Introduction to Astrophysics 0321.3108
Exercise 4

1. For radiation we know that the pressure is $P = u/3$ where u is the radiation energy density. Assume that u is a function of the temperature only, i.e. $u = u(T)$ (this fact was discovered by Kirchoff).

   a. Show that from the second law of thermodynamics

   $$Tds = d(uV) + PdV, \tag{1}$$

   (where V is the volume) that it follows that $u(T) = aT^4$ where a is an undetermined constant. This thermodynamic proof was first given by Boltzmann (prior to Planck's discovery of his formula). *Hint: Find what are the derivatives of the entropy with respect to the volume and with the pressure, i.e.,* $\left(\frac{\partial s}{\partial V}\right)_P$ *and* $\left(\frac{\partial s}{\partial P}\right)_V$. *Than use the symmetry of second the derivatives.*

   b. Show that the entropy per unit volume is: $s = \frac{4}{3}aT^3$.

2. **Kelvin-Helmholtz**.

   a. When the Sun evolved to become a main sequence star it contracted slowly because of gravity (but always close to hydrostatic equilibrium). The inner temperature rose from $30,000$ K to 6×10^6 K, during this Kelvin-Helmholtz stage. What was the energy radiated during this stage?

   b. Show that the rate that the radius changes can be written as

   $$\frac{1}{t_{KH}} = \frac{1}{R_\odot}\frac{dR_\odot}{dt}, \tag{2}$$

   where t_{KH} is the instantaneous Kelvin-Helmholtz time. Calculate the rate that the radius changes for the Sun if it is contracting gravitationally.

   c. In class we discussed the photon diffusion time scale, t_d. Show that

   $$\frac{t_d}{t_{KH}} = 2\frac{P_{rad}}{P_{thermal}}. \tag{3}$$

   If all internal sources of energy in the Sun were turned off, by what fraction $\Delta R/R$ would the Sun shrink in one diffusion time?

3. For a given ionized atmosphere of hydrogen, assume that the gravitation is constant (the problem is in one dimension) $\vec{g} = -g\hat{z}$. Consider only Thompson scattering and radiative transfer, and assume an ideal gas.

   a. Find the gas pressure as a function of the density and the temperature.

b. Assume that the flux, f is constant. What is dT/dz?

c. What is dT/dP?

d. Show that $\rho \sim T^3$

e. How does the relation between the gas pressure and the radiation pressure change with the hight?

4. Consider a source of radiation emitting photons with energy ϵ, and total luminosity L.

a. What is the number density of photons n_γ at distance r from the source? *Hint:* $n_\gamma = \frac{flax}{velocity \times energy}$

b. What is the rate of Thompson scattering of an **electron** by the photons, at distance r from the source?

c. In each electron-photon Thompson scattering, a photon with energy ϵ transfers an average energy of ϵ/c to the electron. What is the rate of momentum transfer to the electrons, i.e., the force on the electrons due to scattering with the radiation?

d. **Eddington Limit**.If the mass of the central source is M,

- Find the gravitational force acting on a proton at a distance r.
- This force, due to Coulomb attraction, is effectively acting on each electron. Compare between this force and the force you found in the previous sub-question, and find the maximum luminosity that a source with mass M produce such that gas is not lost due to outward radiation pressure. This is called Eddington Limit.

e. What is the value of the Eddington Limit for one solar mass.

Chag Pesach Sameach