College of Engineering
School of Aerospace Engineering

AE6210 Advanced Dynamics

Problem Set 1

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Problem Formulation

Consider a satellite orbiting a planet. For this problem, we consider the orbit of the Hubble Space Telescope orbiting around the Earth.

Planet/Satellite	Radius [km]	Standard Gravitaional Parameter, μ [km^{3}s^{-2}]	Distance from Earth [km]
Earth	6378	3.9860×10^5	0
Moon	1738.1	4.9×10^3	384400
Hubble	-	$(\mu_h \lll \mu_\oplus)$	6925

Table 1: Data table for relevant planets and satellites

From Newton's law of universal gravitation we know that gravity, g can be expressed as

$$g = \frac{g_0}{r^2} r'^2.$$

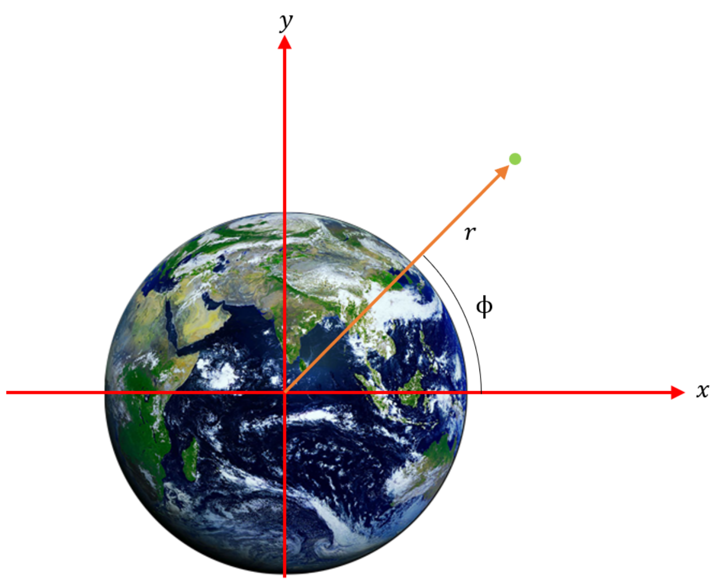

Figure 1: Orbit of the Hubble Space Telescope about the Earth

Assume, that the orbit of Hubble is circular and if $r_\oplus$ is the radius of the Earth and (x, y) indicate the Cartesian 2D-plane of the Hubble's position w.r.t the Earth we have

$$g = \frac{g_\oplus r_\oplus^2}{x^2 + y^2} = \left(\frac{\mu_\oplus}{r_\oplus^2}\right)\left(\frac{r_\oplus^2}{x^2 + y^2}\right) = \frac{\mu_\oplus}{x^2 + y^2}.$$

From Fig 1, we can say that the force exerted on the Earth by the Sun's gravity is

$$F_x = -m_h g \cos\phi$$
$$F_y = -m_h g \sin\phi$$

where m_h is the mass of Hubble, and also,

$$\sin\phi = \frac{y}{r} = \frac{y}{\sqrt{x^2+y^2}}$$
$$\cos\phi = \frac{x}{r} = \frac{x}{\sqrt{x^2+y^2}}$$

Hence, from Newton's law of motion we can define the eqution of motion for this system

$$m_h \ddot{x} = -m_h \frac{\mu_\oplus}{x^2+y^2} \frac{x}{\sqrt{x^2+y^2}}$$
$$m_h \ddot{y} = -m_h \frac{\mu_\oplus}{x^2+y^2} \frac{y}{\sqrt{x^2+y^2}}$$

Hence,

$$\boxed{\begin{aligned} \ddot{x} &= -\frac{\mu_\oplus x}{(x^2+y^2)^{\frac{3}{2}}} \\ \ddot{y} &= -\frac{\mu_\oplus y}{(x^2+y^2)^{\frac{3}{2}}} \end{aligned}}$$

Simulation

Since we are assuming a circular orbit for this simulation we must use a reasonable initial velocity that allows the satellite to remain in a circular orbit. For a circular orbit the velocity is constant and that value is

$$v_0 = \sqrt{\frac{\mu_\oplus}{6925}} = 7.5868 \text{ km/s}.$$

Furthermore, the initial position will be

$$x_0 = 6925 \text{ km}, \qquad y_0 = 0 \text{ km}.$$

If we simulate the equations of motion using this initial condition, we obtain such results as follows.

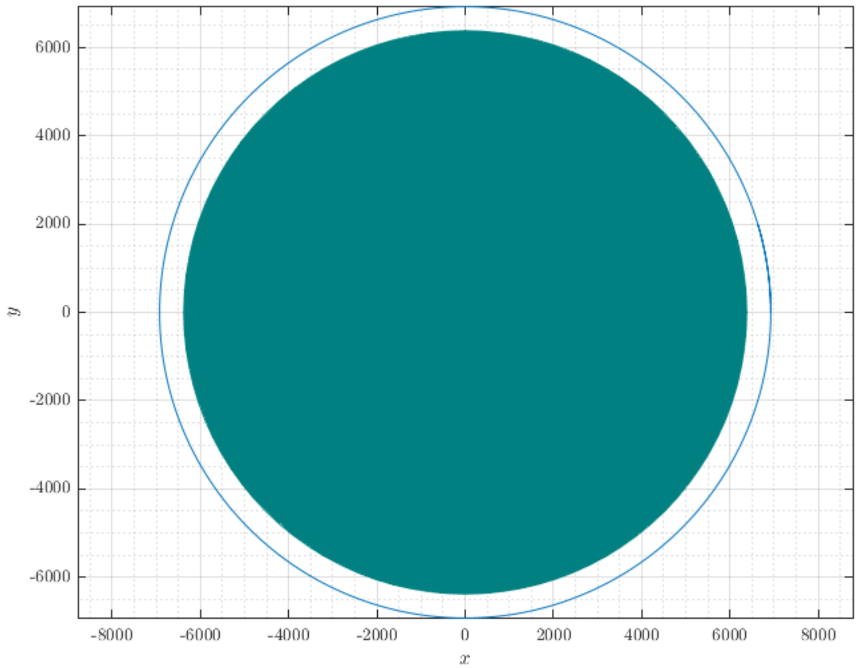

Figure 2: Circular orbit of the Hubble Space Telescope

Verification

Next, we verify that our simulation result is legitimate by assessing the specific orbital energy which is expressed as the below to be conserved.

$$\epsilon = \epsilon_k + \epsilon_p = \frac{v_x^2 + v_y^2}{2} - \frac{\mu_\oplus + \mu_h}{\sqrt{x^2 + y^2}} \approx \frac{v_x^2 + v_y^2}{2} - \frac{\mu_\oplus}{\sqrt{x^2 + y^2}} = \text{const.}$$

Observing the results portrayed in Fig 3, it might not be obvious that the energy is conserved; however, in actuality, the fluctuation is occurring within a very small range in that we are able to approximate this to be constant. Therefore, we have proved that the energy is conserved which shows that our simulation is valid.

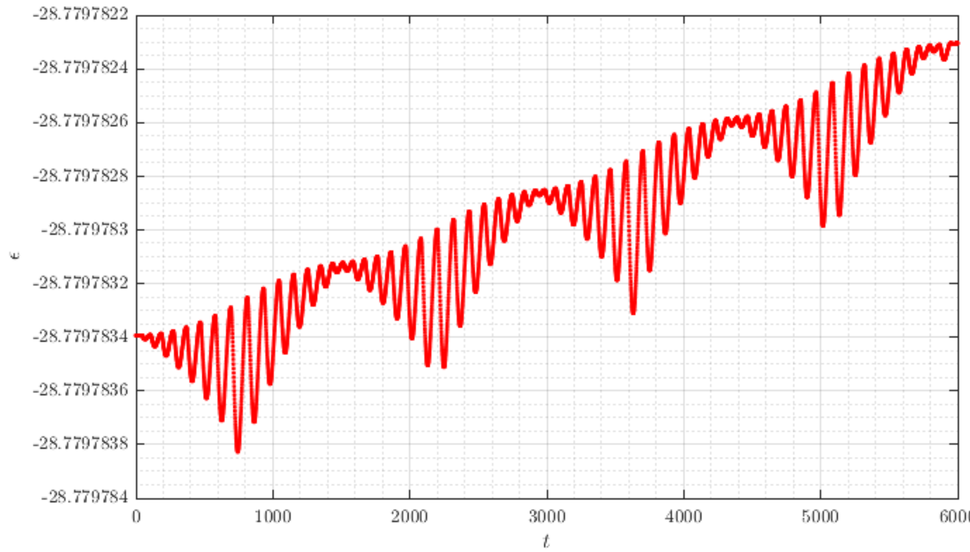

Figure 3: Conservation of the specific orbital energy

Discussion

In reality, currently the Hubble Space Telescope is not perfectly a circle but has an eccentricity, $e_h = 0.0002377$ with a apogee, $r_{ha} = 534$ km and perigee, $r_{hp} = 538$ km which implies a semi-major axis of $a_h = 536$ km. Other parameters being

- inclination: $i = 28.4710°$
- right ascension of ascending node: $\Omega = 68.4011°$
- argument of perigee: $\omega = 238.1016°$
- mean anomaly: $\nu = 239.1213°$

Furthermore, we would have a slight effect from the Moon that alters the orbit. If we simulate this exactly using GMAT we retrieve the results as follows.

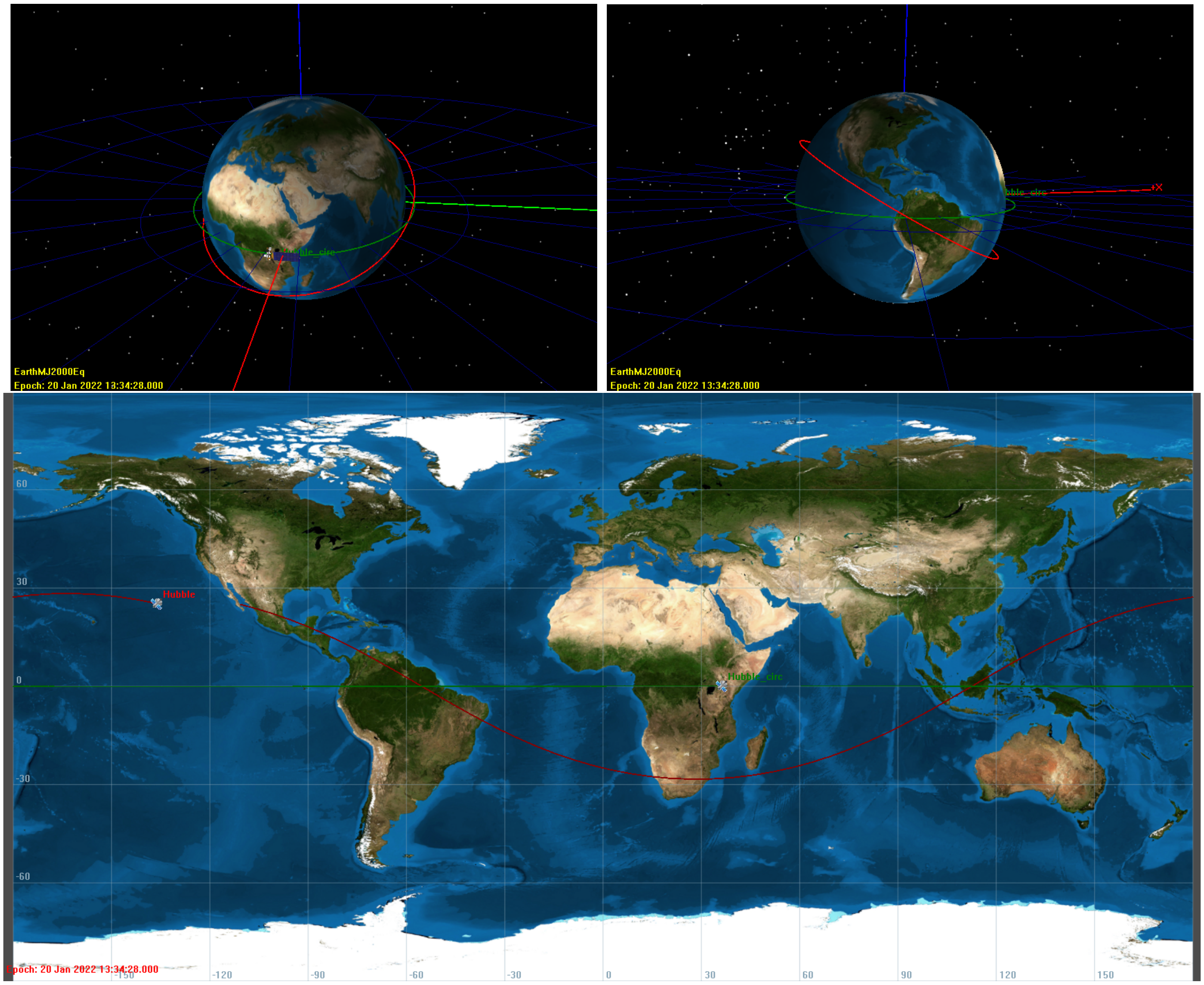

Figure 4: GMAT results of actual Hubble Orbit

The green orbit indicates the circular orbit and the red corresponds to the actual one. As you can see, to visualize the actual orbit of the satellite it is required to consider the inclinations and other parameters that allow the location of the satellite in three dimensional space. For the actual orbit we did consider the effect of the Moon when simulating but it turns out that there is barely any influence which is taken for granted considering the distance of the Moon from the Earth and gravitational force it exerts on the Hubble Space Telescope.