AAE 334: Aerodynamics

HW7: Wing Stability Analysis

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Download the files in the HW7 folder on Blackboard. To review use of XFLR5 for stability analysis see the video 'XFLR5 Demo for AAE 334 Stability Lecture' by Professor Sullivan, which is available on the AAE 334 Course Content page on Blackboard. After performing the calculations or running the analyses described below, enter the results in the appropriate column in the Excel file "AAE334 Homework 7 Data and Answer Sheet". Print a copy of the completed spreadsheet to turn in with your homework.

1. Problem #1
   a. [25 pts] Using the formulas in the notes, calculate the stability quantities on the "Homework sheet" tab of the Excel file "AAE334 Homework 7 Data and Answer Sheet". Show your handwritten work on separate sheets of paper.
   b. [25 pts] Run XFLR5 using the file "AAE 334 telemaster stability Homework #7.xfl" and compare your answers (i.e., calculate the percent difference). (Note that the xfl file already has the geometry of the airplane included. In addition, the analysis of the wing alone has already been done. However, you will have to run the necessary analysis of the airplane.)
2. Problem #2 - Rerun xFLR5 with a velocity V=20 m/sec.
   a. [25 pts] Trim the aircraft by moving the CG so the aircraft is stable at the proper trim point.
   b. [25 pts] Trim the aircraft by changing the tail angle so the aircraft is stable at the proper trim point. (Use the original CG location.)

*The spreadsheet with the final answers is at the end

Case # 1 Handwritten Solution

- q_∞

$$q_\infty = \frac{1}{2}\rho_\infty V_\infty^2$$
$$= \frac{1}{2}\left(1.225\ \frac{kg}{m^3}\right)\left(13.4\ \frac{m}{s}\right)^2$$
$$= \boxed{109.9805\ Pa}$$

- $Re_{,w}$

$$Re_{,w} = \frac{V_\infty c_w}{\nu} = \frac{(13.4\ \frac{m}{s})(0.36\ m)}{(1.50\times10^{-5}\ m^2/s)} = \boxed{321048.79}$$

- $Re_{,T}$

$$Re_{,T} = \frac{V_\infty c_T}{\nu} = \frac{(13.4\ \frac{m}{s})(0.241\ m)}{(1.50\times10^{-5}\ m^2/s)} = \boxed{215561.33}$$

- C_L

assuming steady-level flight

$$C_L = \frac{W}{q_\infty S_w} = \frac{46.71 N}{(109.9805 Pa)(0.86 m^2)} = \boxed{0.4949}$$

- Angles

from the NACA 2412 lift curve (in Appendix)

$$@\ C_L = \boxed{0.4949} \rightarrow C_l = C_L \cdot b = 1.183$$

$$\Downarrow$$

$$\alpha_{ZL} = -2\ deg = -0.0349\ rad$$

$$\rightarrow a_0 = \boxed{2\pi}\ \text{approximation}$$

DynP : $q_\infty = 109.9805$ Pa

Re # wing : $Re_{,w} = 321048.79$

Re # tail : $Re_{,t} = 215561.33$

lift coeff : $C_L = 0.4949$ → from graph → $\alpha_{ZL} = -2° = -0.0349$ rad

$\hat{x}_{ac}$ wing : $(\hat{x}_{ac})_w = \frac{c_w}{4} = 0.09$ m

$\hat{x}_{ac}$ tail : $(\hat{x}_{ac})_t = \frac{c_t}{4} = 0.0602$ m

a_0 : for $C_L = 0.4949$ → slope ≈ 2π

► x_{cg} : The angle of incidence of the wing (w.r.t to plane)

$$i_w = 4.5° = 0.0785 \text{ rad}$$

The angle of incidence of the tail (w.r.t to mean chord of wing)

$$i_t = i_w - 2° = 2.5° = 0.0436 \text{ rad}$$

Using these the lift coefficients of the wing and tail respectively becomes

$$C_{LW} = 2\pi(\alpha - i_w) \quad \cdots ①$$

$$C_{Lt} = 2\pi(\alpha - i_t) \quad \cdots ②$$

C_{LW}, C_{Lt}, α → unknowns

and from equilibrium

$$W = L_w + L_t$$

$$W = q_\infty S_w C_{LW} + q_\infty S_t C_{Lt}$$

$$W = q_\infty(S_w C_{LW} + S_t C_{Lt}) \quad \cdots ③$$

solve the system equations ①②③ (using MATLAB)

→ $C_{LW} = 0.3550$ → $L_w = q_\infty S_w C_{LW} = 33.5728$ N

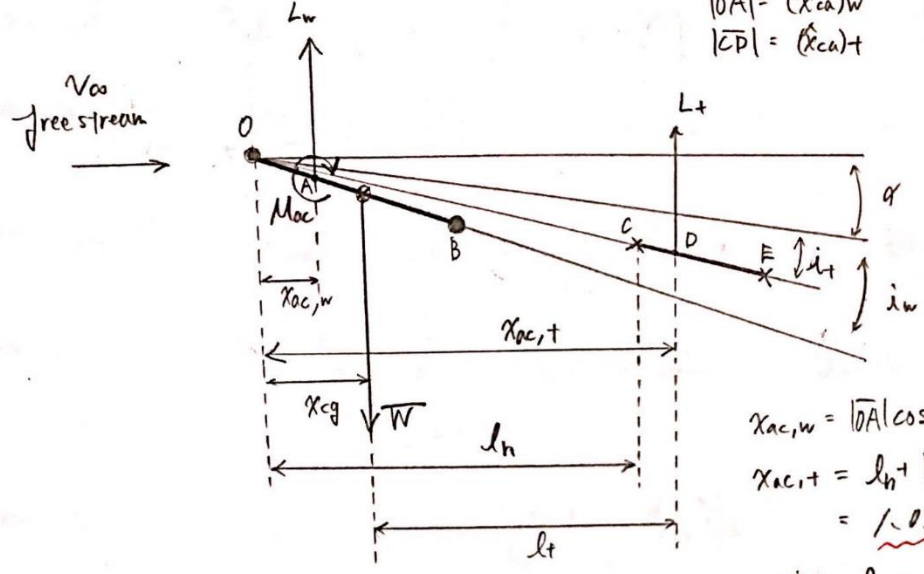
$C_{Lt} = 0.5740$ → $L_t = q_\infty S_t C_{Lt} = 13.1372$ N

and $\alpha = 0.1350$

► $\alpha_a = \alpha - \alpha_{ZL} = 0.1699$ rad (absolute angle of attack)

$|\overline{OA}| = (\hat{x}_{ca})_w \quad |\overline{OB}| = c_w \quad |\overline{OC}| = l$

$|\overline{CD}| = (\hat{x}_{ca})_t \quad |\overline{CE}| = c_t$



{ • wing LE & TE
x tail LE & TE

$x_{ac,w} = |\overline{OA}|\cos(\alpha_a + i_w) = 0.0880\,m$

$x_{ac,t} = l_h + |\overline{CD}|\cos(\alpha_a + i_t)$

$= 1.0591\,m$

since $l_h = l\cos(\alpha_a + i_t)$

given $l = 1.016\,m$

now from the moment about CG

$$\Sigma M_{cg} = M_{ac} + L_w(x_{cg} - x_{ac,w}) - L_t l_t + x_{cg} W$$

since $l_t = (l - x_{cg}) + (x_{ac,t} - l) = x_{ac,t} - x_{cg}$

$$\Sigma M_{cg} = 0, \quad W = 46.71\,N$$

$$W x_{cg} + M_{ac} + L_w(x_{cg} - x_{ac,w}) - L_t(x_{ac,t} - x_{cg}) = 0$$

since $M_{ac} = q_\infty S \bar{c}\, C_{M_{ac}}$ where $C_{M_{ac}} = -0.05$

solving this we get

$$x_{cg} = \boxed{0.1974\,m} \qquad \tilde{x}_{cg} = \frac{x_{cg}}{\bar{c}} = \boxed{0.5483}$$

► $C_{L\alpha}$ since

$$C_{Lw} = C_{L\alpha}\alpha_a \Rightarrow C_{L\alpha} = \frac{C_{Lw}}{\alpha_a} \quad \boxed{2.0675}$$

► $(C_{L\alpha})_t$ also

$$C_{Lt} = (C_{L\alpha})_t(\alpha_a - \epsilon - i_t) \Rightarrow (C_{L\alpha})_t = \frac{C_{Lt}}{\alpha_a - \epsilon - i_t} \quad \boxed{3.3450}$$

- l_t $\quad l_t = x_{ac,t} - x_{cg} = \boxed{0.8603 \text{ m}}$

- V_h $\quad$ since $V_h = \frac{S_t l_t}{S \bar{c}} = \boxed{0.5780}$

- C_{M0} $\quad C_{M0} = C_{Mac} + (C_{L\alpha})_t (\varepsilon_0 + i_t) V_h = \boxed{0.0337}$

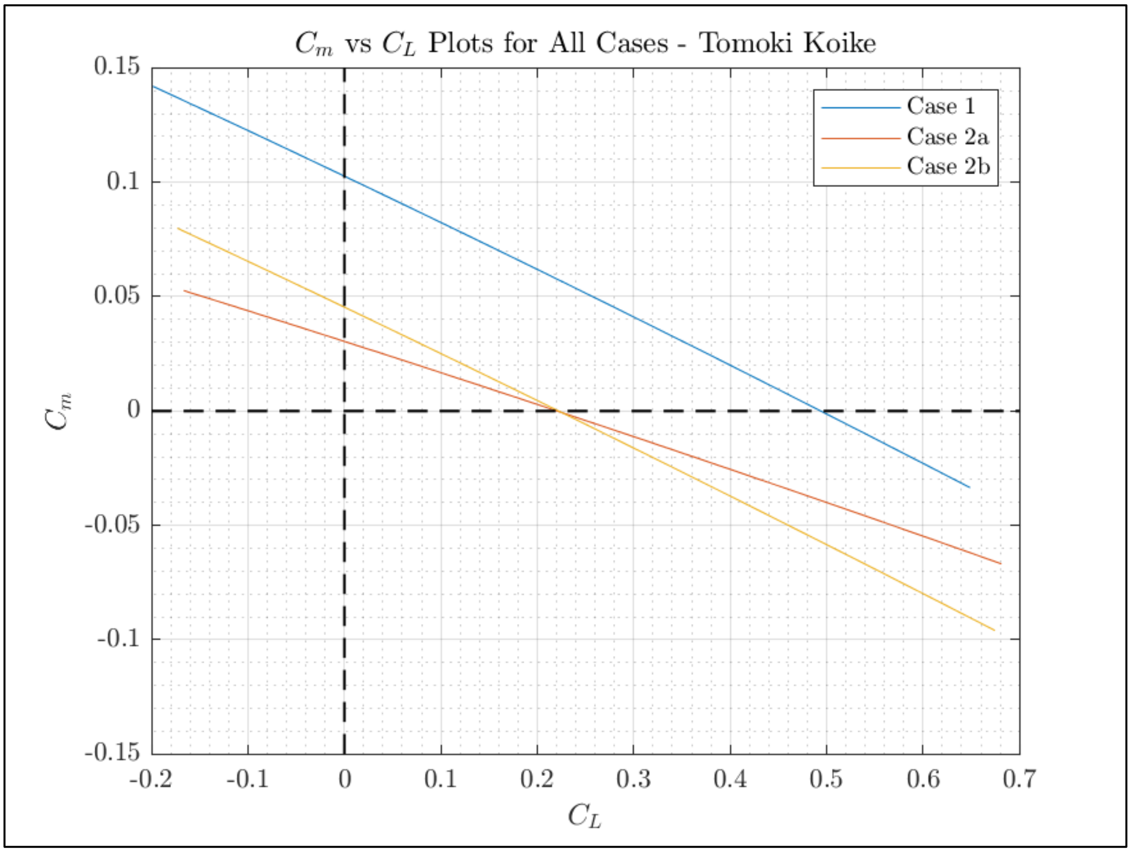
- $C_{M\alpha}$ $\quad C_{M\alpha} = C_{L\alpha}(\tilde{x}_{cg} - \tilde{x}_{ac}) - (C_{L\alpha})_t \left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right) V_h$
$= \boxed{-0.5346}$

- $\tilde{x}_n$ $\quad \tilde{x}_n = \tilde{x}_{ac} + \frac{(C_{L\alpha})_t}{C_{L\alpha}} \left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right) V_h$
$= \boxed{0.8069}$

- x_n $\quad x_n = \tilde{x}_n \cdot \bar{c} = \boxed{0.2905}$

- SM $\quad SM = \tilde{x}_n - \bar{x}_{cg} = \boxed{0.2586}$

PLOT



C_m vs C_L Plots for All Cases - Tomoki Koike
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
C_m
-0.2
-0.1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
C_L
Case 1
Case 2a
Case 2b

AAE334 HW7 MATLAB CODE

```
clear all; close all; clc;

fdir = 'C:\Users\Tomo\Desktop\studies\2020-Spring\AAE334\matlab\outputs\HW7';
set(groot, 'defaulttextinterpreter',"latex");
set(groot, 'defaultAxesTickLabelInterpreter',"latex");
set(groot, 'defaultLegendInterpreter',"latex");
```

problem 1-a

```
% Defining necessary properties
v_inf = 13.4;  % freestream velocity [m/s]
rho_inf = 1.225;  % freestream air density [kg/m3]
q_inf = 0.5*rho_inf*v_inf^2;  % freestream dynamic pressure [Pa]
S_w = 0.86;  % wing area [m2]
S_t = 0.208;  % tail area [m2]
ch_w = 0.36;  % chord length of wing [m]
ch_t = 0.241;  % chord length of tail [m]
l = 1.016;  % distance from the LE of the wing to the LE of the tail [m]
ch_mean = ch_w;  % mean chord length of the wing [m]
epsilon = 0;
e_a = 0.4;  % D(epsilon)/Da
W = 46.71;  % weight of the plane [N]

% Calculating the x_cg (center of gravity of the plane)


% Necessary angles [rad]
a_zl = deg2rad(-2.1)  % zero lift angle
i_w = deg2rad(4.5)  % incident angle of the wing
i_t = i_w - deg2rad(2)  % incident angle of the tail wrt the plane


% find a, C_Lw, and C_Lt
[C_Lw, C_Lt, a] = CLw_CLt_AoA(W, i_w, i_t, q_inf, S_w, S_t)
a_a = a - a_zl  % absolute angle of attack


% the lifts for the wing and tail
L_w = q_inf*S_w*C_Lw  % wing [N]
L_t = q_inf*S_t*C_Lt  % tail [N]


% aerodynamic center positions (wrt the LE of the wing)
xhat_ac_w = ch_w/4  % wing [m]
xhat_ac_t = ch_t/4  % tail [m]
```

```
lh = l*cos(a_a + i_t);
x_ac_w = xhat_ac_w*cos(a_a + i_w)
x_ac_t = lh + xhat_ac_t*cos(a_a + i_t)
x_ac_til = 0.25;


% moment coefficient about the aerodynamic center of the wing
C_Mac_w = -0.05;
% moment about the aerodynamic center of the wing [N-m]
Mac_w = q_inf*S_w*ch_mean*C_Mac_w


% solve
syms var1
eqn1 = Mac_w + L_w*(var1 - x_ac_w) - L_t*(x_ac_t - var1) + W*var1 == 0;
x_cg = double(solve(eqn1, var1))


x_cg_til = x_cg/ch_mean

% C_La
C_La_w = C_Lw/a_a
% C_La_t
C_La_t = C_Lt/(a_a)

% l_t distance from the x_cg to the aerodynamic center of the tail
l_t = x_ac_t - x_cg

% Vh
Vh = S_t*l_t/S_w/ch_mean

% C_Mo
C_Mo = C_Mac_w + C_La_t*(epsilon + i_t)*Vh

% C_Ma
C_Ma = C_La_w*(x_cg_til - x_ac_til) - C_La_t*(1 - e_a)*Vh

% x_n
x_n_til = x_ac_til + C_La_t/C_La_w*(1 - e_a)*Vh
x_n = x_n_til*ch_mean

% SM
SM = x_n_til - x_cg_til
```

problem 1-b

```
% importing data
data_1b = readmatrix("inputs\hw7\hw7_Cm_vs_Cl_case1_2_2.csv");
% Calculating the static margin
SM_1b = calc_SM_from_xflr(data_1b)
```

```
% Importing another data
```

```
data_1b2 = readmatrix("inputs\hw7\hw7_Cm_vs_alpha_case1_2.csv");
% Calculating the C_M0 (*for the angle of attack add 2.1 degrees to adjust
% to the absolute angle of attack)
[C_Mo_1b, C_Ma_1b] = calc_CMo_and_CMa_from_xlfr(data_1b2, -2.1, rad2deg(i_w))
```

problem 2-a

```
% importing data
data_2a = readmatrix("inputs\hw7\hw7_Cm_vs_Cl_case2a.csv");
% Calculating the static margin
SM_2a = calc_SM_from_xflr(data_2a)

% Importing another data
data_2a2 = readmatrix("inputs\hw7\hw7_Cm_vs_alpha_case2a.csv");
% Calculating the C_M0 (*for the angle of attack add 2.1 degrees to adjust
% to the absolute angle of attack)
[C_Mo_2a, C_Ma_2a] = calc_CMo_and_CMa_from_xlfr(data_2a2, -2.1, rad2deg(i_w))

% find a, C_Lw, and C_Lt
q_inf_2a = 245;
[C_Lw_2a, C_Lt_2a, a_2a] = CLw_CLt_AoA(W, i_w, i_t, q_inf_2a, S_w, S_t)
a_a_2a = a_2a - a_zl  % absolute angle of attack


% the lifts for the wing and tail
L_w = q_inf_2a*S_w*C_Lw_2a  % wing [N]
L_t = q_inf_2a*S_t*C_Lt_2a  % tail [N]
```

problem 2-b

```
% importing data
data_2b = readmatrix("inputs\hw7\hw7_Cm_vs_Cl_case2b.csv");
% Calculating the static margin
SM_2b = calc_SM_from_xflr(data_2b)

% Importing another data
data_2b2 = readmatrix("inputs\hw7\hw7_Cm_vs_alpha_case2b.csv");
% Calculating the C_M0 (*for the angle of attack add 2.1 degrees to adjust
% to the absolute angle of attack)
[C_Mo_2b, C_Ma_2b] = calc_CMo_and_CMa_from_xlfr(data_2b2, -2.1, rad2deg(i_w))

% find a, C_Lw, and C_Lt
q_inf_2b = 245;
[C_Lw_2b, C_Lt_2b, a_2b] = CLw_CLt_AoA(W, i_w, i_t, q_inf_2b, S_w, S_t)
a_a_2b = a_2b - a_zl  % absolute angle of attack


% the lifts for the wing and tail
L_w = q_inf_2b*S_w*C_Lw_2b  % wing [N]
L_t = q_inf_2b*S_t*C_Lt_2b  % tail [N]
```

Plotting

```
fig1 = figure("Renderer","painters");
```

```
plot(data_1b(:,1),data_1b(:,2))
title({'$C_m$ vs $C_L$ Plots for All Cases - Tomoki Koike'})
xlabel('$C_L$')
ylabel('$C_m$')
hold on
plot(data_2a(:,1),data_2a(:,2))
plot(data_2b(:,1),data_2b(:,2))
plot(linspace(-0.2,0.7,100),zeros([1,length(linspace(-0.2,0.7,100))]),'--
k',"LineWidth",1.0)
plot(zeros([1,length(linspace(-0.15,0.15,100))]),linspace(-0.15,0.15,100),'--
k',"LineWidth",1.0)
hold off
grid on; grid minor; box on;
legend('Case 1','Case 2a','Case 2b')
saveas(fig1, fullfile(fdir,'Cm_vs_Cl.png'))
```

FUNCTIONS

```
function SM = calc_SM_from_xflr(data)
    % data: data points
    CL_1b = data(:,1);
    Cm_1b = data(:,2);

    % SM
    if Cm_1b(CL_1b == min(abs(CL_1b)))
        Cm_1b_0 = Cm_1b(CL_1b == min(abs(CL_1b)));
    else
        Cm_1b_0 = Cm_1b(CL_1b == -(min(abs(CL_1b))));
    end

    if CL_1b(Cm_1b == -(min(abs(Cm_1b))))
        CL_1b_0 = CL_1b(Cm_1b == -(min(abs(Cm_1b))));
    else
        CL_1b_0 = CL_1b(Cm_1b == (min(abs(Cm_1b))));
    end
    SM = Cm_1b_0/CL_1b_0;
end


function [C_Mo, C_Ma] = calc_CMo_and_CMa_from_xlfr(data, a_zl, i_w)
    % data: data points
    % a_zl: zero lift angle of attack
    % i_w: wing incidence angle
    Cm = data(:,2);
    a = data(:,1) - a_zl + i_w;

    P = polyfit(a,Cm,1);
    C_Ma = P(1);
    C_Mo = P(2);
end
```

```matlab
function [C_Lw, C_Lt, a] = CLw_CLt_AoA(W, i_w, i_t, q_inf, S_w, S_t)
    % system equations to find a, C_Lw, and C_Lt
    syms x1 x2 x3
    eqn1 = x1 == 2*pi*(x3 - i_w);
    eqn2 = x2 == 2*pi*(x3 - i_t);
    eqn3 = W == q_inf*(S_w*x1 + S_t*x2);
    res = solve([eqn1 eqn2 eqn3],[x1 x2 x3]);
    % the lift coefficients for the wing and tail respectively
    C_Lw = double(res.x1);
    C_Lt = double(res.x2);
    % angle of attack
    a = double(res.x3);
end
```

				Change CG from case #1	Change Tail Angle From case #1
	Hand Calculation	**XFLR5**	**% Difference**	**XFLR5**	**XFLR5**
	Case #1	**Case #1**		**Case #2a**	**Case #2b**
Velocity (m/sec)	13.40	13.4	0	20	20
q	109.9805	109.9805	0	245.0000000000	245.0000000000
Re wing	321048.79	321048.79	0	479177.3049645390	479177.3049645390
Re tail	215561.33	215561.33	0	321733.3333333330	321733.3333333330
CL	0.4949	0.4949	1.1216E-14	0.2221725496	0.2221725496
Xac/c (empirical)	0.25	0.25	0	0.2500000000	0.2500000000
Xac	0.087955196	0.08956878	1.834552988	0.0895687802	0.0895687802
Vh	0.578010478	1.443285235	149.6988012	0.9456384323	1.1561750221
Xac/c	0.25	0.25	0	0.2500000000	0.2500000000
a0 (rad-1)	6.283185307	6.283185307	0	6.2831853072	6.2831853072
CL_alpha wing C_{L_α}	2.067484131	2.067484131	3.65155E-13	1.0054767944	1.0054767944
CL_alpha tail $C_{L_{\alpha_t}}$	3.344969336	3.344969336	3.31908E-13	2.6293871060	2.6293871060
Xn/c	0.806946537	0.327539669	59.40999143	0.2809485467	0.3208589791
Xn	0.290500753	0.117712182	59.47956059	0.1009681256	0.1153112556
Xcg	0.197413448	0.120000000	39.21386759	0.1440000000	0.1200000000
Xcg/c	0.548370689	0.333905630	39.10950445	0.4006867563	0.3339056302
AoA	0.13503269	0.13503269	0	0.1001529319	0.1001529319
CL wing	0.354955195	0.354955195	0	0.1357992101	0.1357992101
CL tail	0.574279738	0.574279738	0	0.3551237524	0.3551237524
L wing	33.49718418	33.49718418	0	28.5484413717	28.5484413717
L tail	13.16161366	13.16161366	0	18.1307496389	18.1307496389
a zero lift	-0.036651914	-0.036651914	0	-0.0366519143	-0.0366519143
a absolute	0.171684605	0.171684605	0	0.1350595169	0.1350595169
Static Margin	0.258575848	0.207539669	19.73741152	0.1369485467	0.2008589791
$C_{M_{ac}}$	-0.05	-0.05	0	-0.0500000000	-0.0500000000
C_{M_0}	0.033737852	0.118520086	251.2970676	0.0367934609	0.0561171249
C_{M_α}	-0.534601462	-0.017941016	-96.64403907	-0.0122134878	-0.0179822689
Provide C_L versus $C_{M_{cg}}$	Graph with 3 curves for Case #1, Case #2a, and Case #2b				