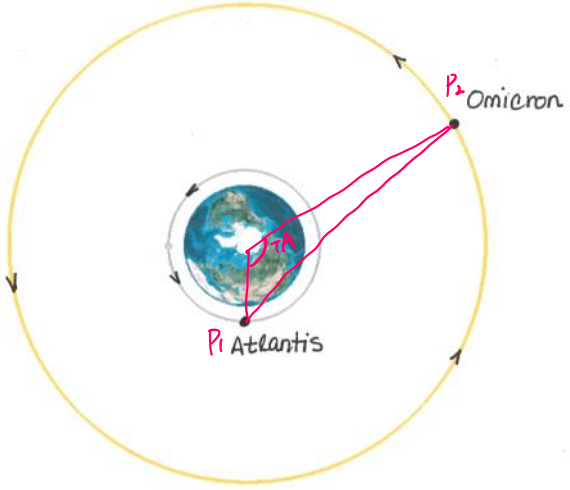
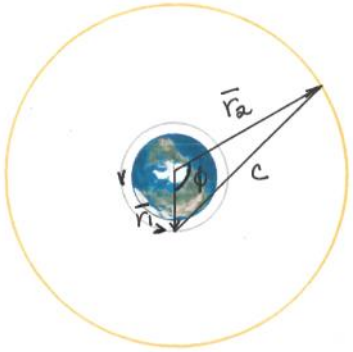
Example:

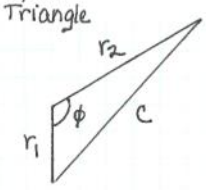
A space facility (Omicron) is currently in a circular Earth orbit at $4R_\oplus$. The spaceship Atlantis is currently in a coplanar, circular orbit at $1.25R_\oplus$. You are asked to provide information on possible transfer orbits for Atlantis to rendezvous with Omicron.

Consider the potential transfer angle: 120°
Try TOF = 15 hr





Space Triangle



(a) $r_1 = 1.25 R_\oplus$

$r_2 = 4 R_\oplus$

$TA = 120^\circ = \phi$

$c^2 = r_1^2 + r_2^2 - 2 r_1 r_2 \cos\phi$

$c = 4.75 R_\oplus \quad s = 5 R_\oplus$

$a_{min} = 2.5 R_\oplus$

(b) Conditions on original orbit/final orbit

$r_1 = 1.25 R_\oplus$

$V_{1\,(original\ orbit)} = \sqrt{\frac{\mu}{r_1}} = 7.071 \text{ Km/s} \quad 7.0707685$ } info for dep

$\gamma_1 = 0^\circ$

$r_2 = 4 R_\oplus$

$V_2 = \sqrt{\frac{\mu}{r_2}} = 3.953 \text{ Km/s}$ } info for arr

$\gamma_2 = 0^\circ$

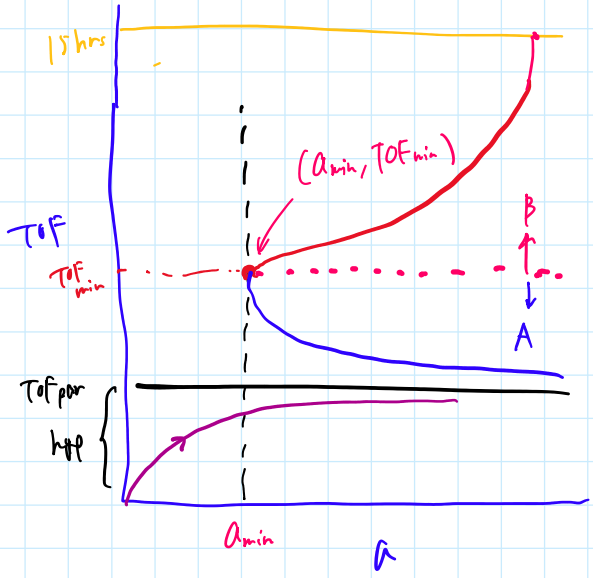
(c) Transfer: elliptical or hyperbolic?

1. Check parabolic (type I)

$$TOF_{par} = \frac{1}{3}\sqrt{\frac{2}{\mu_\oplus}}\left[s^{3/2} - (s-c)^{3/2}\right]$$

$TOF > TOF_{par} \Rightarrow$ elliptical arc

transfer 1A or 1B



2. 1A OR 1B → check minimum energy path

$a_{min} = \frac{s}{2} = 2.5 R_\oplus$

$\alpha_0 = 2\sin^{-1}\sqrt{\frac{s}{2a_{min}}} = \pi\ (180^\circ)$

$\beta_0 = 2\sin^{-1}\sqrt{\frac{s-c}{2a_{min}}} = 25.84^\circ$

$$TOF = \sqrt{\frac{a^3}{\mu}}\left[(\alpha - s_\alpha) - (\beta - s_\beta)\right] = 2.7697 \text{ hrs}$$

$TOF > TOF_{min} \Rightarrow$ 1B

3. Iterate on 'a'

1B → $\alpha = 2\pi - \alpha_0$, $\beta = \beta_0$ → use Lambert's equation

$a = 32,815.7$ km = 5.145 $R_\oplus$

$$p = \frac{4a(s-r_1)(s-r_2)}{c^2}\sin^2\left(\frac{\alpha \pm \beta}{2}\right)$$

1B is this one!

smaller or larger a?

$$p = \frac{4a(s-r_1)(s-r_2)}{c^2}\sin^2\left(\frac{\alpha \pm \beta}{2}\right)$$

smaller or larger p ?

$p = 7258.1$ km or $p = 13,923$ km

which one?

$2ae = |\bar{OF}| \rightarrow$ larger $e \Rightarrow$ smaller $p = a(1-e^2)$

check $\phi = \theta_A^* - \theta_D^* = TA$

$\theta_D^* = 95.825°$ $\quad \gamma_D = +43.96°$ $\quad V_p = 9.3726$ km/s → required to be on transfer

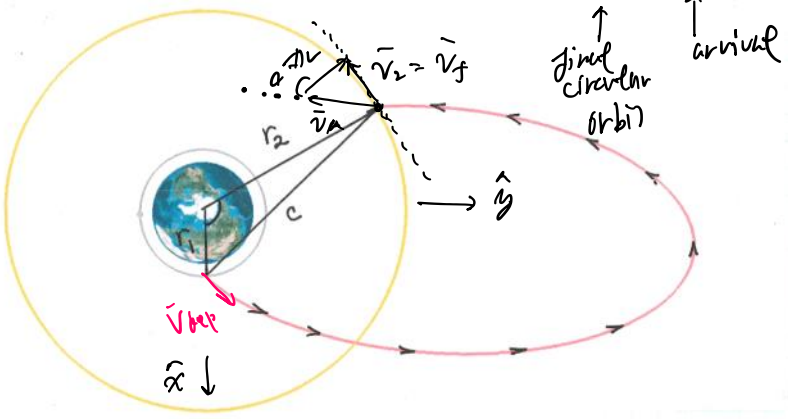
$\theta_A^* = 215.83°$ $\quad \gamma_A = -61.16°$ $\quad V_A = 4.3704$ km/s

check $r_p = 0.6097 R_\oplus$

$r_a = 9.686 R_\oplus$

$$\Delta\bar{v} = \bar{v}_f - \bar{v}_A$$

($\bar{v}_f$: final circular orbit; $\bar{v}_A$: arrival)



4. $\bar{r}_A = f\bar{r}_D + g\bar{v}_D \Rightarrow \bar{v}_D = \dfrac{\bar{r}_A - f\bar{r}_D}{g}$

$f = \left\{1 - \frac{r_A}{p}[1 - \cos TA]\right\} = -4.2726$

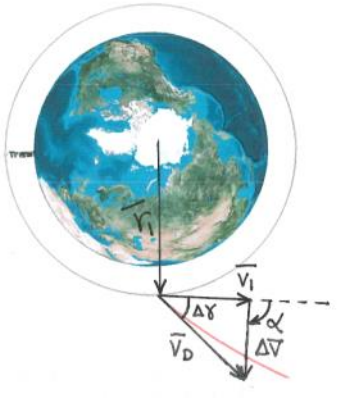
$g = \dfrac{r_D r_A}{\sqrt{\mu p}}\sin TA = 3274.98$ s

$\bar{v}_p = 6.5062\hat{x} + 6.7469\hat{y}$ km/s

$\Delta\bar{v}_p = \bar{v}_D - \bar{v}_1 = 6.506\hat{x} - 0.3243\hat{y}$ km/s

($\hat{\beta}$, $\hat{v}$)

$v_1 = 9.373$ km/s



$V_D = 9.373$ Km/s

$$\alpha = \cos\left(\frac{\Delta\bar{V}_D \cdot \bar{V}_0}{|\Delta\bar{V}_D||\bar{V}_0|}\right)$$

$\alpha = \pm 91.983°$

solve for $\Delta\bar{V}_2$

5. $\bar{V}_A = \dot{f}\,\bar{r}_0 + \dot{g}\,\bar{V}_D$

$$\dot{f} = \frac{\bar{r}_D \cdot \bar{V}_D}{P\,r_D}(1-\cos TA) - \frac{1}{r_D}\sqrt{\frac{\mu}{P}}\sin TA = 5.396 \times 10^{-4}\ /s$$

$$\dot{g} = 1 - \frac{r_D}{P}(1-\cos TA) = -0.6497$$

$\bar{V}_A = .088336\,\hat{x} - 4.3695\,\hat{y}$ Km/s $\quad V_A = 4.3704$ Km/s

$\Delta\bar{V}_A = -3.5115\,\hat{x} + 2.3931\,\hat{y}$ Km/s $\quad \Delta V_A = 4.244$ Km/s

$\alpha = \pm 125.90$