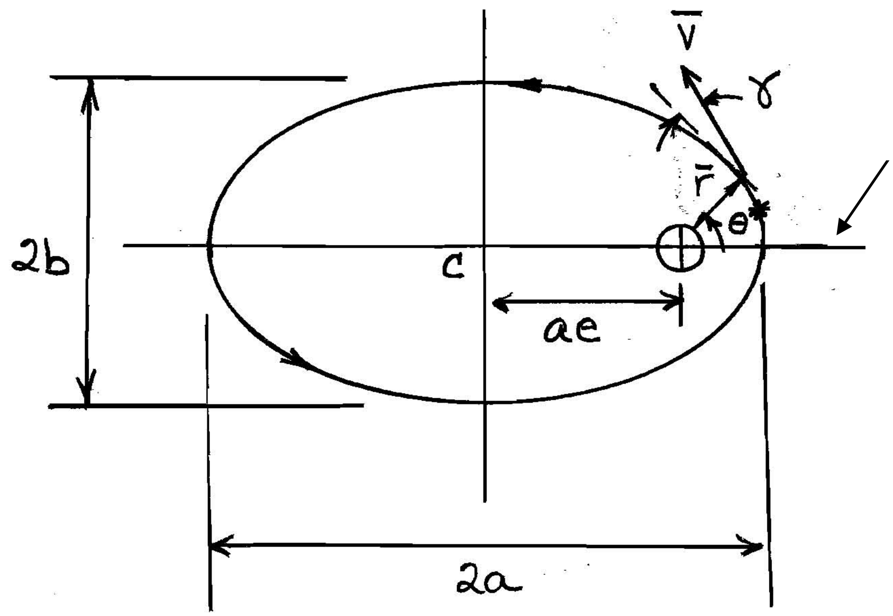
Conic Sections

Ellipse $0 \le e \le 1$ $a > 0$ $\mathcal{E} < 0$



$$r = \frac{p}{1 + e\cos\theta^*}$$

periapsis $\theta^* = 0$ $r_p = \dfrac{p}{1+e} =$

apoapsis $\theta^* = 180^\circ$ $r_a = \dfrac{p}{1-e} =$

$a = \dfrac{1}{2}\left(r_p + r_a\right)$ also known as mean distance

Circle (special case; $e = 0$)

$a = r = p$ $\mathcal{E} = -\dfrac{\mu}{2a} = \dfrac{v^2}{2} - \dfrac{\mu}{r}$

General Ellipse:

$$\frac{v^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2a}$$

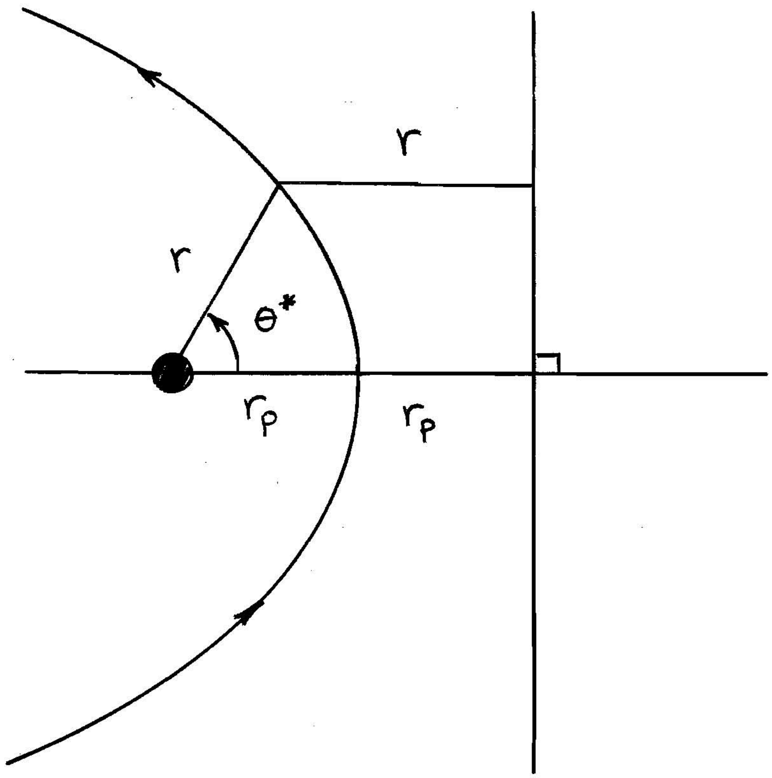
$$v^2 = \frac{2\mu}{r} - \frac{\mu}{a} = 2v_c^2 \underbrace{- \frac{\mu}{a}}_{\text{positive by definition}}$$

$$\frac{dA}{dt} = \frac{h}{2} \qquad \rightarrow \qquad dt = \frac{2}{h}\,dA$$

area of ellipse

$$IP = \frac{2}{h}\left(\pi ab\right)$$

Parabola $e = 1$ $a = \infty$ $\mathcal{E} = 0$



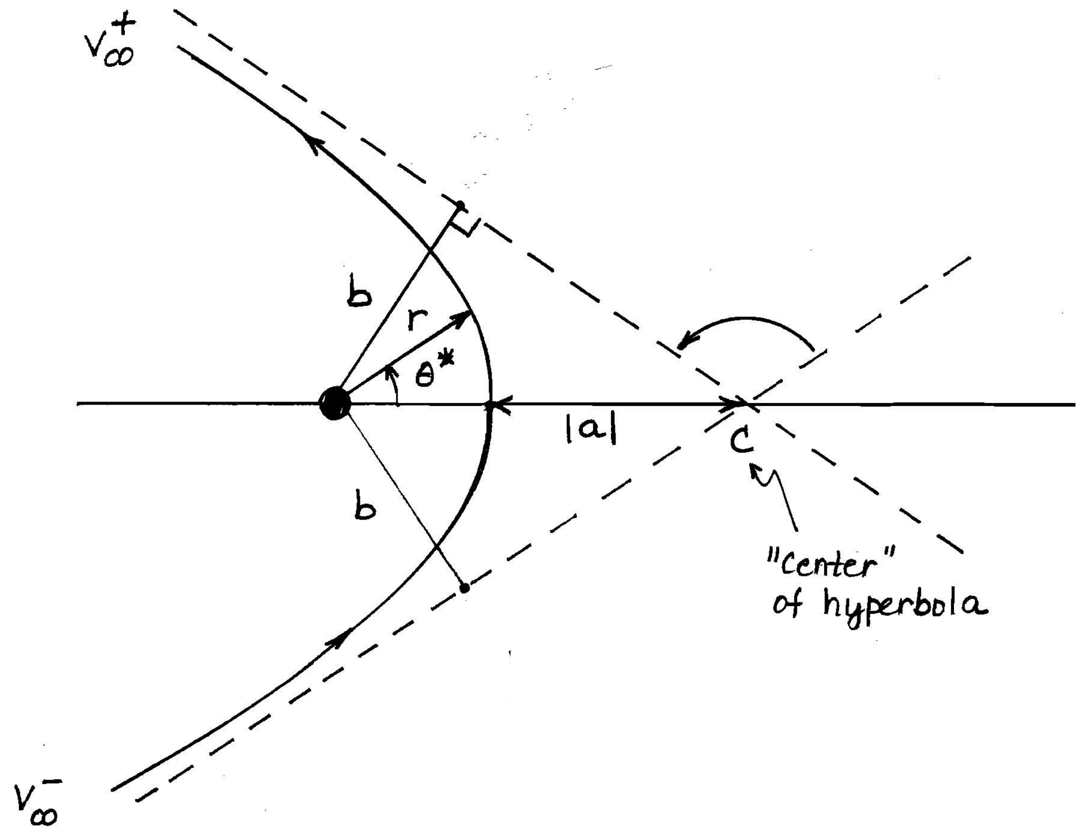
Orbit NOT closed; particle leaves vicinity of attracting body

$$\frac{v^2}{2} - \frac{\mu}{r} = 0 \qquad \rightarrow$$

$$\frac{v_\infty^2}{2} - \frac{\mu}{r_\infty} = 0 \quad \rightarrow \qquad v_\infty = 0$$

Hyperbola $e > 1$ $a < \infty$ (by convention) $\mathcal{E} > 0$

(one branch)



$$\mathcal{E} = -\frac{\mu}{2a} = +\frac{\mu}{2|a|} = \frac{v_\infty^2}{2} - \frac{\mu}{r_\infty} = \frac{v_\infty^2}{2}$$

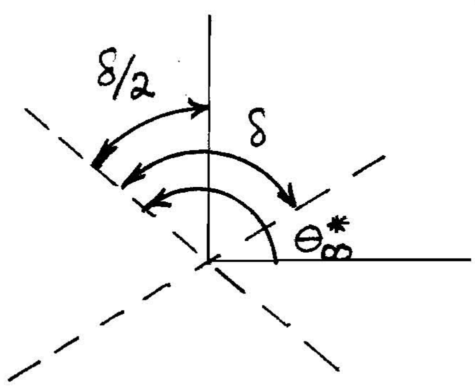
$$r_p = a(1-e) = |a|\,(e-1)$$

$$p = a\left(1-e^2\right) = |a|\left(e^2-1\right)$$

parallel to asymptote $$r_\infty = \frac{|a|\left(e^2-1\right)}{1+e\cos\theta_\infty^*}$$

OR

$$1+e\cos\theta_\infty^* = \cancelto{\text{zero}}{\frac{|a|\left(e^2-1\right)}{r_\infty}}$$



$$\mathcal{E} = \frac{\mu}{2|a|} = \frac{v^2}{2} - \frac{\mu}{r}$$

$$v^2 = \frac{2\mu}{r} + \underbrace{\frac{\mu}{|a|}}_{\text{positive}} \qquad \Longrightarrow$$