

# NYU Computer Science Bridge HW2

Summer 2023 Name: Jacky Choi

## Question 5:

### Exercise 1.12.2 B

$$\frac{p \rightarrow (q \wedge r) \quad \neg q}{\therefore \neg p}$$

1.  $\neg q$  Hypothesis
2.  $\neg q \vee \neg r$  Addition, 1
3.  $\neg(q \wedge r)$  De Morgan, 2
4.  $p \rightarrow (q \wedge r)$  Hypothesis
5.  $\neg p$  Modus tollens, 3 4

■

### Exercise 1.12.2 E

$$\frac{p \vee q \quad \neg p \vee r \quad \neg q}{\therefore r}$$

1.  $p \vee q$  Hypothesis
2.  $\neg p \vee r$  Hypothesis
3.  $q \vee r$  Resolution, 1 2
4.  $\neg q$  Hypothesis
5.  $r$  Disjunctive Syllogism, 3 4

■

### Exercise 1.12.3 C

$$\frac{p \vee q}{\neg p} \quad \frac{}{\therefore q}$$

1.  $p \vee q$  Hypothesis
2.  $\neg(\neg p) \vee q$  Double Negation
3.  $\neg p \rightarrow q$  Conditional Identity, 2
4.  $\neg p$  Hypothesis
5.  $q$  Modus Ponens, 3 4

■

### Exercise 1.12.5 C

I will buy a new car and a new house only if I get a job.

I am not going to get a job.

---

$\therefore$  I will not buy a new car.

c: I will buy a new car  
h: I will buy a new house  
j: I get a job

$$\frac{(c \wedge h) \rightarrow j}{\neg j} \quad \frac{}{\therefore \neg c}$$

Argument is not valid. When both hypotheses are true the conclusion is false:  $c = \text{True}$ ,  $h = \text{True}$  and  $j = \text{False}$

■

### Exercise 1.12.5 D

I will buy a new car and a new house only if I get a job.  
 I am not going to get a job.  
 I will buy a new house.  


---

 $\therefore$  I will not buy a new car.

c: I will buy a new car  
 h: I will buy a new house  
 j: I get a job

$$\begin{array}{l} j \rightarrow (c \wedge h) \\ \neg j \\ h \\ \hline \therefore \neg c \end{array}$$

The argument is valid

1.  $(c \wedge h) \rightarrow j$  Hypothesis
2.  $\neg(c \wedge h) \vee j$  Conditional Identity
3.  $j \vee \neg(c \wedge h)$  Commutative law, 2
4.  $\neg j$  Hypothesis
5.  $\neg(c \wedge h)$  Disjunctive Syllogism, 3 4
6.  $\neg c \vee \neg h$  De Morgans, 5
7.  $\neg h \vee \neg c$  Commutative
8.  $h$  Hypothesis
9.  $\neg\neg h$  Double Negation
10.  $\neg c$  Disjunctive Syllogism, 7 9

### Exercise 1.13.3 B

$$\begin{array}{l} \exists x(P(x) \vee Q(x)) \\ \exists x\neg Q(x) \\ \hline \therefore \exists x\neg P(x) \end{array}$$

	P	Q
a	F	T
b	F	F

When  $x = a$   $\exists x(P(x) \vee Q(x))$  is true and when  $x = b$   $\exists x\neg Q(x)$  is also true, but  $\exists x\neg P(x)$  is false proving it is not valid



**Exercise 1.13.5 D**

Every student who missed class got a detention.  
 Penelope is a student in the class.  
 Penelope did not miss class.  


---

 $\therefore$  Penelope did not get a detention

$M(x)$ :  $x$  missed class  
 $D(x)$ :  $x$  got a detention

$\forall x(M(x) \rightarrow D(x))$   
 Penelope is a student in the class  
 $\neg M(Penelope)$   


---

 $\therefore \neg D(Penelope)$

The argument is invalid. If  $M(Penelope)$  is false and  $D(Penelope)$  is true, then the hypothesis are true and conclusion is false.

■

**Exercise 1.13.5 E**

Every student who missed class or got a detention did not get an A.  
 Penelope is a student in the class.  
 Penelope got an A.  


---

 $\therefore$  Penelope did not get a detention

$M(x)$ :  $x$  missed class  
 $A(x)$ :  $x$  got an A  
 $D(x)$ :  $x$  got a detention

$\forall x(M(x) \vee D(x) \rightarrow A(x))$   
 Penelope is a student in the class  
 $A(Penelope)$   


---

 $\therefore \neg D(Penelope)$

- |    |                                                             |                         |
|----|-------------------------------------------------------------|-------------------------|
| 1. | Penelope is a student in the class                          | Hypothesis              |
| 2. | $\forall x(M(x) \vee D(x) \rightarrow \neg A(x))$           | Hypothesis              |
| 3. | $M(Penelope) \vee D(Penelope) \rightarrow \neg A(Penelope)$ | Universal Instantiation |
| 4. | $A(Penelope)$                                               | Hypothesis              |
| 5. | $\neg(\neg A(Penelope))$                                    | Double negation, 4      |
| 6. | $\neg(M(Penelope) \vee D(Penelope))$                        | Modus Tollens, 3 5      |
| 7. | $\neg M(Penelope) \wedge \neg D(Penelope)$                  | De Morgan               |
| 8. | $\neg D(Penelope) \wedge \neg M(Penelope)$                  | Cummutative             |
| 9. | $\neg D(Penelope)$                                          | Simplification          |

■

**Question 6:**

Exercise 2.4.1 D

The product of two odd integers is an odd integer.

*Proof.* By Direct proof: Let  $x, y$  be odd integers. Then  $\exists x | x = 2k + 1$  for some integer  $k$  and  $\exists y | y = 2j + 1$  for some integer  $j$ . Then

$$\begin{aligned} xy &= (2k + 1)(2j + 1) \\ &= 4kj + 2k + 2j + 1 \\ &= 2(2kj + k + j) + 1 \end{aligned}$$

Since  $(2kj + k + j)$  is an integer in  $xy = 2(2kj + k + j) + 1$ , the product of  $xy$  is also an odd integer. ■

**Exercise 2.4.3 B**

If  $x$  is a real number and  $x \leq 3$ , then  $12 - 7x + x^2 \geq 0$ .

*Proof.* By direct proof: Let  $x$  be a real number and  $x \leq 3$

$$\begin{aligned} 12 - 7x + x^2 &= x^2 - 7x + 12 \geq 0 \\ &= (x - 3)(x - 4) \geq 0 \end{aligned}$$

Since  $x \leq 3$ ,  $x - 3 \leq 0$  and  $x - 4 \leq 0$ , then  $(x - 3)(x - 4) \geq 0$  ■

**Question 7:****Exercise 2.5.1 D**

For every integer  $n$ , if  $n^2 - 2n + 7$  is even, then  $n$  is odd

*Proof.* By Contrapositive: Assume  $n$  is an even integer such that  $n = 2k$  for some integer  $k$ . Show  $n^2 - 2n + 7$  is an odd integer.

$$\begin{aligned} n^2 - 2n + 7 &= (2k)^2 - 2(2k) + 7 \\ &= 4k^2 - 4k + 7 \\ &= 2(2k^2 - 2k + 3) + 1 \end{aligned}$$

Since  $k$  is an integer, then  $(2k^2 - 2k + 3)$  is also an integer in  $2(2k^2 - 2k + 3)$ . Therefore,  $n^2 - 2n + 7 = 2k + 1$  is an odd integer. ■

**Exercise 2.5.4 A**

For every pair of real numbers  $x$  and  $y$ , if  $x^3 + xy^2 \leq x^2y + y^3$  then  $x \leq y$

*Proof.* By contrapositive: Assume for every pair of real numbers  $x$  and  $y$ ,  $x > y$ . Show  $x^3 + xy^2 > x^2y + y^3$

$$\begin{aligned} x^3 + xy^2 &= x(x^2 + y^2) \\ &> y(x^2 + y^2) \text{ by substitution } x > y \\ &= x^2y + y^3 \end{aligned} \quad \blacksquare$$

**Exercise 2.5.4 B**

For every pair of real numbers  $x$  and  $y$ , if  $x + y > 20$  then  $x > 10$  or  $y > 10$ .

*Proof.* By Contrapositive: Assume for every pair of real numbers  $x$  and  $y$ ,  $x \leq 10$  and  $y \leq 10$ . Show  $x + y \leq 20$

$$\begin{aligned} x &\leq 10 + y \leq 10 \\ &= x + y \leq 20 \end{aligned} \quad \blacksquare$$

**Exercise 2.5.5 C**

For every non-zero real number  $x$ , if  $x$  is irrational then  $1/x$  is irrational

*Proof.* By Contrapostive: Assume  $x$  is a real number and  $1/x$  is not irrational. Show that  $x$  is rational for every non zero real number.

Since  $1/x$  is a real number and not irrational, then it has to be a rational number. There exists an  $a$  and  $b$  such that  $a$  and  $b$  are integers.

$$\begin{aligned}1/x &= a/b \text{ (} a \neq 0 \text{ and } b \neq 0\text{)} \\ x &= b/a\end{aligned}$$

Since  $x$  is a ratio of two integers,  $a$  and  $b$ ,  $x$  is rational.

■

**Question 8:****Exercise 2.6.6 C**

The average of three real numbers is greater than or equal to at least one of the numbers.

*Proof.* By Contradiction: Assume  $x, y, z$  are real numbers. Show the average of the three real numbers is less than all three numbers.

$$\frac{x+y+z}{3} < x, \frac{x+y+z}{3} < y, \frac{x+y+z}{3} < z$$

$$\frac{x+y+z}{3} + \frac{x+y+z}{3} + \frac{x+y+z}{3} < x+y+z$$

$$\frac{3x+3y+3z}{3} < x+y+z$$

$$x+y+z < x+y+z$$

Since  $x+y+z \not< x+y+z$ , then the average of three real numbers is greater than or equal to at least one of the three numbers. ■

**Exercise 2.6.6 D**

There is no smallest integer

*Proof.* By Contradiction: Assume There is a smallest integer. Let smallest integer be  $k$ . Since  $k$  is an integer, subtracting  $k$  by 1 will give us an integer  $k-1$ . Since  $(k-1) < k$ , there exists a smaller integer.

∴ There is no smallest integer. ■



**Question 9:**  
**Exercise 2.7.2 B**

If integers  $x$  and  $y$  have the same parity, then  $x + y$  is even. The parity of a number tells whether the number is odd or even. If  $x$  and  $y$  have the same parity, they are either both even or both odd.

*Proof.* By Cases:

**Case 1:  $x$  and  $y$  are even integers.** If  $x$  and  $y$  are even,  $x = 2k$  and  $y = 2j$  for some integer  $k, j$

$$\begin{aligned}x + y &= 2k + 2j \\ &= 2(k + j)\end{aligned}$$

Since  $k$  and  $j$  are integers, there exists an integer  $l$  such that  $l = k + j$ . Since  $x + y = 2l$ ,  $x + y$  is even

**Case 2:  $x$  and  $y$  are odd integers.** If  $x$  and  $y$  are odd,  $x = 2m + 1$  and  $y = 2n + 1$  for some integer  $m, n$

$$\begin{aligned}x + y &= 2m + 1 + 2n + 1 \\ &= 2m + 2n + 2 \\ &= 2(m + n + 1)\end{aligned}$$

Since  $m, n$ , and  $1$  are integers, there exists an integer  $z$ , such that  $z = (m + n + 1)$ , Since  $x + y = 2z$ ,  $x + y$  is even. ■