NYU Computer Science Bridge HW2

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Question 5:
a) Solve the following questions from the Discrete Math zyBook:
1. Exercise 1.12.2, sections b, e

b.

$$\begin{array}{l} p \to (q \land r) \\ \neg q \\ \hline \therefore \neg p \end{array}$$

1.	$\neg q$	Hypothesis
2.	$\neg q \lor \neg r$	Addition, 1
3.	$\neg(q \land r)$	De Morgan, 2
4.	$p \to (q \land r)$	Hypothesis
5.	$\neg p$	Modus tollens, 3 4

e.

$$\begin{array}{l} p \lor q \\ \neg p \lor r \\ \neg q \\ \hline \therefore r \end{array}$$

1.	$p \lor q$	Hypothesis
2.	$\neg p \lor r$	Hypothesis
3.	$q \lor r$	Resolution, 1 2
4.	$\neg q$	Hypothesis
5.	r	Disjunctive Syllogism, 3 4

Exercise 1.12.3, section c

$$\begin{array}{l} p \lor q \\ \neg p \\ \hline \therefore q \end{array}$$

1.	$p \lor q$	Hypothesis
2.	$\neg(\neg p) \lor q$	Double Negation
3.	$\neg p \to q$	Conditional Identity, 2
4.	$\neg p$	Hypothesis
5.	q	Modus Ponens, 3 4

Exercise 1.12.5, section c

I will buy a new car and a new house only if I get a job.
I am not going to get a job.

$\therefore$ I will not buy a new car.

c: I will buy a new car
h: I will buy a new house
j: I get a job

$$\begin{array}{l} (c \land h) \to j \\ \neg j \\ \hline \therefore \neg c \end{array}$$

Argument is not valid. When both hypotheses are true the conclusion is false: c = True, h and j = False

Exercise 1.12.5, section d

I will buy a new car and a new house only if I get a job.
I am not going to get a job.
I will buy a new house.

$\therefore$ I will not buy a new car.

c: I will buy a new car
h: I will buy a new house
j: I get a job

$$\begin{array}{l} j \to (c \land h) \\ \neg j \\ h \\ \hline \therefore \neg c \end{array}$$

The argument is valid

1.	$(c \wedge h) \to j$	Hypothesis
2.	$\neg(c \wedge h) \vee j$	Conditional Identity
3.	$j \vee \neg(c \wedge h)$	Commutative law, 2
4.	$\neg j$	Hypothesis
5.	$\neg(c \wedge h)$	Disjunctive Syllogism, 3 4
6.	$\neg c \vee \neg h$	De Morgans, 5
7.	$\neg h \vee \neg c$	Commutative
8.	h	Hypothesis
9.	$\neg\neg h$	Double Negation
10.	$\neg c$	Disjunctive Syllogism, 7 9

b)
Exercise 1.13.3, section b

$$\begin{array}{l} \exists x(P(x) \vee Q(x)) \\ \exists x \neg Q(x) \\ \hline \therefore \exists x \neg P(x) \end{array}$$

	P	Q
a	F	T
b	F	F

When x = a$\exists x(P(x) \vee Q(x))$ is true and when x = b $\exists x \neg Q(b)$ is also true, but $\exists x P(x)$is false proving it is not valid

Exercise 1.13.5, section d

Every student who missed class got a detention.
Penelope is a student in the class.
Penelope did not miss class.

$\therefore$ Penelope did not get a detention

M(x): x missed class
D(x); x got a detention

$\forall x(M(x) \to D(x))$
Penelope is a student in the class
$\neg M(Penelope)$

$\therefore \neg D(Penelope)$

The argument is invalid. If M(Penelope) is false and D(Penelope) is true, then the hypothesis are true and conclusion is false.

Exercise 1.13.5, section e

Every student who missed class or got a detention did not get an A.
Penelope is a student in the class.
Penelope got an A.

$\therefore$ Penelope did not get a detention

M(x): x missed class
A(x): x got an A
D(x); x got a detention

$\forall x(M(x) \lor D(x)) \to A(x)$
Penelope is a student in the class
$A(Penelope)$

$\therefore \neg D(Penelope)$

1.	Penelope is a student in the class	Hypothesis
2.	$\forall x(M(x) \lor D(x) \to \neg A(x))$	Hypothesis
3.	$M(Penelope) \lor D(Penelope) \to \neg A(Penelope)$	Universal Insatntiation
4.	$A(Penelope)$	Hypothesis
5.	$\neg(\neg A(Penelope))$	Double negation, 4
6.	$\neg(M(Penelope) \lor D(Penelope))$	Modus Tollens, 3 5
7.	$\neg M(Penelope) \land \neg D(Penelope)$	De Morgan
8.	$\neg D(Penelope) \land \neg M(Penelope)$	cumutative
9.	$\neg D(Penelope)$	simplification

Question 6:
Exercise 2.4.1, section d
The product of two odd integers is an odd integer.

Proof. By Direct proof: Let x, y be odd integers. Then $\exists x | x = 2k + 1$ for some integer k and $\exists y | y = 2j + 1$ for some integer j. Then

$$\begin{aligned} xy &= (2k + 1)(2j + 1) \\ &= 4kj + 2k + 2j + 1 \\ &= 2(2kj + k + j) + 1 \end{aligned}$$

Since (2kj + k + j) is an integer in $xy = 2(2kj + k + j) + 1$, the product of xy is also an odd integer. ■

Exercise 2.4.3, section b
If x is a real number and $x \leq 3, \text{then} 12 - 7x + x^2 \geq 0$.

Proof. By direct proof: Let x be a real number and $x \leq 3$

$$\begin{aligned} 12 - 7x + x^2 = x^2 - 7x + 12 &\geq 0 \\ = (x - 3)(x - 4) &\geq 0 \end{aligned}$$

Since $x \leq 3$, $x - 3 \leq 0$ and $x - 4 \leq 0$, then $(x - 3)(x - 4) \geq 0$ ■

Question 7:

Exercise 2.5.1 section d

For every integer n, if $n^2 - 2n + 7$ is even, then n is odd

Proof. By Contrapositive: Assume n is an even integer such that n = 2k for some integer k. Show $n^2 - 2n + 7$ is an odd integer.

$$\begin{aligned} n^2 - 2n + 7 &= (2k)^2 - 2(2k) + 7 \\ &= 4k^2 - 4k + 7 \\ &= 2(2k^2 - 2k + 3) + 1 \end{aligned}$$

Since k is an integer, then $(2k^2 - 2k + 3)$ is also an integer in $2(2k^2 - 2k + 3)$. Therefore, $n^2 - 2n + 7 = 2k + 1$ is an odd integer.

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Exercise 2.5.4 section a

For every pair of real numbers x and y, if $x^3 + xy^2 \leq x^2y + y^3$ then $x \leq y$

Proof. By contrapositive: Assume for every pair of real numbers x and y, x > y. Show $x^3 + xy^2 > x^2y + y^3$

$$\begin{aligned} x^3 + xy^2 &= x(x^2 + y^2) \\ &> y(x^2 + y^2) \text{ by substitution x > y} \\ &= x^2y + y^3 \end{aligned}$$

■

Exercise 2.5.4 section b

For every pair of real numbers x and y, if $x + y > 20$ then $x > 10$ or $y > 10$.

Proof. By Contrapositive: Assume for every pair of real numbers x and y, $x \leq 10$ and $y \leq 10$. Show $x + y \leq 20$

$$\begin{aligned} x \leq 10 + y \leq 10 \\ = x + y \leq 20 \end{aligned}$$

■

Exercise 2.5.5 section c

For every non-zero real number x, if x is irrational then $1/x$ is irrational

Proof. By Contrapostive: Assume x is a real number and $1/x$ is not irrational. Show that x is rational for every non zero real number.

Since $1/x$ is a real number and not irrational, then it has to be a rational number.

There exists an a and b such that a and b are integers.

$$1/x = a/b \ (a \neq 0 \text{ and } b \neq 0)$$
$$x = b/a$$

Since x is a ratio of two integers, a and b, x is rational.

■

Question 8:

Exercise 2.6.6 sections c

The average of three real numbers is greater than or equal to at least one of the numbers.

Proof. By Contradiction: Assume x, y, z are real numbers. Show the average of the three real numbers is less than all thee numbers.

$$\frac{x+y+z}{3} < x, \frac{x+y+z}{3} < y, \frac{x+y+z}{3} < z$$

$$\frac{x+y+z}{3} + \frac{x+y+z}{3} + \frac{x+y+z}{3} < x+y+z$$
$$\frac{3x+3y+3z}{3} < x+y+z$$
$$x+y+z < x+y+z$$

Since $x+y+z \not< x+y+z$, then the average of three real numbers is greater than or equal to at least one of the three numbers. ■

Exercise 2.6.6 sections d

There is no smallest integer

Proof. By Contradiction: Assume There is a smallest integer. Let smallest integer be k.
Since k is an integer, subtracting k by 1 will give us an integer $k-1$. Since $(k-1) < k$, there exists a smaller integer.
$\therefore$ There is no smallest integer.

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Question 9:
Exercise 2.7.2 section b

If integers x and y have the same parity, then $x+y$ is even. The parity of a number tells whether the number is odd or even. If x and y have the same parity, they are either both even or both odd.

Proof. By Cases:
Case 1: x and y are even integers. If x and y are even, x = 2k and y = 2j for some integer k, j

$$\begin{aligned} x + y &= 2k + 2j \\ &= 2(k + j) \end{aligned}$$

Since k and j are integers, there exists an integer l such that $l = k + j$. Since $x + y = 2l$, $x + y$ is even

Case 2: x and y are odd integers. If x and y are odd, x = 2m + 1and y = 2n + 1 for some integer m, n

$$\begin{aligned} x + y &= 2m + 1 + 2n + 1 \\ &= 2m + 2n + 2 \\ &= 2(m + n + 1) \end{aligned}$$

Since m, n, and 1 are integers, there exists an integer z, such that $z = (m + n + 1)$, Since $x + y = 2z$, x + y is even. ■