September 15, 2020

AAE 564 Fall 2020

Homework Three

Due: Friday, September 18

Exercise 1 Obtain the transfer function of the following system

$$\ddot{q}(t) + 4\dot{q} + q(t) = u(t) + \dot{u}(t - T)$$
$$y(t) = q(t)$$

where the constant $T > 0$ represents a constant time delay.

Exercise 2 The two pendulum cart system. Unless otherwise specified, from now on, we will consider the two pendulum cart system as an input-output system with input u and output y described by

$$\begin{array}{rcrcl} (m_0 + m_1 + m_2)\ddot{y} - m_1 l_1 \cos\theta_1\, \ddot{\theta}_1 - m_2 l_2 \cos\theta_2\, \ddot{\theta}_2 & + & m_1 l_1 \sin\theta_1 \dot{\theta}_1^2 + m_2 l_2 \sin\theta_2 \dot{\theta}_2^2 & = & u \\ -m_1 l_1 \cos\theta_1\, \ddot{y} + m_1 l_1^2 \ddot{\theta}_1 & + & m_1 l_1 g \sin\theta_1 & = & 0 \\ -m_2 l_2 \cos\theta_2\, \ddot{y} + m_2 l_2^2 \ddot{\theta}_2 & + & m_2 l_2 g \sin\theta_2 & = & 0 \end{array}$$

(a) For what constant values of u does the system have equilibrium states?

(b) Consider the equilibrium configurations defined by $u^e = 0$ and

$$E1: \quad (y^e, \theta_1^e, \theta_2^e) = (0, 0, 0)$$
$$E2: \quad (y^e, \theta_1^e, \theta_2^e) = (0, \pi, \pi)$$

Using MATLAB, obtain the A, B, C, D matrices for state space representations of the linearizations corresponding to the following combinations of parameters and equilibrium conditions:

L1	P1	E1
L2	P1	E2
L3	P4	E1
L4	P4	E2

Exercise 3 Poles and zeros of the two pendulum cart system. Using MATLAB, obtain the poles and zeros for $L1$-$L4$.

Exercise 4 Obtain a state space realization of the transfer function,

$$\hat{G}(s) = \frac{s^2 + 4s + 4}{s^2 + 3s + 2}\,.$$

Is your realization minimal?

Exercise 5 Obtain a state space representation of the following transfer function.

$$\hat{G}(s) = \begin{pmatrix} \dfrac{s^2+1}{s^2-1} \\ \\ \dfrac{2}{s^2+1} \end{pmatrix}$$

Exercise 6 Obtain a state space realization of the transfer function,

$$\hat{G}(s) = \begin{pmatrix} \dfrac{s^2}{s^2-4} & \dfrac{s}{s-2} \\ \\ \dfrac{1}{s+2} & -\dfrac{1}{s} \end{pmatrix}.$$

Exercise 7 (a) Obtain a state space realization of the following single-input single-output system.

$$\ddot{y} - 3\dot{y} - 4y = \ddot{u} - 2\dot{u} - 8u$$

(b) Is your realization minimal?

Exercise 8 Obtain a state space realization of the following input-output system.

$$\begin{aligned} \dot{y}_1 + y_2 &= \dot{u}_2 + u_1 \\ \dot{y}_2 + y_1 &= \dot{u}_1 + u_2 \end{aligned}$$

Exercise 9 Obtain a linearized state space description of the following system about $u(t) \equiv 0$ and $q(t) \equiv 0$.

$$\begin{aligned} \ddot{q} + \sin q &= u + \dot{u} \\ y + y^3 &= q + \cos u - 1 \end{aligned}$$