Homework 4 AAE 421 Due 11/03 5:00 pm **Name___________________**

Problem 1. (20pts)

The longitudinal dynamics of a plane are described by:

$$\begin{bmatrix} \Delta\dot{u} \\ \Delta\dot{w} \\ \Delta\dot{q} \\ \Delta\dot{\theta} \end{bmatrix} = \begin{bmatrix} X_u & X_w & 0 & -g \\ Z_u & Z_w & u_0 & 0 \\ M_u & M_w & M_q & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta u \\ \Delta w \\ \Delta q \\ \Delta\theta \end{bmatrix} + \begin{bmatrix} X_{\delta_e} \\ Z_{\delta_e} \\ M_{\delta_e} \\ 0 \end{bmatrix} \Delta\delta_e .$$

The phugoid mode has the approximate 2-D dynamics described by:

$$\begin{bmatrix} \Delta\dot{u} \\ \Delta\dot{\theta} \end{bmatrix} = \begin{bmatrix} X_u & -g \\ -Z_u/u_0 & 0 \end{bmatrix} \begin{bmatrix} \Delta u \\ \Delta\theta \end{bmatrix} + \begin{bmatrix} X_{\delta_e} \\ Z_{\delta_e}/u_0 \end{bmatrix} \Delta\delta_e$$

and the short period mode has the approximate 2-D dynamics described by:

$$\begin{bmatrix} \Delta\dot{w} \\ \Delta\dot{q} \end{bmatrix} = \begin{bmatrix} Z_w & u_0 \\ M_w & M_q \end{bmatrix} \begin{bmatrix} \Delta w \\ \Delta q \end{bmatrix} + \begin{bmatrix} Z_{\delta_e} \\ M_{\delta_e} \end{bmatrix} \Delta\delta_e$$

The relative parameters of a fixed-wing aircraft are listed in the following table.

X_u	X_w	Z_u	Z_w	M_u	M_w	M_q	u_0
-0.045 s^{-1}	0.036 s^{-1}	-0.369 s^{-1}	-2.02 s^{-1}	0	-0.05 $\frac{1}{ft \cdot s}$	-2.05 s^{-1}	176 $\frac{ft}{s}$

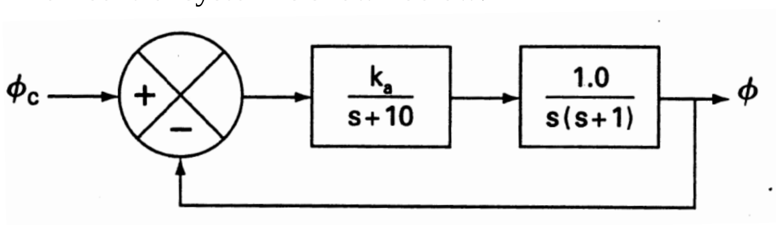
(1) Plot the state response using both the state space and Simulink methods for the original model and approximate model, respectively, assuming initial conditions $\vec{x}_0^{tr} = [10 \quad 0 \quad \pi/3 \quad 0]^{tr}$.

(2) Compute the % error of the approximation model to the original model in terms of the damped natural period $T_d = 2\pi/\omega_d$. [Note: define a % error to be $[(*_{2D} - *_{4D})/*_{4D}] \times 100\%$.]

(3) If you are allowed to change two parameters in the coefficient matrices, what will you choose and how will you change them to decrease the damped natural period?

Problem 2. (20pts)

A roll control system is shown below.



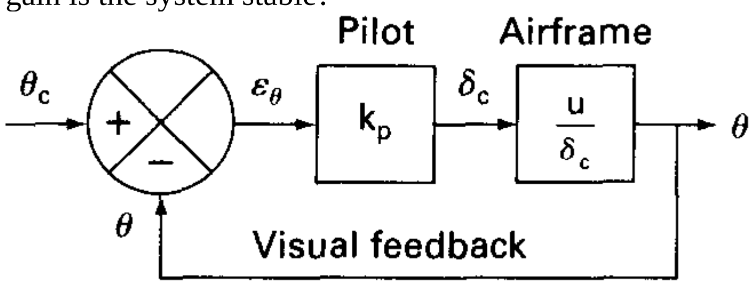
(1) Sketch the root locus diagram for this system.
(2) Determine the value of the gain, K_a, so that control system has a damping ratio = 0.707.
(3) What is the steady-state error for a step and ramp input with the designed system?
(4) Sketch the response of the control system to a 5^o step change in bank angle command.
(5) Repeat this problem using control design software in MATLAB

Problem 3. (20pts)

The Wright Flyer was statically and dynamically unstable. However, because the Wright brothers incorporated sufficient control authority into their design they were able to fly their airplane successfully. Although the airplane was difficult to fly, the combination of the pilot and airplane could be a stable system. The closed-loop pilot is represented as a pure gain, K_p, and the pitch attitude to canard deflection is given as follows:

$$\frac{\theta}{\delta_e} = \frac{11.0(s+0.5)(s+3.0)}{(s^2+0.72s+1.44)(s^2+5.9s-11.9)}$$

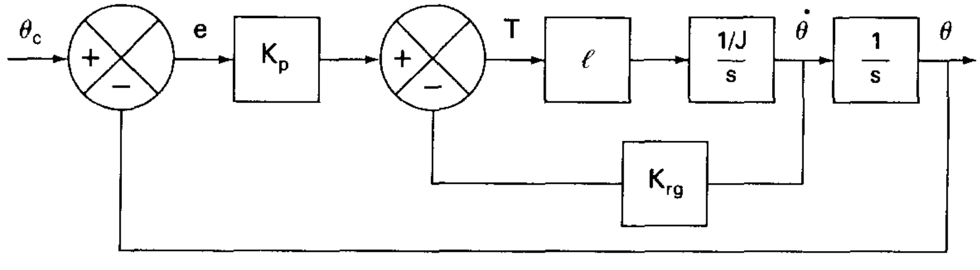
Determine the root locus plot of the closed-loop system shown below. For what range of pilot gain is the system stable?



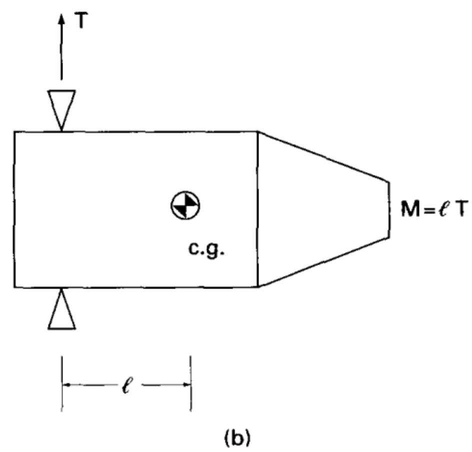
Problem 4. (20pts)

The block diagram for a pitch attitude control system of a spacecraft is shown in Figure (a). Control of the spacecraft is achieved through thrusters located on the side of the spacecraft as illustrated in Figure (b).

(1) Determine the root locus plot for the control system if the rate loop is disconnected ($K_{rg} = 0$). Comment on the potential performance of this system for controlling the pitch attitude.
(2) Determine the rate gain K_{rg}, and the outer loop gyro gain K_p, so that the system has a damping ratio = 0.707 and a settling time $t_s \leq 1.5$ s.
(3) Verify your design by providing the plot of system unit step response.



(a)



Problem 5. (20pts)

A wind-tunnel model is constrained so that it can rotate only about the *z* axis; that is, pure yawing motion. The equation of motion for a constrained yawing motion was shown as follows:

$$\Delta\ddot{\psi} - N_r \Delta\dot{\psi} + N_\beta \Delta\psi = N_{\delta_r} \Delta\delta_r$$

where $N_\beta = 2.0\ s^{-2}$, $N_r = -0.5\ s^{-1}$ and $N_{\delta_r} = -10\ s^{-2}$. Design a heading control system so that the model has the following closed-loop performance characteristics:

$\zeta = 0.6$, $t_s \leq 2.5$. Assume that the rudder servo transfer function can be represented as $\frac{\Delta\delta_r}{e} = \frac{k}{s+10}$.

(Hint: A possible concept for a heading control system is shown below)