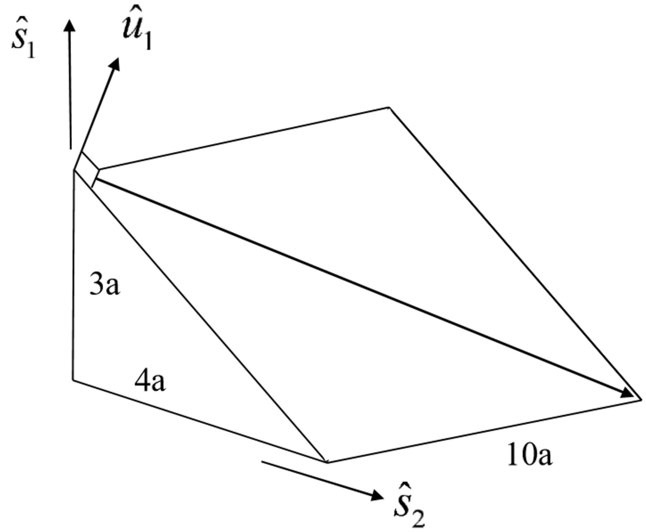
HW2

Saturday, January 25, 2020 10:38 PM

Spacecraft Attitude Dynamics
Problem Set 2
Due: 1/31/20

1. The block below is a right triangular wedge. Vector bases $\hat{u}$ and $\hat{s}$ are dextral, orthonormal triads and fixed in the block. Note that $\bar{H} = H\,\hat{u}_3$ and $\hat{u}_1$ is normal to the top surface.

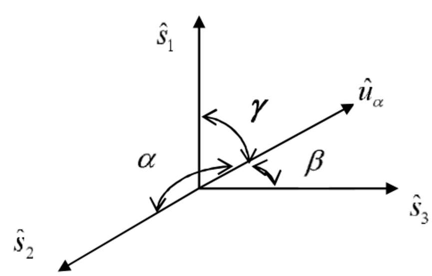


(a) Determine the direction cosine matrix that relates $\hat{u}$ and $\hat{s}$. Note that the relationship can be written in the forms

$$\begin{bmatrix} \hat{s}_1 \\ \hat{s}_2 \\ \hat{s}_3 \end{bmatrix} = L \begin{bmatrix} \hat{u}_1 \\ \hat{u}_2 \\ \hat{u}_3 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} \hat{s}_1 & \hat{s}_2 & \hat{s}_3 \end{bmatrix} = \begin{bmatrix} \hat{u}_1 & \hat{u}_2 & \hat{u}_3 \end{bmatrix} C$$

Write out both L and C. Evaluate the measure numbers and test the orthogonality conditions.

(b) Determine the three (direction cosine) angles for each $\hat{u}_\alpha$ relative to $\hat{s}_i$.



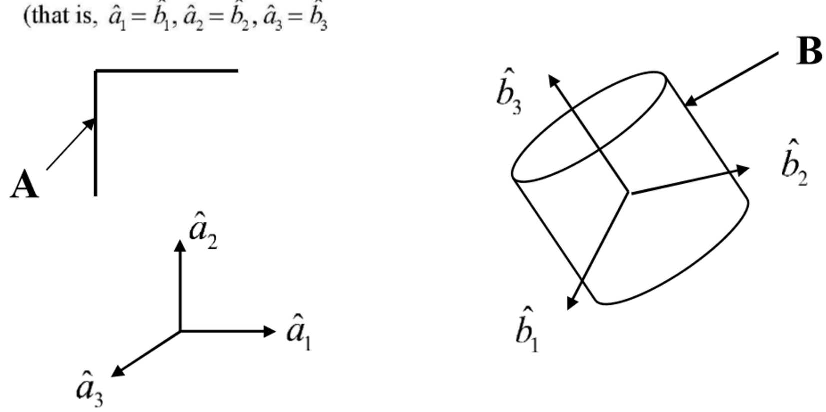
Problem 2: Given that the rigid body B from problem 3 moves relative to A, suppose its orientation is known at a given instant. Let this new orientation be given as

$$ {}^{A}C^{B} = \begin{bmatrix} .4638 & .3607 & .8091 \\ -.6082 & & -.0052 \\ -.6442 & -.4897 & .5876 \end{bmatrix} $$

(a) Determine the missing element in the ${}^{A}C^{B}$
Verify that all orthogonality conditions are satisfied. What is the accuracy?

(b) Determine the equivalent representation ${}^{A}\bar{\varepsilon}^{B}$, ${}^{A}\varepsilon_4^{B}$

Problem 3: Assume that a rigid body B (e.g., a rigid spacecraft) can move with respect to a frame A. Let unit vectors $\hat{b}$ be fixed in body B; unit vectors $\hat{a}$ are fixed in A. At the initial time, $\hat{a}_i = \hat{b}_i$ (that is, $\hat{a}_1 = \hat{b}_1, \hat{a}_2 = \hat{b}_2, \hat{a}_3 = \hat{b}_3$



At some later time, the orientation of B in A is described in terms of the simple rotation:

$$ {}^A\bar{L}^B = -1\hat{a}_1 + 2\hat{a}_2 + 2\hat{a}_3 $$
$$ {}^A\theta^B = -120^\circ $$

(a) Sketch $\hat{\lambda}$ (and $\bar{L}$) in 3-D. Add the direction of θ to the sketch.
(b) Express the orientation as a direction cosine matrix ${}^A C^B$
simple rotation dyadic $\bar{\bar{R}}$. ← in dyadic form $\hat{a}, \hat{b}$
(c) Define a vector $\bar{k}$ that is fixed in $\hat{a}$ initially such that $\bar{k} = 1\hat{a}_1 - 2\hat{a}_2$. Label $\bar{k} = \bar{k}_a$ to reflect the vector prior to the rotation; then, $\bar{k} = \bar{k}_b$ represents the vector after the rotation. The vector $\bar{k}_b$ is fixed in the body B such that $\bar{k}_a = \bar{k}_b$ at the initial time. <u>After</u> the simple rotation, express $\bar{k}_b$ in terms of unit vectors $\hat{b}$; $\hat{a}$. (<u>Use the simple rotation theorem</u>!)
Since $\bar{k}_a = 1\hat{a}_1 - 2\hat{a}_2$, does $\bar{k}_b = 1\hat{b}_1 - 2\hat{b}_2$ $\quad$ $\bar{k}_b = 1\hat{a}_1 - 2\hat{a}_2$?
After the rotation, should the measure numbers be the same in both $\hat{a}$ and $\hat{b}$?