

NENR - 6. domaća zadaća: ANFIS

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1 Izvod pravila učenja

- izlaz sustava za k -ti uzorak: $o_k = \frac{\sum_{j=1}^m w_j z_j}{\sum_{j=1}^m w_j}$, $w_i = A_i(x)B_i(y)$
- funkcija pripadnosti: $A(x) = \sigma(x) = \frac{1}{1+e^{b(x-a)}}$
- pogreška uzorka: $E_k = \frac{1}{2}(t_k - o_k)^2$
- ažuriranje parametra ψ (općenito): $\psi(t+1) = \psi(t) - \eta \frac{\partial E_k}{\partial \psi}$

1.1 Pomoćne parcijalne derivacije

$$\frac{\partial E_k}{\partial o_k} = \frac{1}{2}2(t_k - o_k)(-1) = -(t_k - o_k) \quad (1)$$

$$\frac{\partial o_k}{\partial w_i} = \frac{\sum_{j=1}^m w_j(z_i - z_j)}{(\sum_{j=1}^m w_j)^2} \quad (2)$$

$$\frac{\partial o_k}{\partial z_i} = \frac{w_i}{\sum_{j=1}^m w_j} \quad (3)$$

$$\frac{\partial \sigma}{\partial x} = \sigma(x)(1 - \sigma(x)) \quad (4)$$

1.2 Ažuriranje parametra a_i

$$\frac{\partial E_k}{\partial a_i} = \frac{\partial E_k}{\partial o_k} \frac{\partial o_k}{\partial w_i} \frac{\partial w_i}{\partial a_i} \quad (5)$$

$$\frac{\partial w_i}{\partial a_i} = B_i(y)A_i(x)(1 - A_i(x))b_i \quad (6)$$

$$\frac{\partial E_k}{\partial a_i} = -(t_k - o_k) \frac{\sum_{j=1}^m w_j(z_i - z_j)}{(\sum_{j=1}^m w_j)^2} A_i(x)(1 - A_i(x))B_i(y)b_i \quad (7)$$

1.3 Ažuriranje parametra b_i

$$\frac{\partial E_k}{\partial b_i} = \frac{\partial E_k}{\partial o_k} \frac{\partial o_k}{\partial w_i} \frac{\partial w_i}{\partial b_i} \quad (8)$$

$$\frac{\partial w_i}{\partial b_i} = B_i(y)A_i(x)(1 - A_i(x))(a_i - x) \quad (9)$$

$$\frac{\partial E_k}{\partial b_i} = -(t_k - o_k) \frac{\sum_{j=1}^m w_j(z_i - z_j)}{(\sum_{j=1}^m w_j)^2} A_i(x)(1 - A_i(x))B_i(y)(a_i - x) \quad (10)$$

1.4 Ažuriranje parametra c_i

$$\frac{\partial E_k}{\partial c_i} = \frac{\partial E_k}{\partial o_k} \frac{\partial o_k}{\partial w_i} \frac{\partial w_i}{\partial c_i} \quad (11)$$

$$\frac{\partial w_i}{\partial c_i} = A_i(x)B_i(y)(1 - B_i(y))d_i \quad (12)$$

$$\frac{\partial E_k}{\partial c_i} = -(t_k - o_k) \frac{\sum_{j=1}^m w_j(z_i - z_j)}{(\sum_{j=1}^m w_j)^2} A_i(x)B_i(y)(1 - B_i(y))d_i \quad (13)$$

1.5 Ažuriranje parametra d_i

$$\frac{\partial E_k}{\partial d_i} = \frac{\partial E_k}{\partial o_k} \frac{\partial o_k}{\partial w_i} \frac{\partial w_i}{\partial d_i} \quad (14)$$

$$\frac{\partial w_i}{\partial d_i} = A_i(x)B_i(y)(1 - B_i(y))(c_i - y) \quad (15)$$

$$\frac{\partial E_k}{\partial d_i} = -(t_k - o_k) \frac{\sum_{j=1}^m w_j(z_i - z_j)}{(\sum_{j=1}^m w_j)^2} A_i(x)B_i(y)(1 - B_i(y))(c_i - y) \quad (16)$$

1.6 Ažuriranje parametra p_i

$$\frac{\partial E_k}{\partial p_i} = \frac{\partial E_k}{\partial o_k} \frac{\partial o_k}{\partial z_i} \frac{\partial z_i}{\partial p_i} \quad (17)$$

$$\frac{\partial z_i}{\partial p_i} = x \quad (18)$$

$$\frac{\partial E_k}{\partial p_i} = -(t_k - o_k) \frac{w_i}{\sum_{j=1}^m w_j} x \quad (19)$$

1.7 Ažuriranje parametra q_i

$$\frac{\partial E_k}{\partial q_i} = \frac{\partial E_k}{\partial o_k} \frac{\partial o_k}{\partial z_i} \frac{\partial z_i}{\partial q_i} \quad (20)$$

$$\frac{\partial z_i}{\partial q_i} = y \quad (21)$$

$$\frac{\partial E_k}{\partial q_i} = -(t_k - o_k) \frac{w_i}{\sum_{j=1}^m w_j} y \quad (22)$$

1.8 Ažuriranje parametra r_i

$$\frac{\partial E_k}{\partial r_i} = \frac{\partial E_k}{\partial o_k} \frac{\partial o_k}{\partial z_i} \frac{\partial z_i}{\partial r_i} \quad (23)$$

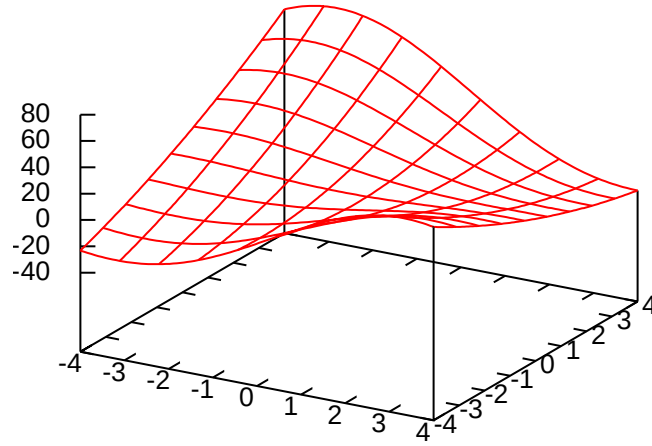
$$\frac{\partial z_i}{\partial r_i} = 1 \quad (24)$$

$$\frac{\partial E_k}{\partial r_i} = -(t_k - o_k) \frac{w_i}{\sum_{j=1}^m w_j} \quad (25)$$

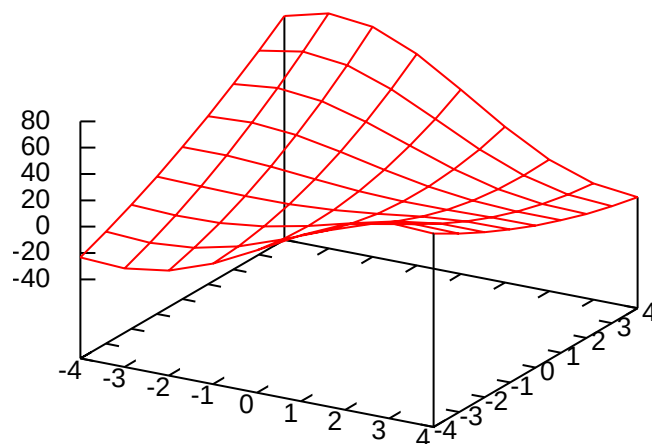
2 Varijante algoritma

- grupni gradijentni spust - ažuriranje parametara nakon izračuna sume gradijenata po svim uzorcima
- stohastički gradijentni spust - ažuriranje parametara nakon izračuna gradijenata za k -ti uzorak

3 Primjeri za učenje



Slika 1: Funkcija $((x-1)^2 + (y+2)^2 - 5xy + 3) * \cos^2(\frac{\pi}{5})$



Slika 2: Skup primjera za učenje generiran na intervalu $[-4, 4] \times [-4, 4] \subset Z \times Z$

4 Provedeni postupci učenja

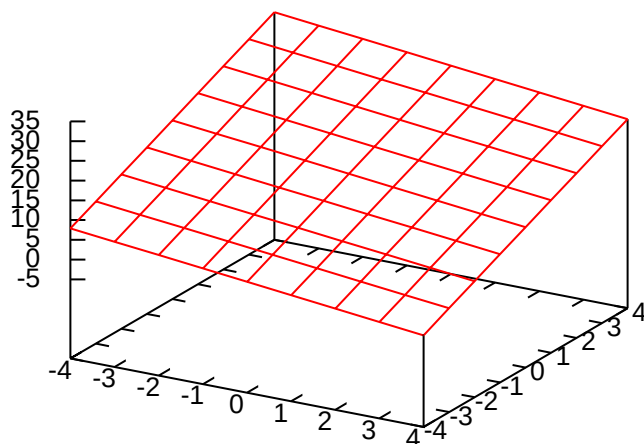
Provedeni su postupci učenja za sustav od jedno, dva i pet pravila. U svim postupcima korišteni su jednaki parametri:

- broj epoha (10 000),
- stopa učenja parametara antecedenata ($1e-5$) te
- stopa učenja parametara konzekventa ($1e-3$).

Na slikama 3, 4 i 5 prikazane su naučene funkcije, a na slikama 6, 7 i 8 njihova odgovarajuća odstupanja od skupa primjera za učenje. Porastom broja pravila, raste ekspresivnost i točnost sustava, a padaju iznosi pogreške uzorka.

4.1 Kretanje pogreške kroz epohe

Na slikama 9, 10 i 11 prikazano je kretanje srednje kvadratne pogreške po epohama za postupke učenja sustava sa jednim, dva i pet pravila. Za svaki od postupaka učenja prikazana su dva grafa - jedan za grupni, a drugi za stohastički postupak učenja.



Slika 3: Naučena funkcija u sustavu s jednim pravilom

5 Interpretacija pravila

Na slikama 12, 13, 14, 15 i 16 prikazane su funkcije pripadnosti neizrazitih skupova A i B te su dane njihove moguće interpretacije.

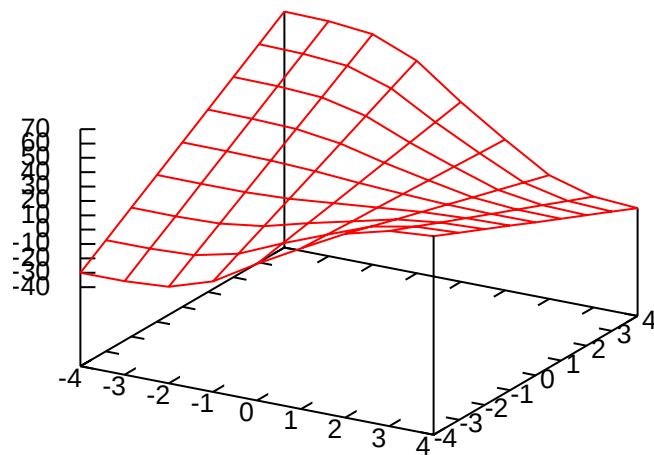
6 Odabir stope učenja

Na slici 17 prikazano je kretanje srednje kvadratne pogreške prilikom grupnog postupka učenja za:

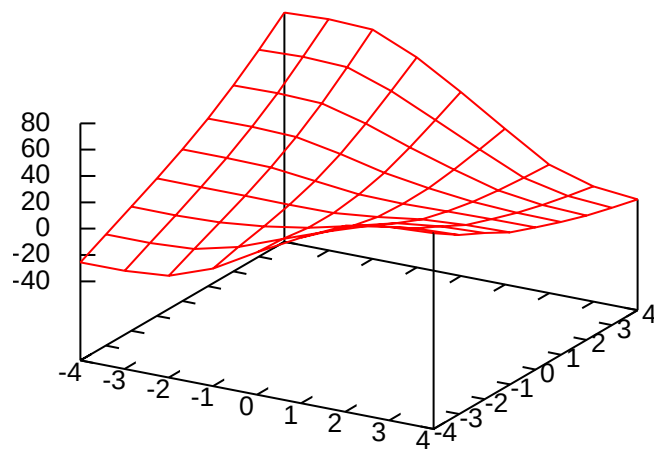
- premalu ($\eta_a = 1e-8$, $\eta_c = 1e-6$),
- prikladnu ($\eta_a = 1e-5$, $\eta_c = 1e-3$) te
- preveliku ($\eta_a = 1e-3$, $\eta_c = 1e-1$) stopu učenja.

Na slici 18 prikazano je kretanje srednje kvadratne pogreške prilikom stohastičkog postupka učenja za:

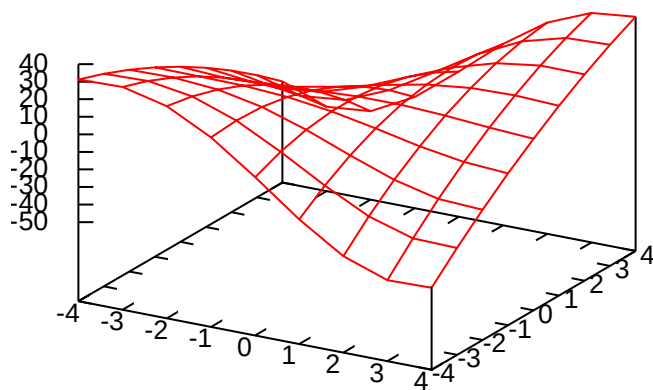
- premalu ($\eta_a = 1e-8$, $\eta_c = 1e-6$),
- prikladnu ($\eta_a = 1e-3$, $\eta_c = 1e-1$) te
- preveliku ($\eta_a = 1e-1$, $\eta_c = 5e-1$) stopu učenja.



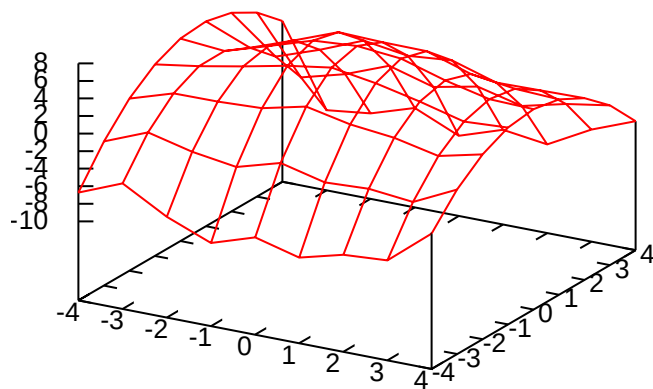
Slika 4: Naučena funkcija u sustavu s dvama pravilima



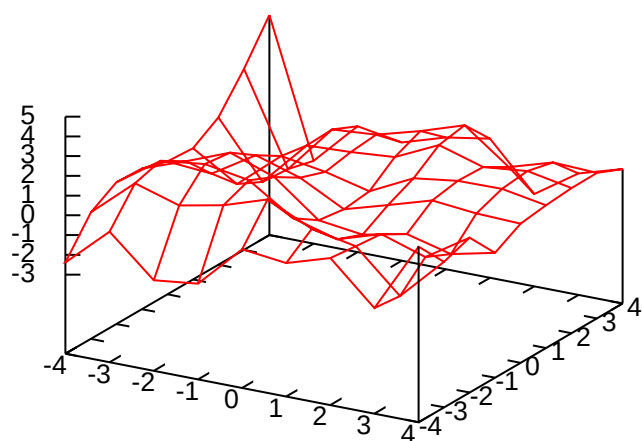
Slika 5: Naučena funkcija u sustavu s pet pravila



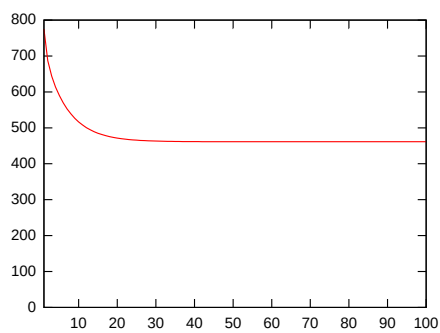
Slika 6: Pogreška δ u sustavu s jednim pravilom



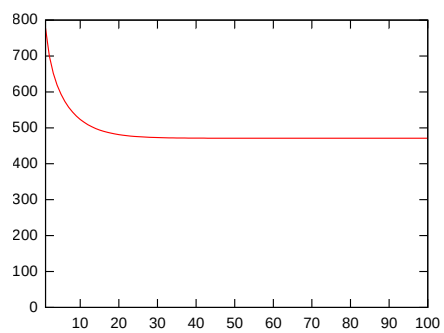
Slika 7: Pogreška δ u sustavu s dvama pravilima



Slika 8: Pogreška δ u sustavu s pet pravila

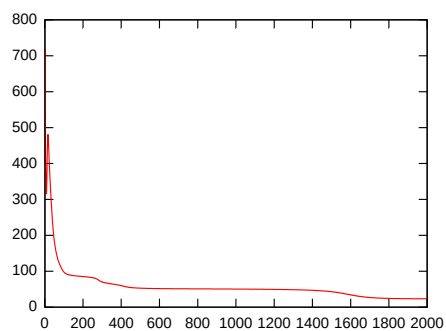


(a) grupni postupak

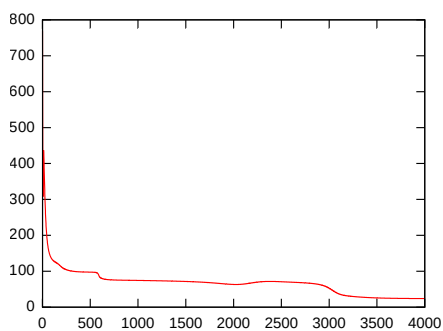


(b) stohastički postupak

Slika 9: Kretanje srednje pogreške u sustavu s jednim pravilom

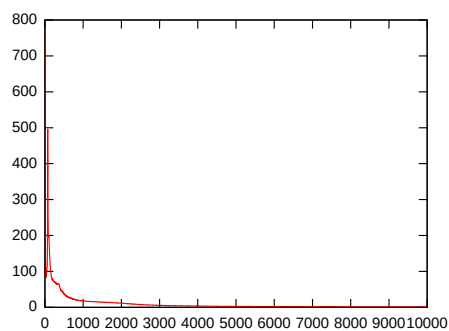


(a) grupni postupak

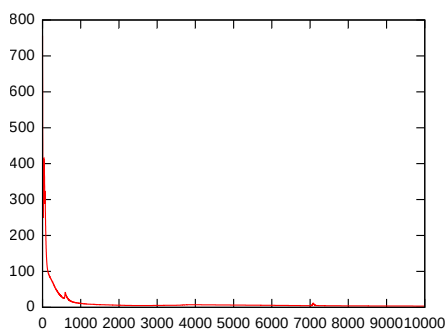


(b) stohastički postupak

Slika 10: Kretanje srednje pogreške u sustavu s dvama pravilima

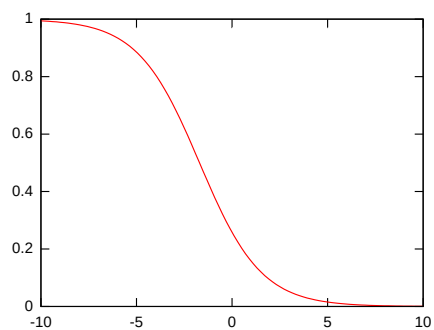


(a) grupni postupak

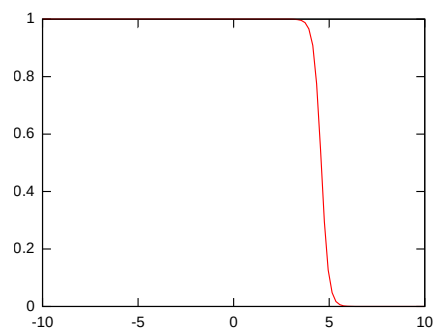


(b) stohastički postupak

Slika 11: Kretanje srednje pogreške u sustavu s pet pravila

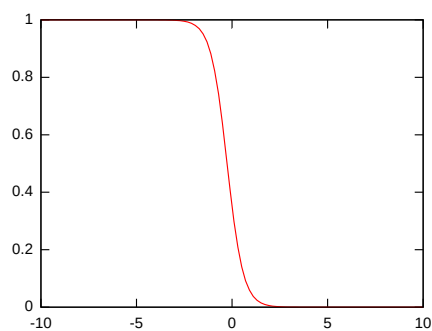


(a) $a \approx -1.70$, $b \approx 0.62$

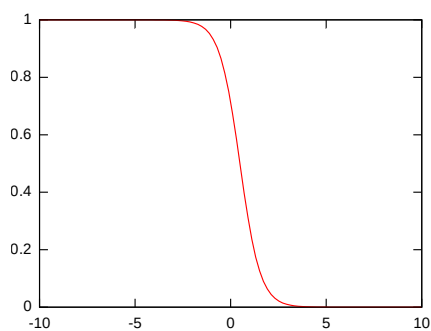


(b) $a \approx 4.58$, $b \approx 5.22$

Slika 12: Pravilo 1 - ako x je dosta manji od 5 i y je otprilike manji od 5

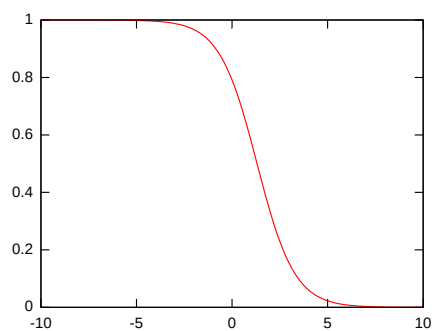


(a) $a \approx -0.26$, $b \approx 2.36$

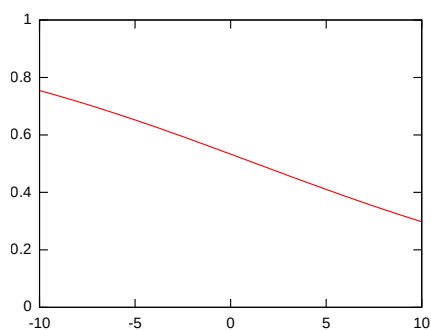


(b) $a \approx 0.49$, $b \approx 1.88$

Slika 13: Pravilo 2 - ako x je otprilike negativan i y je otprilike negativan

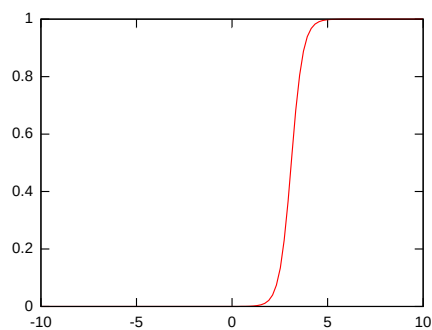


(a) $a \approx 1.31$, $b \approx 1.02$

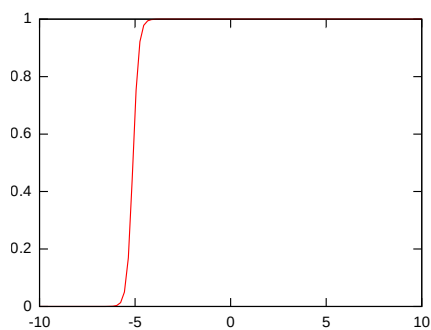


(b) $a \approx 1.34$, $b \approx 0.10$

Slika 14: Pravilo 3 - ako x je dosta manji od 5 i y je dosta manji od 30

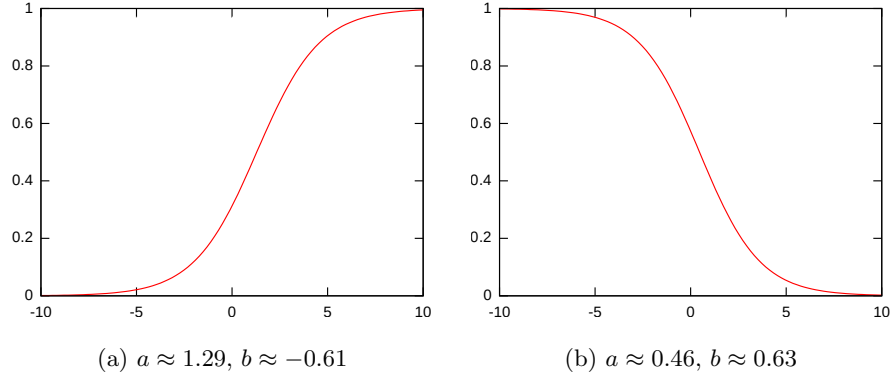


(a) $a \approx 3.10$, $b \approx -3.25$

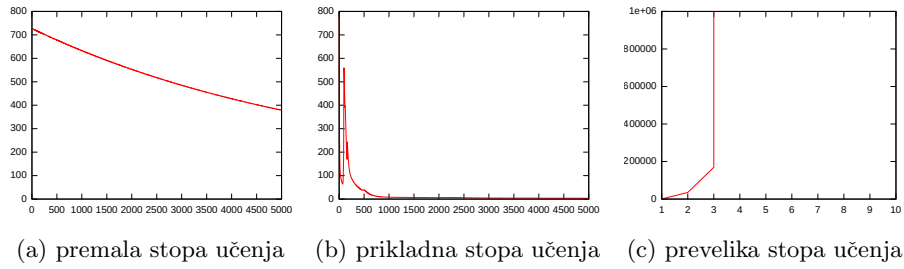


(b) $a \approx -5.12$, $b \approx -6.69$

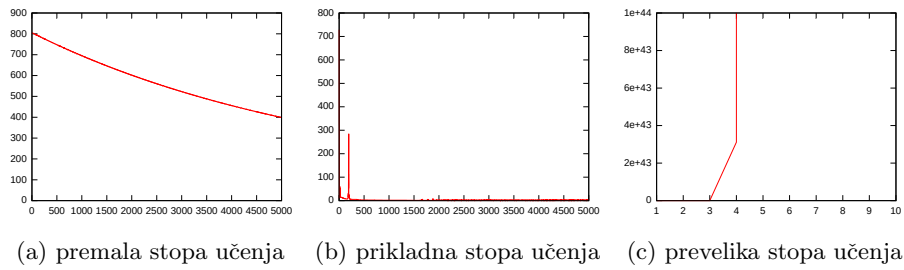
Slika 15: Pravilo 4 - ako x je otprilike veći od 3 i y je otprilike veći od -5



Slika 16: Pravilo 5 - ako x je dosta veći od -5 i y je dosta manji od 5



Slika 17: Kretanje srednje pogreške u sustavu s pet pravila - grupni postupak



Slika 18: Kretanje srednje pogreške u sustavu s pet pravila - stohastički postupak