[illegible]

1/2

1. In a fully degenerate gas, all the particles have energies lower than the Fermi energy. For such a gas we found (Eq. 4.19) the relation between the density n_e and the Fermi momentum p_f:

$$n_e = \frac{8\pi}{3h^3} p_f^3.$$

a. For a nonrelativistic electron gas, use the relation $p_f = \sqrt{2m_e E_f}$ between the Fermi momentum, the electrom mass m_e, and the Fermi energy E_f, to express E_f in terms of n_e and m_e.
b. Estimate a characteristic n_e under typical conditions inside a white dwarf. Using the result of (a), and assuming a temperature $T = 10^7$ K, evaluate numerically the ratio E_{th}/E_f, where E_{th} is the characteristic thermal energy of an electron in a gas of temperature T, to see that the electrons inside a white dwarf are indeed degenerate.

[illegible]

3. Most of the energy released in the collapse of a massive star to a neutron star (a core-collapse supernova) is in the form of neutrinos.
a. If the just-formed neutron star has a mass $M = 1.4M_\odot$ and a radius $R = 10$ km, estimate the mean nucleon density, in cm^{-3}. Find the mean free path, in cm, of a neutrino inside the neutron star, assuming the density you found and a cross section for scattering of neutrinos on neutrons of $\sigma_{\nu n} = 10^{-42}$ cm^2.
b. How many seconds does it take a typical neutrino to emerge from the neutron star in a random walk?
Hint: neutrinos travel at a velocity close to c. Recall that the radial distance d covered in a random walk of N steps, each of length l, is $d = \sqrt{N}l$.
c. Twelve electron antineutrinos from Supernova 1987A were detected by the Kamiokande neutrino detector in Japan. This experiment consisted of a tank filled with 3 kton of water, and surrounded by photomultiplier tubes. The photomultipliers detect the Cerenkov radiation emitted by a recoiling positron that is emitted after a proton absorbs an antineutrino from the supernova. Estimate how many people on Earth could have perceived a flash of light, due to the Cerenkov radiation produced by the same process, when an antineutrino from the supernova traveled through their eyeball. Assume that eyeballs are composed primarily of water, each weighs about 10 g, and that the Earth's population was 5 billion in 1987.

4. Type-Ia supernovae are probably thermonuclear explosions of accreting white dwarfs that have approached or reached the Chandrasekhar limit.
a. Use the virial theorem to obtain an expression for the mean pressure inside a white dwarf of mass M and radius R.
b. Use the result of (a) to estimate, to an order of magnitude, the speed of sound, $v_s = \sqrt{dP/d\rho} \sim \sqrt{P/\rho}$, inside a white dwarf. In an accreting white dwarf with a carbon core that has reached nuclear ignition temperature, a nuclear burning "flame" encompasses the star at the sound velocity or faster. Within how much time, in seconds, does the flame traverse the radius of the white dwarf, assuming $R = 10^4$ km, $M = 1.4M_\odot$? Note that this **sound-crossing timescale** is $\sim (G\rho)^{-1/2}$, which is also the free-fall timescale (Eq. 3.15.)
c. Calculate the total energy output, in ergs, of the explosion, assuming the entire mass of the white dwarf is synthesized from carbon to nickel, with a mass-to-energy conversion efficiency of 0.1%. Compare this energy to the gravitational binding energy of the white dwarf, to demonstrate that the white dwarf explodes completely, without leaving any remnant.
d. Gamma rays from the radioactive decays $^{56}\mathrm{Ni} \rightarrow ^{56}\mathrm{Co} + \gamma \rightarrow ^{56}\mathrm{Fe} + \gamma$ drive most of the optical luminosity of the supernova. The atomic weights of ^{56}Ni and ^{56}Fe are 55.942135 and 55.934941, respectively. Calculate the total energy radiated in the optical range during the event. Given that the characteristic times for the two radioactive decay processes are 8.8 days and 111 days, respectively, show that the typical luminosity is $\sim 10^{10} L_\odot$.

5. קבלי (כולל השלבים שלא נעשו בכיתה) את היחס רדיוס מסה עבור ננס לבן במקרה הלא יחסותי,
הראי ש $P_c = 0.77 \frac{GM^2}{R^4}$, $\rho_c = 1.43 \frac{M}{R^3}$, ובמקרה האולטרה
יחסותי את מסת צ'נדרסקר הראי ש $P_c = 11.0 \cdot \frac{GM^2}{R^4}$, $\rho_c = 12.94 \frac{M}{R^3}$.