NYU Computer Science Bridge HW 11

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Question 5

Exercise a.
Use mathematical induction to prove that for any positive integer n, 3 divide n^3 + 2n

BC: $n = 1 : 1^3 + 2(1) = 3 = 3$
IH: Assume 3 divides $k^3 + 2(k)$ $3m = k^3 + 2(k)$
Show that 3 divides $(k + 1)^3 + 2(k + 1)$
Expanding gives us $k^3 + 2k + 3k^2 + 3k + 3$
Using Inductive Hypothesis gives us $3m + 3k^2 + 3k + 3$
Simplifying to $3(m + k^2 + k + 1)$
Since $(m + k^2 + k + 1)$ is an integer, we have shown that 3 divides $n^3 + 2n$

Exercise b.
Use strong induction to prove that any positive integer $n(n \geq 2)$ can be written as a product of primes

BC: n = 2. 2 is a prime number
IH: For $k \geq 2$. Assume that k can be represented as a product of primes.
Show that $k + 1$ can be represented as a product of primes.
If $k + 1$ is not prime, it is composite and can be written as a product of 2 primes
$2 \leq a, b \leq k + 1$ where a and b are integers.
$\therefore$ Any positive integer $n \geq 2$ can be written as a product of primes.

Question 6

Exercise 7.4.1.a.
$P(3)\ 1^2 + 2^2 + 3^2 = 3((3 + 1)(2 * 3 + 1))/6$

Exercise 7.4.1.b.
$P(k) = (k(k + 1)(2 * k + 1))/6$

Exercise 7.4.1.c.
$P(k + 1) = (k + 1(k + 2)(2k + 3))/6$

Exercise 7.4.1.d.
We must prove that P(1) is true

Exercise 7.4.1.e.
We must assume P(k) to be true and show that P(k+1) is true

Exercise 7.4.1.f.
The inductive hypothesis is P(k)

Exercise 7.4.1.g.
BC: $n = 1\ \sum_{j=1}^{n} j^2 = \frac{n(n+1)(2n+1)}{6}$
$\sum_{j=1}^{1} 1^2 = \frac{1(1+1)(2(1)+1)}{6}$ is 1 = 1
IH: Assume P(k) $\sum_{j=1}^{k} j^2 = \frac{k(k+1)(2k+1)}{6}$ is true so P(k+1) is also true
$\sum_{j=1}^{k+1} j^2 = \sum_{j=1}^{k} j^2 + (k + 1)^2$
Since $\sum_{j=1}^{k} j^2 = \frac{k(k+1)(2k+1)}{6}$ we can use the inductive hypothesis
Now we have $\frac{k(k+1)(2k+1)}{6} + (k + 1)^2$
By Simplifying we will have $\frac{(k+1)(k+2)(2k+3)}{6}$
$\therefore$ P(k+1) is true

Exercise 7.4.3c

For $n \geq 1$ Prove $\sum_{j=1}^{n} \frac{1}{j^2} \leq 2 - \frac{1}{n}$
BC: n = 1 $\frac{1}{1^2} = 2 - \frac{1}{1}$
IH: Assume $\sum_{j=1}^{k} \frac{1}{k^2} \leq 2 - \frac{1}{k}$
Show $\sum_{j=1}^{k+1} \frac{1}{(k+1)^2} \leq 2 - \frac{1}{k+1} + \frac{1}{(k+1)^2}$
$2 - \frac{1}{k+1} + \frac{1}{(k+1)^2}$
$2 - \frac{1}{k+1} + \frac{1}{k(k+1)}$
$2 - \frac{k}{k(k+1)} = 2 - \frac{1}{k+1}$
Exercise 7.5.1a

For $n > 0$ 4 evenly divides $3^{2n} - 1$
BC: n = 1. $3^{2(1)} - 1 = 8$ 8 is divisible by 4
IH: $p(k) = 3^{2k} - 1\ 4m + 1 = 3^{2k}$
Show $3^{2(k+1)} - 1$ is divisible by 4
$3^(2k) * 3^2 - 1$
Using the inductive hypothesis $(4m + 1) * 3^2 - 1$
$4(9m + 2)$ where 9m + 2 is an integer proving it is divisible by 4.