AAE 666 Spring 2021

Homework Two

Friday, February 5

Exercise 1 Determine the nature (node, focus, etc) of each equilibrium state of the damped duffing system

$$\ddot{y} + 0.1\dot{y} - y + y^3 = 0$$

Numerically obtain the phase portrait.

Exercise 2 Determine the nature (if possible) of each equilibrium state of the simple pendulum.

$$\ddot{y} + \sin y = 0$$

Numerically obtain the phase portrait.

Exercise 3 For each of the following systems, determine (from the state portrait) the stability properties of each equilibrium state. For AS equilibrium states, give the region of attraction.

(a)

$$\dot{x} = -x - x^3$$

(b)

$$\dot{x} = -x + x^3$$

(c)

$$\dot{x} = x - 2x^2 + x^3$$

Exercise 4 Show that all non-zero solutions of the following differential "blow up" in a finite time. Compute the "blow-up" time as a function of initial state.

$$\dot{x} = x^3$$

Exercise 5 Prove that no solution of the following differential equation can "blow up" in a finite time.

$$\dot{x} = \frac{x}{1 + x^2} + \sin(x)$$

Exercise 6 What what initial states x_0 can you guarantee that the following equation has a unique solutions with $x(0) = x_0$. Justify your answer.

$$\dot{x} = -\sqrt{(1 - x)^2}$$