Problem 1. (10pts) Show that $\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) + \mathbf{v} \times (\mathbf{w} \times \mathbf{u}) + \mathbf{w} \times (\mathbf{u} \times \mathbf{v}) = \mathbf{0}$.

Problem 2. (20pts) Two particles moving with constant velocity are described by the position vectors:
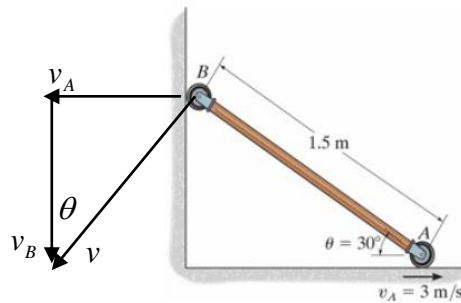$$\mathbf{p} = \mathbf{p_0} + \mathbf{v}t, \quad \mathbf{s} = \mathbf{s_0} + \mathbf{w}t$$
**a)** Show that the shortest distance between their trajectories is given by
$$d = \frac{|(\mathbf{s_0} - \mathbf{p_0}) \cdot (\mathbf{w} \times \mathbf{v})|}{|\mathbf{w} \times \mathbf{v}|}$$
**b)** Find the shortest distance between the particles themselves.

Problem 3. (15pts) If a roller A moves to the right with a constant velocity of v_A=3m/s, determine the angular velocity of the link and the velocity of the roller B when θ=30 deg.


Problem 4. (15pts) Determine the angular acceleration of link BC and the acceleration of the piston C at the instant shown below where the wheel has angular velocity of $\omega = 6\text{rad/s}$ and angular acceleration $\alpha = 3rad/s^2$.
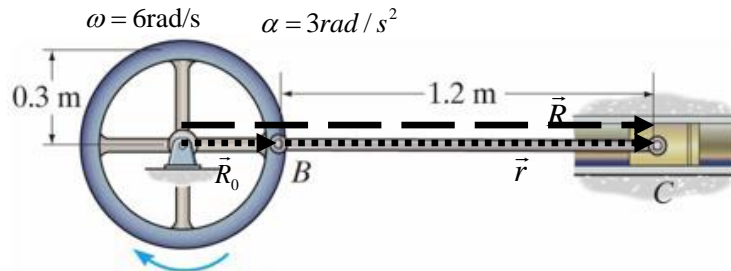
Problem 5. (20pt) An observer positioned at 30 degrees North latitude on the surface of the earth tracks a satellite.

z, local North

x Moving Frame
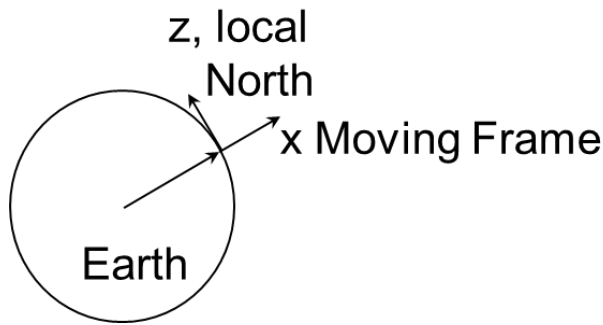
Earth

You may assume that the inertial frame of reference for the problem is attached to the center of the earth, but doesn't rotate with the earth. The z_f axis passes through the North Pole of the earth. The x_f axis passes through the equator at a point nearest the observer. You may assume that the radius of the earth is 4000 miles.

z_f Inertial Frame

z, local North

x Moving Frame

Earth

x_f Inertial Frame
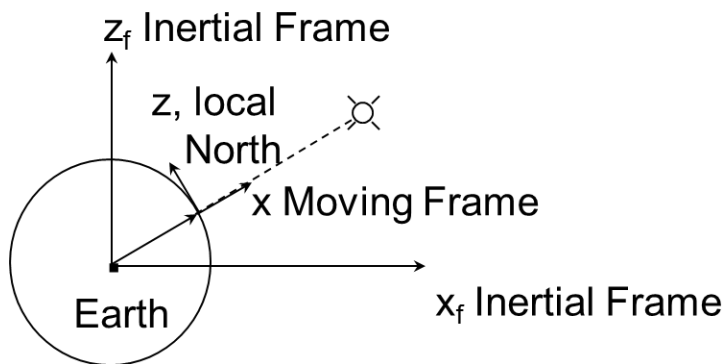
At a particular instant, the satellite appears to be 1000 miles directly above him and, by his observations, appears to be traveling 5000 miles per hour due North (local North). What is the absolute velocity of the satellite in terms of the unit vectors of the moving coordinate system within which the observer resides.

Problem 6. (20pt) Expand the equations of motion for translation $m(\dot{V}|_{body} + \omega \times V) = F$ and rotation $I\dot{\omega} + \omega \times (I\omega) = M$ into components in the body-fixed reference frame where $V = u\hat{i} + v\hat{j} + w\hat{k}$, $\omega = p\hat{i} + q\hat{j} + r\hat{k}$, $F = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$, $M = L\hat{i} + M\hat{j} + N\hat{k}$ and I is the moment of inertia matrix given by

$$I = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{yz} & I_{yy} & -I_{yz} \\ -I_{zx} & -I_{zy} & I_{zz} \end{bmatrix}.$$