College of Engineering
School of Aeronautics and Astronautics

AAE 564
System Analysis and Synthesis

Homework 6
System and Matrices

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Exercise 1

Consider a mechanical/aerospace system described by

$$M\ddot{q} + C\dot{q} + Kq = 0$$

where $q(t)$ is an N-vector and M, C, and K are square matrices. Suppose λ is a complex number which satisfies

$$det(\lambda^2 M + \lambda C + K) = 0 \ .$$

Show that the above system has a solution of the form

$$q(t) = e^{\lambda t} v$$

where v is a constant N-vector.

From what we are given,

$$q = e^{\lambda t} v$$

$$\dot{q} = \lambda e^{\lambda t} v$$

$$\ddot{q} = \lambda^2 e^{\lambda t} v$$

Plug these into the equation

$$M\ddot{q} + C\dot{q} + Kq = 0$$

$$M\left(\lambda^2 e^{\lambda t} v\right) + C\left(\lambda e^{\lambda t} v\right) + e^{\lambda t} v = 0 \ .$$

This becomes

$$(M\lambda^2 + C\lambda + K)e^{\lambda t} v = 0 \ .$$

Since, v is a non-zero vector and $e^{\lambda t}$ cannot be zero. Thus, the matrix $(M\lambda^2 + C\lambda + K)$ is equal to zero and makes it singular.

Then,

$$det(\lambda^2 M + \lambda C + K) = 0 \ .$$

Thus, have proven that

$$q(t) = e^{\lambda t} v$$

is a solution of $M\ddot{q} + C\dot{q} + Kq = 0$.

$q.e.d$

Exercise 2

Suppose A is a 3 x 3 matrix and

$$det(sI - A) = s^3 + 2s^2 + s + 1$$

(a) Express A^3 in terms of I, A, A^2.
(b) Express A^5 in terms of I, A, A^2.
(c) Express A^{-1} in terms of I, A, A^2.

(a).

The roots of the polynomial are

$$s^3 + 2s^2 + s + 1 = 0$$

From Cayley-Hamilton Theorem

$$I + A + 2A^2 + A^3 = 0$$

Thus,

$$\boxed{\therefore\ A^3 = -I - A - 2A^2\ .}$$

(b).

From the result of part (a),

$$AA^3 = A(-I - A - 2A^2)$$

$$A^4 = -A - A^2 - 2A^3$$

$$A^4 = -A - A^2 - 2(-I - A - 2A^2)$$

$$A^4 = 2I + A - A^2 + 2A^2$$

$$A^4 = 2I + A + 3A^2.$$

Then,

$$\therefore A^5 = A(2I + \mathrm{A} + 3A^2) = 2A + A^2 + 3A^3 = \boxed{-3I - A - 5A^2}$$

(c).

From the result of part (a),

$$A^{-1}A^3 = A^{-1}(-I - A - 2A^2)$$

$$A^2 = -A^{-1} - I - 2A$$

$$\boxed{\therefore A^{-1} = -I - 2A - A^2}$$

Exercise 3

Without doing any matrix multiplications, compute A^4 for

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 564 & 1 & 0 & 0 \end{pmatrix}$$

Justify your answer.

Looking at the matrix, we can tell that the matrix is a companion matrix. Thus, the characteristic polynomial is going to be

$$\begin{pmatrix} -s & 1 & 0 & 0 \\ 0 & -s & 1 & 0 \\ 0 & 0 & -s & 1 \\ 564 & 1 & 0 & -s \end{pmatrix}$$

$$s^4 - s - 564 = 0 \ .$$

Using the Cayley-Hamilton Theorem, we have

$$p(A) = 0$$

$$A^4 - A - 564I = 0$$

$$A^4 = A + 564I$$

$$\boxed{A^4 = \begin{pmatrix} 564 & 1 & 0 & 0 \\ 0 & 564 & 1 & 0 \\ 0 & 0 & 564 & 1 \\ 564 & 1 & 0 & 564 \end{pmatrix}}$$