Paper Review: Network Flows That Solve Least Squares for Linear Equations

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***Abstract*—We will analyze and dissect a paper by Yang Liu, Youcheng Lou, Brian D.O. Anderson, and Guodong Shi [1]. This paper introduces a novel first-order distributed continuous-time algorithm that solves a linear equation for a network consisting of multiple agents. Once this algorithm is proposed, the paper analyzes the convergence of this algorithm for various step-sizes and solutions. Different types of graphs are investigated to reveal different results that may be drawn using the algorithm. For example, convergence rate, uniqueness of solution, etc.**

***Index Terms*—distributed algorithms; least square; multi-agent system; graph theory**

I. Introduction and Motivation

Linear equations over a network is a heating topic nowadays due to many applications where heavy loads of computation cannot be handled by one agent but can be handled by a network of agents. Complex system, optimization, distributed estimation, filtering and many other problems are solvable using distributed computation. This leads to why this paper focuses specifically on proposing a novel approach to solve the linear equations over a network using distributed algorithms to obtain a higher convergence rate which as a result can improve systems in real life applications.

One noteworthy case for linear equations over networks is that regardless of implementing a distributed algorithm there are ones that are non-solvable. When this is the case the most popular approach taken is to solve the least-squares solution that minimizes objective function which is often times the error between the actual solution and estimate. The least-squares approach has limitations in itself involving mismatch of equations and nodes and others, and in order to overcome this several new theories such as second order algorithms, distributed discrete-time inexact gradient algorithms, and discrete-time distributed algorithms for nonconvex constrained optimization over direct graphs. This paper manages to borrow these preexisting theories and propose a first-order continuous-time algorithm for least-squares problem for solving network linear equations.

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II. Problem Formulation

For the problem formulation, the preliminary math for the network linear equation is discussed to

$$z = Hy \tag{1}$$

and we solve the minimization of $||z - Hy||^2$ for the least-squares. Furthermore, the paper incorporates graph theory and utilizes adjacency and Laplacian matrices for analysis. A single node or agent i in this graph has a single linear equation associated to it which updates its state as in the form of $z_i = h_i^T y$. And for a single node the following contiuous-time network linear equation is given

$$\dot{x}_i(t) = K \sum_{j \in \mathcal{N}_i(t)} [A(t)]_{ij}(x(t) - x_i(t)) - \frac{\alpha(t)}{2} \nabla f_i(x_i(t)) \tag{2}$$

where K is a positive constant and α is the step size. The final equation is formulated by considering a local averaging consensus and diminishing local objective as a continuous time function where the step size is changing.

That being said, the least square solution is solved by the equation of

$$y^* = (H^T H)^{-1} H^T z. \tag{3}$$

III. Main Results

For a fixed and connected network, with the assumption of an integrable step size, it is possible to obtain a unique least-squares solution using the proposed algorithm. It was also proven for this system to be convergent. The step size and bounds of the convergence speed is what gave guidance for the convergence speed. For a connected switching graph, a square integrability step size assumption was made and that allowed the algorithm to acquire a convergence result for the least-squares problem.

IV. Your Ideas of Further Improvements

For future improvements, the authors mention to investigate the application of their theory for networks where instantaneous connectivity is absent as well as the exact convergence rate which was not discussed in this paper.

This paper kindled my thoughts on how to formulate the network linear equation that I would like to solve for my current research for this class.

REFERENCES

[1] Yang Liu, Youcheng Lou, Brain D.O. Anderson, and Duodong Shi. Network flows that solve least squares for linear equations. *Automatica*, 120, 2020.