Passage through "Local" Gravity Fields

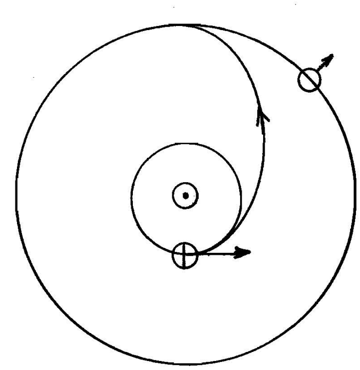
To examine interplanetary transfers completely, it would be necessary to consider all gravitational influences at all times. However, that is an inconvenient approach and can be solved only numerically. But, it is possible to obtain a pretty good approximation to the $\Delta\bar{v}$ requirements by considering the transfer in three phases, each of which involves only a two-body problem, for which there are a large number of analysis techniques.

For an example, consider that you are planning an Earth-to-Mars mission. Assume that the planets are in circular orbits about the Sun and move in the same plane. To determine the $\Delta\bar{v}$, examine the mission as 3 two-body problems:

I.

II.

III.

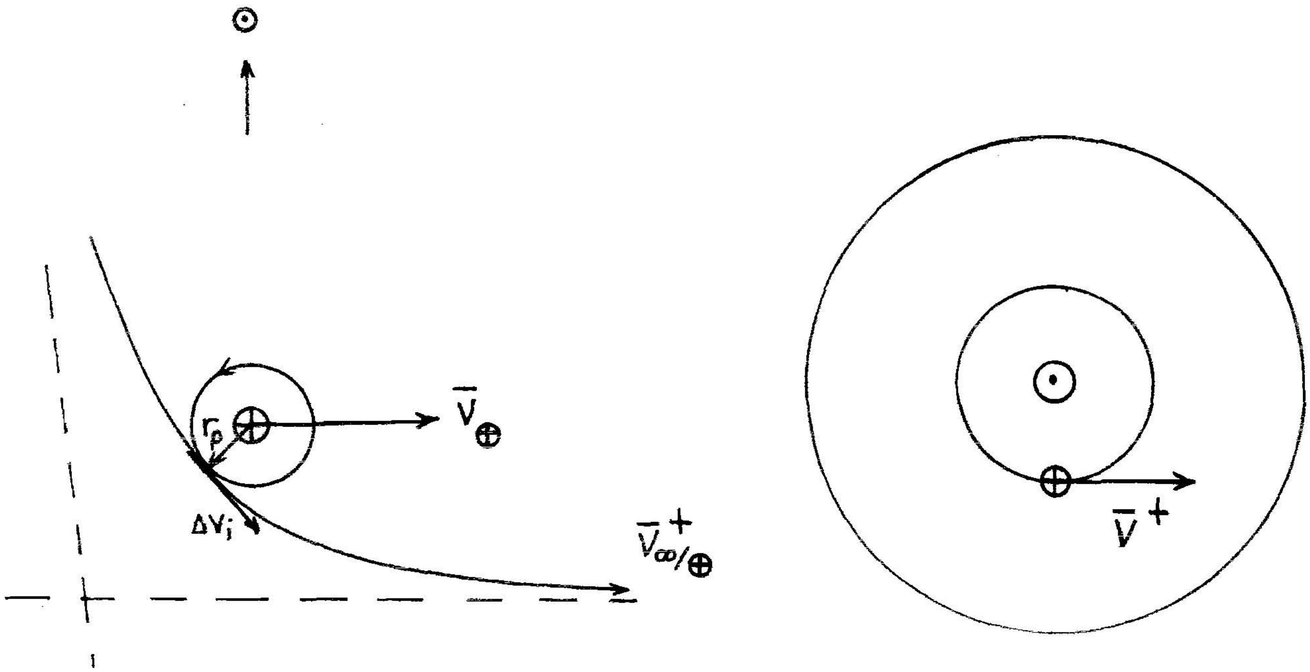


Denoted the "patched conic" approach.
Approximate but yields a good guess of the requirements. Consider each phase separately.

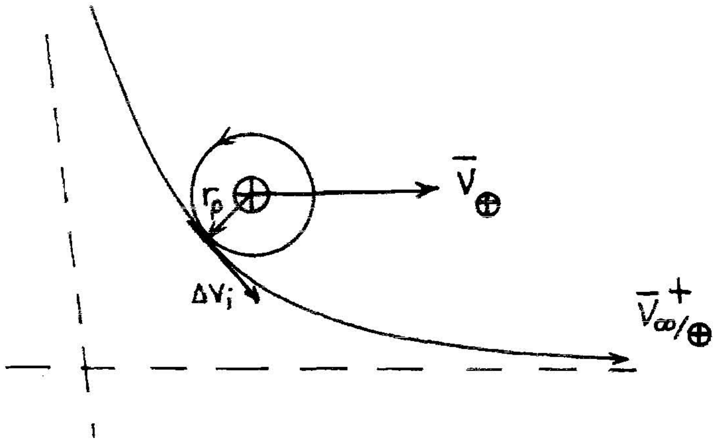
Two-Body Problem #1 (near $\oplus$)

Assume that the vehicle is originally in a circular "parking" orbit around the Earth. Motion is influenced solely by the $\oplus$, totally neglecting the Sun since, at that close range, the Earth is certainly dominant. The vehicle will transfer "instantaneously" from the influence of the $\oplus$ to the influence of the $\odot$. To escape the $\oplus$, the spacecraft must be on a parabolic or hyperbolic orbit <u>with respect</u>

<u>to the Earth</u>. Once "escaped", the vehicle must be moving on the correct transfer orbit about the ⊙ to enable arrival at ♂. Consider two views of the situation.



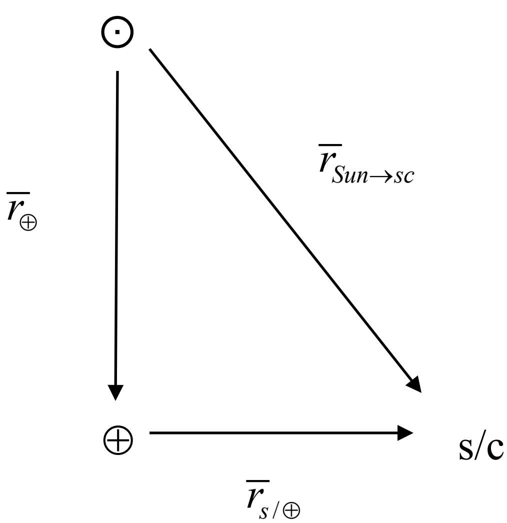
- Circular parking at ⊕ (could be any orbit actually)
- No effect of ⊙
- Transfer "instantaneously" from influence of ⊕ to influence of ⊙
- To escape ⊕, must depart on parabola or hyperbola
- Once escaped, possess exactly correct velocity for transfer orbit about ⊙
- For trip to ♂, s/c velocity wrt ⊙ must be > velocity of Earth wrt ⊙ (gain energy) ⟶



Geocentric: maneuver $\Delta\overline{v}_i$ (tangential) so spacecraft jumps from circular $\oplus$ orbit to hyperbolic orbit to escape $\oplus$. After escape, velocity is $\overline{v}^+_{\infty/\oplus}$. Note that this is a vector quantity. To determine s/c velocity with respect to $\odot$, relate vectors

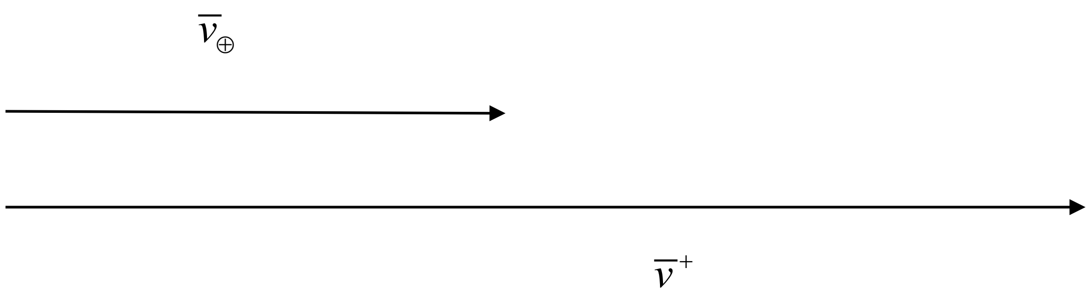
$$\overline{v}^+ = \overline{v}^+_{\infty/\oplus} + \overline{v}_\oplus$$

NOTE:



$$\boxed{\begin{aligned} \overline{r}_s &= \overline{r}_{s/\oplus} + \overline{r}_\oplus \\ \overline{v}_s &= \underbrace{\overline{v}_{s/\oplus}}_{i\ \frac{d\,\overline{r}_{s/\oplus}}{dt}} + \overline{v}_\oplus \\ \overline{v}^+ &= \overline{v}^+_{\infty/\oplus} + \overline{v}_\oplus \end{aligned}}$$

<u>VECTOR Equation</u> $\Rightarrow$ requires vector diagram



Assuming knowledge of the required $\overline{v}^{+}$ to transfer to ♂, solve for the $\overline{v}^{+}_{\infty/\oplus}$ vector required to escape

Use excess velocity to compute the exact vector $\Delta\overline{v}_i$ to jump from the parking orbit to the required hyperbola.

$\Downarrow$

Jump at hyperbolic perigee most effective

At perigee, velocity on hyperbola: $v_c + \Delta v_i$

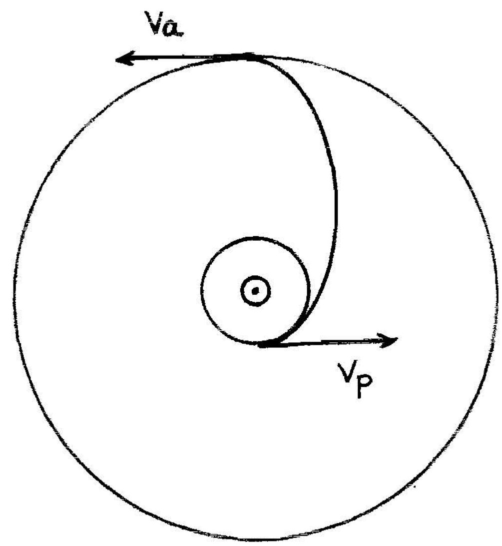
$$\mathcal{E} = \frac{v_\infty^2}{2} = \frac{(v_c + \Delta v_i)^2}{2} - \frac{\mu_\oplus}{r_p}$$

$\leftarrow$ magnitude of Δv_i to depart $\oplus$ orbit

Δv_i: initial burn required to place s/c on heliocentric ellipse with correct v^{+}

Two-Body Problem #2 (influence of ☉)

- assume: leaving and approaching massless planets
- $v^{+} = v_p$ i.e., perihelion on Hohmann transfer ellipse
- easily compute velocity wrt ☉ at ♂ arrival, i.e., the apohelion velocity on Hohmann transfer ellipse
- $v_a = v^{-}$ i.e., heliocentric velocity for ♂ approach



Two-Body Problem #3 (near ♂)

- Assume that the goal is capture into a circular orbit about ♂ at a given radius r_f; also consider the circular orbit must be defined in a particular <u>direction</u>
- Since s/c at apohelion on transfer ellipse, s/c will be moving

- s/c will enter Martian vicinity on hyperbola and – at appropriate time – add Δv_f to slow s/c for capture

- final velocity

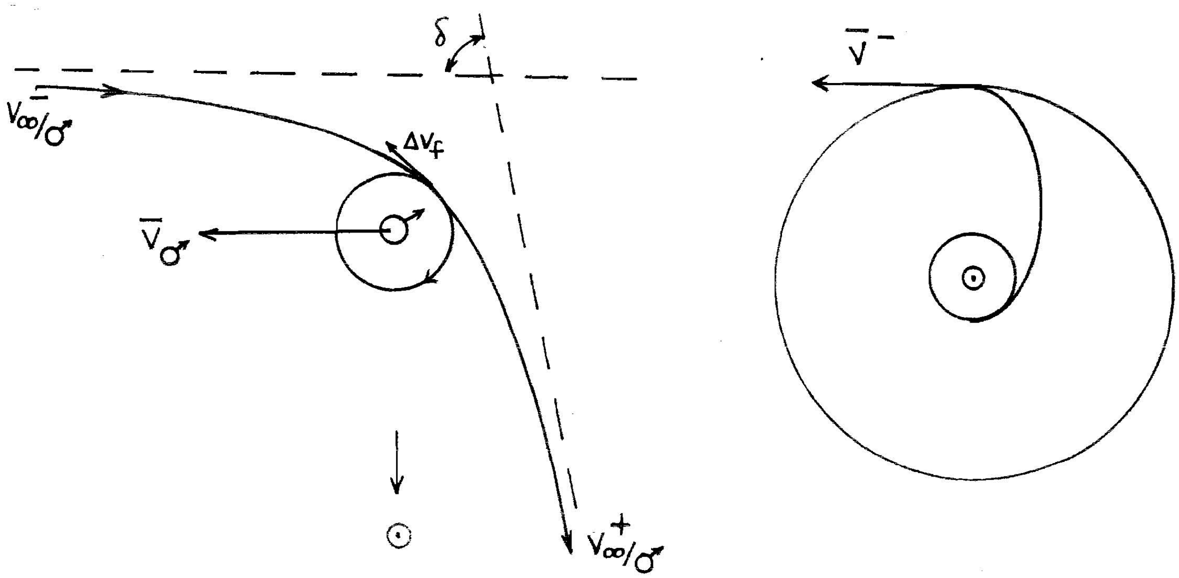


Figure 3

Use heliocentric velocity to determine the velocity on the hyperbola:

$$\overline{v}^{-}_{\infty/♂} = \overline{v}^{-} - \overline{v}_{♂}$$

<u>Vector Diagram</u>

CAPTURE

Solve the vector equation for $\overline{v}^{-}_{\infty/♂}$ and determine Δv_f

Total: $\Delta v_{total} = \Delta v_i + \Delta v_f$

SWINGBY

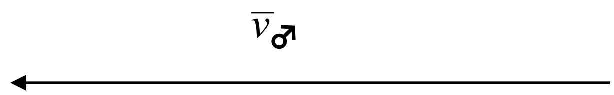
Rather than capture into ♂ orbit, assume pass by of planet at a closest approach of r_f

⇒ same approach hyperbola, but continues past planet on outbound leg of hyperbola

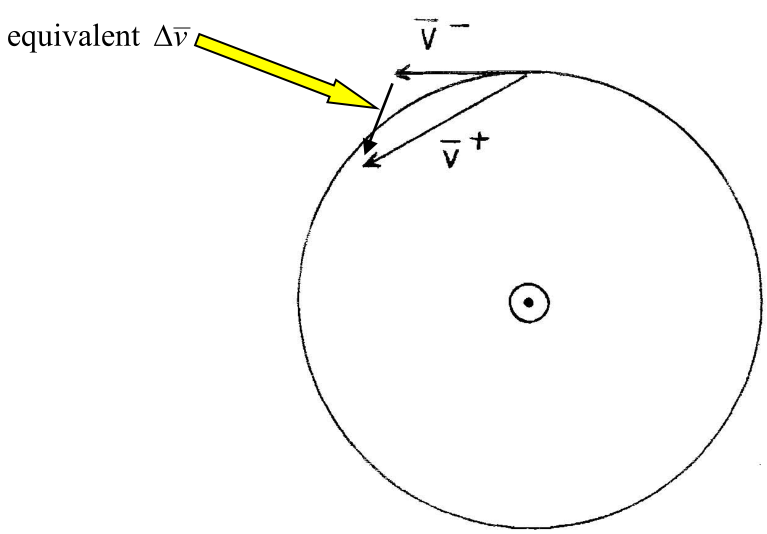
Below: passes on the "dark" side i.e., "behind" Mars

Use vector relationships to determine impact of passage on spacecraft heliocentric velocity

<u>Vector Diagram</u>



passage through the local gravity field of Mars instantaneously changes the **magnitude** and **direction** of the spacecraft heliocentric velocity



Obtained a change in heliocentric velocity for “free” !!!
Can calculate new r, v, γ for elliptical heliocentric orbit

Consider the following:

How can you calculate $\left|\bar{v}^{+}\right|$ and α?

Does the spacecraft gain or lose energy via the Mars passage?

How would $\left|\bar{v}^{+}\right|$ and α change if the spacecraft passed on the “light” side of the planet?

Recall that arrival occurred tangentially. What if arrival occurred at an arbitrary point on the transfer ellipse?

How can the departure from Earth be timed such that Mars is in the assigned position at the proper time?

Amount of "gravity assist" generated depends on closest pass to Mars (or planet)

Example:

$v^{-} = 21.48\,\text{km/s}$ $\qquad\qquad$ $v_{\text{♂}} = 24.13\text{ km/s}$

$$\boxed{\begin{aligned} a &= \frac{1}{2}\left(r_{\oplus} + r_{\text{♂}}\right) = 1.888\times10^{8}\text{ km } \left(1.26\,\text{AU}\right) \\ \mathcal{E} &= \frac{v_{\infty}^{2}}{2} - \frac{\mu_{\odot}}{r_{\text{♂}}} \overset{\odot}{=} -\frac{\mu}{2a} \Rightarrow v_{a} = 21.48\text{ km/s} \end{aligned}}$$

$v_{\infty/\text{♂}}^{-} =$

Assume closest approach to ♂

$r_{f} = R_{\text{♂}} + 1500\,\text{km} \qquad (4.91\times10^{3}\text{ km})$

$\delta =$

$$\boxed{\begin{aligned} \mathcal{E} &= \frac{v_{\infty}^{2}}{2} = \frac{\mu_{\text{♂}}}{2|a|} \Rightarrow |a| = 6.1132\times10^{3}\text{ km} \\ e &= \frac{r_{p}}{|a|} + 1 \\ \sin\frac{\delta}{2} &= \frac{1}{e} \end{aligned}}$$

$\overline{v}_{\text{♂}}$

Alternatively:

$$r_f = R_{♂} + 200\,\text{km} \qquad (3.61 \times 10^3 \text{ km})$$

- Of course, a sunside passage will yield same v^+ but now γ^+ positive → changes orbit relative to Sun
  But $v^+ > v^-$ →

- Project Galileo: at one time, plans included a flyby of ♂ on way to Jupiter

- Patched-conic method for calculating trajectories (or Δv estimates) yields pretty accurate thrust requirements (surprisingly) → pretty accurate nominal trajectories
  [some error in transfer time because only time along actual Hohmann is incorporated]

- Greatest difficulty in practice is <u>implementation</u>
  Hyperbolic excess speed, v_∞, is extremely sensitive to small errors in injection velocity

  Example:

$$\overline{v}_c + \Delta\overline{v} = \overline{v}_p$$

Velocity required at periapsis of hyperbola

added thrust

from energy $\mathcal{E}$:

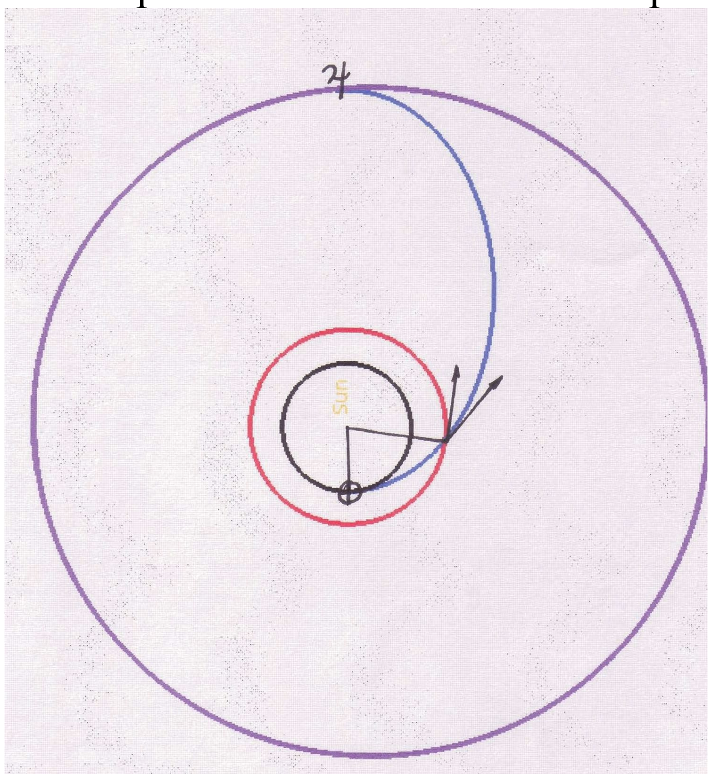
$$v_\infty^2 = v_p^2 - \frac{2\mu}{r_p}$$ first-order difference eqn for small errors in v_p

Hohmann to Mars → $v_p \cong 11.5\,\text{km/s}$ $v_\infty = 3\,\text{km/s}$

Example: Swingby when Arrival not Tangential

Assume Hohmann transfer to Jupiter and intercept Mars
How will Mars swingby affect orbit?

Assume Mars and Jupiter orbits are circular and coplanar

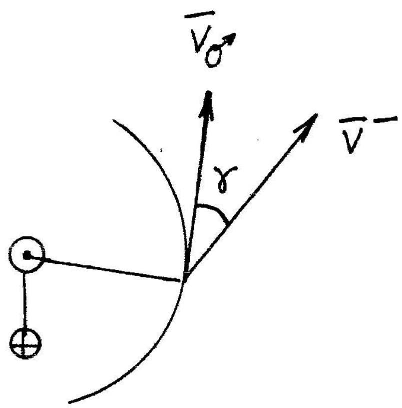


Determine conditions as Mars arrival

$$a = \frac{1}{2}\left(r_{\oplus} + r_{Jup}\right) = 3.1\,\text{AU}$$

$$r_{\oplus} = r_p = a(1-e) \Rightarrow e = .677 \qquad (\,p = 1.68\,\text{AU}\,)$$

$$r_{♂} = \frac{a\left(1-e^2\right)}{1+e\cos\theta^*} \Rightarrow$$



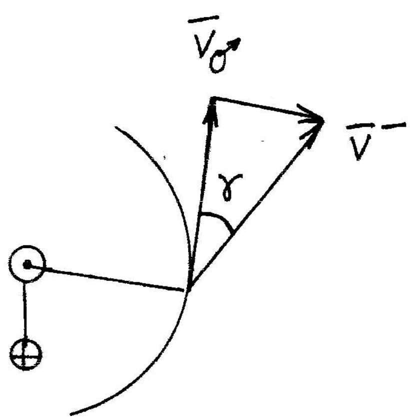
$$\boxed{\bar{v}_♂ = 24.187\,\text{km/s}}$$

$$\frac{\left(v^-\right)^2}{2} - \frac{\mu_\odot}{r_♂} = -\frac{\mu_\odot}{2a}$$

$$\longrightarrow \boxed{v^- = 29.7\,\text{km/s}}$$

$$\frac{v_p^2}{2} - \frac{\mu_\odot}{r_p} = -\frac{\mu_\odot}{2a} \longrightarrow v_p = 38.66\,\text{km/s}$$

$$h = \sqrt{\mu_\odot p} = r_♂ v^- \cos\gamma \longrightarrow \gamma = 31.3^\circ \qquad \text{(why +?)}$$



$$\bar{v}^- = \bar{v}_♂ + \bar{v}^-_{\infty/♂}$$

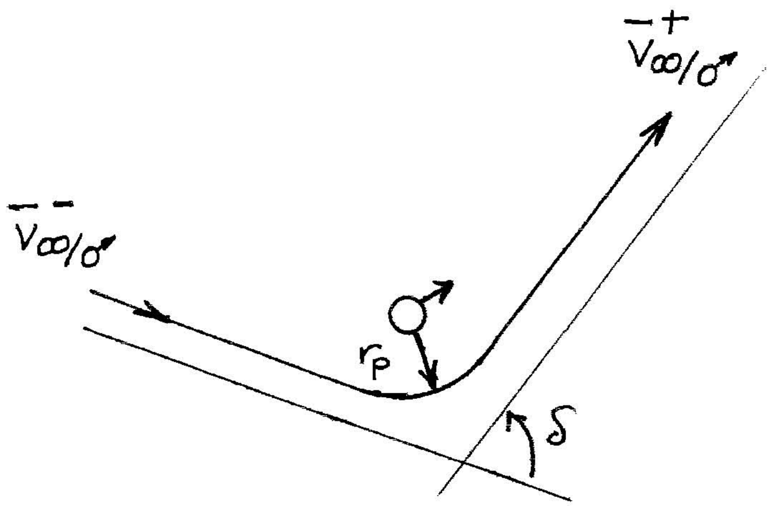
$$\bar{v}^-_{\infty/♂} = \bar{v}^- - \bar{v}_♂$$

Solve for $v_{\infty/♂}$ from cosine law

$$v^2_{\infty/♂} = v^2_♂ + \left(v^-\right)^2 - 2v_♂ v^- \cos\gamma$$

$$\boxed{}$$

which way to pass to increase or decrease energy?



"leading" or "trailing"

"ahead" or "behind"

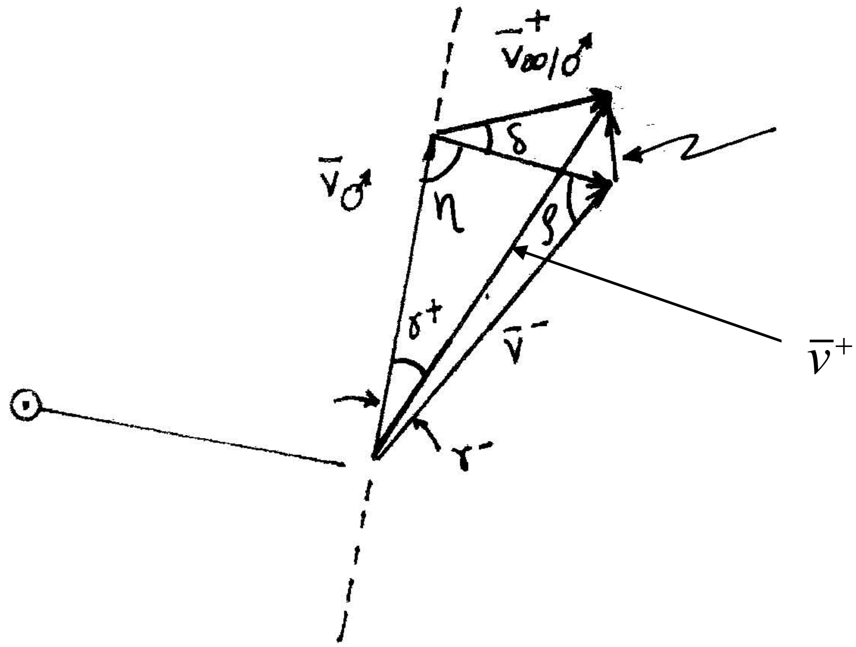
GAIN energy?
Pass behind
$r_f = R_{♂} + 300\,\text{km}$

Hyperbolic orbit wrt ♂

$$\mathcal{E} = \frac{v_\infty^2}{2} = \frac{\mu_{\odot}}{2|a|} \longrightarrow |a| = 179.536\,\text{km}$$

$$e = \frac{r_p}{|a|} + 1 = 21.55$$

$$\sin\frac{\delta}{2} = \frac{1}{e} \longrightarrow \delta = 5.3^\circ$$



$$\frac{v^-}{\sin\eta} = \frac{v^-_{\infty/♂}}{\sin\gamma^-} \qquad \eta =$$

$$\frac{v_{♂}}{\sin\varsigma} = \frac{v^-_{\infty/♂}}{\sin\gamma^-} \qquad \varsigma =$$

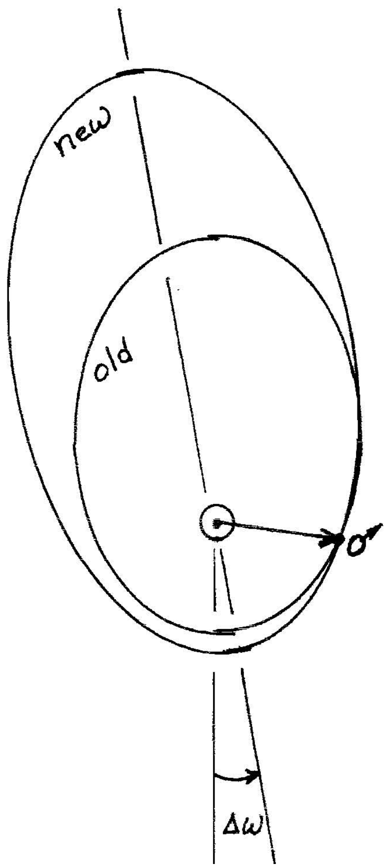
$$\left(v^-\right)^2 = \underbrace{\left(v^-_{\infty/♂}\right)^2 + v_{♂}^2}_{\text{Need to }\underline{\underline{\text{add}}}\text{ to this term}} - 2v^-_{\infty/♂}v_{♂}\cos\eta \longrightarrow \cos\eta \ \ \text{negative}$$

$$\left(v^+\right)^2 = v_{♂}^2 + \left(v^+_{\infty/♂}\right)^2 - 2v_{♂}v^+_{\infty/♂}\cos\left(\eta+\delta\right) \qquad \boxed{v^+ = \qquad\qquad}$$

Increased velocity $\longrightarrow$ higher energy orbit (pass ahead, drop vel?)

$$\frac{v^+_{\infty/♂}}{\sin\gamma^+} = \frac{v^+}{\sin\left(\eta+\delta\right)} \longrightarrow \boxed{\gamma^+ = 29.63^\circ}$$

$$\tan\theta^* = \frac{\left(\frac{rv^2}{\mu}\right)\sin\gamma\cos\gamma}{\left(\frac{rv^2}{\mu}\right)\cos^2\gamma - 1} \longrightarrow \theta^{*+} = 71.9^\circ \;\text{ or }\; 252^\circ \quad (\gamma > 0)$$



$\Delta\omega =$

perihelion advances

	before encounter	after encounter
r	1.52 AU	1.52 AU
v	29.7 km/s	30.84 km/s
γ	31.3°	29.63°
e	.677	.7348
θ^*	81.3°	
a	3.07 AU	
r_p	1 AU	
r_a	5.2 AU	