

3a) Express the following numbers in the form $re^{i\theta}$

i) i^3

$$i^3 = i^2 \cdot i = -i = 0 - 1 \cdot i$$

$$r = |0 - 1 \cdot i| = \sqrt{0^2 + 1^2} = 1, \quad \theta = \tan^{-1}\left(\frac{-1}{0}\right) = \lim_{x \rightarrow 0} \tan^{-1}\left(\frac{-1}{x}\right) = \frac{\pi}{2}$$

$$\Rightarrow i^3 = 1e^{i(\frac{\pi}{2})}$$

ii) $1 - i$

$$r = |1 - i| = \sqrt{1^2 + 1^2} = \sqrt{2}, \quad \theta = \tan^{-1}\left(\frac{-1}{1}\right) = \frac{-\pi}{4} \Rightarrow 1 - i = \sqrt{2}e^{i(\frac{-\pi}{4})}$$

iii) $\sqrt{3} - i$

$$r = |\sqrt{3} - i| = \sqrt{3 + 1} = \sqrt{4} = 2, \quad \theta = \tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) = \frac{-\pi}{6} \Rightarrow \sqrt{3} - i = 2e^{i(\frac{-\pi}{6})}$$

iv) $2 - 2\sqrt{3}i$

$$r = |2 - 2\sqrt{3}i| = \sqrt{4 + 12} = \sqrt{16} = 4, \quad \theta = \tan^{-1}\left(\frac{-2\sqrt{3}}{2}\right) = \frac{-\pi}{3} \Rightarrow 2 - 2\sqrt{3}i = 4e^{i(\frac{-\pi}{3})}$$

b) Express the following numbers in the form $x + iy$

i) $e^{i(\frac{\pi}{4})}$

$$x = \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}, \quad y = \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} \Rightarrow e^{i(\frac{\pi}{4})} = \frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}$$

ii) $5e^{-\pi i}$

$$x = 5\cos(-\pi) = -5, \quad y = 5\sin(-\pi) = 0 \Rightarrow 5e^{-\pi i} = -5 + 0i$$

iii) $2e^{i(\frac{3\pi}{2})}$

$$x = 2\cos\left(\frac{3\pi}{2}\right) = 0, \quad y = 2\sin\left(\frac{3\pi}{2}\right) = -2 \Rightarrow 2e^{i(\frac{3\pi}{2})} = 0 - 2i$$

iv) $e^{i(\frac{4\pi}{3})}$

$$x = \cos\left(\frac{4\pi}{3}\right) = -\frac{1}{2}, \quad y = \sin\left(\frac{4\pi}{3}\right) = -\frac{\sqrt{3}}{2} \Rightarrow e^{i(\frac{4\pi}{3})} = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$$