P1: B BP

2020年1月24日 金曜日　　午後0:31

Solving/Analyzing a spacecraft dynamics problem requires

1. Construction of a kinematical representation of the motion (numerous options available)

2. Using principles of mechanics to formulate equations of motion that govern the quantities appearing in the kinematical representation

→ critical – no use for #3 if step 2 incorrect

3. Extracting useful info from equations

Since every problem in spacecraft dynamics analysis requires the use of various kinematical relationships (some important only since the space age)

⇨ Here we start!!

Rotational Kinematics

To investigate spacecraft attitude, we need effective ways to describe attitude / orientation; rigid bodies

Begin with the most basic concept

Simple Rotation

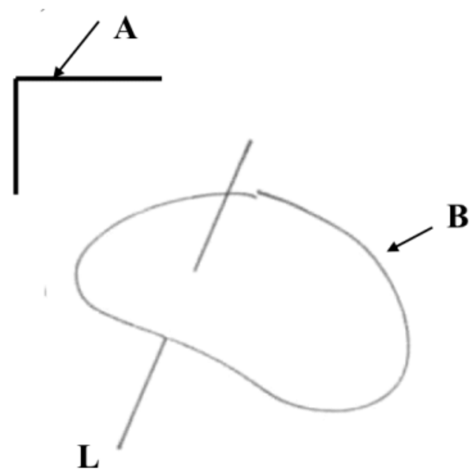
How to describe orientation of a rigid body B relative to some reference orientation?

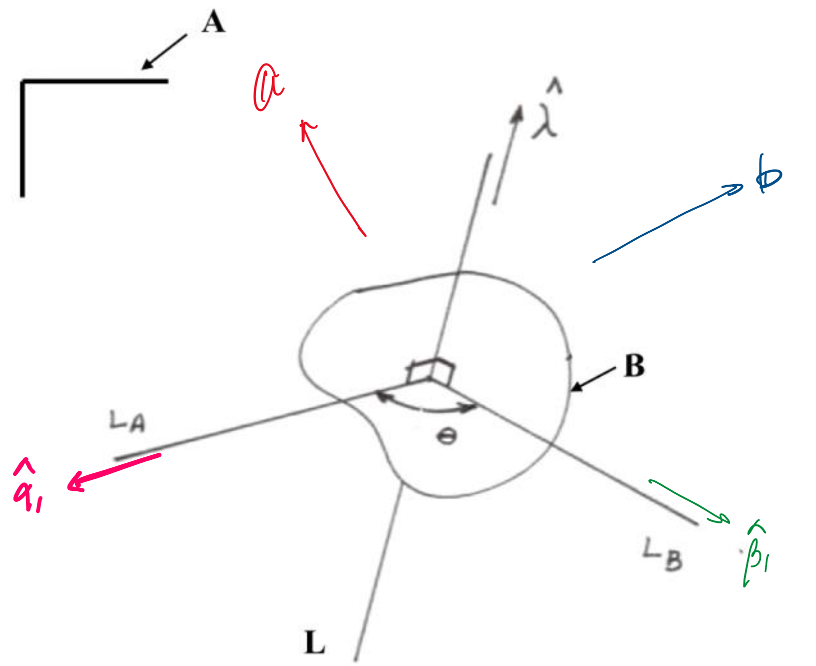
Note: Description is mathematical and for analysis
NOT necessarily how the attitude is achieved

ANY change in orientation is mathematically accomplished via a simple rotation

Define:
Coordinate frame A, B
A may be inertial
B is body-fixed

Simple rotation of B in A:
There exists a line L (axis of rotation) whose orientation relative to both A and B remains fixed





To visualize the rotation

Define: $L_A \perp L$

vector basis $\hat{\alpha}_1, \hat{\alpha}_2, \hat{\lambda}$ fixed in A

Define: $L_B \perp L$

vector basis $\hat{\beta}_1, \hat{\beta}_2, \hat{\lambda}$ fixed in B

such that initially

$L_B = L_A$

$\hat{\beta}_1 = \hat{a}_1$

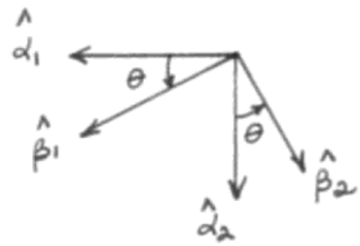
$\hat{\beta}_2 = \hat{a}_2$

Rotate B about L $\left(\hat{\lambda}\right)$ through θ to achieve the above orientation

Note: $\hat{\lambda}$ is fixed in both A and B

Orientation of B in A is understood by knowing L $\left(\hat{\lambda}\right)$ and θ

Of course, can also represent information in the form of a direction cosine matrix



$\hat{\beta}_1 = \cos\theta\,\hat{\alpha}_1 + \sin\theta\,\hat{\alpha}_2$

$\hat{\beta}_2 = -\sin\theta\,\hat{\alpha}_1 + \cos\theta\,\hat{\alpha}_2$

Note: not symmetric

$$\begin{bmatrix}\hat{\beta}_1 & \hat{\beta}_2 & \hat{\lambda}\end{bmatrix} = \begin{bmatrix}\hat{\alpha}_1 & \hat{\alpha}_2 & \hat{\lambda}\end{bmatrix} \underbrace{\begin{bmatrix}\cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1\end{bmatrix}}_{\text{Direction Cosine Matrix } {}^{A}C^{B}}$$

Add an arbitrary vector $\bar{a}$ fixed in A; define $\bar{b}$ such that $\bar{b} = \bar{a}$ initially

$\bar{a} = p\,\hat{\alpha}_1 + q\,\hat{\alpha}_2 + r\,\hat{\lambda}$

From the way the unit vectors are defined

$b = p\hat{\beta}_1 + q\hat{\beta}_2 + r\hat{\lambda}$

b is always going to be a func. of initial cond. a + representation of the rotation $\hat{\lambda}, \theta$

Describe $\bar{b}$ as a function of $\bar{a}$ (its initial value) and the "rotation parameters" $\hat{\lambda}, \theta$

$$\boxed{b = a\cos\theta - a\times\hat{\lambda}\sin\theta + a\cdot\underbrace{\hat{\lambda}\hat{\lambda}}_{\text{dyad}}(1-\cos\theta)}$$

Proof: LHS (write $\bar{b}$ in terms of $\hat{\alpha}_i$'s)

$\bar{b} = p\,\hat{\beta}_1 + q\,\hat{\beta}_2 + r\,\hat{\lambda}$

$= p(C_\theta\hat{a}_1 + S_\theta\hat{a}_2) + q(-S_\theta\hat{a}_1 + C_\theta\hat{a}_2) + r\hat{\lambda}$

$= (pC_\theta - qS_\theta)\hat{a}_1 + (pS_\theta + qC_\theta)\hat{a}_2 + r\hat{\lambda}$

RHS (perform operations)

$$\bar{b} = \left(p\,\hat{\alpha}_1 + q\,\hat{\alpha}_2 + r\,\hat{\lambda}\right)\cos\theta$$
$$-\left(p\,\hat{\alpha}_1 + q\,\hat{\alpha}_2 + r\,\hat{\lambda}\right)\times\hat{\lambda}\sin\theta$$
$$+\left(p\,\hat{\alpha}_1 + q\,\hat{\alpha}_2 + r\,\hat{\lambda}\right)\bullet\hat{\lambda}\,\hat{\lambda}(1-\cos\theta)$$

$$\bar{b} = (p\cos\theta - q\sin\theta)\hat{\alpha}_1 + (q\cos\theta + p\sin\theta)\hat{\alpha}_2 + r\,\hat{\lambda}$$

QED

We know that the theorem is correct but it is not a particularly convenient representation and does not generalize well

➡ More convenient to use a dyadic format

Define: $\bar{\bar{R}} = \bar{\bar{U}}\cos\theta - \bar{\bar{U}} \times \hat{\lambda}\sin\theta + \hat{\lambda}\hat{\lambda}(1-\cos\theta)$

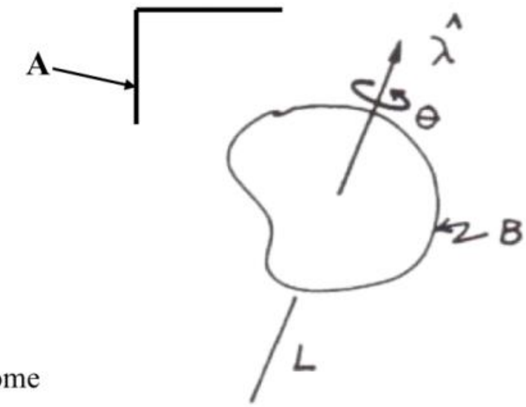
simple rotation dyadic

Unit or identity dyadic

Then rewrite theorem:

$$\boxed{b = a \cdot \bar{\bar{R}}}$$

Simple rotation is the most basic way to think of a change in orientation and easiest to visualize



But, really need to relate initial/final orientation to some reference

requires unit vectors and a set of variables that are common to all; these are variables we can use to describe orientation; variables that can be used to write EOMs for attitude motion

diff var.
each w/ version of simple rot. (SRT)

Many variables to choose from; all have advantages / disadvantages

$\Downarrow$

Examples (certainly not all-inclusive)

3 rotational DOF

To actually use these variables, 4 quantities must be defined (λ_i, θ)

1. Euler axis / angle $\hat{\lambda}, \theta$ (4)

   Pro: Physical meaning

   Con: · one redundant variable

   · trig funcs. $\left\{ \begin{array}{l} \text{non-linear} \\ \text{double-valued} \end{array} \right.$

   Common application: slew maneuvers

2. Euler angles (3) roll, pitch, yaw

   (already familiar with these; in class consider 24 different sets)

   Pro: no redundant parameters

   physical interpretation in some cases.

   Con: · trig funcs

   · singularities

   · no convenient rule for successive rotations

   Common applications: analytical studies
   input / output
   attitude control of 3-axis stabilized spacecraft

   (Some suggest that Euler angles overemphasized in textbooks)

3. Direction cosines elements C_{ij} (9)

   Pro: no singularities

   no trig func.

   convenient rule for successive approx.

   Con: 6 redundant parameters

   Common applications: analysis
   vector transformations

4. Euler parameters – also called quaternions (4)

Pro: no singularities
no trig
convenient rule for successive approx.

Con: one redundant parameter
no obvious physical interpretation

Common applications: onboard inertial navigation
attitude analysis and control

5. Rodrigues parameters – also called Gibbs vector (3)

Pro: no redundant parameter
no trigs
convenient rule for successive approx.

Con: singularities

Common application: analytical studies

6. ……

All are ways of describing orientation; each set of variables are related to each other; each set are therefore related to $\hat{\lambda}, \theta$

We will not actually work with $\hat{\lambda}, \theta$ as kinematic variables; we will use $\hat{\lambda}, \theta$ to help visualize problem; use $\hat{\lambda}, \theta$ to relate variable sets to each other

Examine the new variable sets one at a time $\longrightarrow$ start with <u>direction cosines</u> (already somewhat familiar)

Four important truths

(1) A minimum of 3 coord are required to describe relative ang. orientation between two frames

(2) Any minimal set 3 orientation

(2) Any minimal set 3 orientation
include at lease 1 geometrical attitude

(3) singularity

4) regularization → at least one redundant parameter