

Platelet

Team Reference Material

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Chapter 1

Graph Theory

1.1 Dijkstra

1.2 2-SAT

1.3 双连通分量

1.3.1 点双连通分量

1.3.2 边双连通分量

1.4 K 短路

1.5 最大团

1.6 一般图最大匹配

1.7 树

1.7.1 虚树

1.7.2 矩阵树定理

1.7.3 点分治

1.7.4 Prufer 编码

1.7.5 Link-Cut Tree

1.7.6 树上倍增

1.7.7 数链剖分

1.8 仙人掌

1.9 带花树

1.10 KM 算法

1.11 支配树

1.11.1 DAG

1.11.2 一般图

1.12 弦图

1.13 网络流

Chapter 2

Math

2.1 int64 相乘取模

2.2 扩展欧几里得 (gy)

```
1 // return gcd(a, b)
2 // ax+by=gcd(a,b)
3 template <typename T>
4 T extend_gcd(T a, T b, T &x, T &y) {
5     if (b == 0) {
6         x = 1, y = 0;
7         return a;
8     }
9     T res = extend_gcd(b, a % b, x, y);
10    T t = y;
11    y = x - a / b * y;
12    x = t;
13    return res;
14 }
15
16 // return minimal positive integer x so that ax+by=c
17 // or -1 if such x does not exist
18 template <typename T>
19 inline T solve_equ(T a, T b, T c) {
20     T x, y, d;
21     d = extend_gcd(a, b, x, y);
22     if (c % d)
23         return -1;
24     T t = c / d;
25     x *= t;
26     y *= t;
27     T k = b / d;
28     x = (x % k + k) % k;
29     return x;
30 }
31 // return minimal positive integer x so that ax==b(mod p)
32 // or -1 if such x does not exist
33 template <typename T>
34 inline T solve(T a, T b, T p) {
35     a = (a % p + p) % p;
36     b = (b % p + p) % p;
37     return solve_equ(a, p, b);
38 }
```

2.3 中国剩余定理

2.4 组合数

2.4.1 Lucas 定理

2.4.2 组合数合数取模

2.5 高斯消元

2.6 Miller Rabin & Pollard Rho (gy)

```

1  /*
2  * In Java, use BigInteger.isProbablePrime(int certainty) to replace miller_rabin(BigInteger
   ↪ number)
3  * Test Set / First Wrong Answer
4  * 2 / 2,047
5  * 2, 3 / 1,373,653
6  * 31, 73 / 9,080,191
7  * 2, 3, 5 / 25,326,001
8  * 2, 3, 5, 7 / 3,215,031,751 (> Int.MAX_VALUE)
9  * 2, 7, 61 / 4,759,123,141
10 * 2, 13, 23, 1662803 / 1,122,004,669,633
11 * 2, 3, 5, 7, 11 / 2,152,302,898,747
12 * 2, 3, 5, 7, 11, 13 / 3,474,749,660,383
13 * 2, 3, 5, 7, 11, 13, 17 / 341,550,071,728,321
14 * 2, 3, 5, 7, 11, 13, 17, 19, 23 / 3,825,123,056,546,413,051
15 * 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37 / 318,665,857,834,031,151,167,461 (> Long.MAX_VALUE)
16 * 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41 / 3,317,044,064,679,887,385,961,981
17 */
18 #include <cstdint>
19 #include <cstdlib>
20 #include <vector>
21
22 const int test_case_size = 12;
23 const int test_cases[test_case_size] = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37};
24
25 int64_t multiply_mod(int64_t x, int64_t y, int64_t p) {
26     int64_t t = (x * y - (int64_t) ((long double) x / p * y + 1e-3) * p) % p;
27     return t < 0 ? t + p : t;
28 }
29
30 int64_t add_mod(int64_t x, int64_t y, int64_t p) {
31     return (0ull + x + y) % p;
32 }
33
34 int64_t power_mod(int64_t x, int64_t exp, int64_t p) {
35     int64_t ans = 1;
36     while (exp) {
37         if (exp & 1)
38             ans = multiply_mod(ans, x, p);
39         x = multiply_mod(x, x, p);
40         exp >>= 1;
41     }
42     return ans;
43 }
44
45 bool miller_rabin_check(int64_t prime, int64_t base) {
46     int64_t number = prime - 1;

```



```

47     for (; ~number & 1; number >>= 1)
48         continue;
49     int64_t result = power_mod(base, number, prime);
50     for (; number != prime - 1 && result != 1 && result != prime - 1; number <<= 1)
51         result = multiply_mod(result, result, prime);
52     return result == prime - 1 || (number & 1) == 1;
53 }
54
55 bool miller_rabin(int64_t number) {
56     if (number < 2)
57         return false;
58     if (number < 4)
59         return true;
60     if (~number & 1)
61         return false;
62     for (int i = 0; i < test_case_size && test_cases[i] < number; i++)
63         if (!miller_rabin_check(number, test_cases[i]))
64             return false;
65     return true;
66 }
67
68 int64_t gcd(int64_t x, int64_t y) {
69     return y == 0 ? x : gcd(y, x % y);
70 }
71
72 int64_t pollard_rho_test(int64_t number, int64_t seed) {
73     int64_t x = rand() % (number - 1) + 1, y = x;
74     int head = 1, tail = 2;
75     while (true) {
76         x = multiply_mod(x, x, number);
77         x = add_mod(x, seed, number);
78         if (x == y)
79             return number;
80         int64_t answer = gcd(std::abs(x - y), number);
81         if (answer > 1 && answer < number)
82             return answer;
83         if (++head == tail) {
84             y = x;
85             tail <<= 1;
86         }
87     }
88 }
89
90 void factorize(int64_t number, std::vector<int64_t> &divisor) {
91     if (number > 1) {
92         if (miller_rabin(number)) {
93             divisor.push_back(number);
94         } else {
95             int64_t factor = number;
96             while (factor >= number)
97                 factor = pollard_rho_test(number, rand() % (number - 1) + 1);
98             factorize(number / factor, divisor);
99             factorize(factor, divisor);
100         }
101     }
102 }
103
104 #include <cstdio>
105 int main() {
106     static int64_t tar;
107     static std::vector<int64_t> factors;

```

```

108     while (scanf("%lld", &tar)) {
109         printf("%d\n", miller_rabin(tar));
110         factors.clear();
111         factorize(tar, factors);
112         for (int64_t i : factors)
113             printf("%lld\n", i);
114     }
115     return 0;
116 }

```

2.7 $O(m^2 \log n)$ 线性递推

2.8 Polynomial

2.8.1 FFT

2.8.2 NTT & 多项式求逆

2.9 拉格朗日插值

2.10 杜教筛

2.11 BSGS

2.11.1 BSGS

2.11.2 扩展 BSGS

2.12 直线下整点个数

2.13 单纯形

2.14 辛普森积分

2.15 常用数列定理

- 第一类 Stirling Number
- 第二类 Stirling Number
- Catalan Number
 c_n 表示长度为 $2n$ 的合法括号序的数量

$$c_1 = 1, c_{n+1} = \sum_{i=1}^n c_i \times c_{n+1-i}$$

$$c_n = \frac{\binom{2n}{n}}{n+1}$$
- Bell Number
- Bernoulli Number

2.16 积分表

Chapter 3

Geometry

3.1 点、直线、圆

```
1  #include <cmath>
2  #include <algorithm>
3
4  using number = long double;
5  const number eps = 1e-8;
6
7  number _sqrt(number x) {
8      return std::sqrt(std::max(x, (number) 0));
9  }
10 number _asin(number x) {
11     x = std::min(x, (number) 1), x = std::max(x, (number) -1);
12     return std::asin(x);
13 }
14 number _acos(number x) {
15     x = std::min(x, (number) 1), x = std::max(x, (number) -1);
16     return std::acos(x);
17 }
18
19 int sgn(number x) {
20     return (x > eps) - (x < -eps);
21 }
22 int cmp(number x, number y) {
23     return sgn(x - y);
24 }
25
26 struct point {
27     number x, y;
28     point() {}
29     point(number x, number y) : x(x), y(y) {}
30
31     number len2() const {
32         return x * x + y * y;
33     }
34     number len() const {
35         return _sqrt(len2());
36     }
37     point unit() const {
38         return point(x / len(), y / len());
39     }
40     point rotate90() const {
41         return point(-y, x);
42     }
43 }
```

```

43
44     friend point operator+(const point &a, const point &b) {
45         return point(a.x + b.x, a.y + b.y);
46     }
47     friend point operator-(const point &a, const point &b) {
48         return point(a.x - b.x, a.y - b.y);
49     }
50     friend point operator*(const point &a, number b) {
51         return point(a.x * b, a.y * b);
52     }
53     friend point operator/(const point &a, number b) {
54         return point(a.x / b, a.y / b);
55     }
56     friend number dot(const point &a, const point &b) {
57         return a.x * b.x + a.y * b.y;
58     }
59     friend number det(const point &a, const point &b) {
60         return a.x * b.y - a.y * b.x;
61     }
62     friend number operator==(const point &a, const point &b) {
63         return cmp(a.x, b.x) == 0 && cmp(a.y, b.y) == 0;
64     }
65 };
66
67 number dis2(const point &a, const point &b) {
68     return (a - b).len2();
69 }
70 number dis(const point &a, const point &b) {
71     return (a - b).len();
72 }
73
74 struct line {
75     point a, b;
76     line() {}
77     line(point a, point b) : a(a), b(b) {}
78     point value() const {
79         return b - a;
80     }
81 };
82
83 bool point_on_line(const point &p, const line &l) {
84     return sgn(det(p - l.a, p - l.b)) == 0;
85 }
86 // including endpoint
87 bool point_on_ray(const point &p, const line &l) {
88     return sgn(det(p - l.a, p - l.b)) == 0 &&
89         sgn(dot(p - l.a, l.b - l.a)) >= 0;
90 }
91 // including endpoints
92 bool point_on_seg(const point &p, const line &l) {
93     return sgn(det(p - l.a, p - l.b)) == 0 &&
94         sgn(dot(p - l.a, l.b - l.a)) >= 0 &&
95         sgn(dot(p - l.b, l.a - l.b)) >= 0;
96 }
97 bool seg_has_intersection(const line &a, const line &b) {
98     if (point_on_seg(a.a, b) || point_on_seg(a.b, b) ||
99         point_on_seg(b.a, a) || point_on_seg(b.b, a))
100         return /* including endpoints */ true;
101     return sgn(det(a.a - b.a, b.b - b.a)) * sgn(det(a.b - b.a, b.b - b.a)) < 0
102         && sgn(det(b.a - a.a, a.b - a.a)) * sgn(det(b.b - a.a, a.b - a.a)) < 0;
103 }

```

```

104 point intersect(const line &a, const line &b) {
105     number s1 = det(a.b - a.a, b.a - a.a);
106     number s2 = det(a.b - a.a, b.b - a.a);
107     return (b.a * s2 - b.b * s1) / (s2 - s1);
108 }
109 point projection(const point &p, const line &l) {
110     return l.a + (l.b - l.a) * dot(p - l.a, l.b - l.a) / (l.b - l.a).len2();
111 }
112 number dis(const point &p, const line &l) {
113     return std::abs(dot(p - l.a, l.b - l.a)) / (l.b - l.a).len();
114 }
115 point symmetry_point(const point &a, const point &o) {
116     return o + o - a;
117 }
118 point reflection(const point &p, const line &l) {
119     return symmetry_point(p, projection(p, l));
120 }
121
122 struct circle {
123     point o;
124     number r;
125     circle() {}
126     circle(point o, number r) : o(o), r(r) {}
127 };
128
129 bool intersect(const line &l, const circle &a, point &p1, point &p2) {
130     number x = dot(l.a - a.o, l.b - l.a);
131     number y = (l.b - l.a).len2();
132     number d = x * x - y * ((l.a - a.o).len2() - a.r * a.r);
133     if (sgn(d) < 0) return false;
134     point p = l.a - (l.b - l.a) * (x / y), delta = (l.b - l.a) * (_sqrt(d) / y);
135     p1 = p + delta, p2 = p - delta;
136     return true;
137 }
138 bool intersect(const circle &a, const circle &b, point &p1, point &p2) {
139     if (a.o == b.o && cmp(a.r, b.r) == 0)
140         return /* value for coincident circles */ false;
141     number s1 = (b.o - a.o).len();
142     if (cmp(s1, a.r + b.r) > 0 || cmp(s1, std::abs(a.r - b.r)) < 0)
143         return false;
144     number s2 = (a.r * a.r - b.r * b.r) / s1;
145     number aa = (s1 + s2) / 2, bb = (s1 - s2) / 2;
146     point p = (b.o - a.o) * (aa / (aa + bb)) + a.o;
147     point delta = (b.o - a.o).unit().rotate90() * _sqrt(a.r * a.r - aa * aa);
148     p1 = p + delta, p2 = p - delta;
149     return true;
150 }
151 bool tangent(const point &p0, const circle &c, point &p1, point &p2) {
152     number x = (p0 - c.o).len2();
153     number d = x - c.r * c.r;
154     if (sgn(d) < 0) return false;
155     if (sgn(d) == 0)
156         return /* value for point_on_line */ false;
157     point p = (p0 - c.o) * (c.r * c.r / x);
158     point delta = ((p0 - c.o) * (-c.r * _sqrt(d) / x)).rotate90();
159     p1 = c.o + p + delta;
160     p2 = c.o + p - delta;
161     return true;
162 }
163 bool ex_tangent(const circle &a, const circle &b, line &l1, line &l2) {
164     if (cmp(std::abs(a.r - b.r), (b.o - a.o).len()) == 0) {

```

```

165     point p1, p2;
166     intersect(a, b, p1, p2);
167     l1 = l2 = line(p1, p1 + (a.o - p1).rotate90());
168     return true;
169 } else if (cmp(a.r, b.r) == 0) {
170     point dir = b.o - a.o;
171     dir = (dir * (a.r / dir.len())).rotate90();
172     l1 = line(a.o + dir, b.o + dir);
173     l2 = line(a.o - dir, b.o - dir);
174     return true;
175 } else {
176     point p = (b.o * a.r - a.o * b.r) / (a.r - b.r);
177     point p1, p2, q1, q2;
178     if (tangent(p, a, p1, p2) && tangent(p, b, q1, q2)) {
179         l1 = line(p1, q1);
180         l2 = line(p2, q2);
181         return true;
182     } else {
183         return false;
184     }
185 }
186 }
187 bool in_tangent(const circle &a, const circle &b, line &l1, line &l2) {
188     if (cmp(a.r + b.r, (b.o - a.o).len()) == 0) {
189         point p1, p2;
190         intersect(a, b, p1, p2);
191         l1 = l2 = line(p1, p1 + (a.o - p1).rotate90());
192         return true;
193     } else {
194         point p = (b.o * a.r + a.o * b.r) / (a.r + b.r);
195         point p1, p2, q1, q2;
196         if (tangent(p, a, p1, p2) && tangent(p, b, q1, q2)) {
197             l1 = line(p1, q1);
198             l2 = line(p2, q2);
199             return true;
200         } else {
201             return false;
202         }
203     }
204 }

```

3.2 点到凸包切线

3.3 直线凸包交点

3.4 凸包游戏

3.5 半平面交

3.6 旋转卡壳

3.7 判断圆是否有交

3.8 最小圆覆盖

3.9 最小球覆盖

3.10 $O(n^2 \log n)$ 圆交面积和重心

3.11 圆与多边形交

3.12 $O(n \log n)$ 凸多边形内的最大圆

3.13 三角形的五心

3.14 三维凸包

3.15 三维绕轴旋转

3.16 几何公式

Chapter 4

String

4.1 KMP

4.2 AC 自动机

4.3 后缀数组

4.4 后缀自动机

4.5 Manacher

4.6 回文自动机

4.7 最小表示法

Chapter 5

Data Structure

5.1 莫队 (ct)

```
1 //
2 // Title: Modui
3 // Date: 26.02.2016
4 // Test:BZOJ-2038
5 // Complexity:  $O(n^{3/2})$ 
6 //
7 /*
8     莫队算法——将所有询问储存起来，然后分块暴力处理。
9     时间复杂度为  $O(n \times \sqrt{n})$ 。
10 */
11 #include <cstdio>
12 #include <cstring>
13 #include <algorithm>
14 #include <cmath>
15
16 #ifdef WIN32
17     #define LL "%I64d"
18 #else
19     #define LL "%lld"
20 #endif
21
22 #ifdef CT
23     #define debug(...) printf(__VA_ARGS__)
24 #else
25     #define debug(...)
26 #endif
27
28 #define R register
29 #define getc() (S==T&&(T=(S=B)+fread(B,1,1<<15,stdin),S==T)?EOF:*S++)
30 #define gmax(_a, _b) ((_a) > (_b) ? (_a) : (_b))
31 #define gmin(_a, _b) ((_a) < (_b) ? (_a) : (_b))
32 #define cmax(_a, _b) (_a < (_b) ? _a = (_b) : 0)
33 #define cmin(_a, _b) (_a > (_b) ? _a = (_b) : 0)
34 char B[1<<15], *S=B, *T=B;
35 inline int FastIn()
36 {
37     R char ch; R int cnt=0; R bool minus=0;
38     while (ch=getc(), (ch < '0' || ch > '9') && ch != '-') ;
39     ch == '-' ? minus=1: cnt=ch-'0';
40     while (ch=getc(), ch >= '0' && ch <= '9') cnt = cnt * 10 + ch - '0';
41     return minus?-cnt:cnt;
42 }
```

```

43 #define maxn 50010
44 int col[maxn], num[maxn], size, pos[maxn];
45 long long up[maxn], dw[maxn], ans;
46 struct Query{
47     int l, r, id;
48 }q[maxn];
49 inline bool cmp(const Query &i, const Query &j){
50     return pos[i.l] != pos[j.l] ? (i.l < j.l) : (pos[i.l] & 1 ? i.r < j.r : i.r > j.r);
51 }
52 inline long long gcd(R long long a, R long long b){
53     R long long tmp;
54     while (b){
55         tmp = b;
56         b = a % b;
57         a = tmp;
58     }
59     return a;
60 }
61 inline void update(R int x, R int d){
62     ans -= num[col[x]] * num[col[x]];
63     num[col[x]] += d;
64     ans += num[col[x]] * num[col[x]];
65 }
66 int main()
67 {
68     R int n = FastIn(), m = FastIn(); size = (int)sqrt(n * 1.0);
69     for (R int i = 1; i <= n; i++) col[i] = FastIn(), pos[i] = (i - 1) / size + 1;
70     for (R int i = 1; i <= m; i++){
71         q[i].l = FastIn(); q[i].r = FastIn(); q[i].id = i;
72     }
73     std::sort(q + 1, q + m + 1, cmp);
74     R int l = 1, r = 0;
75     for (R int i = 1; i <= m; i++){
76         R int id_now = q[i].id;
77         if (q[i].l == q[i].r){
78             up[id_now] = 0; dw[id_now] = 1; continue;
79         }
80         for (; r < q[i].r; r++) update(r + 1, 1);
81         for (; r > q[i].r; r--) update(r, -1);
82         for (; l < q[i].l; l++) update(l, -1);
83         for (; l > q[i].l; l--) update(l - 1, 1);
84         R long long aa, bb, cc;
85         aa = ans - q[i].r * q[i].l - 1;
86         bb = (long long)(q[i].r - q[i].l + 1) * (q[i].r - q[i].l);
87         cc = gcd(aa, bb); aa /= cc; bb /= cc;
88         up[id_now] = aa; dw[id_now] = bb;
89     }
90     for (R int i = 1; i <= m; i++) printf("%lld/%lld\n", up[i], dw[i]);
91     return 0;
92 }

```

5.2 ST 表 (ct)

```

1 #include <stdio>
2
3 #define dmax(_a, _b) ((_a) > (_b) ? (_a) : (_b))
4
5 #define maxn 200010
6 int a[maxn], f[20][maxn], n;

```

```

7  int Log[maxn];
8
9  void build()
10 {
11     for (int i = 1; i <= n; ++i) f[0][i] = a[i];
12
13     int lim = Log[n];
14     for (int j = 1; j <= lim; ++j)
15     {
16         int *fj = f[j], *fj1 = f[j - 1];
17         for (int i = 1; i <= n - (1 << j) + 1; ++i)
18             fj[i] = dmax(fj1[i], fj1[i + (1 << (j - 1))]);
19     }
20 }
21 int Query(int l, int r)
22 {
23     int k = Log[r - l + 1];
24     return dmax(f[k][l], f[k][r - (1 << k) + 1]);
25 }
26 int main()
27 {
28     scanf("%d", &n);
29     Log[0] = -1;
30     for (int i = 1; i <= n; ++i)
31     {
32         scanf("%d", &a[i]);
33         Log[i] = Log[i >> 1] + 1;
34     }
35     build();
36     int q;
37     scanf("%d", &q);
38     for (; q; --q)
39     {
40         int l, r; scanf("%d%d", &l, &r);
41         printf("%d\n", Query(l, r));
42     }
43 }

```

5.3 可并堆 (ct)

```

1  struct Node {
2      Node *ch[2];
3      ll val; int size;
4      inline void update()
5      {
6          size = ch[0] -> size + ch[1] -> size + 1;
7      }
8  } mem[maxn], *rt[maxn];
9  Node *merge(Node *a, Node *b)
10 {
11     if (a == mem) return b;
12     if (b == mem) return a;
13     if (a -> val < b -> val) std::swap(a, b);
14     std::swap(a -> ch[0], a -> ch[1]);
15     a -> ch[1] = merge(a -> ch[1], b);
16     a -> update();
17     return a;
18 }

```

5.4 线段树 (ct)

5.4.1 ZKW 线段树

```

1 <<<<<< HEAD
2 //
3 // Title:ZKW Segment Tree
4 // Date:19.11.2015
5 // Complexity:
6 // Build Tree: $O(N)$ 
7 // Query: $O(\log N)$ 
8 // Change: $O(\log N)$ 
9
10 #include<cstdio>
11 #include<cmath>
12 #define maxn 100000
13 #define R register
14 int T[1<<18|1],n,m,M;
15
16 inline int FastIn()
17 {
18     R char ch=getchar();R int cnt=0;R bool minus=0;
19     while ((ch<'0' || ch>'9') && ch!='-') ch=getchar();
20     if (ch=='-') minus=1,ch=getchar();
21     while (ch>='0' && ch<='9') cnt=cnt*10+ch-'0',ch=getchar();
22     return minus?-cnt:cnt;
23 }
24
25 inline void Build_Tree()
26 {
27     for (R int i=M-1;i>=1;i--)
28         T[i]=T[2*i]+T[2*i+1];
29 }
30
31 inline int Query(int s,int t)
32 {
33     R int Ans;
34     for (Ans=0,s=s+M-1,t=t+M+1;s^t^1;s>>=1,t>>=1)
35     {
36         if (~s&1) Ans+=T[s^1];
37         if (t&1) Ans+=T[t^1];
38     }
39     return Ans;
40 }
41
42 inline void Change(int x,int NewValue)
43 {
44     R int i=M+x;
45     for (T[i]=NewValue,i>>=1;i;i>>=1)
46         T[i]=T[2*i]+T[2*i+1];
47 }
48
49 int main()
50 {
51     n=FastIn();m=FastIn();
52     for (M=1;M<=n;M<<=1);
53     for (R int i=0;i<n;i++)
54         T[M+i]=FastIn();
55     Build_Tree();
56     for (R int i=1;i<=m;i++)

```

```

57     {
58         R char cmd=getchar();
59         if (cmd=='Q')
60         {
61             R int a=FastIn()-1,b=FastIn()-1;
62             printf("%d\n",Query(a,b));
63         }
64         if (cmd=='M')
65         {
66             R int a=FastIn()-1,b=FastIn();
67             Change(a,b);
68         }
69     }
70     return 0;
71 }
72 =====
73 //
74 // Title:ZKW Segment Tree
75 // Date:19.11.2015
76 // Complexity:
77 // Build Tree:O(N)
78 // Query:O(logN)
79 // Change:O(logN)
80
81 #include<cstdio>
82 #include<cmath>
83 #define maxn 100000
84 #define R register
85 int T[1<<18|1],n,m,M;
86
87 inline int FastIn()
88 {
89     R char ch=getchar();R int cnt=0;R bool minus=0;
90     while ((ch<'0' || ch>'9') && ch!='-') ch=getchar();
91     if (ch=='-') minus=1,ch=getchar();
92     while (ch>='0' && ch<='9') cnt=cnt*10+ch-'0',ch=getchar();
93     return minus?-cnt:cnt;
94 }
95
96 inline void Build_Tree()
97 {
98     for (R int i=M-1;i>=1;i--)
99         T[i]=T[2*i]+T[2*i+1];
100 }
101
102 inline int Query(int s,int t)
103 {
104     R int Ans;
105     for (Ans=0,s=s+M-1,t=t+M+1;s^t^1;s>>=1,t>>=1)
106     {
107         if (~s&1) Ans+=T[s^1];
108         if (t&1) Ans+=T[t^1];
109     }
110     return Ans;
111 }
112
113 inline void Change(int x,int NewValue)
114 {
115     R int i=M+x;
116     for (T[i]=NewValue,i>=1;i>>=1)
117         T[i]=T[2*i]+T[2*i+1];

```

```

118 }
119
120 int main()
121 {
122     n=FastIn();m=FastIn();
123     for (M=1;M<=n;M<=1);
124     for (R int i=0;i<n;i++)
125         T[M+i]=FastIn();
126     Build_Tree();
127     for (R int i=1;i<=m;i++)
128     {
129         R char cmd=getchar();
130         if (cmd=='Q')
131         {
132             R int a=FastIn()-1,b=FastIn()-1;
133             printf("%d\n",Query(a,b));
134         }
135         if (cmd=='M')
136         {
137             R int a=FastIn()-1,b=FastIn();
138             Change(a,b);
139         }
140     }
141     return 0;
142 }
143 >>>>>> 49188fa6ef8b175c2f4a6388509d8dc5116ebccd

```

5.4.2 主席树

```

1 <<<<<<< HEAD
2 //
3 // Title: Functional Segment Tree
4 // Date:16.12.2015
5 // Complexity:O((n+m)logn)
6 // Test:YZ0J-1991
7 #include<cstdio>
8 #include<algorithm>
9 #define maxt 2000010
10 #define maxn 100010
11 #define R register
12 inline int FastIn(){
13     R char ch=getchar();R int cnt=0;
14     while (ch<'0' || ch>'9') ch=getchar();
15     while (ch>='0' && ch<='9') cnt=cnt*10+ch-'0',ch=getchar();
16     return cnt;
17 }
18
19 int ls[maxt],
20     rs[maxt],
21     count[maxt],
22     root[maxn],
23     tot;
24
25 int num[maxn],rank[maxn],n,m,r[maxn];
26
27 bool cmp(const int &i,const int &j){
28     return num[i]<num[j];
29 }
30
31 inline void Insert(int last,int left,int right,int pre)

```

```

32 {
33     count[++tot]=count[last]+1;
34     if (left==right) return;
35     R int mid=(left+right)>>1;
36     if (pre>mid){
37         rs[tot]=tot+1;
38         Insert(rs[last],mid+1,right,pre);
39     }
40     else{
41         ls[tot]=tot+1;
42         rs[tot]=rs[last];
43         Insert(ls[last],left,mid,pre);
44     }
45 }
46
47 inline int Query(int a,int b,int k)
48 {
49     R int l=1,r=n,mid,f1=a,f2=b,cnt,kk=k;
50     while (l<r){
51         mid=(l+r)>>1;cnt=count[ls[f2]]-count[ls[f1]];
52         if (cnt>=kk) f1=ls[f1],f2=ls[f2],r=mid;
53         else f1=rs[f1],f2=rs[f2],l=mid+1,kk-=cnt;
54     }
55     return l;
56 }
57
58 int main()
59 {
60     n=FastIn();m=FastIn();R int i,a,b,k;
61     for (i=1;i<=n;i++) num[i]=FastIn(),rank[i]=i;
62     std::sort(rank+1,rank+n+1,cmp);
63     std::sort(num+1,num+n+1);
64     for (i=1;i<=n;i++) r[rank[i]]=i;
65     for (i=1;i<=n;i++) {
66         root[i]=tot+1;
67         Insert(root[i-1],1,n,r[i]);
68     }
69     for (i=1;i<=m;i++){
70         a=FastIn();b=FastIn();k=FastIn();
71         printf("%d\n",num[Query(root[a-1],root[b],k)]);
72     }
73     return 0;
74 }
75 =====
76 //
77 // Title: Functional Segment Tree
78 // Date:16.12.2015
79 // Complexity:O((n+m)logn)
80 // Test:YZ0J-1991
81 #include<cstdio>
82 #include<algorithm>
83 #define maxt 2000010
84 #define maxn 100010
85 #define R register
86 inline int FastIn(){
87     R char ch=getchar();R int cnt=0;
88     while (ch<'0' || ch>'9') ch=getchar();
89     while (ch>='0'&&ch<='9') cnt=cnt*10+ch-'0',ch=getchar();
90     return cnt;
91 }
92

```

```

93 int ls[maxt],
94     rs[maxt],
95     count[maxt],
96     root[maxn],
97     tot;
98
99 int num[maxn],rank[maxn],n,m,r[maxn];
100
101 bool cmp(const int &i,const int &j){
102     return num[i]<num[j];
103 }
104
105 inline void Insert(int last,int left,int right,int pre)
106 {
107     count[++tot]=count[last]+1;
108     if (left==right) return;
109     R int mid=(left+right)>>1;
110     if (pre>mid){
111         rs[tot]=tot+1;
112         Insert(rs[last],mid+1,right,pre);
113     }
114     else{
115         ls[tot]=tot+1;
116         rs[tot]=rs[last];
117         Insert(ls[last],left,mid,pre);
118     }
119 }
120
121 inline int Query(int a,int b,int k)
122 {
123     R int l=1,r=n,mid,f1=a,f2=b,cnt,kk=k;
124     while (l<r){
125         mid=(l+r)>>1;cnt=count[ls[f2]]-count[ls[f1]];
126         if (cnt>=kk) f1=ls[f1],f2=ls[f2],r=mid;
127         else f1=rs[f1],f2=rs[f2],l=mid+1,kk-=cnt;
128     }
129     return l;
130 }
131
132 int main()
133 {
134     n=FastIn();m=FastIn();R int i,a,b,k;
135     for (i=1;i<=n;i++) num[i]=FastIn(),rank[i]=i;
136     std::sort(rank+1,rank+n+1,cmp);
137     std::sort(num+1,num+n+1);
138     for (i=1;i<=n;i++) r[rank[i]]=i;
139     for (i=1;i<=n;i++) {
140         root[i]=tot+1;
141         Insert(root[i-1],1,n,r[i]);
142     }
143     for (i=1;i<=m;i++){
144         a=FastIn();b=FastIn();k=FastIn();
145         printf("%d\n",num[Query(root[a-1],root[b],k)]);
146     }
147     return 0;
148 }
149 >>>>>> 49188fa6ef8b175c2f4a6388509d8dc5116ebccd

```


5.5 平衡树 (ct)

5.5.1 Splay

```

1 //
2 // Title : Splay Tree
3 // Date : 11.01.2016
4 // Complexity :  $O(n \log n)$  (期望)
5 // Test : BZOJ-1251
6 /*
7 */
8 #include <cstdio>
9 #include <cstring>
10 #include <algorithm>
11 #include <cmath>
12
13 #ifdef WIN32
14     #define LL "%I64d"
15 #else
16     #define LL "%lld"
17 #endif
18
19 #ifdef CT
20     #define debug(...) printf(__VA_ARGS__)
21 #else
22     #define debug(...)
23 #endif
24
25 #define R register
26 #define getc() (S==T&&(T=(S=B)+fread(B,1,1<<15,stdin),S==T)?EOF:*S++)
27 #define gmax(_a, _b) ((_a) > (_b) ? (_a) : (_b))
28 #define gmin(_a, _b) ((_a) < (_b) ? (_a) : (_b))
29 #define cmax(_a, _b) (_a < (_b) ? _a = (_b) : 0)
30 #define cmin(_a, _b) (_a > (_b) ? _a = (_b) : 0)
31 char B[1<<15], *S=B, *T=B;
32 inline int FastIn()
33 {
34     R char ch; R int cnt=0; R bool minus=0;
35     while (ch=getc(), (ch < '0' || ch > '9') && ch != '-') ;
36     ch == '-' ? minus=1 : cnt=ch-'0';
37     while (ch=getc(), ch >= '0' && ch <= '9') cnt = cnt * 10 + ch - '0';
38     return minus?-cnt:cnt;
39 }
40 #define maxn 50010
41 int n, Q, root;
42 int fa[maxn], ch[maxn][2], id[maxn], size[maxn];
43 int tag[maxn], mx[maxn], num[maxn];
44 bool rev[maxn];
45 inline void update(int x){
46     R int ls=ch[x][0], rs=ch[x][1];
47     mx[x]=num[x];
48     cmax(mx[x], mx[ls]); cmax(mx[x], mx[rs]);
49     size[x]=size[ls]+size[rs]+1;
50 } //更新
51 void build(int l, int r, int rt){
52     if (l>r) return ;
53     R int mid=l+r>>1;
54     fa[mid]=rt;
55     if (mid<rt) ch[rt][0]=mid;
56     else ch[rt][1]=mid;

```

```

57     build(l,mid-1,mid);
58     build(mid+1,r,mid);
59     update(mid);
60 }//建树
61 inline void pushdown(int x){
62     R int ls=ch[x][0],rs=ch[x][1];
63     if (tag[x]){
64         R int lazy=tag[x];
65         if (ls) tag[ls]+=lazy,num[ls]+=lazy,mx[ls]+=lazy;
66         if (rs) tag[rs]+=lazy,num[rs]+=lazy,mx[rs]+=lazy;
67         tag[x]=0;
68     }
69     if (rev[x]){
70         if (ls) rev[ls]^=1;
71         if (rs) rev[rs]^=1;
72         ch[x][1]=ls,ch[x][0]=rs;
73         rev[x]=0;
74     }
75 }//具体下传的过程
76 inline void rotate(int x){//把 x 向上旋转到 x 的父亲
77     R int f=fa[x],gf=fa[f],d=(ch[f][1]==x); //f 表示 x 的父亲, gf 是祖父, d 是 x 在其父亲的位置
78     if (f==root) root=x,ch[0][0]=x;
79     (ch[f][d]=ch[x][d^1])>0 ? fa[ch[f][d]]=f : 0; //把 x 的儿子中与 d 相反的节点来代替 x 的位置
80     (fa[x]=gf)>0 ? ch[gf][ch[gf][1]==f]=x : 0; //把 x 代替 f 的位置
81     fa[ch[x][d^1]=f]=x; //把 f 接到 x 的下面
82     update(f); //更新 f 节点
83 }
84 inline void splay(int x,int rt){//把 x 旋转到 rt
85     while (fa[x]!=rt){
86         R int f=fa[x],gf=fa[f];
87         if (gf!=rt) rotate((ch[gf][1]==f)^(ch[f][1]==x) ? x : f); //如果祖孙三代是相同方向就转
            // 父亲, 不然转自己
88         rotate(x);
89     }
90     update(x);
91 }
92 int find(int x,int rank){
93     if (tag[x]||rev[x]) pushdown(x);
94     R int ls=ch[x][0],rs=ch[x][1],lsize=size[ls];
95     if (lsize+1==rank) return x;
96     if (lsize>=rank) return find(ls,rank);
97     else return find(rs,rank-lsize-1);
98 }//找第 k 小
99 inline int prepare(int l,int r){
100     R int x=find(root,l-1);
101     splay(x,0);
102     x=find(root,r+1);
103     splay(x,root);
104     return ch[x][0];
105 }//把 l-1 旋到根, r+1 旋到右儿子, 然后返回 r+1 的左儿子, 返回一个包含 [l, r] 的节点
106 inline void add(int l,int r,int w){
107     R int x=prepare(l,r);
108     tag[x]+=w,num[x]+=w,mx[x]+=w;
109 }//区间加
110 inline void rever(int l,int r){
111     R int x=prepare(l,r);
112     rev[x]^=1;
113 }//区间翻转
114 inline void query(int l,int r){
115     R int x=prepare(l,r);
116     printf("%d\n",mx[x] );

```

```

117 }//区间查询最大值
118 inline int split(R int k){
119     R int ls;
120     if (k<size[root])
121     {
122         R int kth=find(root,k+1);
123         splay(kth);ls=ch[kth][0];
124         fa[ls]=0;ch[kth][0]=0;
125         size[kth]-=size[ls];
126     }
127     else{
128         ls=root;root=0;
129     }
130     return ls;
131 }//删除数列
132 inline void merge(R int nwrt){
133     if (!root) {root=nwrt;return;}
134     R int nw=find(root,1);
135     splay(nw);fa[nwrt]=nw;ch[nw][0]=nwrt;
136     size[nw]+=size[nwrt];
137 }//合并数列
138 int main()
139 {
140     n=FastIn()+2;Q=FastIn();R int i,l,r,v,cmd;mx[0]=-233333333;
141     build(1,n,0);root=(1+n)>>1;
142     for (;Q--;){
143         cmd=FastIn();l=FastIn()+1;r=FastIn()+1;
144         if (cmd==1) v=FastIn(),add(l,r,v);
145         else if (cmd==2) rever(l,r);
146         else query(l,r);
147     }
148     return 0;
149 }

```

5.5.2 非旋转 Treap

```

1  //
2  //  Title : Treap (unrotated)
3  //  Date : 13.04.2016
4  //  Test : BZOJ-3224
5  //  Complexity :  $O(n \log n)$  (期望)
6  //
7  /*
8      对于序列上的一些操作的问题——
9      解决办法：平衡树 Treap
10 */
11 #include <cstdio>
12 #include <cstring>
13 #include <algorithm>
14 #include <cmath>
15
16 #ifdef WIN32
17     #define LL "%I64d"
18 #else
19     #define LL "%lld"
20 #endif
21
22 #ifdef CT
23     #define debug(...) printf(__VA_ARGS__)
24     #define setfile()

```

```

25 #else
26     #define debug(...)
27     #define filename ""
28     #define setfile() freopen(filename".in", "r", stdin); freopen(filename".out", "w", stdout);
29 #endif
30
31 #define R register
32 #define getc() (S == T && (T = (S = B) + fread(B, 1, 1 << 15, stdin), S == T) ? EOF : *S++)
33 #define dmax(_a, _b) ((_a) > (_b) ? (_a) : (_b))
34 #define dmin(_a, _b) ((_a) < (_b) ? (_a) : (_b))
35 #define cmax(_a, _b) (_a < (_b) ? _a = (_b) : 0)
36 #define cmin(_a, _b) (_a > (_b) ? _a = (_b) : 0)
37 char B[1 << 15], *S = B, *T = B;
38 inline int FastIn()
39 {
40     R char ch; R int cnt = 0; R bool minus = 0;
41     while (ch = getc(), (ch < '0' || ch > '9') && ch != '-') ;
42     ch == '-' ? minus = 1 : cnt = ch - '0';
43     while (ch = getc(), ch >= '0' && ch <= '9') cnt = cnt * 10 + ch - '0';
44     return minus ? -cnt : cnt;
45 }
46 const int Ta = 1 << 16 | 3, Tb = 33333331;
47 int Tc;
48 inline int randint() {return Tc = Ta * Tc + Tb;}
49 struct Treap
50 {
51     int data, key, size;
52     Treap *ls, *rs;
53     Treap(int _val):data(_val), key(randint()), ls(NULL), rs(NULL), size(1){}
54     inline void update()
55     {
56         size = (ls ? ls -> size : 0) + (rs ? rs -> size : 0) + 1;
57     }
58 }*root;
59 inline int Size(Treap *x)
60 {
61     return x ? x -> size : 0;
62 }
63 //为了防止访问到空节点, 定义一个函数来访问 size
64 struct Pair
65 {
66     Treap *fir, *sec;
67 };
68 Treap *Merge(Treap *a, Treap *b)
69 {
70     if (!a) return b;
71     if (!b) return a;
72     if (a -> key < b -> key)
73     {
74         a -> rs = Merge(a -> rs, b);
75         a -> update();
76         return a;
77     }
78     else
79     {
80         b -> ls = Merge(a, b -> ls);
81         b -> update();
82         return b;
83     }
84 }
85 //按照 a, b 的顺序来合并两棵 Treap

```

```

86 Pair Split(Treap *x, int k)
87 {
88     if (!x) return (Pair){NULL, NULL};
89     Pair y; y.fir = NULL; y.sec = NULL;
90     if (Size(x -> ls) >= k)
91     {
92         y = Split(x -> ls, k);
93         x -> ls = y.sec;
94         x -> update();
95         y.sec = x;
96     }
97     else
98     {
99         y = Split(x -> rs, k - Size(x -> ls) - 1);
100        x -> rs = y.fir;
101        x -> update();
102        y.fir = x;
103    }
104    return y;
105 }
106 //将前 k 个的点分离出来
107 inline int Find(R int k)
108 {
109     Pair x = Split(root, k - 1);
110     Pair y = Split(x.sec, 1);
111     Treap *ans = y.fir;
112     root = Merge(Merge(x.fir, ans), y.sec);
113     return ans -> data;
114 }
115 //找到第 k 小的 data 值
116 int Get(Treap *x, R int val)
117 {
118     if (!x) return 0;
119     return val < x -> data ? Get(x -> ls, val) : Get(x -> rs, val) + Size(x -> ls) + 1;
120 }
121 //找到 val 的排名
122 inline void Insert(R int val)
123 {
124     R int k = Get(root, val);
125     Pair x = Split(root, k);
126     Treap *pre = new Treap(val);
127     root = Merge(Merge(x.fir, pre), x.sec);
128 }
129 //插入
130 inline void Delete(R int val)
131 {
132     R int k = Get(root, val);
133     Pair x = Split(root, k - 1);
134     Pair y = Split(x.sec, 1);
135     root = Merge(x.fir, y.sec);
136 }
137 //单点删除
138 inline int upper(R int val)
139 {
140     R int ans = 1e9;
141     Treap *tmp = root;
142     while (tmp)
143     {
144         if (tmp -> data > val)
145         {
146             cmin(ans, tmp -> data);

```

```

147         tmp = tmp -> ls;
148     }
149     else
150         tmp = tmp -> rs;
151 }
152 return ans;
153 }
154 inline int lower(R int val)
155 {
156     R int ans = -1e9;
157     Treap *tmp = root;
158     while (tmp)
159     {
160         if (tmp -> data < val)
161         {
162             cmax(ans, tmp -> data);
163             tmp = tmp -> rs;
164         }
165         else tmp = tmp -> ls;
166     }
167     return ans;
168 }
169 void print(Treap *x)
170 {
171     if (!x) return;
172     print(x -> ls);
173     printf("%d ", x -> data );
174     print(x -> rs);
175 }
176 int main()
177 {
178     root = NULL;
179     for (R int Q = FastIn(); Q; --Q)
180     {
181         R int opt = FastIn(), x = FastIn();
182         if (opt == 1) Insert(x);
183         else if (opt == 2) Delete(x);
184         else if (opt == 3)
185         {
186             R int ans = Get(root, x);
187             while (ans > 1 && Find(ans - 1) == x) ans--;
188             printf("%d\n", ans );
189         }
190         else if (opt == 4) printf("%d\n", Find(x) );
191         else if (opt == 5) printf("%d\n", lower(x) );
192         else printf("%d\n", upper(x) );
193     }
194     return 0;
195 }
196 /*
197 input:
198 10
199 1 106465
200 4 1
201 1 317721
202 1 460929
203 1 644985
204 1 84185
205 1 89851
206 6 81968
207 1 492737

```

```

208 5 493598
209
210 output:
211 106465
212 84185
213 492737
214
215 input2:
216 5
217 1 1
218 1 1
219 1 1
220 1 2
221 3 1
222 output2:
223 1
224 */

```

5.5.3 可持久化平衡树

```

1 //
2 // Title: Functional Treap
3 // Date: 16.04.2016
4 // Test: YZOJ-1620
5 // Complexity:  $O(n \log n)$  (期望)
6 //
7 /*
8     可持久化 Treap:
9     用来解决超级编辑器等问题。
10    优势: 好写好调好理解的平衡树
11    缺点: 写不好看的话常数大。(相较于 SBT 来说, 甚至有可能会比 splay 慢), 需手写 rand
12 */
13 #include <cstdio>
14 #include <cstring>
15 #include <algorithm>
16 #include <cmath>
17
18 #ifdef WIN32
19     #define LL "%I64d"
20 #else
21     #define LL "%lld"
22 #endif
23
24 #ifdef CT
25     #define debug(...) printf(__VA_ARGS__)
26     #define setfile()
27 #else
28     #define debug(...)
29     #define filename ""
30     #define setfile() freopen(filename".in", 'r', stdin); freopen(filename".out", 'w', stdout)
31 #endif
32
33 #define R register
34 // #define getc() (S==EOF?(T=(S=B)+fread(B,1,1<<15,stdin),S==T)?EOF:*S++):getchar())
35 #define getc() getchar()
36 #define dmax(_a, _b) ((_a) > (_b) ? (_a) : (_b))
37 #define dmin(_a, _b) ((_a) < (_b) ? (_a) : (_b))
38 #define cmax(_a, _b) (_a < (_b) ? _a = (_b) : 0)
39 #define cmin(_a, _b) (_a > (_b) ? _a = (_b) : 0)
40 #define cabs(_x) ((_x) < 0 ? (-_x) : (_x))

```

```

41 char B[1<<15],*S=B,*T=B;
42 inline int FastIn()
43 {
44     R char ch;R int cnt=0;R bool minus=0;
45     while (ch=getc(),(ch < '0' || ch > '9') && ch != '-') ;
46     ch == '-' ? minus=1:cnt=ch-'0';
47     while (ch=getc(),ch >= '0' && ch <= '9') cnt = cnt * 10 + ch - '0';
48     return minus?-cnt:cnt;
49 }
50 #define maxn 100010
51 char str[maxn];
52 struct Treap
53 {
54     char data;
55     int size;
56     Treap *ls, *rs;
57     Treap(char _ch): data(_ch), size(1), ls(NULL), rs(NULL){}
58     inline void update()
59     {
60         size = (ls ? ls -> size : 0) + (rs ? rs -> size : 0) + 1;
61     }
62 }*root[maxn];
63 inline int Size(Treap *x)
64 {
65     return x ? x -> size : 0;
66 }
67 struct Pair
68 {
69     Treap *fir, *sec;
70 };
71 inline Treap *copy(Treap *x)
72 {
73     if (!x) return NULL;
74     Treap *nw = new Treap(x -> data);
75     nw -> ls = x -> ls;
76     nw -> rs = x -> rs;
77     nw -> size = x -> size;
78     return nw;
79 }
80 Pair Split(Treap *x, int k)
81 {
82     if (!x) return (Pair){NULL, NULL};
83     Pair y; y.fir = NULL; y.sec = NULL;
84     Treap *nw = copy(x);
85     if (Size(nw -> ls) >= k)
86     {
87         y = Split(nw -> ls, k);
88         nw -> ls = y.sec;
89         nw -> update();
90         y.sec = nw;
91     }
92     else
93     {
94         y = Split(nw -> rs, k - Size(nw -> ls) - 1);
95         nw -> rs = y.fir;
96         nw -> update();
97         y.fir = nw;
98     }
99     return y;
100 }
101 const int Ta = 1 << 16 | 3, Tb = 33333331;

```



```

102 unsigned int Tc;
103 inline unsigned int randint(){return Tc = Ta * Tc + Tb;}
104 Treap *Merge(Treap *a, Treap *b)
105 {
106     Treap *nw;
107     if (!a) return nw = copy(b);
108     if (!b) return nw = copy(a);
109     if (randint() % (Size(a) + Size(b)) < Size(a))
110     {
111         nw = copy(a);
112         nw -> rs = Merge(nw -> rs, b);
113     }
114     else
115     {
116         nw = copy(b);
117         nw -> ls = Merge(a, nw -> ls);
118     }
119     nw -> update();
120     return nw;
121 }
122 Treap *Build(int l, int r)
123 {
124     if (l > r) return NULL;
125     R int mid = l + r >> 1;
126     Treap *nw = new Treap(str[mid]);
127     nw -> ls = Build(l, mid - 1);
128     nw -> rs = Build(mid + 1, r);
129     nw -> update();
130     return nw;
131 }
132 int now;
133 inline void Insert(R int k, R char ch)
134 {
135     Pair x = Split(root[now], k);
136     Treap *nw = new Treap(ch);
137     root[++now] = Merge(Merge(x.fir, nw), x.sec);
138 }
139 inline void Del(R int l, R int r)
140 {
141     Pair x = Split(root[now], l - 1);
142     Pair y = Split(x.sec, r - l + 1);
143     root[++now] = Merge(x.fir, y.sec);
144 }
145 inline void Copy(R int l, R int r, R int ll)
146 {
147     Pair x = Split(root[now], l - 1);
148     Pair y = Split(x.sec, r - l + 1);
149     Pair z = Split(root[now], ll);
150     Treap *ans = y.fir;
151     root[++now] = Merge(Merge(z.fir, ans), z.sec);
152 }
153 inline void Print(Treap *x, R int l, R int r)
154 {
155     if (!x) return ;
156     if (l > r) return;
157     R int mid = Size(x -> ls) + 1;
158     if (r < mid)
159     {
160         Print(x -> ls, l, r);
161         return ;
162     }

```

```

163     if (l > mid)
164     {
165         Print(x -> rs, l - mid, r - mid);
166         return ;
167     }
168     Print(x -> ls, l, mid - 1);
169     printf("%c", x -> data );
170     Print(x -> rs, 1, r - mid);
171 }
172 inline void Printtree(Treap *x)
173 {
174     if (!x) return;
175     Printtree(x -> ls);
176     printf("%c", x -> data );
177     Printtree(x -> rs);
178 }
179 int main()
180 {
181     // setfile();
182     R int n = FastIn();
183     gets(str + 1);
184     R int len = strlen(str + 1);
185     root[0] = Build(1, len);
186     while (1)
187     {
188         R char opt = getc();
189         while (opt < 'A' || opt > 'Z')
190         {
191             if (opt == EOF) return 0;
192             opt = getc();
193         }
194         if (opt == 'I')
195         {
196             R int x = FastIn();
197             R char ch = getc();
198             Insert(x, ch);
199         }
200         else if (opt == 'D')
201         {
202             R int l = FastIn(), r = FastIn();
203             Del(l, r);
204         }
205         else if (opt == 'C')
206         {
207             R int x = FastIn(), y = FastIn(), z = FastIn();
208             Copy(x, y, z);
209         }
210         else if (opt == 'P')
211         {
212             R int x = FastIn(), y = FastIn(), z = FastIn();
213             // printf("%d %d %d\n", x, y, z );
214             Print(root[now - x], y, z);
215             puts("");
216         }
217         // Printtree(root[now]);
218         // puts("");
219     }
220     return 0;
221 }

```

5.6 CDQ 分治 (ct)

```

1 //
2 // Title : cdq 分治
3 // Date : 18.04.2016
4 // Test : BZOJ-1176
5 // Complexity :  $O(n \log^2 n)$ 
6 //
7 /*
8     对于三维偏序等问题——
9     解决办法：离线询问，分治降维，剩下一维用随便什么树乱搞。这样就不用写树套树啦！
10 */
11 #include <cstdio>
12 #include <cstring>
13 #include <algorithm>
14 #include <cmath>
15
16 #ifdef WIN32
17     #define LL "%I64d"
18 #else
19     #define LL "%lld"
20 #endif
21
22 #ifdef CT
23     #define debug(...) printf(__VA_ARGS__)
24     #define setfile()
25 #else
26     #define debug(...)
27     #define filename ""
28     #define setfile() freopen(filename".in", "r", stdin); freopen(filename".out", "w", stdout);
29 #endif
30
31 #define R register
32 #define getc() (S == T && (T = (S = B) + fread(B, 1, 1 << 15, stdin), S == T) ? EOF : *S++)
33 #define dmax(_a, _b) ((_a) > (_b) ? (_a) : (_b))
34 #define dmin(_a, _b) ((_a) < (_b) ? (_a) : (_b))
35 #define cmax(_a, _b) (_a < (_b) ? _a = (_b) : 0)
36 #define cmin(_a, _b) (_a > (_b) ? _a = (_b) : 0)
37 char B[1 << 15], *S = B, *T = B;
38 inline int FastIn()
39 {
40     R char ch; R int cnt = 0; R bool minus = 0;
41     while (ch = getc(), (ch < '0' || ch > '9') && ch != '-') ;
42     ch == '-' ? minus = 1 : cnt = ch - '0';
43     while (ch = getc(), ch >= '0' && ch <= '9') cnt = cnt * 10 + ch - '0';
44     return minus ? -cnt : cnt;
45 }
46 #define maxn 200010
47 #define maxm 2000010
48 struct event
49 {
50     int x, y, pos, opet, ans;
51     inline bool operator < (const event &that) const {return pos < that.pos ;}
52 }t[maxn], q[maxn];
53 #define lowbit(_x) ((_x) & -(_x))
54 int bit[maxm], last[maxm], s, w, cnt, now;
55 inline void add(R int x, R int val)
56 {
57     for (; x <= w; x += lowbit(x))
58         {

```

```

59         if (last[x] != now)
60             bit[x] = 0;
61         bit[x] += val;
62         last[x] = now;
63     }
64 }
65 inline int query(R int x)
66 {
67     R int ans = 0;
68     for (; x ; x -= lowbit(x))
69     {
70         if (last[x] == now)
71             ans += bit[x];
72     }
73     return ans;
74 }
75 void cdq(R int left, R int right)
76 {
77     if (left == right) return ;
78     R int mid = left + right >> 1;
79     cdq(left, mid); cdq(mid + 1, right);
80     //分成若干个子问题
81     ++now;
82     for (R int i = left, j = mid + 1; j <= right; ++j)
83     {
84         for (; i <= mid && q[i].x <= q[j].x; ++i)
85             if (!q[i].opet)
86                 add(q[i].y, q[i].ans);
87         //考虑前面的修改操作对后面的询问的影响
88         if (q[j].opet)
89             q[j].ans += query(q[j].y);
90     }
91     R int i, j, k = 0;
92     //以下相当于归并排序
93     for (i = left, j = mid + 1; i <= mid && j <= right; )
94     {
95         if (q[i].x <= q[j].x)
96             t[k++] = q[i++];
97         else
98             t[k++] = q[j++];
99     }
100     for (; i <= mid; )
101         t[k++] = q[i++];
102     for (; j <= right; )
103         t[k++] = q[j++];
104     for (R int i = 0; i < k; ++i)
105         q[left + i] = t[i];
106 }
107 int main()
108 {
109     // setfile();
110     s = FastIn();
111     w = FastIn();
112     while (1)
113     {
114         R int opt = FastIn();
115         if (opt == 1)
116         {
117             R int x = FastIn(), y = FastIn(), a = FastIn();
118             q[++cnt] = (event){x, y, cnt, 0, a};
119         }

```

```

120         if (opt == 2)
121         {
122             R int x = FastIn() - 1, y = FastIn() - 1, a = FastIn(), b = FastIn();
123             q[++cnt] = (event) {x, y, cnt, 1, x * y * s};
124             q[++cnt] = (event) {a, b, cnt, 2, a * b * s};
125             q[++cnt] = (event) {x, b, cnt, 2, x * b * s};
126             q[++cnt] = (event) {a, y, cnt, 2, a * y * s};
127         }
128         if (opt == 3) break;
129     }
130     cdq(1, cnt);
131     std::sort(q + 1, q + cnt + 1);
132     for (R int i = 1; i <= cnt; ++i)
133         if (q[i].opet == 1)
134             printf("%d\n", q[i].ans + q[i + 1].ans - q[i + 2].ans - q[i + 3].ans ), i +=
                ↪ 3;
135     return 0;
136 }

```

Chapter 6

Others

6.1 vimrc (gy)

```
1 se et ts=4 sw=4 sts=4 nu sc sm lbr is hls mouse=a
2 sy on
3 ino <tab> <c-n>
4 ino <s-tab> <tab>
5 au winnew * winc L
6
7 nm <f6> ggVG"+y
8 nm <f7> :w<cr>:make<cr>
9 nm <f8> :!@@<cr>
10 nm <f9> :!@@ < in<cr>
11 nm <s-f9> :!(time @@ < in &>> out) &>> out<cr>:sp out<cr>
12
13 au filetype cpp cm @@ ./a.out | se cin fdm=syntax mp=g++\ %\ -std=c++11\ -Wall\ -Wextra\ -O2
14
15 map <c-p> :ha<cr>
16 se pheader=%n\ %f
17
18 au filetype java cm @@ java %< | se cin fdm=syntax mp=javac\ %
19 au filetype python cm @@ python % | se si fdm=indent
20 au bufenter *.kt setf kotlin
21 au filetype kotlin cm @@ kotlin _%<Kt | se si mp=kotlinc\ %
```

6.2 Java Template (gy)

```
1 import java.io.BufferedReader;
2 import java.io.IOException;
3 import java.io.InputStreamReader;
4 import java.math.BigDecimal;
5 import java.math.BigInteger;
6 import java.math.RoundingMode;
7 import java.util.ArrayDeque;
8 import java.util.ArrayList;
9 import java.util.Arrays;
10 import java.util.Comparator;
11 import java.util.Deque;
12 import java.util.LinkedList;
13 import java.util.List;
14 import java.util.Scanner;
15 import java.util.StringTokenizer;
16
```

```

17 public class Template {
18     // Input
19     private static BufferedReader reader;
20     private static StringTokenizer tokenizer;
21
22     private static String next() {
23         try {
24             while (tokenizer == null || !tokenizer.hasMoreTokens())
25                 tokenizer = new StringTokenizer(reader.readLine());
26         } catch (IOException e) {
27             // do nothing
28         }
29         return tokenizer.nextToken();
30     }
31
32     private static int nextInt() {
33         return Integer.parseInt(next());
34     }
35
36     private static double nextDouble() {
37         return Double.parseDouble(next());
38     }
39
40     private static BigInteger nextBigInteger() {
41         return new BigInteger(next());
42     }
43
44     public static void main(String[] args) {
45         reader = new BufferedReader(new InputStreamReader(System.in));
46         Scanner scanner = new Scanner(System.in);
47         while (scanner.hasNext())
48             scanner.next();
49     }
50
51     // BigInteger & BigDecimal
52     private static void bigDecimal() {
53         BigDecimal a = BigDecimal.valueOf(1.0);
54         BigDecimal b = a.setScale(50, RoundingMode.HALF_EVEN);
55         BigDecimal c = b.abs();
56         // if scale omitted, b.scale is used
57         BigDecimal d = c.divide(b, 50, RoundingMode.HALF_EVEN);
58         // since Java 9
59         BigDecimal e = d.sqrt(new MathContext(50, RoundingMode.HALF_EVEN));
60         BigDecimal x = new BigDecimal(BigInteger.ZERO);
61         BigInteger y = BigDecimal.ZERO.toBigInteger(); // RoundingMode.DOWN
62         y = BigDecimal.ZERO.setScale(0, RoundingMode.HALF_EVEN).unscaledValue();
63     }
64
65     // sqrt for Java 8
66     private static BigDecimal sqrt(BigDecimal a, int scale, RoundingMode mode) {
67         if (a.equals(BigDecimal.ZERO))
68             return BigDecimal.ZERO;
69         a = a.setScale(scale, mode);
70         BigDecimal ans = a;
71         BigDecimal TWO = BigDecimal.valueOf(2L);
72         for (int i = 1; i <= scale; i++)
73             ans = ans.add(a.divide(ans, scale, mode)).divide(TWO, scale, mode);
74         return ans;
75     }
76
77     private static BigInteger sqrt(BigInteger a) {

```

```

78     BigInteger about = BigInteger.ZERO.setBit(a.bitLength() / 2);
79     return sqrt(new BigDecimal(a.toString()), new BigDecimal(about.toString()).setScale(0,
    ↪ RoundingMode.FLOOR).unscaledValue());
80 }
81
82 private static BigDecimal sqrt(BigDecimal a, BigDecimal initial) {
83     if (a.equals(BigDecimal.ZERO))
84         return BigDecimal.ZERO;
85     a = a.setScale(50, RoundingMode.HALF_EVEN);
86     BigDecimal ans = initial;
87     for (int i = 1; i <= 10; i++)
88         ans = ans.add(a.divide(ans, RoundingMode.HALF_EVEN)).divide(BigDecimal.valueOf(2),
    ↪ RoundingMode.HALF_EVEN);
89     return ans;
90 }
91
92 // ArrayList
93 private static void arrayList() {
94     List<Integer> list = new ArrayList<>();
95     // Generic array is banned
96     List[] lists = new List[100];
97     lists[0] = new ArrayList<Integer>();
98     // for List<Integer>, remove(Integer) stands for element, while remove(int) stands for
    ↪ index
99     list.remove(list.get(1));
100    list.remove(list.size() - 1);
101    list.clear();
102 }
103
104 // Queue
105 private static void queue() {
106     LinkedList<Integer> queue = new LinkedList<>();
107     // return the value without popping
108     queue.peek();
109     // pop and return the value
110     queue.poll();
111     Deque<Integer> deque = new ArrayDeque<>();
112     deque.peekFirst();
113     deque.peekLast();
114     deque.pollFirst();
115 }
116
117 // Others
118 private static void others() {
119     Arrays.sort(new int[10]);
120     Arrays.sort(new Integer[10], (a, b) -> {
121         if (a.equals(b)) return 0;
122         if (a > b) return -1;
123         return 1;
124     });
125     Arrays.sort(new Integer[10], Comparator.comparingInt((a) -> (int) a).reversed());
126     long a = 1_000_000_000_000_000L;
127     int b = Integer.MAX_VALUE;
128     int c = 'a';
129 }
130 }

```


6.3 Big Fraction (gy)

```

1 fun gcd(a: Long, b: Long): Long = if (b == 0L) a else gcd(b, a % b)
2
3 class Fraction(val a: BigInteger, val b: BigInteger) {
4     constructor(a: Long, b: Long) : this(BigInteger.valueOf(a / gcd(a, b)), BigInteger.valueOf(b /
5         ↳ gcd(a, b)))
6
7     operator fun plus(o: Fraction): Fraction {
8         var gcd = b.gcd(o.b)
9         val tempProduct = (b / gcd) * (o.b / gcd)
10        var ansA = a * (o.b / gcd) + o.a * (b / gcd)
11        val gcd2 = ansA.gcd(gcd)
12        ansA /= gcd2
13        gcd /= gcd2
14        return Fraction(ansA, gcd * tempProduct)
15    }
16
17    operator fun minus(o: Fraction): Fraction {
18        var gcd = b.gcd(o.b)
19        val tempProduct = (b / gcd) * (o.b / gcd)
20        var ansA = a * (o.b / gcd) - o.a * (b / gcd)
21        val gcd2 = ansA.gcd(gcd)
22        ansA /= gcd2
23        gcd /= gcd2
24        return Fraction(ansA, gcd * tempProduct)
25    }
26
27    operator fun times(o: Fraction): Fraction {
28        val gcd1 = a.gcd(o.b)
29        val gcd2 = b.gcd(o.a)
30        return Fraction((a / gcd1) * (o.a / gcd2), (b / gcd2) * (o.b / gcd1))
31    }
32 }

```

6.4 模拟退火 (ct)

```

1 #include <stdio>
2 #include <cmath>
3 #include <stdlib>
4 #include <ctime>
5
6 #define R register
7 #define cmax(_a, _b) (_a < (_b) ? _a = (_b) : 0)
8 #define maxn 10010
9 struct Poi {
10     double x, y, m;
11 }p[maxn];
12 double ans_x, ans_y, fans;
13 int n;
14 inline double rand01() {return rand() / 2147483647.0;}
15 inline double randp() {return (rand() & 1 ? 1 : -1) * rand01();}
16 inline double sqr(R double x) {return x * x;}
17 inline double f(R double x, R double y)
18 {
19     R double maxx = 0;
20     for (R int i = 1; i <= n; ++i)
21         maxx += sqrt(sqr(x - p[i].x) + sqr(y - p[i].y)) * p[i].m;

```

```

22     if (maxx < fans) {fans = maxx; ans_x = x; ans_y = y;}
23     return maxx;
24 }
25 int main()
26 {
27     srand(time(NULL) + clock());
28     scanf("%d", &n);
29     R double x = 0, y = 0, tot = 0;
30     for (R int i = 1; i <= n; ++i)
31         scanf("%lf%lf%lf", &p[i].x, &p[i].y, &p[i].m), x += p[i].x * p[i].m, y += p[i].y *
            ↪ p[i].m, tot += p[i].m;
32     fans = 1e30; x /= tot; y /= tot;
33     R double fnow = f(x, y);
34     for (R double T = 1e4; T > 1e-4; T *= 0.997)
35     {
36         R double nx = x + randp() * T, ny = y + randp() * T, fnext = f(nx, ny);
37         R double delta = fnext - fnow;
38         if (delta < 1e-9 || exp(-delta / T) > rand01())
39         {
40             x = nx; y = ny; fnow = fnext;
41         }
42     }
43     printf("%.3lf %.3lf\n", ans_x, ans_y);
44     return 0;
45 }

```

6.5 三分 (ct)

```

1  #define maxn 200010
2  #define inf 1e9
3  int a[maxn], n;
4  inline double check(R double x)
5  {
6      R double tmp, tmp1 = 0, tmp2 = 0, maxx = -inf, minn = -inf;
7      for (R int i = 1; i <= n; ++i)
8      {
9          tmp = (double) a[i] - x;
10
11          tmp1 += tmp;
12          cmax(maxx, tmp1);
13          tmp1 < 0 ? tmp1 = 0 : 0;
14
15          tmp2 -= tmp;
16          cmax(minn, tmp2);
17          tmp2 < 0 ? tmp2 = 0 : 0;
18      }
19      return dmax(maxx, minn);
20 }
21 int main()
22 {
23     n = F();
24     for (R int i = 1; i <= n; ++i) a[i] = F();
25     R double l = -1e4, r = 1e4;
26     for (R int i = 1; i <= 100; ++i)
27     {
28         R double ll = (l + r) * 0.5;
29         R double rr = (l + r) * 0.5;
30         if (check(ll) < check(rr)) r = rr;
31         else l = ll;

```

```

32     }
33     printf("%.6lf\n", check((l + r) * 0.5));
34     return 0;
35 }

```

6.6 博弈论模型 (gy)

- Wythoff's game
 给定两堆石子，每次可以从任意一堆中取至少一个石子，或从两堆中取相同的至少一个石子，取走最后石子的胜
 先手胜当且仅当石子数满足：
 $\lfloor (b - a) \times \phi \rfloor = a, (a \leq b, \phi = \frac{\sqrt{5}+1}{2})$
 先手胜对应的石子数构成两个序列：
 Lower Wythoff sequence: $a_n = \lfloor n \times \phi \rfloor$
 Upper Wythoff sequence: $b_n = \lfloor n \times \phi^2 \rfloor$
- Fibonacci nim
 给定一堆石子，第一次可以取至少一个、少于石子总数数量的石子，之后每次可以取至少一个、不超过上次取石子数量两倍的石子，取走最后石子的胜
 先手胜当且仅当石子数为斐波那契数