College of Engineering
School of Aeronautics and Astronautics

AAE 421
Flight Dynamics and Controls

EXAM 2

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I certify that I have neither given help to, nor received help from, any individual in matters relating to this examination.

Signature:

Problem 1 (25 pts)

Given the following set of nonlinear differential equations:

$$\dot{V} = (T - D - mg sin\gamma)/m$$

$$\dot{\gamma} = (L - mg cos\gamma)/mV$$

$$\dot{h} = V sin\gamma$$

Where

$$L = \frac{1}{2}\rho V^2 S C_L \quad and \quad C_L = C_{L_\alpha}\alpha$$

$$D = \frac{1}{2}\rho V^2 S\left(C_{D_0} + \epsilon C_L^2\right)$$

The parameters are set as

$$\epsilon = 0.9, \quad AR = 0.86$$

$$m = 0.003\ kg, \qquad S = 0.017\ m^2$$

$$C_{D_0} = 0.02, \qquad \rho = 0.41405\ kg/m^2$$

$$h = 10{,}000\ m, \qquad C_{L_\alpha} = 1.2936$$

(a) Find the trim condition for a leveled flight at altitude of $h = 10{,}000\ m$ flying at a speed of $V_e = V_0 = 3.7\ m/s$, where the state $\bar{x} = \begin{bmatrix} V \\ \gamma \\ h \end{bmatrix}$ and the control is denoted by $\bar{u} = \begin{bmatrix} \alpha \\ T \end{bmatrix}$. (Hint: to maintain a trim condition at level flight, we must have $\dot{V} = \dot{\gamma} = \dot{h} = 0$ and $\gamma = 0$)

For the trim conditions we solve the following two equations by plugging in all the given parameters as well as trim conditions.

$$\dot{V} = (T - D - mg sin\gamma)/m$$

$$\dot{\gamma} = (L - mg cos\gamma)/mV$$

That gives the following equations

$$0 = -\frac{\rho S}{2m}\left(c_{D_0} + \epsilon C_{L_\alpha}^2 \alpha_e^2\right)V_e^2 - g sin\gamma_e + \frac{T_e}{m} \tag{1}$$

$$0 = \frac{\rho S}{2m} C_{L_\alpha} \alpha_e V_e - \frac{g cos\gamma_e}{V_e} \tag{2}$$

Solving the second equation gives us

At trim conditions

$\dot{V}_e = \dot{\gamma}_e = \dot{h}_e = 0$ & $\gamma_e = 0$ & $V_e = 3.7\,m/s$

plug these into eqns ① ~ ②

① $\Rightarrow$ $0 = -\frac{\rho S}{2m}(C_{D0} + \epsilon C_{L\alpha}^2 \alpha_e^2) V_e^2 - g\sin\cancelto{0}{\gamma_e} + \frac{T_e}{m}$

$$0 = -\frac{\rho S}{2m} C_{D0} V_e^2 - \frac{\rho S \epsilon C_{L\alpha}^2 V_e^2}{2m} \alpha_e^2 + \frac{T_e}{m}$$

$\approx>$ $0 = -0.32121 - 24.18780\, \alpha_e^2 + \frac{T_e}{0.003}$ ... ④

② $\Rightarrow$ $0 = \frac{1}{2m} \rho S C_{L\alpha} \alpha_e V_e - \frac{g \cancelto{1}{\cos\gamma_e}}{V_e}$

$$0 = \frac{1}{2m} \rho S C_{L\alpha} V_e \alpha_e - \frac{g}{V_e}$$

$\alpha_e = \frac{2mg}{\rho S C_{L\alpha} V_e^2} = 0.47219$ rad ... ⑤

plug ⑤ into ④ and we obtain

$$T_e = 0.003\left[24.18780(0.47219)^2 + 0.32121\right]$$

$$T_e = 0.01714\ N$$

$$\alpha_e = 0.4722\, rad\ .$$

And plugging this into equation (1) gives the trim value of T

$$T_e = 0.0171\, N\ .$$

Also,

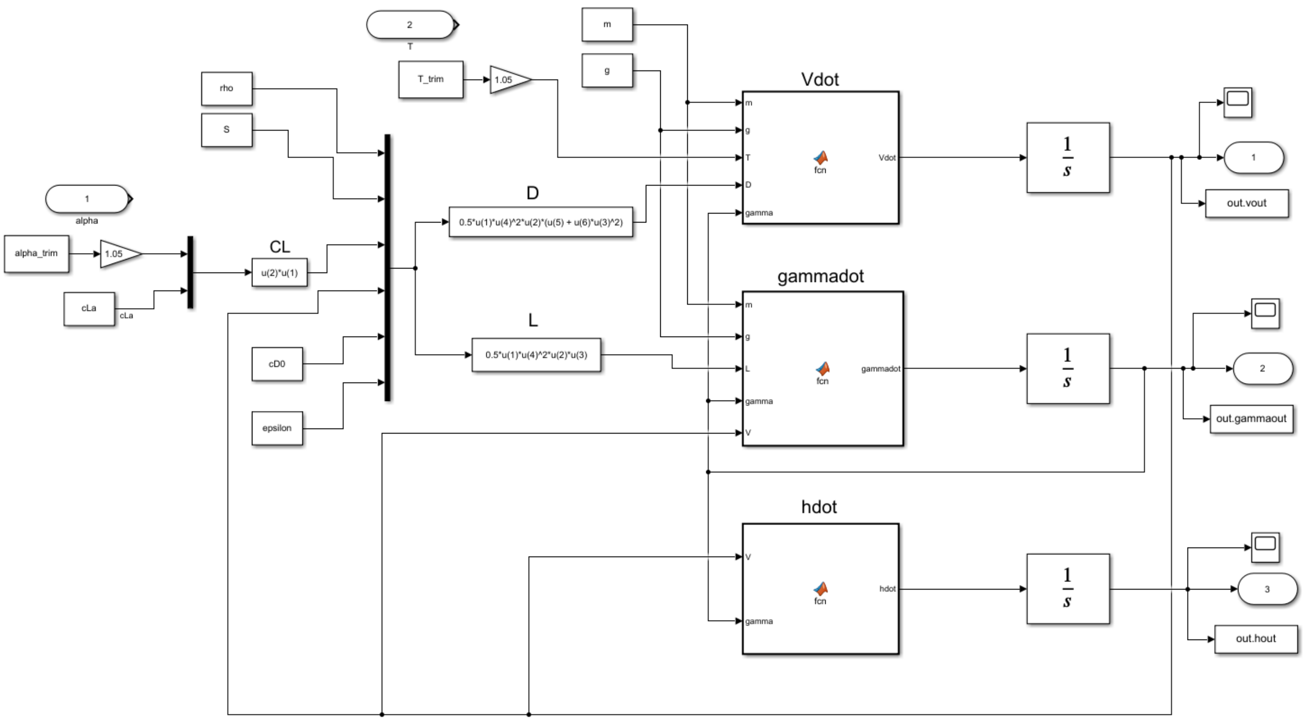
$$V_e = 3.7\ m/s$$

$$\gamma_e = 0\ \ rad$$

$$h_e = 10{,}000\ m\ .$$

Develop a Simulink model to simulate the system state response for t = 10 sec with the initial condition set as the trim condition in Part (a) and input $\bar{u} = \begin{bmatrix} \alpha_{trim} \\ T_{trim} \end{bmatrix} \times 105\%$.

The Simulink model is the following



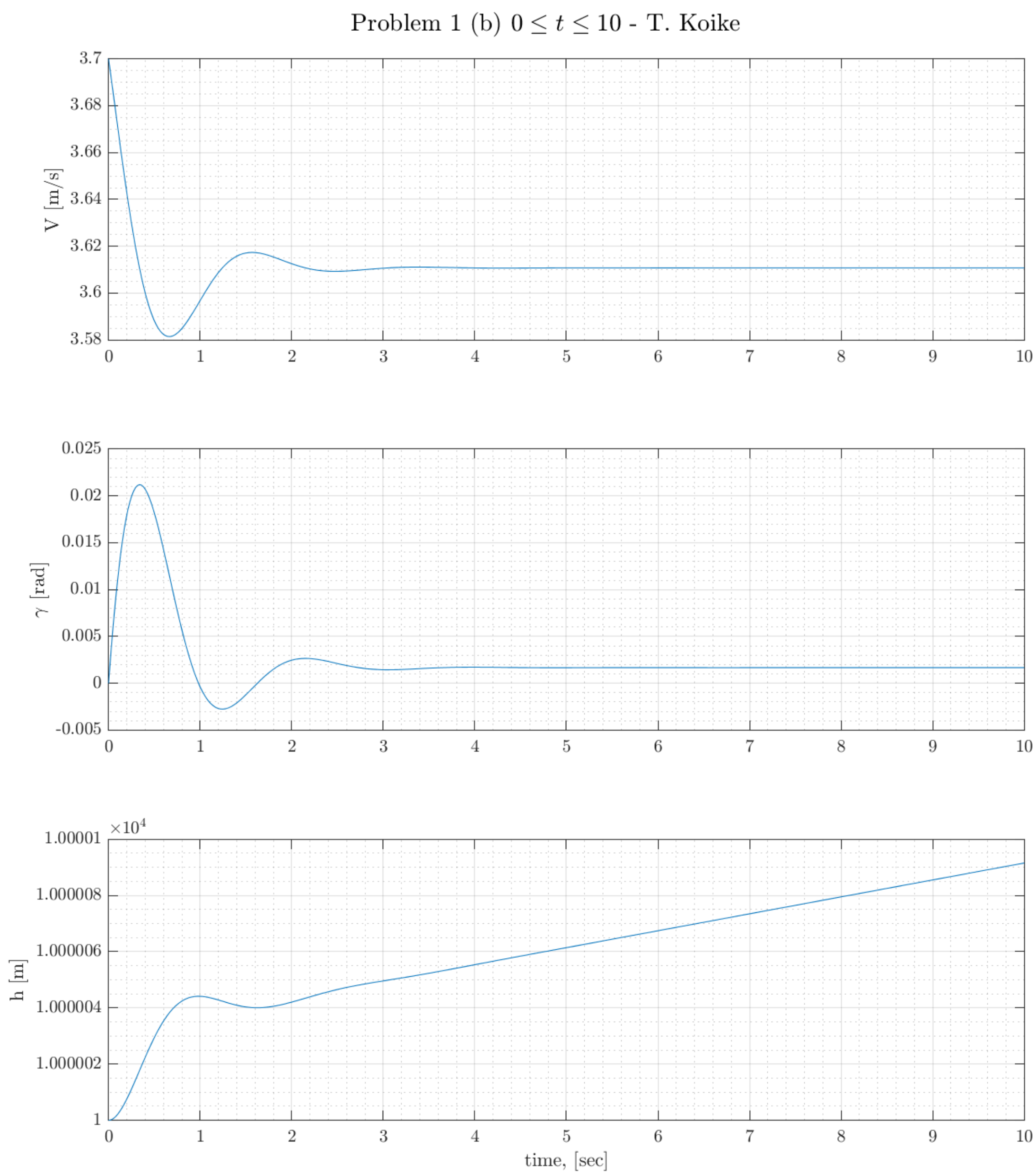
The Embedded MATLAB Blocks have the following functions defined

```
function Vdot = fcn(m, g, T, D, gamma)
Vdot = (T - D - m * g * sin(gamma)) / m;
end
```

```
function gammadot = fcn(m, g, L, gamma, V)
gammadot = (L - m * g * cos(gamma)) / m / V;
end
```

```
function hdot = fcn(V, gamma)
hdot = V * sin(gamma);
end
```

The Simulation results is the following



Running the following commands give us the simulation

```
% Problem 1
% Parameters
epsilon = 0.9;
AR = 0.86;
m = 0.003;
S = 0.017;
cD0 = 0.02;
rho = 0.41405;
h = 10000;
cLa = 1.2936;
g = 9.81;

% (a)
% Equilibrium conditions
he = h;
Ve = 3.7;
gamma = 0;

% Find trim conditions
syms alpha T
cL = cLa * alpha;
L = 0.5*rho*Ve^2*S*cL;
D = 0.5*rho*Ve^2*S*(cD0 + epsilon*cL^2);
eqn1 = 0 == (T - D - m*g*sin(gamma)) / m;
eqn2 = 0 == (L - m*g*cos(gamma)) / m / Ve;
res = solve([eqn1, eqn2], [alpha, T]);

% (b)
alpha_trim = double(res.alpha)
T_trim = double(res.T)
simout = sim("e2_p1_model.slx");
t = simout.tout;
Vsim = simout.vout.signals.values;
gammasim = simout.gammaout.signals.values;
hsim = simout.hout.signals.values;

% Plotting
fig1 = figure('Renderer',"painters", 'Position', [10 10 900 1000]);
    subplot(3,1,1)
    plot(t,Vsim)
    grid on; grid minor; box on;
    ylabel('V [m/s]')
    subplot(3,1,2)
    plot(t,gammasim)
    grid on; grid minor; box on;
    ylabel('$\gamma$ [rad]')
    subplot(3,1,3)
    plot(t,hsim)
    grid on; grid minor; box on;
    ylabel('h [m]')
    xlabel('time, [sec]')
    sgtitle('Problem 1 (b) $0\leq t \leq 10$ - T. Koike')
saveas(fig1, fullfile(fdir, "p1_2.png"));
```

(b) Make use of the MATLAB command 'linmod.m' to find the linearized state space model about the trim condition found in part (a), assuming the output $\bar{y} = \bar{x}$.

The following command gives us the A, B, C, and D matrices

```
[A, B, C, D] = linmod('e2_p1_model',[Ve;gamma;he],[alpha_trim; T_trim])
```

$$A = \begin{bmatrix} -3.0889 & -9.8100 & 0 \\ 1.4332 & 0 & 0 \\ 0 & 3.7000 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} -22.8430 & 333.3333 \\ 5,6150 & 0 \\ 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Problem 2 (25 pts)

Using Dutch roll approximation, determine the state feedback gains so that the damping ratio and frequency of the Dutch roll are 0.3 and 1.0 rad/s, respectively. Assume the airplane has the following characteristics:

$$Y_\beta = -19.5\ ft/s^2$$

$$Y_r = 1.3\ ft/s$$

$$N_\beta = 1.5\ s^{-1}$$

$$N_r = -0.21\ s^{-1}$$

$$U_0 = 400\ ft/s\ .$$

We are given that

$$\zeta = 0.3 \quad \& \quad \omega_n = 1.0 \text{ rad/s}$$

The desired poles are

$$S_d = -\zeta\omega_n \pm \omega_n^2\sqrt{1-\zeta^2}\,j$$
$$S_d = -0.3000 \pm 0.9539\,j$$

For the Dutch roll approximation the following is used

$$\begin{bmatrix} \dot{\beta} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} Y_\beta/U_0 & Y_r/U_0 \\ N_\beta & N_r \end{bmatrix}\begin{bmatrix} \beta \\ r \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix}\Delta\delta_r$$

Then

$$A = \begin{bmatrix} -19.5/400 & 1.3/400 \\ 1.5 & -0.21 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$A = \begin{bmatrix} -0.0488 & -0.0033 \\ 1.5000 & -0.2100 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

The characteristic equation for the desired poles are

$$\alpha_d(s) = (s - s_d^+)(s - s_d^-) = s^2 + 0.6s + 1$$

$$\alpha_d(A) = A^2 + 0.6A + I = \begin{bmatrix} 0.9683 & -0.0011 \\ 0.5119 & 0.9132 \end{bmatrix}$$

The controllability matrix is

$$Q_c = [B \quad AB] = \begin{bmatrix} 1.0000 & -0.0520 \\ 1.0000 & 1.2900 \end{bmatrix}$$

Now if $u = -Kx$

$$K = [k_1 \quad k_2] = [0 \quad 1] \begin{bmatrix} 1.0000 & -0.0520 \\ 1.0000 & 1.2900 \end{bmatrix}^{-1} \begin{bmatrix} 0.9683 & -0.0011 \\ 0.5119 & 0.9132 \end{bmatrix}$$

$$K = [k_1 \quad k_2] = \boxed{[-0.3401 \quad 0.6813]}$$

To check

$$A_d = A - BK = \begin{bmatrix} 0.2913 & -0.6846 \\ 1.3401 & -0.8913 \end{bmatrix}$$

The eigenvalues of A_d are

$$\lambda_{1,2} = -0.3000 \pm 0.9539j$$

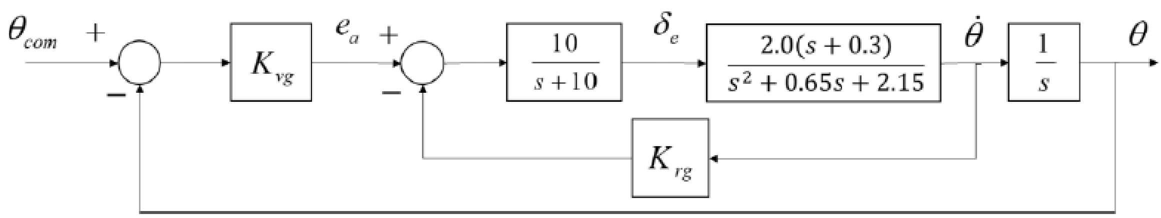
If

$$u = -Kx$$

$$K = [-0.3401 \quad 0.6813]$$

Problem 3 (25 pts)

For the pitch displacement autopilot system shown below



(a) Determine the gain necessary to improve the system characteristic so that the control system has the following performance: $\zeta = 0.3$ and $\omega_n = 2.0$ rad/s. Verify your solution by providing the root locus plot of the overall system and plot of system response to a 5° step change in the commanded pitch attitude.

Use the Control Systems Designer to tune the gyro gains for the given requirements

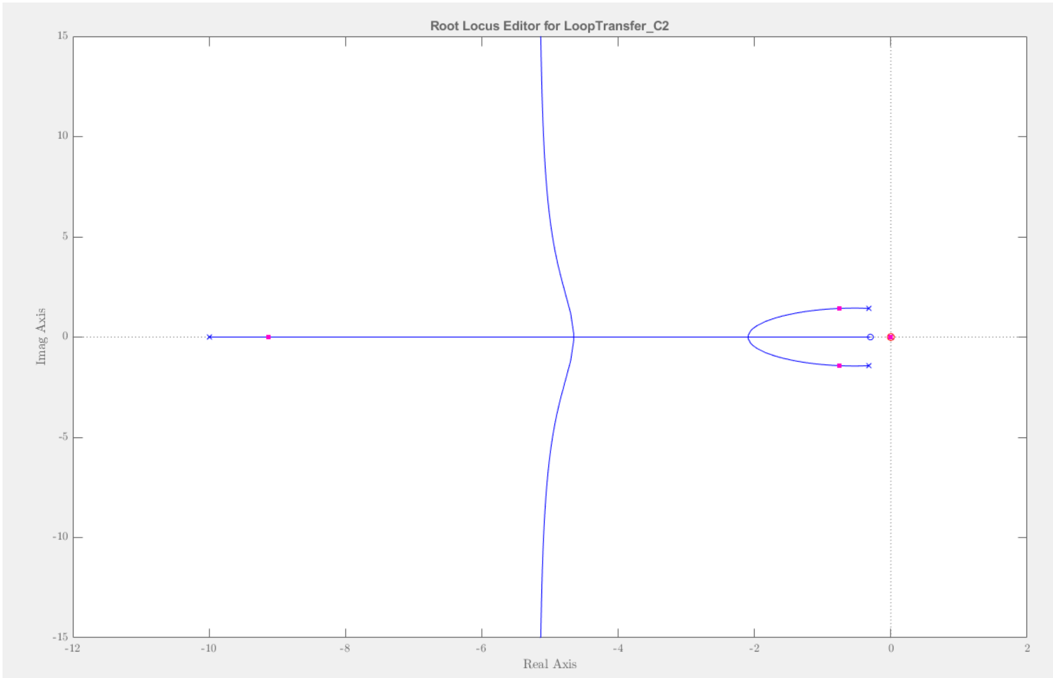
```
num = conv([0, 10], 2.0*[1, 0.3]);
den = conv([1, 10], [1, 0.65, 2.15]);
sys = tf(num, den);
s = tf("s");
H = 1*s;
controlSystemDesigner(sys);
```

The control system tuner gives us the following gains

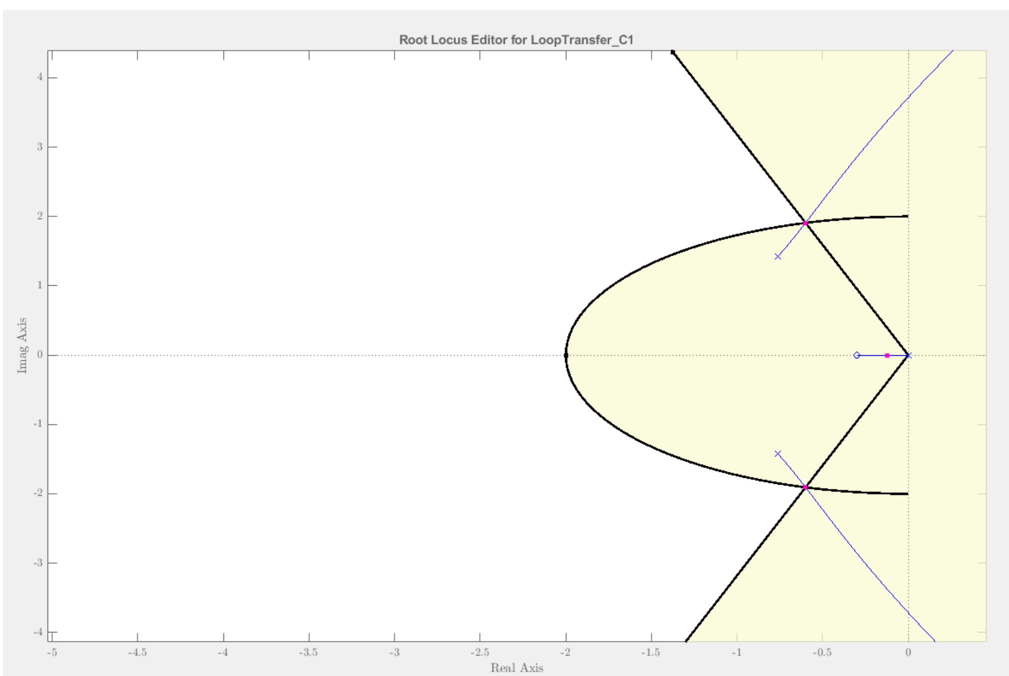
$$K_{vg} = 0.76561$$

$$K_{rg} = 0.392$$

The Root locus of the inner loop system becomes

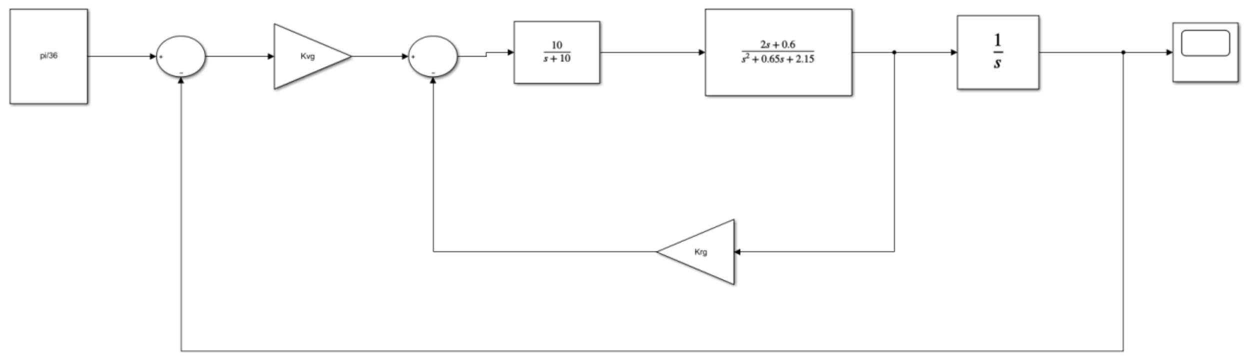


The Root Locus of the outer loop system becomes

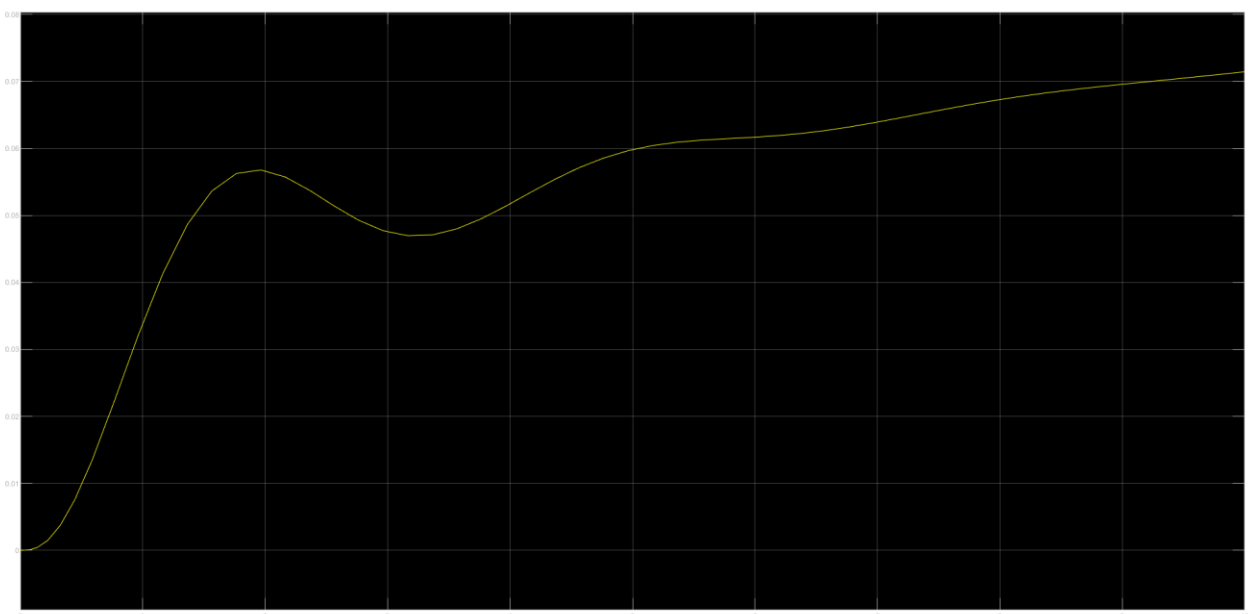


Then we get the response for 5-degree pitch attitude change

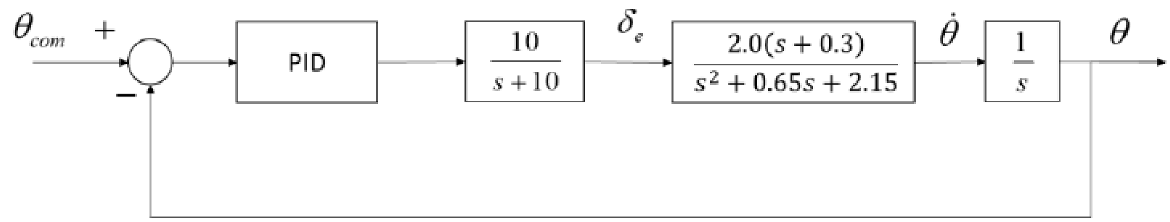
```
num = conv([0, 10], 2.0*[1, 0.3]);
den = conv([1, 10], [1, 0.65, 2.15]);
Krg = 0.392;
Kvg = 0.76561;
```



The response to 5-degree ($\pi/36$) change in pitch is



(b) Replace the rate gyro and amplifier with a PID controller shown below:



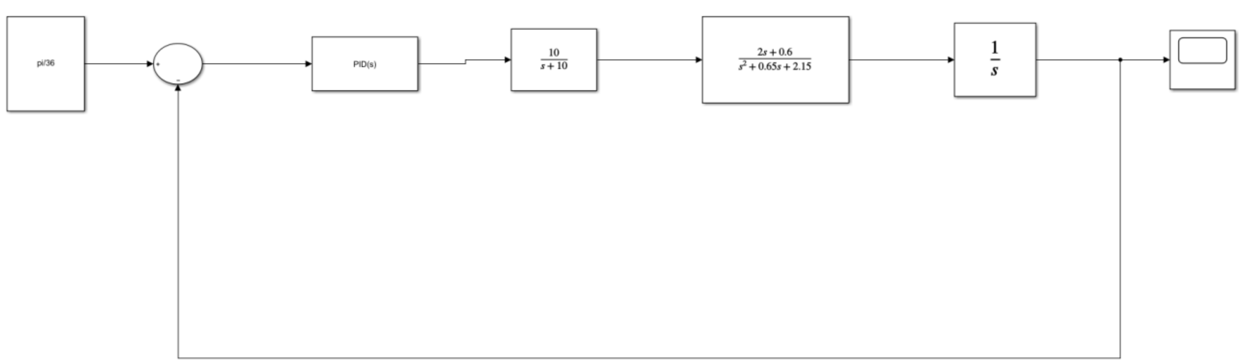
Design the PID gains using MATLAB Control System Tuner. Compare the design results with Part (a) by providing the plot of system response to a 5-degree step change in the commanded pitch altitude.

The requirements are

$$\%OS = exp\left(\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right) \times 100 \ = 32.23\%$$

$$timeconstant \coloneqq \tau = \frac{1}{\zeta\omega_n} = 1.6667$$

The model is as follows.

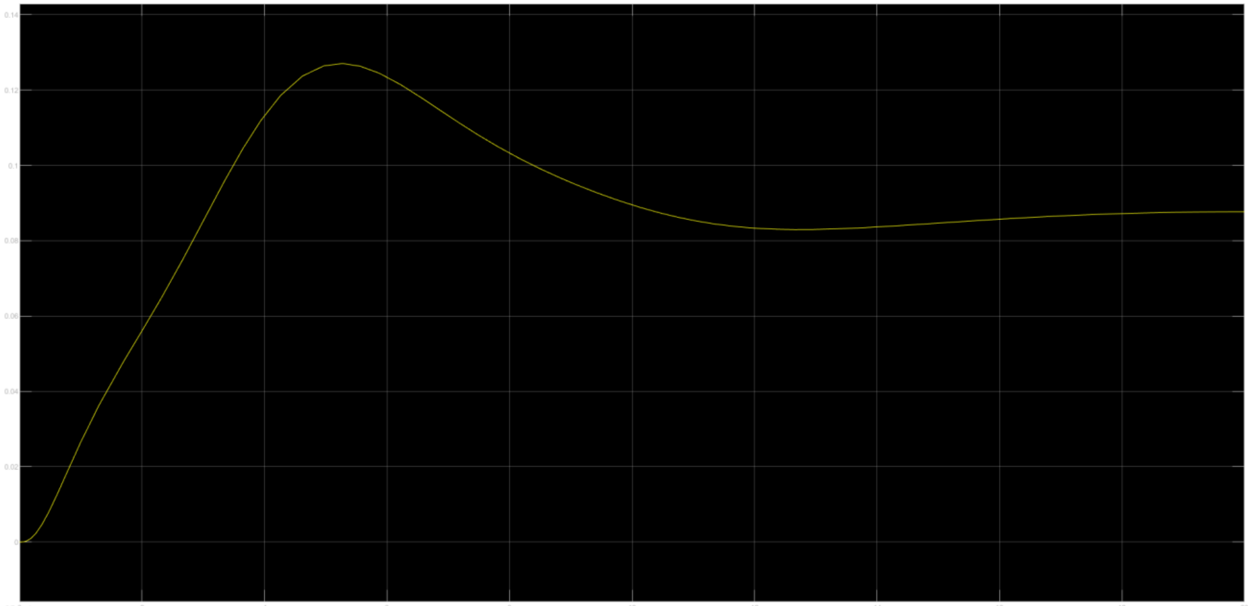


The results are

$$K_p + K_i \times \frac{1}{s} + K_d \times \frac{s}{T_f + 1}$$

$$K_p = -0.148, \qquad K_i = 0.541, \qquad K_d = 0.35, \qquad T_f = 0.325$$

The response to a 5-degree step change is as follows.



The PID tuned results seem to be more stable than the one tuned by 2 proportional controllers inside feedback loops.

Problem 4 (25 pts)

The equations of motion governing the aircraft's motion are

$$\Delta\dot{\alpha} = \frac{Z_\alpha}{U_0}\Delta\alpha - \Delta q$$

$$\Delta\dot{q} = M_\alpha\Delta\alpha + M_q\Delta q + M_\delta\Delta\delta_e$$

The performance index is set as

$$\mathrm{J} = \int_0^\infty \left[\left(\frac{\alpha}{\alpha_{max}}\right)^2 + \left(\frac{\delta_e}{\delta_{e_{max}}}\right)^2 + \left(\frac{q}{q_{max}}\right)^2\right]\mathrm{dt}$$

where α_{max}, $\delta_{e_{max}}$, q_{max} are given parameters. Derive the nonlinear algebraic equations to satisfy the algebraic Riccati equation that is required to be solved to design an LQR controller.

$$\begin{cases} \Delta\dot{\alpha} = \dfrac{Z_\alpha}{V_0}\Delta\alpha - \Delta q \\ \Delta\dot{q} = M_\alpha \Delta\alpha + M_q \Delta q + M_\delta \Delta\delta_e \end{cases}$$

The state matrices are

$$A = \begin{bmatrix} \dfrac{Z_\alpha}{V_0} & -1 \\ M_\alpha & M_q \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ M_\delta \end{bmatrix}$$

The weight matrices for LQR are

$$Q = \begin{bmatrix} 1/\alpha_{max}^2 & 0 \\ 0 & 1/q_{max}^2 \end{bmatrix} \qquad R = \frac{1}{\delta_{e\,max}^2}$$

say $\bar{P} = \begin{bmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{bmatrix}$

The algebraic ricatti equation becomes

$$0 = \bar{P}A + A'\bar{P} - \bar{P}BR^{-1}B'\bar{P} + Q$$

$$0 = \begin{bmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{bmatrix}\begin{bmatrix} Z_\alpha/V_0 & -1 \\ M_\alpha & M_q \end{bmatrix} + \begin{bmatrix} Z_\alpha/V_0 & M_\alpha \\ -1 & M_q \end{bmatrix}\begin{bmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{bmatrix}$$

$$- \delta_{e\,max}^2 \begin{bmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{bmatrix}\begin{bmatrix} 0 \\ M_\delta \end{bmatrix}\begin{bmatrix} 0 & M_\delta \end{bmatrix}\begin{bmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{bmatrix} + \begin{bmatrix} \dfrac{1}{\alpha_{max}^2} & 0 \\ 0 & \dfrac{1}{q_{max}^2} \end{bmatrix}$$

$$0 = \begin{bmatrix} \frac{Z_q}{V_0} P_{11} + M_q P_{12} & -P_{11} + M_q P_{12} \\ \frac{Z_q}{V_0} P_{12} + M_q P_{22} & -P_{12} + M_q P_{22} \end{bmatrix} + \begin{bmatrix} \frac{Z_q}{V_0} P_{11} + M_q P_{12} & \frac{Z_q}{V_0} P_{12} + M_q P_{22} \\ -P_{11} + M_q P_{12} & -P_{12} + M_q P_{22} \end{bmatrix}$$

$$- \delta_{e_{max}}^2 \begin{bmatrix} M_\delta P_{12} \\ M_\delta P_{22} \end{bmatrix} \begin{bmatrix} 0 & M_\delta \end{bmatrix} \begin{bmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{bmatrix} + \begin{bmatrix} \frac{1}{\alpha_{max}^2} & 0 \\ 0 & \frac{1}{q_{max}^2} \end{bmatrix}$$

$$= -\delta_{e_{max}}^2 \begin{bmatrix} 0 & M_\delta^2 P_{12} \\ 0 & M_\delta^2 P_{22} \end{bmatrix} \begin{bmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{bmatrix}$$

$$= -\delta_{e_{max}}^2 \begin{bmatrix} M_\delta^2 P_{12}^2 & M_\delta^2 P_{12} P_{22} \\ M_\delta^2 P_{12} P_{22} & M_\delta^2 P_{22}^2 \end{bmatrix}$$

$$\Longrightarrow \quad 0 = \begin{bmatrix} 2\left(\frac{Z_q}{V_0} P_{11} + M_q P_{12}\right) & -P_{11} + \left(M_q + \frac{Z_q}{V_0}\right) P_{12} + M_q P_{22} \\ -P_{11} + \left(M_q + \frac{Z_q}{V_0}\right) P_{12} + M_q P_{22} & 2\left(-P_{12} + M_q P_{22}\right) \end{bmatrix}$$

$$+ \begin{bmatrix} \frac{1}{\alpha_{max}^2} - \delta_{e_{max}}^2 M_\delta^2 P_{12}^2 & -\delta_{e_{max}} M_\delta^2 P_{12} P_{22} \\ -\delta_{e_{max}}^2 M_\delta^2 P_{12} P_{22} & \frac{1}{q_{max}^2} - \delta_{e_{max}}^2 M_\delta^2 P_{22}^2 \end{bmatrix}$$

The equations are

$$\boxed{\begin{aligned}
&2\left(\frac{Z_q}{v_0}P_{11} + M_q P_{12}\right) + \frac{1}{q_{max}^2} - \delta_{e_{max}}^2 M_\delta^2 P_{12}^2 = 0 \\
&-P_{11} + \left(M_q + \frac{Z_q}{v_0}\right)P_{12} + M_q P_{22} - \delta_{e_{max}}^2 M_\delta^2 P_{12} P_{22} = 0 \\
&2\left(-P_{12} + M_q P_{22}\right) + \frac{1}{q_{max}^2} - \delta_{e_{max}}^2 M_\delta^2 P_{22}^2 = 0
\end{aligned}}$$

Bonus (5 pts)

Use state feedback to design an altitude hold control system. Assume the forward speed is held constant and the longitudinal equation can be modeled using the short-period approximation. The short-period equations are

$$\begin{bmatrix}\Delta\dot{\alpha}\\ \Delta\dot{q}\\ \Delta\dot{\theta}\end{bmatrix} = \begin{bmatrix}-1.5 & 1 & 0\\ -4.0 & -1.0 & 0\\ 0 & 1 & 0\end{bmatrix}\begin{bmatrix}\Delta\alpha\\ \Delta\, q\\ \Delta\theta\end{bmatrix} + \begin{bmatrix}-0.2\\ -8.0\\ 0\end{bmatrix}\Delta\delta_e$$

Assume the $\Delta\dot{h} = u_0(\Delta\theta - \Delta\alpha)$ where $u_0 = 200\ ft/s$. Determine the state feedback gain if the closed-loop eigenvalues are located at

$$\lambda_1 = -1.5 \pm 2.5j$$

$$\lambda_2 = -0.75 \pm 1.0j$$

The state space equation can be remodeled by including the altitude change, which becomes as follows.

$$\begin{bmatrix}\Delta\dot{\alpha}\\ \Delta\dot{q}\\ \Delta\dot{\theta}\\ \Delta\dot{h}\end{bmatrix} = \begin{bmatrix}-1.5 & 1 & 0 & 0\\ -4.0 & -1.0 & 0 & 0\\ 0 & 1 & 0 & 0\\ -200 & 0 & 200 & 0\end{bmatrix}\begin{bmatrix}\Delta\alpha\\ \Delta q\\ \Delta\theta\\ \Delta h\end{bmatrix} + \begin{bmatrix}-0.2\\ -8.0\\ 0\\ 0\end{bmatrix}\Delta\delta_e$$

First, we have to find out if this system is controllable or not.

```
% Setup
A = [-1.5, 1, 0, 0; -4, -1, 0, 0; 0, 1, 0, 0; -200, 0, 200, 0];
B = [-0.2; -8; 0; 0];
% Check controllability
Qc = ctrb(A, B);
Qc_rref = rref(Qc);
```

```
Qc = 4×4
10³ ×
   -0.0002   -0.0077    0.0204   -0.0085
   -0.0080    0.0088    0.0220   -0.1034
         0   -0.0080    0.0088    0.0220
         0    0.0400   -0.0600   -2.3100
```

```
Qc_rref = 4×4
     1     0     0     0
     0     1     0     0
     0     0     1     0
     0     0     0     1
```

The controllability matrix has full rank, and therefore, this system is controllable.

Now we find the controller gains using the Brogan's Algorithm

Step 1:

Find

$$\phi = (xI_{n\times n} - A)^{-1}$$

$$\Rightarrow \begin{bmatrix} \frac{2\,(x+1)}{2x^2+5x+11} & \frac{2}{2x^2+5x+11} & 0 & 0 \\ -\frac{8}{2x^2+5x+11} & \frac{2\,x+3}{2x^2+5x+11} & 0 & 0 \\ -\frac{8}{x\,(2x^2+5x+11)} & \frac{2\,x+3}{x\,(2x^2+5x+11)} & \frac{1}{x} & 0 \\ -\frac{400\,(x^2+x+4)}{x^2(2x^2+5x+11)} & \frac{600}{x^2\,(2x^2+5x+11)} & \frac{200}{x^2} & \frac{1}{x} \end{bmatrix}$$

Step 2:

Compute

$$\psi = \phi B$$

$$\Rightarrow \begin{bmatrix} -\frac{0.4000\,(x+1)}{2x^2+5x+11} & -\frac{16}{2x^2+5x+11} \\ \frac{1.6000}{2x^2+5x+11} & -\frac{8\,(2\,x+3)}{2x^2+5x+11} \\ \frac{1.6000}{x(2x^2+5x+11)} & -\frac{8\,(2\,x+3)}{x\,(2x^2+5x+11)} \\ \frac{80\,(x^2+x+4)}{x^2\,(2x^2+5x+11)} & -\frac{4800}{x^2(2x^2+5x+11)} \end{bmatrix}$$

Step 3:

Calculate

$$\bar{\psi} = [\psi_1(\lambda_1) \quad \psi_1(\lambda_2) \quad \psi_1(\lambda_3) \quad \psi_1(\lambda_4)]$$

Where $\psi_1(x), \psi_2(x)$ correspond to the columns of ψ.

$$\bar{\psi} = \begin{bmatrix} 2.7774 - 1.3208\,j & 2.7774 + 1.3208\,j & -2.3171 + 0.6642\,j & -2.3171 - 0.6642\,j \\ 3.5019 + 6.9434\,j & 3.5019 - 6.9434\,j & -2.2020 - 1.8190\,j & -2.2020 + 1.8190\,j \\ 1.4242 - 2.2553\,j & 1.4242 + 2.2553\,j & -0.1072 + 2.2824\,j & -0.1072 - 2.2824\,j \\ -7.2129 + 112.5808\,j & -7.2129 - 112.5808\,j & -5.0239 - 438.2218\,j & -5.0239 + 438.2218\,j \end{bmatrix}$$

Step 4:

Find the gains with

$$K = -E\bar{\Psi}^{-1}$$

Where

$$E = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}$$

Thus,

$$K = \begin{bmatrix} 0.7734 & -0.2693 & -1.5781 & -0.0059 \end{bmatrix}$$

And

$$A_{cl} = A - BK = \begin{bmatrix} -1.3453 & 0.9461 & -0.3156 & -0.0012 \\ 2.1870 & -3.1547 & -12.6248 & -0.0474 \\ 0 & 1 & 0 & 0 \\ -200 & 0 & 200 & 0 \end{bmatrix}.$$

$$eig(A_{cl}) = \begin{bmatrix} -1.5000 - 2.5000\,\mathrm{i} \\ -1.5000 + 2.5000\,\mathrm{i} \\ -0.7500 - 1\,\mathrm{i} \\ -0.7500 + 1\,\mathrm{i} \end{bmatrix}$$

Thus, our gains are correct.

```
% Brogan's Algorithm with desired poles
lambda = [-1.5+2.5j, -1.5-2.5j, -0.75+1j, -0.75-1j];
syms x
[n, m] = size(B);
phi = inv(x*eye(n)-A);
psi = phi * B;
psibar = ([subs(psi(:), lambda(1)), subs(psi(:), lambda(2)),...
   subs(psi(:), lambda(3)), subs(psi(:), lambda(4))]);
K = -[1, 1, 1, 1]*inv(psibar);
Acl = A - B*K
eig(Acl)
```