College of Engineering
School of Aeronautics and Astronautics

AAE 532
Orbital Mechanics

PS 6
In-Plane Maneuver

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Problem 1: Assume a relative two-body model and a space vehicle that is currently in a Mars orbit with $r_p = 1.1R_♂$ and $r_a = 6.0R_♂$. The spacecraft is currently located at $\theta_c^* = 90°$ at time t_c. A single in-plane adjustment will be employed to circularize the orbit.

(a) At what true anomaly values does $r = 4.5R_♂$? (Note that two locations exist!) Select the location that is the earliest opportunity after t_c to reach $r = 4.5R_♂$. let this time be t_1. Determine $v_1, \gamma_1, E_1, (t_1 - t_p)$ at this location.

From the radius of periapsis and apoapsis we can find the orbital parameters to define the orbit.

$$a = 0.5(r_p + r_a) = 0.5(1.1R_♂ + 6.0R_♂) = 4R_♂ \ .$$

Since, $R_♂ = 3397\ km$, the semi-major axis becomes

$$a = 1.2059e + 4\ km \ .$$

Next, from the periapsis and apoapsis we can calculate the eccentricity

$$e = \frac{r_a - r_p}{r_a + r_p} = 0.6901 \ .$$

Then, the semi-latus rectum, p becomes

$$p = a(1 - e^2) = 6.3155e + 3\ km \ .$$

When the gravitational parameter, $\mu \cong 42828\ km^3/s^2$, the specific angular momentum becomes

$$h = \sqrt{\mu p} = 1.6446e + 4\ \frac{km^2}{s} \ .$$

Then, at $r = 4.5R_♂$ we can find the true anomaly by

$$\theta_1^* = arccos\left(\frac{1}{e}\left(\frac{p}{4.5R_♂} - 1\right)\right)$$

$$\theta_1^* = \pm 148.2486°$$

The earliest to $\theta_c^* = 90°$ is the positive value. Thus,

$$\theta_1^* = 148.2486° \ .$$

The velocity at this point is

$$v_1 = \sqrt{\mu\left(\frac{2}{r_1} - \frac{1}{a}\right)} = 1.4325\ km/s \ .$$

The flight path angle is positive because at time t_1 it is ascending,

$$\gamma_1 = arccos\left(\frac{h}{r_1 v_1}\right) = 41.317° \ .$$

Using the equation below the eccentric anomaly becomes,

$$tan\frac{\theta^*}{2} = \left(\frac{1+e}{1-e}\right)^{1/2} tan\frac{E}{2}$$

$$E_1 = 112.81° \ .$$

Using, the equation on notes G7 we can find the elapsed time as the following

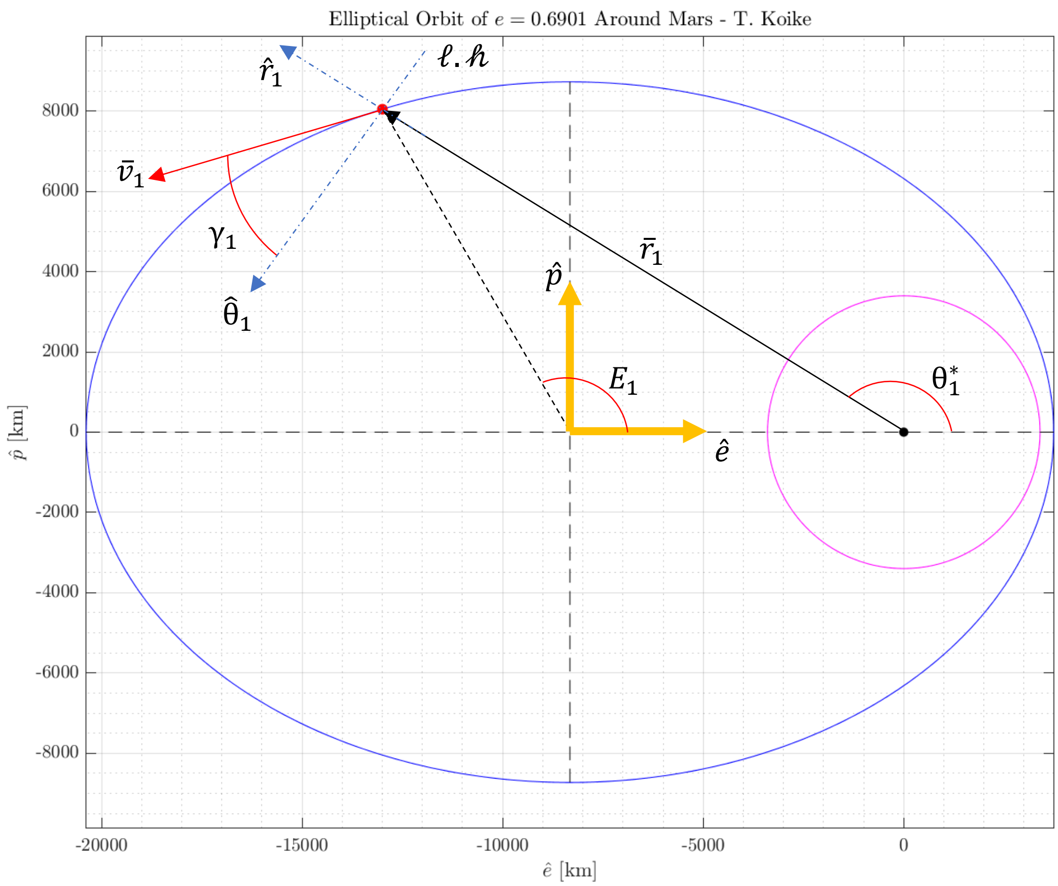
$$\sqrt{\frac{\mu}{a^3}}\left(t_1 - t_p\right) = E_1 - esinE_1$$

$$(t_1 - t_c) = \sqrt{\frac{a^3}{\mu}}(E_1 - esinE_1) = 7.1785e+5 \ \ s \ = \ 8.3084 \ days \ .$$

We check if this is an elliptical orbit

$$v_1 = 1.4325 \ \ km/s < 2.3672 \ \ km/s = \sqrt{\frac{2\mu}{r_1}} = \sqrt{2}v_c.$$

(b) Sketch the orbit. Mark the usual quantities at the time t_1: $\bar{r}_1, \bar{v}_1, \gamma_1, \ell.\hbar., E_1$; also add appropriate unit vectors $\hat{e}, \hat{p}, \hat{r}_1, \hat{\theta}_1$.



(c) What is the "wait time" till the maneuver $(t_1 - t_c)$?

Similar to E_1, we can find E_c using θ_c^*.

$$E_c = 46.359° \ .$$

Then, like in the previous part (a), where we found the time difference for point 1, we can find,

$$(t_c - t_p) = \sqrt{\frac{a^3}{\mu}}(E_c - e\sin E_c) = 2.9346e+5 \ s = 3.3965 \ days \ .$$

Thus, the wait time becomes

$$(t_1 - t_p) - (t_c - t_p) = (t_1 - t_c) = 4.2439e+5 \ s = 4.9119 \ days \ .$$

(d) Determine r_1^+, v_1^+, γ_1^+ after the maneuver. Compute the required maneuver $(\Delta v, \alpha)$. Recall that $\Delta v = |\Delta\bar{v}|$. [Include VECTOR diagrams!!!!]

Since after the maneuver the spacecraft is going to enter a circular orbit, we know that the radius of this circle is going to be equivalent with the distance at the point at time t_1. Hence,

$$r_1^+ = r_1 = 4.5R_♂ = 1.5287e+4 \ km \ .$$

And the velocity after the maneuver is going to be equal to the circular velocity,

$$v_1^+ = v_c = \sqrt{\frac{\mu}{r_1}} = 1.6738 \ km/s \ .$$

Now, since the new velocity is tangent to a circle the direction of it is parallel to $\hat{\theta}_1$, its vector form is

$$\bar{v}_1^+ = v_1^+\hat{\theta}_1 \ .$$

And because $\bar{v}_1$ can be expressed as

$$\bar{v}_1 = v_1(\sin\gamma_1\hat{r}_1 + \cos\gamma_1\hat{\theta}_1) \ .$$

Thus, $\Delta\bar{v}$ becomes

$$\Delta\bar{v} = v_1^+\hat{\theta}_1 - v_1(sin\gamma_1\hat{r}_1 + cos\gamma_1\hat{\theta}_1) = -v_1 sin\gamma_1\hat{r}_1 + (v_1^+ - v_1 cos\gamma_1)\hat{\theta}_1$$

$$\Delta\bar{v} = -0.94575\hat{r}_1 + (1.6738 - 1.0759)\hat{\theta}_1 = (-0.94575\hat{r}_1 + 0.59795\hat{\theta}_1)\ km/s\ .$$

Thus,

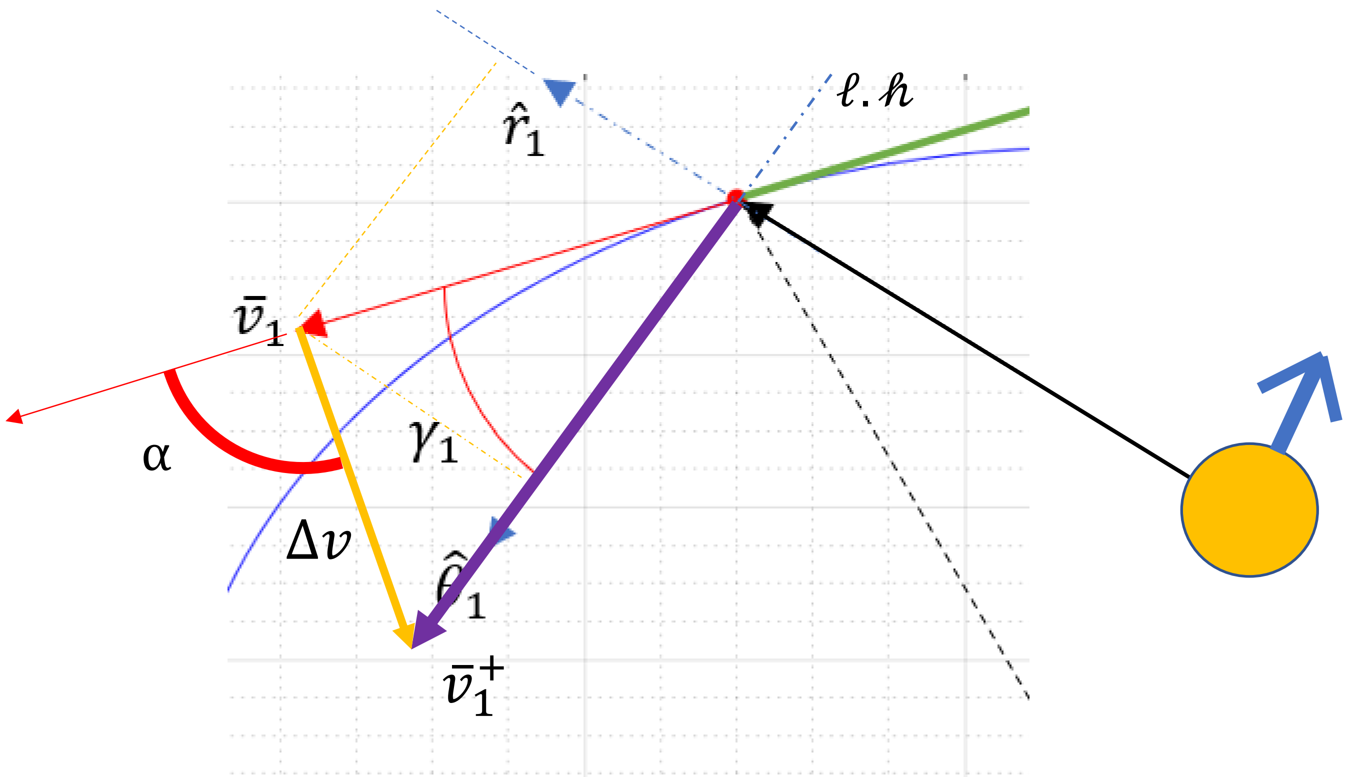
$$\Delta v = 1.1189\ km/s\ .$$

Then the angle α can be found by the dot product rule and geometry (negative sign since it is moving away from the center of orbit),

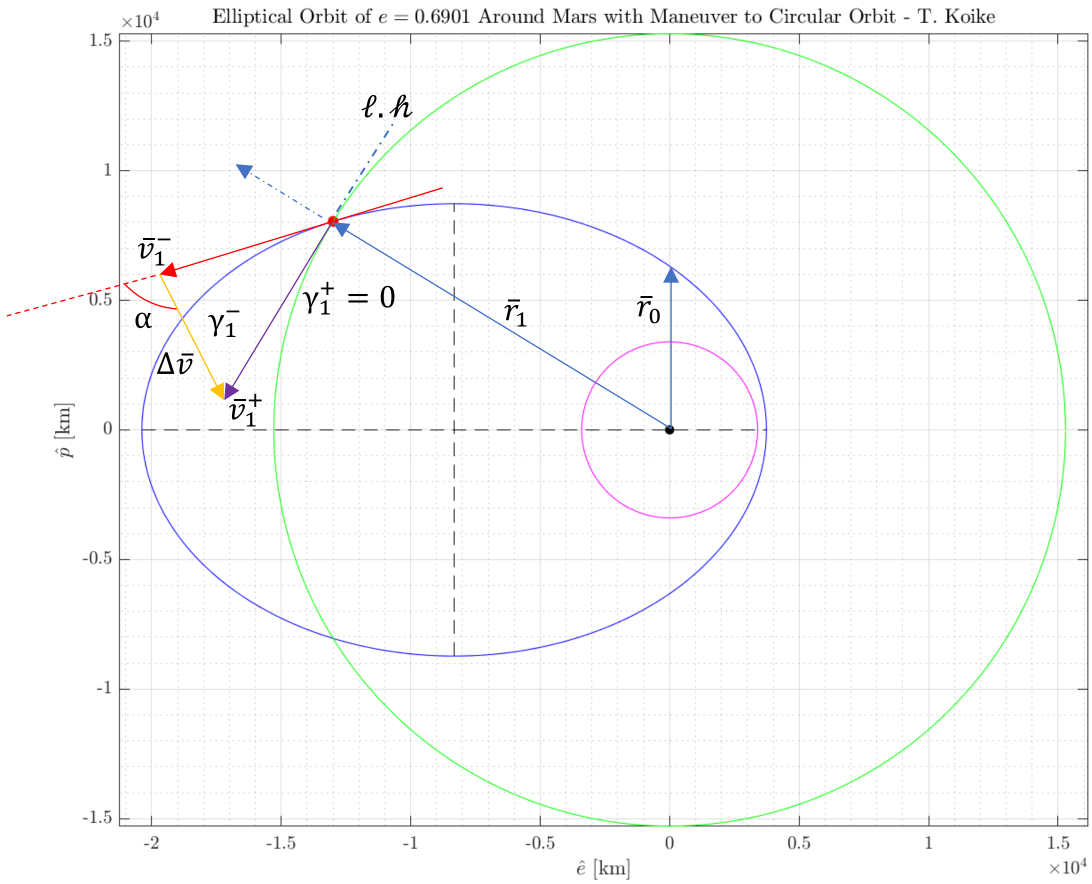
$$\alpha = arccos\left(\frac{\bar{v}_1^+ \cdot \Delta\bar{v}}{|\bar{v}_1^+||\Delta\bar{v}\,|}\right) + \gamma_1$$

$$\alpha = -99.014°\ .$$

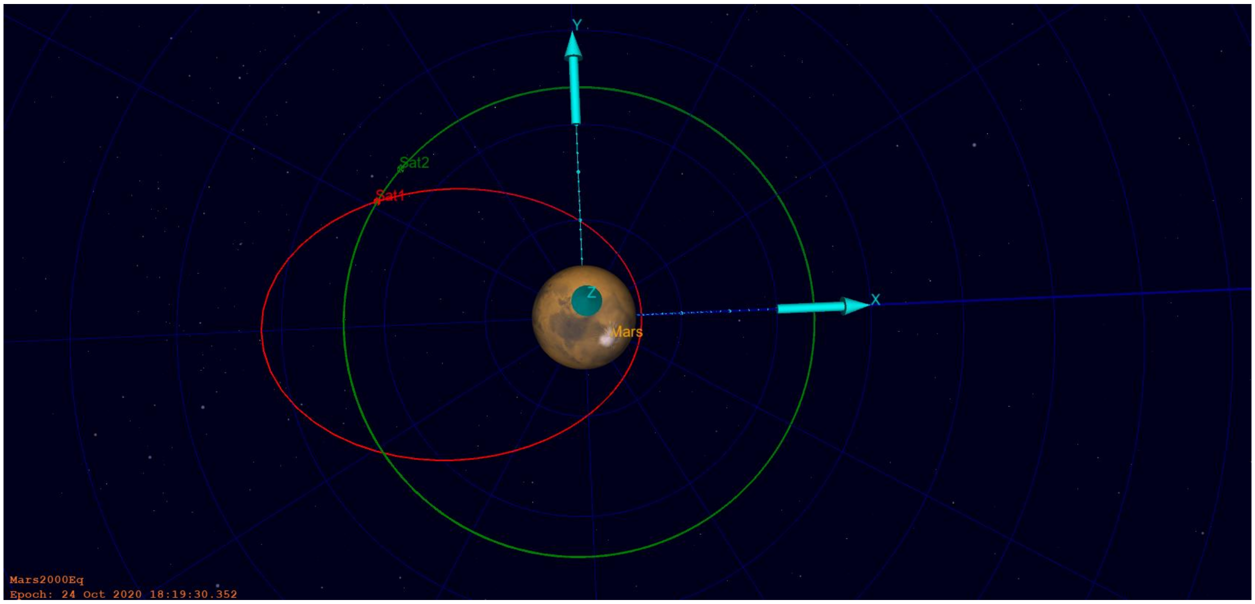
The angle alpha is equal to the flight path angle since the flight path angle is the angle between $\hat{\theta}_1$ and $\bar{v}_1$.



(e) Plot the old and new orbits on the same figure using your MATLAB script. On the plot, mark $\bar{r}_0, \bar{r}_1, \bar{v}_1^-, \ell.\hbar., \gamma_1^-, \bar{v}_1^+, \gamma_1^+, \Delta\bar{v}, \alpha$.



(f) Plot the two orbits in GMAT using Mars as the central body. At the maneuver time, use a report to list $\bar{r}, \bar{v}$ in each orbit at the maneuver time. Choose a convenient set of unit vectors (coordinate frame). Subtracting the velocity vectors should yield your $\Delta v=|\Delta\bar{v}|$. Does it?



From the GMAT report we have the following results for the position and velocity vectors at the maneuver point for the two orbits. The raw data is the following

Sat1.Mars2000Eq.X	Sat1.Mars2000Eq.Y	Sat1.Mars2000Eq.VX	Sat1.Mars2000Eq.VY	Sat2.Mars2000Eq.X	Sat2.Mars2000Eq.Y	Sat2.Mars2000Eq.VX	Sat2.Mars2000Eq.VY
-13000	8044	-1.37	-0.4173	-13000	8045	-0.8808	-1.423
-13080	8019	-1.361	-0.423	-13000	8045	-0.8808	-1.423
-13280	7954	-1.338	-0.4371	-13000	8045	-0.8808	-1.423

Elliptical orbit:

	$\hat{e}$	$\hat{p}$
$\bar{r}$	-1.3e+4 km	8044 km
$\bar{v}$	-1.37 km/s	-0.4173 km/s

Circular orbit:

	$\hat{e}$	$\hat{p}$
$\bar{r}$	-1.3e+4 km	8044 km
$\bar{v}$	-0.8808 km/s	-1.4230 km/s

Thus, the velocity difference from the GMAT results become

$$\Delta v = (-0.8808 + 1.37)\hat{e} + (-1.423 + 0.4173)\hat{p} \quad km/s$$

$$\Delta v = 0.4892\hat{e} - 1.0057\hat{p} \quad km/s \ .$$

Now since

$$\hat{r} = cos\theta^{*}\hat{e} + sin\theta^{*}\hat{p}$$

$$\hat{\theta} = cos\left(\theta^{*} + \frac{\pi}{2}\right)\hat{e} + sin\left(\theta^{*} + \frac{\pi}{2}\right)\hat{p} = -sin\theta^{*}\hat{e} + cos\theta^{*}\hat{p}$$

$$\Delta v_{orbital} = \Delta v \begin{pmatrix} cos\theta^{*} & -sin\theta^{*} \\ sin\theta^{*} & cos\theta^{*} \end{pmatrix}$$

$$\Delta v_{orbital} = (0.4892 \quad -1.0057) \begin{pmatrix} cos\theta^{*} & -sin\theta^{*} \\ sin\theta^{*} & cos\theta^{*} \end{pmatrix}$$

where $\theta^{*} = 148.2486°$, therefore

$$\Delta v_{orbital} = (-0.94575\hat{r}_1 + 0.59795\hat{\theta}_1) \ \ km/s \ .$$

And,

$$\Delta v_{orbital} = 1.1189 km/s \ .$$

This agrees with our result in part (d). Thus, we have verified our result.

Problem 2: given a two-body model, vehicle is successfully launched into an Earth orbit with $e = 0.4$ and $a = 4R_{\oplus}$. A single in-plane maneuver will be implemented when $\theta^* = 135°$. Let the maneuver be defined as $|\Delta\bar{v}| = 0.90\ km/s,\ \alpha = +45°$.

(a) Determine $\bar{r}.\bar{v}^-, \gamma^-$ at the moment of maneuver point.

Firstly, the semi latus rectum is

$$p = a(1 - e^2) = 21431\ \ km\ \ .$$

Then, we can find the magnitude of $\bar{r}$

$$r = \frac{p}{1 + ecos\theta^*} = 29883\ \ km\ \ .$$

Since, $\mu = 398600.4415\ \ km^3/s^2$, the magnitude of the velocity vector is

$$v = \sqrt{\mu\left(\frac{2}{r} - \frac{1}{a}\right)} = 3.3248\ \ km/s\ \ .$$

The specific angular momentum is

$$h = \sqrt{\mu p} = 92424\ \ km^2/s\ \ .$$

The flight path angle is

$$\gamma^- = arccos\left(\frac{h}{rv}\right) = 21.524° \quad (\because \theta^* < 180°).$$

The position vector can be expressed by its magnitude and true anomaly

$$\bar{r} = r(cos\theta^*\hat{e} + sin\theta^*\hat{p}) = -21130\hat{e} + 21130\hat{p}\ \ km$$

or

$$\bar{r} = 29883\hat{r}\ \ km$$

Then the velocity vector becomes

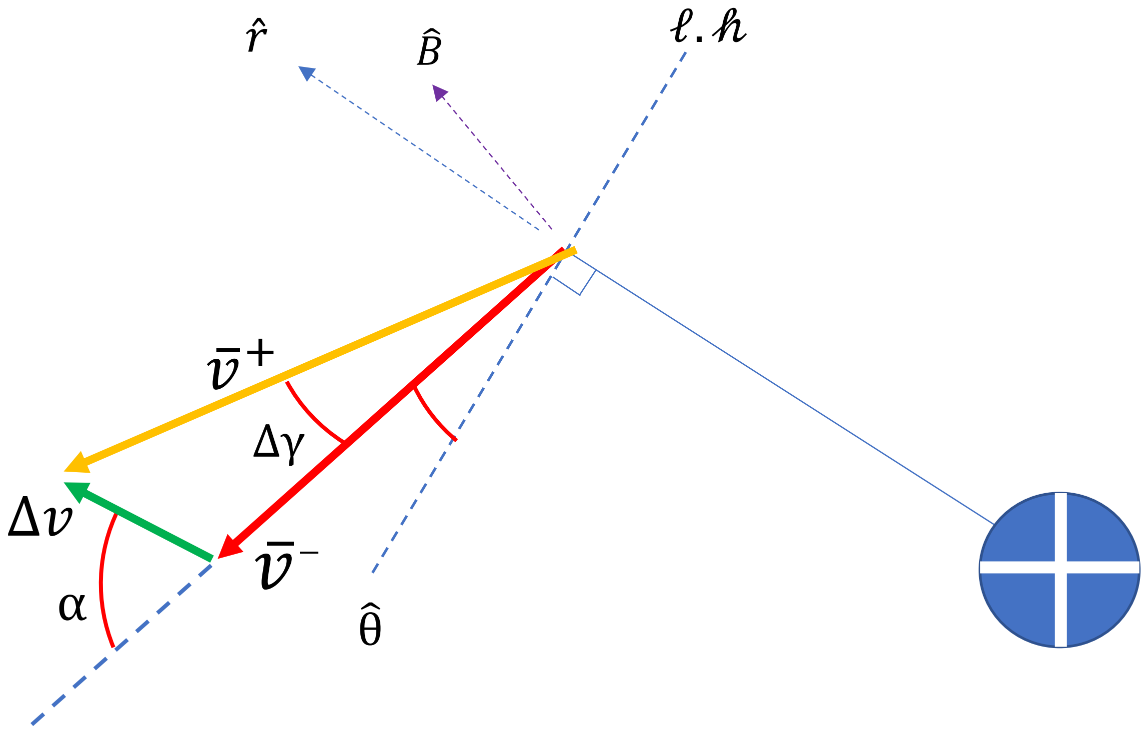
$$\bar{v} = v\left(sin\gamma\ \hat{r} + cos\gamma\hat{\theta}\right) = 1.2198\hat{r}\ + 3.09929\hat{\theta}\ \ km/s$$

or in the $\hat{e} - \hat{p}$ frame

$$\bar{v} = \bar{v}_{orbital}\begin{pmatrix} cos\theta^* & sin\theta^* \\ -sin\theta^* & cos\theta^* \end{pmatrix} = -3.0496\hat{e} - 1.3245\hat{p}\ \ km/s$$

(b) Express the maneuver in both $\hat{r}, \hat{\theta}$ and $\hat{e}, \hat{p}$ unit vectors. Also determine the maneuver in the VNB set of coordinates.
(c) Prepare any VECTOR diagrams!!!! Determine r^+, v^+, γ^+ in the new orbit immediately after the maneuver.

The vector diagram at the maneuver point is



From the cosine law

$$v_N^2 = (\Delta v)^2 + v^2 - 2(\Delta v)v\cos(180° - 45°)$$

$$v_N = v^+ = 4.012 \ \ km/s$$

The corresponding flight path angle is

$$\gamma^+ = \arccos\left(\frac{h}{rv_N}\right) = 39.563°$$

$$\Delta\gamma = \gamma^+ - \gamma^- = 18.039°$$

The vector of v_N becomes,

$$\bar{v}^+ = v^+(\sin\gamma^+\hat{r} + \cos\gamma^+\hat{\theta}) = 2.5553\hat{r} + 3.0929\hat{\theta} \ \ km\backslash s \ .$$

Thus, the velocity vector of the difference of the velocity vectors become

$$\overline{\Delta v} = \bar{v}^+ - \bar{v}^- = (2.5553\hat{r} + 3.0929\hat{\theta} \ km\backslash s) - (1.2198\hat{r} + 3.0929\hat{\theta} \ km/s)$$

$$\overline{\Delta v} = 1.3355\hat{r} \quad km/s$$

In $\hat{e}, \hat{p}$ unit vectors the two become

$$\bar{v}^+ = (2.5553 \quad 3.0929)\begin{pmatrix} cos\theta^* & sin\theta^* \\ -sin\theta^* & cos\theta^* \end{pmatrix} = -3.9939\hat{e} - 0.38013\hat{p} \;\; km/s$$

$$\overline{\Delta v} = (1.3355 \quad 0)\begin{pmatrix} cos\theta^* & sin\theta^* \\ -sin\theta^* & cos\theta^* \end{pmatrix} = -0.94434\hat{e} + 0.94434\hat{p} \;\; km/s$$

In the VNB coordinate system,

$$\hat{V} = \hat{v}^-$$

and $\hat{B}$ is normal to $\hat{V}$ on the same plane depicted in the vector diagram on the previous page.

$$\bar{v}^- = 3.3248\hat{V} \;\; km/s$$

$$\bar{v}^+ = v^+\left(cos\Delta\gamma\hat{V} + sin\Delta\gamma\hat{B}\right) = 3.8147\hat{V} + 1.2424\hat{B} \;\; km/s \;.$$

$$\overline{\Delta v} = \bar{v}^+ - \bar{v}^- = 0.48998\hat{V} + 1.2424\hat{B} \;\; km/s \;.$$

And finally,

$$r^+ = r^- = 29883km.$$

(d) To determine the impact that such a maneuver creates on the orbital characteristics, compute the following characteristics of the new orbit: $a, e, h, \mathcal{P}, \mathcal{E}, \theta^*, E, \gamma, (t - t_p), r_p, \Delta\omega$.

Now, the semi major axis is

$$a = \frac{\frac{-\mu}{2}}{\left(\frac{(v^+)^2}{2} - \frac{\mu}{r^+}\right)} = 37668 \ km \ .$$

The eccentricity becomes from notes JS3

$$e = \sqrt{\left(\frac{r^+(v^+)^2}{\mu} - 1\right)^2 cos^2\gamma^+ + sin^2\gamma^+}$$

$$e \ = \ 0.65656 \ .$$

Then the semi latus rectum becomes

$$p = a(1 - e^2) = 21431 \ km \ .$$

Then the specific angular momentum becomes

$$h = \sqrt{\mu p} = 92424 \ km^2/s \ .$$

Then the period becomes

$$\mathcal{P} = 2\pi\sqrt{\frac{a^3}{\mu}} \ = \ 72756 \, s \ = \ 0.84208 \, days \ .$$

The specific energy becomes

$$\mathcal{E} = -\frac{\mu}{2a} = -5.291 \, km^2/s^2 \ .$$

We know from the previous problem that the flight path angle is

$$\gamma = \gamma^+ = 39.563° \ .$$

The true anomaly is positive since the flight path angle is positive, and therefore,

$$\theta^* = +arccos\left(\frac{1}{e}\left(\frac{p}{r^+} - 1\right)\right) = 115.52° \ .$$

The eccentric anomaly is computed by

$$tan\frac{\theta^*}{2} = \left(\frac{1+e}{1-e}\right)^{1/2} tan\frac{E}{2}$$

$$E = 71.651° \ .$$

Then, the time elapsed from periapsis is

$$(t - t_p) = (E - sinE)\sqrt{\frac{a^3}{\mu}} = 3490\ s \ .$$

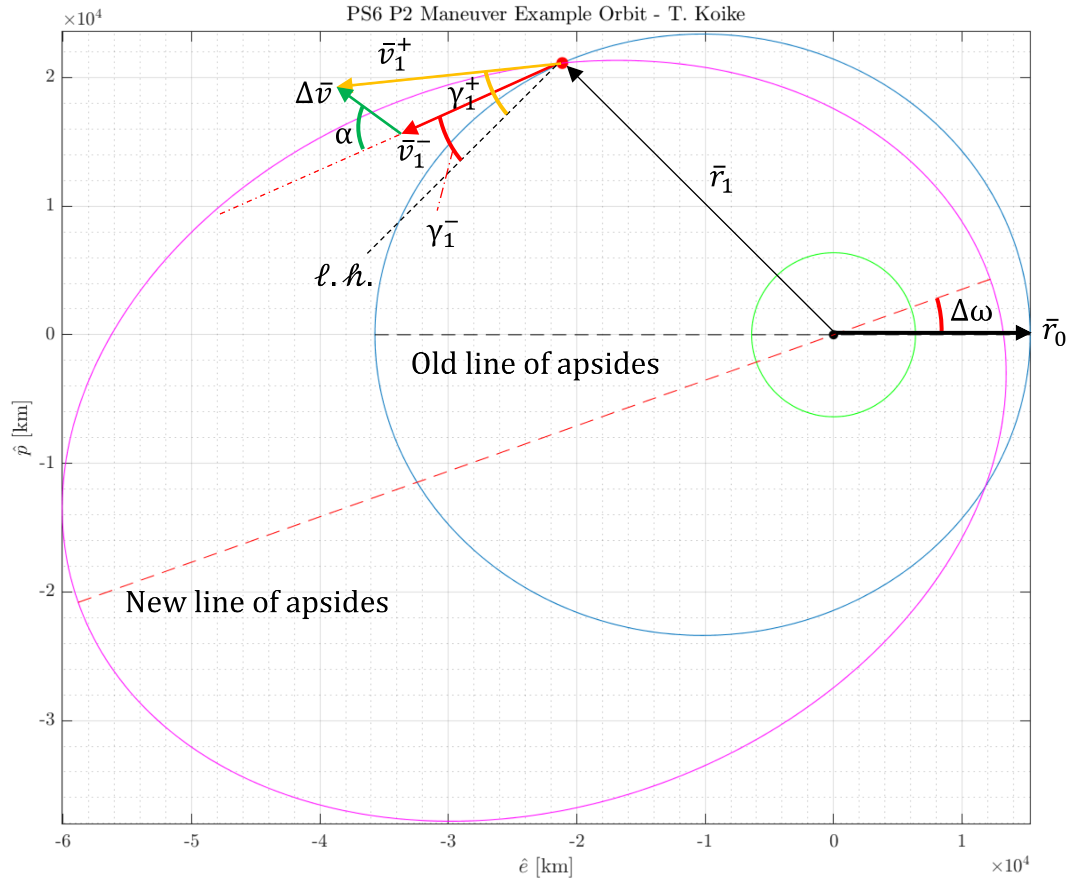
The radius of periapsis is

$$r_p = a(1-e) = 12937\ km \ .$$

Finally,

$$\Delta\omega = 135° - 115.52° = 19.482° \ .$$

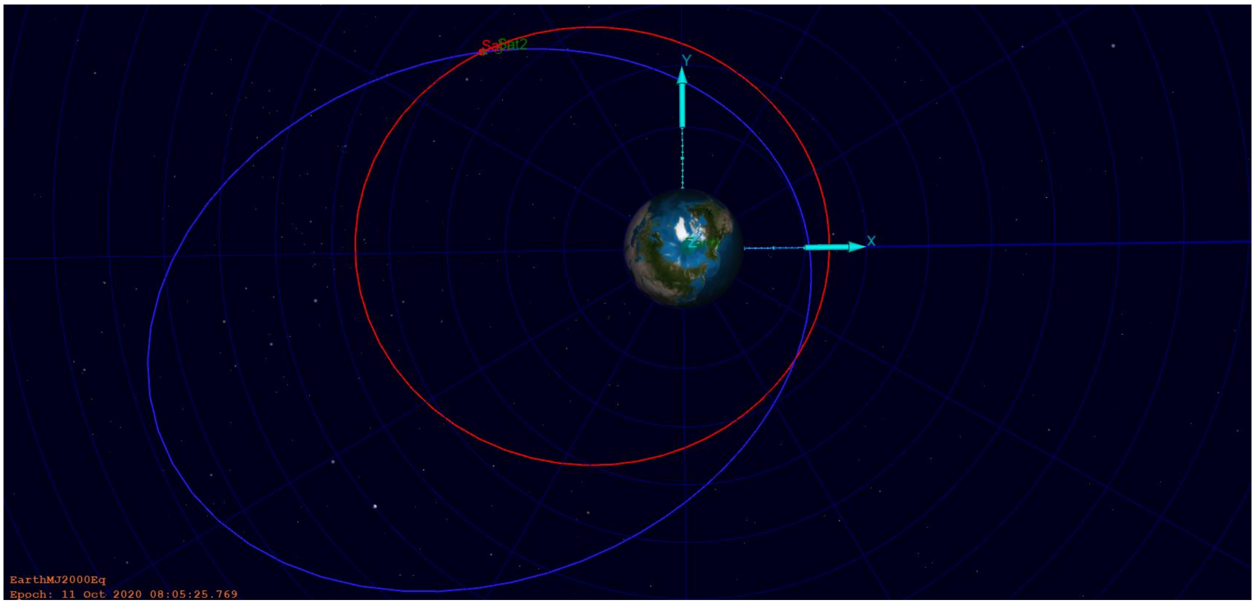
(e) Plot the new and the old orbits in Matlab on the same figure using your Matlab script. On the plot, mark $\bar{r}_0, \bar{r}_1, \bar{v}_1^-, \ell.h., \gamma_1^-, \bar{v}_1^+, \gamma_1^+, \Delta\bar{v}, \alpha$. Also indicate the new and old lines of apsides and the shift, i.e., $\Delta\omega$. Is it positive or negative? Why?



$\Delta\omega$ is a positive value because this value is the difference between the true anomaly of the new orbit and the old orbit, and that calculation yields a positive value. Also, this value indicates the angle in which the line of apsides rotates and from the old orbit to the new one the line of apsides rotates in a positive angle with respect to the $\hat{h}$ vector which is coming out of the page.

(f) Bonus → You can use GMAT in two ways to check the maneuver and the new orbit. Use either method to check your results:

Use a start date October 10, 2020 12:00:00 to propagate the satellites. Put in the old and new orbits with 2 satellites and compare the velocities at the intersection point to assess if the difference equals the required $\Delta\bar{v}$;



Sat1 in the figure orbits the old orbit and Sat2 orbits the new one.

The summary of Sat1 and Sat2 at the maneuver point is the following

```
Propagate Command: Propagate1
Spacecraft       : Sat1
Coordinate System: EarthMJ2000Eq

Time System   Gregorian                     Modified Julian
----------------------------------------------------------------------
UTC Epoch:    10 Oct 2020 23:15:55.660      29133.4693942075
TAI Epoch:    10 Oct 2020 23:16:32.660      29133.4698224483
TT  Epoch:    10 Oct 2020 23:17:04.844      29133.4701949483
TDB Epoch:    10 Oct 2020 23:17:04.842      29133.4701949292

Cartesian State                       Keplerian State
---------------------------           --------------------------------
X  =  -21130.064944958 km             SMA  =   25513.000000033 km
Y  =   21130.802534376 km             ECC  =   0.4000000000002
Z  =   0.0000000000000 km             INC  =   0.0000000000000 deg
VX =  -3.0495872416288 km/sec         RAAN =   0.0000000000000 deg
VY =  -1.3244038463119 km/sec         AOP  =   360.00000000000 deg
VZ =   0.0000000000000 km/sec         TA   =   134.99900000255 deg
                                      MA   =   94.643047549128 deg
                                      EA   =   115.35387465803 deg

Spherical State                       Other Orbit Data
---------------------------           --------------------------------
RMAG =   29882.945978014 km           Mean Motion        =   1.549268155e-04 deg/sec
RA   =   134.99900000253 deg          Orbit Energy       =  -7.8117124897009 km^2/s^2
```

```
      DEC  =   0.0000000000000 deg          C3                  =  -15.623424979402 km^2/s^2
      VMAG =   3.3247598247740 km/s         Semilatus Rectum    =   21430.920000024 km
      AZI  =   90.000000000000 deg          Angular Momentum    =   92424.965100133 km^2/s
      VFPA =   68.475802105204 deg          Beta Angle          =  -6.9917202653511 deg
      RAV  =  -156.52519789227 deg          Periapsis Altitude =   8929.6637000150 km
      DECV =   0.0000000000000 deg          VelPeriapsis        =   6.0377693136860 km/s
                                            VelApoapsis         =   2.5876154201499 km/s
                                            Orbit Period        =   40555.828160079 s

      Planetodetic Properties
      ---------------------------
      LST       =   135.26514928876 deg
      MHA       =   8.9964759666473 deg
      Latitude  =  -0.0802677349499 deg
      Longitude =   126.26867332211 deg
      Altitude  =   23504.809719853 km


      Spacecraft Properties
      ------------------------------
      Cd                    =   2.200000
      Drag area             =   15.00000 m^2
      Cr                    =   1.800000
      Reflective (SRP) area =   1.000000 m^2
      Dry mass              =   850.00000000000 kg
      Total mass            =   850.00000000000 kg
      SPADDragScaleFactor   =   1.000000
      SPADSRPScaleFactor    =   1.000000


 ========================================================================

      Spacecraft       : Sat2
      Coordinate System: EarthMJ2000Eq

      Time System   Gregorian                     Modified Julian
      ----------------------------------------------------------------------
      UTC Epoch:    10 Oct 2020 12:00:00.000      29133.0000000000
      TAI Epoch:    10 Oct 2020 12:00:37.000      29133.0004282407
      TT  Epoch:    10 Oct 2020 12:01:09.184      29133.0008007407
      TDB Epoch:    10 Oct 2020 12:01:09.182      29133.0008007216

      Cartesian State                       Keplerian State
      ---------------------------           --------------------------------
      X  =  -21131.423961286 km             SMA  =   37668.000000000 km
      Y  =   21129.948761085 km             ECC  =   0.6565600000000
      Z  =   0.0000000000000 km             INC  =   0.0000000000000 deg
      VX =  -3.9938238313617 km/sec         RAAN =   0.0000000000000 deg
      VY =  -0.3802198116453 km/sec         AOP  =   19.482000000000 deg
      VZ =   0.0000000000000 km/sec         TA   =   115.52000000000 deg
                                            MA   =   35.947110547546 deg
                                            EA   =   71.653010021515 deg

      Spherical State                       Other Orbit Data
      ---------------------------           --------------------------------
      RMAG =   29883.303252446 km           Mean Motion         =   8.635948526e-05 deg/sec
      RA   =   135.00200000000 deg          Orbit Energy        =  -5.2909690121589 km^2/s^2
      DEC  =   0.0000000000000 deg          C3                  =  -10.581938024318 km^2/s^2
      VMAG =   4.0118818403737 km/s         Semilatus Rectum    =   21430.416306355 km
      AZI  =   90.000000000000 deg          Angular Momentum    =   92423.878955830 km^2/s
      VFPA =   50.436279597976 deg          Beta Angle          =  -6.8144880605859 deg
      RAV  =  -174.56172040202 deg          Periapsis Altitude =   6558.5616200000 km
      DECV =   0.0000000000000 deg          VelPeriapsis        =   7.1443176247429 km/s
                                            VelApoapsis         =   1.4811684726431 km/s
                                            Orbit Period        =   72756.169034291 s

      Planetodetic Properties
      ---------------------------
      LST       =   135.26813282482 deg
      MHA       =   199.55190411210 deg
      Latitude  =  -0.0804990060739 deg
```

```
        Longitude =  -64.283771287277 deg
        Altitude  =   23505.166994526 km


        Spacecraft Properties
        ------------------------------
        Cd                       =   2.200000
        Drag area                =   15.00000 m^2
        Cr                       =   1.800000
        Reflective (SRP) area =   1.000000 m^2
        Dry mass                 =   850.00000000000 kg
        Total mass               =   850.00000000000 kg
        SPADDragScaleFactor      =   1.000000
        SPADSRPScaleFactor       =   1.000000
```

The red bolded values are the velocities for Sat1 and Sat2 at the maneuver point.

For Sat1

$\boldsymbol{V_x}$	-3.0495872416288 km/sec
$\boldsymbol{V_y}$	-1.3244038463119 km/sec
$\boldsymbol{V_z}$	0.0000000000000 km/sec

For Sat2

$\boldsymbol{V_x}$	-3.9938238313617 km/sec
$\boldsymbol{V_y}$	-0.3802198116453 km/sec
$\boldsymbol{V_z}$	0.0000000000000 km/sec

Then the difference of these two vectors become

$$\Delta\bar{V} = -0.9442365897\hat{e} + 0.9441840347\hat{p}$$

Which is approximately

$$\Delta\bar{V} = -0.9443\hat{e} + 0.9443\hat{p}$$

This agrees with out results on page 12. Thus, we have verified our results using GMAT.

Problem 3: Assume a relative two-body model and a space vehicle that is currently located in an Earth orbit with $a = 3R_{\oplus}$ and $e = 0.6$. A single in-plane adjustment is to be implemented for a perigee-raise maneuver. The goal is a new periapsis distance such the new $r_p = 2R_{\oplus}$. At the same time, it is desired to produce an orbit that is less eccentric, i.e., the new eccentricity is $e = 0.4$ and both goals are accomplished using the same maneuver. The maneuver will take place when the spacecraft is located at the end of the minor axis and descending.

(a) Determine $\bar{r}_1^-, \bar{v}_1^-, \gamma_1^-, (t_1 - t_p)$ at the maneuver point. Sketch or plot the orbit in MATLAB. Mark the usual quantities at the maneuver location prior to the maneuver: $\bar{r}^-, \bar{v}^-, \gamma^-, \ell.\hbar., E^-$; also add appropriate unit vectors $\hat{e}, \hat{p}; \hat{r}, \hat{\theta}$.

Plugging in the radius of the Earth we have

$$a = 19134 \ km$$

And we are given that

$$E^- = -90° = 270°$$

Using the following relationship

$$tan\frac{\theta^*}{2} = \left(\frac{1+e}{1-e}\right)^{1/2} tan\frac{E}{2}$$

We find that the true anomaly is

$$\theta^* = -126.87° = 233.13°$$

the semi latus rectum is

$$p = a(1-e^2) = 12246 \ km \ .$$

Then, we can find the magnitude of $\bar{r}$

$$r_1^- = \frac{p}{1 + ecos\theta^*} = 19134 \ km.$$

Since, $\mu = 398600.4415 \ km^3/s^2$, the magnitude of the velocity vector is

$$v_1^- = \sqrt{\mu\left(\frac{2}{r} - \frac{1}{a}\right)} = 4.5642 \ km/s.$$

The specific angular momentum is

$$h = \sqrt{\mu p} = 69866 \ km^2/s \ .$$

The flight path angle is

$$\gamma_1^- = arccos\left(\frac{h}{r_1^- v_1^-}\right) = -36.870° \quad (\because \theta^* > 180°)\ .$$

The mean anomaly becomes

$$M = E - esinE$$

$$M = -32.704° = 327.30°$$

The time elapse from periapsis is

$$\left(t_1 - t_p\right) = M\sqrt{\frac{a^3}{\mu}} = 23948\ s = 0.27718\ days$$

Using the radial distance, we can express the r vector in the orbital frame

$$\bar{r}_1^- = r_1^- \hat{r} = 19134\ km\ \hat{r}\ .$$

In the other frame this is

$$\bar{r}_1^- = r_1^-(cos\theta^*\hat{e} + sin\theta^*\hat{p})$$

$$\bar{r}_1^- = -11481\hat{e} - 15308\hat{p}\ \ km\ .$$

Then, the velocity vector becomes

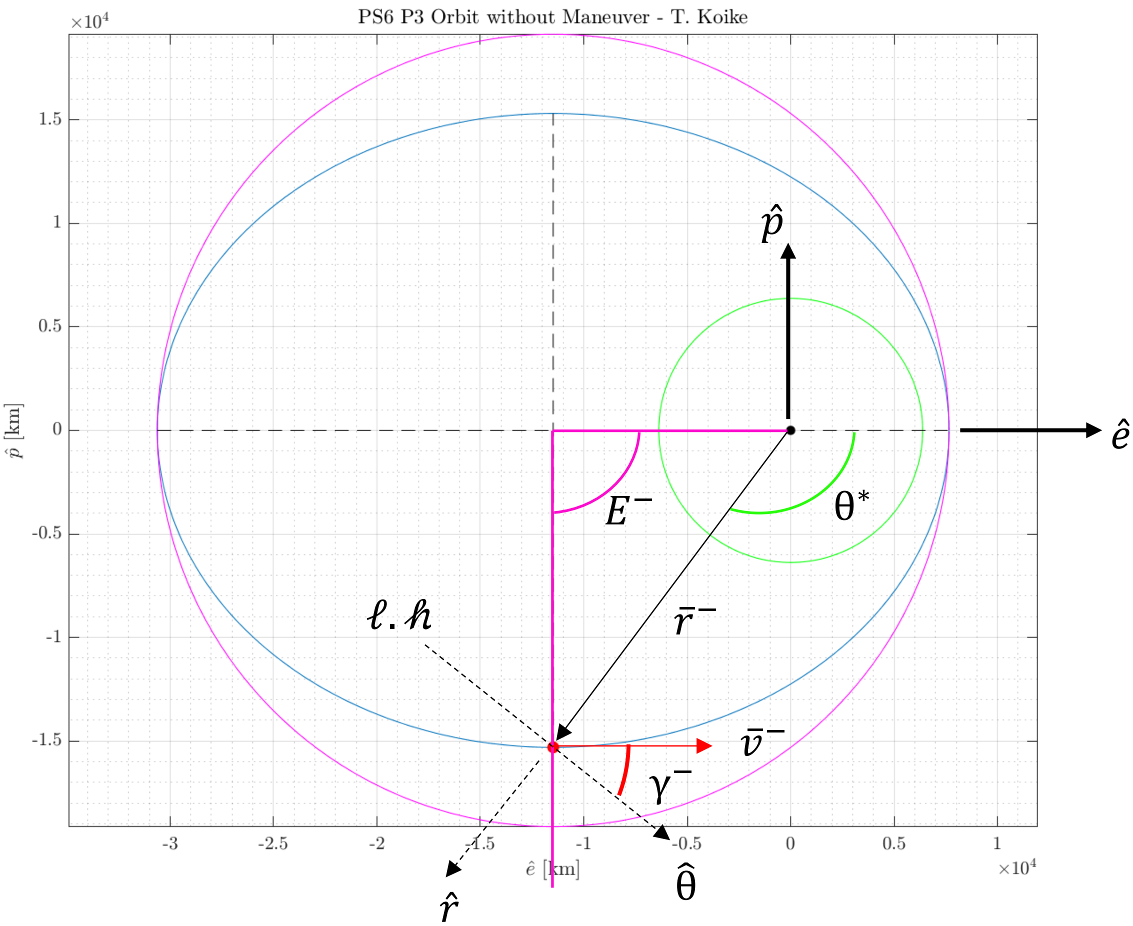
$$\bar{v}^- = v_1^-\left(sin(\gamma_1^-)\hat{r} + cos(\gamma_1^-)\hat{\theta}\right)$$

$$\bar{v}^- = \left(-2.7385\hat{r} + 3.65113\hat{\theta}\right)\ km/s\ .$$

The velocity vector is also represented as

$$\bar{v}^- = \left(-2.7385\hat{r} + 3.65113\hat{\theta}\right)\begin{pmatrix} cos\theta^* & sin\theta^* \\ -sin\theta^* & cos\theta^* \end{pmatrix} = 4.5642\hat{e}\ \ km/s\ .$$

The orbit is plotted on the next page



PS6 P3 Orbit without Maneuver - T. Koike
$\times 10^4$
1.5
1
0.5
0
-0.5
-1
-1.5
$\hat{p}$ [km]
-3
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
$\hat{e}$ [km]
$\times 10^4$
$\hat{p}$
$\hat{e}$
E^-
θ^*
$\ell.h$
$\bar{r}^-$
$\bar{v}^-$
γ^-
$\hat{r}$
$\hat{\theta}$

(b) Determine $\bar{r}^+, \bar{v}^+, \gamma^+$ at the maneuver point. [Include the vector diagrams!!!!] Compute the required maneuver $(\Delta v, \alpha)$. Express that maneuver in terms of $\hat{r}, \hat{\theta}, \hat{V}, \hat{B}$ sets of unit vectors.

From the requirements of the new orbit we can compute the new semi major axis

$$a^+ = \frac{r_p^+}{1-e^+} = \frac{2R_\oplus}{1-0.4} = 12756 \ \ km \ .$$

Then, the new semi latus rectum becomes

$$p^+ = a^+(1-e^{+2}) = 17859 \ \ km \ .$$

Then distance from Earth is the same as r^-

$$r^+ = r^- = 19134 \ \ km \ .$$

Thus, we can get the true anomaly for the new orbit

$$\theta^{*+} = \pm arccos\left(\frac{1}{e}\left(\frac{p}{r^+} - 1\right)\right) = \pm 99.594°$$

The new velocity becomes

$$v^+ = \sqrt{\mu\left(\frac{2}{r^+} - \frac{1}{a^+}\right)} = 4.7869 \ \ km/s \ .$$

Then the new flight path angle is going to become

$$\gamma^+ = arccos\left(\frac{\sqrt{\mu p^+}}{r^+ v^+}\right) = \pm 22.908°$$

If $\gamma^+ = 22.908°$,

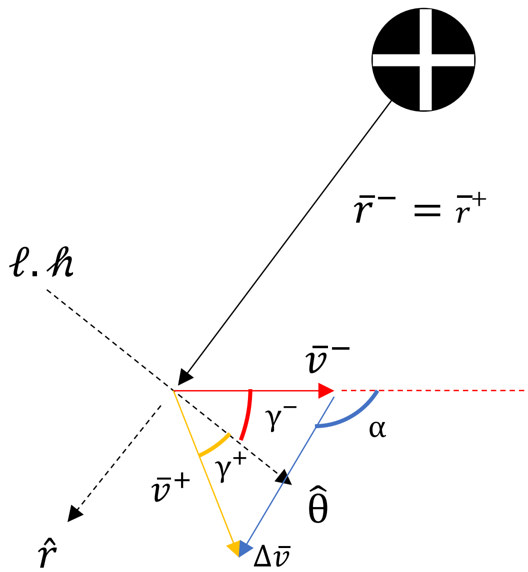
$$tan\theta^{*+} = \frac{\left(\frac{r^+ v^{+2}}{\mu}\right) cos\gamma^+ sin\gamma^+}{\left(\frac{r^+ v^{+2}}{\mu}\right) cos^2\gamma^{\pm}1}$$

$$\theta^{*+} = arccos\left(\frac{\left(\frac{r^+ v^{+2}}{\mu}\right) cos\gamma^+ sin\gamma^+}{\left(\frac{r^+ v^{+2}}{\mu}\right) cos^2\gamma^{\pm}1}\right) = -80.406° \ \text{ or } 99.594°$$

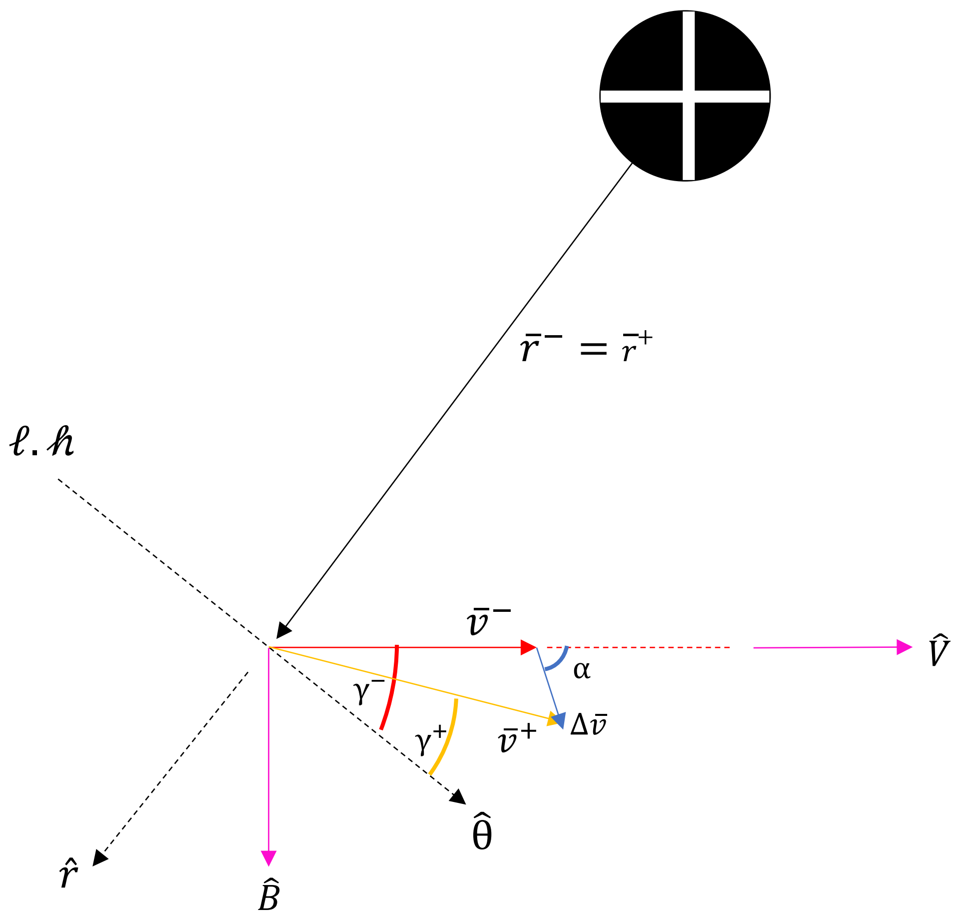
Similarly, if $\gamma^+ = -22.908°$

$$\theta^{*+} = 80.406° or - 99.594°$$

If we choose a positive flight path angle,



If we choose a negative flight path angle,



We want to choose a Δv that is smaller so we define

$$\gamma^{+} = -22.908°$$

$$\theta^{*+} = -99.594°$$

Thus, the new orbit is descending as well. Hence the position and velocity vectors will become $\bar{r}^{+}, \bar{v}^{+}, \gamma^{+}$

$$\bar{r}^{+} = r^{+}\hat{r} = 19134 \ \ km \ \ \hat{r} \ .$$

$$\bar{v}^{+} = v^{+}\left(sin(\gamma^{+})\hat{r} + cos(\gamma^{+})\hat{\theta}\right)$$

$$\bar{v}^{+} = \left(-1.8633\hat{r} + 4.4094\hat{\theta}\right) \ km/s \ .$$

The maneuver becomes

$$\Delta\bar{v} = \bar{v}^{+} - \bar{v}^{-} = 0.87519\hat{r} + 0.75807\hat{\theta} \ \ km/s \ .$$

$$\Delta v = 1.1579 \ \ km/s \ .$$

and the angle becomes

$$\alpha = arccos\left(\frac{\bar{v}^{-} \cdot \Delta\bar{v}}{|\bar{v}^{-}||\Delta\bar{v}|}\right) = 85.971°$$

The angle alpha is negative since the Earth is oriented above the local horizon and the $\Delta\bar{v}$ is moving downwards (moving away from the Earth).

In the VNB coordinate system, we can define the velocity change as

$$\Delta\bar{v} = \Delta v\left(cos\alpha\hat{V} + sin\alpha\hat{B}\right) = 0.081344\hat{V} + 1.1550\hat{B} \ \ km/s \ .$$

(c) Determine the characteristics of the new orbit: $a, e, r_p, r_a, \mathcal{P}, \mathcal{E}; \theta^*, E, \gamma, (t - t_p), \Delta\omega$. How long till the spacecraft reaches perigee in the new orbit?

From the calculations of the previous parts of this problem we know that

$$a^+ = 12756 \;\; km$$

$$e^+ = 0.4$$

$$\theta^{*+} = -99.594°$$

$$\gamma^+ = -22.908°$$

The radius of perigee and apogee become

$$r_p^+ = a(1 - e^+) = 12756 \;\; km$$

$$r_a^+ = a(1 + e^+) = 29765 \;\; km \;\; .$$

The period is

$$\mathcal{P} = 2\pi \sqrt{\frac{a^{+3}}{\mu}} \; = \; 30851 \, s \; = \; 0.35707 \, days \;\; .$$

The specific energy is

$$\mathcal{E} = -\frac{\mu}{2a^+} = \;\; -9.3742 \; km^2/s^2 \;\; .$$

From the relationship

$$tan\frac{\theta^*}{2} = \left(\frac{1+e}{1-e}\right)^{1/2} tan\frac{E}{2}$$

the eccentric anomaly becomes,

$$E^+ = -75.522° \;\; .$$

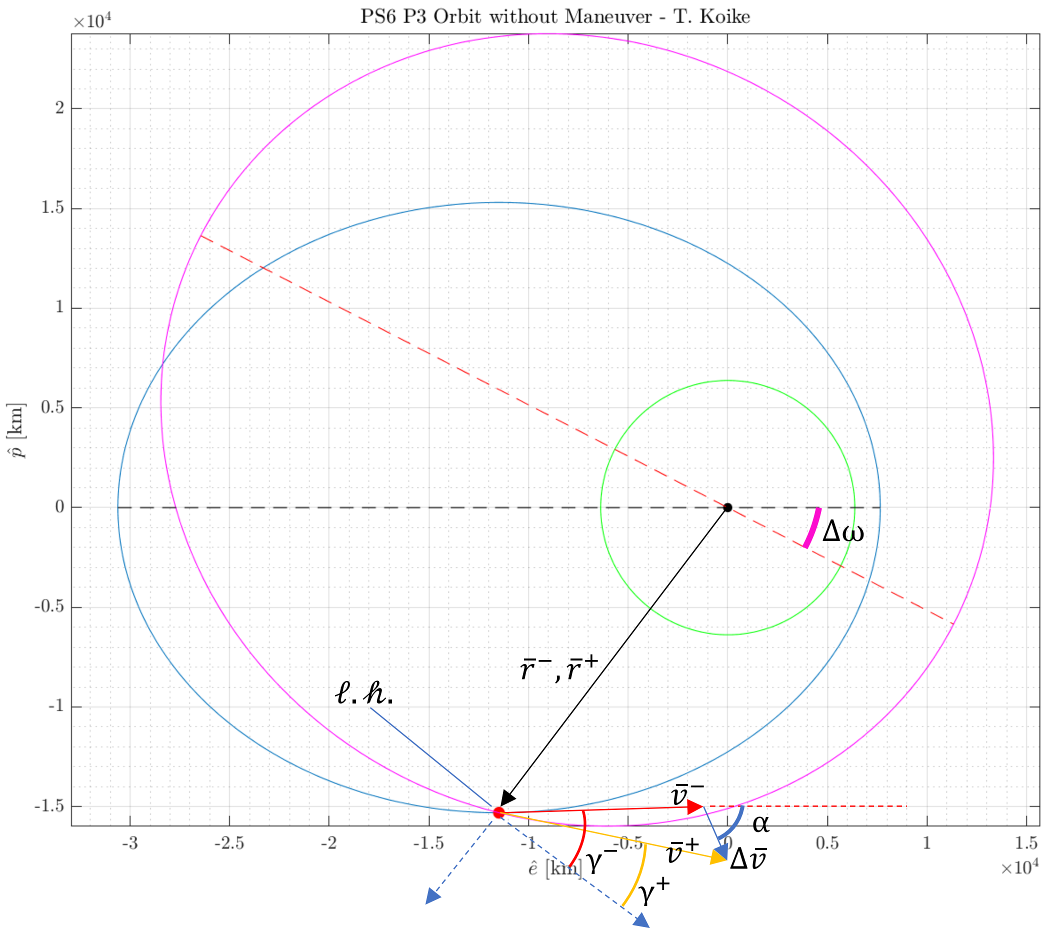
The elapsed time from the perigee is

$$(t - t_p) = (E^+ - sinE^+)\sqrt{\frac{a^{+3}}{\mu}} = \; 29133 \;\; s \; = \; 0.33719 \, days \;\; .$$

Finally,

$$\Delta\omega = \theta^{*-} - \theta^{*+} = (-126.87°) - (-99.594°) = -27.276°$$

(d) Plot the old and new orbits on the same figure using your MATLAB script. On the plot, mark $\bar{r}^-, \bar{r}^+, \bar{v}^-, \ell.\hbar., \gamma^-, \bar{v}^+, \gamma^+, \Delta\bar{v}, \alpha$.



Appendix

MATLAB Code

Problem 1

```
% AAE 532 HW 6 Problem 1
% Tomoki Koike
close all; clear all; clc;
fdir = 'C:\Users\Tomo\Desktop\studies\2020-Fall\AAE532\matlab\outputs\ps6';
set(groot, 'defaulttextinterpreter','latex');
set(groot, 'defaultAxesTickLabelInterpreter','latex');
set(groot, 'defaultLegendInterpreter','latex');
format shortG;

% Set constants
planet_consts = setup_planetary_constants();  % Function that sets up all the
constants in the table
sun = planet_consts.sun;  % structure of sun
earth = planet_consts.earth;  % structure of earth
mars = planet_consts.mars;  % structure of mars
G = 6.6743015e-20;  % Gravitational constant [km^3/kg/s^2]

% (a)
Rm = mars.mer;
mu = mars.gp;
rp = 1.1 * Rm;  % radius of periapsis
ra = 6.0 * Rm;  % radius of apoapsis
a = 0.5 * (rp + ra);  % semi major axis
e = (ra - rp) / (ra + rp);  % eccentricity
p = a * (1 - e^2);  % semi laturs rectum
h = sqrt(mu * p);  % specific angular momentum

% At time t1
TA_c = 90;  % at time tc
r_1 = 4.5 * Rm;
TA_1 = acos_dbval(1/e * (p/r_1 - 1), "deg");
TA_1 = TA_1(find(min(TA_1 - TA_c)));
v_1 = vis_viva(r_1, a, mu);
FPA_1 = acos_dbval(h / r_1 / v_1, "deg");
FPA_1 = FPA_1(FPA_1 > 0);
EA_1 = T2E_anomaly(e, TA_1, "deg");
EA_c = T2E_anomaly(e, TA_c, "deg");
dt_p1 = sqrt(a^3 / mu) * (EA_1 - e * sind(EA_1));
dt_p1_day = dt_p1 / 60 / 60 / 24;

% (b)
% Ellipse
theta = 0:0.01:2*pi;
R = p ./ (1 + e * cos(theta));
X = R .* cos(theta); Y = R .* sin(theta);
```

```
% Mars
Xm = Rm * cos(theta); Ym = Rm * sin(theta);

% At time t1
X1 = r_1 * cosd(TA_1); Y1 = r_1 * sind(TA_1);

% Plot
fig = figure("Renderer","painters","Position",[10 10 900 700]);
    plot(X, Y, '-b')
    title('Elliptical Orbit of $e=0.6901$ Around Mars - T. Koike')
    hold on; grid on; grid minor; box on; axis equal;
    plot(X1, Y1, '.r', 'MarkerSize', 18)

    % Mars
    plot(Xm, Ym, '-m')
    plot(0, 0, '.k', 'MarkerSize', 15)
    % Axes
    nanikore = -ra:rp; korenani = -a*sqrt(1-e^2):a*sqrt(1-e^2);
    plot(nanikore, zeros(size(nanikore)), '--k')
    plot(-a*e*ones(size(korenani)), korenani, '--k')

    hold off
    xlabel('$\hat{e}$ [km]')
    ylabel('$\hat{p}$ [km]')
saveas(fig, fullfile(fdir, 'p1_orbit1.png'));

% (c)
EA_c = T2E_anomaly(e, TA_c, "deg");
dt_pc = sqrt(a^3 / mu) * (EA_c - e * sind(EA_c));
dt_pc_day = dt_pc / 60 / 60 / 24;
dt_c1 = dt_p1 - dt_pc;
dt_c1_day = dt_c1 / 60 / 60 / 24;

% (d)
r_new = r_1;
v_new = sqrt(mu / r_1);
v_1_vec = v_1 * [sind(FPA_1), cosd(FPA_1)];
v_new_vec = v_new * [0, 1];
dv_vec = v_new_vec - v_1_vec;
dv = norm(dv_vec);
syms alpha
eqn = v_new / sind(180-alpha) == dv / sind(FPA_1);
alpha = solve(eqn, alpha)
temp_ang = acosd(dot(v_new_vec, dv_vec)/ dv / v_new) + FPA_1
% alpha = acosd(dot(v_new_vec, v_1_vec) / v_new / v_1);

% (e)
% Ellipse
theta = 0:0.01:2*pi;
R = p ./ (1 + e * cos(theta));
X = R .* cos(theta); Y = R .* sin(theta);
```

```
% Mars
Xm = Rm * cos(theta); Ym = Rm * sin(theta);

% At time t1
X1 = r_1 * cosd(TA_1); Y1 = r_1 * sind(TA_1);

% New circular orbit
Xnew = r_1 * cos(theta); Ynew = r_1 * sin(theta);

% Plot
fig = figure("Renderer","painters","Position",[10 10 900 700]);
    plot(X, Y, '-b')
    title('Elliptical Orbit of $e=0.6901$ Around Mars with Maneuver to Circular
Orbit - T. Koike')
    hold on; grid on; grid minor; box on; axis equal;
    plot(X1, Y1, '.r', 'MarkerSize', 18)

    % Mars
    plot(Xm, Ym, '-m')
    plot(0, 0, '.k', 'MarkerSize', 15)
    % Axes
    nanikore = -ra:rp; korenani = -a*sqrt(1-e^2):a*sqrt(1-e^2);
    plot(nanikore, zeros(size(nanikore)), '--k')
    plot(-a*e*ones(size(korenani)), korenani, '--k')

    % New orbit
    plot(Xnew, Ynew, '-g')

    hold off
    xlabel('$\hat{e}$ [km]')
    ylabel('$\hat{p}$ [km]')
saveas(fig, fullfile(fdir, 'p1_orbit2.png'));

% (f)
vvec1 = [-1.37, -0.4173];
vvec2 = [-0.8808, -1.4230];
Dv = vvec2 - vvec1;
Dv_orbital = Dv * [cosd(TA_1), -sind(TA_1); sind(TA_1), cosd(TA_1)];
```

Problem 2

```
% AAE 532 HW 6 Problem 2
% Tomoki Koike
close all; clear all; clc;
fdir = 'C:\Users\Tomo\Desktop\studies\2020-Fall\AAE532\matlab\outputs\ps6';
set(groot, 'defaulttextinterpreter','latex');
set(groot, 'defaultAxesTickLabelInterpreter','latex');
set(groot, 'defaultLegendInterpreter','latex');
format shortG;
```

```
% Set constants
planet_consts = setup_planetary_constants();  % Function that sets up all the
constants in the table
sun = planet_consts.sun;  % structure of sun
earth = planet_consts.earth;  % structure of earth
mars = planet_consts.mars;  % structure of mars
G = 6.6743015e-20;  % Gravitational constant [km^3/kg/s^2]

% (a)
Re          = earth.mer;  % Earth radius
mu          = earth.gp;  % gravitational parameter
a           = 4*Re;  % semi major axis
e           = 0.4;  % eccentricity
p           = a * (1 - e^2);  % semi latus rectum
TA          = 135;  % true anomaly at maneuver point
r1          = p / (1 + e * cosd(TA));
v1          = vis_viva(r1, a, mu);
h           = sqrt(mu * p);  % specific angular momentum
FPA1        = acosd(h / r1 / v1);  % flight path angle
r1vec_i      = r1 * [cosd(TA), sind(TA)];
v1vec_o     = v1 * [sind(FPA1), cosd(FPA1)]
v1vec_i     = v1vec_o * [cosd(TA), sind(TA); -sind(TA), cosd(TA)]

% Maneuver
dv = 0.90;  % km/s
alpha = 45;  % degrees
v2 = sqrt(dv^2 + v1^2 - 2*dv*v1*cosd(180 - alpha));
FPA2 = acosd(h / r1 / v2)
dFPA = FPA2 - FPA1
v2vec_o = v2 * [sind(FPA2), cosd(FPA2)]
dvvec_o = v2vec_o - v1vec_o
v2vec_i = v2vec_o * [cosd(TA), sind(TA); -sind(TA), cosd(TA)]
dvvec_i = dvvec_o * [cosd(TA), sind(TA); -sind(TA), cosd(TA)]

v1vec_vbn = v1 * [1, 0];
v2vec_vbn = v2 * [cosd(dFPA), sind(dFPA)];
dvvec_vbn = v2vec_vbn - v1vec_vbn;

% (d)
r2 = r1;
a_new = (-mu / 2) / (v2^2/2 - mu/r2);
e_new = calc_e_posVel(r2, v2, FPA2, mu, "deg")
p_new = a_new*(1 - e_new^2)
h = sqrt(mu * p_new)
IP = 2*pi*sqrt(a_new^3 / mu)
IP_day = IP / 60 /60 / 24
En = -mu / 2 / a_new
TA_new = acos_dbval(1/e_new *(p_new/r2 - 1), "deg")
TA_new = TA_new(TA_new >0)
E_new = T2E_anomaly(e_new, TA_new, "deg")
dt = (deg2rad(E_new) - sind(E_new)) * sqrt(a_new^3 / mu)
rp_new = a_new*(1 - e_new)
```

```
Domega = TA - TA_new
ra_new = a_new*(1 + e_new)
rp = a*(1-e);
ra = a*(1+e);

% Plotting for visualization
% old orbit
angles = 0:0.01:2*pi;
RR = p ./ (1 + e*cos(angles)); XX = RR.*cos(angles); YY = RR.*sin(angles);

% new orbit
RR_new = p_new ./ (1 + e_new*cos(angles - deg2rad(Domega)));
XX_new = RR_new.*cos(angles);
YY_new = RR_new.*sin(angles);
rp_vec = rp_new*[cosd(Domega), sind(Domega)];
ra_vec = ra_new*[cosd(Domega+180), sind(Domega+180)];

Xearth = Re*cos(angles); Yearth = Re*sin(angles);
fig = figure("Renderer","painters","Position",[10, 10, 900, 700]);
plot(XX,YY)
hold on; grid on; grid minor; box on; axis equal;
plot(r1vec_i(1), r1vec_i(2), '.r', 'MarkerSize', 20)
plot(0,0,'.k', 'MarkerSize', 15)
plot(Xearth, Yearth, '-g')
plot(XX_new, YY_new, '-m')
plot([-ra, rp], [0, 0], '--k')
plot([ra_vec(1), rp_vec(1)],[ra_vec(2), rp_vec(2)], '--r')
hold off
title('PS6 P2 Maneuver Example Orbit - T. Koike')
xlabel('$\hat{e}$ [km]')
ylabel('$\hat{p}$ [km]')
saveas(fig, fullfile(fdir, "p2-orbit.png"))


function e = calc_e_posVel(r, v, gamma, mu, unit)
    A = (r*v^2 / mu - 1)^2;
    if unit == "rad"
        B = cos(gamma)^2;
        C = sin(gamma)^2;
    else
        B = cosd(gamma)^2;
        C = sind(gamma)^2;
    end
    e = sqrt(A*B+C);
end
```

Problem 3

```
% AAE 532 HW 6 Problem 2
% Tomoki Koike
close all; clear all; clc;
fdir = 'C:\Users\Tomo\Desktop\studies\2020-Fall\AAE532\matlab\outputs\ps6';
set(groot, 'defaulttextinterpreter','latex');
set(groot, 'defaultAxesTickLabelInterpreter','latex');
set(groot, 'defaultLegendInterpreter','latex');
format shortG;

% Set constants
planet_consts = setup_planetary_constants();  % Function that sets up all the
constants in the table
sun = planet_consts.sun;  % structure of sun
earth = planet_consts.earth;  % structure of earth
mars = planet_consts.mars;  % structure of mars
G = 6.6743015e-20;  % Gravitational constant [km^3/kg/s^2]

% (a)
Re          = earth.mer;  % Earth radius
mu          = earth.gp;  % gravitational parameter
a           = 3*Re;  % semi major axis
e           = 0.6;  % eccentricity
p           = a * (1 - e^2);  % semi latus rectum
E           = -90;  % true anomaly at maneuver point
M           = rad2deg(deg2rad(E) - sind(E))
TA          = E2T_anomaly(e, E, "deg")
r1          = p / (1 + e * cosd(TA));
v1          = vis_viva(r1, a, mu);
h           = sqrt(mu * p);  % specific angular momentum
FPA1        = -acosd(h / r1 / v1)  % flight path angle
dt_1p       = deg2rad(360+M)*sqrt(a^3 / mu)
dt_1p_day   = dt_1p / 60 / 60 / 24
ra          = a*(1+e); rp = a*(1-e); b = a*sqrt(1-e^2);

r1vec_o     = r1 * [1, 0];
r1vec_i     = r1 * [cosd(TA), sind(TA)];
v1vec_o     = v1 * [sind(FPA1), cosd(FPA1)];
v1vec_i     = v1vec_o * [cosd(TA), sind(TA); -sind(TA), cosd(TA)];

% Plotting for visualization
% old orbit
angles = 0:0.01:2*pi;
RR = p ./ (1 + e*cos(angles)); XX = RR.*cos(angles); YY = RR.*sin(angles);

% Reference circle
Xref = a*cos(angles)-a*e; Yref = a*sin(angles);

% new orbit
% RR_new = p_new ./ (1 + e_new*cos(angles - deg2rad(Domega)));
% XX_new = RR_new.*cos(angles);
```

```
% YY_new = RR_new.*sin(angles);
% rp_vec = rp_new*[cosd(Domega), sind(Domega)];
% ra_vec = ra_new*[cosd(Domega+180), sind(Domega+180)];

% Earth
Xearth = Re*cos(angles); Yearth = Re*sin(angles);

fig = figure("Renderer","painters","Position",[10, 10, 900, 700]);
plot(XX,YY)
hold on; grid on; grid minor; box on; axis equal;
% Maneuver point
plot(r1vec_i(1), r1vec_i(2), '.r', 'MarkerSize', 20)
% Center of orbit
plot(0,0,'.k', 'MarkerSize', 15)
% Reference circle
plot(Xref, Yref, '-m')
% Semi minor axis
plot([-a*e, -a*e], [-b, b], '--k')
% Earth
plot(Xearth, Yearth, '-g')
% plot(XX_new, YY_new, '-m')
plot([-ra, rp], [0, 0], '--k')
% plot([ra_vec(1), rp_vec(1)],[ra_vec(2), rp_vec(2)], '--r')
hold off
title('PS6 P3 Orbit without Maneuver - T. Koike')
xlabel('$\hat{e}$ [km]')
ylabel('$\hat{p}$ [km]')
saveas(fig, fullfile(fdir, "p3-orbit-noManeuver.png"))

% New orbit after maneuver
rp2 = 2*Re
e2 = 0.4
a2 = rp2 / (1 - e2)
p2 = a2 * (1 - e2^2)
r2 = r1
TA2 = acos_dbval(1/e2 * (p2/r2 - 1), "deg")
v2 = vis_viva(r2, a2, mu)
FPA2 = acos_dbval(sqrt(mu*p2)/r2/v2, "deg")
TA2_eval1 = TAfromRVGamma(r2, v2, FPA2(1), mu, "deg")
TA2_eval2 = TAfromRVGamma(r2, v2, FPA2(2), mu, "deg")
FPA2 = FPA2(FPA2 < 0)
TA2 = TA2(TA2 < 0)

r2vec_o = r2 * [1, 0];
v2vec_o = v2 * [sind(FPA2), cosd(FPA2)]
dvvec_o = v2vec_o - v1vec_o
dv = norm(dvvec_o)
alpha = acos_dbval(dot(v1vec_o, dvvec_o)/v1/norm(dvvec_o), "deg")
alpha = alpha(alpha > 0)
dvvec_VBN = dv * [cosd(alpha), sind(alpha)]

rp2 = a2*(1-e2)
```

```
ra2 = a2*(1+e2)
IP = 2*pi*sqrt(a2^3/mu)
IP_day = IP / 60 / 60 / 24
En = -mu/2/a2
E2 = T2E_anomaly(e2, TA2, "deg")
dt_2p = (deg2rad(360 + E2) - sind(360 + E2))*sqrt(a2^3/mu)
dt_2p_day = dt_2p / 60/60/24
Domega = (TA-360) - TA2

% Plotting for visualization
% old orbit
angles = 0:0.01:2*pi;
RR = p ./ (1 + e*cos(angles)); XX = RR.*cos(angles); YY = RR.*sin(angles);

% Reference circle
% Xref = a*cos(angles)-a*e; Yref = a*sin(angles);

% new orbit
RR_new = p2 ./ (1 + e2*cos(angles - deg2rad(Domega)));
XX_new = RR_new.*cos(angles);
YY_new = RR_new.*sin(angles);
rp_vec = rp2*[cosd(Domega), sind(Domega)];
ra_vec = ra2*[cosd(Domega+180), sind(Domega+180)];

% Earth
Xearth = Re*cos(angles); Yearth = Re*sin(angles);

fig = figure("Renderer","painters","Position",[10, 10, 900, 700]);
plot(XX,YY)
hold on; grid on; grid minor; box on; axis equal;
% Maneuver point
plot(r1vec_i(1), r1vec_i(2), '.r', 'MarkerSize', 20)
% Center of orbit
plot(0,0,'.k', 'MarkerSize', 15)

% Reference circle
% plot(Xref, Yref, '-m')

% Semi minor axis
% plot([-a*e, -a*e], [-b, b], '--k')

% Earth
plot(Xearth, Yearth, '-g')
plot(XX_new, YY_new, '-m')
plot([-ra, rp], [0, 0], '--k')
plot([ra_vec(1), rp_vec(1)],[ra_vec(2), rp_vec(2)], '--r')
hold off
title('PS6 P3 Orbit without Maneuver - T. Koike')
xlabel('$\hat{e}$ [km]')
ylabel('$\hat{p}$ [km]')
saveas(fig, fullfile(fdir, "p3-orbit-withManeuver.png"))
```

```
function TA = TAfromRVGamma(r, v, gamma, mu, unit)
    A = r*v^2/mu;
    if unit == "rad"
        gamma = rad2deg(gamma);
    end
    num = A*cosd(gamma)*sind(gamma);
    den = A*cosd(gamma)^2 - 1;
    TA = atan_dbval(num/ den, unit);
end
```

Functions

```
function v = vis_viva(r, a, mu)
 %{
       NAME:    VIS_VIVA
       AUTHOR:  TOMOKI KOIKE
       INPUTS:  (1) r:  POSITION (LENGTH) ON ORBIT
                (2) a:  SEMI MAJOR AXIS
                (3) mu: GRAVITATIONAL PARAMETER
       OUTPUTS: (1) v:  VELOCITY AT THE POSITION
       DESCRIPTION: CALCULATES THE VELOCITY FOR A CERTAIN POSITION ON A
       CONIC ORBIT.
 %}

    v = sqrt(mu * (2/r - 1/a));
end
```

```
function E = T2E_anomaly(e, theta_star, unit)
%{
       NAME:    T2E_anomaly
       AUTHOR:  TOMOKI KOIKE
       INPUTS:  (1) e:          ECCENTRICITY
                (2) theta_star: TRUE ANOMALY
                (3) unit:       DEGREES OR RADIANS
       OUTPUTS: (1) E:          ECCENTRIC ANOMALY
       DESCRIPTION: CALCULATES THE ECCENTRIC ANOMALY FROM THE TRUE ANOMALY.
 %}

    ee = sqrt((1 - e) / (1 + e));
    if unit == "deg"
        E = 2*atand(ee * tand(theta_star / 2));
    else
        E = 2*atan(ee * tan(theta_star / 2));
    end
end
```

```
function theta_star = E2T_anomaly(e, E, unit)
%{
       NAME:    E2T_anomaly
```

```
        AUTHOR:   TOMOKI KOIKE
        INPUTS:   (1) e:     ECCENTRICITY
                  (2) E:     ECCENTRIC ANOMALY
                  (3) unit: DEGREES OR RADIANS
        OUTPUTS: (1) theta_star: TRUE ANOMALY
        DESCRIPTION: CALCULATES THE TRUE ANOMALY FROM THE ECCENTRIC ANOMALY.
 %}

    ee = sqrt((1 + e) / (1 - e));
    if unit == "deg"
        theta_star = 2*atand(ee * tand(E / 2));
        if theta_star < 0
            theta_star = 360 + theta_star;
        end
    else
        theta_star = 2*atan(ee * tan(E/ 2));
        if theta_star < 0
            theta_star = 2*pi + theta_star;
        end
    end
end
```

```
function res = atan_dbval(x, unit)
    if unit == "deg"
        ang1 = atand(x); % -90 to 90
        ang2 = 180 - (-ang1);
        if ang2 > 180
            ang2 = ang2 - 360;
        end
    else
        ang1 = atan(x);
        ang2 = pi - (-ang1);
        if ang2 > pi
            ang2 = ang2 - 2*pi;
        end
    end
    res = [ang1, ang2];
end
```

```
function res = asin_dbval(x, unit)
    if unit == "deg"
        ang1 = asind(x);
        if (0<=ang1 && ang1<=180)
            ang2 = 180 - ang1;
        elseif -90<=ang1 && ang1<0
            ang2 = -ang1 - 180;
        else
            ang2 = 540 - ang1;
        end
    else
```

```
        ang1 = asin(x);
        if (0<=ang1 && ang1<=pi)
            ang2 = pi - ang1;
        elseif -pi/2<=ang1 && ang1<0
            ang2 = -ang1 - pi;
        else
            ang2 = 3*pi - ang1;
        end
    end
    res = [ang1, ang2];
end
```

```
function res = acos_dbval(x, unit)
    if unit == "deg"
        ang1 = acosd(x);
        if (0<=ang1 && ang1<=180)
            ang2 = -ang1;
        else
            ang2 = 360 - ang1;
        end
    else
        ang1 = asin(x);
        if (0<=ang1 && ang1<pi)
            ang2 = -ang1;
        else
            ang2 = 2*pi - ang1;
        end
    end
    res = [ang1, ang2];
end
```

Auxiliary MATLAB Setup File

```
%% Table of Constants

function planets = setup_planetary_constants()

    %{
      arp  : Axial Rotational Period (Rev / Day)
      mer  : Mean Equatorial Radius (km)
      gp   : Gravitational Parameter, mu (km^3 / s^2)
      smao : Semi-Major Axis of Orbit (km)
      op   : Orbital Period (s)
      eo   : Eccentricity of Orbit
      ioe  : Inclination of Orbit to Ecliptic (deg)
    %}

    % Sun
    sun.arp = 0.0394011;
    sun.mer = 695990;
    sun.gp  = 132712440017.99;
    sun.smao = NaN;
```

```
    sun.op   = NaN;
    sun.eo   = NaN;
    sun.ioe  = NaN;

    % Moon
    moon.arp  = 0.0366004;
    moon.mer  = 1738.2;
    moon.gp   = 4902.8005821478;
    moon.smao = 384400;
    moon.op   = 2360592;
    moon.eo   = 0.0554;
    moon.ioe  = 5.16;

    % Mercury
    mercury.arp  = 0.0170514;
    mercury.mer  = 2439.7;
    mercury.gp   = 22032.080486418;
    mercury.smao = 57909101;
    mercury.op   = 7600537;
    mercury.eo   = 0.20563661;
    mercury.ioe  = 7.00497902;

    % Venus
    venus.arp  = 0.0041149;  % retrograde
    venus.mer  = 6051.9;
    venus.gp   = 324858.59882646;
    venus.smao = 108207284;
    venus.op   = 19413722;
    venus.eo   = 0.00676399;
    venus.ioe  = 3.39465605;

    % Earth
    earth.arp  = 1.0027378;
    earth.mer  = 6378.1363;
    earth.gp   = 398600.4415;
    earth.smao = 149597898;
    earth.op   = 31558205;
    earth.eo   = 0.01673163;
    earth.ioe  = 0.00001531;

    % Mars
    mars.arp  = 0.9747000;
    mars.mer  = 3397;
    mars.gp   = 42828.314258067;
    mars.smao = 227944135;
    mars.op   = 59356281;
    mars.eo   = 0.09336511;
    mars.ioe  = 1.84969142;

    % Jupiter
    jupiter.arp  = 2.4181573;
    jupiter.mer  = 71492;
    jupiter.gp   = 126712767.8578;
    jupiter.smao = 778279959;
    jupiter.op   = 374479305;
    jupiter.eo   = 0.04853590;
    jupiter.ioe  = 1.30439695;

    % Saturn
    saturn.arp  = 2.2522053;
    saturn.mer  = 60268;
    saturn.gp   = 37940626.061137;
```

```
    saturn.smao = 1427387908;
    saturn.op   = 930115906;
    saturn.eo   = 0.05550825;
    saturn.ioe  = 2.48599187;

    % Uranus
    uranus.arp  = 1.3921114;  % retrograde
    uranus.mer  = 25559;
    uranus.gp   = 5794549.0070719;
    uranus.smao = 2870480873;
    uranus.op   = 2652503938;
    uranus.eo   = 0.04685740;
    uranus.ioe  = 0.77263783;

    % Neptune
    neptune.arp  = 1.4897579;
    neptune.mer  = 25269;
    neptune.gp   = 6836534.0638793;
    neptune.smao = 4498337290;
    neptune.op   = 5203578080;
    neptune.eo   = 0.00895439;
    neptune.ioe  = 1.77004347;

    % Pluto
    pluto.arp  = -0.1565620;  % retrograde
    pluto.mer  = 1162;
    pluto.gp   = 981.600887707;
    pluto.smao = 5907150229;
    pluto.op   = 7830528509;
    pluto.eo   = 0.24885238;
    pluto.ioe  = 17.14001206;

    % Return
    planets.sun = sun;
    planets.moon = moon;
    planets.mercury = mercury;
    planets.venus = venus;
    planets.earth = earth;
    planets.mars = mars;
    planets.jupiter = jupiter;
    planets.saturn = saturn;
    planets.uranus = uranus;
    planets.neptune = neptune;
    planets.pluto = pluto;
end
```