AE 6230 – HW2: Response of SDOF Systems and Equations of MDOF Systems

Out: September 29, 2022; **Due:** October 6, 2022 by 11:59 PM ET in Canvas

Guidelines

- Read each question carefully before doing any work;
- If you find yourself doing pages of math, pause and consider if there is an easier approach;
- You can consult any relevant materials;
- You can discuss solution approaches with others, but your submission must be your own work;
- If you have doubts, please ask questions in class, during office hours, and/or Piazza (no questions via email);
- The solution to each question should concisely and clearly show the steps;
- Simplify your results as much as possible;
- Box the final answer for each question;
- Submit any code with the solution (but remember to also submit all relevant plots).

Problem 1 – 40 points

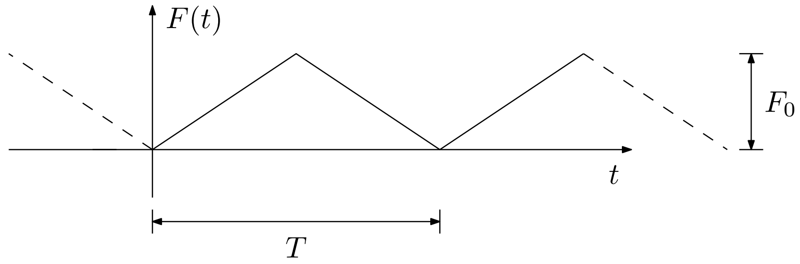

Figure 1: Periodic excitation applied to a single-degree-of-freedom system.

Consider a single-degree-of-freedom system subject to the periodic excitation in Fig. 1, with parameters given in Table 1. Answer the following questions:

1. Determine the expressions of the coefficients of the Fourier series representation of $F(t)$

$$F(t) = \frac{a_0}{2} + \sum_{p=1}^{\infty} a_p \cos p\omega_0 t + \sum_{p=1}^{\infty} b_p \sin p\omega_0 t \tag{1}$$

2. Plot the discrete frequency spectrum associated with Eq. (1)

$$c_p = \sqrt{a_p^2 + b_p^2} \ \text{ vs. } \ p \tag{2}$$

   for $p = 0, \ldots, 12$;

3. Determine how many terms must be kept in Eq. (1) such that the highest-order harmonic has an amplitude below $0.05F_0$, $0.025F_0$, and $0.005F_0$;

4. Plot the truncated Fourier series representations of $F(t)$ identified via the convergence study in Question 3 against the true function in Fig. 1 for $t \in [0, T]$;

5. Determine the expression of the steady-state response of the system $x(t)$ subject to $F(t)$;

6. Plot the discrete frequency spectrum for with $x(t)$ (that is, the amplitude of each harmonic) for $p = 0, \ldots, 12$;

7. Plot $x(t)$ for each truncated Fourier series representation of $F(t)$ identified in Question 3 for $t \in [0, T]$;

8. Motivate the trends observed in the plots for Questions 2, 4, 6, 7 for increasing p.

Guidelines:

- **Questions 1 and 5:** do not substitute the values of the parameters for these questions;
- **Question 3:** you can solve this analytically or numerically (or both).

Table 1: Parameter values for Problem 1.

Parameter	Symbol	Value
Excitation peak value	F_0	1 N
Excitation period	T	0.2 s
Natural frequency	ω_n	$5\omega_0$
Viscous damping factor	ζ	0.05
Stiffness constant	k	10 N/m

Problem 2 – 35 points

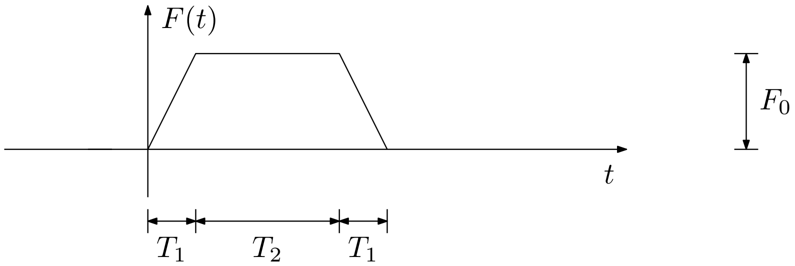

Figure 2: Trapezoidal input applied to a single-degree-of-freedom system.

Consider a single-degree-of-freedom system subject to the input in Fig. 2, with parameters in Table 2. Assuming the system at rest for $t \leq 0$ and neglecting damping, answer the following questions:

1. Using the convolution integral, show that the response for $0 \leq t \leq T_1$ is given by

$$x(t) = \frac{x_s}{T_1 \omega_n} \left(\omega_n t - \sin \omega_n t \right) \tag{3}$$

   where $\omega_n = 2\pi / T_n$ is the natural frequency of the system and $x_s = F_0/k$ is the response for a static input having the same amplitude as the trapezoidal input in Fig. 2;

2. Considering the other time intervals $T_1 \leq t \leq T_1 + T_2$, $T_1 + T_2 \leq t \leq 2T_1 + T_2$, and $t \geq 2T_1 + T_2$
   (a) Explain the approach you pursue to determine $x(t)$;
   (b) Derive the expression of $x(t)$ specialized to each time interval;

3. Plot $x(t)/x_s$ for $T_1 = 0.1T_n, 0.5T_n, T_n, 1.5T_n, 2T_n, 2.5T_n$ for $t \in [0, 1.5]$ s and $x(t)/x_s \in [-2, 2]$;

4. Determine the maximum value of $x(t)/x_s$ in the time interval $T_1 \leq t \leq T_1 + T_2$ as a function of T_1/T_n;

5. Plot the result from Question 4 for $T_1/T_n \in [0, 4]$;

6. Discuss the trends in the results for Question 3 and 5.

Guidelines:

- **Question 4:** you can use the plots from Question 3 to check the results for this question.

Table 2: Parameter values for Problem 2.

Parameter	Symbol	Value
Time length of constant input	T_2	0.5 s
Natural frequency of the system	ω_n	20π rad/s

Problem 3 – 25 points

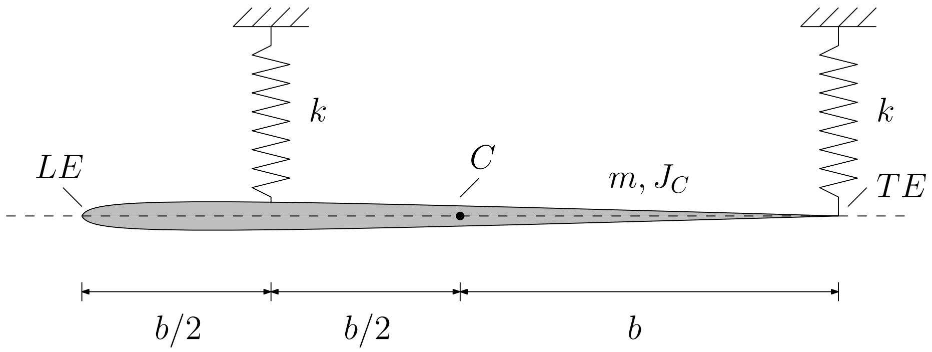

Figure 3: Schematic of wind-tunnel wing model undergoing plunge and pitch vibrations.

Consider a rigid wing mounted in a wind-tunnel test section (Fig. 3). The wing undergoes plunge (vertical translation) and pitch vibrations, which are restrained by two springs attached to the quarter-chord and trailing-edge points as shown in Fig. 3. The wing has mass m and pitch moment of inertia J_C about the center of mass C, located at the half-chord point. The chord has length $2b$ and the two springs both have spring constant k. The motion is described by choosing the vertical translations of the leading-edge and trailing-edge points, denoted by $h_{LE}(t)$ and $h_{TE}(t)$, as the generalized coordinates. The traslations are assumed to be positive in the upward direction and are measured from the horizontal configuration in Fig. 3. Neglecting gravity and assuming small-amplitude motions, answer the following questions:

1. Write the kinetic and potential energies of the system as functions of $h_{LE}(t)$ and $h_{TE}(t)$;
2. Derive the equations of motion in the matrix form

$$\mathbf{M}\begin{Bmatrix}\ddot{h}_{LE}(t)\\ \ddot{h}_{TE}(t)\end{Bmatrix}+\mathbf{K}\begin{Bmatrix}h_{LE}(t)\\ h_{TE}(t)\end{Bmatrix}=\begin{Bmatrix}0\\ 0\end{Bmatrix} \tag{4}$$

   using Lagrange's equations;
3. Define a coordinate transformation that results in inertial decoupling (but not necessarily elastic decoupling) and derive the corresponding transformation matrix $\mathbf{T}$;
4. Obtain the new mass and stiffness matrices based on the transformation matrix from Question 3;
5. Derive the equations of motion using the Netwonian approach based on the free-body diagram for the system (to be included in the solution) and compare the results with Question 4.

Guidelines:

- **Question 2:** show the steps in the process, not only the final form of the matrices;
- **Question 4:** you can verify the results by obtaining the new matrices directly from Lagrange's equations.