

**AAE 532 – Orbit Mechanics**  
**Problem Set 9**  
**Due: JD 2459167.1875 (UT)**

**Problem 1:** As noted last week in Problem 2 in PS 8, the US is currently planning for humans to reach the Moon in 2024. Return to consideration of a trajectory to the Moon and its return. Assume departure from a 190 km altitude circular Earth parking orbit; include the local gravity field at the Moon.

- (a) The path to the Moon last week was planned as a Hohmann transfer so the outbound leg was a  $180^\circ$  transfer. But, the passage by the Moon modified the orbit relative to the Earth. Recall the conditions immediately after the lunar encounter, i.e.,  $r^+, v^+, \gamma^+, \theta^+$ .

Assume that the goal is to return to the Earth orbit. Should the vehicle pass on the light side or the dark side? Was the lunar passage a light side or dark side pass in PS8?

If the goal is to return to the Earth, which pass is better, light-side or dark-side? Why?

Assume that the pass occurs with a plan to return to Earth; what are the post-encounter conditions? To immediately return to Earth orbit, assume that a maneuver is implemented to offset the lunar gravity and to return the crew to the second half of the Hohmann transfer path and to the original Earth orbit. Determine the maneuver  $|\Delta \bar{v}|, \alpha$  that would be required to immediately return the vehicle to the Hohmann transfer path for the return/inbound arc back to the Earth parking orbit.

Does this maneuver seem reasonable? Recall the analysis from last week, if this return maneuver is missed, can crew return?

- (b) To avoid reliance on a return maneuver, reconsider the transfer as a free-return. Assume a circular, coplanar lunar orbit. Assume that the vehicle departs the same 190-km Earth parking orbit. However, rather than a Hohmann transfer, design a free-return trajectory such that the transfer angle is  $173.8^\circ$ . Determine the transfer orbit characteristics:

$a, e, r_p, r_a, period, \mathcal{E}$ .

The  $\oplus \rightarrow \mathbb{C}$  time-of-flight on the outbound leg?

Phase angle at departure from the parking orbit?

- (c) What is the pass distance  $r_p$  at the Moon to ensure a free-return? Altitude? Is this altitude reasonable?

What are the orbital characteristics, relative to the Earth, after the lunar encounter, i.e.,

$r^+, v^+, \gamma^+, \theta^{*+}, a, e, r_p, r_a, period, energy, \Delta \omega$  in the new orbit?

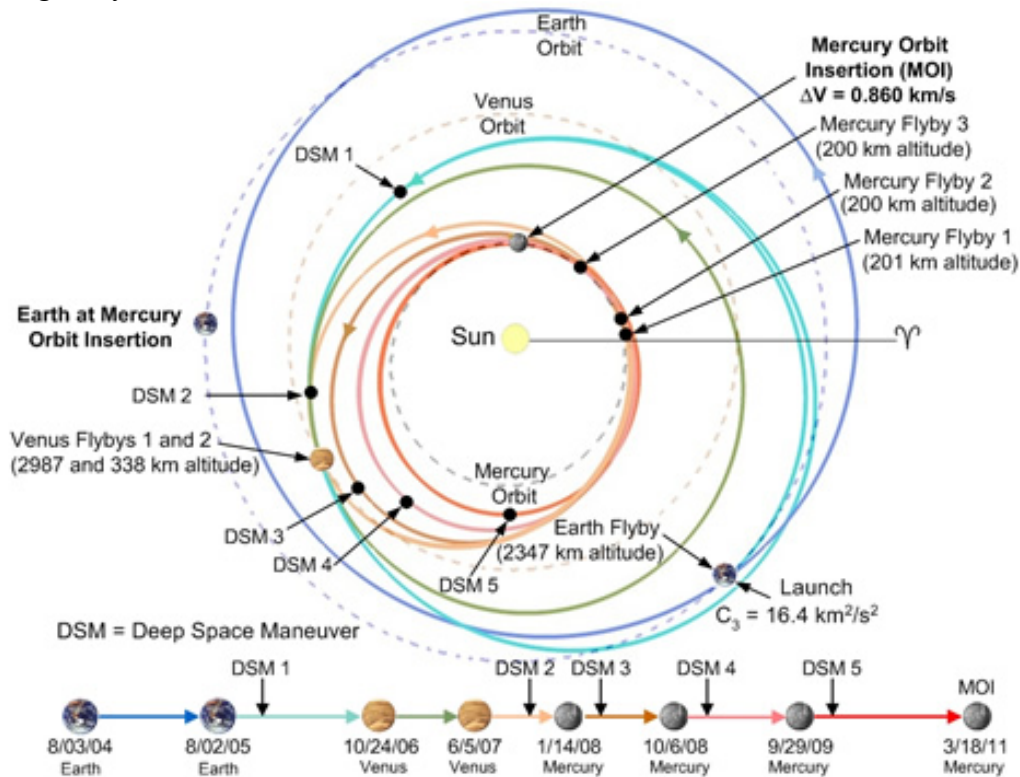
Compute is the  $|\Delta \bar{v}_{eq}|, \alpha$  that corresponds to the free-return?

- (d) For the free-return, plot the orbit in Matlab: (i) in the Earth centered frame, plot the parking orbit, then the outbound and the return arcs only. On the plot, add  $\bar{v}^-, \bar{v}^+$ , local horizon,

$\Delta \bar{v}_{eq}, \alpha$  . (ii) in the Moon centered frame, plot the hyperbola. Add the asymptotes, the vectors  $\bar{v}_{\infty}^{-}, \bar{v}_{\infty}^{+}$  and the flyby angle  $\delta$  .

[Optional: You can use GMAT to check your results. Plot the Earth orbits in GMAT assuming a two-body Earth propagator. Add the equivalent gravity assist maneuver.]

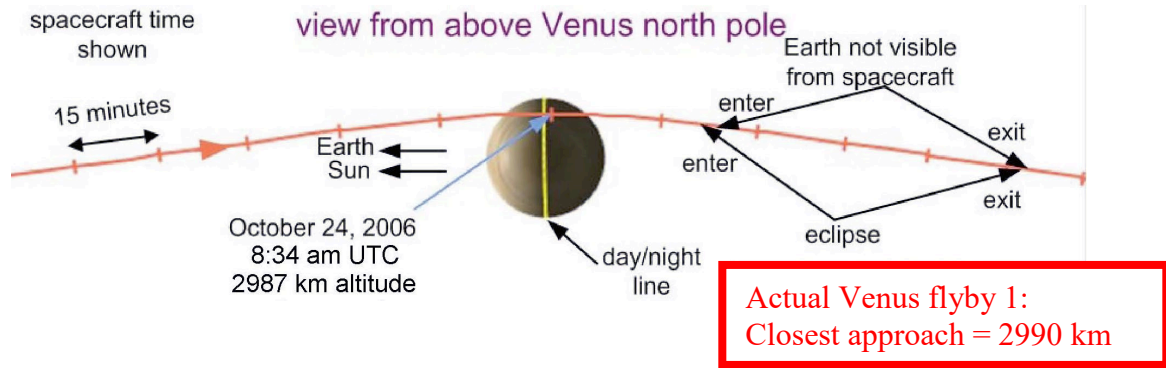
**Problem 2:** The Messenger spacecraft offered exploration of the planet Mercury after launch in 2004 with Mercury orbit insertion in March 2011. The transfer to Mercury employed more than one Venus gravity assist.



- Determine the actual values for TOF for the following:
  - Earth launch to the first Venus flyby.
  - Earth launch to first Mercury flyby.
  - Earth launch to MOI (Mercury Orbit Insertion).
- Assume that you are completing a preliminary analysis for such a Mercury mission but assume that all planetary orbits are coplanar and circular and use patched conics. A Venus gravity assist will aid in reducing the launch maneuver. So, to assess the possible gravity assist, consider a Hohmann transfer; but, let the transfer path possess a **perihelion distance equal to 0.50 AU**. As a result, the transfer path does not reach Mercury; however, along the path the spacecraft encounters Venus. Examine a Venus gravity assist and explore whether Venus can deliver the spacecraft to Mercury.
 

Sketch the heliocentric view to describe the path and identify the location for the Venus encounter. Compute the  $\theta^*$  along the heliocentric path at which the Venus encounter occurs. TOF from Earth to Venus?
- For the actual Venus Flyby 1, the flyby distance was 2990 km altitude. How many Venus radii was the actual encounter? Assume that your Venus encounter uses the same pass distance.

To continue down to Mercury most efficiently, is it desirable to gain or lose energy? Should the spacecraft pass 'ahead' or 'behind' Venus? Determine the following quantities for the post-encounter heliocentric orbit:  $a, e, r, v, \gamma, \theta^*, r_p, r_a$ , period,  $\Delta\omega$ . {Don't forget the Venus-centered vector diagram!]



Determine the equivalent  $\Delta\bar{v}_{eq}$  due to the Venus encounter. What is the magnitude and direction, i.e.,  $|\Delta\bar{v}_{eq}|$  and  $\alpha$ ?

- (d) Plot the old and the new orbits. (Use either Matlab or GMAT.) Identify  $\bar{v}^-$ ,  $\bar{v}^+$ ,  $\Delta\bar{v}_{eq}$ ,  $\alpha$ , line of apsides,  $\Delta\omega$ . Add Mercury's orbit to the plot. Does the s/c now reach the orbit of Mercury? If it does, mark the crossing.

If the orbit does cross Mercury's orbit, you could further reduce the launch cost by launching into a smaller heliocentric orbit (selecting a larger value of the perihelion in part (b)). If your resulting orbit does NOT reach Mercury, you will need to increase the launch maneuver cost by selecting a smaller perihelion value. Discuss: if you try a new initial heliocentric orbit for the transfer, what perihelion distance will you try? Why?

[Note: no more calculations! Just discuss what you might select and why.]

**Problem 3:** The Juno spacecraft remains in orbit about Jupiter until at least 2021; now consider a follow-up mission to the Jovian system. Currently, 79 moons are orbiting Jupiter and the number is increasing as sky searches continue. Assume there exists a new Jovian moon (Remus) with the following characteristics:

$$\begin{aligned} a_R &= 15R_{Jup} & R_R &= 3000 \text{ km} \\ e_R &= 0.25 & \mu_R &= 1 \times 10^5 \text{ km}^3 / \text{s}^2 \end{aligned}$$

The spacecraft orbit is in the same plane as Remus with

$$\begin{aligned} r_p &= 7.5R_{Jup} \\ e &= 0.5 \end{aligned}$$

- (a) The spacecraft encounters the moon when Remus is at the end of the minor axis and is ascending. The spacecraft is outbound in its orbit. Compare the orientation of the s/c orbit line of apsides with that of Remus prior to the encounter. Sketch the orbits. What is the angle between the lines of apsides?
- (b) For the spacecraft, determine  $r^-, v^-, \gamma^-, \theta^{*-}$  just prior to the encounter.
- (c) Sketch the vector diagram for the encounter and the appropriate Remus-centered trajectory.

The Remus gravity assist will be used to change the spacecraft orbit and the goal is to decrease the s/c orbital energy. Should the spacecraft pass “ahead” or “behind” the moon? Why?

- (d) The closest approach during the encounter with Remus is 1500 km altitude. Compute the new spacecraft orbit relative to Jupiter, i.e., determine  $r^+, v^+, \gamma^+, \theta^{*+}$ .

Also determine the new orbital characteristics:  $a, e, r_p, r_a, \text{period}, \mathcal{E}, \Delta\omega$

Use your vector diagram and determine the equivalent  $\Delta\bar{v}_{eq}$ , i.e., compute the  $|\Delta v_{eq}|$  and angle  $\alpha$ . Express  $\Delta\bar{v}_{eq}$  in VNB coordinates.

- (e) Plot the old and new spacecraft orbits: (i) in the Jupiter centered frame, plot old and new spacecraft orbits. On the plot, add  $\bar{v}^-, \bar{v}^+$ , local horizon,  $\Delta\bar{v}_{eq}, \alpha$ . (ii) in the Remus centered frame, plot the hyperbola. Add the asymptotes, the vectors  $\bar{v}_\infty^-, \bar{v}_\infty^+$  as well as  $a, b, r_p$ , the aim point, and the flyby angle  $\delta$ .