Name	Team Number	
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# AAE 251: Introduction to Aerospace Design

Assignment 8—Aircraft Performance I

# Due Tuesday April 2, 10:00 am on Blackboard

### Instructions

Write or type your answers into the appropriate boxes. **Make sure you submit a single PDF on Blackboard.** 

Make sure you keep a record of submission receipts or the confirmation emails after each submission as a proof that your submission was accepted.

	Score	Max
Question 1		20
Question 2		4
Question 3		4
Question 4		16
Question 5		16
Question 6		10
TOTAL		70

Many aircraft performance questions require the same calculations. Creating the calculations as Matlab functions (not scripts) can save you effort in the homework and also help to hone your Matlab skills.

Create a set of well commented Matlab functions to implement the equations we gave or derived in class, i.e. :

- Lift, and drag as a function of velocity
- Thrust required in SLUF as a function of velocity for the jet from the case study
- Power required in SLUF as a function of velocity for the prop from the case study
- Thrust required in level-flight as a function of velocity, mass, surface area, e, AR, rho and zero-lift drag coefficient
- Power required in level-flight as a function of velocity, mass, surface area, e, AR, rho and zero-lift drag coefficient
- Any other functions you find you need in this homework

Now use your Matlab functions to answer the following questions. Attach your <u>function and</u> <u>script</u> files and write your answers if needed as your response for each question. Make sure to show which functions you use, the values used in the function, and your final response. <u>Start each new question on a new page.</u>

### **FUNC: Thrust and Power Required for GA Aircraft Propeller Driven**

#### >>Details

- (1) Description: This program uses predefined equations with provided coefficients to calculate the power for a certain general aviation propeller driven aircraft.
- (2) Input values:
  - 1. vel: free stream velocity [m/s] or [ft/s]
- (3) Output values:
  - power: power [W]
     thrust: thrust [N]

```
function [power, thrust] = GAprop powerThrust cal(vel)
% Preparations
mass = 1315; % Mass of the aircraft [kg]
wing_area = 16.3; % Wing surface area [m^3]
grav = 9.81; % Gravitational acceleration [m/s^2]
rho = 1.225; % Air density at sea level [kg/m^3]
C D0 = 0.026; % Zero lift drag coefficient
K = 0.054; % Drag polar coefficient
% Calculate
% First calculate lift coefficient
C_L = 2 * mass * grav / rho ./ vel.^2 / wing_area;
% Then calculate the drag coefficient with drag polar
C_D = C_{D0} + K .* C_L.^2;
% Thrust is approximated to be equal to drag
thrust = 0.5 * rho .* vel.^2 * wing_area .* C_D;
% Than power is the product of thrust and velocity
power = thrust .* vel;
end
```

# FUNC: LIFT & DRAG CALCULATOR (level flight)

#### >> Details

- (1) Description: This program defines the equations for the lift and drag (level flight), and calculates them using the variables involved in the equation as inputs.
- (2) Input values:
  - 1. den: atmospheric denstiy [kg/m^3] or [slug/ft^3]
  - 2. vel: free stream velocity [m/s] or [ft/s]
  - 3. area: wing area [m^2] or [ft^2]
  - 4. D\_o: lift zero drag coefficient
  - 5. weight: the weight of the aircraft [N] or [lb]
  - 6. AR: aspect ratio of the wing
  - 7. e\_o: the Oswald efficiency factor
- (3) Output values:
  - 1. L: lift [N] or [lb]
  - 2. D: drag [N] or [lb]
- (4) Theory

$$Lift = L = \frac{1}{2} \rho_{\infty} V_{\infty}^2 SC_L$$

$$Drag = D = \frac{1}{2} \rho_{\infty} V_{\infty}^2 SC_D$$

at level flight

$$L \approx \mathrm{mg} \Longrightarrow C_L = \frac{2\mathrm{mg}}{\rho_\infty V_\infty^2 S}$$

$$C_D = C_{\text{Do}} + \frac{C_L^2}{\pi e A R}$$

```
function [L, D] = lift_drag_cal(den, vel, area, D_o, weight, AR, e_o)
% Calculate the coefficients
L_const = 2 * weight / den ./ vel.^2 / area;
D_const = D_o + L_const.^2 / pi / e_o / AR;
% Calculate the list and drag
L = 0.5 * den .* vel.^2 * area .* L_const;
D = 0.5 * den .* vel.^2 * area .* D_const;
end
```

## **FUNC: POWER CALCULATOR (level flight)**

#### >> Details

- (1) Description: This program defines the equations for the lift and drag (level flight), and calculates them using the variables involved in the equation as inputs.
- (2) Input values:
  - 1. den: atmospheric denstiy [kg/m^3] or [slug/ft^3]
  - 2. vel: free stream velocity [m/s] or [ft/s]
  - 3. area: wing area [m^2] or [ft^2]
  - 4. D\_o: lift zero drag coefficient
  - 5. weight: the weight of the aircraft [N] or [lb]
  - 6. AR: aspect ratio of the wing
  - 7. e\_o: the Oswald efficiency factor
- (3) Output values:
  - 1. P: power [W] or [lb-ft/s]
  - 2. T: Thrust [N] or [lb]
  - 3. drag coefficient
  - 4. lift coefficient

```
function [power, thrust] = power_cal(rho, vel, wing_area, C_D0, weight, AR, e_o)
% Calculate the coefficients
K = 1 / pi / e_o / AR;
% Calculate
% First calculate lift coefficient
C_L = 2 * weight / rho ./ vel.^2 / wing_area;
% Then calculate the drag coefficient with drag polar
C_D = C_D0 + K .* C_L.^2;
% Thrust is approximated to be equal to drag
thrust = 0.5 * rho .* vel.^2 * wing_area .* C_D;
% Than power is the product of thrust and velocity
power = thrust .* vel;
end
```

### **FUNC: Thrust and Power Required for Executive Jet Example**

#### >>Details

- (1) Description: This program uses predefined equations with provided coefficients to calculate the thrust for a certain executive jet.
- (2) Input values:
  - 1. vel: free stream velocity [m/s] or [ft/s]
- (3) Output values:
  - thrust: thrust [N]
     power: power [W]

```
function [thrust, power] = ExecJet_thrustPower_cal(vel)
% Preparations
mass = 33100; % Mass of the aircraft [kg]
wing area = 88.2; % Wing surface area [m^3]
grav = 9.81; % Gravitational acceleration [m/s^2]
rho = 1.225; % Air density at sea level [kg/m^3]
C D0 = 0.015; % Zero lift drag coefficient
K = 0.05; % Drag polar coefficient
% Calculate
% First calculate lift coefficient
C_L = 2 * mass * grav / rho ./ vel.^2 ./ wing_area;
% Then calculate the drag coefficient with drag polar
C_D = C_{D0} + K * C_L.^2;
% Thrust is approximated to be equal to drag
thrust = 0.5 * rho .* vel.^2 * wing_area .* C_D;
% Than power is the product of thrust and velocity
power = thrust .* vel;
end
```

# **FUNC: Thrust CALCULATOR (level flight)**

#### >> Details

- (1) Description: This program defines the equations for the lift and drag (level flight), and calculates them using the variables involved in the equation as inputs.
- (2) Input values:
  - 1. den: atmospheric denstiy [kg/m^3] or [slug/ft^3]
  - 2. vel: free stream velocity [m/s] or [ft/s]
  - 3. area: wing area [m^2] or [ft^2]
  - 4. D\_o: lift zero drag coefficient
  - 5. weight: the weight of the aircraft [N] or [lb]
  - 6. AR: aspect ratio of the wing
  - 7. e\_o: the Oswald efficiency factor
- (3) Output values:
  - 1. T: thrust [N] or [lb]
- (4) Theory

$$Lift = L = \frac{1}{2} \rho_{\infty} V_{\infty}^2 SC_L$$

$$Drag = D = \frac{1}{2} \rho_{\infty} V_{\infty}^2 SC_D$$

at level flight

$$L \approx \text{mg} \Longrightarrow C_L = \frac{2\text{mg}}{\rho_{\infty}V_{\infty}^2S}$$

$$C_D = C_{\text{Do}} + \frac{C_L^2}{\pi e A R}$$

thrust =  $T \approx D$ 

```
function [T] = thrust_cal(den, vel, area, D_o, weight, AR, e_o)
% Calculate the coefficients
L_const = 2 * weight / den / vel^2 / area;
D_const = D_o + L_const^2 / pi / e_o / AR;
% Calculate the list and drag
L = 0.5 * den * vel^2 * area * L_const;
D = 0.5 * den * vel^2 * area * D_const;
% Thrust output
T = D;
end
```

Plot the thrust and power required curves in SLUF for the two example aircraft we use in class (see the handout from class, or on BB). You may find Anderson Examples 6.1 and 6.3 useful in responding to this question. For the jet airplane, use  $V_{\infty} = 50$  m/s to 300 m/s, and use  $V_{\infty} = 20$  m/s to 150 m/s for the GA aircraft.

#### >> Solution

To solve this problem we will use the function **GAprop powerThrust cal.mlx** and **ExecJet thrustPower cal.mlx** to calculate corresponding thrust and power value for a given velocity for 2 example aircrafts.

#### >> Code

### **Preparations**

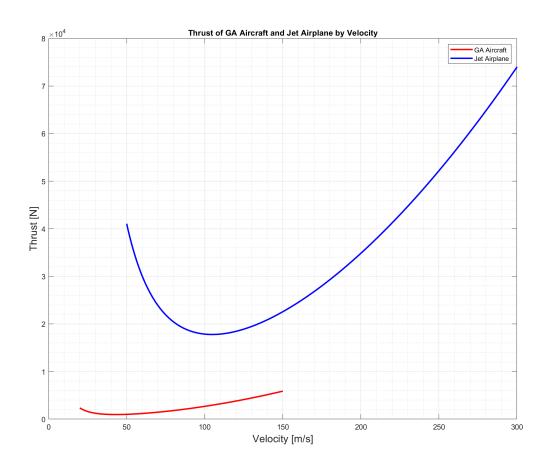
```
% For GA propeller type
vel_prop = 20:150; % velocity [m/s]
% For jet airplane
vel_jet = 50:300; % velocity [m/s]
```

# **Calling Fucntions**

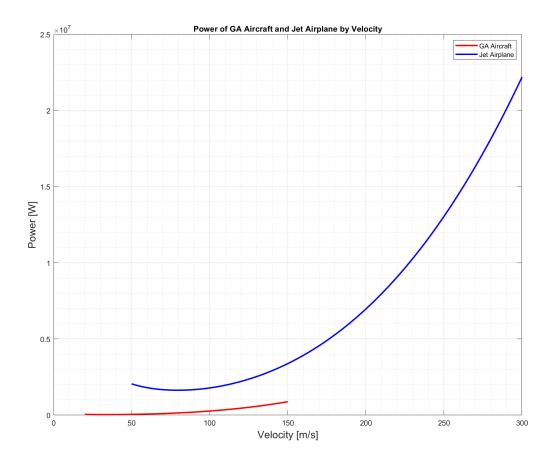
```
% For GA propeller type
[propPower, propThrust] = GAprop_powerThrust_cal(vel_prop);
% For jet airplane
[jetThrust, jetPower] = ExecJet_thrustPower_cal(vel_jet);
```

## **Plotting**

```
% Adjusting fontsize and linewidth
fontsize = 14;
linewidth = 2;
% GA aircraft & jet airplane Thrust
plot(vel_prop, propThrust, '-r', 'LineWidth', linewidth)
title('Thrust of GA Aircraft and Jet Airplane by Velocity')
xlabel('Velocity [m/s]', 'FontSize', fontsize)
ylabel('Thrust [N]', 'FontSize', fontsize)
grid on
grid minor
box on
hold on
plot(vel_jet, jetThrust, '-b', 'LineWidth', linewidth)
hold off
legend('GA Aircraft', 'Jet Airplane')
% Positioning
set(gcf, 'PaperPositionMode', 'auto', 'Position', [0 0 1100 850])
```



```
% GA aircraft & jet airplane power
figure(2)
plot(vel_prop, propPower, '-r', 'LineWidth', linewidth)
title('Power of GA Aircraft and Jet Airplane by Velocity')
xlabel('Velocity [m/s]', 'FontSize',fontsize)
ylabel('Power [W]', 'FontSize',fontsize)
grid on
grid minor
box on
hold on
plot(vel_jet, jetPower, '-b','LineWidth',linewidth)
hold off
legend('GA Aircraft', 'Jet Airplane')
```



```
% Positioning
set(gcf,'PaperPositionMode','auto','Position',[0 0 1100 850])
```

Consider an aircraft modelled after the Cessna Citation X medium business jet aircraft. Its weight is 12000 N, wing area is 50  $m^2$ , aspect ratio is 7.8, Oswald efficiency factor is 0.86, and zero-lift drag coefficient is  $C_{D0}=0.027$ . Calculate the thrust required to fly at a velocity of 720 km/hr at the height of:

- (a) 4 km
- (b) 10 km

Assume SLUF.

#### >> Solution

To solve this problem I will be using the function **thrust cal.mlx** to find the thrust for a Cessna Citation X medium business jet aircraft at different altitudes.

#### >> Code

## **Preparations**

```
weight = 12000; % Weight [N]
wing_area = 50; % Wing area [m^2]
AR = 7.8; % Aspect ratio
e_oswald = 0.86; % Oswald efficiency constant
C_D0 = 0.027; % Zero lift drag coefficient
vel = 720; % Velocity [km/hr]
rho_4km = 0.81935; % Density at altitude 4km [kg/m^3]
rho_10km = 0.41351; % Density at altitude 10km [kg/m^3]
% Convert velocity to [m/s]
vel = vel * 1000 / 3600;
```

# **Calling Function (Calculate Thrust)**

```
thrust_4km = thrust_cal(rho_4km, vel, wing_area, C_D0, weight, AR, e_oswald);
thrust_10km = thrust_cal(rho_10km, vel, wing_area, C_D0, weight, AR, e_oswald);
```

#### **Answers**

```
fprintf('The thrust required for a Citation X medium business jet aircraft at an altitude of')
The thrust required for a Citation X medium business jet aircraft at an altitude of

fprintf('4 km: %.2f N', thrust_4km);

4 km: 22130.79 N

fprintf('10 km: %.2f N', thrust_10km);
```

10 km: 11181.29 N

An airplane weighing 5000 lb is flying at standard sea level with a velocity of 200 mi/hr. At this velocity, the L/D is a maximum. The wing area and aspect ratio are 200 ft $^2$  and 8.5, respectively. The Oswald efficiency factor is 0.93. Calculate the total drag on the airplane.

#### >> Solution

To solve this problem I will be using the function <u>lift\_drag\_cal.mlx</u> to find the drag for the certain airplane given in the instruction.

#### >> Code

## **Preparation**

```
weight = 5000; % weight of the aircraft [lb]
vel = 200; % velocity [mi/hr]
wing_area = 200; % wing surface area [ft^2]
AR = 8.5; % Aspect ratio of wing
e_oswald = 0.93; % Oswald efficiency factor
rho = 2.3769 * 10^(-3); % air density [slug/ft^3]

% Converting the velocity to ft/s
vel = 293.333; % [ft/s]

% Because the condition is that L/D is maximum
% the zero lift drag coefficient is calculated using the following relation
```

at 
$$\left(\frac{L}{D}\right)_{\text{MAX}}$$
 or in other words  $\left(\frac{C_L}{C_D}\right)_{\text{MAX}}$ 

$$V_{\text{Tmin}} = \sqrt{2 \frac{W}{\rho S} \sqrt{\frac{K}{C_{\text{D0}}}}}$$
, where  $K = \frac{1}{\pi \text{eAR}}$ 

thus,

$$C_{\rm D0} = \frac{1}{\pi \rm eAR} \left( \frac{W}{\frac{1}{2} \rho V^2 S} \right)^2$$

```
% Thererfore zero lift drag coefficient is
C_D0 = (1 / pi / e_oswald / AR) * (weight / 0.5 / rho / vel^2 / wing_area)^2
```

```
C D0 = 0.0024
```

# **Calling Function**

```
[lift, drag] = lift_drag_cal(rho, vel, wing_area, C_D0, weight, AR, e_oswald);
```

# Answer

```
fprintf('The drag on the airplane is: %.6f lb', drag);
```

The drag on the airplane is: 98.443149 lb

Consider an airplane patterned after the A-10. The airplane has the following characteristics: wing area =  $505.6 \text{ ft}^2$ , aspect ratio = 6.5, Oswald efficiency factor = 0.87, weight = 23,105 lb, and zero lift drag coefficient = 0.032. The airplane is equipped with two jet engines with 9035 lb of static thrust *each* at sea level.

- a. Calculate and plot the power required at sea level as a function of velocity.
- b. Calculate the maximum velocity that can be achieved at sea level at maximum thrust.
- c. Calculate and plot the power required at 5 km above sea level as a function of velocity.
- d. Calculate the maximum velocity that can be achieved at 5 km above sea level at maximum thrust. (Assume that the jet engine thrust will be proportional to free-stream air density)

#### >> Solution

To solve this problem I will be using the function **power cal.mlx** to find the power and thrust for the certain airplane given in the instruction.

#### >> Code

## **Preparation**

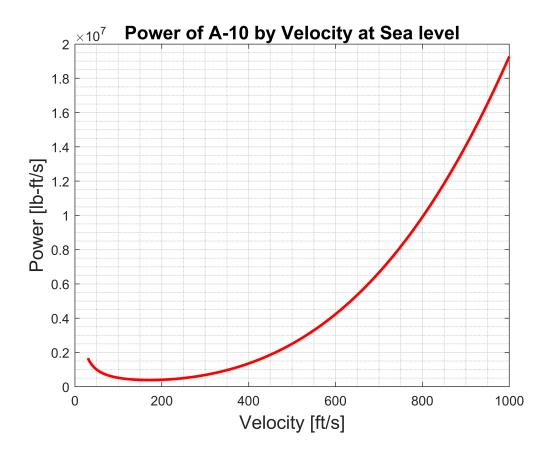
```
weightQ4 = 23105; % weight [lb]
wing_areaQ4 = 505.6; % [ft^2]
ARQ4 = 6.5; % Aspect Ratio
e_oswaldQ4 = 0.87; % Oswald efficiency factor
C_D0Q4 = 0.032; % zero lift drag ratio
rho_seaLevelQ4 = 2.3769 * 10^(-3); % [slug/ft^3]
rho_5kmQ4 = 1.4244 * 10^(-3); % [slug/ft^3]
KQ4 = 1 / pi / e_oswaldQ4 / ARQ4;

% velocity
velQ4 = 30:1000; % [ft/s]
```

# Part (a)

```
% sea level
[Power_0, Thrust_0] = power_cal(rho_seaLevelQ4, velQ4, wing_areaQ4, C_D0Q4, weightQ4, ARQ4, e_d
% Plotting
% Adjusting fontsize and linewidth
fontsize = 14;
linewidth = 2;

figure(1)
plot(velQ4, Power_0, '-r', 'LineWidth',linewidth)
title('Power of A-10 by Velocity at Sea level', 'FontSize',fontsize)
xlabel('Velocity [ft/s]', 'FontSize',fontsize)
ylabel('Power [lb-ft/s]', 'FontSize',fontsize)
grid on
grid minor
box on
```



# Part (b)

since the static thrust produced by the 2 jet engines are

$$max(T) = T_A = 2 \times 9035 \text{ lb} = 18070 \text{ lb}$$

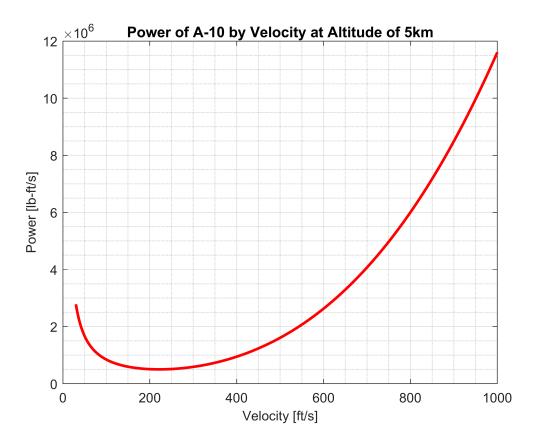
```
% Maximum thrust is
T_A = 18070;
% The max velocity for this is
V_max_sealevel1 = sqrt((T_A - sqrt(T_A^2 - 4*KQ4*C_D0Q4))/rho_seaLevelQ4/wing_areaQ4/C_D0Q4);
V_max_sealevel2 = sqrt((T_A + sqrt(T_A^2 - 4*KQ4*C_D0Q4))/rho_seaLevelQ4/wing_areaQ4/C_D0Q4);
if V_max_sealevel1 > V_max_sealevel2
    V_max_sealevel = V_max_sealevel1;
else
    V_max_sealevel = V_max_sealevel2;
end
% Print out result
fprintf('The max V at max thrust (%.2f lb)is %.2f ft/s.', T_A, V_max_sealevel);
```

The max V at max thrust (18070.00 lb)is 969.42 ft/s.

# Part (c)

```
% sea level
[Power_5km, Thrust_5km] = power_cal(rho_5kmQ4, velQ4, wing_areaQ4, C_D0Q4, weightQ4, ARQ4, e_o
% Plotting
% Adjusting fontsize and linewidth
fontsize = 10;
linewidth = 2;

figure(2)
plot(velQ4, Power_5km, '-r', 'LineWidth',linewidth)
title('Power of A-10 by Velocity at Altitude of 5km', 'FontSize',11)
xlabel('Velocity [ft/s]', 'FontSize',fontsize)
ylabel('Power [lb-ft/s]', 'FontSize',fontsize)
grid on
grid minor
box on
```



# Part (d)

since the static thrust produced by the 2 jet engines are

$$\max(T) = T_A = 2 \times 9035 \text{ lb} = 18070 \text{ lb}$$

$$\max(T) = T_{A,\text{alt}} = \frac{\rho}{\rho_0} T_A$$

The max V at max thrust (10828.77 lb)is 750.45 ft/s.

Consider an airplane patterned after the Beechcraft Bonanza V-tailed single-engine light private airplane. The characteristics of this airplane are as follows: aspect ratio = 6.2, wing area =  $16.82 \, m^2$ , Oswald efficiency factor = 0.91, weight =  $13380 \, \text{N}$ , and zero-lift drag coefficient = 0.027. The airplane is powered by a single piston engine capable of producing 345 hp at sea level. Assume that the power of the engine is proportional to free-stream air density and that the two-bladed propeller has an efficiency of 0.83.

- a. Calculate and plot the power required at sea level as a function of velocity.
- b. Calculate the maximum velocity that can be achieved at sea level at maximum power.
- c. Calculate and plot the power required at 3.66 km above sea level as a function of velocity.
- d. Calculate the maximum velocity that can be achieved at 3.66 km above sea level at maximum power. For this problem, assume that the jet engine thrust will be proportional to free-stream air density.

#### >> Given

Consider an airplane patterned after the Beechcraft Bonanza V-tailed single-engine light private airplane. The characteristics of this airplane are as follows: aspect ratio = 6.2, wing area = 16.82 m $^2$ , Oswald efficiency factor = 0.91, weight = 13380 N, and zero-lift drag coefficient = 0.027. The airplane is powered by a single piston engine capable of producing 345 hp at sea level. Assume that the power of the engine is proportional to free-stream air density and that the two-bladed propeller has an efficiency of 0.83.

#### >> Solution

To solve this problem we will use the function **power\_cal.mlx** to calculate corresponding thrust and power value for a given velocity for 2 example aircrafts.

#### >> Code

### **Preparation**

```
ARQ5 = 6.2;

wing_areaQ5 = 16.82;

e_oQ5 = 0.91;

weightQ5 = 13380;

C_D0Q5 = 0.027;

P_max = 257266; % converted to Watts

prop_eff = 0.83;

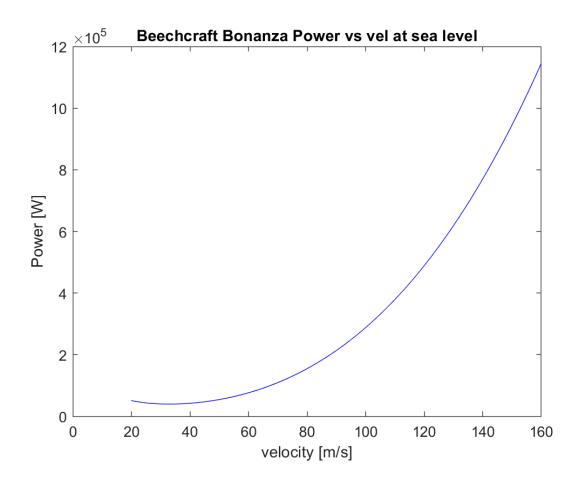
rho_sealevelQ5 = 1.225;

rho_alt = 0.84557; % at 3.66 km

velQ5 = 20:160;
```

### Part (a)

```
% Calculate and plot the power required at sea level as
% a function of velocity.
[powerQ5, thrustQ5] = power_cal(rho_sealevelQ5, velQ5, wing_areaQ5, C_D0Q5, weightQ5, ARQ5, e_G
% Plot
figure(1)
plot(velQ5, powerQ5, '-b')
xlim([0, 160])
title('Beechcraft Bonanza Power vs vel at sea level')
xlabel('velocity [m/s]')
ylabel('Power [W]')
```



# Part (b)

```
% Calculate the maximum velocity that can be achieved at sea
% level at maximum power.
P_limit_0 = P_max * prop_eff;
syms V
a = 0.5 * rho_sealevelQ5 * wing_areaQ5 * C_D0Q5;
b = 2 * (1/e_oQ5/pi/ARQ5) * weightQ5^2 / rho_sealevelQ5 / wing_areaQ5;
eqn = round(a,4) * V^4 + round(b,1) == round(P_limit_0,1) * V;
V_max_sealevelQ5 = solve(eqn, V);

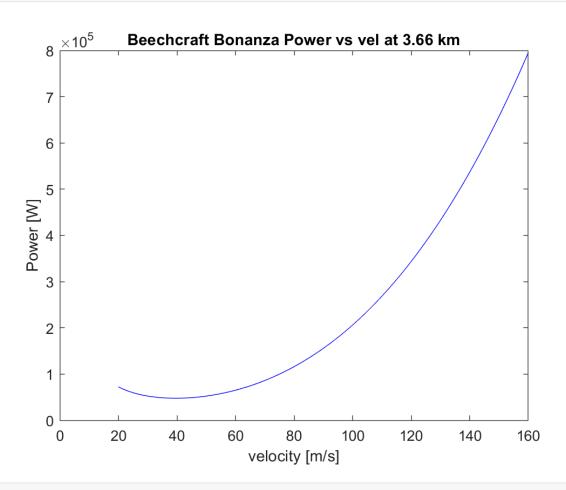
V_max_Q5b = double(max(V_max_sealevelQ5(1:2,1)));
fprintf('The maximum velocity is %f m/s', V_max_Q5b);
```

The maximum velocity is 89.974360 m/s

# Part (c)

```
% Calculate and plot the power required at 3.66 km above sea level as a function of velocity.
[powerQ52, thrustQ52] = power_cal(rho_alt, velQ5, wing_areaQ5, C_D0Q5, weightQ5, ARQ5, e_oQ5);
% Plot
```

```
figure(1)
plot(velQ5, powerQ52, '-b')
xlim([0, 160])
title('Beechcraft Bonanza Power vs vel at 3.66 km')
xlabel('velocity [m/s]')
ylabel('Power [W]')
```



# Part (d)

```
P_limit_02 = (rho_alt / rho_sealevelQ5 * P_max) * prop_eff;
syms V
a = 0.5 * rho_sealevelQ5 * wing_areaQ5 * C_D0Q5;
b = 2 * (1/e_oQ5/pi/ARQ5) * weightQ5^2 / rho_sealevelQ5 / wing_areaQ5;
eqn = round(a,4) * V^4 + round(b,1) == round(P_limit_02,1) * V;
V_max_sealevelQ52 = solve(eqn, V);

V_max_Q5d = double(max(V_max_sealevelQ52(1:2,1)));
fprintf('The maximum velocity is %f m/s', V_max_Q5d);
```

The maximum velocity is 78.565648 m/s

Span loading is defined as the ratio of aircraft's weight to its wing span. For an airplane in subsonic steady level flight, the drag force due to lift depends directly on the square of span loading through the relation

$$D_i = \frac{1}{\pi e q_{\infty}} \left(\frac{W}{b}\right)^2$$

where W is the weight of the airplane, b is the wing span, e is the Oswald efficiency factor, and  $q_{\infty}$  is the dynamic pressure.

- a) Derive this relation.
- b) Consider an aircraft with a wingspan of 37 ft and a gross weight of 10,000 lb. Assume that the Oswald efficiency factor is 0.8. The airplane is flying in steady, level flight at a velocity of 300 mi/h at a standard altitude of 5000 ft. Calculate the drag due to lift using the relation you derived.
- c) What happens to the drag due to lift if the wing span is reduced to 34 ft? Why do you think the wingspan has an effect on the drag due to lift?

SLUF (a)

thus
$$b_{3} = \frac{g_{0}s}{\pi e b^{2}} \left( \frac{s}{q_{0}^{2}s^{2}} \right) \left( \frac{w^{2}}{q_{0}^{2}s^{2}} \right)$$

$$= \left| \frac{1}{\pi e g_{0}} \left( \frac{w}{b} \right)^{2} \right|$$

b, Pli-5000 += 2.0482 x/6-3 V= 440 71/3 9.0. = (2.0412×10-3) (440) = 198,27 16/4.

\$:= \frac{1}{72(0.8)(198.27)}\left(\frac{10000}{37}\right)^2 \approx \frac{146.59}{46.59}\left(\frac{1}{6}\right)

if b=34 ft

Do = 173, 60 16 it 12 reases /

It has effect on the induced drag because the turbulence produced by air flowing on the ning is distributed on the length of the oving span.