$\mathbb{Q}^+$

POSITIVE RATIONALS

DEFINITION

$\frac{1}{q}$
- number you add q times to get 1.
- solution to $qx=1$

Ex. $\frac{1}{3}$ is soln to $3x=1$

$\frac{p}{q}$
- repeated add^n of $\frac{1}{q}$, p times

$p, q \in \mathbb{N}, q \neq 0$

MULTIPLICATION

Example	Meaning
One Third 5's	Solution to $3x=5$
Two thirds 5's	(One Third 5's) + (One Third 5's)
Five $\frac{1}{3}$'s	Repeated Add^n $\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=\frac{5}{3}$ (by def^n of $\frac{p}{q}$)

MULT^n IS COMMUTATIVE

Why Five $\frac{1}{3}$'s = One Third 5's

Because $\frac{5}{3}$ is a solution to $3x=5$

$$3 \cdot \frac{5}{3} = \frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3} + \frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3} + \frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3} = 1+1+1+1+1 = 5$$ ✓

EXPONENTIATION

Example	Meaning
$a^{\frac{1}{3}}$	Soln to $x^3=a$
$a^{\frac{2}{3}}$	$a^{\frac{1}{3}} \cdot a^{\frac{1}{3}}$
$(a^{\frac{1}{3}})^5$	Repeated mult^n $a^{\frac{1}{3}} \cdot a^{\frac{1}{3}} \cdot a^{\frac{1}{3}} \cdot a^{\frac{1}{3}} \cdot a^{\frac{1}{3}}$
$(a^5)^{\frac{1}{3}}$	Soln to $x^3=a^5$ (cube root of a^5)

POWER of POWER

Why $(a^{\frac{1}{3}})^5 = (a^5)^{\frac{1}{3}}$

Because $(a^{\frac{1}{3}})^5$ is a cube root of a^5

$$((a^{\frac{1}{3}})^5)^3 = a^{\frac{1}{3}} \cdot a^{\frac{1}{3}} \cdot a^{\frac{1}{3}} \cdot a^{\frac{1}{3}} \cdot a^{\frac{1}{3}} \cdot a^{\frac{1}{3}} \cdot a^{\frac{1}{3}} \cdot a^{\frac{1}{3}} \cdot a^{\frac{1}{3}} \cdot a^{\frac{1}{3}} \cdot a^{\frac{1}{3}} \cdot a^{\frac{1}{3}} \cdot a^{\frac{1}{3}} \cdot a^{\frac{1}{3}} \cdot a^{\frac{1}{3}} = a \cdot a \cdot a \cdot a \cdot a = a^5$$ ✓

So $(a^{\frac{1}{3}})^5 = (a^5)^{\frac{1}{3}} = a^{\frac{5}{3}}$

(OTHER PROPERTIES STILL HOLD, TOO.)