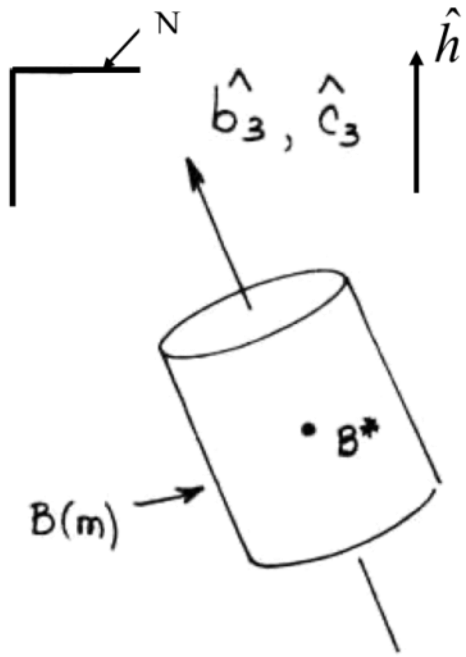
P3: Ob

2020年3月13日 金曜日　　午後0:58

Axisymmetric RB; Torque-free



Model

Mass properties:

I transverse moment of inertia

J axial moment of inertia

$\hat{b}_i$ central, principal

$$\bar{\bar{I}}^{B/B^*} = I\left(\hat{b}_1\hat{b}_1 + \hat{b}_2\hat{b}_2\right) + J\,\hat{b}_3\hat{b}_3$$

$$\bar{M}^{B^*} = \frac{{}^N d\, {}^N\bar{H}^{B/B^*}}{dt} = \bar{0} \quad \Longrightarrow \quad \bar{H}^{B/B^*} \text{ constant}$$

$${}^N\bar{H}^{B/B^*} = H\,\hat{h}$$

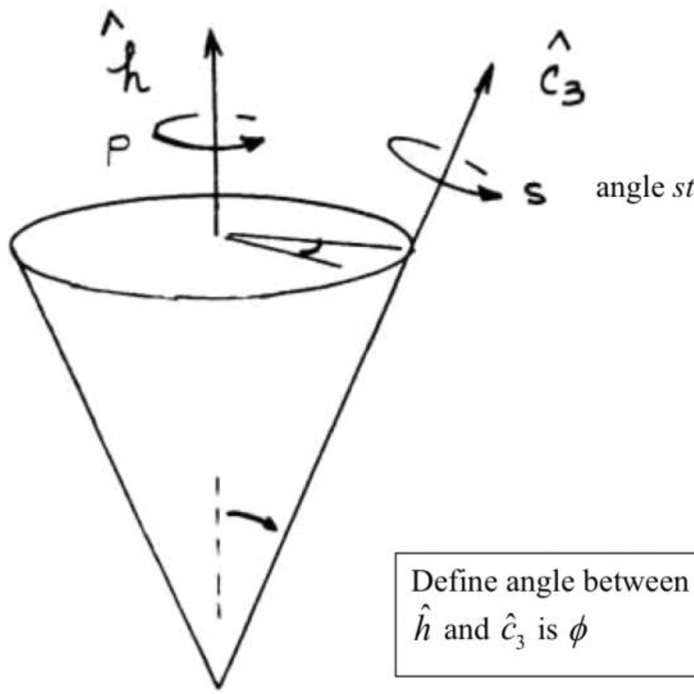
Motion of B in N can be described as the sum of 2 rotations

Summarize: Rotational motion of B in N can be described as a sum of two simple rotations

$^{N}\bar{\omega}^{C} = p\,\hat{h}$ constant mag; constant direction

$^{C}\bar{\omega}^{B} = s\,\hat{c}_3$ constant mag

if $s = \left(\frac{I-J}{I}\right)\omega_3; \quad p = \frac{H}{I}$



Define angle between $\hat{h}$ and $\hat{c}_3$ is ϕ

Geometric Solution – Poinsot Construction

Solution rests on the observation that there are 2 integrals of the motion

1. Torque-free motion $\frac{{}^{N}d\bar{H}}{dt} = \bar{0}$ (most obvious)

$\Rightarrow$

2. Conservative System?

Check forces: no external forces
internal forces for a RB do no work

Total energy constant

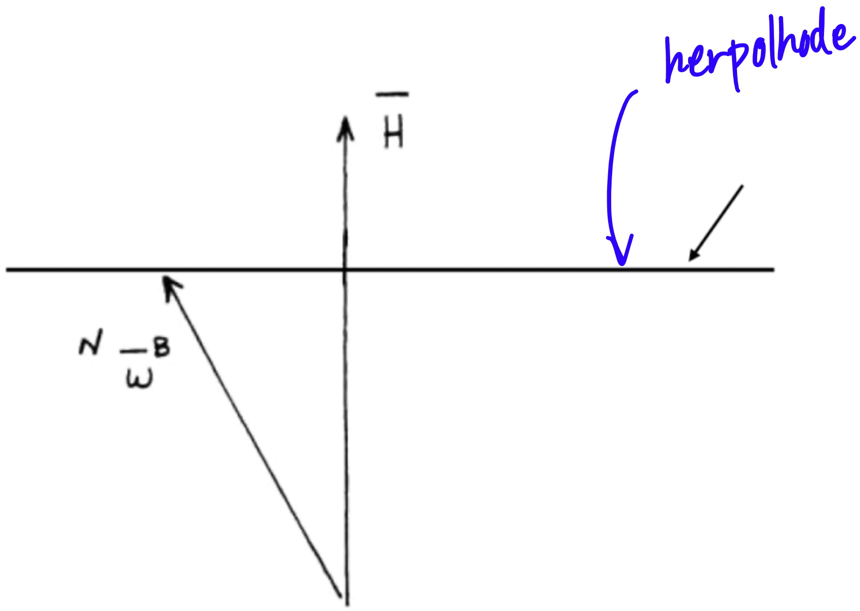
$\Downarrow$

Total kinetic energy constant

$\Downarrow$

O11

$$T_{ROT} = \frac{1}{2}\, {}^N\bar{\omega}^B \cdot \underbrace{\bar{\bar{I}}^{B/B^*} \cdot {}^N\bar{\omega}^B}_{{}^N\bar{H}^{B/B^*}}$$



${}^N\bar{\omega}^B \cdot \bar{H}$ constant $\longrightarrow$

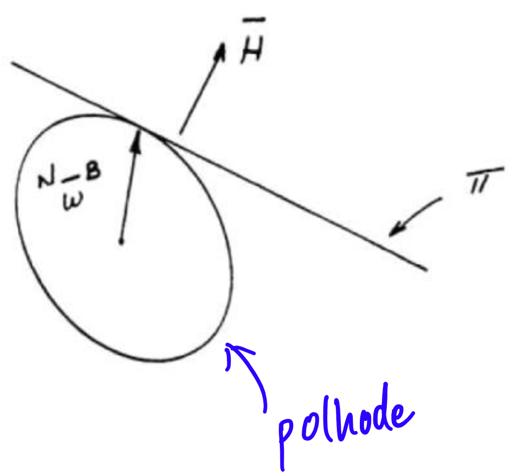
${}^N\bar{\omega}^B$ need <u>not</u> be constant

So ${}^N\bar{\omega}^B$ (tip) must move on a plane $\perp$ to $\bar{H}$ (the invariable plane)

Further expand $\underbrace{\bar{\omega} \bullet \bar{H} = 2T_{ROT}}$

$$\boxed{\frac{\omega_1^2}{(cI_1^{-\frac{1}{2}})^2} + \frac{\omega_2^2}{(cI_2^{-\frac{1}{2}})^2} + \frac{\omega_3^2}{(cI_3^{-\frac{1}{2}})^2} = 1}$$

Equation of ellipsoid in terms of body fixed axes where tip of $\bar{\omega}$ develops the ellipsoid surface



Energy Ellipsoid