Q. Back Savers is a company that produces backpacks primarily for students. They are considering offering some combination of two different models—the Collegiate and the Mini. Both are made out of the same rip-resistant nylon fabric. Back Savers has a long-term contract with a supplier of the nylon and receives a 5000 square-foot shipment of the material each week. Each Collegiate requires 3 square feet while each Mini requires 2 square feet. The sales forecasts indicate that at most 1000 Collegiates and 1200 Minis can be sold per week. Each Collegiate requires 45 minutes of labor to produce and generates a unit profit of $32. Each Mini requires 40 minutes of labor and generates a unit profit of $24. Back Savers has 35 laborers that each provides 40 hours of labor per week. Management wishes to know what quantity of each type of backpack to produce per week.

   a. Clearly define the decision variables
   b. What is the objective function?
   c. What are the constraints?
   d. Write down the full mathematical formulation for this LP problem.

ANSWER

a. **Clearly define the decision variables.**

Number of Collegiate backpacks made per week and **Number of Mini backpacks made per week** are the two decision variables with which we can maximize the profit of back savers.

Let us assume two variables to assign decision variables,

X = Total number of Collegiate produced per week.
Y = Total number of Mini produced per week.

b. **What is the objective function?**

Objective function is to maximize the profits of the company. We obtain objective function by taking in count of profit gained by each backpack.

Objective function can be given by equation

$$\mathbf{Z_{max} = 32\ X + 24\ Y}$$ (Z is the maximum profit obtained)

c. **What are the constraints?**

There are two constraints in the problem than can be imposed on decision variables used in equation.

1. Material availability: -

X need 3 sq feet for one quantity and Y need 2 sq feet for one quantity and they have supplies of 5000 sq feet per week.

$$\mathbf{3\ X + 2\ Y \leq 5000}$$

2. Working hours of labors: -

35 laborers can work for 40 hours each in a week. X need 45 mins and Y needs 40 mins for one quantity.

$$\mathbf{45\ X + 40\ Y \leq (35*40) * 60}$$

d. **Write down the full mathematical formulation for this LP problem.**

X: Number of colligate made per week.
Y: Number of Mini made per week.
Zmax: Maximum profit per week.

$$\mathbf{Zmax = 32\ X + 24\ Y}$$

Constraints:

$$\mathbf{3\ X + 2\ Y \leq 5000}$$
$$\mathbf{45\ X + 40\ Y \leq (35*40) * 60}$$

And

$$\mathbf{0 \leq X \leq 1000}$$
$$\mathbf{0 \leq Y \leq 1200}$$