AAE 339: Aerospace Propulsion

HW5: Turbojet and Turbofan Cycles

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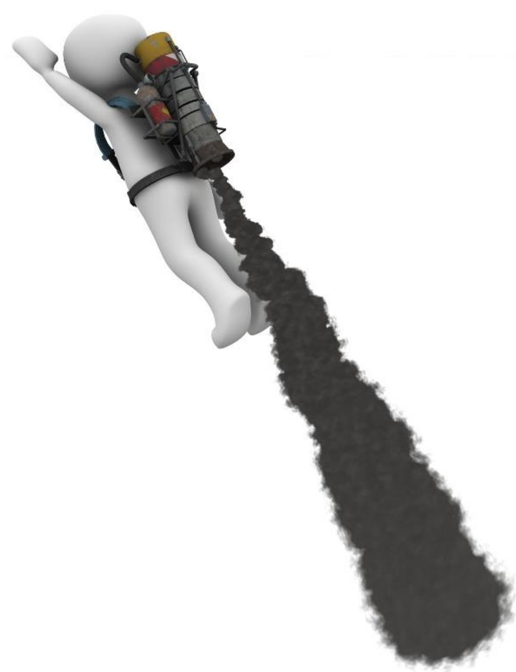

1. Until very recently, the Rolls-Royce Trent-900 was the largest commercial turbofan in the world. As per the engine specifications, the pressure ratio $p_{03}/p_{02} = 42$ and the turbine inlet temperature $T_{04} = 1800$ K. Like we did in class, let's perform a cycle analysis of a hypothetical pure turbojet version of the Trent-900, but at a different flight condition.

Do the analysis for a cruise Mach number of 0.85 at an altitude of 40,000 ft. Use a heating value $Q = 45{,}000$ kJ/kg for the fuel, and R=287 J/kg-K and $\gamma = 1.4$ for air. For a bit more accuracy, let's use $c_p = 1.10$ kJ/kg-K for the flow starting at the compressor inlet. Assume a mechanical efficiency of 1.0 ($w_t = -w_c$). Use adiabatic efficiencies from the notes - $\eta_d = 0.97$, $\eta_c = 0.85$, $\eta_t = 0.90$, and $\eta_n = 0.98$. Let the burner efficiency $\eta_b = 0.98$ and furthermore assume the flow across the combustor incurs a 2.0% pressure loss ($p_{04}/p_{03} = 0.98$). Use 1.0 kg/s of air as a basis.

(a) Calculate stagnation temperature and stagnation pressure of the free stream before it enters the engine (a), at the compressor inlet (2), and at the combustor inlet (3).
(b) Determine the stagnation pressure at the combustor exit/turbine inlet (4), and the fuel-air ratio in the combustor.
(c) Calculate the stagnation temperature and pressure at the turbine exit (5).
(d) Consider two different exit conditions – (1) where the flow is expanded to atmospheric pressure $p_7 = p_a$ with a converging-diverging nozzle, and (2) where a simple converging nozzle is used to produce sonic flow with $M_e = 1.0$ at the exit plane. Calculate u_e for both cases. Using the appropriate form of the thrust equation for each, calculate the specific thrust and specific fuel consumption for each case.
(e) If the thrust requirement of each of the Airbus-380 engines is 80 kN, what is the rate of fuel consumption per engine during Mach 0.85 cruise for cases 1 and 2?
(f) Calculate the overall efficiency, η_o, for each case.

$M_a = 0.85$, $\frac{P_{03}}{P_{02}} = 42$, $T_{04} = 1800\,K$, $Q = 45{,}000\ kJ/kg$

$R = 287\ J/kg{\cdot}K$, $\gamma = 1.4$, $C_p = 1.10\ kJ/kg{\cdot}K$, $1.0\ (w_t = -w_c)$

$$\eta_d = \frac{h_{02s} - h_a}{h_{02} - h_a} = \frac{T_{02s} - T_a}{T_{02} - T_a} = 0.97, \quad \eta_c = \frac{h_{03s} - h_{02}}{h_{03} - h_{02}} = \frac{T_{03s} - T_{02}}{T_{03} - T_{02}} = 0.85$$

$$\eta_t = \frac{h_{04} - h_{05}}{h_{04} - h_{05s}} = \frac{T_{04} - T_{05}}{T_{04} - T_{05s}} = 0.90, \quad \eta_n = \frac{h_{06} - h_7}{h_{06} - h_{7s}} = \frac{T_{06} - T_7}{T_{06} - T_{7s}} = 0.98$$

Burner Energy Equation

$$(1+f)h_{04} = h_{03} + f\eta_b Q \quad \text{or} \quad (1+f)C_p T_{04} = C_p T_{03} + f\eta_b Q \quad \text{where } f = \frac{\dot m_f}{\dot m_a}$$

and $P_{04}/P_{03} = 0.98$ $\quad \dot m_a = 1.0\ kg/s$

(a)

<i> free-stream (a)

At an altitude of 40,000 ft the atmospheric conditions are

$$T_a = 389.97 \ R = 216.65 \ K$$
$$P_a = 3.9312 \times 10^2 \ lb/ft^2 = 18823 \ Pa$$

(from textbook : "Mechanics and Thermodynamics")

then,

$$P_{0a} = P_a \left[1 + \frac{\gamma - 1}{2} M_a^2 \right]^{\frac{\gamma}{\gamma - 1}}$$
$$P_{0a} = (18823 Pa) \left[1 + \frac{1.4-1}{2} (0.85)^2 \right]^{\frac{1.4}{1.4-1}} \cong 30189 \ Pa$$
$$T_{0a} = T_a \left[1 + \frac{\gamma - 1}{2} M_a^2 \right]$$
$$T_{0a} \cong 247.96 \ K$$

$$\boxed{\begin{aligned} P_{0a} &= 30189 \ Pa \\ T_{0a} &= 247.96 \ K \end{aligned}}$$

<ii> compressor inlet (2)

the stagnation temperature and pressure does not change from when it entered the engine, thus

$$P_{02} = 30189 \ Pa$$
$$T_{02s} = 247.96 K$$

now, from η_d , we know that

$$T_{02} = T_a + \frac{T_{02s} - T_a}{\eta_d}$$
$$T_{02} = (216.65K) + \frac{(247.96K) - (216.65K)}{0.97} \cong 248.93K$$

$$\boxed{\begin{aligned} P_{02} &= 30189\ Pa \\ T_{02} &= 248.93\ K \end{aligned}}$$

<iii> Combustion inlet (3)

using $\dfrac{P_{03}}{P_{02}} = 42 \iff P_{03} = 42 P_{02} = 42(30189\ Pa) = 1.2679\ MPa$

then from isentropic relations

$$\frac{T_{03s}}{T_{02}} = \left(\frac{P_{03}}{P_{02}}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_{03s} = T_{02}\left(\frac{P_{03}}{P_{02}}\right)^{\frac{\gamma-1}{\gamma}} = (248.93\ K)(42)^{\frac{0.4}{1.4}} = 724.207\ K$$

using η_c

$$T_{03} = T_{02} + \frac{T_{03s} - T_{02}}{\eta_c} = 248.93K + \frac{(724.207K) - (248.93K)}{0.85}$$

$$T_{03} = 808.08\ K$$

$$\boxed{\begin{aligned} P_{03} &= 1.2679\ MPa \\ T_{03} &= 808.08\ K \end{aligned}}$$

(b) @ combustor exit/turbine inlet

$$\frac{P_{04}}{P_{03}} = 0.98 \iff P_{04} = (0.98)(1.2679\ MPa) = 1.2425\ MPa$$

then using the Burner Energy Equation
(given $T_{04} = 1800K$)

$$(1+f)C_pT_{04} = C_pT_{03} + f\eta_b Q$$

$$(\eta_b Q - C_pT_{04})f = C_p(T_{04} - T_{03})$$

$$f = \frac{C_p(T_{04} - T_{03})}{\eta_b Q - C_pT_{04}}$$

$$f = \frac{\left(\frac{1.10\,kJ}{kg\text{-}K}\right)(1800K - 808.08K)}{0.98\left(\frac{45000\,kJ}{kg}\right) - \left(\frac{1.10\,kJ}{kg\text{-}K}\right)(1800K)} = 0.025991$$

$$\boxed{\begin{array}{l} P_{04} = 1.2425\ MPa \\ f = 0.02590 \end{array}}$$

(c) @ the turbine exit (5)

for a turbine from $W_t = -W_c$

since

$$\Delta U = \cancel{\dot{Q}}^{\,0} - \dot{W}$$

$$W_t = C_p(T_{05s} - T_{04})$$

$$-W_c = -C_p(T_{03} - T_{02})$$

thus $\quad T_{05s} - T_{04} = T_{02} - T_{03}$

$$T_{05s} = (T_{02} - T_{03}) + T_{04} \cong 1240.85K$$

now using η_t

$$T_{05} = T_{04} - \eta_t(T_{04} - T_{05s})$$

$$= 1800K - (0.90)(1800K - 1240.85K)$$

$$\cong 1296.765\ K$$

from isentropic relations

$$\frac{P_{05}}{P_{04}} = \left(\frac{T_{05}}{T_{04}}\right)^{\frac{\gamma}{\gamma-1}}$$

$$P_{05} = P_{04}\left(\frac{T_{05}}{T_{04}}\right)^{\frac{\gamma}{\gamma-1}}$$

$$P_{05} = (1.2425\ MPa)\left(\frac{1296.8K}{1800K}\right)^{\frac{1.4}{0.4}} = 0.3943\ MPa$$

$$\boxed{\begin{array}{l} P_{05} = 0.3943\ MPa \\ T_{05} = 1296.8K \end{array}}$$

(d) Case (1)

$P_7 = P_a = 18823\ Pa$

and stagnation pressure and temperature are constant from the turbine exit.

$P_{07} = 0.39757\ MPa$, $T_{07} = 1299.8\ K$

now

$$\frac{P_{07}}{P_7} = \left[1 + \frac{\gamma-1}{2}M_7^2\right]^{\frac{\gamma}{\gamma-1}}$$

$$M_7 = \sqrt{\frac{\left(\frac{P_{07}}{P_7}\right)^{\frac{\gamma-1}{\gamma}} - 1}{\frac{\gamma-1}{2}}} = \boxed{2.633}$$

next

$$\frac{T_{07}}{T_{7s}} = \left[1 + \frac{\gamma-1}{2}M_7^2\right]$$

$$T_{7s} = \frac{T_{07}}{1 + \frac{\gamma-1}{2}M_7^2} = \frac{1299.8\ K}{1 + 0.2\,(2.6368)^2}$$

$$\boxed{T_{7s} = 543.37\ K}$$

using η_n

$$T_7 = T_{05} - \eta_n(T_{05} - T_{7s})$$

$$T_7 = (1299.8\ K) - 0.98\,(1299.8\ K - 543.73\ K)$$

$$\boxed{T_7 = 558.44\ K}$$

thus,

$$u_{e,1} = u_7 = M_7\sqrt{\gamma R T_7} = 2.6368\sqrt{(1.4)\left(287\ \frac{J}{kg\cdot K}\right)(558.44\ K)}$$

$$u_{e,1} \cong 1247.2\ m/s$$

$$\boxed{u_{e,1} = 1247.2\ m/s}$$

ST (specific thrust)

$$ST_1 = \frac{T}{\dot{m}_a} = (1+f)\,u_{e,1} - u$$

$$ST_1 = (1+f)u_{e,1} - M_a\sqrt{\gamma R T_a}$$

$$= (1+0.02590)(1247.2\ \text{m/s}) - \underbrace{0.85\sqrt{(1.4)\left(\frac{287\,\text{J}}{\text{kg K}}\right)(216.65\,\text{K})}}_{u\,=\,250.79\ \text{m/s}}$$

$$\approx 1028.7\ \text{m/s}$$

SFC (specific fuel consumption)

$$SFC_1 = \frac{\dot{m}_f}{T} = \frac{f}{(1+f)u_e - u} = \frac{0.02590}{1.025991 \times 1249.5 - 250.79} = 2.5177 \times 10^{-5}\ \text{s/m}$$

$$\boxed{\begin{array}{l} ST_1 = 1028.7\ \text{m/s} \\ SFC_1 = 2.5177 \times 10^{-5}\ \text{s/m} \end{array}}$$

Case (2)

if $M_e = 1.0$

$$P_7 = \frac{P_{07}}{\left[1 + \frac{\gamma-1}{2}M_e^2\right]^{\frac{\gamma}{\gamma-1}}} = \frac{0.39757\ \text{MPa}}{\left[1 + 0.2(1.0)^2\right]^{1.4/0.4}}$$

$$\boxed{P_7 = 0.21003\ \text{MPa}}$$

$$T_{7s} = \frac{T_{07}}{1 + \frac{\gamma-1}{2}M_e^2} = \boxed{1081.6\ \text{K}}$$

using η_n

$$T_7 = T_{05} - \eta_n(T_{05} - T_{7s})$$

$$T_7 = (1299.8\,\text{K}) - 0.98(1299.8\,\text{K} - 1080.6\,\text{K})$$

$$\boxed{T_7 = 1084.96\ \text{K}}$$

thus

$$u_{e,2} = M_e\sqrt{\gamma R T_7}$$

$$u_{e,2} = (1.0)\sqrt{(1.4)\left(\frac{287\,\text{J}}{\text{kg K}}\right)(1084.96\,\text{K})}$$

$$u_{e,2} = 660.26\ \text{m/s}$$

$$\boxed{u_{e,2} = 660.26\ \text{m/s}}$$

now

$$ST_2 = (1+f)u_{e,2} - u$$

$$= (1 + 0.025991)(660.26\ \text{m/s}) - 250.79\ \text{m/s}$$

$$= 426.56\ \text{m/s}$$

and

$$SFC_2 = \frac{f}{(1+f)u_{e,2} - u} = \frac{0.025991}{1.025991 \times 660.24 - 250.79} = 6.072 \times 10^{-5}\ \mathrm{s/m}$$

$$\boxed{\begin{aligned} ST_2 &= 426.56\ \mathrm{m/s} \\ SFC_2 &= 6.072 \times 10^{-5}\ \mathrm{s/m} \end{aligned}}$$

(e) Case (1)

$$\dot{m}_f = F\,(SFC_1)$$
$$= \left(80 \times 10^3\ \frac{\mathrm{kg \cdot m}}{\mathrm{s^2}}\right)\left(2.5171 \times 10^{-5}\ \frac{\mathrm{s}}{\mathrm{m}}\right)$$
$$= 2.0141\ \mathrm{kg/s}$$

$$\boxed{\dot{m}_{f,1} = 2.0141\ \mathrm{kg/s}}$$

Case (2)

$$\dot{m}_f = F\,(SFC_2)$$
$$= 4.8576\ \mathrm{kg/s}$$

$$\boxed{\dot{m}_{f,2} = 4.8576\ \mathrm{kg/s}}$$

(f) Case (1)

$$\eta_{0,1} = \eta_{T,1} \cdot \eta_{P,1}$$
$$= \frac{0.5[(1+f)u_{e,1}^2 - u_a^2]}{fQ} \cdot \frac{T u_a}{0.5\,\dot{m}_a[(1+f)u_{e,1}^2 - u_a^2]}$$
$$= \frac{T u_a}{fQ\,\dot{m}_a} = ST_1 \cdot \frac{u_a}{fQ}$$
$$= \frac{\left(\frac{1031.2\ \mathrm{m}}{\mathrm{s}}\right)\left(\frac{250.79\ \mathrm{m}}{\mathrm{s}}\right)}{(0.025991)\left(45000 \times 10^3\ \frac{\mathrm{J}}{\mathrm{kg}}\right)} = 0.2214$$

Case (2)

$$\eta_{o,2} = \eta_{T,2} \cdot \eta_{P,2}$$

$$= \xi T_2 \cdot \frac{v_a}{f Q}$$

$$= \frac{\left(\frac{427.42\,m}{s}\right)\left(\frac{250.79\,m}{s}\right)}{(0.025991)\left(45000 \times 10^3 \frac{J}{kg}\right)} = 0.0918$$

$$\boxed{\begin{aligned} \eta_{o,1} &= 0.2214 \\ \eta_{o,2} &= 0.0918 \end{aligned}}$$

2. Repeat the thermodynamic cycle analysis a – d above for the actual turbofan version of the Trent-900. Per the engine specifications, the bypass ratio $\beta = 8.5$. Let the fan pressure ratio $p_{08}/p_{02} = 1.5$. The pressure ratio p_{03}/p_{02} is still equal to 42, but since the fan is upstream of the compressor it does some work on the air before it enters the compressor, ie, the stagnation pressure of the air entering the compressor is $1.5p_{02}$. Use the same values as in Problem 1 for the component efficiencies for the main engine. Let the fan have the same adiabatic efficiency as the compressor, and let the fan nozzle have an efficiency of 0.98. Like in (1.d-1) above, assume both core flow and fan flow are expanded to the ambient pressure, and calculate specific thrust.* Compare specific thrust, specific fuel consumption, and overall efficiency to the results from the turbojet analysis. Estimate the cruise (steady flight) range of the A-380 – you will have to use external sources to find some of the necessary information to make this calculation.

$$M_a = 0.85, \quad \frac{P_{03}}{P_{02c}} = 42, \quad T_{04} = 1800\,K, \quad Q = 45{,}000\ kJ/kg$$

$$\beta = 8.5, \quad \frac{P_{08}}{P_{02}} = 1.5, \quad P_{02c} = 1.5P_{02}$$

$$w_t = -w_c$$

$$\eta_d = \frac{h_{02s} - h_a}{h_{02} - h_a} = \frac{T_{02s} - T_a}{T_{02} - T_a} = 0.97, \quad \eta_c = \frac{h_{03s} - h_{02}}{h_{03} - h_{02}} = \frac{T_{03s} - T_{02}}{T_{03} - T_{02}} = 0.85$$

$$\eta_t = \frac{h_{04} - h_{05}}{h_{04} - h_{05s}} = \frac{T_{04} - T_{05}}{T_{04} - T_{05s}} = 0.90, \quad \eta_n = \frac{h_{06} - h_7}{h_{06} - h_{7s}} = \frac{T_{06} - T_7}{T_{06} - T_{7s}} = 0.98$$

$$\frac{P_{04}}{P_{03}} = 0.98 \qquad \dot{m}_a = 1.0\ kg/s$$

(a)

(i) @ freestream.

same as p1 the properties are

$$T_a = 389.97\ R = 216.65\ K$$

$$P_a = 3.9312 \times 10^2\ lb/ft^2 = 18823\ Pa$$

then

$$P_{0a} = P_a \left[1 + \frac{\gamma-1}{2} M_a^2\right]^{\frac{\gamma}{\gamma-1}}$$

$$P_{0a} = (18823\,Pa)\left[1 + \frac{1.4-1}{2}(0.85)^2\right]^{\frac{1.4}{1.4-1}} \simeq 30189\ Pa$$

$$T_{0a} = T_a \left[1 + \frac{\gamma-1}{2} M_a^2\right]$$

$$T_{0a} \simeq 247.96\ K$$

$$\boxed{\begin{aligned} P_{0a} &= 30189\ Pa \\ T_{0a} &= 247.96\ K \end{aligned}}$$

<ii> compressor inlet (2)

the stagnation temperature and pressure does not change from when it entered the engine, thus

$$P_{02} = 30189\ Pa$$

$$T_{02s} = 247.96\,K$$

now, from η_d, we know that

$$T_{02} = T_a + \frac{T_{02s} - T_a}{\eta_d}$$

$$T_{02} = (216.65K) + \frac{(247.96K)-(216.65K)}{0.97} \simeq 248.93K$$

$$\boxed{\begin{aligned} P_{02} &= 30189\ Pa \\ T_{02} &= 248.93\,K \end{aligned}}$$

the fan changes the result from P1 with some work

$$P_{02c} = 1.5\,P_{02} = \boxed{45283.5\ Pa}$$

then,

$$\frac{T_{02cs}}{T_{02}} = \left(\frac{P_{02c}}{P_{02}}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_{02cs} = 279.5 \text{ K}$$

fan has adiabatic efficiency of $\eta_c = 0.85$

$$T_{02c} = T_{02} + \frac{T_{02cs} - T_{02}}{\eta_c}$$

$$T_{02c} = 248.93\text{K} + \frac{279.5 - 248.93\text{ K}}{0.85}$$

$$\boxed{T_{02c} = 284.90\text{K}}$$

<iii> Combustion inlet (3)

using $\frac{P_{03}}{P_{02}} = 42 \iff P_{03} = 42 P_{02c} = 42(45283.5\text{Pa}) = 1.90191 \text{ MPa}$

then from isentropic relations

$$\frac{T_{03s}}{T_{02c}} = \left(\frac{P_{03}}{P_{02}}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_{03s} = T_{02c}\left(\frac{P_{03}}{P_{02}}\right)^{\frac{\gamma-1}{\gamma}} = (284.90\text{K})(42)^{\frac{0.4}{1.4}} = 828.85 \text{ K}$$

using η_c

$$T_{03} = T_{02c} + \frac{T_{03s} - T_{02}}{\eta_c} = 284.9 \text{ K} + \frac{(828.85\text{K}) - (284.9\text{ K})}{0.85}$$

$$T_{03} = 924.85 \text{ K}$$

$$\boxed{\begin{aligned} P_{03} &= 1.90191 \text{ MPa} \\ T_{03} &= 924.85 \text{ K} \end{aligned}}$$

(b) @ combustor exit/turbine inlet

$$\frac{P_{04}}{P_{03}} = 0.98 \iff P_{04} = (0.98)(1.90191\ MPa) = 1.86387\ MPa$$

then using the Burner Energy Equation
(given $T_{04} = 1800K$)

$$(1+f)C_p T_{04} = C_p T_{03} + f\eta_b Q$$

$$(\eta_b Q - C_p T_{04})f = C_p(T_{04} - T_{03})$$

$$f = \frac{C_p(T_{04} - T_{03})}{\eta_b Q - C_p T_{04}}$$

$$f = \frac{\left(\frac{1.10\ kJ}{kg\text{-}K}\right)(1800K - 924.85K)}{0.98\left(\frac{45000\ kJ}{kg}\right) - \left(\frac{1.10\ kJ}{kg\text{-}K}\right)(1800K)} = 0.02286$$

$$\boxed{\begin{array}{l} P_{04} = 1.8639\ MPa \\ f = 0.02286 \end{array}}$$

(c) @ the turbine exit (5)
for a turbine from $W_t = -W_c$
since

$$\Delta U = \cancel{\dot{Q}}^{\,0} - \dot{W}$$

$$W_t = C_p(T_{05s} - T_{04})$$

$$-W_c = -C_p(T_{03} - T_{02})$$

thus $\quad T_{05s} - T_{04} = T_{02} - T_{03}$

$$T_{05s} = (T_{02} - T_{03}) + T_{04} \cong 1160.05\ K$$

now using η_t

$$T_{05} = T_{04} - \eta_t(T_{04} - T_{05s})$$

$$= 1800K - (0.90)(1800K - 1160.5K)$$

$\simeq 1224.045\ K$

from isentropic relations

$$\frac{P_{05}}{P_{04}} = \left(\frac{T_{05}}{T_{04}}\right)^{\frac{\gamma}{\gamma-1}}$$

$$P_{05} = P_{04}\left(\frac{T_{05}}{T_{04}}\right)^{\frac{\gamma}{\gamma-1}}$$

$$P_{05} = (1.86387\,MPa)\left(\frac{1224.04\,K}{1600\,K}\right)^{\frac{1.4}{0.4}} = 0.4833\ MPa$$

$$\boxed{\begin{aligned} P_{05} &= 0.4833\ MPa \\ T_{05} &= 1224.04\ K \end{aligned}}$$

d) Case (1)

$P_7 = P_a = 18823\ Pa$

and stagnation pressure and temperature are constant from the turbine exit.

$P_{07} = 0.4833\ MPa \quad , \quad T_{07} = 1224.045\ K$

now

$$\frac{P_{07}}{P_7} = \left[1 + \frac{\gamma-1}{2}M_7^2\right]^{\frac{\gamma}{\gamma-1}}$$

$$M_7 = \sqrt{\frac{\left(\frac{P_{07}}{P_7}\right)^{\frac{\gamma-1}{\gamma}} - 1}{\frac{\gamma-1}{2}}} = \boxed{2.764}$$

next $\quad \dfrac{T_{07}}{T_{7s}} = \left[1 + \dfrac{\gamma-1}{2}M_7^2\right]$

$$T_{7s} = \frac{T_{07}}{1 + \frac{\gamma-1}{2} M_7^2} = \frac{1224.04 K}{1 + 0.2 (2.764)^2}$$

$$\boxed{T_{7s} = 484.21 K}$$

using η_n

$$T_7 = T_{05} - \eta_n (T_{05} - T_{7s})$$
$$T_7 = (1224.05 K) - 0.98 (1224.05 K - 484.21 K)$$

$$\boxed{T_7 = 499.004 K}$$

thus,

$$u_{e,1} = u_7 = M_7 \sqrt{\gamma R T_7} = 2.764 \sqrt{(1.4)(287 \frac{J}{kg\text{-}K})(499.00 K)}$$
$$u_{e,1} \cong 1237.64 m/s$$

$$\boxed{u_{e,1} = 1237.64 \, m/s}$$

Fan Flow

$$P_{02} = 30189 Pa \qquad T_{02C} = 247.96 K \quad (\text{post fan})$$

now from assumption

$$P_{08} = P_{02C} = 45283.5 Pa$$
$$T_{08} = T_{02C} = 284.90 K$$

and

$$P_9 = P_a = 30189 Pa$$
$$T_9 = T_{08} \times \left(\frac{P_9}{P_{08}}\right)^{\frac{\gamma-1}{\gamma}} = 253.74 K$$

Fan exit velocity

$$T_{0q} = T_{0r} = 253.94\,K$$

$$\frac{T_{0q}}{T_q} = \left[1 + \frac{\gamma-1}{2}M_q^2\right] = \frac{287.90}{253.74}$$

$$\Rightarrow \quad M_q = 0.7836$$

$$u_q = M_q\sqrt{\gamma R T_q}$$

$$= 0.7836\sqrt{1.4 \times 287 \times 253.74}$$

$$= 250.204 \text{ m/s}$$

$$\boxed{u_f = u_q = 250.20 \text{ m/s}}$$

$$ST_1 = \frac{T}{\dot{m}_a} = (1+f)u_e + \beta u_f - (1+\beta)u_a$$

$$= \boxed{1010.16 \text{ m/s}}$$

$\because u_a = 0.85\sqrt{\gamma R T_a}$
$= 250.79$

$$TSFC_1 = \frac{f}{ST_1}$$

$$= \boxed{2.2630 \times 10^{-5} \text{ s/m}}$$

$$\eta_{o1} = (1+\beta)\frac{(\bar{u}_e - u)u}{f Q_R}$$

$$\because \quad \bar{u}_e = \frac{\dot{m}_a u_e + \dot{m}_b u_c}{\dot{m}_a + \dot{m}_b} = \frac{u_e + \beta u_f}{1+\beta} = 354.15 \text{ m/s}$$

$$= 9.5\,\frac{(354.15 - 250.78)(250.78)}{0.02286 \times 45000 \times 10^3}$$

$$= \boxed{0.2394}$$

Case (2)

if $M_e = 1.0$

$$T_{7s} = \frac{T_{07}}{1 + \frac{\gamma-1}{2}M_e^2} = \boxed{1020.04\,K}$$

using η_n

$$T_7 = T_{05} - \eta_n (T_{05} - T_{7s})$$

$$T_7 = (1224.0\,K) - 0.98(1224.0\,K - 1021.04\,K)$$

$$\boxed{T_7 = 1024.12\,K}$$

thus

$$u_{e,2} = M_e\sqrt{\gamma R T_7}$$

$$u_{e,2} = (1.0)\sqrt{(1.4)\left(\frac{287\,J}{kg\,K}\right)(1024.1\,K)}$$

$$u_{e,2} = 641.475 \text{ m/s}$$

$$\boxed{u_{e,2} = 641.48 \text{ m/s}}$$

$$FT_2 = \frac{T}{\dot{m}_a} = (1+f)u_e + \beta u_f - (1+\beta)u_a$$

$$= \boxed{400.37 \text{ m/s}}$$

$$TSFC_2 = \frac{f}{FT_2}$$

$$= \boxed{5.7097 \times 10^{-5}\ s/m}$$

$$\eta_{o2} = (1+\beta)\frac{(\bar{u}_e - u)u}{f Q_R}$$

$$\because\ \bar{u}_e = \frac{\dot{m}_a u_e + \dot{m}_b u_f}{\dot{m}_a + \dot{m}_b} = \frac{u_e + \beta u_f}{1+\beta} = 241.390\ m/s$$

$$\therefore\ \boxed{\eta_{o2} = 0.0940}$$

Comparison

- Case1 $P_7 = P_a$

	Turbojet	Turbofan
FT [m/s]	1028.73	1010.26
TSFC [s/m]	2.518×10^{-5}	2.263×10^{-5}
η_o	0.2214	0.2394

- Case2

$M_e = 1$

	Turbojet	Turbofan
FT [m/s]	426.57	400.37
TSFC [s/m]	6.072×10^{-5}	5.7097×10^{-5}
η_o	0.0918	0.0940

The specific thrust of the turbofan is lower than the turbojet which is in actuality a strange result.

But the TSFC has a slightly better performance.

verall efficiency is slightly higher but is almost the same.

Range A380

from online and this HW

$\eta_0 = 0.2394$ $Q_R = 45000$ kJ/kg $g = 9.81$ m/s²

$m_2 \cong$ OEW + Max payload $= 277t + 84t = 361000$ kg

$m_1 =$ MTOW $= 575$

$\frac{L}{D} = 19 \sim 20$

$$R = \eta_0 \frac{L}{D} \frac{Q_R}{g} \ln\left(\frac{m_1}{m_2}\right)$$

$$= (0.2394)(20)\frac{\left(45000\ \frac{kJ}{kg}\right)}{\left(9.81\ \frac{m}{s^2}\right)} \ln\left(\frac{575000\ kg}{361000\ kg}\right)$$

$$= 10223.7\ km$$

$$\boxed{R = 10223\ km}$$

might be different