AAE 339: Aerospace Propulsion

HW9: Space Mission Planning

Dr. Anderson

School of Aeronautical and Astronautical

Purdue University

Tomoki Koike

Friday April 17th, 2020

Problem 1

To launch a spacecraft into geosynchronous orbit, two-stage rockets use a geostationary transfer orbit (GTO). The rocket's upper stage releases the spacecraft at the apogee of the elliptical orbit, which is tangent to the circular GEO. At that point a propulsive maneuver provided by an apogee system or on-board propulsion slows the spacecraft for transfer to GEO. A common GTO has a perigee of h_p = 185 km and apogee h_a = 35,786 km. Hence, to achieve the GTO, the rocket must achieve a velocity equal to the orbital velocity at perigee, v_p. Consider a two-stage rocket that is used to deliver a 4500 kg spacecraft (m_{PL} = 4500 kg) into the elliptical transfer orbit. It is launched from Cape Canaveral (latitude = 27° N). Assume the first stage has an average $I_{sp,1}$ = 310 s, and a propellant mass fraction λ_1 = 0.94. The second stage has $I_{sp,2}$ = 450 s, and λ_2 = 0.92. Thrust losses due to gravity, drag, and required steering maneuvers add up to 20% of the ideal mission velocity increment, ie $\Delta V_{req} = 1.2\ \Delta V_{ideal}$.

a) Calculate the semi-major axis, a, the velocity of the rocket at perigee, v_p, and the velocity increment ΔV_{req} required to insert the rocket into the geostationary orbit.

Semi-major axis

$$a = \frac{r_p + r_a}{2} = \frac{(R_\oplus + h_p) + (R_\oplus + h_a)}{2}$$

$$a = \frac{(6371 + 185) + (6371 + 35786)\ \text{km}}{2}$$

$$\boxed{a = 24356.5\ \text{km}}$$

velocity at perigee, v_p

$$v_p = \sqrt{\mu\left(\frac{2}{r_p} - \frac{1}{a}\right)}$$

$$\because \mu = GM_\oplus \qquad G: 6.67\times10^{-11}\ [m^3 kg^{-1} s^{-2}]$$

$$M_\oplus : 5.972\times10^{24}\ [kg]$$

$$\boxed{v_p = 10258\ m/s}$$

ΔV required

from Earth's rotation

$$V_i = \frac{2\pi R_\oplus}{T_{rot}} \cos\theta$$

$\because$ T_{rot} : 23 hrs 56 mins 4 sec, latitude : $\theta = 27°$

$$V_i = 414.318 \text{ m/s}$$

then
$$\Delta V_{ideal} = V_p - V_i$$
$$= 9843.7 \text{ m/s}$$

$$\Delta V_{req} = 1.2\,\Delta V_{ideal}$$

$$\boxed{\Delta V_{req} = 11812.44 \text{ m/s}}$$

b) The rocket is designed so that the first stage provides 30% of the required ΔV and the second stage provides the rest. Calculate the inert mass and the propellant mass of both stages. The masses of the interstage structure and the payload fairing (m = 1000 kg each) are not included in λ for either stage. Include them in your calculations and assume they are both jettisoned between the end of the first stage burn and the start of the second stage burn.
Like the example, start the calculations with the second stage.

Second Stage

from the ideal rocket equation

$$\Delta V_2 = g_0 I_{sp_2} \ln \frac{m_{02}}{m_{f2}} = \Delta V_{req} \times 0.7$$

$$\because \; g_0 = 9.81 \text{ m/s}^2, \quad I_{sp2} = 450 \text{ s}$$

$$\frac{m_{02}}{m_{f2}} = \frac{m_{in2} + m_{p2} + m_{PL}}{m_{in2} + m_{PL}} = \exp\left(\frac{\Delta V_2}{g_0 I_{sp2}}\right)$$

$$\because \; m_{PL} = 4500 \text{ kg}$$
$$m_2 = m_{in2} + m_{02}$$
$$m_{p2} = \lambda_2 m_2$$
$$m_{in2} = (1-\lambda_2) m_2 = \frac{1-\lambda_2}{\lambda_2} m_{p2}$$

$$\exp\left(\frac{\Delta V_2}{g_0 I_{sp2}}\right) = \frac{\frac{1-\lambda_2}{\lambda_2} m_{p2} + m_{p2} + m_{PL}}{\frac{1-\lambda_2}{\lambda_2} m_{p2} + m_{PL}}$$

Compute using MATLAB (code in Appendix)

$$\boxed{\begin{aligned} m_{p2} &= 47375 \text{ kg} \\ m_{in2} &= \frac{1-\lambda_2}{\lambda_2} m_{p2} = 4137 \text{ kg} \end{aligned}}$$

First Stage

Similar to second stage

$$\Delta V_1 = 0.3\,\Delta V_{req} = g_0 I_{sp1} \ln \frac{m_{01}}{m_{f1}}$$

$$\frac{m_{01}}{m_{f1}} = \frac{m_{in1} + m_{p1} + m_{12} + m_2 + m_{PL}}{m_{in1} + m_{12} + m_2 + m_{PL}} = \exp\left(\frac{\Delta V_1}{g_0 I_{sp1}}\right)$$

$$\because \quad m_2 = m_{p2} + m_{in2}$$

$$m_{12} = 2000 \text{ kg}$$

$$I_{sp1} = 310 \text{ s}$$

$$m_1 = m_{p1} + m_{in1}$$

$$m_{p1} = \lambda_1 m_1$$

$$m_{in1} = (1-\lambda_1) m_1 = \frac{1-\lambda_1}{\lambda_1} m_{p1}$$

$$\therefore \quad \exp\left(\frac{\Delta V_1}{g_0 I_{sp1}}\right) = \frac{\frac{1-\lambda_1}{\lambda_1} m_{p1} + m_{p1} + m_{12} + m_2 + m_{PL}}{\frac{1-\lambda_1}{\lambda_1} m_{p1} + m_{12} + m_2 + m_{PL}}$$

Compute using MATLAB (code in Appendix)

$$\boxed{\begin{aligned} m_{p1} &= 149525.7 \text{ kg} \\ m_{in1} &= \frac{1-\lambda_1}{\lambda_1} m_{p1} = 9544.2 \text{ kg} \end{aligned}}$$

c) Calculate the gross lift-off weight (GLOW) of the rocket. Assume $F/W = 1.3$ to calculate thrust. For convenience, use the average I_{sp} of the first stage to calculate the propellant flowrate of the first stage engine.

The total mass

$$m_{tot} = m_1 + m_2 + m_{12} + m_{PL}$$

$$= 149525.7 + 9544.2 + 47575 + 4137 + 2000 + 4500$$

$$= 217282 \text{ kg}$$

$$\boxed{GLOW = m_{tot} g_0 = 2131535.7 \text{ N}}$$

$$\boxed{\text{Thrust} = F_t = GLOW \times 1.3 = 2770996.4 \text{ N}}$$

$\dot{m}_p$: propellant flow rate

$$g_0 I_{sp1} = \frac{F_t}{\dot{m}_1} \Rightarrow \boxed{\dot{m} = \frac{F_t}{g_0 I_{sp1}} = 911.1823 \text{ kg/s}}$$

Problem 2

Read Sections 1 and 2 of the ULA Users Guide https://www.ulalaunch.com/docs/default-source/rockets/atlasvusersguide2010.pdf. Determine the minimal Atlas V 400 Series rocket configuration that can meet the requirements of the mission described in Problem 1.

a) Using the Atlas V data for the minimal configuration, calculate the propellant mass fractions of each stage, and the ideal ΔV (no losses) that is provided by each stage. For the first stage engine, estimate the average specific impulse by using the rule of thumb $I_{sp,av} = 1/3\ I_{sp,SL} + 2/3\ I_{sp,vac}$.

b) Calculate the GLOW of this Atlas V rocket (including payload) and its thrust-to-weight at liftoff.

mission requirements

m_{PL} = 4500 kg , GTO Apogee Height = 35786 km, lattitude = θ = 27°

Table 2.6-1: Atlas V 400/500 Series and HLV Performance Capabilities Summary

Orbit Type (ΔV to GSO)	400 Series				500 Series						HLV
	Number of Solid Rocket Boosters										
	0	1	2	3	0	1	2	3	4	5	N/A
	Payload Systems Weight (PSW), kg (lb)										
GTO (1804 m/s)	4,750 (10,470)	5,950 (13,110)	6,890 (15,180)	7,700 (16,970)	3,775 (8,320)	5,250 (11,570)	6,475 (14,270)	7,475 (16,470)	8,290 (18,270)	8,900 (19,620)	13,000 (28,660)
GTO (1500 m/s)	3,460 (7,620)	4,450 (9,810)	5,210 (11,480)	5,860 (12,910)	2,690 (5,930)	3,900 (8,590)	4,880 (10,750)	5,690 (12,540)	6,280 (13,840)	6,860 (15,120)	--
GSO	--	--	--	--	--	--	2,632 (5,802)	3,192 (7,037)	3,630 (8,003)	3,904 (8,608)	6,454 (14,229)
LEO I =28.5 deg	9,797* (21,598)	12,150* (26,787)	14,067* (31,012)	15,718* (34,653)	8,123 (17,908)	10,986 (24,221)	13,490 (29,741)	15,575 (34,337)	17,443 (38,456)	18,814 (41,478)	29,400* (64,816)*
LEO Sun-sync	7,724 (17,028)	8,905 (19,633)	10,290 * (22,687)	11,704 * (25,803)	6,424 (14,163)	8,719 (19,223)	10,758 (23,717)	12,473 (27,498)	14,019 (30,908)	15,179 (33,464)	--

Atlas V 400 Series	Atlas V 500 Series and HLV
• All Performance is SEC • Quoted Performance is with 4-m EPF	• All Performance is SEC • Quoted Performance is with 5-m Short PLF • HLV LEO Performance is DEC • HLV Quoted Performance is with 5-m Long PLF

* For 400 series, PSW above 9,072 kg (20,000 lb) may require mission-unique accommodations. For 500 series and HLV, PSW above 19,051 kg (42,000 lb) may require mission-unique accommodations.

Notes:

GTO (1804 m/s): ≥185 x 35,786 km (≥ 100 x 19,323 nmi), Inclination = 27.0 deg, Argument of Perigee = 180 deg, CCAFS

GTO (1500 m/s): Apogee Height = 35,786 km (19,323 nmi), Argument of Perigee = 180 deg, CCAFS

GSO: 35,786 km Circular (19,323 nmi Circular), Inclination = 0 deg, CCAFS

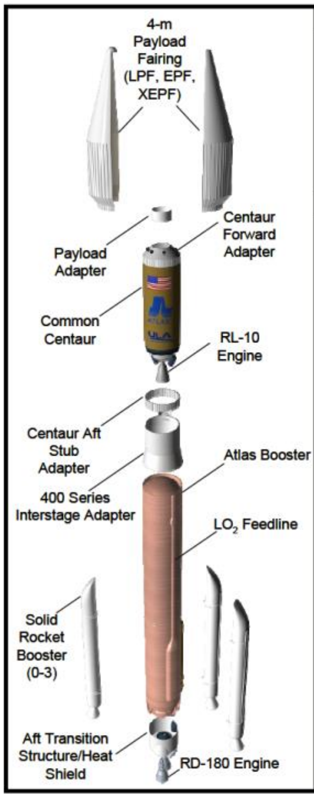
LEO 28.5 deg: 200 km (108 nmi) Circular, CCAFS

LEO Sun-sync: 200 km (108 nmi) Circular, VAFB

GCS: Guidance Commanded Shutdown, 2.33 sigma for CCAFS, and for VAFB

The minimal Atlas V 400 Series Rocket design is $\boxed{\text{Atlas V 401}}$

Figure 1.4.1-3: Atlas V 400 Series Launch System



PAYLOAD FAIRING (PLF)

Feature	LPF	EPF	XEPF
Diameter:	4.2-m (165.4-in)	4.2-m (165.4-in)	4.2-m (165.4-in)
Length:	12.0-m (472.4-in)	12.9-m (507.9-in)	13.8-m (543.3-in)
Mass:	2,127 kg (4,689 lbs)	2,305 kg (5,082 lbs)	2,487 kg (5,483 lbs)

Subsystems
Fairing: Aluminum Skin Stringer & Frame Clamshell
Boattail: Aluminum Skin Stringer & Frame Clamshell
Separation: Pyro Bolts & Spring Jettison Actuators

COMMON CENTAUR

Features
Size: 3.05-m (120-in) Dia. x 12.68-m (499-in) Length with Extended Nozzle
Inert Mass: 4X1: 2,243 kg (4,945 lbs)
4X2: 2,458 kg (5,419 lbs)
Propellant: 20,830-kg (45,922 lbs) LH2 & LO2
Guidance: Inertial

Subsystems
Structure: Pressure Stabilized Stainless Steel Tanks Separated by Common Ellipsoidal Bulkhead
Propulsion: One or Two Pratt & Whitney Restartable Engine(s)
— Model: RL 10A-4-2
— Thrust: 99.2 kN (22,300 lbf) (SEC)
198.4 kN (44,600 lbf) (DEC)
— ISP: 450.5 s
(SEC) One Electromechanically Actuated 51-cm Columbium Fixed Nozzle
Four 27-N Hydrazine Thrusters
Eight 40.5-N Lateral Hydrazine Thrusters
(DEC) Two Hydraulically Actuated 51-cm Columbium Extendible Nozzles
Eight 40-N Lateral Hydrazine Thrusters
Four 27-N Hydrazine Thrusters
Pneumatics: Common with Atlas V 400/500 Series
Avionics: Common with Atlas V 400/500 Series
Insulation: Foam with Optional Radiation Shields

SOLID ROCKET BOOSTERS (SRB)

Zero-to-Three Ground-Lit
Size: 158-cm (62.2-in) Dia. x 20-m (787-in) Length
Mass: 46,697 kg (102,949 lbs) (Including SRB Attach Kit, SRB Nose Fairing & Instrumentation)
Thrust: 1688.4 kN (379,550 lbf) (Each)
Vac ISP: 279.3 s
Nozzle Cant: 3°

AFT STUB ADAPTER (ASA)

Features
Size: 3.05-m (120-in) Dia. x 0.65-m (25.6-in) Length
Mass: 181.7 kg (400.6 lbs)

Subsystems
Structure: Aluminum Monocoque
Separation: Frangible Joint Assembly (FJA) Separation System

400 SERIES INTERSTAGE ADAPTER (400-ISA)

Features
Size: 3.05-m (120-in) Dia. x 2.52-m (99.2-in) Length (Cylinder Section)
3.81-m (150-in) Dia. x 1.61-m (63.4-in) Length (Conic Section)
Mass: 4X1: 947 kg (2,087.8 lbs)
4X2: 962 kg (2,120.8 lbs)
Structure: Composite (Graphite Epoxy) with Aluminum Ring Frames Fwd & Aft

ATLAS BOOSTER

Features
Size: 3.81-m (150-in) Dia x 32.46-m (1278-in) Length
Propellant: 284,089-kg (626,309 lbs) LO2 & RP-1
Inert Mass: 21,054 kg (46,416 lbs)
Guidance: From Upper Stage

Subsystems
Structure: Structurally Stable Aluminum Isogrid Tanks; Integrally Machined Aft Transition Structure Composite Heat Shield
Separation: 8 Retro Rockets
Propulsion: Pratt & Whitney/NPO Energomash RD-180 Booster Engine (2 Chambers)
SL 100% Thrust = 3,827 kN (860,309 lbf), ISP = 311.3 s
Vac 100% Thrust = 4,152 kN, (933,369 lbf) ISP = 337.8 s
Pneumatics: Helium for Tank Pressurization, Computer-Controlled Pressurization System
Hydraulics: Fluid —Integral with Engine Provides Gimbal Control
Avionics: Flight Control, Flight Termination, Telemetry, Redundant Rate Gyros, Electrical Power

AVUG11_F010401_03h

Masses

$m_{PL} = 4500 \text{ kg}$ (from problem 1)

$m_{p2} = 20830 \text{ kg}$

$m_{in2} = 2243 + 2458 = 4701 \text{ kg}$

$m_{12} = 181.7 + 947 + 962 = 2090.7 \text{ kg}$

$m_{p1} = 284089 \text{ kg}$

$m_{in1} = 21054 \text{ kg}$

Propellant Mass Fraction

$$\boxed{\begin{aligned} \lambda_1 &= \frac{m_{p1}}{m_{p1} + m_{in1}} = 0.9310 \\ \lambda_2 &= \frac{m_{p2}}{m_{p2} + m_{in2}} = 0.8159 \end{aligned}}$$

Isp

$$I_{sp2} = 450.5 \ s$$

$$I_{vac1} = 337.8 \ s$$

$$I_{SL1} = 311.3 \ s$$

$$I_{sp1} = I_{vac1} \times \frac{2}{3} + I_{SL1} \times \frac{1}{3} = 328.97 \ s$$

ΔV for each stage

$$\Delta V_2 = g_0 I_{sp2} \ln\left(\frac{m_{02}}{m_{f2}}\right)$$

$$= g_0 I_{sp2} \ln\left(\frac{m_{PL} + m_{p2} + m_{in2}}{m_{PL} + m_{in2}}\right)$$

$$= \boxed{5227.8 \ m/s}$$

$$\Delta V_1 = g_0 I_{sp1} \ln\left(\frac{m_{01}}{m_{f1}}\right)$$

$$= g_0 I_{sp1} \ln\left(\frac{m_{PL} + m_{p1} + m_{in1} + m_{12} + m_{in2} + m_{p2}}{m_{PL} + m_{in1} + m_{12} + m_{in2} + m_{p2}}\right)$$

$$= \boxed{5961.4 \ m/s}$$

$$GLOW = m_{tot} \, g_0$$

$$= (m_{PL} + m_{p1} + m_{in1} + m_{12} + m_{p2} + m_{in2}) \, g_0$$

$$= \boxed{3.3086 \ MN}$$

from table

Thrust at SL $\Rightarrow$ $F_t = 3827$ kN

$$\frac{F_t}{GLOW} = \frac{3827 \text{ kN}}{3.3058 \text{ MN}} = \boxed{1.1564}$$

Problem 3

The 4500 kg spacecraft in Problem 1 has its own integrated propulsion system (including propellant) that will be used to provide the necessary ΔV for transfer from the elliptical orbit at apogee to GEO. Evaluate two options for propulsion. Option 1 is a bipropellant thruster using nitrogen tetroxide and monomethyl hydrazine (NTO/MMH), with an I_{sp} = 340 s. Option 2 is an electrostatic ion thruster, with I_{sp} = 3000 s. Calculate the ΔV required for this maneuver, and the required mass of propellant for each option.

velocity at apogee

$$v_a = \sqrt{\mu\left(\frac{2}{r_a} - \frac{1}{a}\right)}$$

μ, r_a, a are same value in problem 1

$$v_a = 1595.3 \text{ m/s}$$

velocity in GEO

$$v_f = \sqrt{\frac{\mu}{r_a}} = 3074.8 \text{ m/s}$$

$$\boxed{\Delta V_{req} = v_f - v_a = 1479.6 \text{ m/s}}$$

Propellant mass

(i) NTO/MMH bi-propellant thruster

$$\Delta V = g_0 I_{sp1} \ln \frac{m_0}{m_0 - m_{p1}}$$

$$m_{p1} = m_0 - \frac{m_0}{\exp\left(\frac{\Delta V}{g_0 I_{sp1}}\right)}$$

$m_0 = 4500$ kg

$I_{sp1} = 340$ s

$$\boxed{m_{p1} = 1612.2 \text{ kg}}$$

(ii) Electric ion thruster

$$m_{p2} = m_0 - \frac{m_0}{\exp\left(\frac{\Delta v}{g_0 I_{sp2}}\right)} \qquad I_{sp2} = 4000 \text{ s}$$

$$\boxed{m_{p2} = 166.52 \text{ kg}}$$

Analysis

Option 2 is preferred over 1 because there is much less propellant used.

Appendix

```
clear all; close all; clc;
```

Problem 1

(a)

```
% Defining Constants
G           = 6.67408e-11; % m3 kg-1 s-2
m_e         = 5.972e24; % kg
h_p         = 185000;   % m
h_a         = 35786000; % m
R_e         = 6371000;  % m
m_pl        = 4500;     % kg
Isp_1       = 310;      % s
Isp_2       = 450;      % s
lambda_1    = 0.94;
lambda_2    = 0.92;
T_loss      = 0.2;
lattitude   = 27;       % deg

% Semi major axis
r_p = R_e + h_p
r_a = R_e + h_a
a   = (r_p+r_a)/2

% velocity at perigee
mu  = G*m_e
v_p = sqrt(mu*(2/r_p - 1/a))

% Delta V
v_i     = 465*cos(deg2rad(lattitude))
DVideal = v_p - v_i
DVreq   = (1 + T_loss)*DVideal
```

(b)

```
m_mid   = 2000;     % kg
g0      = 9.81;     % m s-2
DVreq1  = DVreq*0.3;
DVreq2  = DVreq*0.7;

syms m_p m_in
eqn1 = (m_in+m_p+m_pl)/(m_in+m_pl) == exp(DVreq2/g0/Isp_2);
eqn2 = (1-lambda_2)/lambda_2*m_p == m_in;
sol     = solve([eqn1,eqn2], [m_p,m_in]);
m_p_2   = double(sol.m_p);
m_in_2  = double(sol.m_in);
m_2     = m_p_2 + m_in_2;
```

```
syms m_p m_in
eqn1 = (m_in+m_p+m_2+m_mid+m_pl)/(m_in+m_2+m_mid+m_pl) == exp(DVreq1/g0/Isp_1)
eqn2 = (1-lambda_1)/lambda_1*m_p == m_in
sol     = solve([eqn1,eqn2], [m_p,m_in])
m_p_1   = double(sol.m_p)
m_in_1  = double(sol.m_in)
m_1     = m_p_1 + m_in_1
```

(c)

```
m_tot   = m_1 + m_2 + m_mid + m_pl
W       = m_tot*g0
F       = 1.3*W
m_dot   = F/(Isp_1*g0)
```

Problem 2

(a)

```
% Atlas V 401 masses [kg]
m_p_2   = 20830;
m_p_1   = 284089;
m_in_2  = 4701;
m_in_1  = 21054;
m_1     = m_p_1 + m_in_1;
m_2     = m_p_2 + m_in_2;
m_mid   = 2090.7;
m_pl    = 4500;

% Isp [s]
Isp_vac = 337.8;
Isp_SL  = 311.3;
Isp_1   = 1/3*Isp_SL + 2/3*Isp_vac;
Isp_2   = 450.5;

% Propellant Mass Fractions
lambda_1 = m_p_1/m_1
lambda_2 = m_p_2/m_2

% DeltaV_2
m0_2        = m_pl + m_2
mf_2        = m0_2 - m_p_2
deltaV_2    = g0*Isp_2*log(m0_2/mf_2)

% DeltaV_1
m0_1        = m_pl + m_2 + m_mid + m_1
mf_1        = m0_1 - m_p_1
deltaV_1    = g0*Isp_1*log(m0_1/mf_1)

% GLOW
m_tot = m0_1
W = m_tot*g0
```

```
F = 3.826e6
FtoW = F/W
```

Problem 3

```
v_a     = sqrt(mu*(2/r_a - 1/a))
v_f     = sqrt(mu/r_a)
deltaV  = v_f - v_a

% Propellant Mass
Isp_bi  = 340;
Isp_ion = 4000;
m_p_bi  = m_pl - m_pl/(exp(deltaV/g0/Isp_bi))
m_p_ion = m_pl - m_pl/(exp(deltaV/g0/Isp_ion))
```