

Convexity - 2nd derivative of bond's Price with respect to "i" - Interest rate. or just take derivative of modified duration (MD)

$$MD = \frac{1}{(1+i)} \left[\sum_{t=1}^n -t \cdot \frac{C_t}{(1+i)^t} \right] \text{ rewrite as:}$$

$$MD = (1+i)^{-1} \left[\sum_{t=1}^n -t \cdot C_t (1+i)^{-t} \right]$$

using product rule & chain rule

* We can just take the 1st derivative of MD since it's already a derivative

$$\frac{\partial^2 P}{\partial i^2} = (1+i)^{-1} \left[\sum_{t=1}^n t^2 \cdot C_t (1+i)^{-t-1} \right] +$$

$$\sum_{t=1}^n -t \cdot C_t (1+i)^{-t} \cdot -1 (1+i)^{-2} \cdot 1$$

pull out $(1+i)^{-1}$ and rewrite as $\frac{1}{(1+i)}$

$$= \frac{1}{(1+i)} \left[\sum_{t=1}^n t^2 \cdot \frac{C_t}{(1+i)^{t+1}} \right] + \sum_{t=1}^n -t \cdot \frac{C_t}{(1+i)^t} \left[\frac{-1}{(1+i)^2} \right]$$

$$= \sum_{t=1}^n t^2 \cdot \frac{C_t}{(1+i)^{t+2}} + t \cdot \frac{C_t}{(1+i)^{t+2}}$$