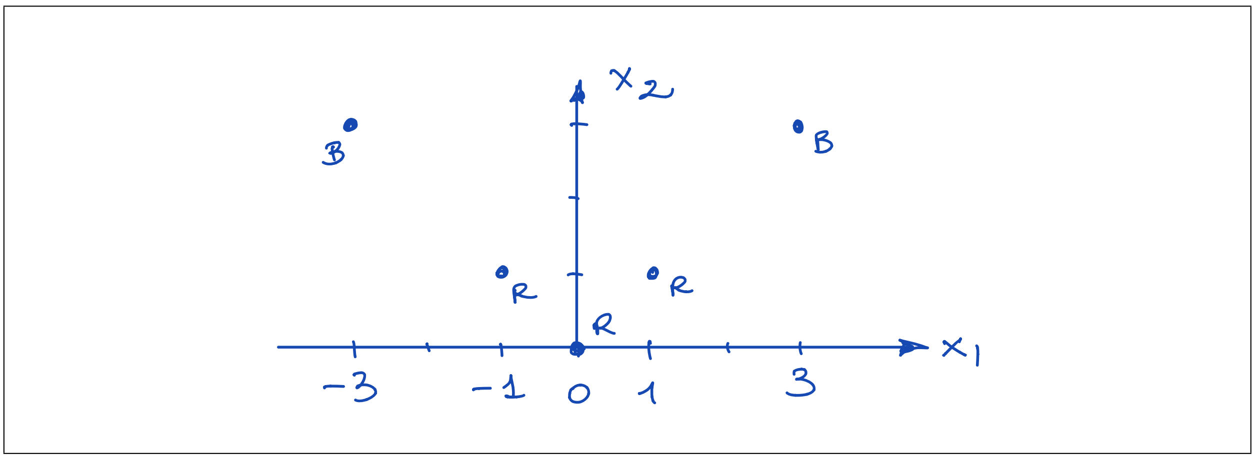
ESE 541 Week 11 & 12 Practice Problems - Solutions

Problem 1. In this problem, we consider the following dataset with $n = 5$, $p = 1$, and categorical outputs {Red, Blue}:

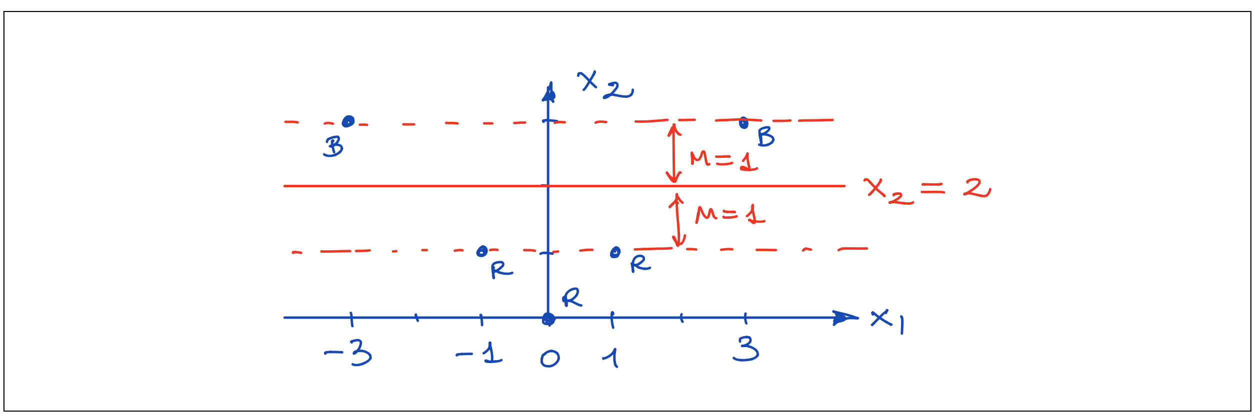
Obs.	X	Y
1	0	Red
2	1	Red
3	3	Blue
4	-1	Red
5	-3	Blue

Lift the feature space from $p = 1$ to $p = 2$ by defining the variables $X_1 = X$ and $X_2 = |X|$ (the absolute value) and answer the questions below:

1. Sketch the observations on the plane X_1/X_2.



2. On your sketch, draw the maximum margin hyperplane (in a solid line) and indicate the margins (in two dashed lines). What is the value of the maximum margin?



3. Find the values β_0, β_1, and β_2 corresponding to the maximum margin classifier $f(X_1, X_2) = \beta_0 + \beta_1 X_1 + \beta_2 X_2$, such that $f(X_1, X_2) > 0$ for the Blue class and $f(X_1, X_2) < 0$ for the Red class.

We have that the Blue class satisfies $f(X_1, X_2) = -2 + X_2 > 0$ and the Red class $-2 + X_2 < 0$; hence we have that $\beta_0 = -2$, $\beta_1 = 0$, and $\beta_2 = 1$.

4. Find the classifier associated to this maximum margin hyperplane in the original space of features, i.e., a function such that $f(X) > 0$ for the Blue class and $f(X) < 0$ for the Red class.

   In the lifted space, we have that $f(X_1, X_2) = -2 + X_2$; hence, in the original space, we have $f(X) = -2 + |X|$

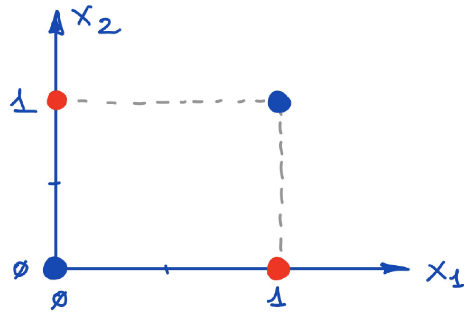
5. Using this classifier, assign a label to the points $X = 0.5$ and $X = -4$.

   The first point satisfies $f(X) = -2 + |0.5| = -1.5 < 0$; hence, it is Red. The second point satisfies $f(X) = -2 + |-4| = 2 > 0$; hence, it is Blue.

6. If you run a Support Vector Classifier in this dataset, what would be the values of ε_i for $i = 1, \ldots, 5$, i.e., the values of all the slack variables. Explain your answer.

   All the points are in the right side of the hyperplane and outside the margins; hence, all the ε_i are equal to zero

Problem 2. Consider a dataset with $N = 4$ points indicated in the figure:



1. Is this data linearly seperable?

   No, this data is not linearly separable.

2. We are now proposing a lifting of the input space by a adding a third coordinate X_3 to the given two-dimensional inputs. Which one of the following liftings render a separable dataset in three dimensions?

   c) $X_3 = (X_1 - X_2)^2$ is the correct answer.
   The blue datapoints have $X_3 = (0-0)^2 = 0$ and $X_3 = (1-1)^2 = 0$, while the red datapoints have $X_3 = (0-1)^2 = 1$ and $X_3 = (1-0)^2 = 1$.

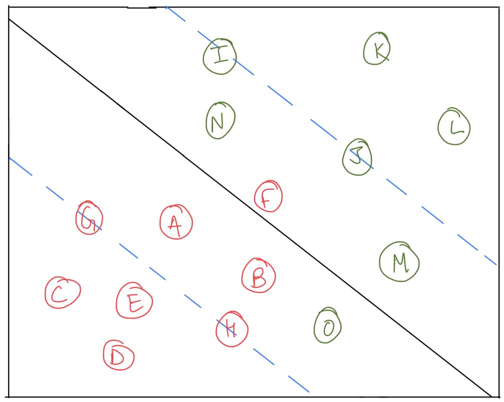
3. Which one of the following equations represent a separating hyperplane for the lifted dataset?

   c) $X_3 - \frac{1}{2} = 0$ is the correct answer.
   As we saw above in part 2, we have both blue datapoints at $X_3 = 0$ and both red datapoints at $X_3 = 1$. Hence, $X_3 - \frac{1}{2} = 0$ would be a separating hyperplane the lifted dataset.

4. Is this a maximum margin separating hyperplane?

   Yes, $X_3 - \frac{1}{2} = 0$ is a maximum margin separating hyperplane for the lifted dataset.
   The value between 0 and 1 with the maximal distance from each of the endpoints would be $\frac{1}{2}$, so our maximum margin separating hyperplane should intersect this point.

Problem 3. The figure below depicts SVM classifier with blue dashed lines referring to the soft margins. The original points either belong to the red category or the green category.



i List the points for which $0 < \epsilon_i < 1$, where ϵ_i is the i-th slack variable associated with the soft margin.

ii List the points for which $\epsilon_i \geq 1$, where ϵ_i is the slack variable associated with the soft margin.

iii List the points that are the support vectors, i.e., points that lie on top of the margin (dashed line) or for which $\epsilon_i > 0$.

i A,B,M,N as these are are within the margin and not misclassified

ii F,O as these are misclassified

iii A,B,F,G,H,I,J,M,N,O as these lie within or on top of the margin