October 16, 2020

AAE 564 Fall 2020

# Homework Eight

Due: Friday, October 23

**Exercise 1** Determine (by hand) whether each of the following systems is asymptotically stable, stable, or unstable.

$$\begin{aligned} \dot{x}_1 &= -x_1 + 2001x_2 \\ \dot{x}_2 &= -x_1 \end{aligned}$$

$$\begin{aligned} \dot{x}_1 &= -x_1 \\ \dot{x}_2 &= x_2 \end{aligned}$$

$$\begin{aligned} \dot{x}_1 &= jx_1 + x_2 \\ \dot{x}_2 &= jx_2 \end{aligned}$$

$$\begin{aligned} x_1(k+1) &= -2x_1(k) \\ x_2(k+1) &= 0.5x_2(k) \end{aligned}$$

**Exercise 2** Determine (by hand) the stability properties of a linear continuous-time system with

$$A = \begin{pmatrix} \frac{1}{2} & 1 & -\frac{1}{2} & 0 \\ -1 & \frac{1}{2} & 0 & -\frac{1}{2} \\ \frac{1}{2} & 0 & -\frac{1}{2} & 1 \\ 0 & \frac{1}{2} & -1 & -\frac{1}{2} \end{pmatrix}$$

Using the eig command on MATLAB, what would your stability conclusion be?

**Exercise 3** Using linearization determine (if possible) the stability properties of each of the following systems about their corresponding specified equilibrium solution $q^e$. If not possible, provide a reason.

(a)

$$\ddot{q} + (\dot{q} - 1)|\dot{q} - 1| + 2\sin q = 0$$

and $q^e = \pi/6$.

(b)

$$\begin{aligned} \dot{q}_1 &= e^{q_1} q_2 - q_1^3 \\ \dot{q}_2 &= -q_1 \cos q_2 \end{aligned}$$

and $q^e = [0 \;\; 0]'$.

(c)

$$\begin{aligned} \ddot{q}_1 &= q_2 \\ \ddot{q}_2 &= \sin q_1 \end{aligned}$$

and $q^e = [0 \;\; 0]'$.

**Exercise 4** Determine the stability properties of the following system about the zero solution.

$$\begin{aligned} x_1(k+1) &= x_1(k)^3 + \sin(x_2(k)) \\ x_2(k+1) &= -\frac{1}{2}\cos(x_2(k))x_1(k) + x_2(k)^3 \end{aligned}$$

**Exercise 5** Stability properties of the two pendulum cart system. Using MATLAB, determine the stability properties of the linearizations $L1$ and $L2$. What can you say about the stability properties of the nonlinear system about the corresponding equilibrium states?