

# Complex Analysis Homework 5

Colin Williams

October 7, 2020

## Question 5

Show that

$$\int_{\gamma} (z - z_0)^{n-1} dz = 0, \quad n \in \mathbb{N} \setminus \{0\}$$

where  $\gamma$  is any closed path that does not pass through  $z_0$ .

**Answer.**

I will use antiderivatives to prove this. Note that  $\frac{d}{dz} \left( \frac{1}{n} (z - z_0)^n \right) = (z - z_0)^{n-1}$ . I will assume that since  $\gamma$  is closed, that it starts and ends at the point  $w \neq z_0$ . Thus,

$$\begin{aligned} \int_{\gamma} (z - z_0)^{n-1} dz &= \frac{1}{n} (z - z_0)^n \Big|_{z=w}^{z=w} \\ &= \frac{1}{n} (w - z_0)^n - \frac{1}{n} (w - z_0)^n \\ &= 0 \end{aligned}$$

The only problem here would be if  $n = 0$ , but we excluded that from consideration in the statement of the problem. Thus, it has been shown.