P1: H BP

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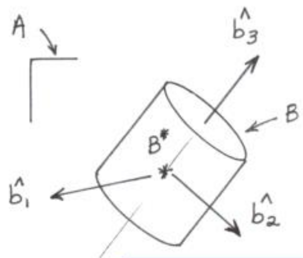
Determination of attitude and certainly controlling it means understanding orientation. Thus far, we have been describing orientation. We also introduced angular velocity and the relationships for changes in orientation.

Example **"Spin-up" problem; axially symmetric spacecraft**
Find the time history of B in A of the rigid body B is subject to a constant torque



center of mass

Define
A inertial $\hat{a}_i$
B $\hat{b}_i$ body-fixed; central, principal axes; arbitrarily select $\hat{b}_3$ parallel to axis of symmetry, center of mass

$\bar{\bar{I}}^{B/B^*} = I\hat{b}_1\hat{b}_1 + I\hat{b}_2\hat{b}_2 + I\hat{b}_3\hat{b}_3$

${}^A\bar{\omega}^B = \omega_1\hat{b}_1 + \omega_2\hat{b}_2 + \omega_3\hat{b}_3$

3 ROT DOF
ROTATIONAL MOTION

(torques/moments) Dynamics, kinetics → Moments → $\bar{M}$

(description) orientation variables; rate of chg. of orientation → $\bar{H}$

$$\underbrace{\bar{M} \qquad \bar{H}}$$

$\bar{M} = \frac{d\bar{H}}{dt} \Rightarrow$ Rot DOF $\Rightarrow \omega$

process to determine impact of various moments

Assume: System of forces acting on B with the resultant moment $M\hat{b}_3$ (equivalent to $\bar{M}$ about B^*)

$\bar{M} = M\hat{b}_3$ ← const.

Initially $\omega_1(0) = \omega_{10}$
$\omega_2(0) = \omega_3(0) = 0$

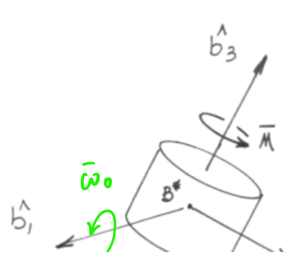
Determine: Orientation of B in A for $t > 0$ for some specified length of time

Approach: determination of orientation → rotational DOF
FBD → EOM (moment/angular momentum)

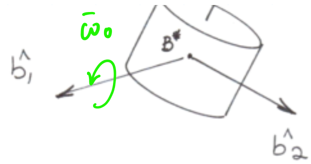
$\bar{M}^{B^*} = \frac{{}^A d\,{}^A\bar{H}^{B^*}}{dt} \longrightarrow$ Rot EOM

Solve if possible to determine motion history

FBD



$\bar{M}^{B^*} = M\hat{b}_3$ ← const.



J const.

H3

$${}^A\bar{H}^{B^*} = \bar{\bar{I}}^{B/B^*} \bullet {}^A\bar{\omega}^B$$

$$= \left(I\hat{b}_1\hat{b}_1 + I\hat{b}_2\hat{b}_2 + J\hat{b}_3\hat{b}_3\right) \bullet \left(\omega_1\hat{b}_1 + \omega_2\hat{b}_2 + \omega_3\hat{b}_3\right)$$

$$= \bar{\bar{I}}_{ij}\,\hat{b}_i\hat{b}_j \cdot \omega_k\hat{b}_k \qquad \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & J \end{bmatrix}$$

$$= \bar{\bar{I}}_{ij}\,\omega_k\hat{b}_i\,\delta_{jk}$$

$$= \bar{\bar{I}}_{ij}\,\omega_k\hat{b}_i = I\omega_1\hat{b}_1 + I\omega_2\hat{b}_2 + J\omega_3\hat{b}_3$$

Note option here: if write ω's in terms of some set of angles, $\dot{\theta}$'s appear. Then, must differentiate so $\ddot{\theta}$ will appear in equations. Leave angular momentum in terms of ω, will retain first-order equations. Also keeps kinematic and dynamics equations separate.

$$\frac{{}^A d\,{}^A\bar{H}^{B^*}}{dt} = \frac{{}^B d\,{}^A\bar{H}^{B^*}}{dt} + {}^A\bar{\omega}^B \times {}^A\bar{H}^{B^*} \qquad \text{BKE}$$

$$= I\dot{\omega}_1\hat{b}_1 + I\dot{\omega}_2\hat{b}_2 + J\dot{\omega}_3\hat{b}_3$$

$$+ \underbrace{\left(\omega_i\hat{b}_i \times H_j\hat{b}_j\right)}_{\omega_i H_j\,\varepsilon_{ijk}\,\hat{b}_k}$$

$$\frac{{}^A d\,{}^A\bar{H}^{B^*}}{dt} = \left[I\dot{\omega}_1 + (J-I)\,\omega_2\omega_3\right]\hat{b}_1$$

$$+ \left[I\dot{\omega}_2 - (J-I)\,\omega_1\omega_3\right]\hat{b}_2$$

$$+ J\dot{\omega}_3\hat{b}_3$$

H4

Diff Eqn $\Rightarrow$ Dynamic DE + Kinematic DE

Dynamical Differential Equations

$\omega_i\hat{b}_i$

$$\boxed{\begin{aligned} {}^A\dot{\omega}_1^B &= \frac{(I-J)}{I}\,\omega_2\omega_3 \\ \dot{\omega}_2 &= -\frac{(I-J)}{I}\,\omega_1\omega_3 \\ \dot{\omega}_3 &= \frac{M}{J} \end{aligned}}$$

const.

In general, the $\boxed{\text{first-order}}$ EOM in terms of ω are NOT solvable analytically.

(Usually, the moment is also a function of the orientation variables.)

This sample prob. dyn/kine DE decouple

In this special case, can SOLVE ANALYTICALLY
- 3 dependent variables
- three differential equations
- coupled/non-constant "coefficients"

Number of different ways to solve, but use a simple method here

Solution:

integration constant

$$\omega_3 = \frac{M}{J}t + C_3$$

indep. var.

ω_3 increases with time

$\omega_3(0) = 0 \Rightarrow C_3 = 0$

$$\bar{\bar{I}} = J\hat{b}_1\hat{b}_1 + I\hat{b}_2\hat{b}_2 + I\hat{b}_3\hat{b}_3$$

$${}^A\bar{H}^B = \bar{\bar{I}}^{B/B^*} \cdot {}^A\bar{\omega}^B$$

$$= J\omega_1\hat{b}_1 + I\omega_2\hat{b}_2 + I\omega_3\hat{b}_3$$

$$\frac{{}^A d\,{}^A\bar{H}^B}{dt} = \frac{{}^B d\,{}^A\bar{H}^B}{dt} + {}^A\bar{\omega}^B \times {}^A\bar{H}^B$$

$$= J\dot{\omega}_1\hat{b}_1 + I\dot{\omega}_2\hat{b}_2 + I\dot{\omega}_3\hat{b}_3$$

$$(\omega_1\hat{b}_1 + \omega_2\hat{b}_2 + \omega_3\hat{b}_3)$$

$$\times (J\omega_1\hat{b}_1 + I\omega_2\hat{b}_2 + I\omega_3\hat{b}_3)$$

$$= // + I\omega_1\omega_2\hat{b}_3 - I\omega_1\omega_3\hat{b}_2$$

$$- J\omega_1\omega_2\hat{b}_3 + \cancel{I\omega_2\omega_3}\hat{b}_1$$

$$+ J\omega_1\omega_3\hat{b}_2 - \cancel{I\omega_2\omega_3\hat{b}_1}$$

$$= J\dot{\omega}_1\hat{b}_1$$

$$+ \left[I\dot{\omega}_2 - (I-J)\,\omega_1\omega_3\right]\hat{b}_2$$

$$+ \left[I\dot{\omega}_3 + (I-J)\,\omega_1\omega_2\right]\hat{b}_3$$

$T_1 =$

$T_2 =$

$T_3 =$

$$\omega_3 = \frac{M}{J}t + C_3$$

indep. var.

ω_3 increases with time

$\omega_3(0) = 0 \Rightarrow C_3 = 0$

$$\left.\begin{aligned}\dot{\omega}_1 &= \left[\frac{I-J}{I}\frac{M}{J}\right] t\,\omega_2 = K\,t\,\omega_2 \\ \dot{\omega}_2 &= -\left[\frac{I-J}{I}\frac{M}{J}\right] t\,\omega_1 = -K\,t\,\omega_1\end{aligned}\right\}$$

coupled
1st order
Non-constant coefficients
(not even periodic coefficients)

$$\begin{cases}\dot{\omega}_1 = Kt\omega_2 \\ \dot{\omega}_2 = (-Kt)\omega_1\end{cases} \Longrightarrow$$ standard general form for soln.

$$\ddot{\omega}_1 = K\omega_2 + Kt\dot{\omega}_2 = K\omega_2 + Kt(-Kt)\omega_1 = K\omega_2 - K^2t^2\omega_1$$

$$\omega_1^{(3)} = K\dot{\omega}_2 - 2K^2t\,\omega_1 - K^2t^2\dot{\omega}_1 = K(-Kt)\omega_1 - 2K^2t\omega_1 - K^2t^2\dot{\omega}_1$$

H5

General form of the solution

$$\omega_1 = A\sin\left(\frac{Kt^2}{2}\right) + B\cos\left(\frac{Kt^2}{2}\right)$$

$$\omega_2 = D\sin\left(\frac{Kt^2}{2}\right) + E\cos\left(\frac{Kt^2}{2}\right)$$

$$\omega_1^{(3)} = -K^2t\omega_1 - 2K^2t\omega_1 - K^2t^2\dot{\omega}_1$$

$$\omega_1^{(3)} + K^2t^2\dot{\omega}_1 + 3K^2t\,\omega_1 = 0$$

$A = E$ $\quad\quad B = -D$

Initial conditions:

$$\boxed{\begin{aligned}\omega_1 &= \omega_{10}\cos\left(\frac{Kt^2}{2}\right) \\ \omega_2 &= -\omega_{10}\sin\left(\frac{Kt^2}{2}\right) \\ \omega_3 &= \frac{M}{J}t\end{aligned}}$$

So, we have a time history $(t > 0)$ for ${}^A\bar{\omega}^B$ that results from an applied moment for a given set of initial conditions

Is this what we want?
What does it mean for a change in orientation?
How can you find out?

We actually need to know how orientation – defined in terms of some set of orientation variables – is influenced by ${}^{A}\bar{\omega}^{B}$

⇒ Need a set of relationships between the change in orientation variables and the angular velocity

Kinematic Differential Equations

solve 1st order Kin DE to obtain the history for orientation.

" Describe Rates of change of orientation parameters in terms of angular velocity

Angular Velocity and Directions Cosines

$$\dot{C}_{11} = C_{12}\omega_3 - C_{13}\omega_2 = C_{12}\frac{M}{J}t + C_{13}\omega_{10}\sin\left(\frac{kt^2}{2}\right)$$
$$\dot{C}_{12} = C_{13}\omega_1 - C_{11}\omega_3$$
$$\dot{C}_{13} = C_{11}\omega_2 - C_{12}\omega_1$$
$$\dot{C}_{21} = C_{22}\omega_3 - C_{23}\omega_2$$
$$\dot{C}_{22} = C_{23}\omega_1 - C_{21}\omega_3$$
$$\dot{C}_{23} = C_{21}\omega_2 - C_{22}\omega_1$$
$$\dot{C}_{31} = C_{32}\omega_3 - C_{33}\omega_2$$
$$\dot{C}_{32} = C_{33}\omega_1 - C_{31}\omega_3$$
$$\dot{C}_{33} = C_{31}\omega_2 - C_{32}\omega_1$$

1-st order coupled non-linear

Angular Velocity and Euler Parameters

$$\dot{\varepsilon}_1 = \frac{1}{2}(\omega_1\varepsilon_4 - \omega_2\varepsilon_3 + \omega_3\varepsilon_2) = \frac{1}{2}\left[\omega_{10}\cos\left(\frac{kt^2}{2}\right)\varepsilon_4 + \omega_{10}\sin\left(\frac{kt^2}{2}\right)\varepsilon_3 + \frac{M}{J}t\,\varepsilon_2\right]$$
$$\dot{\varepsilon}_2 = \frac{1}{2}(\omega_1\varepsilon_3 + \omega_2\varepsilon_4 - \omega_3\varepsilon_1)$$
$$\dot{\varepsilon}_3 = \frac{1}{2}(-\omega_1\varepsilon_2 + \omega_2\varepsilon_1 + \omega_3\varepsilon_4)$$
$$\dot{\varepsilon}_4 = -\frac{1}{2}(\omega_1\varepsilon_1 + \omega_2\varepsilon_2 + \omega_3\varepsilon_3)$$

dependent variables? Potentially could include

D.C. independent var t
dependent $C_{11}, C_{12}, C_{13}, C_{21}, C_{22}, C_{23}, C_{31}, C_{32}, C_{33}$
$\omega_1, \omega_2, \omega_3$

OR

ε independent var t
dependent $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \omega_1, \omega_2,$

have a solution for ω as a function of time (the independent variable); substitute the solution for ω to reduce the number of dependent variables by three but, still no known method to solve analytically!

Options

assumptions

use numerical integration	make further approximations to simplify equations
(NOT a complete solution; may give us SOME help in understanding the motion)	seek approximate analytical solutions
must specify or assume some set of initial conditions on kinematic variables $(C \text{ or } \varepsilon)$; integrate for some predetermined time; select some output information to analyze	(NOT a complete solution; may offer SOME help in understanding the motion)

some insights

some insights

output - C_{ij}'s over time
ε_{ij}'s over time

No other assumptions reasonable so select numerical integration:

1. Propagate differential equations to produce a time history for the kinematic variables → maybe direction cosines
2. Produce table or plots of C_{ij} over time

Will this actually give you any insight into the orientation history? Does a plot of C_{32} as a function of time really help understand the motion?
What else could you plot?

Angles? Which ones might be useful?
How will you compute them?

take output (C or ε)

$\Longrightarrow$ useful output

(angles – set to use)

$\Downarrow$

w.r.t inertial frame

orbit frame?

$\Downarrow$

$\hat{\lambda}, \theta$