AE 6230 – HW3: Mode Shapes and Responses of MDOF Systems

Out: November 1, 2022; **Due:** November 8, 2022 by 11:59 PM ET in Canvas

Guidelines

- Read each question carefully before doing any work;
- If you find yourself doing pages of math, pause and consider if there is an easier approach;
- You can consult any relevant materials;
- You can discuss solution approaches with others, but your submission must be your own work;
- If you have doubts, please ask questions in class, during office hours, and/or Piazza (no questions via email);
- The solution to each question should concisely and clearly show the steps;
- Simplify your results as much as possible;
- Box the final answer for each question;
- Submit any code with the solution (but remember to also submit all relevant plots).

Problem 1 – 40 points

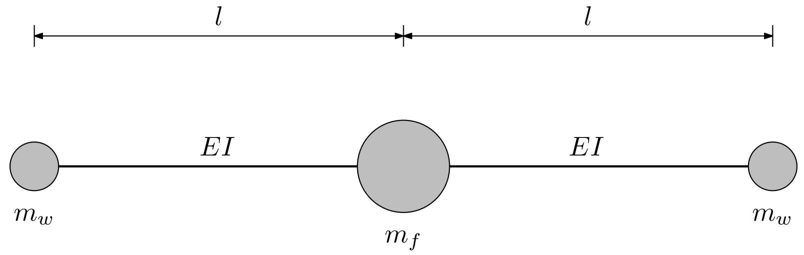

Figure 1: Schematic of an aircraft undergoing out-of-plane (vertical) bending vibrations in free flight.

Table 1: Parameter values for Problem 1.

Parameter	Symbol	Value
Half-wing mass	m_w	750 kg
Fuselage mass	m_f	$5m_w$
Wing semispan	l	10 m
Wing out-of-plane bending stiffness	EI	5×10^6 Nm^2

Figure 1 shows a simplified model for the out-of-plane (vertical) bending vibrations of a free-flying aircraft. The aircraft inertia is modeled by a concentrated mass m_f at the fuselage centerline and two concentrated masses m_w at the wing tips. The elasticity of each half wing is modeled by a beam of negligible mass with out-of-plane bending stiffness EI and length l, which behaves as a spring $k = 3EI/l^3$. The aircraft motion is described in terms of the vertical translations of the left, center, and right masses, denoted by $h_{wl}(t)$, $h_f(t)$, and $h_{wr}(t)$, respectively. These translations are positive upward and measured from the undeformed configuration of the aircraft in Fig. 1. Assuming small-amplitude vibrations and neglecting gravity, answer the following questions:

1. Considering the equations of motion

$$\begin{bmatrix} m_f & 0 & 0 \\ 0 & m_w & 0 \\ 0 & 0 & m_w \end{bmatrix} \begin{Bmatrix} \ddot{h}_f \\ \ddot{h}_{wl} \\ \ddot{h}_{wr} \end{Bmatrix} + \frac{3EI}{l^3} \begin{bmatrix} 2 & -1 & -1 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{Bmatrix} h_f \\ h_{wl} \\ h_{wr} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{1}$$

   evaluate the natural frequencies for the parameters in Table 1 (in ascending order);

2. Evaluate the corresponding mode shapes normalized to have unit maximum displacement;

3. Plot the mode shapes from Question 2 and interpret their meaning;

4. Evaluate the inverse of the modal matrix $\mathbf{U}$ for the assumed mode shape normalization[1];

5. Assuming that a wind gust causes the initial conditions

$$\mathbf{q}(0) = \mathbf{q}_0 = \begin{Bmatrix} 0.5 \\ 0.0 \\ 0.0 \end{Bmatrix} \qquad \dot{\mathbf{q}}(0) = \dot{\mathbf{q}}_0 = 0 \tag{2}$$

   determine the initial conditions for the modal equations;

6. Write the analytical expression of the damped free response in the form

$$\mathbf{q}(t) = \mathbf{U}\boldsymbol{\eta}(t) \tag{3}$$

   considering the modal viscous damping factors $\zeta_1 = 0, \zeta_2 = \zeta_3 = 0.04$;

7. Plot the components of $\mathbf{q}(t)$ and $\boldsymbol{\eta}(t)$ for $0 \le t \le 20$ s;

8. Explain the results from Question 7 (motivate the contribution from each mode).

[1]Note that the assumed mode shape normalization yields non-unit modal mass.

Problem 2 – 30 points

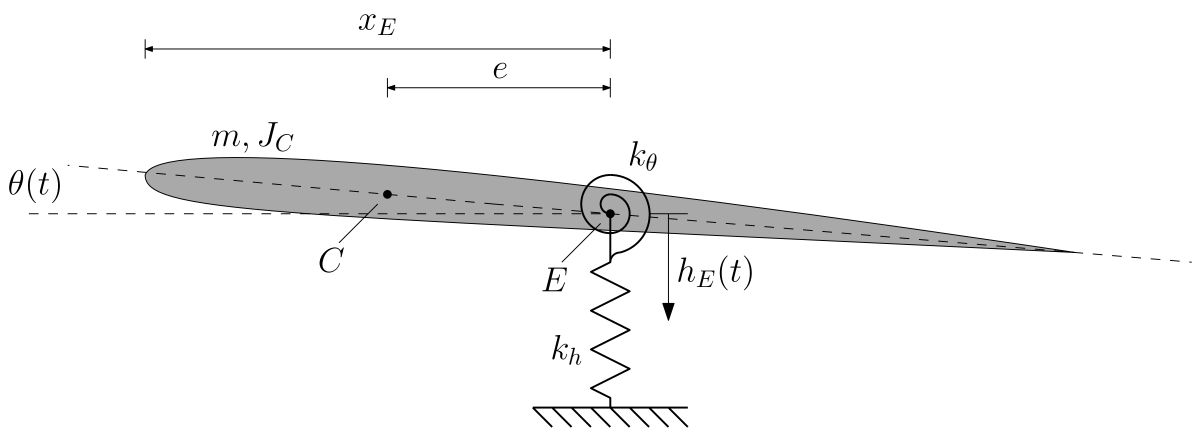

Figure 2: Schematic of typical section model.

Table 2: Parameter values for Problem 2.

Parameter	Symbol	Value
Mass	m	10 kg
Moment of inertia about E	J_E	0.08 kg·m^2
Chord	c	0.2 m
Offset of C from E (positive as in Fig. 2)	e	$-0.2c$
Position of E along the chord (positive as in Fig. 2)	x_E	$0.4c$
Translational spring stiffness	k_h	1000 N/m
Rotational spring stiffness	k_θ	200 Nm/rad

Consider the typical section model in Fig. 2, which is an abstraction for the cross section of a wing undergoing out-of-plane (vertical) bending and torsion. The typical section is subject to the excitation

$$\mathbf{Q}(t) = \mathbf{Q}_0 \sin \omega_0 t \tag{4}$$

with $Q_{0_1} = -10$ N, $Q_{0_2} = 1.5$ Nm, and $\omega_0 = 15$ rad/s. The modal mass and stiffness matrices along with the natural frequencies and mode shapes (normalized to have unit modal mass) can be computed using the script

`AE6230_Fall2022_L17_MDOF_Free_TypicalSection.m`

available in Canvas. Damping effects are captured by the modal viscous damping factors $\zeta_1 = \zeta_2 = 0.02$. Assuming small-amplitude vibrations and neglecting gravity, answer the following questions:

1. Determine the modal excitation $\mathbf{N}(t)$;
2. Considering the frequency response functions $H_1(\omega)$ and $H_2(\omega)$ associated with the modal coordinates $\boldsymbol{\eta}(t)$
   (a) Evaluate their magnitudes at the excitation frequency ω_0;
   (b) Evaluate their phase delays at that frequency;
3. Write the analytical expression of the damped steady-state response in the form of Eq. (3);
4. Plot the components of $\mathbf{q}(t)$ and $\boldsymbol{\eta}(t)$ for $0 \leq t \leq 2$ s;
5. Explain the results from Question 4 (motivate the contribution from each mode).

Problem 3 – 30 points

Consider the same typical section model as in Problem 2. The model experiences the step excitation

$$\mathbf{Q}(t) = \mathbf{Q}_0 \mathrm{u}(t) \tag{5}$$

with $Q_{0_1} = -10$, $Q_{0_2} = 1.5$ Nm, and zero initial conditions. Damping is captured by the proportional model

$$\mathbf{C} = \alpha \mathbf{M} + \beta \mathbf{K} \tag{6}$$

where $\alpha = 1.0\ \mathrm{s}^{-1}$ and $\beta = 1 \times 10^{-5}$ s. Answer the following questions:

1. Evaluate the modal viscous damping factors ζ_1 and ζ_2;
2. Evaluate the damped frequencies ω_{d_1} and ω_{d_2};
3. Write the analytical expression of the damped response in the form of Eq. 3;
4. Plot the components of $\mathbf{q}(t)$ and $\boldsymbol{\eta}(t)$ for $0 \leq t \leq 10$ s;
5. Explain the results from Question 4 (motivate the contribution from each mode);
6. Obtain the results from Question 4 for $e = -0.05c$ and explain any qualitative changes.