Astro-286 - Week 3

1. **Properties of the particles in a disk** (8pt)
   (a) **Thermal velocity of molecules** (5pt) Given a Maxwell–Boltzmann distribution for the speed of the molecules in the gas, calculate the average (i.e., expected value) of the speed distribution and the rms value.
   (b) **Mean free path** (3pt) Calculate the mean free path of a hydrogen molecule at 10AU. The effective molecular diameter of a hydrogen molecule H_2 is $\sim 10^{-10}$ m.

2. **Dust Settling** (10pt, 5pt each)
   (a) Find the general expression for the dust settling time $t_{settle} = z/v_{settle}$ as a function of the following disk parameters: Σ, Ω and the dust properties s, ρ_s times the vertical density profile. *Hint: use week 2 problem set to remind yourself the vertical profile.* Do we recover the qualitative solution we discussed in class?
   (b) **Dust Settling in the presence of turbulence** Given the characteristic diffusion timescale is similar to the characteristic settling timescale, find an expression (in terms of the kinematic viscosity) to the diffusion coefficient of a dust settling in the presence of turbulence at the scale hight of the disk.

3. **Sedimentation** (10pt) In class we talked about dust settling in the Epstein regime. Lets now consider somewhat larger spherical bodies; the excess force due to gravity and buoyancy is balanced by the Stokes drag and a typical velocity. Find an expression for this velocity assuming small (< 1) Reynolds number.

4. **Coagulation** (24pt, 12pt each)
   (a) Estimate the growth rate of an ice particle at 5 AU (the temperature is 75 K), specifically estimate the value of the size growth rate (i.e., $\dot{s}$, where s is the radius of the ice particle). Assume that the particle accretes water molecules from the gas phase. A that you may find useful is the density of water vapors in a solar like nebula is 9.4×10^{-14} g cm^{-3}.
   Hint 1: assume a perfect efficiency for the accretion, i.e., each water molecule that collides with the ice particle sticks to it with - explain why its a ok approximation.
   Hint 2: assume that the ice particle and the water molecule is dominated by the thermal speed of the molecules
   Hint 3: what is the density of ice?
   (b) Is the above calculation promising or discouraging for growing grains in the solar nebula, given that the lifetime of the solar nebula is estimated to be 10^7 yr?

5. **The good (referee) the bad (author) and the ugly (truth about this process?)** (24pt, 6pt each) An author has submitted a paper to a refereed journal. After few weeks he received a very thorough referee report.

(a) In one of the manuscript section the author calculated the coagulation of a large particle (mass m size s) in a sea of small particles. The author wrote:

$$\frac{dm}{dt} = \frac{3}{4}\frac{\Omega^2 f}{v_{th}} zm \ , \tag{1}$$

(as we got in class). Then the he did an integral and found

$$m(t) \sim \exp\left(\frac{3}{4}\frac{\Omega^2 f}{v_{th}} zt\right) \ . \tag{2}$$

The referee wrote that the author should specify his assumptions. What did she mean?, i.e., what is the assumption that is missing in the transition from eq. (1) to (4)?

(b) In another of the section he wrote that the the equation for the settling of dust in the pretense of turbulence can be written as:

$$\rho_d v + D\rho \frac{\partial}{\partial z}\frac{\rho_d}{\rho} = 0 \ . \tag{3}$$

Again the referee requested that the author will specify the assumption here, what was it?

(c) The author than wrote that he can assume that the gas density doesn't change significantly and wrote the following solution to the above equation:

$$\rho_d = \rho_{0,d} \exp\left(-\frac{vz}{D}\rho_0\sqrt{\pi} h Erf\left[\frac{z}{\sqrt{2}h}\right]\right) \ . \tag{4}$$

All of the authors numerical simulations were based on this equation. The referee wrote that the paper cannot be accepted to publication unless the author will correct his mistake in the above equation. What were the author mistakes? (note that there are more than one problem in this equation). What did he do to reach that equation?

(d) Given $v_{settle} = t_{fric}\Omega^2 z$ and under the assumptions that the author should have specified the referee also wrote the solution to the equation that the author should have got. Derive her solution.

6. **Derivation of the disruption relation** (24pt, 12pt each)

(a) Derive the following equations for the $\hat{\mathbf{r}}$ and $\hat{\phi}$ components from the close equations we wrote in class (i.e., momentum, continuity and poisson).

$$\hat{\mathbf{r}} : \quad \frac{\partial v_r}{\partial t} - 2\Omega\delta v_\phi = \frac{1}{\Sigma_0}\frac{dP_1}{dr} - \frac{\partial \Phi}{\partial r} \ . \tag{5}$$

$$\hat{\phi} : \quad \frac{\partial \delta v_\phi}{\partial t} v_r \left(\Omega + \frac{d(\Omega r)}{dr}\right) = 0 \ . \tag{6}$$

(b) Using these equations find the dispersion relation we wrote in class.