Astro-286 - Week 7

1. **Close Encounters of the Liquid Kind** (40pt)
During an international conference about Jupiter and Saturn a spaceship had landed and two aliens disembark and demanded to file an official complaint because their home planet Saturn is called on Earth "gas giant" but it should be called "Liquid giant". Are they correct?

   Hint: To help their argument (and to help you solve this puzzle) they handed out shiny cards that stated that the temperature on Saturn at 0.5 *bar (*1 *bar* = 10^6 *dyne* cm^{-2}*) is* 110 *K and Hydrostatic equilibrium. Below this level the atmosphere is convecting and the temperature increases with depth s at the quasi-adiabatic rate of* $dT/dr \sim 0.7$ *K* km^{-1}*. Also remember that gas liquifies once its density approaches that of liquid, of the order of* ~ 1 *g* cm^{-3}*, you can assume that Saturn is made of pure hydrogen.*

2. **The Great Escape** (40pt)

   (a) (7pt) Assume a planet with mass M_p, that holds an atmosphere. Assume that the atmosphere is isothermal and in hydrostatic equilibrium and ideal gas, and show that the gas number density has the following profile:

   $$n(r) = n_0(r_0) \exp\left(\frac{r}{H(r)} - \frac{r_0}{H(r_0)}\right) . \tag{1}$$

   where r_0 is some reference frame and

   $$H(r) = \frac{k_B T r^2}{G M_p m} = \frac{k_B T}{m g(r)} , \tag{2}$$

   where m is the gas molecules mass and $g(r)$ is the gravitational acceleration.

   (b) (7pt) Assume that r represent a small displacement from r_0, i.e., $r = r_0 + z$ where $z < r_0$, so expand the density profile to the linear order of z in the exponent and show that

   $$n(r) = n_0(r_0) \exp\left(-\frac{z}{H(r_0)}\right) , \tag{3}$$

   (c) (10pt) Assume that the gas is well approximated with Maxwellian distribution for particle velocities

   $$f(v) = n \left(\frac{m}{2\pi k_B T}\right)^{3/2} \exp\left(-\frac{m v^2}{2 k_B T}\right) . \tag{4}$$

   If the tail of this distribution is larger than the escape velocity from the planet ($v_{esc} = \sqrt{2GM_p/r}$) we can estimate the escape flux of the atmosphere by calculating the following flux (also known as Jeans flux, or Jeans escape)

   $$F_J = 2\pi \int_{v_{esc}}^{\infty} \int_{\theta=\pi/2}^{\theta=0} f(v) v^3 dv \cos\theta d(\cos\theta) , \tag{5}$$

The number density at the surface is n_c.Show that the flax can be written as

$$F_J = \frac{n_c}{2}\sqrt{\frac{2k_BT}{m\pi}}\,(1+\lambda_c)\,e^{-\lambda_c}\ , \tag{6}$$

where

$$\lambda_c = \frac{mv_{esc}^2/2}{k_BT}\ . \tag{7}$$

(d) (6pt) How much of atmosphere will a planet at 5 AU with a mass of 0.2 M_E will lose this way in 1 day? (express your answer in scale hight of the planet, assume that the gas composed from hydrogen molecules and that the sounds speed of the gas is about 7×10^4 cm s^{-1}). - The value that you will get represent an extreme value for a case of isothermal atmosphere and a simplified model for an atmosphere.

(e) (10pt) If you got the last question correctly, please explain then how core planets can retain their atmosphere?

3. **Convection** (20pt)

(a) Assume a core plant with an envelope initially at rest with the following profiles: $dP/dr|_{ev}$, $d\rho/dr|_{ev}$ and $dT/dr|_{ev}$. We displace a blob of fluid upward slowly (which keeps the pressure equilibrium) and adiabatically (so no energy exchange takes place between the blob and its surrounding). In class we showed that the if the density change in the blob is larger than the envelop density change, the envelop will be stable (the blob will sink down). This is expressed as:

$$\left.\frac{d\rho}{dr}\right|_{blob} > \left.\frac{d\rho}{dr}\right|_{env}\ . \tag{8}$$

Show that this is equivalent to the following condition (for a blob and an envelope with the same composition):

$$\left.\frac{d\ln T}{d\ln P}\right|_{blob} > \left.\frac{d\ln T}{d\ln P}\right|_{env}\ . \tag{9}$$

Hint: Consider the derivative of the equation of state $d\rho/\rho$ which can be written as $\rho = \rho(P, T, \mu)$, where μ is the molecule weight

(b) A student wrote in homework that for the adiabatic condition we stated above, assuming a small displacement of the blob, the condition in equation (8) can be written as

$$-\frac{1}{\gamma P}\frac{\partial P}{\partial r} < -\frac{1}{\rho}\frac{\partial \rho}{\partial r}\ , \tag{10}$$

where γ is the adiabatic index. Is she correct? If yes please show her derivation, if not please explain why.