AAE 364: Control Systems Analysis

HW 11

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B–7–3. Using MATLAB, plot Bode diagrams of $G_1(s)$ and $G_2(s)$ given below.

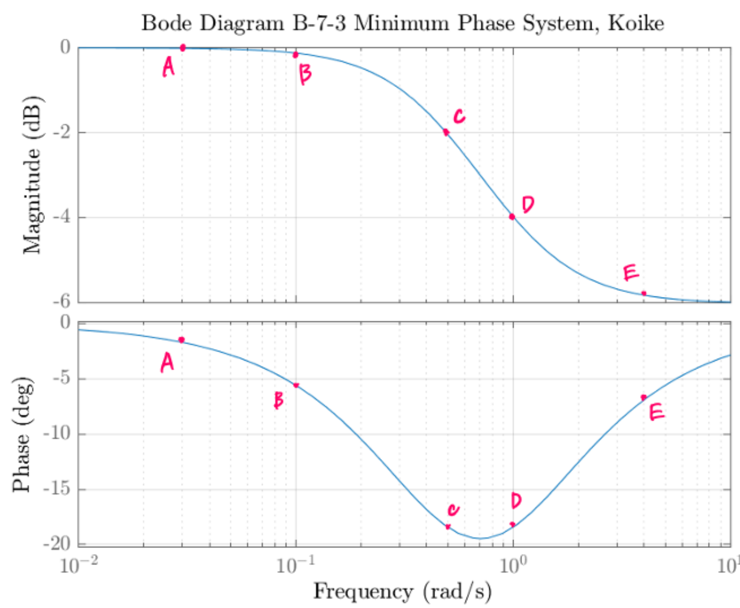
$$G_1(s) = \frac{1 + s}{1 + 2s}$$

$$G_2(s) = \frac{1 - s}{1 + 2s}$$

$G_1(s)$ is a minimum-phase system and $G_2(s)$ is a nonminimum-phase system.

<u>Minimum Phase System</u>

Bode Plot (from HW10)



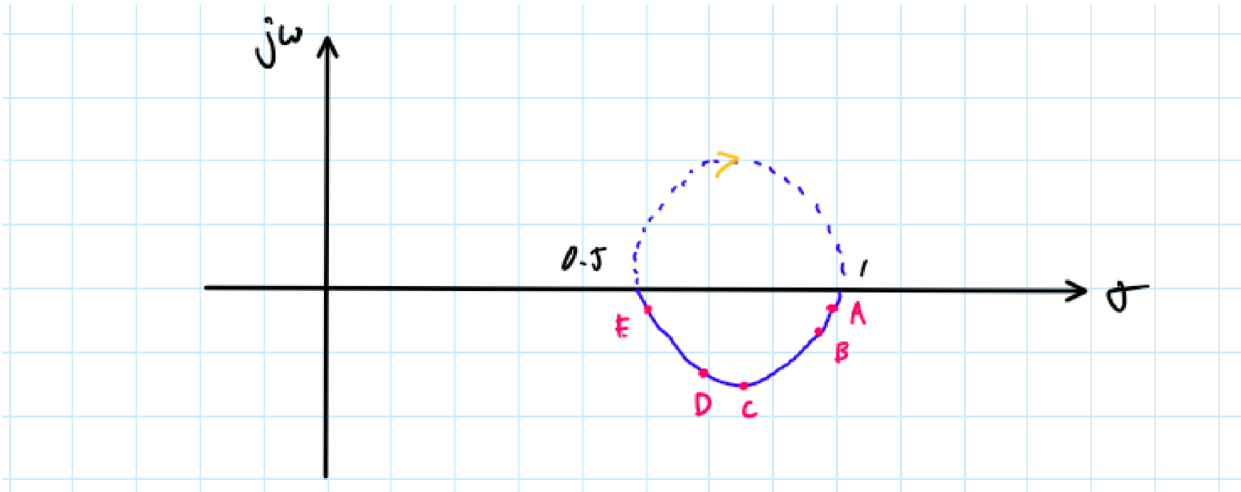
Point	ω [rad/s]	$\angle G$ [deg]	$-20\log_{10}\lvert G\rvert$ [dB]	$\lvert G\rvert$
A	0.02	-1	0	1
B	0.1	-6	-0.15	0.9829
C	0.5	-18	-2	0.7943
D	1	-19	-4	0.6310
E	4	-7	-5.9	0.5129

$$G_1(j\omega) = \frac{1+j\omega}{1+2j\omega}$$

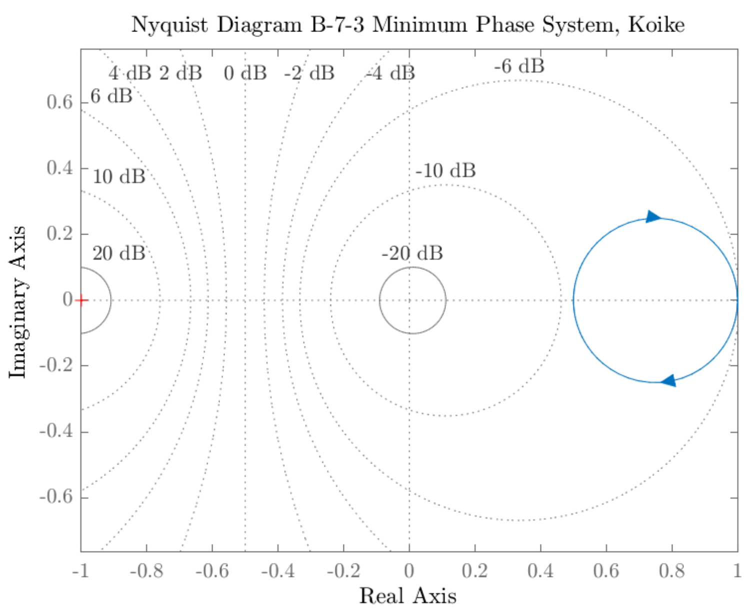
$$\omega \to 0 : \; G_1(j\omega) = 1 \angle 0^\circ$$

$$\omega \to \infty : \; G_1(j\omega) = \tfrac{1}{2} \angle 0^\circ$$

Nyquist Plot Sketch

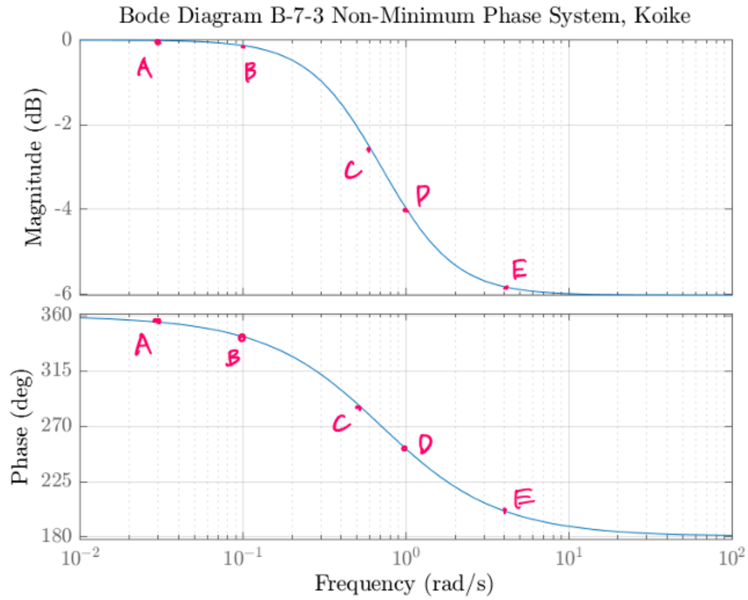


Nyquist Plot (MATLAB)



Non-Minimum Phase System

Bode Plot (from HW 10)



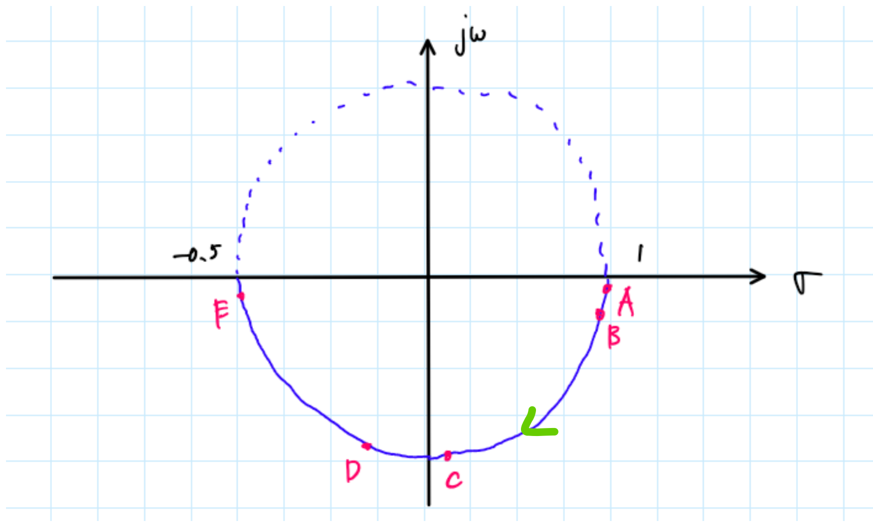
Point	ω [rad/s]	$\angle G$ [deg]	$-20\log_{10}\lvert G\rvert$ [dB]	$\lvert G\rvert$
A	0.02	352	0	1
B	0.1	343	-0.02	0.9988
C	0.5	281	-2.5	0.7480
D	1	255	-4	0.6310
E	4	198	-5.9	0.5070

$$G_2(j\omega) = \frac{1-j\omega}{1+2j\omega}$$

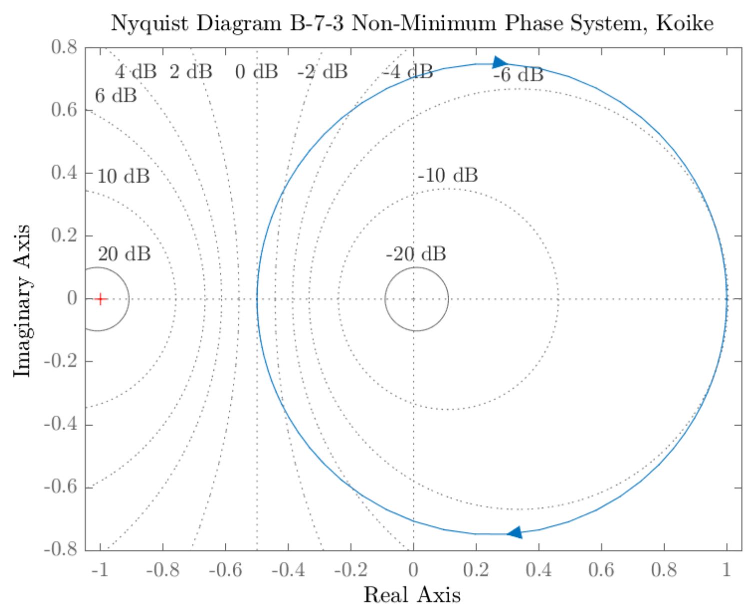
$$\omega \to 0 : G_2(j\omega) = 1 \angle 0^\circ$$

$$\omega \to \infty : G_2(j\omega) = -\frac{1}{2} \angle 180^\circ$$

Nyquist Plot Sketch



Nyquist Plot (MATLAB)

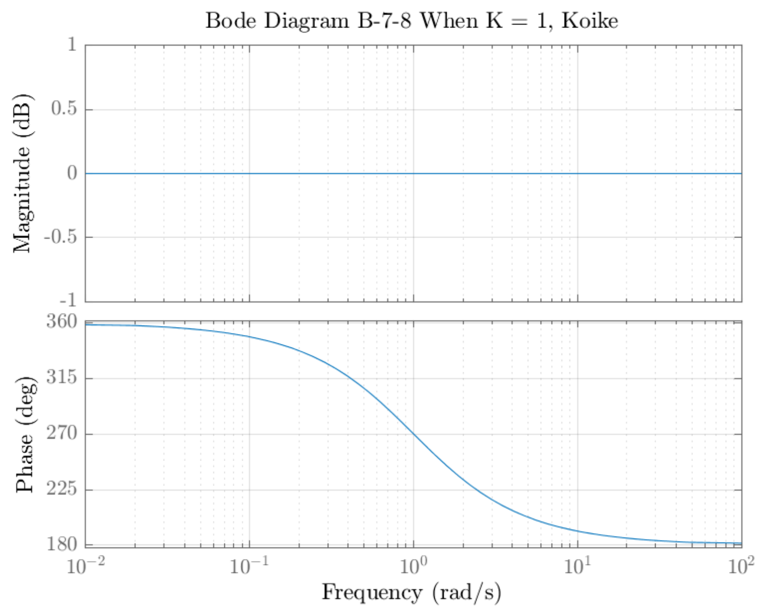


B–7–8. Draw a Nyquist locus for the unity-feedback control system with the open-loop transfer function

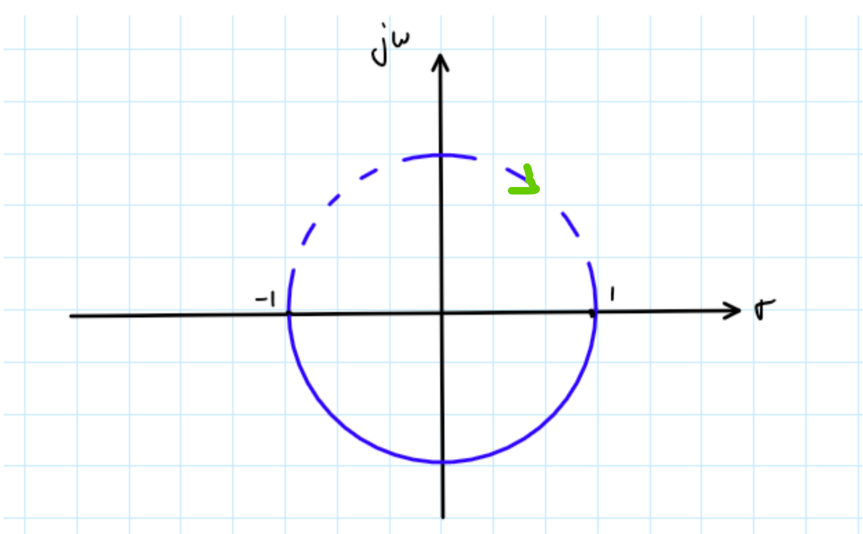
$$G(s) = \frac{K(1 - s)}{s + 1}$$

Using the Nyquist stability criterion, determine the stability of the closed-loop system.

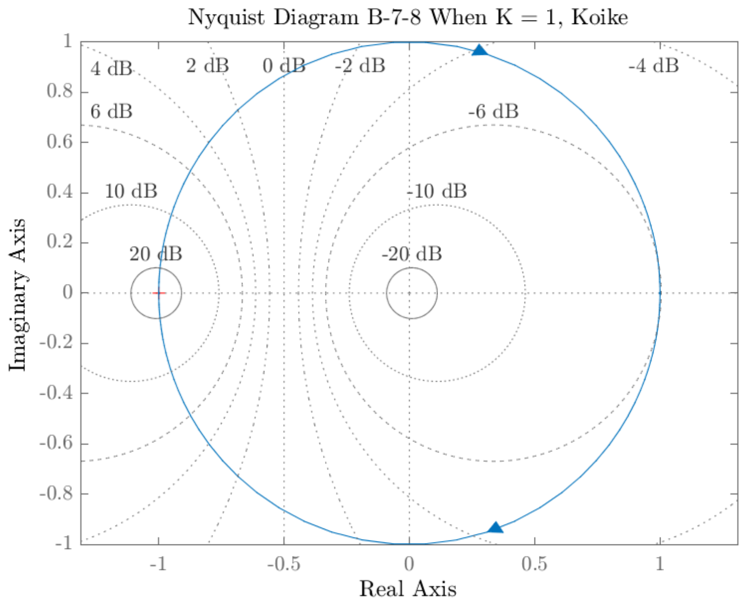
Bode Plot (from HW10)



Nyquist Plot Sketch



Nyquist Plot (MATLAB)

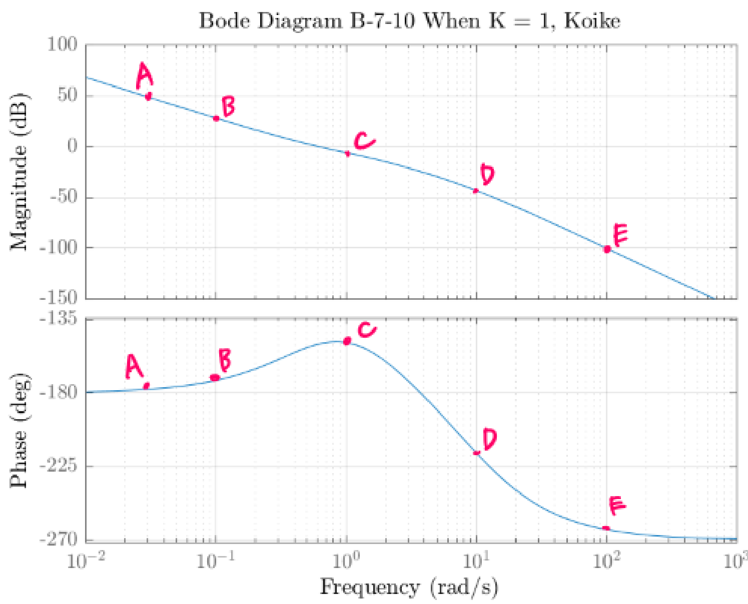


B–7–10. Consider the closed-loop system with the following open-loop transfer function:

$$G(s)H(s) = \frac{10K(s+0.5)}{s^2(s+2)(s+10)} = \frac{2K(0.5s+1)}{20s^2(2s+1)(10s+1)}$$

Plot both the direct and inverse polar plots of $G(s)H(s)$ with $K = 1$ and $K = 10$. Apply the Nyquist stability criterion to the plots, and determine the stability of the system with these values of K.

Bode Plot (from HW10)



Point	ω [rad/s]	$\angle$G [deg]	$-20\log_{10}$|G| [dB]	|G|
A	0.02	-178	55	562.3412
B	0.1	-171	28	25.1189
C	1	-150	-7	0.4467
D	10	-215	-48	0.0004
E	100	-261	-100	0

$$G(j\omega)H(j\omega) = \frac{-(2j\omega+1)}{4\omega^2\left(\frac{j\omega}{2}+1\right)\left(\frac{j\omega}{10}+1\right)}$$

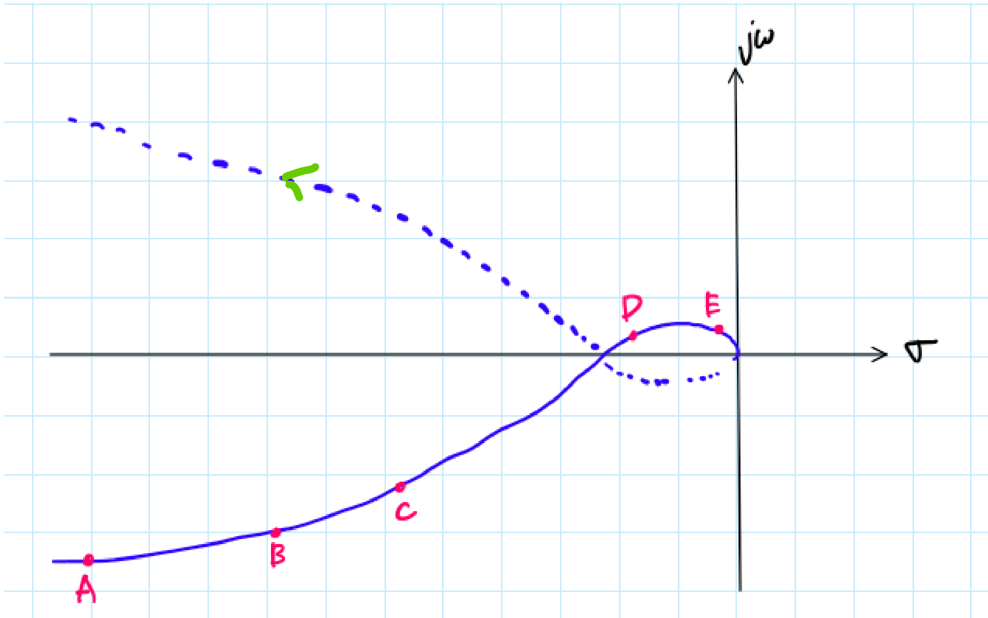
$\omega \to 0$

$$G(j\omega)H(j\omega) = -\infty\angle -180^\circ$$

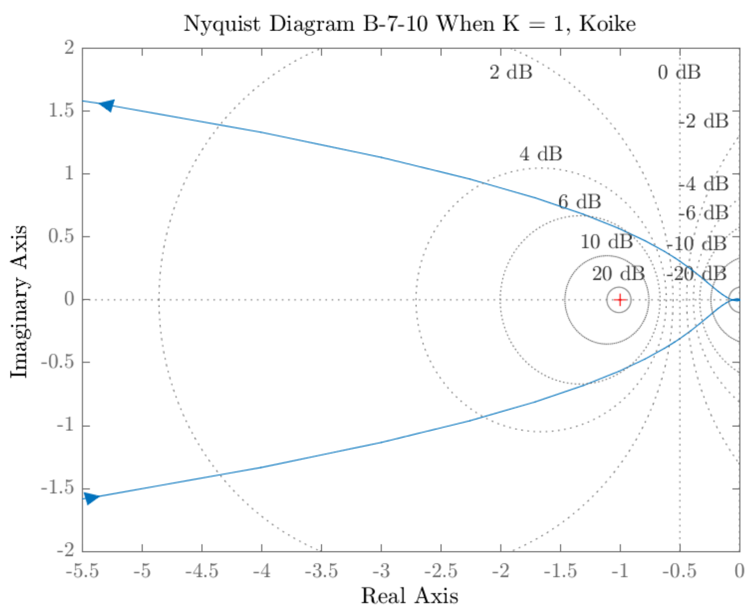
$\omega \to \infty$

$$G(j\omega)H(j\omega) = -\frac{2j\omega}{4\omega^2 \frac{\omega^2}{20}} = 0\angle 90^\circ$$

Nyquist Plot Sketch



Nyquist Plot (MATLAB)

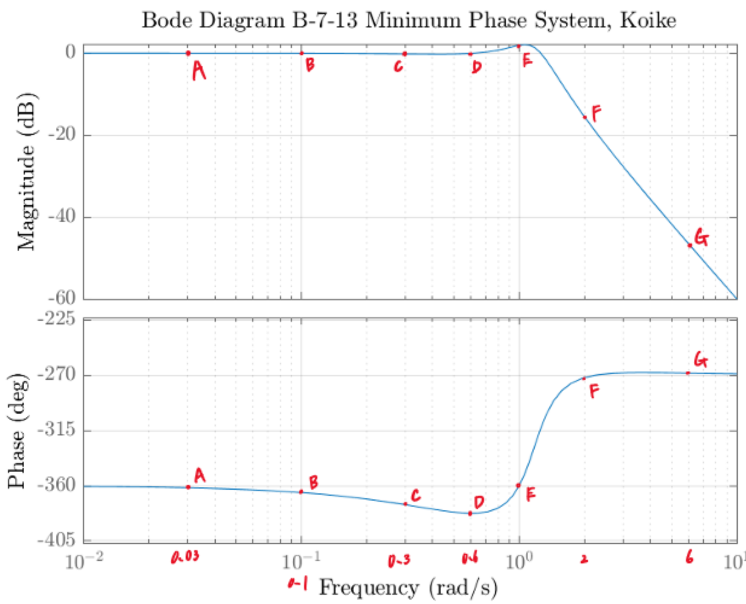


B–7–13. Consider a unity-feedback control system with the following open-loop transfer function:

$$G(s) = \frac{1}{s^3 + 0.2s^2 + s + 1}$$

Draw a Nyquist plot of $G(s)$ and examine the stability of the system.

Bode Plot (from HW10)



Point	ω [rad/s]	$\angle$G [deg]	$-20\log_{10}\lvert G\rvert$ [dB]	$\lvert G\rvert$
A	0.03	-360	0	1
B	0.1	-364	0	1
C	0.3	-375	0	1
D	0.6	-382	0	1
E	1	-360	2	1.2589
F	2	-271	-18	0.1259
G	6	-270	-46	0.00501

$$G(j\omega) = \frac{1}{-j\omega^3 - 0.2\omega^2 + j\omega + 1}$$

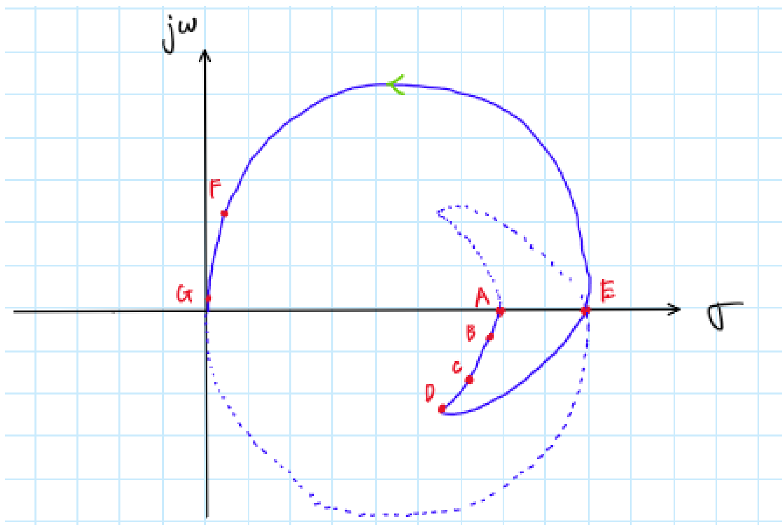
$\omega \to 0$

$$G(j\omega) = 1\angle 0^\circ$$

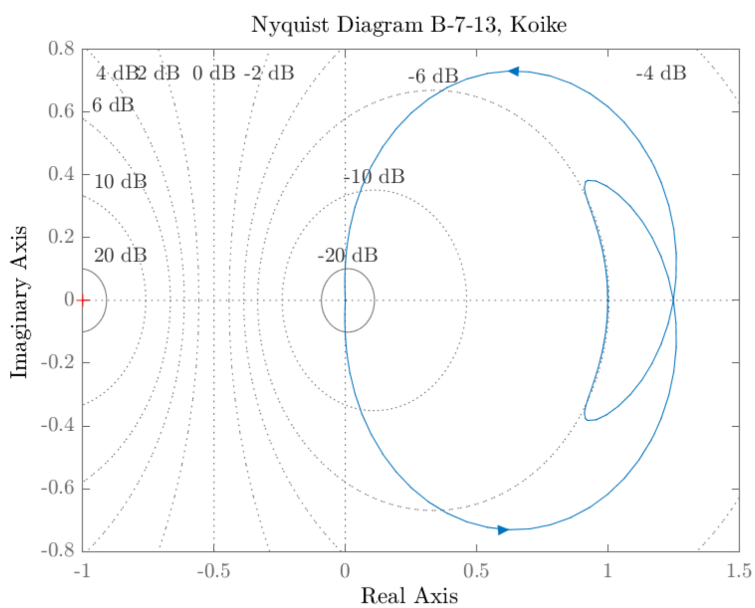
$\omega \to \infty$

$$G(j\omega) = -\frac{1}{j\omega^3} = \frac{j}{\omega^3} = 0\angle 90^\circ$$

Nyquist Plot Sketch



Nyquist Plot (MATLAB)



Problem 2

1. Sketch the Bode plots of the following three systems:

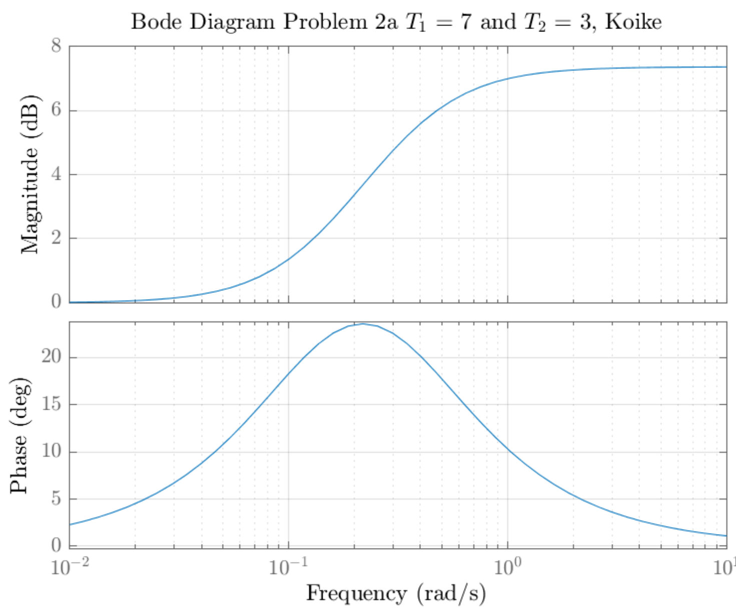
   (a) $G(s) = \frac{T_1 s+1}{T_2 s+1}$, $(T_1 > T_2 > 0)$

   (b) $G(s) = \frac{T_1 s-1}{T_2 s+1}$, $(T_1 > T_2 > 0)$

   (c) $G(s) = \frac{-T_1 s+1}{T_2 s+1}$, $(T_1 > T_2 > 0)$

(a)

Bode Plot (from HW10)



$$G(j\omega) = \frac{T_1 j\omega + 1}{T_2 j\omega + 1}$$

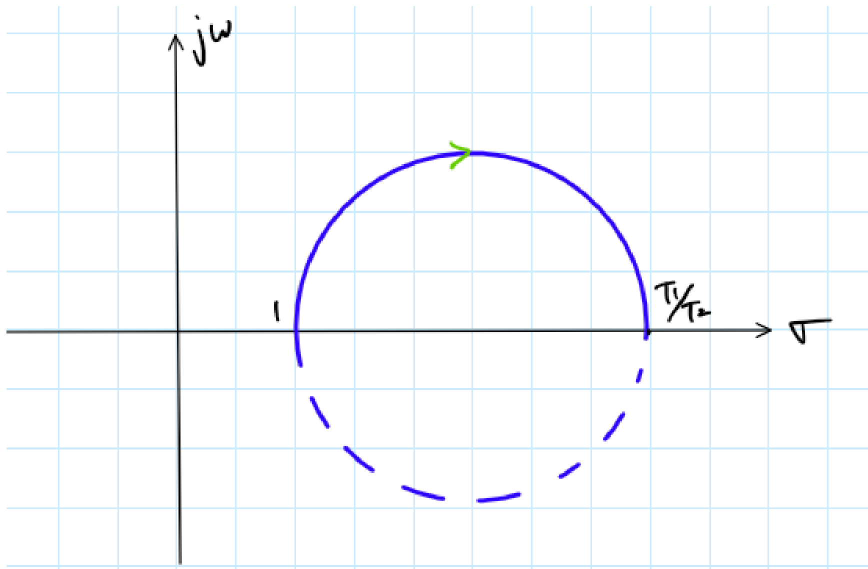
$\omega \to 0$

$$G(j\omega) = 1\angle 0^\circ$$

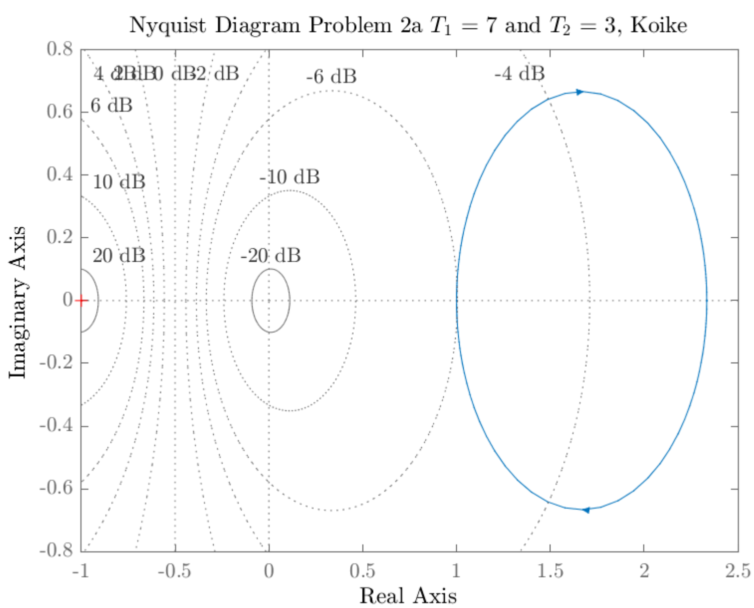
$\omega \to \infty$

$$G(j\omega) = \frac{T_1}{T_2}\angle 0^\circ$$

Nyquist Plot Sketch

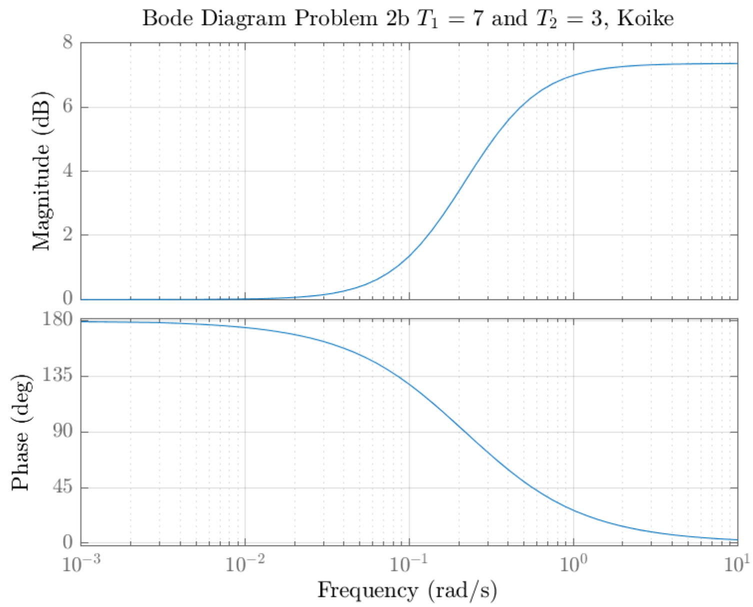


Nyquist Plot (MATLAB)



(b)

Bode Plot (from HW10)



$$G(j\omega) = \frac{T_1 j\omega - 1}{T_2 j\omega + 1}$$

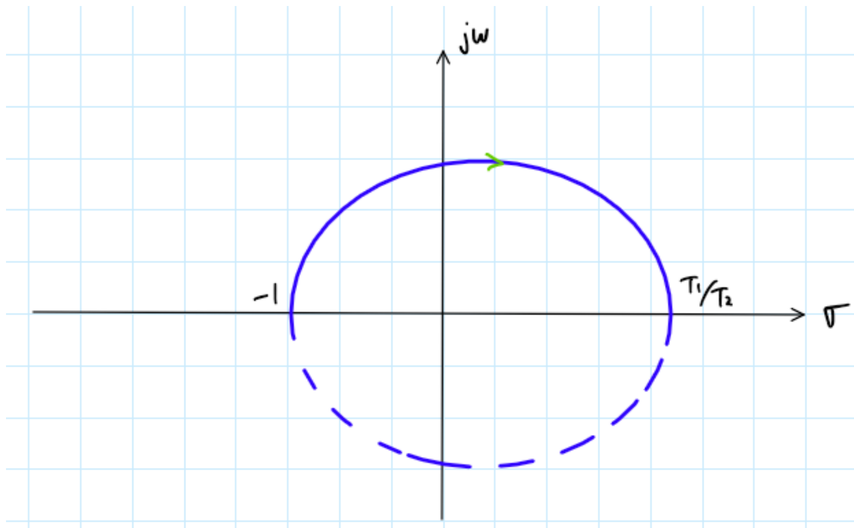
$\omega \to 0$

$$G(j\omega) = -1 \angle 180^\circ$$

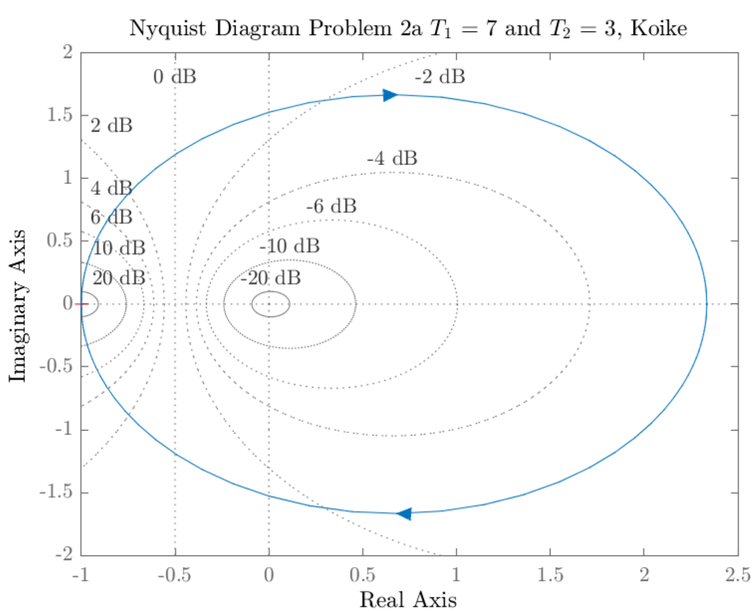
$\omega \to \infty$

$$G(j\omega) = \frac{T_1}{T_2} \angle 0^\circ$$

Nyquist Plot Sketch

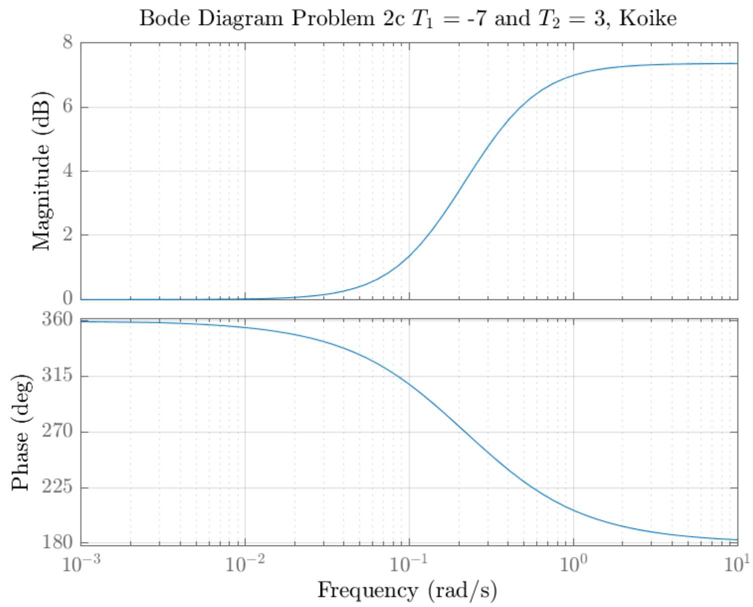


Nyquist Plot (MATLAB)



(c)

Bode Plot (from HW10)



$G(j\omega) = \frac{-T_1 j\omega + 1}{T_2 j\omega - 1}$

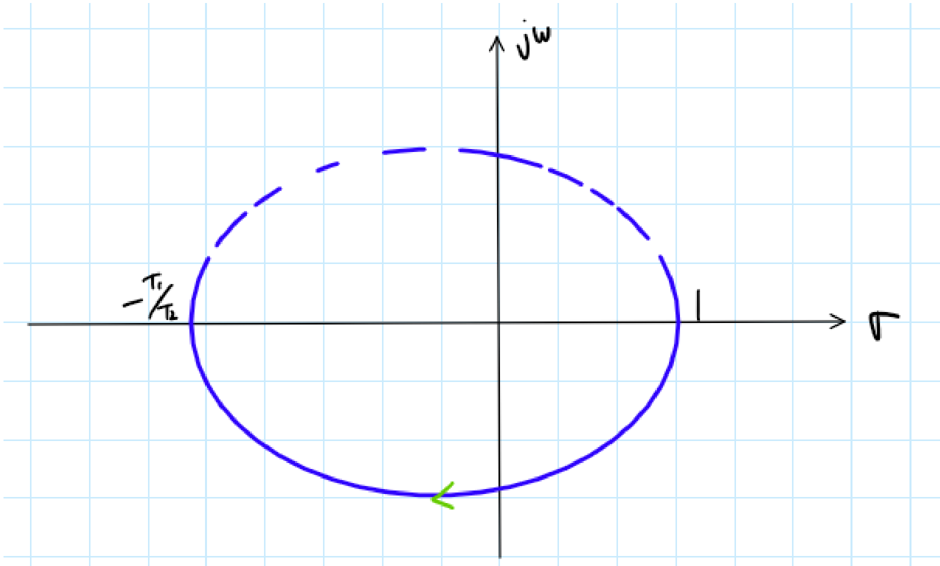
$\omega \to 0$

$G(j\omega) = 1 \angle 0^\circ$

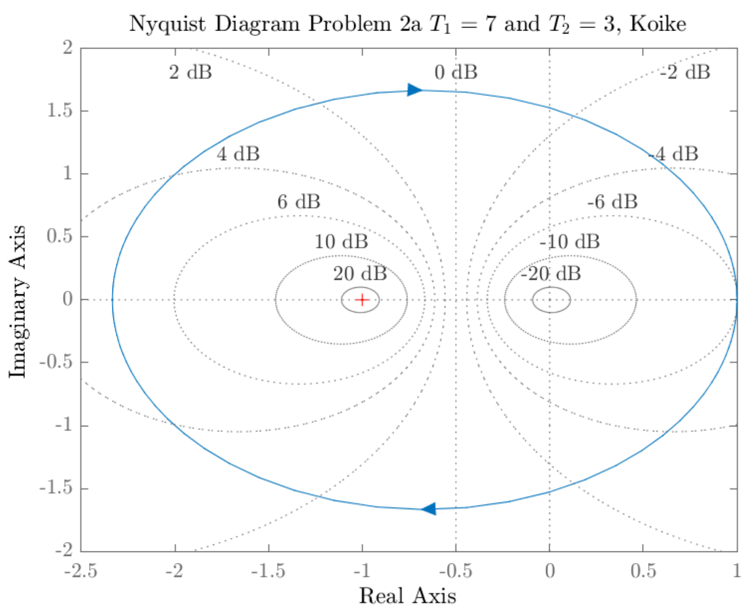
$\omega \to \infty$

$G(j\omega) = \frac{-T_1}{T_2} \angle 180^\circ$

Nyquist Plot Sketch



Nyquist Plot (MATLAB)



Problem 2: Aircraft Example

The following figure shows the coordinate axes and forces acting on the aircraft in the longitudinal plane of motion. Assuming that the aircraft is cruising at constant velocity and altitude.

Plot the Bode diagram of the following $G(s)$:

1. $G(s)$ representing the aircraft pitch angle response output to the elevator deflection input:

$$G(s) = \frac{\Theta(s)}{\Delta(s)} = \frac{1.1057s + 0.1900}{s^3 + 0.7385s^2 + 0.8008s}$$

Poles and Zeros

i	Poles, P_i	Zeros, Z_i
1	0 + 0j	-0.1718
2	-0.3693 + 0.8151j	
3	-0.3693 - 0.8151j	

Corner Frequencies

$$\omega_1 = P_1 = 0$$

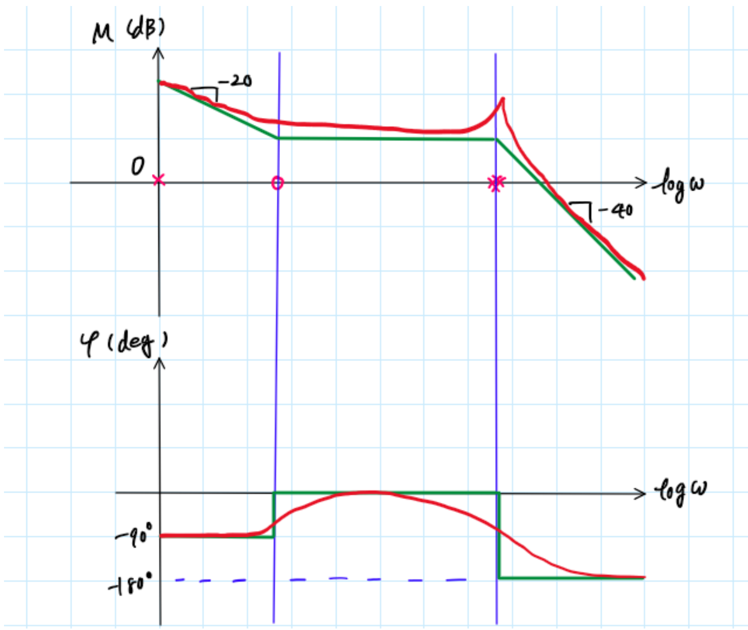
$$\omega_2 = -Z_1 = 0.1718$$

$$\omega_3 = \| P_2 \| = \| P_3 \| = \sqrt{0.3693^2 + 0.8151^2} = 0.8949$$

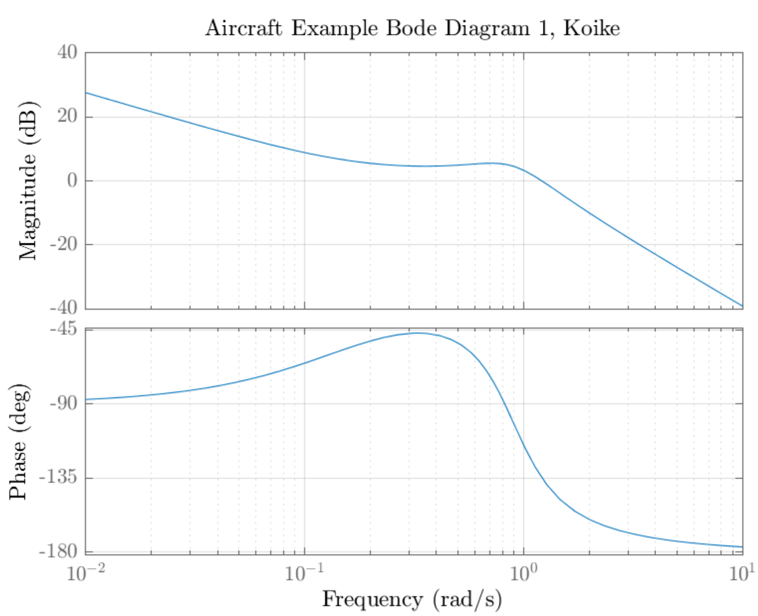
$$\lim_{\omega \to 0} G_1(j\omega) = \lim_{\omega \to 0} \frac{1.1057j\omega + 0.1900}{(j\omega)^3 + 0.7385(j\omega)^2 + 0.8008j\omega}$$

$$\longrightarrow \infty \angle -90^0$$

Bode Plot Sketch



Bode Plot (MATLAB)



2. $G(s)$ representing the aircraft altitude response output to the elevator deflection input:

$$G(s) = \frac{H(s)}{\Delta(s)} = \frac{1.1057s - 0.1900}{s^5 + 17.95s^4 + 123.3s^3 + 366.3s^2 + 112.2s}$$

Poles and Zeros

i	Poles, P_i	Zeros, Z_i
1	0 + 0j	0.1718
2	-8.0992	
3	-4.7533 + 4.2012j	
4	-4.7533 - 4.2012j	
5	-0.3442	

Corner Frequencies

$$\omega_1 = P_1 = 0$$

$$\omega_2 = Z_1 = 0.1718$$

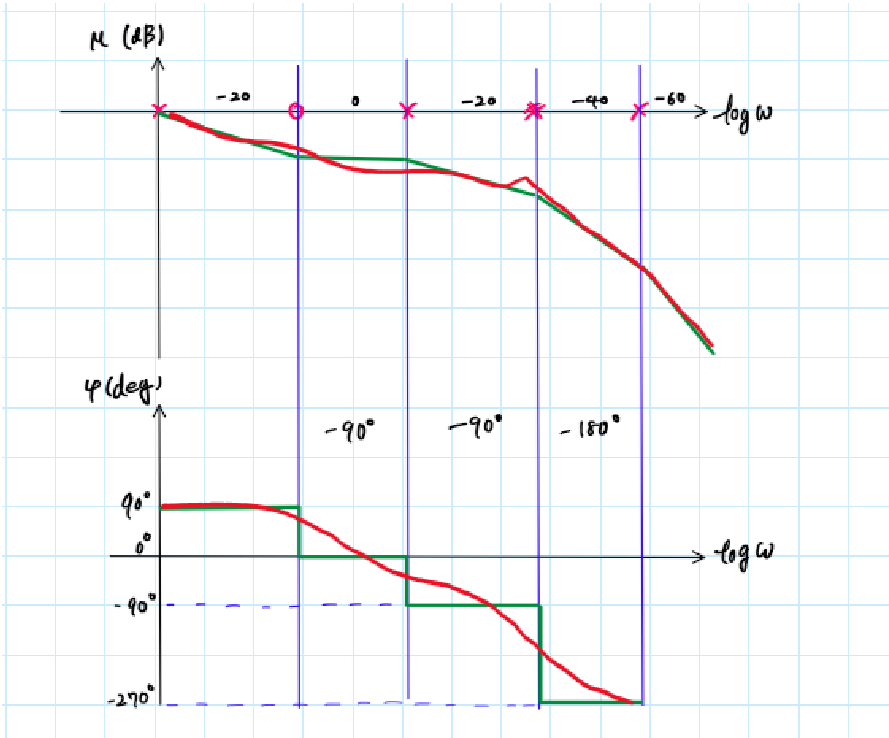
$$\omega_3 = -P_5 = 0.3442$$

$$\omega_4 = \| P_3 \| = \| P_4 \| = \sqrt{4.7533^2 + 4.2012^2} = 6.3438$$

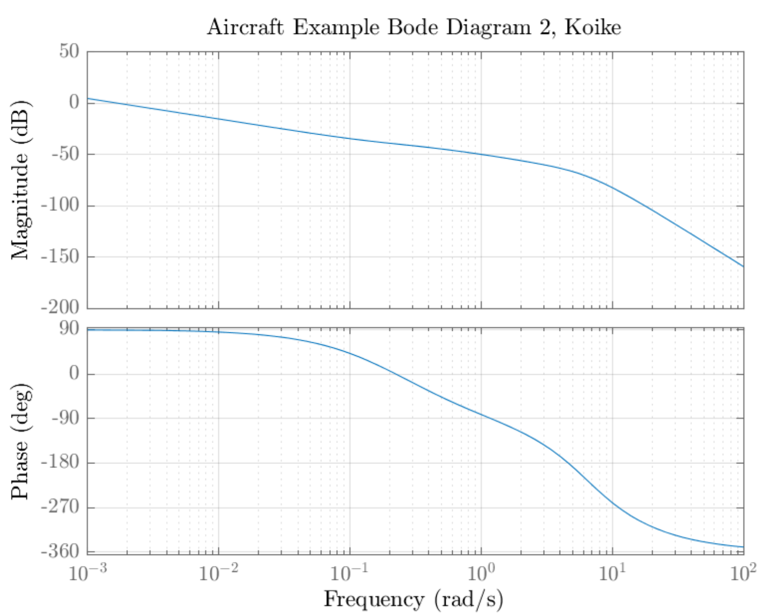
$$\omega_5 = 8.0992$$

$$\lim_{\omega \to 0} G(j\omega) = \lim_{\omega \to 0} \frac{1.1057j\omega - 0.1900}{j\omega^5 + 17.95\omega^4 - 123j\omega^3 - 366.3\omega^2 + 112.2j\omega} = \infty\angle 90^\circ$$

Bode Plot Sketch



Bode Plot (MATLAB)



Problem 3: Spacecraft

Consider the plant $G(s)$ representing the spacecraft attitude dynamics shown in Figure 2:

$$G(s) = \frac{\theta(s)}{T_c(s)} = \frac{0.036(s+25)}{s^2(s^2+0.04s+1)}$$

Plot the Bode diagram of $G(s)$.

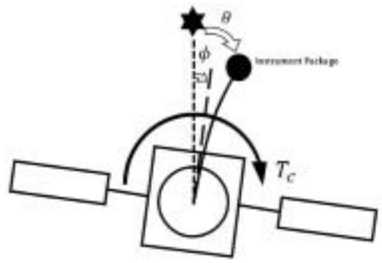


Figure 2: Two-body Model of Satellite

Poles and Zeros

i	Poles, P_i	Zeros, Z_i
1	0 + 0j	-25
2	0 + 0j	
3	-0.02+0.9998j	
4	-0.02-0.9998j	

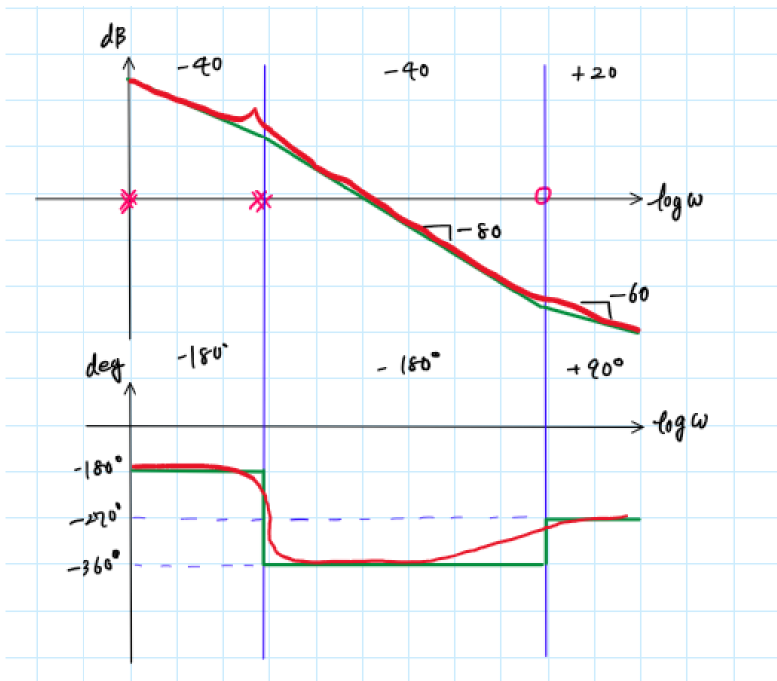
$$\omega_1 = P_1 = P_2 = 0$$

$$\omega_2 = \|P_3\| = \|P_4\| = \sqrt{0.02^2 + 0.9998^2} = 1$$

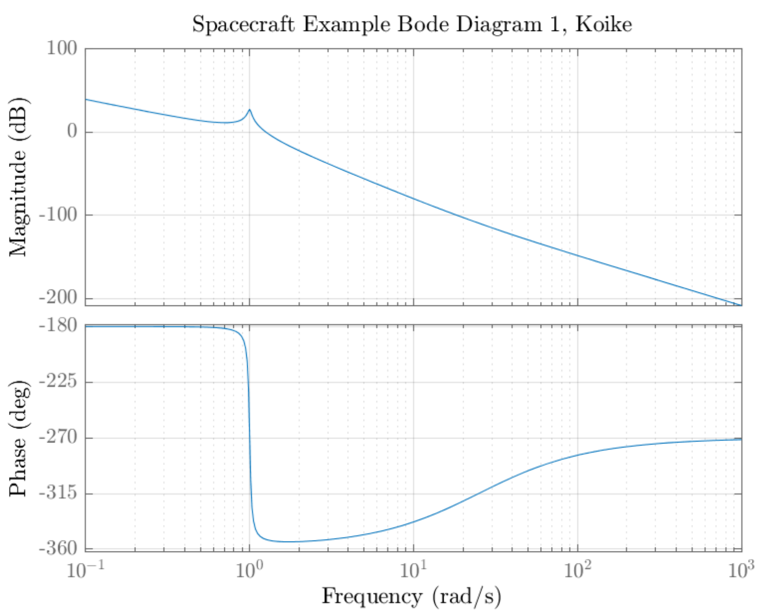
$$\omega_3 = 25$$

$$\lim_{\omega \to 0} G(j\omega) = \lim_{\omega \to 0} \frac{0.036 \times 25}{-\omega^2} = -\infty \angle -180^\circ$$

Bode Plot Sketch



Bode Plot (MATLAB)



Appendix

MATLAB CODE

AAE 364 HW11

```
clear all; close all; clc;
fdir = 'C:\Users\Tomo\Desktop\studies\2020-
Spring\AAE364\matlab\matlab_output\hw11';
set(groot, 'defaulttextinterpreter','latex');
set(groot, 'defaultAxesTickLabelInterpreter','latex');
set(groot, 'defaultLegendInterpreter','latex');

% Bode plot options
opts_bd = bodeoptions('cstprefs');
opts_bd.Title.Interpreter = "latex";
opts_bd.XLabel.Interpreter = "Latex";
opts_bd.YLabel.Interpreter = "Latex";
opts_bd.Grid = 'on';

% Nyquist plot options
opts_nq = nyquistoptions("cstprefs");
opts_nq.Title.Interpreter = 'latex';
opts_nq.XLabel.Interpreter = "Latex";
opts_nq.YLabel.Interpreter = "Latex";
opts_nq.Grid = 'on';
```

B-7-3

```
% Minimum Phase System
num = [1 1];
den = [2 1];
G = tf(num,den);
arr_log = [0 -0.15 -2 -4 -5.8];
arr = 10.^(arr_log/20);
% Nyquist Plot
fig = figure("Renderer","painters");
    opts_nq.Title.String = "Nyquist Diagram B-7-3 Minimum Phase System, Koike";
    nyquistplot(G,opts_nq);
    axis equal;
saveas(fig,fullfile(fdir,"B-7-3_min_nyquist.png"));

% Non-minimum Phase System
num = [-1 1];
den = [2 1];
G = tf(num,den);
arr_log = [0 -0.01 -2.5 -4 -5.9];
arr = 10.^(arr_log/20);
% Nyquist Plot
fig = figure("Renderer","painters");
    opts_nq.Title.String = "Nyquist Diagram B-7-3 Non-Minimum Phase System,
Koike";
```

```
    nyquistplot(G,opts_nq);
    axis equal;
saveas(fig,fullfile(fdir,"B-7-3_nonmin_nyquist.png"));
```

B-7-8

```
K = 1;
num = K*[-1 1];
den = [1 1];
G = tf(num,den);
% Bode Plot
fig = figure("Renderer","painters");
    opts_bd.Title.String = "Bode Diagram B-7-8 When K = 1, Koike";
    bodeplot(G,opts_bd);
saveas(fig,fullfile(fdir,"B-7-8_bode_K=1.png"));
% Nyquist Plot
fig = figure("Renderer","painters");
    opts_nq.Title.String = "Nyquist Diagram B-7-8 When K = 1, Koike";
    nyquistplot(G,opts_nq);
    axis equal;
saveas(fig,fullfile(fdir,"B-7-8_nyquist_K=1.png"));
```

B-7-10

```
% Define the OL transfer function
num = 5*[2 1];
den = conv(2*[0.5 1 0 0],10*[0.1 1]);
K = 1;
G = tf(K*num,den);
% Bode Plot
fig = figure("Renderer","painters");
    opts_bd.Title.String = "Bode Diagram B-7-10 When K = 1, Koike";
    bodeplot(G,opts_bd);
saveas(fig,fullfile(fdir,"B-7-10_bode_K=1.png"));
arr_log = [55 28 -7 -48 -100];
arr = 10.^(arr_log/20);
% Nyquist Plot
fig = figure("Renderer","painters");
    title_txt = sprintf("Nyquist Diagram B-7-10 When K = 1, Koike");
    opts_nq.Title.String = title_txt;
    nyquistplot(G,opts_nq);
    xlim([-5.5 0])
    file_txt = sprintf("B-7-10_nyquist_K=1.png");
saveas(fig,fullfile(fdir,file_txt));
```

B-7-13

```
% Define the transfer function
num = [0 1];
den = [1 0.2 1 1];
G = tf(num,den);
% Bode Plot
```

```
fig = figure("Renderer","painters");
    opts_bd.Title.String = "Bode Diagram B-7-13 Minimum Phase System, Koike";
    bodeplot(G,opts_bd);
saveas(fig,fullfile(fdir,"B-7-13_bode.png"));
% Nyquist Plot
fig = figure("Renderer","painters");
    opts_nq.Title.String = "Nyquist Diagram B-7-13, Koike";
    nyquistplot(G,opts_nq);
saveas(fig,fullfile(fdir,"B-7-13_nyquist.png"));
```

P2

```
% (a)
num = [7 1];
den = [3 1];
G = tf(num,den);
% Bode Plot
fig = figure("Renderer","painters");
    opts_bd.Title.String = "Bode Diagram Problem 2a $T_1$ = 7 and $T_2$ = 3,
Koike";
    bodeplot(G,opts_bd);
saveas(fig,fullfile(fdir,"P2-a_bode.png"));
% Nyquist Plot
fig = figure("Renderer","painters");
    opts_nq.Title.String = "Nyquist Diagram Problem 2a $T_1$ = 7 and $T_2$ = 3,
Koike";
    nyquistplot(G,opts_nq);
saveas(fig,fullfile(fdir,"P2-a_nyquist.png"));

% (b)
num = [7 -1];
den = [3 1];
G = tf(num,den);
% Bode Plot
fig = figure("Renderer","painters");
    opts_bd.Title.String = "Bode Diagram Problem 2b $T_1$ = 7 and $T_2$ = 3,
Koike";
    bodeplot(G,opts_bd);
saveas(fig,fullfile(fdir,"P2-b_bode.png"));
% Nyquist Plot
fig = figure("Renderer","painters");
    opts_nq.Title.String = "Nyquist Diagram Problem 2a $T_1$ = 7 and $T_2$ = 3,
Koike";
    nyquistplot(G,opts_nq);
saveas(fig,fullfile(fdir,"P2-b_nyquist.png"));

% (c)
num = [-7 1];
den = [3 1];
G = tf(num,den);
% Bode Plot
fig = figure("Renderer","painters");
```

```
    opts_bd.Title.String = "Bode Diagram Problem 2c $T_1$ = -7 and $T_2$ = 3,
Koike";
    bodeplot(G,opts_bd);
saveas(fig,fullfile(fdir,"P2-c_bode.png"));
% Nyquist Plot
fig = figure("Renderer","painters");
    opts_nq.Title.String = "Nyquist Diagram Problem 2a $T_1$ = 7 and $T_2$ = 3,
Koike";
    nyquistplot(G,opts_nq);
saveas(fig,fullfile(fdir,"P2-c_nyquist.png"));
```

P3 Aircraft Example

```
% 1
num = [1.1057 0.19];
den = [1 0.7385 0.8008 0];
G = tf(num,den);

pls = roots(den)
zrs = roots(num)
cornFreq = corner_freq(num,den)

fig = figure("Renderer","painters");
    opts_bd.Title.String = "Aircraft Example Bode Diagram 1, Koike";
    bodeplot(G,opts_bd);
saveas(fig,fullfile(fdir,"P3-1_bode.png"));
```

```
% 2
num = [1.1057 -0.19];
den = [1 17.95 123.3 366.3 112.2 0];

pls = roots(den)
zrs = roots(num)
cornFreq = corner_freq(num,den)

G = tf(num,den);
fig = figure("Renderer","painters");
    opts_bd.Title.String = "Aircraft Example Bode Diagram 2, Koike";
    bodeplot(G,opts_bd);
saveas(fig,fullfile(fdir,"P3-2_bode.png"));
```

P4 Spacecraft Example

```
num = 0.036*[1 25];
den = [1 0.04 1 0 0];

pls = roots(den)
zrs = roots(num)
cornFreq = corner_freq(num,den)

G = tf(num,den);
fig = figure("Renderer","painters");
```

```
    opts_bd.Title.String = "Spacecraft Example Bode Diagram 1, Koike";
    bodeplot(G,opts_bd);
saveas(fig,fullfile(fdir,"P4_bode.png"));
```

```
function w_i = corner_freq(num,den)
    %{
      Function:    corner_freq()
      Author:      Tomoki Koike
      Description: Computes the corner frequencies for a Bode Plot.
      >>Inputs
          num: the numerator of the open-loop transfer function
          den: the denominator of the open-loop transfer function
      Outputs<<
          w_i: the table with the corner frequencies for poles and zeros
    %}

    pls = roots(den);
    zrs = roots(num);
    cornP = unique(abs(pls));
    cornZ = unique(abs(zrs));
    if length(cornP) > length(cornZ)
        cornZ = [cornZ; NaN([(length(cornP) - length(cornZ)), 1])];
    else
        cornP = [cornP; NaN([(length(cornZ) - length(cornP)), 1])];
    end
    w_i = array2table([cornP, cornZ],"VariableNames",{'Poles','Zeros'});
end
```