**GIVEN**

power cycle ($m_{CO_2} = 2$ kg of $CO_2(g)$)

>> initially $p_1 = 1$ bar, $T_1 = 300$ K

>> processes

* $1 \to 2$ : isovol $p_2 = 4$ bar
* $2 \to 3$ : polytropic expansion $p v^k = $ const. $k = 1.28$
* $3 \to 1$ : isobar compression.

**FIND**

(a) sketch p–v diagram

(b) $\dot{Q}$ and $\overline{W}$ for each process, in kJ

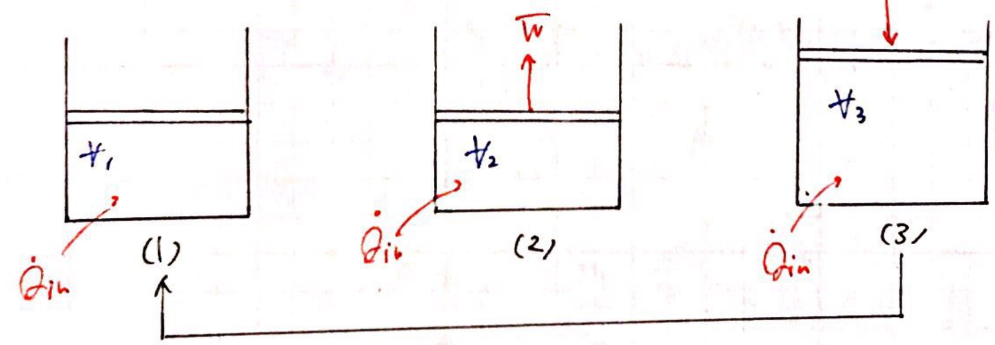
**ASSUMP**

- closed sys. · no friction
- ideal gas
- Quasiequilibrium
- $\Delta PE = \Delta KE = 0$
- molar mass $\equiv M_{CO_2} : 44.01 \frac{kg}{kmol}$
- $R_{CO_2} = 188.92$ J/kg·K
- $\bar{R} = 8.314$ kJ/kmol·K

**EQN**

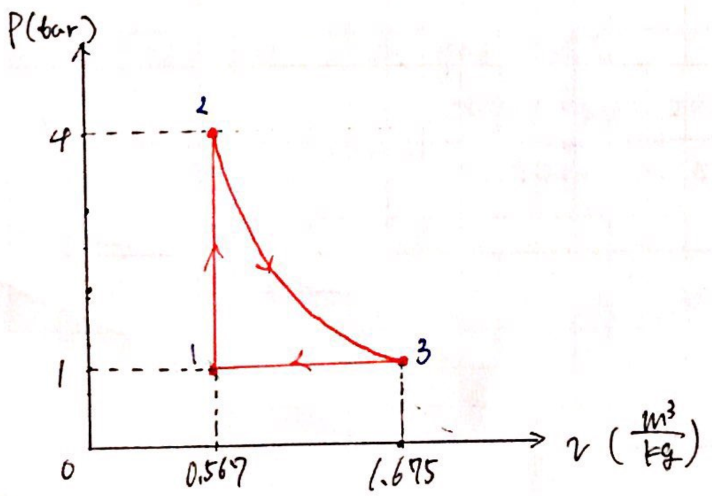
- $\frac{dm}{dt}\Big|_{sys} = \cancel{\dot{m}_{in}} - \cancel{\dot{m}_{out}}$ ; $\Delta U = \int c_v dT$
- $Q = \Delta U + W + \cancel{\Delta PE} + \cancel{\Delta KE}$ ; $\frac{P_0 V_0}{T_0} = \frac{PV}{T}$
- $P v = RT$ ; $PV = mRT = n\bar{R}T$
- $W = \int P dV$ ; $c_p = \frac{kR}{k-1}$ ; $c_v = \frac{R}{k-1}$
- $PV^k = C$, $TV^{k-1} = C$, $TP^{\frac{1-k}{k}} = C$

**EFD**




**SOLN**

(a)




moles $\equiv n = \frac{m_{CO_2}}{M_{CO_2}}$

$= \frac{2 \times 10^3 \text{ g}}{44.01 \text{ g/mol}}$

$\approx 45.44$ mol ... (i)

$v_1 = \frac{R_{CO_2} \cdot T_1}{P_1}$

$= \frac{(188.92 \text{ J/kg·K})(300 \text{ K})}{1 \times 10^5 \text{ Pa}} \approx 0.567 \text{ m}^3/\text{kg}$

(b) $1 \to 2$:

isovol thus $W_{12} = 0$ and $Q_{12} = \Delta U_{12}$

using $\frac{P_1}{T_1} = \frac{P_2}{T_2}$ $\therefore T_2 = \frac{(4\,bar)}{(1\,bar)}(300.15) = 1200.6\,K$

from ideal gas table and (P)

@ $T_2$ . $U_2 = u_2 \cdot n = \left(\frac{43892.8\,J}{mol}\right)(45.44\,mol) \cong 1994.5\,kJ$

@ $T_1$ $U_1 = u_1 \cdot n = \left(\frac{6939\,J}{mol}\right)(45.44\,mol) \cong 315.3\,kJ$

$\therefore \Delta U_{12} = U_2 - U_1 = 1679.2\,kJ$

$$\boxed{\begin{aligned} W_{12} &= 0 \\ \Delta U_{12} &= 1680\,kJ \end{aligned}}$$

$2 \to 3$: $V_2 = v_2 \cdot m_{CO_2} = \left(0.567\,\frac{m^3}{kg}\right)(2\,kg) = 1.134\,m^3$

from polytropic characteristic $P_2 V_2^k = PV^k$

$P_2 V_2^k = (4 \times 10^5\,Pa)(1.134\,m^3)^{1.28} \cong 469855.9$

$V_3 = \left(\frac{P_2 V_2^k}{P_3}\right)^{\frac{1}{k}} \cong 3.3494\,m^3$ $\therefore v_3 = \frac{V_3}{m_{CO_2}} = 1.6747\,\frac{m^3}{kg}$

also $W_{23} = \int_{V_2}^{V_3} p\,dV = \frac{P_3 V_3 - P_2 V_2}{1-k} = \frac{(1 \times 10^5\,Pa)(3.3494\,m^3) - (4 \times 10^5\,Pa)(1.134\,m^3)}{1 - 1.28}$

$\cong 423786\,J \cong 423.79\,kJ$

and $T_3 = \frac{P_3 v_3}{R_{CO_2}} = \frac{(1 \times 10^5\,Pa)\left(1.6747\,\frac{m^3}{kg}\right)}{(188.92\,J/kg \cdot K)} \cong 886.46\,K$

interpolating values on ideal gas table

$u_3 = (886.46\,K - 880\,K)\left(\frac{29461\,J/mol - 29017\,J/mol}{890\,K - 880\,K}\right) + 29017\,J/mol$

$\cong 29303.824\,J/mol$

$U_3 = \left(\frac{29303.824\,J}{mol}\right)(45.44) \cong 1331.57\,kJ$

$\Delta U_{23} = U_3 - U_2 = -661.63\,kJ$

$Q_{23} = \Delta U_{23} + W_{23} = -237.84\,kJ$

$$\boxed{\begin{aligned} W_{23} &= 424\,kJ \\ Q_{23} &= -238\,kJ \end{aligned}}$$

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3 → 1 :

$$W_{31} = \int_{V_3}^{V_1} P\,dV$$

$$= (1\times10^5\,Pa)(1.134\,m^3 - 3.3494\,m^3)$$

$$\cong -221.54\,kJ$$

$$\Delta U_{31} = U_1 - U_3$$

$$= 315.3\,kJ - 1331.57\,kJ$$

$$= -1016.47\,kJ$$

$$\therefore\ Q_{31} = \Delta U_{31} + W_{31}$$

$$\cong -1230\ \ l$$

$$\boxed{\begin{aligned} W_{31} &= -222\,kJ \\ Q_{31} &= -1230\,kJ \end{aligned}}$$