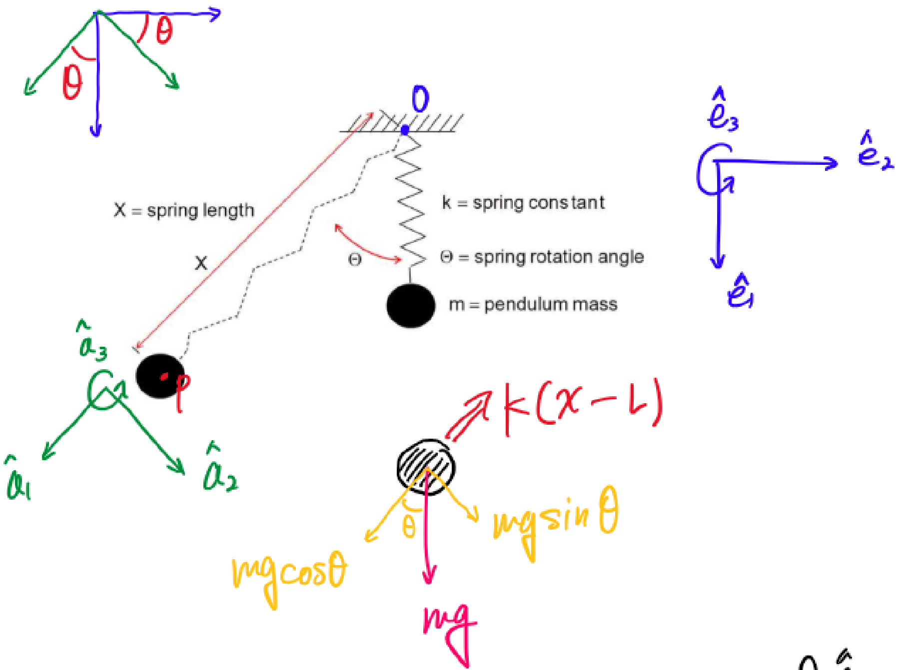

$$\Sigma \mathbb{F} = [mg\cos\theta - k(x-L)]\,\hat{a}_1 + mg\sin\theta\,\hat{a}_2$$

$$\bar{r}^{op} = x\,\hat{a}_1$$

$${}^{e}\bar{v}^{op} = \frac{{}^{e}d\bar{r}^{op}}{dt} = \frac{{}^{a}d\bar{r}^{op}}{dt} + {}^{e}\bar{\omega}^{a} \times \bar{r}^{op} = \dot{x}\,\hat{a}_1 + \dot{\theta}\,\hat{a}_3 \times x\,\hat{a}_1$$

$$= \dot{x}\,\hat{a}_1 + x\dot{\theta}\,\hat{a}_2$$

$${}^{e}\bar{a}^{op} = \frac{{}^{e}d\,{}^{e}\bar{v}^{op}}{dt} = \frac{{}^{a}d\,{}^{e}\bar{v}^{op}}{dt} + \dot{\theta}\,\hat{a}_3 \times {}^{e}\bar{v}^{op} = \ddot{x}\,\hat{a}_1 + \dot{x}\dot{\theta}\,\hat{a}_2 + x\ddot{\theta}\,\hat{a}_2 + \dot{\theta}\,\hat{a}_3 \times (\dot{x}\,\hat{a}_1 + x\dot{\theta}$$

$$= \ddot{x}\,\hat{a}_1 + \dot{x}\dot{\theta}\,\hat{a}_2 + x\ddot{\theta}\,\hat{a}_2 + \dot{x}\dot{\theta}\,\hat{a}_2 - x\dot{\theta}^2\,\hat{a}_1$$

Thus,

$$m\,{}^{e}\bar{a}^{op} = \Sigma \mathbb{F}$$

$$m(\ddot{x} - x\dot{\theta}^2)\hat{a}_1 + m(x\ddot{\theta} + 2\dot{x}\dot{\theta})\hat{a}_2$$

$$= [mg\cos\theta - k(x-L)]\hat{a}_1 + mg\sin\theta\,\hat{a}_2$$

$$m\ddot{x} - mx\dot{\theta}^2 = mg\cos\theta - k(x-L)$$

$$m\ddot{x} + k(x-L) - mg\cos\theta - mx\dot{\theta}^2 = 0$$

$$\ddot{x} + \frac{k}{m}(x-L) - g\cos\theta - x\dot{\theta}^2 = 0$$

$$m(x\ddot{\theta} + 2\dot{x}\dot{\theta}) = mg\sin\theta$$

$$x\ddot{\theta} - g\sin\theta + 2\dot{x}\dot{\theta} = 0$$