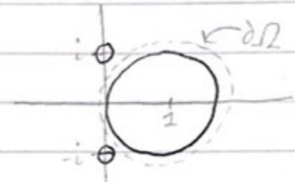
Question 2

Suppose $\gamma(a,r) = a + re^{it}$, $t \in [0, 2\pi]$ is a positively oriented circle centered at a with radius r. Evaluate the following.

a.) $\displaystyle\int_{\gamma(1,1)} \frac{e^{iz^2}}{z^2+1}\,dz$



Let $f(z) = e^{iz^2}/z^2+1$, then we can see that f is undefined at $z=i$ and $z=-i$. However, if we consider the set $\Omega = D(1, 1.1)$, then f is analytic on Ω and $\gamma^* \subset \Omega$. Therefore, by Cauchy's Integral Theorem,

$$\int_{\gamma(1,1)} \frac{e^{iz^2}}{z^2+1}\,dz = 0$$

b.) $\displaystyle\int_{\gamma(0,1)} |z|^2\,dz$

By parametrizing γ as $\gamma(t) = e^{it}$, $t \in [0, 2\pi]$, we can use the definition of the path integral to get:

$$\int_{\gamma(0,1)} |z|^2\,dz = \int_0^{2\pi} |e^{it}|^2 \cdot ie^{it}\,dt = i\int_0^{2\pi} 1^2 \cdot e^{it}\,dt = \frac{i}{i} e^{it}\Big|_0^{2\pi} = \boxed{0}$$