

על פי המידע שניתן, נניח שהכוכב הוא כדור אחיד (1)

הכוכב הוא כדור אחיד. $\bar{\rho}_{\text{gas}} = 10^{15} \frac{\text{dyne}}{\text{cm}^2}$ $T = 8 \cdot 10^6 \text{ K}$

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הכוכב.

4. Consider a hypothetical star of radius R , with density ρ that is constant, i.e., independent of radius. The star is composed of a classical, non-relativistic, ideal gas of fully ionized hydrogen.

a. Solve the equations of stellar structure for the pressure profile, $P(r)$, with the boundary condition $P(R) = 0$.

Answer: $P(r) = (2\pi/3)G\rho^2(R^2 - r^2)$

b. Find the temperature profile, $T(r)$.

c. Assume the nuclear energy production rate depends on temperature as $\epsilon \sim T^4$. (This is the approximate dependence of the rate for the pp chain at the temperature in the core of the Sun.) At what radius does ϵ decrease to 0.1 of its central value, and what fraction of the star's volume is included within this radius?

5. Suppose a star of total mass M and radius R has a density profile $\rho = \rho_c(1 - r/R)$, where ρ_c is the central density.

a. Find $M(r)$.

b. Express the total mass M in terms of R and ρ_c .

c. Solve for the pressure profile, $P(r)$, with the boundary condition $P(R) = 0$.

Answer:

$$P(r) = -4\pi G\rho_c^2 R^2 \left[\frac{5}{36} - \frac{2}{3} \left(\frac{r}{R} \right)^2 + \frac{7}{9} \left(\frac{r}{R} \right)^3 - \frac{1}{4} \left(\frac{r}{R} \right)^4 \right].$$

6. Consider a star of mass $M = 10M_\odot$, composed entirely of fully ionized ^{12}C . Its core temperature is $T_c = 6 \times 10^8 \text{ K}$ (compared to $T_{c,\odot} = 1.5 \times 10^7 \text{ K}$ for the Sun).

a. What is the mean particle mass \bar{m} , in units of m_H ?

Answer: $12/7$.

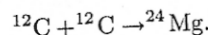
b. Use the classical ideal gas law, the dimensional relation between mass, density, and radius, and the virial theorem, to find the scaling of the stellar radius r_* with total mass M , mean particle mass \bar{m} , and core temperature T_c . Using the values of these parameters for the Sun, derive the radius of the star.

Answer: $0.86r_\odot$.

c. If the luminosity of the star is $L = 10^7 L_\odot$, what is the effective surface

temperature?

d. Suppose the star produces energy via the reaction



The atomic weight of ^{12}C is 12, and that of ^{24}Mg is 23.985. (The atomic weight of a nucleus is defined as the ratio of its mass to 1/12 the mass of a ^{12}C nucleus). What fraction of the star's mass can be converted into thermal energy?

Answer: 6.3×10^{-4} .

e. How much time does it take for the star to use up 10% of its carbon?

Answer: 950 yr.