Tomoki Koike | AAE339 | HW2 | 1

Q1. (problem #1 from text book p.56)

Given: A "perfect" gas

» $\bar{M} = 20$

» $\gamma = 1.2$

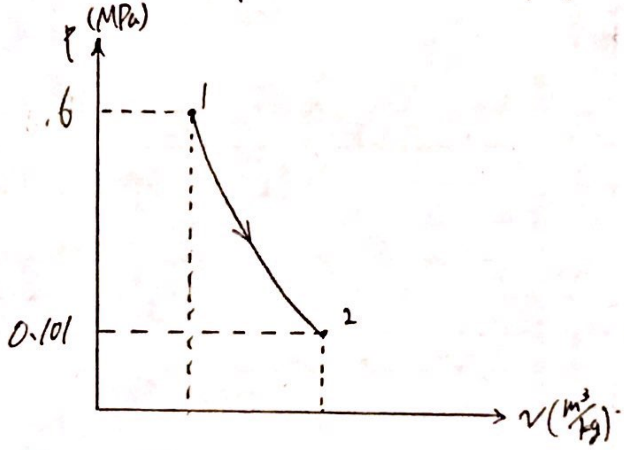
» states

(1) $P_1 = 6\,MPa$, $T_1 = 3000K$, velocity $= u_1 = 200\,m/s$

↓ expands adiabatically

(2) $P_2 = 0.101\,MPa$

(a) Find: if $T_2 = 1800K$ and v_2 is negligible
how much work per unit mass of gas w has been done?



$$R = \frac{\bar{R}}{\bar{M}} \left(\frac{8314.472\,J}{kmol \cdot K}\right) \cdot \left(\frac{kmol}{20\,kg}\right) = 415.7236\,\frac{J}{kg \cdot K}$$

- from energy conservation

$$-\left(C_p T_1 + \frac{u_1^2}{2}\right) + \left(C_p T_2 + \cancel{\frac{u_2^2}{2}}^{0}\right) = \cancel{q}^{0} - w$$

$$w = C_p T_1 + \frac{u_1^2}{2} - C_p T_2$$

$$= \frac{\gamma}{\gamma - 1} R (T_1 - T_2) + \frac{u_1^2}{2}$$

$$= \frac{1.2}{0.2}\left(\frac{415.7236\,J}{kg \cdot K}\right)(3000K - 1800K) + \frac{1}{2}\left(\frac{200\,m}{s}\right)^2$$

$$= \boxed{3013.21\,\frac{kJ}{kg}}$$

(b) Find: For adiabatic expansion is it possible for T_2 to equal 450K?

using equation

$$ds = c_p\frac{dT}{T} - R\frac{dp}{p}$$

$$\int_1^2 ds = c_p\int_1^2\frac{dT}{T} - R\int_1^2\frac{dp}{p}$$

$$\Delta S_{12} = \frac{\gamma}{\gamma-1}R\ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}$$

$$= \left(\frac{1.2}{1.2-1}\right)\left(\frac{415.7236\ J}{kg\cdot K}\right)\ln\left(\frac{450\ K}{3000\ K}\right) - \left(\frac{415.7236\ J}{kg\cdot K}\right)\ln\left(\frac{0.101\ MPa}{6\ MPa}\right)$$

$$= \boxed{-3034.09\ \frac{J}{kg\cdot K}}$$

from the assumption of adiabatic process

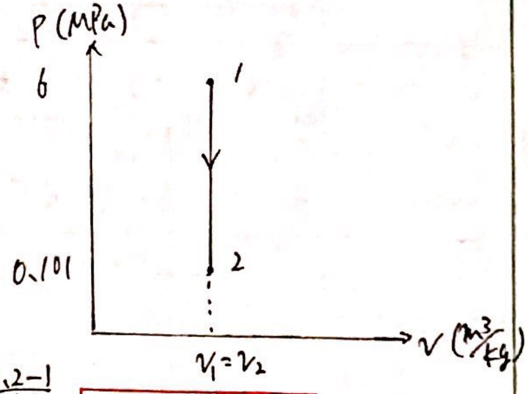
$$ds \geq \frac{dQ}{T} = 0$$

however, $\boxed{\Delta S_{12} < 0 \quad \text{thus } T_2 \text{ cannot be as low as } 450K}$

(c) if no work is done, what is $(u_2)_{max}$ and T_2?

no work & no heat transfer

$\Rightarrow$ isentropic = $\Delta S = 0$



$$0 = \int_{T_1}^{T_2} c_p\frac{dT}{T} - \int_{P_1}^{P_2} R\frac{dp}{p}$$

$$0 = c_p\ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}$$

$$\left(\frac{T_2}{T_1}\right)^{c_p} = \left(\frac{P_2}{P_1}\right)^{R} \iff T_2 = T_1\left(\frac{P_2}{P_1}\right)^{R/c_p}$$

$$\frac{R}{c_p} = R\cdot\frac{\gamma-1}{\gamma R} = \frac{\gamma-1}{\gamma} \qquad = (3000K)\left(\frac{0.101\ MPa}{6\ MPa}\right)^{\frac{1.2-1}{1.2}} = \boxed{1518.74K}$$

$$\rightarrow \quad h_0 = h + \frac{u^2}{2} = const$$

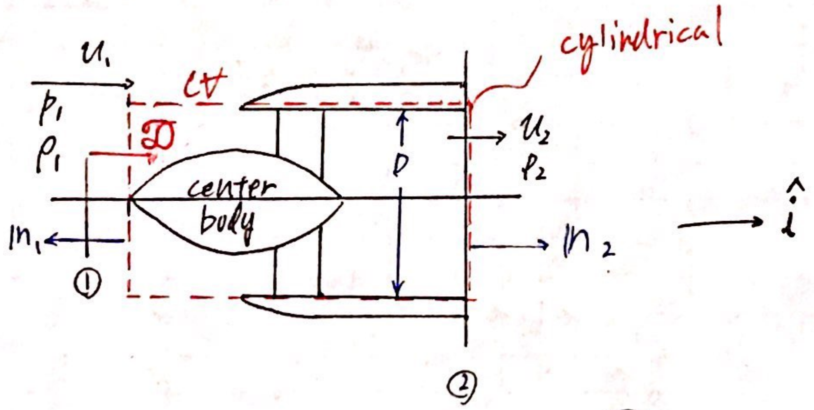
$$\iff \quad c_p\,\Delta T_{12} + \left(\frac{u_2^2}{2} - \frac{u_1^2}{2}\right) = 0 \qquad \text{since } c_p = \frac{dh}{dT}$$

$$\frac{\gamma R}{\gamma-1}(T_2 - T_1) + \frac{u_2^2}{2} - \frac{u_1^2}{2} = 0$$

$$\therefore (u_2)_{max} = \left[u_1^2 - \frac{2R\gamma}{\gamma-1}(T_2-T_1)\right]^{1/2} = \left[\left(\frac{200m}{s}\right)^2 - \left(\frac{2\cdot1.2}{1.2-1}\right)\left(\frac{415.7236J}{kg\cdot K}\right)(1518.74K - 3000K)\right]^{1/2} = \boxed{2725.72\ m/s}$$

Q2. (Problem #8 of text book p.58 ~ 59)

Given: idealized supersonic ramjet diffuser



Find: show that drag on center body is $\mathcal{D} = -\frac{\pi}{4}D^2[\rho_1 u_1(u_2-u_1)+p_2-p_1]$

assume steady, uniform, incompressible flow

then

$$\rho_1 = \rho_2 \quad \cdots ①$$

From conservation of momentum

$$\cancel{\frac{\partial}{\partial t}\int_{CV}\rho \mathbf{u}\, d\forall}^{\,0} + \int_{CS}\rho \mathbf{u}(\mathbf{u}\cdot\mathbf{n})dA = \Sigma \mathbf{F}$$

$$\dot{m}(u_2-u_1) = \rho_1 A u_1(u_2-u_1) = \Sigma \mathbf{F}$$

now surface is cylindrical

$$\because \quad A = \pi\left(\frac{D}{2}\right)^2 = \frac{\pi}{4}D^2 \quad \longrightarrow \quad \int_1 dA = \int_2 dA = \frac{\pi}{4}D^2$$

$$\therefore \quad \rho_1\left(\frac{\pi}{4}D^2\right)u_1(u_2-u_1) = -\mathcal{D} + p_1A - p_2A - p_a(A_{cv}-A) + p_a(A_{cv}-A)$$

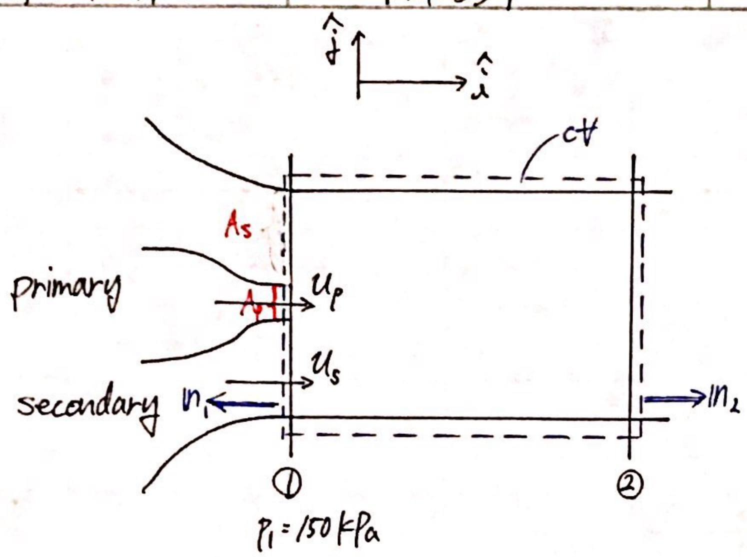
$\because$ ①

$$\frac{\pi}{4}D^2\rho_1 u_1(u_2-u_1) = -\mathcal{D} + (p_1-p_2)\left(\frac{\pi}{4}D^2\right)$$

$$\therefore \quad \boxed{\mathcal{D} = -\frac{\pi}{4}D^2[\rho_1 u_1(u_2-u_1)+p_2-p_1]}$$

Q3.

Given:



$u_p = 250\,m/s$
$T_p = 600\,K$
$u_s = 30\,m/s$
$T_s = 300\,K$
$A_s = 3A_p$

$P_p = P_s = P_1 = 150\,kPa$

$A_1 = A_2 = A_s + A_p = 4A_p$

Assumptions: 1-D uniform flow, steady, isentropic, ideal gas, no work

Find: u_2, T_2, P_2

from continuity

$$0 = \cancel{\frac{\partial}{\partial t}\int_{cv} \rho\, d\forall}^{\,0} + \int_{cs} \rho(\mathbf{u}\cdot\mathbf{n})\,dA$$

$$0 = -\rho_p u_p A_p - \rho_s u_s A_s + \rho_2 u_2 A_2 \quad \cdots ①$$

from momentum conservation

$$\cancel{\frac{\partial}{\partial t}\int_{cv} \rho \mathbf{u}\, d\forall}^{\,0} + \int_{cs} \rho\mathbf{u}(\mathbf{u}\cdot\mathbf{n})\,dA = \Sigma \mathbf{F}$$

$$-\rho_p u_p^2 A_p - \rho_s u_s^2 A_s + \rho_2 u_2^2 A_2 = -p_2 A_2 + p_1 A_1 \quad \cdots ②$$

from energy conservation

$$\cancel{\frac{\partial}{\partial t}\int_{cv} (e + \tfrac{u^2}{2} + gz)\rho\, d\forall}^{\,0} + \int_{cs} (h + \tfrac{u^2}{2} + \cancel{gz})\rho(\mathbf{u}\cdot\mathbf{n})\,dA = \cancel{\dot{Q}}^{\,0} - \cancel{\dot{W}}^{\,0} = 0$$

$$-\rho_p(h_p + \tfrac{u_p^2}{2})u_p A_p - \rho_s(h_s + \tfrac{u_s^2}{2})u_s A_s + \rho_2(h_2 + \tfrac{u_2^2}{2})u_2 A_2 = 0$$

$$\because h = c_p T \Rightarrow -\rho_p(c_p T_p + \tfrac{u_p^2}{2})u_p A_p - \rho_s(c_p T_s + \tfrac{u_s^2}{2})u_s A_s + \rho_2(c_p T_2 + \tfrac{u_2^2}{2})u_2 A_2 = 0 \quad \cdots ③$$

using the ideal gas eqn. $\rho = \frac{P}{RT}$

$$① \rightarrow \quad \frac{P_2}{RT_2}u_2 A_2 = \frac{P_p}{RT_p}u_p A_p + \frac{P_s}{RT_s}u_s A_s$$

$$\frac{4P_2}{RT_2}u_2 A_p = \frac{P_1}{RT_p}u_p A_p + \frac{3P_1}{RT_s}u_s A_p$$

$$\frac{4P_2 u_2}{T_2} = \frac{P_1 u_p}{T_p} + \frac{3P_1 u_s}{T_s} \quad \cdots ④$$

$$② \rightarrow -P_2A_1 + P_1A_1 = -\frac{P_p}{RT_p}u_p^2A_p - \frac{P_s}{RT_s}u_s^2A_s + \frac{P_2}{RT_2}u_2^2A_2$$

$$-4P_2A_p + 4P_1A_p = -\frac{P_1}{RT_p}u_p^2A_p - \frac{3P_1}{RT_s}u_s^2A_p + \frac{4P_2}{RT_2}u_2^2A_p$$

$$-4P_2 + 4P_1 = -\frac{P_1}{RT_p}u_p^2 - \frac{3P_1}{RT_s}u_s^2 + \frac{4P_2}{RT_2}u_2^2$$

$$4P_1 + \frac{P_1}{RT_p}u_p^2 + \frac{3P_1}{RT_s}u_s^2 = 4P_2 + \frac{4P_2}{RT_2}u_2^2 \quad \cdots ⑤$$

$$③ \rightarrow \frac{P_p}{RT_p}\left(C_pT_p + \frac{u_p^2}{2}\right)u_pA_p + \frac{3P_s}{RT_s}\left(C_pT_s + \frac{u_s^2}{2}\right)u_sA_p = \frac{4P_2}{RT_2}\left(C_pT_2 + \frac{u_2^2}{2}\right)u_2A_p$$

$$\frac{P_1}{RT_p}\left(C_pT_p + \frac{u_p^2}{2}\right)u_p + \frac{3P_1}{RT_s}\left(C_pT_s + \frac{u_s^2}{2}\right)u_s = \frac{4P_2}{RT_2}\left(C_pT_2 + \frac{u_2^2}{2}\right)u_2$$

$$\frac{P_1}{R}C_pu_p + \frac{P_1u_p^3}{2RT_p} + \frac{3P_1}{R}C_pu_s + \frac{3P_1u_s^3}{2RT_s} = \frac{4P_2}{R}C_pu_2 + \frac{2P_2u_2^3}{RT_2}$$

$$P_1C_pu_p + \frac{P_1u_p^3}{2T_p} + 3P_1C_pu_s + \frac{3P_1u_s^3}{2T_s} = 4P_2C_pu_2 + \frac{2P_2u_2^3}{T_2} \quad \cdots ⑥$$

plug in numbers

$$④ \rightarrow \frac{4P_2u_2}{T_2} = \frac{(150\times10^3 Pa)(\frac{250m}{s})}{600K} + \frac{3(150\times10^3 Pa)(\frac{30m}{s})}{300K}$$

$$\frac{4P_2u_2}{T_2} = 107500$$

$$\frac{P_2u_2}{T_2} = 26875 \quad \cdots ⑦$$

$$⑤ \rightarrow 4(150\times10^3 Pa) + \frac{(150\times10^3 Pa)(\frac{250m}{s})^2}{(\frac{287.05J}{kg\cdot K})(600K)} + \frac{3(150\times10^3 Pa)(\frac{30m}{s})^2}{(\frac{287.05J}{kg\cdot K})(300K)} = 4P_2 + \frac{4P_2}{RT_2}u_2^2$$

$$4P_2 + \frac{4P_2}{RT_2}u_2^2 = 659136.039 \quad \cdots ⑧$$

$$C_p = \frac{\gamma}{\gamma-1}R = \frac{1.4}{0.4}\left(\frac{287.05J}{kg\cdot K}\right) = 1004.675\frac{J}{kg\cdot K}$$

$$⑥ \rightarrow (150\times10^3 Pa)\left(\frac{1004.675J}{kg\cdot K}\right)\left(\frac{250m}{s}\right) + \frac{(150\times10^3 Pa)(\frac{250m}{s})^3}{2(600K)} + 3(150\times10^3 Pa)\left(\frac{1004.675J}{kg\cdot K}\right)\left(\frac{30m}{s}\right)$$

$$+ \frac{3(150\times10^3 Pa)(\frac{30m}{s})^3}{2(300K)} = 4C_pP_2u_2 + \frac{2P_2u_2^3}{T_2}$$

$$4C_pP_2u_2 + \frac{2P_2u_2^3}{T_2} = 5.32118\times10^{10} \quad \cdots ⑨$$

using MATLAB, solve for non-linear system ⑦⑧⑨ and the answers are

$$\boxed{\begin{aligned} P_2 &= 156.94 \text{ kPa} \\ T_2 &= 489.20 \text{ K} \\ u_2 &= 83.772 \text{ m/s} \end{aligned}}$$

Question 3 - Calculation

main

```
clear all; close all; clc

% Non-linear system
R = 287.05;   % gas constant [J/kg-K]
Cp = 1004.675;  % Cp constant [J/kg-K]
syms u T P
eqn1 = 26875 - P*u/T;
eqn2 = 659136.039 - 4*P - 4*P*u^2/R/T;
eqn3 = 5.32118*10^10 - 4*Cp*P*u - 2*P*u^3/T;

soln = solve(eqn1, eqn2, eqn3);
ans = struct2table(soln)
```

ans = 2×3 table

	P	T	u
1	1×1 sym	1×1 sym	1×1 sym
2	1×1 sym	1×1 sym	1×1 sym

```
ans = table2array(ans);
ans = vpa(ans, 10);
disp(ans);
```

$$\begin{pmatrix} 156940.9128 & 489.1976513 & 83.77157137 \\ -19620.90467 & -1437.97761 & 1969.616025 \end{pmatrix}$$

Q4.

Given: Combustion of kerosene ($\approx C_{11.8}H_{23.0}$) and air yields a stream of CO, CO_2, H_2O, and N_2

mass fractions are $x_{CO} = 0.06$, $x_{CO_2} = 0.14$, $x_{H_2O} = 0.08$, and $x_{N_2} = 0.72$
$P = 1.0 MPa$, $T = 2200K$

(a) Find: mole fraction y_i of each species.

$M_{CO} = 28 \text{ g/mol}$, $M_{CO_2} = 44 \text{ g/mol}$, $M_{H_2O} = 18 \text{ g/mol}$, $M_{N_2} = 28 \text{ g/mol}$

say total mass is M
then there are the following moles of each species

$$\frac{x_{CO}M}{M_{CO}}, \quad \frac{x_{CO_2}M}{M_{CO_2}}, \quad \frac{x_{H_2O}M}{M_{H_2O}}, \quad \frac{x_{N_2}M}{M_{N_2}} \quad \text{(moles)}$$

$= 2.1429\times10^{-3}M$, $3.1818\times10^{-3}M$, $4.4444\times10^{-3}M$, $0.025714M$ respectively

$$y_{CO} = \frac{\text{moles for } CO}{\text{total moles}} = \frac{2.1429\times10^{-3}M}{0.0354831M} = \boxed{0.060392}$$

likewise,

$$y_{CO_2} = \boxed{0.089671}$$

$$y_{H_2O} = \boxed{0.125254}$$

$$y_{N_2} = \boxed{0.724683}$$

(b) Find: average values of MW, C_p, γ, speed of sound a

$$\overline{MW} = y_{co}M_{co} + y_{co_2}M_{co_2} + y_{H_2O}M_{H_2O} + y_{N_2}M_{N_2}$$
$$= \boxed{28.182196 \text{ g/mol}}$$

From appendix 2 of textbook $\left(\theta = \frac{kelvin}{100}\right)$ $\quad \bar{C}_{po} : \frac{J}{mol \cdot K}$ $\quad \rightarrow \theta = 22$

$$\bar{C}_{po}(CO) = 69.145 - 0.704630\theta^{0.75} - 200.77\theta^{-0.5} + 176.76\theta^{-0.75}$$
$$= 36.5836$$

$$\bar{C}_{po}(CO_2) = -3.7357 + 30.529\theta^{0.5} - 4.1034\theta + 0.024198\theta^2$$
$$= 82.8950$$

$$\bar{C}_{po}(H_2O) = 143.05 - 183.54\theta^{0.25} + 82.751\theta^{0.5} - 3.6989\theta$$
$$= 52.3115$$

$$\bar{C}_{po}(N_2) = 39.060 - 512.79\theta^{-1.5} + 1072.7\theta^{-2} - 820.40\theta^{-3}$$
$$= 36.2299$$

$$(\bar{C}_{po})_{avg} = y_{co}\bar{C}_{po}(CO) + y_{co_2}\bar{C}_{po}(CO_2) + y_{H_2O}\bar{C}_{po}(H_2O) + y_{N_2}\bar{C}_{po}(N_2)$$
$$= \boxed{42.4500 \frac{J}{mol \cdot K}}$$

$$R = \frac{\bar{R}}{\overline{MW}} = \left(\frac{8.31446 \text{ J}}{mol \cdot K}\right)\left(\frac{mol}{28.182196 \text{ g}}\right) = 0.295025 \frac{J}{g \cdot K}$$

$$\bar{R} = C_p - C_v \quad \because C_p = (\bar{C}_{po})_{avg}$$
$$C_v = C_p - \bar{R} = 34.13554 \frac{J}{mol \cdot K}$$

then

$$\gamma = \frac{C_p}{C_v} = \frac{42.4500 \frac{J}{mol K}}{34.13554 \frac{J}{mol K}} = \boxed{1.2436}$$

finally,

$$a = \sqrt{\gamma R T}$$
$$= \sqrt{(1.2436)\left(\frac{0.295025 \text{ J}}{g \cdot K}\right)\left(\frac{1000 \text{ g}}{1 \text{ kg}}\right)(2200 \text{ K})}$$
$$= \boxed{898.42 \text{ m/s}}$$

(c) Check ideal gas assumption by comparing the mixture T and p with the pseudo-critical properties of the mixture.

– from lecture notes

$P_c(CO) = 35.0 \text{ bar} = 3.5 \text{ MPa}$ $\quad T_c(CO) = 133 K$

$P_c(CO_2) = 73.9 \text{ bar} = 7.39 \text{ MPa}$ $\quad T_c(CO_2) = 304 K$

$P_c(H_2O) = 220.9 \text{ bar} = 22.09 \text{ MPa}$ $\quad T_c(H_2O) = 647.3 K$

$P_c(N_2) = 33.9 \text{ bar} = 3.39 \text{ MPa}$ $\quad T_c(N_2) = 126 K$

then

$$P_{c,mixture} = P_c(CO) y_{CO} + P_c(CO_2) y_{CO_2} + P_c(H_2O) y_{H_2O} + P_c(N_2) y_{N_2}$$
$$= 6.0976 \text{ MPa}$$

$$T_{c,mixture} = T_c(CO) y_{CO} + T_c(CO_2) y_{CO_2} + T_c(H_2O) y_{H_2O} + T_c(N_2) y_{N_2}$$
$$= 207.68 K$$

$$P_{reduced} = \frac{P}{P_{c,mixture}} = \boxed{0.164}$$

$$T_{reduced} = \frac{T}{T_{c,mixture}} = \boxed{10.59}$$

then from the compressibility factor Z graph on Lecture 3 slide $Z \approx 1$ for the computed $P_{reduced}$ & $T_{reduced}$ thus it is adequate to assume ideal gas