**AAE 532 – Orbit Mechanics**
**Problem Set 10**
**Due: 2459194.1875 (UT)**

**Problem 1:** Recall the example problem that was discussed in class concerning a small robotic explorer sent to the Martian system to observe and characterize the two moons – Phobos and Deimos. The Martian moons Phobos and Deimos are assumed to be in circular and coplanar about Mars, with a radius equal to the semi-major axis listed in the Table of Constants for moons and dwarfs under Supplementary Documents on Brightspace. Let's again assume that the spacecraft has completed its observations in the orbit of Phobos and must transfer to the orbit of Deimos. But, now an option for a transfer with different characteristics is sought. [Assume that it is reasonable to assume a relative two-body problem and consider only the gravity of Mars.]

(a) In the class example, recall that the planned transfer is based on a $240^{\circ}$ transfer angle and a minimum energy transfer. But, recall that a wide variety of elliptical arcs could be used to connect these two orbits. Perhaps the maneuver costs could be improved by extending the transfer time. Use the space triangle, but try to extend the transfer time to 15 hours.
Produce the transfer and include the following: $type, a, p, e, \boldsymbol{\mathcal{E}}, v_{dep}, v_{arr}, \theta^*_{dep}, \theta^*_{arr}, \gamma_{dep}, \gamma_{arr}$. As usual, supply all the appropriate justifications for these results. Include the $r_p$ and $r_a$ distances for the transfer ellipse. Does the difference in the true anomalies equal the transfer angle?

(b) Determine the maneuvers at departure and arrival, i.e., $|\Delta\bar{v}|$ and $\alpha$. Transform the maneuvers to VNB coordinates. How do the maneuvers compare to the minimum energy transfer in terms of time and total maneuver cost?

(c) Plot the transfer in GMAT. Plot a full revolution of the spacecraft orbit as the circular orbit of Phobos. Then apply the departure maneuver. Upon arrival, implement the arrival maneuver; end with a complete revolution in the final orbit (i.e., the circular orbit of Deimos.

Does the transfer pass through periapsis or apoapsis?

(d) To implement such a transfer and rendezvous with Deimos, it is necessary to phase the departure correctly. What is the required phase angle between Phobos and Deimos at departure? How often does the correct phase angle recur (in hours)? Compare this result to the periods of Phobos and Deimos.

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**Problem 2:** Recall the 2015 movie "The Martian" where the character Mark Watney must be rescued from Mars. The astrodynamicist designs the critical transfer trajectory to send the spacecraft Hermes from Earth to Mars and enable the rescue. [Are there any other movies where the astrodynamicist is the star and 'saves the day'?!!]

Recall that Rich Purnell (the astrodynamicist) spent extensive time exploring Lambert arcs and incorporating an Earth gravity assist that could satisfy all the requirements! Let's explore the possible Earth-to-Mars transfer arcs. Assume that Earth and Mars move along circular coplanar orbits. For the Earth-to-Mars transfer, initially ignore the local fields; the relative two-body problem involves only solar gravity.

(a) Consider first a transfer with a transfer angle of 120 degrees and a time of flight of 160 days. Given this space triangle, is the transfer elliptic or hyperbolic? A transfer of what type then emerges?

(b) Produce the transfer and include the following:
$type, a, p, e, \boldsymbol{\mathcal{E}}, v_{dep}, v_{arr}, \theta^*_{dep}, \theta^*_{arr}, \gamma_{dep}, \gamma_{arr}$. As usual, supply all the appropriate justifications for these results. Include the $r_p$ and $r_a$ distances for the transfer ellipse. Does the difference in the true anomalies equal the transfer angle?

(c) Determine the maneuvers at departure $|\Delta\bar{v}_{dep}|, \alpha_{dep}$ and arrival $|\Delta\bar{v}_{arr}|$, $\alpha_{arr}$. Transform the maneuvers to VNB coordinates.

(d) Plot the transfer using either GMAT or Matlab. (Recall that GMAT gives you a chance to check your results.) Include the orbit of Earth; then apply the maneuver. After the suitable transfer angle, apply the second maneuver. Include the Mars orbit as well.

(f) Now consider the Earth local field. [In the movie, a vehicle was launched from Earth to rendezvous with the Hermes rescue vehicle at Earth closest approach to receive additional supplies for the return trip to Mars.]
The transfer computed in (b)-(c) requires a $\bar{v}_{\infty/\oplus}$ relative to Earth in the Earth local view. What is the magnitude of this $\bar{v}_{\infty/\oplus}$? Assume that the pass distance at the Earth is required to be at 1000 km altitude. What is the velocity magnitude at closest approach along the hyperbolic path?
Of course, a diagram of the local view is necessary. Should the rescue vehicle pass ahead or behind Earth?
Is it reasonable to attempt to rendezvous with the rendezvous vehicle moving at such a speed?

(g) In the Mars local field, what is the speed at periapsis if the Mars pass distance is 500 km altitude? [Recall that Mark Watney was required to achieve nearly this necessary speed as a result of his launch from the Martian surface to be rescued…hmmmm…..good thing that it is only a movie!]

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**Problem 3:** Recall Problem 2. We are still trying to get to Mars to save Mark Watney—yep, we are still using the 2015 movie to explore that challenges of getting to Mars! Following up on Problem 2, consider the fact that the trip requires a significant time interval and survival on the Mars surface while awaiting rescue, Watney is running out of food. This time try to speed up the trip to Mars. Still assume all the same conditions as in Problem 2, i.e., use the SAME space triangle. But reduce the trip time--

(a) Consider a transfer with a transfer angle of 120 degrees and a time of flight of 92 days. Given this space triangle, is the transfer elliptic or hyperbolic? A transfer of what type then emerges?

Repeat the steps in Prob 2 for this new time:

(b) Produce the transfer and include the following:
$type, a, p, e, \boldsymbol{\mathcal{E}}, v_{dep}, v_{arr}, \theta^*_{dep}, \theta^*_{arr}, \gamma_{dep}, \gamma_{arr}$. As usual, supply all the appropriate justifications for these results. Include the $r_p$ and $r_a$ distances as determined. Does the difference in the true anomalies equal the transfer angle?

(c) Determine the maneuvers at departure $|\Delta\bar{v}_{dep}|, \alpha_{dep}$ and arrival $|\Delta\bar{v}_{arr}|$, $\alpha_{arr}$. Transform the maneuvers to VNB coordinates.

(d) Plot the transfer using either GMAT or Matlab. (Recall that GMAT gives you a chance to check your results.) Include the orbit of Earth; then apply the maneuver. After the suitable transfer angle, apply the second maneuver. Include the Mars orbit as well.

(f) Now consider the Earth local field. [In the movie, a vehicle was launched from Earth to rendezvous with the Hermes rescue vehicle at Earth closest approach to receive additional supplies for the return trip to Mars.] The transfer computed in (b)-(c) requires a $\bar{v}_{\infty/\oplus}$ relative to Earth in the Earth local view. What is the magnitude of this $\bar{v}_{\infty/\oplus}$? Assume that the pass distance at the Earth is required to be at 1000 km altitude. What is the velocity magnitude at closest approach along the hyperbolic path?
Of course, a diagram of the local view is necessary. Should the rescue vehicle pass ahead or behind Earth to pick up the supplies?
Is it reasonable to attempt to rendezvous with the rescue vehicle moving at such a speed?

(g) In the Mars local field, what is the speed at periapsis if the Mars pass distance is 500 km altitude? Discuss: Is it likely that Watney could have reached this speed in his launch from the Mars surface?

Is it reasonable to try to reach Mars in 92 days?

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**LAST Problem for the Semester**

**Problem 4:** Recall Problems 2 and 3. Both problems considered the challenge of an Earth-to-Mars transfer that was reasonably efficient in terms of DV but also arriving as quickly as possible by reducing TOF. Both Problems 2 and 3 employed the same space triangle but used different TOFs.

For the final Problem of the semester, try to further explore the DV vs TOF trade-off. Examine one more transfer and assess whether you can improve on the DV cost for an different TOF.

(a) Focus on one of two options: (a) You can define the same space triangle and use a new TOF; OR (b) try a new space triangle and one of the TOFs in Prob 2 or Prob 3. [You cannot use a Hohmann or bielliptic transfer!]

Justify your space triangle / TOF combination. Why do you think your choices might yield a better result?

(b) Produce results consistent with the results in the earlier problems. Assess and discuss your results:

Did your new combination produce 'improved' results?

Why did the DV improve? What dynamically changed about the transfer that lead to improvements?

OR

Why was there not an improvement? What dynamic conditions during the transfer likely resulted in a less desirable transfer?

(c) What is the next step? What combination would you try next time? Why?

**[Note: You only have to try one more transfer and demonstrate the results. *To support an actual mission, many combinations are compared!*]**