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# *Platelet*

Team Reference Material  
(unlimited version)

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# Chapter 1

## Graph Theory

### 1.1 2-SAT

### 1.2 双连通分量

#### 1.2.1 点双连通分量

#### 1.2.2 边双连通分量

### 1.3 K 短路 (lhy)

```
1  const int MAXNODE = MAXN + MAXM * 2;
2
3  bool used[MAXN];
4  int n, m, cnt, S, T, Kth, N, TT;
5  int rt[MAXN], seq[MAXN], adj[MAXN], from[MAXN], dep[MAXN];
6  LL dist[MAXN], w[MAXM], ans[MAXK];
7
8  struct GivenEdge{
9      int u, v, w;
10     GivenEdge() {}
11     GivenEdge(int _u, int _v, int _w) : u(_u), v(_v), w(_w){};
12 }edge[MAXM];
13
14 struct Edge{
15     int v, nxt, w;
16     Edge() {}
17     Edge(int _v, int _nxt, int _w) : v(_v), nxt(_nxt), w(_w) {};
18 }e[MAXM];
19
20 inline void addedge(int u, int v, int w)
21 {
22     e[++cnt] = Edge(v, adj[u], w); adj[u] = cnt;
23 }
24
25 void dij(int S)
26 {
27     for(int i = 1; i <= N; i++)
28     {
29         dist[i] = INF;
30         dep[i] = 0x3f3f3f3f;
31         used[i] = false;
32         from[i] = 0;
33     }
```

```

29 static priority_queue<pair<LL, int>, vector<pair<LL, int> >, greater<pair<LL, int> > > hp;
30 while(!hp.empty())hp.pop();
31 hp.push(make_pair(dist[S] = 0, S));
32 dep[S] = 1;
33 while(!hp.empty())
34 {
35     pair<LL, int> now = hp.top();
36     hp.pop();
37     int u = now.second;
38     if(used[u])continue;
39     else used[u] = true;
40     for(int p = adj[u]; p; p = e[p].nxt)
41     {
42         int v = e[p].v;
43         if(dist[u] + e[p].w < dist[v])
44         {
45             dist[v] = dist[u] + e[p].w;
46             dep[v] = dep[u] + 1;
47             from[v] = p;
48             hp.push(make_pair(dist[v], v));
49         }
50     }
51 }
52 for(int i = 1; i <= m; i++) w[i] = 0;
53 for(int i = 1; i <= N; i++)
54     if(from[i])w[from[i]] = -1;
55 for(int i = 1; i <= m; i++)
56 {
57     if(~w[i] && dist[edge[i].u] < INF && dist[edge[i].v] < INF)
58     {
59         w[i] = -dist[edge[i].u] + (dist[edge[i].v] + edge[i].w);
60     }
61     else
62     {
63         w[i] = -1;
64     }
65 }
66 }

67 inline bool cmp_dep(int p, int q)
68 {
69     return dep[p] < dep[q];
70 }

71 struct Heap{
72     LL key;
73     int id, lc, rc, dist;
74     Heap() {}
75     Heap(LL k, int i, int l, int r, int d) : key(k), id(i), lc(l), rc(r), dist(d) {};
76     inline void clear()
77     {
78         key = 0;
79         id = lc = rc = dist = 0;
80     }
81 }hp[MAXNODE];

82 inline int merge_simple(int u, int v)
83 {
84     if(!u)return v;
85     if(!v)return u;
86     if(hp[u].key > hp[v].key)

```

```

87     {
88         swap(u, v);
89     }
90     hp[u].rc = merge_simple(hp[u].rc, v);
91     if(hp[hp[u].lc].dist < hp[hp[u].rc].dist)
92     {
93         swap(hp[u].lc, hp[u].rc);
94     }
95     hp[u].dist = hp[hp[u].rc].dist + 1;
96     return u;
97 }

98 inline int merge_full(int u, int v)
99 {
100     if(!u) return v;
101     if(!v) return u;
102     if(hp[u].key > hp[v].key)
103     {
104         swap(u, v);
105     }
106     int nownode = ++cnt;
107     hp[nownode] = hp[u];
108     hp[nownode].rc = merge_full(hp[nownode].rc, v);
109     if(hp[hp[nownode].lc].dist < hp[hp[nownode].rc].dist)
110     {
111         swap(hp[nownode].lc, hp[nownode].rc);
112     }
113     hp[nownode].dist = hp[hp[nownode].rc].dist + 1;
114     return nownode;
115 }

116 priority_queue<pair<LL, int>, vector<pair<LL, int> >, greater<pair<LL, int> > > Q;

117 int main()
118 {
119     while(scanf("%d%d", &n, &m) != EOF)
120     {
121         scanf("%d%d%d", &S, &T, &Kth, &TT);
122         for(int i = 1; i <= m; i++)
123         {
124             int u, v, w;
125             scanf("%d%d%d", &u, &v, &w);
126             edge[i] = {u, v, w};
127         }
128         N = n;
129         memset(adj, 0, sizeof(*adj) * (N + 1));
130         cnt = 0;
131         for(int i = 1; i <= m; i++)
132             addedge(edge[i].v, edge[i].u, edge[i].w);
133         dijkstra(T);
134         if(dist[S] > TT)
135         {
136             puts("Whitesnake!");
137             continue;
138         }
139         for(int i = 1; i <= N; i++)
140             seq[i] = i;
141         sort(seq + 1, seq + N + 1, cmp_dep);

142         cnt = 0;
143         memset(adj, 0, sizeof(*adj) * (N + 1));

```

```

144     memset(rt, 0, sizeof(*rt) * (N + 1));
145     for(int i = 1; i <= m; i++)
146         addedge(edge[i].u, edge[i].v, edge[i].w);
147     rt[T] = cnt = 0;
148     hp[0].dist = -1;
149     for(int i = 1; i <= N; i++)
150     {
151         int u = seq[i], v = edge[from[u]].v;
152         rt[u] = 0;
153         for(int p = adj[u]; p; p = e[p].nxt)
154         {
155             if(-w[p])
156             {
157                 hp[++cnt] = Heap(w[p], p, 0, 0, 0);
158                 rt[u] = merge_simple(rt[u], cnt);
159             }
160         }
161         if(i == 1) continue;
162         rt[u] = merge_full(rt[u], rt[v]);
163     }
164     while(!Q.empty()) Q.pop();
165     Q.push(make_pair(dist[S], 0));
166     edge[0].v = S;
167     for(int kth = 1; kth <= Kth; kth++)
168     {
169         if(Q.empty())
170         {
171             ans[kth] = -1;
172             continue;
173         }
174         pair<LL, int> now = Q.top(); Q.pop();
175         ans[kth] = now.first;
176         int p = now.second;
177         if(hp[p].lc)
178         {
179             Q.push(make_pair(+hp[hp[p].lc].key + now.first - hp[p].key, hp[p].lc));
180         }
181         if(hp[p].rc)
182         {
183             Q.push(make_pair(+hp[hp[p].rc].key + now.first - hp[p].key, hp[p].rc));
184         }
185         if(rt[edge[hp[p].id].v])
186         {
187             Q.push(make_pair(hp[rt[edge[hp[p].id].v]].key + now.first, rt[edge[hp[p].id].v]));
188         }
189     }
190     if(ans[Kth] == -1 || ans[Kth] > TT)
191     {
192         puts("Whitesnake!");
193     }
194     else
195     {
196         puts("Yareyaredawa");
197     }
198 }
199 }

```



## 1.4 最大团

## 1.5 一般图最大匹配

## 1.6 树

### 1.6.1 虚树

### 1.6.2 矩阵树定理

### 1.6.3 点分治

### 1.6.4 Prufer 编码

### 1.6.5 Link-Cut Tree (ct)

```

1 struct Node *null;
2 struct Node {
3     Node *ch[2], *fa, *pos;
4     int val, mn, l, len; bool rev;
5     // min_val in chain
6     inline bool type()
7     {
8         return fa -> ch[1] == this;
9     }
10    inline bool check()
11    {
12        return fa -> ch[type()] == this;
13    }
14    inline void pushup()
15    {
16        pos = this; mn = val;
17        ch[0] -> mn < mn ? mn = ch[0] -> mn, pos = ch[0] -> pos : 0;
18        ch[1] -> mn < mn ? mn = ch[1] -> mn, pos = ch[1] -> pos : 0;
19        len = ch[0] -> len + ch[1] -> len + 1;
20    }
21    inline void pushdown()
22    {
23        if (rev)
24        {
25            ch[0] -> rev ^= 1;
26            ch[1] -> rev ^= 1;
27            std::swap(ch[0], ch[1]);
28            rev ^= 1;
29        }
30    }
31    inline void pushdownall()
32    {
33        if (check()) fa -> pushdownall();
34        pushdown();
35    }
36    inline void rotate()
37    {
38        bool d = type(); Node *f = fa, *gf = f -> fa;
39        (fa = gf, f -> check()) ? fa -> ch[f -> type()] = this : 0;
40        (f -> ch[d] = ch[!d]) != null ? ch[!d] -> fa = f : 0;
41        (ch[!d] = f) -> fa = this;
42        f -> pushup();
43    }
44    inline void splay(bool need = 1)

```

```

45 {
46     if (need) pushdownall();
47     for (; check(); rotate())
48         if (fa -> check())
49             (type() == fa -> type() ? fa : this) -> rotate();
50     pushup();
51 }
52 inline Node *access()
53 {
54     Node *i = this, *j = null;
55     for (; i != null; i = (j = i) -> fa)
56     {
57         i -> splay();
58         i -> ch[1] = j;
59         i -> pushup();
60     }
61     return j;
62 }
63 inline void make_root()
64 {
65     access();
66     splay();
67     rev ^= 1;
68 }
69 inline void link(Node *that)
70 {
71     make_root();
72     fa = that;
73     splay(0);
74 }
75 inline void cut(Node *that)
76 {
77     make_root();
78     that -> access();
79     that -> splay(0);
80     that -> ch[0] = fa = null;
81     that -> pushup();
82 }
83 } mem[maxn];
84 inline Node *query(Node *a, Node *b)
85 {
86     a -> make_root(); b -> access(); b -> splay(0);
87     return b -> pos;
88 }
89 inline int dist(Node *a, Node *b)
90 {
91     a -> make_root(); b -> access(); b -> splay(0);
92     return b -> len;
93 }

```

## 1.6.6 树上倍增

## 1.6.7 数链剖分

## 1.7 仙人掌

## 1.8 带花树

## 1.9 KM 算法

## 1.10 支配树

## 1.10.1 DAG

## 1.10.2 一般图

## 1.11 弦图

## 1.12 网络流

## 1.13 最小割

## 1.14 最大流

## 1.15 费用流

## 1.16 有上下界的网络流 (Durandal)

$B(u, v)$  表示边  $(u, v)$  流量的下界,  $C(u, v)$  表示边  $(u, v)$  流量的上界, 设  $F(u, v)$  表示边  $(u, v)$  的实际流量  
 设  $G(u, v) = F(u, v) - B(u, v)$ , 则  $0 \leq G(u, v) \leq C(u, v) - B(u, v)$

- 无源汇的上下界可行流  
 建立超级源点  $S^*$  和超级汇点  $T^*$ , 对于原图每一条边  $(u, v)$  在新网络中连如下三条边:  $S^* \rightarrow v$ , 容量为  $B(u, v)$ ;  $u \rightarrow T^*$ , 容量为  $B(u, v)$ ;  $u \rightarrow v$ , 容量为  $C(u, v) - B(u, v)$ 。最后求新网络的最大流, 判断从超级源点  $S^*$  出发的边是否都满流即可, 边  $(u, v)$  的最终解中的实际流量为  $G(u, v) + B(u, v)$ 。
- 有源汇的上下界可行流  
 从汇点  $T$  到源点  $S$  连一条上界为  $\infty$ , 下界为 0 的边。按照无源汇的上下界可行流一样做即可, 流量即为  $T \rightarrow S$  边上的流量。
- 有源汇的上下界最大流
  - 在有源汇的上下界可行流中, 从汇点  $T$  到源点  $S$  的边改为连一条上界为  $\infty$ , 下界为  $x$  的边。  $x$  满足二分性质, 找到最大的  $x$  使得新网络存在有源汇的上下界可行流即为原图的最大流。
  - 从汇点  $T$  到源点  $S$  连一条上界为  $\infty$ , 下界为 0 的边, 变成无源汇的网络。按照无源汇的上下界可行流的方法, 建立超级源点  $S^*$  与超级汇点  $T^*$ , 求一遍  $S^* \rightarrow T^*$  的最大流, 再将从汇点  $T$  到源点  $S$  的这条边拆掉, 求一次  $S \rightarrow T$  的最大流即可。
- 有源汇的上下界最小流
  - 在有源汇的上下界可行流中, 从汇点  $T$  到源点  $S$  的边改为连一条上界为  $x$ , 下界为 0 的边。  $x$  满足二分性质, 找到最小的  $x$  使得新网络存在有源汇的上下界可行流即为原图的最大流。
  - 按照无源汇的上下界可行流的方法, 建立超级源点  $S^*$  与超级汇点  $T^*$ , 求一遍  $S^* \rightarrow T^*$  的最大流, 但是注意不加上汇点  $T$  到源点  $S$  的这条边, 即不使之改为无源汇的网络去求解。求完后, 再加上那条汇点  $T$  到源点  $S$  的边, 上界为  $\infty$  的边。因为这条边的下界为 0, 所以  $S^*, T^*$  无影响, 再求

一次  $S^* \rightarrow T^*$  的最大流。若超级源点  $S^*$  出发的边全部满流，则  $T \rightarrow S$  边上的流量即为原图的最小流，否则无解。

#### 1.16.1 zkw 费用流

### 1.17 差分约束

# Chapter 2

## Math

### 2.1 int64 相乘取模 (Durandal)

```
1 int64_t mul(int64_t x, int64_t y, int64_t p) {
2     int64_t t = (x * y - (int64_t) ((long double) x / p * y + 1e-3) * p) % p;
3     return t < 0 ? t + p : t;
4 }
```

### 2.2 扩展欧几里得 (gy)

```
1 // return gcd(a, b)
2 // ax+by=gcd(a,b)
3 int extend_gcd(int a, int b, int &x, int &y) {
4     if (b == 0) {
5         x = 1, y = 0;
6         return a;
7     }
8     int res = extend_gcd(b, a % b, x, y);
9     int t = y;
10    y = x - a / b * y;
11    x = t;
12    return res;
13 }
14 // return minimal positive integer x so that ax+by=c
15 // or -1 if such x does not exist
16 int solve_equ(int a, int b, int c) {
17     int x, y, d;
18     d = extend_gcd(a, b, x, y);
19     if (c % d)
20         return -1;
21     int t = c / d;
22     x *= t;
23     y *= t;
24     int k = b / d;
25     x = (x % k + k) % k;
26     return x;
27 }
28 // return minimal positive integer x so that ax==b(mod p)
29 // or -1 if such x does not exist
30 int solve(int a, int b, int p) {
31     a = (a % p + p) % p;
32     b = (b % p + p) % p;
```

```

33     return solve_equ(a, p, b);
34 }

```

## 2.3 中国剩余定理 (Durandal)

返回是否可行, 余数和模数结果为  $r_1, m_1$

```

1 bool CRT(int &r1, int &m1, int r2, int m2) {
2     int x, y, g = extend_gcd(m1, m2, x, y);
3     if ((r2 - r1) % g != 0) return false;
4     x = 1ll * (r2 - r1) * x % m2;
5     if (x < 0) x += m2;
6     x /= g;
7     r1 += m1 * x;
8     m1 *= m2 / g;
9     return true;
10 }

```

## 2.4 线性同余不等式 (Durandal)

必须满足  $0 \leq d < m, 0 \leq l \leq r < m$ , 返回  $\min\{x \geq 0 \mid l \leq x \cdot d \bmod m \leq r\}$ , 无解返回  $-1$

```

1 int64_t calc(int64_t d, int64_t m, int64_t l, int64_t r) {
2     if (l == 0) return 0;
3     if (d == 0) return -1;
4     if (d * 2 > m) return calc(m - d, m, m - r, m - l);
5     if ((l - 1) / d < r / d) return (l - 1) / d + 1;
6     int64_t k = calc((-m % d + d) % d, d, l % d, r % d);
7     if (k == -1) return -1;
8     return (k * m + l - 1) / d + 1;
9 }

```

## 2.5 组合数

## 2.6 高斯消元

## 2.7 Miller Rabin & Pollard Rho (gy)

```

1 /*
2  * In Java, use BigInteger.isProbablePrime(int certainty) to replace miller_rabin(BigInteger
3  *   ↪ number)
4  * Test Set / First Wrong Answer
5  * 2 / 2,047
6  * 2, 3 / 1,373,653
7  * 31, 73 / 9,080,191
8  * 2, 3, 5 / 25,326,001
9  * 2, 3, 5, 7 / 3,215,031,751 (> Int.MAX_VALUE)
10 * 2, 7, 61 / 4,759,123,141
11 * 2, 13, 23, 1662803 / 1,122,004,669,633
12 * 2, 3, 5, 7, 11 / 2,152,302,898,747
13 * 2, 3, 5, 7, 11, 13 / 3,474,749,660,383
14 * 2, 3, 5, 7, 11, 13, 17 / 341,550,071,728,321
15 * 2, 3, 5, 7, 11, 13, 17, 19, 23 / 3,825,123,056,546,413,051
16 * 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37 / 318,665,857,834,031,151,167,461 (> Long.MAX_VALUE)
17 * 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41 / 3,317,044,064,679,887,385,961,981

```

```

17  */
18  const int test_case_size = 12;
19  const int test_cases[test_case_size] = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37};

20  int64_t multiply_mod(int64_t x, int64_t y, int64_t p) {
21      int64_t t = (x * y - (int64_t) ((long double) x / p * y + 1e-3) * p) % p;
22      return t < 0 ? t + p : t;
23  }

24  int64_t add_mod(int64_t x, int64_t y, int64_t p) {
25      return (0ull + x + y) % p;
26  }

27  int64_t power_mod(int64_t x, int64_t exp, int64_t p) {
28      int64_t ans = 1;
29      while (exp) {
30          if (exp & 1)
31              ans = multiply_mod(ans, x, p);
32          x = multiply_mod(x, x, p);
33          exp >>= 1;
34      }
35      return ans;
36  }

37  bool miller_rabin_check(int64_t prime, int64_t base) {
38      int64_t number = prime - 1;
39      for (; ~number & 1; number >>= 1)
40          continue;
41      int64_t result = power_mod(base, number, prime);
42      for (; number != prime - 1 && result != 1 && result != prime - 1; number <<= 1)
43          result = multiply_mod(result, result, prime);
44      return result == prime - 1 || (number & 1) == 1;
45  }

46  bool miller_rabin(int64_t number) {
47      if (number < 2)
48          return false;
49      if (number < 4)
50          return true;
51      if (~number & 1)
52          return false;
53      for (int i = 0; i < test_case_size && test_cases[i] < number; i++)
54          if (!miller_rabin_check(number, test_cases[i]))
55              return false;
56      return true;
57  }

58  int64_t gcd(int64_t x, int64_t y) {
59      return y == 0 ? x : gcd(y, x % y);
60  }

61  int64_t pollard_rho_test(int64_t number, int64_t seed) {
62      int64_t x = rand() % (number - 1) + 1, y = x;
63      int head = 1, tail = 2;
64      while (true) {
65          x = multiply_mod(x, x, number);
66          x = add_mod(x, seed, number);
67          if (x == y)
68              return number;
69          int64_t answer = gcd(std::abs(x - y), number);
70          if (answer > 1 && answer < number)

```

```

71     return answer;
72     if (++head == tail) {
73         y = x;
74         tail <<= 1;
75     }
76 }
77 }

78 void factorize(int64_t number, std::vector<int64_t> &divisor) {
79     if (number > 1) {
80         if (miller_rabin(number)) {
81             divisor.push_back(number);
82         } else {
83             int64_t factor = number;
84             while (factor >= number)
85                 factor = pollard_rho_test(number, rand() % (number - 1) + 1);
86             factorize(number / factor, divisor);
87             factorize(factor, divisor);
88         }
89     }
90 }

```

## 2.8 $O(m^2 \log n)$ 线性递推

## 2.9 Polynomial

### 2.9.1 FFT

### 2.9.2 NTT & 多项式求逆

## 2.10 拉格朗日插值

## 2.11 杜教筛

## 2.12 BSGS

### 2.12.1 BSGS

### 2.12.2 扩展 BSGS

## 2.13 直线下整点个数 (gy)

必须满足  $a \geq 0, b \geq 0, m > 0$ , 返回  $\sum_{i=0}^{n-1} \frac{a+bi}{m}$

```

1 int64_t count(int64_t n, int64_t a, int64_t b, int64_t m) {
2     if (b == 0)
3         return n * (a / m);
4     if (a >= m)
5         return n * (a / m) + count(n, a % m, b, m);
6     if (b >= m)
7         return (n - 1) * n / 2 * (b / m) + count(n, a, b % m, m);
8     return count((a + b * n) / m, (a + b * n) % m, m, b);
9 }

```



## 2.14 单纯形

## 2.15 辛普森积分

# Chapter 3

## Geometry

### 3.1 点、直线、圆 (gy)

```
1 using number = long double;
2 const number eps = 1e-8;

3 number _sqrt(number x) {
4     return std::sqrt(std::max(x, (number) 0));
5 }
6 number _asin(number x) {
7     x = std::min(x, (number) 1), x = std::max(x, (number) -1);
8     return std::asin(x);
9 }
10 number _acos(number x) {
11     x = std::min(x, (number) 1), x = std::max(x, (number) -1);
12     return std::acos(x);
13 }

14 int sgn(number x) {
15     return (x > eps) - (x < -eps);
16 }
17 int cmp(number x, number y) {
18     return sgn(x - y);
19 }

20 struct point {
21     number x, y;
22     point() {}
23     point(number x, number y) : x(x), y(y) {}

24     number len2() const {
25         return x * x + y * y;
26     }
27     number len() const {
28         return _sqrt(len2());
29     }
30     point unit() const {
31         return point(x / len(), y / len());
32     }
33     point rotate90() const {
34         return point(-y, x);
35     }

36     friend point operator+(const point &a, const point &b) {
37         return point(a.x + b.x, a.y + b.y);
```

```

38     }
39     friend point operator-(const point &a, const point &b) {
40         return point(a.x - b.x, a.y - b.y);
41     }
42     friend point operator*(const point &a, number b) {
43         return point(a.x * b, a.y * b);
44     }
45     friend point operator/(const point &a, number b) {
46         return point(a.x / b, a.y / b);
47     }
48     friend number dot(const point &a, const point &b) {
49         return a.x * b.x + a.y * b.y;
50     }
51     friend number det(const point &a, const point &b) {
52         return a.x * b.y - a.y * b.x;
53     }
54     friend number operator==(const point &a, const point &b) {
55         return cmp(a.x, b.x) == 0 && cmp(a.y, b.y) == 0;
56     }
57 };

58 number dis2(const point &a, const point &b) {
59     return (a - b).len2();
60 }
61 number dis(const point &a, const point &b) {
62     return (a - b).len();
63 }

64 struct line {
65     point a, b;
66     line() {}
67     line(point a, point b) : a(a), b(b) {}
68     point value() const {
69         return b - a;
70     }
71 };

72 bool point_on_line(const point &p, const line &l) {
73     return sgn(det(p - l.a, p - l.b)) == 0;
74 }
75 // including endpoint
76 bool point_on_ray(const point &p, const line &l) {
77     return sgn(det(p - l.a, p - l.b)) == 0 &&
78         sgn(dot(p - l.a, l.b - l.a)) >= 0;
79 }
80 // including endpoints
81 bool point_on_seg(const point &p, const line &l) {
82     return sgn(det(p - l.a, p - l.b)) == 0 &&
83         sgn(dot(p - l.a, l.b - l.a)) >= 0 &&
84         sgn(dot(p - l.b, l.a - l.b)) >= 0;
85 }
86 bool seg_has_intersection(const line &a, const line &b) {
87     if (point_on_seg(a.a, b) || point_on_seg(a.b, b) ||
88         point_on_seg(b.a, a) || point_on_seg(b.b, a))
89         return /* including endpoints */ true;
90     return sgn(det(a.a - b.a, b.b - b.a)) * sgn(det(a.b - b.a, b.b - b.a)) < 0
91         && sgn(det(b.a - a.a, a.b - a.a)) * sgn(det(b.b - a.a, a.b - a.a)) < 0;
92 }
93 point intersect(const line &a, const line &b) {
94     number s1 = det(a.b - a.a, b.a - a.a);
95     number s2 = det(a.b - a.a, b.b - a.a);

```

```

96     return (b.a * s2 - b.b * s1) / (s2 - s1);
97 }
98 point projection(const point &p, const line &l) {
99     return l.a + (l.b - l.a) * dot(p - l.a, l.b - l.a) / (l.b - l.a).len2();
100 }
101 number dis(const point &p, const line &l) {
102     return std::abs(dot(p - l.a, l.b - l.a)) / (l.b - l.a).len();
103 }
104 point symmetry_point(const point &a, const point &o) {
105     return o + o - a;
106 }
107 point reflection(const point &p, const line &l) {
108     return symmetry_point(p, projection(p, l));
109 }
110 struct circle {
111     point o;
112     number r;
113     circle() {}
114     circle(point o, number r) : o(o), r(r) {}
115 };
116 bool intersect(const line &l, const circle &a, point &p1, point &p2) {
117     number x = dot(l.a - a.o, l.b - l.a);
118     number y = (l.b - l.a).len2();
119     number d = x * x - y * ((l.a - a.o).len2() - a.r * a.r);
120     if (sgn(d) < 0) return false;
121     point p = l.a - (l.b - l.a) * (x / y), delta = (l.b - l.a) * (_sqrt(d) / y);
122     p1 = p + delta, p2 = p - delta;
123     return true;
124 }
125 bool intersect(const circle &a, const circle &b, point &p1, point &p2) {
126     if (a.o == b.o && cmp(a.r, b.r) == 0)
127         return /* value for coincident circles */ false;
128     number s1 = (b.o - a.o).len();
129     if (cmp(s1, a.r + b.r) > 0 || cmp(s1, std::abs(a.r - b.r)) < 0)
130         return false;
131     number s2 = (a.r * a.r - b.r * b.r) / s1;
132     number aa = (s1 + s2) / 2, bb = (s1 - s2) / 2;
133     point p = (b.o - a.o) * (aa / (aa + bb)) + a.o;
134     point delta = (b.o - a.o).unit().rotate90() * _sqrt(a.r * a.r - aa * aa);
135     p1 = p + delta, p2 = p - delta;
136     return true;
137 }
138 bool tangent(const point &p0, const circle &c, point &p1, point &p2) {
139     number x = (p0 - c.o).len2();
140     number d = x - c.r * c.r;
141     if (sgn(d) < 0) return false;
142     if (sgn(d) == 0)
143         return /* value for point_on_line */ false;
144     point p = (p0 - c.o) * (c.r * c.r / x);
145     point delta = ((p0 - c.o) * (-c.r * _sqrt(d) / x)).rotate90();
146     p1 = c.o + p + delta;
147     p2 = c.o + p - delta;
148     return true;
149 }
150 bool ex_tangent(const circle &a, const circle &b, line &l1, line &l2) {
151     if (cmp(std::abs(a.r - b.r), (b.o - a.o).len()) == 0) {
152         point p1, p2;
153         intersect(a, b, p1, p2);
154         l1 = l2 = line(p1, p1 + (a.o - p1).rotate90());

```

```

155     return true;
156 } else if (cmp(a.r, b.r) == 0) {
157     point dir = b.o - a.o;
158     dir = (dir * (a.r / dir.len())).rotate90();
159     l1 = line(a.o + dir, b.o + dir);
160     l2 = line(a.o - dir, b.o - dir);
161     return true;
162 } else {
163     point p = (b.o * a.r - a.o * b.r) / (a.r - b.r);
164     point p1, p2, q1, q2;
165     if (tangent(p, a, p1, p2) && tangent(p, b, q1, q2)) {
166         l1 = line(p1, q1);
167         l2 = line(p2, q2);
168         return true;
169     } else {
170         return false;
171     }
172 }
173 }
174 bool in_tangent(const circle &a, const circle &b, line &l1, line &l2) {
175     if (cmp(a.r + b.r, (b.o - a.o).len()) == 0) {
176         point p1, p2;
177         intersect(a, b, p1, p2);
178         l1 = l2 = line(p1, p1 + (a.o - p1).rotate90());
179         return true;
180     } else {
181         point p = (b.o * a.r + a.o * b.r) / (a.r + b.r);
182         point p1, p2, q1, q2;
183         if (tangent(p, a, p1, p2) && tangent(p, b, q1, q2)) {
184             l1 = line(p1, q1);
185             l2 = line(p2, q2);
186             return true;
187         } else {
188             return false;
189         }
190     }
191 }

```

- 3.2 点到凸包切线
- 3.3 直线凸包交点
- 3.4 凸包游戏
- 3.5 半平面交
- 3.6 旋转卡壳
- 3.7 判断圆是否有交
- 3.8 最小圆覆盖
- 3.9 最小球覆盖
- 3.10  $O(n^2 \log n)$  圆交面积和重心
- 3.11 圆与多边形交
- 3.12  $O(n \log n)$  凸多边形内的最大圆
- 3.13 三角形的五心
- 3.14 三维凸包
- 3.15 三维绕轴旋转
- 3.16 几何公式

## Chapter 4

# String

4.1 KMP

4.2 AC 自动机

4.3 后缀数组

4.4 后缀自动机

4.5 Manacher

4.6 回文自动机

4.7 最小表示法

## Chapter 5

# Data Structure

### 5.1 莫队 (ct)

```
1 int size;
2 struct Query {
3     int l, r, id;
4     inline bool operator < (const Query &that) const {return l / size != that.l / size ? l < that.l
5         ↪ : ((l / size) & 1 ? r < that.r : r > that.r);}
6 } q[maxn];
7 int main()
8 {
9     size = (int) sqrt(n * 1.0);
10    std::sort(q + 1, q + m + 1);
11    int l = 1, r = 0;
12    for (int i = 1; i <= m; ++i)
13    {
14        for (; r < q[i].r; ) add(++r);
15        for (; r > q[i].r; ) del(r--);
16        for (; l < q[i].l; ) del(l++);
17        for (; l > q[i].l; ) add(--l);
18        /*
19         * write your code here.
20         */
21    }
22    return 0;
23 }
```

### 5.2 ST 表 (ct)

```
1 int a[maxn], f[20][maxn], n;
2 int Log[maxn];
3 void build()
4 {
5     for (int i = 1; i <= n; ++i) f[0][i] = a[i];
6
7     int lim = Log[n];
8     for (int j = 1; j <= lim; ++j)
9     {
10        int *fj = f[j], *fj1 = f[j - 1];
11        for (int i = 1; i <= n - (1 << j) + 1; ++i)
12            fj[i] = dmax(fj1[i], fj1[i + (1 << (j - 1))]);
13    }
14 }
```



```

14 int Query(int l, int r)
15 {
16     int k = Log[r - l + 1];
17     return dmax(f[k][l], f[k][r - (1 << k) + 1]);
18 }
19 int main()
20 {
21     scanf("%d", &n);
22     Log[0] = -1;
23     for (int i = 1; i <= n; ++i)
24     {
25         scanf("%d", &a[i]);
26         Log[i] = Log[i >> 1] + 1;
27     }
28     build();
29     int q;
30     scanf("%d", &q);
31     for (; q; --q)
32     {
33         int l, r; scanf("%d%d", &l, &r);
34         printf("%d\n", Query(l, r));
35     }
36 }

```

### 5.3 可并堆 (ct)

```

1 struct Node {
2     Node *ch[2];
3     ll val; int size;
4     inline void update()
5     {
6         size = ch[0] -> size + ch[1] -> size + 1;
7     }
8 } mem[maxn], *rt[maxn];
9 Node *merge(Node *a, Node *b)
10 {
11     if (a == mem) return b;
12     if (b == mem) return a;
13     if (a -> val < b -> val) std::swap(a, b);
14     // a -> pushdown();
15     std::swap(a -> ch[0], a -> ch[1]);
16     a -> ch[1] = merge(a -> ch[1], b);
17     a -> update();
18     return a;
19 }

```

### 5.4 zkw 线段树 (ct)

```

1 // must be 0-based !
2 inline void build()
3 {
4     for (int i = M - 1; i; --i) tr[i] = dmax(tr[i << 1], tr[i << 1 | 1]);
5 }
6 inline void Change(int x, int v)
7 {
8     x += M; tr[x] = v; x >>= 1;
9     for (; x; x >>= 1) tr[x] = dmax(tr[x << 1], tr[x << 1 | 1]);
10 }

```

```

11 inline int Query(int s, int t)
12 {
13     int ret = -0x7fffffff;
14     for (s = s + M - 1, t = t + M + 1; s ^ t ^ 1; s >>= 1, t >>= 1)
15     {
16         if (~s & 1) cmax(ret, tr[s ^ 1]);
17         if (t & 1) cmax(ret, tr[t ^ 1]);
18     }
19     return ret;
20 }
21 int main()
22 {
23     int n; scanf("%d", &n);
24     for (M = 1; M < n; M <= 1) ;
25     for (int i = 0; i < n; ++i)
26         scanf("%d", &tr[i + M]);
27     for (int i = n; i < M; ++i) tr[i + M] = -0x7fffffff;
28     build();
29     int q; scanf("%d", &q);
30     for (; q; --q)
31     {
32         int l, r; scanf("%d%d", &l, &r); --l, --r;
33         printf("%d\n", Query(l, r));
34     }
35     return 0;
36 }

```

## 5.5 主席树

## 5.6 Splay (ct)

指针版

```

1 struct Node *null;
2 struct Node {
3     Node *ch[2], *fa;
4     int val; bool rev;
5     inline bool type()
6     {
7         return fa -> ch[1] == this;
8     }
9     inline void pushup()
10    {
11    }
12    inline void pushdown()
13    {
14        if (rev)
15        {
16            ch[0] -> rev ^= 1;
17            ch[1] -> rev ^= 1;
18            std::swap(ch[0], ch[1]);
19            rev ^= 1;
20        }
21    }
22    inline void rotate()
23    {
24        bool d = type(); Node *f = fa, *gf = f -> fa;
25        (fa = gf, f -> fa != null) ? fa -> ch[f -> type()] = this : 0;
26        (f -> ch[d] = ch[!d]) != null ? ch[!d] -> fa = f : 0;

```

```

27     (ch[!d] = f) -> fa = this;
28     f -> pushup();
29 }
30 inline void splay()
31 {
32     for (; fa != null; rotate())
33         if (fa -> fa != null)
34             (type() == fa -> type() ? fa : this) -> rotate();
35     pushup();
36 }
37 } mem[maxn];

```

## 数组版

```

1 // BZOJ - 1500 维修数列
2 int fa[maxn], ch[maxn][2], a[maxn], size[maxn], cnt;
3 int sum[maxn], lmx[maxn], rmx[maxn], mx[maxn], v[maxn], id[maxn], root;
4 bool rev[maxn], tag[maxn];
5 inline void update(R int x)
6 {
7     R int ls = ch[x][0], rs = ch[x][1];
8     size[x] = size[ls] + size[rs] + 1;
9     sum[x] = sum[ls] + sum[rs] + v[x];
10    mx[x] = gmax(mx[ls], mx[rs]);
11    cmax(mx[x], lmx[rs] + rmx[ls] + v[x]);
12    lmx[x] = gmax(lmx[ls], sum[ls] + v[x] + lmx[rs]);
13    rmx[x] = gmax(rmx[rs], sum[rs] + v[x] + rmx[ls]);
14 }
15 inline void pushdown(R int x)
16 {
17     R int ls = ch[x][0], rs = ch[x][1];
18     if (tag[x])
19     {
20         rev[x] = tag[x] = 0;
21         if (ls) tag[ls] = 1, v[ls] = v[x], sum[ls] = size[ls] * v[x];
22         if (rs) tag[rs] = 1, v[rs] = v[x], sum[rs] = size[rs] * v[x];
23         if (v[x] >= 0)
24         {
25             if (ls) lmx[ls] = rmx[ls] = mx[ls] = sum[ls];
26             if (rs) lmx[rs] = rmx[rs] = mx[rs] = sum[rs];
27         }
28         else
29         {
30             if (ls) lmx[ls] = rmx[ls] = 0, mx[ls] = v[x];
31             if (rs) lmx[rs] = rmx[rs] = 0, mx[rs] = v[x];
32         }
33     }
34     if (rev[x])
35     {
36         rev[x] ^= 1; rev[ls] ^= 1; rev[rs] ^= 1;
37         swap(lmx[ls], rmx[ls]); swap(lmx[rs], rmx[rs]);
38         swap(ch[ls][0], ch[ls][1]); swap(ch[rs][0], ch[rs][1]);
39     }
40 }
41 inline void rotate(R int x)
42 {
43     R int f = fa[x], gf = fa[f], d = ch[f][1] == x;
44     if (f == root) root = x;
45     (ch[f][d] = ch[x][d ^ 1]) > 0 ? fa[ch[f][d]] = f : 0;
46     (fa[x] = gf) > 0 ? ch[gf][ch[gf][1] == f] = x : 0;
47     fa[ch[x][d ^ 1] = f] = x;

```

```

48     update(f);
49 }
50 inline void splay(R int x, R int rt)
51 {
52     while (fa[x] != rt)
53     {
54         R int f = fa[x], gf = fa[f];
55         if (gf != rt) rotate((ch[gf][1] == f) ^ (ch[f][1] == x) ? x : f);
56         rotate(x);
57     }
58     update(x);
59 }
60 void build(R int l, R int r, R int rt)
61 {
62     if (l > r) return ;
63     R int mid = l + r >> 1, now = id[mid], last = id[rt];
64     if (l == r)
65     {
66         sum[now] = a[l];
67         size[now] = 1;
68         tag[now] = rev[now] = 0;
69         if (a[l] >= 0) lmx[now] = rmx[now] = mx[now] = a[l];
70         else lmx[now] = rmx[now] = 0, mx[now] = a[l];
71     }
72     else
73     {
74         build(l, mid - 1, mid);
75         build(mid + 1, r, mid);
76     }
77     v[now] = a[mid];
78     fa[now] = last;
79     update(now);
80     ch[last][mid >= rt] = now;
81 }
82 int find(R int x, R int rank)
83 {
84     if (tag[x] || rev[x]) pushdown(x);
85     R int ls = ch[x][0], rs = ch[x][1], lsize = size[ls];
86     if (lsize + 1 == rank) return x;
87     if (lsize >= rank)
88         return find(ls, rank);
89     else
90         return find(rs, rank - lsize - 1);
91 }
92 inline int prepare(R int l, R int tot)
93 {
94     R int x = find(root, l - 1), y = find(root, l + tot);
95     splay(x, 0);
96     splay(y, x);
97     return ch[y][0];
98 }
99 std::queue<int> q;
100 inline void Insert(R int left, R int tot)
101 {
102     for (R int i = 1; i <= tot; ++i) a[i] = FastIn();
103     for (R int i = 1; i <= tot; ++i)
104         if (!q.empty()) id[i] = q.front(), q.pop();
105         else id[i] = ++cnt;
106     build(1, tot, 0);
107     R int z = id[(1 + tot) >> 1];
108     R int x = find(root, left), y = find(root, left + 1);

```

```

109     splay(x, 0);
110     splay(y, x);
111     fa[z] = y;
112     ch[y][0] = z;
113     update(y);
114     update(x);
115 }
116 void rec(R int x)
117 {
118     if (!x) return ;
119     R int ls = ch[x][0], rs = ch[x][1];
120     rec(ls); rec(rs); q.push(x);
121     fa[x] = ch[x][0] = ch[x][1] = 0;
122     tag[x] = rev[x] = 0;
123 }
124 inline void Delete(R int l, R int tot)
125 {
126     R int x = prepare(l, tot), f = fa[x];
127     rec(x); ch[f][0] = 0;
128     update(f); update(fa[f]);
129 }
130 inline void Makesame(R int l, R int tot, R int val)
131 {
132     R int x = prepare(l, tot), y = fa[x];
133     v[x] = val; tag[x] = 1; sum[x] = size[x] * val;
134     if (val >= 0) lmx[x] = rmx[x] = mx[x] = sum[x];
135     else lmx[x] = rmx[x] = 0, mx[x] = val;
136     update(y); update(fa[y]);
137 }
138 inline void Reverse(R int l, R int tot)
139 {
140     R int x = prepare(l, tot), y = fa[x];
141     if (!tag[x])
142     {
143         rev[x] ^= 1;
144         swap(ch[x][0], ch[x][1]);
145         swap(lmx[x], rmx[x]);
146         update(y); update(fa[y]);
147     }
148 }
149 inline void Query(R int l, R int tot)
150 {
151     R int x = prepare(l, tot);
152     printf("%d\n", sum[x] );
153 }
154 #define inf ((1 << 30))
155 int main()
156 {
157     R int n = FastIn(), m = FastIn(), l, tot, val;
158     R char op, op2;
159     mx[0] = a[1] = a[n + 2] = -inf;
160     for (R int i = 2; i <= n + 1; i++)
161     {
162         a[i] = FastIn();
163     }
164     for (R int i = 1; i <= n + 2; ++i) id[i] = i;
165     n += 2; cnt = n; root = (n + 1) >> 1;
166     build(1, n, 0);
167     for (R int i = 1; i <= m; i++)
168     {
169         op = getc();

```

```

170     while (op < 'A' || op > 'Z') op = getc();
171     getc(); op2 = getc();getc();getc();getc();getc();
172     if (op == 'M' && op2 == 'X')
173     {
174         printf("%d\n",mx[root] );
175     }
176     else
177     {
178         l = FastIn() + 1; tot = FastIn();
179         if (op == 'I') Insert(l, tot);
180         if (op == 'D') Delete(l, tot);
181         if (op == 'M') val = FastIn(), Makesame(l, tot, val);
182         if (op == 'R')
183             Reverse(l, tot);
184         if (op == 'G')
185             Query(l, tot);
186     }
187 }
188 return 0;
189 }

```

## 5.7 Treap (ct)

```

1 struct Treap {
2     Treap *ls, *rs;
3     int size;
4     bool rev;
5     inline void update()
6     {
7         size = ls -> size + rs -> size + 1;
8     }
9     inline void set_rev()
10    {
11        rev ^= 1;
12        std::swap(ls, rs);
13    }
14    inline void pushdown()
15    {
16        if (rev)
17        {
18            ls -> set_rev();
19            rs -> set_rev();
20            rev = 0;
21        }
22    }
23 } mem[maxn], *root, *null = mem;
24 struct Pair {
25     Treap *fir, *sec;
26 };
27 Treap *build(R int l, R int r)
28 {
29     if (l > r) return null;
30     R int mid = l + r >> 1;
31     R Treap *now = mem + mid;
32     now -> rev = 0;
33     now -> ls = build(l, mid - 1);
34     now -> rs = build(mid + 1, r);
35     now -> update();

```

```

36     return now;
37 }
38 inline Treap *Find_kth(R Treap *now, R int k)
39 {
40     if (!k) return mem;
41     if (now -> ls -> size >= k) return Find_kth(now -> ls, k);
42     else if (now -> ls -> size + 1 == k) return now;
43     else return Find_kth(now -> rs, k - now -> ls -> size - 1);
44 }
45 Treap *merge(R Treap *a, R Treap *b)
46 {
47     if (a == null) return b;
48     if (b == null) return a;
49     if (rand() % (a -> size + b -> size) < a -> size)
50     {
51         a -> pushdown();
52         a -> rs = merge(a -> rs, b);
53         a -> update();
54         return a;
55     }
56     else
57     {
58         b -> pushdown();
59         b -> ls = merge(a, b -> ls);
60         b -> update();
61         return b;
62     }
63 }
64 Pair split(R Treap *now, R int k)
65 {
66     if (now == null) return (Pair) {null, null};
67     R Pair t = (Pair) {null, null};
68     now -> pushdown();
69     if (k <= now -> ls -> size)
70     {
71         t = split(now -> ls, k);
72         now -> ls = t.sec;
73         now -> update();
74         t.sec = now;
75     }
76     else
77     {
78         t = split(now -> rs, k - now -> ls -> size - 1);
79         now -> rs = t.fir;
80         now -> update();
81         t.fir = now;
82     }
83     return t;
84 }
85 inline void set_rev(int l, int r)
86 {
87     R Pair x = split(root, l - 1);
88     R Pair y = split(x.sec, r - l + 1);
89     y.fir -> set_rev();
90     root = merge(x.fir, merge(y.fir, y.sec));
91 }

```

## 5.8 可持久化平衡树 (ct)

```

1  char str[maxn];
2  struct Treap
3  {
4      Treap *ls, *rs;
5      char data; int size;
6      inline void update()
7      {
8          size = ls -> size + rs -> size + 1;
9      }
10 } *root[maxn], mem[maxcnt], *tot = mem, *last = mem, *null = mem;
11 inline Treap* new_node(char ch)
12 {
13     *++tot = (Treap) {null, null, ch, 1};
14     return tot;
15 }
16 struct Pair
17 {
18     Treap *fir, *sec;
19 };
20 inline Treap *copy(Treap *x)
21 {
22     if (x == null) return null;
23     if (x > last) return x;
24     *++tot = *x;
25     return tot;
26 }
27 Pair Split(Treap *x, int k)
28 {
29     if (x == null) return (Pair) {null, null};
30     Pair y;
31     Treap *nw = copy(x);
32     if (nw -> ls -> size >= k)
33     {
34         y = Split(nw -> ls, k);
35         nw -> ls = y.sec;
36         nw -> update();
37         y.sec = nw;
38     }
39     else
40     {
41         y = Split(nw -> rs, k - nw -> ls -> size - 1);
42         nw -> rs = y.fir;
43         nw -> update();
44         y.fir = nw;
45     }
46     return y;
47 }
48 Treap *Merge(Treap *a, Treap *b)
49 {
50     if (a == null) return b;
51     if (b == null) return a;
52     Treap *nw;
53     if (rand() % (a -> size + b -> size) < a -> size)
54     {
55         nw = copy(a);
56         nw -> rs = Merge(nw -> rs, b);
57     }
58     else

```



```

59     {
60         nw = copy(b);
61         nw -> ls = Merge(a, nw -> ls);
62     }
63     nw -> update();
64     return nw;
65 }
66 Treap *Build(int l, int r)
67 {
68     if (l > r) return null;
69     R int mid = l + r >> 1;
70     Treap *nw = new_node(str[mid]);
71     nw -> ls = Build(l, mid - 1);
72     nw -> rs = Build(mid + 1, r);
73     nw -> update();
74     return nw;
75 }
76 int now;
77 inline void Insert(int k, char ch)
78 {
79     Pair x = Split(root[now], k);
80     Treap *nw = new_node(ch);
81     root[++now] = Merge(Merge(x.fir, nw), x.sec);
82 }
83 inline void Del(int l, int r)
84 {
85     Pair x = Split(root[now], l - 1);
86     Pair y = Split(x.sec, r - l + 1);
87     root[++now] = Merge(x.fir, y.sec);
88 }
89 inline void Copy(int l, int r, int ll)
90 {
91     Pair x = Split(root[now], l - 1);
92     Pair y = Split(x.sec, r - l + 1);
93     Pair z = Split(root[now], ll);
94     Treap *ans = y.fir;
95     root[++now] = Merge(Merge(z.fir, ans), z.sec);
96 }
97 void Print(Treap *x, int l, int r)
98 {
99     if (!x) return ;
100    if (l > r) return;
101    R int mid = x -> ls -> size + 1;
102    if (r < mid)
103    {
104        Print(x -> ls, l, r);
105        return ;
106    }
107    if (l > mid)
108    {
109        Print(x -> rs, l - mid, r - mid);
110        return ;
111    }
112    Print(x -> ls, l, mid - 1);
113    printf("%c", x -> data );
114    Print(x -> rs, 1, r - mid);
115 }
116 void Printtree(Treap *x)
117 {
118     if (!x) return;
119     Printtree(x -> ls);

```

```

120     printf("%c", x -> data );
121     Printtree(x -> rs);
122 }
123 int main()
124 {
125     srand(time(0) + clock());
126     null -> ls = null -> rs = null; null -> size = 0; null -> data = 0;
127     int n = F();
128     gets(str + 1);
129     int len = strlen(str + 1);
130     root[0] = Build(1, len);
131     while (1)
132     {
133         last = tot;
134         R char opt = getc();
135         while (opt < 'A' || opt > 'Z')
136         {
137             if (opt == EOF) return 0;
138             opt = getc();
139         }
140         if (opt == 'I')
141         {
142             R int x = F();
143             R char ch = getc();
144             Insert(x, ch);
145         }
146         else if (opt == 'D')
147         {
148             R int l = F(), r = F();
149             Del(l, r);
150         }
151         else if (opt == 'C')
152         {
153             R int x = F(), y = F(), z = F();
154             Copy(x, y, z);
155         }
156         else if (opt == 'P')
157         {
158             R int x = F(), y = F(), z = F();
159             Print(root[now - x], y, z);
160             puts("");
161         }
162     }
163     return 0;
164 }

```

## 5.9 CDQ 分治 (ct)

```

1 struct event
2 {
3     int x, y, id, opt, ans;
4 } t[maxn], q[maxn];
5 void cdq(int left, int right)
6 {
7     if (left == right) return ;
8     R int mid = left + right >> 1;
9     cdq(left, mid);
10    cdq(mid + 1, right);
11    //分成若干个子问题

```

```

12 ++now;
13 for (int i = left, j = mid + 1; j <= right; ++j)
14 {
15     for (; i <= mid && q[i].x <= q[j].x; ++i)
16         if (!q[i].opt)
17             add(q[i].y, q[i].ans);
18     //考虑前面的修改操作对后面的询问的影响
19     if (q[j].opt)
20         q[j].ans += query(q[j].y);
21 }
22 R int i, j, k = 0;
23 //以下相当于归并排序
24 for (i = left, j = mid + 1; i <= mid && j <= right; )
25 {
26     if (q[i].x <= q[j].x)
27         t[k++] = q[i++];
28     else
29         t[k++] = q[j++];
30 }
31 for (; i <= mid; )
32     t[k++] = q[i++];
33 for (; j <= right; )
34     t[k++] = q[j++];
35 for (int i = 0; i < k; ++i)
36     q[left + i] = t[i];
37 }

```

## 5.10 Bitset (ct)

```

1 namespace Game {
2     #define maxn 300010
3     #define maxs 30010
4     uint b1[32][maxs], b2[32][maxs];
5     int popcnt[256];
6     inline void set(R uint *s, R int pos)
7     {
8         s[pos >> 5] |= 1u << (pos & 31);
9     }
10    inline int popcount(R uint x)
11    {
12        return popcnt[x >> 24 & 255]
13            + popcnt[x >> 16 & 255]
14            + popcnt[x >> 8 & 255]
15            + popcnt[x & 255];
16    }
17    void main() {
18        int n, q;
19        scanf("%d%d", &n, &q);
20
21        char *s1 = new char[n + 1];
22        char *s2 = new char[n + 1];
23        scanf("%s%s", s1, s2);
24
25        uint *anss = new uint[q];
26
27        for (R int i = 1; i < 256; ++i) popcnt[i] = popcnt[i >> 1] + (i & 1);
28
29        #define modify(x, _p)\
30        {\

```

```

27     for (R int j = 0; j < 32 && j <= _p; ++j)\
28         set(b##x[j], _p - j);\
29 }
30 for (R int i = 0; i < n; ++i)
31     if (s1[i] == '0') modify(1, 3 * i)
32     else if (s1[i] == '1') modify(1, 3 * i + 1)
33     else modify(1, 3 * i + 2)
34
35 for (R int i = 0; i < n; ++i)
36     if (s2[i] == '1') modify(2, 3 * i)
37     else if (s2[i] == '2') modify(2, 3 * i + 1)
38     else modify(2, 3 * i + 2)
39
40 for (int Q = 0; Q < q; ++Q) {
41     R int x, y, l;
42     scanf("%d%d%d", &x, &y, &l); x *= 3; y *= 3; l *= 3;
43     uint *f1 = b1[x & 31], *f2 = b2[y & 31], ans = 0;
44     R int i = x >> 5, j = y >> 5, p, lim;
45     for (p = 0, lim = l >> 5; p + 8 < lim; p += 8, i += 8, j += 8)
46     {
47         ans += popcount(f1[i + 0] & f2[j + 0]);
48         ans += popcount(f1[i + 1] & f2[j + 1]);
49         ans += popcount(f1[i + 2] & f2[j + 2]);
50         ans += popcount(f1[i + 3] & f2[j + 3]);
51         ans += popcount(f1[i + 4] & f2[j + 4]);
52         ans += popcount(f1[i + 5] & f2[j + 5]);
53         ans += popcount(f1[i + 6] & f2[j + 6]);
54         ans += popcount(f1[i + 7] & f2[j + 7]);
55     }
56     for (; p < lim; ++p, ++i, ++j) ans += popcount(f1[i] & f2[j]);
57     R uint S = (1u << (l & 31)) - 1;
58     ans += popcount(f1[i] & f2[j] & S);
59     anss[Q] = ans;
60 }
61 }

```

output\_arr(anss, q \* sizeof(uint));

## Chapter 6

# Others

### 6.1 vimrc (gy)

```
1 se et ts=4 sw=4 sts=4 nu sc sm lbr is hls mouse=a
2 sy on
3 ino <tab> <c-n>
4 ino <s-tab> <tab>
5 au bufwinenter * winc L

6 nm <f6> ggVG"+y
7 nm <f7> :w<cr>:make<cr>
8 nm <f8> :!@<cr>
9 nm <f9> :!@ < in<cr>
10 nm <s-f9> :!(time @ < in &> out) &>> out<cr>:sp out<cr>

11 au filetype cpp cm @ ./a.out | se cin fdm=syntax mp=g++\ \% -std=c++11\ -Wall\ -Wextra\
    ↪ -Wconversion\ -O2

12 map <c-p> :ha<cr>
13 se pheader=%N@%F popt=number:y

14 au filetype java cm @ java %< | se cin fdm=syntax mp=javac\ %
15 au filetype python cm @ python % | se si fdm=indent
16 au bufenter *.kt setf kotlin
17 au filetype kotlin cm @ kotlin _%<Kt | se si mp=kotlinc\ %
```

### 6.2 STL 释放内存 (Durandal)

```
1 template <typename T>
2 __inline void clear(T &container) {
3     container.clear();
4     T(container).swap(container);
5 }
```

### 6.3 开栈 (Durandal)

```
1 register char *_sp __asm__("rsp");
2 int main() {
3     const int size = 400 << 20; // 400 MB
4     static char *sys, *mine(new char[size] + size - 4096);
5     sys = _sp; _sp = mine;
6     _main(); // main method
```

```

7   _sp = sys;
8   return 0;
9 }

```

## 6.4 Java Template (gy)

```

1  import java.io.*;
2  import java.math.*;
3  import java.util.*;

4  public class Template {
5      // Input
6      private static BufferedReader reader;
7      private static StringTokenizer tokenizer;
8      private static String next() {
9          try {
10             while (tokenizer == null || !tokenizer.hasMoreTokens())
11                 tokenizer = new StringTokenizer(reader.readLine());
12             } catch (IOException e) {
13                 // do nothing
14             }
15             return tokenizer.nextToken();
16         }
17         private static int nextInt() {
18             return Integer.parseInt(next());
19         }
20         private static double nextDouble() {
21             return Double.parseDouble(next());
22         }
23         private static BigInteger nextBigInteger() {
24             return new BigInteger(next());
25         }

26         public static void main(String[] args) {
27             reader = new BufferedReader(new InputStreamReader(System.in));
28             Scanner scanner = new Scanner(System.in);
29             while (scanner.hasNext())
30                 scanner.next();
31         }

32         // BigInteger & BigDecimal
33         private static void bigDecimal() {
34             BigDecimal a = BigDecimal.valueOf(1.0);
35             BigDecimal b = a.setScale(50, RoundingMode.HALF_EVEN);
36             BigDecimal c = b.abs();
37             // if scale omitted, b.scale is used
38             BigDecimal d = c.divide(b, 50, RoundingMode.HALF_EVEN);
39             // since Java 9
40             BigDecimal e = d.sqrt(new MathContext(50, RoundingMode.HALF_EVEN));
41             BigDecimal x = new BigDecimal(BigInteger.ZERO);
42             BigInteger y = BigDecimal.ZERO.toBigInteger(); // RoundingMode.DOWN
43             y = BigDecimal.ZERO.setScale(0, RoundingMode.HALF_EVEN).unscaledValue();
44         }

45         // sqrt for Java 8
46         // can solve scale=100 for 10000 times in about 1 second
47         private static BigDecimal sqrt(BigDecimal a, int scale) {
48             if (a.compareTo(BigDecimal.ZERO) < 0)
49                 return BigDecimal.ZERO.setScale(scale, RoundingMode.HALF_EVEN);
50             int length = a.precision() - a.scale();

```

```

51     BigDecimal ret = new BigDecimal(BigInteger.ONE, -length / 2);
52     for (int i = 1; i <= Integer.highestOneBit(scale) + 10; i++)
53         ret = ret.add(a.divide(ret, scale,
54             ↪ RoundingMode.HALF_EVEN)).divide(BigDecimal.valueOf(2), scale,
55             ↪ RoundingMode.HALF_EVEN);
56     return ret;
57 }
58 // can solve a=2^10000 for 100000 times in about 1 second
59 private static BigInteger sqrt(BigInteger a) {
60     int length = a.bitLength() - 1;
61     BigInteger l = BigInteger.ZERO.setBit(length / 2), r = BigInteger.ZERO.setBit(length / 2);
62     while (!l.equals(r)) {
63         BigInteger m = l.add(r).shiftRight(1);
64         if (m.multiply(m).compareTo(a) < 0)
65             l = m.add(BigInteger.ONE);
66         else
67             r = m;
68     }
69     return l;
70 }
71
72 // Collections
73 private static void arrayList() {
74     List<Integer> list = new ArrayList<>();
75     // Generic array is banned
76     List[] lists = new List[100];
77     lists[0] = new ArrayList<Integer>();
78     // for List<Integer>, remove(Integer) stands for element, while remove(int) stands for
79     ↪ index
80     list.remove(list.get(1));
81     list.remove(list.size() - 1);
82     list.clear();
83     Queue<Integer> queue = new LinkedList<>();
84     // return the value without popping
85     queue.peek();
86     // pop and return the value
87     queue.poll();
88     Queue<Integer> priorityQueue = new PriorityQueue<>();
89     Deque<Integer> deque = new ArrayDeque<>();
90     deque.peekFirst();
91     deque.peekLast();
92     deque.pollFirst();
93     TreeSet<Integer> set = new TreeSet<>();
94     TreeSet<Integer> anotherSet = new TreeSet<>(Comparator.reverseOrder());
95     set.ceiling(1);
96     set.floor(1);
97     set.lower(1);
98     set.higher(1);
99     set.contains(1);
100     HashSet<Integer> hashSet = new HashSet<>();
101     HashMap<String, Integer> map = new HashMap<>();
102     map.put("", 1);
103     map.get("");
104     map.forEach((string, integer) -> System.out.println(string + integer));
105     TreeMap<String, Integer> treeMap = new TreeMap<>();
106     Arrays.sort(new int[10]);
107     Arrays.sort(new Integer[10], (a, b) -> {
108         if (a.equals(b)) return 0;
109         if (a > b) return -1;
110         return 1;
111     });

```

```

108     Arrays.sort(new Integer[10], Comparator.comparingInt((a) -> (int) a).reversed());
109     long a = 1_000_000_000_000_000L;
110     int b = Integer.MAX_VALUE;
111     int c = 'a';
112 }
113 }

```

## 6.5 Big Fraction (gy)

```

1 fun gcd(a: Long, b: Long): Long = if (b == 0L) a else gcd(b, a % b)
2 class Fraction(val a: BigInteger, val b: BigInteger) {
3     constructor(a: Long, b: Long) : this(BigInteger.valueOf(a / gcd(a, b)), BigInteger.valueOf(b /
4         ↪ gcd(a, b)))
5
6     operator fun plus(o: Fraction): Fraction {
7         var gcd = b.gcd(o.b)
8         val tempProduct = (b / gcd) * (o.b / gcd)
9         var ansA = a * (o.b / gcd) + o.a * (b / gcd)
10        val gcd2 = ansA.gcd(gcd)
11        ansA /= gcd2
12        gcd /= gcd2
13        return Fraction(ansA, gcd * tempProduct)
14    }
15
16    operator fun minus(o: Fraction): Fraction {
17        var gcd = b.gcd(o.b)
18        val tempProduct = (b / gcd) * (o.b / gcd)
19        var ansA = a * (o.b / gcd) - o.a * (b / gcd)
20        val gcd2 = ansA.gcd(gcd)
21        ansA /= gcd2
22        gcd /= gcd2
23        return Fraction(ansA, gcd * tempProduct)
24    }
25
26    operator fun times(o: Fraction): Fraction {
27        val gcd1 = a.gcd(o.b)
28        val gcd2 = b.gcd(o.a)
29        return Fraction((a / gcd1) * (o.a / gcd2), (b / gcd2) * (o.b / gcd1))
30    }
31 }

```

## 6.6 模拟退火 (ct)

```

1 db ans_x, fans;
2 inline double rand01() {return rand() / 2147483647.0;}
3 inline double randp() {return (rand() & 1 ? 1 : -1) * rand01();}
4 inline double f(double x)
5 {
6     /*
7      * write your function here.
8      */
9     if (maxx < fans) {fans = maxx; ans_x = x;}
10    return maxx;
11 }
12 int main()
13 {
14    srand(time(NULL) + clock());

```



```

15 db x = 0, fnow = f(x);
16 fans = 1e30;
17 for (db T = 1e4; T > 1e-4; T *= 0.997)
18 {
19     db nx = x + randp() * T, fnext = f(nx);
20     db delta = fnext - fnow;
21     if (delta < 1e-9 || exp(-delta / T) > rand01())
22     {
23         x = nx;
24         fnow = fnext;
25     }
26 }
27 return 0;
28 }

```

## 6.7 三分 (ct)

```

1 inline db cubic_search()
2 {
3     double l = -1e4, r = 1e4;
4     for (int i = 1; i <= 100; ++i)
5     {
6         double ll = (l + r) * 0.5;
7         double rr = (ll + r) * 0.5;
8         if (check(ll) < check(rr)) r = rr;
9         else l = ll;
10    }
11    return (l + r) * 0.5;
12 }

```

## 6.8 Zeller Congruence

```

1 inline db cubic_search()
2 {
3     double l = -1e4, r = 1e4;
4     for (int i = 1; i <= 100; ++i)
5     {
6         double ll = (l + r) * 0.5;
7         double rr = (ll + r) * 0.5;
8         if (check(ll) < check(rr)) r = rr;
9         else l = ll;
10    }
11    return (l + r) * 0.5;
12 }

```

## 6.9 博弈论模型 (gy)

- Wythoff's game

给定两堆石子，每次可以从任意一堆中取至少一个石子，或从两堆中取相同的至少一个石子，取走最后石子的胜

先手胜当且仅当石子数满足：

$$\lfloor (b - a) \times \phi \rfloor = a, (a \leq b, \phi = \frac{\sqrt{5}+1}{2})$$

先手胜对应的石子数构成两个序列：

Lower Wythoff sequence:  $a_n = \lfloor n \times \phi \rfloor$

Upper Wythoff sequence:  $b_n = \lfloor n \times \phi^2 \rfloor$

- Fibonacci nim

给定一堆石子，第一次可以取至少一个、少于石子总数数量的石子，之后每次可以取至少一个、不超过上次取石子数量两倍的石子，取走最后石子的胜  
先手胜当且仅当石子数为斐波那契数

## 6.10 积分表

- $\sin x \rightarrow -\cos x$
- $\cos x \rightarrow \sin x$
- $\tan x \rightarrow -\ln \cos x$
- $\sec x \rightarrow \ln \left| \sin \frac{x}{2} + \cos \frac{x}{2} \right| - \ln \left| -\sin \frac{x}{2} + \cos \frac{x}{2} \right|$
- $\csc x \rightarrow \ln \left| \sin \frac{x}{2} \right| - \ln \left| \cos \frac{x}{2} \right|$
- $\sin^2 x \rightarrow \frac{x}{2} - \frac{1}{2} \sin x \cos x$
- $\cos^2 x \rightarrow \frac{x}{2} + \frac{1}{2} \sin x \cos x$
- $\tan^2 x \rightarrow \tan x - x$
- $\sec^2 x \rightarrow \tan x$
- $\csc^2 x \rightarrow -\tan x$
- $\arcsin x \rightarrow \frac{1}{\sqrt{1-x^2}}$
- $\arccos x \rightarrow -\frac{1}{\sqrt{1-x^2}}$
- $\arctan x \rightarrow \frac{1}{1+x^2}$
- $a^x \rightarrow \frac{a^x}{\ln a}$
- $\frac{1}{x^2+a^2} \rightarrow \frac{1}{|a|} \arctan \frac{x}{|a|}$
- $\frac{1}{x^2-a^2} \rightarrow \frac{1}{2} \ln |x-a| - \frac{1}{2} \ln |x+a|$
- $\frac{x}{ax+b} \rightarrow \frac{x}{a} - \frac{b}{a^2} \ln |ax+b|$
- $\frac{x}{ax^2+c} \rightarrow \frac{1}{2a} \ln |ax^2+c|$
- $\sqrt{c+x^2} \rightarrow \frac{x}{2} \sqrt{c+x^2} + \frac{c}{2} \ln |x+\sqrt{c+x^2}|$
- $\sqrt{c-x^2} \rightarrow \frac{x}{2} \sqrt{c-x^2} + \frac{c}{2} \arctan \frac{x}{\sqrt{c-x^2}}$
- $\frac{1}{\sqrt{c+x^2}} \rightarrow \ln |x+\sqrt{c+x^2}|$
- $\frac{1}{\sqrt{c-x^2}} \rightarrow \arctan \frac{x}{\sqrt{c-x^2}}$

## 6.11 公式、数列、定理

- 求和公式

$$\begin{aligned}
 & - \sum_{k=1}^n (2k-1)^2 = \frac{1}{3}n(4n^2-1) \\
 & - \sum_{k=1}^n k^3 = \frac{1}{4}n^2(n+1)^2
 \end{aligned}$$

$$\begin{aligned}
& - \sum_{k=1}^n (2k-1)^3 = n^2(2n^2-1) \\
& - \sum_{k=1}^n k^4 = \frac{1}{30}n(n+1)(2n+1)(3n^2+3n-1) \\
& - \sum_{k=1}^n k^5 = \frac{1}{12}n^2(n+1)^2(2n^2+2n-1) \\
& - \sum_{k=1}^n k(k+1) = \frac{1}{3}n(n+1)(n+2) \\
& - \sum_{k=1}^n k(k+1)(k+2) = \frac{1}{4}n(n+1)(n+2)(n+3) \\
& - \sum_{k=1}^n k(k+1)(k+2)(k+3) = \frac{1}{5}n(n+1)(n+2)(n+3)(n+4)
\end{aligned}$$

- **错排公式**

$D_n$  表示  $n$  个元素错位排列的方案数

$$D_1 = 0, D_2 = 1$$

$$D_n = (n-1)(D_{n-2} + D_{n-1}), n \geq 3$$

$$D_n = n! \cdot (1 - \frac{1}{1!} + \frac{1}{2!} - \cdots + (-1)^n \frac{1}{n!})$$

- **Fibonacci sequence**

$$F_0 = 0, F_1 = 1$$

$$F_n = F_{n-1} + F_{n-2}$$

$$F_{n+1} \cdot F_{n-1} - F_n^2 = (-1)^n$$

$$F_{-n} = (-1)^n F_n$$

$$F_{n+k} = F_k \cdot F_{n+1} + F_{k-1} \cdot F_n$$

$$\gcd(F_m, F_n) = F_{\gcd(m, n)}$$

$$F_m \mid F_n^2 \Leftrightarrow n F_n \mid m$$

$$F_n = \frac{\varphi^n - \Psi^n}{\sqrt{5}}, \varphi = \frac{1+\sqrt{5}}{2}, \Psi = \frac{1-\sqrt{5}}{2}$$

$$F_n = \lfloor \frac{\varphi^n}{\sqrt{5}} + \frac{1}{2} \rfloor, n \geq 0$$

$$n(F) = \lfloor \log_{\varphi}(F \cdot \sqrt{5} + \frac{1}{2}) \rfloor$$

- **第一类 Stirling number**

用  $s(n, k) = (-1)^{n-k} \begin{bmatrix} n \\ k \end{bmatrix}$  表示第一类 Stirling number

$$\begin{bmatrix} n+1 \\ k \end{bmatrix} = n \begin{bmatrix} n \\ k \end{bmatrix} + \begin{bmatrix} n \\ k-1 \end{bmatrix}, k > 0$$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = 1, \begin{bmatrix} n \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ n \end{bmatrix} = 0, n > 0$$

$\begin{bmatrix} n \\ k \end{bmatrix}$  为将  $n$  个元素分成  $k$  个环的方案数

- **第二类 Stirling number**

用  $S(n, k) = \left\{ \begin{smallmatrix} n \\ k \end{smallmatrix} \right\}$  表示第二类 Stirling number

$$\left\{ \begin{smallmatrix} n+1 \\ k \end{smallmatrix} \right\} = k \left\{ \begin{smallmatrix} n \\ k \end{smallmatrix} \right\} + \left\{ \begin{smallmatrix} n \\ k-1 \end{smallmatrix} \right\}, k > 0$$

$$\left\{ \begin{smallmatrix} 0 \\ 0 \end{smallmatrix} \right\} = 1, \left\{ \begin{smallmatrix} n \\ 0 \end{smallmatrix} \right\} = \left\{ \begin{smallmatrix} 0 \\ n \end{smallmatrix} \right\} = 0, n > 0$$

$$\left\{ \begin{smallmatrix} n \\ k \end{smallmatrix} \right\} = \frac{1}{k!} \sum_{j=0}^k (-1)^{k-j} \binom{k}{j} j^n$$

$\left\{ \begin{smallmatrix} n \\ k \end{smallmatrix} \right\}$  为将  $n$  个元素划分成  $k$  个非空集合的方案数

- **Catalan number**

$c_n$  表示长度为  $2n$  的合法括号序的数量

$$c_1 = 1, c_{n+1} = \sum_{i=1}^n c_i \times c_{n+1-i}$$

$$c_n = \frac{\binom{2n}{n}}{n+1}$$

- **Bell number**

$B_n$  表示基数为  $n$  的集合的划分方案数

$$B_i = \begin{cases} 1 & i = 0 \\ \sum_{k=0}^n \binom{n}{k} B_k & i > 0 \end{cases}$$

$$B_n = \sum_{k=0}^n \{n \atop k\}$$

- **五边形数定理**

$p(n)$  表示将  $n$  划分为若干个正整数之和的方案数

$$p(n) = \sum_{k \in \mathbb{N}^*} (-1)^{k-1} p(n - \frac{k(3k-1)}{2})$$

- **Bernoulli number**

$$\sum_{j=0}^m \binom{m+1}{j} B_j = 0, m > 0$$

$$B_i = \begin{cases} 1 & i = 0 \\ -\frac{\sum_{j=0}^{i-1} \binom{i+1}{j}}{i+1} & i > 0 \end{cases}$$

$$\sum_{k=1}^n k^m = \frac{1}{m+1} \sum_{k=0}^m \binom{m+1}{k} B_k n^{m+1-k}$$

- **Möbius function**

$$\mu(n) = \begin{cases} 1 & n \text{ is a square-free positive integer with an even number of prime factors} \\ -1 & n \text{ is a square-free positive integer with an odd number of prime factors} \\ 0 & n \text{ has a squared prime factor} \end{cases}$$

$$\sum_{d|n} \mu(d) = \begin{cases} 1 & n = 1 \\ 0 & n > 1 \end{cases}$$

$$g(n) = \sum_{d|n} f(d) \Leftrightarrow f(n) = \sum_{d|n} \mu(d) g(\frac{n}{d})$$

- **Lagrange polynomial**

给定次数为  $n$  的多项式函数  $L(x)$  上的  $n+1$  个点  $(x_0, y_0), (x_1, y_1), \dots, (x_n, y_n)$

$$\text{则 } L(x) = \sum_{j=0}^n y_j \prod_{0 \leq m \leq n, m \neq j} \frac{x - x_m}{x_j - x_m}$$

- **树的计数**

- **有根树计数**

$$a_1 = 1$$

$$a_{n+1} = \frac{\sum_{j=1}^n j \cdot a_j \cdot S_{n,j}}{n}$$

$$S_{n,j} = \sum_{i=1}^{n/j} a_{n+1-ij} = S_{n-j,j} + a_{n+1-j}$$

- **无根树计数**

$$\begin{cases} a_n - \sum_{i=1}^{n/2} a_i a_{n-i} & n \text{ is odd} \\ a_n - \sum_{i=1}^{n/2} a_i a_{n-i} + \frac{1}{2} a_{\frac{n}{2}} (a_{\frac{n}{2}} + 1) & n \text{ is even} \end{cases}$$

- **完全图生成树计数**

$$n^{n-2}$$

- **矩阵-树定理**

设  $\mathbf{A}[G]$  为图  $G$  的邻接矩阵、 $\mathbf{D}[G]$  为图  $G$  的度数矩阵，则图  $G$  的不同生成树的个数为  $\mathbf{C}[G] = \mathbf{D}[G] - \mathbf{A}[G]$  的任意一个  $n-1$  阶主子式的行列式值。

- **Euler characteristic**

平面图形的顶点个数  $V$ ，边数  $E$ ，平面被划分的区域数  $F$ ，组成图形的连通部分的数目  $C$  满足：

$$V - E + F = C + 1$$

- **Pick theorem**

顶点为整点的简单多边形，其面积  $A$ ，内部格点数  $i$ ，边上格点数  $b$  满足：

$$A = i + \frac{b}{2} - 1$$

- **平面几何公式**

- **三角形**

半周长  $p = \frac{a+b+c}{2}$

面积  $S = \frac{1}{2}aH_a = \frac{1}{2}ab \cdot \sin C = \sqrt{p(p-a)(p-b)(p-c)} = pr = \frac{abc}{4R}$

中线长  $M_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2} = \frac{1}{2}\sqrt{b^2 + c^2 + 2bc \cdot \cos A}$

角平分线长  $T_a = \frac{\sqrt{bc((b+c)^2 - a^2)}}{b+c} = \frac{2bc}{b+c} \cos \frac{A}{2}$

高  $H_a = b \sin C = \sqrt{b^2 - \left(\frac{a^2 + b^2 - c^2}{2a}\right)^2}$

内切圆半径  $r = \frac{S}{p} = 4R \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2} = \sqrt{\frac{(p-a)(p-b)(p-c)}{p}} = p \tan \frac{A}{2} \tan \frac{B}{2} \tan \frac{C}{2}$

外接圆半径  $R = \frac{abc}{4S} = \frac{a}{2 \sin A}$