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Complex Analysis Exam 1

Question 1

Perform the operations and express result in rectangular coordinates

(a) $\left(\frac{1-i}{1+i}\right)^7$

First, note that $\frac{1-i}{1+i} = \frac{(1-i)(1-i)}{(1+i)(1-i)} = \frac{1-i-i+i^2}{1-i+i-i^2} = \frac{-2i}{2} = -i$

Thus, $\left(\frac{1-i}{1+i}\right) = (-i)^7 = (-1)^7(i)^7 = -1 i^4 \cdot i^3 = -i^3 = -i \cdot i^2 = i$

$\Rightarrow \left(\frac{1-i}{1+i}\right)^7 = i$

(b) $(-i)^{1/3}$

First, note that $-i = e^{\frac{-\pi}{2}i + 2\pi i K}$ for $K \in \mathbb{Z}$

$\Rightarrow (-i)^{1/3} = \left(e^{\frac{-\pi}{2}i + 2\pi i K}\right)^{1/3} = e^{\frac{-\pi}{6}i + \frac{2\pi}{3}iK}$

This only has 3 distinct values, so for simplicity, choose $K = 0, 1, 2$

$K = 0$

$(-i)^{1/3} = e^{\frac{-\pi}{6}i} = \cos\left(\frac{-\pi}{6}\right) + i\sin\left(\frac{-\pi}{6}\right) = \frac{\sqrt{3}}{2} - \frac{i}{2}$

$K = 1$

$(-i)^{1/3} = e^{\frac{-\pi}{6}i + \frac{2\pi}{3}i} = e^{\frac{3\pi}{6}i} = e^{\frac{\pi}{2}i} = \cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right) = i$

$K = 2$

$(-i)^{1/3} = e^{\frac{-\pi}{6} + \frac{4\pi}{3}i} = e^{\frac{7\pi}{6}i} = \cos\left(\frac{7\pi}{6}\right) + i\sin\left(\frac{7\pi}{6}\right) = \frac{-\sqrt{3}}{2} - \frac{i}{2}$