

- 1) John C. Mitchell, "Concepts in Programming Languages", Cambridge University Press, 2006
- 2) Benjamin Pierce, "Types and Programming Languages", MIT Press, 2002
- 3) Kenneth Slanenge and Barry L. Kutz, "Formal Syntax and Semantics of Programming Languages", Addison Wesley, 1995

Lisp, Algol, ML, C, Simula, C++, Java

OCAML Functional Programming with types

- Programming Languages are the medium of expression in the art of abstraction.

Computer Programming

- To write programs quickly vs. harder to design, easier to optimize

vs. to make programming cumbersome. Correctness vs. complicated type.

Specific Themes:

- Computability (Gödel's) \rightarrow Computable Functions in Computability

Computability is undecidable for Turing machines

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static analysis (analysis) → Compile time optimization for

Compile time Run time

(ribbed) conservative

more accurate, more expensive, more time-consuming, more difficult to implement

$P_{\text{err}} = 4\%$

more accurate, more expensive, more time-consuming, more difficult to implement

syntax check

more accurate, more expensive, more time-consuming, more difficult to implement

more accurate, more expensive, more time-consuming, more difficult to implement

- Expressiveness vs. efficiency

(high)

(low)

(high)

(low)

History:

1958 - 1960: Algol, Cobol, Fortran, Lisp

1970: methods for analyzing data

1980 - 1990 Abstract data type

21st Century: - greater diversity computing devices

- correctness

- security

Definition:

- A Computational model is a collection of value & operations.

intrinsic

in code, structures MAHAN

value $\xrightarrow{\text{2+3}}$ operation

Computing is the art of manipulating data

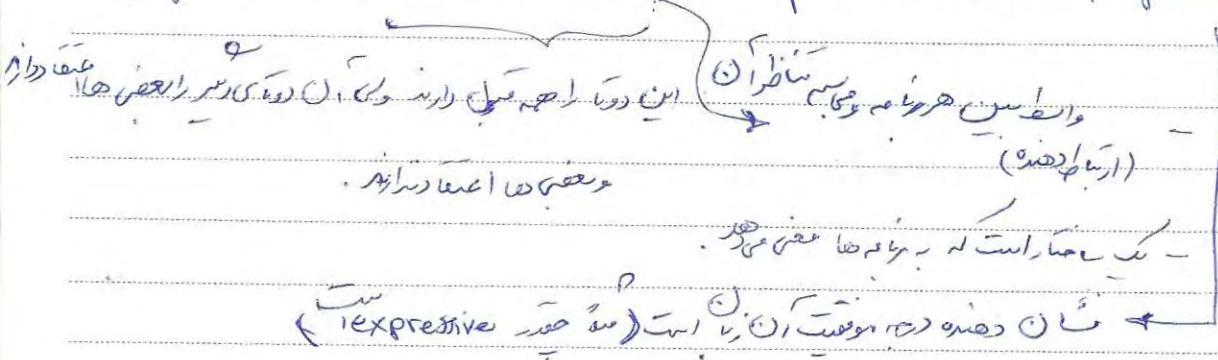
- A Computation is the application of a sequence of operations to a value to yield another value.
- A Program is a specification of a computation.
- A Programming language is a notation for writing programs.

Computing is the art of manipulating data. This notation is the programming language.

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Language = Syntax + Semantics + Computational model + pragmatics



Computational Models:

- 1) Functional 2) Logic 3) Imperative

(SMT)
Declarative

(SMT)

Functional:

value = functional , operations = function application

(In Functional lang.) is functional & ML , Lisp

(In assignment lang.)

$$\sigma = \sqrt{\frac{\sum_{i=1}^n x_i^2}{n} - \left(\frac{\sum_{i=1}^n x_i}{n}\right)^2}$$

(for functional lang.)

$$sd(xs) = \sqrt{v}$$

where

$$n = \text{length}(xs)$$

$$v = \frac{\text{fold}(\text{plus}, \text{map}(\text{sqr}, xs))}{n} - \text{sqr}(\text{fold}(\text{plus}, xs)/n)$$

(In assignment). Ext definition (i.e., \equiv " = " talk)

$$sd \stackrel{\text{def}}{=} \sqrt{v}$$

✓ Logic:

عنوان: ما هي المنطق؟

Logic ما هي المنطق؟

human (Socrates)

human (prolope)

1. mortal(x) if human(x) → mortal(y) if human(y)
 $\frac{\text{mortal}(x) \text{ if } \text{human}(x)}{\text{mortal}(y) \text{ if } \text{human}(y)}$ $\frac{\text{mortal}(x) \text{ if } \text{human}(x)}{\text{mortal}(y) \text{ if } \text{human}(y)}$

1a: human (Socrates) Fact

1b: human (prolope) Fact

2: mortal(x) if human(x) Rule

3: $\neg \text{mortal}(y)$ assumption

4a: $x = y$ 2, 3, unification

4b: $\neg \text{human}(y)$ modus tollens

5a: $y = \text{Socrates}$ 1, 4, unification

5b: $y = \text{prolope}$ 1, 4, unification

6: Contradiction

لذلك لا يمكن أن يكون $\neg \text{mortal}(y)$ صحيحًا.

Value = fact and relation

Operation = inference rule \rightarrow ما هي المضاعف؟

✓ Imperative: Value = state, operation = transition (assignment)

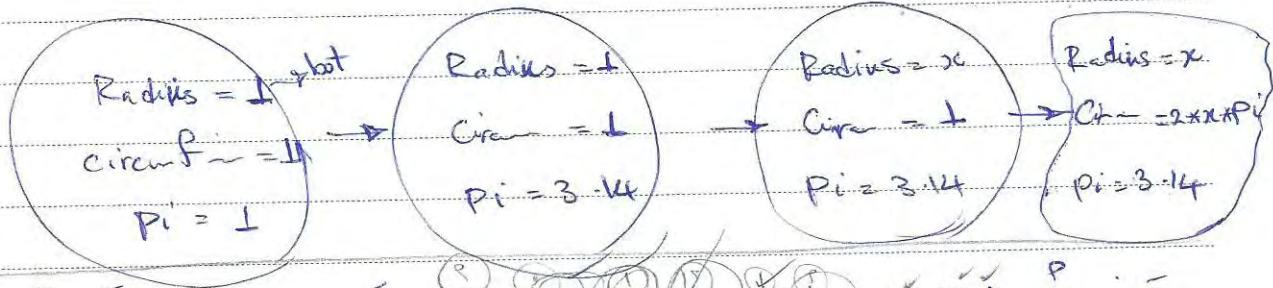
ما هي المضاعف؟ assign to state

MAHAN $s_0 \xrightarrow{o_0} s_1 \xrightarrow{o_1} \dots \xrightarrow{o_{n-2}} s_{n-1} \xrightarrow{o_{n-1}} s_n$

PL29031806.PDF
 Year: Month: Day:
 Due Date: PDF (zip) deadline: Jun 15, 2023
 Subject: Imperative program
 Constant pi = 3.14;

input (Radius); circumference = 2 * pi * Radius;

output (circumference);



→ The series of steps from stage 1 to stage 4 is valid.

Syntax:

(formal syntax = w, l)

Grammar Symbol

valid phrase

Definition: A grammar (Σ, N, P, S) consists of four parts

1. A finite set Σ of terminal symbols, the alphabet of the language
 (atomic) words, i.e., letters, digits, punctuation marks, etc.
2. A finite set of nonterminal symbols, or syntactic categories, each of which represents some collection of subphrases of the sentences.

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3. A finite set P of production rule or productions that

describe how each nonterminal is defined in terms of terminal symbols and nonterminal.

4. A distinguished nonterminal S , the start symbol, the specification the principle category being defined.

$\vdash P$, the start symbol, where \vdash

$\vdash = \rightarrow$ is defined to be $\xrightarrow{\text{derivation}}$

$\langle \text{declaration} \rangle \rightarrow \text{nonterminal}^*$

$\langle \text{declaration} \rangle \vdash = \underline{\text{var}} \ \& \ \underline{\text{variable list}} \ \vdash \langle \text{type} \rangle ;$

terminal nonterminal
 $\text{var } x, y : \text{int};$

into BNF (Backus-Naur Form) $\vdash \text{int} ;$

and vocabulary \rightarrow nonterminal symbol, terminal symbol

Type 3, Type 2, Type 1, Type 0. $\vdash \text{int} ;$

Type 0 (Unrestricted grammar) $\vdash \text{int} ;$

Non-terminal \rightarrow Production rule \rightarrow $\alpha ::= \beta$

MAHAN $\vdash \text{int} ;$

Sequence of terminal & nonterminal

$a \langle \text{thing} \rangle b ::= b \langle \text{another thing} \rangle$

Type 0 grammar $\xleftarrow{\text{in rules}} \xrightarrow{\text{Turing}}$
 \downarrow Expressing $\text{NPDA} \subset \text{Turing}$

\checkmark Type 1 (Context Sensitive grammars) $\xrightarrow{\text{via}}$

$\langle \text{thing} \rangle b ::= b \langle \text{thing} \rangle$

$\langle \text{thing} \rangle c ::= \lambda \langle \text{another thing} \rangle d$

Type 1 \longleftrightarrow LBA (Linear Bounded Automata)

Type 2 (Context-free grammar) $\xrightarrow{\text{via}}$

$\langle A \rangle ::= \alpha$

β non-terminal $\in \Sigma^*$ (غير مinal)

Type 2 \longleftrightarrow PDA (Push-Down Automata)

(BPF Grammar) \hookrightarrow just one position + Syntax \hookrightarrow first of

(!) (!) (!) static semantics \hookrightarrow Context-Sensitive

Type 3 (Regular grammars)

$\langle A \rangle ::= \alpha$

$\langle A \rangle ::= \alpha \langle B \rangle$

$\langle \text{binary numerical} \rangle ::= 0$

: نیکیتہ بولٹھی (Jee)

$\langle \text{binary numerical} \rangle ::= 1$

$\langle \text{binary num} \rangle ::= 0 \quad \langle \text{binary num} \rangle$

$\langle \text{binary num} \rangle ::= 1 \quad \langle \text{binary num} \rangle$

regular B.N.F Grammar

Type 3 \longleftrightarrow FSA

(Finite state Automata)

Context-free Grammars:

$\langle \text{Sentence} \rangle ::= \langle \text{noun phrase} \rangle \langle \text{verb phrase} \rangle$

$\langle \text{noun phrase} \rangle ::= \langle \text{determiner} \rangle \langle \text{noun} \rangle \mid \langle \text{determiner} \rangle \langle \text{noun} \rangle \langle \text{prepositional phrase} \rangle$

$\langle \text{verb phrase} \rangle ::= \langle \text{verb} \rangle \mid \langle \text{verb} \rangle \langle \text{noun phrase} \rangle$

$\langle \text{verb} \rangle \langle \text{noun phrase} \rangle \langle \text{prepositional phrase} \rangle$

$\langle \text{prepositional phrase} \rangle ::= \langle \text{preposition} \rangle \langle \text{noun phrase} \rangle$

$\langle \text{noun} \rangle ::= \underline{\text{boy}} \mid \underline{\text{girl}} \mid \underline{\text{cat}} \mid \underline{\text{telescope}} \mid \underline{\text{song}} \mid \underline{\text{feather}}$

$\langle \text{determiner} \rangle ::= \underline{\text{all}} \mid \underline{\text{the}}$

$\langle \text{verb} \rangle ::= \underline{\text{saw}} \mid \underline{\text{song}} \mid \underline{\text{surprised}} \mid \underline{\text{touched}}$

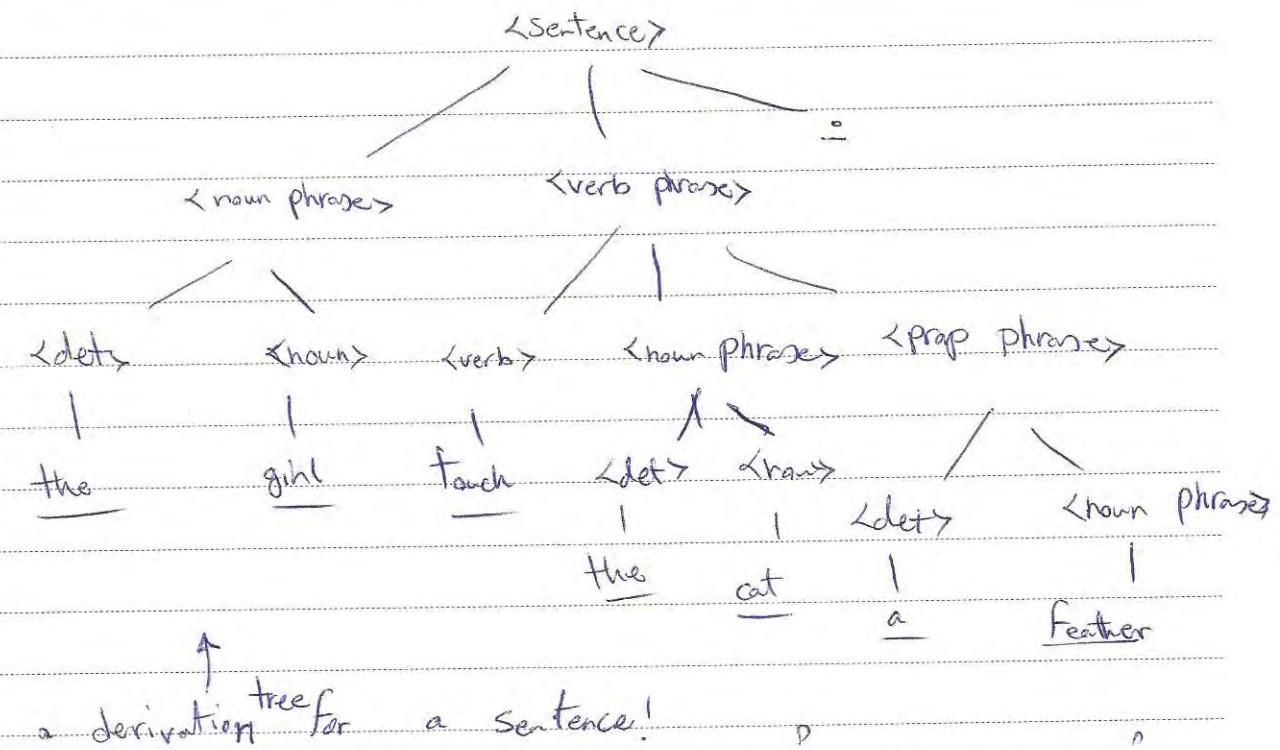
$\langle \text{preposition} \rangle = \underline{\text{by}} \mid \underline{\text{with}}$

a boy surprised.

the girl with the cat sang feather.

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Semantic grammar [boy! song! feathers! girl! with! by!]



a derivation tree for a sentence!

the girl touch the cat a feather

the girl touch the cat a feather

ambiguity the girl touch the cat a feather

the girl touch the cat a feather

loop (cycle) the girl touch the cat a feather

valid phrase, valid word

Definition - A grammar is ~~ob~~ ambiguous if some phrase

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in the language generated by the grammar has more than one

derivation tree.

Derivations
non-leftmost

Context-sensitive grammars:

$\langle \text{Sentence} \rangle ::= abc \mid a \langle \text{thing} \rangle bc$

$\langle \text{thing} \rangle b ::= b \langle \text{thing} \rangle$

$\langle \text{thing} \rangle c ::= \langle \text{others} \rangle bcc$

$a \langle \text{others} \rangle ::= aa \mid aa \langle \text{thing} \rangle$

$b \langle \text{others} \rangle ::= \langle \text{others} \rangle b$

$\langle \text{Sentence} \rangle \Rightarrow a \langle \text{thing} \rangle bc \Rightarrow ab \langle \text{thing} \rangle c \Rightarrow ab \langle \text{others} \rangle bcc \Rightarrow$

$a \langle \text{others} \rangle bcc \Rightarrow aabbcc$ } $a^nb^n \mid n \in \mathbb{N}^+$

wren:

wren \cup \emptyset

Control structures \rightarrow if, while

typed \rightarrow boolean, integer

strongly typed

(if, type) is well-typed (typed safe)

terminal symbol \rightarrow reserved words, +, -, /, ., !, ?, *, &

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(empty string) \cup \emptyset

Learn. - Month. - Day. : In C/C++ there are two syntax

Two parts of syntax → lexical syntax → token
(lexer)

→ phrase-structure syntax → tree
(parser)

(no tokenize lexer) (no parser) → tokens

<token> ::= <identifier> | <numerical> : In this when we token

<reserved words> | <relations> | <weak opr> | <strong opr> () + = ! , ;

no tokens of tag → no tokens in parser even in text for lexer

no tokens in lexical syntax (no tokens in parser)

no tokens in phrase → phrase-structure syntax (no tokens in parser)

involving tokens in parser parser : syntactic analyzer

Given : BNF Formal syntax

<program> ::= Program <identifier> is <blocks>

<block> ::= Declaration sequence begin <command seq> end

<declaration seq> ::= ε | <declarations> <declaration seq>

<declaration> ::= var <variable list> : <type>;

$\langle \text{types} \rangle ::= \underline{\text{integer}} \mid \underline{\text{boolean}}$

$\langle \text{Variable list} \rangle ::= \langle \text{variable} \rangle \mid \langle \text{variable} \rangle, \langle \text{Variable list} \rangle$

$\langle \text{Command seq} \rangle ::= \langle \text{Command} \rangle \mid \langle \text{Command} \rangle ; \langle \text{Command seq} \rangle$

$\langle \text{Command} \rangle ::= \langle \text{variable} \rangle := \langle \text{expression} \rangle$ assignment
 $\mid \underline{\text{skip}} \mid \underline{\text{read}} \langle \text{variable} \rangle \mid \underline{\text{write}} \langle \text{integer expr} \rangle \mid \underline{\text{while}} \langle \text{boolean expr} \rangle$
 $\quad \quad \quad \text{if } \langle \text{boolean expr} \rangle \text{ then } \langle \text{Command seq} \rangle \text{ end if } \mid \underline{\text{do}} \langle \text{Command seq} \rangle$
 $\quad \quad \quad \text{if } \langle \text{boolean expr} \rangle \text{ then } \langle \text{Command seq} \rangle \text{ else } \langle \text{Command seq} \rangle \text{ end if } \mid \underline{\text{endwhile}}$

$\langle \text{Variable} \rangle ::= \langle \text{identifier} \rangle$

$\langle \text{Relation} \rangle ::= \langle = \rangle \mid \langle \neq \rangle \mid \langle < \rangle \mid \langle > \rangle \mid \langle \leq \rangle \mid \langle \geq \rangle$

$\langle \text{weak op} \rangle ::= + \mid -$

$\langle \text{Strong op} \rangle ::= * \mid /$

$\langle \text{Identifier} \rangle ::= \langle \text{letters} \rangle \mid \langle \text{Identifier} \rangle \langle \text{letters} \rangle \mid \langle \text{Identifier} \rangle \langle \text{digit} \rangle$

$\langle \text{letters} \rangle ::= \text{a} \mid \text{b} \mid \dots \mid \text{z}$

$\langle \text{numerals} \rangle ::= \langle \text{digit} \rangle \mid \langle \text{digit} \rangle \langle \text{numerals} \rangle$

$\langle \text{digit} \rangle ::= 0 \mid 1 \mid \dots \mid 9$

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wren

<Command> := if <boolean expr> then <Command> : Pascal

| if <boolean expr> then <Command> else <Command>

control

if expr1 then if expr2 then cond1 else cond2

~~several if~~

end if, then end if

logics operators (with scope) (strong operation) \rightarrow strong rule

1/8
operator with scope (strong operation) (weak operation) (weak operation)

(weak operation) : weak operation (weak operation) 2+3+5
2+(3+5) = (2+3)+5

Context Constraints in Wren:

expressions (phrase structure, lexical), Syntax errors, etc.

(b)

Program illegal is ? Syntactic / semantic / illegal

Var a:=boolean;

begin

a:=5 / runtime error / semantic / illegal

end

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arithmetic expressions, function definitions, assignment statements, control structures (loop, condition, etc.)

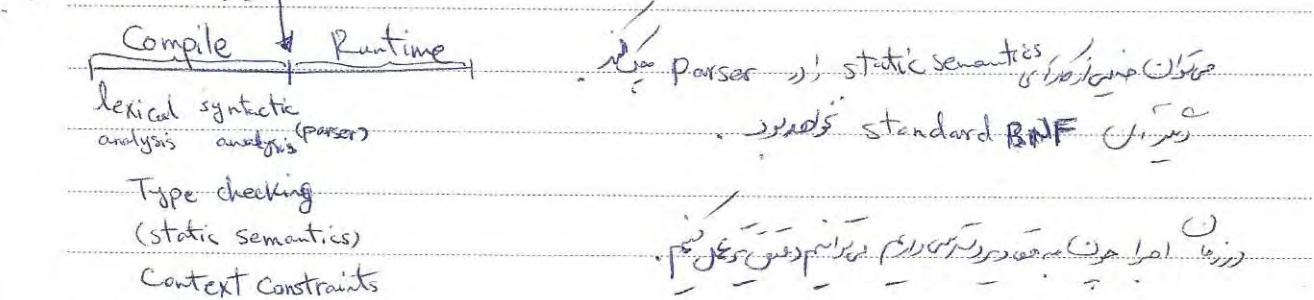
أولاً دوافع إثبات البرمجة

پـ> First semantics for runtime analysis, پـ> static semantics

and static semantic analysis can be implemented in parallel with syntactic analysis

(I-II) Context conditions (بن توافق شرطی) Context Condition, when, then for

BNF is too simple Context-sensitive languages



Semantic Errors:

(Executive Programmer) must specify what is expected

what is not allowed to specify the language requirement

2D Error must terminate as soon as possible

- An attempt is to divide by zero.

Compile time error exception for illegal null pointer

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thread will start executing when

① ~~Runtime error~~ / ~~Compile time error~~ in Routine ~~variables~~

② Runtime checking / static checking! ~~Runtime~~ ~~Compile time~~

A variable that hasn't been initialized! ~~Runtime checking~~

An iteration command doesn't terminate!

③ ~~Illegal value~~ ~~variables~~

$\text{exp} ::= \text{num} \mid \text{lit} \mid \text{id} \mid \text{LP exp RP} \mid \text{exp APP exp} \mid \text{exp CAT exp} \mid \text{exp MUL exp}$

$\text{VB exp VB } \mid \text{LET }$
~~LP~~ id BE exp IN exp

$\text{num} ::= \text{Num}[n]$ (n is a natural number)

$\text{lit} ::= \text{LIT}[s]$ (s is a string)

$\text{id} ::= \text{ID}[s]$

$\text{Num}[1] \text{ APP Num}[2] \text{ MUL Num}[3]$

↑
 Ambiguity & Interambiguity

Associativity: $a b c$ is flat, over exp $\frac{a}{b} c$

Descriptive vs. Syntax \rightarrow MAHAN

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 Date _____
 No-Terminal

(Formal Syntax & semantics ...)

fct ::= numlit | id | LP exp RP

trm ::= fct | fct mul fct | VB fct VB

exp ::= trm CAT exp | trm APP exp

Prg ::= exp | LET id BE exp IN prg

النحو البرمجي للبيانات
 البرمجي للبيانات = البرمجي للبيانات
 البرمجي للبيانات < exp < trm < fct

النحو البرمجي للبيانات
 البرمجي للبيانات = البرمجي للبيانات
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 البرمجي للبيانات = البرمجي للبيانات
 البرمجي للبيانات < exp < trm < fct

sebastien
 chapter 3 - 19 & 20
 8P
 16P
 9, 7, 6 ← after check

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Abstract Syntax:

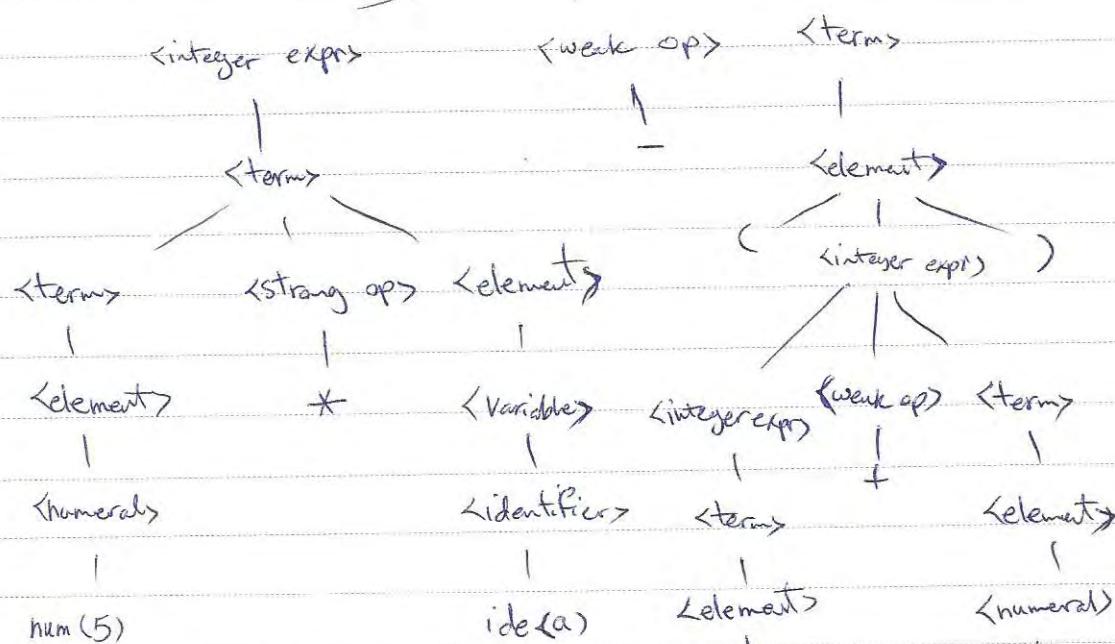
(Abstract & Concrete)

physical text: $5 * a - (b + 1)$ concrete syntax: $5 * a - (b + 1)$

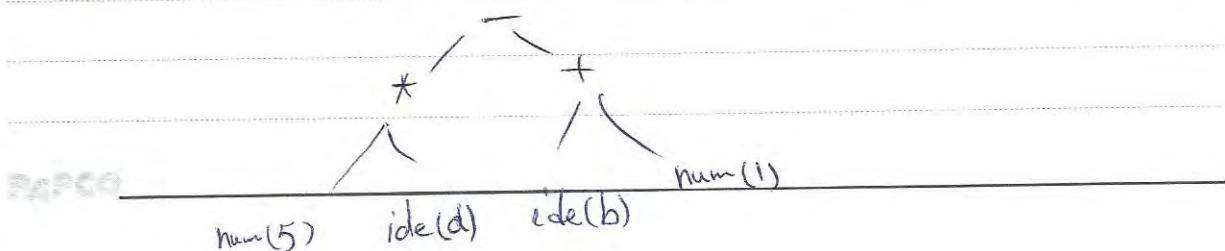
in abstract representation, $*$ is a symbol of Abstract

$5 * a - (b + 1)$

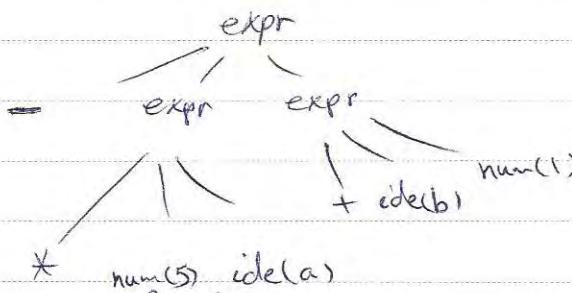
<integer expr>



in abstract representation, $*$ is a symbol of Abstract



$$\left. \begin{array}{l} \text{line} \rightarrow \text{Wife} \\ \text{line} \rightarrow \text{OCML} \end{array} \right\}$$



الرسالة الأولى توضح أن First Compiler هو مترجم من الأدلة إلى الأكواد.

(abstraction). إنها توضح أن الرسالة الأولى توضح كم عدد الأكواد.

Concrete \Rightarrow Wren \Rightarrow abstract. إن abstraction هي العملية التي تغير الأكواد.

الرسالة الثانية توضح أن abstraction هي عملية التي تغير الأكواد.

الرسالة الثالثة توضح أن abstraction هي عملية التي تغير الأكواد.

~~Abstract~~ \Rightarrow (introduction). إن introduction هي عملية التي تغير الأكواد.

Abstract Syntax. \Rightarrow ~~Abstract~~ \Rightarrow (introduction). \Rightarrow ~~Abstract~~ \Rightarrow syntax tree \Rightarrow ~~Abstract~~ \Rightarrow Wren.

arg argument \Rightarrow operation \Rightarrow

Abstract non-terminals, Abstract productions

is Wren \Rightarrow

expression \Rightarrow numerical, boolean, constant, identifier, operations

PAGE 2 (true, false)

Subject
Date

Abstract Syntax Tree

Expression, Numeral, Identifier

$\text{expr} ::= \langle \text{integer expr} \rangle \mid \langle \text{boolean expr} \rangle$

$\langle \text{integer expr} \rangle ::= \langle \text{term} \rangle \mid \langle \text{integer expr} \rangle + \langle \text{term} \rangle \mid \langle \text{integer expr} \rangle - \langle \text{term} \rangle$

$\langle \text{term} \rangle ::= \langle \text{element} \rangle \mid \langle \text{term} \rangle * \langle \text{element} \rangle \mid \langle \text{term} \rangle / \langle \text{element} \rangle$

$\langle \text{element} \rangle ::= \langle \text{numeral} \rangle \mid \langle \text{identifier} \rangle \mid (\langle \text{integer expr} \rangle) \mid - \langle \text{element} \rangle$

:

$\langle \text{comparison} \rangle ::= \langle \text{integer expr} \rangle \leq \langle \text{integer expr} \rangle$

:

|
 $\langle \text{integer expr} \rangle \geq \langle \text{integer expr} \rangle$

$\text{expr} \Rightarrow \langle \text{integer expr} \rangle \rightarrow \langle \text{term} \rangle \rightarrow \langle \text{element} \rangle \Rightarrow \langle \text{numeral} \rangle$

↑ ↑ ↑ ↑

$\text{expr} \Rightarrow \langle \text{integer expr} \rangle \rightarrow \langle \text{term} \rangle \rightarrow \langle \text{element} \rangle \Rightarrow \langle \text{numeral} \rangle$

in PDA Basic Components $\langle \text{integer expr} \rangle$, $\langle \text{term} \rangle$, $\langle \text{element} \rangle$ to (red, red) Unit Rule

↓ ↓

$\langle \text{identifier} \rangle \rightarrow \langle \text{numeral} \rangle$

$\dots + \dots$ Essential non-terminals

$\langle \text{integer expr} \rangle + \langle \text{term} \rangle$

relaxed grammar

$\langle \text{integer expr} \rangle - \langle \text{term} \rangle$

$\langle \text{term} \rangle * \langle \text{element} \rangle$

$\langle \text{term} \rangle / \langle \text{element} \rangle$

$\langle \text{numerical} \rangle$

$\langle \text{identifier} \rangle$

$\langle \text{element} \rangle$

:

$\langle \text{integer expr} \rangle \Leftarrow \langle \text{integer expr} \rangle$

:

$\langle \text{integer expr} \rangle \Leftrightarrow \langle \text{integer expr} \rangle$

↓ Parse Tree → Abstract Syntax Tree ↗

Abstract non-terminal

Expression ::= Numerical | Identifier | true | false | Expression operator Expression

- Expression

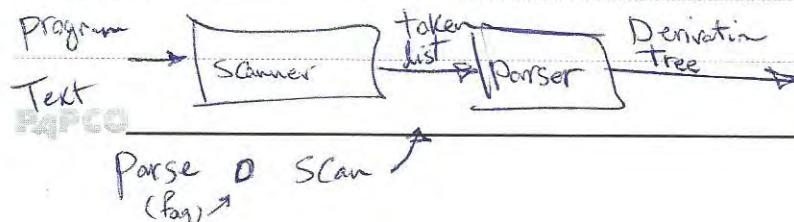
Operator ::= + | - | * | / | V | A | < | <= | = | > | = | >

↑ Original Syntax ↗ Abstract Syntax

↓ Abstract Syntax Tree ↗ Abstract Semantics

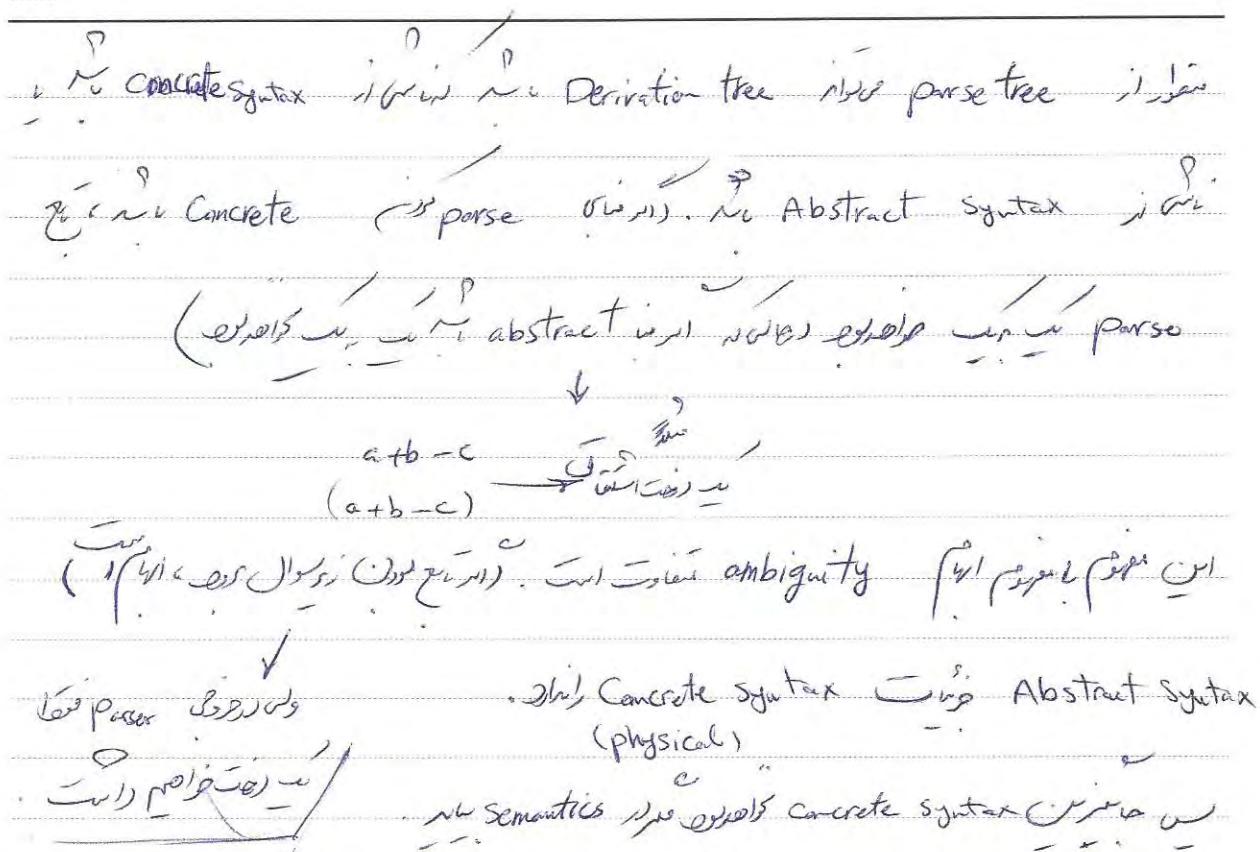
Scanner: character* → Token*

Parser: Token* → Derivation tree



Subject

Date



Attribute Grammars:

Handle context-sensitive grammar (BNF) via static semantics
(static semantics)

Physical attribute vs. Context-free attribute

Parallel grammar . Knuth

1) Attributes \rightarrow Structured
declaration, tail recursive

2) Attribute computation functions \rightarrow attribute rule (semantic function)

3) Predicate Functions \rightarrow ~~more general, more complex~~

(i) static semantic rules, ~~involves mirror~~

Grammar symbol $X \rightarrow A(X)$ of attributes

$$A(X) = S(X) \cup I(X)$$

~~Synthesized
origin~~

~~Inherited
origin~~

~~synthesized and synthesized origin, object in parse tree~~

~~Inherited~~

$$X_0 ::= X_1 \dots X_n$$

$$S(X_0) = f(A(X_1), \dots, A(X_n))$$

$$I(X_j) = f(A(X_0), \dots, A(X_{j-1}))$$

$$I(X_j) = f(A(X_0), A(X_1), \dots, A(X_{j-1}))$$

A predicate function is ~~in~~ in the form of boolean expression on

the union of the attribute sets $\{A(X_0), A(X_1), \dots, A(X_n)\}$ and a set of literal attribute values.

Subject _____

Date _____

• Rightmost parser, start with fully attributed parse tree (right)

• Given Fully attributed tree

intrinsic

attribute \rightarrow ~~attribute~~ basic attribute (User-defined)

• synthesized attribute

$\langle \text{proc_def} \rangle ::= \underline{\text{Procedure}} \; \langle \text{proc_name} \rangle [1] \; \langle \text{proc_body} \rangle \; \underline{\text{end}} \; \langle \text{proc_name} \rangle [2]$

$\langle \text{proc_name} \rangle [2] \downarrow$

ADA

Predicate: $\langle \text{proc_name} \rangle [1] == \langle \text{proc_name} \rangle [2]. \text{string}$

• string
• attribute \downarrow Compiler, no user

synthesized attribute, \downarrow string \downarrow $\langle \text{proc_name} \rangle [2]$

$\langle \text{proc_name} \rangle \downarrow$ string

$\langle \text{Assign} \rangle ::= \langle \text{Var} \rangle = \langle \text{expr} \rangle$

$\langle \text{expr} \rangle ::= \langle \text{Var} \rangle + \langle \text{Var} \rangle \mid \langle \text{Var} \rangle$

$\langle \text{Var} \rangle ::= A \mid B \mid C$

static semantic

Variables are int or real

- If real, places value $\langle \text{Var} \rangle + \text{val}$ into

actual_type : A synthesized attribute associated with <var> & <expr>.

$\frac{\text{Intrinsic attribute}}{\text{It is an intrinsic attribute}}$

expected_type : An inherited attribute for <expr>

$\frac{\text{It is an expected type}}{\text{It is an expected type}}$

: This Semantic Rule is BNF Syntax Rule or is

$\frac{\text{Semantic rule}}{\rightarrow}$

<assign> ::= <var> = <expr> $\frac{\langle \text{expr} \rangle \cdot \text{expected_type} \leftarrow \langle \text{var} \rangle \cdot \text{actual_type}}$

①

$\langle \text{expr} \rangle ::= \langle \text{var} \rangle [2] + \langle \text{var} \rangle [3]$ $\frac{\langle \text{expr} \rangle \cdot \text{actual_type} \leftarrow \text{if}(\langle \text{var} \rangle [2].\text{actual_type} = \text{int})}{\text{and}(\langle \text{var} \rangle [3].\text{actual_type} = \text{int})}$

then int

else real

end if

Predicate : $\langle \text{expr} \rangle \cdot \text{actual_type} == \langle \text{expr} \rangle \cdot \text{expected_type}$

$\langle \text{expr} \rangle ::= \langle \text{vars} \rangle$

② $\langle \text{expr} \rangle \cdot \text{actual_type} \leftarrow \langle \text{vars} \rangle \cdot \text{actual_type}$

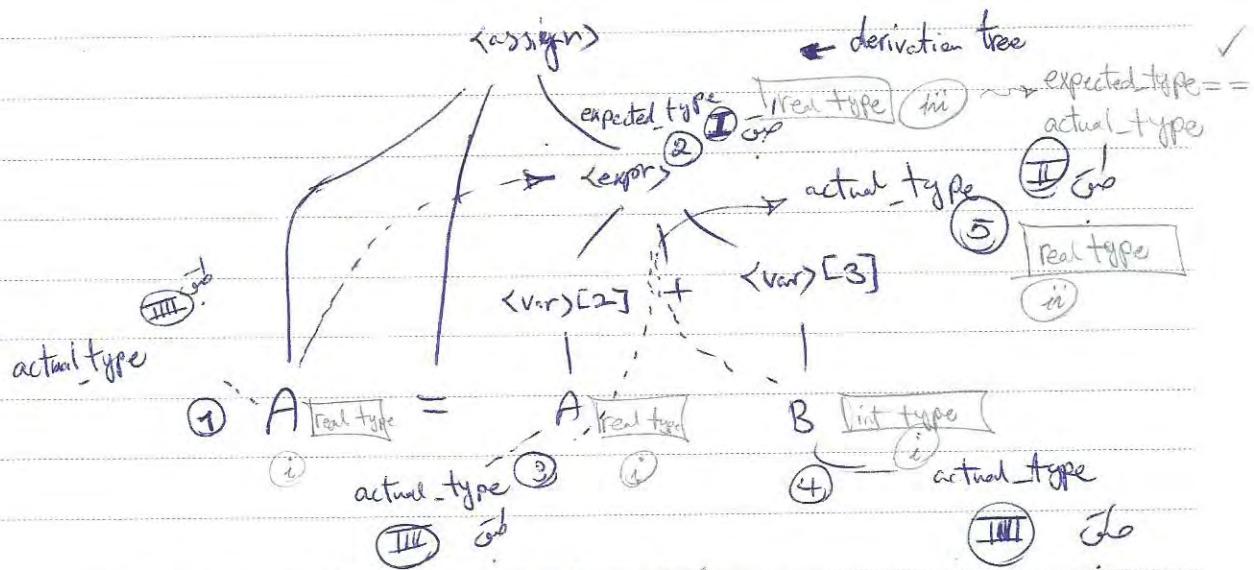
Predicate : $\langle \text{expr} \rangle \cdot \text{actual_type} == \langle \text{expr} \rangle \cdot \text{expected_type}$

$\langle \text{vars} \rangle ::= A | B | C$

③ $\langle \text{var} \rangle \cdot \text{actual_type} \leftarrow \text{look_up}(\langle \text{var} \rangle \cdot \text{string})$

RPCE

. $\langle \text{var} \rangle \cdot \text{type} \leftarrow \text{look_up}(\langle \text{var} \rangle \cdot \text{string})$

$A = A + B$ 

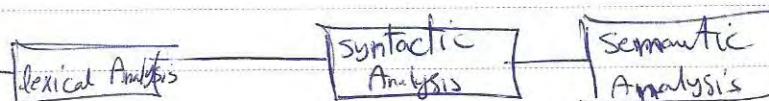
↑ درسترهای کاربردی، بNF بNF

ویژگی های این سیستمها دلیل

این سیستمها برای ایجاد پروتکل های امنیتی و داده های

این سیستمها برای ایجاد پروتکل های امنیتی و داده های

F: bit



Semantics:

(semantics: تئیین / Semantic: معنی)

1) Operational semantics → تئیین عملیاتی
 دوستی کاربردی برای اینجا

برای Turing Machine و abstract Machine

Operational Semantics: نهایی و نهایی ایجاد شوند

Int Abstract Machine پردازش سیمبل

2) Denotational semantics (تئیین معنی)

این تئیین دلیل رفعت داشت (من می‌توانم...)
 این تئیین دلیل رفعت داشت (من می‌توانم...)

3) Axiomatic Semantics (تئیین اثباتی) → تعریف را بگیرید

برای اینجا کسری و کسری و کسری

axiomatic, denotational, imperative و operational semantics

formal semantics کسری (کسری) کسری (کسری) کسری (کسری)

formal semantics Java کسری کسری کسری کسری کسری کسری

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...، تعریف (Def) متادل (Well-formed) semantics Object semantics (متادل) (Well-formed)

$t := \text{true} | \text{false} | \text{if } t \text{ then } t \text{ else } t | 0 | \text{succ}(t) | \text{pred}(t) | \text{iszero } t$ (متادل)

as Base Case اساس as Base Case اساس via inductive

$$f^0 = f$$

$$f^{n+1} = f \circ f^n$$

$$(f^0)$$

$$(f^n)$$

$$f^0 = g$$

$$f^{n+1} = fog$$

proper rule اصح برهان

: Rule algorithm

proper rule

$$\alpha : \text{int} \quad (*)$$

$$\text{zero} : \text{int} \quad (*)$$

$$\text{succ}(\alpha) : \text{int}$$

$$\text{zero int} \quad (*)$$

t

~~اندیشه~~ Natural numbers
Definition of

(*)

zero int

(**)

succ(zero) int

(**)

succ(succ(zero)) int

successor zero برهان درخت

proof برهان judgment برهان judgment برهان برهان

(\vdash)

$b \triangleright a : \text{int} \vdash t \rightarrow t' \vdash f = g$ برهان برهان judgment

برهان برهان برهان برهان برهان برهان برهان برهان

f_1, f_2, \dots, f_k

f

~~Value~~ \rightarrow term \rightarrow operational

~~abstract machine~~, term \rightarrow operational

$v ::= \text{true} \mid \text{false} \mid \text{nr}$

$v ::= \text{value}$ \rightarrow semantics

$\text{nr} ::= 0 \mid \text{succ } \text{nr}$

\rightarrow semantics

\rightarrow transition judgment

1)

Pred $0 \rightarrow^0$

\rightarrow zero (true) and false

\rightarrow zero (false)

2)

$t_1 \rightarrow t_1'$

7)

$\text{succ } t_1 \rightarrow \text{succ } t_1'$

is zero succ(nr) \rightarrow False

3)

$t_1 \rightarrow t_1'$

8)

Pred $t_1 \rightarrow \text{pred } t_1'$

if true then t_2 else $t_3 \rightarrow t_2$

4.)

Pred $\text{succ } \text{nr}_1 \rightarrow \text{nr}_1$

9)

if false then t_2 else $t_3 \rightarrow t_3$

5.)

iszero $0 \rightarrow \text{true}$

10)

$t_1 \rightarrow t_1'$

if t_1 then t_2 else $t_3 \rightarrow$ if t_1' then t_2

$t_1 \rightarrow t_1'$

else t_3

iszero $t_1 \rightarrow \text{iszero } t_1'$

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(pierce)
(types & pm - اورجینیز)

Pred(Succ(pred o)) : on Pierce rule \rightarrow $\exists \forall$ $\exists \forall$ $\exists \forall$ $\exists \forall$

$$\begin{array}{c} \text{Pred } o \rightarrow o \\ \text{Succ } (\text{pred } o) \rightarrow \text{Succ } o \\ \text{Pred } (\text{succ } (\text{pred } o)) \rightarrow \text{Pred } (\text{succ } o) \end{array} \quad \begin{array}{l} (1) \\ (2) \\ (3) \end{array}$$

$$\frac{\exists \forall}{\exists} \quad \text{pred } (\text{succ } o) \rightarrow o \quad (4)$$

$\xrightarrow{\text{pierce rule}} \text{pred } (\text{succ } (\text{pred } o)) \rightarrow \text{pred } (\text{succ } o)$

$\xrightarrow{\text{pred rule}} \text{pred } (\text{succ } (\text{pred } o)) \rightarrow \text{pred } (\text{succ } o) \rightarrow o$

پیویسیون (پیویسیون) \rightarrow evaluation semantics (تعریف کرنے والے موارد)

↑ In operational semantics \rightarrow دو سال بعد پڑھو
evaluate a value \rightarrow term \rightarrow term \rightarrow term
one value \rightarrow one term \rightarrow term

Denotational Semantics :

denote (represent) meaning \rightarrow (connote \rightarrow pronunciation)

essentially mathematical

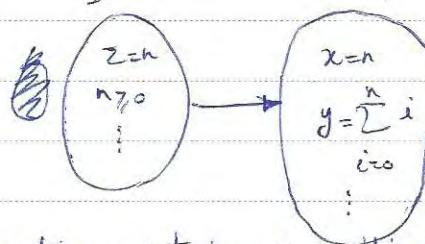
The denotation of a program is a mathematical object.

Engaging discussion

$x := 0; y := 0; \text{while } x \leq z \text{ do } (y := y + x; x := x + 1)$

program: states \rightarrow states

$\begin{cases} \text{state} \rightarrow \text{state} & ; \text{Engaging program} \\ \text{var} : \text{type name} & ; \text{Engaging state} \end{cases}$



denotational semantics must be compositional.

if B then p else q

denotes

$$B \vdash B'$$

$p = p' \Rightarrow \text{if } B \text{ then } p \text{ else } q = \text{if } B' \text{ then } p' \text{ else } q'$

$$Q = Q'$$

page 1 part (ii) class discussion

: Meta language vs Object language

metalinguistic objects

for example (), if, or,

(mathematics \vdash)

Proposition

Meta lang. \vdash is used writing object language in it

$$3+6-4 \rightarrow 5$$

↓ Propositional logic

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Denotational semantics of binary numbers:

$e ::= n \mid e_1 + e_2 \mid e_1 - e_2$

$\llbracket [e] \rrbracket \triangleq$ The parse tree of expression e

$n ::= b \mid nb$

$b ::= 0 \mid 1$

$\llbracket [e_1 + e_2] \rrbracket \triangleq \begin{array}{c} e \\ / \backslash \\ \llbracket [e_1] \rrbracket + \llbracket [e_2] \rrbracket \end{array}$

$E[\llbracket e \rrbracket] \triangleq$ The meaning of e according to the parse tree $\llbracket [e] \rrbracket$

$E[\llbracket 0 \rrbracket] = 0$ number

! Expresses a meaning

$E[\llbracket 1 \rrbracket] = 1$ number

$E[\llbracket nb \rrbracket] = E[\llbracket n \rrbracket] \otimes 2 + E[\llbracket b \rrbracket]$

in meta lang? ↪

↓ operation?

$E[\llbracket e_1 + e_2 \rrbracket] = E[\llbracket e_1 \rrbracket] \oplus E[\llbracket e_2 \rrbracket]$

$E[\llbracket e_1 - e_2 \rrbracket] = E[\llbracket e_1 \rrbracket] \square E[\llbracket e_2 \rrbracket]$

denotational semantics ↪ operational semantics ↪ sound

operational semantics ↪ denotational semantics ↪ complete

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$$e ::= v \mid n \mid e_1 e_2 \mid e - e$$

(J=)

$$n ::= d \mid n d$$

~~variable~~ / state ~~values~~ ~~variables~~

$$x + 25 - y$$

$$d ::= 0 \mid 1 \mid \dots \mid 9$$

state = variables \rightarrow values

$$V ::= x \mid y \mid z \mid \dots$$

~~variables~~ \rightarrow state ~~variables~~

$$E[e](s) = s(x)$$

↓

~~abs~~ $s \rightarrow$ ~~variables~~

$$E[0](s) = 0$$

$$E[nd](s) = E[n](s) * (\oplus E[d](s))$$

$$E[e_1 \pm e_2](s) = E[e_1](s) \pm E[e_2](s)$$

E : parse trees \times states $\rightarrow \mathbb{N}$
 meaning \in $(\dots, 1, 0)$ ~~order~~

: type $list$

$$E = Tree \rightarrow (State \rightarrow \mathbb{N})$$

associate x : state \in $Tree$, y : $Tree$, z : $Tree$ $\in E$ via
 words

words, \in $Tree$ $\in E$ \in

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E: Parse Trees \rightarrow {P: state \rightarrow N}

interpret meaning $\in \Sigma^*$

While Programs:

variables, assignment, loops into Turing Complete

P ::= $x := e \mid P; P \mid \text{if } e \text{ then } p \text{ else } p' \mid \text{while } e \text{ do } P$

(Turing complete). In arithmetic \vdash additive

$x := 0; y := 0; \text{while } x \leq z \text{ do } (y := y + x; x := x + 1)$

initially $x=0$ and $y=0$, z unknown, x increases by 1
step by step, y increases by 1 //

state: variable \rightarrow value

Command: states \rightarrow states

(initial: $x = 0$) $\xrightarrow{\text{loop}} (y = 0, x = 1)$ initial command

modify (s, x, a) = $\lambda v \in \text{Variable}. (\text{if } v=x \text{ then } a \text{ else } s(v))$

$$\left\{ \begin{array}{l} P: \mathbb{R} \rightarrow \mathbb{R} \\ f(x) = x + 1 \end{array} \right. \quad \left. \begin{array}{l} \text{interpret: } x+1 \\ \text{if } v=x \text{ then } v+1 \end{array} \right\}$$

lambda

Ex) λ functional programming
(Core)

$[P]$

C : Tree \rightarrow state \rightarrow state

meaning $C[x:=e](s) = \text{modify}(s, x, E[e](s))$

Interpreting s as a sequence of state \rightarrow state, x is a variable

$C[P_1 ; P_2](s) = C[P_2](C[P_1](s))$

$C[\text{if } e \text{ then } p_1 \text{ else } p_2](s) = \text{if } E[e](s) \text{ then } C[p_1](s) \text{ else } C[p_2](s)$

$C[\text{while } e \text{ do } p](s) = \text{if not } E[e](s) \text{ then } s$

else $C[\text{while } e \text{ do } p](C[p](s))$

if e is true
 \vdash
if e is false
then p is complete
but e is not complete
so p is incomplete
and e is diverge
in general partial

while directed
Syntax ~~semantics~~ \rightarrow ~~semantics~~ \rightarrow ~~syntax~~
Syntax \rightarrow Semantics \rightarrow Syntax

else semantics \rightarrow Syntax \rightarrow Syntax

$C[\text{while } x=x \text{ do } x:=x](s) : \text{state} \rightarrow \text{states}$

↓
Partially \rightarrow \perp (always diverging)
Totally partial

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$C[\text{while } x \leq y \text{ do } x := y](S)$ states \rightarrow states

$$S = \begin{cases} \text{undefined} & S(x) \neq S(y) \\ + & \text{otherwise} \end{cases}$$

~~definition~~ ~~def~~ (JL)

$x := 0; y := 0; \text{while } x \leq z \text{ do } (y := y + x, x := x + 1)$

$$S_0(z) = 1$$

$$S_1 = C[\{x=0\}](S_0)$$

$$S_2 = C[\{y=0\}](S_1)$$

$$S_2 = \{(x_0), (y_0), (z_1)\}$$

$$C[\{\dots\}](S_2) = C[\{\text{while } x \leq z \text{ do } (y := y + x, x := x + 1)\}](S_2)$$

= if not $E[x \leq z](S_2)$ then S_2

else $C[\text{while } x \leq z \text{ do } (y := y + x, x := x + 1)](C[\{y := y + x; x := x + 1\}](S_2))$

= if not $E[x \leq z](S_3)$ then else $\dots (C[\dots](S_3))$

$$S_4 = \{(y_1), (x_2), (z_1)\}$$

= if not $E[x \leq z](S_4)$ then \dots

$$= S_4$$

$\therefore S_4 \vdash S_0 \text{ in } \mathcal{C}$

! If semantics is fine, then denotational semantics is also fine

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(Mitchell - i)

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imperative, functional \rightarrow side effect

statement \rightarrow Expressive side effect

use expression instead of i for side effect phrase

(i is a side effect)

what programming does is to have parallel with no variables or

new expression \rightarrow side effect

(Haskell) in functional side effect statement

imperative (isn't)

طلب واجب

declarative (is)

Sentences

interrogative (isn't)

exclamatory (is) \rightarrow wow!

(isn't) \rightarrow interrogative (isn't) \rightarrow asking for something

\downarrow
expression

\downarrow
Statement

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$x := 5$ imperative \rightarrow نهاد ا Imperative, نهاد ا Imperative
(forall (in state $x := 5$)

Function f (int x) { return $x + 5$ } declarative \rightarrow نهاد ا Imperative
نهاد ا Imperative, نهاد ا Imperative, نهاد ا Imperative

Functional Language:

Functional Programming can specify computation *

نهاد ا Imperative, نهاد ا Imperative, نهاد ا Imperative *

(No assignment) in Functional programming scheme, Lisp & ML *

(Partial function)

by higher lambda, Haskell is into pure functional language

state, local bind, the side effect of binding into functional

pure functional

Test of being pure functional:

Declaration language Test: within the scope of specific declarations

of x_1, \dots, x_n , all occurrences of an expression in e ~~are~~ containing only

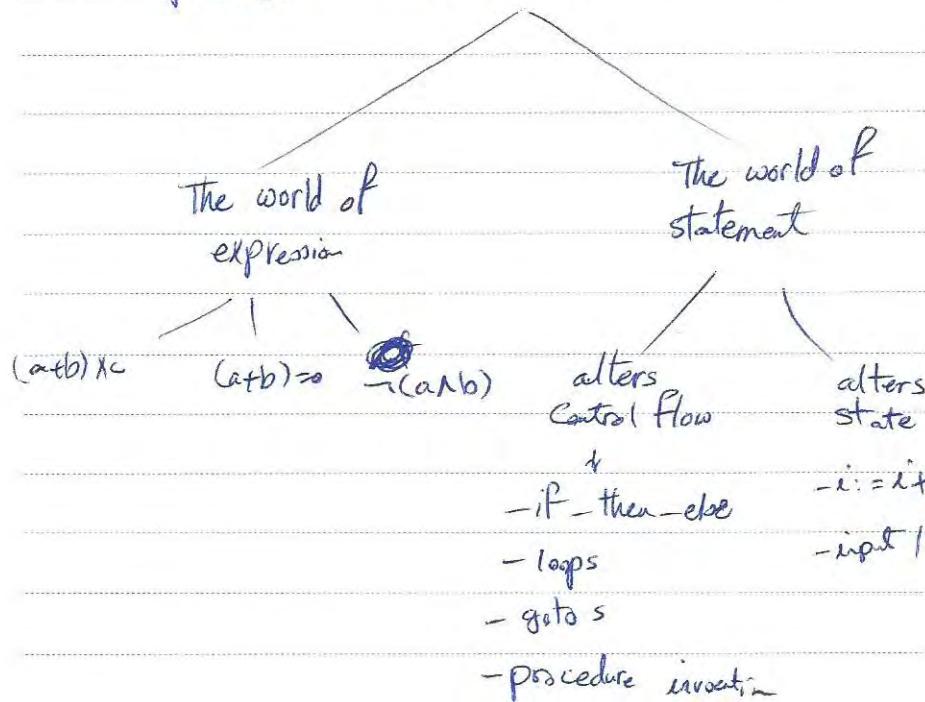
variables x_1, \dots, x_n have the same value.

In declarative C, Java or C++ ~~we can't change the control flow~~
 (good!!)

the ~~control flow~~ is the control

to begin (2) beginning (1) ~~is called Programming Language~~ ~~is good~~

expressions statement



(Order is relevant). ~~beginning (1) is good~~ ~~beginning (2) is good~~

~~beginning (1) is good~~ ~~beginning (2) is good~~

(Common expression cannot be eliminated.)

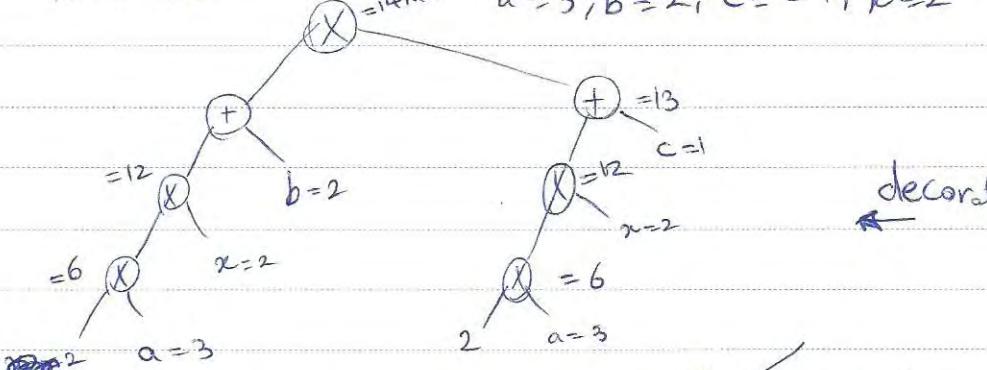
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$$(2ax+b)(2ax+c)$$

With Parallelism Using Object-Oriented

$$= 14x^3 + 4x^2 - 1 \quad a=3, b=2, c=-1, x=2$$



decoration

Parallel processing without losing parallelism

No matter what the order of decoration, as long as we obey the

structure of the tree, we will always get the same answer.

* Church-Rosser property (Confluence)

Function $F(x: \text{integer}) : \text{integer}$

begin

$a := a + 1;$

$F := x + F;$

end

↑ def'ns in this order

(LB first)

$$a+2+F(b) \quad a=1, b=1$$

$$a+2+F(b)$$

$$\text{II} = 3 \quad \text{III} = 4$$

Context is the O. Other's meaning context changes its meaning over time

(J. Weizenbaum) (Referential Transparency) Intensional

I saw ~~walter~~ walter get into his car.

I saw walter get into his Ferrari. (Explicit meaning)

! Context changes meaning

He was called William Rufus because of his red hair.
Reddish

He was called William II because of his red hair.



Context is just in referential transparency

2 - Calculus:

pure functional logic

(lambda) Church-Turing Thesis

Church-Turing Thesis: Church-Turing Thesis

Church-Turing Thesis: Church-Turing Thesis

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lambda term

Syntax of λ -calculus: $t := x \mid \lambda x. t \mid t t$

lambda variable lambda abstraction lambda application

Origin: Slavutin, Functional programming in λ -calculus
Church

Church's design idea concept into Core language

Carl Π -Calculus (with types) Milner, ...

Carl Object-calculus (with object-oriented)

Abadi & Cardelli

Untyped λ -calculus, core object-oriented model language

(object feature). Carl gunther

Untyped λ -calculus:

Programming language is set of λ -calculus

$\lambda x. t \stackrel{\text{def}}{=} f(x) = t$

$\lambda n. \text{if } n=0 \text{ then } 1 \text{ else } n*(n-1)$

object

object oriented programming (first * else, then if, λ is)

String, ... \rightarrow A term

A3

Subject

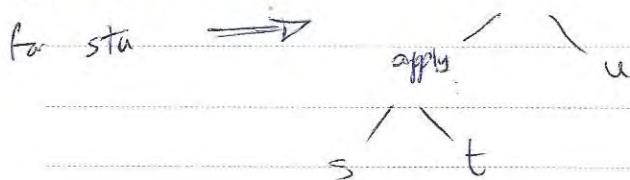
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-^o associate ~~right associativity~~ ^o left associativity

o

st application / $\lambda x.t$ abstraction / $stu = (st)u$

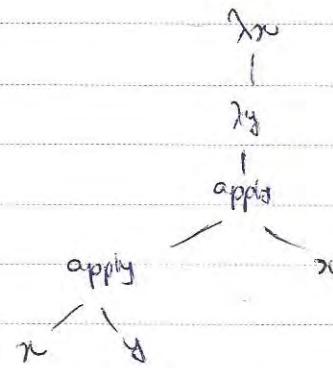
abstract syntax tree apply



$$\lambda x. \lambda y. xyz = \lambda x. (\lambda y. (xy)z) . \xrightarrow{\text{assoc left}} \text{associate left}$$

abstract syntax

for abstract



Scope :

- An occurrence of the variable x is bound when it occurs in the body of an abstraction $\lambda x.t$. (λx is "binder")

In "free" x x says nothing to us

$\lambda x.x \rightarrow x$ is bound

$\lambda z. \lambda x. \lambda y. xyz \rightarrow x$ is bound

Scope

y , z
 z , z

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this x is bound

$(\lambda x.x) x$

this x is free

(Combinator) closed \leftarrow no free var term

$\text{id} = \lambda x.x$ is a combinator

identity

Operational Semantics:

Substitution by currying \rightarrow $t_1 t_2 \rightarrow (\lambda x.t_1)x t_2$

$(\lambda x.t_{12}) t_2 \rightarrow [x \mapsto t_2] t_{12}$ (β -reduction)

Redex (reducible expression) \rightarrow Free occurrence of x in t_2

$(\lambda x.((\lambda x.x).x))(\lambda y.y) \rightarrow (\lambda x.x)(\lambda y.y)$

Self redex containing

Evaluation Strategies:

eval order

1) Full beta-reduction \rightarrow explicit reduction via redex w/

$(\lambda x.x)((\lambda x.x)(\lambda z.(\lambda x.x)z))$ (bottom up traversal)

id(id(id x. id z))

redex

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$$\underline{id(id(\lambda z.z))} \rightarrow id(\underline{id(\lambda z.z)}) \rightarrow$$

$$\underline{id(\lambda z.z)} \rightarrow \lambda z.z \rightarrow$$

$$\underline{id(id(\lambda z.id z))} \rightarrow id(\underline{\lambda z.id z}) \rightarrow \lambda z.\underline{id z} \rightarrow \lambda z.z \rightarrow$$

↑
is in redex

2) Normal Order:

(أولاً نصل إلى redex) ثم نطبق (نؤدي إلى ناتج)

3) Call-by-name:

عملية abstraction \rightarrow reduction \rightarrow

$$\underline{id(id(\lambda z.id z))} \rightarrow id(\underline{\lambda z.id z}) \rightarrow \lambda z.id z \rightarrow$$

(Lazy Semantics!) give name (أي اعطاء قيمة) = value (قيمة)

\hookrightarrow $\lambda z.z$ gives value $\lambda z.z$ (أي $\lambda z.z$ يعطي قيمة $\lambda z.z$)

it's always $\lambda z.z$ which is call-by-name \rightarrow call-by-need

4) Call-by-value:

value way, apply! redex \rightarrow value (أي قيمة)

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(Pierce فصل خامس)

$$\text{id}(\text{id}(\underline{\lambda z. \text{id} z})) \rightarrow \underline{\text{id}(\lambda z. \text{id} z)} \rightarrow \lambda z. \text{id} z \rightarrow$$

inhabited by a Value
(abstraction)

{ strict \rightarrow Call-by-value
non-strict \rightarrow Call-by-name

in λ -calculus, it's O.K. to do this

Multiple arguments:

$$\lambda(x,y). \underline{s} \quad \Leftarrow \text{fix or Peano-style}$$

مثلاً في المجموعات يمكننا تعيين متغير مرتين في f أو higher-order functions

$$f_{(x,y)} = (f_{(x)})_{(y)}$$

ذلك يعني second order E لـ $f_{(x)}$

$$\lambda(x,y). s = \lambda x. \lambda y. s \quad (\text{so apply before } \lambda)$$

$$(\lambda x. \lambda y. s) t r \rightarrow ([x \mapsto t](\lambda y. s)) r \rightarrow [y \mapsto r] [x \mapsto t] s$$

Currying

given λ -expression $\lambda x. f(x)$, Boolean b

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④ Church Boolean:

$$\text{tru} = \lambda t. \lambda f. t$$

$$\text{fls} = \lambda t. \lambda f. f$$

: view to true, not to fls, ok!

⑤ test = $\lambda l. \lambda m. \lambda n. \lambda mn$

$$\text{test } b \vee w$$

$$\left\{ \begin{array}{l} \text{if } b \\ \quad \text{then } v \end{array} \right. \quad \text{else } w$$

→ pb, ok!

$$\text{test tru } v \vee w = (\lambda l. \lambda m. \lambda n. \lambda mn) \text{ tru } v \vee w \rightarrow (\lambda m. \lambda n. \text{trumn}) \vee w$$

$$\rightarrow (\lambda n. \text{tru } v \vee n) \ w \rightarrow \text{tru } v \vee w = (\lambda t. \lambda f. t) \vee w \rightarrow$$

$$(\lambda f. v) \ w \rightarrow v$$

⑥ and = $\lambda b. \lambda c. b \ c$

if (b=true) then c
if (b=false) then fls

⑦ or = $\lambda b. \lambda c. b \ \text{tru } c$

⑧ neg = $\lambda b. b \ \text{fls } \text{tru}$

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Pair :

(Body of the definition of pair)

وهي تعرف بـ pair وهي تكتب

~~pair = $\lambda p. \lambda s. \lambda b. b\ f\ s$~~

$fst = \lambda p. p\ true$

$snd = \lambda p. p\ false$

$fst(pair\ t_1\ t_2) \rightarrow (\lambda p. p\ true)(\lambda b. b\ t_1\ t_2) \rightarrow t_1$

$\xrightarrow{*} (\lambda p. p\ true)(\lambda b. b\ t_1\ t_2) \rightarrow (\lambda b. b\ t_1\ t_2)\ true \rightarrow$

$true\ t_1\ t_2 \rightarrow t_1$

Church Numeral:

والتي هي succ, zero و one

$c_0 = \lambda s. \lambda z. z \rightarrow$ هي λ -closure

$c_1 = \lambda s. \lambda z. sz \rightsquigarrow \text{succ}(z)$

$c_2 = \lambda s. \lambda z. s(sz)$

$c_3 = \lambda s. \lambda z. s(s(sz))$

$scc = \lambda n. \lambda s. \lambda z. s(nsz)$



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d. $\Sigma^+ = \{m, \bar{m}, \bar{s}, \bar{z}, m.s (n \geq 0)\}$

$$\text{plus } c_m c_s = p_{mn}$$

d. $\rightarrow \text{times} = \{m, \bar{m}, m.s (n \geq 0), c\}$

\vdots Σ^+ is Turing Complete by recursion

Recursion

factorial(n) = if $n=0$ then 1 else $n \cdot \text{factorial}(n-1)$

$$f: \mathbb{N} \rightarrow \mathbb{N}$$

$$f(0) = 1$$

$$\begin{cases} f(0) \\ f(n) = n \cdot f(n-1) \end{cases}$$

$$f(n+1) = (n+1) \cdot f(n)$$

\Rightarrow

$$f(n) = \begin{cases} 1 & \text{if } n=0 \\ n \cdot f(n-1) & \text{if } n \neq 0 \end{cases}$$

\Rightarrow

$$f: \mathbb{N} \rightarrow \begin{cases} 1 & \text{if } n=0 \\ n \cdot f(n-1) & \text{if } n \neq 0 \end{cases}$$

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$R^{\infty}(F(S)) = R$

$F: g \rightarrow g'$

$\text{Since } F \text{ fix } g, g'$

$g': n \mapsto \begin{cases} 1 & \text{if } n=0 \\ \ln(g(n)), & \text{if } n=n'+1 \end{cases}$

($\ln(g(n))$ is it correct? \Rightarrow ?)

\therefore if f mapping g we have $f \circ g = g \circ f$

$F(g) = g$

\leftarrow first we prove F is a functional

$\exists x F \in \text{Fixed point of } F$

F is a functional

function, Functional \rightarrow Fixed point

$\exists x \in F, \text{ Fixed point}$

$f(0)=1$

$f(1)=1$

$f(n) = f(n-1) + f(n-2)$

Ques 8.4.5

$\rightarrow F: g \mapsto g'$

$g': n \mapsto \begin{cases} 1 & n=0 \\ g(n') + g(n'-1) & n=n'+1 \end{cases}$

Follow up: the fixed point of F

$F(g) = g$

281 (first) recursion of i fixed point

Fix F = the fixed point of F

↳ λ -term

$$\boxed{F(\text{Fix } F) = \text{Fix } F} \quad \text{Fix} = ?$$

$$\Omega_\alpha = (\lambda x. x x)(\lambda x. x x) \rightarrow (\lambda x. x x)(\lambda x. x x) \rightarrow \dots$$

normal form (11)-abstraction → λ -term

into diverge $\rightarrow \perp$

$$\textcircled{1} \quad y = \lambda f. (\lambda x. f(x x))(\lambda x. f(x x))$$

$$y g = (\lambda x. g(x x))(\lambda x. g(x x))$$

$$= g(\lambda x. g(x x))(\lambda x. g(x x)) = g(\perp)$$

\perp is fixed pt. $y g$

$\Rightarrow \text{fix} = y \rightarrow \text{call-by-name fixed-point combinator}$

$$\textcircled{2} \quad z = \text{fix} = \lambda f. (\lambda x. f(\lambda y. xxy))(\lambda x. f(\lambda y. xxy))$$



Call-by-value fixed-point combinator

$$zg = g(zg) \rightarrow \eta\text{-reduction}$$

$F = \lambda g. \lambda n. \text{if } n=0 \text{ then } 1 \text{ else } n*g(n-1)$

Factorial = fix F

$\text{fix } A \text{ is } \lambda x. \text{let } f = x \text{ in } f(x)$

$\text{fix } (\lambda g. \lambda n. \text{if } n=0 \text{ then } 1 \text{ else } n*g(n-1)) c_5 = c_{120}$

$\text{fix } (\lambda g. \lambda n. \text{if } n=0 \text{ then } 1 \text{ else if } n=1 \text{ then } 1 \text{ else plus}(g(n-1))(g(n-2)))$

fibonacci

(Gibby values), $y.g(y) \rightarrow y.g(y) \rightarrow \dots$ (call-by-value) $\rightarrow y \rightarrow \dots$

$$y.g = g(yg) = g(g(yg)) = \dots$$

$$(w.r.t z.g = g(zg) \text{ let's do})$$

pierce

5.2.2

(in, redex)

5.2.3

5.2.4

5.2.7

5.2.8

5.2.10

5.2.11

$\cancel{\lambda} n\text{-reduction} \rightarrow \cancel{\lambda} n.Mx \xrightarrow{n} M$



$$z.g \rightarrow g(y.(\lambda x. g(y.x)yg)(\lambda x. g(y.x)yg))y$$

$$\rightarrow g(y. \cancel{z.g} y) \rightarrow g(zg) \oplus$$

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$$f = \lambda x. (\lambda x. P_{\text{even}})(\lambda x. f_{\text{even}}) \rightarrow fg = g(fg)$$

$g = \lambda x. \lambda n. \text{if } n=0 \text{ then } 1 \text{ else } n * f(n-1)$

↑ functional

$$Yg \rightarrow \text{eval}(g)$$

↑
g apply (Yg) g, g

$$\text{factorial}(2) = Yg(2) = g(Yg)(2) = \text{eval}(\text{if } n=0 \text{ then } 1 \text{ else } (n * (Yg)(n-1)))$$

$$= f_{2=0} \text{ then } 1 \text{ else } 2 * (Yg)(1) = 2 * (Yg)(1) =$$

$$\text{apply } 2 * g(Yg)(1) = 2 * 1 * (Yg)(0) = 2 * 1 * 1$$

↓
eval recursion

Definition - The set of free variables of a term t , written $\text{FV}(t)$, is

defined as follows:

$$\text{FV}(x) = \{x\}$$

↓
eval

$$\text{FV}(\lambda x. t_1) = \text{FV}(t_1) \setminus \{x\}$$

$$\text{FV}(t_1 t_2) = \text{FV}(t_1) \cup \text{FV}(t_2)$$

(Free variable system) is a syntax directed tree or

Substitution:

$$[x_1 \rightarrow s]t$$

Replace t *(inductively)*

(inductively)

$$\text{I)} [x_1 \rightarrow s]x = s$$

$$\text{II)} [x_1 \rightarrow s]y = y \quad (\text{if } x \neq y)$$

$$\text{III)} [x_1 \rightarrow s](\lambda y. t_1) = \lambda y. [x_1 \rightarrow s]t_1$$

$$\text{IV)} [x_1 \rightarrow s](t_1 t_2) = ([x_1 \rightarrow s]t_1)([x_1 \rightarrow s]t_2)$$

$$[x_1 \rightarrow (\lambda z. zw)](\lambda y. y)$$

$$= \lambda y. [\lambda z. zw]x = \lambda y. \lambda z. zw$$

$$[\alpha \rightarrow y](\lambda x. x) = \lambda x. y$$

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$\vdash \alpha \rightarrow \beta, \Gamma \vdash s : \gamma \vdash t : \delta$

$$\text{IV) } [\alpha \rightarrow s](\lambda y.t_i) = \begin{cases} \lambda y.t_i & \text{if } y=x \\ \lambda y. [\alpha \rightarrow s]t_i & \text{if } y \neq x \end{cases}$$

$$[\alpha \rightarrow z][\lambda z.x] = xz$$

(α is not free in x) $\vdash s : \gamma \vdash t : \delta$

free (α -variable) (Variable Capturing). α -capture. z is α -bound

Free Variable Capture Avoiding Substitution

$$\text{III) } \dots = \begin{cases} \dots & \dots \\ \dots & \text{if } y \neq x \text{ and } y \notin FV(s) \end{cases}$$

α -conversion α -renaming α -simplification

freshen! A term α -bound α is y if y is fresh α -renamed

$$[\alpha \rightarrow z](\lambda w.x) = xw$$

*) $\text{IV) } [\alpha \rightarrow s](\lambda y.t_i) = \lambda y. [\alpha \rightarrow s]t_i \text{ if } y \neq x \text{ and } y \notin FV(s)$

*) α -conversion α -renaming α -simplification

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Operational Semantics (for λ -calculus)

(\rightarrow Transition) \rightarrow judgment

$t \rightarrow t'$

$t_1 \rightarrow t'_1$

1) $t_1 t_2 \rightarrow t'_1 t'_2$

3) $(\lambda x.t_{12}) v_2 \rightarrow [x \mapsto v_2] t_{12}$

2) $t_2 \rightarrow t'_2$

$v_1 t_2 \rightarrow v_1 t'_2$

↗
Axiom

$(v_2 = \lambda x.t)$

↙ abstraction

Value \equiv value by Axiom \rightarrow (call-by-value) \rightarrow Operational Semantics

5.3.6

(piece - i)

[i-6]

5.3.7

5.3.8

(patchell - i)

4.4

4.5

✓ 4.6

121 Shadab m

4.8

4.9

4.11

4.13

Non-Standard Semantics: (Abstract Interpretation)

(Analysed)

with abstract interpretation

- A program is said to be safe if it is initialized before use

state = {error} ∪ (Variables → {init, uninited})

 $E[e](s) = \text{error}$ if e contains any variable with $s(v) = \text{uninit}$

= O.K. otherwise

 $\hookrightarrow \leftarrow \text{expr}(e)$ $C[x := e](s) = \text{if } E[e](s) = \text{OK}$ then modify(s, x, init)

else error

for state

 $\hookrightarrow \text{state}$

initial (0,0)

state

 $s_v = \lambda v. \text{unit} \rightarrow$ $\hookrightarrow \text{unit}$

variable

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10K w. Cursive

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$c[\![x := \text{init}]\!](s) = \text{modify}(s, x, \text{init})$

$c[\![p_1; p_2]\!](s) = \begin{cases} c[\![p_1]\!](s) & \text{if } c[\![p_1]\!](s) = \text{error} \text{ then error} \\ \text{else } c[\![p_2]\!](c[\![p_1]\!](s)) \end{cases}$

$s_1 + s_2 = \lambda v. \text{if } s_1(v) = s_2(v) = \text{init} \text{ then init else uninit}$

Initial state \rightarrow initial state \rightarrow initial state

$c[\![\text{if } e \text{ then } p_1 \text{ else } p_2]\!](s) = \begin{cases} \text{if } E[\![e]\!](s) = \text{error} \text{ or} \\ c[\![p_1]\!] = \text{error} \text{ or} \end{cases}$

Initial state
 $c[\![p_2]\!] = \text{error}$

then error

else $c[\![p_1]\!](s) + c[\![p_2]\!](s)$

Initial state \rightarrow Initial state \rightarrow Initial state

Initial state

$c[\![\text{if } o=1 \text{ then } x:=o \text{ else } x:=t; y:=2]\!](s) = \text{modify}(s, x, \text{init})$

either constant or initial if o then x else y

Mitchell -
Subject: Mitchell -
Year. Month. Date. ()

Complete of, I am present and the Sound is
in

الآن أنا هنا وصوتي واضح

Lisp: (Mitchell -
)

What is Lisp? Lisp is a programming language.

Lisp is created by John McCarthy.

LISP processor

convenience, flexibility, simplicity.

exploratory programming ← Lisp

Implementation: LISP, XEmacs, GTK, Emacs

Scheme in MacLisp, Interlisp, LISP

Recursive functions of symbolic
expressions and their computation
by machine

Computational
Symbolic Computation
LISP

I - motivating application

Digitizing

- Advice taken over common sense reasoning

↓
(منطقی، ساده)

کوئٹہ ملک

- Symbolic expression

$$\int x \, dx = \frac{x^2}{2} + C \quad (\text{معکوس تابع})$$

Symbolic Computation

II - Abstract Machine

(Abstract machine, or IBM 704, hardware)

→ Parallel portability is achieved in hardware

But parallel efficiency in general, abstract world

III - Theoretical Foundation:

partial Function, Turing Complete

! (Practical) and Axiomatic

Overview:

(40 mins) ~~Lisp 15~~ 12

~~prefix notation:~~

(+ 1 2 3 4 5) $\Rightarrow 1 + 2 + 3 + 4 + 5$

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$$(*(+23)(+45)) \rightarrow (2+3)* (4+5)$$

$$(f x y) \rightarrow f(x y)$$

Atoms:

? first, top down up to find

integer, floating point, symbolic

2 2.1 duck

(first, top down to find)

<atom> ::= <symbol> | <num>

<symbol> ::= <char> | <symbol> <char> | <symbol> <digit>

<num> ::= <digit> | <num> <digit>

(first, top down to find "nil" object)

S-Expression:

is a recursive data structure object
dotted pairs basic

<S-exp> ::= <atom> | (<exp> . <exp>)

to Sep; i.e.

Functions of special forms:

functions

cons, car, cdr, eq, atom

concatenate

consolidate

read! separator (Delimited)

Content of address register Content of device register

(in linked list words, in pointer w/o)

special form ✓

cond, lambda, define, quote, eval

no evaluate

for loop (evaluate) (eval, to do, to go)

(lambda (x))

Special Form (function, procedure, lambda, eval)

Procedural and functional eval

+, -, *

arithmetic

Expression ? Functional in pure LISP

functional Side effect in LISP in side effect

purely functional LISP

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replace, replaced, set,setq

✓

Lisp \vdash Impure \vdash

replace car

Assignment

* $(\text{cond } (p_1 e_1) (p_2 e_2) \dots (p_n e_n))$

if p_1 then e_1 else if p_2 then e_2 ...

... result is, first is true, so TRUE is no result. p_i is to special form

} T for TRUE

} nil for FALSE

(quote cons) \rightarrow ~~explic Cons~~ \rightarrow Cons

(cons a b) \rightarrow ~~explic Cons~~ \rightarrow Cons

↳ b is cdr & a is car

(cons (quote A) (quote B)) \rightarrow "B,A" \rightarrow

(quote (+ 1 2)) \rightarrow (+ 1 2)

{ (+ 1 2) \rightarrow (+ 1 2)

121 Shadab

Shadab

quote \vdash

Subject:

Year 1392 Month 07 Date 30 (J)

لisp لغة برمجة تصنف ضمن لغات الـ functional، وهي لغة برمجة متعددة الأوجه.

(define find (lambda (x y))

{
ز&f
ز&f

(cond ((equal y nil) nil))

{
nil
ز&f
False

((equal x (car y)) x))

(true (find x (cdr y)))

))))

ز&f
else

(find 'Apple '(pear peach Apple Fig Banana))

ز&f
"Apple"

Dynamic Scope - Historical LISP *

(1960)

Activation Record ز&f
ز&f

Local Scope - It's called static scope

Subject:

Year 392 Month 08 Date 05 (S)

: LTSp (لُغَةِ سَمْبَل)

Cond (Condition) expression , statement , declaration

(if, jump, set) (if, (if), right (else)) Conditional Expression

(Cond ((< 2+) 2)((< 1 2) 1))) \Rightarrow 1

(Cond ((< 2 1) 2)((< 3 2) 3))) \Rightarrow undefined

(Cond (true 0) (diverge 1)) \Rightarrow 0

↑
Final value

(Cond (diverge 1) (true 1)) \Rightarrow undefined

Value is undefined or entity : expression
statement

? state : statement

x := y + 3

first new identifier : declaration

expression: if f(1)=2 or f(3)=3 then 4 else 4

(if (f(1)=2) (f(3)=3) then 4 else 4)

if expression (if expression)

if (condition) goto 112 → strict functional Lisp

The Lisp Abstract Machine:

Workings of a Lisp Abstract Machine → LISP

Lisp Abstract machine

A Lisp expression to be evaluated.

A continuation, which is a function representing to Remaining program to evaluate when done with the current expression.

An association list (A-list) or runtime stack.

in Abstract Machine, a semantics

A heap which is a set of cons cells.

is in Abstract Machine a state. Abstract Machine is a cycle.

find(x,y)

A-list

x		y
---	--	---

apple

(pear peach ... banana)

(! (let scheme))

In heap with cons x,y

→ cdr b1

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Subject:

Year 3912 Month 8 Date 05 (S)

equal y nil

nil



Continuation

equal x (Car y)

nil



Continuation

Find b (cdr y)



changing A-list



(apple) (peach ... banana)

Cons cells:

احجز Cons cells. انت Cons cell . Lisp يطلب منك ذلك

cons cell

address
(Car)

decrement

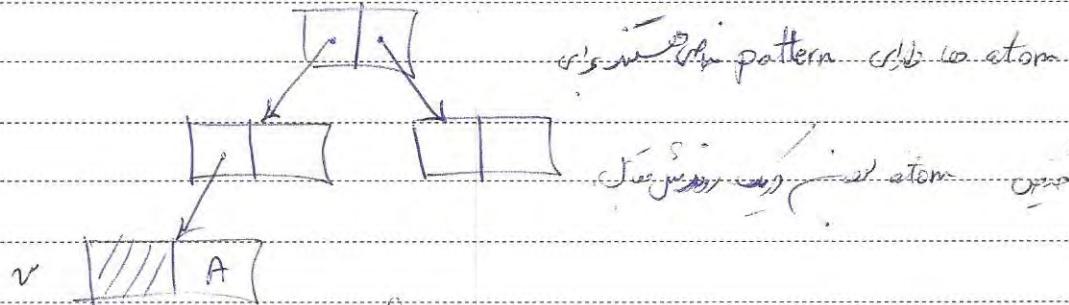
(cdr)

inhibit - بـ inhibit car, cdr

Subject:

Year 1342 Month 08 Date 05

Notes on the concept of Lisp



atom

Functions:

1. atom v → \boxed{v} (atom)

2. eq x y → $\boxed{x} = \boxed{y}$ (atom)

3. (cons x y)

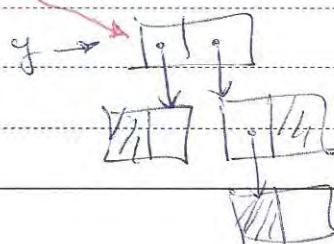
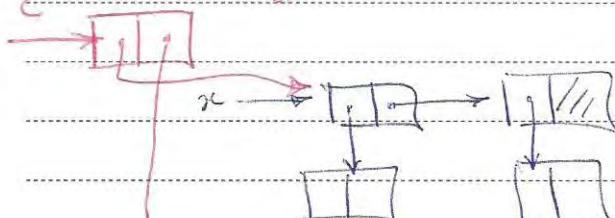
(x is not equal to y)

(eq x y) is atom (x is not equal to y)

After 2nd slide of lisp

(cons x y)

: cons (x,y)

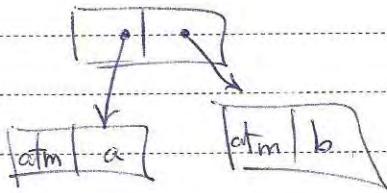


4. Car 20

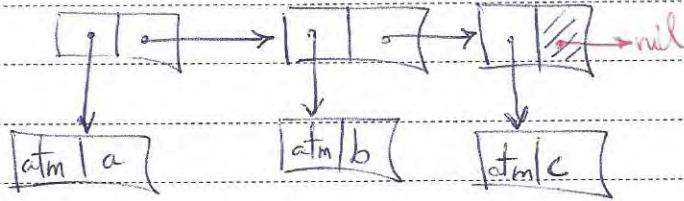
▷ (Cons 'a' 'b')

(a, b)

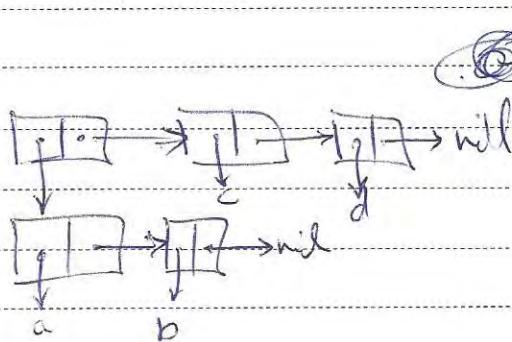
- list



▷ '(a b c)'



(a . (b . (c . nil))) : dotted pair list

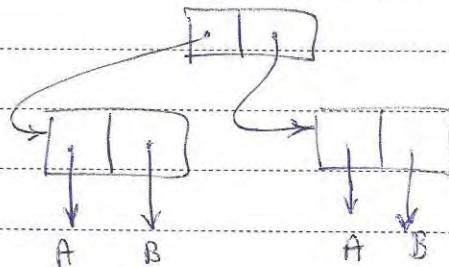


((a b) c d)

((((a . (b . nil)) . c) . d) . nil)

Env. Env.
 $(\text{Cons} (\text{Cons} 'A' 'B')) (\text{Cons} 'A' 'B')$

Q



$((A, B), (A, B))$

Env. list

$((A, B), (A, B))$

B, A Env. Env. Env.

▷ $(\text{set} 'a' \text{post} '(math mark peter))$

$(math mark peter)$

you

open apost & close Ctl

▷ $(\text{Cons} 'john' \text{apst})$

$(john math mark peter)$

free

→ Ctl apst Ctl Ctl

▷ $(\text{Cons} 'a' \text{ nil})$

(a)

free

▷ $(\text{Cons} ('ab') 'c(de))$

$((ab) c(de))$

free

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→ (car lisp)

John ← lisp

→ (cdr (abc))
 b

(bc) ← lisp

→ (cdr apst)

(math mark peter) ← lisp

pitchell

Exercise: 3.1, 3.2, 3.4, 3.6, 3.8 (all 8.7 is all solved)

visit program as data in Lisp & eval it
(special form)

To substitute expression x for all occurrences of y in

expression z and then evaluate the resulting expression.

(define substitute (lambda (exp1 var exp2)

(cond ((atom exp2) (cond ((eq exp2 var) exp1)
(true exp2)))

(true (cons (substitute exp1 var (car exp2))

(substitute exp1 var (cdr exp2)))))))

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define substitute eval (lambda (x y z) (eval (substitute x y z)))

"program as a data"

(lambda (<parameters>) <function body>) كائنات

(lambda (x) (+ (square x) y))

Formal parameter

can't apply

(lambda (x) (+ (square x) y) 4) $\Rightarrow 16 + y$

} { binder

local variable

bounded variable

global variable

(not bound variable)

(Free Variable)

Implementation of LISP in Scheme recursion

Recursion:

abstraction, label is from McCarthy

(label f (lambda (x) (and ((eq x 0) 0) (true (+ x (f (- x 1)))))))

insert define if label or label

(define f (lambda ...))

(define f (lambda ...))

: expand definition

Higher-order functions calculate

functions from state via various Functional OOP

In observable state via Imperative vs OOP

Higher order Functions:

Lambda calculus is first-order

(variables + terms) with function application. Higher-order

is MAX

(in first-order, in second-order)

Third-order Function

$$(P_{fg})(x) = P(g(x))$$

(define compose (lambda (f g) (lambda (x) (f (g (x))))))

lambda (anonymous)

(compose (lambda (x) (+ x x)) (lambda (x) (* x x)))

lambda (x) (+ (* x x) (* x x))

(define maplist (lambda (f x)) *function definition*)

(cond ((eq x nil) nil) *case 1*)

(true (cons (f (car x)) (maplist f (cdr x))))) *is apply*

(maplist square '(1 2 3 4 5)) \Rightarrow (1 4 9 16 25)

Garbage Collection:

a) deallocate *old unused memory* *allocate new memory*

b) *incremental garbage collection* *incremental GC*

c) *whole system garbage collection* *global GC*

i) portable interpreter (e.g., msp), LISP *garbage collector*
interpreter overhead 5% *overhead*

host garbage collector

At a given point in the execution of a program p, a memory

location m is garbage if no completed execution of p from

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Garbage collection garbage collected language

(Car (Cans e1 e2))



garbage evaluate tree

((lambda (x) (car (Cans x x)))) '(A B)



garbage collection tree

garbage collection tree

I. mark and sweep.

new. object. pointer tag. old. old.

garbage collecting tag. forward tag

(forward)

backward

II. reference counting.

Counting the references to objects, incrementing and decrementing the count.

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(define select (lambda (x list))

(cond ((equal list nil) nil)

((equal x (car list)) (cdr list)))

(else (select x (cdr list))))

)))

typedef struct cell cell;

struct cell {

cell * char, * cdr;

};

cell * select (cell * x, cell * list);

cell * ptr;

for (ptr = list; ptr != 0;) {

if (ptr->Car == x) return (ptr->cdr);

else ptr = ptr->cdr;

};

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if all elements of first are in second list, then

cell * struct (cell * a, cell * list);

cell * ptr, * previous;

for (ptr = list; ptr != 0;)

if (ptr -> car == ~) return (ptr -> cdr);

else { previous = ptr; }

else
ptr = ptr -> cdr;

free (previous);

{ $x / \lambda y \rightarrow p(x)$ }

list

! Indirection

no side effect observed

(replace x y) : replace the address field of cons cell x with y

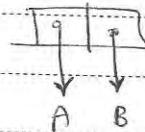
(replace x y) : replace the decrement of cons cell x with y.

field

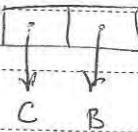
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(replace (cons 'A 'B) 'C)



~~store~~ ↓



↓
Mut. assignment

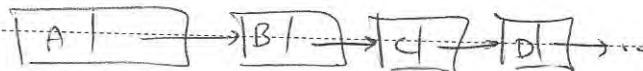
~~or functional~~ (or ~~declarative~~ LISP)

(lambda (x) (cons (car x) (cons (replace x c) (cdr x)))) (Cons
↓ a ↓ b ↓ c)

Side effect is. It adds 'C' to expression

∴ replace →

→ list → list



∴ list → list (or LISP)

(Cons (Car x) (Cons (Cadr x) (Cons y (Cddr x)))))

A

B

pri

word

Car (cdr x) ← syntactic sugar

cdr (cdr (cdr x))

syntactic sugar

why Pure LISP

(replace (cddr x) y)

(cdr (cdr x))

Syntactic
Sugar

! Encapsulated

! Car + Cdr + Cons + Cons = Side effect

Encapsulate pure lisp by correctness

! Impure lisp → (Pseudo) func. manageable

! Impure lisp → parallelism

Subject:

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Algol

Soldat, Pascal, C & ML, Algol-like

Statically typed operation, type (Turing complete)

Refined (with type system) to be type safe

Correctness and Safety type system, with type safety

Incorrectness, with respect to type system

Cast rule: type safety (flat)

In Safety via abstraction

(closure)

new type

In Safe \leftrightarrow Lisp, ML, Java

Dynamic pointer (C) In Safe \leftarrow C
boundary

(Algol pointer with)

O.

dynamic (2) static (1) \rightarrow Safe (no error)

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safe, well-typed, well-formed \rightarrow type safe, statically \rightarrow Java

well-typed

safe, well-typed, well-formed \rightarrow type safe, dynamically \rightarrow Lisp

safe, \rightarrow statically \rightarrow type system \rightarrow well-typed

well-typed

well-typed \rightarrow well-typed
(well-typed)

well-typed \rightarrow ill-typed

well-typed

type, type system \rightarrow type checker

لهم

(well-typed), well-typed type \rightarrow \checkmark

Value

(well-typed) \rightarrow \checkmark

Strong normalization property

well-typed \rightarrow preserve type \rightarrow \checkmark

Preservation

preservation

well-typed \rightarrow well-typed value \rightarrow type \rightarrow \checkmark

progress

Type safety is in progress → Preservation rule (✓)

→ implies static type safety → Type safety

(→ exception handling, Thread, runtime, process)

[dynamic semantics, rules, static type safety]

(built-in) implies the best security & well-typed

JIF (Java Information Flow) JIF

(→ Information Security, Confidentiality)

! Security, Correctness, Completeness

→ static type system, type checking

→ dynamic type

- naming and organizing Concepts

- making sure that bit sequences in computer memory are interpreted

Consistently

- Providing information to the compiler about data manipulated by the program.

- Provability correctness (verifiability)

→ prototype -> specifies what the program will do requirement principle

Correctness is checked statically

of types. Runtime error in untyped language

(incorrect type) static with type system is Dynamic type

Definition: In C/C++ int is specified documentation

parrot type checking error in type

variable declared (Entity) or is illegal type error

operand operation function for int value

int x(); Hardware Errors (file corruption etc.)

operator overloading (operator overloading)

static student = {name: string, number: int} (no record
(struct))

مكتبة المعرفة والبيانات

: type safety

if we use OOPs then type safety implies that whenever type abstraction is used

then it is type safe

A Programming Language is type-safe if no program is allowed to violate its type distinctions.

C is type-safe in its function interface values are checked

safety over type safety

Safety	Example language	Explanation
Not safe	C and C++	Type casts, Pointer arithmetic
Almost safe	Pascal	Explicit deallocation, dangling pointer
safe	Lisp, ML, Smalltalk, Java	Complete type checking

$t ::= \text{true} | \text{false} | \text{if } t_1 \text{ then } t_2 \text{ else } t_3 | \text{of } s \in t_1 \text{ pred } t_2 | \text{iszero } t_1 | \text{if } t_1$

$v ::= \text{true} | \text{false} | n, v$

$n ::= 0 | \text{succ } n, v$

↳ if term t_1 . t_1 's value is (normal) evaluate it if

(exists, error) prob., stuck (prob. t_1 's value)

its value to term t_2 via normalization step

meaningless to term t_1 . its stuck & pred false

↳ t_1 does not have type system Γ

" t has type T " = " t belongs to T " = " t is an element of T "

$t : T \rightarrow$ Typing Relation

Typing Statement

: Γ type system

$T ::= \text{Bool} \mid \text{Nat}$ $\xrightarrow{\text{into}} \text{BaseType}$

(Axiom) $\vdash \text{Bool} \in \text{type}$ if true is

true : Bool

False : Bool

$t_1 : \text{Bool} \quad t_2 : T \quad t_3 : T \quad \xrightarrow{\text{in type}} T_2 \cap T_3$

if t_1 , then t_2 else $t_3 : T$

0 : Nat

t : Nat

succ : Nat

t : Nat

Pred t : Nat

t : Nat

iszero t : Bool

is Typing Rule

In Syntax directed type system

It is type system because

general type is in pred

it's stuck in typable if term

if iszero 0 then 0 else Pred 0

\Rightarrow

0 : Nat

iszero 0 : Bool

0 : Nat

Pred 0 : Nat

if iszero 0 then 0 else Pred 0 : Nat

Type safety (Type soundness)

A term is well-typed \Rightarrow the term does not go wrong



(get stuck)

progress: A well-typed term is not stuck.

preservation: If a well-typed term takes a step of evaluation,

then the resulting term is also well-typed

Curry-Howard
Curry-Howard-Lambek

(Curry Howard-Lambek)

Simply typed lambda calculus:

• Substitution Type System (Substitution) Turing Completeness, Church-Turing Thesis

[Church] recursive functions, lambda-type theory, Church-Turing Thesis

• Church recursive functions, lambda-type theory, Church-Turing Thesis

• Church & Fixed point combinators, primitive recursion

plotting's PCF



Programming computing



3N

Subject : PL

Date : 92/8/19

Page : 1

elementary

- elementary \vdash Bad \vdash عنصری که نتیجه از مکانیزم

محج

Inclusion or Log rule

- elementary \vdash Good \vdash نحوی که نتیجه از مکانیزم

eliminatory

= Type safety (Type Soundness) \vdash (در خود ری)

(get stuck)

- A term is well-typed \Rightarrow The term does not go wrong

- ای دلیل بالا داده شده sound است. توجه کنید نکس عبارت با sound نیست. آنرا

گویی محقق نشوند complete \vdash Type sys.

- از دلایلی که sound نیست \vdash non-terminating در این حالت Type sys نیست

- این است که تجزیه ار بر عکس شرط Bad باشد. در این حالت Type sys sound نیست

- conservative & restrictive \vdash non-restrictive

Value \vdash evaluate \vdash آخر دلیل \vdash normal \vdash Term \vdash good

- stuck \vdash آخر دلیل \vdash get stuck \vdash bad

progress : a well-typed term is not stuck.

- evaluate \vdash آخر دلیل \vdash , not stuck \vdash آخر دلیل \vdash

Preservation : 1. Well-typed \vdash evaluate \vdash آخر دلیل \vdash well-typed \vdash term

- If a well-typed term takes a step of evaluation, then the resulting term is also well-typed.

- این مفهومی است که در نظر گیری آن را نمی‌دانیم. دسته ای است. استرای استند. البرهان می‌دانیم.

- برای اثبات مراجعه کنید.



IRAN

Subject _____

Date _____

أ.ن.ت : \rightarrow (single arrow)

$T_1 \rightarrow T_2$

$T ::= \text{Bool} \mid T \rightarrow T$

$\text{Bool} \rightarrow \text{Bool}$

function type function

$\text{Bool} \rightarrow \text{Bool} \rightarrow \text{Bool}$

two argument function

in Right associative (\rightarrow) arrow

$(\text{Bool} \rightarrow \text{Bool}) \rightarrow \text{Bool} \rightarrow \text{Bool}$

one argument function

construct one type constructor over arrow

type constructor is a type of type

A.N.T \rightarrow function type

function type type Type annotation ($:=$)
Explicitly

Annotations

A.N.T \downarrow

parameters

first parameter with annotations

Subject

Date 1392.08.19

be explicitly-typed

be originally-untyped

be interpreted as typed, so want to keep original type

Ex. $x : \text{A} \rightarrow \text{B}$

be implicitly-typed

x is Bool

original type-reconstructing Type-inference algorithm

(ML style). Intrinsic Polymorphism of original

original lambda calculus. λ -typing via rule

$x : T_1 \vdash t_2 : T_2$ $\frac{x : T_1 \vdash t_2 : T_2}{\vdash \lambda x : T_1. t_2 : T_1 \rightarrow T_2}$

$\vdash \lambda x : T_1. t_2 : T_1 \rightarrow T_2 \quad \text{Hypothetical judgment}$
 $(\alpha \rightarrow \beta) \rightarrow \gamma$

$\vdash T_1 \rightarrow T_2 \quad \vdash \lambda x : T_1. t_2 : T_1 \rightarrow T_2$ $\frac{\vdash T_1 \rightarrow T_2 \quad \vdash \lambda x : T_1. t_2 : T_1 \rightarrow T_2}{\vdash \lambda x : T_1. t_2 : T_1 \rightarrow T_2}$ Substitution rule

Original lambda calculus, typing \rightarrow original

typing context

inhabited

$\Gamma, x : T_1 \vdash t : T_2$

(I)

$\vdash \lambda x : T_1. t_2 : T_1 \rightarrow T_2$

PPCO

no outer boundary point, it has no boundary points

Subject PL

Date 13.2.08.19

Type Context (local env) Γ for declarations $x : T$

Global Type environment

(abstraction rule)

$$(II) \frac{x:T \in \Gamma}{\Gamma \vdash x:T} \quad (\text{reflexivity})$$

: $\frac{\rho}{\rho}$

$$\Gamma, x:T \vdash x:T$$

: $\frac{\rho}{\rho}$

$$(III) \frac{\Gamma \vdash t_1:T_1 \rightarrow T_2 \quad \Gamma \vdash t_2:T_1}{\Gamma \vdash t_1 t_2:T_2}$$

: $\frac{\rho}{\rho}$

: $\frac{\rho}{\rho}$

$$\Gamma \vdash t_1 t_2:T_2$$

: syntax directed, $\frac{\rho}{\rho}$ is rule

Env

application (III), variable (II), abstraction (I)

for : Bool. f : Bool - and x y

(Qs)

PAPCQ : 1. Bool (Qs) 2. (Qs) (Qs)

$\text{Bool} \rightarrow \text{Bool} \rightarrow \text{Bool}$

لـ أيضاً كـ

$x: \text{Bool}, y: \text{Bool} \vdash \text{and } xy : \text{Bool}$

رـ مـ

$\vdash x: \text{Bool} \vdash y: \text{Bool}, \text{and } xy =? \text{Bool} \rightarrow \text{Bool}$

$\vdash \lambda x: \text{Bool}, y: \text{Bool} \text{ and } xy =? \text{Bool} \rightarrow \text{Bool} \rightarrow \text{Bool}$

حـ أـ بـ جـ دـ هـ (I) عـ فـ كـ

(\rightarrow إـ نـ أـ يـ عـ يـ أـ يـ أـ يـ أـ يـ أـ يـ أـ يـ)

يـ وـ

(a, n أـ نـ أـ يـ أـ يـ)

يـ وـ

ـ نـ كـ نـ كـ

The Alge Family and ML :

: أـ لـ جـ أـ لـ جـ أـ لـ جـ

- Colon - separated statements

- block structure

- static type

- function and procedure

Subject PL
Date 1992.08.21

Algol 60 → A general-purpose language

scientific computation

— simple statement-oriented syntax (imperative)

— begin...end (from C)

— recursive functions (stack storage allocation)

— fewer ad hoc restrictions

variables

— Procedures with procedure parameters

indexable

→ high-order functions

Real procedure Average (A, n);

Real array A; integer n;

↓ explicitly typed Algol

begin

↓ function definition

Real sum; sum:=0;

For i=1 step 1 until n do

sum := sum + A[i];

average := sum / n → recursive U, i.e., procedure!

end;

Subject _____
Date _____

Algorithmic Discipline

- shortcoming in its type discipline \rightarrow type discipline is not strict

• side-effect type is changing (procedure) or temporary

- The type of a procedure parameter to a procedure

doesn't include the type of parameters.

- array bounds

- Parameter passing mechanism

- pass-by-name \rightarrow expression? object? copy rule

if variable is mutable (side effect) \rightarrow changes don't procedure,
but value

- pass-by-value \rightarrow copy, no sharing

value, copy?

Subject _____

Date _____

begin integer i;

Jensen's Device (D¹)

integer procedure sum(i,j);

integer i,j;

Comment parameters passed by name;

begin integer sm; sm:=0;

i:=1 step 1 until 100 do sm:=sm+i;

Sum:=sm

end;

print(sum(i , i*10))

end

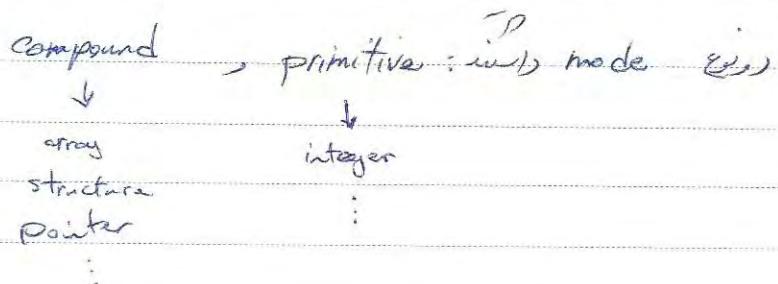
↳ difference between Algol 60 & Algol 68

$$\Rightarrow \sum_{i=1}^{100} i*10 = 10 \frac{100(100+1)}{2} \neq 100 * 100$$

↓
pass by value

↳ difference between Algol 60 & Algol 68 → Algol 60 uses
(Terminology)

↳ object type descriptive vs. generic mode ; is type



↓
Subs over type set cur

an array of pointers to procedures

! we expect w. side effects



Algol 68

- type system

- memory management

Stack (local variables)

Heap (other variables)

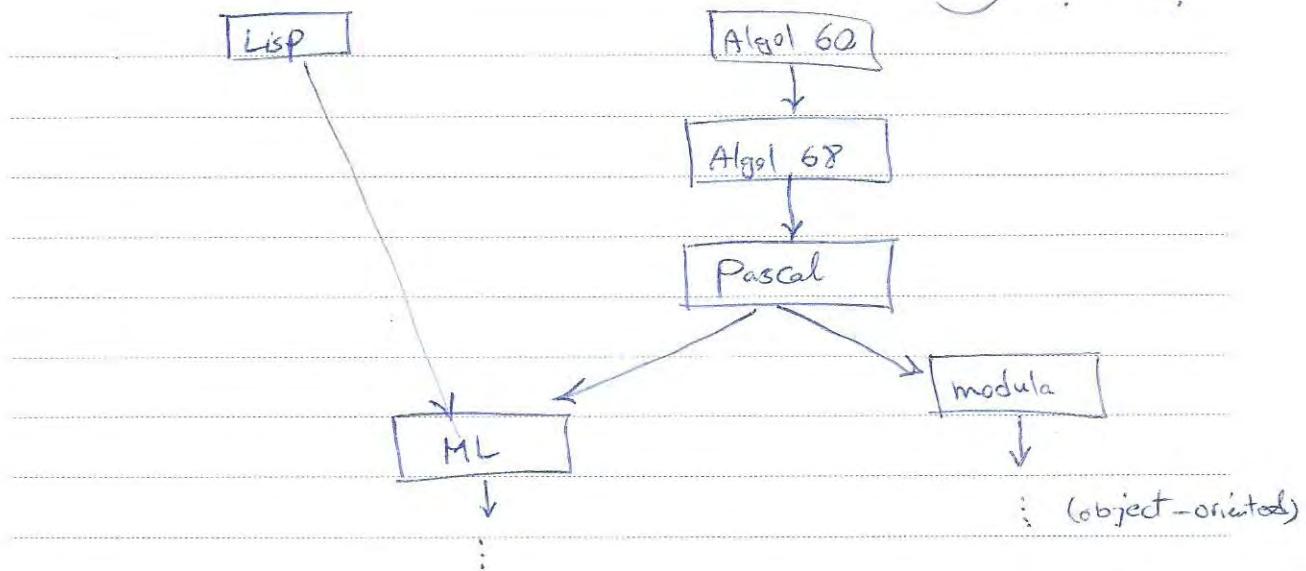
global

garbage collector, deallocation, free pointers, allocation

- Pass-by-value

Pass-by-reference

(1) Second order function
 (2) ~~function~~, new type system, Pascal →



C:

sim Unix je pascal c C inter

system call, interface C Unix

ML:

extensible type system

mostly functional & Function-Oriented

imperative discipline

(higher order func). support Lisp via

functional, & full featured

PPCO

Date 1392.08.25

Unprovable \rightarrow correctness \vdash . Inside Out Analyze \rightarrow proof w/
سیستم اثباتی

in λ -calculus \rightarrow Milner (Pure LCF)
(Logic for Computable Functions)

ML \rightarrow Meta Language

in Meta language \rightarrow with higher-order ML (LCF)

Curry-Howard Isomorphism:

inference rules, Gödel's logic
(programming language)

$(\lambda x:p.x) : p \rightarrow p$

دینامیک
این دینامیک را می‌توان در میان این دو نظریه از جمله

بررسی کرد که این دینامیک را می‌توان در میان این دو نظریه از جمله

Standard ML 97 (SML)

Expressions:

def = <expression>

P4PSC

def \rightarrow Val it = <print_value : <type>

Subject _____

Date _____

- $(5+3) = 2;$

→ Val it = 6 : int

↳ first inference of type typing error fix

- if true then 1 else 3;

→ Val it = 1 : int

↳ execute, compile, type checker parse 1, case

- if true then 3 else False;

→ error (type checking)

Declarations

- Val <identifier> = <expression>;

→ Val <identifier> = <print values> : type

- Val x = 7 + 2; → value object

→ Val x = a : int → alias x

- Initialize or declare the variable C with 0 (0)

integer pointer reference input parameter assignment operator

→ Val $y = x + 3;$

→ Val $y = 12 : \text{int};$

→ Val $f = \text{fn } x \Rightarrow x + 5;$ → function declaration

→ Val $f = \text{fn} : \text{int} \rightarrow \text{int}$ ~~return value expression~~
new type fn

→ Fun Identifier ~~Let~~ ^{arguments} = Expression;

→ Fun $f(x) = x + 2;$

val ~~f~~ $f = \text{fn} : \text{int} \rightarrow \text{int}$

Basic types:

↳ (integer, float, string, unit) introduction new type is fn

(integer, float, string, unit) elimination

unit → ~~unit~~

() : unit ~~unit~~ type following

Subject

Date 192.08.26

(unit type), in ML \rightarrow unit ~~in C~~ void

unit type given void in ML or

in unit type \rightarrow side effect

~~type unit~~

true : bool

introduction form

elimination form



False : bool

in type \rightarrow unit

if e_1 then e_2 else e_3

in type \rightarrow unit

and also \rightarrow in and

\Rightarrow in bool, type i

in bool

orelse

not

- fun equiv(x, y) = (x andalso y) orelse ((not x) andalso (not y))

\rightarrow val equiv = fn : bool * bool \rightarrow bool

in type \rightarrow unit

- equiv (true, false); if x is False, x is True

andalso, y , not true

\rightarrow val it = False : bool

((true, ~~orelse~~),

: val it

Subject _____
Date _____

~~bool ← no implicit type conversion type → Basic type~~

bool * bool ← int → " " Compound type

bool * bool → bool
is not a basic type - why?

0, 1, 2, ..., -1, -2, ... : int

+, -, /, * : int → int (overloading)

float & int & string are objects

"William" : string

"Boris" : string

- "chelsey" & " " & "clinton" ;

↳ concatenation

→ val it = "chelsey clinton" : string

1.0, 2.0, 3.1415 : real

- 3 + 4

→ val it = 7 : int

- 3.0 + 4.0;

→ val it = 7.0 : real

Subject _____

Date _____

real(3) → real int or

floor(), ceil() → int real

round(), trunc()

: Compound Or/Is type

Tuple →

Compound Or/Is pair

- $(3, 4)$;

→ val it = $(3, 4)$: int * int

↓
product type (or is)

- $(4, 5, \text{true})$;

→ val it = $(4, 5, \text{true})$: int * int * bool

↑ introduction
↓ elimination

- #2 $(3, 4)$; tuple type or

→ val it = 4 : int

Record type:

val name : string tuple in record

- { First_name = "Donald", Last_name = "Knuth" };

→ val it ... : { first_name : String, last_name : String }

↑ type record with names

- # first_name ({ first_name = "Donald", last_name = "Knuth" });

→ val it = "Donald" : string

List:

- [1, 2, 3, 4];

→ draw in paper

→ val it = [1, 2, 3, 4] : int list

↑ in type list

- [f_n x → x+1, f_n x → x+2];

→ val it = [f_n, f_n] : (int → int) list

- 3 :: nil

↑ introduction
↓ elimination

→ val it = 3 [3] : int

Subject PL

Date 13.08.26

- 4 :: 5 :: it

→ val it = [4, 5, 3] : int list

Value Declarations:

via pattern match

Logic: Pattern Matching

A pattern is an expression containing variables (x, y, ...)

and constants (true, false, 1, 2, ...) combined by certain

forms such as tupling, record expressions, and a form of

operation called a constructor.

<pattern> ::= <id> | <tuple> | <cons> | <records> | <consts>

<tuple> ::= (<pattern>, ..., <pattern>)

<cons> ::= <patterns> :: <pattern>

<records> ::= { <id> = <pattern>, ..., <id> = <pattern> }

<consts> ::= <id>(<pattern>, ..., <pattern>)

- val <pattern> = <exp>;

- val t = (1, 2, 3);

PPCQ

→ val ~~t~~ = (1, 2, 3) : int * int * int

- val $(x, y, z) = t;$ ↳ first pattern matching in t

→ Val $x = 1 : \text{int}$
 Val $y = 2 : \text{int}$
 Val $z = 3 : \text{int}$ $\left\{ \begin{array}{l} p_1 \\ p_2 \\ p_3 \end{array} \right. \text{in } \text{first pattern matching}$

- Val $t = [1, 2, 3, 4];$

- Val $x :: y = t$

→ Val $x = 1 : \text{int}$
 Val $y = [2, 3, 4] : \text{int list}$

- Val $x :: y :: z :: t$

→ Val $x :: z = 1 : \text{int}$

Val $z :: z = 2 : \text{int}$

Val $z :: z = [3, 4] : \text{int list}$

~~(HKT, α , β , γ) in the book of mitchell~~ ↳ ~~signature~~

- fun $f(\langle \text{pattern} \rangle) = \langle \text{expr} \rangle | f(\langle \text{pattern} \rangle) = \langle \text{exp}_2 \rangle | \dots |$

$\delta f(\langle \text{pattern} \rangle) = \langle \text{exp} n \rangle$

- Funco $f(x, y) = x + y;$

Subject PL

Date 13a2.09.03

- fun length(nil) = 0 | length(x::xs) = 1 + length(xs)

→ val length = fn : 'a list → int

type variable
(polymorphism)
← (inference) ∴ type inference, is ML
[x] = [a]
[xs] = [as]

is also first type of length ∴ is

'a list → int' gives int, ∴ type

'a → obj' ∴ Universal Polymorphism, ML

- fun f(x, (y, z)) = y;

→ val f = fn = 'a * ('b * 'c) → 'b

- fun h({a=x, b=y, c=z}) = {d=y, e=z};

→ val h=fn: {a:'a, b:'b, c:'c} → {d:'b, e:'c}

- fun f(x,y) = x | f(0,y) = y | f(x,y) = x+y;

→ type declaration Obj to pattern

Type declaration :

$\langle \text{constructor_clauses} \rangle \leftarrow \underline{\underline{\text{constructor_clauses}}}$

$\text{datatype } \langle \text{type_name} \rangle = \langle \text{constructor_clauses} \rangle | \langle \text{constructor_clauses} \rangle \dots |$

$\langle \text{constructor_clauses} \rangle = \langle \text{constructors} \rangle | \langle \text{constructor} \rangle \text{ of } \langle \text{arg_types} \rangle$

$\langle \text{constructor} \rangle \leftarrow \underline{\underline{\text{constructor}}}$

Example - An enumerated data type

- $\text{datatype color} = \text{Red} | \text{Blue} | \text{Green};$

\swarrow

$\text{in Color, type } (\text{Value})$

$\text{General Constructor in initial type - } \underline{\underline{\text{initial type}}}$

Example - A Tagged union data type

$A = \{1, 2\}$

$B = \{2, 3, 4\}$

$A \cup B = \{1, 2, 3, 4\}$

$A \underset{T}{\cup} B = \{(L, 1), (L, 2), (R, 2), (R, 3), (R, 4)\}$

$A \leftarrow \underline{\underline{\text{tagged type}}}$

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→ ~~disjoint Union~~ → Tagged Union ?

- datatype student = BS of name | MS of name * school | PhD of
name * faculty

↓
name

fun name(BS(n)) = n | name(MS(n,s)) = n | name(PhD(n,f)) = n
↓ ↓
pattern expression

→ val name : fn : student → name
 ↓ ↓
 type type

Example - A Recursive type

infinite datatype is. recursive type with, for Recursive Function

↓
int

The set of trees with integer labels at leaves.

datatype tree = LEAF of int | NODE of tree * tree ;

↓
else tree type is (int) (constructor)
(int type is)

→ [↑] _↓
else (tree) is

? else or all cases or else // is optional

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Subject

Date

FALSE \leftarrow , TRUE : \rightarrow زن است : \downarrow

- Fun inTree ($x, \text{LEAF}(y)$) = $x = y$ | inTree ($x, \text{NODE}(y, z)$) = \rightarrow

inTree (x, y) or else inTree (x, z) :

\rightarrow val inTree = fn : int * Tree \rightarrow bool

L-values vs. R-values:

int x;

int y;

$x = y + 3$,

لهم x لـ y \rightarrow x \leftarrow y \rightarrow x \leftarrow $y + 3$

لـ x \leftarrow $y + 3$

ارسال x \leftarrow y \rightarrow ارسال x

لـ x \leftarrow y

ارسال x \leftarrow y \rightarrow ارسال x

لـ x

لـ x \leftarrow y \rightarrow ارسال x \leftarrow y \rightarrow ارسال x \rightarrow ارسال y \rightarrow ارسال x

ii) ref v : creates a reference cell containing value v.

لـ v \leftarrow $\underline{\underline{v}}$ \rightarrow ارسال v \leftarrow $\underline{\underline{v}}$ \rightarrow ارسال v



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2) $!r$: returns the value contained in reference cell r .

r reference cell (عنصر مرجع)

3) $r := v$: places value v in reference cell r .

- $\text{val } x = \text{ref } 0;$
 \rightarrow initialization (initialization) x \rightarrow reference cell, value 0

$\rightarrow \text{val } x = \text{ref } 0: \text{int ref}$ $x \neq 0$

- $x := 3 * (!x) + 5;$ \rightarrow expression (x)

$\rightarrow \text{val } it = () : \text{unit}$ \rightarrow unit type, statement (assignment)

(unit) \rightarrow type conversion (x), to assignment

$!x;$

$\rightarrow \text{val } it = 5 : \text{int}$

(is not positive). \rightarrow error (عنصر غير موجب)

The user uses it as Error, $x := 3 * x + 5$ must be

$\rightarrow \text{val } y = \text{ref } "Apple";$ y \rightarrow "Apple"

$\rightarrow \text{val } y = \text{ref } "Apple": \text{string ref. } y$ \rightarrow mutable assignable

III

- $y := "Fried green tomatoes"$
- $\rightarrow \text{val } it = () : \text{unit}$



- $!y;$

- $\rightarrow \text{val } it = "Fried green tomatoes" : \text{string}$

. if it is String ML $\rightarrow (\text{String})$ (وَسْتِرِن)

\downarrow
 either unit either String
 either pointer (أو إما
 إما مُنطَبِّعٌ إما مُنطَبِّعٌ)

↑ رُسْفَجَانِي

Type : Mitchell \rightarrow pre-jon

. \rightarrow each type inference is using type inference
 $\text{(ML) دليل)$

Type Inference (Type Reconstruction):

جُنْدِي : Inference

(Type declaration) \hookrightarrow compiler guideline \rightarrow Type checking

وَسْلَبِي

وَسْلَبِي \rightarrow type context

وَسْلَبِي \rightarrow declarations \rightarrow (inforce) \rightarrow type system \rightarrow precise

الهدف من دروسنا هو التعرف على ال_declarations (بيانات) وال_expressions (expressions).

في الواقع، هناك نوعان من Type ، Type checking . و Type Inference

الـ_type errors (أخطاء النوع)، Type Checking ، Type Inference

الـ_type inference (الـ_inference)، type ، Type Variable

الـ_type expression (الـ_expression)، Type Expression

الـ_type = Val f = fn : 'a list * 'a list → bool

الـ_Polyorphism ، Type Inference (الـ_inference)

- fun f2 (g,h) = g(h(0)) ;

→ val f2 = fn : ('a → 'b) * (int → 'a) → 'b

$g(h(0))$



حيث int هو h :

$h : \text{int} \rightarrow 'a$

و $g : 'a \rightarrow 'b$

الـ_Type Inference (الـ_inference)

الـ_Type Inference (الـ_inference)

1. assigning type variables to expression and subexpression.

$g : 'a \rightarrow 'b$

2. Deriving a number of equations on type variables based on the parse tree of the expression and typing rules (constraints) (constraint-based typing)

3 - Solving the system of equations by substitution (unification)

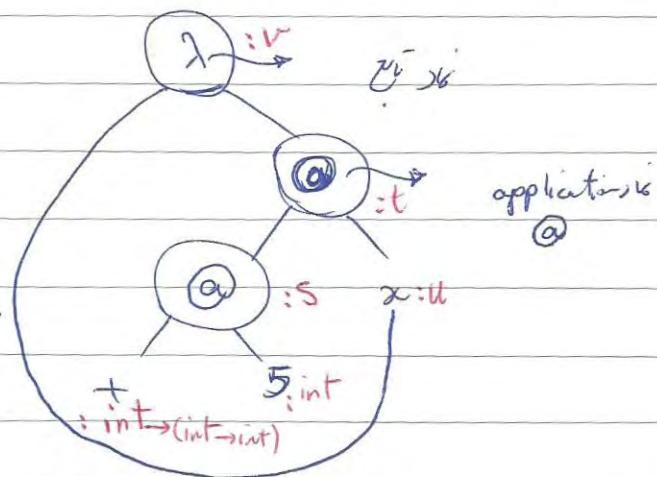
- fun $g(x) = x + 5$
 $\rightarrow \text{val } g = \text{fn} : \text{int} \rightarrow \text{int}$

$\rightarrow \text{int} \rightarrow (\text{int} \rightarrow \text{int})$

work types $\frac{\text{fun}}{\text{int}}$ for out $\frac{\text{int}}{\text{int}}$

پیش فکر

Parse tree :



دالجی \Rightarrow میتوانیم این دلایل را برای type

پیش فکر

$$\Gamma, x:T_1 \vdash e:T_2$$

: پیش فکر از اینجا شروع می شود
Abstraction rule

$$\Gamma \vdash \lambda x:T_1. e : T_1 \rightarrow T_2$$

$$\frac{\Gamma \vdash e_1 : T_1 \rightarrow T_2 \quad \Gamma \vdash e_2 : T_1}{\Gamma \vdash e_1 e_2 : T_2} \text{ Application rule}$$

ML

$$f: a \rightarrow b \Rightarrow f e: a \rightarrow b \quad (\text{App})$$

$$e: a$$

$$x: a \Rightarrow \lambda x. e: a \rightarrow b \quad (\text{Abst})$$

$$(1) \text{ int} \rightarrow (\text{int} \rightarrow \text{int}) = \text{int} \rightarrow \text{s}$$

$$(2) \text{ s} = u \rightarrow t \quad \xrightarrow{\text{App on } \bar{c}} \text{ App on } \bar{c}$$

$$(3) \text{ v} = u \rightarrow t \quad \xrightarrow{\text{Abst on } \bar{c}} \text{ Abst on } \bar{c}$$

تام عرب: μf

$$\xrightarrow{(1)} s = \text{int} \rightarrow \text{int}$$

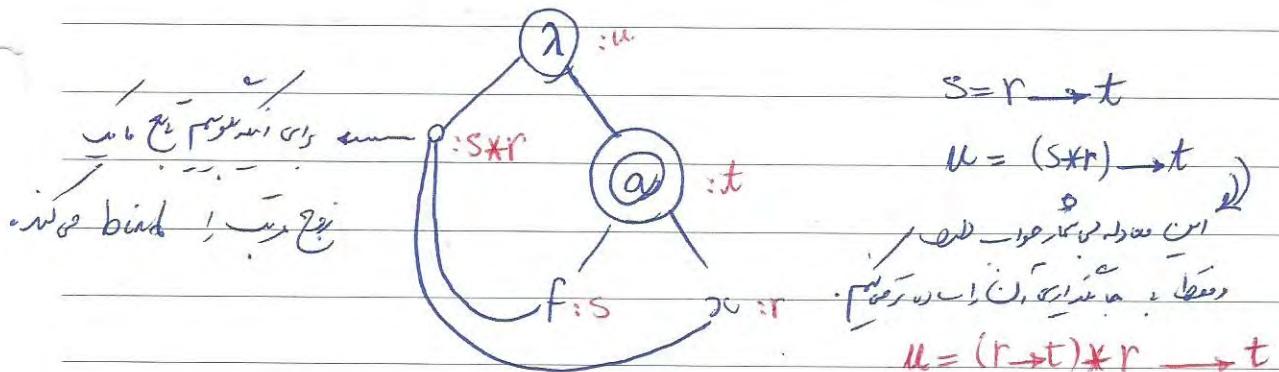
$$\xrightarrow{(1),(2)} u = \text{int}, t = \text{int}$$

$$\xrightarrow{(1),(2),(3)} v = \text{int} \rightarrow \text{int}$$

$\rightarrow g(\text{false})$: مرض، ارتباط

\rightarrow Type Error!

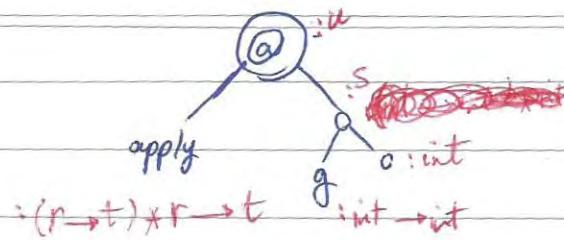
- Fun apply (F, x) = $F(x)$;



\rightarrow val apply = Fn: ($'a \rightarrow 'b$) * $'a \rightarrow 'b$

- apply (g, x) \rightsquigarrow \rightarrow Type Inference

\rightarrow Val it = 5: int



$$s = (\text{int} \rightarrow \text{int}) * \text{int}$$

$$s = (r \rightarrow t) * r \Rightarrow r \rightarrow t = \text{int} \rightarrow \text{int}$$

$$\Rightarrow r = \text{int}, t = \text{int}$$

$$\Rightarrow u = t \Rightarrow u = \text{int}$$

Type Inference \hookrightarrow Type Inference \hookrightarrow Type checking \hookrightarrow Checking

Checking \hookrightarrow Checking

undecidable \hookrightarrow Type Inference

Type annotation \hookrightarrow ML

CBT (Constraint-Based Typing) :
 (Constraint) \hookrightarrow Type Inference
 Type Inference \hookrightarrow Type Inference
 Type Inference \hookrightarrow Type Guard

Type Inference \hookrightarrow Type Guard

(Type Guard) \hookrightarrow Type Guard

Type Inference \hookrightarrow Polymorphism
 Polymorphism \hookrightarrow Type Guard

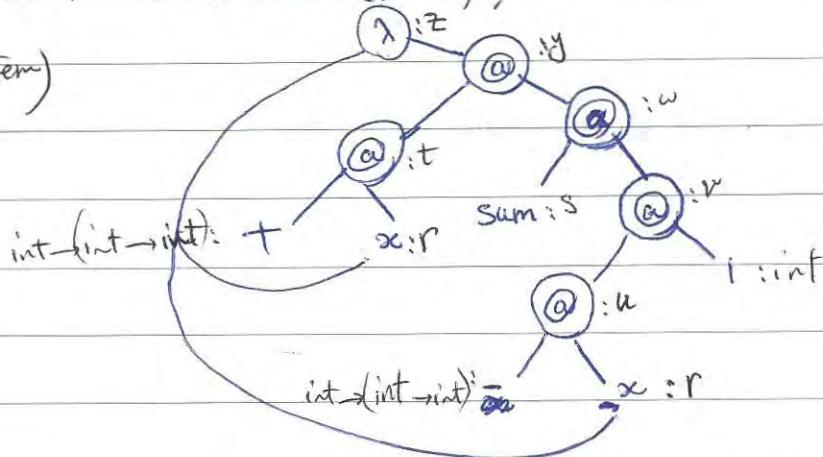
Example A recursive Function

- fun sum(x) = x + sum(x-1);

(ML Runtimesystem)

newCell b/c

2



↓ Type ↓ decorate ↓ parseTree

$$(1) \text{ int} \rightarrow (\text{int} \rightarrow \text{int}) = r \rightarrow u$$

$$(2) \text{ int} \rightarrow \text{int} \rightarrow \text{int} = r \rightarrow t$$

$$(3) u = \text{int} \rightarrow v$$

$$(4) s = v \rightarrow w$$

$$(5) t = w \rightarrow y$$

$$(6) z = r \rightarrow y$$

$$(7) z = s$$

..... (1) recursive def?

!Circular definition

$r = \text{int}$, $u = \text{int} \rightarrow \text{int}$, $t = \text{int} \rightarrow \text{int}$, $v = \text{int}$, $s = \text{int} \rightarrow w$,

$t = w \rightarrow y \Rightarrow w = \text{int}$, $y = \text{int}$, $z = r \rightarrow y \Rightarrow z = \text{int} \rightarrow \text{int}$, $z = s$, $s = \text{int} \rightarrow \text{int}$

→ Val sum = fn: int → int

: need to find all fixpoints of ML w

- Fun append (nil, l) = l

| append ($x :: xs, l$) = $x :: \text{append}(xs, l)$;

Type Inference

append: 'a list * 'b → 'b

: $c b = j o h i a s s t$

append: 'c list * 'd → 'c list

: $c b = j o h i a s s t$

$= 'c$ list $* 'c$ list $\rightarrow 'c$ list

: $c b = j o h i a s s t$

'a list * 'b → 'b = 'c list * 'd → 'c list

⇒ append: 'a list * 'a list → 'a list

- Fun reverse (nil) = nil

| reverse ($x :: lst$) = lst (دومین)

reverse = 'a list → 'b list

: $c b = j o h i a s s t$

reverse = 'c list → 'd

: $c b = j o h i a s s t$

⇒ 'a list → 'b list = 'c list → 'd ⇒

reverse: 'a list → 'b list

← $c b = j o h i a s s t$ (ML (F9))

\hookrightarrow (Control Semantic rule) \rightarrow Type CML (Control rule)

جواب با متن است و در تفاوت است. و سه در زیر نوشته اند که نوعی کوئی میگویند است باز هم در هر چند تفاوت

ویکی خبر و معرفت داشم این تصور نباید بسیار روزگار باشد بلطفاً است

most semantic order عاقلهٔ بیرونی خواهد بود و معرفت است Type Consistency

ML type system دادهٔ Type Recursion محدود

طبقه‌واریتی محدود است بروز آن می‌شود

Polyorphism :

پلی‌مورفیسم (Polyorphism) یعنی "چندین شکل" یا "چندین شکل" morph

برای هر یک دادهٔ داشت، چندین شکل دارد که در آن دادهٔ داشت مانند

insert ... insert sort, addition

Abstraction

پلی‌مورفیسمی که برای هر دادهٔ داشت چندین شکل داشته باشد

برای هر دادهٔ داشت چندین شکل داشته باشد

appending, concatenation apparent

- append([2,3], [0,1]);

→ val it = [2,3,0,1] : int list

- append([1,2],[2,3]), [0,5],[1,2]);

→ val it = [[1,2],[2,3],[0,5],[1,2]] : int list list

: Polymorphism ابراع

- Parametric polymorphism :

parametric polymorphism (parametric type) .
 ابراعیتیپ (parametric type) .
 first class Type Application
 first class Sub Application

- Ad Hoc Polymorphism (overloading) :

(sum of) two OOPs implementations in overloaded function
 float, int etc -

Ad Hoc Polymorphism (ML)

- Subtype Polymorphism :

SuperType type < SubType same in Type inheritance
 Superclass type , subclass type is object

! SuperType type is SubType in Type Inheritance

- ابراعیتیپ جوں

Parametric polymorphism :

$\lambda \text{ sort } (\text{a} * \text{a} \rightarrow \text{bool}) * \text{a list} \rightarrow \text{a list}$

first class
اول کلاس

(٦) (٧). Insert $\in \text{insert}(\text{list}, \text{value})$ is a permutation

Explicit Parametric Polymorphism \rightarrow (عکس این دستوراتی است) (نوعی دستوراتی است) $\Rightarrow C++$

Implicit Parametric Polymorphism \rightarrow (عکس این دستوراتی است) $\Rightarrow ML$

void swap (int& x, int& y) {

: C++ \rightarrow (جاء)

int tmp = x;

x = y;

y = tmp;

برای جایگزینی مترزیم

template <typename T>

void swap (T& x, T& y) {

T tmp = x;

x = y;

y = tmp;

برای جایگزینی

Type Application \rightarrow (دستوراتی است)

(Type Application)

برای Explicit parametric polymorphism

برای درستی از type

برای Activation Record \rightarrow (دستوراتی است) برای template type چیزی داشتیم

برای درستی از type

برای Type Application Link Time \rightarrow C++

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Subject: PL

Year: 1392 Month: 09 Date: 11 ت

$e : \tau \quad \Gamma : \tau'$

(subsumption)

: subtype "

$e : \tau'$

$\tau <: \tau' \rightarrow \tau, \text{subtype}, \tau$

وَيُعَدُّ اَنْ تَعْلَمُ اَنْ

اَنْ $\tau' \leq \tau$ اَنْ τ' اَسْبُطَ τ

τ' [Name:] [Scope:]

(τ' supsumes τ !)

وَيُعَدُّ اَنْ τ اَسْبُطَ τ' اَنْ τ يَعْلَمُ اَنْ τ' اَسْبُطَ τ

وَيُعَدُّ اَنْ τ اَسْبُطَ τ' , polymorphism &

وَيُعَدُّ اَنْ τ اَسْبُطَ τ' , Type Application

وَيُعَدُّ اَنْ τ اَسْبُطَ τ' , Impredicative

وَيُعَدُّ اَنْ τ اَسْبُطَ τ' , Predicative

وَيُعَدُّ اَنْ τ اَسْبُطَ τ' , Polymorphism

(τ لا يَعْلَمُ اَنْ τ' اَسْبُطَ τ)

(polymorphism inheritance)

وَيُعَدُّ اَنْ τ اَسْبُطَ τ' , C++

وَيُعَدُّ اَنْ τ اَسْبُطَ τ' , module

وَيُعَدُّ اَنْ τ اَسْبُطَ τ' , Linker

وَيُعَدُّ اَنْ τ اَسْبُطَ τ' , C++

- (1) ویرایشی. Sub polymorphic Function Overloading (فرم) in ML type system
- (2) isomorphism pointer i.e. (isomorphic) representation in C++ & ML
- (3) Fun swap (x, y) = let val tmp = x in
 $x := !y ; y := !tmp$ end;
- (4) \rightarrow val swap = Fn: a ref * a ref \rightarrow unit
- (5) inise reference by operation $\underline{x} \leftarrow x := !y$
- (6) ref assign لذوقی ایجاد
- (7) بازگشتی و ایندکسی، Polymorphism overafifc ML در روزه.
- (8) پسندیدنی uniform data representation یوسف
- (9) بیتotype روزگاری ویکی ویکی ویکی ویکی ویکی ویکی ویکی
- (10) اندازه لذوقی لذوقی لذوقی لذوقی لذوقی لذوقی لذوقی لذوقی لذوقی
- (11) ML و C++ و ML و C++ و ML

System F Gerard J. S. Almeida Abstract: parametric polymorphism

(overloading) (Polymorphismによる多義化)

+ (plus): int + int \rightarrow int

: ML //

+ (plus): real + real \rightarrow real

int や float など、異なる型の数値を加算する場合、
実数 (real) が int や float など、異なる型の数値と
(inference!) オーバロード (overload) される。
(overload) が int (real) の場合、

Type declarations and Type equality:

type declaration は assignment など、複数の型を定義する
(Objetif: type checker) ためのエラー

type equality は type declaration の間の比較

Type declaration $\begin{cases} \text{Transparent} \Rightarrow \text{int} \text{ などの alternative name} \\ \text{Opaque} \Rightarrow \text{no type} \end{cases}$ など、no type

no type など

Subject :

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Type <identifier> = <type_expression> : Transparent

type Celsius = real; : ML ~ ; ~

type Fahrenheit = real;

- fun toCelsius(x) = ((x - 32.0) * 0.55556);

→ Val toCelsius = fn : real → real

- fun toCelsius(x: Fahrenheit) = ((x - 32.0) * 0.55556); Celsius;

→ val toCelsius = fn : Fahrenheit → Celsius

Type Equality

Name type equality → Two types have same name

Structural type equality → One type has same structure

Integrate with ML

integrate with C++ Type checking

typedef char byte;

typedef byte ten_byte[10]; : C++

char equal byte

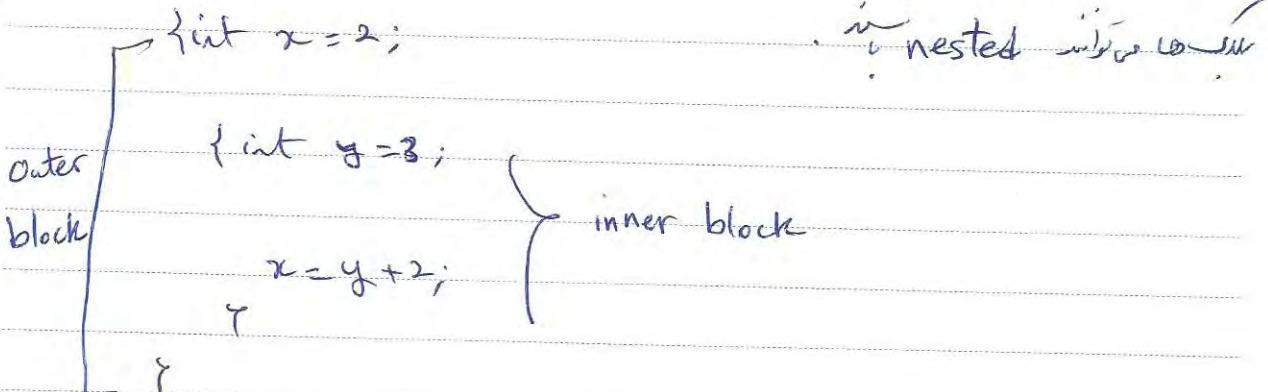
char[10] equal byte[10] equal ten_byte[10]

PAPCO

introduction Name for C++

Subject:

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in local , local variable (declaration)

(This will be global in environment of declaration)

assign ① initial global is static scope

but variables are dynamic scope,

: Block-structured language,
it declares variables newly

in each block newly declaration

In nested block int overlap not int

(local), variables, are declared newly in block

or deallocated when block

لیک اسٹاک پروجئی

global variables

(3)

parameters

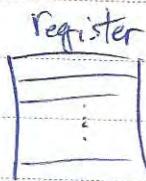
(2)

local Variables

(1)

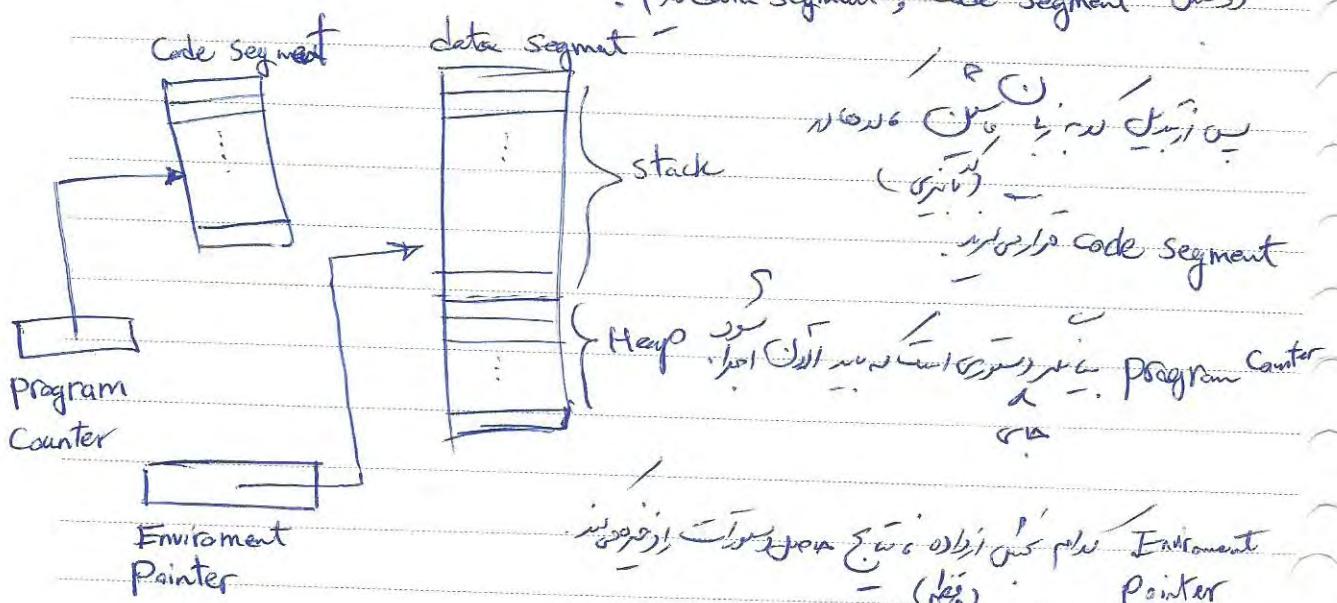
↓

میکریٹ



میکریٹ کی رکھیں

پر دیتا سگمنٹ، کوڈ سگمنٹ میں



Reference Implementation

پر دیتا سگمنٹ، کوڈ سگمنٹ میں

پر دیتا سگمنٹ، کوڈ سگمنٹ میں

stack outer block

stack inner block

stack outer block

stack inner block

stack discipline

Subject: _____
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(^{وَجْهِ} outer ^{وَجْهِ} stack ^{وَجْهِ} inner ^{وَجْهِ})

inhibit declaration \leftarrow In-line blocks \leftarrow ^{فِي} ^{السُّلْطَانِ}

Function call procedure call

inhibit \rightarrow right hand

int $x = 0;$

int $y = x + 1;$

int $z = (x + y) * (x - y);$

;

;

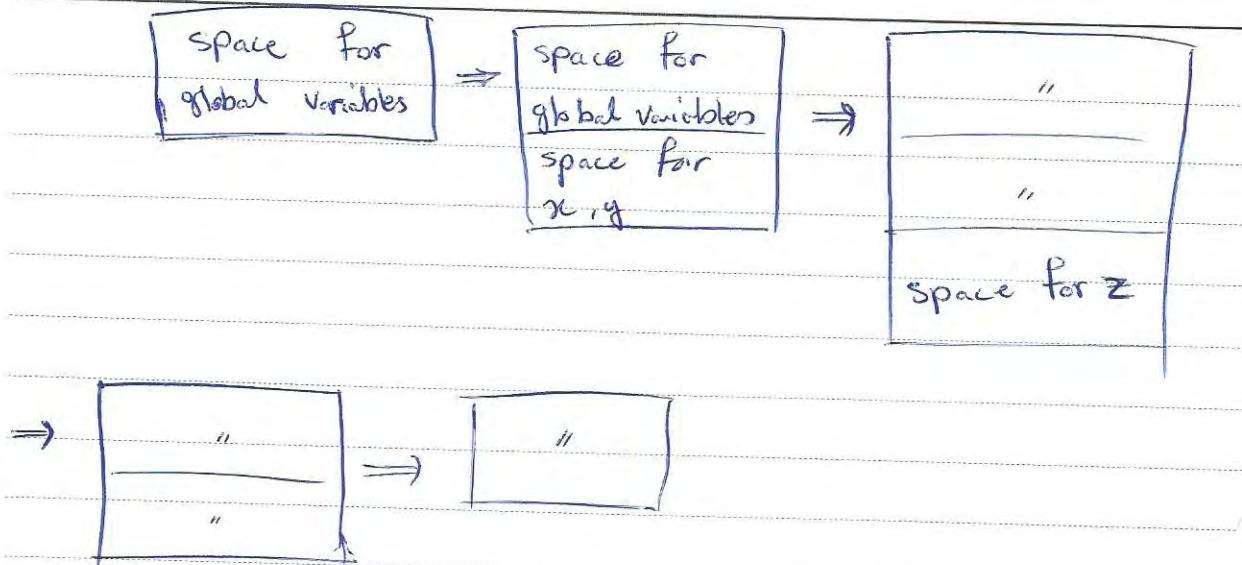
new activation record \rightarrow new activation record

old activation record \rightarrow old activation record

deallocates end \leftarrow inner activation record \leftarrow beginning \leftarrow " { "

new activation record
Stack Frame activation record

new stack \rightarrow old



• ترتیب اعلانات مت�یر در کد اهم است

• اگر کوئی متغیر مطابق با اعلان پیش از این متغیرها باشد، این متغیر را باید در نظر بگیریم

Scope vs. lifetime:

• محدوده و مدت زمانی که یک متغیر در کد قابل مشاهده و دسترسی است

• محدوده و مدت زمانی که یک متغیر در کد قابل مشاهده و دسترسی است

• میتواند از اعلان تا انتها کاربرد داشته باشد / محدوده و مدت زمانی که یک متغیر در کد قابل مشاهده و دسترسی است

• جزو محدوده و مدت زمانی که یک متغیر در کد قابل مشاهده و دسترسی است

{ int x = ... ; }

lifetime x

{ int y = ... ; }

PCPQO

y | lifetime x | { int x = ... ; } | → x, scope (only y)

y; (only y) scope (only). in

- Fun $f(x) = x + 1;$

Fun $g(y) = f(y) + 2;$

$g(3);$

let Fun $g(y) = y + 3$

Fun $h(z) = g(g(z))$

declarations(s) in, in

in $h(3)$

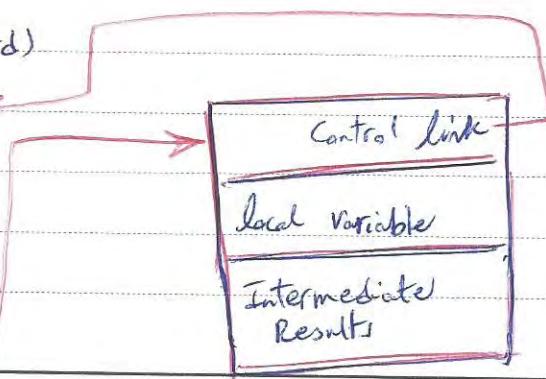
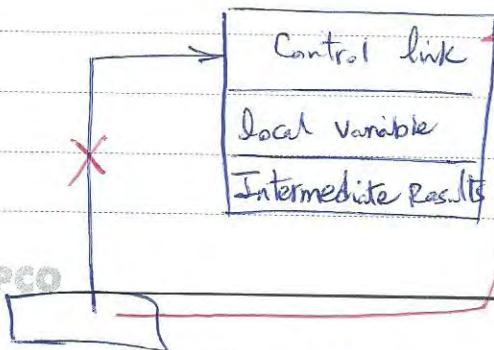
expression(s)

end

global variables:

activation record
new

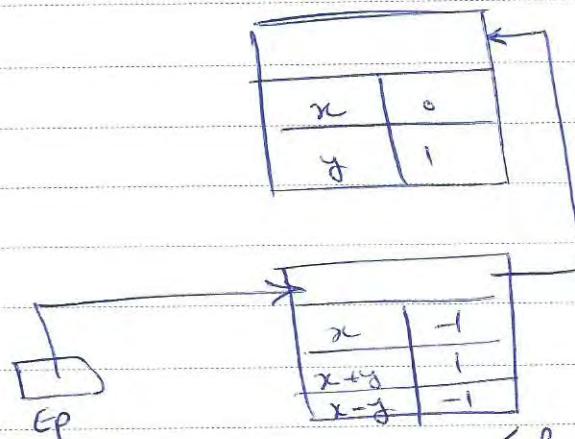
s(activation record)



EP(Environment pointer)

activation record (new memory). (EP) is initial control link : EP (Jiffy)

new control link \rightarrow control link, new control link : EP
(Jiffy)



: Jiffy

new activation record, new frame, recursive function

stack \rightarrow pop, push previous activation record in stack

new activation record's function call position. Fortran

overwrite, initialize, new activation record. In recursive call

Procedures & Functions:

(no return) no side effects to procedure

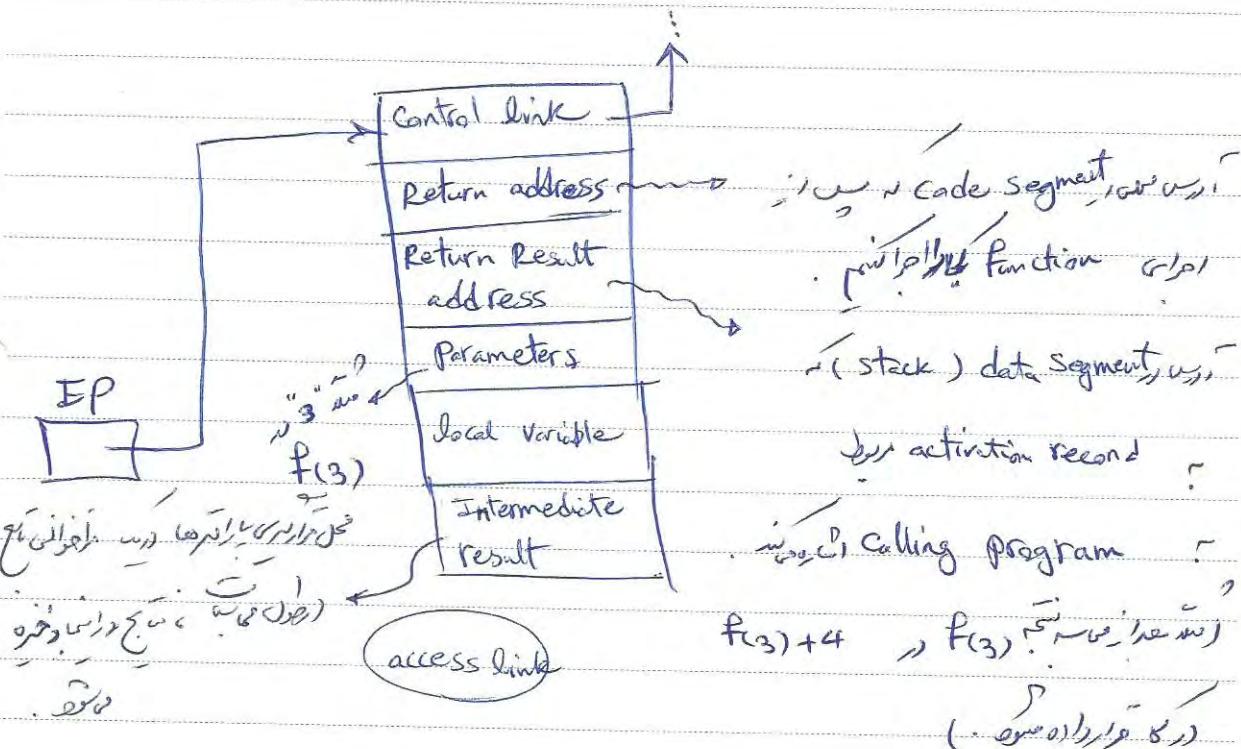
(no return) initializes side effect into function

PARCO

(return) input output (variables)

Q. (Function call)

write ~~function~~ function using activation record



After we call a function using activation record

global (obj), escape (obj), import access link (program)

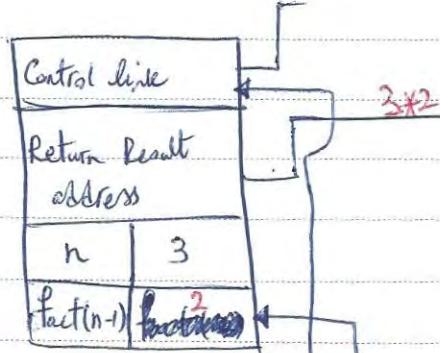
(ex (main block) function obj)

Fun fact (n) = if $n=1$ then 1 else $n \times \text{fact}(n-1)$

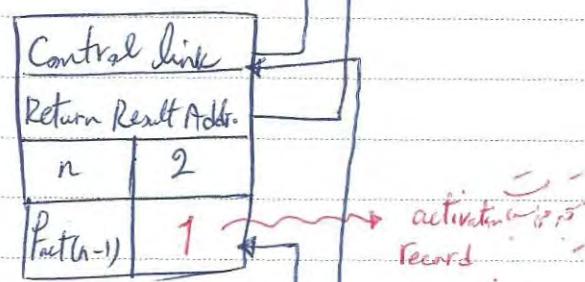
$\text{fact}(3) + 1$

Calling Program
Calling block
Calling Function

Fact(3) :



Fact(2) :



Fact(1) :



activation record Fact(3) activation record

no deallocate activation record

Call by value activation record (Call by value) (Call by value) (Call by value)

Parameter Passing

Arg P Call-by-value Call-by-name

Subject :

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Formal parameters:

Temporary variables associated with the formal parameters.

Actual parameters:

Actual values corresponding to the formal parameters.

Most formal parameters are actual parameters.

proc p (int x, int y)

if (x > y) then

else

:
x := y * 2 + 5;

p(2, 4 * 2 + 1),

actual arguments

↳ 2, 4 * 2 + 1, 2, y, x are actual arguments.

↳ Actual parameters are stored in activation record in memory.

I The time that the actual parameter is calculated.

II The location used to store the parameter value.

میں اسی طریقہ سے اس کا پاس کرنے کا فہرست ہے

لیکن اس کو اپنے لئے اپنے اس طریقہ سے اس کا پاس کرنے کا فہرست ہے

1) Pass-by-reference: pass by L-value

\rightarrow اسی ملے

2) Pass-by-value: pass by R-value

ایکی ایسے پڑھے اور

I-side effect \rightarrow II-side effect, pass-by-value

II-Aliasing \rightarrow (بے ایڈنٹیفائر) \Rightarrow II-Aliasing, \rightarrow pass-by-reference

روزگاری، ایڈنٹیفائر، نام جیسا جو \rightarrow call-by-value وارے
روزگاری، ایڈنٹیفائر، نام جیسا جو \rightarrow call-by-value وارے

III-Efficiency \rightarrow اسی طریقے پر Pass-by-value

(کوئی ایڈنٹیفائر)
(structure)

ایک طریقہ سے structure with pass-by-reference

ایک طریقہ سے pass-by-reference کی کسی ایسی

روزگاری اسی طریقہ سے structure with pass-by-reference

Subject:

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Semantics of pass-by-value:

(Translational Semantics)

function $f(x) = \{ x := x + 1; \text{return } x \}$

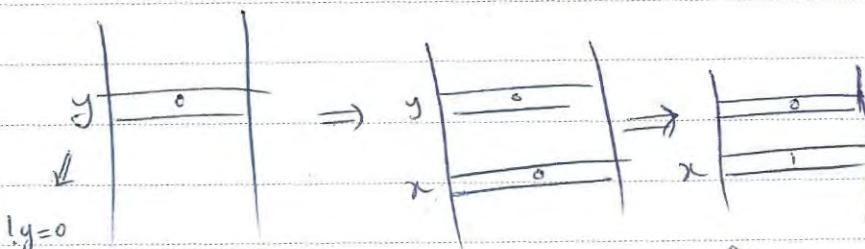
... $f(y)$... ↓
 Val
 \rightarrow $y := !x + 1$ in y shows in pass-by-value position

fun $f(z: \text{int}) = \text{let } x = \text{ref } z \text{ in } x := !x + 1; !x \text{ end}$

... $f(!y)$...

↓ \rightarrow $x := !y + 1$

Later pass changing value place



→ pass-by-value Aliasing, side effect actions, y becomes

Semantics of pass-by-reference:

fun $f(x: \text{int ref}) = (x := !x + 1; !x)$

... $f(y)$...

→ Causal Aliasing (w/)

Referring to same memory location in two environments

$y \boxed{0} \Rightarrow y \boxed{1} \Rightarrow$ side effect
 Einzelne Werte von y , keine α -Copy

fun f (pass-by-ref x: int, pass-by-value y: int) (x)

begin

$x := 2;$

$y := 1;$

if $x = 1$ then return 1 else return 2;

end

var z: int;

$z := 0;$

print f(z);

fun f (x: int ref, y: int)

let val yy = ref y in

$x := 2;$

$yy := 1;$

if (! $x = 1$) then 1 else 2;

end;

val z: ref 0;
 $f(z, !z);$

• In C/C++, function reference is passed by-value parameter.

• In "!" we want to write by-reference



↳ Aliasing \rightarrow reference

↳ Control flow inspection! Control flow aliasing is just

Global Variables:

C/C++, Java \rightarrow static scoping \Rightarrow Text of program

Lisp \rightarrow dynamic scoping \Rightarrow stack frames

(other Lisp)

static scope \rightarrow closest enclosing block in program text containing

variables, procedures, functions, etc.

dynamic scope

the declaration of the global

variable.

activation record

declaration

The most recent activation record containing the global variables.

• not in the stack frame or heap

• static linkage

• PAPC

older Lisp, TEX, LATEX, Exceptions in languages, macros \rightarrow dynamic scope

Java, C, ML, Algol, \rightarrow static scope

($\cup_{i=1}^n \cup_{j=1}^{m_i}$)

Example -

int $x = 1;$

function $g(z) = x + z;$

function $f(y) = \{$

int $x = y + 1;$ $\rightarrow x = 4$

return $g(y + x)$ $\rightarrow g(12)$

$y;$

$f(3);$

$$1 + 12 = 13 \quad (\text{static scope}) \rightarrow \text{static scope} \rightarrow$$

$$4 + 12 = 16 \quad (\text{activation record}) \rightarrow \text{dynamic scope} \rightarrow$$

(static scope) \rightarrow activation record \rightarrow dynamic scope \rightarrow static \rightarrow x in y

example access link \rightarrow Activation Record \rightarrow static \rightarrow dynamic

↓

Static Scope \rightarrow Activation Record \rightarrow static \rightarrow dynamic

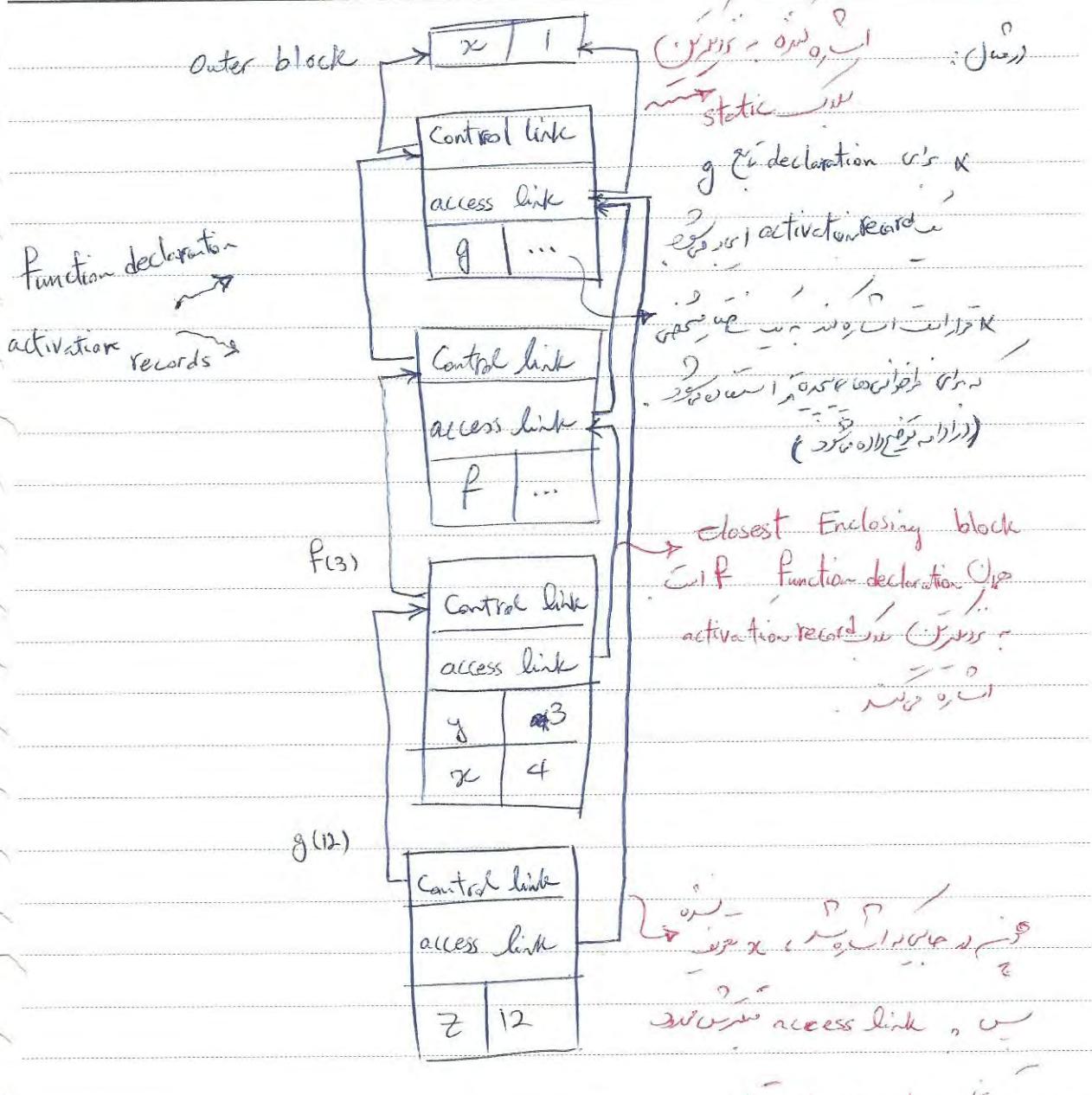
↓

↓

Static Link Access Link \rightarrow ↓

↓

↓



activation record
optimization recursive algorithm in optimize

اگر سرو و میرت شود

Tail Recursion:

(without the activation record with return address)

Tail Recursion →

tail call:

آخر دفعه من التكاليف

tail call after C!

آخر دفعه من التكاليف

fun f(x) = if $x=0$ then $g(x)$ else $g(x)+2$

↓
Tail Call

tail recursive: tail call \rightarrow self invocation

(in Tail Call \rightarrow no local vars)

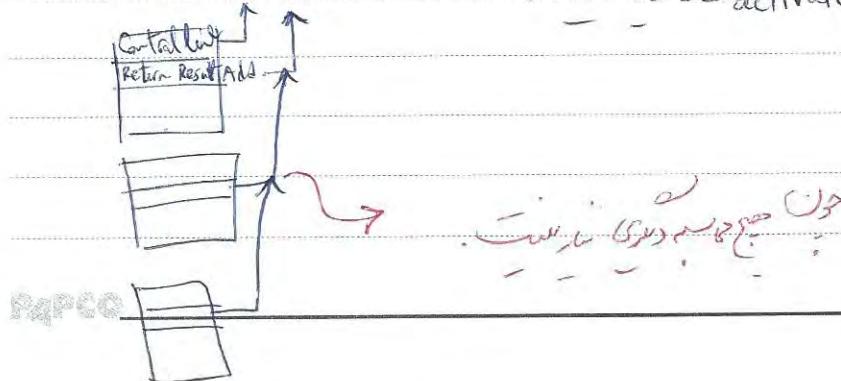
: Tail Recursive \rightarrow (in Tail Call \rightarrow no local vars)

fun $tlfact(n, a) =$ if $n \leq 1$ then a else $tlfact(n-1, n*a)$;

$tlfact(3, 1) \rightarrow tlfact(2, 3*1) \rightarrow tlfact(1, 2*3*1) \rightarrow 2*3*1$

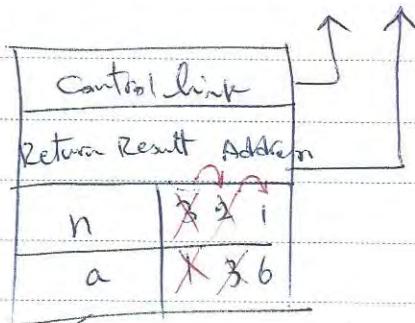
31

in the activation record of function 31



Subject:

Year 1392 Month 09 Date 25



الدالة السابقة

رسالة: مفهوم الـ Tail Recursion

Recursive vs. Iterative

الـ Tail Recursion هي طريقة إنشاء دالة متتالية (Iterative) من دالة متursive (Recursive).

fun tlFact(n,a) = (while !n > 0 do (a := !n*a, n := !n-1); !a);

Activation record (Call stack frame)

n	X 32 i
a	X 36

activation record

Higher-order functions:

Second-order function

First class functions:

Second-order

activation record

closure

Rapco

closure

fun map (f , nil) = nil

(JUN)

| map (f , $x::xs$) = $f(x) :: \text{map}(f, xs)$

↑ fold & unfold

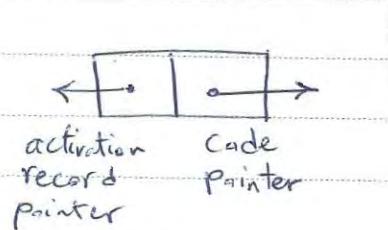
? Closure as (!) $\lambda x. x + 1$ is a first class citizen

7, 1, 2, 4, 6, 8, 10, 13, 15

: $\lambda x. x + 1$
(fold / unfold)

downward funarg problem:

Closure



(environment pointer)

val x = 4;

(JUN)

{ Fun f(y) = x * y;

{ Fun g(h) = let val x = 7 in h(3) + x;

FAPCO

g(f);

Code Segment

Activation Record

x	4
---	---

access	
f	

closure

	1
	1

code	
for f	

access	
g	

	1
	1

code	
for g	

g(f)

access	
h	

x 7

h(3)

access	
y	3

h is formal parameter of g.

well access link is not present in stack discipline

so it access link will closure

(closure is present in access link)

regions, loops, in function to return value value of stack discipline

(nested scope)

PAPCO PAPCO PAPCO PAPCO PAPCO PAPCO PAPCO PAPCO PAPCO PAPCO

١٤٥

// Function make_counter (Counter)

مُفْعِلٌ مُحَاجِرٌ مُنْهَجٌ (ج)

يُكَوِّنُ (أ) يُؤْخِذُ increment function و Counter و \rightarrow private integer value
inc_fn.

(In state - Full (أ) increment

Fun make_counter (init : int) =

let val count = ref init \rightarrow or initialize counter

Fun counter (inc : int) = (count := !count + inc; !count)

in

counter .

end;

val c = make_counter (1);

c(2) + c(2);

\rightarrow val makeCounter = fn : int \rightarrow (int \rightarrow int)

counter \approx (C)

\rightarrow val c = fn : int \rightarrow int

\rightarrow 8 : int

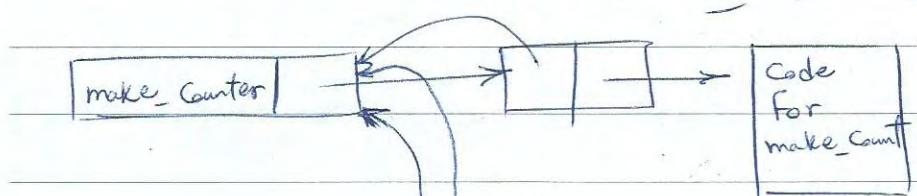
و $c(2)$ \rightarrow initial $c(2)$ \rightarrow in state full

و $c(2) = 3 = 1+2$

و $c(2) = 5 = 3+2$

و $c(2)$ \rightarrow in state full \rightarrow state full , In statement \approx و $c(2)$

(Calculated object predicting)



access	
C	

Code
for
make_Count

(make_Counter())

make_Counter()

access	
init	1
Count	
Counter	

Code
for
Counter

C(2)

access	
inc	2

(With access = 2)