LL ex2

Monday, November 23, 2020 01:32 PM

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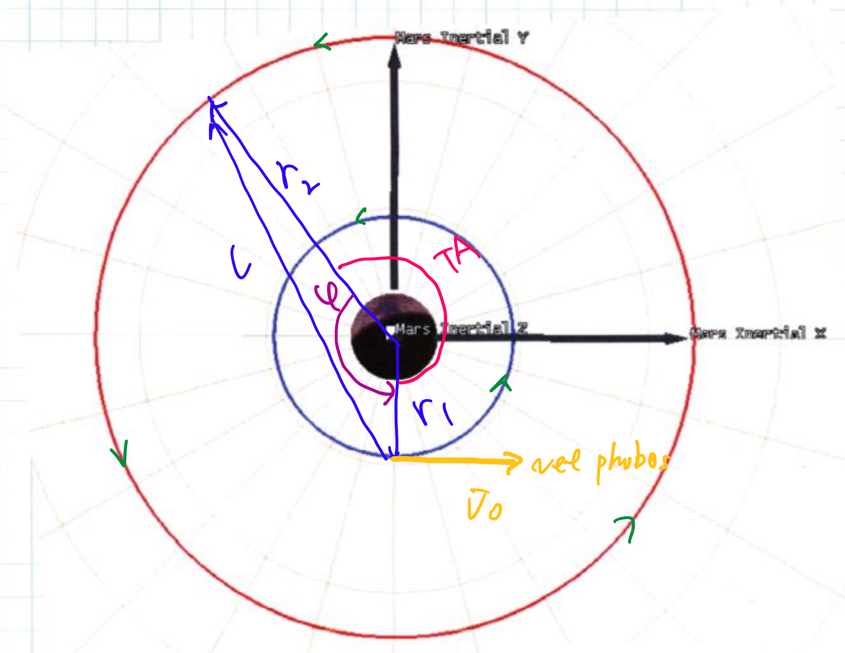
Mars Explorer

A small robotic explorer has been sent to the Martian system to observe and characterize the two moons – Phobos and Deimos, whose orbits are assumed to be circular and coplanar about Mars, with a radius equal to the semi-major axis listed in the constants table for moons and dwarfs. Let's assume that the spacecraft has completed its observations in the orbit of Phobos and must transfer to the orbit of Deimos. Consider only the gravity of Mars.

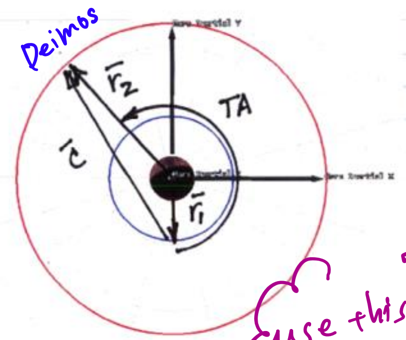
a) Assume that the spacecraft departs from Phobos to rendezvous with Deimos using an elliptical minimum energy transfer arc and a transfer angle of 240 degrees. Determine the following quantities: $a, TOF, p, e, \mathcal{E}, type$ (1 or 2), $v_D, v_A, \theta_D^*, \theta_A^*, \gamma_D, \gamma_A$.

min $\mathcal{E}$



$\mu_♂ = 42828.37\ Km^3/s^2$ $R_♂ = 3397\ Km$

$a_{phobos} = 9376\ Km$ $a_{Deimos} = 23,458\ Km$



Min Energy Transfer

TA=240° type 2

$c = 2.9294 \times 10^4$ Km
$= 8.6 R_♂$

$s = 3.1064 \times 10^4$ Km
$= 9.144 R_♂$

{use this a!} → $a_{min} = 15{,}532$ Km

$$\alpha_0 = 2\sin^{-1}\sqrt{\frac{s}{2a_m}} = 180°$$

$$\beta_0 = 2\sin^{-1}\sqrt{\frac{s-c}{2a_m}} = 27.62°$$

Both 2A/2B Lambert TOF eqns deliver same TOF$_{min}$ only for a_{min}!

$$TOF = TOF_{min} = \sqrt{\frac{a^3}{\mu}}\left[(\alpha_0 - S_{\alpha_0}) + (\beta_0 - S_{\beta_0})\right] =$$

TA > 180° → Type 2

p = 11,262 Km   Both p's are same cuz min & case 4-π

$p = a(1-e^2)$ ⟹ $e = 0.5243$ $\xi = -1.3784\ \frac{km^2}{s^2}$

Departure

on transfer path → $V_{dep} = 2.526$ km/s

$\theta^*_{dep} = \pm 67.437°$

Arrival

$V_{arr} = 0.9455$ km/s

$\theta^*_{arr} = \pm 172.563°$

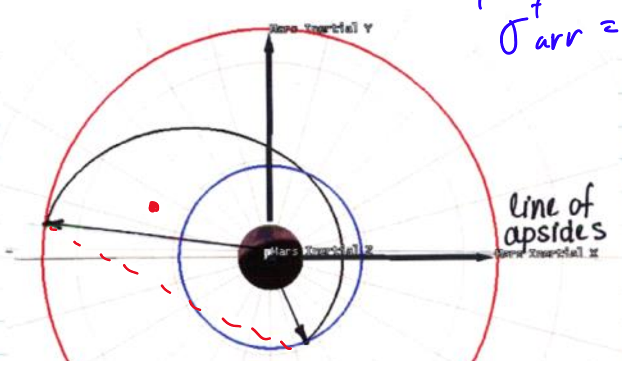
☆ $\theta^*_{arr} - \theta^*_{dep} = TA$!

$\theta^*_{dep} + TA = \theta^*_{arr}$

$\theta^*_{arr} = 172.563°$
$\theta^*_{dep} = -67.437°$

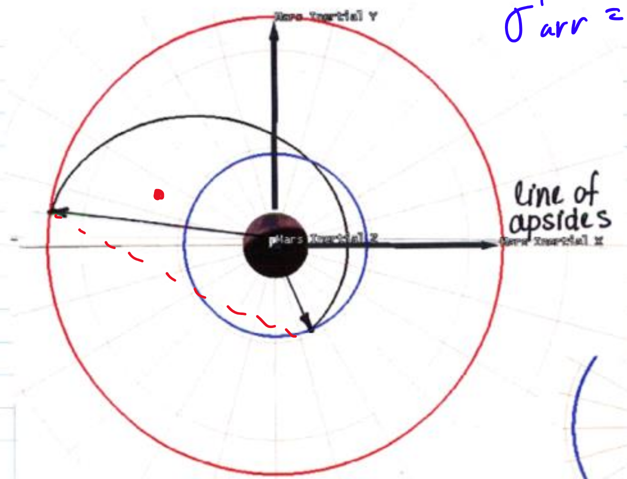
$\gamma^-_{dep} = -21.95°$

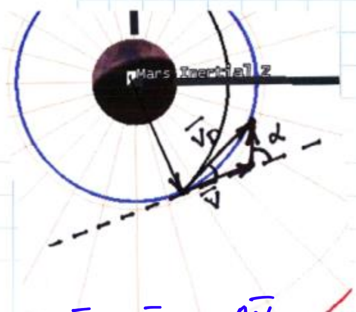
$\gamma^+_{arr} = +8.05°$

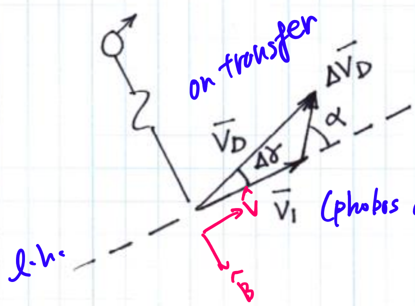


$\gamma^-_{arp} = -21.95^\circ$ LLex2 3

$\gamma^+_{arr} = +8.05^\circ$







$\longrightarrow$ $\bar{V}_P = \underbrace{\bar{V}_1}_{\bar{V}_{Ph}} + \Delta\bar{V}_D$

vel of phobos $\bar{V}_1 = \sqrt{\frac{\mu_{♂}}{r_D}} = 2.137$ Km/s

$V_D = 2.525$ Km/s

$\Delta\gamma = \gamma_D = -21.954^\circ$

vector triangle

$\Delta v_P = 0.966$ km/s

$\alpha = -77.7^\circ$

$\downarrow$

$\hat{v}$ $\hat{\beta}$ components

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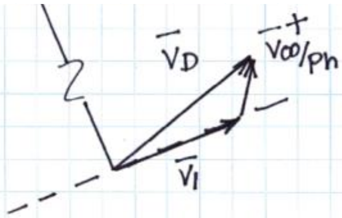
phobos-centered view

Phobos $R_{Ph} = 11.1$ Km

$\mu_{Ph} = 7.112 \times 10^{-4}$ Km^3/s^2

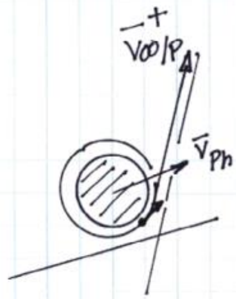
$\bar{V}_D = \bar{V}^+ = \bar{V}_1 + \bar{V}^+_{\infty/Ph}$

$V_D = V = V_1 + V_{\infty/Ph}$



$\bar{v}^{+}_{\infty/Ph} = 0.966$ km/s

vel at periapsis of hyp is higher



V_c at ~20 Km altitude

$V_c = 4.87 \times 10^{-3}$ Km/s

$\Downarrow$

$\bar{v}_{peri}$ on hyp

$\Delta \bar{v}$ at periapsis

$\Delta \bar{v} = \bar{v}_{peri} - \bar{v}_c$

Arrival

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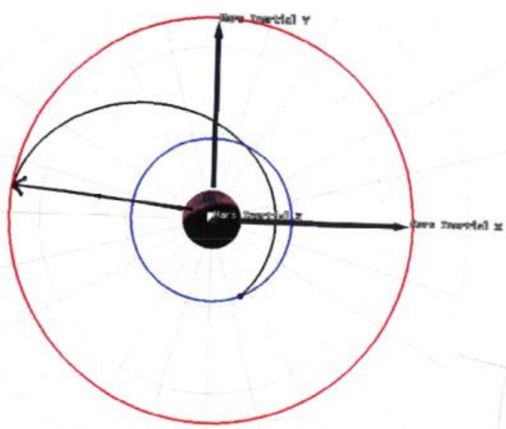

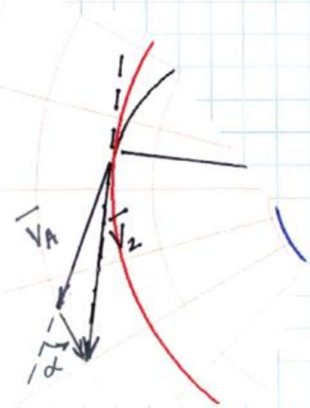

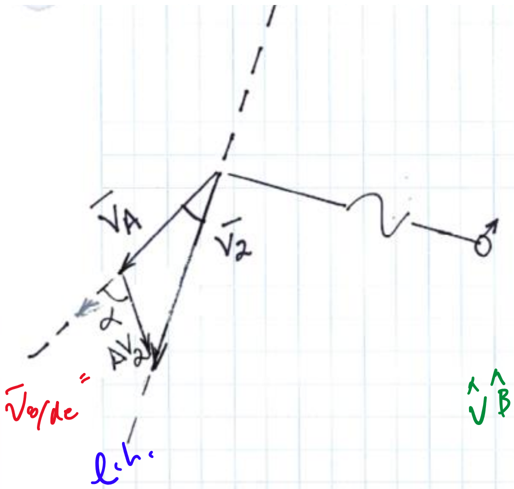

$$V_2 = \sqrt{\frac{\mu_{♂}}{r_{Deimos}}} = 1.3512 \text{ Km/s}$$

$V_A = .9455$ Km/s } on transfer

$\gamma_{arr}^{-} = +8.05°$

$\hat{v}\ \hat{B}$ $\begin{cases} \Delta V_2 = 0.436 \text{ km/s} \\ \alpha = -25.74° \end{cases}$

min Energy Transfer

$$|\Delta\bar{V}_{TOTAL}| = |\Delta\bar{V}_D| + |\Delta\bar{V}_A| = 1.402 \text{ km/s}$$