Staubli 机器人逆运动学通解

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列举 **RX60、RX160L、TX90** 机器人 DH 参数(基本涵盖了 Staubli 六轴机器人所有 DH 类型) :

注：此处已简化 GetDefaultDH()函数获取的 DH 参数　i=1,2,3,4,5,6

机器人型号	i	RZ(度)θ	X(mm) a	Z(mm) d	RX(度)α
Staubli RX60	1	θ1	0	0	-90
	2	θ2-90°	290	0	0
	3	θ3+90°	0	49	90
	4	θ4	0	310	-90
	5	θ5	0	0	90
	6	θ6	0	65	0

机器人型号	i	RZ(度)θ	X(mm) a	Z(mm) d	RX(度)α
RX160L	1	θ1	150	0	-90
	2	θ2-90°	825	0	0
	3	θ3+90°	0	0	90
	4	θ4	0	925	-90
	5	θ5	0	0	90
	6	θ6	0	110	0

机器人型号	i	RZ(度)θ	X(mm) a	Z(mm) d	RX(度)α
Staubli TX90	1	θ1	50	0	-90
	2	θ2-90°	425	0	0
	3	θ3+90°	0	50	90
	4	θ4	0	425	-90
	5	θ5	0	0	90
	6	θ6	0	100	0

写出 DH 参数通式：

机器人型号	i	RZ(度)θ	X(mm) a	Z(mm) d	RX(度)α
Staubli 机器人	1	θ1	a1	0	-90
	2	θ2-90°	a2	0	0
	3	θ3+90°	0	d3	90
	4	θ4	0	d4	-90
	5	θ5	0	0	90
	6	θ6	0	d6	0

把关节轴处的坐标系写成矩阵：

$$trsf_1 = Rz(\theta_1)\cdot Dx(a_1)\cdot Dz(d_1)\cdot Rx(\alpha_1) = \begin{bmatrix} c1 & 0 & -s1 & a1\cdot c1 \\ s1 & 0 & c1 & a1\cdot s1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$trsf_2 = Rz(\theta_2)\cdot Dx(a_2)\cdot Dz(d_2)\cdot Rx(\alpha_2) = \begin{bmatrix} s2 & c2 & 0 & a2\cdot s2 \\ -c2 & s2 & 0 & -a2\cdot c2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$trsf_3 = Rz(\theta_3)\cdot Dx(a_3)\cdot Dz(d_3)\cdot Rx(\alpha_3) = \begin{bmatrix} -s3 & 0 & c3 & 0 \\ c3 & 0 & s3 & 0 \\ 0 & 1 & 0 & d3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$trsf_4 = Rz(\theta_4)\cdot Dx(a_4)\cdot Dz(d_4)\cdot Rx(\alpha_4) = \begin{bmatrix} c4 & 0 & -s4 & 0 \\ s4 & 0 & c4 & 0 \\ 0 & -1 & 0 & d4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$trsf_5 = Rz(\theta_5)\cdot Dx(a_5)\cdot Dz(d_5)\cdot Rx(\alpha_5) = \begin{bmatrix} c5 & 0 & s5 & 0 \\ s4 & 0 & -c5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$trsf_6 = Rz(\theta_6)\cdot Dx(a_6)\cdot Dz(d_6)\cdot Rx(\alpha_6) = \begin{bmatrix} c6 & -s6 & 0 & 0 \\ s6 & c6 & 0 & 0 \\ 0 & 0 & 1 & d6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

已知 $P_1 = \{x_1, y_1, z_1, rx_1, ry_1, rz_1\}$ 代表 flange 记录的一点

$P_2 = \{x_2, y_2, z_2, rx_2, ry_2, rz_2\}$ 代表 flange 根部的一点与 P_1 在工具坐标系 Z 方向相差 d6

所以 $P_2 = appro(P_1, \{0, 0, -d6, 0, 0, 0\})$ 从 P_2 计算逆解会相对容易

如果用矩阵表达 P_2 与 P_1 关系：

$$\left[P_2\right] = \left[P_1\right]\cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -d6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

注：P_2 与 P_1 都为 4x4 矩阵，由 $\{x, y, z, rx, ry, rz\}$ 六个参数转化而来

解θ_1： $trsf_1^{-1}\cdot trsfP_2 = trsf_2\cdot trsf_3\cdot trsf_4\cdot trsf_5\cdot trsf_6$

$$trsf_1^{-1}\cdot trsfP_2 = \begin{bmatrix} \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & c1p_y - s1p_x \\ \cdots & \cdots & \cdots & \cdots \end{bmatrix}$$

$$trsf_2\cdot trsf_3\cdot trsf_4\cdot trsf_5\cdot trsf_6 = \begin{bmatrix} \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & d3 \\ \cdots & \cdots & \cdots & \cdots \end{bmatrix}$$

两边矩阵元素（3,4）相等得到：$c1p_y - s1p_x = d3$

三角恒等变换：

令 $p_y = A\sin\phi$

令 $p_x = A\cos\phi$

$A = \sqrt{p_x^2 + p_y^2}$

$\phi = a\tan 2(p_y, p_x)$

$As\phi c1 - Ac\phi s1 = d3$

$$s(\phi - \theta_1) = \frac{d3}{A}$$

$$c(\phi - \theta_1) = \pm\sqrt{1 - \left(\frac{d3}{A}\right)^2}$$

$$\phi - \theta_1 = a\tan 2\left(\frac{d3}{A}, \pm\sqrt{1 - \left(\frac{d3}{A}\right)^2}\right)$$

$$\theta_1 = a\tan 2(p_y, p_x) - a\tan 2\left(\frac{d3}{A}, \pm\sqrt{1 - \left(\frac{d3}{A}\right)^2}\right)$$

解θ_2： $trsf_2^{-1}\cdot trsf_1^{-1}\cdot trsfP_2 = trsf_3\cdot trsf_4\cdot trsf_5\cdot trsf_6$

$$trsf_2^{-1}\cdot trsf_1^{-1}\cdot trsfP_2 = \begin{bmatrix} \cdots & \cdots & \cdots & p_zc2 + p_xc1s2 + p_ys1c2 - a1s2 - a2 \\ \cdots & \cdots & \cdots & -p_zs2 + p_xc1c2 + p_ys1c2 - a1c2 \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{bmatrix}$$

$$trsf_3\cdot trsf_4\cdot trsf_5\cdot trsf_6 = \begin{bmatrix} \cdots & \cdots & \cdots & c3\cdot d4 \\ \cdots & \cdots & \cdots & s3\cdot d4 \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{bmatrix}$$

两边矩阵元素（1,4）和（2,4）分别相等

$$\begin{matrix} (1)\rightarrow \\ (2)\rightarrow \end{matrix}\begin{cases} p_zc2 + p_xc1s2 + p_ys1s2 - a1s2 - a2 = c3d4 \\ -p_zs2 + p_xc1c2 + p_ys1c2 - a1c2 = s3d4 \end{cases}$$

$$\begin{aligned}(1)\cdot s2 &\rightarrow \\ (2)\cdot c2 &\rightarrow\end{aligned}\begin{cases} p_z s2c2 + p_x c1\left(s2\right)^2 + p_y s1\left(s2\right)^2 - a1\left(s2\right)^2 - a2s2 = s2c3d4 \\ \quad -p_z s2c2 + p_x c1\left(c2\right)^2 + p_y s1\left(c2\right)^2 - a1\left(c2\right)^2 = c2s3d4 \end{cases}$$

$(1)\cdot s2 + (2)\cdot c2$

$p_x c1 + p_y s1 - a_1 = d4s23 + a2s2$

令 $B = p_x c1 + p_y s1 - a_1$

所以 $B = d4s23 + a2s2$

$$s23 = \frac{B - a2s2}{d4}$$

$$\begin{aligned}(1)\cdot c2 &\rightarrow \\ (2)\cdot s2 &\rightarrow\end{aligned}\begin{cases} p_z\left(c2\right)^2 + p_x c1c2s2 + p_y s1c2s2 - a1c2s2 - a2c2 = c2c3d4 \\ \quad -p_z\left(s2\right)^2 + p_x c1c2s2 + p_y s1c2s2 - a1c2s2 = s2s3d4 \end{cases}$$

$(1)\cdot c2\text{-}(2)\cdot s2$

$p_z - a2c2 = d4c23$

$$c23 = \frac{p_z - a2c2}{d4}$$

$\because \left(s23\right)^2 + \left(c23\right)^2 = 1$

$$\therefore \left(\frac{B - a2s2}{d4}\right)^2 + \left(\frac{P_z - a2c2}{d4}\right)^2 = 1$$

$$B^2 + 2B(a2)(s2) + (a2)^2(s2)^2 + p_z^2 - 2p_z a2c2 + (a2)^2(c2)^2 = (d4)^2$$

$$B(s2) + p_z(c2) = \frac{B^2 + p_z^2 + (a2)^2 - (d4)^2}{2(a2)}$$

三角恒等变换：

$B = C\sin\phi$

$p_z = C\cos\phi$

$C = \sqrt{B^2 + p_z^2}$

$\phi = a\tan 2(B, p_z)$

$$Cs\phi s2 + Cc\phi c2 = \frac{B^2 + p_z^2 + (a2)^2 - (d4)^2}{2(a2)}$$

$$c(\phi - \theta_2) = \frac{B^2 + p_z^2 + (a2)^2 - (d4)^2}{2(a2)C} = D$$

$s(\phi - \theta_2) = \pm\sqrt{1 - D^2}$

$\phi - \theta_2 = a\tan 2(\pm\sqrt{1 - D^2}, D)$

$\theta_2 = a\tan 2(B, p_z) - a\tan 2(\pm\sqrt{1 - D^2}, D)$

解θ_3 ：

$$\theta_2+\theta_3=a\tan 2(s23,c23)$$
$$\theta_3=a\tan 2(s23,c23)-\theta_2$$
$$\theta_3=a\tan 2(\frac{B-a2s2}{d4},\frac{p_z-a2c2}{d4})-\theta_2$$

解θ_5 ： $trsf_3^{-1}\cdot trsf_2^{-1}\cdot trsf_1^{-1}\cdot trsfP_2=trsf_4\cdot trsf_5\cdot trsf_6$

$$trsf_3^{-1}\cdot trsf_2^{-1}\cdot trsf_1^{-1}\cdot trsfP_2=\begin{bmatrix} \dots & \dots & -r33\cdot s23+r13\cdot c1\cdot c23+r23\cdot s1\cdot c23 & \dots \\ \dots & \dots & r23\cdot c1-r13\cdot s1 & \dots \\ r11\cdot c1\cdot s23+r31\cdot c23+r21\cdot s1\cdot s23 & r12\cdot c1\cdot s23+r32\cdot c23+r22\cdot s1\cdot s23 & r13\cdot c1\cdot s23+r33\cdot c23+r23\cdot s1\cdot s23 & \dots \\ \dots & \dots & \dots & \dots \end{bmatrix}$$

$$trsf_4\cdot trsf_5\cdot trsf_6=\begin{bmatrix} \dots & \dots & s5\cdot c4 & \dots \\ \dots & \dots & s5\cdot s4 & \dots \\ -s5\cdot c6 & s5\cdot s6 & c5 & \dots \\ \dots & \dots & \dots & \dots \end{bmatrix}$$

令 $E=-r33\cdot s23+r13\cdot c1\cdot c23+r23\cdot s1\cdot c23$

令 $F=r23\cdot c1-r13\cdot s1$

令 $G=r11\cdot c1\cdot s23+r31\cdot c23+r21\cdot s1\cdot s23$

令 $H=r12\cdot c1\cdot s23+r32\cdot c23+r22\cdot s1\cdot s23$

令 $I=r13\cdot c1\cdot s23+r33\cdot c23+r23\cdot s1\cdot s23$

$$c5=I$$
$$s5=\pm\sqrt{1-I^2}$$
$$\theta_5=a\tan 2(s5,c5)$$
$$\theta_5=a\tan 2(\pm\sqrt{1-I^2},I)$$

解θ_4 ：

$$\begin{cases} s5\cdot c4=E \\ s5\cdot s4=F \end{cases}$$
$$s4=\frac{F}{s5}$$
$$c4=\frac{E}{s5}$$
$$\theta_4=a\tan 2(s4,c4)$$
$$\theta_4=a\tan 2(\frac{F}{s5},\frac{E}{s5})$$

解θ_6：

$$\begin{cases} s5 \cdot s6 = H \\ -s5 \cdot c6 = G \end{cases}$$

$$s6 = \frac{H}{s5}$$

$$c6 = -\frac{G}{s5}$$

$$\theta_6 = a\tan 2(s6, c6)$$

$$\theta_6 = a\tan 2(\frac{H}{s5}, \frac{G}{s5})$$

心得：

一个工具坐标系{x,y,z,rx,ry,rz}，理论上有 8 种手臂的姿态可以到达，还需要肩部、肘部、腕部来定义。Shoulder（Lefty、Righty） Elbow（Positive、Negative） Wrist（Positive、Negative）。

问题：

在算逆解的时候，8 种手臂形态对应的关节值已经求出且验证，但存在一些问题。

比如 4 轴运动范围为-270°—270°，逆解的时候 matlab 算出的关节值是 120°，Val3 算出的是-240°。虽然手臂形态是一样的。