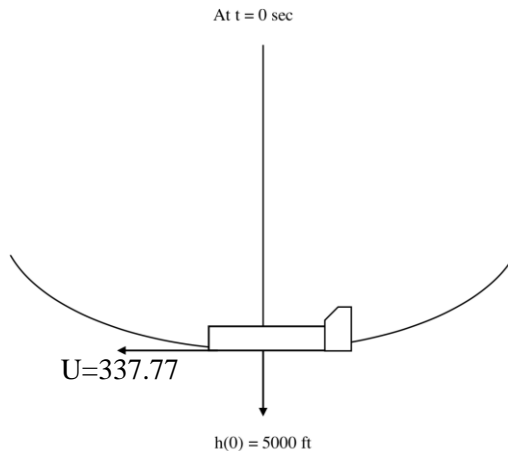


Problem 1. (15pts)

An aircraft is flying straight and level at a constant velocity of 337.77 ft/sec, and then performs a symmetric pull up such that $\dot{\Theta} = 0.05 \text{ rad/s} = \text{constant}$. Assume the aircraft's x-axis is aligned with the flight path throughout the motion and that at $t = 0$, the aircraft's location in North-East-Altitude coordinate is $p_N = 0$, $p_E = 0$, and $h = 5000 \text{ ft}$. Find the position coordinates (p_N , p_E , h) at $t = 5 \text{ sec}$. Assume $\Psi = 0$.

**Problem 2. (10pts)**

The aircraft velocity vector expressed in the Earth-fixed reference frame is

$$\bar{V}_I = U\mathbf{I} + V\mathbf{J} + W\mathbf{K} = 6.6637\mathbf{I} + 289.1164\mathbf{J} - 407.8815\mathbf{K} \text{ (ft/sec)}$$

and in the aircraft fixed body reference frame it is given by

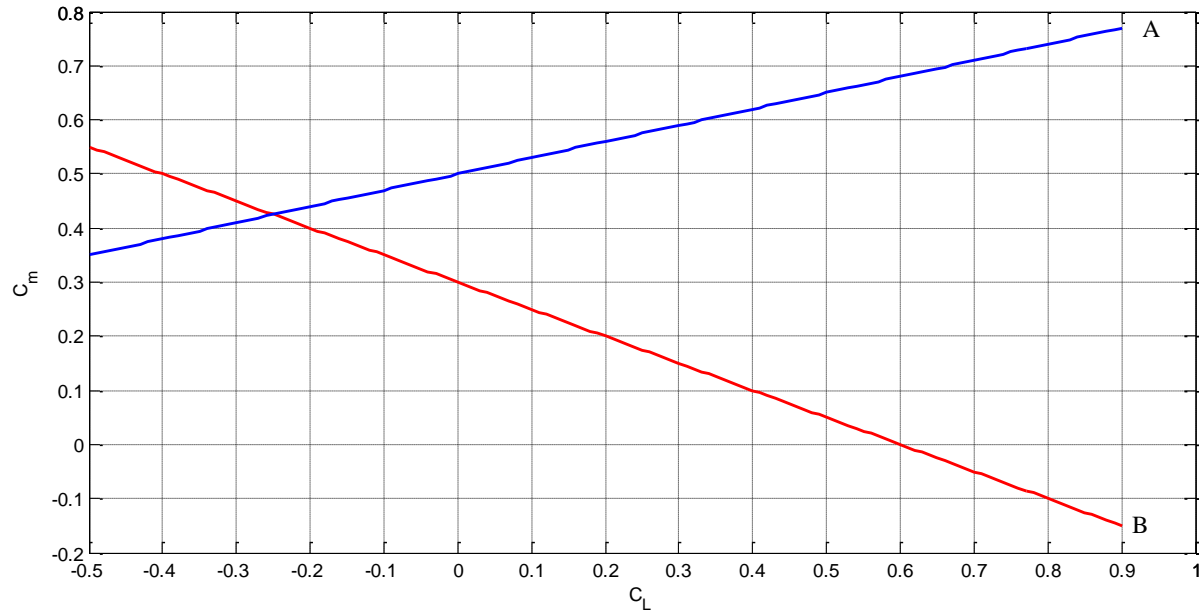
$$\bar{V}_b = u\mathbf{i} + v\mathbf{j} + w\mathbf{k} = 497.7939\mathbf{i} + 17.4497\mathbf{j} + 43.5513\mathbf{k} \text{ (ft/sec)}$$

Find the attitude of the aircraft in terms of its Euler angles (Ψ , Θ , Φ). Is your answer unique?

Problem 3 (10pts)

For the C_L and C_m relationship shown in the following plots

- (1) Find the linear expressions of C_m in term of C_L for line A and B, respectively.
- (2) To obtain a trim condition, which line should be selected?
- (3) Assuming $\frac{x_{cg}}{\bar{c}} = 0.6$, how to relocate the a.c center ($\frac{x_{ac}}{\bar{c}}$) to obtain a new $C_{L,trim} = 0.8$?



Problem 4. (15pts)

Wind tunnel test on a full-scale flying wing yielded the following database

Angle of Attack, deg	C_L	$C_{m_{cg}}$
8.0	0.64	-0.014
5.0	0.40	0.010
2.0	0.16	0.034
-3.0	-0.24	0.074

The configuration c.g is located 0.58 ft from the leading edge of the chord and the chord length is 2.6 ft.

- Estimate the configuration lift curve slope
- Is the configuration, as tested, statically stable? Explain your answer.
- Estimate values for C_m at the aerodynamic center and aerodynamic center location.
- Can this configuration, as tested, be trimmed in a steady glide condition? Explain your answer.
- If the answer to part (d) is yes, then estimate values for trim angle of attack and trim lift coefficient

Problem 5. (10pts)

Consider the following nonlinear 2nd-order system

$$\ddot{y} + a_1 \dot{y}^2 + \frac{a_0}{y^2} = f$$

where a_0 and a_1 are constant, and $a_0 > 0$

- (1) For a constant input $f=f_0 > 0$, determine the equilibrium points of the system
- (2) Obtain the linearized equations of the system at the equilibrium points
- (3) Express the linearized model in state equations, choosing $x_1 = \Delta y, x_2 = \Delta \dot{y}, u = \Delta f$

Problem 6. (10pts)

Consider an airplane in constant-altitude, straight-line flight. The velocity equation is

$$\dot{V} = T - \frac{1}{2}kV^2$$

where the second term represents aerodynamic drag, and assume $k = \text{constant}$, and T is the engine thrust acceleration. Treat T as the control (input). Let V^* be a given constant cruise speed. Obtain the linearized differential equation for the velocity around V^* .

Problem 7. (15pts)

From the nonlinear flight dynamics model, derive the following linear perturbation equations for Y force

$$m(\dot{v} + u_0 r) = \Delta Y + mg \cos(\theta_0) \phi$$

and moments:

$$\begin{aligned} \Delta L &= I_{xx} \dot{p} - I_{xz} \dot{r} \\ \Delta M &= I_{yy} \dot{q} \\ \Delta N &= -I_{xz} \dot{p} + I_{zz} \dot{r} \end{aligned}$$

Show all the steps!

Problem 8. (15pts)

Consider the 2-degree-of-freedom spring mass pendulum shown below (All motion is in the plane of the picture shown). The nonlinear equations of motion are given by

$$(1) \ddot{X} + \frac{k}{m}(X - L) - g \cos \theta - X \dot{\theta}^2 = 0$$

$$(2) X^2 \ddot{\theta} + g X \sin \theta + 2 \dot{\theta} X \dot{X} = 0$$

where L is the original spring length.

Linearize the equations of motion for this system. Let the reference condition be the equilibrium (no motion) state for the pendulum mass. In particular

- (a) Define a set of perturbation variables
- (b) Substitute the results of Part (a) into the equations of motion
- (c) Expand the equations and discard appropriate terms (show the terms that are to be discarded)

