AAE 532 – Orbit Mechanics
Problem Set 6
Due: 10/16/20

Problem 1: Assume a relative two-body model and a space vehicle that is currently in a Mars orbit with $r_p = 1.1\, R_{♂}$ and $r_a = 6.0\, R_{♂}$. The spacecraft is currently located at $\theta_c^* = 90^\circ$ at time t_c. A single in-plane adjustment will be employed to circularize the orbit.

(a) At what true anomaly values does $r = 4.5\, R_{♂}$? (Note that two locations exist!)
Select the location that is the earliest opportunity after t_c to reach $r = 4.5\, R_{♂}$. Let this time be t_1. Determine $v_1, \gamma_1, E_1, \left(t_1 - t_p\right)$ at this location.

(b) Sketch the orbit. Mark the usual quantities at the time t_1: $\bar{r}_1, \bar{v}_1, \gamma_1, l.h., E_1$; also add appropriate unit vectors $\hat{e}, \hat{p}; \hat{r}_1, \hat{\theta}_1$.

(c) What is the "wait time" till the maneuver $\left(t_1 - t_c\right)$?

(d) Determine r_1^+, v_1^+, γ_1^+ after the maneuver.
Compute the required maneuver ($\Delta v, \alpha$). Recall that $\Delta v = \left|\Delta \bar{v}\right|$. [Include VECTOR diagrams!!!!]

(e) Plot the old and new orbits on the same figure using your Matlab script. On the plot, mark $\bar{r}_0, \bar{r}_1, \bar{v}_1^-$, local horizon, γ_1^-, $\bar{v}_1^+, \gamma_1^+, \Delta\bar{v}, \alpha$.

(f) Plot the two orbits in GMAT using Mars as the central body. At the maneuver time, use a report to list $\bar{r}, \bar{v}$ in each orbit at the maneuver time. Choose a convenient set of unit vectors (coordinate frame). Subtracting the velocity vectors should yield your $\Delta v = \left|\Delta \bar{v}\right|$. Does it?

Problem 2: Given a two-body model, a vehicle is successfully launched into an Earth orbit with $e = 0.4$ and $a = 4R_\oplus$. A single <u>in-plane</u> maneuver will be implemented when $\theta^* = 135^\circ$. Let the maneuver be defined as $|\Delta\bar{v}| = 0.90$ km/s, $\alpha = +45^\circ$.

(a) Determine $\bar{r}, \bar{v}^-, \gamma^-$ at the maneuver point.

(b) Express the maneuver in both $\hat{r}, \hat{\theta}$ and $\hat{e}, \hat{p}$ unit vectors. Also determine the maneuver in the VNB set of coordinates.

(c) Prepare any VECTOR diagrams!!!! Determine r^+, v^+, γ^+ in the new orbit immediately after the maneuver.

(d) To determine the impact that such a maneuver creates on the orbital characteristics, compute the following characteristics of the new orbit: $a, e, h, period, \boldsymbol{\mathcal{E}}; \theta^*, E, \gamma, IP, (t-t_p), r_p, \Delta\omega$.

(e) Plot the new and the old orbits in Matlab on the same figure using your Matlab script. On the plot, mark $\bar{r}_0, \bar{r}_1, \bar{v}_1^-$, local horizon, γ_1^-, $\bar{v}_1^+, \gamma_1^+, \Delta\bar{v}, \alpha$. Also indicate the new and old lines of apsides and the shift, i.e., $\Delta\omega$. Is it positive or negative? Why?

(f) Bonus → You can use GMAT in two ways to check the maneuver and the new orbit. Use <u>either</u> method to check your results:

(i) Use a start date October 10, 2020 12:00:00 to propagate the satellites. Put in the old and new orbits with 2 satellites and compare the velocities at the intersection point to assess if the difference equals the required $\Delta\bar{v}$;

(ii) Under GMAT Tips is a new document titled "Implement Maneuvers in GMAT". Use a start date October 10, 2020 12:00:00 to propagate and plot the old and new orbit but use the option to insert a maneuver. The $\Delta\bar{v}$ can also be expressed in terms of $\hat{V}, \hat{N}, \hat{B}$. Confirm that adding this maneuver yields the orbit that you calculated.

Problem 3: Assume a relative two-body model and a space vehicle that is currently located in an Earth orbit with $a = 3R_\oplus$ and $e = 0.6$. A single in-plane adjustment is to be implemented for a perigee-raise maneuver. The goal is a new periapsis distance such the new $r_p = 2R_\oplus$. At the same time, it is desired to produce an orbit that is less eccentric, i.e., the new eccentricity is $e = 0.4$ and both goals are accomplished using the same maneuver. The maneuver will take place when the spacecraft is located at the end of the minor axis and <u>descending</u>.

(a) Determine $\bar{r}_1, \bar{v}_1^-, \gamma_1^-, (t - t_p)$ at the maneuver point.

Sketch or plot the orbit in Matlab. Mark the usual quantities at the maneuver location prior to the maneuver: $\bar{r}^-, \bar{v}^-, \gamma^-, l.h., E^-$; also add appropriate unit vectors $\hat{e}, \hat{p}; \hat{r}, \hat{\theta}$.

(b) Determine $\bar{r}^+, \bar{v}^+, \gamma^+$ at the maneuver point. [Include VECTOR diagrams!!!!]

Compute the required maneuver ($\Delta v, \alpha$). Express that maneuver in terms of $\hat{r}, \hat{\theta}; \hat{V}, \hat{B}$ sets of unit vectors.

(c) Determine the characteristics of the <u>new</u> orbit:

$a, e, r_p, r_a, period, \mathcal{E}; \theta^*, E, \gamma, IP, (t - t_p), r_p, \Delta\omega$

How long till the spacecraft reaches perigee in the new orbit?

(d) Plot the old and new orbits on the same figure using your Matlab script. On the plot, mark $\bar{r}_0, \bar{r}_1, \bar{v}_1^-$, local horizon, γ_1^-, $\bar{v}_1^+, \gamma_1^+, \Delta\bar{v}, \alpha$.

Practice Problem (no submission): The relationship between sets of unit vectors is clearly important!

(a) Derive the direction cosine matrix, i.e., the transformation matrix, that relates the inertial unit vectors $\hat{x}, \hat{y}, \hat{z}$ to the rotating set of unit vectors $\hat{r}, \hat{\theta}, \hat{h}$.

(b) A vehicle is moving in some Earth orbit. At a certain time, the following information is given

$$\bar{r}_1 = 2.12\ R_\oplus \hat{x} + 2.73\ R_\oplus \hat{y} - 0.6\ R_\oplus \hat{z}$$
$$\bar{v}_1 = -3.4\hat{x} + 1.62\hat{y} + 2.9\hat{z} \text{ km/s}$$

Could you express $\bar{r}, \bar{v}$ in terms of other sets of unit vectors: $\hat{n}_x, \hat{n}_y, \hat{n}_z; \hat{q}_x, \hat{q}_y, \hat{q}_z; \hat{r}, \hat{\theta}, \hat{h}$?

(c) Determine the following orbital characteristics: $a, e, p, i, \omega, \Omega, r, v, \gamma, \theta^*, M, E, (t - t_p)$. [Be sure to include the appropriate quadrant checks!!]

Are you sure the orbit is an ellipse? How do you know?

(d) Plot/sketch the orbit in its plane; mark all the usual quantities as well as the nodal line and the orientation angle ω.

Plot/sketch the 3D orbit and add AN, DN, fundamental plane, unit vectors $\hat{x}, \hat{y}, \hat{z}; \hat{r}, \hat{\theta}, \hat{h}$ as well as the nodal axis.