Name	Team Number
Solution	

AAE 251: Introduction to Aerospace Design

Assignment 1—Pitot Tubes

Due Tuesday 22 January, 10:00 am on Blackboard

Instructions

This assignment has two parts—the derivation and application of Pitot tubes, and a Matlab assignment. Start now, or you will run out of time!

In part 1 of the assignment we will apply Bernoulli's equation to understand how a Pitot tube works. You may find Anderson Chapter 4 very helpful, especially the examples.

In part 2 of the assignment we will write a code to reproduce the standard atmosphere. You will find the code useful in future Homeworks and your projects.

Write or type your answers into the appropriate boxes. Make sure you submit a single PDF, with responses to both parts on Blackboard. Your homework will be a handy study guide.

Step 0: Watching old TV

Here is a clear albeit old video discussing static and dynamic pressure, and, right at the end, pitot tubes. Take a look and marvel at how TV has improved and the laws of physics have remained true:

https://youtu.be/dTXHoIs64hw

Step 1: Define Stagnation Pressure

When a stream of uniform velocity flows into a blunt body like an airfoil, the stream lines are as shown in Figure 1.

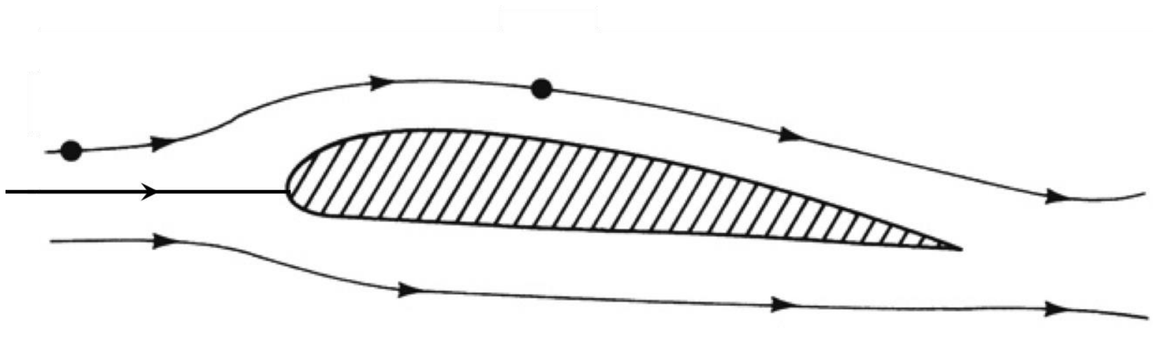

Figure 1: Stagnation Point

Some of the streamlines move over the airfoil, and some of them move under. But one streamline goes to the tip of the airfoil and then stops. At this *stagnation point*, the velocity is zero.

Let's use the Bernoulli equation we derived in class to calculate the pressure at this point, or the *stagnation pressure*.

First, indicate your velocities and pressures on Figure 1: Designate a point upstream where the flow is undisturbed as having velocity and pressure (u_1, p_1), and the stagnation point as (u_2, p_2).

What assumption do we have to make about the fluid to apply Bernoulli's equation?

Answer: Incompressible flow

Now write down Bernoulli's equation to relate these two pairs of values, and solve for the stagnation pressure:

<u>Answer</u>:

$$\frac{p_1}{\rho g} + \frac{u_1^2}{2g} = \frac{p_2}{\rho g} + \frac{u_2^2}{2g}$$

Assume flow at position 2 is at rest, so $u_2^2 = 0$. Thus, p_2 will be the stagnation pressure.

Also, divide out gravitational acceleration term to get:

$$\frac{p_1}{\rho} + \frac{u_1^2}{2} = \frac{p_2}{\rho}$$

Multiply by density to solve for p_2

$$p_2 = p_1 + \frac{1}{2}\rho u_1^2$$

What is the difference between p_2 and p_1 called?

<u>Answer</u>: Dynamic pressure

Is p_2 larger or smaller than p_1, and why would you expect or not expect that from a physical point of view?

<u>Answer</u>:

The dynamic pressure is positive. The stagnation pressure is larger, because the fluid has gone from a positive velocity to zero velocity.

Step 2: Create a simple velocity measurement device

The body stopping the fluid does not have to be solid—it can also be a static column of fluid (including air). So, if we want to measure the velocity of an airstream, we just need to find a way to bring some of that air to rest. Figure 2 shows a very simple way of doing this.

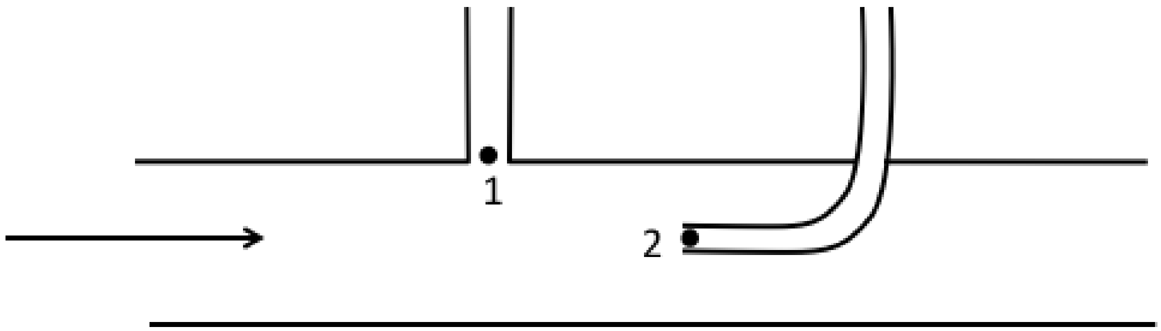

Figure 2: A piezometer and Pitot tube combination

What is the velocity at point 2?

<u>Answer</u>: zero

On Figure 2, shade in the fluid, and indicate its relative height in the two tubes. Which tube is the fluid higher in, and why?

<u>Answer</u>: The fluid in the bent tube is higher because the stagnation pressure is larger.

Now derive an expression for the free stream velocity in the tube, using the pressure measurements at points 1 and 2:

<u>Answer</u>:

Start with equation from step 1:

$$p_2 = p_1 + \frac{1}{2}\rho u_1^2$$

This relates the static pressure, the stagnation pressure, and the velocity.

Solve for the velocity term u_1 through algebriaic manipulation:

$$2(p_2 - p_1) = \rho u_1^2$$

Then

$$u_1 = \sqrt{\frac{2(p_2 - p_1)}{\rho}}$$

Step 3: A better device—The Pitot Static Tube

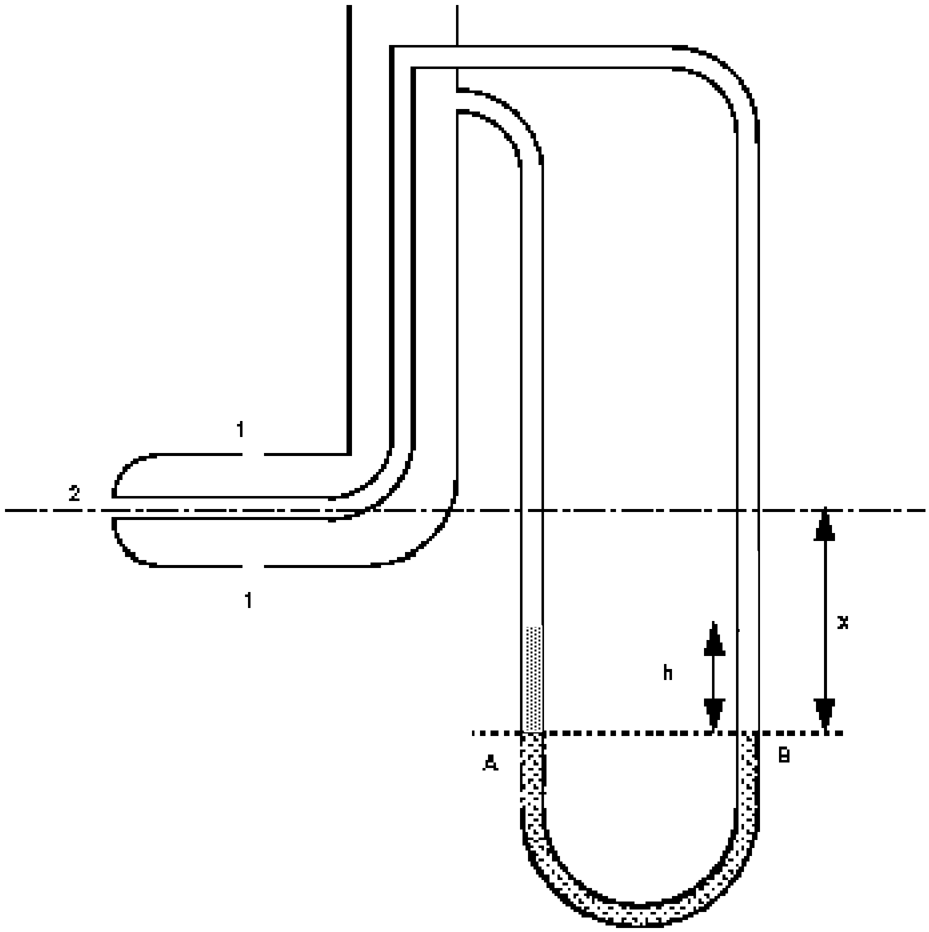

Figure 3: A Pitot Static Tube

Figure 3 shows a slightly more elegant arrangement, where we use a single manometer rather than two piezometers. Write an expression for the free stream velocity in terms of the height difference, h, on the figure. Assume that the density of the fluid in the manometer is given by ρ_{man}.

Answer:

The pressure at point A will be the pressure at point 1 plus the pressure of the air column above the point and the pressure of the water column above the point

$$p_A = p_1 + \rho_{air} g(x - h) + \rho_{man} gh$$

The pressure at point B will be the pressure at point 2 plus the pressure of the air column above the point and the water column above the point:

$$p_B = p_2 + \rho_{air} gx$$

Given the low density of air relative to the water, the expressions can be simplified by assuming that the contribution from the air column is negligible. This will then give:

$$p_A = p_1 + \rho_{man} gh$$

$$p_B = p_2$$

Since point A and B are located at the same vertical height and since the system is connected and in equilibrium, the pressures at these two points will be equal. Thus:

$$p_2 = p_1 + \rho_{man} gh$$

From above, we know that:

$$u_1 = \sqrt{\frac{2(p_2 - p_1)}{\rho}}$$

In this case the density in the denominator is the density of the air. By plugging in the expression from above, the expression for velocity becomes:

$$u_1 = \sqrt{\frac{2(p_1 + \rho_{man} gh - p_1)}{\rho_{air}}}$$

This simplifies to

$$u_1 = \sqrt{\frac{2\rho_{man} gh}{\rho_{air}}}$$

The static Pitot tube has some practical limitations:

At low velocities:

The pressure difference we're trying to measure may be smaller than the measurement error. This will lead to erroneous or unreliable velocity values.

At supersonic velocities:

We can no longer assume incompressibility.

In very cold and humid conditions:

The tube may become partially or completely clogged with ice.

Step 4: Application

Let's consider now the last set of limitations for pitot tubes—clogging up with ice.

1. An aircraft is performing aerobatic maneuvers when the static port suddenly becomes clogged with ice, unbeknownst to the pilot. If the pilot is in a fast dive, will his airspeed reading be affected, and, if so, how? Explain your answer using appropriate equations.

Yes, it will be affected.

The pitot tube used for measuring air speed consists of two ports – the pitot port and the static port. The pitot port, port 2 in Figure 3, measures the ram pressure caused by air entering the port as the aircraft moves forward. The static port, port 1, measures the outside atmospheric pressure.

Applying Bernoulli's equation at the two ports (similar to Step 2), we get

$$u_1 = \sqrt{\frac{2(p_2 - p_{1c})}{\rho_{air}}}$$

Here, p_2 is the ram pressure caused by air entering the port as the aircraft moves forward. The velocity at port 2 is zero.

$p_{1c}$is the reading from the clogged static port.

As the pilot dives, p_2 increases due to decrease in altitude but p_{1c} stays the same because it is clogged. Therefore, $u_1$will be faster speed than the actual speed.

You may find the following video helpful in understanding how an airspeed indicator works:

https://www.youtube.com/watch?v=wxzEQ48CJac

2. The pilot now begins a dramatic climb, still unaware that the static port is clogged. What do you expect the airspeed reading to indicate now?

The reading will be faster than the actual airspeed until the airplane climbs to the altitude where the static port clogged. If the airplane keeps climbing, the reading will be slower than the actual airspeed.

3. Another pilot at the same ill-fated air show experiences a sudden blockage in her pitot port. If she is accompanying our first pilot on a rapid dive, how would her airspeed reading be affected?

From step 2, u_1 is:

$$u_1 = \sqrt{\frac{2(p_{2c} - p_1)}{\rho}}$$

where p_{2c} is the reading from the clogged pitot port.

As the pilot dives, p_1 increases but p_{2c} stays the same because it is clogged. Therefore, u_1 will be slower speed than the actual speed.

4. Unbelievably enough, yet another aircraft at the same air show is also having problems. In this case though, the pitot port becomes gradually clogged over the course of the day. How will this pilot's airspeed reading be affected? Why might landing this aircraft be particularly challenging?

Indicated airspeed will generally be lower than actual, and difference will increase as the clog gets worse. When they come in to land, the pilot will have no idea (underestimate when descending and the error will be bigger as clog gets worse).

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Matlab Assignment

Write a **Matlab** program to reproduce the standard atmosphere in Appendix B of Anderson. Recreate this data, and also calculate the speed of sound at each altitude. Show your results using a set of appropriate graphs. Show your data in SI and English units. Make sure your graphs are properly labeled.

Use equations from Anderson Ch 3.4. No credit will be given for simply entering the data from the tables. Program your code to produce correctly labeled plots for altitude vs. temperature, pressure, density, and the speed of sound similar to Figure 4 (up to 100,000 ft or 30.48 km).

Your HW file must contain the following:

- This page, with your name filled in.
- Your MATLAB code (including comments).
- A write-up of the formulas you used to create your table.
- A set of graphs showing how pressure, temperature, and density, and the speed of sound vary with altitude, plotted in SI and English units [8 graphs in total].

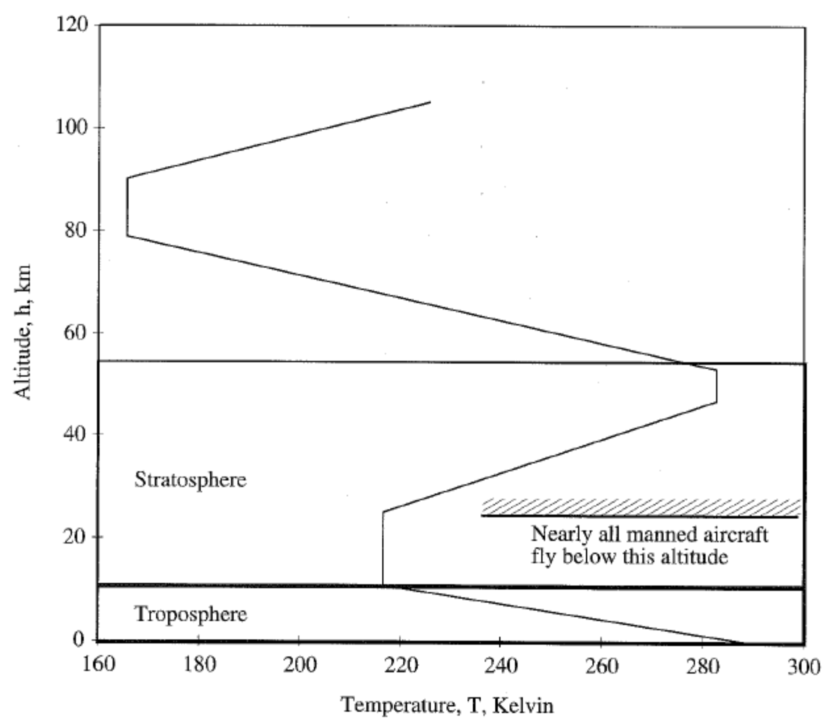


Figure 4 Standard Atmosphere temperature model (Brandt, 2004)

Sample Code


```
clc
clear all
close all
```

Constants SI Units

```
P_s = 1.01325e5; % Sea Level Pressure (kPa (N/m^2))
rho_s = 1.2250; % Sea Level Density (kg/m^3)
T_s = 288.16; % Sea Level Temperature (deg K)
g0 = 9.8; % Standard Gravity (m/s^2)
R = 287; % Gas Constant for Air(J/kg*K)
gamma = 1.4; % Specific Heat cp/cv (Unitless)

h = 0:30480; % Altitude Range of Interest (0 m to 30,480 m)

i = 1; % Parameter Loop Index

% pre allocating arrays
l = length([0:10:30480]);
T_SI(1:l,1:2) = 0;
P_SI(1:l,1:2) = 0;
rho_SI(1:l,1:2) = 0;
a_SI(1:l,1:2) = 0;
```

Atmospheric Parameter Calculation Loop

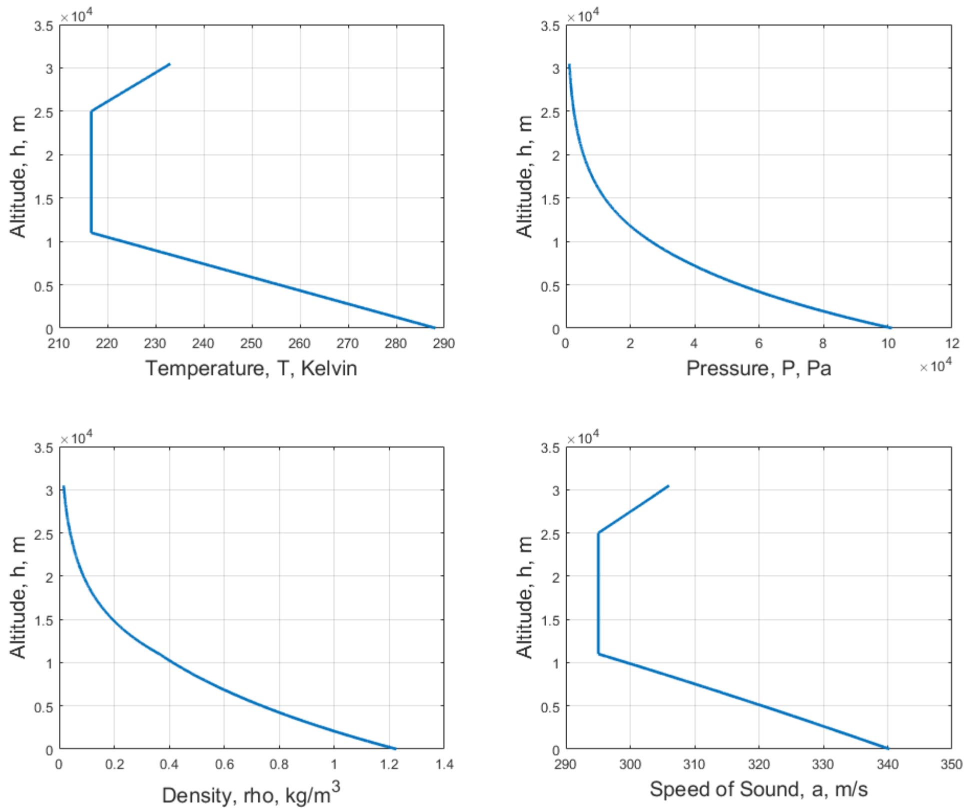
```
for h = 0:10:30480 % Move up in altitude by 10m jumps

    h_Eng(i) = h*3.28084; % SI to English Units (m -> ft)

    if h <= 11000 % First Altitude Block (0 m to 11,000 m)
        a = - 6.5e-3; % Temperature Lapse Rate (deg K) (T2-T2 / (h2 -  h1))
        h0 = 0; % sea level (m)
        T_SI(i,:)=[h, T_s+a*(h-h0)]; % Creates column array [alt, temp]in SI units from
(0 m - 11,000 m) (deg K)
        P_SI(i,:)=[h, P_s*(T_SI(i,2)/T_s)^(-g0/(a*R))]; % Creates column array
[alt,pressure] in SI units from (0 m - 11,000 m) (kPa)
        rho_SI(i,:)=[h rho_s*(T_SI(i,2)/T_s)^(-(g0/(a*R)+1))]; % Creates column array
[alt, density] in SI units from (0 m - 11,000 m) (kg/m^3)
        a_SI(i,:)=[h sqrt(gamma*R*T_SI(i,2))]; % Creates column array [alt, speed of
```

```
sound] in SI units from (0 m - 11,000 m) (m/s)

        if h == 11000 % at atmospheric transition point the final parameter value becomes
the base value of the next atmospheric zone
            h1=h;
            T1=T_SI(i,2);
            P1=P_SI(i,2);
            rho1=rho_SI(i,2);
        end
        i=i+1; % Next Altitude Index

    elseif h > 11000 && h <= 25000 % Second Atmospheric block (11,010 m to 25,000)
        % No temperature lapse rate as temp is constant with altitude
        T_SI(i,:)=[h T1];
        P_SI(i,:)=[h P1*exp(-g0/(R*T_SI(i,2))*(h-h1))];
        rho_SI(i,:)=[h rho1*exp(-g0/(R*T_SI(i,2))*(h-h1))];
        a_SI(i,:)=[h sqrt(gamma*R*T_SI(i,2))];

        if h == 25000 % at atmospheric transition point the final parameter value becomes
the base value of the next atmospheric zone
            h2=h;
            T2=T_SI(i,2);
            P2=P_SI(i,2);
            rho2=rho_SI(i,2);
        end

        i=i+1;

    elseif h > 25000 && h <= 47000 % Third Atmospheric block (25,010 m to 47,000) (up to
47 km because it includes 30.48 km)
        a=3e-3; % Temperature lapse rate (note that it is positive as temp is increasing
with altitude)
        T_SI(i,:)=[h T2+a*(h-h2)];
        P_SI(i,:)=[h P2*(T_SI(i,2)/T2)^(-g0/(a*R))];
        rho_SI(i,:)=[h rho2*(T_SI(i,2)/T2)^(-(g0/(a*R)+1))];
        a_SI(i,:)=[h sqrt(gamma*R*T_SI(i,2))];

        i=i+1;

    end
end
```

Plotting Results (SI)

```
% Adjusting Plot axis font size and plot line width
Fontsize=15;
Linewidth=2;

figure(1)
% Because the Parameters and Altitudes are in vector forms, plotting the
% Alt(y-axis) against the Parameter (x_axis)
```

```
subplot(2,2,1)
plot(T_SI(:,2),T_SI(:,1),'Linewidth',Linewidth)
xlabel('Temperature, T, Kelvin','FontSize',Fontsize);
ylabel('Altitude, h, m','FontSize', Fontsize);
% set(gca,'FontSize',Fontsize)
box on
grid on

subplot(2,2,2)
plot(P_SI(:,2),P_SI(:,1),'Linewidth',Linewidth)
ylabel('Altitude, h, m','FontSize', Fontsize);
xlabel('Pressure, P, Pa','FontSize', Fontsize);
box on
grid on

subplot(2,2,3)
plot(rho_SI(:,2),rho_SI(:,1),'Linewidth',Linewidth)
ylabel('Altitude, h, m','FontSize', Fontsize);
xlabel('Density, rho, kg/m^3','FontSize', Fontsize);
box on
grid on

subplot(2,2,4)
plot(a_SI(:,2),a_SI(:,1),'Linewidth',Linewidth)
ylabel('Altitude, h, m','FontSize', Fontsize);
xlabel('Speed of Sound, a, m/s','FontSize', Fontsize);
set(gca,'XLim',[290 350])
box on
grid on
set(gcf,'PaperPositionMode','auto','Position',[0 0 1100 850]) % Control where plots are
positioned
```



Plotting Results (English)

```
% Convert SI units to English units
T_Eng=1.8.*T_SI(:,2);

P_Eng=0.020885438.*P_SI(:,2);

rho_Eng=rho_SI(:,2)./515.3788184;

a_Eng=3.28084.*a_SI(:,2);


figure(2)
subplot(2,2,1)
plot(T_Eng,h_Eng,'Linewidth',Linewidth)
xlabel('Temperature, T, \circR','FontSize',Fontsize);
ylabel('Altitude, h, ft','FontSize', Fontsize);
box on
grid on

subplot(2,2,2)
plot(P_Eng,h_Eng,'Linewidth',Linewidth)
ylabel('Altitude, h, ft','FontSize', Fontsize);
xlabel('Pressure, P, lb/ft^2','FontSize', Fontsize);
```

```
box on
grid on

subplot(2,2,3)
plot(rho_Eng,h_Eng,'Linewidth',Linewidth)
ylabel('Altitude, h, ft','FontSize', Fontsize);
xlabel('Density, rho, slug/ft^3','FontSize', Fontsize);
box on
grid on

subplot(2,2,4)
plot(a_Eng,h_Eng,'Linewidth',Linewidth)
ylabel('Altitude, h, ft','FontSize', Fontsize);
xlabel('Speed of Sound, a, ft/s','FontSize', Fontsize);
% set(gca,'XLim',[290 350])
box on
grid on

set(gcf,'PaperPositionMode','auto','Position',[0 0 1100 850])
```