

AAE 666 Spring 2021

HOMEWORK FIVE

Due: Friday, February 26

Exercise 1 Using linearization, determine (if possible) the stability properties of the following system about the zero solution.

$$\frac{d^4 q}{dt^4} - \sin(q) = 0.$$

If not possible, explain why.

Exercise 2 Using linearization, determine (if possible) the stability properties of the following system about the zero solution.

$$\ddot{q} + \dot{q} - q^3 = 0$$

If not possible, explain why.

Exercise 3 If possible, use linearization to determine the stability properties of each of the following systems about the zero equilibrium state.

(i)

$$\begin{aligned}\dot{x}_1 &= (1 + x_1^2)x_2 \\ \dot{x}_2 &= -x_1^3\end{aligned}$$

(ii)

$$\begin{aligned}\dot{x}_1 &= \sin x_2 \\ \dot{x}_2 &= (\cos x_1)x_3 \\ \dot{x}_3 &= e^{x_1}x_2\end{aligned}$$

Exercise 4 If possible, use linearization to determine the stability properties of the following system about the zero equilibrium state.

$$\begin{aligned}x_1(k+1) &= x_1(k)^2 + \sin(x_2(k)) \\ x_2(k+1) &= 0.4 \cos(x_2(k)) x_1(k)\end{aligned}$$

Exercise 5 If possible, use linearization to determine the stability properties of the following system about the zero equilibrium state.

$$\begin{aligned}x_1(k+1) &= (1 + x_1(k)^3)x_2(k) \\ x_2(k+1) &= x_1(k)^3 + x_2(k)^5\end{aligned}$$

Exercise 6 If possible, use linearization to determine the stability properties of the following system about the zero equilibrium state.

$$\begin{aligned}x_1(k+1) &= x_2(k) \\x_2(k+1) &= \sin(x_1(k)) + x_2(k)^5\end{aligned}$$

Exercise 7 Recall the Lorenz system

$$\begin{aligned}\dot{x}_1 &= \sigma(x_2 - x_1) \\ \dot{x}_2 &= rx_1 - x_2 - x_1x_3 \\ \dot{x}_3 &= -bx_3 + x_1x_2\end{aligned}$$

with $\sigma, r, b > 0$. Prove that all solutions of this system are bounded. (Hint: Consider $V(x) = rx_1^2 + \sigma x_2^2 + \sigma(x_3 - 2r)^2$.)