NYU Computer Science Bridge HW2

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Question 5:
a) Solve the following questions from the Discrete Math zyBook:
1. Exercise 1.12.2, sections b, e

b.

$$\begin{array}{l} p \to (q \land r) \\ \neg q \\ \hline \therefore \neg p \end{array}$$

1.	$\neg q$	Hypothesis
2.	$\neg q \lor \neg r$	Addition, 1
3.	$\neg(q \land r)$	De Morgan, 2
4.	$p \to (q \land r)$	Hypothesis
5.	$\neg p$	Modus tollens, 3 4

e.

$$\begin{array}{l} p \lor q \\ \neg p \lor r \\ \neg q \\ \hline \therefore r \end{array}$$

1.	$p \lor q$	Hypothesis
2.	$\neg p \lor r$	Hypothesis
3.	$q \lor r$	Resolution, 1 2
4.	$\neg q$	Hypothesis
5.	r	Disjunctive Syllogism, 3 4

Exercise 1.12.3, section c

$$\begin{array}{l} p \vee q \\ \neg p \\ \hline \therefore q \end{array}$$

1.	$p \vee q$	Hypothesis
2.	$\neg(\neg p) \vee q$	Double Negation
3.	$\neg p \to q$	Conditional Identity, 2
4.	$\neg p$	Hypothesis
5.	q	Modus Ponens, 3 4

Exercise 1.12.5, section c

$$\begin{array}{l} \text{I will buy a new car and a new house only if I get a job.} \\ \text{I am not going to get a job.} \\ \hline \therefore \text{I will not buy a new car.} \end{array}$$

c: I will buy a new car
h: I will buy a new house
j: I get a job

$$\begin{array}{l} (c \wedge h) \to j \\ \neg j \\ \hline \therefore \neg c \end{array}$$

Argument is not valid. When both hypotheses are true the conclusion is false: c = True, h and j = False

Exercise 1.12.5, section d

$$\begin{array}{l} \text{I will buy a new car and a new house only if I get a job.} \\ \text{I am not going to get a job.} \\ \text{I will buy a new house.} \\ \hline \therefore \text{I will not buy a new car.} \end{array}$$

c: I will buy a new car
h: I will buy a new house
j: I get a job

$$\begin{array}{l} j \to (c \wedge h) \\ \neg j \\ h \\ \hline \therefore \neg c \end{array}$$

The argument is valid

1.	$(c \wedge h) \to j$	Hypothesis
2.	$\neg(c \wedge h) \vee j$	Conditional Identity
3.	$j \vee \neg(c \wedge h)$	Commutative law, 2
4.	$\neg j$	Hypothesis
5.	$\neg(c \wedge h)$	Disjunctive Syllogism, 3 4
6.	$\neg c \vee \neg h$	De Morgans, 5
7.	$\neg h \vee \neg c$	Commutative
8.	h	Hypothesis
9.	$\neg\neg h$	Double Negation
10.	$\neg c$	Disjunctive Syllogism, 7 9

b)
Exercise 1.13.3, section b

$$\begin{array}{l} \exists x(P(x) \vee Q(x)) \\ \exists x \neg Q(x) \\ \hline \therefore \exists x \neg P(x) \end{array}$$

	P	Q
a	F	T
b	F	F

When x = a$\exists x(P(x) \vee Q(x))$ is true and when x = b $\exists x \neg Q(b)$ is also true, but $\exists x P(x)$is false proving it is not valid

Exercise 1.13.5, section d

Every student who missed class got a detention.
Penelope is a student in the class.
Penelope did not miss class.

$\therefore$ Penelope did not get a detention

M(x): x missed class
D(x); x got a detention

$$\begin{array}{l} \forall x(M(x) \to D(x)) \\ \text{Penelope is a student in the class} \\ \neg M(Penelope) \\ \hline \therefore \neg D(Penelope) \end{array}$$

The argument is invalid. If M(Penelope) is false and D(Penelope) is true, then the hypothesis are true and conclusion is false.

Exercise 1.13.5, section e

Every student who missed class or got a detention did not get an A.
Penelope is a student in the class.
Penelope got an A.

$\therefore$ Penelope did not get a detention

M(x): x missed class
A(x): x got an A
D(x); x got a detention

$$\begin{array}{l} \forall x(M(x) \lor D(x)) \to A(x) \\ \text{Penelope is a student in the class} \\ A(Penelope) \\ \hline \therefore \neg D(Penelope) \end{array}$$

1.	Penelope is a student in the class	Hypothesis
2.	$\forall x(M(x) \lor D(x) \to \neg A(x))$	Hypothesis
3.	$M(Penelope) \lor D(Penelope) \to \neg A(Penelope)$	Universal Insatntiation
4.	$A(Penelope)$	Hypothesis
5.	$\neg(\neg A(Penelope))$	Double negation, 4
6.	$\neg(M(Penelope) \lor D(Penelope))$	Modus Tollens, 3 5
7.	$\neg M(Penelope) \land \neg D(Penelope)$	De Morgan
8.	$\neg D(Penelope) \land \neg M(Penelope)$	cumutative
9.	$\neg D(Penelope)$	simplification

Question 6:
Exercise 2.4.1, section d
The product of two odd integers is an odd integer.

Proof. Let x, y be an odd integer

□

Exercise 2.4.3, section b
If x is a real number and $x \leq 3, \text{then} 12 - 7x + x^2 \geq 0$.

Proof. Let x be a real number$\land x \leq 3$ □

Question 7:

Exercise 2.5.1 section d

For every integer n, if $n^2 - 2n + 7$ is even, then n is odd

Proof. Let $t, u \in \mathbb{R}$ where $t = xy$ and $u = zw$. So,

$$\begin{aligned}
4xyzw &= 2 \cdot 2tu \\
&\leq 2 \cdot (t^2 + u^2) \\
&= 2 \cdot ((xy)^2 + (zw)^2) && \text{(substituting variables)} \\
&= 2 \cdot (x^2y^2 + z^2w^2) \\
&= 2x^2y^2 + 2z^2w^2 \\
&\leq ((x^2)^2 + (y^2)^2) + ((z^2)^2) + (w^2)^2) \\
&= x^4 + y^4 + z^4 + w^4
\end{aligned}$$

□

Exercise 2.5.4 section a

For every pair of real numbers x and y, if $x^3 + xy^2 \leq x^2y + y^3$ then $x \leq y$

Exercise 2.5.4 section b

For every pair of real numbers x and y, if $x + y > 20$

Exercise 2.5.5 section c

For every non-zero real number x, if x is irrational then $1/x$ is irrational

Question 8:

Exercise 2.6.6 sections c

The average of three real numbers is greater than or equal to at least one of the numbers.

Exercise 2.6.6 sections d

There is no smallest integer

Question 9:
Exercise 2.7.2 section b

If integers x and y have the same parity, then $x + y$ is even. The parity of a number tells whether the number is odd or even. If x and y have the same parity, they are either both even or both odd.