Introduction to Astrophysics 0321.3108
Exercise 6

1. Measurements of the radial recession velocities of five galaxies in a cluster give velocities of 9700, 8600, 8200, 8500, and 10,000 km s^{-1}.

   a. What is the distance to the cluster if the Hubble parameter is $H_0 = 72$ km s^{-1} Mpc^{-1}? *Hint: Use the Hubble law for the average velocity of the members in the cluster.*

   b. Estimate, to an order of magnitude, the mass of the cluster if every galaxy is projected roughly half a degree from the cluster center. *Hint: Use the virial theorem*

2. **The age of the Universe** For a Hubble constant of $H_0 = 72$ km s^{-1} Mpc^{-1} , calculate the Hubble time $t_H = H_0^{-1}$.

3. **The critical density**

   a. Consider the Friedmann equation: $\left(\frac{\dot{R}}{R}\right)^2 = H^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{R^2}$. What is the critical density ρ_c that gives a marginally bound Universe. Assume: $H_0 = 72$ km s^{-1} Mpc^{-1} and flat Universe.

   b. Estimate the stellar mass density $\rho_\star$. Assume that the density of galaxies is 2×10^{-2} Mpc^{-3} and that in each galaxy there are 5×10^{10} stars, and that each star has an average mass of $0.5M_\odot$. What is the ratio $\frac{\rho_\star}{\rho_c}$.

4. The proper distance to a source is rR_0 where r is the comoving distance and R is the scale factor.

   a. Use the relation between redshift and the scale factor, i.e., $1 + z = \frac{R_0}{R(t)}$ and show that $\frac{1}{R(t)} = \frac{1}{R_0} - \frac{1}{R_0}H_0(t - t_0)$. *Hint: Use Taylor series about the point $t = t_0$ (the age of the Universe today) to the first order*

   b. Consider the geodesic expression: $cdt = R(t)\frac{dr}{\sqrt{1-kr^2}}$, and assume a flat Universe. Calculate to the first order the physical distance today, rR_0 in terms of z and Hubble time t_0. *Hint: Use the above approximation, and approximate to the first order.*