22/02/2018 Implementation of A*

Implementation of A*

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This article is a companion guide to my introduction to A*. I show how to implement Breadth-First Search, Dijkstra's Algorithm, Greedy Best-First Search, and A*. I focused on keeping things simple.

Graph search is a family of related algorithms. There are *lots* of variants of the algorithms, and lots of variants in implementation. Treat the code on this page as a starting point, not as a final version of the algorithm that works for all situations.

1 Python Implementation

#

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I explain most of the code below. There are a few extra bits that you can find in implementation.py. These use **Python 3** so if you use Python 2, you will need to change the super() call and the print function to the Python 2 equivalents.

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#

1.1 Breadth First Search

Let's implement Breadth First Search in Python. The main article shows the Python code for the search algorithm, but we also need to define the graph it works on. These are the abstractions I'll use:

Graph

a data structure that can tell me theneighbors for each location (see<u>this</u> <u>tutorial</u>). A *weighted* graph can also tell me the cost of moving along an edge.

Locations

a simple value (int, string, tuple, etc.) that labels locations in the graph

Search

an algorithm that takes a graph, a starting location, and optionally a goal location, and calculates some useful information (visited, parent pointer distance) for some or all locations

Queue

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a data structure used by the search algorithm to decide the order in which to process the locations

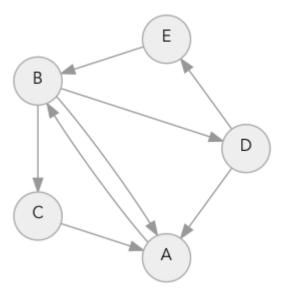
In the main article, I focused on **search**. On this page, I'll fill in the rest of the details to make complete working programs. Let's start with a **graph**:

```
class SimpleGraph:
    def __init__(self):
        self.edges = {}

    def neighbors(self, id):
        return self.edges[id]
```

Yep, that's all we need! You may be asking, where's the Node object? The answer is: I rarely use a node object. I find it simpler to use integers, strings, or tuples as locations, and then use arrays or hash tables that use locations as an index.

Note that the edges are *directed*: we can have an edge from A to B without also having an edge from B to A. In game maps most edges are bidirectional but sometimes there are one-way doors or jumps off cliffs that are expressed as directed edges. Let's make an example graph, where the **locations** are letters A-E.



For each location I need a list of which locations it leads to:

```
example_graph = SimpleGraph()
example_graph.edges = {
    'A': ['B'],
    'B': ['A', 'C', 'D'],
    'C': ['A'],
    'D': ['E', 'A'],
    'E': ['B']
}
```

Before we can use it with a search algorithm, we need to make **queue**:

```
import collections
class Queue:
```

```
def __init__(self):
    self.elements = collections.deque()

def empty(self):
    return len(self.elements) == 0

def put(self, x):
    self.elements.append(x)

def get(self):
    return self.elements.popleft()
```

This queue class is just a wrapper around the built-in collections.deque class. Feel free to use deque directly in your own code.

Let's try the example graph with this queue and the breadth-first search algorithm code from the main article:

```
from implementation import *

def breadth_first_search_1(graph, start):
    # print out what we find
    frontier = Queue()
    frontier.put(start)
    visited = {}
    visited[start] = True

while not frontier.empty():
    current = frontier.get()
    print("Visiting %r" % current)
    for next in graph.neighbors(current):
        if next not in visited:
            frontier.put(next)
            visited[next] = True
```

```
breadth_first_search_1(example_graph, 'A')

Visiting 'A'
Visiting 'B'
Visiting 'C'
Visiting 'D'
Visiting 'E'
```

Grids can be expressed as graphs too. I'll now define a new **graph** called SquareGrid, with **locations** tuples (int, int). Instead of storing the edges explicitly, I'll calculate them in the neighbors function.

```
class SquareGrid:
    def __init__(self, width, height):
        self.width = width
        self.height = height
        self.walls = []
    def in_bounds(self, id):
        (x, y) = id
        return 0 <= x < self.width and 0 <= y < self.height
    def passable(self, id):
        return id not in self.walls
    def neighbors(self, id):
        (x, y) = id
        results = [(x+1, y), (x, y-1), (x-1, y), (x, y+1)]
       if (x + y) % 2 == 0: results.reverse() # aesthetics
       results = filter(self.in_bounds, results)
        results = filter(self.passable, results)
        return results
```

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Let's try it out with the first gridin the main article:

```
from implementation import *
g = SquareGrid(30, 15)
g.walls = DIAGRAM1_WALLS # long list, [(21, 0), (21, 2), ...]
draw_grid(g)
      . . . . . . . #### . . . . . . . . .
 ####. . . . . . . . ####. . . . . . ####. . . . . . . .
```

In order to reconstruct paths we need to store the location of where we came from, so I've renamed visited (True/False) to came_from (location):

```
from implementation import *

def breadth_first_search_2(graph, start):
    # return "came_from"
    frontier = Queue()
    frontier.put(start)
    came_from = {}
    came_from[start] = None
```

```
while not frontier.empty():
    current = frontier.get()
    for next in graph.neighbors(current):
        if next not in came_from:
            frontier.put(next)
            came_from[next] = current

return came_from

g = SquareGrid(30, 15)
g.walls = DIAGRAM1_WALLS

parents = breadth_first_search_2(g, (8, 7))
draw_grid(g, width=2, point_to=parents, start=(8, 7))
```

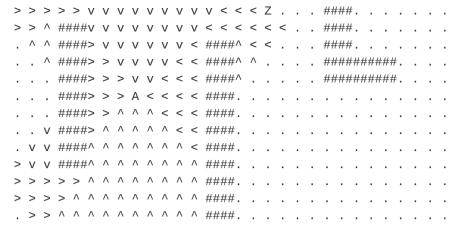
Some implementations use *internal storage*, creating a Node object to hold came_from and other values for each graph node. I've instead chosen to use *external storage*, creating a single hash table to store the came_from for all graph nodes. If you know your map locations have integer indices, another option is to use an array to store came_from.

1.2 Early Exit



Following the code from the main article, all we need to do is add an *if* statement in the main loop. This test is optional for Breadth First Search or Dijkstra's Algorithm and effectively required for Greedy Best-First Search and A*:

```
from implementation import *
def breadth_first_search_3(graph, start, goal):
   frontier = Queue()
   frontier.put(start)
   came_from = {}
   came_from[start] = None
   while not frontier.empty():
       current = frontier.get()
       if current == goal:
          break
       for next in graph.neighbors(current):
          if next not in came_from:
              frontier.put(next)
              came_from[next] = current
   return came_from
g = SquareGrid(30, 15)
g.walls = DIAGRAM1_WALLS
parents = breadth_first_search_3(g, (8, 7), (17, 2))
draw_grid(g, width=2, point_to=parents, start=(8, 7), goal=(17, 2))
```



You can see that the algorithm stops when it finds the goal.

1.3 Dijkstra's Algorithm

#

This is what adds complexity to graph search, because we're going to start processing locations in a better order than "first in, first out". What do we need to change?

- 1. The *graph* needs to know cost of movement.
- 2. The *queue* needs to return nodes in a different order.
- 3. The *search* needs to keep track of these costs from the graph and give them to the queue.

1.3.1 Graph with weights

A regular graph tells me the neighbors of each node. A *weighted* graph also tells me the cost of moving along each edge. I'm going to add a cost(from_node, to_node) function that tells us the cost of moving from location from_node to its neighbor to_node. In this forest map I chose to make movement depend only on to_node, but there are other types of movement that use both nodes. An alternate implementation would be to mege this into the neighbors function.

```
class GridWithWeights(SquareGrid):
    def __init__(self, width, height):
        super().__init__(width, height)
        self.weights = {}

    def cost(self, from_node, to_node):
        return self.weights.get(to_node, 1)
```

1.3.2 Queue with priorities

A priority queue associates with each item a number called a "priority". When returning an item, it picks the one with the lowest number

insert

Add item to queue

remove

Remove item with the lowest number

reprioritize

(optional) Change an existing item's priority to a lower number

Here's a reasonably fast priority queue that uses *binary heaps*, but does not support reprioritize. To get the right ordering, we'll use tuples (priority, item). When an element is inserted that is already in the queue, we'll have a duplicate; I'll explain why that's ok in the Optimization section.

```
import heapq

class PriorityQueue:
    def __init__(self):
        self.elements = []

    def empty(self):
        return len(self.elements) == 0

    def put(self, item, priority):
        heapq.heappush(self.elements, (priority, item))

    def get(self):
        return heapq.heappop(self.elements)[1]
```

1.3.3 Search

Here's a tricky bit about the implementation: once we add movement costs it's possible to visit a location again, with a better cost_so_far. That means the line if next not in came_from won't work. Instead, have to check if the cost has gone down since the last time we visited. (In the original version of the article I

wasn't checking this, but my code worked anyway; <u>I wrote some notes about that</u> bug.)

This forest map is from the main page.

```
def dijkstra_search(graph, start, goal):
   frontier = PriorityQueue()
    frontier.put(start, 0)
    came_from = {}
    cost_so_far = {}
    came_from[start] = None
    cost_so_far[start] = 0
   while not frontier.empty():
        current = frontier.get()
        if current == goal:
            break
        for next in graph.neighbors(current):
            new_cost = cost_so_far[current] + graph.cost(current, next)
            if next not in cost_so_far or new_cost < cost_so_far[next]:</pre>
                cost_so_far[next] = new_cost
                priority = new_cost
                frontier.put(next, priority)
                came_from[next] = current
    return came_from, cost_so_far
```

Finally, after searching I need tobuild the path:

```
def reconstruct_path(came_from, start, goal):
    current = goal
```

```
path = []
while current != start:
    path.append(current)
    current = came_from[current]
path.append(start) # optional
path.reverse() # optional
return path
```

Although paths are best thought of as a sequence of edges, it's convenient to store them as a sequence of nodes. To build the path, start at the end and follow the came_from map, which points to the previous node. When we reach start, we're done. It is the **backwards** path, so call reverse() at the end of reconstruct_path if you need it to be stored forwards. Sometimes it's actually more convenient to store it backwards. Sometimes it's useful to also store the start node in the list.

Let's try it out:

```
< < < < < <
       9
         14 15 12 13 14
       8 13 18 17 14 .
       11 16 . . . .
    7 12 17 .
#######14 19 18
#######15 16 13 Z . .
    9 10 11 12 13 14 .
```

The line if next not in cost_so_far or new_cost < cost_so_far[next] could be simplified to if new_cost < cost_so_far.get(next, Infinity) but I didn't want to explain Python's get() in the main article so I left it as is. Another approach would be to use collections.defaultdict defaulting to infinity.

1.4 A* Search

Both Greedy Best-First Search and A* use a heuristic function. The only difference is that A* uses both the heuristic and the ordering from Dijkstra's Algorithm. I'm going to show A* here.

```
def heuristic(a, b):
    (x1, y1) = a
    (x2, y2) = b
    return abs(x1 - x2) + abs(y1 - y2)
def a_star_search(graph, start, goal):
    frontier = PriorityQueue()
    frontier.put(start, 0)
    came_from = {}
    cost_so_far = {}
    came_from[start] = None
    cost_so_far[start] = 0
    while not frontier.empty():
        current = frontier.get()
        if current == goal:
            break
        for next in graph.neighbors(current):
            new_cost = cost_so_far[current] + graph.cost(current, next)
            if next not in cost_so_far or new_cost < cost_so_far[next]:</pre>
                cost_so_far[next] = new_cost
                priority = new_cost + heuristid(goal, next)
                frontier.put(next, priority)
                came_from[next] = current
    return came_from, cost_so_far
```

Let's try it out:

```
from implementation import *
  start, goal = (1, 4), (7, 8)
  came_from, cost_so_far = a_star_search(diagram4, start, goal)
  draw_grid(diagram4, width=3, point_to=came_from, start=start, goal=goal)
  print()
  draw_grid(diagram4, width=3, number=cost_so_far, start=start, goal=goal)
  print()
```

1.4.1 Straighter paths

If you implement this code in your own project you might find that some of the paths aren't as "straight" as you'd like. **This is normal**. When using *grids*, especially grids where every step has the same movement cost, you end up with **ties**: many paths have exactly the same cost. A* ends up picking one of the many

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short paths, and very often **it doesn't look good to you**. The quick hack is to <u>break the ties</u>, but it's not entirely satisfactory. The better approach is to <u>change</u> the map representation, which makes A* a lot faster, and also produces straighter, better looking paths. However, that only works for mostly-static maps where every step has the same movement cost. For the demos on my page, I'm using a quick hack, but it only works with my slow priority queue. If you switch to a faster priority queue you'll need a different quick hack.

2 C++ Implementation

<u>#</u>

Note: some of the sample code needs to include <u>redblobgames/pathfinding/a-star/implementation.cpp</u> to run. I am using **C++11** for this code so some of it will need to be changed if you use an older version of the C++ standard.

The code here is meant for the tutorial and is not production-quality; there's a section at the end with tips on making it better

2.1 Breadth First Search

#

Let's implement Breadth First Search in C++. These are the components we need:

Graph

a data structure that can tell me theneighbors for each location (see<u>this</u> <u>tutorial</u>).. A *weighted* graph can also tell me the cost of moving along an edge.

Locations

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a simple value (int, string, tuple, etc.) that labels locations in the graph

Search

an algorithm that takes a graph, a starting location, and optionally a goal location, and calculates some useful information (visited, parent pointer distance) for some or all locations

Queue

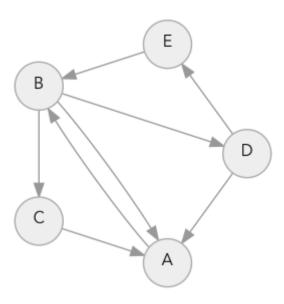
a data structure used by the search algorithm to decide the order in which to process the locations

In the main article, I focused on **search**. On this page, I'll fill in the rest of the details to make complete working programs. Let's start with a **graph** where the labels are char:

```
struct SimpleGraph {
  std::map<char, std::vector<char> > edges;

std::vector<char> neighbors(char id) {
  return edges[id];
  }
};
```

Here's an example:



The C++ standard library already includes a queue class. We now have a graph (SimpleGraph), locations (char), and a queue (std::queue). Now we can try Breadth First Search:

```
#include "redblobgames/pathfinding/a-star/implementation.cpp"
void breadth_first_search(SimpleGraph graph, char start) {
  std::queue<char> frontier;
  frontier.push(start);
   std::set<char> visited;
  visited.insert(start);
  while (!frontier.empty()) {
    char current = frontier.front();
     frontier.pop();
     std::cout << "Visiting " << current << '\n';</pre>
    for (char next : graph.neighbors(current)) {
      if (visited.find(next) == visited.end()) {
         frontier.push(next);
         visited.insert(next);
int main() {
  breadth_first_search(example_graph, 'A');
}
Visiting A
Visiting B
```

Visiting C Visiting D Visiting E

How about square grids? This time, locations will be labeled with x,y, and I'll make another graph class that keeps track of the walls in the grid:

```
struct GridLocation {
  int x, y;
};
struct SquareGrid {
  static std::array<GridLocation, 4> DIRS;
  int width, height;
  std::set<GridLocation→ walls;
  SquareGrid(int width_, int height_)
     : width(width_), height(height_) {}
  bool in_bounds(GridLocation id) const {
    return 0 <= id.x && id.x < width</pre>
        && 0 <= id.y && id.y < height;
  }
  bool passable(GridLocation id) const {
    return walls.find(id) == walls.end();
  }
  std::vector<GridLocation> neighbors(GridLocation id) const {
    std::vector<GridLocation> results;
    for (GridLocation dir : DIRS) {
      GridLocation next{id.x + dir.x, id.y + dir.y};
      if (in_bounds(next) && passable(next)) {
        results.push_back(next);
      }
    }
    if ((id.x + id.y) % 2 == 0) {
      // aesthetic improvement on square grids
      std::reverse(results.begin(), results.end());
    return results;
  }
};
```

```
std::array<GridLocation, 4> SquareGrid::DIRS =
    {GridLocation{1, 0}, GridLocation{0, -1}, GridLocation{-1, 0}, GridLocation{0, 1}};
```

In the helper file implementation.cpp I defined a function to make grids:

```
#include "redblobgames/pathfinding/a-star/implementation.cpp"
int main() {
SquareGrid grid = make_diagram1();
draw_grid(grid, 2);
              . . . . . ####. . . . . . .
       . . . . . . . . . . . . ####. . . . .
  . . . . . . . . ####. . . . . .
   . . . . . . . . ####. . . . . . . . .
   . . . . . . . . ####. . . . . .
```

Let's try Breadth First Search again, keeping track of came_from:

```
#include "redblobgames/pathfinding/a-star/implementation.cpp"

template<typename Location, typename Graph>
std::map<Location, Location>
```

```
breadth_first_search(Graph graph, Location start) {
  std::gueue<Location> frontier;
 frontier.push(start);
  std::map<Location, Location> came_from;
  came from[start] = start;
  while (!frontier.empty()) {
   Location current = frontier.front();
    frontier.pop();
    for (Location next : graph.neighbors(current)) {
      if (came_from.find(next) == came_from.end()) {
        frontier.push(next);
        came_from[next] = current;
  return came from;
}
int main() {
  SquareGrid grid = make_diagram1();
  auto parents = breadth_first_search(grid, GridLocation{7, 8});
  draw_grid(grid, 2, nullptr, &parents);
```

Some implementations use *internal storage*, creating a Node object to hold came_from and other values for each graph node. I've instead chosen to use *external storage*, creating a single std::map to store the came_from for all graph nodes. If you know your map locations have integer indices, another option is to use a 1D or 2D array/vector to storecame_from and other values.

2.2 Early Exit

#

Breadth First Search and Dijkstra's Algorithm will explore the entire map by default. If we're looking for a path to a single, point we can add if (current == goal) to exit the loop as soon as we find the path.

```
#include "redblobgames/pathfinding/a-star/implementation.cpp"

template<typename Location, typename Graph>
std::map<Location, Location>
breadth_first_search(Graph graph, Location start, Location goal) {
   std::queue<Location> frontier;
   frontier.push(start);

   std::map<Location, Location> came_from;
   came_from[start] = start;

while (!frontier.empty()) {
   Location current = frontier.front();
   frontier.pop();

   if (current == goal) {
        break;
    }
}
```

```
for (Location next : graph.neighbors(current)) {
    if (came_from.find(next) == came_from.end()) {
        frontier.push(next);
        came_from[next] = current;
    }
    }
    return came_from;
}

int main() {
    GridLocation start{8, 7};
    GridLocation goal{17, 2};
    SquareGrid grid = make_diagram1();
    auto came_from = breadth_first_search(grid, start, goal);
    draw_grid(grid, 2, nullptr, &came_from);
}
```

In the output we can see that the algorithm did not explore the entire map, but stopped early.

2.3 Dijkstra's Algorithm



This is what adds complexity to graph search, because we're going to start processing locations in a better order than "first in, first out". What do we need to change?

- 1. The *graph* needs to know cost of movement.
- 2. The *queue* needs to return nodes in a different order.
- 3. The *search* needs to keep track of these costs from the graph and give them to the queue.

2.3.1 Graph with weights

A regular graph tells me the neighbors of each node. A weighted graph also tells me the cost of moving along each edge. I'm going to add a cost(from_node, to_node) function that tells us the cost of moving from location from_node to its neighbor to_node. In this forest map I chose to make movement depend only on to_node, but there are other types of movement that use both nodes. An alternate implementation would be to mege this into the neighbors function. Here's a grid with a list of forest tiles, which will have movement cost 5:

```
struct GridWithWeights: SquareGrid {
  std::set<GridLocation> forests;
```

```
GridWithWeights(int w, int h): SquareGrid(w, h) {}
double cost(GridLocation from_node, GridLocation to_node) const {
   return forests.find(to_node) != forests.end()? 5 : 1;
}
};
```

2.3.2 Queue with priorities

We need a priority queue. C++ offers a priority_queue class that uses a binary heap but not the reprioritize operation. I'll use a pair (priority, item) for the queue elements to get the right ordering. By default, the C++ priority queue returns the maximum element first, using the std::less comparator; we want the minimum element instead, so I'll use the std::greater comparator.

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```
};
```

In this sample code I'm wrapping the C++ std::priority_queue class but I think it'd be reasonable to use that class directly without the wrapper

2.3.3 Search

See the forest map from the main page

```
template<typename Location, typename Graph>
void dijkstra_search
  (Graph graph,
  Location start,
  Location goal,
  std::map<Location, Location>& came_from,
   std::map<Location, double>& cost_so_far)
 PriorityQueue<Location, double> frontier;
 frontier.put(start, 0);
 came_from[start] = start;
 cost_so_far[start] = 0;
 while (!frontier.empty()) {
   Location current = frontier.get();
   if (current == goal) {
      break;
   for (Location next : graph.neighbors(current)) {
      double new_cost = cost_so_far[current] + graph.cost(current, next);
```

The types of the cost variables should all match the types used in the graph. If you use int then you can use int for the cost variable and the priorities in the priority queue; if you use double then you should use double for these. In this code I used double but I could've used int and it would've worked the same. However, if your graph edge costs are doubles or if your heuristic uses doubles, then you'll need to use doubles here.

Finally, after searching I need tobuild the path:

```
template<typename Location>
std::vector<Location> reconstruct_path(
    Location start, Location goal,
    std::map<Location, Location> came_from
) {
    std::vector<Location> path;
    Location current = goal;
    while (current != start) {
        path.push_back(current);
        current = came_from[current];
    }
    path.push_back(start); // optional
```

```
std::reverse(path.begin(), path.end());
return path;
}
```

Although paths are best thought of as a sequence of edges, it's convenient to store them as a sequence of nodes. To build the path, start at the end and follow the came_from map, which points to the previous node. When we reach start, we're done. It is the **backwards** path, so call reverse() at the end of reconstruct_path if you need it to be stored forwards. Sometimes it's actually more convenient to store it backwards. Sometimes it's useful to also store the start node in the list.

Let's try it out:

```
#include "redblobgames/pathfinding/a-star/implementation.cpp"

int main() {
    GridWithWeights grid = make_diagram4();
    GridLocation start{1, 4};
    GridLocation goal{8, 5};
    std::map<GridLocation, GridLocation> came_from;
    std::map<GridLocation, double> cost_so_far;
    dijkstra_search(grid, start, goal, came_from, cost_so_fai);
    draw_grid(grid, 2, nullptr, &came_from);
    std::cout << '\n';
    draw_grid(grid, 3, &cost_so_far, nullptr);
    std::cout << '\n';
    std::vector<GridLocation> path = reconstruct_path(start, goal, came_from);
    draw_grid(grid, 3, nullptr, nullptr, &path);
}
```

```
v v < < < < < < < <
\vee \vee < < < \wedge \wedge < <
\vee \vee < < < < \wedge \wedge < <
\vee \vee < < < < \wedge \wedge <
> * < < < < < > ^ <
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   ########14 19 18 15 16 .
   ########15 16 13 14 15 16
      8 9 10 11 12 13 14 15
   ########.
```

The results are not exactly the same as the Python version because I'm using the built-in priority queues in C++ and Python may order equal-valued nodes differently.

2.4 A* Search



A* is almost exactly like Dijkstra's Algorithm, except we add in a heuristic. Note that the code for the algorithm *isn't specific to grids*. Knowledge about grids is in the graph class (SquareGrids in this case), the locations (Location struct), and in the heuristic function. Replace those three and you can use the A* algorithm code with any other graph structure.

```
inline double heuristic(GridLocation a, GridLocation b) {
  return std::abs(a.x - b.x) + std::abs(a.y - b.y);
template<typename Location, typename Graph>
void a_star_search
  (Graph graph,
   Location start,
   Location goal,
   std::map<Location, Location>& came_from,
   std::map<Location, double>& cost_so_far)
  PriorityQueueLocation, double> frontier;
  frontier.put(start, 0);
  came_from[start] = start;
  cost_so_far[start] = 0;
  while (!frontier.empty()) {
    Location current = frontier.get();
    if (current == goal) {
      break;
    for (Location next : graph.neighbors(current)) {
```

```
double new_cost = cost_so_far[current] + graph.cost(current, next);
   if (cost_so_far.find(next) == cost_so_far.end()
        || new_cost < cost_so_far[next]) {
        cost_so_far[next] = new_cost;
        double priority = new_cost + heuristid(next, goal);
        frontier.put(next, priority);
        came_from[next] = current;
    }
}
}</pre>
```

The type of the priority values including the type used in the priority queue should be big enough to include both the graph costs (cost_t) and the heuristic value. For example, if the graph costs are into and the heuristic returns a double, then you need the priority queue to accept doubles. In this sample code I use double for all three (cost, heuristic, and priority), but I could've used int because my costs and heuristics are integer valued.

Minor note: It would be more correct to write frontier.put(start, heuristic(start, goal)) than frontier.put(start, 0) but it makes no difference here because the start node's priority doesn't matter. It is the only node in the priority queue and it is selected and removed before anything else is put in there.

```
#include "redblobgames/pathfinding/a-star/implementation.cpp"
int main() {
```

```
GridWithWeights grid = make_diagram4();
GridLocation start{1, 4};
GridLocation goal{8, 5};
std::map<GridLocation, GridLocation> came_from;
std::map<GridLocation, double> cost_so_far;
a_star_search(grid, start, goal, came_from, cost_so_fa);
draw_grid(grid, 2, nullptr, &came_from);
std::cout << '\n';
draw_grid(grid, 3, &cost_so_far, nullptr);
std::cout << '\n';
std::vector<GridLocation> path = reconstruct_path(start, goal, came_from);
draw_grid(grid, 3, nullptr, nullptr, &path);
}
```

```
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```

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2.4.1 Straighter paths

If you implement this code in your own project you might find that some of the paths aren't as "straight" as you'd like. **This is normal**. When using *grids*, especially grids where every step has the same movement cost, you end up with **ties**: many paths have exactly the same cost. A* ends up picking one of the many short paths, and very often **it doesn't look good to you**. The quick hack is to break the ties, but it's not entirely satisfactory. The better approach is to change the map representation, which makes A* a lot faster, and also produces straighter, better looking paths. However, that only works for mostly-static maps where every step has the same movement cost. For the demos on my page, I'm using a quick hack, but it only works with my slow priority queue. If you switch to a faster priority queue you'll need a different quick hack.

2.5 Production code



The C++ code I've shown above is simplified to make it easier to follow the algorithm and data structures. In practice there are many things you'd want to do differently:

- inlining small functions
- the Location parameter should be part of the Graph
- the cost could be int or double, and should be part of the Graph
- use unordered_set or array instead of set
- use unordered_map or array instead of map
- pass larger data structures by reference instead of by value
- return larger data structures in out parameters instead of returning them, or use move constructors (for example, the vector returned from the neighbors function)
- the heuristic can vary and should be a parameter to the A* function

Here's how the A* code might look different with some of these changes:

```
template<typename Graph>
void a_star_search
  (Graph graph,
    typename Graph::Location start,
    typename Graph::Location goal,
    std::function<typename Graph::Location, typename Graph::Location>& came_from,
    std::unordered_map<typename Graph::Location, typename Graph::Location>& came_from,
    std::unordered_map<typename Graph::Location, typename Graph::cost_t>& cost_so_far)
{
    typedef typename Graph::Location Location;
    typedef typename Graph::cost_t cost_t;
    PriorityQueu<Location, cost_t> frontier;
    std::vector<Location> neighbors;
    frontier.put(start, cost_t(0));

    came_from[start] = start;
    cost_so_far[start] = cost_t(0);
```

I wanted the code on this page to be about the algorithms and data structures and not about the C++ optimizations so I tried to show simple code instead of fast or abstract code.

3 C# Implementation

<u>#</u>

These were my first C# programs so they might not be idiomatic or stylistically proper. These examples aren't as complete as the Python and C++ sections, but I

hope they're helpful.

Here's a simple graph, and Breadth First Search:

```
using System;
using System.Collections.Generic;
public class Graph<Location>
    // NameValueCollection would be a reasonable alternative here, if
    // you're always using string location types
    public Dictionary<Location, Location[]> edges
        = new Dictionary<Location, Location[]>();
    public Location[] Neighbors(Location id)
        return edges[id];
};
class BreadthFirstSearch
    static void Search(Graph<string> graph, string start)
        var frontier = new Queue<string>();
        frontier.Enqueue(start);
        var visited = new HashSet<string>();
        visited.Add(start);
        while (frontier.Count > 0)
            var current = frontier.Dequeue();
            Console.WriteLine("Visiting {0}", current);
            foreach (var next in graph.Neighbors(current))
```

```
if (!visited.Contains(next)) {
                    frontier.Enqueue(next);
                    visited.Add(next);
                }
            }
        }
    static void Main()
        Graph<string> g = new Graph<string>();
        g.edges = new Dictionary<string, string[]>
            { "A", new [] { "B" } },
            { "B", new [] { "A", "C", "D" } },
            { "C", new [] { "A" } },
            { "D", new [] { "E", "A" } },
            { "E", new [] { "B" } }
       };
        Search(g, "A");
    }
}
```

Here's a graph representing a grid with weighted edges (the forest and walls example from the main page):

```
using System.Collections.Generic;

// A* needs only a WeightedGraph and a location type L, and does *not*
// have to be a grid. However, in the example code I am using a grid.
public interface WeightedGraph<L>
{
    double Cost(Location a, Location b);
    IEnumerable<Location> Neighbors(Location id);
```

```
}
public struct Location
    // Implementation notes: I am using the default Equals but it can
    // be slow. You'll probably want to override both Equals and
    // GetHashCode in a real project.
    public readonly int x, y;
    public Location(int x, int y)
        this.x = x;
        this.y = y;
    }
public class SquareGrid : WeightedGraph<Location>
    // Implementation notes: I made the fields public for convenience,
    // but in a real project you'll probably want to follow standard
    // style and make them private.
    public static readonly Location[] DIRS = new []
        {
            new Location(1, 0),
            new Location(0, -1),
            new Location(-1, 0),
            new Location(0, 1)
        };
    public int width, height;
    public HashSet<Location> walls = new HashSet<Location>();
    public HashSet<Location> forests = new HashSet<Location>();
    public SquareGrid(int width, int height)
        this.width = width;
        this.height = height;
```

```
public bool InBounds(Location id)
        return 0 <= id.x && id.x < width
           && 0 <= id.y && id.y < height;
   }
    public bool Passable(Location id)
        return !walls.Contains(id);
    public double Cost(Location a, Location b)
        return forests.Contains(b) ? 5 : 1;
    public IEnumerable<Location> Neighbors(Location id)
        foreach (var dir in DIRS) {
           Location next = new Location(id.x + dir.x, id.y + dir.y);
           if (InBounds(next) && Passable(next)) {
               vield return next;
           }
        }
public class PriorityQueue<T>
   // I'm using an unsorted array for this example, but ideally this
   // would be a binary heap. There's an open issue for adding a binary
   // heap to the standard C# library: https://github.com/dotnet/corefx/issues/574
   //
   // Until then, find a binary heap class:
   // * https://github.com/BlueRaja/High-Speed-Priority-Queue-for-C-Sharp
   // * http://visualstudiomagazine.com/articles/2012/11/01/priority-queues-with-c.aspx
   // * http://xfleury.github.io/graphsearch.html
   // * http://stackoverflow.com/questions/102398/priority-queue-in-net
    private List<Tuple<T, double>> elements = new List<Tuple<T, double>>();
```

```
public int Count
        get { return elements.Count; }
    public void Enqueue(T item, double priority)
        elements.Add(Tuple.Create(item, priority));
    public T Dequeue()
        int bestIndex = 0;
        for (int i = 0; i < elements.Count; <math>i++) {
            if (elements[i].Item2 < elements[bestIndex].Item2) {</pre>
                bestIndex = i;
            }
        }
       T bestItem = elements[bestIndex].Item1;
        elements.RemoveAt(bestIndex);
        return bestItem;
/* NOTE about types: in the main article, in the Python code I just
 * use numbers for costs, heuristics, and priorities. In the C++ code
* I use a typedef for this, because you might want int or double or
* another type. In this C# code I use double for costs, heuristics,
* and priorities. You can use an int if you know your values are
* always integers, and you can use a smaller size number if you know
 * the values are always small. */
public class AStarSearch
    public Dictionary<Location, Location> cameFrom
        = new Dictionary<Location, Location>();
    public Dictionary<Location, double> costSoFar
        = new Dictionary<Location, double>();
```

```
// Note: a generic version of A* would abstract over Location and
    // also Heuristic
   static public double Heuristic(Location a, Location b)
        return Math.Abs(a.x - b.x) + Math.Abs(a.y - b.y);
   public AStarSearch(WeightedGraph<Location> graph, Location start, Location goal)
       var frontier = new PriorityQueue<Location>();
        frontier.Enqueue(start, 0);
        cameFrom[start] = start;
        costSoFar[start] = 0;
        while (frontier.Count > 0)
            var current = frontier.Dequeue();
            if (current.Equals(goal))
                break;
            foreach (var next in graph.Neighbors(current))
                double newCost = costSoFar[current]
                    + graph.Cost(current, next);
                if (!costSoFar.ContainsKey(next)
                    || newCost < costSoFar[next])</pre>
                    costSoFar[next] = newCost;
                    double priority = newCost + Heuristic(next, goal);
                    frontier.Enqueue(next, priority);
                    cameFrom[next] = current;
            }
        }
public class Test
```

```
static void DrawGrid(SquareGrid grid, AStarSearch astar) {
    // Print out the cameFrom array
    for (var y = 0; y < 10; y++)
    {
        for (var x = 0; x < 10; x++)
            Location id = new Location(x, y);
            Location ptr = id;
            if (!astar.cameFrom.TryGetValue(id, out ptr))
                ptr = id;
            if (grid.walls.Contains(id)) { Console.Write("##"); }
            else if (ptr.x == x+1) { Console.Write("\u2192 "); }
            else if (ptr.x == x-1) { Console.Write("\u2190 "); }
            else if (ptr.y == y+1) { Console.Write("\u2193 "); }
           else if (ptr.y == y-1) { Console.Write("\u2191 "); }
            else { Console.Write("* "); }
        }
        Console.WriteLine();
    }
}
static void Main()
    // Make "diagram 4" from main article
    var grid = new SquareGrid(10, 10);
    for (var x = 1; x < 4; x++)
        for (var y = 7; y < 9; y++)
        {
            grid.walls.Add(new Location(x, y));
        }
    }
    grid.forests = new HashSet<Location>
        {
            new Location(3, 4), new Location(3, 5),
            new Location(4, 1), new Location(4, 2),
            new Location(4, 3), new Location(4, 4),
            new Location(4, 5), new Location(4, 6),
            new Location(4, 7), new Location(4, 8),
```

```
new Location(5, 1), new Location(5, 2),
new Location(5, 3), new Location(5, 4),
new Location(5, 5), new Location(5, 6),
new Location(5, 7), new Location(5, 8),
new Location(6, 2), new Location(6, 3),
new Location(6, 4), new Location(6, 5),
new Location(6, 6), new Location(6, 7),
new Location(7, 3), new Location(7, 4),
new Location(7, 5)
};

// Run A*
var astar = new AStarSearch(grid, new Location(1, 4),
new Location(8, 5));
DrawGrid(grid, astar);
```

4 Algorithm changes

<u>#</u>

The version of Dijkstra's Algorithm and A* on my pages is slightly different from what you'll see in an algorithms or AI textbook.

The pure version of Dijkstra's Algorithm starts the priority queue with all nodes, and does not have early exit. It uses a "decrease-key" operation in the queue. It's fine in theory. But in practice…

1. By starting the priority with only the start node, we can keep it small, which makes it faster and use less memory

- 2. With early exit, we almost never need to insert all the nodes into the queue, and we can return the path as soon as its found.
- 3. By not putting all nodes into the queue at the start, most of the time we can use a cheap insert operation instead of the more expensive decrease-key operation.
- 4. By not putting all nodes into the queue at the start, we can handle situations where we do not even know all the nodes, or where the number of nodes is infinite.

This variant is sometimes called "Uniform Cost Search". See <u>Wikipedia</u> to see the pseudocode, or read<u>this paper</u> [PDF] to see justifications for these changes.

There are three further differences between my version and what you might find elsewhere. These apply to both Dijkstras Algorithm and A*:

5. I eliminate the check for a node being in the frontier with a higher cost. By not checking, I end up with duplicate elements in the frontier. *The algorithm still works*. It will revisit some locations more than necessary (but rarely, in my experience). The code is simpler and it allows me to use a simpler and faster priority queue that does not support the decrease-key operation. The paper "Priority Queues and Dijkstra's Algorithm" suggests that this approach is faster in practice.

- 6. Instead of storing both a "closed set" and an "open set", I have a visited flag that tells me whether it's in *either* of those sets. This simplifies the code further.
- 7. I don't need to store a separate open or closed set explicitly because the set is *implicit* in the keys of the came_from and cost_so_far tables. Since we always want one of those two tables, there's no need to store the open/closed sets separately.
- 8. I use hash tables instead of arrays of node objects. This eliminates the rather expensive *initialize* step that many other implementations have. For large game maps, the initialization of those arrays is often slower than the rest of A*.

If you have more suggestions for simplifications that preserve performance, please let me know!

5 Optimizations

#

For the code I present here, I've been focusing on simplicity and generality rather than performance. **First make it work, then make it fast.** Many of the optimizations I use in real projects are specific to the project, so instead of presenting optimal code, here are some ideas to pursue for your own project:

5.1 Graph



The biggest optimization you can make is to explore fewer nodes. My #1 recommendation is that if you're using a grid map, consider using a non-grid pathfinding graph. It's not always feasible but it's worth looking at.

If your graph has a simple structure (e.g. a grid), calculate the neighbors in a function. If it's a more complex structure (either a non-grid, or a grid with lots of walls, like a maze), store the neighbors in a data structure.

You can also save a bit of copying by reusing the neighbors array. Instead of *returning* a new one each time, allocate it once in the search code and pass it into the graph's neighbors method.

5.2 Queue



Breadth First Search uses a simple queue instead of the priority queue needed by the other algorithms. Queues are simpler and faster than priority queues. In exchange, the other algorithms usually explore fewer nodes. In most game maps, exploring fewer nodes is worth the slowdown from the other algorithms. There are some maps though where you don't save much, and it might be better to use Breadth First Search.

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For queues, use a deque instead of an array. A deque allows fast insertion and removal on either end, whereas an array is fast only at one end. In Python, see <u>collections.deque</u> in C++, see the <u>deque</u> container. However, breadth first search doesn't even need a queue; it can use two vectors, swapping them when one is empty.

For priority queues, use a binary heap instead of an array or sorted array. A binary heap allows fast insertion and removal, whereas an array is fast at one or the other but not both. In Python, see<u>heapq</u>; in C++, see the <u>priority_queue</u> container.

In Python, the Queue and PriorityQueue classes I presented above are so simple that you might consider inlining the methods into the search algorithm. I don't know if this buys you much; I need to measure it. The C++ versions are going to be inlined.

In Dijkstra's Algorithm, note that the priority queue's priority is stored twice, once in the priority queue and once in cost_so_far, so you could write a priority queue that gets priorities from elsewhere. I'm not sure if it's worth it.

The paper <u>"Priority Queues and Dijkstra's Algorithm"</u> by Chen, Chowdhury, Ramachandran, Lan Roche, Tong suggests optimizing the structure of Dijkstra's Algorithm by not reprioritizing, and it also suggests looking at <u>pairing heaps</u> and other data structures.

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If you're considering using something other than a binary heap, first measure the size of your frontier and how often you reprioritize. Profile the code and see if the priority queue is the bottleneck.

My gut feeling is that *bucketing* is promising. Just as bucket sort and radix sort can be useful alternatives to quicksort when the keys are integers, we have an even better situation with Dijkstra's Algorithm and A*. The priorities in Dijkstra's Algorithm are *incredibly narrow*. If the lowest element in the queue has priority f, then the highest element has priority f+e where e is the maximum edge weight. In the forest example, I have edge weights 1 and 5. That means all the priorities in the queue are going to be between f and f+5. Since they're all integers, *there are only six different priorities*. We could use six buckets and not sort anything at all! A* produces a wider range of priorities but it's still worth looking at. And there are fancier bucketing approaches that handle a wider range of situations.

Note that if all the edge weights are 1, the priorities will all be between f and f+1. This yields a variant of Breadth First Search that uses two arrays instead of a queue, which I used <u>on my hex grid page</u>. If the weights are 1 or 2, you'll have three arrays; if the weights are 1, 2, or 3, you'll have four arrays; and so on.

I have more note about priority queue data structures here

5.3 Search



The heuristic adds complexity and cpu time. The goal though is to explore fewer nodes. In some maps (such as mazes), the heuristic may not add much information, and it may be better to use a simpler algorithm without a heuristic guide.

If your graph uses integers as locations, consider using a simple array instead of a hash table for cost_so_far, visited, came_from, etc. Since visited is an array of booleans, you can use a bit vector. Initialize the bit vector for all ids, but leave cost_so_far and came_from uninitialized (if your language allows). Then only initialize on the first visit.

If you run only one search at a time, you can statically allocate and then reuse data structures from one invocation to the next. Instead of initializing them on entry, reset them on exit. You can use an array to keep track of which locations were modified, then reset only those. For example, if you're using a visited[] array initialized for 1000 nodes, but most searches visit fewer than 100 nodes, then you can keep an array of indices modified, and then reset only those 100 nodes on exiting the function instead of re-initializing all 1000 nodes when you enter the function.

Some people use an *inadmissible* (overestimating) heuristic to speed up A* search. This seems reasonable. I haven't looked closely into its implications though. I believe (but don't know for sure) that some already-visited elements may need to be visited again even after they've been taken out of the frontier

Some implementations *always* insert a new node into the open set, even if it's already there. You can avoid the potentially expensive step of checking whether the node is already in the open set. This will make your open set bigger/slower and you'll also end up evaluating more nodes than necessary. If the open-set test is expensive, it might still be worth it. However, in the code I've presented, I made the test cheap and I don't use this approach.

Some implementations *don't test* whether a new node is better than an existing node in the open set. This avoids a potentially expensive check. However, it also *can lead to a bug*. For some types of maps, you will not find the shortest path when you skip this test. In the code I've presented, I check this (new_cost < cost_so_far). The test is cheap because I made it cheap to look upcost_so_far.

6 Troubleshooting

<u>#</u>

6.1 Wrong path

#

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If you're not getting a shortest path, try testing:

- Does your priority queue work correctly? Try stopping the search and dequeuing all the elements. They should all be in order
- Does your heuristic ever overestimate the true distance? The priority of a new node should never be lower than the priority of its parent, unless you are overestimating the distance (you can do this but you won't get shortest paths anymore).
- In a statically typed language, the cost, heuristic, and priority values need to have compatible types. The sample code on this page works with either integers or floating point types, but not all graphs and heuristics are limited to integer values. Since priorities are the sum of costs and heuristics, the priorities will need to be floating point if *either* costs or heuristics are floating point.

6.2 Ugly path



The most common question I get when people run A* on a grid is *why don't my paths look straight?* If you've told A* that all grid movements are equal, then there are lots of shortest paths of the same length, and it's going to pick one arbitrarily. The path is *short* but it doesn't *look* good.

- One solution is to *straighten* the paths afterwards, using a "string pulling" algorithm.
- One solution is to *guide* the paths in the right direction, by adjusting the heuristic. There are some cheap tricks that don't work in all situations; <u>read</u> more here.
- One solution is to *not use a grid*. Tell A* just the places where you might turn, instead of every grid square; read more here.
- One solution is a hack, but it works some of the time. When iterating through neighbors, instead of always using the same ordering (such as north, east, south, west), change the ordering on "odd" grid nodes (those where (x+y) % 2 == 1). **I use this trick on these tutorial pages.**

7 More reading

<u>#</u>

- Algorithms textbooks often use mathematical notation with single-letter variable names. In these pages I've tried to use more descriptive variable names. Correspondences:
 - ∘ cost is sometimes written as *w* or *d* or *l* or length
 - cost_so_far is usually written as *q* or *d* or distance
 - heuristic is usually written as *h*
 - In A*, the priority is usually written as f, where f = g + h
 - \circ came_from is sometimes written as π or parent or previous or prev