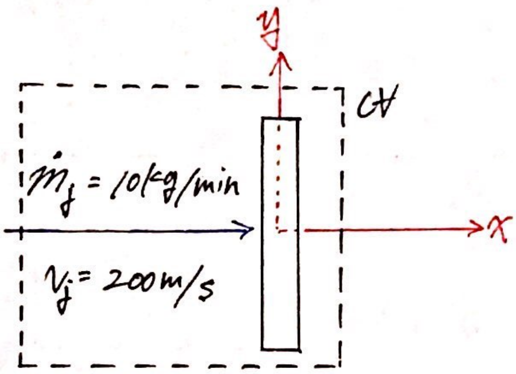
Q1.

Given:



jet impinged directly on normally oriented plate.

Assumptions: steady, uniform flow
1D flow.

Find: Calculate the force, F on plate

Soln → First convert the units of the mass flow rate to be congruent w/ the velocity.

$$\dot{m}_j = \frac{10\ kg}{min} \cdot \frac{1\ min}{60\ s} = \frac{1}{6} \frac{kg}{s}$$

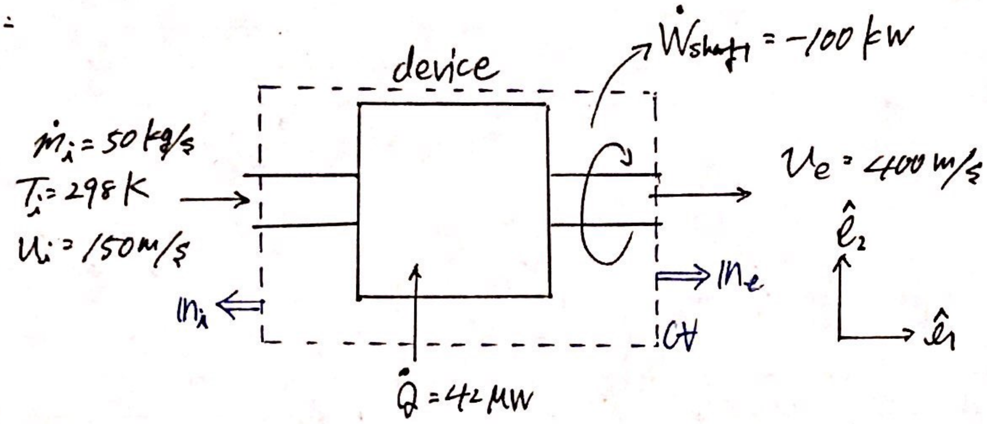
force $F = \dot{m}_j \cdot v_j$

$$\therefore F = \left(\frac{1\ kg}{6\ s}\right)\left(\frac{200\ m}{s}\right) = 33.33 \frac{kg \cdot m}{s^2} = \boxed{33.33\ N}$$

Scanned with CamScanner

Q2.

Given:



Assumptions: 100% efficiency of shaft, steady, uniform 1-D flow, $c_p = 1.0\,\frac{kJ}{kg\cdot K}$, ρ = const

Find: (1) specific stagnation enthalpy @ inlet $h_{0,i}$
(2) specific stagnation enthalpy @ outlet $h_{0,e}$
(3) temperature @ exit T_e
(4) How does c_p vary over this temperature range?
→ was the assumption of const. c_p adequate?

(1) Use mass conservation

$$\cancelto{0}{\frac{\partial}{\partial t}\int_{CV}\rho\, dV} + \int_{CS}\rho\, \mathbf{u}\cdot\mathbf{n}\, dA = 0$$

$$\int_i \mathbf{u}_i\cdot\mathbf{n}_i\,\rho\, dA + \int_e \mathbf{u}_e\cdot\mathbf{n}_e\,\rho\, dA = 0$$

$$-\dot{m}_i + \dot{m}_e = 0$$

$$\therefore \dot{m}_i = \dot{m}_e \quad \cdots ①$$

Use energy conservation.

$$\cancelto{0}{\frac{\partial}{\partial t}\int_{CV}\left(e + \frac{u^2}{2} + gz\right)\rho\, dV} + \int_{CS}\left(h + \frac{u^2}{2} + \cancelto{0}{gz}\right)\rho(\mathbf{u}\cdot\mathbf{n})dA = \dot{Q} - \dot{W}_{shaft}$$

$$\int_i \left(h_i + \frac{u_i^2}{2}\right)\rho(\mathbf{u}_i\cdot\mathbf{n}_i)dA + \int_e\left(h_e + \frac{u_e^2}{2}\right)\rho(\mathbf{u}_e\cdot\mathbf{n}_e)dA = \dot{Q} - \dot{W}_{shaft}$$

$$-\dot{m}_i\left(h_i + \frac{u_i^2}{2}\right) + \dot{m}_e\left(h_e + \frac{u_e^2}{2}\right) = \dot{Q} - \dot{W}_{shaft}$$

$$\because ① \qquad -h_i - \frac{u_i^2}{2} + h_e + \frac{u_e^2}{2} = \frac{\dot{Q} - \dot{W}_{shaft}}{\dot{m}_i}$$

$$\therefore h_{0,e} = \frac{\dot{Q} - \dot{W}_{shaft}}{\dot{m}_i} + h_i + \frac{u_i^2}{2} \quad \cdots ②$$

from the eqn. $dh = c_p dT \Leftrightarrow h = c_p T$

$$\therefore h_i = c_p T_i = \left(1.0\,\frac{kJ}{kg\cdot K}\right)(298\,K) = 298\,\frac{kJ}{kg} \quad \cdots ③$$

then stagnation enthalpy @ inlet $h_{o,i}$ is

$$h_{o,i} = h_i + \frac{u_i^2}{2}$$
$$= \left(298 \frac{kJ}{kg}\right) + \left(\frac{150\,m}{s}\right)^2 (0.5) \left(\frac{1\,kJ}{1000\,J}\right)$$
$$= 309.25 \frac{kJ}{kg}$$

$$\boxed{h_{o,i} = 309.25 \frac{kJ}{kg}}$$

(2) substituting ③ into ②

$$h_{o,e} = \left[\left(\frac{42\times10^3\,kJ}{s}\right) - \left(\frac{-100\,kJ}{s}\right)\right]\left(\frac{s}{50\,kg}\right) + 298 \frac{kJ}{kg} + \left(\frac{150\,m}{s}\right)^2 (0.5)\left(\frac{1\,kJ}{1000\,J}\right)$$
$$= 1151.25 \frac{kJ}{kg}$$

$$\boxed{h_{o,e} = 1151.25 \frac{kJ}{kg}}$$

(3) $h_e = h_{o,e} - \frac{u_e^2}{2}$

$$= \left(1151.25 \frac{kJ}{kg}\right) - \left(\frac{400\,m}{s}\right)^2 (0.5)\left(\frac{1\,kJ}{1000\,J}\right) = 1071.25 \frac{kJ}{kg}$$

since $h = c_p T$

$$T_e = \frac{h_e}{c_p} = \left(\frac{1071.25\,kJ}{kg}\right)\left(\frac{kg\text{-}K}{1.0\,kJ}\right) = 1071.25 \approx 1071\,K$$

$$\boxed{T_e = 1071\,K}$$

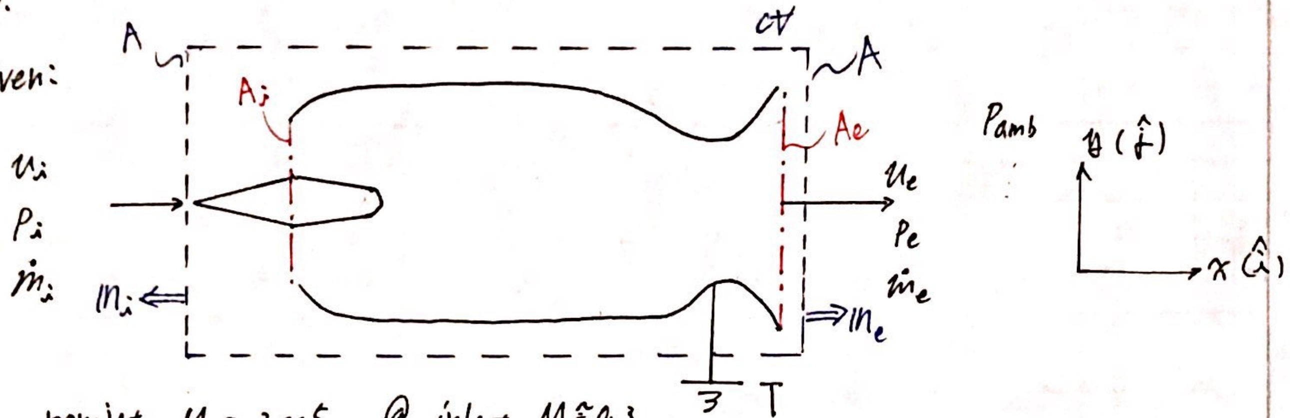
(4) Use 1st Law

$$c_p' = \frac{h_i - h_e}{T_i - T_e} = \frac{\left(\frac{298\,kJ}{kg}\right) - \left(\frac{1071.25\,kJ}{kg}\right)}{298\,K - 1071.25\,K}$$
$$= 1.0 \frac{kJ}{kg\text{-}K}$$

the temperature gradient is constant for c_p and this satisfies the assumption of $c_p = const.$

Q3.

Given:



ramjet $M = 2 \sim 5$ @ inlet $M \doteq 0.3$

assumptions: 1-D uniform flow, isentropic, steady

Find: (a) Derive thrust equation.

(b) $h = 15000\,m$, $M = 2.0$ $\gamma = const. = 1.4$

calculate (i) T_0 @ inlet

(ii) p_0 @ inlet

(iii) U_∞ up the inlet

(iv) if $M_e = 2.0$ what is P_e

(V) $\frac{T_0}{T_e}$

(a) Use momentum conservation

$$\cancel{\frac{\partial}{\partial t}}\int_{cv}^{0} \rho \mathbf{u}\, d\forall + \int_{cs} \rho \mathbf{u}(\mathbf{u}\cdot\mathbf{n})\,dA = \Sigma \mathbf{F}$$

L.H.S $\rightarrow$ $\int_i \rho u_i[(u_i\hat{i})\cdot(-\hat{i})]\,dA + \int_e \rho u_e[(u_e\hat{i})\cdot\hat{i}]\,dA$

$= -\dot{m}_i u_i + \dot{m}_e u_e$

also one inlet and outlet implies $\dot{m}_i = \dot{m}_e = \dot{m}$

$\therefore$ L.H.S $= \dot{m}(u_e - u_i)$ $\cdots$ ①

R.H.S $\rightarrow$ $\Sigma F_x = T - P_{amb}(A - A_e) - P_e A_e + P_{amb}(A - A_i) + P_i A_i$

$= T + P_{amb} A_e - P_e A_e - P_{amb} A_i + P_i A_i$

$= T - P_{amb}(A_i - A_e) + P_i A_i - P_e A_e$ $\cdots$ ②

① = ②

$$\therefore \boxed{T = \dot{m}(u_e - u_i) + P_{amb}(A_i - A_e) - P_i A_i + P_e A_e}$$

(b) from ICAO standard atmosphere table

$T = 216.650\,K$ $\qquad \rho = 0.19475\ \frac{kg}{m^3}$

$P = 1.2112 \times 10^4\ Pa$

from online compressible flow calculator @ $M = 2.0$ $h = 15000$ $\gamma = 1.4$

$P/P_0 = 0.12780452$

$T/T_0 = 0.55555555$

(i) $\therefore\ P_0 = \left(\frac{P_0}{P}\right)P = \left(\frac{1.2112\times10^4\,Pa}{0.12780452}\right) = \boxed{94769.73\ Pa}$

(ii) $\therefore\ T_0 = \left(\frac{T_0}{T}\right)T = \left(\frac{216.650K}{0.55555555}\right) = \boxed{389.97\ K}$

(iii) $M = \frac{u_\infty}{\sqrt{\gamma R T}}$

$$\therefore\ u_\infty = M\sqrt{\gamma R_{air} T}$$
$$= (2.0)\sqrt{(1.4)\left(\frac{287.05 J}{kg\cdot K}\right)(216.650K)}$$
$$= \boxed{590.14\ m/s}$$

(iv) since $M_e = 2.0$ $\quad P_e = P = \boxed{1.2112\times10^4\ Pa}$

(v) $\left(\frac{T_0}{T}\right)_e = 1 + \frac{1}{2}(\gamma-1)M_e^2$

$$= 1 + \frac{1}{2}(0.4)(2.0)^2 = \boxed{1.8}$$