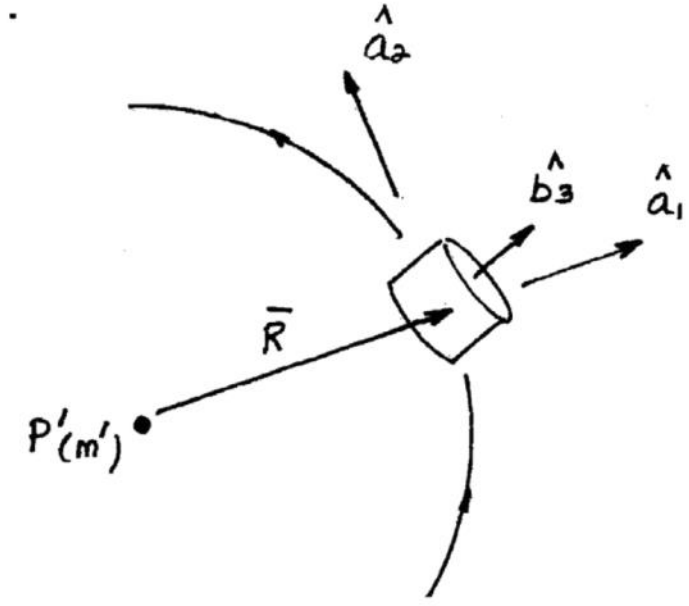
P3: R

Monday, March 23, 2020 01:53 PM

Gravitational Moment on an Axisymmetric Body in Circular Orbit



$C_{\omega}^{\beta} = S\hat{C}_2$

$N \rightarrow A \rightarrow C \rightarrow B$

Equations of Motion

$$2\dot{\varepsilon}_1 = \varepsilon_2(\omega_3 - s + \Omega) - \varepsilon_3\omega_2 + \varepsilon_4\omega_1$$

$$2\dot{\varepsilon}_2 = \varepsilon_3\omega_1 + \varepsilon_4\omega_2 - \varepsilon_1(\omega_3 - s + \Omega)$$

$$2\dot{\varepsilon}_3 = \varepsilon_4(\omega_3 - s - \Omega) + \varepsilon_1\omega_2 - \varepsilon_2\omega_1$$

$$2\dot{\varepsilon}_4 = -\varepsilon_1\omega_1 - \varepsilon_2\omega_2 - \varepsilon_3(\omega_3 - s - \Omega)$$

other kinematic variables?

$$\dot{\omega}_1 = -s\omega_2 + \left(1 - \frac{J}{I}\right)\left[\omega_2\omega_3 - 12\Omega^2(\varepsilon_1\varepsilon_2 - \varepsilon_3\varepsilon_4)(\varepsilon_3\varepsilon_1 + \varepsilon_2\varepsilon_4)\right]$$

$$\dot{\omega}_2 = s\omega_1 - \left(1 - \frac{J}{I}\right)\left[\omega_1\omega_3 - 6\Omega^2(\varepsilon_3\varepsilon_1 + \varepsilon_2\varepsilon_4)(1 - 2\varepsilon_2^2 - 2\varepsilon_3^2)\right]$$

NB

$$\dot{\omega}_3 = 0$$

$$\dot{\omega}_3 = 0$$

expressed in $\hat{c}_i$

Numerical Investigation

Set up an investigation of the impact of the gravity torque; axisymmetric rigid body – circular orbit

First consider what information available

Torque-free

1. EOM
2. Analytical Solution — particularly easy to obtain with proper choice of s
3. Solution is complete
   (no specific additional inquiry required)

→ constant precession rate
= nutation angle
= spin rate

Gravity Torque place spacecraft in orbit to create torque

1. EOM
2. NOT solvable analytically (in general)

   May be a particular soln that is of interest
   ↳ can base investigation on such a particular soln.
3. Plan
   (i) determine a particular solution (use *s* to help process)
   (ii) investigate stability of the particular solution
   (iii) consider analytical approximation for motion with

(ii) investigate stability of the particular solution
(iii) consider analytical approximation for motion with respect to the particular solution
(iv) numerical investigation