Astro-286 - Week 1

1. **Radial Velocity** (10 pt)
   What is the expected amplitude of velocity oscillations of 1 $M_\odot$ star that is orbited by a Jupiter mass planet ($m_J = 0.001$ $M_\odot$) at 1 AU separation? What is the expected Doppler shift ?

2. **Transit** (20 pt, 4 pt each)
   Given the parameters in Figures 1 and 2 and the radius of the star as $R_\star$, and assuming the common case $R_\star << a$, where a is the separation between the star and a planet. The planets path across (or behind) the stellar disk is approximately a straight line between the points (x_1, y_1) and (x_2, y_2).

   (a) Show that the transit duration for a non-central transit (see Figures) is:

   $$t_T = t_4 - t_1 = \frac{PR_\star}{\pi a}\sqrt{\left(1+\frac{R_p}{R_\star}\right)^2 - \left(\frac{a\cos i}{R_\star}\right)^2}\,, \tag{1}$$

   where P is the period of the planet, a is the semi major axis of he planet orbit and i is the inclination between the system and the line of sight.
   Hint 1: use the simple geometry shown in the Figure
   Hint 2: note that the impact parameter b for a circular orbit is defined as the projected distance between the planet and star centers during midtransit in units of $R_\star$ and thus can be written as:

   $$b = \frac{a\cos i}{R_\star}. \tag{2}$$

   (b) Show that

   $$t_{Full} = t_3 - t_2 = \frac{PR_\star}{\pi a}\sqrt{\left(1-\frac{R_p}{R_\star}\right)^2 - \left(\frac{a\cos i}{R_\star}\right)^2}\,, \tag{3}$$

   (c) For a plant on a circular orbit, with separation a from its star, what is the probability of observing a transit (including grazing encounters)?
   Hint: very simple calculation, and assume an ensemble of such systems has random inclination distribution.

   (d) What is the probability to observe an Earth like planet in Earth position?

   (e) How long will this transit take (i.e., calculate t_T)?

3. **Microlensing** (30 pt, 10 pt each)

   (a) Calculate the Einstein radius (note: not the angle but radius) for a lens with mass 0.3 $M_\odot$ and a source at a distance of 8 kpc. We also know that for this system the $d_l/d_s = 0.5$.

(b) What is the typical timescale for a microlensing event? Calculate the typical timescale for the above Einstein radius and for a proper motion of the source relative to the lens at a value of 10 milliarcsecond per year what is the time of the event? *Remember that the proper motion of a star is its rate of angular change in position over time, as observed from the center of mass of the Solar System.*

(c) If that star hosted an earth mass planet, what is the duration of an microlensing event of the "blip" associated with that planet?

4. **Direct Imaging** (40 pt, 5 pt each)

(a) In class we used **Wien's law** which states that $h\nu_{max} = 2.8K_BT$. Derive that law.

(b) Derive the Stefan–Boltzmann law, i.e., $f = \sigma T^4$.

(c) Calculate Earth's effective temperature (also known as the equilibrium temperature, not to be confused with the effective temperature of a star). Using the Stefan–Boltzmann law the basic assumption is that the flux that comes in is the flux that comes out. However, Earth's albedo, A_{alb} is about 0.3 so in fact the actual flux that is absorbed by the Earth is about

$$f = \frac{A_{abs}L_{\odot}(1 - A_{alb})}{4\pi a^2} \ , \tag{4}$$

where $L_{\odot} = 3.8 \times 10^{33}$ erg sec^{-1} and A_{abs} is the area of the planet that absorbs the power from the star out of the total area and a is Earth's semi major axis. The StefanBoltzmann law gives the expression for the radiation emitted from the planet $A_{rad}\sigma T^4$, where A_{rad} is the area over which the plane will radiate at temperature T. (Did I forget something in the latter equation? if yes what is it, and why can I neglect it?). Calculate the effective temperature of the Earth, assuming that $A_{abs}/A_{rad} = 1/4$ which is a common assumption for a rapidly rotating body like the Earth.

(d) Professor X asked wrote in her lecture that the equilibrium temperature of a planet is:

$$T = \left(\frac{A_{abs}}{A_{rad}}\right)^{1/4} T_{eff,\star} \left(\frac{R_\star}{a}\right)^{1/2} \left[(1 - A_{alb})\right]^{1/4} \ , \tag{5}$$

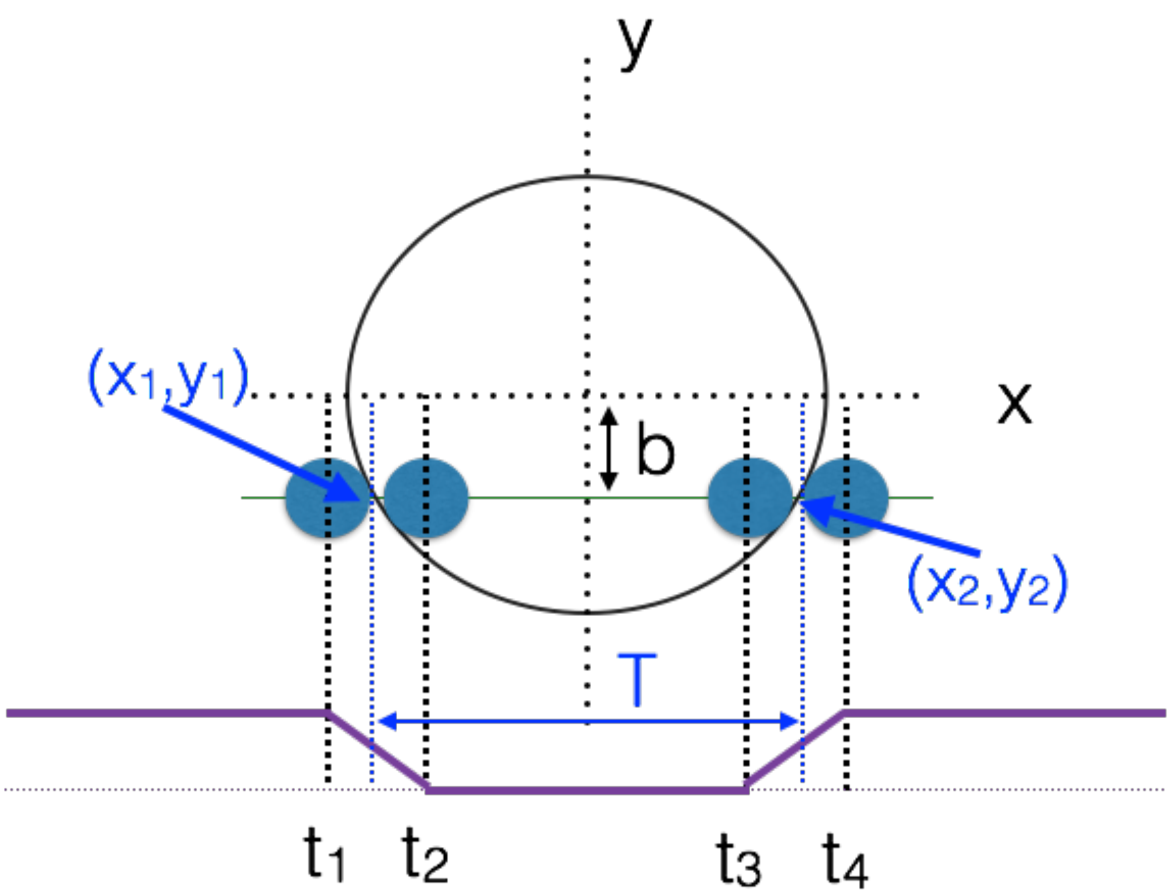
where $T_{eff,\star}$ and $R_\star$ are the star effective temperature and radius respectively, and a is the planet's semi-major axis. This equation is different from your above result. Was the professor wrong? Are you wrong? Explain!

(e) In class we said that Earth's average temperature is about 290 K. If you did the above calculation correct you will find that there is a slight discrepancy between the effective temperature and the average temperature. Why do you think there is a difference?

(f) Beta Pictoris b is a planet located about 9 AU from its host star. The star Beta Pictoris has a luminosity of $\sim$ 8.7 $L_{\odot}$. Assume a fast rotator and an albedo of about 0.5 what is the effective temperature of the planet?

(g) Given that Boltzman constant is: 1.38×10^{-16} erg K^{-1} calculate the wavelength associated with this.

(h) The planet was detected in mid infrared associated with few μm. One of the detections was done in fact in ~ 3 μm. How do you explain that?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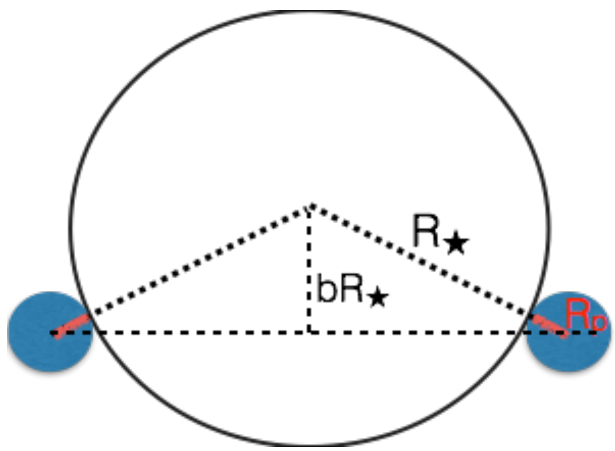
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