NYU Computer Science Bridge HW5

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Question 3

Exercise 4.1.3 b
$f(x) = 1/(x^2 - 4)$
$\frac{1}{(x+2)(x-2)}$ shows this function is not defined on x = 2 and x = -2

Exercise 4.1.3 c
$f(x) = \sqrt{(x^2)}$
This is a well-defined function. The range is all $\mathbb{R}$ >= 0

Exercise 4.1.5 b
$Let A = \{2, 3, 4, 5\}$
$f : A \to Z$ such that $f(x) = x^2$
Z = {4, 9, 16, 25}

Exercise 4.1.5 d
$f : \{0, 1\}5 \to Z$. For $x \in 0, 15$, f(x) is the number of 1's that occur in x. For example f(01101) = 3, because there are three 1's in the string "01101".
f(00000) = 0, f(00001) = 1, ... f(11111) = 5. Z = {0,1,2,3,4,5} since the max number of 1s can appear is 5 and min is 0.

Exercise 4.1.5 h
$Let A = \{1, 2, 3\}$
$f : AxA \to \mathbb{Z}x\mathbb{Z}$ where $f(x, y) = (y, x)$
= {(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3)}
Exercise 4.1.5 i
$Let A = \{1, 2, 3\}$
$f : AxA \to \mathbb{Z}x\mathbb{Z}$ where $f(x, y) = (x, y + 1)$
{ (1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,2), (3,3), (3,4) }

Exercise 4.1.5 l
$Let A = \{1, 2, 3\}$
$f : P(A) \to P(A)$ For $X \subseteq A, f(X) = X - \{1\}$
{ {}, {2}, {3}, {2,3} }

Question 4 Part I

Exercise 4.2.2 c
$h : \mathbb{Z} \to \mathbb{Z}.h(x) = x^3$
h(x) cannot = 5 One to one but not onto.

Exercise 4.2.2 g
$f : \mathbb{Z}x\mathbb{Z} \to \mathbb{Z}x\mathbb{Z}.f(x, y) = (x + 1, 2y)$
f(x,y) cannot = (0,5) One to one but no onto.
Exercise 4.2.2 k
$f : \mathbb{Z}^+x\mathbb{Z}^+ \to \mathbb{Z}^+, f(x, y) = (2^x + y)$
Neither one to one or onto. f(1,4) and f(2,2) both = 6. f(x,y) cannot = 1

Exercise 4.2.4 b
$f : 0, 13 \to 13$. The output of f is obtained by taking the input string and replacing the first bit by 1, regardless of whether the first bit is a 0 or 1. For example, $f(001) = 101$ and $f(110) = 110$.

Neither one to one or onto. f(010) and f(110) = 110 and f(x) also cannot be 000

Exercise 4.2.4 c

$f : 0, 13 \to 13$. The output of f is obtained by taking the input string and reversing the bits. For example $f(011) = 110$.

Both one to one and onto.

Exercise 4.2.4 d
$f : \{0, 1\}^3 \to \{0, 4\}^4$. The output of f is obtained by taking the input string and adding an extra copy of the first bit to the end of the string. For example, $f(100) = 1001$.

One to one, but not onto. Adding first bit doesn't allow us to hit 1000 f(100) = 1001

Exercise 4.2.4 g
Let A be defined to be the set {1,2,3,4,5,6,7,8} and let B = {1}. $f : P(A) \to P(A)$. For $X \subseteq A, f(X) = X - B$. Recall that for a finite set A, P(A) denotes the power set of A which is the set of all subsets of A.

Neither one to one or onto. If X1 = {1,2,3} and X2 = {2,3} then they are both {2,3} f(X) also cannot be {1}

Question 4 Part II

Give an example of a function from the set of integers to the set of positive integers that is:
a. one-to-one but not onto

$f(z) = |x| + 5$

b. onto but not one to one

$f(x) = x^2$

c. one-to-one and onto

$f(x) = x + 1$

d. neither one-to-one nor onto

$f(x) = 5$

Question 5

Exercise 4.3.2 c
$f : \mathbb{R} \to \mathbb{R}. f(x) = 2x + 3$

$f^{-1}(x) = \frac{x-3}{2}$

Exercise 4.3.2 d
Let A be defined to the set {1,2,3,4,5,6,7,8}
$f : P(A) \to \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$
For $X \subseteq |X|$

Not a well defined function. It is not one to one as they can map to a single output.

Exercise 4.3.2 g
$f : \{0, 1\}^3 \to \{0, 1\}^3$
$f^{-1}(x) = f(x)=$
$\{0, 1\}^3 = \{0, 1\}^3$

Exercise 4.3.2 i
$f : \mathbb{Z}x\mathbb{Z} \to \mathbb{Z}x\mathbb{Z}, f(x, y) = (x + 5, y - 2)$

$f^{-1}(x, y) = (x - 5, y + 2)$

Exercise 4.4.8
$f(x) = 2x + 3$ $g(x) = 5x + 7$ $h(x) = x^2 + 1$

Exercise 4.4.8 c
$f \circ h$

$f \circ h(x) = 2x^2 + 5$

Exercise 4.4.8 d
$h \circ f$

$=((2x + 3)^2 + 1)$
$h \circ f(x) = 4x^2 + 12x + 10$

Exercise 4.4.2
$f(x) = x^2$ $g(x) = 2^x$ $h(x) = \lceil \frac{x}{5} \rceil$

Exercise 4.4.2 b
Evaluate $(f \circ h)(52)$

$f \circ h(52) = (\lceil \frac{52}{5} \rceil)^2$
$=11^2 = 121$

Exercise 4.4.2 c
Evaluate $(g \circ h \circ f)(4)$
f(4) = 16
$=g \circ h(16) = g(4) = 16$
Exercise 4.4.2 d
Give mathematical expression for h o f.

$h \circ f = \lceil \frac{x^2}{5} \rceil$

Exercise 4.4.6 c
What is (h o f)(010)?

$h \circ f(010)$ = h(110) = 111

Exercise 4.4.6 d
Range of h o f: {101, 111}

Exercise 4.4.6 e

Range of g o f: {001, 011, 101, 111}

Extra Credit:

Exercise 4.4.4 c
Is it possible that f is not one-to-one and g o f is one-to-one? Justify your answer. If the answer is "yes", give a specific example for f and g.

No.

Proof. If f is not one to one, then $g \circ f$ must not be one to one
There exists an x_1 and $x_2 \in X$ such that $x_1 \not\models x_2 \wedge f(x_1) = f(x_2)$ and $g(f(x_1)) = g(f(x_2))$.
And since $x_1 \not\models x_2$
$\therefore g \circ f$ is not one to one.

$\blacksquare$

Exercise 4.4.4 d
Is it possible that g is not one-to-one and g o f is one-to-one? Justify your answer. If the answer is "yes", give a specific example for f and g.

yes

Proof. If $f(x_1) =$ a and g(a) $=$ 1 and $f(x_2 =$ b and g(b) $=$ 2, there exists a c such that f(x) is not one to one but g(f(c) $=$ 2. Since g(f(c) $=$ 2 and g(b) $=$ 2
$\therefore g \circ f$ is one to one

$\blacksquare$