Homework 5   AAE 421   Due 11/17 5:00 pm   **Name___________________**

Problem 1. (20pts)
For the differential equations that follow, rewrite the equations in the state-space formulation.

(1) $\frac{d^2c}{dt^2} + 2\zeta\omega_n \frac{dc}{dt} + \omega_n^2 c = r$

(2) $\frac{d^2\theta}{dt^2} + 3\frac{d\theta}{dt} + 2\frac{d\alpha}{dt} + 5\alpha = -6\delta_e$

$$\frac{d\alpha}{dt} + 4\alpha - 15\frac{d\theta}{dt} = -3\delta_e$$

Problem 2. (20pts)

The transfer functions for a feedback control system follow. Determine the states pace equations for the closed-loop system.

(1) $G(s) = \frac{k}{s(s+2)(s+3)}, H(s) = 1$

(2) $G(s) = \frac{k}{s(s^2+8s+10)}, H(s) = 1$

Problem 3. (20pts)

The state-space equations are given as follows:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 0 \\ 1 & -4 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u$$

The output is: $y = [1 - 1\ \ 1] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

determine the following:

(1) Is the system controllable?
(2) Is the system observable?

Problem 4. (20pts)
The longitudinal equations for an airplane having poor handling qualities follow. Use state feedback to provide stability augmentation so that the augmented aircraft has the following short- and long-period (phugoid) characteristics:

$$\begin{bmatrix} \Delta\dot{u} \\ \Delta\dot{w} \\ \Delta\dot{q} \\ \Delta\dot{\theta} \end{bmatrix} = \begin{bmatrix} -0.01 & 0.1 & 0 & -32.2 \\ -0.40 & -0.8 & 180 & 0 \\ 0 & -0.003 & -0.5 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta u \\ \Delta w \\ \Delta q \\ \Delta\theta \end{bmatrix} + \begin{bmatrix} 0 \\ -10 \\ -2.8 \\ 0 \end{bmatrix} \delta_e,$$

$\zeta_{sp} = 0.6, \qquad \omega_{n_{sp}} = 3.0\, rad/s$

$\zeta_p = 0.05, \qquad \omega_{n_{sp}} = 0.1\, rad/s$

using the Ackermann's Formula to design the feedback gain to locate the closed-loop eigenvalues.

Problem 5. (20pts)

The rolling motion of an aerospace vehicle is given by these state equations:

$$\begin{bmatrix} \dot{\delta}_a \\ \dot{p} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} -1/\tau & 0 & 0 \\ L_{\delta_a} & L_p & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \delta_a \\ p \\ \phi \end{bmatrix} + \begin{bmatrix} 1/\tau \\ 0 \\ 0 \end{bmatrix} \delta_v$$

Where δ_a, p, ϕ, and δ_v are the aileron deflection angle, roll rate, roll angle, and voltage input to the aileron actuator motor. Note that in this problem the aileron angle is considered a state and the control voltage, δ_v, is the input. Determine the optimal control law that minimizes the performance index, J, as follows:

$$J = \int_0^\infty (x^T Q x + u^T R u)\, dt$$

where $Q = \begin{bmatrix} 1/\delta_{a_{max}} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1/\phi_{max}^2 \end{bmatrix}$, $R = [1/\delta_{v\,max}^2]$. For the problem, assuming the following: $\tau = 0.1\ s, L_{\delta_a} = 30/s^2, L_p = -1.0\,\frac{rad}{s},\ \delta_{a_{max}} = \pm 25^o = 0.436\ rad, \phi_{max} = \pm 45^o = 0.787\ rad, \delta_{v_{max}} = 10$ volts