Measurements of the radial recession velocities of five galaxies in a cluster give velocities of 9700, 8600, 8200, 8500, and 10,000 km s⁻¹. What is the distance to the cluster if the Hubble parameter is $H_0 = 70 \text{ km s}^{-1}\text{Mpc}^{-1}$? Estimate, to an order of magnitude, the mass of the cluster if every galaxy is projected roughly half a degree from the cluster center. Answers: 130 Mpc; $10^{14}M_{\odot}$.

Modified Newtonian Dynamics (MoND) proposes that, for small accelerations, Newton's third Law, F=ma approaches the form $F=ma^2/a_0$, where a_0 is a constant (see Eq. 6.50).

a. Show how such an acceleration law can lead to flat rotation curves, without the need for dark matter.

b. Alternatively, propose a new law of gravitation to replace $F=GMm/r^2$ at distances greater than some characteristic radius r_0 , so as to produce flat rotation curves without dark matter. Make sure your modified law has the right dimensions.

c. Modify further the gravitation law you proposed in (b) with some mathematical formulation (many different formulations are possible), so that the law is Newtonian on scales much smaller than r_0 , with a continuous transition to the required behavior at $r\gg r_0$.

(4)

In the Sunyaev-Zeldovich effect, photons from the cosmic microwave background radiation are Compton scattered by hot electrons in a cluster along the line of sight. Assume 0.001 of the photons are scattered, and the mass of the cluster is $2 \times 10^{14}~M_{\odot}$, of which 15% is in the hot gas (fully ionized hydrogen).

a. Use the Thomson cross section to represent the cross section for Compton scattering, and assume the cluster is spherical and of constant density, to find the diameter of the cluster (assume the photons pass through one diameter).

b. If the angular diameter of the cluster is 1°, what is its distance?
c. If the cluster velocity of recession is 8400 km s⁻¹, what is the Hubble parameter, in units of km s⁻¹ Mpc⁻¹?

parameter, in units of km s $^{-1}$ Mpc; $^{-1}$ Answers: diameter 2.1 Mpc; distance 120 Mpc; $H_0 = 70 \text{ km s}^{-1}$ Mpc $^{-1}$.