhaps can be a more simplified expression

   (b) will their estimation results in larger or smaller eccentricities?

2. **Jacobi Intergal**

   (a) Show that the Jacobi integral we found in class can be written as:

   $$\mathcal{H}_J = \frac{1}{2}|\dot{\vec{x}}|^2 + \Phi_{eff} \ , \tag{4}$$

   where

   $$\Phi_{eff} = \Phi - \frac{1}{2}|\vec{\Omega}_p \times \vec{x}|^2 \ , \tag{5}$$

   (b) In a homework problem set students were requested to find the acceleration in the inertial frame $\ddot{\vec{x}}$. One student immediately wrote the following answer

   $$\ddot{\vec{x}} = -\nabla\Phi_{eff} \ . \tag{6}$$

   Is he correct? if yes explain why, if no than what is the correct answer? Explain your answer in details.

3. **Terrestrial planets**

   (a) **Packed up planets** In a homework question the students were asked what is the maximum number of Earth-like planets you can pack up to $1AU$ (while ignoring tides). One student found that you can pack about ~ 1158 Earths between the sun and 1 AU. Another student found that the maximum number is 192. What is the correct answer? What did each student considered?
   Hint: think what are the different approaches to solve this problem

(b) **Out of time** The last stages of terrestrial planet formation are thought to occur in a gas-free environment after the gaseous protoplanetary disk has been dispersed. The usual scenario is that coagulation of solids has led to the build-up of a few fairly massive protoplanets that are widely spaced. Substantial orbit crossing between these protoplanets must arise before they can undergo collisions that lead to the final accumulation of earth-like planets. Estimate how long this final stage of terrestrial planet formation will take at 1 AU.
Hint 1: use your result from the pervious question from the number of planets
Hint 2: remember that surface area can be written as the number of planets over the area of the disk that the planet occupy

(c) Based on that result and because gaseous protoplanetary disks are observed to have life times between $10^6 - 10^7$ yr, is the process involved in producing the cores of gas giant planets must differ from that outlined for the formation of terrestrial planets?

4. **Feeling the heat** I was asked in class what is the typical temperature for the different cases. Assuming small bodies are water molecules and big bodies are at the isolation mass in 1 AU (but of course can be much less than that) which is 0.07 M_E I answered that when $u \sim v_H$ the thermal temperature of the small bodies is about 40 K and if $u \sim v_{esc}$ the thermal temperature is about 9000 K. How did I find this?