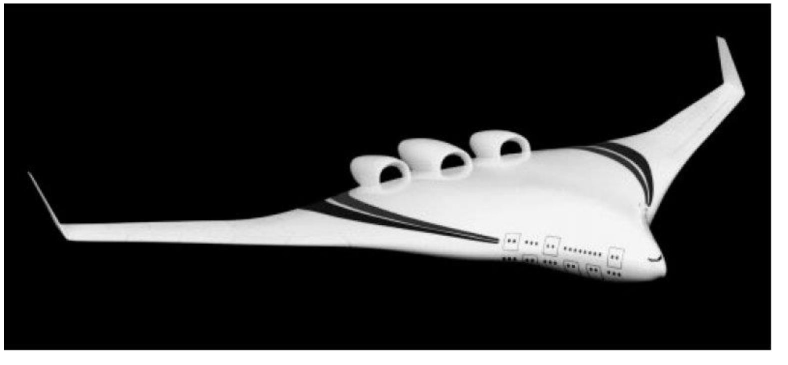
Review for Midterm 1

AAE 33400 Aerodynamics

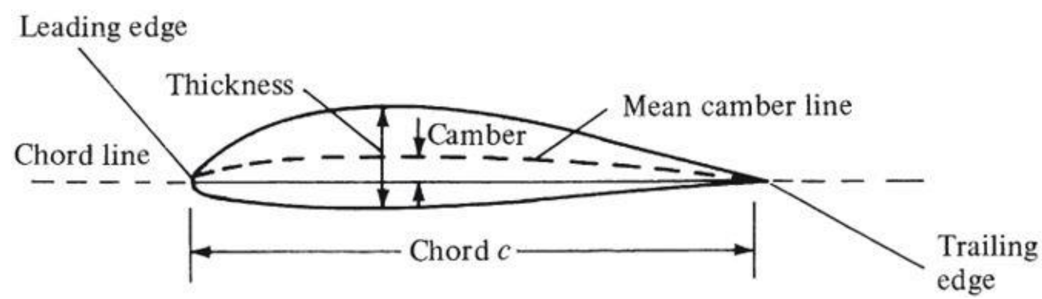


Midterm Exam 1

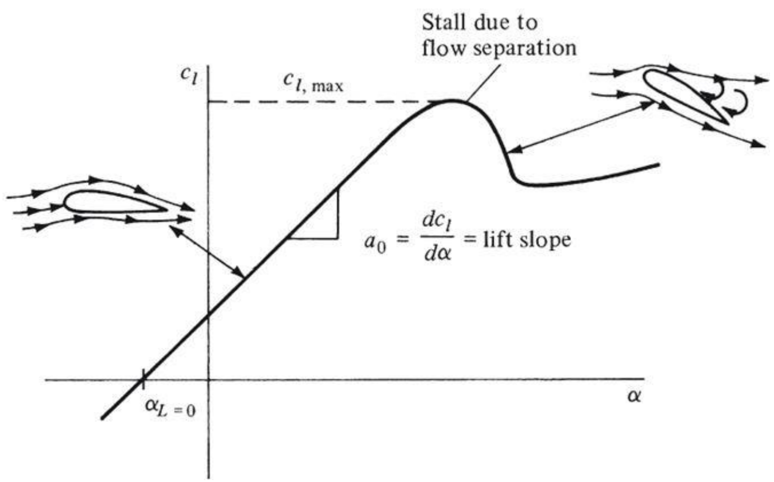
- Closed book and notes; simple calculator (TI-30Xa or TI-30XIIS – see syllabus)
- All necessary formulas given
- Emphasis is on 2D airfoils (Lectures 1-11)
- 15 short questions (multiple choice, etc., no explanation needed): 30 points
- 3 short qualitative questions: 20 points
- 2 quantitative problems: 50 points
- Be able to sketch and understand curves such as drag polar, lift curve etc.
- Know origin and consequences of flow separation and laminar-turbulent transition
- “Know this figure”
- Study solutions for HW1-HW4 (especially HW4 in detail)
- Practice cambered thin airfoil theory calculations

Definitions

- Mean camber line
- Leading edge
- Trailing edge
- Chord line, chord
- Camber
- Thickness

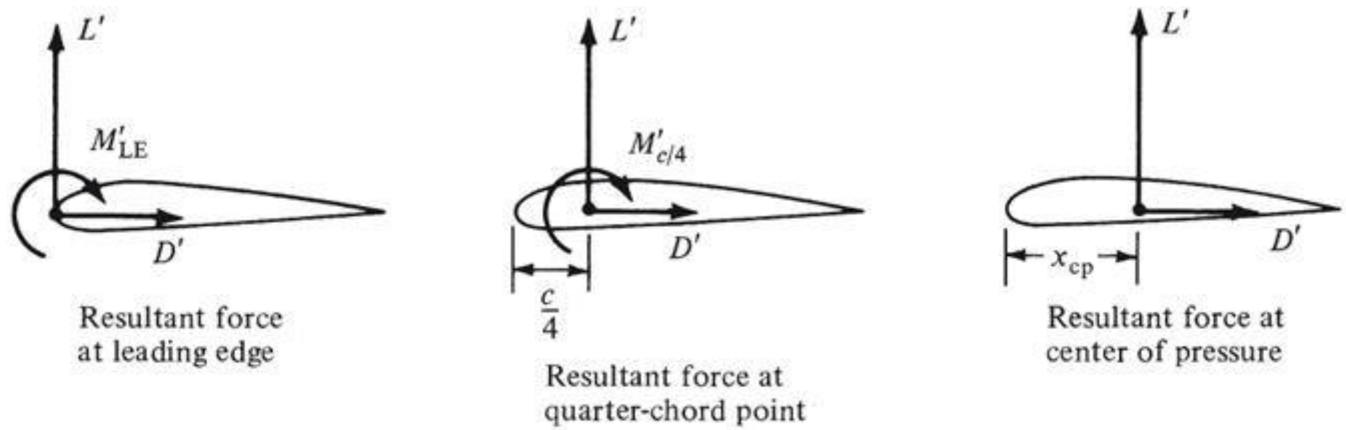


Definitions



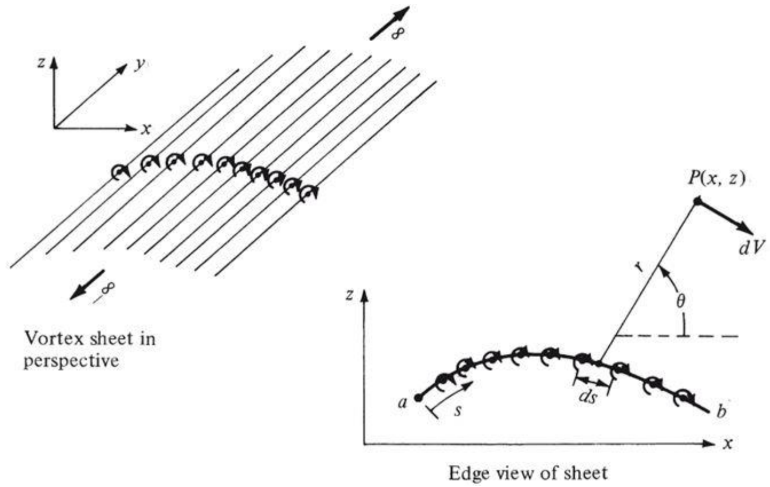
- Lift coefficient
- Angle-of-attack
- Lift slope
- Stall
- Zero-lift angle of attack

Definitions



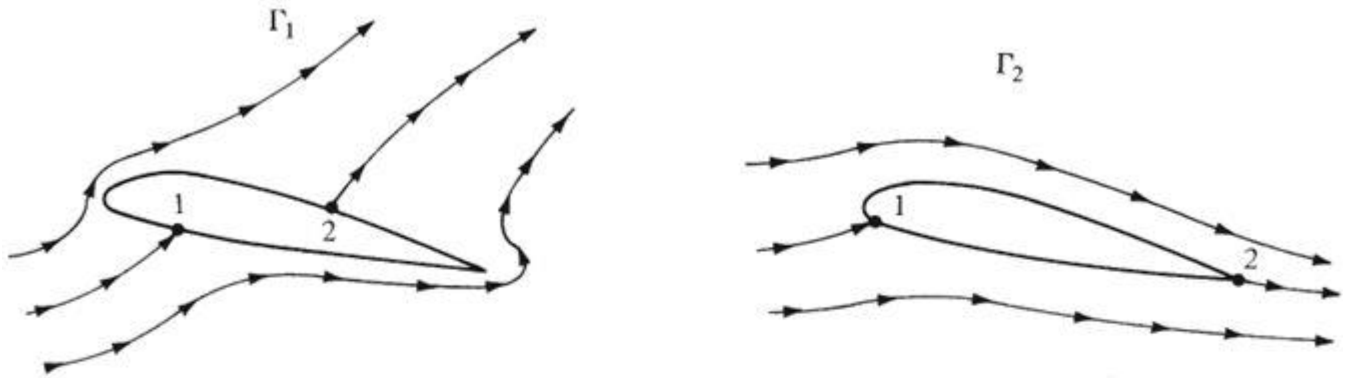
- Center of pressure
- Aerodynamic center
- Know how to shift moment from leading edge to another point

Vortex Sheets



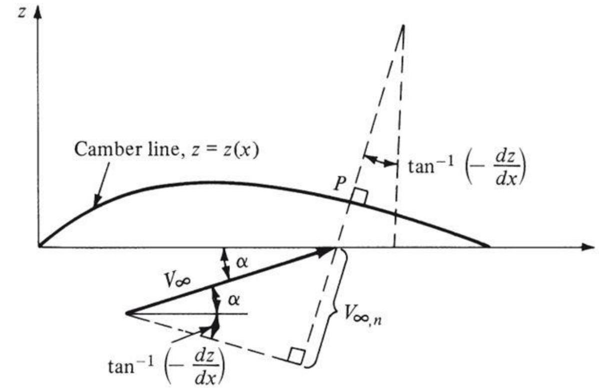
- Distribute vortices over curve
- Strength of sheet per unit length: γ(s)
- Circulation: $\Gamma = \int_a^b \gamma(s)ds$
- Lift per unit span: $L' = \rho_\infty V_\infty \Gamma$

Kutta Condition



- Multiple solutions possible
- Additional condition to get one solution
- Kutta condition: flow leaves the trailing edge smoothly
- Finite trailing edge: $V_1 = V_2 = 0$, $\gamma(TE) = 0$

Thin Airfoil Theory



- Freestream velocity normal to camber line:

$$V_{\infty,n} = V_\infty(\alpha - \frac{dz}{dx})$$

- The corresponding velocity induced by the vortex sheet is:

$$w(x) = -\frac{1}{2\pi}\int_0^c \frac{\gamma(\xi)}{x-\xi}d\xi$$

- Fundamental equation of thin airfoil theory:

$$\frac{1}{2\pi}\int_0^c \frac{\gamma(\xi)}{x-\xi}d\xi = V_\infty(\alpha - \frac{dz}{dx})$$

- This makes the camber line a streamline

Symmetric Airfoils

- Lift coefficient:

$$c_l = 2\pi\alpha$$

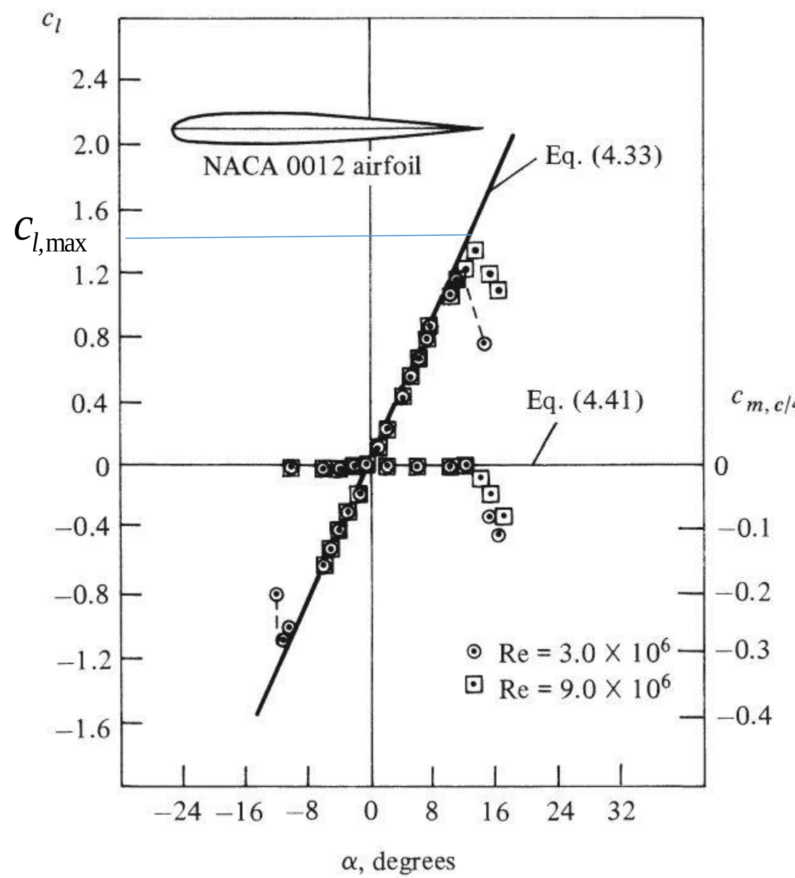
- Moment coefficient:

$$c_{m,le} = -\frac{c_l}{4}$$

- Center of pressure and aerodynamic center:

$$x_{ac}/c = 1/4$$

$$c_{mac} = c_{m,c/4} = 0$$



Cambered Airfoils

- Camber line: z(x), $dz/dx \neq 0$
- Solution:

$$\gamma(\theta) = 2V_\infty \left[A_0 \frac{1+\cos\theta}{\sin\theta} + \sum_{n=1}^{\infty} A_n \sin n\theta\right]$$

- Coefficients:

$$A_0 = \alpha - \frac{1}{\pi}\int_0^\pi \frac{dz}{dx} d\theta_0$$

- Coefficients:

$$A_n = \frac{2}{\pi}\int_0^\pi \frac{dz}{dx} \cos n\theta_0 \, d\theta_0$$

- Remember to find dz/dx, then convert x to θ_0

$$x = \frac{c}{2}(1 - \cos\theta_0)$$

Cambered Airfoils

- Lift coefficient:

$$c_l = 2\pi\left(A_0 + \frac{A_1}{2}\right)$$
$$= 2\pi(\alpha - \alpha_{L=0})$$

- Moment coefficient:

$$c_{m,le} = -\frac{c_l}{4} + \frac{\pi}{4}(A_2 - A_1)$$

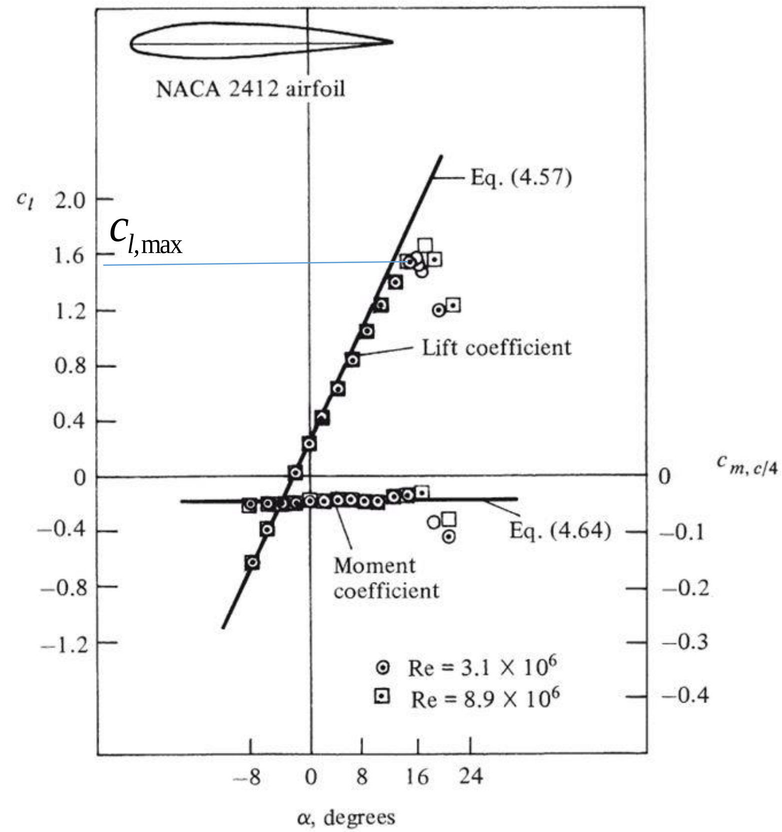
- Center of pressure

$$\frac{x_{cp}}{c} = -\frac{c_{m,le}}{c_l} = \frac{1}{4}\left[1 + \frac{\pi}{c_l}(A_1 - A_2)\right]$$

- Aerodynamic center

$x_{ac}/c = 1/4$

$$c_{mac} = c_{m,c/4} = \frac{\pi}{4}(A_2 - A_1)$$



Example (from lecture) of application of thin airfoil theory for cambered airfoil

Consider an NACA 23012 airfoil. The mean camber line for this airfoil is given by

$$\frac{z}{c} = 2.6595\left[\left(\frac{x}{c}\right)^3 - 0.6075\left(\frac{x}{c}\right)^2 + 0.1147\left(\frac{x}{c}\right)\right] \qquad \text{for } 0 \le \frac{x}{c} \le 0.2025$$

$$\frac{z}{c} = 0.02208\left(1 - \frac{x}{c}\right) \qquad \text{for } 0.2025 \le \frac{x}{c} \le 1.0$$

Calculate: (a) the zero-lift angle of attack, (b) the lift coefficient when α=4º, (c)$c_{m,c/4}$, (d) x_{cp}/c at α=4º.

Solution:

First, we calculate *dz/dx*

$$\frac{dz}{dx} = \frac{d(z/c)}{d(x/c)} = 2.6595\left[3\left(\frac{x}{c}\right)^2 - 1.215\left(\frac{x}{c}\right) + 0.1147\right] \qquad \text{for } 0 \le \frac{x}{c} \le 0.2025$$

$$\frac{dz}{dx} = -0.02208 \qquad \text{for } 0.2025 \le \frac{x}{c} \le 1.0$$

Now we transform *x* to θ: $\quad x = (c/2)(1 - \cos\theta)$

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$$\frac{dz}{dx} = 0.6840 - 2.3736\cos\theta + 1.995\cos^2\theta \qquad \text{for } 0 \le \theta \le 0.9335\,\text{rad}$$

$$\frac{dz}{dx} = -0.02208 \qquad \text{for } 0.9335 \le \theta \le \pi$$

Now we calculate (a) the zero-lift angle of attack:

$$\alpha_{L=0} = -\frac{1}{\pi}\int_0^{\pi}\frac{dz}{dx}(\cos\theta - 1)d\theta = -\frac{1}{\pi}\int_0^{0.9335}\left(-0.6840 + 3.0576\cos\theta - 4.3686\cos^2\theta + 1.995\cos^3\theta\right)d\theta$$

$$-\frac{1}{\pi}\int_{0.9335}^{\pi}\left(0.02208 - 0.02208\cos\theta\right)d\theta$$

$$\alpha_{L=0} = -0.0191\,\text{rad} = -1.09^{\circ}$$

(b) $\alpha = 4^{\circ} = 0.0698\,\text{rad}$

Therefore,

$$c_l = 2\pi\left(\alpha - \alpha_{L=0}\right) = 0.559$$

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$$\frac{dz}{dx} = 0.6840 - 2.3736\cos\theta + 1.995\cos^2\theta \qquad \text{for } 0 \le \theta \le 0.9335\,\text{rad}$$

$$\frac{dz}{dx} = -0.02208 \qquad \text{for } 0.9335 \le \theta \le \pi$$

(c) Now we calculate $c_{m,c/4}$ for which we need the two Fourier coefficients A_1 and A_2

$$A_1 = \frac{2}{\pi}\int_0^{\pi} \frac{dz}{dx}\cos\theta\, d\theta = 0.0954$$

$$A_2 = \frac{2}{\pi}\int_0^{\pi} \frac{dz}{dx}\cos 2\theta\, d\theta = 0.0792$$

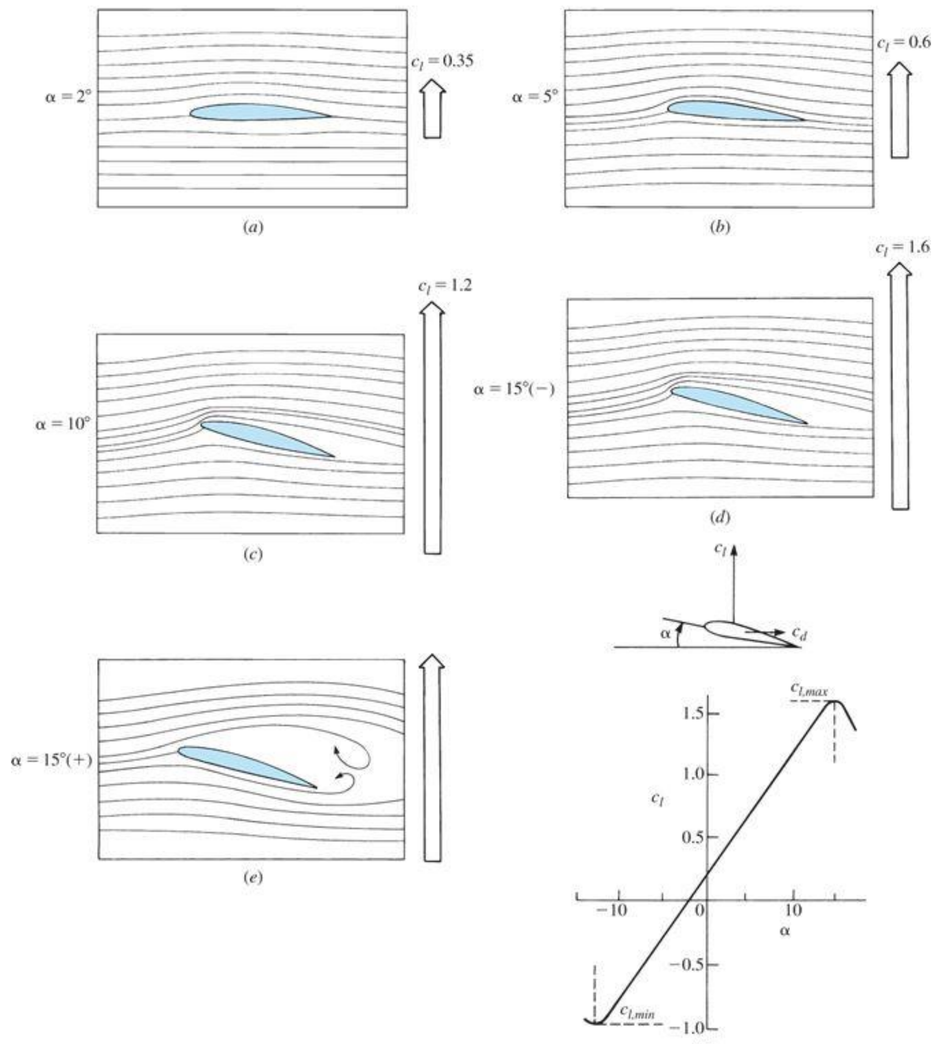
Therefore,

$$c_{m,c/4} = \frac{\pi}{4}\left(A_2 - A_1\right) = -0.0127$$

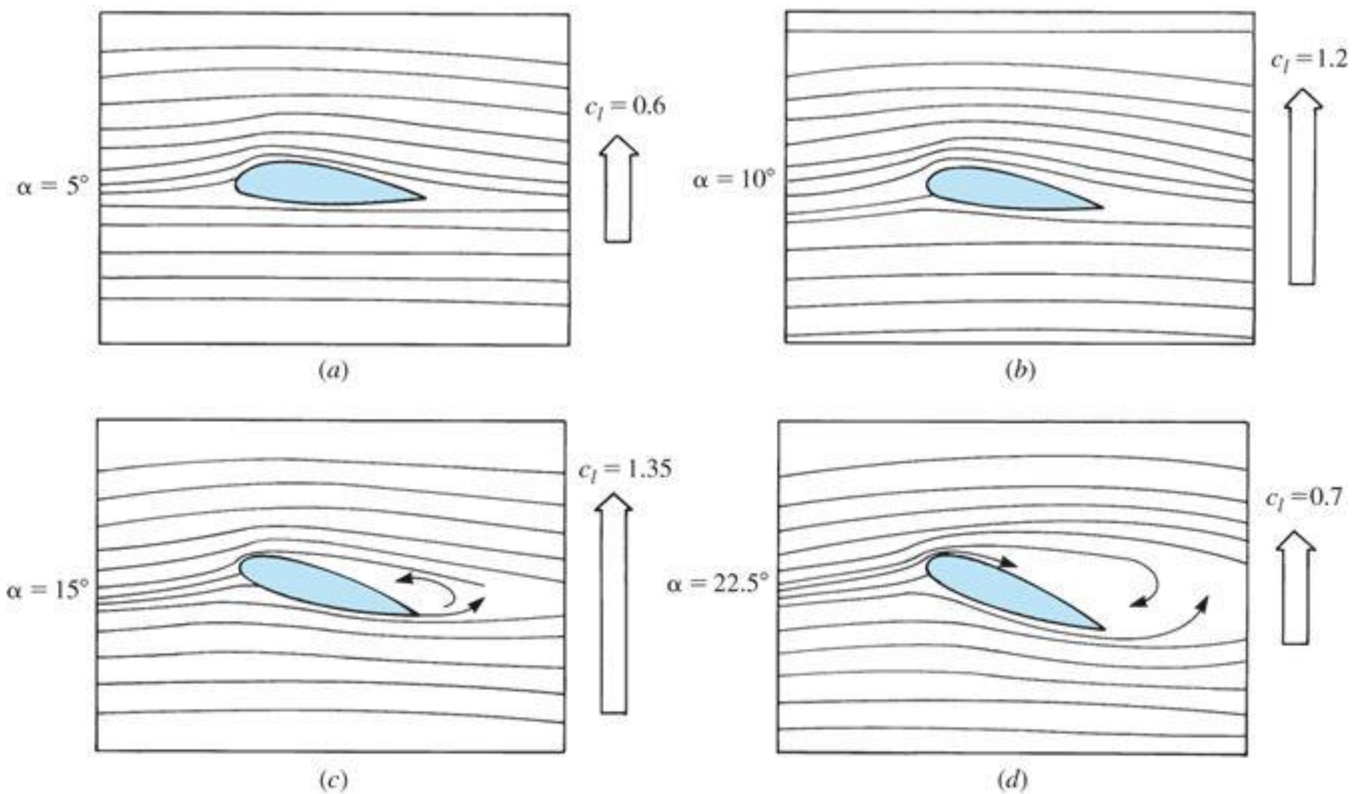
(d) The center of pressure:

$$\frac{x_{cp}}{c} = \frac{1}{4}\left[1 + \frac{\pi}{c_l}\left(A_1 - A_2\right)\right] = 0.273$$

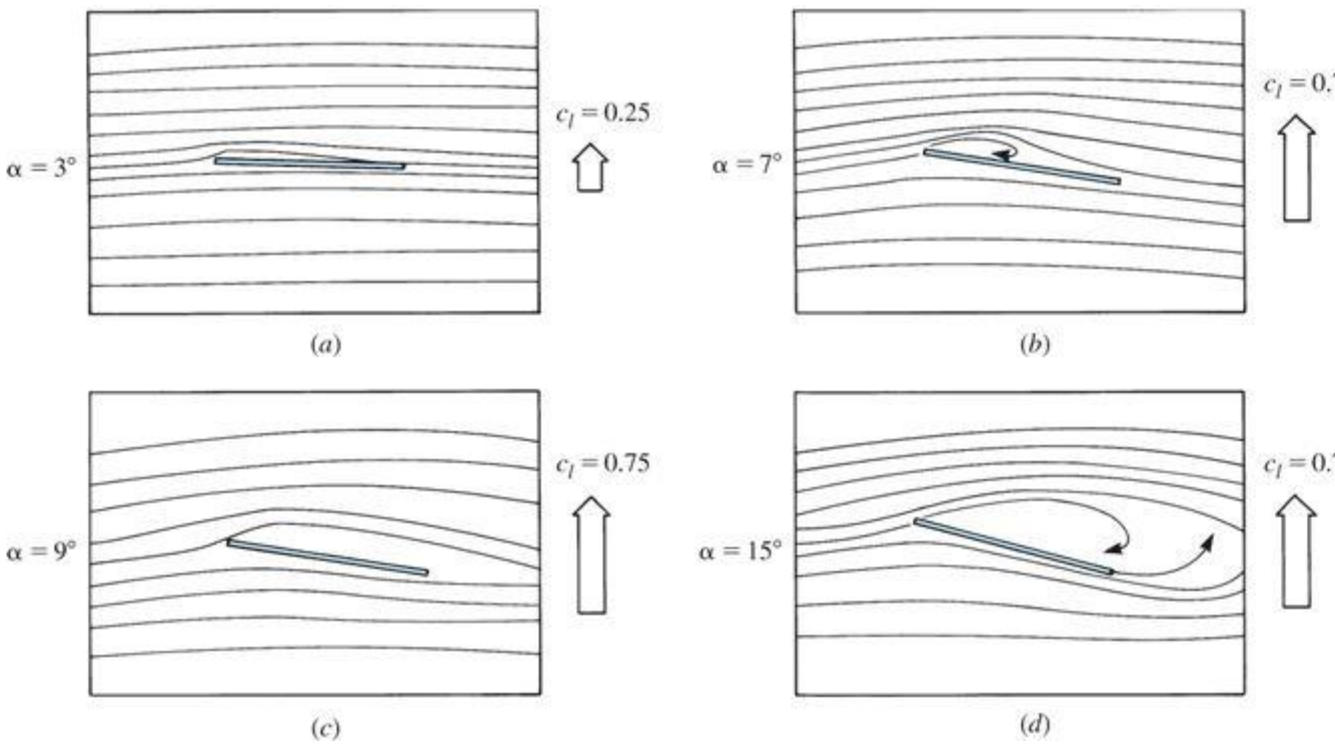
Leading-Edge Stall



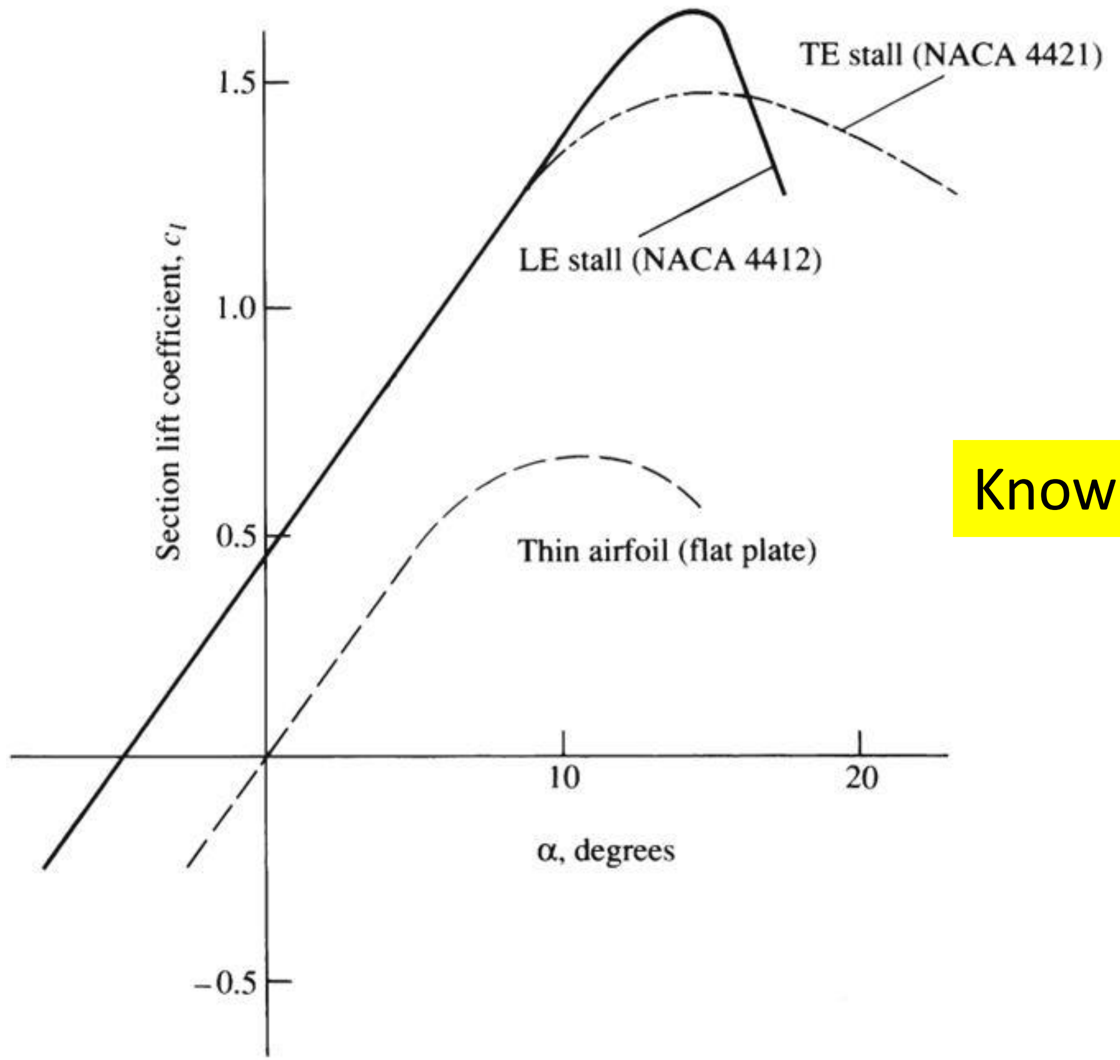
Trailing-Edge Stall



Flat Plate Stall

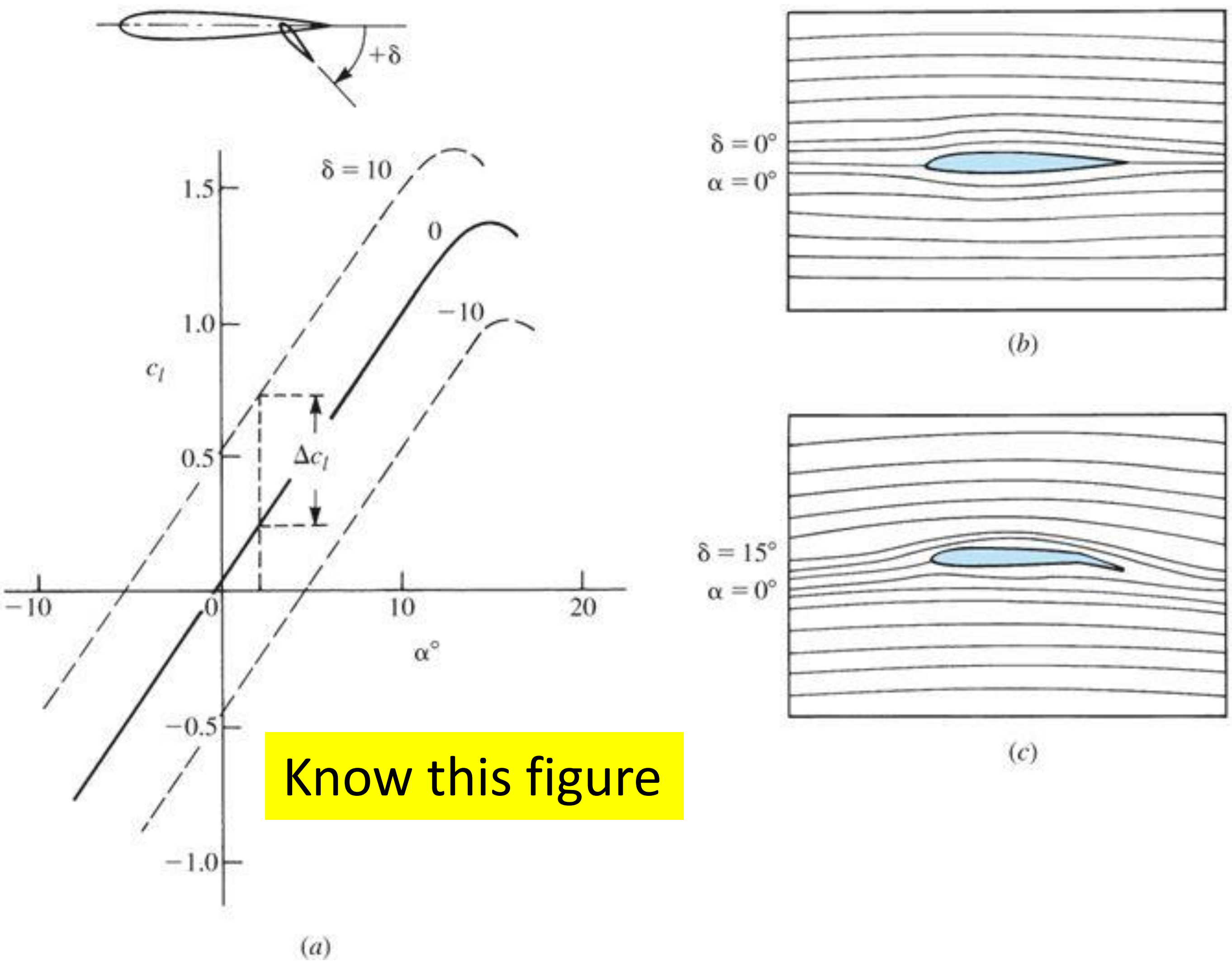


Effect of Thickness on Stall



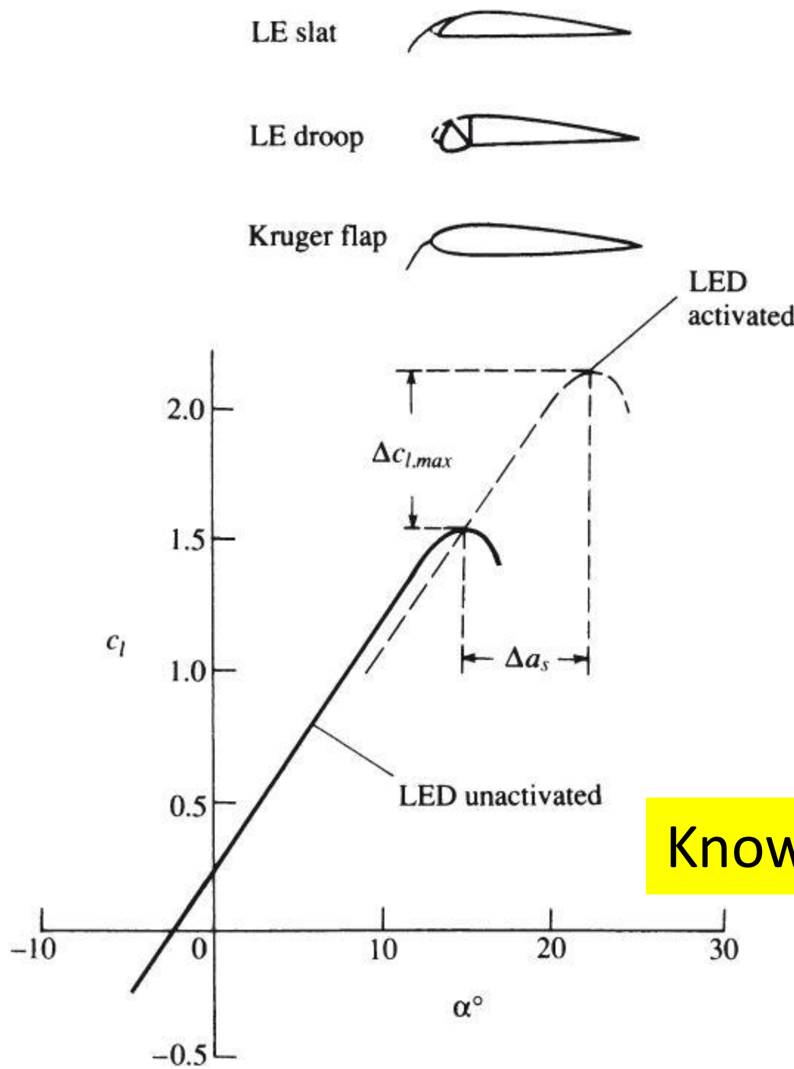
Know this figure

Effect of Flap Deflection or Camber



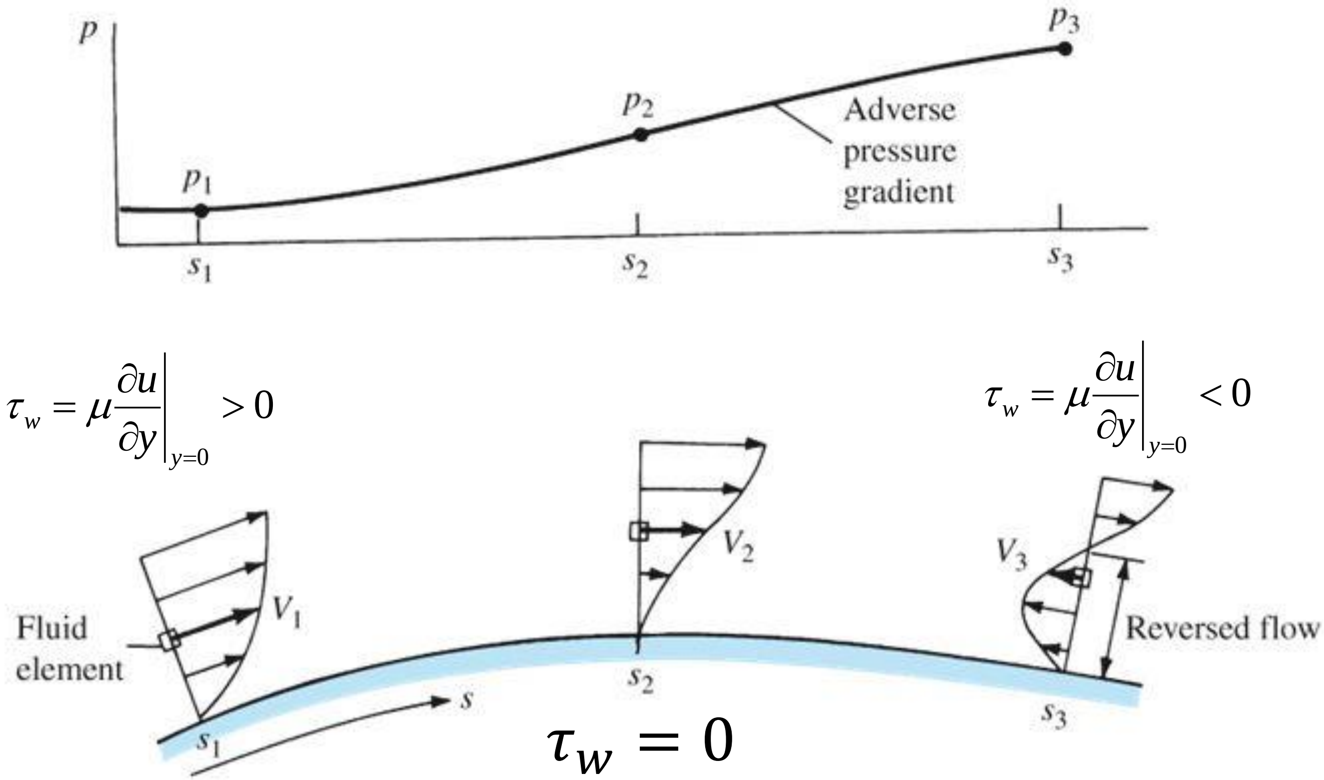
Know this figure

Effect of Leading Edge Devices



Know this figure

Separation is caused by adverse pressure gradient



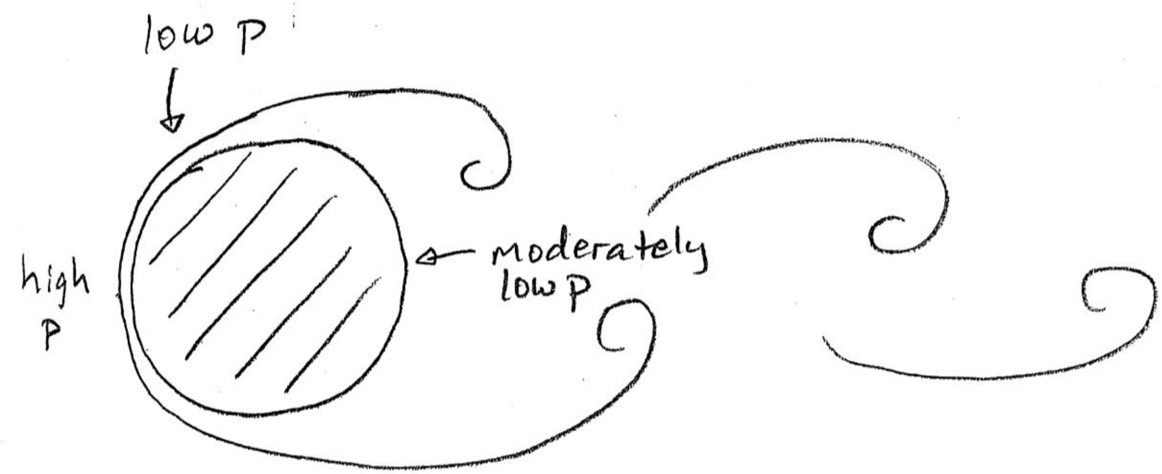
$$\tau_w = \mu \frac{\partial u}{\partial y}\bigg|_{y=0} > 0$$

$$\tau_w = 0$$

$$\tau_w = \mu \frac{\partial u}{\partial y}\bigg|_{y=0} < 0$$

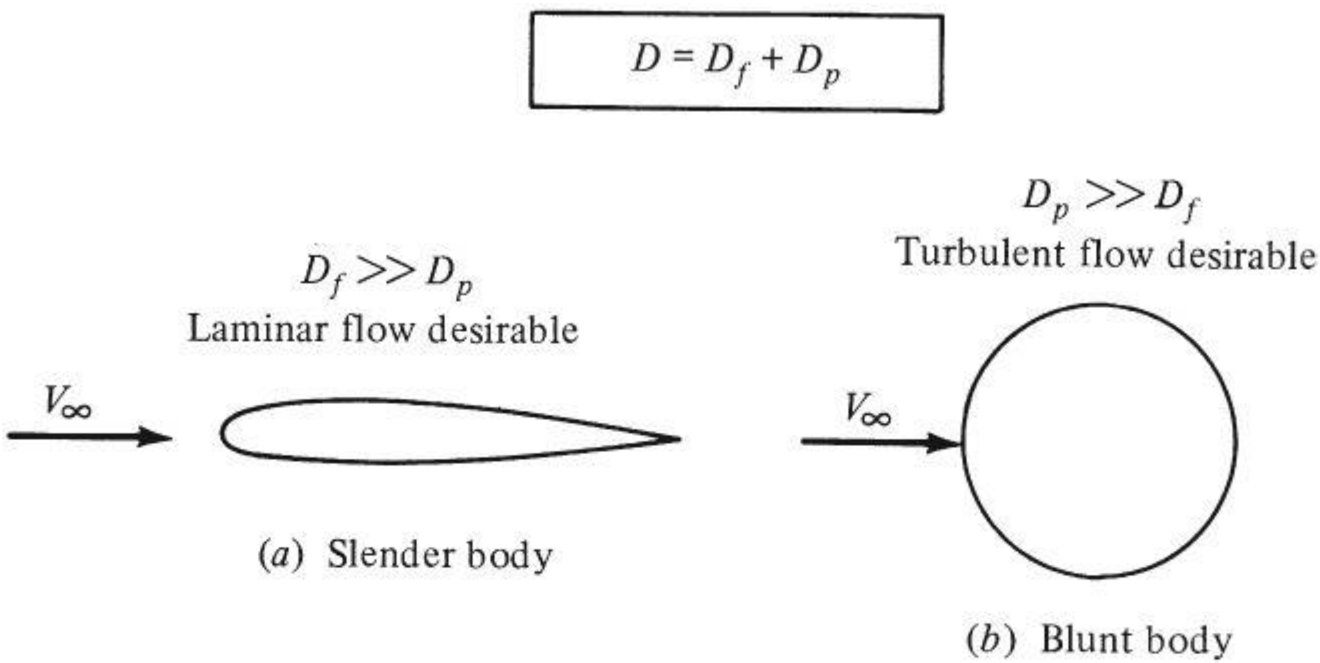
Separation: going from $\tau_w > 0$ to $\tau_w < 0$.
Reattachment: from negative to positive τ_w

Form Drag

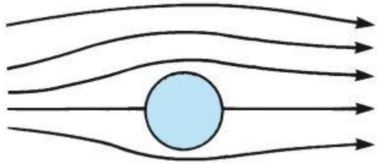


- Pressure difference implies form drag
- Separation not required to have form drag

Sometimes Transition is Helpful

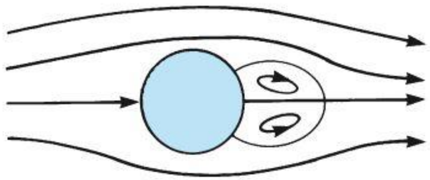


Cylinder Flow



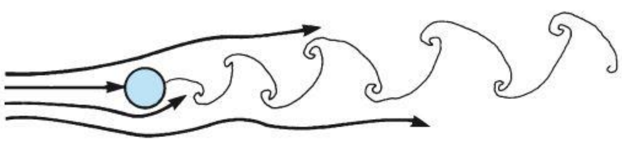
Re < 4

(a)



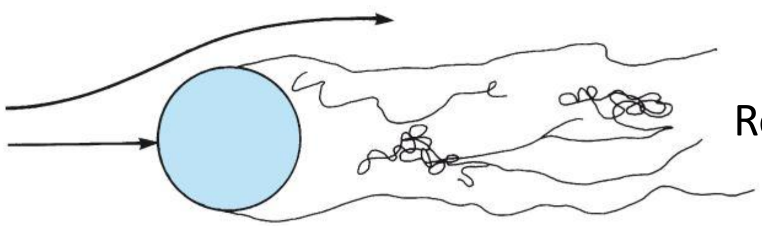
4 < Re < 40

(b)



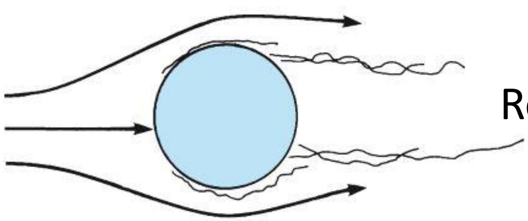
40 < Re < 10^3

(c)



Re = 1 x 10^5

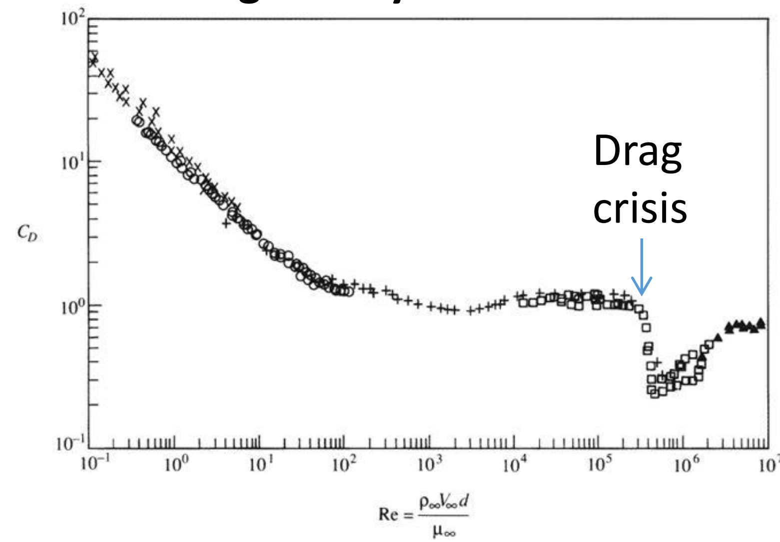
(d)



Re > 3 x 10^6

(e)

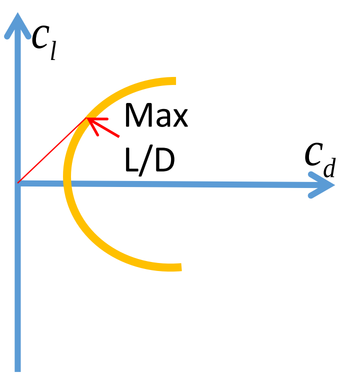
Drag vs. Reynolds Number

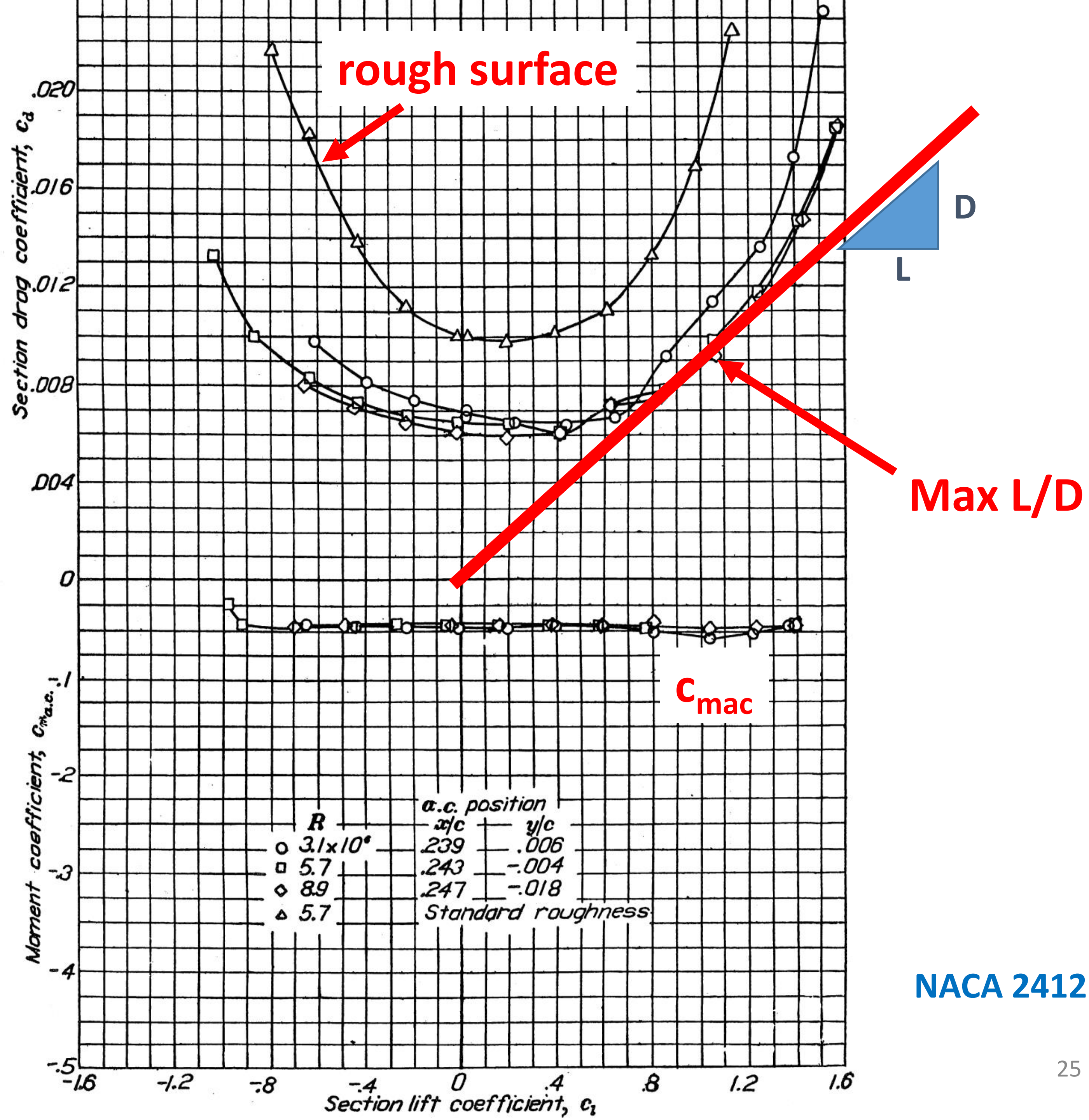


Know these figures

Drag Polar

Drag polar is often plotted "turned and flipped"





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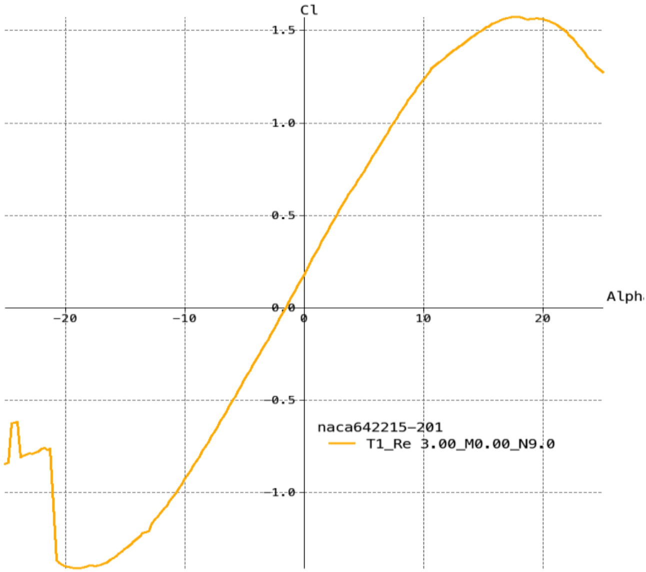
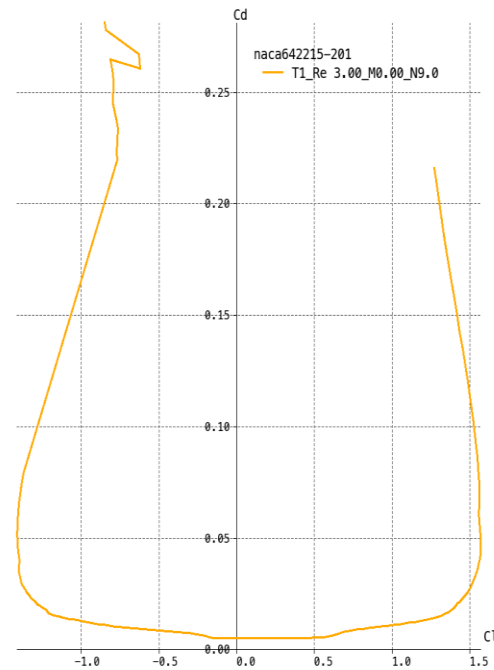
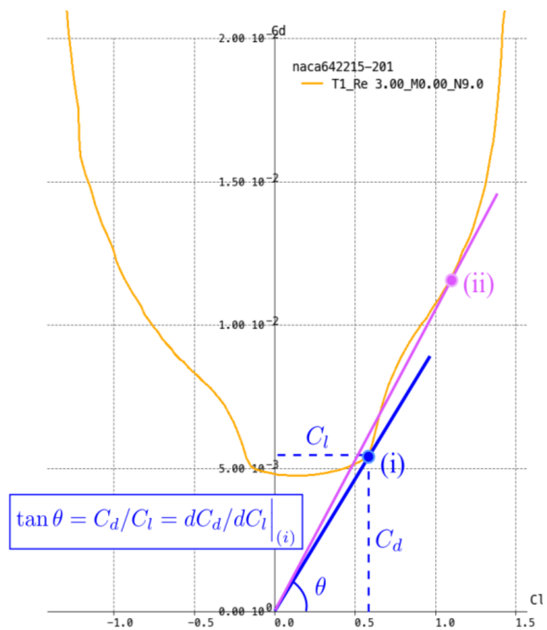
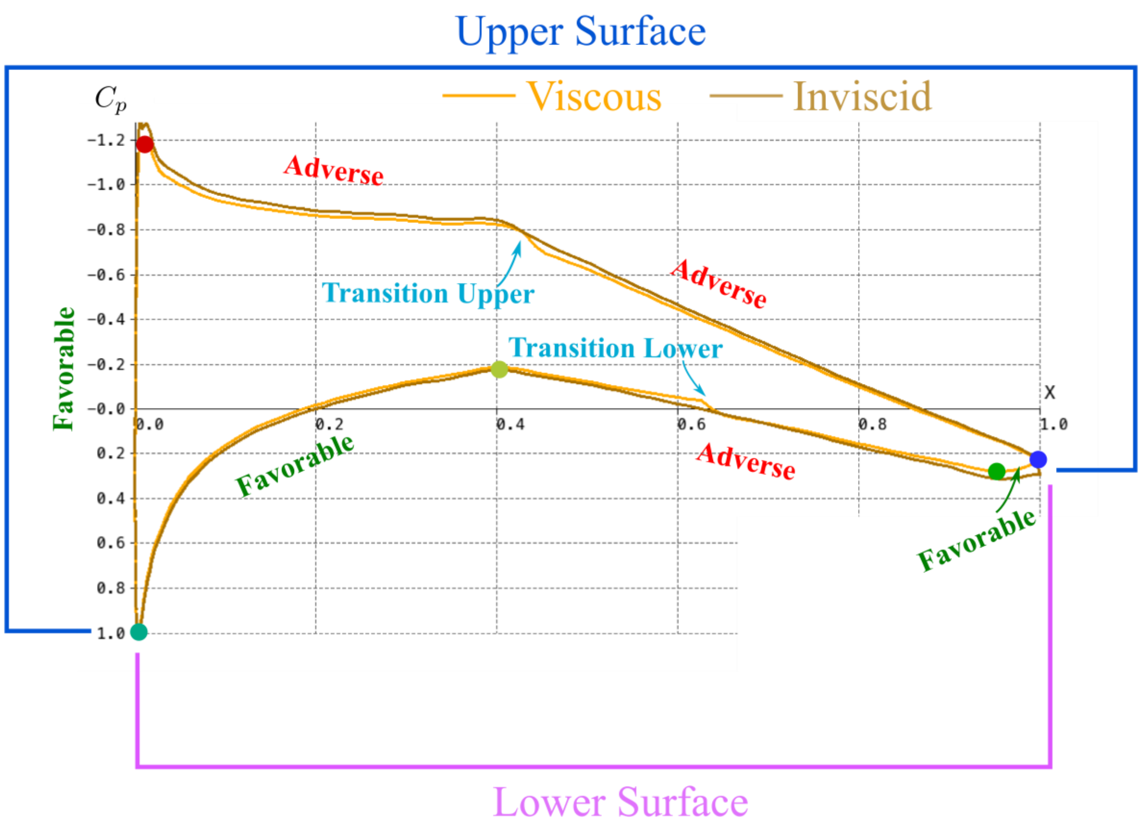
Study solution of HW4





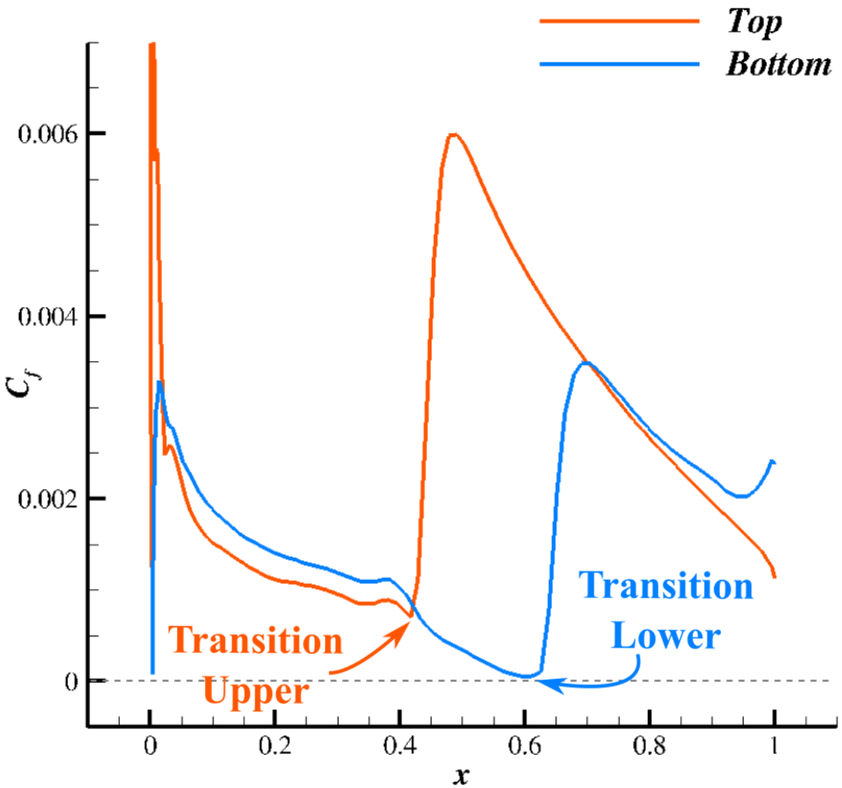


Study solution of HW4



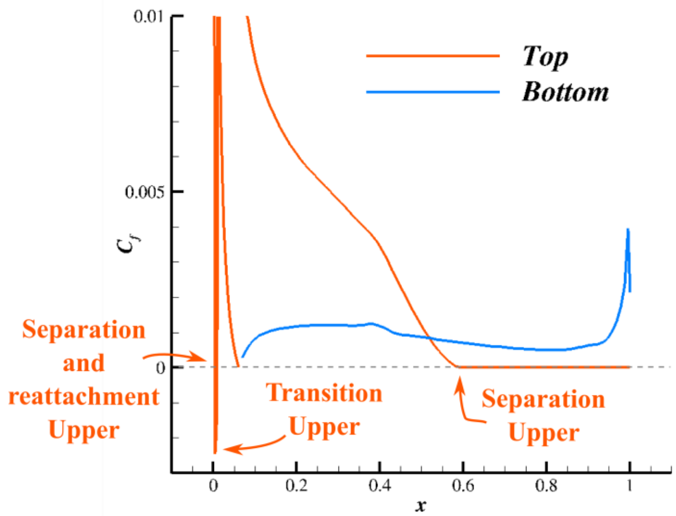
Study solution of HW4

The figure below shows the skin friction coefficient, c_f, which is just a constant times the wall shear stress, τ_w. The figures for c_f and τ_w will only differ by a scaling factor



Transition to turbulence is accompanied by a large jump in τ_w

Study solution of HW4



Here α = 17.75° at which C_l attains its maximum value. On the upper surface, C_f increases rapidly away from the front stagnation point. It then goes through a sudden drop to negative values for a short length, indicating separation and a short recirculation zone near the leading edge. Then C_f shows a sudden rise to positive values at about x/c = 0.009. This bump occurs due to the boundary layer transitioning from laminar to turbulent nature and reattaching to the upper surface. This is called a separation bubble. Beyond the sudden rise after transition, C_f gradually decreases and becomes 0 at x/c = 0.58. This location marks the flow separation on the upper surface. Thus, the main flow separation happens after transition on the upper surface.

The lower surface does not show a sudden increase in C_f except very near the trailing edge. Therefore, the flow does not transition on the lower surface. The increase in C_f on the lower surface near the trailing edge is due to the favorable pressure gradient that exists for a short distance before the trailing edge. Since $C_f > 0$ everywhere on the lower surface, the flow does not separate at this α on the lower surface.