Student Name____________________________________

Student ID______________________________________

Orbit Mechanics
10/23/20

Exam 2

Please read the problems carefully.

Write clearly and use diagrams when necessary.

Use the following constant values when appropriate

Body	GM (km^3/s^2)	Radius (km)
Earth	4.0000×10^5	6400.0
Moon	5.000×10^3	1500.0
Earth-Moon Distance 4.0000×10^5 km		

Purdue Honor Pledge "As a Boilermaker pursuing academic excellence, I pledge to be honest and true in all that I do. Accountable together—We Are Purdue."

For this exam, I understand it is a take-home exam with the following requirements:

1. I can use my own class notes and my own previously completed assignments.
2. I am not allowed to search for any resources online.
3. I can use my own calculator. I cannot use Matlab or other commercial software.
4. I am expected to work the exam on my own. I am not allowed to work with another person. I am not allowed to contact another person for help while completing the exam.
5. If I have any questions during the exam period, I will email Prof Howell (howell@purdue.edu) AND the TAs Beom Park (park1103@purdue.edu) + Nadia Numa (nnuma@purdue.edu). Given this exam period is 24 hours, we will answer as soon as possible.

Signature ____

(40 Points)

Problem 1: It is now 2050 and under the new Global Lunar Vision – an international commercial enterprise for development of lunar resources – facilities are based at various locations on the Moon's surface. The Lunar Transport Service uses vehicles (LTVs) in various orbits to facilitate operations. Note $\mu_{LTV} <<< \mu_{☾}$.

(a) Analysis for lunar activities requires a valid model. Assume that a vehicle is located $50R_{☾}$ from the Moon. Complete a quick look by placing Earth—LTV—Moon along a line and in this order; assume that the LTV is at a radius distance $50R_{☾}$.

What equation might give you insight to decide if a relative two-body model is adequate at this distance?

Compare the various terms in an equation for motion of the LTV with respect to the Moon; is the relative two-body model reasonable at this distance? Why or why not?

(b) Assume that a relative 2B model is adequate. A Lunar Transport Service vehicle (LTV) is currently in a lunar orbit with $p = 4R_{☾}$ and $e = \dfrac{1}{\sqrt{3}}$. (See the figure on the next page.)

When the LTV is located at the end of the latus rectum and ascending, an in-plane maneuver will shift the vehicle into a new orbit for a run to Earth such that $\boldsymbol{\mathcal{E}}_N = +\dfrac{1}{6}\,\text{km}^2/\text{s}^2$ and $h_N = 8200\,\text{km}^2/\text{s}$.

(i) What is the escape speed for this departure location?

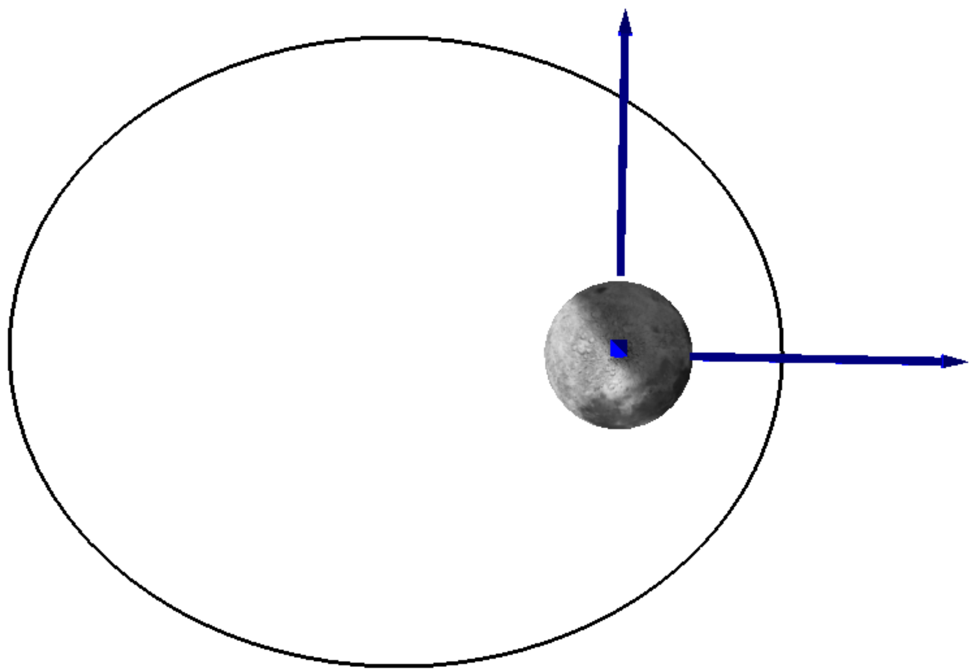
(ii) Check the vector diagram. Two reasonable options exist to accomplish this task. Describe these options; which one offers the smallest $|\Delta\bar{v}|$? Why?

(iii) Determine the $\Delta\bar{v}$ (magnitude $\Delta\bar{v}$ and angle α) to accomplish the adjustment. Also express it in terms of $\hat{V}\hat{N}\hat{B}$ coordinates.

(iv) For the new orbit, determine $e_N, \theta_N^*, \Delta\omega$.

Sketch the old and new orbits; mark the old and new line of apsides as well as $\Delta\omega$.

(c) Discuss: How are the results from (a) and (b) related? What are your conclusions and/or observations?



(40 points)

Problem 2: A vehicle is currently in Earth orbit. Assume that it is reasonable to employ a relative two-body model. At time t_1, the vehicle is located at position $\bar{r}_1$ and later, at t_2, it is observed at location $\bar{r}_2$. The orbit semi-major axis is given as $a = a_n$. In terms of inertial Earth equatorial coordinates, the locations are given as:

$$\bar{r}_1 = a_n \left(\cos 45^\circ \, \hat{x} + \sin 45^\circ \, \hat{z} \right)$$
$$\bar{r}_2 = a_n \left(\cos 45^\circ \, \hat{y} + \sin 45^\circ \, \hat{z} \right)$$

It is known that the spacecraft passes through apoapsis between t_1 and t_2.

(a) Make two sketches: (i) the general scenario in 3D; (ii) the scenario in the orbit plane. (Your sketches should be reasonably accurate!)

(b) Determine the angle between the vectors.

Is the specific angular momentum vector above or below the fundamental plane? Why?

(c) Determine the following orbital characteristics: Ω, i, ω, e, p

(d) Determine conditions at the given locations: $\theta_1^*, \theta_2^*, \gamma_1, \gamma_2$

(e) Velocity $\bar{v}_1$ can be written $\bar{v}_1 = k_1 v_c \hat{v}_1$ where v_c is the corresponding circular speed. Determine k_1.

Use the f and g functions to determine the velocity direction in terms of the inertial coordinates $\hat{x}\,\hat{y}\,\hat{z}$. Also express the velocity direction in terms of $\hat{e}\,\hat{p}\,\hat{h}$ and $\hat{r}, \hat{\theta}, \hat{h}$.

(f) Return to a sketch of the orbit plane. Mark $\bar{r}_1, \bar{r}_2, \bar{v}_1, \bar{v}_2$, local horizons, flight path angles. Also identify the arc between t_1 and t_2.

(20 points)

Problem 3: Assume a relative two-body model is appropriate. In an elliptical Earth orbit, a spacecraft is located such that the true anomaly is $\theta_1^* = 90^\circ$; at a later time $\theta_2^* = 225^\circ$. If $r_p = 2R_\oplus$ and $r_a = 10R_\oplus$, determine the time $(t_2 - t_1)$.

Sketch the orbit and identify the arc of interest.