Question 3

- Find the Laurent series expansion for the function

$$f(z) = \frac{1}{z(3+3z)}$$

around the point $z=0$, and specify where it is valid.

Answer

- First, note the following

$$f(z) = \frac{1}{z(3+3z)} = \frac{1}{3}\left(\frac{1}{z(1+z)}\right) = \frac{1}{3z}\left(\frac{1}{1-(-z)}\right)$$

- Also, recall the formula for a geometric series:

$$\sum_{n=0}^{\infty} w^n = \frac{1}{1-w} \quad \text{for all } |w| < 1.$$

- Thus,

$$f(z) = \frac{1}{3z}\sum_{n=0}^{\infty}(-z)^n = \frac{1}{3z}\sum_{n=0}^{\infty}(-1)^n \cdot z^n = \sum_{n=0}^{\infty}\frac{(-1)^n}{3}z^{n-1}$$

$$\Rightarrow f(z) = \sum_{n=-1}^{\infty}\frac{(-1)^{n+1}}{3}z^n$$

- This series was obtained from geometric series on $(-z)$, thus, it is valid whenever $|-z| < 1 \Leftrightarrow |z| < 1$