

(2.179) the poincare variables ①  
 The transformation of the Delaunay variables.

$$\Gamma = \alpha (1 - \sqrt{1 - e^2}) \quad \gamma = -\bar{\omega} - \Omega$$

$$R = K_1 e^2 + K_2 e'^2 + K_3 e e' \cos(\bar{\omega} - \bar{\omega}')$$

$$\left\{ \begin{array}{l} \frac{d\epsilon}{dt} \\ \frac{d\bar{\omega}}{dt} \end{array} \right. \quad \begin{array}{l} \gamma = -\bar{\omega} \\ \Gamma = \epsilon^2/2 \quad \text{Taylor expansion} \end{array}$$

$$\frac{d\delta}{dt} = \partial R / \partial \Gamma \quad \frac{\partial \Gamma}{\partial t} = - \partial R / \partial \gamma$$

$$-\frac{d\bar{\omega}}{dt} = \frac{1}{\epsilon} \partial R / \partial \epsilon$$

$$\epsilon \frac{d\epsilon}{dt} = \partial R / \partial \bar{\omega}$$

Canonical variables

$$P, q \rightarrow \partial H / \partial q = -\dot{P} \quad \partial H / \partial P = \dot{q}$$

complex canonical variables

$$z = \sqrt{P} e^{iq} \quad z^* = z' = \sqrt{P} e^{-iq}$$

$$z = \sqrt{P} e^{i\gamma} \quad dz/dt = i \partial H / \partial z^*$$

$$z = \sqrt{r} e^{i\gamma} = \frac{e}{\sqrt{2}} e^{-i\bar{\omega}} \quad (2)$$

$$\langle R \rangle = 2K|z|^2 + K_3 e' \sqrt{2} |z| \left\{ e^{\frac{-i(\gamma+\bar{\omega}')}{2}} + e^{\frac{i(\gamma+\bar{\omega}')}{2}} \right\} + K_3 e' \frac{\sqrt{2}}{2} (z e^{i\bar{\omega}'} + z' e^{-i\bar{\omega}'})$$

$$\frac{dz}{dt} = i \left[ 2Kz + \frac{\sqrt{2}}{2} K_3 e' \underbrace{e^{-i\bar{\omega}'}}_{z^*} \right]$$

$$\frac{dz}{dt} = i[a z + b z']$$

$$\frac{dz}{dt} = i\omega z \rightarrow z = C_0 e^{i\omega t}$$

$$\frac{dz}{dt} = i(\omega_0 z + b z')$$

$$dz/dt - i\omega_0 z = ib z'$$

$$z = e e^{i\bar{\omega}} \quad \frac{d}{dt} \left\{ z e^{-i\omega_0 t} \right\}$$

$$e^{i\omega_0 t} \left\{ \dot{z} e^{-i\omega_0 t} + z(-i\omega_0) e^{-i\omega_0 t} \right\} = \dot{z} e^{-i\omega_0 t}$$

$$e^{i\omega_0 t} \frac{d}{dt} [z e^{-i\omega_0 t}] = ibz \quad (1)$$

$$= \nabla \frac{d}{dt} [z e^{-i\omega_0 t}] = e^{-i\omega_0 t} [ibz']$$

$$z e^{-i\omega_0 t} = -\frac{b}{\omega_0} z' e^{-i\omega_0 t} + C$$

$$z = -\frac{b}{\omega_0} z' + C e^{i\omega_0 t}$$

$$\begin{bmatrix} \frac{dz_s}{dt} \\ \frac{dz_j}{dt} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} z_s \\ z_j \end{bmatrix}$$