AAE 334: Aerodynamics

HW 9: Nozzle Flow Analysis

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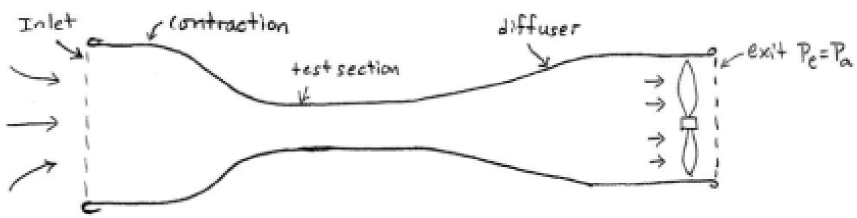
1. [20 pts] An indraft wind tunnel pulls air into an inlet, through a contraction, the test section, and the diffuser, as shown below. The test section has a circular cross section and it has a diameter of 0.5 m. Assume the wind tunnel is at sea level and the temperature and pressure of the atmosphere are given by standard atmospheric conditions.
   (a) If the test section is designed to have a velocity of 200 m/s, determine the Mach number, temperature and pressure in the test section. Is the pressure in the test section greater than or less than the surrounding atmospheric pressure?
   (b) Determine the mass flow rate through the wind tunnel.
   (c) The contraction between the inlet and the test section has an area ratio of 5. Determine the velocity, Mach number, temperature and pressure at the inlet of the wind tunnel.
   (d) The diffuser section downstream of the test section has area ratio of 3. At the exit of the diffuser is the fan that drives the wind tunnel. The air exiting the fan must be at atmospheric pressure, since the flow there is subsonic. Determine the pressure change across the fan.



Given Properties

Standard Sea-level conditions

$P_a = 101.3$ kPa $\qquad \gamma = 1.4$

$T_a = 288.15$ K $\qquad R = 287.05$ J/kg-K

$\rho = 1.225$ kg/m^3

(a) The airflow in the surrounding of the wind tunnel is stagnant far from the wind tunnel, and if we assume the wind tunnel to be isentropic

$$\begin{cases} T_0 = T_a = 288.15\text{K} \\ P_0 = P_a = 101.3\text{ kPa} \end{cases}$$

from the equation $\quad u = M\sqrt{\gamma R T}$

since

$$T = \frac{T_0}{1 + \frac{\gamma - 1}{2} M^2}$$

then, we can see the relation

$$u^2 = M^2 \frac{\gamma R T_0}{1 + \frac{\gamma-1}{2}M^2}$$

solve this for M

$$u^2 + \frac{\gamma-1}{2}u^2 M^2 = \gamma R T_0 M^2$$

$$\left(\gamma R T_0 - \frac{\gamma-1}{2}u^2\right)M^2 = u^2$$

$$M = \sqrt{\frac{u^2}{\gamma R T_0 - \frac{\gamma-1}{2}u^2}}$$

$$M = \sqrt{\frac{(200\ m/s)^2}{(1.4)(287.05\ J/kg{\cdot}K)(288.15K) - 0.2(200\ m/s)^2}}$$

$$\boxed{M_t = 0.6091}$$

then, using isentropic relations

$$P_t = P_0\left(1 + \frac{\gamma-1}{2}M_t^2\right)^{-\gamma/\gamma-1} \Longrightarrow \boxed{P_t = 78.849\ kPa}$$

$$T_t = T_0\left(1 + \frac{\gamma-1}{2}M_t^2\right)^{-1} \Longrightarrow \boxed{T_t = 268.24K}$$

The pressure is smaller than the atmospheric pressure.

(b) using P_t & T_t from (a)

$$\rho_t = \frac{P_t}{R T_t} = 1.0240\ kg/m^3$$

then, $(u_t = u = 200\,m/s)$

$$\dot{m} = \rho_t u_t A_t$$

since $A_t = \pi\left(\frac{D}{2}\right)^2 = 0.1963\ m^2$

$$\therefore \boxed{\dot{m} = 40.2131\ kg/s}$$

(c) The contraction ratio, $\frac{A_i}{A_t} = 5 \Rightarrow A_i = 5A_t$

since, the $\dot{m}$ is constant throughout the wind tunnel

using the following relation

$$\frac{A_i}{A_t} = \frac{M_t}{M_i}\left[\frac{1+(\gamma-1/2)M_i^2}{1+(\gamma-1/2)M_t^2}\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

we can find M_i. Deploy MATLAB to solve this equation.
(*Code in Appendix). We obtain (excluding imaginary values)

$$\boxed{M_i = 0.0989,\ 3.3478}$$

both is possible

$M_i = 0.0989$

$$\boxed{\begin{aligned} T_i &= T_0\left(1+\frac{\gamma-1}{2}M_i^2\right)^{-1} = 287.59\,K \\ P_i &= P_0\left(1+\frac{\gamma-1}{2}M_i^2\right)^{\frac{-\gamma}{\gamma-1}} = 100.61\ kPa \\ u_i &= M_i\sqrt{\gamma R T_i} = 33.61\ m/s \end{aligned}}$$

(d) expansion ratio $\frac{A_e}{A_t} = 3$

using same method as (c)

$$\frac{A_i}{A_t} = \frac{M_t}{M_i}\left[\frac{1+(\gamma-1/2)M_i^2}{1+(\gamma-1/2)M_t^2}\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

from this equation, MATLAB outputs

$$M_e = 0.1665,\ 2.8098$$

Since, the flow is subsonic $M_e = 0.1665$

then, right before the fan

$$p_e' = p_0\left(1 + \frac{\gamma-1}{2}M_e^2\right)^{-\gamma/\gamma-1}$$

$$p_e' = 99.357\,kPa$$

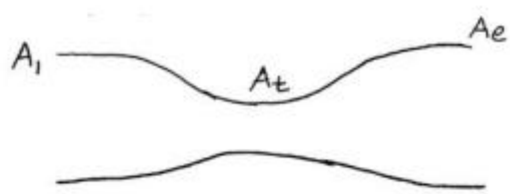
then the pressure difference across the fan

$$\Delta p = p_e - p_e'$$

$$\Delta p = 101.3\,kPa - 99.357\,kPa$$

$$\boxed{\Delta p = 1.9426\,kPa}$$

2. [15 pts] Consider a converging diverging nozzle with an exit-to-throat area ratio of Ae/At = 1.25 as shown below. The stagnation pressure upstream of the throat is 8.5 atm and the stagnation temperature is 1000 K.
   (a) Assume the air is expanded isentropically to supersonic speed at the exit. Determine the following properties at the nozzle exit: M_e, P_e, T_e, ρ_e, u_e, P_{0e}, T_{0e}.
   (b) If the area ratio in the subsonic part of the converging diverging nozzle, A_1/A_t is 1.24, determine the following properties at the upstream station: M_1, P_1, T_1, ρ_1, u_1, P_{01}, T_{01}.



Given Properties

$\gamma = 1.4$ $\qquad \frac{A_e}{A_t} = 1.25$

$R = 287.05\ J/kg\text{-}K$

$P_0 = 8.5\ atm = 861.26\ kPa$

$T_0 = 1000K$

(a) The expansion ratio, $\frac{A_e}{A_t} = 1.25 \Rightarrow A_i = 1.25 A_t$

from the Appendix A of the textbook

when $\frac{A_e}{A_t} = \frac{A_e}{A^*} = 1.25 \Rightarrow \frac{P_0}{P_e} = 4.250$

$\frac{T_0}{T_e} = 1.512$

$\boxed{M_e = 1.6}$

$$p_e = \frac{P_0}{4.250} = 202.65\ kPa$$

$$T_e = \frac{T_0}{1.512} = 661.38\ K$$

$$\rho_e = \frac{p_e}{RT_e} = 1.0674\ kg/m^3$$

$$u_e = M_e\sqrt{\gamma R T_e} = 824.88\ m/s$$

$$P_{0e} = P_0 = 861.26\ kPa$$

$$T_{0e} = T_0 = 1000\ K$$

(b) contraction ratio $\frac{A_i}{A_t} = 1.24$

since the converging nozzle should be a subsonic flow from Appendix A

$$\boxed{M_i = 0.56}$$

$$\frac{P_0}{P_i} = 1.237$$

$$\frac{T_0}{T_i} = 1.063$$

$$P_i = \frac{P_0}{1.237} = 696.25\ kPa$$

$$T_i = \frac{T_0}{1.063} = 940.73\ K$$

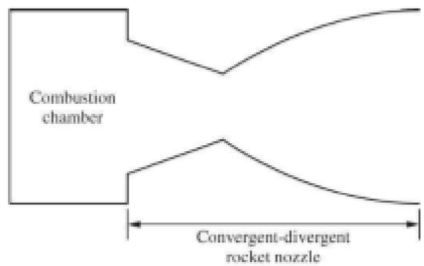
$$\rho_i = \frac{P_i}{RT_i} = 2.5784\ kg/m^3$$

$$u_i = M_i\sqrt{\gamma R T_i} = 344.32\ m/s$$

$$P_{0i} = P_0 = 861.26\ kPa$$

$$T_{0i} = T_0 = 1000\ K$$

3. [35 pts] Consider a rocket that runs on compressed air. The conditions in the reservoir (stagnation chamber) are $p_{01} = 700$ kPa and $T_{01} = 300$ K, and the flow is choked. If the area ratio for the nozzle is $A_e/A_t = 25$ and $A_e = 4.0$ cm^2, compute the exit Mach number, mass flow rate, and exit pressure p_e for the conditions of:
(a) Subsonic, isentropic flow
(b) Supersonic, isentropic flow
(c) A shock at the nozzle exit
Finally, (d) if the exit pressure is $p_e = 60$ kPa, find the exit Mach number M_e and the thrust T.



Given Properties

$\gamma = 1.4$

$R = 287.05$ J/kg-K

$P_0 = 700$ kPa

$T_0 = 300$ K

$\frac{A_e}{A_t} = 25$

$A_e = 4.0 \times 10^{-4}$ m^2

(a) Using the Appendix B of the textbook for the expansion ratio at subsonic condition we get the following properties from interpolation

$$M_e = \frac{0.02 - 0.04}{28.94 - 14.48}(25 - 14.48) + 0.04$$

$$M_e = 0.0254$$

then

$$P_e = P_0\left(1+\frac{\gamma-1}{2}M_e^2\right)^{-\gamma/\gamma-1} = 699.68 \text{ kPa}$$

next

$$\dot{m} = P_0 A\sqrt{\frac{\gamma}{RT_0}}\, M_e\left(1+\frac{\gamma-1}{2}M_e^2\right)^{-(\gamma+1)/2(\gamma-1)}$$

$$\dot{m} = 0.0287 \text{ kg/s}$$

$$\boxed{\begin{aligned} M_e &= 0.0254 \\ P_e &= 699.68 \text{ kPa} \\ \dot{m} &= 0.0287 \text{ kg/s} \end{aligned}}$$

(b) Using the Appendix B of the textbook at supersonic isentropic condition

$$M_e = 5$$

then

$$P_e = P_0\left(1+\frac{\gamma-1}{2}M_e^2\right)^{-\gamma/\gamma-1} = 1.3230 \text{ kPa}$$

next

$$\dot{m} = P_0 A\sqrt{\frac{\gamma}{RT_0}}\, M_e\left(1+\frac{\gamma-1}{2}M_e^2\right)^{-(\gamma+1)/2(\gamma-1)}$$

$\dot{m} = 0.0261 \ kg/s$

$\downarrow$

$T_e = \left(1 + \frac{\gamma - 1}{2} M_e^2\right)$

$T_e = 50 K$

$\downarrow$

$u_e = M_e \sqrt{\gamma R T_e}$

$u_e = 708.76 \ m/s$

$\rho_e = \frac{\dot{m}}{u_e A_e} = 0.0922 \ kg/m^3$

$$\boxed{\begin{aligned} M_e &= 5 \\ p_e &= 1.3230 \ kPa \\ \dot{m} &= 0.0261 \ kg/s \end{aligned}}$$

(c) Shock at nozzle exit

to have a shock wave at the nozzle exit the flow right before the exit is the condition (b) and at the exit shock jump relations are applied.

using Appendix B from textbook

if $M_1 = 5$

$M_2 = 0.4152$

$\therefore M_e = M_2 = 0.4152$

$\frac{P_2}{P_1} = 29.00$

$\therefore p_e = (1.320 \ kPa)(29.00)$

$$P_e = 38.28 \text{ kPa}$$

$$\frac{\rho_2}{\rho_1} = 5.000$$

and since in (b) $\rho_e = 0.0922 \text{ kg/m}^3$

$$\rho_2 = (5.000)(0.0922 \text{ kg/m}^3)$$

$$\rho_e = \rho_2 = 0.461 \text{ kg/m}^3$$

then

$$\frac{T_2}{T_1} = 5.800$$

since $T_1 = 50K$ from (b)

$$T_e = T_2 = (50K)(5.800) = 290K$$

then

$$u_e = M_e \sqrt{\gamma R T_e}$$

$$u_e = 141.74 \text{ m/s}$$

finally

$$\dot{m} = \rho_e u_e A_e$$

$$\dot{m} = (0.461 \text{ kg/m}^3)(141.74 \text{ m/s})(4.0 \times 10^{-4} \text{ m}^2)$$

$$\dot{m} = 0.0261 \text{ kg/s}$$

$$\boxed{\begin{aligned} M_e &= 0.4152 \\ P_e &= 38.28 \text{ kPa} \\ \dot{m} &= 0.0261 \text{ kg/s} \end{aligned}}$$

(d) if $p_e = 60\,kPa$

$$\frac{p_0}{p_e} = \frac{700\,kPa}{60\,kPa} = 11.67$$

from (a)

$$\frac{p_0}{p_b} \approx 1.000$$

from (b) when $\frac{A_e}{A_t} = \frac{A_e}{A^*} = 25$

$$\frac{p_0}{p_e} = 529.1 \Longrightarrow \frac{p_0}{p_b} = 529.1$$

from (c)

$$\frac{p_0}{p_e} = \frac{700\,kPa}{38.28\,kPa} = 18.29 \Longrightarrow \frac{p_0}{p_b} = 18.29$$

now since

$$1.000 < 11.67 < 18.29$$

the flow is to be a "shock in the nozzle" type flow

$$M_e = \sqrt{\frac{1}{\gamma-1}\left\{-1+\left[1+2(\gamma-1)\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\left(\frac{p_0}{p_e}\frac{A_t}{A_e}\right)^2\right]^{1/2}\right\}}$$

since $\frac{p_{01}}{p_e} = 11.67$ & $\frac{A_t}{A_e} = 0.04$

$$\boxed{M_e = 0.2681}$$

then

$$\frac{p_{02}}{p_{01}} = \frac{p_e}{p_{01}} \cdot \frac{p_{02}}{p_e} = \frac{p_e}{p_{01}}\left(1+\frac{\gamma-1}{2}M_e^2\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{p_{02}}{p_{01}} = 0.0901$$

$$\frac{T_{02}}{T_{01}} = \left(\frac{P_{02}}{P_{01}}\right)^{\gamma-1/\gamma} = 0.5028$$

$$T_{02} = (0.5028)(300\,K)$$

$$T_{02} = 150.83\,K$$

from Appendix B at this $\frac{P_{02}}{P_{01}}$

~~from~~ interpolation

extra

$$M_1 = \frac{4.55 - 4.5}{0.08806 - 0.09170}(0.0901 - 0.09170) + 4.5$$

$$M_1 = 4.5219$$

and

$$M_2 = \frac{0.4226 - 0.4236}{0.08806 - 0.09170}(0.0901 - 0.09170) + 0.4236$$

$$M_2 = 0.4232$$

$\longrightarrow$ $P_{02} = (0.0901) P_{01} = 63.074\,kPa$

from appendix A at $M_e = 0.2681$

$$\frac{T_{02}}{T_e} = \frac{1.016 - 1.014}{0.2800 - 0.2600}(0.2681 - 0.2600) + 1.014$$

$$\frac{T_{02}}{T_e} = 1.0148$$

$$\therefore T_e = \frac{T_{02}}{1.0148} = \frac{150.83\,K}{1.0148}$$

$$T_e = 148.63\,K$$

$$u_e = M_e \sqrt{\gamma R T_e}$$

$$u_e = 65.5318 \text{ m/s}$$

$$\rho_e = \frac{p_e}{RT_e} = 1.4064$$

$$\Rightarrow \dot{m} = \rho_e u_e A_e$$

$$\dot{m} = 0.0369 \text{ kg/s}$$

since this is a subsonic jet

$$p_e = p_a$$

thus, thrust F_t will be

$$F_t = \dot{m} u_e + \cancel{(p_e - p_a)A_e}^{\,0}$$

$$F_t = (0.0369 \text{ kg/s})(65.5318 \text{ m/s})$$

$$\boxed{F_t = 2.4158 \text{ N}}$$

4. [30 pts] A rocket engine sketched above is being tested on the ground. Liquid hydrogen and oxygen are burned in the combustion chamber producing a combustion gas pressure $p_0 = 30$ atm and temperature $T_0 = 3500$ K, respectively. The area of the rocket nozzle throat is $A_t = 0.4\ \mathrm{m}^2$. The area of the exit is designed so that the exit pressure exactly matches the ambient pressure at a standard altitude of 20 km, $p_a = 5.5293 \times 10^3$ Pa, and this ambient pressure is maintained in the ground test facility. Assume an isentropic flow through the rocket engine nozzle with an effective value of the ratio of specific heats $\gamma = 1.22$ and a constant value of the specific gas constant $R = 520\ \mathrm{J/(kg{\cdot}K)}$.
   (a) Calculate the thrust of the rocket engine as measured in the test facility.
   (b) Calculate the area of the nozzle exit.
   (c) To test the rocket engine performance at low altitude, the ambient pressure is increased. What is the minimum ambient pressure above which a normal shock would appear in the nozzle?

Given Properties

$P_0 = 30$ atm $= 3.0398$ MPa

$T_0 = 3500$ K

standard altitude 20 km

$P_a = 5.5293$ kPa

$\gamma = 1.22$

$R = 520$ J/kg-K

$A_t = 0.4\ \mathrm{m}^2$

$P_e = P_a$ perfectly expanded

(a)

At the exit the exit pressure equates the ambient pressure. The stagnation and static pressure ratio at the exit becomes

$$\frac{P_0}{P_e} = \frac{3.0398\ \mathrm{MPa}}{5.5293\ \mathrm{kPa}} = 549.7622$$

since this is a perfectly expanded condition there is no shockwave inside the nozzle and the diverging nozzle accelerates to supersonic conditions and choked.

$$\frac{p_0}{p_e} = \left(1 + \frac{\gamma-1}{2} M_e^2\right)^{\gamma/\gamma-1}$$

$$M_e = \sqrt{\frac{2}{\gamma-1}\left[\left(\frac{p_0}{p_e}\right)^{\gamma-1/\gamma} - 1\right]}$$

$$M_e = 4.3899$$

then we can find the corresponding temperatures

$$T_e = T_0 \left(1 + \frac{\gamma-1}{2} M_e^2\right)^{-1}$$

$$T_e = 1121.9 K$$

the velocity

$$u_e = M_e \sqrt{\gamma R T_e}$$

$$u_e = 3.7034 \text{ km/s}$$

since the flow is choked and perfectly expanded the expansion ratio can be found from the appendix A corresponding to M_e

$$M_1 = 4.350 \quad \& \quad M_2 = 4.400$$

$$\frac{A}{A_1^*} = 14.57 \quad \& \quad \frac{A}{A_2^*} = 15.21$$

from interpolation

$$\frac{A_e}{A_t} = \frac{15.21 - 14.57}{4.400 - 4.350}(4.3899 - 4.350) + 14.57$$

$$\frac{A_e}{A_t} = 15.0806$$

thus,

$$A_e = (0.4 m^2)(15.0806) = 6.0322 \text{ m}^2$$

the mass flow rate becomes

$$\dot{m} = P_0 A_e \sqrt{\frac{\gamma}{R T_0}} M_e \left(1 + \frac{\gamma-1}{2} M_e^2\right)^{-(\gamma+1)/2(\gamma-1)}$$

$$\dot{m} = 211.7436 \ kg/s$$

Hence, the thrust becomes

$$F_t = \dot{m} u_e + \cancel{(p_e - p_a)} A_e \rightarrow 0$$

$$F_t = (211.7436 \ kg/s)(3.7034 \ km/s)$$

$$\boxed{F_t = 784.18 \ kN}$$

(b)

Refer back to the calculations in (a)

$$\boxed{A_e = 6.0322 \ m^2}$$

(c)

The minimum ambient increased, P_a' is when there is a shockwave formed at the nozzle exit. Thus, the conditions or calculated properties hold true right close to the nozzle exit; however, at exit we must consider shock jump relations. From shock jump relations

$$M_{e2} = \frac{1 + \frac{\gamma-1}{2} M_e^2}{\gamma M_e^2 - \frac{\gamma-1}{2}} = 0.3651$$

$$\frac{P_{e2}}{P_e} = 1 + \frac{2\gamma}{\gamma+1}(M_e^2 - 1)$$

$$\frac{p_{e2}}{p_e} = 21.0818$$

$$\therefore \quad p_{e2} = (5.5293\ \text{kPa})(21.0818)$$

$$p_{e2} = 116.57\ \text{kPa}$$

for a shockwave to be generated at the nozzle exit

$$\frac{P_0}{P_b} = \frac{P_0}{P_{e2}} = \frac{3.0398\ \text{MPa}}{116.57\ \text{kPa}} = 26.0776$$

must be sufficed thus,

$$P_b = P_{e2} = 116.57\ \text{kPa}$$

$$\boxed{P_a' = 116.57\ \text{kPa}}$$

Appendix

AAE 334 HW9

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clear all; close all; clc;
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1-a

```
% Given properties
Pa = 101.3e3;
Ta = 288.15;
rho_a = 1.225;
gamma = 1.4;
R = 287.05;
D = 0.5;
A_t = pi*(D/2)^2;

% Mach number
P0 = Pa; T0 = Ta;
u_t = 200;
den = u_t^2;
num = gamma*R*T0 - (gamma - 1)/2*u_t^2;
M_t = sqrt(den/num);
% Pressure
P_t = p_from_M_and_gamma(P0,M_t,gamma,"static")
% Temperature
T_t = T_from_M_and_gamma(T0,M_t,gamma,"static")
```

1-b

```
% Mass flow rate
rho_t = P_t/T_t/R;
m_dot = rho_t*u_t*A_t;
```

1-c

```
A_i = 5*A_t;

% Calculate the Mach number at the inlet
syms M_i
assume(M_i,["real","positive"])
a1 = 1 + (gamma - 1)/2*M_i^2;
a2 = 1 + (gamma - 1)/2*M_t^2;
a3 = (gamma + 1)/2/(gamma - 1);
eqn = A_i/A_t == M_t/M_i * (a1/a2)^(a3);
M_i = double(vpasolve(eqn,M_i));
M_i = M_i(M_i == real(M_i));
M_i_sub = min(M_i);
M_i_sup = max(M_i);

% If inlet subsonic
T_i_sub = T_from_M_and_gamma(T0,M_i_sub,gamma,"static");
P_i_sub = p_from_M_and_gamma(P0,M_i_sub,gamma,"static");
```

```
u_i_sub = M_i_sub*sqrt(gamma*R*T_i_sub);

% If inlet is supersonic
T_i_sup = T_from_M_and_gamma(T0,M_i_sup,gamma,"static");
P_i_sup = p_from_M_and_gamma(P0,M_i_sup,gamma,"static");
u_i_sup = M_i_sup*sqrt(gamma*R*T_i_sup);
```

1-d

```
A_e = 3*A_t;

% Calculate the Mach number at the inlet
syms M_e
assume(M_e,["real","positive"])
a1 = 1 + (gamma - 1)/2*M_e^2;
a2 = 1 + (gamma - 1)/2*M_t^2;
a3 = (gamma + 1)/2/(gamma - 1);
eqn = A_e/A_t == M_t/M_e * (a1/a2)^(a3);
M_e = double(vpasolve(eqn,M_e))
M_e = M_e(M_e == real(M_e))
M_e_sub = min(M_e)
M_e_sup = max(M_e)

Pe_b = p_from_M_and_gamma(P0,M_e_sub,gamma,"static")
delta_P = Pa - Pe_b
```

3-a

```
% Given properties
gamma = 1.4;
R = 287.05;
P0 = 700e3;
T0 = 300;
epsilon = 25;
Ae = 4e-4;
At = Ae/25;

Me = two_point_interpolate(25,14.48,28.94,0.04,0.02)
Pe = p_from_M_and_gamma(P0,Me,gamma,"static")
m_dot = mDot_from_M(Me,P0,T0,Ae,gamma,R)
```

3-b

```
Me = 5;
Pe = p_from_M_and_gamma(P0,Me,gamma,"static")
m_dot = mDot_from_M(Me,P0,T0,Ae,gamma,R)

Te = T_from_M_and_gamma(T0,Me,gamma,"static")
ue = Me*sqrt(gamma*R*Te)
rho_e = m_dot/ue/Ae
```

3-d

```
Pe = 60e3;
Me = shockInNozzle_M(P0,Pe,At,Ae,gamma)
```

```
P02_P01 = Pe/P0*(1 + (gamma - 1)/2*Me^2)^(gamma/(gamma - 1))
M1 = two_point_interpolate(P02_P01,0.0917,0.08806,4.5,4.55)
M2 = two_point_interpolate(P02_P01,0.0917,0.08806,0.4236,0.4226)
P02 = P02_P01*P0
T02_T01 = (P02_P01)^((gamma - 1)/gamma)
T02 = T02_T01*T0

T02_Te = two_point_interpolate(Me,0.26,0.28,1.014,1.016)
Te = T02/T02_Te
ue = Me*sqrt(gamma*R*Te)
rho_e = Pe/R/Te
m_dot = rho_e*ue*Ae
Ft = m_dot*ue
```

4-a

```
% Given properties
P0 = 3.0398e6;
T0 = 3500;
Pa = 5.5293e3;
gamma = 1.22;
R = 520;
At = 0.4;
Pe = Pa;
```

```
P0_Pe = P0/Pe
Me = M_from_P_ratio(P0,Pe,gamma)
Te = T_from_M_and_gamma(T0,Me,gamma,"static")
ue = Me*sqrt(gamma*R*Te)
Ae_At = two_point_interpolate(Me,4.35,4.4,14.57,15.21)
Ae = Ae_At*At
m_dot = mDot_from_M(Me,P0,T0,Ae,gamma,R)
Ft = m_dot*ue + (Pe - Pa)*Ae
```

4-c

```
% Shock jump relations
Me2 = shock_jump_M(Me,gamma)
Pe2_Pe = 1 + 2*gamma/(gamma + 1)*(Me^2 - 1)
Pe2 = Pe2_Pe*Pe
P0_Pb = P0/Pe2
```

Function

```
function m_dot = mDot_from_M(M,P0,T0,A,gamma,R)
    a1 = P0*A;
    a2 = sqrt(gamma/R/T0);
    a3 = (1 + (gamma - 1)/2*M^2);
    a4 = -(gamma + 1)/2/(gamma - 1);
    m_dot = a1*a2*M*a3^(a4);
end

function M = shockInNozzle_M(P0,Pe,At,Ae,gamma)
    a1 = (gamma - 1);
    a2 = (gamma + 1);
```

```
    a3 = 2*a1*(2/a2)^(a2/a1);
    a4 = (P0/Pe * At/Ae)^2;
    a5 = 1 + a3*a4;
    M = sqrt(1/a1 * (-1 + (a5)^(0.5)));
end

function M = M_from_P_ratio(P0,P,gamma)
    a1 = 2/(gamma - 1);
    a2 = (P0/P)^((gamma - 1)/gamma);
    M = sqrt(a1*(a2 - 1));
end

function M2 = shock_jump_M(M1,gamma)
    a1 = (gamma - 1)/2;
    M2 = sqrt((1 + a1*M1^2)/(gamma*M1^2 - a1));
end
```

```
function  T2 = T_from_M_and_gamma(T1, M, gamma, type)
    if type == "stagnation"
        T2 = T1 * (1 + (gamma - 1) / 2 * M^2);
    elseif type == "static"
        T2 = T1 / (1 + (gamma - 1) / 2 * M^2);
    else
        disp("Error. Incorrect type. Type can only be 'stagnation' or 'static'.")
    end
end
```

```
function  p2 = p_from_M_and_gamma(p1, M, gamma, type)
    if type == "stagnation"
        p2 = p1 * (1 + (gamma - 1) / 2 * M^2)^(gamma/(gamma - 1));
    elseif type == "static"
        p2 = p1 / (1 + (gamma - 1) / 2 * M^2)^(gamma/(gamma - 1));
    else
        disp("Error. Incorrect type. Type can only be 'stagnation' or 'static'.")
    end
end
```