College of Engineering
School of Aerospace Engineering

AE6210 Advanced Dynamics I

Assignment

JWST Simulation

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I Problem Statement

Figure 1: JWST being assembled and tested. *Credit: NASA*

In this assignment, we shall simulate the Halo orbit of the James Webb Space Telescope (JWST) at L2 with the three-body problem model where the bodies are the Earth, Sun, and JWST. This paper will be organized in the following manner.

1. Showing the data and assumptions used along with their justifications.
2. Formulation of the equations of motion.
3. Simulation results with its verification.
4. Further discussions.

II Problem Formulation

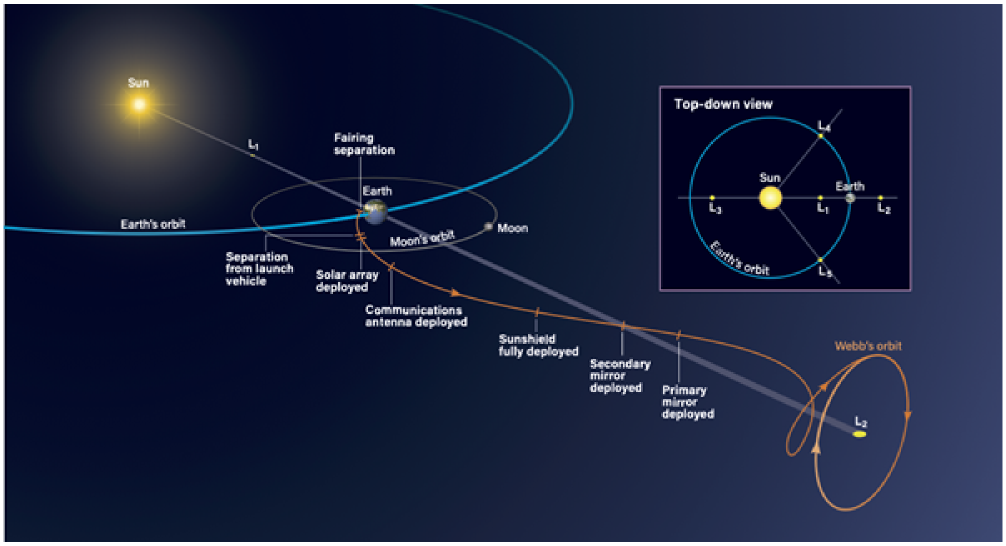

Figure 2: Detailed diagram of JWST's orbit. *Credit: Astronomy.com*

The orbit of the JWST is modelled as a three-body problem consisting of the Earth, Sun, and JWST. This is because the gravitational force exerted on JWST by the Moon is 10^2 to 10^3 order of magnitude smaller compared to that of the Earth and Sun. It is also very important to mention that JWST is on the second Lagrange point or L2 which is a special location in that it allows JWST to orbit the Sun while staying in line with the Earth [1]. Additionally, for the equation of motion we can assume all bodies to be a point mass. In summary, we list the assumptions for the problem formulation.

- All bodies are a point mass where $m_j \ll m_\oplus \ll m_\odot$
- JWST orbits in-plane with the Earth.
- Neglect all exogenous forces that may affect the dynamics such as gravity gradient torque, magnetic torque, solar radiation, aerodynamic effect, etc.
- Assume a circular orbit for the Earth and JWST about the Sun, i.e. $e = 0$.
- The mass of JWST is approximately $m_j = 6200$ kg [2].

For further information, we tabulated the data for the Earth, Sun, Moon, and JWST below [3][4].

Planet/Satellite	Radius [km]	Standard Gravitaional Parameter, μ [km^{3}s^{-2}]	Distance from Sun [km]
Sun	695700	132712×10^6	0
Earth	6378	3.9860×10^5	149.6×10^6
Moon	1738.1	4.9×10^3	384400*
JWST	-	$(\mu_j \lll \mu_\odot)$	1.5×10^6*

Table 1: Data table for the Earth, Sun, Moon, and JWST. (*Distance from the Earth.)

The orbit, to be more exact, is a circular restricted three body problem (CR3BP), and we will derive the equations of motion for this specific orbit.

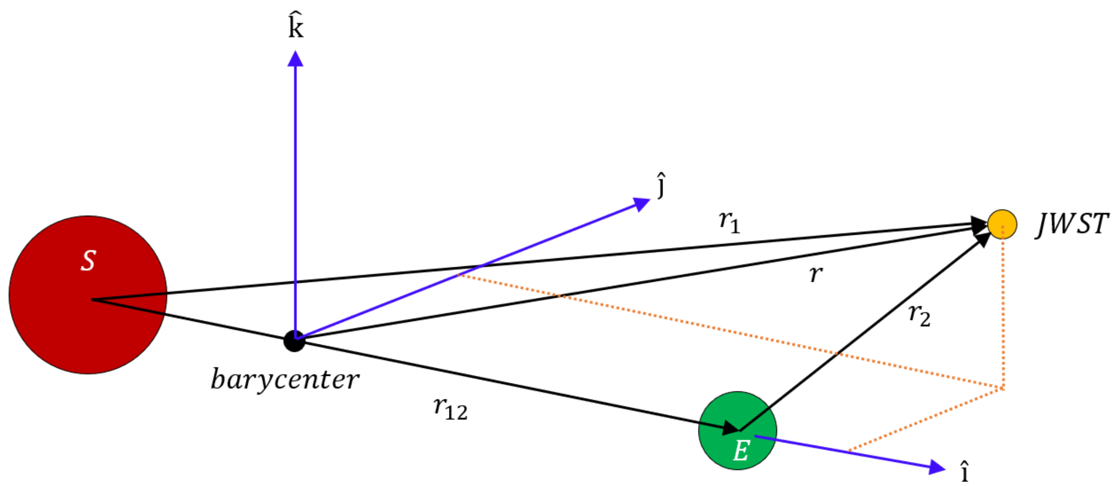

Figure 3: Body diagram of the three body problem.

To begin with, we attach a non-inertial coordinate frame to the barycenter of the system of the masses of the Sun and Earth denoted as $m_\odot$, $m_\oplus$ respectively. The y-axis is in the orbital plane and the z-axis is perpendicular to the orbital plane. The angular momentum vector is in the same direction as the z-axis and the Sun and Earth are stationary with respect to the rotating reference frame. The inertial angular velocity is

$$\mathbf{\Omega} = \Omega \hat{k} \tag{II.1}$$

where $\Omega = 2\pi/T$ and the period of the orbit, T is represented as

$$T = \frac{2\pi}{\sqrt{\mu}} r_{12}^{3/2}. \tag{II.2}$$

Them from (III.1) and (III.2) we have

$$\Omega = \sqrt{\frac{\mu}{r_{12}^3}} \tag{II.3}$$

where the gravitational parameter is $\mu = \mu_\odot + \mu_\oplus$. Since this coordinate system is based on the barycenter or center of gravity we find the positions of the Sun and Earth with respect to it. This easily be done with the equation of the center of mass.

$$m_\odot x_1 + m_\oplus x_2 = 0, \tag{II.4}$$

and since $x_2 = x_1 + r_{12}$, if we let

$$\eta_1 = \frac{m_\odot}{m_\odot + m_\oplus} = \frac{\mu_\odot}{\mu_\odot + \mu_\oplus} \tag{II.5}$$

$$\eta_2 = \frac{m_\oplus}{m_\odot + m_\oplus} = \frac{\mu_\oplus}{\mu_\odot + \mu_\oplus}, \tag{II.6}$$

then we have

$$x_1 = -\eta_2 r_{12} \qquad x_2 = \eta_1 r_{12}. \tag{II.7}$$

Subsequently, we find the orbit of the tertiary mass or JWST with respect to the barycenter. Let the position of JWST be

$$\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}. \tag{II.8}$$

The position of JWST relative to the Earth is

$$\mathbf{r}_1 = (x - x_1)\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}} = (x + \eta_2 r_{12})\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}. \tag{II.9}$$

Similarly, the position of JWST relative to the Sun is

$$\mathbf{r}_2 = (x - x_2)\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}} = (x - \eta_1 r_{12})\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}. \tag{II.10}$$

Considering the inertial rotation of the Earth relative to the Sun, if we take the derivative of the position we obtain

$$\dot{\mathbf{r}} = \mathbf{v}_{cog} + \mathbf{v} + \boldsymbol{\Omega} \times \mathbf{r}, \tag{II.11}$$

where

$$\mathbf{v} = \dot{x}\hat{\mathbf{i}} + \dot{y}\hat{\mathbf{j}} + \dot{z}\hat{\mathbf{k}}, \tag{II.12}$$

and $\mathbf{v}_{cog}$ is the absolute velocity of the barycenter. If we take the derivative again to find the acceleration we have

$$\ddot{\mathbf{r}} = \mathbf{a}_{cog} + \mathbf{a} + \dot{\boldsymbol{\Omega}} \times \mathbf{r} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}) + 2\boldsymbol{\Omega} \times \mathbf{v}, \tag{II.13}$$

where

$$\mathbf{a} = \ddot{x}\hat{\mathbf{i}} + \ddot{y}\hat{\mathbf{j}} + \ddot{z}\hat{\mathbf{k}}, \tag{II.14}$$

This can be simplified because $\mathbf{a}_{cog} = 0$ and $\dot{\boldsymbol{\Omega}} = 0$. Hence, we have

$$\ddot{\mathbf{r}} = \mathbf{a} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}) + 2\boldsymbol{\Omega} \times \mathbf{v}, \tag{II.15}$$

which is

$$\ddot{\mathbf{r}} = (\ddot{x} - 2\Omega\dot{y} - \Omega^2 x)\hat{\mathbf{i}} + (\ddot{y} + 2\Omega\dot{x} - \Omega^2 y)\hat{\mathbf{j}} + \ddot{z}\hat{\mathbf{k}}. \tag{II.16}$$

The forces can be found using Newton's second law where

$$m_j\ddot{\mathbf{r}} = \mathbf{F}_1 + \mathbf{F}_2. \tag{II.17}$$

The two forces are gravitational forces relative to the Sun and Earth, and therefore we have

$$\mathbf{F}_1 = -\frac{\mu_\odot m_j}{r_1^3}\mathbf{r}_1 \tag{II.18}$$

$$\mathbf{F}_2 = -\frac{\mu_\oplus m_j}{r_2^3}\mathbf{r}_2. \tag{II.19}$$

Hence, we get

$$\ddot{\mathbf{r}} = -\frac{\mu_\odot}{r_1^3}\mathbf{r}_1 - \frac{\mu_\oplus}{r_2^3}\mathbf{r}_2. \tag{II.20}$$

Finally, let $\mu_1 = \mu_\odot$, $\mu_2 = \mu_\oplus$, and the equations of motion for the CR3BP orbit becomes

$$\begin{aligned} \ddot{x} - 2\Omega\dot{y} - \Omega^2 x &= -\frac{\mu_1}{r_1^3}(x + \eta_2 r_{12}) - \frac{\mu_2}{r_2^3}(x - \eta_1 r_{12}) \\ \ddot{y} + 2\Omega\dot{x} - \Omega^2 y &= -\frac{\mu_1}{r_1^3}y - \frac{\mu_2}{r_2^3}y \\ \ddot{z} &= -\frac{\mu_1}{r_1^3}z - \frac{\mu_2}{r_2^3}z \end{aligned} \tag{II.21}$$

III Simulation

For the simulation, MATLAB was used to solve the equations of motion with the `ode45` numerical integrator. However, there is one thing to keep in mind before plotting the results from the numerical integration. The equations of motion represents the positions and velocities in the Sun-Earth reference frame in which the x-axis is always passing through the line between the Sun and the Earth. Thus, to fix the values into the inertial frame we must convert the Sun-Earth $\hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}}$ into inertial $\hat{\mathbf{e}}_1, \hat{\mathbf{e}}_2, \hat{\mathbf{e}}_3$ coordinates by considering the angle $\phi = \Omega t$.

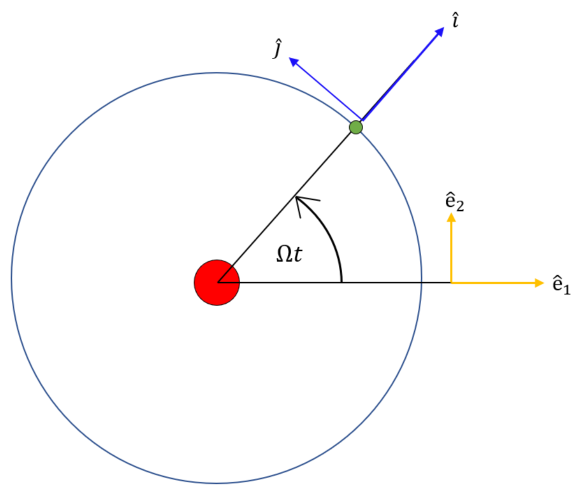

Figure 4: Relation between the Sun-Earth frame and the inertial frame.

$$\begin{bmatrix} \hat{\mathbf{e}}_1 \\ \hat{\mathbf{e}}_2 \\ \hat{\mathbf{e}}_3 \end{bmatrix} = \begin{bmatrix} \cos \Omega t & -\sin \Omega t & 0 \\ \sin \Omega t & \cos \Omega t & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{\mathbf{i}} \\ \hat{\mathbf{j}} \\ \hat{\mathbf{k}} \end{bmatrix}. \tag{III.1}$$

Also, to solve the numerical integration for the CR3BP orbit, the initial conditions is a crucial yet difficult factor to figure out for this problem. The conventional method to figure out the initial conditions is to construct the initial guess with a 3rd order guess for the halo orbit or start with a small Lyapunov orbit and bifurcate into halo as they continue the solution.

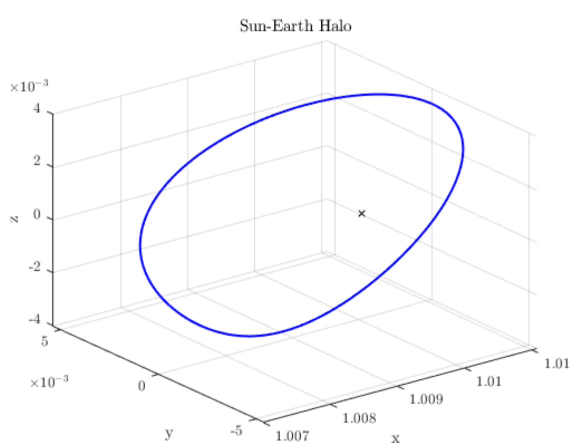

Figure 5: Non-dimensionalized halo orbit of JWST about L2.

Then for the second step, the initial guess is corrected as a root-solving problem or BVP (boundary value problem). With this method, we are able to find the initial conditions fed into the `ode45` which happened to be

$$\begin{bmatrix} x_0 & y_0 & z_0 & v_{x0} & v_{y0} & v_{z0} \end{bmatrix}^T = \begin{bmatrix} 1.5126\text{E+}8 & 0 & 5.1082\text{E+}5 & 0 & -0.3100 & 0 \end{bmatrix}^T. \tag{III.2}$$

The non-dimensionalized halo orbit produced for the JWST for this initial condition is shown in Figure 5.

The visualizations of the simulation results are presented below.

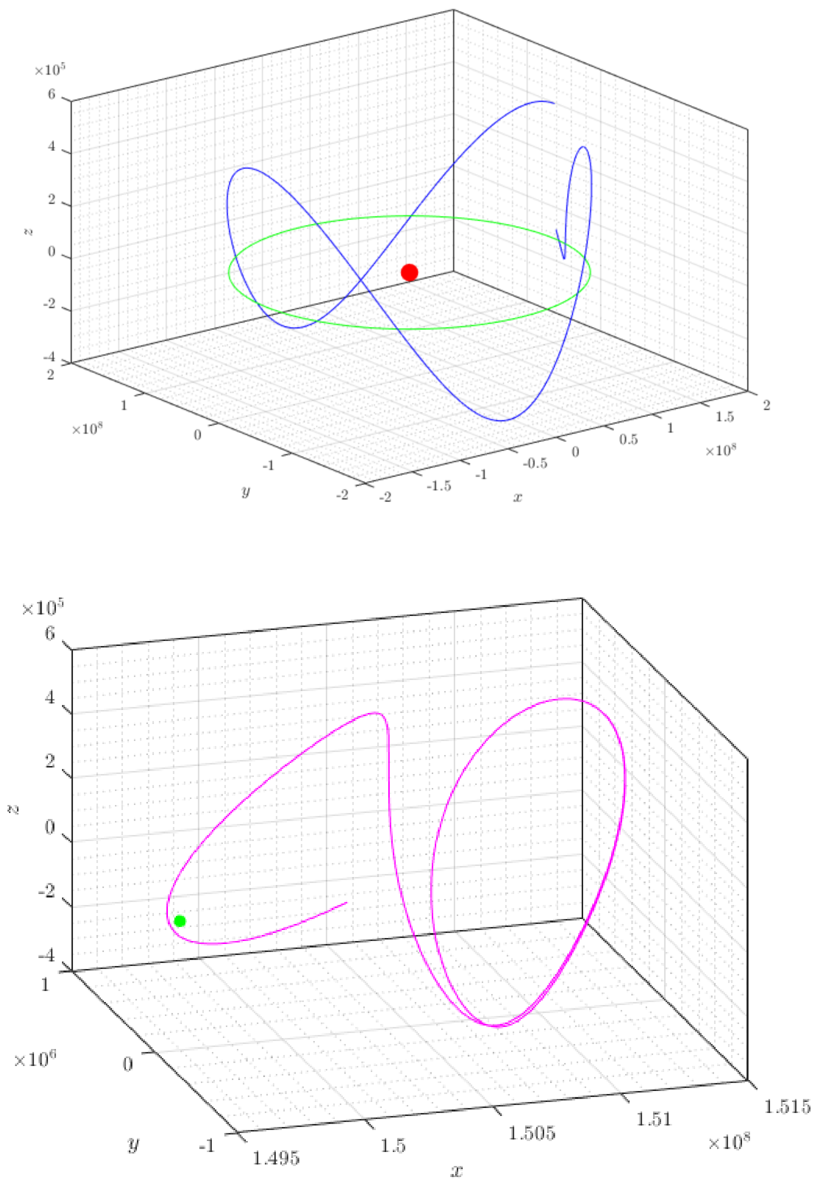

Figure 6: JWST orbit w.r.t the Sun (top) and Earth (bottom).

IV Verification

To verify the accuracy of the simulation, two methods have been conducted. One is by comparing our simulation to an external and credible source/data and the second is to compute the Jacobi constant which is essentially the sum of the kinematic and potential energy of the orbit. For the first method, we took the data from NASA's Horizons System which allows to predict and propagate the past and future orbit of a planet or satellite of which you desire. An example data set is shown in the Appendix (Section VII). After retrieving this data we plot the orbit using the Horizons System data and our simulation together to check to see the validity of our simulation. The plot shows from three different angles.

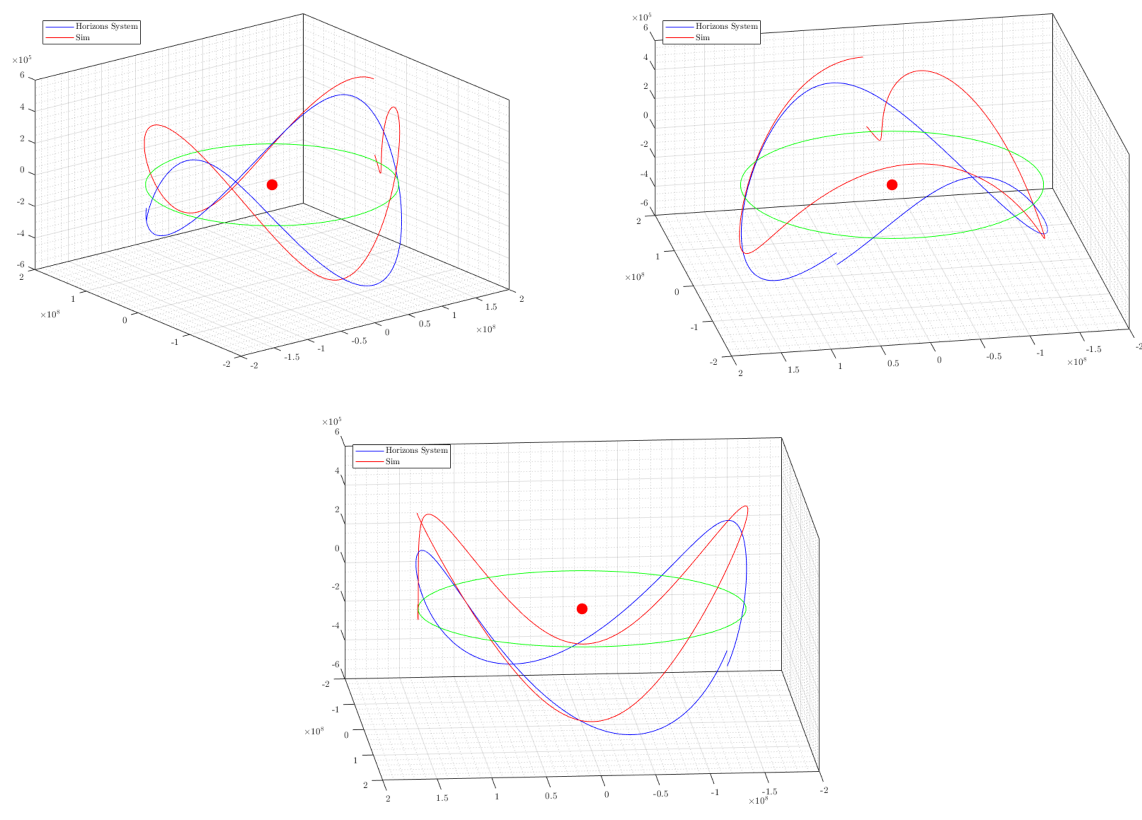

Figure 7: Comparison of Horizons System data and simulation.

It can be observed that the overall shape of the orbit or curve is somewhat similar; however, throughout the entire orbit there are discrepancies between the two that cannot be ignored. Interestingly, there are parts in the two orbits that the differences are small and there are places that it is large. Note that there is a large difference at the end of the orbit of the simulation where the orbit dips and gets extremely close to the Earth. This erroneous behavior will be discussed further in the discussion (section V).

Now, for the second method, we find that the Jacobi constant is computed as follows.

$$J = \frac{1}{2}(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) - \frac{1-\eta_2}{\alpha} - \frac{\eta_2}{\beta} - \frac{1}{2}\left[(1-\eta_2)\,\alpha^2 + \eta_2\beta^2\right]. \tag{IV.1}$$

where

$$\alpha = \frac{r_1}{r_{12}} \qquad \beta = \frac{r_2}{r_{12}}. \tag{IV.2}$$

Theoretically, energy is conserved, and therefore, the Jacobi constant should be stable in order to verify our orbit.

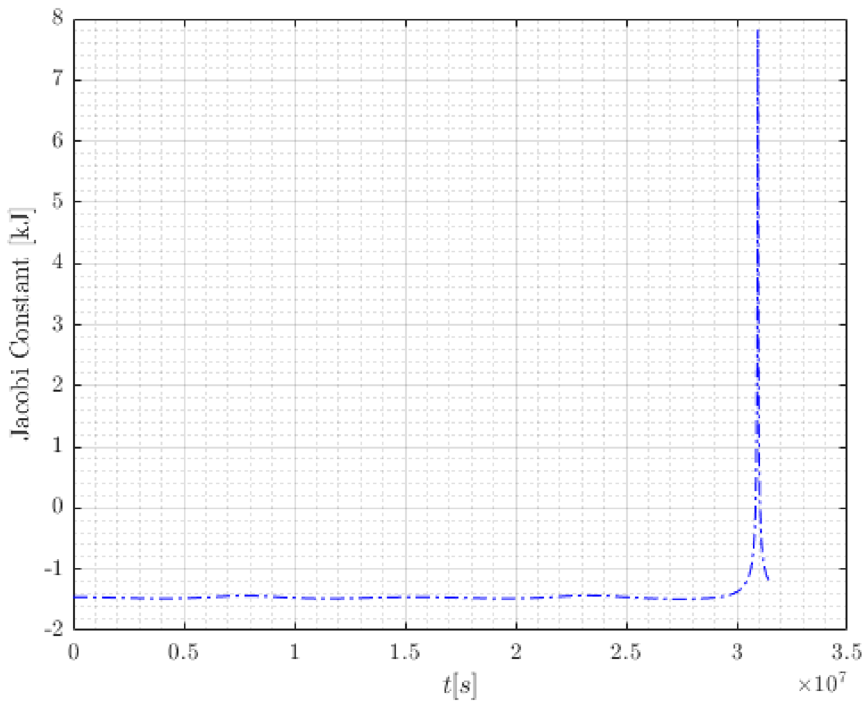

Figure 8: Jacobi constant of the orbit.

As expected, Figure 8 shows that it is constant but at the end of the orbit where we saw an erratic behavior the Jacobi constant seems to spike which shows that there is something wrong with the simulation at the end. However, the overall performance and validity of the orbit is verified through the two methods.

V Discussion

So far we have verified that the results of our simulation is coherent with the actual expected orbit of JWST. However, as aforementioned, there seems to be small and large deviations with the actual data and an erratic behavior of the simulation orbit towards the end.

For the small and large deviations, the possible cause would be the initial conditions. In actuality, JWST is launched from Earth and enters the L2 halo orbit with a number of maneuvers and other disturbances in space. Our simulation, on the other hand, makes an initial guess with JWST already being on the halo orbit without any maneuvers or disturbances. With the initial conditions being different it is clear that the following trajectory will differ for the simulation and the actual data from NASA. To solve this we would have to have access to the actual data from JWST to make a more proper initial guess for the initial conditions of our simulation.

For our next discussion, we will talk about the behavior where JWST diverts from the orbit and dips toward the Earth as observed in Figure 6. To diagnose the cause of this behavior we plot out the position and velocity of JWST in the simulation.

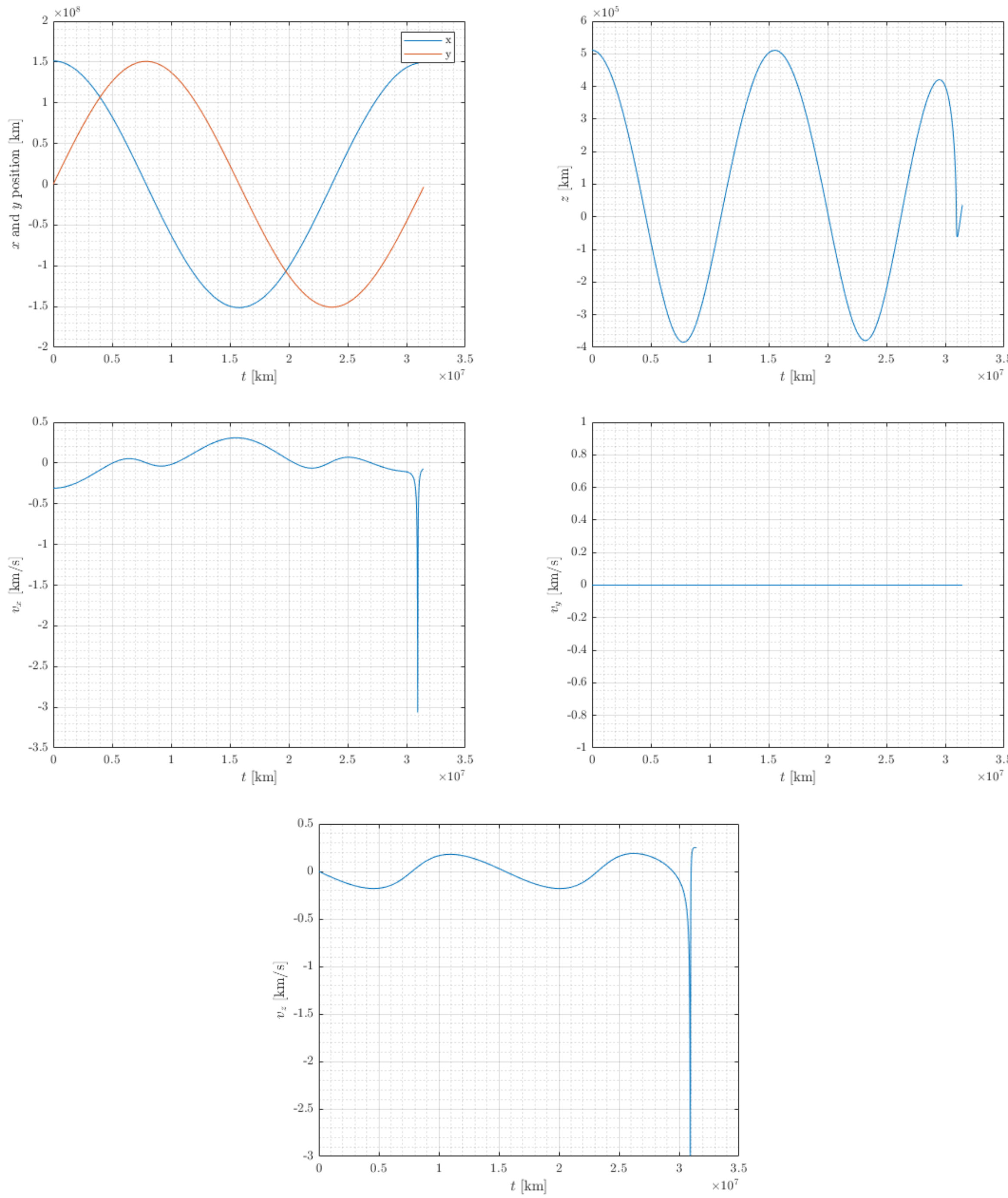

Figure 9: Positions and velocities of JWST in the simulation.

These plots show that the x and z velocity components seem to change drastically at the end of the orbit which causes JWST to change its direction significantly. It is difficult to diagnose the actual cause of this issue, however it may be due to tolerance errors or other error within the numerical integration.

VI References

[1] NASA, "Orbit - webb/nasa." `https://jwst.nasa.gov/content/about/orbit.html`, 2022.

[2] NASA, "Key facts - webb/nasa." `https://webb.nasa.gov/content/about/faqs/facts.html`, 2022.

[3] D. R. Williams, "Planetary fact sheet." `https://nssdc.gsfc.nasa.gov/planetary/factsheet/`, 2018.

[4] D. R. Williams, "Sun fact sheet." `https://nssdc.gsfc.nasa.gov/planetary/factsheet/sunfact.html`, 2018.

VII Appendix

Horizons System Raw Data

```
*******************************************************************************
 Revised: Feb 16, 2022     James Webb Space Telescope / (E-S L2)          -170
                           https://www.jwst.nasa.gov/index.html
                        https://www.youtube.com/watch?v=bTxLAGchWnA

 The James Webb Space Telescope ("JWST" or "Webb") is a space-based infrared
 observatory and NASA's successor to the Hubble Space Telescope.

 Launched by Ariane 5 booster on 2021-Dec-25 @ 12:20 UTC from the ELA-3
 launch complex near Korou, French Guiana.

 After launch, the telescope will deploy during its 30-day, 1.5 million km
 journey to halo orbit at the second Earth-Sun Lagrange point (E-S L2).
 Mission duration is nominally 5-10 years.

 GOALS
  - Search for the first galaxies or luminous objects formed after the Big Bang
  - Determine how galaxies evolved from their formation until now
  - Observe the formation of stars from the first stages to the formation of
      planetary systems
  - Measure the physical and chemical properties of planetary systems,
      including our own Solar System, investigating the potential for life
      in those systems.

 TELESCOPE
  * total launch mass : ~6200 kg (observatory, fuel, launch adaptor)
  * primary mirror    : 25 m^2
             mass     : 705 kg
         material     : beryllium coated w/48.25 grams gold (golf-ball size)
      segment mass    : 20.1 kg, 39.48 kg for entire segment assembly
      No. of segments : 18
  * focal length      : 131.4 meters
  * optical resolution: 0.1 arcseconds
  * wavelength        : 0.6 - 28.5 microns
  * size of sun shield: 21.197 m x 14.162 m
  * Sun shield layers : 1: Max temp 283K, 231 deg. F.
                        5: Max temp 221K,  -80 F
                           Min temp 36K,  -394 F
  * Operating temp    : < 50K (-370 deg. F)

 INSTRUMENTS
  - Near Infrared Camera (NIRCam)
  - Near Infrared Spectrograph (NIRSpec)
  - Mid Infrared Instrument (MIRI)
  - Fine Guidance Sensors/Near Infrared Imager & Slitless Spectrograph
      (FGS/NIRISS)

 TRAJECTORY
 MCC1A (65-minute engine burn) began 2021-Dec-26 12:50 UTC, completed 01:55 UTC.
 MCC1B (09:27 engine burn) began 2021-Dec-28 00:20 UTC, completed 00:29:27 UTC.
 MCC2  (04:57 engine burn) began 2022-Jan-24 19:00 UTC, completed 19:04:57 UTC.

 Post-launch trajectory from Goddard Flight Dynamics Facility (FDF), based on
 data through ~16:00 UTC Feb 14, predicts thereafter.

 Trajectory files                             Start (TDB)        End (TDB)
 ----------------------------------------  -----------------  -----------------
 BURN_TTF_01_2021359124800_02U.OEM.V0.3    2021-Dec-25 12:50  2021-Dec-25 20:01
 BURN_MCC_1A_2021359200000_04U.OEM.V0.1    2021-Dec-25 20:01  2021-Dec-26 15:01
 BURN_MCC_1B_2021360150000_01U.OEM.V0.1    2021-Dec-26 15:01  2021-Dec-27 14:01
 BURN_MCC_1B_2021361140000_02U.OEM.V0.1    2021-Dec-27 14:01  2021-Dec-29 00:01
 NOBURN_2021363-2022017_01U.OEM.V0.1       2021-Dec-29 00:01  2022-Jan-18 00:01
 BURN_MCC_02_2022018000000_01U.OEM.V0.1    2022-Jan-18 00:01  2022-Jan-22 00:01
```

```
 BURN_MCC_02_2022022000000_02U.OEM.V0.1    2022-Jan-22 00:01  2022-Jan-27 00:01
 28D_BURN_PREDICT_SK_2022027-2022038.V0.1  2022-Jan-27 00:01  2022-Mar-14 00:01
 2Y_SCHEDULE_2022045000000.OEM.V0.1        2022-Mar-14 00:01  2024-Feb-14 00:00
*******************************************************************************


*******************************************************************************
Ephemeris / WWW_USER Sat Feb 19 19:42:20 2022 Pasadena, USA      / Horizons
*******************************************************************************
Target body name: James Webb Space Telescope (spacecraft) (-170) {source: JWST_merged}
Center body name: Sun (10)                        {source: DE441}
Center-site name: BODY CENTER
*******************************************************************************
Start time      : A.D. 2021-Dec-25 12:49:09.1840 TDB
Stop  time      : A.D. 2022-Jan-25 12:49:09.1840 TDB
Step-size       : 1440 minutes
*******************************************************************************
Center geodetic : 0.00000000,0.00000000,0.0000000 {E-lon(deg),Lat(deg),Alt(km)}
Center cylindric: 0.00000000,0.00000000,0.0000000 {E-lon(deg),Dxy(km),Dz(km)}
Center radii    : 696000.0 x 696000.0 x 696000.0 k{Equator, meridian, pole}
Output units    : KM-S
Output type     : GEOMETRIC cartesian states
Output format   : 2x (pos. & vel. w/uncertainties: XYZ)
Reference frame : Ecliptic of J2000.0
*******************************************************************************
JDTDB
 State   : X       Y        Z     VX       VY        VZ
 1-sigma : X_s     Y_s      Z_s   VX_s     VY_s      VZ_s
***********************************************************************************************
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 ↪  2.959679472836080E+00  -2.791058207386196E+00
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 ↪  n.a.                  n.a.
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 XYZ   : -1.192523056913623E+07   1.468412611522774E+08  -9.638064091245085E+04  -3.030136274112026E+01
 ↪  -9.553617191599464E-01  -6.651476819212098E-01
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 sigmas:                  n.a.                    n.a.                    n.a.                    n.a.
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 ↪  n.a.                   n.a.
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 ↪  n.a.                   n.a.
2459584.034134074 = A.D. 2022-Jan-04 12:49:09.1840 TDB [del_T=     69.183995 s]
 XYZ   : -3.526410511921029E+07   1.436476708433834E+08  -3.395684917078167E+05  -2.955707841225200E+01
 ↪  -6.756407971872987E+00  -1.690253618897439E-01
 sigmas:                  n.a.                   n.a.                   n.a.                   n.a.
 ↪  n.a.                   n.a.
2459585.034134074 = A.D. 2022-Jan-05 12:49:09.1840 TDB [del_T=     69.184024 s]
 XYZ   : -3.781209428354236E+07   1.430402542201563E+08  -3.534296944836676E+05  -2.942306401251465E+01
 ↪  -7.303286378475656E+00  -1.518012518921763E-01
 sigmas:                  n.a.                   n.a.                   n.a.                   n.a.
 ↪  n.a.                   n.a.
2459586.034134074 = A.D. 2022-Jan-06 12:49:09.1840 TDB [del_T=     69.184054 s]
 XYZ   : -4.034814178961932E+07   1.423858448626229E+08  -3.658503169006929E+05  -2.928026072498833E+01
 ↪  -7.844136941604552E+00  -1.359919681865964E-01
 sigmas:                  n.a.                   n.a.                   n.a.                   n.a.
 ↪  n.a.                   n.a.
2459587.034134074 = A.D. 2022-Jan-07 12:49:09.1840 TDB [del_T=     69.184083 s]
 XYZ   : -4.287147100205635E+07   1.416849489093387E+08  -3.769619219238758E+05  -2.912868202139686E+01
 ↪  -8.379408697564283E+00  -1.214073726547427E-01
 sigmas:                  n.a.                   n.a.                   n.a.                   n.a.
 ↪  n.a.                   n.a.
2459588.034134074 = A.D. 2022-Jan-08 12:49:09.1840 TDB [del_T=     69.184112 s]
 XYZ   : -4.538132627231346E+07   1.409380344687873E+08  -3.868581140281260E+05  -2.896836696682663E+01
 ↪  -8.909425358804581E+00  -1.078290032610045E-01
 sigmas:                  n.a.                   n.a.                   n.a.                   n.a.
 ↪  n.a.                   n.a.
2459589.034134074 = A.D. 2022-Jan-09 12:49:09.1840 TDB [del_T=     69.184141 s]
 XYZ   : -4.787695402919450E+07   1.401455452456198E+08  -3.956185080095157E+05  -2.879936563816431E+01
 ↪  -9.434415042494967E+00  -9.509354291449723E-02
 sigmas:                  n.a.                   n.a.                   n.a.                   n.a.
 ↪  n.a.                   n.a.
2459590.034134074 = A.D. 2022-Jan-10 12:49:09.1840 TDB [del_T=     69.184170 s]
 XYZ   : -5.035760710628101E+07   1.393079080589615E+08  -4.033106929339990E+05  -2.862172239596740E+01
 ↪  -9.954534493617871E+00  -8.308125720664927E-02
 sigmas:                  n.a.                   n.a.                   n.a.                   n.a.
 ↪  n.a.                   n.a.
2459591.034134074 = A.D. 2022-Jan-11 12:49:09.1840 TDB [del_T=     69.184199 s]
 XYZ   : -5.282254031630930E+07   1.384255391461863E+08  -4.099927163494602E+05  -2.843548941919672E+01
 ↪  -1.046988209637903E+01  -7.168897724479528E-02
 sigmas:                  n.a.                   n.a.                   n.a.                   n.a.
 ↪  n.a.                   n.a.
2459592.034134074 = A.D. 2022-Jan-12 12:49:09.1840 TDB [del_T=     69.184228 s]
 XYZ   : -5.527101429636963E+07   1.374988482258360E+08  -4.157140338758454E+05  -2.824072564640830E+01
 ↪  -1.098051333453895E+01  -6.082997761035713E-02
 sigmas:                  n.a.                   n.a.                   n.a.                   n.a.
 ↪  n.a.                   n.a.
2459593.034134074 = A.D. 2022-Jan-13 12:49:09.1840 TDB [del_T=     69.184257 s]
 XYZ   : -5.770229400120717E+07   1.365282416419295E+08  -4.205176674731299E+05  -2.803749662650193E+01
 ↪  -1.148645132898790E+01  -5.043578015044670E-02
 sigmas:                  n.a.                   n.a.                   n.a.                   n.a.
 ↪  n.a.                   n.a.
2459594.034134074 = A.D. 2022-Jan-14 12:49:09.1840 TDB [del_T=     69.184285 s]
 XYZ   : -6.011565184472909E+07   1.355141247894991E+08  -4.244412015195414E+05  -2.782587629801508E+01
 ↪  -1.198769055020314E+01  -4.045184420900849E-02
 sigmas:                  n.a.                   n.a.                   n.a.                   n.a.
 ↪  n.a.                   n.a.
2459595.034134074 = A.D. 2022-Jan-15 12:49:09.1840 TDB [del_T=     69.184314 s]
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 XYZ   : -6.251036604796328E+07   1.344569049975051E+08  -4.275185240446627E+05  -2.760594421587392E+01
 ↪  -1.248419688292345E+01  -3.084257437942828E-02
 sigmas:                  n.a.                   n.a.                   n.a.                   n.a.
 ↪  n.a.                   n.a.
2459596.034134074 = A.D. 2022-Jan-16 12:49:09.1840 TDB [del_T=     69.184342 s]
 XYZ   : -6.488572190211237E+07   1.333569934936531E+08  -4.297809029245377E+05  -2.737777917240954E+01
 ↪  -1.297591362283174E+01  -2.158445918468832E-02
 sigmas:                  n.a.                   n.a.                   n.a.                   n.a.
 ↪  n.a.                   n.a.
2459597.034134074 = A.D. 2022-Jan-17 12:49:09.1840 TDB [del_T=     69.184370 s]
 XYZ   : -6.724100622606170E+07   1.322148092963347E+08  -4.312585283569470E+05  -2.714143785098365E+01
 ↪  -1.346275979473823E+01  -1.266899181528203E-02
 sigmas:                  n.a.                   n.a.                   n.a.                   n.a.
 ↪  n.a.                   n.a.
2459598.034134074 = A.D. 2022-Jan-18 12:49:09.1840 TDB [del_T=     69.184398 s]
 XYZ   : -6.957552764574346E+07   1.310307723051472E+08  -4.319791015310064E+05  -2.689702984375743E+01
 ↪  -1.394466776004450E+01  -4.068485752629414E-03
 sigmas:                  n.a.                   n.a.                   n.a.                   n.a.
 ↪  n.a.                   n.a.
2459599.034134074 = A.D. 2022-Jan-19 12:49:09.1840 TDB [del_T=     69.184426 s]
 XYZ   : -7.188858279007620E+07   1.298053153771014E+08  -4.319702806555182E+05  -2.664458022585779E+01
 ↪  -1.442154533094587E+01   4.223361049395358E-03
 sigmas:                  n.a.                   n.a.                   n.a.                   n.a.
 ↪  n.a.                   n.a.
2459600.034134074 = A.D. 2022-Jan-20 12:49:09.1840 TDB [del_T=     69.184454 s]
 XYZ   : -7.417948169188198E+07   1.285388765463113E+08  -4.312575426776558E+05  -2.638416505931004E+01
 ↪  -1.489330910787330E+01   1.222928408573054E-02
 sigmas:                  n.a.                   n.a.                   n.a.                   n.a.
 ↪  n.a.                   n.a.
2459601.034134074 = A.D. 2022-Jan-21 12:49:09.1840 TDB [del_T=     69.184481 s]
 XYZ   : -7.644753914512824E+07   1.272319012593561E+08  -4.298646888730302E+05  -2.611585525093482E+01
 ↪  -1.535987507106071E+01   1.997022178432140E-02
 sigmas:                  n.a.                   n.a.                   n.a.                   n.a.
 ↪  n.a.                   n.a.
2459602.034134074 = A.D. 2022-Jan-22 12:49:09.1840 TDB [del_T=     69.184509 s]
 XYZ   : -7.869207482144207E+07   1.258848433691440E+08  -4.278144672763720E+05  -2.583972476907254E+01
 ↪  -1.582115347111159E+01   2.746392375253937E-02
 sigmas:                  n.a.                   n.a.                   n.a.                   n.a.
 ↪  n.a.                   n.a.
2459603.034134074 = A.D. 2022-Jan-23 12:49:09.1840 TDB [del_T=     69.184536 s]
 XYZ   : -8.091241961139716E+07   1.244981614359479E+08  -4.251259505874515E+05  -2.555586345857429E+01
 ↪  -1.627706224123290E+01   3.473350083917559E-02
 sigmas:                  n.a.                   n.a.                   n.a.                   n.a.
 ↪  n.a.                   n.a.
2459604.034134074 = A.D. 2022-Jan-24 12:49:09.1840 TDB [del_T=     69.184564 s]
 XYZ   : -8.310790778152201E+07   1.230723244231930E+08  -4.218186804443821E+05  -2.526435484622855E+01
 ↪  -1.672750528306212E+01   4.178901961868409E-02
 sigmas:                  n.a.                   n.a.                   n.a.                   n.a.
 ↪  n.a.                   n.a.
2459605.034134074 = A.D. 2022-Jan-25 12:49:09.1840 TDB [del_T=     69.184591 s]
 XYZ   : -8.527797183795436E+07   1.216078370778281E+08  -4.179148114575073E+05  -2.496668714808940E+01
 ↪  -1.717194217382037E+01   4.857577139170566E-02
 sigmas:                  n.a.                   n.a.                   n.a.                   n.a.
 ↪  n.a.                   n.a.
$$EOE
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TIME

  Barycentric Dynamical Time ("TDB" or T_eph) output was requested. This
continuous relativistic coordinate time is equivalent to the relativistic
proper time of a clock at rest in a reference frame comoving with the
solar system barycenter but outside the system's gravity well. It is the
independent variable in the solar system relativistic equations of motion.

  TDB runs at a uniform rate of one SI second per second and is independent
of irregularities in Earth's rotation.

  Calendar dates prior to 1582-Oct-15 are in the Julian calendar system.
```

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Later calendar dates are in the Gregorian system.

REFERENCE FRAME AND COORDINATES

  Ecliptic at the standard reference epoch

    Reference epoch: J2000.0
    X-Y plane: adopted Earth orbital plane at the reference epoch
               Note: IAU76 obliquity of 84381.448 arcseconds wrt ICRF X-Y plane
    X-axis   : ICRF
    Z-axis   : perpendicular to the X-Y plane in the directional (+ or -) sense
               of Earth's north pole at the reference epoch.

  Symbol meaning:

    JDTDB    Julian Day Number, Barycentric Dynamical Time
    del_T    Time-scale conversion difference TDB - UT (s)
      X      X-component of position vector (km)
      Y      Y-component of position vector (km)
      Z      Z-component of position vector (km)
      VX     X-component of velocity vector (km/sec)
      VY     Y-component of velocity vector (km/sec)
      VZ     Z-component of velocity vector (km/sec)
      X_s    X-direction 1-sigma position uncertainty (km)
      Y_s    Y-direction 1-sigma position uncertainty (km)
      Z_s    Z-direction 1-sigma position uncertainty (km)
      VX_s   X-direction 1-sigma velocity uncertainty (km/sec)
      VY_s   Y-direction 1-sigma velocity uncertainty (km/sec)
      VZ_s   Z-direction 1-sigma velocity uncertainty (km/sec)

   "n.a." indicates the value for that field is not currently available.

ABERRATIONS AND CORRECTIONS

 Geometric state vectors have NO corrections or aberrations applied.

Computations by ...

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    Pasadena, CA  91109   USA

    General site: https://ssd.jpl.nasa.gov/
    Mailing list: https://ssd.jpl.nasa.gov/email_list.html
    System news : https://ssd.jpl.nasa.gov/horizons/news.html
    User Guide  : https://ssd.jpl.nasa.gov/horizons/manual.html
    Connect     : browser        https://ssd.jpl.nasa.gov/horizons/app.html#/x
                  API            https://ssd-api.jpl.nasa.gov/doc/horizons.html
                  command-line   telnet ssd.jpl.nasa.gov 6775
                  e-mail/batch   https://ssd.jpl.nasa.gov/ftp/ssd/hrzn_batch.txt
                  scripts        https://ssd.jpl.nasa.gov/ftp/ssd/SCRIPTS
    Author      : Jon.D.Giorgini@jpl.nasa.gov
********************************************************************************
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