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GERRY-ME NDERING

Thus far we have examined several geometric measures of compactness as they relate to congressional districts. However, there is an additional geographic method of measuring whether or not congressional districts have been gerrymandered that we have not yet explored: meanderingness. Broadly speaking, meanderingness can be described in terms of being able to traverse a shape using straight lines. For example, polygons such as triangles and squares do not meander significantly, as a straight line can travel unimpeded between any two points in the shape. However, more complicated shapes are much harder to cross using straight lines, and thus we say that these shapes meander. (See Figure 1).

On a congressional level, certain districts have garnered attention over the years for how much they seem to meander throughout a state (See Figure 2). It seems apparent to the casual observer that such districts have been drawn with specific intent. Unfortunately, the appearance of gerrymandering is not enough to convince the Supreme Court that it has in fact taken place. Because of this, there has been a recent influx of mathematicians who have attempted to mathematically quantify meanderingness to lend credence to its potential judicial applications. In this chapter we will look at some of these attempts as well as the underlying geometry that makes it possible.

Figure 1: shape that is very difficult to cross using straight lines [4]

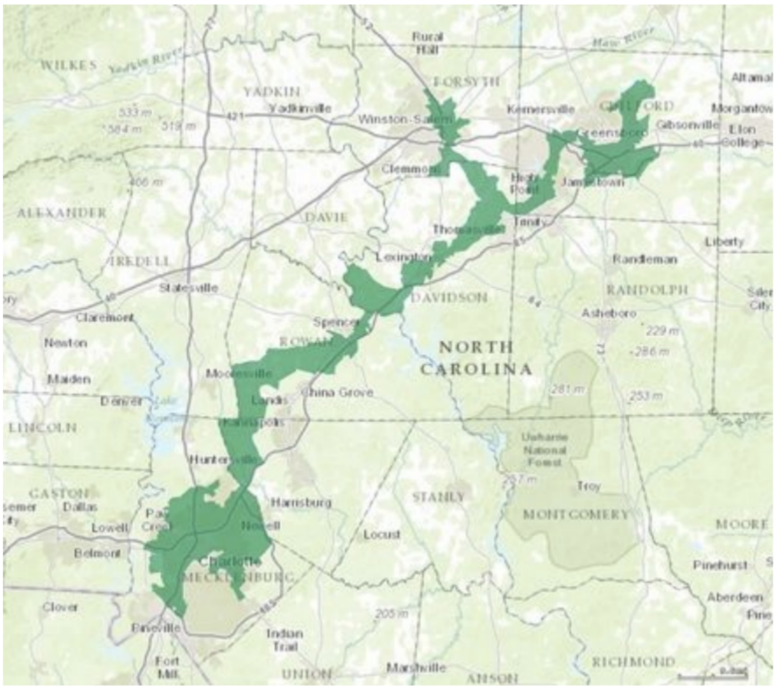

Figure 2: Former North Carolina District 12 [2]

3.1 HANDLEY'S MEANDERINGNESS TEST

ttempts to measure meanderingness– much like many of the other tests we have seen– arose after the Supreme Court communicated that they would be open to accepting mathematical measures of gerrymandering. In *Shaw v. Reno* (1993), the Court ruled that a citizen has the right under the Equal Protection Clause to challenge districts that were strangely shaped as potential racial gerrymanders [**Shaw**]. One of the first suggested approaches to measure meanderingness arose from *Johnson v. Miller*, a 1994 United States district court case in Georgia [**Johnson**]. In *Johnson v. Miller*, Georgia's eleventh congressional district was challenged, and Dr. Lisa Handley was called as an expert witness to propose a metric on how to classify a district as gerrymandered. Her measure is described in [2], and will be paraphrased here.

Dr. Handley's measure utilizes the fact that congressional districts are constructed using census blocks that are constructed by the United States Census Bureau. These blocks are small in area, sometimes only taking up a city block in urban areas. Prior to the 199 census, only select portions of the country were divided into blocks. However, beginning in 199 the Census Bureau began using census blocks for 1 % of its tabulation [3]. This enabled Dr. Handley to develop the following method:

Consider a district D, and a census block B with central point P. Beginning at P, draw a line segment straight upward until it intersects a boundary of D. The ray may intersect multiple boundaries, but we will focus just on the intersection closest to P.

Label this point $_1$. Repeat this process by shifting the line five degrees to the right, and label the new point of intersection $_2$ ($\angle\ _1P\ _2$ should be five degrees). Repeat this process until you have a collection of 72 points around P. Draw closed line segments $\overline{\ _1\ _2}, \overline{\ _2\ _3}, \ldots, \overline{\ _{71}\ _{72}}, \overline{\ _{72}\ _1}$, forming a simple polygon. We call this polygon a *coverage polygon* of D, denoted $C\ D)$. In theory, this coverage polygon approximates the portion of the congressional district that can be reached with straight lines emanating from P. Next, let $\ D)$ equal the area of the district and $\ C\ D))$ denote the area of the coverage polygon of D. Denote the total set of census blocks in this district $\mathcal{B}$. Repeat this process for every 25th census block in $\mathcal{B}$.

Definition 3.1.1. For any district D, the **meanderingness measure** of D is defined to be

$$\mu\ D) = \max_{B\in\mathcal{B}} \left. \frac{\ C\ D))}{\ D)} \right\}$$

Example 3.1.2. What do high and low values of μ indicate about a district's meanderingness? What are the bounds of μ?

High and low values of μ are somewhat counter intuitive. large value of μ indicates that the area of the largest coverage polygon is reasonably close to the area of the district, meaning that a majority of the district can be crossed with straight lines. This implies that the district does not meander significantly, and thus that it is not gerrymandered.

low value of μ indicates that the district twists and turns enough that no census block can accurately be used to estimate its area using straight lines. This implies that the district meanders and was therefore gerrymandered.

s it is impossible for a district or coverage polygon to have negative area the meanderingness measure is bounded below by zero. However, μ technically has no upper bound. This is due to the fact that coverage polygons are constructed using 72 points, and thus districts have the potential to have razor thin divots or tendrils that are missed by the coverage polygon. However, given the nature of census blocks and the construction of congressional districts, the upper bound of μ is functionally 1.

t its core, this measure is attempting to mathematically quantify how much of a congressional district can be crossed using straight lines. Returning to Figure 2, by inspection we can see that any coverage polygon of this district would fail to cover a significant portion of its area, resulting in a low meanderingness measure. s we conjectured that this district was gerrymandered, a low meanderingness measure would support this claim.

Unfortunately, several problems with this measure quickly arise. Firstly, why only use every 25th census block? It seems as though a large amount of potentially crucial data is lost when only a fraction of census blocks are used. The answer lies in computing power. The 199 census consisted of just over seven million census blocks, meaning that each of the 435 congressional districts comprised of an average of over 16, blocks each. Dr.