

Duration Take the 1st derivative of the bond's price with respect to " i ", interest rate.

$$P = \sum_{t=1}^n \frac{C_t}{(1+i)^t} = \sum_{t=1}^n C_t (1+i)^{-t}$$

$$\frac{\partial P}{\partial i} = \sum_{t=1}^n -t \cdot C_t (1+i)^{-t-1} \cdot 1$$

(via the chain rule)

$$= \sum_{t=1}^n -t \cdot \frac{C_t}{(1+i)^{t+1}}$$

$$= \frac{1}{(1+i)} \cdot \left[\sum_{t=1}^n -t \cdot \frac{C_t}{(1+i)^t} \right]$$

for very small $\frac{1}{(1+i)}$ we get:

$$= \sum_{t=1}^n -t \cdot \frac{C_t}{(1+i)^t}$$

Now divide by P and get duration

$$D = \frac{\sum_{t=1}^n -t \frac{C_t}{(1+i)^t}}{P}$$

Modified duration: using the $\frac{1}{(1+i)}$ above we "got rid of" we get

$$MD = \frac{1}{(1+i)} \cdot D$$