מבוא לאסטרו תרגיל 5

1) קיבלנו במקרה של גז לא יחסותי $\bar{P} = \frac{2}{3}\frac{E_k}{V}$ ועבור גז יחסותי $\bar{P} = \frac{1}{3}\frac{E_k}{V}$.
בשביל משק הירגולים: $\bar{P} = -\frac{1}{3}\frac{E_m}{V}$. הסבר מה קורה
לאנרגיה הכללית של כוכב בכל אחד מהמקרים, ודונו בתוצאה.

2) הראו שגם במקרה הכללי של גז אידיאלי מונואטומי
$\bar{P} = \frac{2}{3}\frac{E_k}{V}$.

3) קיבלתם בכיתה שטף הנייטרינו ע"פ כדה"א הינו
$f =$, מתוך הלומי' הבולומטרי של השמש, העריכו
את האנרגיה האופיינית של פוטון ואת שטף מספר הפוטונים
ע"פ כדה"א.

4. We saw that the nuclear reaction rate in a star depends on

$$\langle \sigma v \rangle \propto \int_0^\infty f(E)dE,$$

where

$$f(E) \equiv e^{-E/kT} e^{-\sqrt{E_G/E}},$$

and E_G is the Gamow energy (Eq. 3.134).
a. By taking the derivative of $f(E)$ and equating to zero, show that $f(E)$ has a maximum at

$$E_0 = \left(\frac{kT}{2}\right)^{2/3} E_G^{1/3}.$$

b. Perform a Taylor expansion, to second order, of $f(E)$ around E_0, to approximate $f(E)$ with a Gaussian. Show that the width parameter (i.e., the "σ") of the Gaussian is

$$\Delta \approx \frac{2^{1/6}}{3^{1/2}} E_G^{1/6} (kT)^{5/6}.$$

Hint: Take the logarithm of $f(E)$, before Taylor expanding, and then exponentiate again the Taylor expansion.
c. Show that

$$\int_0^\infty f(E)dE = \sqrt{2\pi} f(E_0)\Delta.$$

5. We saw (Eq. 3.142) that, on Earth, the number flux of Solar neutrinos from the p-p chain is

$$f_{\text{neutrino}} = \frac{2f_\odot}{26.2\ \text{MeV}} = \frac{2 \times 1.4 \times 10^6\ \text{erg s}^{-1}\ \text{cm}^{-2}}{26.2 \times 1.6 \times 10^{-6}\ \text{erg}} = 6.5 \times 10^{10}\ \text{s}^{-1}\ \text{cm}^{-2}.$$

Other nuclear reactions in the Sun supplement this neutrino flux with a small additional flux of higher-energy neutrinos. A neutrino detector in Japan, named SuperKamiokande, consists of a tank of 50 kton of water, surrounded by photomultiplier tubes. The tubes detect the flash of **Cerenkov radiation** emitted by a recoiling electron when a high-energy neutrino scatters on it.
a. How many electrons are there in the water of the detector?
b. Calculate the detection rate for neutrino scattering, in events per day, if 10^{-6} of the Solar neutrinos have a high-enough energy to be detected by this experiment, and each electron poses a scattering cross section $\sigma = 10^{-43}\ \text{cm}^2$.
Hint: Consider the density of neutrino targets "seen" by an individual electron, with a relative velocity of c between the neutrinos and the electron, to obtain the rate at which one electron interacts with the incoming neutrinos, and multiply by the total number of electrons, from (a), to obtain the rate in the entire detector.