WIKIPEDIA

Glossary of set theory

This is a **glossary of set theory**.

Contents:

Greek

α
: Often used for an ordinal

β
1. β*X* is the Stone–Čech compactification of *X*
2. An ordinal

γ
: A gamma number, an ordinal of the form ω^α

Γ
: The Gamma function of ordinals. In particular Γ_0 is the Feferman–Schütte ordinal.

δ
1. A delta number is an ordinal of the form ω^{ω^α}
2. A limit ordinal

Δ (Greek capital delta, not to be confused with a triangle ∆)
1. A set of formulas in the Lévy hierarchy
2. A delta system

ε
: An epsilon number, an ordinal with $\omega^\varepsilon=\varepsilon$

η
1. The order type of the rational numbers
2. An eta set, a type of ordered set
3. η_α is an Erdős cardinal

θ

The order type of the real numbers

Θ

The supremum of the ordinals that are the image of a function from $^{\omega}\omega$ (usually in models where the axiom of choice is not assumed)

κ

1. Often used for a cardinal, especially the critical point of an elementary embedding
2. The Erdős cardinal $\kappa(\alpha)$ is the smallest cardinal such that $\kappa(\alpha) \to (\alpha)^{<\omega}$

λ

1. Often used for a cardinal
2. The order type of the real numbers

μ

A measure

Π

1. A product of cardinals
2. A set of formulas in the Lévy hierarchy

ρ

The rank of a set

σ

countable, as in σ-compact, σ-complete and so on

Σ

1. A sum of cardinals
2. A set of formulas in the Lévy hierarchy

φ

A Veblen function

ω

1. The smallest infinite ordinal
2. ω_α is an alternative name for $\aleph_\alpha$, used when it is considered as an ordinal number rather than a cardinal number
3. An ω-huge cardinal is a large cardinal related to the I_1 rank-into-rank axiom

Ω

1. The class of all ordinals, related to Cantor's absolute
2. Ω-logic is a form of logic introduced by Hugh Woodin

!$@

∈, =, ⊆, ⊇, ⊃, ⊂, ∪, ∩, ∅

Standard set theory symbols with their usual meanings (is a member of, equals, is a subset of, is a superset of, is a proper superset of, is a proper subset of, union, intersection, empty set)

∧ ∨ → ↔ ¬ ∀ ∃

Standard logical symbols with their usual meanings (and, or, implies, is equivalent to, not, for all, there exists)

≡

An equivalence relation

↾

f↾*X* is now the restriction of a function or relation *f* to some set *X*, though its original meaning was the corestriction

1

f 1 *X* is the restriction of a function or relation *f* to some set *X*

Δ (A triangle, not to be confused with the Greek letter Δ)

1. The symmetric difference of two sets
2. A diagonal intersection

◊

The diamond principle

♣

A clubsuit principle

□

The square principle

∘

The composition of functions

⁀

$s^\frown x$ is the extension of a sequence *s* by *x*

+

1. Addition of ordinals
2. Addition of cardinals
3. α^+ is the smallest cardinal greater than α
4. B^+ is the poset of nonzero elements of a Boolean algebra *B*
5. The inclusive or operation in a Boolean algebra. (In ring theory it is used for the exclusive or operation)

~

1. The difference of two sets: *x*~*y* is the set of elements of *x* not in *y*.
2. An equivalence relation

****

The difference of two sets: $x \backslash y$ is the set of elements of x not in y.

−

The difference of two sets: $x-y$ is the set of elements of x not in y.

≈

Has the same cardinality as

×

A product of sets

/

A quotient of a set by an equivalence relation

·

1. $x \cdot y$ is the ordinal product of two ordinals
2. $x \cdot y$ is the cardinal product of two cardinals

*****

An operation that takes a forcing poset and a name for a forcing poset and produces a new forcing poset.

∞

The class of all ordinals, or at least something larger than all ordinals

α^β

1. Cardinal exponentiation
2. Ordinal exponentiation

${}^\beta\alpha$

1. The set of functions from β to α

→

1. Implies
2. $f:X\to Y$ means f is a function from X to Y.
3. The ordinary partition symbol, where $\kappa\to(\lambda)^n_m$ means that for every coloring of the n-element subsets of κ with m colors there is a subset of size λ all of whose n-element subsets are the same color.

f* ' *x

If there is a unique y such that $\langle x,y\rangle$ is in f then f ' x is y, otherwise it is the empty set. So if f is a function and x is in its domain, then f ' x is $f(x)$.

f* “ *X

f “ X is the image of a set X by f. If f is a function whose domain contains X this is $\{f(x):x\in X\}$

[ ]

1. $M[G]$ is the smallest model of ZF containing G and all elements of M.

2. $[\alpha]^\beta$ is the set of all subsets of a set α of cardinality β, or of an ordered set α of order type β
3. [*x*] is the equivalence class of *x*

{ }

1. {*a*, *b*, ...} is the set with elements *a*, *b*, ...
2. {*x* : φ(*x*)} is the set of *x* such that φ(*x*)

⟨ ⟩

⟨*a*,*b*⟩ is an ordered pair, and similarly for ordered *n*-tuples

|*X*|

The cardinality of a set *X*

||φ||

The value of a formula φ in some Boolean algebra

⌜φ⌝

⌜φ⌝ (Quine quotes, unicode U+231C, U+231D) is the Gödel number of a formula φ

⊦

A⊦φ means that the formula φ follows from the theory *A*

⊧

A⊧φ means that the formula φ holds in the model *A*

⊩

The forcing relation

≺

An elementary embedding

⊥

$p \perp q$ means that *p* and *q* are incompatible elements of a partial order

$0^\#$

zero sharp, the set of true formulas about indiscernibles and order-indiscernibles in the constructible universe

$0^\dagger$

zero dagger, a certain set of true formulas

ℵ

The Hebrew letter aleph, which indexes the aleph numbers or infinite cardinals $\aleph_\alpha$

ב

The Hebrew letter beth, which indexes the beth numbers $\beth_\alpha$

ג

A serif form of the Hebrew letter gimel, representing the gimel function $\gimel(\kappa) = \kappa^{\mathrm{cf}\,\kappa}$

ת

The Hebrew letter Taw, used by Cantor for the class of all cardinal numbers

A

$\mathfrak{a}$

The almost disjointness number, the least size of a maximal almost disjoint family of infinite subsets of ω

A

The Suslin operation

absolute

1. A statement is called absolute if its truth in some model implies its truth in certain related models
2. Cantor's absolute is a somewhat unclear concept sometimes used to mean the class of all sets
3. Cantor's Absolute Infinite Ω is a somewhat unclear concept related to the class of all ordinals

AC

1. AC is the Axiom of choice
2. AC_ω is the Axiom of countable choice

AD

The axiom of determinacy

add

additivity

The additivity add(*I*) of I is the smallest number of sets of *I* with union not in *I*

additively

An ordinal is called additively indecomposable if it is not the sum of a finite number of smaller ordinals. These are the same as gamma numbers or powers of ω.

admissible

An admissible set is a model of Kripke–Platek set theory, and an admissible ordinal is an ordinal α such that L_α is an admissible set

AH

Aleph hypothesis, a form of the generalized continuum hypothesis

aleph

1. The Hebrew letter ℵ
2. An infinite cardinal
3. The aleph function taking ordinals to infinite cardinals
4. The aleph hypothesis is a form of the generalized continuum hypothesis

almost universal
A class is called almost universal if every subset of it is contained in some member of it

amenable
An amenable set is a set that is a model of Kripke–Platek set theory without the axiom of collection

analytic
An analytic set is the continuous image of a Polish space. (This is not the same as an analytical set)

analytical
The analytical hierarchy is a hierarchy of subsets of an effective Polish space (such as ω). They are definable by a second-order formula without parameters, and an analytical set is a set in the analytical hierarchy. (This is not the same as an analytic set)

antichain
An antichain is a set of pairwise incompatible elements of a poset

antinomy
paradox

arithmetic
arithmetical
The arithmetical hierarchy is a hierarchy of subsets of a Polish space that can be defined by first-order formulas

Aronszajn
1. Nachman Aronszajn
2. An Aronszajn tree is an uncountable tree such that all branches and levels are countable. More generally a κ-Aronszajn tree is a tree of cardinality κ such that all branches and levels have cardinality less than κ

atom
1. An urelement, something that is not a set but allowed to be an element of a set
2. An element of a poset such that any two elements smaller than it are compatible.
3. A set of positive measure such that every measurable subset has the same measure or measure 0

atomic
An atomic formula (in set theory) is one of the form $x=y$ or $x\in y$

axiom
Aczel's anti-foundation axiom states that every accessible pointed directed graph corresponds to a unique set
AD+ An extension of the axiom of determinacy
Axiom F states that the class of all ordinals is Mahlo
Axiom of adjunction Adjoining a set to another set produces a set
Axiom of amalgamation The union of all elements of a set is a set. Same as axiom of union
Axiom of choice The product of any set of non-empty sets is non-empty
Axiom of collection This can mean either the axiom of replacement or the axiom of separation
Axiom of comprehension The class of all sets with a given property is a set. Usually contradictory.
Axiom of constructibility Any set is constructible, often abbreviated as $V=L$

Axiom of countability Every set is hereditarily countable
Axiom of countable choice The product of a countable number of non-empty sets is non-empty
Axiom of dependent choice A weak form of the axiom of choice
Axiom of determinacy Certain games are determined, in other words one player has a winning strategy
Axiom of elementary sets describes the sets with 0, 1, or 2 elements
Axiom of empty set The empty set exists
Axiom of extensionality or axiom of extent
Axiom of finite choice Any product of non-empty finite sets is non-empty
Axiom of foundation Same as axiom of regularity
Axiom of global choice There is a global choice function
Axiom of heredity (any member of a set is a set; used in Ackermann's system.)
Axiom of infinity There is an infinite set
Axiom of limitation of size A class is a set if and only if it has smaller cardinality than the class of all sets
Axiom of pairing Unordered pairs of sets are sets
Axiom of power set The powerset of any set is a set
Axiom of projective determinacy Certain games given by projective set are determined, in other words one player has a winning strategy
Axiom of real determinacy Certain games are determined, in other words one player has a winning strategy
Axiom of regularity Sets are well founded
Axiom of replacement The image of a set under a function is a set. Same as axiom of substitution
Axiom of subsets The powerset of a set is a set. Same as axiom of powersets
Axiom of substitution The image of a set under a function is a set
Axiom of union The union of all elements of a set is a set
Axiom schema of predicative separation Axiom of separation for formulas whose quantifiers are bounded
Axiom schema of replacement The image of a set under a function is a set
Axiom schema of separation The elements of a set with some property form a set
Axiom schema of specification The elements of a set with some property form a set. Same as axiom schema of separation
Freiling's axiom of symmetry is equivalent to the negation of the continuum hypothesis
Martin's axiom states very roughly that cardinals less than the cardinality of the continuum behave like $\aleph_0$.
The proper forcing axiom is a strengthening of Martin's axiom

B

$\mathfrak{b}$

The bounding number, the least size of an unbounded family of sequences of natural numbers

B

A Boolean algebra

BA

Baumgartner's axiom, one of three axioms introduced by Baumgartner.

BACH
: Baumgartner's axiom plus the continuum hypothesis.

Baire
1. René-Louis Baire
2. A subset of a topological space has the Baire property if it differs from an open set by a meager set
3. The Baire space is a topological space whose points are sequences of natural numbers
4. A Baire space is a topological space such that every intersection of a countable collection of open dense sets is dense

basic set theory
1. Naive set theory
2. A weak set theory, given by Kripke–Platek set theory without the axiom of collection

BC
: Berkeley cardinal

BD
: Borel determinacy

Berkeley cardinal
: A Berkeley cardinal is a cardinal κ in a model of ZF such that for every transitive set *M* that includes κ, there is a nontrivial elementary embedding of *M* into *M* with critical point below κ.

Bernays
1. Paul Bernays
2. Bernays–Gödel set theory is a set theory with classes

Berry's paradox
: Berry's paradox considers the smallest positive integer not definable in ten words

Berkeley cardinal
: A Berkeley cardinal is a cardinal κ in a model of ZF such that for every transitive set *M* that includes κ, there is a nontrivial elementary embedding of *M* into *M* with critical point below κ.

beth
1. The Hebrew letter ב
2. A beth number $\beth_\alpha$

BG
: Bernays–Gödel set theory without the axiom of choice

BGC
: Bernays–Gödel set theory with the axiom of choice

boldface

The boldface hierarchy is a hierarchy of subsets of a Polish space, definable by second-order formulas with parameters (as opposed to the lightface hierarchy which does not allow parameters). It includes the Borel sets, analytic sets, and projective sets

Boolean algebra

A Boolean algebra is a commutative ring such that all elements satisfy $x^2=x$

Borel

1. Émile Borel
2. A Borel set is a set in the smallest sigma algebra containing the open sets

bounding number

The bounding number is the least size of an unbounded family of sequences of natural numbers

BP

Baire property

BS

BST

Basic set theory

Burali-Forti

1. Cesare Burali-Forti
2. The Burali-Forti paradox states that the ordinal numbers do not form a set

C

c
$\mathfrak{c}$

The cardinality of the continuum

$\mathfrak{C}$

Complement of a set

C

The Cantor set

cac

countable antichain condition (same as the countable chain condition)

Cantor

1. Georg Cantor
2. The Cantor normal form of an ordinal is its base ω expansion.
3. Cantor's paradox says that the powerset of a set is larger than the set, which gives a contradiction when applied to the universal set.
4. The Cantor set, a perfect nowhere dense subset of the real line

5. Cantor's absolute infinite Ω is something to do with the class of all ordinals
6. Cantor's absolute is a somewhat unclear concept sometimes used to mean the class of all sets
7. Cantor's theorem states that the powerset operation increases cardinalities

Card
: The cardinality of a set

cardinal
1. A cardinal number is an ordinal with more elements than any smaller ordinal

cardinality
: The number of elements of a set

categorical
1. A theory is called categorical if all models are isomorphic. This definition is no longer used much, as first-order theories with infinite models are never categorical.
2. A theory is called κ-categorical if all models of cardinality κ are isomorphic

category
1. A set of first category is the same as a meager set: a set that is the union of a countable number of nowhere-dense sets, and a set of second category is a set that is not of first category.
2. A category in the sense of category theory.

ccc
: countable chain condition

cf
: The cofinality of an ordinal

CH
: The continuum hypothesis

chain
: A linearly ordered subset (of a poset)

cl
: Abbreviation for "closure of" (a set under some collection of operations)

class
1. A class is a collection of sets
2. First class ordinals are finite ordinals, and second class ordinals are countable infinite ordinals

club
: A contraction of "closed unbounded"
1. A club set is a closed unbounded subset, often of an ordinal
2. The club filter is the filter of all subsets containing a club set
3. Clubsuit is a combinatorial principle similar to but weaker than the diamond principle

coanalytic
A coanalytic set is the complement of an analytic set

cofinal
A subset of a poset is called cofinal if every element of the poset is at most some element of the subset.

cof
confinality

cofinality
1. The cofinality of a poset (especially an ordinal or cardinal) is the smallest cardinality of a cofinal subset
2. The cofinality cof(*I*) of an ideal *I* of subsets of a set *X* is the smallest cardinality of a subset *B* of *I* such that every element of *I* is a subset of something in *B*.

Cohen
1. Paul Cohen
2. Cohen forcing is a method for constructing models of ZFC
3. A Cohen algebra is a Boolean algebra whose completion is free

Col
collapsing algebra
A collapsing algebra Col(κ,λ) collapses cardinals between λ and κ

complete
1. "Complete set" is an old term for "transitive set"
2. A theory is called complete if it assigns a truth value (true or false) to every statement of its language
3. An ideal is called κ-complete if it is closed under the union of less than κ elements
4. A measure is called κ-complete if the union of less than κ measure 0 sets has measure 0
5. A linear order is called complete if every nonempty bounded subset has a least upper bound

Con
Con(*T*) for a theory *T* means *T* is consistent

condensation lemma
Gödel's condensation lemma says that an elementary submodel of an element L_α of the constructible hierarchy is isomorphic to an element L_γ of the constructible hierarchy

constructible
A set is called constructible if it is in the constructible universe.

continuum
The continuum is the real line or its cardinality

core
A core model is a special sort of inner model generalizing the constructible universe

countable antichain condition

A term used for the countable chain condition by authors who think terminology should be logical

countable chain condition

The countable chain condition (ccc) for a poset states that every antichain is countable

cov(*I*)

covering number

The covering number cov(*I*) of an ideal *I* of subsets of *X* is the smallest number of sets in *I* whose union is *X*.

critical

1. The critical point κ of an elementary embedding *j* is the smallest ordinal κ with *j*(κ) > κ
2. A critical number of a function *j* is an ordinal κ with *j*(κ) = κ. This is almost the opposite of the first meaning.

CRT

The critical point of something

CTM

Countable transitive model

cumulative hierarchy

A cumulative hierarchy is a sequence of sets indexed by ordinals that satisfies certain conditions and whose union is used as a model of set theory

D

𝔡

The dominating number of a poset

DC

The axiom of dependent choice

def

The set of definable subsets of a set

definable

A subset of a set is called definable set if it is the collection of elements satisfying a sentence in some given language

delta

1. A delta number is an ordinal of the form $\omega^{\omega^{\alpha}}$
2. A delta system, also called a sunflower, is a collection of sets such that any two distinct sets have intersection *X* for some fixed set *X*

denumerable

countable and infinite

Df

The set of definable subsets of a set

diagonal intersection

If $\langle X_\alpha \mid \alpha < \delta \rangle$ is a sequence of subsets of an ordinal δ, then the diagonal intersection $\Delta_{\alpha<\delta} X_\alpha$, is $\{\beta < \delta \mid \beta \in \bigcap_{\alpha<\beta} X_\alpha\}$.

diamond principle

Jensen's diamond principle states that there are sets $A_\alpha \subseteq \alpha$ for $\alpha < \omega_1$ such that for any subset A of ω_1 the set of α with $A \cap \alpha = A_\alpha$ is stationary in ω_1.

dom

The domain of a function

DST

Descriptive set theory

E

E

E(*X*) is the membership relation of the set *X*

Easton's theorem

Easton's theorem describes the possible behavior of the powerset function on regular cardinals

EATS

The statement "every Aronszajn tree is special"

elementary

An elementary embedding is a function preserving all properties describable in the language of set theory

epsilon

1. An epsilon number is an ordinal α such that $\alpha = \omega^\alpha$
2. Epsilon zero (ε_0) is the smallest epsilon number

Erdos
Erdős

1. Paul Erdős
2. An Erdős cardinal is a large cardinal satisfying a certain partition condition. (They are also called partition cardinals.)
3. The Erdős–Rado theorem extends Ramsey's theorem to infinite cardinals

ethereal cardinal

An ethereal cardinal is a type of large cardinal similar in strength to subtle cardinals

extender

An extender is a system of ultrafilters encoding an elementary embedding

extendible cardinal

A cardinal κ is called extendible if for all η there is a nontrivial elementary embedding of $V_{\kappa+\eta}$ into some V_λ with critical point κ

extension

1. If *R* is a relation on a class then the extension of an element *y* is the class of *x* such that *xRy*
2. An extension of a model is a larger model containing it

extensional

1. A relation *R* on a class is called extensional if every element *y* of the class is determined by its extension
2. A class is called extensional if the relation ∈ on the class is extensional

F

F

An F_σ is a union of a countable number of closed sets

Feferman–Schütte ordinal

The Feferman–Schütte ordinal Γ_0 is in some sense the smallest impredicative ordinal

filter

A filter is a non-empty subset of a poset that is downward-directed and upwards-closed

finite intersection property
FIP

The finite intersection property, abbreviated FIP, says that the intersection of any finite number of elements of a set is non-empty

first

1. A set of first category is the same as a meager set: one that is the union of a countable number of nowhere-dense sets.
2. An ordinal of the first class is a finite ordinal
3. An ordinal of the first kind is a successor ordinal
4. First-order logic allows quantification over elements of a model, but not over subsets

Fodor

1. Géza Fodor
2. Fodor's lemma states that a regressive function on a regular uncountable cardinal is constant on a stationary subset.

forcing

Forcing (mathematics) is a method of adjoining a generic filter *G* of a poset *P* to a model of set theory *M* to obtain a new model *M*[*G*]

formula

Something formed from atomic formulas *x*=*y*, *x*∈*y* using ∀∃∧∨¬

Fraenkel

Abraham Fraenkel

G

$\mathfrak{g}$

The groupwise density number

G

1. A generic ultrafilter
2. A G_δ is a countable intersection of open sets

gamma number

A gamma number is an ordinal of the form ω^α

GCH

Generalized continuum hypothesis

generalized continuum hypothesis

The generalized continuum hypothesis states that $2^{\aleph_\alpha} = \aleph_{\alpha+1}$

generic

1. A generic filter of a poset *P* is a filter that intersects all dense subsets of *P* that are contained in some model *M*.
2. A generic extension of a model *M* is a model *M*[*G*] for some generic filter *G*.

gimel

1. The Hebrew letter gimel $\gimel$
2. The gimel function $\gimel(\kappa) = \kappa^{\mathrm{cf}(\kappa)}$
3. The gimel hypothesis states that $\gimel(\kappa) = \max(2^{\mathrm{cf}(\kappa)}, \kappa^+)$

global choice

The axiom of global choice says there is a well ordering of the class of all sets

Godel
Gödel

1. Kurt Gödel
2. A Gödel number is a number assigned to a formula
3. The Gödel universe is another name for the constructible universe
4. Gödel's incompleteness theorems show that sufficiently powerful consistent recursively enumerable theories cannot be complete
5. Gödel's completeness theorem states that consistent first-order theories have models

H

$\mathfrak{h}$

The distributivity number

H
: Abbreviation for "hereditarily"

H_κ

$H(\kappa)$
: The set of sets that are hereditarily of cardinality less than κ

Hartogs
1. Friedrich Hartogs
2. The Hartogs number of a set *X* is the least ordinal α such that there is no injection from α into *X*.

Hausdorff
1. Felix Hausdorff
2. A Hausdorff gap is a gap in the ordered set of growth rates of sequences of integers, or in a similar ordered set

HC
: The set of hereditarily countable sets

hereditarily
: If *P* is a property the a set is hereditarily *P* if all elements of its transitive closure have property *P*. Examples: Hereditarily countable set Hereditarily finite set

Hessenberg
1. Gerhard Hessenberg
2. The Hessenberg sum and Hessenberg product are commutative operations on ordinals

HF
: The set of hereditarily finite sets

Hilbert
1. David Hilbert
2. Hilbert's paradox states that a Hotel with an infinite number of rooms can accommodate extra guests even if it is full

HS
: The class of hereditarily symmetric sets

HOD
: The class of hereditarily ordinal definable sets

huge cardinal
: A huge cardinal is a cardinal number κ such that there exists an elementary embedding $j : V \to M$ with critical point κ from *V* into a transitive inner model *M* containing all sequences of length $j(\kappa)$ whose elements are in *M*

hyperarithmetic
: A hyperarithmetic set is a subset of the natural numbers given by a transfinite extension of the notion of arithmetic set

hyperinaccessible

hyper-inaccessible
1. "Hyper-inaccessible cardinal" usually means a 1-inaccessible cardinal
2. "Hyper-inaccessible cardinal" sometimes means a cardinal κ that is a κ-inaccessible cardinal
3. "Hyper-inaccessible cardinal" occasionally means a Mahlo cardinal

hyper-Mahlo
A hyper-Mahlo cardinal is a cardinal κ that is a κ-Mahlo cardinal

hyperverse
The hyperverse is the set of countable transitive models of ZFC

I

𝔦
The independence number

I0, I1, I2, I3
The rank-into-rank large cardinal axioms

ideal
An ideal in the sense of ring theory, usually of a Boolean algebra, especially the Boolean algebra of subsets of a set

iff
if and only if

inaccessible cardinal
A (weakly or strongly) inaccessible cardinal is a regular uncountable cardinal that is a (weak or strong) limit

indecomposable ordinal
An indecomposable ordinal is a nonzero ordinal that is not the sum of two smaller ordinals, or equivalently an ordinal of the form ω^α or a gamma number.

independence number
The independence number 𝔦 is the smallest possible cardinality of a maximal independent family of subsets of a countable infinite set

indescribable cardinal
An indescribable cardinal is a type of large cardinal that cannot be described in terms of smaller ordinals using a certain language

individual
Something with no elements, either the empty set or an urelement or atom

indiscernible
A set of indiscernibles is a set *I* of ordinals such that two increasing finite sequences of elements of *I* have the same first-order properties

inductive
A poset is called inductive if every non-empty ordered subset has an upper bound

ineffable cardinal
An ineffable cardinal is a type of large cardinal related to the generalized Kurepa hypothesis whose consistency strength lies between that of subtle cardinals and remarkable cardinals

inner model
An inner model is a transitive model of ZF containing all ordinals

Int
Interior of a subset of a topological space

internal
An archaic term for extensional (relation)

J

j
An elementary embedding

J
Levels of the Jensen hierarchy

Jensen
1. Ronald Jensen
2. The Jensen hierarchy is a variation of the constructible hierarchy
3. Jensen's covering theorem states that if $0^{\#}$ does not exist then every uncountable set of ordinals is contained in a constructible set of the same cardinality

Jónsson
1. Bjarni Jónsson
2. A Jónsson cardinal is a large cardinal such that for every function *f*: $[\kappa]^{<\omega} \to \kappa$ there is a set *H* of order type κ such that for each *n*, *f* restricted to *n*-element subsets of *H* omits at least one value in κ.
3. A Jónsson function is a function $f:[x]^{\omega}\to x$ with the property that, for any subset *y* of *x* with the same cardinality as *x*, the restriction of f to $[y]^{\omega}$ has image x.

K

Kelley
1. John L. Kelley
2. Morse–Kelley set theory, a set theory with classes

KH

Kurepa's hypothesis

kind

Ordinals of the first kind are successor ordinals, and ordinals of the second kind are limit ordinals or 0

KM

Morse–Kelley set theory

Kleene–Brouwer ordering

The Kleene–Brouwer ordering is a total order on the finite sequences of ordinals

KP

Kripke–Platek set theory

Kripke

1. Saul Kripke
2. Kripke–Platek set theory consists roughly of the predicative parts of set theory

Kurepa

1. Đuro Kurepa
2. The Kurepa hypothesis states that Kurepa trees exist
3. A Kurepa tree is a tree (T, <) of height ω_1, each of whose levels is countable, with at least $\aleph_2$ branches

L

L

1. L is the constructible universe, and L_α is the hierarchy of constructible sets
2. $L_{\kappa\lambda}$ is an infinitary language

large cardinal

1. A large cardinal is type of cardinal whose existence cannot be proved in ZFC.
2. A large large cardinal is a large cardinal that is not compatible with the axiom V=L

Laver

1. Richard Laver
2. A Laver function is a function related to supercompact cardinals that takes ordinals to sets

Lebesgue

1. Henri Lebesgue
2. Lebesgue measure is a complete translation-invariant measure on the real line

LEM

Law of the excluded middle

Lévy

1. Azriel Lévy
2. The Lévy collapse is a way of destroying cardinals
3. The Lévy hierarchy classifies formulas in terms of the number of alternations of unbounded quantifiers

lightface

The lightface classes are collections of subsets of an effective Polish space definable by second-order formulas without parameters (as opposed to the boldface hierarchy that allows parameters). They include the arithmetical, hyperarithmetical, and analytical sets

limit

1. A (weak) limit cardinal is a cardinal, usually assumed to be nonzero, that is not the successor κ^+ of another cardinal κ
2. A strong limit cardinal is a cardinal, usually assumed to be nonzero, larger than the powerset of any smaller cardinal
3. A limit ordinal is an ordinal, usually assumed to be nonzero, that is not the successor α+1 of another ordinal α

limited

A limited quantifier is the same as a bounded quantifier

LM

Lebesgue measure

local

A property of a set *x* is called local if it has the form $\exists\delta\ V_\delta \models \varphi(x)$ for some formula φ

LOTS

Linearly ordered topological space

Löwenheim

1. Leopold Löwenheim
2. The Löwenheim–Skolem theorem states that if a first-order theory has an infinite model then it has a model of any given infinite cardinality

LST

The language of set theory (with a single binary relation ∈)

M

m

1. A measure
2. A natural number

$\mathfrak{m}$

The smallest cardinal at which Martin's axiom fails

M

1. A model of ZF set theory

2. M_α is an old symbol for the level L_α of the constructible universe

MA

Martin's axiom

MAD

Maximally Almost Disjoint

Mac Lane

1. Saunders Mac Lane
2. Mac Lane set theory is Zermelo set theory with the axiom of separation restricted to formulas with bounded quantifiers

Mahlo

1. Paul Mahlo
2. A Mahlo cardinal is an inaccessible cardinal such that the set of inaccessible cardinals less than it is stationary

Martin

1. Donald A. Martin
2. Martin's axiom for a cardinal κ states that for any partial order *P* satisfying the countable chain condition and any family *D* of dense sets in *P* of cardinality at most κ, there is a filter *F* on *P* such that *F* ∩ *d* is non-empty for every *d* in *D*
3. Martin's maximum states that if *D* is a collection of $\aleph_1$ dense subsets of a notion of forcing that preserves stationary subsets of ω_1, then there is a *D*-generic filter

meager
meagre

A meager set is one that is the union of a countable number of nowhere-dense sets. Also called a set of first category.

measure

1. A measure on a σ-algebra of subsets of a set
2. A probability measure on the algebra of all subsets of some set
3. A measure on the algebra of all subsets of a set, taking values 0 and 1

measurable cardinal

A measurable cardinal is a cardinal number κ such that there exists a κ-additive, non-trivial, 0-1-valued measure on the power set of κ. Most (but not all) authors add the condition that it should be uncountable

mice

Plural of mouse

Milner–Rado paradox

The Milner–Rado paradox states that every ordinal number α less than the successor κ^+ of some cardinal number κ can be written as the union of sets X1,X2,... where Xn is of order type at most κ^n for n a positive integer.

MK

Morse–Kelley set theory

MM

Martin's maximum

morass

A morass is a tree with ordinals associated to the nodes and some further structure, satisfying some rather complicated axioms.

Morse

1. Anthony Morse
2. Morse–Kelley set theory, a set theory with classes

Mostowski

1. Andrzej Mostowski
2. The Mostowski collapse is a transitive class associated to a well founded extensional set-like relation.

mouse

A certain kind of structure used in constructing core models; see mouse (set theory)

multiplicative axiom

An old name for the axiom of choice

N

N

1. The set of natural numbers
2. The Baire space ω^ω

naive set theory

1. Naive set theory can mean set theory developed non-rigorously without axioms
2. Naive set theory can mean the inconsistent theory with the axioms of extensionality and comprehension
3. Naive set theory is an introductory book on set theory by Halmos

natural

The natural sum and natural product of ordinals are the Hessenberg sum and product

NCF

Near Coherence of Filters

non

non(I) is the uniformity of I, the smallest cardinality of a subset of X not in the ideal I of subsets of X

nonstat

nonstationary

1. A subset of an ordinal is called nonstationary if it is not stationary, in other words if its complement contains a club set
2. The **nonstationary ideal** I_{NS} is the ideal of nonstationary sets

normal

1. A normal function is a continuous strictly increasing function from ordinals to ordinals
2. A normal filter or normal measure on an ordinal is a filter or measure closed under diagonal intersections
3. The Cantor normal form of an ordinal is its base ω expansion.

NS
: Nonstationary

null
: German for zero, occasionally used in terms such as "aleph null" (aleph zero) or "null set" (empty set)

number class
: The first number class consists of finite ordinals, and the second number class consists of countable ordinals.

O

OCA
: The open coloring axiom

OD
: The ordinal definable sets

Omega logic
: Ω-logic is a form of logic introduced by Hugh Woodin

On
: The class of all ordinals

ordinal

1. An ordinal is the order type of a well-ordered set, usually represented by a von Neumann ordinal, a transitive set well ordered by ∈.
2. An ordinal definable set is a set that can be defined by a first-order formula with ordinals as parameters

ot
: Abbreviation for "order type of"

P

$\mathfrak{p}$
: The pseudo-intersection number, the smallest cardinality of a family of infinite subsets of ω that has the strong finite intersection property but has no infinite pseudo-intersection.

P

1. The powerset function
2. A poset

pairing function
: A pairing function is a bijection from $X \times X$ to X for some set X

pantachie
pantachy
: A pantachy is a maximal chain of a poset

paradox
1. Berry's paradox
2. Burali-Forti's paradox
3. Cantor's paradox
4. Hilbert's paradox
5. Milner–Rado paradox
6. Richard's paradox
7. Russell's paradox
8. Skolem's paradox

partial order
1. A set with a transitive antisymmetric relation
2. A set with a transitive symmetric relation

partition cardinal
: An alternative name for an Erdős cardinal

PCF
: Abbreviation for "possible cofinalities", used in PCF theory

PD
: The axiom of projective determinacy

perfect set
: A perfect set is a subset of a topological set equal to its derived set

permutation model
: A permutation model of ZFA is constructed using a group

PFA
: The proper forcing axiom

PM
: The hypothesis that all projective subsets of the reals are Lebesgue measurable

po
: An abbreviation for "partial order" or "poset"

poset
: A set with a partial order

Polish space
A Polish space is a separable topological space homeomorphic to a complete metric space

pow
Abbreviation for "power (set)"

power
"Power" is an archaic term for cardinality

power set
powerset
The powerset or power set of a set is the set of all its subsets

projective
1. A projective set is a set that can be obtained from an analytic set by repeatedly taking complements and projections
2. Projective determinacy is an axiom asserting that projective sets are determined

proper
1. A proper class is a class that is not a set
2. A proper subset of a set *X* is a subset not equal to *X*.
3. A proper forcing is a forcing notion that does not collapse any stationary set
4. The proper forcing axiom asserts that if P is proper and D_α is a dense subset of P for each $\alpha<\omega_1$, then there is a filter $G \subseteq P$ such that $D_\alpha \cap G$ is nonempty for all $\alpha<\omega_1$

PSP
Perfect subset property

Q

Q
The (ordered set of) rational numbers

QPD
Quasi-projective determinacy

quantifier
∀ or ∃

Quasi-projective determinacy
All sets of reals in *L*(*R*) are determined

R

$\mathfrak{r}$

The unsplitting number

R

1. R_α is an alternative name for the level V_α of the von Neumann hierarchy.
2. The real numbers

Ramsey

1. Frank P. Ramsey
2. A Ramsey cardinal is a large cardinal satisfying a certain partition condition

ran

The range of a function

rank

1. The rank of a set is the smallest ordinal greater than the ranks of its elements
2. A **rank** V_α is the collection of all sets of rank less than α, for an ordinal α
3. rank-into-rank is a type of large cardinal (axiom)

reflecting cardinal

A reflecting cardinal is a type of large cardinal whose strength lies between being weakly compact and Mahlo

reflection principle

A reflection principle states that there is a set similar in some way to the universe of all sets

regressive

A function *f* from a subset of an ordinal to the ordinal is called regressive if $f(\alpha)<\alpha$ for all α in its domain

regular

A regular cardinal is one equal to its own cofinality.

Reinhardt cardinal

A Reinhardt cardinal is a cardinal in a model *V* of ZF that is the critical point of an elementary embedding of *V* into itself

relation

A set or class whose elements are ordered pairs

Richard

1. Jules Richard
2. Richard's paradox considers the real number whose *n*th binary digit is the opposite of the *n*th digit of the *n*th definable real number

RO

The regular open sets of a topological space or poset

Rowbottom

1. Frederick Rowbottom
2. A Rowbottom cardinal is a large cardinal satisfying a certain partition condition

rud
: The rudimentary closure of a set

rudimentary
: A rudimentary function is a functions definable by certain elementary operations, used in the construction of the Jensen hierarchy

Russell
1. Bertrand Russell
2. Russell's paradox is that the set of all sets not containing themselves is contradictory so cannot exist

S

$\mathfrak{s}$
: The splitting number

SBH
: Stationary basis hypothesis

SCH
: Singular cardinal hypothesis

SCS
: Semi-constructive system

Scott
1. Dana Scott
2. Scott's trick is a way of coding proper equivalence classes by sets by taking the elements of the class of smallest rank

second
1. A set of second category is a set that is not of first category: in other words a set that is not the union of a countable number of nowhere-dense sets.
2. An ordinal of the second class is a countable infinite ordinal
3. An ordinal of the second kind is a limit ordinal or 0
4. Second order logic allows quantification over subsets as well as over elements of a model

sentence
: A formula with no bound variables

separating set
1. A separating set is a set containing a given set and disjoint from another given set
2. A separating set is a set *S* of functions on a set such that for any two distinct points there is a function in *S* with different values on them.

separative
: A separative poset is one that can be densely embedded into the poset of nonzero elements of a Boolean algebra.

SFIP
: Strong finite intersection property

SH
: Suslin's hypothesis

Shelah
1. Saharon Shelah
2. A Shelah cardinal is a large cardinal that is the critical point of an elementary embedding satisfying certain conditions

shrewd cardinal
: A shrewd cardinal is a type of large cardinal generalizing indecribable cardinals to transfinite levels

Sierpinski
Sierpiński
1. Wacław Sierpiński
2. A Sierpiński set is an uncountable subset of a real vector space whose intersection with every measure-zero set is countable

Silver
1. Jack Silver
2. The Silver indiscernibles form a class I of ordinals such that $I \cap L_\kappa$ is a set of indiscernibles for L_κ for every uncountable cardinal κ

singular
1. A singular cardinal is one that is not regular
2. The singular cardinal hypothesis states that if κ is any singular strong limit cardinal, then $2^\kappa = \kappa^+$.

SIS
: Semi-intuitionistic system

Skolem
1. Thoralf Skolem
2. Skolem's paradox states that if ZFC is consistent there are countable models of it
3. A Skolem function is a function whose value is something with a given property if anything with that property exists
4. The Skolem hull of a model is its closure under Skolem functions

small
: A small large cardinal axiom is a large cardinal axiom consistent with the axiom $V=L$

SOCA
: Semi open coloring axiom

Solovay
1. Robert M. Solovay
2. The Solovay model is a model of ZF in which every set of reals is measurable

special
: A special Aronszajn tree is one with an order preserving map to the rationals

square
The square principle is a combinatorial principle holding in the constructible universe and some other inner models

standard model
A model of set theory where the relation ∈ is the same as the usual one.

stationary set
A stationary set is a subset of an ordinal intersecting every club set

strong
1. The strong finite intersection property says that the intersection of any finite number of elements of a set is infinite
2. A strong cardinal is a cardinal κ such that if λ is any ordinal, there is an elementary embedding with critical point κ from the universe into a transitive inner model containing all elements of V_λ
3. A strong limit cardinal is a (usually nonzero) cardinal that is larger then the powerset of any smaller cardinal

strongly
1. A strongly inaccessible cardinal is a regular strong limit cardinal
2. A strongly Mahlo cardinal is a strongly inaccessible cardinal such that the set of strongly inaccessible cardinals below it is stationary
3. A strongly compact cardinal is a cardinal κ such that every κ-complete filter can be extended to a κ complete ultrafilter

subtle cardinal
A subtle cardinal is a type of large cardinal closely related to ethereal cardinals

successor
1. A successor cardinal is the smallest cardinal larger than some given cardinal
2. A successor ordinal is the smallest ordinal larger than some given ordinal

such that
A condition used in the definition of a mathematical object

sunflower
A sunflower, also called a delta system, is a collection of sets such that any two distinct sets have intersection *X* for some fixed set *X*

Souslin
Suslin
1. Mikhail Yakovlevich Suslin (sometimes written Souslin)
2. A Suslin algebra is a Boolean algebra that is complete, atomless, countably distributive, and satisfies the countable chain condition
3. A Suslin cardinal is a cardinal λ such that there exists a set $P \subset 2^\omega$ such that P is λ-Suslin but P is not λ'-Suslin for any λ' < λ.
4. The Suslin hypothesis says that Suslin lines do not exist
5. A Suslin line is a complete dense unbounded totally ordered set satisfying the countable chain condition
6. The Suslin number is the supremum of the cardinalities of families of disjoint open non-empty sets
7. The Suslin operation, usually denoted by *A*, is an operation that constructs a set from a Suslin scheme
8. The Suslin problem asks whether Suslin lines exist
9. The Suslin property states that there is no uncountable family of pairwise disjoint non-empty open subsets
10. A Suslin representation of a set of reals is a tree whose projection is that set of reals

11. A Suslin scheme is a function with domain the finite sequences of positive integers
12. A Suslin set is a set that is the image of a tree under a certain projection
13. A Suslin space is the image of a Polish space under a continuous mapping
14. A Suslin subset is a subset that is the image of a tree under a certain projection
15. The Suslin theorem about analytic sets states that a set that is analytic and coanalytic is Borel
16. A Suslin tree is a tree of height ω_1 such that every branch and every antichain is at most countable.

supercompact
: A supercompact cardinal is an uncountable cardinal κ such that for every *A* such that Card(*A*) ≥ κ there exists a normal measure over $[A]^{\kappa}$.

super transitive
supertransitive
: A supertransitive set is a transitive set that contains all subsets of all its elements

symmetric model
: A symmetric model is a model of ZF (without the axiom of choice) constructed using a group action on a forcing poset

T

$\mathfrak{t}$
: The tower number

T
: A tree

tall cardinal
: A tall cardinal is a type of large cardinal that is the critical point of a certain sort of elementary embedding

Tarski
1. Alfred Tarski
2. Tarski's theorem states that the axiom of choice is equivalent to the existence of a bijection from *X* to *X*×*X* for all sets *X*

TC
: The transitive closure of a set

total order
: A total order is a relation that is transitive and antisymmetric such that any two elements are comparable

totally indescribable
: A totally indescribable cardinal is a cardinal that is Π^m_n-indescribable for all *m*,*n*

transfinite
1. An infinite ordinal
2. Transfinite induction is induction over ordinals

transitive

1. A transitive relation
2. The transitive closure of a set is the smallest transitive set containing it.
3. A transitive set or class is a set or class such that the membership relation is transitive on it.
4. A transitive model is a model of set theory that is transitive and has the usual membership relation

tree

1. A tree is a partially ordered set (T, <) such that for each $t \in T$, the set $\{s \in T : s < t\}$ is well-ordered by the relation <
2. A tree is a collection of finite sequences such that every prefix of a sequence in the collection also belongs to the collection.
3. A cardinal κ has the tree property if there are no κ-Aronszajn trees

type class

A type class or class of types is the class of all order types of a given cardinality, up to order-equivalence.

U

$\mathfrak{u}$

The ultrafilter number, the minimum possible cardinality of an ultrafilter base

Ulam

1. Stanislaw Ulam
2. An Ulam matrix is a collection of subsets of a cardinal indexed by pairs of ordinals, that satisfies certain properties.

Ult

An ultrapower or ultraproduct

ultrafilter

1. A maximal filter
2. The ultrafilter number $\mathfrak{u}$ is the minimum possible cardinality of an ultrafilter base

ultrapower

An ultraproduct in which all factors are equal

ultraproduct

An ultraproduct is the quotient of a product of models by a certain equivalence relation

unfoldable cardinal

An unfoldable cardinal a cardinal κ such that for every ordinal λ and every transitive model *M* of cardinality κ of ZFC-minus-power set such that κ is in *M* and *M* contains all its sequences of length less than κ, there is a non-trivial elementary embedding *j* of *M* into a transitive model with the critical point of *j* being κ and $j(\kappa) \geq \lambda$.

uniformity

The uniformity non(I) of I is the smallest cardinality of a subset of X not in the ideal I of subsets of X

uniformization

Uniformization is a weak form of the axiom of choice, giving cross sections for special subsets of a product of two Polish spaces

universal
universe

1. The universal class, or universe, is the class of all sets.

A universal quantifier is the quantifier "for all", usually written ∀

urelement

An urelement is something that is not a set but allowed to be an element of a set

V

V

V is the universe of all sets, and the sets V_α form the Von Neumann hierarchy

V=L

The axiom of constructibility

Veblen

1. Oswald Veblen
2. The Veblen hierarchy is a family of ordinal valued functions, special cases of which are called Veblen functions.

von Neumann

1. John von Neumann
2. A von Neumann ordinal is an ordinal encoded as the union of all smaller (von Neumann) ordinals
3. The von Neumann hierarchy is a cumulative hierarchy V_α with $V_{\alpha+1}$ the powerset of V_α.

Vopenka
Vopěnka

1. Petr Vopěnka
2. Vopěnka's principle states that for every proper class of binary relations there is one elementarily embeddable into another
3. A Vopěnka cardinal is an inaccessible cardinal κ such that and Vopěnka's principle holds for V_κ

W

weakly

1. A weakly inaccessible cardinal is a regular weak limit cardinal
2. A weakly compact cardinal is a cardinal κ (usually also assumed to be inaccessible) such that the infinitary language $L_{\kappa,\kappa}$ satisfies the weak compactness theorem
3. A weakly Mahlo cardinal is a cardinal κ that is weakly inaccessible and such that the set of weakly inaccessible cardinals less than κ is stationary in κ

well founded
A relation is called well founded if every non-empty subset has a minimal element

well ordering
A well ordering is a well founded relation, usually also assumed to be a total order

Wf
The class of well-founded sets, which is the same as the class of all sets if one assumes the axiom of foundation

Woodin
1. Hugh Woodin
2. A Woodin cardinal is a type of large cardinal that is the critical point of a certain sort of elementary embedding, closely related to the axiom of projective determinacy

XYZ

Z
Zermelo set theory without the axiom of choice

ZC
Zermelo set theory with the axiom of choice

Zermelo
1. Ernst Zermelo
2. Zermelo−Fraenkel set theory is the standard system of axioms for set theory
3. Zermelo set theory is similar to the usual Zermelo-Fraenkel set theory, but without the axioms of replacement and foundation
4. Zermelo's well-ordering theorem states that every set can be well ordered

ZF
Zermelo−Fraenkel set theory without the axiom of choice

ZFA
Zermelo−Fraenkel set theory with atoms

ZFC
Zermelo−Fraenkel set theory with the axiom of choice

ZF-P
Zermelo−Fraenkel set theory without the axiom of choice or the powerset axiom

Zorn
1. Max Zorn
2. Zorn's lemma states that if every chain of a non-empty poset has an upper bound then the poset has a maximal element

See also

- Glossary of *Principia Mathematica*
- List of topics in set theory
- Set-builder notation

References

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