Introduction to Astrophysics 0321.3108
Exercise 3

1. In class we showed that in thermal equilibrium

$$\frac{N(H^+)}{N(H)} = \frac{1}{N(e)} \left(\frac{2\pi m_e k_B T}{h^2} \right)^{3/2} \exp\left(\frac{-1.6 \times 10^5}{T} \right), \tag{1}$$

where $N(H^+)$ is the proton number density, $N(H)$ is the atomic hydrogen number density, $N(E)$ is the electron number density and T is the gas temperature. For a typical density in a stellar photosphere: $\sim 5 \times 10^{14}$ cm^{-3}, find the temperature such that $N(H^+)/N(H) = 1$. *Guidance: assume that the density of the electrons is the same as the density of the protons. The last equation must be solved numerically!*

2. Find the frequencies of the spectral lines: Lyα, Lyβ, Lyγ, Hα, Hβ and Hγ. Explain your derivations.

3. Suppose a star of total mass M and radius R has a density profile $\rho = \rho_c(1 - r/R)$, where ρ_c is the central density.

   a. Find M(r).

   b. Express the total mass M in terms of R and ρ_c.

   c. Solve for the pressure profile, P(r), with the boundary condition P(R) = 0.

4. Consider a hypothetical star of radius R, with density ρ that is constant, i.e., independent of radius. The star is composed of a classical, non-relativistic, ideal gas of fully ionized hydrogen.

   a. Solve the equation hydrostatic equilibrium for the pressure profile, $P(r)$, with the boundary condition $P(R) = 0$.

   b. Find the temperature profile, $T(r)$.

5. In class we found that the virial temperature of the Sun is: 4×10^6 K, and that the pressure is $\bar{P}_\odot = 10^{15}$ dyne cm^{-2}. What is the radiation pressure compared to the ideal gas pressure?

6. **The cosmic billiard game: Star collisions** In this question we will find what is the estimated rate for star to star collision. For simplicity we assume that all the stars in our galaxy have the same properties as our Sun, i.e., the same solar radius, and the same velocity 200 km sec^{-1}. All of the stars have random direction (this of-course not true, but sufficient for this rough estimation). Assume that the number density of the stars in the galaxy is 10^{-9} pc^{-3}, and neglect the gravitational pull between the stars.

   a. Find the cross section for collision.

   b. Find the mean free path between collision to collision.

c. Finally, find the rate of star to star collision. Are we in any danger?

7. In this problem we estimate the equilibrium surface temperature of the Earth assuming that the Earth is heated by radiation from the Sun, and that the Earth radiates as a blackbody.

  a. What is the flux of Solar radiation reaching the Earth?

  b. What is the geometrical cross section of the Earth given its radius R?

  c. Equate the rate at which the Earth absorbs Solar energy to the rate at which the Earth reradiates energy, and find the Earth's equilibrium temperature T. At what wavelengths is most of the Earth's radiation emitted?

  d. The green-house effect. Now estimate the temperature assuming that a layer of clouds covers the Earth's surface. The clouds are transparent to the sunlight but absorb the radiation emitted from the Earth's surface. For these conditions, what is the Earth's equlibrium surface temperature? *(hint: see figure)*.

Greenhouse effect