AAE 364: Controls System Analysis

HW 6: Steady State Error Evaluation

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(1)

B–5–21. Consider the following characteristic equation:

$$s^4 + 2s^3 + (4 + K)s^2 + 9s + 25 = 0$$

Using the Routh stability criterion, determine the range of K for stability.

using Routh's Stability Criterion (RSC)

s^4	1	$4+K$	25
s^3	2	9	0
s^2	$\frac{2K-1}{2}$	25	
s^1	$\frac{18K-109}{2K-1}$	0	
s^0	25		

$$\frac{2(4+K) - 1\times 9}{2} = \frac{2K-1}{2}$$

$$\frac{\left(\frac{2K-1}{2}\right)9 - 50}{\frac{2K-1}{2}} = \frac{18K-109}{2K-1}$$

now, the 1^{st} column has to all be positive; thus,

$\frac{2K-1}{2} > 0$ and $\frac{18K-109}{2K-1} > 0$

$K > \frac{1}{2} = 0.5$ $\qquad$ $K > \frac{109}{18} \simeq 6.056$

$$\boxed{K \in \left(\frac{109}{18}, \infty\right)}$$

B–5–23. Consider the satellite attitude control system shown in Figure 5–80(a). The output of this system exhibits continued oscillations and is not desirable. This system can be stabilized by use of tachometer feedback, as shown in Figure 5–80(b). If $K/J = 4$, what value of K_h will yield the damping ratio to be 0.6?

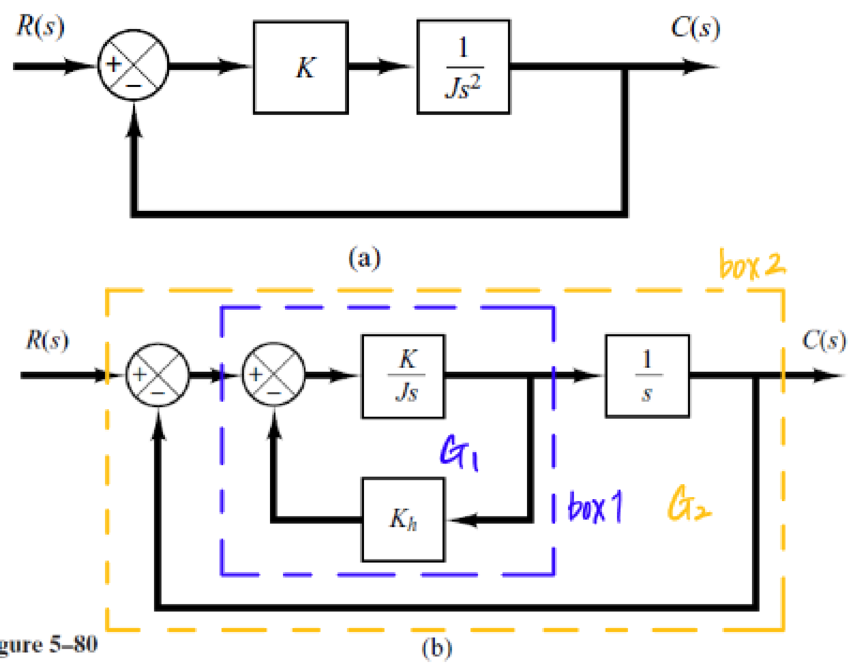

Figure 5–80
(a) Unstable satellite attitude control system;
(b) stabilized system.

from the figure above, by disecting it into 2 boxes

for box 1

$$G_1 = \frac{\frac{K}{Js}}{1+\left(\frac{K}{Js}\right)(K_h)} = \frac{K}{Js + KK_h}$$

then for box 2

$$G_2 = \frac{G_1 \cdot \frac{1}{s}}{1 + G_1 \cdot \frac{1}{s}} = \frac{G_1}{s + G_1} = \frac{\frac{K}{Js + KK_h}}{s + \frac{K}{Js + KK_h}}$$

$$= \frac{K}{Js^2 + KK_h s + K}$$

thus,

$$\frac{C(s)}{R(s)} = G_2 = \frac{K}{Js^2 + KK_h s + K} = \frac{K/J}{s^2 + K_h(K/J)s + K/J}$$

since $K/J = 4$

$$\frac{C(s)}{R(s)} = \frac{4}{s^2 + 4K_h s + 4}$$

here

$$\omega_n = \sqrt{4} = 2 \quad (>0)$$

then

$$4K_h = 2\zeta\omega_n$$

if $\zeta = 0.6$

$$K_h = \frac{2 \times 0.6 \times 2}{4} = 0.6$$

$$\boxed{K_h = 0.6}$$

B–5–26. Consider a unity-feedback control system with the closed-loop transfer function

$$\frac{C(s)}{R(s)} = \frac{Ks + b}{s^2 + as + b}$$

Determine the open-loop transfer function $G(s)$.

Show that the steady-state error in the unit-ramp response is given by

$$e_{ss} = \frac{1}{K_v} = \frac{a - K}{b}$$

the open-loop tf can be deduced from the following relation

$$\frac{G(s)}{1+G(s)} = \frac{Ks+b}{s^2+as+b}$$

$$G(s)(s^2+as+b) = Ks+b + (Ks+b)G(s)$$

$$G(s)(s^2+as+\cancel{b}-Ks-\cancel{b}) = Ks+b$$

$$G(s)[s^2+(a-K)s] = Ks+b$$

$$\therefore\ G(s) = \frac{Ks+b}{s^2+(a-K)s} \qquad \boxed{G(s) = \frac{Ks+b}{s(s+a-K)}}$$

now

$$\frac{E(s)}{R(s)} = \frac{R(s)-C(s)}{R(s)} = 1 - \frac{C(s)}{R(s)} = 1 - \frac{Ks+b}{s^2+as+b}$$

$$= \frac{s^2+as+b-Ks-b}{s^2+as+b} = \frac{s^2+(a-K)s}{s^2+as+b}$$

thus, if $R(s) \longrightarrow$ unit ramp input $\frac{1}{s^2}$

$$\boxed{e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} s\left(\frac{1}{s^2}\right)\left[\frac{s^2+(a-K)s}{s^2+as+b}\right] = \lim_{s\to 0}\frac{s+(a-K)}{s^2+as+b} = \frac{a-K}{b}}$$

(2)

2. Consider the unity-feedback control system in Figure 1 with the following open-loop transfer function:

$$G(s) = \frac{2}{(s+10)^2}$$

Compute the steady state error of the closed-loop system with respect to the step input $r(t) = 5$.

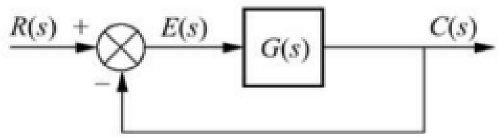

Figure 1: Unity-feedback system

The CLTF is

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{\frac{2}{(s+10)^2}}{1+\frac{2}{(s+10)^2}} = \frac{2}{s^2+20s+102}$$

then

$$\frac{E(s)}{R(s)} = 1 - \frac{C(s)}{R(s)} = \frac{s^2+20s+102-2}{s^2+20s+102} = \frac{s^2+20s+100}{s^2+20s+102}$$

if $r(t) = 5 \xrightarrow{\mathcal{L}} R(s) = \frac{5}{s}$

thus

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s)$$

$$= \lim_{s\to 0} s\left(\frac{5}{s}\right)\left(\frac{s^2+20s+100}{s^2+20s+102}\right)$$

$$= \lim_{s\to 0} \frac{5(s^2+20s+100)}{s^2+20s+102}$$

$$= \frac{500}{102}$$

$$\cong 4.9020$$

$$\boxed{e_{ss} = 4.9020}$$

(3)

3. Consider the unity-feedback control system in Figure 1 with the following open-loop transfer function:

$$G(s) = \frac{3}{s(s+1)(s+2)}$$

Compute the steady state error of the closed-loop system with respect to the ramp input $r(t) = 3t$.

the CLTF becomes

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{\frac{3}{s(s+1)(s+2)}}{\frac{s(s+1)(s+2)+3}{s(s+1)(s+2)}} = \frac{3}{s(s+1)(s+2)+3}$$

$$= \frac{3}{s^3+2s^2+s^2+2s+3} = \frac{3}{s^3+3s^2+2s+3}$$

if $r(t) = 3t \xrightarrow{\mathcal{L}} R(s) = \frac{3}{s^2}$

$$E(s) = 1 - \frac{C(s)}{R(s)}$$

$$= 1 - \frac{3}{s^3+3s^2+2s+3} = \frac{s^3+3s^2+2s}{s^3+3s^2+2s+3}$$

thus,

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s)$$

$$= \lim_{s\to 0} s\left(\frac{3}{s^2}\right)\left(\frac{s^3+3s^2+2s}{s^3+3s^2+2s+3}\right)$$

$$= \lim_{s\to 0} \frac{3(s^2+3s+2)}{s^3+3s^2+2s+3}$$

$$= \frac{6}{3} = 2$$

$$\boxed{e_{ss} = 2}$$