HW1

Sunday, January 19, 2020 12:32 AM

AAE 440 – Spacecraft Attitude Dynamics
Problem Set 1
Due: 1/24/20

Problem 1: (a) The scalar triple product $\left(\bar{a}\times\bar{b}\right)\bullet\bar{c}$

scalar and vector product operations or by a change in the order of $\bar{a},\bar{b},\bar{c}$
are in cyclic order, i.e.,

$$\left(\bar{a}\times\bar{b}\right)\bullet\,\bar{c}=\bar{a}\bullet\left(\bar{b}\times\bar{c}\right)=\bar{b}\bullet\left(\bar{c}\times\bar{a}\right)=\left(\bar{c}\times\bar{a}\right)\bullet\,\bar{b}$$

Express these in subscript format to demonstrate that this statement is true.

(b) Use the permutation symbol with subscript format and prove that the following identity is also true

$$\left(\bar{a}\times\bar{b}\right)\times\left(\bar{c}\times\bar{d}\right)=\left[\left(\bar{a}\times\bar{b}\right)\bullet\bar{d}\right]\bar{c}-\left[\left(\bar{c}\times\bar{a}\right)\bullet\bar{b}\right]\bar{d}$$

Problem 2: Assume that vectors $\bar{q},\bar{r},\bar{s}$
orthonormal triad $\hat{n}_1,\hat{n}_2,\hat{n}_3$

$$\bar{q}=q_i\hat{n}_i$$
$$\bar{r}=r_j\hat{n}_j$$
$$\bar{s}=s_\alpha\hat{n}_\alpha$$

(a) Evaluate the following quantities and express each in summation (subscript) format, in terms of the vector basis $\hat{n}_i$.

$$\begin{aligned} \textit{Example}:\quad \bar{v}&=\bar{q}\bullet\bar{r}\;\bar{q}\times\bar{s}\\ &=q_i\,\hat{n}_i\bullet r_j\,\hat{n}_j\;q_\ell\,\hat{n}_\ell\times s_m\,\hat{n}_m\\ &=q_i\,r_j\,\delta_{ij}\,q_\ell\,s_m\,\varepsilon_{\ell mp}\,\hat{n}_p\\ \bar{v}&=q_i\,r_i\;q_\ell\,s_m\,\varepsilon_{\ell mp}\,\hat{n}_p \end{aligned}$$

(i) $\bar{\bar{G}}=\bar{r}\,\bar{s}+\left(\bar{r}+\bar{s}\right)\bar{q}$

(ii) $\bar{a}=\bar{r}\bullet\bar{\bar{G}}+\bar{q}\times\bar{s}$

(iii) $\bar{\bar{H}}=\bar{\bar{G}}-\bar{r}\,\bar{a}\bullet\bar{s}\,\bar{q}$

(iv) $\bar{\bar{R}}=\bar{\bar{G}}\times\bar{s}-\bar{\bar{H}}$

(v) $c=\bar{q}\bullet\bar{\bar{R}}\bullet\bar{s}+\bar{q}\bullet\bar{r}\times\bar{\bar{G}}\bullet\bar{s}$

(b) Let $\bar{q}, \bar{r}, \bar{s}$

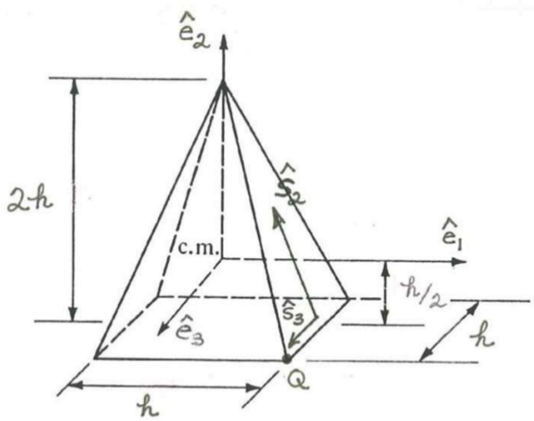
$\bar{q} = \hat{n}_2 - 2\hat{n}_3$

$\bar{r} = 3\hat{n}_1 + 0.5\hat{n}_2 - \hat{n}_3$

$\bar{s} = 4\hat{n}_1 - 2\hat{n}_2$

Evaluate each of the quantities in (a) using the subscript definitions.

Problem 3: The inertia matrix can be written in dyadic form which is particularly useful when inertia information is required in various vector bases. Below is a right rectangular pyramid of total mass *m*.



Right rectangular pyramid

(a) Determine the inertia dyadic for the pyramid P, relative to point Q, i.e. $\bar{\bar{I}}^{P/Q}$, for unit vectors $\hat{e}_1, \hat{e}_2, \hat{e}_3$

(b) Use the similarity transformation to transform direction cosine matrix $\bar{\bar{I}}^{P/Q}$ to vector basis $\hat{s}$; write the corresponding dyadic.

(c) An inertia element in vector basis. relative to the <u>same point</u>, can also be determined through the relation

$$I_{ij} = \hat{s}_i \bullet \bar{\bar{I}} \bullet \hat{s}_j$$

Demonstrate that this expression produces the same result as the similarity transformation.