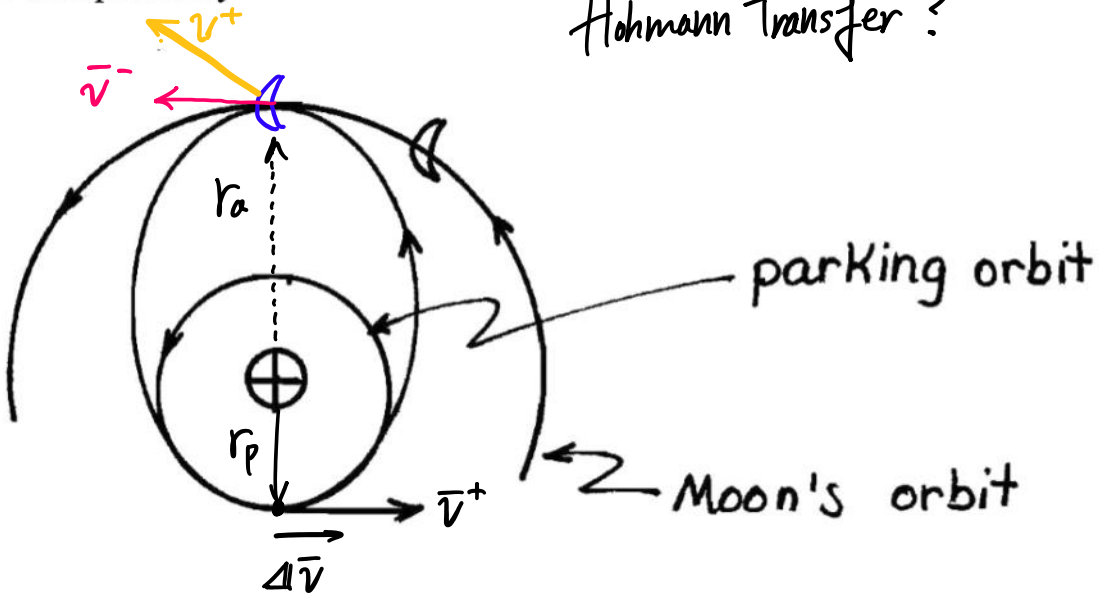
Example: Free-Return Trajectories

Consider circumlunar trajectories

Assume ⟹
- Coplanar orbits
- Moon's orbit circular
- Sun is off

Note: patched-conic approach less accurate in this problem than for interplanetary

Hohmann Transfer?



If ☾ has no gravity
1. Jump to ellipse from parking orbit
2. At ☾, with no $\Delta\bar{v}$, remains on transfer ellipse
3. Returns to ⊕ at radius of parking orbit

☾ does possess gravity

dominant grav. field

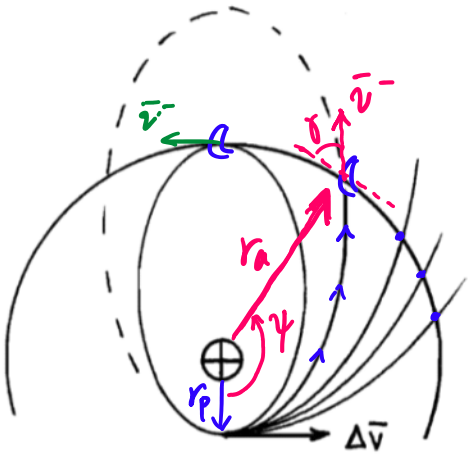
1. Pass through local field of ☾

Change vel mag. & vel dir. ⟶ w.r.t Earth } $\bar{v}^+$

Change vel mag. & vel dir. → w.r.t Earth } $\bar{v}^+$

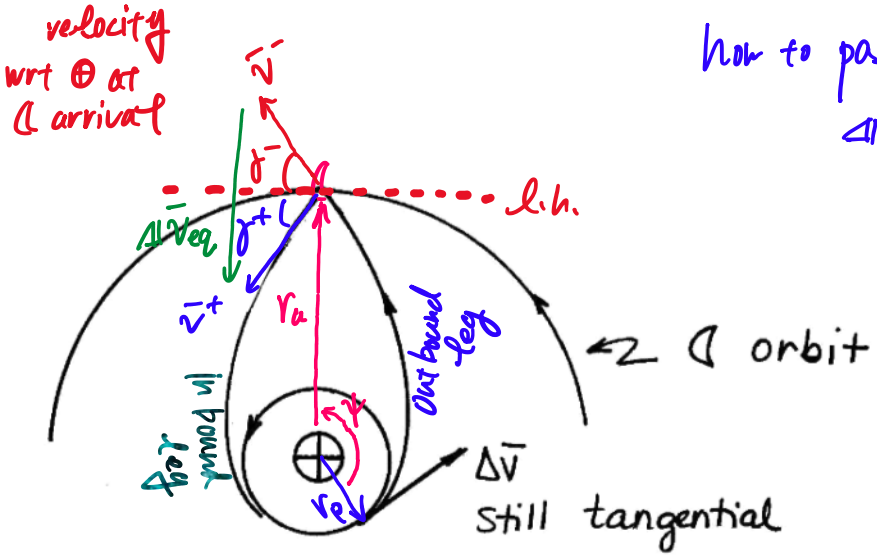
2. No return to ⊕ unless introduce Δv to transfer ellipse

3. Can s/c pass ☾ such that no Δv?



Consider:

1. make transfer ellipse larger
2. $\Delta\bar{v}$ still tangential (most efficient)
3. apogee $\geq r_{\mathbb{C}}$
4. reach $\mathbb{C}$ sooner at different angle



how to pass ℂ to get Δv_{eq} ?

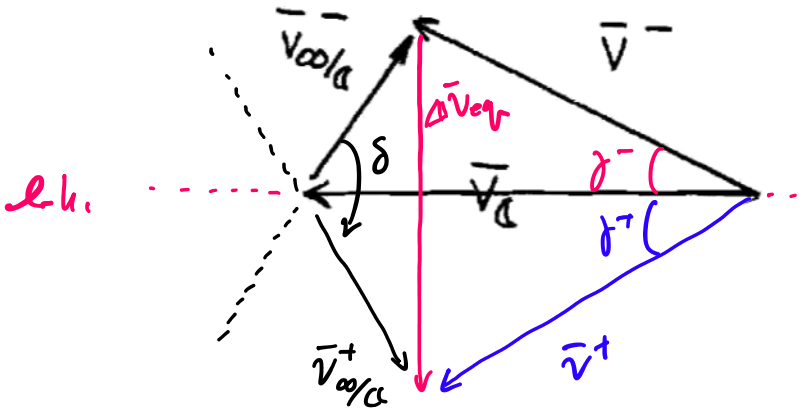
If pass $\mathbb{C}$ such that

$$v_r^+ = -v_r^-$$
$$v_\theta^+ = +v_\theta^-$$

→ end up on same trajectory for $\oplus$ return

$|\bar{v}_c| > |\bar{v}^-|$

Vector Diagram



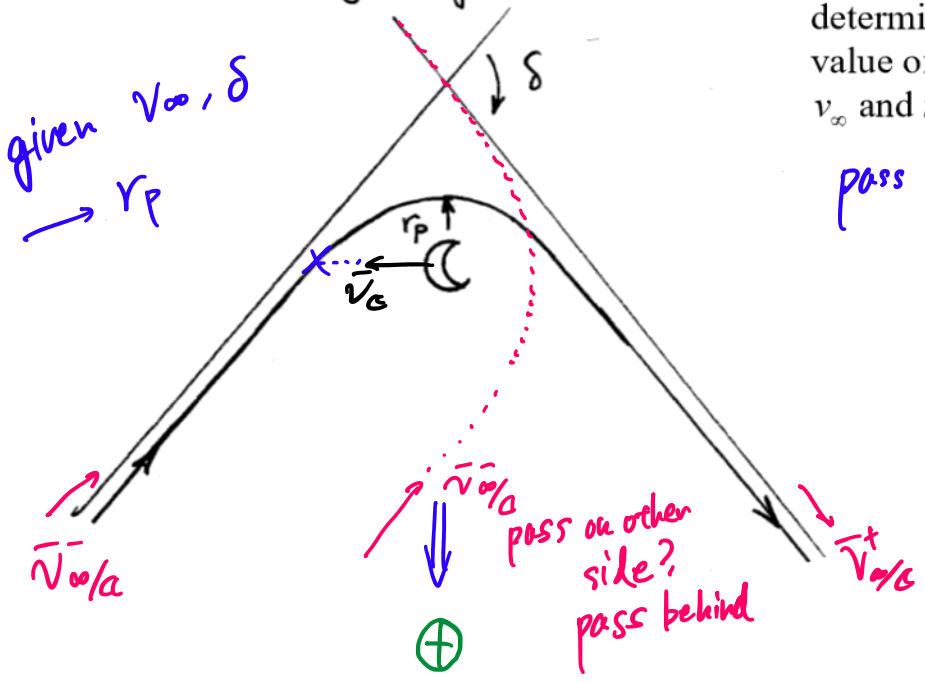
Same relative velocity equation:

$$\bar{v}^- = \bar{v}_{\infty/c}^- + \bar{v}_c$$

solve for $\bar{v}_{\infty/c}^-$

$\bar{v}^+ = \bar{v}_c + \bar{v}_{\infty/c}^+$

Local trajectory



Flyby angle determined by value of v_∞ and r_p

pass ahead

Notes:

1. Early Apollo flights → free-return
   Typical 3-day outbound leg
   Pass ahead of ☾ so s/c could enter 3-day return leg if failure occurs
   Apollo 11 (for landing) 3 day out; 2.5 day return (if insertion did not occur would not return to vicinity of Earth)

2. Apollo 17 altered its initial free-return translunar trajectory to get a more precise landing
   $r_{p/☾} = 1849$ (111 altitude) / passed ahead

   Entered lunar orbit

3. Apollo 13
   Had made a mid-course correction to leave free-return path before experiencing failure that aborted ☾ landing
   Lunar module engines used after explosion to modify trajectory and return to ⊕

   Apollo 13 → same trajectory / failure enroute to ☾ / lunar module descent engines did fire
   Lunar approach: $v_r = +.244 \quad v_\theta = 0$