NYU Computer Science Bridge HW3

Summer 2023 Name: Jacky Choi

Question 7 Part a:

Exercise 3.1.1 a
$27 \in A$ True 3 x 9 = 27. 27 is an integer multiple of 3

Exercise 3.1.1 b
$27 \in B$ False 27 is not a perfect square

Exercise 3.1.1 c
$100 \in B$ True 10 x 10 = 100

Exercise 3.1.1 d
$E \subseteq C$ or $C \subseteq E$ False

Exercise 3.1.1 e
$E \subseteq A$ True

Exercise 3.1.1 f
$A \subset E$ False

Exercise 3.1.1 g
$E \in A$ True

Question 7 Part b:

Exercise 3.1.2 a
$15 \subset A$ False

Exercise 3.1.2 b
$\{15\} \subset A$ True

Exercise 3.1.2 c
$\emptyset \subset C$ True

Exercise 3.1.2 d
$D \subseteq D$ True

Exercise 3.1.2 e
$\emptyset \in B$ False

Question 7 Part c:

Exercise 3.1.5 b
{3,6,9,12, ...} $= \{x \in N$:x is a multiple of 3} Set is infinite

Exercise 3.1.5 d
{0, 10, 20, 30, ..., 1000} $= \{x \in N : 0 \leq x \leq 1000 \wedge$x is a multiple of 10}
Cardinality 101

Question 7 Part d:

Let X $= \{1, \{1\}, \{1, 2\}, 2, \{3\}, 4\}$

Exercise 3.2.1 a
$2 \in X$ True

Exercise 3.2.1 b
$\{2\} \subseteq X$ True

Exercise 3.2.1 c
$\{2\} \in X$ False

Exercise 3.2.1 d
$3 \in X$ False

Exercise 3.2.1 e
$\{1, 2\} \in X$ True

Exercise 3.2.1 f
$\{1, 2\} \subseteq X$ True

Exercise 3.2.1 g
$\{2, 4\} \subseteq X$ True

Exercise 3.2.1 h
$\{2, 4\} \in X$ False

Exercise 3.2.1 i
$\{2, 3\} \subseteq X$ False

Exercise 3.2.1 j
$\{2, 3\} \in X$ False

Exercise 3.2.1 k
$|\mathrm{X}| = 7$ False

Question 8:

Exercise 3.2.4 b
Let A = $\{1, 2, 3\}$. What is $\{X \in P(A) : 2 \in X\}$?
{2, {2,3}} $\blacksquare$

Question 9 Part a:

Exercise 3.3.1 c
$(A \cap C)$ = {-3, 1, 17}

Exercise 3.3.1 d
$A \cup (B \cap C)$ = {-5, -3, 0, 1, 4, 17}

Exercise 3.3.1 e

$A \cap B \cap C = \{1\}$

Question 9 Part b:

Exercise 3.3.3 a
$\bigcap_{i=2}^{5} A_i = A_2 \cap A_3 \cap A_4 \cap A_5$
$= \{1, 2, 4\} \cap \{1, 3, 9\} \cap \{1, 4, 16\} \cap \{1, 5, 25\} = \{1\}$
Exercise 3.3.3 b
$\bigcup_{i=2}^{5} A_i = A_2 \cup A_3 \cup A_4 \cup A_5$
$= \{1, 2, 4\} \cup \{1, 3, 9\} \cup \{1, 4, 16\} \cup \{1, 5, 25\} = \{1, 2, 3, 4, 5, 9, 16, 25\}$
Exercise 3.3.3 e
$\bigcap_{i=1}^{100} C_i = C_1 \cap C_2 \cap C_3 \cap .... \cap C_100$
$= \{x : x \in \mathbb{R} : \frac{-1}{100} \leq x \leq \frac{1}{100}\}$

Exercise 3.3.3 f
$\bigcup_{i=1}^{100} C_i = C_1 \cup C_2 \cap C_3 \cup .... \cup C_100$
$= \{x : x \in \mathbb{R} : -1 \leq x \leq 1\}$

Question 9 Part c:

Exercise 3.3.4 b
$P(A \cup B) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}$

Exercise 3.3.4 d
$P(A) \cup P(B)$
P(A) $= \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$
P(B) $= \{\emptyset, \{b\}, \{c\}, \{b, c\}\}$
$P(A) \cup P(B) =$$\{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}\}$ ■

Question 10 Part a:

Exercise 3.5.1 b
One element from: B x A x C = (no-foam, venti, whole)

Exercise 3.5.1 c

Roster Notation: B x C = {(foam, nonfat), (foam, whole), (no-foam, nonfat), (no foam, whole)}

Question 10 Part b:

Exercise 3.5.3 b
$\mathbb{Z}^2 \subseteq \mathbb{R}^2$ True

Exercise 3.5.3 c
$\mathbb{Z}^2 \bigcap \mathbb{Z}^3 = \emptyset$ False

Exercise 3.5.3 e
For any three sets A, B, C, if $A \subseteq B$, then A x C $\subseteq$B x C
Example: Let A = {a,b}, B = {a,b,c}, and c = {d}
A x C = {(a,d), (b,d)}
B x C = {(a,d),(b,d),(c,d)}
$\therefore$ A x B $\subseteq$ B x C True ■

Question 10 Part c:

Exercise 3.5.6 d
{xy: where x $\in \{0\} \cup \{0\}^2$ and y $\in \{1\} \cup \{0,1\}^2$}
x = {0, 00} and y = {1, 11}
xy = {01, 011, 001, 0011} ■

Exercise 3.5.6 e

{xy: where x $\in$ {aa, bb} and y $\in$ {a} $\cup$ $\{a\}^2$}
x = {aa, bb} and y = {a, aa}
xy = {aaa, aaaa, aba, abaa} ■

Question 10 Part d:

Exercise 3.5.7 c
$(A \times B) \cup (A \times C)$
A x B = {ab, ac}
A x C = {aa, ab, ad}
$(A \times B) \cup (A \times C)$ = {aa, ab, ac, ad} $\blacksquare$

Exercise 3.5.7 f
P(A x B) = {$\emptyset$, {ab}, {ac}, {ab, ac}} $\blacksquare$

Exercise 3.5.7 g
P(A) x P(B) Use ordered pair notation for elements of the Cartesian product
P(A) = {$\emptyset$, {a}}
P(B) = {$\emptyset$, {b}, {c}, {b, c}}
P(A) x P(B) = { {$\emptyset$, $\emptyset$}, {$\emptyset$, {b}}, {$\emptyset$, {c}}, {$\emptyset$, {b, c}}, {{a},$\emptyset$},
{{a}, {b}}, {{a}, {c}}, {{a}, {b,c}} } $\blacksquare$

Question 11 Part a:

Exercise 3.6.2 b

$$B \cup A \cap (\overline{B} \cup A) = A$$

$B \cup A \cap (\overline{B} \cup A) = A$	
$(B \cap \overline{B}) \cup A$	Distributive Law
$A \cup (B \cap \overline{B})$	Commutative Law
$A \cup \emptyset$	Complement
A	Identity Law

■

Exercise 3.6.2 c

$$\overline{A \cup \overline{B}} = \overline{A} \cup B$$

$\overline{A \cup \overline{B}} = \overline{A} \cup B$	
$\overline{A} \cup B$	De Morgans

■

Question 11 Part b:

Exercise 3.6.3 b
$A - (B \cap A) = A$
If A = {1,2} and B = {2}, then A - (B ∩ A) is {1} ≠ {2} ■

Exercise 3.6.3 d

(B - A) ∪ A = A
if A = {1} and B = {1,2}, then (B - A) ∪ A is {1, 2} ≠ {1} ■

Question 11 Part c:

Exercise 3.6.4 b

$$A \cap (B - A) = \emptyset$$

$A \cap (B - A)$	
$A \cap (B \cap \overline{A})$	Set Subtraction
$(B \cap \overline{A}) \cap A$	Commutative Law
$B \cap (\overline{A} \cap A)$	Associative
$\emptyset \cap B$	Complement
$\emptyset$	Dominaton Law

■

Exercise 3.6.4 c

$$A \cup (B - A) = A \cup B$$

$A \cup (B - A)$	
$A \cup (B \cap \overline{A})$	Set Subtraction
$(A \cup B) \cap (A \cup \overline{A})$	Distributive Law
$(A \cup B) \cap U$	Complement
$A \cup B$	Identity Law

■