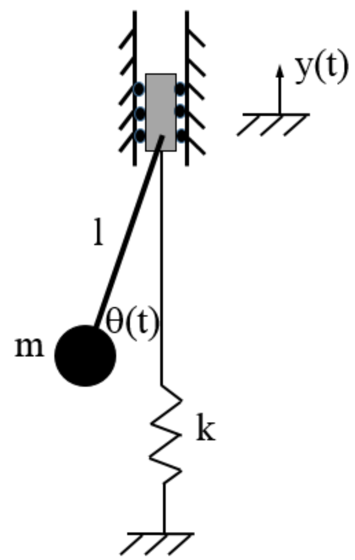
HW # 2   ME 6444 Nonlinear Systems
Fall 2021

Due Date: Tuesday, September 21

1. (10 PTS) 2DOF Sprung Pendulum



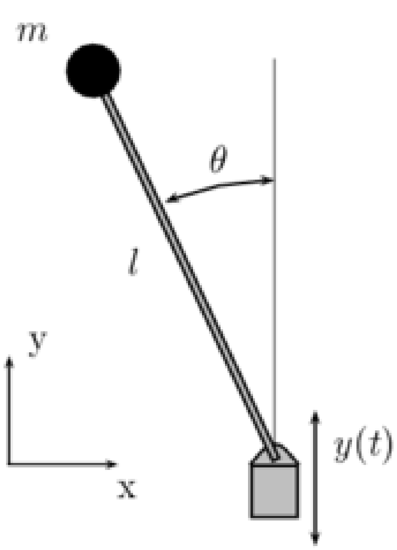
In the sprung pendulum shown, the only appreciable mass comes from that shown at the end of the pendulum. The cart shown is free to move in the vertical direction. There is no friction or other source of dissipation in the problem. There is no forcing – i.e., non-trivial initial conditions are required for there to be motion.

a. Using Lagrange's equations, derive the two equations of motion governing y(t) and θ(t).

b. Identify extra terms in the equation governing θ(t) absent from a simple pendulum.

c. Identify extra terms in the equation governing y(t) absent from a simple linear oscillator.

2. (10 PTS) Forced Inverted Pendulum



a. Derive the equations of motion for a forced inverted pendulum of end-mass *m* and length *l*. Neglect the mass of the rod. Assume base-excitation in the form of $y = A\cos\Omega t$. Keep the full nonlinear equation – i.e., do not expand sine.

b. Explore the behavior of the system, for a parameter set of your choice, by plotting trajectories in the phase plane (use Maple or another mathematics package). Demonstrate that low frequency forcing leads to unstable response, while high frequency forcing can stabilize the pendulum about the θ=0 position.

3. (10 PTS) Variational Operator

a. Show that for small virtual displacements, the variation of the Lagrangian $L = L(u, u', u'', \ddot{u})$ as defined by

$$L(u + \delta u, u' + \delta u', u'' + \delta u'', \ddot{u} + \delta \ddot{u}) - L(u, u', u'', \ddot{u})$$

is given to first order by

$$\delta L = \frac{\partial L}{\partial u}\delta u + \frac{\partial L}{\partial u'}\delta u' + \frac{\partial L}{\partial u''}\delta u'' + \frac{\partial L}{\partial \ddot{u}}\delta \ddot{u}.$$

b. For larger virtual displacements a second-order term can be defined such that

$$L(u + \delta u, u' + \delta u', u'' + \delta u'', \ddot{u} + \delta \ddot{u}) - L(u, u', u'', \ddot{u}) = \delta L + \frac{1}{2}\delta^2 L + \mathrm{OH}.$$

Find an expression for $\delta^2 L$ and show that $\delta^2 L = \delta(\delta L)$ - i.e. is equivalent to two operations of the variational operator.

Hint: for both a. and b. it is helpful to introduce a small parameter by letting $u + \delta u = u + \varepsilon\phi(x,t)$ and to then use a Taylor expansion on ε. Alternatively, you can use a multivariable Taylor expansion as done in class.