Astro-286 - Week 2

1. **Spherical collapse** (30pt, 10pt each)

   (a) In class I said that spherical collapse conserves angular momentum. Show that this is true.

   (b) A typically observed radius of a collapsing and rotating cloud of a mass 1 $M_\odot$ is $R_c \sim 0.1$ pc with an angular velocity of 10^{-14} rad sec^{-1}. What is the associated disk radius?

   (c) Student Y missed the class and when trying to solve the above question he did the following: He derived the angular momentum of the cloud and found (like we did in class) that $J_c = \delta m R_c^2 \Omega$, he then said that since $\Omega^2 = GM/R_c^3$, so the specific angular momentum of the cloud is $j_c = J_c/\delta m = \sqrt{GMR_c}$. Similarly he found that the disk's specific angular momentum is $j_D = J/\delta m = \sqrt{GMR_D}$. From conservation of angular momentum $j_c = j_D$ he found that $R_D = R_c$ and his answer to the above question was 0.1 pc. What was his mistake?

2. **Disks** (70pt, 14pt each)

   (a) Show that $\Sigma = \rho_0 \sqrt{2\pi} h$ (*hint: remember what is the connection between* Σ *and the vertical density distribution*).

   (b) In class we found that for a Keplerian motion

   $$\frac{\partial \Sigma}{\partial t} = \frac{3}{R}\frac{\partial}{\partial R}\left(\sqrt{R}\frac{\partial}{\partial R}\left[\nu\Sigma\sqrt{R}\right]\right) \ , \tag{1}$$

   Find the expression for v_R.

   (c) **Steady state:** In class we saw that the equation for the conservation of angular momentum is:

   $$\frac{\partial(\Sigma R^3 \Omega)}{\partial t} = -\frac{\partial}{\partial R}\left(R v_R \Sigma R^2 \Omega\right) + \frac{1}{2\pi}\frac{\partial G_T}{\partial R} \ . \tag{2}$$

   Assuming a steady state solution find the analytical expression the disk surface density, for a Keplerian disk, as a function of R in terms of the mass accretion rate $\dot{M}$, the viscosity coefficient ν and the radius of the star $R_\star$.
   Hint 1: Note that the accretion rate is defined as $\dot{M} = -2\pi R \Sigma v_R$.
   Hint 2: Note that where $d\Omega/dR = 0$ *the viscous stress vanishes. In a good approximation this can be set as a condition at the surface of the star.*

   (d) **Disk surface temperature:** The transport of energy, associated with viscous torque through in annulus is simply $G_T d\Omega/dR$, where G_T is the torque we found in class. On one hand the dissipation rate per unit surface area of the disk, $D(R)$ is simply the viscous torque through in annulus over the two sided circumference (remember that the disk has two sides - what does it mean???). On the other hand for black body emission $D(R) = \sigma T_{disk}^4$. Assume a Keplerian disk, and use the result for Σ you obtain to find the temperature profile of the disk as a function of R in term of $\sigma, M_\star, R_\star, G$ and $\dot{M}$.

(e) **Disk central temperature:** The vertical energy flux $F(z)$ for an optically thick disk is given by the equation of radiative diffusion, i.e.,

$$F(z) = -\frac{16\sigma T^3}{3\kappa\rho}\frac{dT}{dz} \ . \tag{3}$$

Assume that all the energy dissipation happens at $z = 0$, and in that case $F(z) = \sigma T_{disk}^4$, which doesn't depend on z. Find what is the temperature at the central of the disk T_c. Is the central temperature smaller or larger than T_{disk}?