AAE 339: Aerospace Propulsion

HW 8: Introduction to Rocket Propulsion

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Friday April 10th, 2020

Problem 1

A rocket operates with a throat stagnation pressure $p_{0,t}$ = 20 MPa, and produces 200 kN of thrust during sea level tests. The combustion products have a molecular weight of 22 kg/kg-mole, a specific heat ratio γ = 1.2, and a temperature T_0 = 3200 K. During the sea level tests (problem parts a and b), the chamber ends at the throat ($A_t = A^*$), and there is no diverging nozzle to further expand the combustion products. Determine the following:

a) The characteristic velocity (c^*), the thrust coefficient (c_f), and the specific impulse (I_{sp}).

b) The throat area and the mass flowrate, and the static pressure and temperature at the throat.

c) Now consider the case where a converging-diverging nozzle is used. The nozzle provides optimal expansion, i.e., the static pressure at the nozzle exit is equal to atmospheric pressure, or $p_e = p_a$. Determine the expansion ratio ($\varepsilon = A_e/A_t$), c^*, c_f, I_{sp}, and thrust produced with the added expansion section.

d) Rockets used to launch spacecraft spend most of their time at high altitude where ambient pressure is small, hence rocket nozzles on first stages are usually designed so that p_e < 1 atm. Repeat the calculations of part (c), but use a nozzle that produces p_e = 0.4 atm.

e) For the expansion ratios of parts c and d, calculate the momentum thrust term and the pressure thrust term for three values of p_a – 0.1 MPa, 0.04 MPa, and 0 (vacuum condition).

Given Properties

$$P_{0t} = 20\ \text{MPa}$$

$F_T = 200$ kN at sea-level condition

$$\mathcal{M}_w = 22\ \text{kg/kmol} \Rightarrow R = \frac{R_u}{\mathcal{M}_w} = \frac{8314.5\ \text{J/kmol-K}}{22\ \text{kg/kmol}} = 377.93\ \frac{\text{J}}{\text{kg-K}}$$

$$\gamma = 1.2$$

$$T_0 = 3200\ \text{K}$$

a) Characteristic velocity

$$c^* = \frac{P_{0t}A_t}{\dot{m}} = \left[\frac{1}{\gamma}\left(\frac{\gamma+1}{2}\right)^{\frac{\gamma+1}{\gamma-1}}\frac{R_u T_0}{\mathcal{M}_w}\right]^{1/2}$$

$$c^* = \left[\frac{1}{1.2}\left(\frac{2.2}{2}\right)^{\frac{2.4}{0.4}}\frac{(8.314\ \text{kJ/kmol-K})(3200\ \text{K})}{\mathcal{M}_w\ \text{kg/kmol}}\right]^{1/2}$$

$$\boxed{c^* = 1695.7\ \text{m/s}}$$

Thrust coefficient

since $A_t = A^* \;\&\; A_e = A_t \Rightarrow M_t = 1$ (Mach 1 at throat)

then from isentropic relations

$$T = T_0 \Big/ \left(1 + \frac{\gamma-1}{2} M_t^2\right) = 2909.1 \text{ K}$$

$$p = P_0 \Big/ \left(1 + \frac{\gamma-1}{2} M_t^2\right)^{\gamma/\gamma-1} = 11.289 \text{ MPa}$$

thus,

$$\rho = \frac{p}{RT} = \frac{11.289 \text{ MPa}}{\left(377.93 \frac{J}{kg\,K}\right)(2909.1 K)} = 10.2684 \text{ kg/m}^3$$

and

$$u = M_t \sqrt{\gamma R T} = 1148.6 \text{ m/s}$$

now

$$\varepsilon = \frac{A_e}{A_t} = 1 \quad \& \quad P_e = p = 11.289 \text{ MPa}$$

and at sea-level $P_a = 101.3$ kPa

thus,

$$C_f = \left\{ \frac{2\gamma^2}{\gamma-1} \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}} \left[1 - \left(\frac{P_e}{P_{0t}}\right)^{\frac{\gamma-1}{\gamma}} \right] \right\}^{1/2} + \frac{P_e - P_a}{P_{0t}} \varepsilon$$

$$\boxed{C_f = 1.2368}$$

Specific Impulse

$$I_{sp} = C_f \cdot C^* = (1.2368)(1695.7 \text{ m/s})$$

$$\boxed{I_{sp} = 2097.2 \text{ m/s}}$$

b)

Throat Area

from the thrust coefficient

$$\boxed{A_t = \frac{F_t}{P_{0t}\, C_f} = 0.008086\ m^2}$$

Mass flow Rate

$$\boxed{\dot{m} = \frac{F_t}{c^* C_f} = 95.3647\ kg/s}$$

Static Pressure

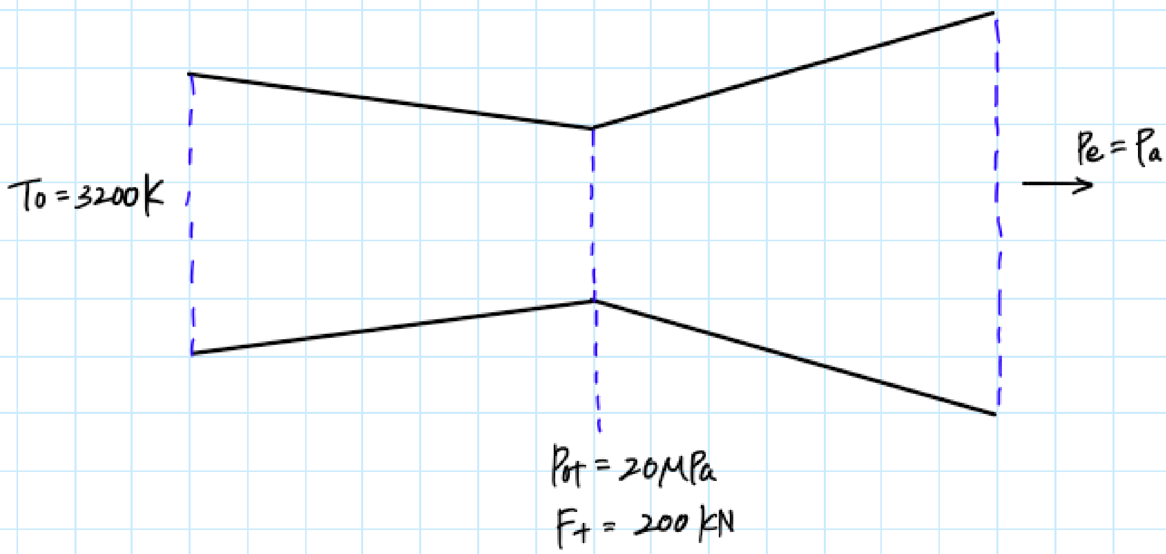
Calculated in a) $\rightarrow$ $\boxed{P = 11.289\ MPa}$

Static Temperature

Calculated in a) $\rightarrow$ $\boxed{T = 2909.1\ K}$

c)

There is now a diverging nozzle attached after the throat



Expansion Ratio

Since $P_e = P_a = 101.3 kPa$
from isentropic relations

$$M_e = \left\{ \frac{2}{\gamma-1}\left[\left(\frac{P_{0t}}{P_e}\right)^{\frac{\gamma-1}{\gamma}} - 1 \right] \right\}^{1/2}$$

$$M_e = 3.7591$$

now, using the following equation

$$\frac{A_e}{A_t} = \left\{ \frac{1}{M_e^2}\left[\frac{2}{\gamma+1}\left(1 + \frac{\gamma-1}{2}M_e^2\right)\right]^{\gamma+1/\gamma-1} \right\}^{1/2}$$

$$\boxed{\varepsilon = \frac{A_e}{A_t} = 20.0168}$$

Characteristic velocity

$$c^* = \frac{P_{0t} A_t}{\dot{m}} = \left[\frac{1}{\gamma}\left(\frac{\gamma+1}{2}\right)^{\frac{\gamma+1}{\gamma-1}} \frac{R_u T_0}{\mathcal{M}_w}\right]^{1/2}$$ does not change

$$\boxed{c^* = 1695.7 \ \text{m/s}}$$

Thrust Coefficient

$$c_f = \left\{\frac{2\gamma^2}{\gamma-1}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\left[1-\left(\frac{P_e}{P_{0t}}\right)^{\frac{\gamma-1}{\gamma}}\right]\right\}^{1/2} + \cancelto{0}{\frac{P_e - P_a}{P_{0t}}}\,\varepsilon$$

$$\boxed{c_f = 1.7192}$$

Specific Impulse

$$I_{sp} = c_f \cdot c^*$$

$$\boxed{I_{sp} = 2915.2 \ \text{m/s}}$$

Additional Thrust

the static temperature at the exit becomes

$$T_e = T_0\left(1 + \frac{\gamma-1}{2}M_e^2\right)^{-1}$$

$$T_e = 1326.1 \ \text{K}$$

the static pressure at the exit is

$$P_e = P_{ot}\left(1+\frac{\gamma-1}{2}M_e^2\right)^{-\gamma/\gamma-1}$$

$$P_e = 101.3 \text{ kPa}$$

the density then becomes

$$\rho_e = \frac{P_e}{RT_e} = 0.2021 \text{ kg/m}^3$$

then the exit velocity becomes

$$u_e = M_e\sqrt{\gamma R T_e}$$

$$u_e = 2915.2 \text{ m/s}$$

so

$$A_e = \frac{\dot{m}}{\rho_e u_e} = 0.1618 \text{ m}^2$$

since $\dot{m} = 95.3647 \text{ kg/s}$

thus

$$F_T = \dot{m}u_e + \cancel{(P_e - P_a)}^{0} A_e$$

since $\dot{m} = 95.3647 \text{ kg/s}$

$$F_t' = 278.01 \text{ kN}$$

thus, the additional thrust introduced by the diffuser

$$\Delta F_t = F_t' - F_t$$

$$\Delta F_t = 278.01 \text{ kN} - 200 \text{ kN}$$

$$\boxed{\Delta F_t = 78.01 \text{ kN}}$$

d)

if $P_e = 0.4$ atm $= 40.53$ kPa

Expansion Ratio

Since $P_e = 40.53$ kPa

from isentropic relations

$$M_e = \left\{ \frac{2}{\gamma-1} \left[\left(\frac{P_{ot}}{P_e} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] \right\}^{1/2}$$

$$M_e = 4.2557$$

now, using the following equation

$$\frac{A_e}{A_t} = \left\{ \frac{1}{M_e^2} \left[\frac{2}{\gamma+1} \left(1 + \frac{\gamma-1}{2} M_e^2 \right) \right]^{\gamma+1/\gamma-1} \right\}^{1/2}$$

$$\boxed{\varepsilon = \frac{A_e}{A_t} = 40.9435}$$

Characteristic velocity

$$c^* = \frac{P_{ot} A_t}{\dot{m}} = \left[\frac{1}{\gamma} \left(\frac{\gamma+1}{2} \right)^{\frac{\gamma+1}{\gamma-1}} \frac{R_u T_o}{\mu_w} \right]^{1/2}$$ does not change

$$\boxed{c^* = 1695.7 \text{ m/s}}$$

Thrust Coefficient

$$C_f = \left\{ \frac{2\gamma^2}{\gamma-1}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}} \left[1-\left(\frac{P_e}{P_{0t}}\right)^{\frac{\gamma-1}{\gamma}}\right]\right\}^{1/2} + \frac{P_e-P_a}{P_{0t}}\varepsilon$$

$$\boxed{C_f = 1.6788}$$

Specific Impulse

$$I_{sp} = C_f \cdot c^*$$

$$\boxed{I_{sp} = 2846.8 \ m/s}$$

Additional Thrust

the static temperature at the exit becomes

$$T_e = T_0\left(1+\frac{\gamma-1}{2}M_e^2\right)^{-1}$$

$$T_e = 1138.3 \ K$$

the static pressure at the exit is

$$P_e = P_{0t}\left(1+\frac{\gamma-1}{2}M_e^2\right)^{-\gamma/\gamma-1}$$

$$P_e = 40.53 \ kPa$$

the density then becomes

$$\rho_e = \frac{P_e}{RT_e} = 0.0942 \ kg/m^3$$

then the exit velocity becomes

$$u_e = M_e\sqrt{\gamma R T_e}$$

$$u_e = 3057.8 \text{ m/s}$$

so

$$A_e = \frac{\dot{m}}{\rho_e u_e} = 0.3310 \text{ m}^2$$

since $\dot{m} = 95.3647$ kg/s

thus

$$F_T = \dot{m} u_e + (p_e - p_a) A_e$$

$$F_T' = 271.49 \text{ kN}$$

thus, the additional thrust introduced by the diffuser

$$\Delta F_T = F_T' - F_T$$

$$\Delta F_T = 271.49 \text{ kN} - 200 \text{ kN}$$

$$\boxed{\Delta F_T = 71.49 \text{ kN}}$$

e)

for (c) $\rightarrow$ $\varepsilon = 20.0168$

p_e remains $p_e = 101.3$ kPa

moment thrust

$$= \dot{m} u_e = (95.3647 \text{ kg/s})(2915.2 \text{ m/s})$$

$$= 278.01 \text{ kN}$$

this is constant throughout any P_{atm}

since

$\varepsilon = \frac{A_e}{A_t}$ is only a function of M_e

and M_e is only on function of P_e

$\Rightarrow P_e$ cannot be changed

$\therefore P_e = 101.3$ kPa

$P_{atm} = 0.1$ MPa

pressure thrust $= (P_e - P_{atm}) A_e = 210.40$ N

$\because A_e = 0.1618\ m^2$

$P_{atm} = 0.04$ MPa

$(P_e - P_{atm}) A_e = 9.9213$ kN

$P_{atm} = 0$

$(P_e - P_{atm}) A_e = 16.395$ kN

(c)

P_{atm} (MPa)	Momentum Thrust (kN)	Pressure Thrust (kN)
0.1	278.01	0.21040
0.04	278.01	9.9212
0	278.01	16.395

for (d), similar to (c)

P_e remains the same $\Rightarrow P_e = 40.53$ kPa

$\therefore$ momentum thrust $= \dot{m} u_e = 291.60$ kN

and

$p_{atm} = 0.1$ MPa

$$(p_e - p_{atm})A_e = (90.53\text{ kPa} - 0.1\text{ MPa})(0.3310\text{ m}^2)$$
$$= -19.688\text{ kN}$$

$p_{atm} = 0.04$ MPa

$$(p_e - p_{atm})A_e = 0.17546\text{ kN}$$

$p_{atm} = 0$

$$(p_e - p_{atm})A_e = 13.417\text{ kN}$$

(d)

p_{atm} (MPa)	Momentum Thrust (kN)	Pressure Thrust (kN)
0.1	291.60	−19.688
0.04	291.60	0.17546
0	291.60	13.417

Problem 2

Use CEA to compare the performance of three common propellant combinations: liquid oxygen and Rocket Propellant 1 (LOX/RP-1), LOX and liquid hydrogen (LOX/LH2), and nitrogen tetroxide and monomethyl hydrazine (NTO/MMH). LOX/RP-1 is typically used in booster applications, like the Merlin engines used in the Falcon 9 and the RD-180 engine used in the Atlas V. LOX/LH2 is typically used in upper stage engines like the RL-10, and NTO/MMH is typically used in spacecraft or missile applications. Use a common equivalence ratio (ϕ = 1.2, rockets tend to operate at fuel-rich conditions), chamber pressure (p_c = 1470 psia, note CEA assumes $p_{0t} = p_c$), and chamber pressure to nozzle exit pressure ratio (p_c / p_e = 100). Note CEA produces two values of specific impulse: I_{sp}, where $p_e = p_a$; and I_{vac}, where p_a = 0. For a thrust of 10 kN, calculate the propellant flowrate and oxidizer-to-fuel mass ratio, O/F for each combination. Tabulate and compare c^*, c_f, I_{sp}, $\dot{m}$, and O/F for the three cases.

liquid Oxygen & RP-1

CEA gives the following results

$O/F = 2.83806$ $\quad c^* = 1781.6 \text{ m/s}$

$C_f = 1.7029$ $\quad I_{sp} = 3033.8 \text{ m/s}$

$I_{vac} = 3285.8$

if thrust, $F_t = 10 \text{ kN}$

$$\dot{m} = \frac{F_t}{I_{sp}} = \frac{10 \text{ kN}}{3033.8 \text{ m/s}} = 3.2962 \text{ kg/s}$$

liquid Oxygen & liquid hydrogen

CEA gives the following results

$O/F = 6.61390$ $\qquad$ $C^* = 2265.0 \ m/s$

$C_f = 1.6862$ $\qquad$ $I_{sp} = 3819.4 \ m/s$

$I_{vac} = 4117.6 \ m/s$

if thrust, $F_t = 10 \ kN$

$$\dot{m} = \frac{F_t}{I_{sp}} = \frac{10 \ kN}{3819.4 \ m/s} = 2.6182 \ kg/s$$

nitrogen tetroxide & monomethyl hydrazine

CEA gives the following results

$O/F = 2.08034$ $\qquad$ $C^* = 1755.9 \ m/s$

$C_f = 1.6608$ $\qquad$ $I_{sp} = 2916.2 \ m/s$

$I_{vac} = 3127.8$

if thrust, $F_t = 10 \ kN$

$$\dot{m} = \frac{F_t}{I_{sp}} = \frac{10 \ kN}{2916.2 \ m/s} = 3.4291 \ kg/s$$

property	LOX/RP-1	LOX/LH2	NTO/MMH
c^* [m/s]	1781.6	2265.0	1755.9
C_f	1.7029	1.6862	1.6608
I_{sp} [m/s]	3033.8	3819.4	2916.2
I_{vac} [m/s]	3285.3	4117.6	3127.8
$\dot{m}$ [kg/s]	3.2962	2.6182	3.4291
O/F	2.83806	6.61390	2.08034

Discussion

∇ From the table above we can tell that the LOX/LH2 has the best performance within the 3 for its high C^* & I_{sp} value. However, the downside to this is that the O/F is outstandingly high; although considering that its primary composition is H & O the fuel mass is relatively light and is not much of a concern.

∇ LOX/RP-1 has the second best performance with a higher C_f value than LOX/LH2. This indicates that the thrust per unit chamber pressure & throat area is higher. And this combination has the highest propellent flow rate.

∇ NTO/MMH on the other hand has the worst performance with lowest I_{sp}, C^*, & C_f. Hydrazine is also not a safe substance to deal with; thus would place this combination the least preferred.

Appendix

AAE 339 HW 8

```
close all; clear all; clc;
```

P1

a)

```
% Define the given properties
P0t   = 20e6;    % throat stagnation pressure [Pa]
Ft    = 200e3;   % thrust [N] @ sea-level conditions
Mw    = 22;      % molecular weight of the product gas [kg/kmol]
gamma = 1.2;     % specific hear ratio
T0    = 3200;    % temperature at the chamber [K]
Ru    = 8314.5;  % universal gas constant [J/kmol-K]
R     = Ru/Mw;   % specific gas constant [J/kg-K]
Patm  = 101.3e3; % atmospheric pressure [Pa]
% Characteristic velocity
a1 = 1/gamma*((gamma + 1)/2)^((gamma + 1)/(gamma - 1));  % intermediate var 1
a2 = R*T0;                                                % intermediate var 2
c_star = sqrt(a1*a2)

% Specific thrust
Mt = 1.0;
T = T_from_M_and_gamma(T0,Mt,gamma,"static")
P = p_from_M_and_gamma(P0t,Mt,gamma,"static")
rho = P/R/T
u = Mt*sqrt(gamma*R*T)

epsilon = 1.0;
a1 = 2*gamma^2/(gamma - 1);                        % intermediate var 1
a2 = (2/(gamma + 1))^((gamma + 1)/(gamma - 1));   % intermediate var 2
a3 = 1 - (P/P0t)^((gamma - 1)/gamma);              % intermediate var 3
a4 = (P - Patm)/P0t*epsilon;                       % intermediate var 4
cf = sqrt(a1*a2*a3) + a4

% Specific impulse
Isp = cf*c_star
```

b)

```
% Throat area
At = Ft/P0t/cf
% Mass flow rate
m_dot = Ft/c_star/cf
```

c)

```
% Calculate the exit Mach number
Pe = Patm;
a1 = 2/(gamma - 1);    % intermediate var 1
```

```
a2 = (P0t/Pe)^((gamma - 1)/gamma) - 1
Me = sqrt(a1*a2)

% Expansion ratio
a1 = 2/(gamma + 1);                % intermediate var 1
a2 = (1 + (gamma - 1)/2*Me^2);     % intermediate var 2
a3 = (gamma + 1)/(gamma - 1);      % intermediate var 3
epsilon = sqrt(1/Me^2*(a1*a2)^(a3))
% Thrust coefficient
a1 = 2*gamma^2/(gamma - 1);                          % intermediate var 1
a2 = (2/(gamma + 1))^((gamma + 1)/(gamma - 1));  % intermediate var 2
a3 = 1 - (Pe/P0t)^((gamma - 1)/gamma);            % intermediate var 3
a4 = (Pe - Patm)/P0t*epsilon;                      % intermediate var 4
cf = sqrt(a1*a2*a3) + a4

% Specific impulse
Isp = cf*c_star

% Additional thrust
Te = T_from_M_and_gamma(T0,Me,gamma,"static")
Pe = p_from_M_and_gamma(P0t,Me,gamma,"static")
rho_e = Pe/R/Te
ue = Me*sqrt(gamma*R*Te)
Ae = m_dot/rho_e/ue
Ft_new = m_dot*ue + (Pe - Patm)*Ae
delta_Ft = Ft_new - Ft

% PROBELM (e)
Patm = 0.1e6;
M_thrust = m_dot*ue
P_thrust = (Pe - Patm)*Ae

Patm = 0.04e6;
M_thrust = m_dot*ue
P_thrust = (Pe - Patm)*Ae

Patm = 0;
M_thrust = m_dot*ue
P_thrust = (Pe - Patm)*Ae
```

d)

```
% Calculate the exit Mach number
Pe = 40.53e3;
a1 = 2/(gamma - 1);                      % intermediate var 1
a2 = (P0t/Pe)^((gamma - 1)/gamma) - 1;  % intermediate var 2
Me = sqrt(a1*a2)

% Expansion ratio
a1 = 2/(gamma + 1);                % intermediate var 1
a2 = (1 + (gamma - 1)/2*Me^2);     % intermediate var 2
a3 = (gamma + 1)/(gamma - 1);      % intermediate var 3
epsilon = sqrt(1/Me^2*(a1*a2)^(a3))
```

```
% Thrust coefficient
a1 = 2*gamma^2/(gamma - 1);                     % intermediate var 1
a2 = (2/(gamma + 1))^((gamma + 1)/(gamma - 1));  % intermediate var 2
a3 = 1 - (Pe/P0t)^((gamma - 1)/gamma);           % intermediate var 3
a4 = (Pe - Patm)/P0t*epsilon;                    % intermediate var 4
cf = sqrt(a1*a2*a3) + a4

% Specific impulse
Isp = cf*c_star

% Additional thrust
Te = T_from_M_and_gamma(T0,Me,gamma,"static")
Pe = p_from_M_and_gamma(P0t,Me,gamma,"static")
rho_e = Pe/R/Te
ue = Me*sqrt(gamma*R*Te)
Ae = m_dot/rho_e/ue
Ft_new = m_dot*ue + (Pe - Patm)*Ae
delta_Ft = Ft_new - Ft

% PROBELM (e)
Patm = 0.1e6;
M_thrust = m_dot*ue
P_thrust = (Pe - Patm)*Ae

Patm = 0.04e6;
M_thrust = m_dot*ue
P_thrust = (Pe - Patm)*Ae

Patm = 0;
M_thrust = m_dot*ue
P_thrust = (Pe - Patm)*Ae
```

```
function  T2 = T_from_M_and_gamma(T1, M, gamma, type)
    if type == "stagnation"
        T2 = T1 * (1 + (gamma - 1) / 2 * M^2);
    elseif type == "static"
        T2 = T1 / (1 + (gamma - 1) / 2 * M^2);
    else
        disp("Error. Incorrect type. Type can only be 'stagnation' or 'static'.")
    end
end
```

```
function  p2 = p_from_M_and_gamma(p1, M, gamma, type)
    if type == "stagnation"
        p2 = p1 * (1 + (gamma - 1) / 2 * M^2)^(gamma/(gamma - 1));
    elseif type == "static"
        p2 = p1 / (1 + (gamma - 1) / 2 * M^2)^(gamma/(gamma - 1));
    else
        disp("Error. Incorrect type. Type can only be 'stagnation' or 'static'.")
    end
end
```