AAE 334: Aerodynamics

HW 5: 3D Wing Theories and Computations

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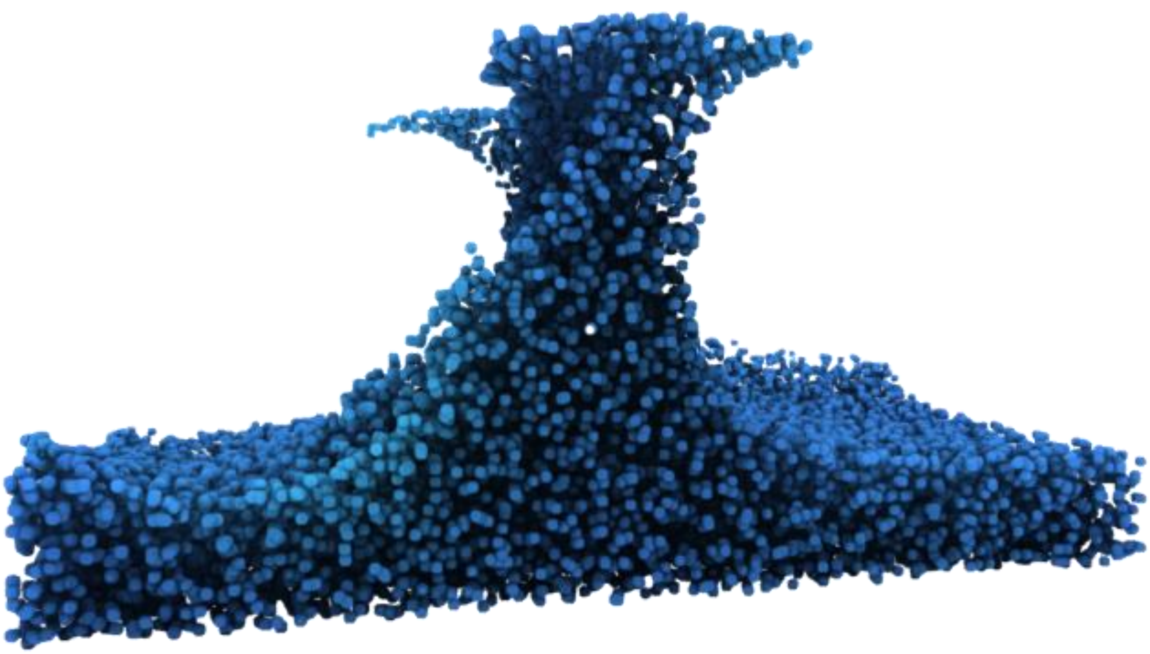
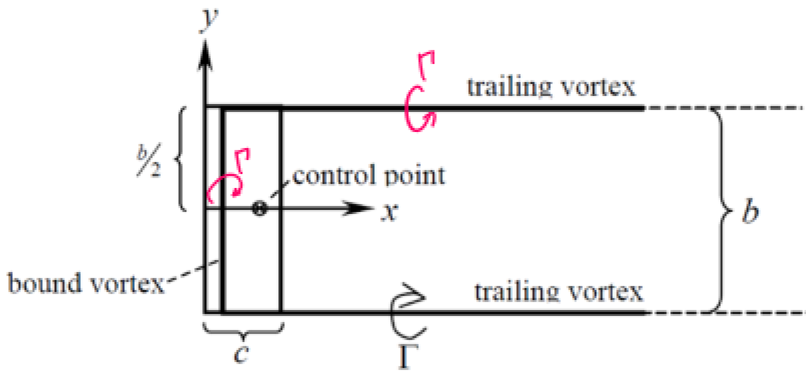
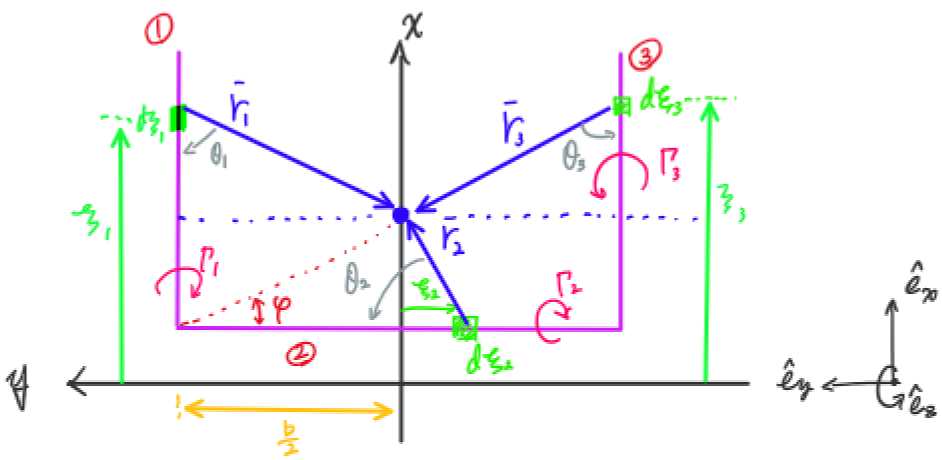




1. [20 points] Assuming incompressible flow, consider an uncambered, rectangular wing with span b and chord c as shown in the figure above. Model this wing using a single horseshoe vortex, with its bound vortex at the quarter-chord line of the wing ($x = c/4$), as shown in the figure.

(a) Use the Biot-Savart law to calculate the velocity induced by the horseshoe vortex at the control point, which is located at (3/4 c, 0, 0). Remember that the bound vortex is finite and the trailing vortices are semi-infinite.



say the velocity at the control point is $\bar{V}_{cp}$
cut the bound vortex and trailing into segments as the figure above.

Now, each position vectors become

$$\begin{cases} \bar{r}_1 = \frac{3c}{4}\hat{e}_x - \left(\xi_1\hat{e}_x + \frac{b}{2}\hat{e}_y\right) = \left(\frac{3}{4}c - \xi_1\right)\hat{e}_x - \frac{b}{2}\hat{e}_y \\ \bar{r}_2 = \frac{3c}{4}\hat{e}_x - \left(\frac{c}{4}\hat{e}_x - \xi_2\hat{e}_y\right) = \frac{c}{2}\hat{e}_x + \xi_2\hat{e}_y \\ \bar{r}_3 = -\left(\xi_3 - \frac{3c}{4}\right)\hat{e}_x + \frac{b}{2}\hat{e}_y = \left(\frac{3c}{4} - \xi_3\right)\hat{e}_x + \frac{b}{2}\hat{e}_y \end{cases}$$

for linear integration

$$\begin{cases} d\bar{l}_1 = d\xi_1 \hat{e}_x \\ d\bar{l}_2 = d\xi_2 \hat{e}_y \\ d\bar{l}_3 = d\xi_3 \hat{e}_x \end{cases} \qquad \Gamma_1 = \Gamma_2 = -\Gamma \quad \Gamma_3 = \Gamma$$

w.r.t CP

using Biot-Savart law

$$\bar{v}_{cp} = \frac{\Gamma}{4\pi}\int_C \frac{d\bar{l}\times\bar{r}}{r^3} = \frac{\Gamma_1}{4\pi}\int_1 \frac{d\bar{l}_1\times\bar{r}_1}{r_1^3} + \frac{\Gamma_2}{4\pi}\int_2 \frac{d\bar{l}_2\times\bar{r}_2}{r_2^3} + \frac{\Gamma_3}{4\pi}\int_3 \frac{d\bar{l}_3\times\bar{r}_3}{r_3^3}$$

$$= -\frac{\Gamma}{4\pi}\int_{\frac{c}{4}}^{\infty} \frac{d\xi_1 \hat{e}_x \times\left[\left(\frac{3c}{4}-\xi_1\right)\hat{e}_x - \frac{b}{2}\hat{e}_y\right]}{r_1^3} - \frac{\Gamma}{4\pi}\int_{-\frac{b}{2}}^{\frac{b}{2}} \frac{d\xi_2 \hat{e}_y \times\left(\frac{c}{2}\hat{e}_x + \xi_2\hat{e}_y\right)}{r_2^3}$$

$$+ \frac{\Gamma}{4\pi}\int_{\frac{c}{4}}^{\infty} \frac{d\xi_3 \hat{e}_x \times\left[\left(\frac{3c}{4}-\xi_3\right)\hat{e}_x + \frac{b}{2}\hat{e}_y\right]}{r_3^3}$$

$$= -\frac{\Gamma}{4\pi}\int_{\frac{c}{4}}^{\infty}\left(\frac{-b}{2r_1^3}\right)d\xi_1\,\hat{e}_z - \frac{\Gamma}{4\pi}\int_{-\frac{b}{2}}^{\frac{b}{2}} \frac{c}{2r_2^3}\,d\xi_2(-\hat{e}_z)$$

$$+ \frac{\Gamma}{4\pi}\int_{\frac{c}{4}}^{\infty}\left(\frac{b}{2r_3^3}\right)d\xi_3\,\hat{e}_z$$

with the following relations

$$\begin{cases} r_1 = \dfrac{\frac{b}{2}}{\sin\theta_1} = \dfrac{b}{2\sin\theta_1} \\ r_2 = \dfrac{c}{2\sin\theta_2} \\ r_3 = \dfrac{b}{2\sin\theta_3} \end{cases}$$

and

$$\tan\theta_1 = \frac{\frac{b}{2}}{\xi_1 - \frac{3}{4}c} \iff \xi_1 - \frac{3c}{4} = \frac{b}{2}\cot\theta_1$$

$$d\xi_1 = -\frac{b}{2}\csc^2\theta_1 d\theta = -\frac{b}{2}\frac{d\theta_1}{\sin^2\theta_1}$$

samely

$$\tan\theta_2 = \frac{\frac{c}{2}}{\xi_2} \iff \xi_2 = \frac{c}{2}\cot\theta_2$$

$$d\xi_2 = -\frac{c}{2}\csc^2\theta_2 d\theta = -\frac{c}{2}\frac{d\theta_2}{\sin^2\theta_2}$$

$$\tan\theta_3 = \frac{\frac{b}{2}}{\xi_3 - \frac{3}{4}c} \iff \xi_3 - \frac{3}{4}c = \frac{b}{2}\tan\theta_3$$

$$d\xi_3 = -\frac{b}{2}\csc^2\theta_3\, d\theta = -\frac{b}{2}\frac{d\theta_3}{\sin^2\theta_3}$$

then,

$$\bar{V}_{cp} = -\frac{\Gamma}{4\pi}\int_{\varphi+\frac{\pi}{2}}^{0}\left(-\frac{b}{2}\right)\left(\frac{b}{2\sin\theta_1}\right)^{-3}\left(-\frac{b}{2}\right)\frac{d\theta_1}{\sin^2\theta_1}\,\hat{e}_3$$

$$-\frac{\Gamma}{4\pi}\int_{\varphi}^{\pi-\varphi}\frac{c}{2}\left(\frac{c}{2\sin\theta_2}\right)^{-3}\left(-\frac{c}{2}\right)\frac{d\theta_2}{\sin^2\theta_2}(-\hat{e}_3)$$

$$+\frac{\Gamma}{4\pi}\int_{\varphi+\frac{\pi}{2}}^{0}\left(\frac{b}{2}\right)\left(\frac{b}{2\sin\theta_3}\right)^{-3}\left(-\frac{b}{2}\right)\frac{d\theta_3}{\sin^2\theta_3}\,\hat{e}_2$$

$$= -\frac{\Gamma}{4\pi}\int_{\varphi+\frac{\pi}{2}}^{0}\left(\frac{2}{b}\right)\sin\theta_1\, d\theta_1\,\hat{e}_3$$

$$-\frac{\Gamma}{4\pi}\int_{\varphi}^{\pi-\varphi}\left(\frac{2}{c}\right)\sin\theta_2\, d\theta_2\,\hat{e}_3$$

$$-\frac{\Gamma}{4\pi}\int_{\varphi+\frac{\pi}{2}}^{0}\left(\frac{2}{b}\right)\sin\theta_3\, d\theta_3\,\hat{e}_3$$

$$= -\frac{\Gamma}{2\pi b}\left[-\cos\theta_1\right]_{\varphi+\frac{\pi}{2}}^{0}$$

$$-\frac{\Gamma}{2\pi c}\left[-\cos\theta_2\right]_{\varphi}^{\pi-\varphi}$$

$$-\frac{\Gamma}{2\pi b}\left[-\cos\theta_3\right]_{\varphi+\frac{\pi}{2}}^{0}$$

$$= -\frac{\Gamma}{2\pi b}\left[-1+\cos\left(\varphi+\frac{\pi}{2}\right)\right]$$

$$-\frac{\Gamma}{2\pi c}\left[-\cos(\pi-\varphi)+\cos\varphi\right]$$

$$-\frac{\Gamma}{2\pi b}\left[-1+\cos\left(\varphi+\frac{\pi}{2}\right)\right]$$

$$\because \cos\left(\varphi+\frac{\pi}{2}\right) = \cos\left[\frac{\pi}{2}-(-\varphi)\right] = \sin(-\varphi) = -\sin\varphi$$

$$\cos(\pi-\varphi) = -\cos\varphi$$

$$= -\frac{\Gamma}{2\pi b}\left(-1-\sin\varphi\right)\hat{e}_3$$
$$-\frac{\Gamma}{2\pi c}\left(\cos\varphi+\cos\varphi\right)\hat{e}_3$$
$$-\frac{\Gamma}{2\pi b}\left(-1-\sin\varphi\right)\hat{e}_3$$

also

$$\sin\varphi = \frac{\frac{c}{2}}{\sqrt{\left(\frac{b}{2}\right)^2+\left(\frac{c}{2}\right)^2}} = \frac{c}{\sqrt{b^2+c^2}}$$

$$\cos\varphi = \frac{\frac{b}{2}}{\sqrt{\left(\frac{b}{2}\right)^2+\left(\frac{c}{2}\right)^2}} = \frac{b}{\sqrt{b^2+c^2}}$$

thus

$$\bar{v}_{cp} = -\frac{\Gamma}{\pi b}\left(-1-\frac{c}{\sqrt{b^2+c^2}}\right)\hat{e}_3$$
$$-\frac{\Gamma}{\pi c}\left(\frac{b}{\sqrt{b^2+c^2}}\right)\hat{e}_3$$

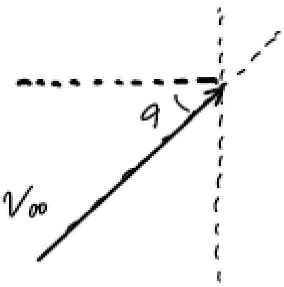
$$\bar{v}_{cp} = -\frac{\Gamma}{\pi}\left(\frac{b}{c\sqrt{b^2+c^2}} - \frac{c}{b\sqrt{b^2+c^2}} - \frac{1}{b}\right)\hat{e}_3$$

$$\boxed{\bar{v}_{cp} = -\frac{\Gamma}{\pi}\left(\frac{b^2-c^2-c\sqrt{b^2+c^2}}{bc\sqrt{c^2+b^2}}\right)\hat{e}_3}$$

(b) Determine the strength of the vortex required to enforce the flow tangency condition (no flow through the airfoil) at the control point for a given angle of attack α. Assume that the thickness of the wing and the angle of attack are small.

the resultant velocity from free-stream and downwash will be zero.

the relation can be expressed as



$$V_\infty \sin\alpha + w_{cp} = 0$$

since $\alpha \ll 1 \iff \sin\alpha \approx \alpha$

thus,

$$V_\infty \alpha - \frac{\Gamma}{\pi}\left(\frac{b^2 - c^2 - c\sqrt{b^2+c^2}}{bc\sqrt{b^2+c^2}}\right) = 0$$

$$\boxed{\Gamma = \frac{\pi V_\infty \alpha \left(bc\sqrt{b^2+c^2}\right)}{b^2 - c^2 - c\sqrt{b^2+c^2}}}$$

(c) Find the lift on the wing for an angle of attack α.

(d) Find the slope of the lift curve $dC_L/d\alpha$ for a wing with aspect ratio AR=10. Compare the computed lift curve slope to that given by the lifting line theory for an elliptically loaded wing with the same aspect ratio.

(c) from (b) we know Γ

using Kutta-Joukowsky Theorem

$$L' = \rho V_\infty \Gamma$$

$$\frac{L}{b} = \rho V_\infty \Gamma$$

$$L = \rho V_\infty \Gamma b$$

$$\boxed{L = \rho V_\infty b \frac{\pi V_\infty \alpha \left(bc - \sqrt{b^2 + c^2} \right)}{b^2 - c^2 - c\sqrt{b^2 + c^2}}}$$

(d) when $AR = 10 \Leftrightarrow \frac{b}{c} = 10 \Leftrightarrow b = 10c$

now since

$$C_L = \frac{L}{\frac{1}{2}\rho V_\infty^2 bc}$$

$$= \frac{2}{\cancel{\rho V_\infty^3 bc}}\left[\cancel{\rho V_\infty b}\frac{\pi V_\infty q\,(bc\sqrt{b^2+c^2})}{b^2+c^2-c\sqrt{b^2+c^2}}\right]$$

$$= 2\pi q\frac{b\sqrt{b^2+c^2}}{b^2+c^2-c\sqrt{b^2+c^2}}$$

divide denominator and numerator by c^2

$$= 2\pi q\frac{\frac{b}{c}\sqrt{\frac{b^2}{c^2}+1}}{\frac{b^2}{c^2}+1-\frac{c}{c}\sqrt{\frac{b^2}{c^2}+1}}$$

$$= 2\pi q\frac{10\sqrt{10^2+1}}{10^2+1-\sqrt{10^2+1}} = 2\pi q\frac{10\sqrt{101}}{101-\sqrt{101}}$$

thus,

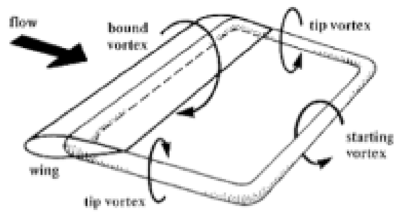
$$\boxed{\frac{dC_L}{dq} = 2\pi\frac{10\sqrt{101}}{101-\sqrt{101}} \simeq 6.9428}$$

for elliptically loaded wing

$$\frac{dC_L}{dq} = \frac{2\pi}{1+\frac{2}{AR}} = \frac{2\pi}{1+\frac{2}{10}} = \frac{2\pi}{1+0.2} = \frac{2\pi}{1.2}$$

$$\boxed{\left(\frac{dC_L}{dq}\right)_{eliptic} = 5.2360}$$

The lift curve slope computed from horseshoe vortice assumption and Biot-Savart law is substantially larger than the elliptical load case.



2. [20 pts] When an airplane takes off it sheds a starting vortex. We have been using a horseshoe vortex model for the finite wing, but a square closed loop might be more realistic. Estimate the downwash velocity induced at the center of the wing by the starting vortex for a small plane (mass 1000 kg, wingspan 11 m, speed at takeoff 25 m/s) for a distance from the starting vortex to the wing of 10 m, 100 m, and 1000 m. Comment on the magnitude of the effect. Assume standard atmospheric conditions and neglect the ground effect.

We have computed the downwash equation in problem 1.
So we will use that.

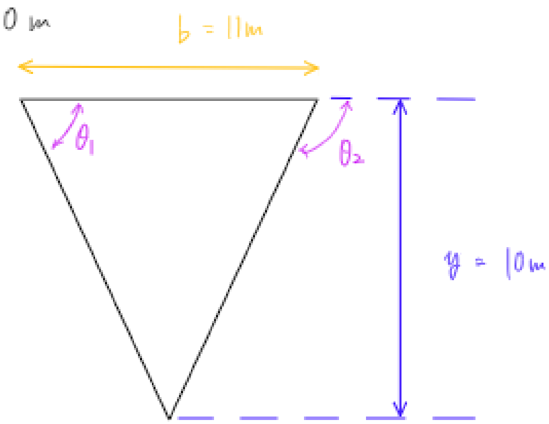
$$w(y) = -\frac{\Gamma}{4\pi y}\left(\cos\theta_2 - \cos\theta_1\right)$$

assuming steady level flight @ vicinity of sea level

$$L = mg = \rho_\infty V_\infty \Gamma b$$

$$\Gamma = \frac{mg}{\rho_\infty V_\infty b} = \frac{(1000\ \mathrm{kg})(9.81\ \frac{\mathrm{m}}{\mathrm{s}^2})}{(1.225\ \frac{\mathrm{kg}}{\mathrm{m}^3})(25\ \frac{\mathrm{m}}{\mathrm{s}})(11\ \mathrm{m})} = 29.121\ \frac{\mathrm{m}^2}{\mathrm{s}}$$

(i) $y = 10$ m



$$\tan\theta_1 = \frac{y}{b/2} \iff \theta_1 = \arctan\left(\frac{2y}{b}\right)$$

since this is an isosceles triangle

$$\theta_2 = \pi - \theta_1 \iff \cos\theta_2 = \cos(\pi - \theta_1) = -\cos\theta_1$$

$$\cos\theta_1 = \cos\left[\arctan\left(\frac{2y}{b}\right)\right] = \cos\left[\arctan\left(\frac{2\times 10\text{m}}{11\text{ m}}\right)\right] \approx 0.481919$$

thus

$$w_{10} = -\frac{\Gamma}{4\pi y}(\cos\theta_2 - \cos\theta_1) = -\frac{\Gamma}{4\pi y}(-\cos\theta_1 - \cos\theta_1)$$

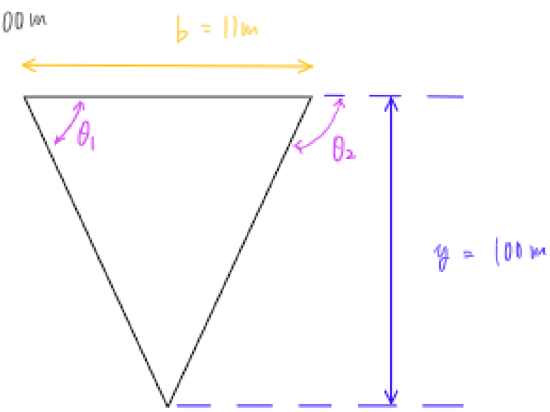
$$= \frac{\Gamma}{2\pi y}\cos\theta_1$$

$$= \frac{(29.121\ \frac{\text{m}^2}{\text{s}})}{2\pi(10\text{m})}(0.481919)$$

$$\approx 0.22336\ \frac{\text{m}}{\text{s}}$$

$$\boxed{w_{10} = 0.22336\ \frac{\text{m}}{\text{s}}}$$

(i) $y = 100\text{m}$



$$\tan\theta_1 = \frac{y}{b/2} \iff \theta_1 = \arctan\left(\frac{2y}{b}\right)$$

since this is an isosceles triangle

$$\theta_2 = \pi - \theta_1 \iff \cos\theta_2 = \cos(\pi - \theta_1) = -\cos\theta_1$$

$$\cos\theta_1 = \cos\left[\arctan\left(\frac{2y}{b}\right)\right] = \cos\left[\arctan\left(\frac{2\times 100\text{m}}{11\text{ m}}\right)\right] \approx 0.054917$$

thus

$$w_{100} = -\frac{\Gamma}{4\pi y}\left(\cos\theta_2 - \cos\theta_1\right) = -\frac{\Gamma}{4\pi y}\left(-\cos\theta_1 - \cos\theta_1\right)$$

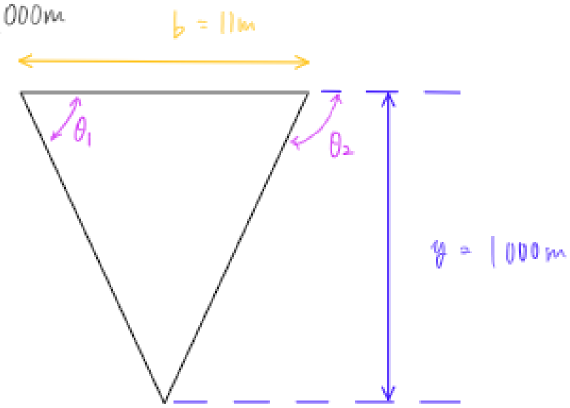
$$= \frac{\Gamma}{2\pi y}\cos\theta_1$$

$$= \frac{\left(29.121\ \frac{m^2}{s}\right)}{2\pi\,(100m)}\left(0.054917\right)$$

$$\cong 2.5453\times10^{-3}\ \frac{m}{s}$$

$$\boxed{w_{100} = 2.5453\times10^{-3}\ \frac{m}{s}}$$

(ii) $y = 1000m$



$$\tan\theta_1 = \frac{y}{b/2} \iff \theta_1 = \arctan\left(\frac{2y}{b}\right)$$

Since this is an isosceles triangle

$$\theta_2 = \pi - \theta_1 \iff \cos\theta_2 = \cos(\pi - \theta_1) = -\cos\theta_1$$

$$\cos\theta_1 = \cos\left[\arctan\left(\frac{2y}{b}\right)\right] = \cos\left[\arctan\left(\frac{2\times1000m}{11m}\right)\right] \approx 5.49992\times10^{-3}$$

thus

$$w_{1000} = -\frac{\Gamma}{4\pi y}\left(\cos\theta_2 - \cos\theta_1\right) = -\frac{\Gamma}{4\pi y}\left(-\cos\theta_1 - \cos\theta_1\right)$$

$$= \frac{\Gamma}{2\pi y}\cos\theta_1$$

$$= \frac{\left(29.121 \frac{m^2}{s}\right)}{2\pi (1000 m)} \left(5.49992 \times 10^{-3}\right)$$

$$\cong 2.5491 \times 10^{-5} \frac{m}{s}$$

$$\boxed{w_{1000} = 2.5491 \times 10^{-5} \frac{m}{s}}$$

Discussion

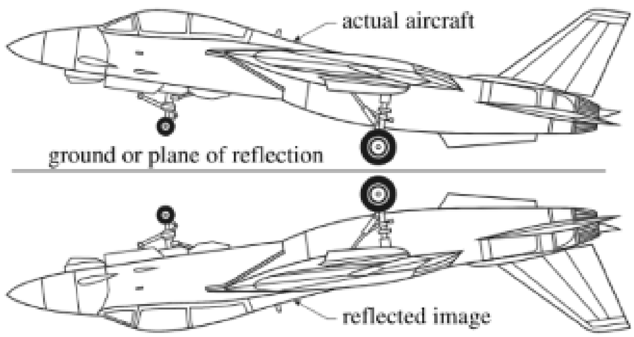
The relative magnitude ratio to the take-off velocity for each case is

$$10m : \frac{0.22336 \, m/s}{25 \, m/s} = 8.9344 \times 10^{-3}$$

$$100m : \frac{2.5453 \times 10^{-3} \, m/s}{25 \, m/s} = 1.01812 \times 10^{-4}$$

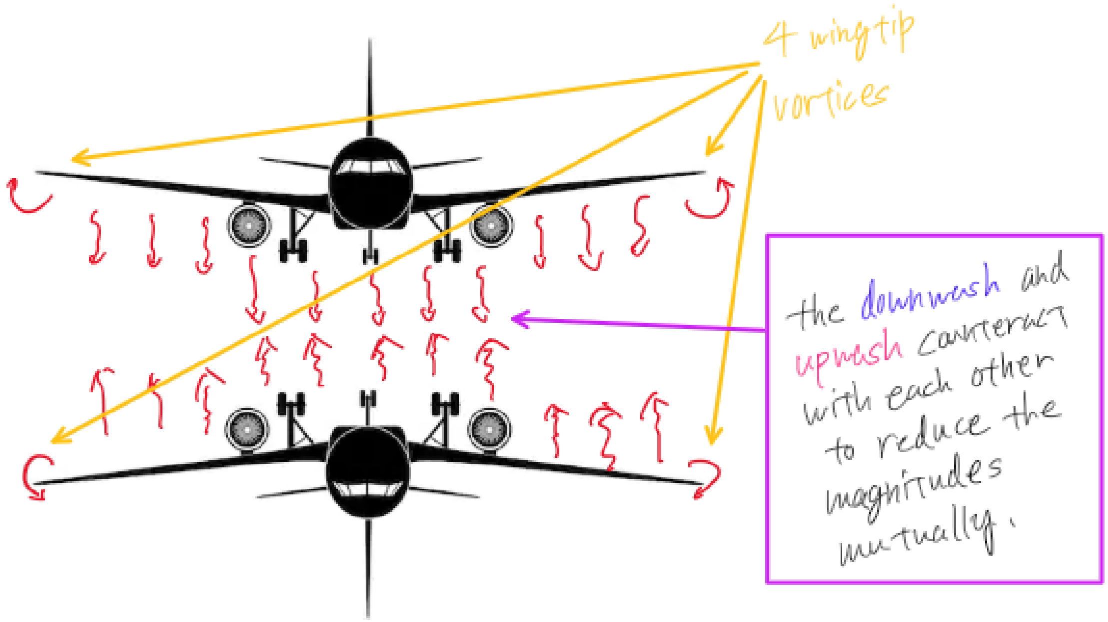
$$1000m : \frac{2.5491 \times 10^{-5} \, m/s}{25 \, m/s} = 1.01964 \times 10^{-6}$$

Thus, the farther the starting vortex is the smaller the downwash becomes (evident from the equation) for this finite wing at takeoff.

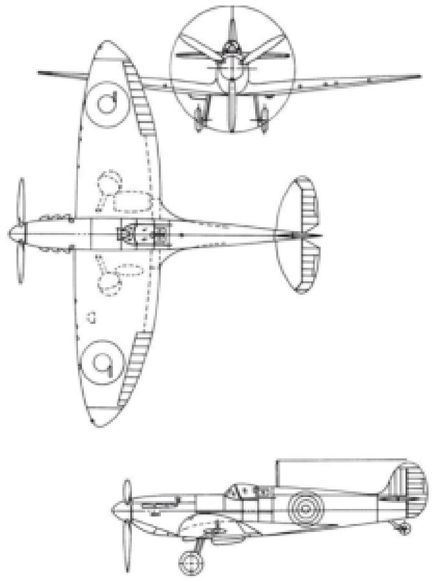


Mirror-image model used to simulate ground effect in inviscid flow calculations (Phillips and Hunsaker, *Journal of Aircraft, v. 50, n. 4, pp. 1226-1233, 2013).*

3. [20 pts] There is an aerodynamic phenomenon called the *ground effect,* in which the induced drag on an aircraft drops significantly when it is close to the ground. One convenient way to account for the influence of the ground in an inviscid model is to replace the surface of the ground with a mirror image of the aircraft, making the ground a stream-surface of the flow. Use this approach with the single horseshoe vortex model of a finite wing to explain the ground effect qualitatively. Don't compute anything! Just draw a diagram and explain.



When considering a mirror image, we can see that the downwash for the actual aircraft and mirror image counteract with each other to reduce the magnitude. Thus, the reduction of downwash leads to reduction of induced drag.



4. [40 pts] The Supermarine Spitfire shown in the figure had elliptical cord distribution and no twist, resulting in elliptical wing loading. The airplane had a maximum velocity of $V_\infty = V_{max} = 362$ mph at an altitude of 18,500 ft. Its weight was 5820 lb, wing area was 242 ft^2, and wing span was 36.1 ft.

(a) Calculate the induced drag coefficient of the Spitfire at $V_{max} = 362$ mph and 18,500 ft altitude.

provided properties

$V_\infty = V_{max} = 362$ mph $= 161.83$ m/s
altitude $= h = 5638.8$ m
weight $= W = 5820$ lb $= 25888.7$ N
wing area $= S = 242$ ft^2 $= 22.483$ m^2
wing span $= b = 36.1$ ft $= 11.003$ m

from the slides we know that

the elliptic distribution is

$$\Gamma(y) = \Gamma_0 \sqrt{1 - \left(\frac{2y}{b}\right)^2}$$

and the lift coeff. can be computed as

$$C_L = \frac{2\Gamma_0}{V_\infty S} \int_{-b/2}^{b/2} \sqrt{1 - \left(\frac{2y}{b}\right)^2}\, dy = \frac{2\Gamma_0}{V_\infty S} \frac{b}{2} \int_0^{\pi} \sin^2\theta \, d\theta$$

$$= \frac{\Gamma_0 b}{V_\infty S}\frac{\pi}{2} = \pi \frac{\Gamma_0}{2bV_\infty}\frac{b^2}{S} = \pi \alpha_i AR$$

and induced drag is computed as

$$C_{Di} = \frac{2}{V_\infty S}\int_{-b/2}^{b/2} \Gamma(y)\alpha_i(y)\,dy$$

$$= \frac{2}{V_\infty S}\alpha_i \int_{-b/2}^{b/2} \Gamma(y)\,dy = \alpha_i C_L$$

$$= \frac{C_L}{\pi AR}\cdot C_L = \frac{C_L^2}{\pi AR} \quad \cdots ①$$

now assuming steady-level flight.

$$L = mg$$

$$\frac{1}{2}\rho_\infty V_\infty^2 S C_L = mg$$

(from "engineering toolbox")

Geo potential Altitude above Sea Level - h - (m)	Temperature - t - (^oC)	Acceleration of Gravity - g - (m/s^2)	Absolute Pressure - p - $(10^4\ N/m^2)$	Density - ρ - (kg/m^3)	Dynamic Viscosity - μ - $(10^{-5}\ N\ s/m^2)$
-1000	21.50	9.810	11.39	1.347	1.821
0	15.00	9.807	10.13	1.225	1.789
1000	8.50	9.804	8.988	1.112	1.758
2000	2.00	9.801	7.950	1.007	1.726
3000	-4.49	9.797	7.012	0.9093	1.694
4000	-10.98	9.794	6.166	0.8194	1.661
5000	-17.47	9.791	5.405	0.7364	1.628
6000	-23.96	9.788	4.722	0.6601	1.595
7000	-30.45	9.785	4.111	0.5900	1.561
8000	-36.94	9.782	3.565	0.5258	1.527

$* \rho_\infty$

$\rightarrow$ by interpolating

$$\rho_\infty = \frac{(0.6601 - 0.7364)\frac{kg}{m^3}}{(6000 - 5000)\ m}(5638.8\,m - 5000\,m) + 0.7364\frac{kg}{m^3}$$

$$\rho_\infty = 0.68766\ kg/m^3$$

$$* \ g = 9.81\ \frac{m}{s^2}$$

$$\therefore \quad \frac{1}{2}\rho_\infty V_\infty^2 S C_L = mg = W$$

$$\Longleftrightarrow C_L = \frac{2W}{\rho_\infty V_\infty^2 S} = \frac{2(25888.7N)}{(0.68766\frac{kg}{m^3})(161.83\frac{m}{s})^2(22.483m^2)}$$

$$C_L \approxeq 0.127878$$

plug this value into ①, then we get

$$C_{Di} = \frac{C_L^{\,2}}{\pi AR} = \frac{C_L^2 S}{\pi b^2}$$

$$C_{Di} = \frac{(0.127878)^2(22.483\,m^2)}{\pi(11.003\,m)^2} \approxeq 9.6666 \times 10^{-4}$$

$$\boxed{C_{Di} = 9.6666 \times 10^{-4}}$$

(b) What fraction of the total drag is the induced drag at these conditions? (To calculate the total drag, note that in steady, level flight the drag D equals the thrust T which, in turn, is related to the power P: P=TV$_\infty$. The Spitfire had a supercharged Merlin engine that produced 1050 hp at 18,500 ft, and the propeller efficiency can be assumed to be 0.9. Since 1 hp = 550 ft·lb/s, the thrust power is P = 1050×550×0.9 ft·lb/s = 519750 ft·lb/s.)

$$P = P_{out} = 1050\ hp = 782985\ W$$

$$\eta_{prop} = 0.9$$

now from these

$$\eta_{prop} P = T V_\infty$$

$$\therefore\ T = \frac{\eta_{prop} P}{V_\infty} = \frac{0.9 \times 782985\ W}{161.83\ \frac{m}{s}} = 4354.49\ N$$

at steady-level flight

$$D = T$$

$$\frac{1}{2}\rho_\infty V_\infty^2 S C_D = T$$

$$C_D = \frac{2T}{\rho_\infty V_\infty^2 S} = \frac{2(4354.49\ N)}{(0.68766\ \frac{kg}{m^3})(161.83\ \frac{m}{s})^2(22.483\ m^2)}$$

$$C_D = 0.021509$$

thus, the fraction of the overall drag coeff. is

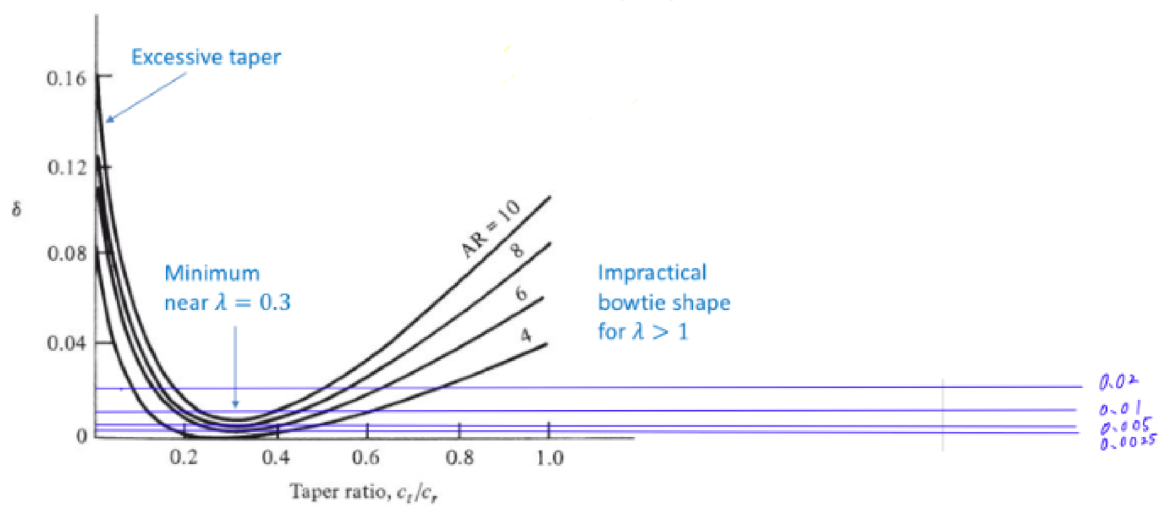
$$\boxed{\frac{C_{Di}}{C_D} = \frac{9.6666\times10^{-4}}{0.021509} = 0.044942}$$

(c) If the elliptical wing of the Spitfire were replaced by a tapered wing with a taper ratio of 0.4, and everything else remained the same, what would the induced drag coefficient be? Compare this with the actual Spitfire induced drag coefficient obtained in part (a) and comment on the effect of planform shape on the drag at high speed. For tapered wing,

you can use the figure on slide 14 of Lecture 15 to estimate the induced drag factor δ.

from slide 14 of lecture 15

@ taper ratio $= \lambda = \frac{C_t}{C_r} = 0.4$ and $AR = \frac{b^2}{S} \cong 5.3848$



interpolate induced drag factor w/ $\delta - AR$

$$\delta = \frac{0.005 - 0.0025}{6 - 4}(5.3848 - 4) + 0.0025$$

$$\delta = 0.004231$$

then

$$C_{D_i,tap} = \frac{C_L^2(1+\delta)}{\pi AR} = \frac{0.127878^2(1+0.004231)}{\pi(5.3848)}$$

$$\boxed{C_{D_i,tap} = 9.7075\times10^{-4}}$$

Discussion

for the tapered wing there was a

$$\frac{9.7075\times10^{-4} - 9.6666\times10^{-4}}{9.6666\times10^{-4}}\times100 = 0.42311\,\%$$

increase in the induced drag coeff.
This implies that when tapered with a taper ratio, λ of $0 < \lambda < 1$, the induced drag increases compared to the original wing with an elliptical load distribution.

(d) Assume that the Spitfire's landing velocity at sea level is 70 mph. Calculate the induced drag coefficient in this case and compare the result with that in high-speed case of part (a). Comment on the relative importance of the induced drag coefficient at low speed versus that at high speed.

we are given that

$$V_{land} = 70 \text{ mph} = 31.293 \text{ m/s}$$
$$\rho_0 = 1.225 \text{ kg/m}^3$$

by tracing the procedure in part (a), the lift coeff. becomes

$$C_{L,land} = \frac{2W}{\rho_0 V_{land}^2 S} = \frac{2(25888.7\text{ N})}{(1.225\frac{\text{kg}}{\text{m}^3})(31.293\frac{\text{m}}{\text{s}})^2(22.483\text{ m}^2)}$$

$$C_{L,land} = 1.91980$$

and induced drag coeff. becomes

$$C_{D_i,land} = \frac{C_{L,land}^2}{\pi AR} = \frac{1.91980^2}{\pi(5.3848)}$$

$$\boxed{C_{D_i,land} = 0.21787}$$

Discussion

At low-speed conditions the induced drag coeff. becomes significantly higher since to operate a safe landing the aircraft requires a high lift, that is a high lift coefficient; and therefore, leads to a higher induced drag coefficient. The induced drag is by definition induced by the lift and induced angle, so this result is congruent with the theory and proves it numerically.