(i) GIVEN

Piston-Cylinder

>> 2 processes A & B, 2 states 1 & 2

>> $p_1 = 10$ bar $= 10\times10^5$ Pa, $V_1 = 0.1\,m^3$, $U_1 = 400$ kJ,
and $p_2 = 1$ bar $= 1\times10^5$ Pa, $V_2 = 1.0\,m^3$, $U_2 = 200$ kJ

>> Processes

*A: $1 \rightarrow 2$ polytropic $PV = $ const. $= C$

*B: $1 \rightarrow 3$ ($p_3 = 2$ bar $= 2\times10^5$ Pa, isovolumetric)

$3 \rightarrow 2$ linear P-V process

>> KE & PE ignored

FIND For each processes A & B

(a) sketch p-V diagram

(b) Evaluate work, kJ

(c) Evaluate $\dot{Q}$ in kJ

EQN

$\Delta Q = \Delta U + \Delta W$

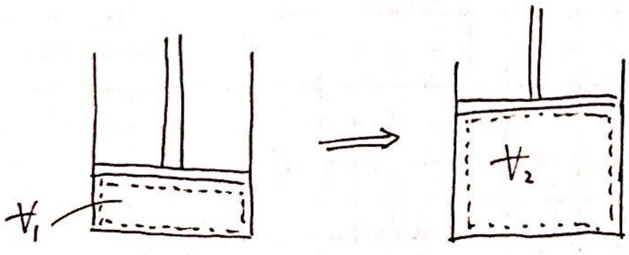
$\frac{dm}{dt}\Big|_{sys} = \sum_{in}\cancel{\dot{m}_{in}} - \sum_{out}\cancel{\dot{m}_{out}}$

$W_{boundary} = \int P dV$

ASSUMP

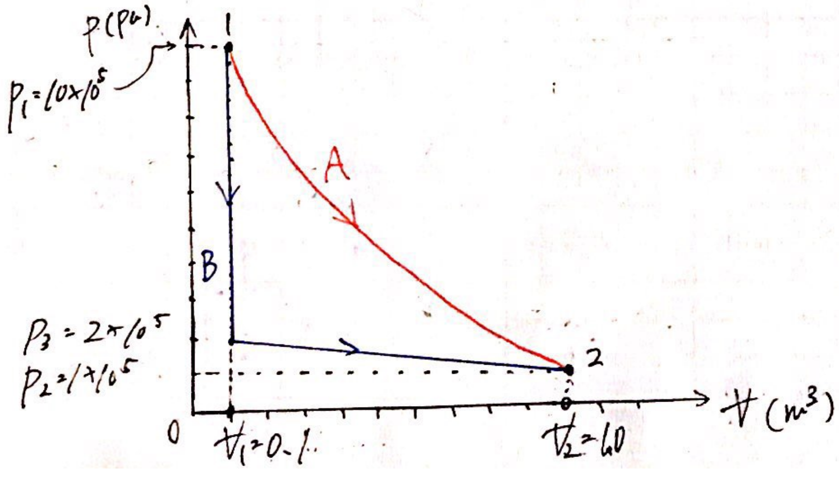
- Quasi equilibrium
- steady state
- closed system
- frictionless
- PE = KE = 0

EFD



SOLN

(a)



(b)

process A:

$$pV = C = p_1V_1 = (10\times10^5 Pa)(0.1 m^3) = 1\times10^5\ Pa\cdot m^3$$

$$p = \frac{C}{V}$$

$$W_A = \int_1^2 p\,dV = \int_{V_1}^{V_2} \frac{C}{V}dV = C\ln\frac{V_2}{V_1} = (1\times10^5\ Pa\cdot m^3)\ln\left(\frac{1.0 m^3}{0.1 m^3}\right)$$

$$\approx 2.302\times10^5\ J = 230.2\ kJ$$

$$\boxed{W_A = 230\ kJ}$$

process B:

$1\to3$ isovolumetric so no work

$3\to2$ $\quad P(V) = \alpha V + \beta \quad \cdots ①$

$$\alpha = \frac{(2\times10^5 - 1\times10^5)Pa}{(0.1 - 1.0)m^3} \approx -1.11\times10^5 \frac{Pa}{m^3}$$

plug in p_2, V_2 into ①

$$\beta = 1\times10^5 + \left(1.11\times10^5\frac{Pa}{m^3}\right)(1.0 m^3) = 2.11\times10^5\ Pa$$

$$P(V) = \alpha V + \beta$$

$$W_B = \int_3^2 P(V)dV = \int_{V_1}^{V_2}(\alpha V+\beta)dV = \frac{\alpha}{2}(V_2^2 - V_1^2) + \beta(V_2 - V_1)$$

$$= \frac{1}{2}\left(-1.11\times10^5\frac{Pa}{m^3}\right)(1.0^2 - 0.1^2)(m^6) + (2.11\times10^5 Pa)(1.0 - 0.1)(m^3)$$

$$= 134955\ J = 134.955\ kJ$$

$$\boxed{W_B = 135\ kJ}$$

(c)

$$Q_A = \Delta U + W_A = (U_2 - U_1) + W_A$$

$$= (200 kJ - 400 kJ) + 230\ kJ$$

$$= -30.0\ kJ$$

$$Q_B = \Delta U + W_B = (U_2 - U_1) + W_B$$

$$= -200 kJ + 135\ kJ$$

$$= -65.0\ kJ$$

$$\boxed{\begin{aligned} Q_A &= -30.0\ kJ \\ Q_B &= -65.0\ kJ \end{aligned}}$$

(ii) GIVEN

Piston-cylinder

>> shaft cross-sectional Area ≡ $A = 0.8\ cm^2 = 0.8\times10^{-4}\ m^2$

>> diameter of piston top $D = 10\ cm = 0.1\ m$

>> piston-shaft mass ≡ $m = 25\ kg$

>> *(heated slowly) internal EN increase $\Delta U = 0.1\ kJ$

* PE increase $\Delta PE = 0.2\ kJ$

* force ≡ $F = 1334\ N$ exerted on shaft

>> piston poor conductor & no friction

>> $g = 9.81\ m/s^2$, $P_{atm} = 1\ bar = 1\times10^5\ Pa$

FIND

(a) the work done by shaft (kJ)

(b) work done in displacing atmosphere (kJ)

(c) $\dot{Q}$ to gas (kJ)

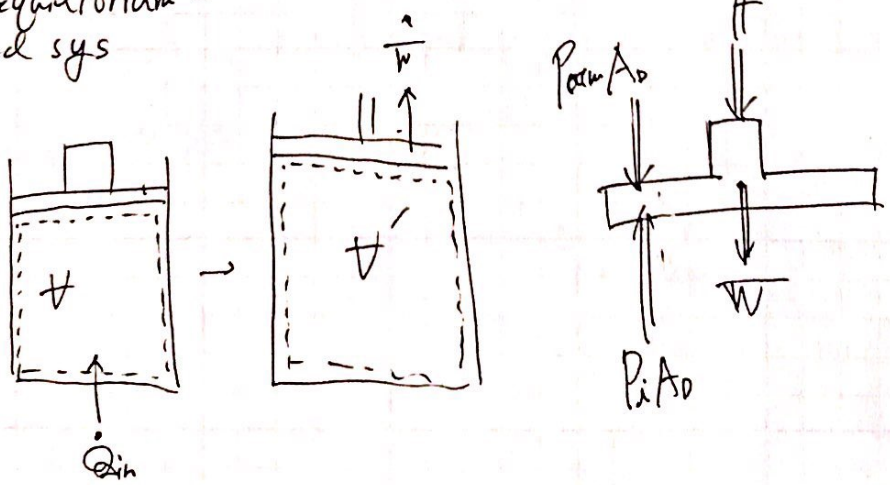
EQN

$$\frac{dE}{dt}\bigg|_{sys} = \sum_{in}\dot{m}_{in}(h+KE+PE)_{in} - \sum_{out}\dot{m}_{out}(h+KE+PE)_{out} + \dot{Q} - \dot{W}$$

ASSUMP

- Quasiequilibrium - frictionless
- closed sys

EFD



SOLN

$\Delta PE = mg\Delta h$

$\therefore \Delta h = (0.2\times10^3\ J)/(25\ kg)(9.81\ \frac{m}{s^2}) \cong 0.8155\ m$

$\cong 0.816$

∴ thus, work done by shaft is

$$W_{shaft} = F\Delta h = (1334\,N)(0.816\ m)\times10^{-3}$$
$$\simeq 1.088\ kJ$$

$$\boxed{W_{shaft} = 1.09\ kJ}$$

(b) work done by atmosphere being displaced is

$$W_{atm} = P_{atm} A_{net} \Delta h$$

the net area ≡ $A_{net} = \pi\left(\frac{D}{2}\right)^2 - A$

$$= (7.85\times10^{-3} - 0.08\times10^{-3})\,m^2$$
$$= 7.774\times10^{-3}\,m^2$$

$$\therefore W_{atm} = (1\times10^5\ Pa)(7.774\times10^{-3}\ m^2)(0.8155\ m)$$
$$\cong 634.4\ J$$

$$\boxed{W_{atm} = 0.634\ kJ}$$

(c) $\dot{Q} = \Delta U + W + \Delta PE = \Delta U + (W_{shaft} + W_{atm}) + \Delta PE$

$$= 0.1\,kJ + (1.09\,kJ + 0.634\,kJ) + 0.2\,kJ$$

$$= 2.024\ kJ$$

$$\boxed{\dot{Q} = 2.02\ kJ}$$