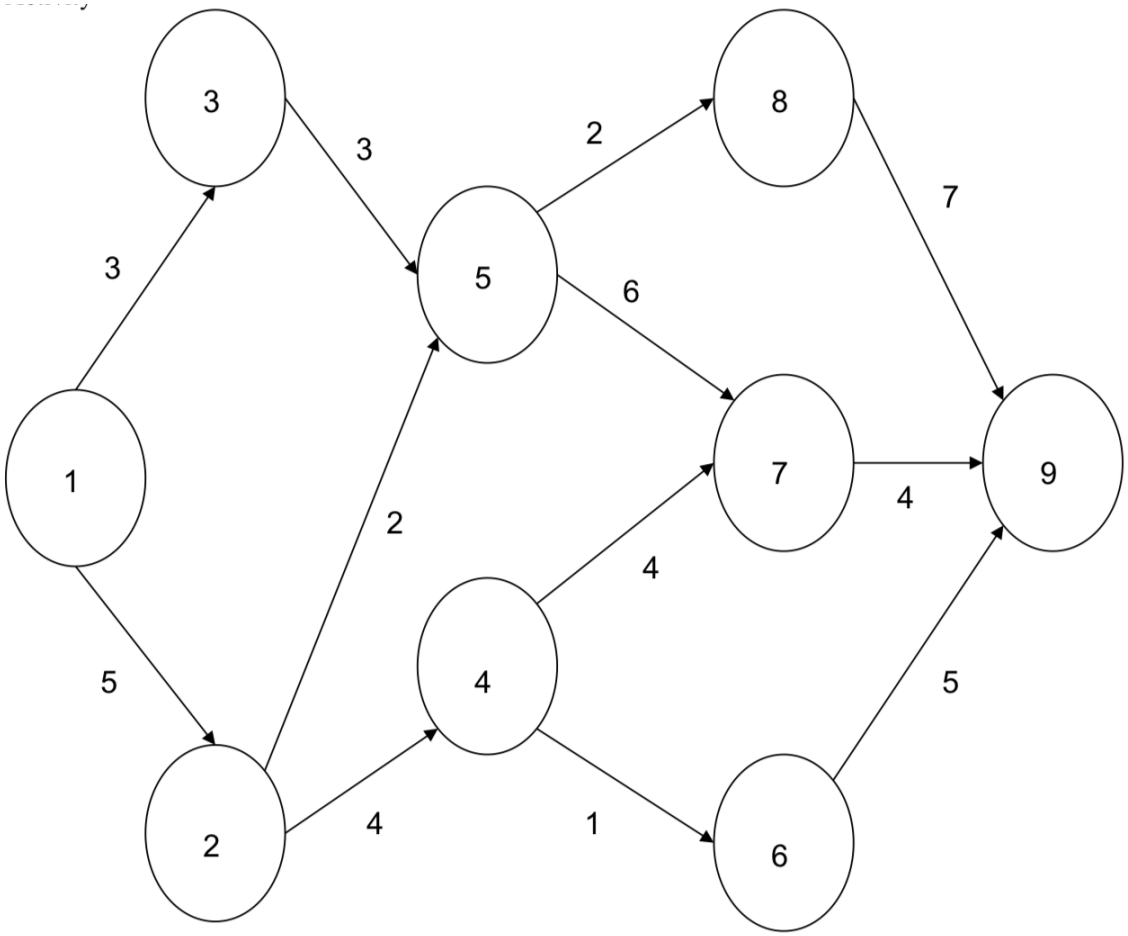
Assignment – 5

1. Consider the following activity-on-arc project network, where the 12 arcs (arrows) represent the 12 activities (tasks) that must be performed to complete the project and the network displays the order in which the activities need to be performed. The number next to each arc (arrow) is the time required for the corresponding activity. Consider the problem of finding the longest path (the largest total time) through this network from start (node 1) to finish (node 9), since the longest path is the critical path.



Formulate and solve the binary integer programming (BIP) model for this problem using library *lpsolve* or equivalent in R.

SOLUTION

The longest path is the critical path and objective function is given by

$$\mathbf{Zmax= 3\,X_{13} + 5\,X_{12} + 3\,X_{35} + 2\,X_{25} + 2\,X_{58} + 4\,X_{24} + 6\,X_{57} + 4\,X_{47} + 1\,X_{46} + 7\,X_{89} + 4\,X_{79} + 5\,X_{69}}$$

Starting node:

$$\mathbf{3\,X_{13} + 5\,X_{12} = 1}$$

Intermediate nodes:

$$\mathbf{5\,X_{12} - 2\,X_{25} - 4\,X_{24} = 0}$$

$$\mathbf{3\,X_{13} - 3\,X_{35} = 0}$$

$$\mathbf{4\,X_{24} - 1\,X_{46} - 4\,X_{47} = 0}$$

$$\mathbf{3\,X_{35} + 2\,X_{25} - 2\,X_{58} - 6\,X_{57} = 0}$$

$$\mathbf{1\,X_{46} - 5\,X_{69} = 0}$$

$$\mathbf{6\,X_{57} + 4\,X_{47} - 4\,X_{79} = 0}$$

$$\mathbf{2\,X_{58} - 7\,X_{89} = 0}$$

Ending node:

$$\mathbf{7\,X_{89} + 4\,X_{79} + 5\,X_{69} = 1}$$

Where $\boldsymbol{X_{IJ}}$ are binary

2. **Selecting an Investment Portfolio** An investment manager wants to determine an optimal portfolio for a wealthy client. The fund has $2.5 million to invest, and its objective is to maximize total dollar return from both growth and dividends over the course of the coming year. The client has researched eight high-tech companies and wants the portfolio to consist of shares in these firms only. Three of the firms (S1 – S3) are primarily software companies, three (H1–H3) are primarily hardware companies, and two (C1–C2) are internet consulting companies. The client has stipulated that no more than 40 percent of the investment be allocated to any one of these three sectors. To assure diversification, at

least $100,000 must be invested in each of the eight stocks. Moreover, the number of shares invested in any stock must be a multiple of 1000.

The table below gives estimates from the investment company's database relating to these stocks. These estimates include the price per share, the projected annual growth rate in the share price, and the anticipated annual dividend payment per share.

	Stock							
	S1	S2	S3	H1	H2	H3	C1	C2
Price per share	$40	$50	$80	$60	$45	$60	$30	$25
Growth rate	0.05	0.10	0.03	0.04	0.07	0.15	0.22	0.25
Dividend	$2.00	$1.50	$3.50	$3.00	$2.00	$1.00	$1.80	$0.00

1) Determine the maximum return on the portfolio. What is the optimal number of shares to buy for each of the stocks? What is the corresponding dollar amount invested in each stock?
2) Compare the solution in which there is no integer restriction on the number of shares invested. By how much (in percentage terms) do the integer restrictions alter the value of the optimal objective function? By how much (in percentage terms) do they alter the optimal investment quantities?

SOLUTION

2a:

According to the problem returns can be given by

Returns = (Price per share) * (Growth rate of share) + (Dividend per share)

Hence the objective function is

$$\mathbf{Zmax = 4 X_{S1} + 6.5 X_{S2} + 5.9 X_{S3} + 5.4 X_{H1} + 5.15 X_{H2} + 10 X_{H3} + 8.4 X_{C1} + 6.25 X_{C2}}$$

Constraints,

Investment constraint:

$$\mathbf{40 X_{S1} + 50 X_{S2} + 80 X_{S3} + 60 X_{H1} + 45 X_{H2} + 60 X_{H3} + 30 X_{C1} + 25 X_{C2} <= 2500000}$$

The number of shares invested in any stock must be a multiple of 1000

$1000\ X_{S1} >= 0; 1000\ X_{S2} >= 0; 1000\ X_{S3} >= 0$

$1000\ X_{H1} >= 0; 1000\ X_{H2} >= 0; 1000\ X_{H3} >= 0$

$1000\ X_{C1} >= 0; 1000\ X_{C2} >= 0$

At least $100,000 must be invested in each of the eight stocks

$40\ X_{S1} >= 100000; 50\ X_{S2} >= 100000; 80\ X_{S3} >= 100000;$

$60\ X_{H1} >= 100000; 45\ X_{H2} >= 100000; 60\ X_{H3} >= 100000;$

$30\ X_{C1} >= 100000; 25\ X_{C2} >= 100000$

No more than 40 percent of the investment constraints

$40\ X_{S1} + 50\ X_{S2} + 80\ X_{S3} <= 1000000$

$60\ X_{H1} + 45\ X_{H2} + 60\ X_{H3} <= 1000000$

$30\ X_{C1} + 25\ X_{C2} <= 1000000$

Where ***X_{SJ}, X_{HJ}, X_{CJ}*** **$>= 0$** are integers.

Using lpsolve with integer restriction we get the objective function,

maximum returns as **487145.2**

number of stocks are

S1= **2500,** S2= **6000,** S3= **1250;**

H1= **1667,** H2= **2223,** H3= **3332;**

C1= **30000,** C2= **4000;**

The amount invested in each stock

S1= **100000**, S2= **300000**, S3= **100000**;

H1= **100020**, H2= **100035**, H3= **799920**;

C1= **900000**, C2= **100000**;

2b:

Using lpsolve without integer restriction we get the objective function,

maximum returns = **487152.8**

number of stocks:

S1= **2500.0,** S2= **6000.0,** S3= **1250.0;**

H1= **1667.667,** H2= **2222.222,** H3= **13333.333;**

C1= **30000.0,** C2= **4000.0;**

The amount invested in each stock

S1= **100000**, S2= **300000**, S3= **100000**;

H1= **100000**, H2= **100000**, H3= **800000**;

C1= **900000**, C2= **100000**;

the integer restrictions alter the value of the optimal objective function by **0.00156**