NYU Computer Science Bridge HW1

Summer 2023 Name: Jacky Choi

Question 1:

A:
1. $10011011_2 = (1*2^7)+0+0+(1*2^4)+(1*2^3)+0+(1*2^1)+(1*2^0) = 128+16+8+2+1 = 155_{10}$

2. $456_7 = (4*7^2)+(5*7^1)+(6*7^0) = 237_{10}$

3. $38A_{16} = (3*16^2)+(8*16^1)+(10*16^0) = 906_{10}$

4. $2214_5 = (2*5^3)+(2*5^2)+(1*5^1)+(4*5^0) = 309_{10}$

B:
256 128 64 32 16 8 4 2 1
1. $69_{10} = 64+4+1 = 01000101_2$

2. $485_{10} = 256+128+64+32+4+1$
$= 111100101_2$

3. $6D1A_{16} = (6 = 0+4+2+0 = 0110_2|D = 8+4+0+1 = 1101_2$
$1 = 0+0+0+1 = 0001_2|A = 8+0+2+0 = 1010_2) = 0110110100011010_2$

C:
1. $1101011_2 = 01101011 = 6+11 = 6B$

Decimal:Hexadecimal Pairs 0:0 1:1 2:2 3:3 4:4 5:5 6:6 7:7 8:8 9:9 10:A 11:B 12:C 13:D 14:E 15:F

2. $895_{10} = 895/16 = 55R15 = F$
$55/16 = 3R7$
$3/16 = 0R3 = 37F_{16}$

Question 2:

Solve the following, do all calculation in the given base. Show your work.

1. $7566_8 + 4515_8 =$
$6_8 + 5_8 = 3_8\ carry 1$
$7_8 + 1_8 = 0_8\ carry 1$
$6_8 + 5_8 = 3_8\ carry 1$
$8_8 + 4_8 = 4_8\ carry 1$
$1_8 + 0_8 = 1_8$
$= 14303_8$

2. $10110011_2 + 1101_2 =$
$1_2 + 1_2 = 0\ carry 1$
$2_2 + 0_2 = 0\ carry 1$
$1_2 + 1_2 = 0\ carry 1$
$1_2 + 1_2 = 0\ carry 1$
$2_2 + 0_2 = 0\ carry 1$
$2_2 + 0_2 = 0\ carry 1$
$1_2 + 0_2 = 1$
$1_2 + 0_2 = 1$
$= 11000000_2$

3. $7A66_{16} + 45C5_{16} =$
$6_{16} + 5_{16} = 11_{16} = B$
$6_{16} + 12_{16} = 2_{16} = 2\ carry 1$
$11_{16} + 5_{16} = 0_{16} = 0$
$8_{16} + 4_{16} = 12_{16} = C$
$= C02B_{16}$

4. $3022_5 - 2433_5 =$
$7_5 - 3_5 = 4_5 |\ (2 - 1 = 1)$
$6_5 - 3_5 = 3_5 |\ (5 - 1 = 4)$
$4_5 - 4_5 = 0_5 |\ (3 - 1 = 2)$
$2_5 - 2_5 = 0_5$
$= 34_5$

Question 3:

256 128 64 32 16 8 4 2 1
A. Convert the following numbers to their 8-bits two's complement representation. Show your work.

1. $124_{10} = 0 + 64 + 32 + 16 + 8 + 4 + 0 + 0 = 01111100_2$

2. $-124_{10} = 10000011_8 + 00000001_8 = 10000100_2$ (flipped bits 0 - 1 and added 1)

3. $109_{10} = 0 + 64 + 32 + 0 + 8 + 4 + 0 + 1 = 01101101_2$

4. $-79_{10} =$
$79_{10} = 0 + 64 + 0 + 0 + 8 + 4 + 2 + 1 = 01001111_2$
$= 10110000 + 00000001 = 10110001_2$

B. Convert the following numbers represented as 8-bit two's complement) to their decimal representation. Show your work.

1. $00011110_{8bit2'scomp} = 0 + 0 + 0 + 16 + 8 + 4 + 2 + 0$
$= 30_{10}$

2. $11100110_{8bit2'scomp} = 00011001_2 + 00000001_2$
$= 00011010_2 = 0 + 0 + 0 + 16 + 8 + 0 + 2 + 0$
$= -26_{10}$

3. $00101101_{8bit2'scomp} = 0 + 0 + 32 + 0 + 8 + 4 + 0 + 1$
$= 45_{10}$

4. $10011110_{8bit2'scomp} = 01100001_2 + 00000001_2$
$= 01100010_2 = 0 + 64 + 32 + 0 + 0 + 0 + 2 + 0$
$= -98_{10}$

Question 4:

Solve the following question from the Discrete Math Zybook:

1. Exercise 1.2.4, sections b, c

b.$\neg p \vee q$

p	q	$\neg(p \vee q)$
T	T	F
T	F	F
F	T	F
F	F	T

c. $r \vee (p \wedge \neg q)$

p	q	r	$\neg q$	$(p \wedge \neg q)$	$(r \vee (p \wedge \neg q))$
T	T	T	F	F	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	F	F	T
F	T	F	F	F	F
F	F	T	T	F	T
F	F	F	T	F	F

2. Exercise 1.3.4, sections b, d

b. $(p \rightarrow q) \rightarrow (q \rightarrow p)$

p	q	$p \rightarrow q$	$(q \rightarrow p)$	$(p \rightarrow q) \rightarrow (q \rightarrow p)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

d. $(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$

p	q	$p \leftrightarrow q$	$p \leftrightarrow \neg q$	$(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	T	F	T

Question 5:

Solve the following question from the Discrete Math Zybook:

1. Exercise 1.2.7, sections b, c

b. The applicant must present at least two of the following forms of identification: birth certificate, driver's license, marriage license.

$$(B \wedge D) \vee (B \wedge M) \vee (D \wedge M)$$

c. Applicant must present either a birth certificate or both a driver's license and a marriage license.

$$B \vee (D \wedge M)$$

2. Exercise 1.3.7, sections b - e

b. A person can park in the school parking lot if they are a senior or at least seventeen years of age.

$$(s \vee y) \rightarrow p$$

c. Being 17 years of age is a necessary condition for being able to park in the school parking lot.

$$p \rightarrow y$$

d. A person can park in the school parking lot if and only if the person is a senior and at least 17 years of age.

$$p \leftrightarrow (s \wedge y)$$

e. Being able to park in the school parking lot implies that the person is either a senior or at least 17 years old.

$$p \rightarrow (s \vee y)$$

3. Exercise 1.3.9, sections c, d

c. The applicant can enroll in the course only if the applicant has parental permission.

$$c \rightarrow p$$

d. Having parental permission is a necessary condition for enrolling in the course.

$$c \rightarrow p$$

Question 6:

Solve the following question from the Discrete Math Zybook:

1. Exercise 1.3.6, sections b - d

b. Maintaining a B average is necessary for Joe to be eligible for the honors program.

If Joe wants to be eligible for the honors program then he must maintain a B average.

c. Rajiv can go on the roller coaster only if he is at least four feet tall

If Rajiv can go on the rollercoaster, then he is at least four feet tall.

d. Rajiv can go on the roller coaster if he is at least four feet tall.

If Rajiv is at least four feet tall, then he can go on the roller coaster

2. Exercise 1.3.10, sections c - f

p is true, q is false, r is unknown
c. $(p \vee r) \leftrightarrow (q \wedge r)$

False - $(p \vee r)$ is true and $(q \wedge r)$ is false

d. $(p \wedge r) \leftrightarrow (q \wedge r)$

Unknown - Since r is unknown if it is true, then the expression will be true, but false if r is false.

e. $p \rightarrow (r \vee q)$

Unknown - Since r is unknown if it is true, then the expression will be true, but false if r is false.

f. $(p \wedge q) \rightarrow r$

True - Since $(p \wedge q)$ is false, the expression is true even if r is true or if it is false

Question 7:

Solve the following question from the Discrete Math Zybook:

1. Exercise 1.4.5, sections b - d

b. If Sally did not get the job, then she was late for her interview or did not update her resume.
If Sally updated her resume and was not late for her interview, then she got the job.

j	l	r	$\neg j \to (l \vee \neg r)$	$(r \wedge \neg l) \to j$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	T	T
F	F	T	F	F
F	F	F	T	T

$$\neg j \to (l \vee \neg r) \equiv (r \wedge \neg l) \to j$$

c. If Sally got the job then she was not late for her interview.
If Sally did not get the job, then she was late for her interview.

j	l	$j \to \neg l$	$\neg j \to l$
T	T	F	T
T	F	T	T
F	T	T	T
F	F	T	F

$$j \to \neg l \not\equiv \neg j \to j$$

d. If Sally updated her resume or she was not late for her interview, then she got the job.
If Sally got the job, then she updated her resume and was not late for her interview.

j	l	r	$(r \vee \neg l) \to j$	$j \to (r \wedge \neg l)$
T	T	T	T	F
T	T	F	F	T
T	F	T	T	T
T	F	F	F	T
F	T	T	T	F
F	T	F	F	T
F	F	T	T	F
F	F	F	T	T

$$(r \vee \neg l) \to j \not\equiv j \to (r \wedge \neg l)$$

Question 8:

Solve the following question from the Discrete Math Zybook:

1. Exercise 1.5.2, sections c, f, i

c. $(p \to q) \land (r \to r) \equiv p \to (q \land r)$

$(p \to q) \land (r \to r) \equiv p \to (q \land r)$	
$(\neg p \lor q) \land (\neg p \lor r)$	Conditional Identity
$(\neg p \lor (q \lor r)$	Distributive
$p \to (q \lor r)$	Conditional Identity

f. $\neg(p \lor (\neg(p \land q)) \equiv \neg p \land \neg q$

$\neg(p \lor (\neg(p \land q)) \equiv \neg p \land \neg q$	
$\neg p \land \neg(\neg p \land q)$	De Morgan's
$\neg p \land \neg\neg p \lor \neg q)$	De Morgan's
$\neg p \land (p \lor \neg q)$	Double Negation
$(\neg p \land p) \lor (\neg p \land \neg q)$	Distributive Law
$F \lor (\neg p \land \neg q)$	Complement Law
$(\neg p \land \neg q) \lor F$	Commutative Law
$\neg p \land \neg q$	Identity Law

i. $(p \land q) \to r \equiv (p \land \neg r) \to \neg q$

$(p \land q) \to r$	
$\neg(p \land q) \lor r$	Conditional Identity
$(\neg p \lor \neg q) \lor r$	De Morgan's
$r \lor (\neg p \lor \neg q)$	Commutative
$(r \lor \neg p) \lor \neg q$	Associative
$\neg\neg(r \lor \neg p) \lor \neg q$	Double Negation
$\neg(r \lor \neg p) \to \neg q$	Conditional Identity
$\neg(\neg p \lor r) \to \neg q$	Commutative
$(\neg\neg p \land \neg r) \to \neg q$	DeMorgans
$(p \land \neg r) \to \neg q$	Double Negation

Question 8:

2. Exercise 1.5.3, sections c, d

c. $\neg r \vee (\neg r \to p)$

$\neg r \vee (\neg r \to p)$	
$\neg r \vee (\neg\neg r \vee p)$	Conditional Identity
$\neg r \vee (r \vee p)$	Double Negation
$(\neg r \vee r) \vee p$	Associative
$(r \vee \neg r) \vee p$	Commutative
$T \vee p$	Complement
$p \vee T$	Commutative
T	Domination

d. $\neg(p \to q) \to \neg q$

$\neg(p \to q) \to \neg q$	
$\neg(\neg p \vee q) \to \neg q$	Conditional Identity
$\neg\neg(\neg p \vee q) \vee \neg q$	Conditional Identity
$(\neg p \vee q) \vee \neg q$	Double Negation
$\neg p \vee (q \vee \neg q)$	Associative
$\neg p \vee T$	Complement
T	Donmination

Question 9:

Solve the following question from the Discrete Math Zybook:

1. Exercise 1.6.3, sections c, d

c. There is a number that is equal to its square.

$$\exists x(x = x^2)$$

d. Every number is less than or equal to its square plus 1

$$\forall x(x \leq x^2 + 1)$$

2. Exercise 1.7.4, sections b - d

b. Everyone was well and went to work yesterday.

$$\forall x(\neg(S(x) \land W(x))$$

c. Everyone who was sick yesterday did not go to work.

$$\forall x(S(x) \rightarrow \neg W(x))$$

d. Yesterday someone was sick and went to work.

$$\exists x(S(x) \land W(x))$$

Question 10:

Solve the following question from the Discrete Math Zybook:

1. Exercise 1.7.9, sections c - i

c. $\exists x((x = c) \rightarrow P(x))$: False

d. $\exists(Q(x) \wedge R(x))$: True

e. $(Q(a) \wedge P(d)$: True

f. $\forall x((x \neq b) \rightarrow Q(x))$

g. $\forall x(P(x) \vee R(x))$: False Row C P(x) is F and R(x) is $F \vee F = F$

h. $\forall x(P(x) \rightarrow R(x))$: True

i. $\exists x(Q(x) \vee R(x))$: True

2. Exercise 1.9.2, sections b - i

b. $\exists x \forall y Q(x, y)$: True Q(2) = all True

c. $\exists y \forall x P(x, y)$: True There is a value for every y

d. $\exists x \exists y S(x, y)$: False All S(x, y) is False

e. $\forall x \exists y Q(x, y)$: False All values of Q(1) are false

f. $\forall x \exists y P(x, y)$: True There is a value for all P(x,y)

g. $\forall x \forall y P(x, y)$: False Some P(x, y) values are false

h. $\exists x \exists Q(x, y)$: True Q(2,2) is true

i. $\forall x \forall y \neg S(x, y)$: True All values of S(x, y) are False and the opposite of that is all true

Question 11:

Solve the following question from the Discrete Math Zybook:

1. Exercise 1.10.4, sections c - g

c. There are two numbers whose sum is equal to their product

$$\exists x \ existsy(x + y = x * y)$$

d. The ratio of every two positive numbers is also positive

$$\forall x \forall y(x > 0 \wedge y > 0) \to (x/y > 0)$$

e. The reciprocal of every positive number less than one is greater than one.

$$\forall x(x < 1 \wedge x > 0) \to ((1/x) > 1)$$

f. There is no smallest number

$$\neg\exists x \forall y(x < y)$$

g. Every number other than 0 has a multiplicative inverse

$$\forall x \exists y(x \neq 0) \to (x * y = 1)$$

2. Exercise 1.10.7, sections c - f

c. There is at least one new employee who missed the deadline

$$\exists x(N(x) \wedge D(x))$$

d. Sam knows the phone number of everyone who missed the deadline

$$\forall y(D(y) \to P(Sam, y))$$

e. There is a new employee who knows everyone's phone number.

$$\exists x \forall y(N(x) \wedge P(x, y))$$

f. Exactly one new employee missed the deadline

$$\exists x \forall y((N(x) \wedge D(x)) \wedge (((x \neq y) \wedge N(y)) \to \neg(D(y))$$

3. Exercise 1.10.10, sections c - f

c. Every student has taken at least one class other than Math101.

$$\forall x \exists y (T(x, y) \land (y \neq Math101))$$

d. There is a student who has taken every math class other than Math 101.

$$\exists x \forall y ((y \neq Math101) \rightarrow (T(x, y))$$

e. Everyone other than Sam has taken at least two different math classes.

$$\forall x \exists y \exists k (((x \neq Sam)) \rightarrow ((y \neq k) \land T(x, y) \land T(x, k)))$$

f. Sam has taken exactly two math classes

$$\exists x \exists y \forall k ((x \neq y) \land T(Sam, x) \land T(Sam, y) \land (((x \neq k) \land (y \neq k)) \rightarrow \neg T(Sam, k)))$$

Question 12:

Solve the following question from the Discrete Math Zybook:

1. Exercise 1.8.2, sections b - e

b. Every patient was given the medication or the placebo or both

$$\forall x(D(x) \lor P(x))$$
$$\neg x(D(x) \lor P(x))$$
$$\exists x(\neg D(x) \land \neg P(x))$$

There is a patient who is not given medication and not given the placebo.

c. There is a patient who took the medication and had migraines

$$\exists x(D(x) \land M(x))$$
$$\neg \exists x(D(x) \land M(x))$$
$$\forall x(\neg D(x) \lor \neg M(x))$$

Every patient was not given the medication or did not have migraines

d. Every patient who took the placebo had migraines

$$\forall x(P(x) \to M(x))$$
$$\neg]forallx(P(x) \to M(x))$$
$$\exists x(P(x) \land \neg M(x))$$

There is one patient who took the placebo and did not have migraines

e. There is a patient who had migraines and was given the placebo

$$\exists x(M(x) \land P(x))\ \neg \exists x(M(x) \land P(x))$$
$$\exists x(P(x) \land \neg P(x))$$

Every patient did not have migraines or was not given the placebo

2. Exercise 1.9.4, sections c - e

c. $\exists x \forall y(P(x,y) \to (Q(x,y))$

$$\forall x \exists y(P(x,y) \land \neg Q(x,y))$$

d. $\exists x \forall y(P(x,y) \leftrightarrow P(y,x))$

$$\forall x \exists y((P(x,y) \land \neg P(y,x) \land \neg P(x,y)))$$

e. $\exists x \exists y(P(x,y) \land \forall x \forall y(Q(x,y)$

$$\forall x \forall y \neg P(x,y) \lor \exists x \exists y \neg Q(x,y)$$