מבוא לאסטרו' תרגיל 7

(1) A spinning neutron star of mass $M = 1.4\ M_\odot$, constant density, and radius $R = 10$ km has a period $P = 1$ s. The neutron star is accreting mass from a binary companion through an accretion disk, at a rate of $\dot{M} = 10^{-9}\ M_\odot\ \mathrm{yr}^{-1}$. Assume the accreted matter is in a circular Keplerian orbit around the neutron star until just before it hits the surface, and once it does then all of the matter's angular momentum is transferred onto the neutron star.

a. Derive a differential equation for $\dot{P}$, the rate at which the neutron-star period decreases.

b. Solve the equation to find how long will it take to reach $P = 1$ ms, the maximal spin rate of a neutron star.

Hint: Calculate the Keplerian velocity of the accreted material a moment before it hits the neutron star surface, and use it to derive the angular momentum per unit mass of this material, J/m. The angular momentum of a rotating object with moment of inertia I is $I\omega$. The rate of change of the star's angular momentum is just the rate at which it receives angular momentum from the accreted matter, i.e.,

$$\frac{d}{dt}(I\omega) = \dot{M}\frac{J}{m}.$$

The moment of inertia of a constant-density sphere is $I = \frac{2}{5}MR^2$. Solve for the angular acceleration $\dot{\omega}$, neglecting changes in the neutron star's mass and radius. (This will be justified by the result). From the relation $P = 2\pi/\omega$, derive $\dot{P}$. This "spin-up" process explains the properties of old, "millisecond pulsars", some of which, indeed, have negative $\dot{P}$.

Answer: 2.6×10^8 yr. Over this time, the neutron star mass increases by 18%, and its radius decreases by 5%, justifying the approximation of constant mass and radius.

(2) A star of mass m and radius r approaches a black hole of mass M to within a distance $d \gg r$.

a. Using Eq. 4.127, express, in terms of m, r, and M, the distance d at which the Newtonian radial tidal force exerted by the black hole on the star equals the gravitational binding force of the star, and hence the star will be torn apart.

b. Find the black-hole mass M above which the tidal disruption distance, d, is smaller than the Schwarzschild radius of the black hole, and evaluate it for a star with $m = M_\odot$ and $r = r_\odot$. Black holes with masses above this value can swallow Sun-like stars whole, without first tidally shredding them.

Answer: $10^8 M_\odot$.

c. Derive a Newtonian expression for the **tangential** tidal force exerted inward on the star, in terms of m, r, M, and d, again under the approximation $r \ll d$. The combined effects of the radial tidal force in (a) and and the tangential tidal force in (c) will lead to "spaghettification" of stars, or other objects that approach the black hole to within the disruption distance.

Hint: Remember that the star is in a *radial* gravitational field, and hence there is a tangential component to the gravitational force exerted on regions of the star that are off the axis defined by the black hole and the center of the star. The tangential component can be found by noting that the small angle between the axis and the edge of the star is $\approx r/d$.

(3) A new star lights up inside a cloud of atomic hydrogen with a constant number density of n atoms per unit volume. The star emits ionizing photons at a rate of Q_* photons per unit time. The ionizing photons begin carving out a growing "Strömgren sphere" of ionized gas inside the neutral gas .

a. At a distance r from the star, what is the timescale $\tau_{\rm ion}$ over which an individual atom gets ionized, if the ionization cross section is $\sigma_{\rm ion}$?

b. If the recombination coefficient is $\alpha \equiv \langle \sigma v \rangle$, what is the timescale $\tau_{\rm rec}$ for an individual proton to recombine with an electron?

c. At a position close to the star, where the ionizing flux is high, and therefore $\tau_{\rm ion} \ll \tau_{\rm rec}$, show that the velocity at which the ionization front that bounds the Strömgren sphere advances is $v_{\rm if} = Q_*/(4\pi r^2 n)$.

Hint: Assume, as usual, that the gas is completely ionized within the front, and completely neutral beyond it. Consider a slab of neutral gas behind the ionization front, with area ΔA and thickness Δr, and find the volume of neutral gas that is ionized during an interval Δt.

d. Evaluate $v_{\rm if}$ for $Q_* = 3 \times 10^{49}\ {\rm s}^{-1}$, $n = 10^4\ {\rm cm}^{-3}$, and for $r = 0.01$ pc, 0.05 pc , and 0.1 pc, respectively. From $v_{\rm if}(r)$, obtain and solve a simple differential equation for $r_{\rm strom}(t)$, and find roughly how long does it take the ionization front to reach the final Strömgren radius.

Answers: At $r = 0.01$ pc, $v_{\rm if} = c$; at 0.05 pc, $v_{\rm if} = 0.3c$ and at 0.1 pc, $2.5 \times 10^4\ {\rm km\ s}^{-1}$; about 10 years to reach $r_{\rm strom}$.

(4) בהרצאה מצאנו את הצפיפות היחסית של אלקטרונים, פרוטונים ונייטרונים עבור כוכב ניוטרונים שבו האלקטרונים אולטרה יחסותיים ושאר החלקיקים קלאסיים. חזרו על החישוב כאשר כל סוגי החלקיקים אולטרה יחסותיים.