

# Complex Analysis Homework 4

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## Question 2

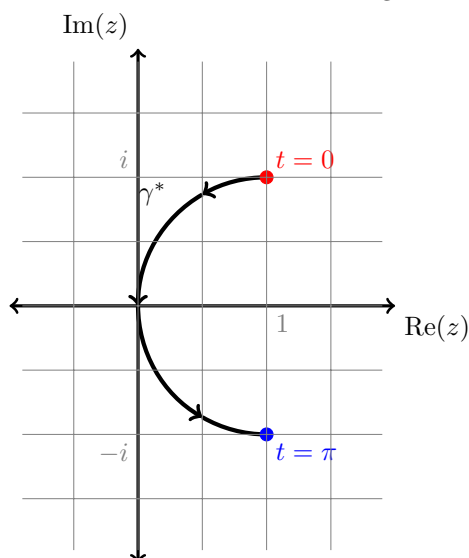
Plot the image  $\gamma^*$  of the curve  $\gamma$  in the following cases, indicating how the image is traced:

(a)  $\gamma(t) = 1 + ie^{it}, t \in [0, \pi]$

(b)  $\gamma$  is the join of three line segments:  $[-1, 1]$ ,  $[1, 1 + i]$ , and  $[1 + i, -1 - i]$

**Answer. (a)**

First, note that the curve  $e^{it}, t \in [0, \pi]$  simply has an image of the upper semi-circle centered at 0 going counter-clockwise. Next, note that multiplying any  $a + bi \in \mathbb{C}$  by  $i$  gives  $-b + ai$  which is a  $90^\circ$  rotation counter-clockwise. Thus, the curve  $ie^{it}, t \in [0, \pi]$  would have the image of the left semi-circle centered at 0 going counter-clockwise. Lastly, adding 1 simply shifts the center of the semi-circle right 1 unit, so  $\gamma(t) = 1 + ie^{it}, t \in [0, \pi]$  have the image  $\gamma^*$  represented as the following:



**Answer. (b)**

The way that  $\gamma$  is defined, it should consist of line segments going to the following points in this order:  $-1, 1, 1 + i, -1 - i$ . Thus,  $\gamma^*$  looks like the following:

