

# AAE 334: Aerodynamics

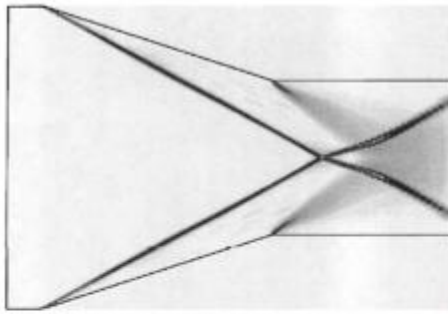
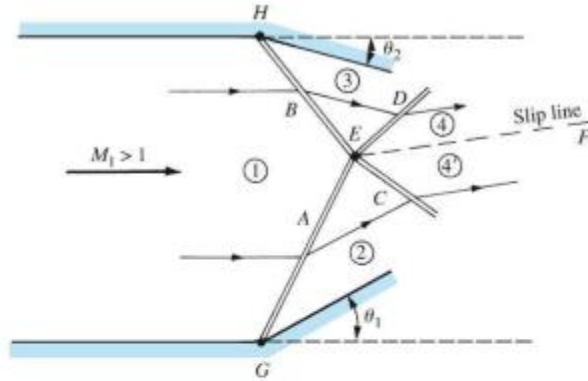
## HW 10: Shockwave Interactions

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Fig. 12 Shock-wave structure in the  $18 \times 18$  deg interaction.

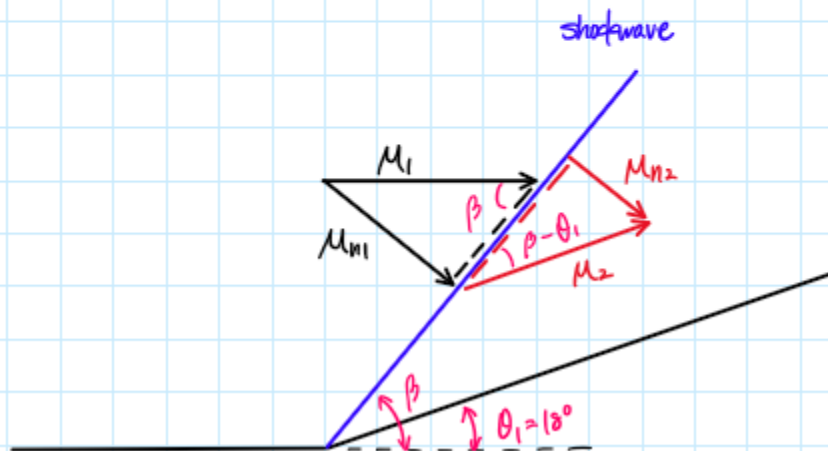
1.) [35 pts] In Lecture Notes 27, we looked at a crossing-shock interaction at Mach 5 with an  $18^\circ$  turning angle. The problem is symmetric as illustrated in the picture on the left, so that the slip line occurs on the axis of symmetry (despite being shown as inclined in the picture on the right) and there is no difference in the flow properties across the axis.

- Compute the static pressure ratio (relative to Region 1) and Mach number in Regions 2, 3, and 4.
- Where is the expansion fan in the figure on the left? (Show the location on a diagram.) What are the pressure ratio and Mach number downstream of the expansion? Why is it OK to leave the expansion fan out of the first part of this problem?

(a)

We are given that  $M_1 = 5$  &  $\theta_1 = \theta_2 = 18^\circ$

Region 2



Calculating the shockwave angle from  $\theta$ - $\beta$ - $M$  relation

$$\tan \theta = 2 \cot \beta \frac{M_1^2 \sin^2 \beta - 1}{M_1^2 (\gamma + \cos 2\beta) + 2} \quad \dots \textcircled{1}$$

using **MATLAB** (code in **Appendix**) we obtain

$$\beta = 27.5495 \text{ deg}$$

then  $M_{n1} = M_1 \sin \beta = 2.3126$

now, from normal shock relations

$$M_{n2}^2 = \frac{1 + \frac{\gamma-1}{2} M_{n1}^2}{\gamma M_{n1}^2 - \frac{\gamma-1}{2}} \quad \dots \textcircled{2}$$

$$M_{n2} = 0.5329$$

So

$$M_2 = \frac{M_{n2}}{\sin(\beta - \theta)} = 3.2123$$

then, since from normal shock relations

$$\frac{P_2}{P_1} = \frac{2\gamma M_{n1}^2 - (\gamma - 1)}{\gamma + 1} \quad \dots \textcircled{3}$$

$$\frac{P_2}{P_1} = 6.0727$$

Region 3

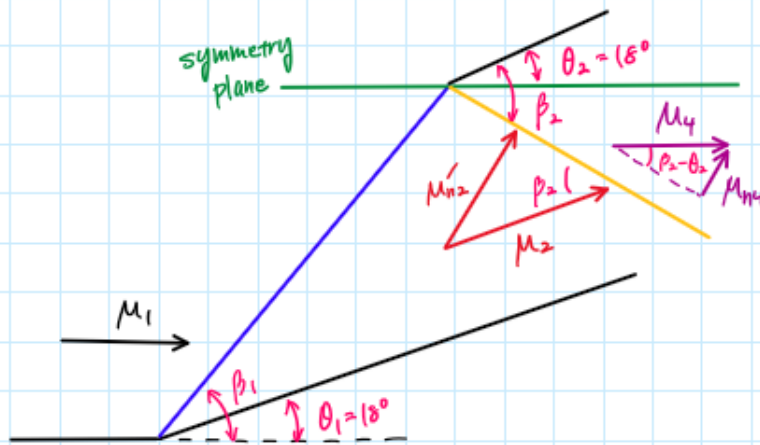
since the structure is symmetric everything is the same as Region 2

$$M_3 = 3.2123$$

$$\frac{P_3}{P_1} = 6.0727$$

Region 4

For this region we acknowledge that the oblique shock occurring at region 3 (or 2) reflect off at the symmetry plane which is parallel to the freestream flow in region 1.



since we know  $M_2$  and  $\theta_2$  from equation ①

$$\beta_2 = 33.9926 \text{ deg}$$

then

$$M_{n2} = M_2 \sin \beta_2 = 1.7960$$

now from equation ②

$$M_{n4} = 0.6174$$

thus,

$$M_4 = \frac{M_{n4}}{\sin(\beta_2 - \theta_2)}$$

$$M_4 = 2.2409$$

finally, from equation ③ using  $M_{n2}$

$$\frac{p_4}{p_2} = 3.5964$$

$$\therefore \frac{p_4}{p_1} = \frac{p_4}{p_2} \cdot \frac{p_2}{p_1} = (3.5964)(6.0727)$$

$$\frac{p_4}{p_1} = 21.8395$$

(b)

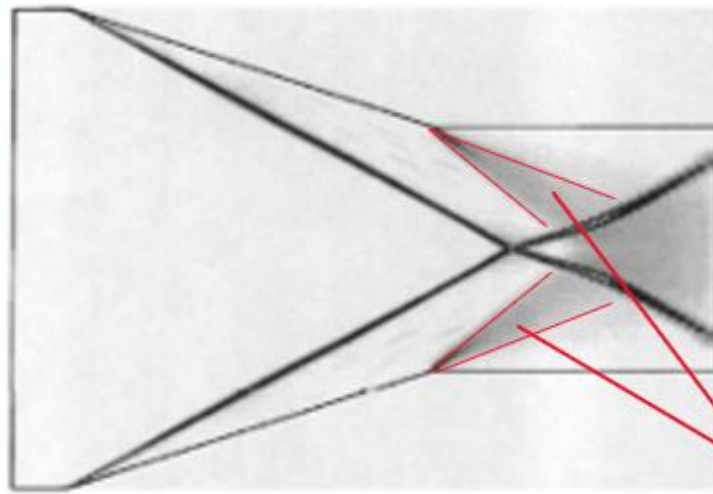
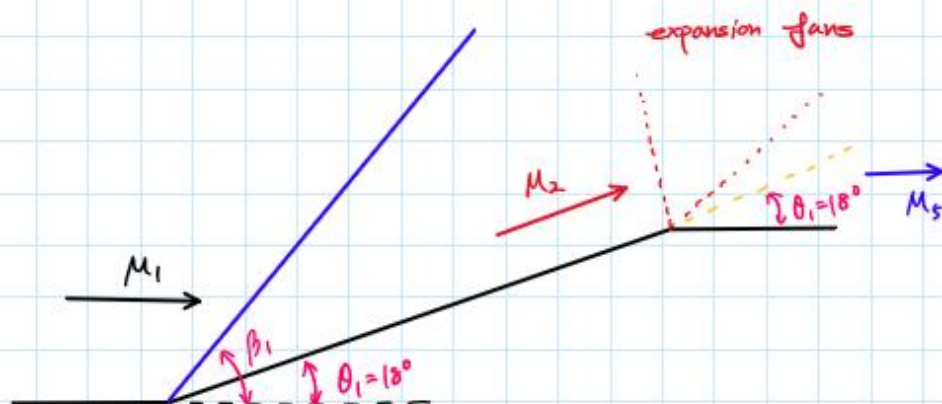


Fig. 12 Shock-wave structure in the 18 x 18 deg interaction.

Expansion Fans



using Prandtl-Meyer function

$$\omega(M_2) = \sqrt{\frac{\gamma+1}{\gamma-1}} \arctan \left[ \sqrt{\frac{\gamma-1}{\gamma+1}} (M_2^2 - 1) \right] - \arctan(M_2^2 - 1)$$

$$\omega(M_2) = 53.6896^\circ$$

then

$$\theta_1 = \omega(M_5) - \omega(M_2)$$

$$\omega(M_5) = 18^\circ + 53.6896^\circ$$

$$\omega(M_5) = 71.6896^\circ$$

now solve the equation above for  $M_5$  &  $z(M_5)$   
then we obtain

$$M_5 = 4.4872$$

and

$$\frac{P_5}{P_2} = \frac{P_0/P_2}{P_0/P_5} = \left( \frac{1 + \frac{\gamma-1}{2} M_2^2}{1 + \frac{\gamma-1}{2} M_5^2} \right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{P_5}{P_2} = \frac{50.3388}{284.8171} = 0.1767$$

$$\therefore \frac{P_5}{P_1} = \frac{P_5}{P_2} \cdot \frac{P_2}{P_1} = (0.1767)(6.0727)$$

$$\frac{P_5}{P_1} = 1.0733$$

### Analysis

It is okay to leave out the expansion fan because under the assumption of the tunnel being large enough the interactions between the oblique shocks and expansion fans do not occur.

2.) [15 pts] A  $21^\circ$  wedge and a  $21^\circ$  cone are both tested in a wind tunnel in air at Mach 2.9 with freestream pressure 17.9 kPa. What is the surface pressure for each experiment?

You should either interpolate on the conical shock charts

(<http://hdl.handle.net/2060/19930091059>) or use the online compressible flow

calculator: <http://www.dept.aoe.vt.edu/~devenpor/aoe3114/calc.html>.

### Wedge

For the wedge we can use the oblique shock relation

$$\tan \theta = 2 \cot \beta \frac{M_1^2 \sin^2 \beta - 1}{M_1^2 (\gamma + \cos 2\beta) + 2}$$

since  $\theta = 21^\circ$  &  $M_1 = 2.9$  (Using MATLAB) we obtain

$$\beta = 39.7950^\circ$$

then  $M_{n1} = M_1 \sin \beta = 1.8561$   
from normal shock relations

$$M_{n2}^2 = \frac{1 + \frac{\gamma-1}{2} M_{n1}^2}{\gamma M_{n1}^2 - \frac{\gamma-1}{2}}$$

$$M_{n2} = 0.6044$$

then  $M_2 = \frac{M_{n2}}{\sin(\beta - \theta)} = 1.8760$

thus,

$$\frac{P_2}{P_1} = 3.8527 \quad \because P_1 = 17.9 \text{ kPa}$$

$$P_2 = 68.9638 \text{ kPa}$$

## Cone

Using the online compressible flow calculator from Virginia Tech for conical shocks we get

$$M_2 = 2.1791$$

and

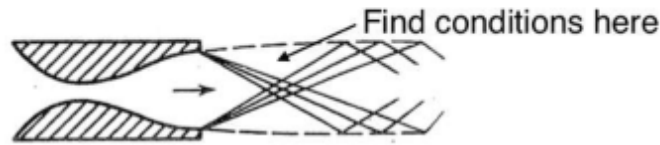
$$\frac{p_2}{p_1} = 2.4456 \quad \therefore p_1 = 17.9 \text{ kPa}$$

$$p_2 = 43.7762 \text{ kPa}$$

## Analysis

The cone has a much smaller pressure distribution meaning that it generates a weaker shock wave compared to a wedge.





3.) [15 pts] A converging-diverging nozzle with a rectangular cross-section and exit area ratio  $A_e/A_t = 4.0$  operates in an under-expanded condition. The working gas is air, the back pressure away from the nozzle exit is  $p_a = 10$  kPa, and the stagnation pressure is  $p_0 = 500$  kPa. The exit pressure is greater than the back pressure ( $p_e > p_a$ ), so expansion waves form outside the exit. For the given conditions, what is the Mach number on the downstream side of the first set of expansion waves, and what is the turning angle?

First, we want to find the properties at the exit.  
Since an underexpanded flow is choked the Mach # at the throat is 1.

$$\frac{A_e}{A_t} = \frac{M_t}{M_e} \left[ \frac{1 + (\frac{\gamma-1}{2})M_e^2}{1 + (\frac{\gamma-1}{2})M_t^2} \right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

Solving this with **MATLAB** (code in **Appendix**) we retrieve

$$M_e = 2.9402$$

For supersonic condition.

Now, from isentropic relations,

$$p_e = p_0 \left[ 1 + \frac{\gamma-1}{2} M_e^2 \right]^{-\frac{\gamma}{\gamma-1}} \quad \because p_0 = 500 \text{ kPa}$$

$$p_e = 14.893 \text{ kPa}$$

$$p_e > p_a = 10 \text{ kPa} \quad \text{is TRUE}$$

The pressure ratio for before and after the expansion fans become

$$\frac{\text{after}}{\text{before}} \Rightarrow \frac{p_a}{p_e} = \frac{10 \text{ kPa}}{14.893 \text{ kPa}} = 0.6714$$

thus, from isentropic relations

$$\frac{P_0}{P_a} = \left(1 + \frac{\gamma-1}{2} M_2^2\right)^{\frac{\gamma}{\gamma-1}}$$

solve this for  $M_2$  and we get the Mach # after the expansion fans, we obtain

$$M_2 = 3.2077$$

now from Prandtl-Meyer function

$$\nu(M) = \sqrt{\frac{\gamma+1}{\gamma-1}} \arctan\left[\sqrt{\frac{\gamma-1}{\gamma+1}} (M^2 - 1)\right] - \arctan(M^2 - 1)$$

$$\nu(M_e) = 48.5898^\circ$$

$$\nu(M_2) = 53.6078^\circ$$

$$\therefore \theta = \nu(M_2) - \nu(M_e)$$

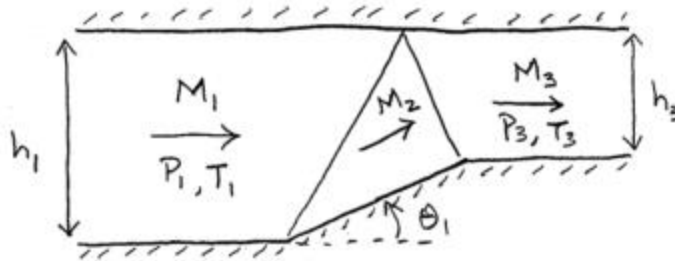
$$\theta = 53.6078^\circ - 48.5898^\circ$$

$$\theta = 5.0180^\circ$$

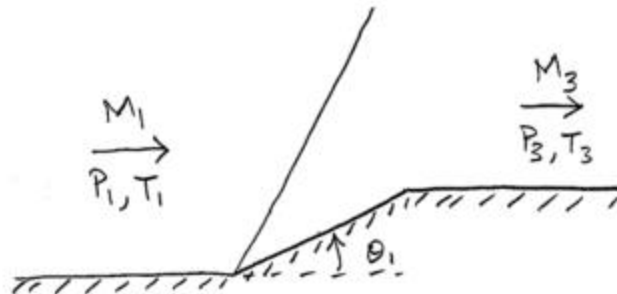
4.) [35 pts]

- (a) Consider an inviscid air flow in a channel. The Mach number is  $M_1 = 2$ , the pressure is  $P_1 = 1$  atm, and the temperature is  $T_1 = 300$  K. The lower wall makes a turn of  $\theta_1 = 10^\circ$ , as shown below. The flow behind the resulting oblique shock has a Mach number  $M_2$ , pressure  $P_2$ , and temperature  $T_2$ . Determine the values of  $M_2$ ,  $P_2$  and  $T_2$ . The oblique shock reflects off the upper wall of the channel, which is parallel to the lower wall. The reflected shock then intersects the turned lower wall at a corner where the flow is turned straight again, as shown below. **Determine** the Mach number,  $M_3$ , pressure,  $P_3$ , the temperature,  $T_3$ , and the stagnation pressure,  $P_{0,3}$ , downstream of the reflected shock.
- (b) The height of the channel upstream of the first oblique shock is  $h_1 = 1$  m. Determine the height of the channel downstream of the reflected shock,  $h_3$ .
- (c) Now assume that the upper wall of the channel is moved very far away so that within the region of interest the shock is not reflected. The flow behind the oblique shock turns a corner and flows straight again, as shown below. Determine the Mach number,  $M_3$ , pressure,  $P_3$ , the temperature,  $T_3$ , and the stagnation pressure,  $P_{0,3}$ , in the straight section downstream of the corner at the top of the ramp made by the lower wall.

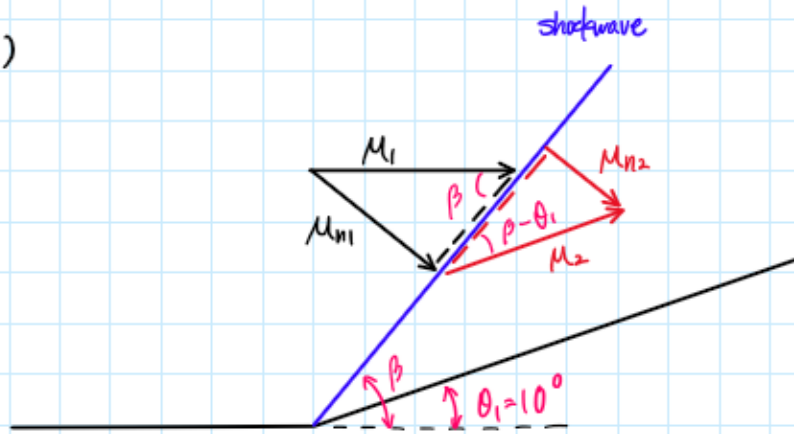
(a), (b)



(c)



(a)



Calculating the shockwave angle from  $\theta$ - $\beta$ - $M$  relation

$$\tan \theta = 2 \cot \beta \frac{M_1^2 \sin^2 \beta - 1}{M_1^2 (\gamma + \cos 2\beta) + 2} \quad \dots \textcircled{1}$$

using **MATLAB** (code in **Appendix**) we obtain

$$\beta = 39.3139 \text{ deg}$$

then  $M_{n1} = M_1 \sin \beta = 1.2671$

now, from normal shock relations

$$M_{n2}^2 = \frac{1 + \frac{\gamma-1}{2} M_{n1}^2}{\gamma M_{n1}^2 - \frac{\gamma-1}{2}} \quad \dots \textcircled{2}$$

$$M_{n2} = 0.8032$$

So

$$M_2 = \frac{M_{n2}}{\sin(\beta - \theta)} = 1.6405$$

then, since from normal shock relations

$$\frac{P_2}{P_1} = \frac{2\gamma M_{n1}^2 - (\gamma - 1)}{\gamma + 1} \quad \dots \textcircled{3}$$

$$\frac{P_2}{P_1} = 1.7066$$

$$\therefore P_2 = (1.7066)(101.325 \text{ kPa})$$

$$P_2 = 172.92 \text{ kPa}$$

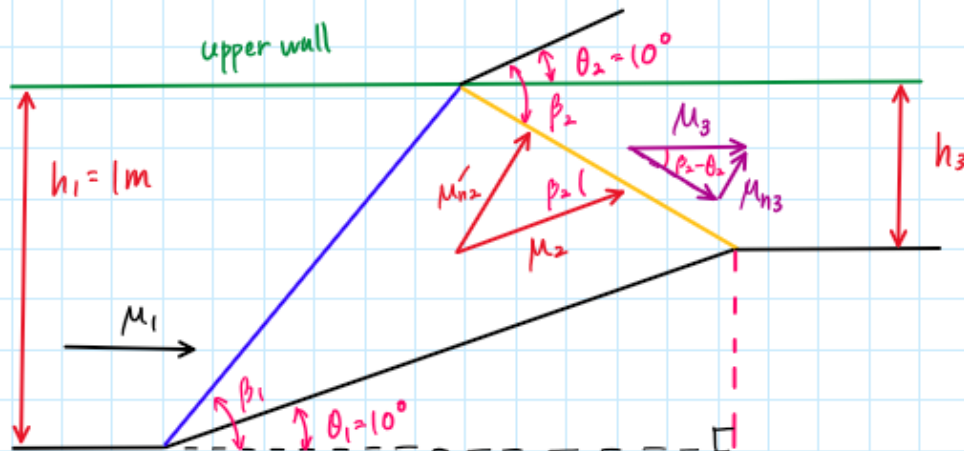
similarly, from normal shock relations

$$\frac{T_2}{T_1} = \left[ 1 + \frac{2\gamma}{\gamma+1} (M_{n1}^2 - 1) \right] \frac{2 + (\gamma-1) M_{n1}^2}{(\gamma+1) M_{n1}^2} \dots \textcircled{4}$$

$$\frac{T_2}{T_1} = 1.1702$$

$$\therefore T_2 = (1.1702)(300 \text{ K})$$

$$T_2 = 351.0454 \text{ K}$$



since we know  $M_2$  and  $\theta_2$ , from equation ①

$$\beta_2 = 49.3840 \text{ deg}$$

then

$$M_{n2}' = M_2 \sin \beta_2 = 1.2453$$

now from equation (2)

$$\mu_{h3} = 0.8153$$

thus,

$$\mu_3 = \frac{\mu_{n3}}{\sin(\beta_2 - \theta_2)}$$

$$\mu_3 = 1.2849$$

next, from equation ③ using  $M_{n2}$

$$\frac{P_3}{P_2} = 1.6426$$

also, from equation ④ using  $M_{n2}$

$$\frac{T_3}{T_2} = 1.1564$$

the flow tangential to the obli  
thus,

$$P_3 = (1.6426)(172.92 \text{ kPa})$$

$$P_3 = 284.03 \text{ kPa}$$

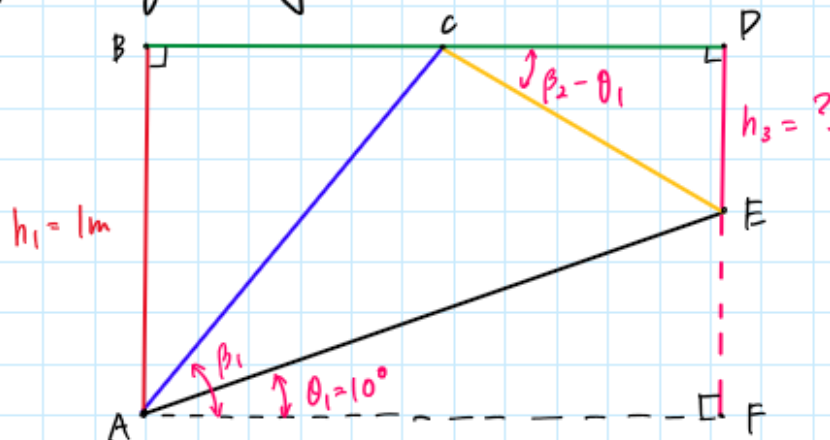
and

$$T_3 = (1.1564)(351.0454 \text{ K})$$

$$T_3 = 405.9576 \text{ K}$$

(b)

Basic Trigonometry



$$\angle BAC = 90^\circ - \beta_1 = 90^\circ - 39.3139^\circ$$

$$\angle BAC = 50.6861^\circ$$

$$\overline{BC} = h_1 \tan \angle BAC = 1.2212 \text{ m}$$

$$\overline{AC} = h_1 / \cos \angle BAC = 1.5784 \text{ m}$$

$$\angle CAE = \beta_1 - \theta_1 = 29.3139^\circ$$

$$\angle ECA = 180^\circ - (\beta_2 - \theta_1) - \beta_1 = 101.3020^\circ$$

$$\angle AEC = (\beta_2 - \theta_1) + \theta_1 = \beta_2 = 49.3840^\circ$$

then,

$$\frac{\overline{AC}}{\sin \angle AEC} = \frac{\overline{AE}}{\sin \angle ECA}$$

$$\overline{AE} = \frac{\sin \angle ECA}{\sin \angle AEC} \overline{AC} = 2.0390 \text{ m}$$

thus,

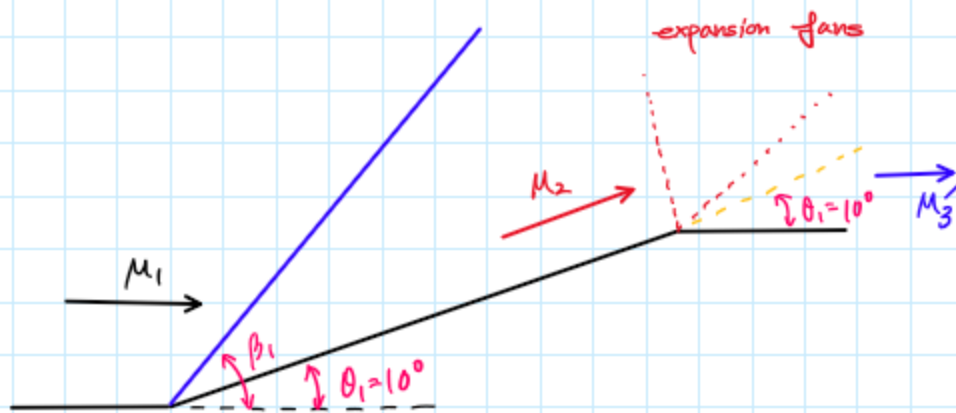
$$\overline{EF} = \overline{AE} \sin \theta_1 = 0.3541 \text{ m}$$

finally,

$$h_3 = h_1 - \overline{EF}$$

$$h_3 = 0.6459 \text{ m}$$

(c)



using Prandtl-Meyer function

$$\omega(M_2) = \sqrt{\frac{\gamma+1}{\gamma-1}} \arctan \left[ \sqrt{\frac{\gamma-1}{\gamma+1}} (M_2^2 - 1) \right] - \arctan(M_2^2 - 1)$$

$$\omega(M_2) = 16.0581^\circ$$

then

$$\theta_1 = \angle(M'_3) - \angle(M_2)$$

$$\angle(M'_3) = \theta_1 + \angle(M_2)$$

$$\angle(M'_3) = 26.0581^\circ$$

now solve the equation above for  $M_3$  &  $\angle(M_3)$   
then we obtain

$$M'_3 = 1.9884$$

and

$$\frac{P'_3}{P_2} = \frac{P_1/P_2}{P_0/P'_3} = \left( \frac{1 + \frac{\gamma-1}{2} M_2^2}{1 + \frac{\gamma-1}{2} M_3'^2} \right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{P'_3}{P_2} = \frac{4.5145}{7.6840} = 0.5875$$

$$\therefore P'_3 = (0.5875)(172.92 \text{ kPa})$$

$$P'_3 = 101.59 \text{ kPa}$$

similarly,

$$\frac{T'_3}{T_2} = \frac{T_0/T_2}{T_0/T'_3} = \left( \frac{1 + \frac{\gamma-1}{2} M_2^2}{1 + \frac{\gamma-1}{2} M_3'^2} \right)$$

$$\frac{T'_3}{T_2} = \frac{1.5383}{1.7907} = 0.8590$$

$$T'_3 = (0.8590)(351.0454 \text{ K})$$

$$T'_3 = 301.5567 \text{ K}$$

$$P_{03}' = \frac{P_0}{P'_3} P'_3 = (7.6840)(101.59 \text{ kPa})$$

$$P_{03}' = 780.64 \text{ kPa}$$



## Appendix

### AAE 334 HW10

```
clear all; close all; clc;
```

```
P1
```

```
% (a)
```

```
% Given Properties
```

```
M1 = 5;
```

```
theta = 18; % [deg]
```

```
gamma = 1.4;
```

```
% Region 2
```

```
beta = theta_beta_M_relation(theta,M1,gamma);
```

```
Mn1 = M1*sind(beta);
```

```
Mn2 = normalShock_jump_M(Mn1,gamma);
```

```
M2 = Mn2/sind(beta-theta);
```

```
P2_P1 = normalShock_jump_P_static(Mn1,gamma)
```

```
% Region 4
```

```
beta2 = theta_beta_M_relation(theta,M2,gamma);
```

```
Mn2_new = M2*sind(beta2);
```

```
Mn4 = normalShock_jump_M(Mn2_new,gamma);
```

```
M4 = Mn4/sind(beta2 - theta);
```

```
P4_P2 = normalShock_jump_P_static(Mn2_new,gamma);
```

```
P4_P1 = P4_P2*P2_P1;
```

```
% (b) Expansion fan
```

```
nu_M2 = Prandtl_Meyer_Expansion(M2,gamma);
```

```
nu_M5 = theta + nu_M2;
```

```
M5 = calc_M_from_PrantlMeyer(nu_M5,gamma);
```

```
P0_P2 = isentropic_relation_P_ratio(M2,gamma);
```

```
P0_P5 = isentropic_relation_P_ratio(M5,gamma);
```

```
P5_P2 = P0_P2/P0_P5;
```

```
P5_P1 = P5_P2*P2_P1;
```

```
P2
```

```
% Given Properties
```

```
M1 = 2.9;
```

```
theta = 21; % [deg]
```

```
gamma = 1.4;
```

```
P1 = 17.9; % [kPa]
```

```
beta = theta_beta_M_relation(theta,M1,gamma)
```

```
Mn1 = M1*sind(beta)
```

```
Mn2 = normalShock_jump_M(Mn1,gamma)
```

```
M2 = Mn2/sind(beta-theta)
```

```
P2_P1 = normalShock_jump_P_static(Mn1,gamma)
```

```
P2 = P2_P1*P1
```

```

P3
% Exit condtions
Ae_At = 4.0;
At = 1;
Ae = Ae_At*At;
Mt = 1;
gamma = 1.4;
P0 = 500e3; % [Pa]
Pa = 10e3; % [Pa]

[Me_sub, Me_sup] = M_for_area_ratio(At,Ae,Mt,gamma);
Me = Me_sup;
Pe = p_from_M_and_gamma(P0,Me,gamma,"static");
Pa_Pe = Pa/Pe;
M2 = M_from_P_ratio(P0,Pa,gamma)

% Expansion angles
nu_Me = Prandtl_Meyer_Expansion(Me,gamma)
nu_M2 = Prandtl_Meyer_Expansion(M2,gamma)
theta = nu_M2 - nu_Me

P4
% Defining the given properties
M1 = 2;
P1 = 101325; % [Pa]
theta1 = 10; % [deg]
T1 = 300;
T0 = 1; % dummy
gamma = 1.4;

% Region 2
beta12 = theta_beta_M_relation(theta1,M1,gamma)
Mn1 = M1*sind(beta12)
Mn2 = normalShock_jump_M(Mn1,gamma)
M2 = Mn2/sind(beta12-theta1)
P2_P1 = normalShock_jump_P_static(Mn1,gamma)
P2 = P1*P2_P1
T2_T1 = normalShock_jump_T_static(Mn1,gamma)
T2 = T2_T1*T1

% Region 3'
theta2 = theta1;
beta23 = theta_beta_M_relation(theta2,M2,gamma)
Mn2p = M2*sind(beta23)
Mn3 = normalShock_jump_M(Mn2p,gamma)
M3 = Mn3/sind(beta23 - theta2)
P3_P2 = normalShock_jump_P_static(Mn2p,gamma)
T3_T2 = normalShock_jump_T_static(Mn2p,gamma)
P3 = P3_P2*P2
T3 = T3_T2*T2
P03 = p_from_M_and_gamma(P3,M3,gamma,"stagnation")

```

```
% (b)
h1 = 1;
ang_BAC = 90 - beta12;
BC = h1*tand(ang_BAC);
AC = h1/cosd(ang_BAC);
ang_CAE = beta12 - theta1;
ang_ECA = 180 - (beta23 - theta1) - beta12;
ang_AEC = beta23;
AE = sind(ang_ECA)/sind(ang_AEC)*AC;
EF = AE*sind(theta1);
h3 = h1 - EF;
```

```
% (c) Expansion fan
nu_M2 = Prandtl_Meyer_Expansion(M2,gamma)
nu_M3p = theta1 + nu_M2
M3p = calc_M_from_PrantlMeyer(nu_M3p,gamma)
P0_P2 = isentropic_relation_P_ratio(M2,gamma)
P0_P3p = isentropic_relation_P_ratio(M3p,gamma)
P3p_P2 = P0_P2/P0_P3p
P3p = P3p_P2*P2

T0_T2 = isentropic_relation_T_ratio(M2,gamma)
T0_T3p = isentropic_relation_T_ratio(M3p,gamma)
T3p_T2 = T0_T2/T0_T3p
T3p = T3p_T2*T2

P03p = P0_P3p*P3p
```

Functions

```
function M = calc_M_from_PrantlMeyer(nu,gamma)
    M = sym('M');
    assume(M,["real","positive"]);
    a1 = sqrt((gamma + 1)/(gamma - 1));
    a2 = atand(a1^(-1)*sqrt(M^2 - 1));
    a3 = atand(sqrt(M^2 - 1));
    eqn = nu == a1*a2 - a3;
    M = double(vpasolve(eqn,M));
    if M < 0
        M = -M;
    end
end

function P_rat = isentropic_relation_P_ratio(M,gamma)
    P_rat = (1 + (gamma - 1)/2*M^2)^(gamma/(gamma - 1));
end

function T_rat = isentropic_relation_T_ratio(M,gamma)
    T_rat = (1 + (gamma - 1)/2*M^2);
end

function [M2_sub, M2_sup] = M_for_area_ratio(A1,A2,M1,gamma)
```

```

% Calculate the Mach number at the inlet
M2 = sym('M2');
assume(M2,["real","positive"])
a1 = 1 + (gamma - 1)/2*M2^2;
a2 = 1 + (gamma - 1)/2*M1^2;
a3 = (gamma + 1)/2/(gamma - 1);
eqn = A2/A1 == M1/M2 * (a1/a2)^(a3);
M2 = double(vpasolve(eqn,M2));
M2 = M2(M2 == real(M2));
M2_sub = min(M2);
M2_sup = max(M2);
end

function M = M_from_P_ratio(P0,P,gamma)
a1 = 2/(gamma - 1);
a2 = (P0/P)^((gamma - 1)/gamma);
M = sqrt(a1*(a2 - 1));
end

function T_rat = normalShock_jump_T_static(M1,gamma)
%{
    Function:    normalShock_jump_T_stati
    Author:      Tomoki Koike
    Description: This function calculates the static pressure ratios
                  before and after a normal shockwave.

    >>Inputs
        M1:      Mach number before shockwave
        gamma:    specific hear ratio
    Outputs<<
        T_rat:    static temperature ratio
%}
a1 = 1 + 2*gamma*(M1^2 - 1)/(gamma + 1);
a2 = 2 + (gamma - 1)*M1^2;
a3 = (gamma + 1)*M1^2;
T_rat = (a1)*(a2)/(a3);
end

function P_rat = normalShock_jump_P_static(M1,gamma)
%{
    Function:    normalShock_jump_P_static
    Author:      Tomoki Koike
    Description: This function calculates the static pressure ratios
                  before and after a normal shockwave.

    >>Inputs
        M1:      Mach number before shockwave
        gamma:    specific hear ratio
    Outputs<<
        P_rat:    static pressure ratio
%}

P_rat = (2*gamma*M1^2 - (gamma - 1))/(gamma + 1);
end

```

```
function M2 = normalShock_jump_M(M1,gamma)
    %{
        Function:    normalShock_jump_M
        Author:      Tomoki Koike
        Description: This function calculates the Mach number jump after a normal
        shockwave.
        >>Inputs
            M1:      Mach number
            gamma:    specific hear ratio
        Outputs<<
            M2: Mach number after shock
    %}
    a1 = (gamma - 1)/2;
    M2 = sqrt((1 + a1*M1^2)/(gamma*M1^2 - a1));
end
```

```
function nu = Prandtl_Meyer_Expansion(M,gamma)
    %{
        Function:    Prandtl_Meyer_Expansion
        Author:      Tomoki Koike
        Description: This function calculates the Prandtl-Meyer function results
        for
            a given flow with given Mach number to find the
            expansion fan relations
        >>Inputs
            M1:      Mach number before expansion fan
            gamma:    specific hear ratio
        Outputs<<
            nu:      Prandtl-Meyer function result [deg]
    %}
    a1 = sqrt((gamma + 1)/(gamma - 1));
    a2 = atand(a1^(-1)*sqrt(M^2 - 1));
    a3 = atand(sqrt(M^2 - 1));
    nu = a1*a2 - a3;
end
```

```
function p2 = p_from_M_and_gamma(p1, M, gamma, type)
    if type == "stagnation"
        p2 = p1 * (1 + (gamma - 1) / 2 * M^2)^(gamma/(gamma - 1));
    elseif type == "static"
        p2 = p1 / (1 + (gamma - 1) / 2 * M^2)^(gamma/(gamma - 1));
    else
        disp("Error. Incorrect type. Type can only be 'stagnation' or 'static'.")
    end
end
```