

Assignment – 4

- Heart Start produces automated external defibrillators (AEDs) in each of two different plants (A and B). The unit production costs and monthly production capacity of the two plants are indicated in the table below. The AEDs are sold through three wholesalers. The shipping cost from each plant to the warehouse of each wholesaler along with the monthly demand from each wholesaler are also indicated in the table. How many AEDs should be produced in each plant, and how should they be distributed to each of the three wholesaler warehouses so as to minimize the combined cost of production and shipping?

	Unit Shipping Cost			Unit	Monthly
	Warehouse 1	Warehouse 2	Warehouse 3	Production Cost	Production Capacity
Plant A	\$22	\$14	\$30	\$600	100
Plant B	\$16	\$20	\$24	\$625	120
Monthly Demand	80	60	70		

Formulate and solve this transportation problem using *lpsolve*, or any other equivalent library in R.

SOLUTION

From the given problem, the minimized cost of production and shipping together is formulated as below:

Objective Function is given by

$$Z_{\min} = 622 X_{11} + 614 X_{12} + 630 X_{13} + 0 X_{14} + 641 X_{21} + 645 X_{22} + 649 X_{23} + 0 X_{24}$$

We have 6 decision variables and 2 dummy variables are to be considered to equalize demand and supply

Constraints:

$$\begin{aligned} X_{11} + X_{12} + X_{13} + X_{14} &= 100 \\ X_{21} + X_{22} + X_{23} + X_{24} &= 120 \end{aligned} \quad \left. \vphantom{\begin{aligned} X_{11} + X_{12} + X_{13} + X_{14} &= 100 \\ X_{21} + X_{22} + X_{23} + X_{24} &= 120 \end{aligned}} \right\} \text{Supply Constraints}$$

$$\begin{aligned} X_{11} + X_{21} &= 80 \\ X_{12} + X_{22} &= 60 \\ X_{13} + X_{23} &= 70 \\ X_{14} + X_{24} &= 10 \end{aligned} \quad \left. \vphantom{\begin{aligned} X_{11} + X_{21} &= 80 \\ X_{12} + X_{22} &= 60 \\ X_{13} + X_{23} &= 70 \\ X_{14} + X_{24} &= 10 \end{aligned}} \right\} \text{Demand Constraints}$$

Where, $X_{ij} \geq 0$ (i (Plant) = 1,2 and j (warehouses) = 1,2,3,4)

2. **Oil Distribution** Texxon Oil Distributors, Inc., has three active oil wells in a west Texas oil field. Well 1 has a capacity of 93 thousand barrels per day (TBD), Well 2 can produce 88 TBD, and Well 3 can produce 95 TBD. The company has five refineries along the Gulf Coast, all of which have been operating at stable demand levels. In addition, three pump stations have been built to move the oil along the pipelines from the wells to the refineries. Oil can flow from any one of the wells to any of the pump stations, and from any one of the pump stations to any of the refineries, and Texxon is looking for a minimum cost schedule. The refineries' requirements are as follows.

Refinery	R1	R2	R3	R4	R5
Requirement (TBD)	30	57	48	91	48

The company cost accounting system recognizes charges by the segment of pipeline that is used. These daily costs are given in the tables below, in dollars per thousand barrels.

To		Pump A	Pump B	Pump C
From	Well 1	1.52	1.60	1.40
	Well 2	1.70	1.63	1.55
	Well 3	1.45	1.57	1.30

To		R1	R2	R3	R4	R5
From	Pump A	5.15	5.69	6.13	5.63	5.80
	Pump B	5.12	5.47	6.05	6.12	5.71
	Pump C	5.32	6.16	6.25	6.17	5.87

- 1) What is the minimum cost of providing oil to the refineries? Which wells are used to capacity in the optimal schedule? Formulation of the problem is enough.
- 2) Show the network diagram corresponding to the solution in (a). That is, label each of the arcs in the solution and verify that the flows are consistent with the given information.

SOLUTION

1) As given in the problem, the minimum objective function is formulated below:

$$\begin{aligned} Z_{\min} = & 1.52 X_{1A} + 1.60 X_{1B} + 1.40 X_{1C} + 1.70 X_{2A} + 1.63 X_{2B} + 1.55 X_{2C} + 1.45 X_{3A} + \\ & 1.57 X_{3B} + 1.30 X_{3C} + 5.15 X_{A4} + 5.12 X_{B4} + 5.32 X_{C4} + 5.69 X_{A5} + 5.47 X_{B5} + 6.16 X_{C5} + \\ & 6.13 X_{A6} + 6.05 X_{B6} + 6.25 X_{C6} + 5.63 X_{A7} + 6.12 X_{B7} + 6.17 X_{C7} + 5.80 X_{A8} + 5.71 X_{B8} + \\ & 5.87 X_{C8} \end{aligned}$$

Constraints:

$X_{1A} + X_{1B} + X_{1C} = 93$ $X_{2A} + X_{2B} + X_{2C} = 88$ $X_{3A} + X_{3B} + X_{3C} = 95$	}	Supply Constraints
$X_{A4} + X_{B4} + X_{C4} = 30$ $X_{A5} + X_{B5} + X_{C5} = 57$ $X_{A6} + X_{B6} + X_{C6} = 48$ $X_{A7} + X_{B7} + X_{C7} = 91$ $X_{A8} + X_{B8} + X_{C8} = 48$ $X_{A9} + X_{B9} + X_{C9} = 2$	}	Demand Constraints

Constraints from pumps to refinery,

$$X_{1A} + X_{2A} + X_{3A} = X_{A4} + X_{A5} + X_{A6} + X_{A7} + X_{A8} + X_{A9}$$

$$X_{1B} + X_{2B} + X_{3B} = X_{B4} + X_{B5} + X_{B6} + X_{B7} + X_{B8} + X_{B9}$$

$$X_{1C} + X_{2C} + X_{3C} = X_{C4} + X_{C5} + X_{C6} + X_{C7} + X_{C8} + X_{C9}$$

Where, $X_{ij} \geq 0$: i (pumps) = (A, B, C), j = 1,2,3(wells), 4:9(refineries)

Using Ipsolve the optimal solution is **1966.68**.

Well 3 has used to the capacity in the optimal schedule.

2: Network Diagram

