Introduction to Astrophysics 0321.3108
Problems Set

1. The virial theorem states that for a star in hydrostatic equilibrium $< P >= -\frac{1}{3}\frac{E_{gr}}{V}$, where $< P >$ is the volume average pressure, V is the volume, and E_{gr} is the gravitational potential energy.

   a. For a star supported by thermal gas pressure $P = nk_BT$, what is the relationship between the internal energy E_K and E_{gr}.

   b. Assume the Sun is supported by thermal gas pressure and is composed entirely of ionized hydrogen. Estimate the typical (virial) temperature $< T >$ in the Sun.

   c. For a star dominated by radiation pressure, what is the relationship between E_k and E_{gr}? What is the total E_{tot} in this case?

   d. Find an expression for the mass of a star at which radiation pressure begins to dominate. Don't worry about numerical factors, express your answer in terms of the physical constants c, h, G and m_H.

2. **Kelvin-Helmholtz time** Derive an expression for the Kelvin Helmholtz time for which the Sun could be shining at its present luminosity due to the release of gravitational energy. Calculate this time in years.

3. Imagine the Sun were to lose all pressure support. Estimate how long it would take to collapse gravitationally?

4. For single-line spectroscopic binary star, the maximal radial velocity v_{1obs} for the observed primary star is given by the expression:

$$(m_1 + m_2)\sin^3 i = \frac{P|v_{1obs}|^3}{2\pi G}\left(1 + \frac{m_1}{m_2}\right)^3, \qquad (1)$$

   where P is the binary period, m_1 is the primary mass, m_2 is the secondary mass and i is the inclination angle. Use this expression to estimate the "wobble velocity" v_{1ons} induced on a $1M_\odot$ star which is orbited by a Jupiter mass $(10^{-3}M_\odot)$ star at distance of 1 AU. What is the size of the Doppler shift $\Delta\lambda/\lambda$ expected for this wobble velocity?

5. The luminosity of the Sun can be written as:

$$L = \frac{U}{t_d}, \qquad (2)$$

   where U is the total energy in radiation in the star, and t_d is the typical time it takes a photon to diffuse in a random-walk from the center of the surface.

   a. Estimate the mean-free path of the photons between scattering in the Sun.

b. Estimate t_d for the Sun, in years.

c. Is t_d larger or smaller than the Kelvin-Helmholtz time t_{kH}? Explain.

6. Assume scattering of photons is via Thomson scattering with free electrons. Estimate the relation between stellar mass and luminosity in this case.

7. Write down the nuclear reaction in the p-p chain.

8. Estimate what fraction of the proton rest mass energy is released per fusion event.

9. Estimate how long the Sun can shine at its present luminosity via p-p reactions.

10. The nuclear reaction rate between two species A and B is:

$$P_{AB} = 6.5 \times 10^{-18} \frac{n_A n_B}{A_r Z_A Z_B} S(E_0) \left(\frac{E_G}{4k_B T}\right)^{2/3} \exp\left\{ -3\left(\frac{E_G}{4k_B T}\right)^{1/3} \right\} \tag{3}$$

where $E_G = (\pi \alpha Z_A Z_B)^2 2\mu c^2$, α is the fine structure constant, A_r is the reduced mass in units of the proton mass, and $\mu = A_r m_p$. Assume the p-p reaction occurs near a core temperature 10^7 K. What is the sensitivity of the stellar luminosity to the core temperature (express as $L \propto T^a$, and find a)?

11. Recall that the Fermi momentum for an electron gas with density n_e is define by the equation:

$$n_e = \frac{8\pi}{3} \frac{P_f^3}{h^3}. \tag{4}$$

a. Derive an expression for the degeneracy pressure for non-relativistic electrons given by n_e.

b. Derive the mass-radius relation for white dwarfs supported by non-relativistic degeneracy pressure. Don't worry about numerical factors, express your answer as a relation between R,M and the constant h,G,m_e and m_p.

12. Derive an expression for the degeneracy pressure for relativistic electrons.

13. Estimate the Chandrasekhar Mass for the maximum mass of white-dwarfs, assuming that electrons become relativistic as $R \to 0$. Again, ignore numerical factors, gust keep physical constants h, c, G, m_p.

14. Estimate the radius of a neutron star supported by degenerate non-relativistic neutrons. How much gravitational energy is released in the formation of a neutron star? If this occurs on a free-fall time what is the luminosity released?

15. The Friedmann equations are:

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{R^2} \tag{5}$$

$$\frac{\ddot{R}}{R} = \frac{8\pi G}{3}\left(\rho + \frac{3P}{c^2}\right) \tag{6}$$

$$0 = \dot{\rho} + 3\frac{\dot{R}}{R}\left(\rho + \frac{P}{c^2}\right) \tag{7}$$

(i) A galaxy is observed in which the hydrogen $Ly\alpha$ line appears at 6000 Angstrom. By what factor has the universe expanded since the light was emitted by the galaxy?

(ii) For "radiation dominated" universe where $P = \frac{1}{3}\rho_r c^2$, how does the scale factor vary with time?

(iii) Assume a flat matter-dominated universe.

a. How does the scale factor vary with time?

b. If the present day Hubble constant equals 72 km s^{-1} Mpc^{-1}, what is the age of the Universe?

c. A galaxy is observed with redshift $z = 2$. How old was the Universe when the light was emitted?