University of Zurich[UZH]

S3IT

Dynamic and Unbounded Sequences of Tasks

Riccardo Murri <riccardo.murri@uzh.ch>

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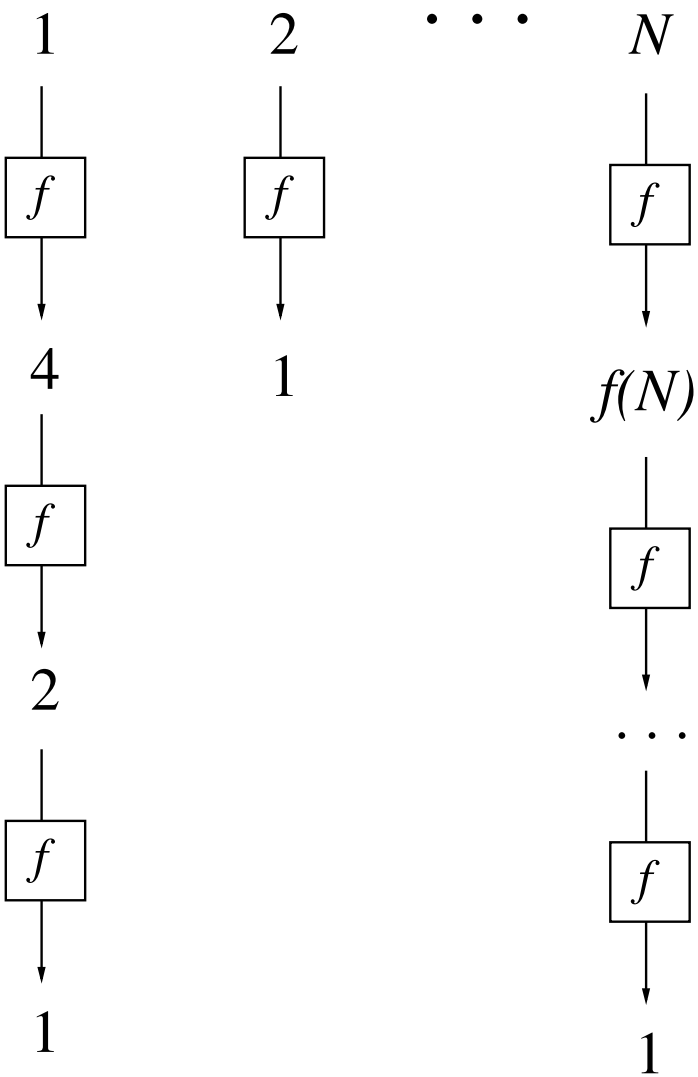
The *3n+1* conjecture, a fictitious use case



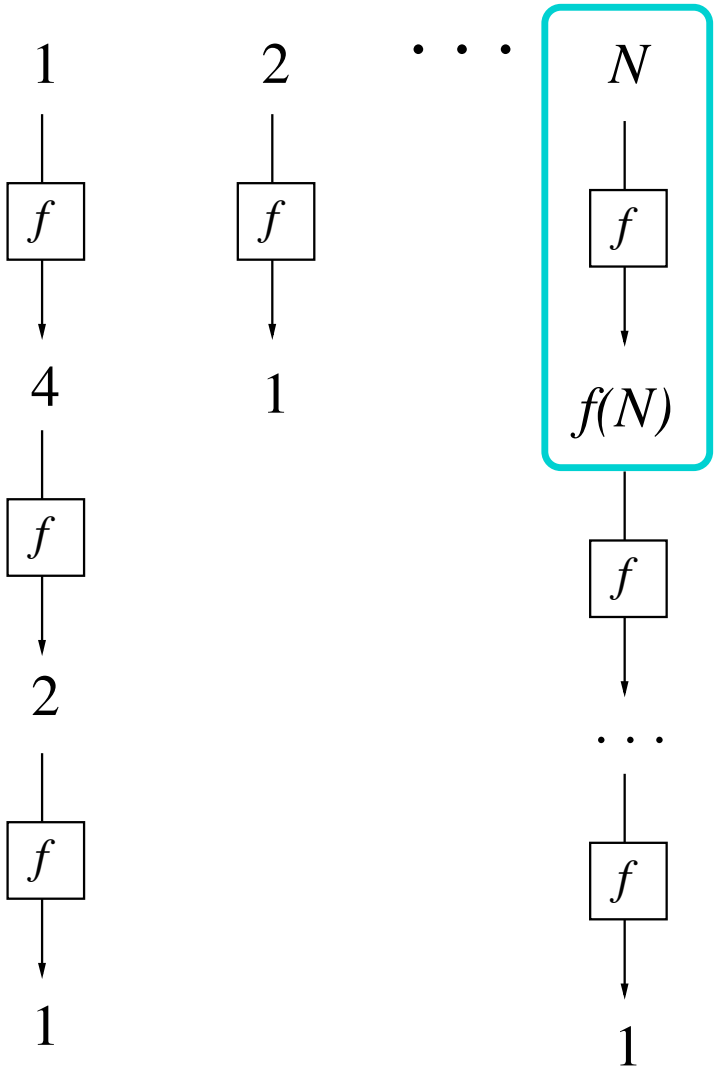
Define a function f, for n positive integer:

- if n is even, then $f(n) = n/2$,
- if n is odd, then $f(n) = 3n + 1$,

For every positive integer n, form the sequence $S(n)$: $n \to f(n) \to f(f(n)) \to f(f(f(n))) \to \ldots$

Conjecture: *For every positive integer n, the sequence $S(n)$ eventually hits 1.*

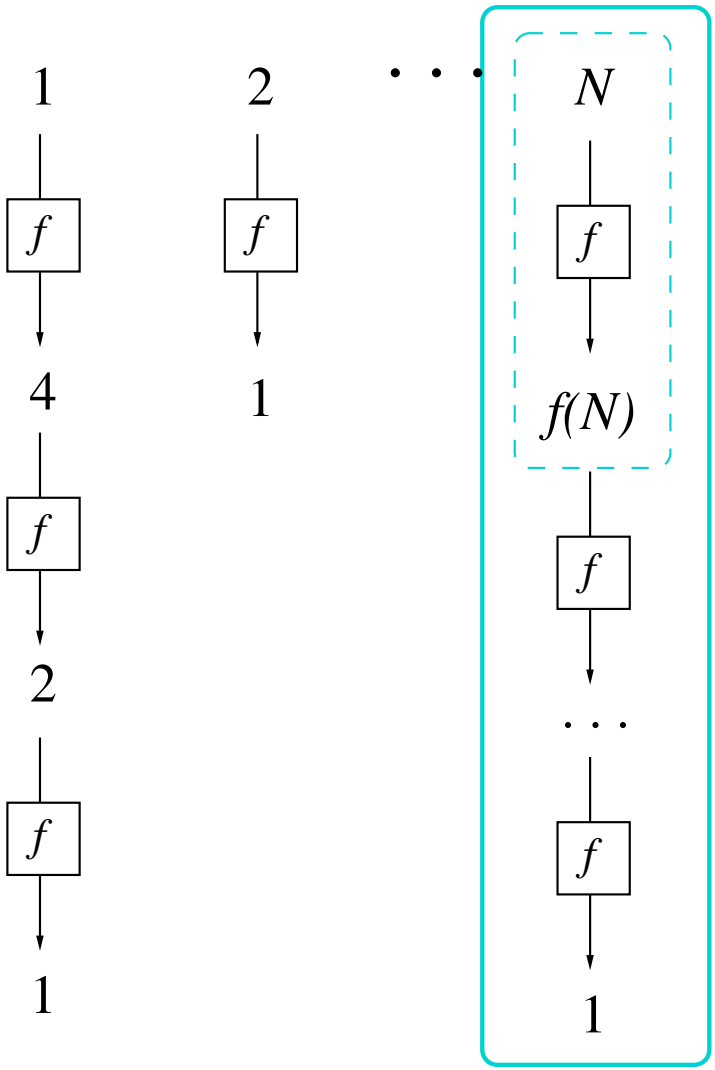
The *3n+1* conjecture, *I*



A computational job $F(n,k)$, applies function f to the result of $F(n,k-1)$.

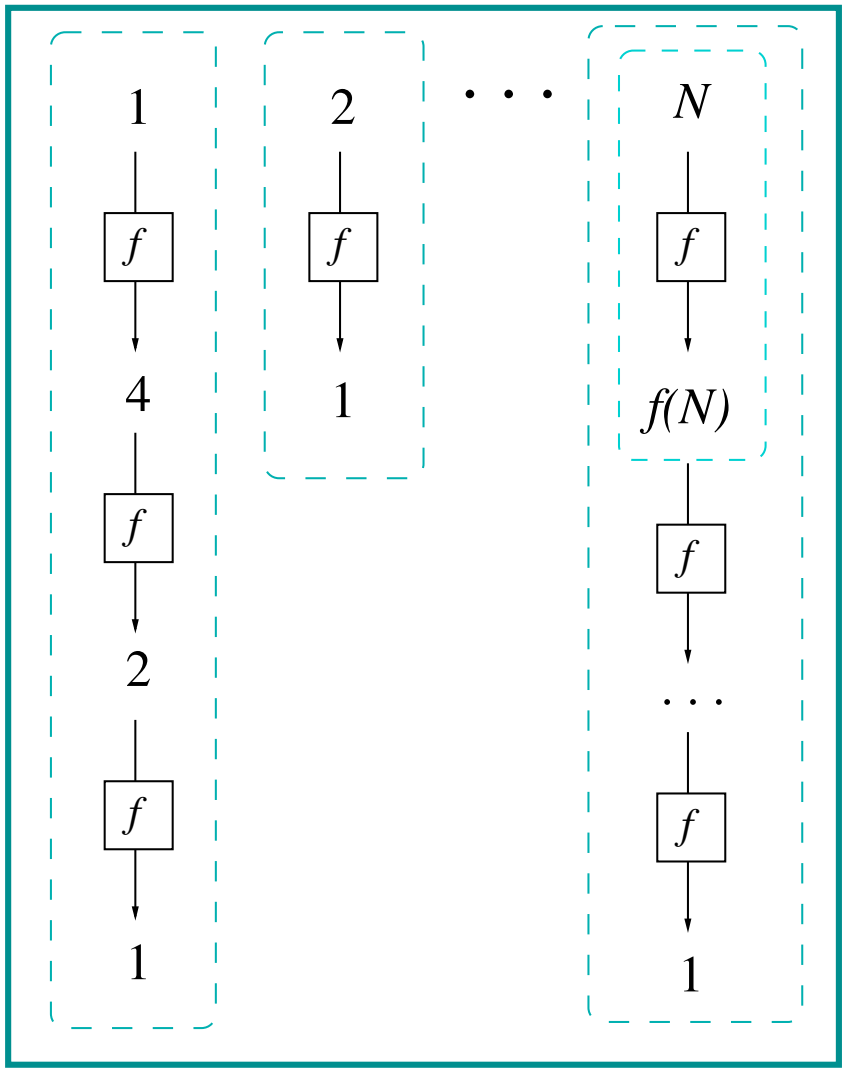
(With $F(n,0) = n$.)

The *3n+1* conjecture, *II*



A sequence $H(n)$ of jobs computes the chain $n \to f(n) \to \ldots \to 1$.

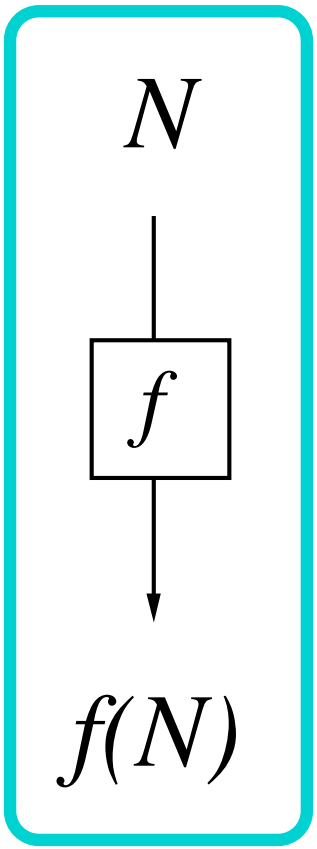
The *3n+1* conjecture, *III*



Run one sequence $H(n)$ per each $n = 1, \ldots, N$.

They all can run in **parallel**.

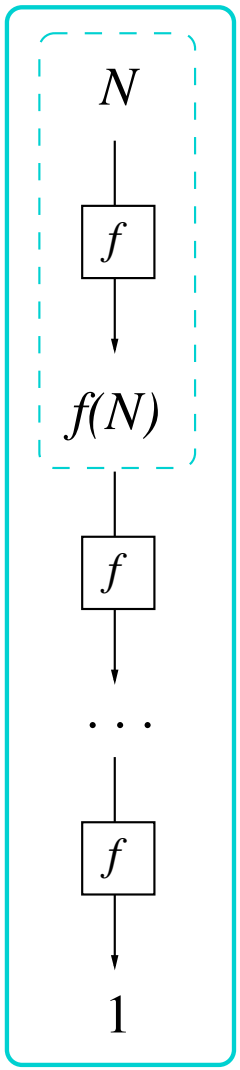
The *3n+1* conjecture, *IV*



Let's define the simple application that computes f:

```
class HotpoApplication(Application):
  def __init__(self, n):
    Application.__init__(
      self,
      arguments = (['/usr/bin/expr'] +
        # run `expr n / 2` if n even
        ([n, '/', n] if n % 2 == 0
        # `expr 1 + 3 * n` if n odd
        else [1, '+', 3, '*', n])),
      stdout = "stdout.txt",
      # ...
    )
```

The *3n+1* conjecture, *V*



Now string together applications to compute a single sequence:

```
from gc3libs.workflow \
  import SequentialTaskCollection as Seq

class HotpoSequence(Seq):

  def __init__(self, n):
    # compute first iteration of f
    SequentialTask.__init__(self,
      [ HotpoApplication(n) ])

  def next(self, k):
    last = self.tasks[k].result
    if last == 1:
      return 'TERMINATED'
    else:
      self.tasks.append(HotpoApplication(last))
      return 'RUNNING'
```

The `next()` method in `SequentialTaskCollection`

The **`next`**`()` method is called whenever a task in the sequence has turned to TERMINATED state.

```
from gc3libs.workflow \
  import SequentialTaskCollection as

class HotpoSequence(Seq):

  # ...

  def next(self, k):
    last = self.tasks[k].result
    if last == 1:
      return 'TERMINATED'
    else:
      self.tasks.append(
        HotpoApplication(last))
      return 'RUNNING'
```

The `next()` method in `SequentialTaskCollection`

The second argument to **`next`**`()` is the index (within `self.tasks`) of the task that just finished.

```
from gc3libs.workflow \
  import SequentialTaskCollection as

class HotpoSequence(Seq):

  # ...

  def next(self, k):
    last = self.tasks[k].result
    if last == 1:
      return 'TERMINATED'
    else:
      self.tasks.append(
        HotpoApplication(last))
      return 'RUNNING'
```

The `next()` method in `SequentialTaskCollection`

You can access all attributes of tasks that are already done.

```
from gc3libs.workflow \
  import SequentialTaskCollection as

class HotpoSequence(Seq):

  # ...

  def next(self, k):
    last = self.tasks[k].result
    if last == 1:
      return 'TERMINATED'
    else:
      self.tasks.append(
        HotpoApplication(last))
      return 'RUNNING'
```

The `next()` method in `SequentialTaskCollection`

Returning the state TERMINATED interrupts the sequence: no other tasks from this collection will be run.

```
from gc3libs.workflow \
  import SequentialTaskCollection as

class HotpoSequence(Seq):

  # ...

  def next(self, k):
    last = self.tasks[k].result
    if last == 1:
      return 'TERMINATED'
    else:
      self.tasks.append(
        HotpoApplication(last))
      return 'RUNNING'
```

The `next()` method in `SequentialTaskCollection`

It is entirely possible to modify the `SequentialTaskCollection` and add (or remove) tasks.

```
from gc3libs.workflow \
  import SequentialTaskCollection as

class HotpoSequence(Seq):

  # ...

  def next(self, k):
    last = self.tasks[k].result
    if last == 1:
      return 'TERMINATED'
    else:
      self.tasks.append(
        HotpoApplication(last))
      return 'RUNNING'
```

The `next()` method in `SequentialTaskCollection`

Returning state `RUNNING` makes the sequence continue with task `k+1`

```
from gc3libs.workflow \
  import SequentialTaskCollection as

class HotpoSequence(Seq):

  # ...

  def next(self, k):
    last = self.tasks[k].result
    if last == 1:
      return 'TERMINATED'
    else:
      self.tasks.append(
        HotpoApplication(last))
      return 'RUNNING'
```

The `next()` method in `SequentialTaskCollection`

Alternatively, you can **return** a number `j` less than `k`, meaning that the sequence will rewind to the `j`-th task and continue running from there.

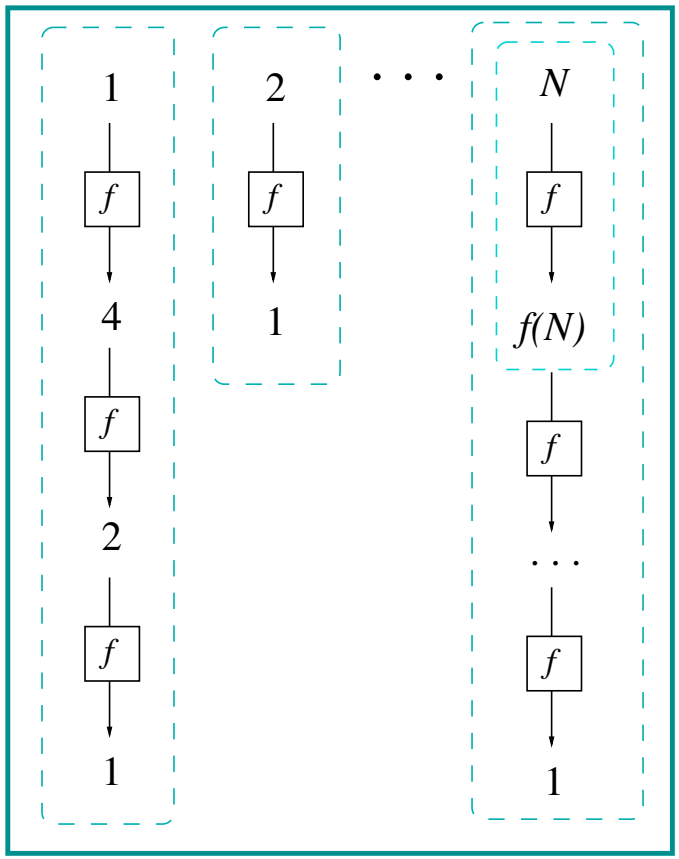
```
from gc3libs.workflow \
  import SequentialTaskCollection as

class HotpoSequence(Seq):

  # ...

  def next(self, k):
    last = self.tasks[k].result
    if last == 1:
      return 'TERMINATED'
    else:
      self.tasks.append(
        HotpoApplication(last))
      return 'RUNNING'
```

The *3n+1* conjecture, *VI*



Parallel tasks are independent by definition, so it's even easier to create a collection:

```
tasks = ParallelTaskCollection([
  HotpoSequence(n)
  for n in range(1, N)
])
```

We can run such a collection like any other `Task`.

Exercise 11.A:

Fill in the missing parts and write a `hotpo.py` session-based script that:

- takes a single integer parameter N on the command-line:

  ```
  $ python hotpo.py 42
  ```

- computes all the "$3n + 1$" sequences of numbers 1 up to N in parallel,
- prints a final statement that the Collatz conjecture is verified up to N (or —who knows— not?)