1. **(Computer Center Staffing)** You are the Director of the Computer Center for Gaillard College and responsible for scheduling the staffing of the center. It is open from 8 am until midnight. You have monitored the usage of the center at various times of the day and determined that the following numbers of computer consultants are required.

|  |  |
| --- | --- |
| Time of day | Minimum number of consultants required to be on duty |
| 8 am–noon | 4 |
| Noon–4 pm | 8 |
| 4 am–8 pm | 10 |
| 8 am–midnight | 6 |

Two types of computer consultants can be hired: full-time and part-time. The full-time consultants work for eight consecutive hours in any of the following shifts: morning (8 am – 4 pm), afternoon (noon – 8 pm), and evening (4 pm – midnight). Full-time consult- ants are paid $14 per hour.

Part-time consultants can be hired to work any of the four shifts listed in the table. Part-time consultants are paid $12 per hour. An additional requirement is that during every time period, at least one full-time consultant must be on duty for every part-time consultant on duty.

1. Determine a minimum-cost staffing plan for the center. In your solution, how many consultants will be paid to work full time and how many will be paid to work part time? What is the minimum cost?
2. After thinking about this problem for a while, you have decided to recognize meal breaks explicitly in the scheduling of full-time consultants. In particular, full-time consultants are entitled to a one-hour lunch break during their eight-hour shift. In addition, employment rules specify that the lunch break can start after three hours of work or after four hours of work, but those are the only alternatives. Part-time consultants do not receive a meal break. Under these conditions, find a minimum-cost staffing plan. What is the minimum cost?

Hint: for this problem, you only need to formulate the LP problem without solving it.

**SOLUTION**

**a)**

Xi = Number of full-time workers working in three shifts (8am - 4pm), (noon – 8pm), (4pm – 8pm) where i = 1,2,3.

Yj= Number of part-time workers working in four shifts (8am - noon), (noon – 4pm), (4pm – 8pm), (8pm - midnight) where j = 1,2,3,4.

Amount paid to full time worker per hour = $14

Amount paid to part time worker per hour = $12

Full time worker work for 8 hrs and part time worker work for 4 hrs

Zmin = Minimum cost required to pay for workers.

**Zmin = 8\*14\*(X1 + X2 + X3) + 4\*12\*(Y1 + Y2 + Y3 + Y4).**

Constraints

X1 + Y1 ≥ 4

X1 + X2 + Y2≥ 8 4,8,10,6 are the minimum required worker to work in four

X2 + X3 + Y3 ≥ 10 shifts (8am - noon), (noon – 4pm), (4pm – 8pm),

X3 + Y4 ≥ 6 (8pm - midnight)

X1 ≥ Y1

X1 + X2 ≥ Y2 Constraints that one full time should always be there for

X2 + X3 ≥ Y3 part time.

X3 ≥ Y4

And

Xi ≥ 0, Yj ≥ 0.

Since, cost of part time workers is less than full time workers, we should try to take more

number of part time workers.

Y1 = 2, Y2 = 4, Y3 = 5, Y4 = 3 and X1 = 2, X2 = 2, X3 = 3

**Zmin = 112\*(7) + 48\*(14) = 1456.**

Full time workers = 7 and Part time workers = 14.

**b).**

As per the problem we need to give 1-hour break to full timers for every three or four hours

and part timers will have no break. So, we must remove 1 hr cost of full timers in three shifts

from the above cost function to obtain new minimum cost function.

Therefore, minimum cost function is given by

**Zmin1 = 8\*14\*(X1 + X2 + X3) – 14\*(X1 + X2 + X3) + 4\*12\*(Y1 + Y2 + Y3 + Y4).**

Constraints remain same as above.

Zmin1 = 112\*(7) – 14\*(7) + 48\*(14) = 1358.

Zmin – Zmin1 = 1456 – 1358

= 98.

We save $98 by giving breaks to Full time workers.

2. Consider the problem from the previous assignment.

Back Savers is a company that produces backpacks primarily for students. They are considering offering some combination of two different models—the Collegiate and the Mini. Both are made out of the same rip-resistant nylon fabric. Back Savers has a long-term contract with a supplier of the nylon and receives a 5000 square-foot shipment of the material each week. Each Collegiate requires 3 square feet while each Mini requires 2 square feet. The sales forecasts indicate that at most 1000 Collegiates and 1200 Minis can be sold per week. Each Collegiate requires 45 minutes of labor to produce and generates a unit profit of $32. Each Mini requires 40 minutes of labor and generates a unit profit of $24. Back Savers has 35 laborers that each provides 40 hours of labor per week. Management wishes to know what quantity of each type of backpack to produce per week. Solve this problem graphically.

**SOLUTION**

Please find the PDF file attached for the solution to above problem.

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3) (**Weigelt Production)** The Weigelt Corporation has three branch plants with excess production capacity. Fortunately, the corporation has a new product ready to begin production, and all three plants have this capability, so some of the excess capacity can be used in this way. This product can be made in three sizes--large, medium, and small--that yield a net unit profit of $420, $360, and $300, respectively. Plants 1, 2, and 3 have the excess capacity to produce 750, 900, and 450 units per day of this product, respectively, regardless of the size or combination of sizes involved.  
 The amount of available in-process storage space also imposes a limitation on the production rates of the new product. Plants 1, 2, and 3 have 13,000, 12,000, and 5,000 square feet, respectively, of in-process storage space available for a day's production of this product. Each unit of the large, medium, and small sizes produced per day requires 20, 15, and 12 square feet, respectively.  
 Sales forecasts indicate that if available, 900, 1,200, and 750 units of the large, medium, and small sizes, respectively, would be sold per day.  
 At each plant, some employees will need to be laid off unless most of the plant’s excess production capacity can be used to produce the new product. To avoid layoffs if possible, management has decided that the plants should use the same percentage of their excess capacity to produce the new product.  
 Management wishes to know how much of each of the sizes should be produced by each of the plants to maximize profit.

* 1. Define the decision variables
  2. Formulate a linear programming model for this problem.
  3. Solve the problem using *lpsolve*, or any other equivalent library in R.

**SOLUTION**

**a. Define the decision variables.**

Xij = Is the amount produced in each of the sizes “ j ” by each of the plant “ i ” in one day.

where j = l, m, s and i = 1, 2,3

Decision Variables are given by

X1l decision variables for plant 1 for different sizes of large(l),

X1m medium(m), small(s).

X1s

X2l decision variables for plant 2 for different sizes of large(l),

X2m medium(m), small(s).

X2s

X3l decision variables for plant 3 for different sizes of large(l),

X3m medium(m), small(s).

X3s

**b. Formulate a linear programming model for this problem.**

Zmax = max profit obtained in producing each sizes of each plants in a day.

Objective function with the above decision variables for the given problem can given by

**Zmax = 420\*(X1l + X2l + X3l) + 360\*(X1m + X2m + X3m) + 300\*(X1s + X2s + X3s)**

$420, $360, $300 are unit profits obtained by three sizes of plants large, medium, small respectively.

Constraints for Max capacity:

X1l + X1m + X1s ≤ 750

X2l + X2m + X2s ≤ 900

X3l + X3m + X3s ≤ 450

Constraints for Storage space:

20\*X1l + 15\*X1m + 12\*X1s ≤ 13000

20\*X2l + 15\*X2m + 12\*X2s ≤ 12000

20\*X3l + 15\*X3m + 12\*X3s ≤ 5000

Constraints for Same percentage of capacity:

900\*(X1l + X1m + X1s) – 750\*(X2l + X2m + X2s) = 0 any two of these

450\*(X2l + X2m + X2s) – 900\*(X3l + X3m + X3s) = 0 can be used.

450\*(X1l + X1m + X1s) – 750\*(X3l + X3m + X3s) = 0

Sales constraints:

X1l + X2l + X3l ≤ 900

X1m + X2m + X3m ≤ 1200

X1s + X2s + X3s ≤ 750

And Xij ≥ 0 where i = 1, 2, 3 and j = l, m, s