College of Engineering
School of Aeronautics and Astronautics

AAE 532
Orbital Mechanics

PS 8
Transfers with Local Gravity Fields

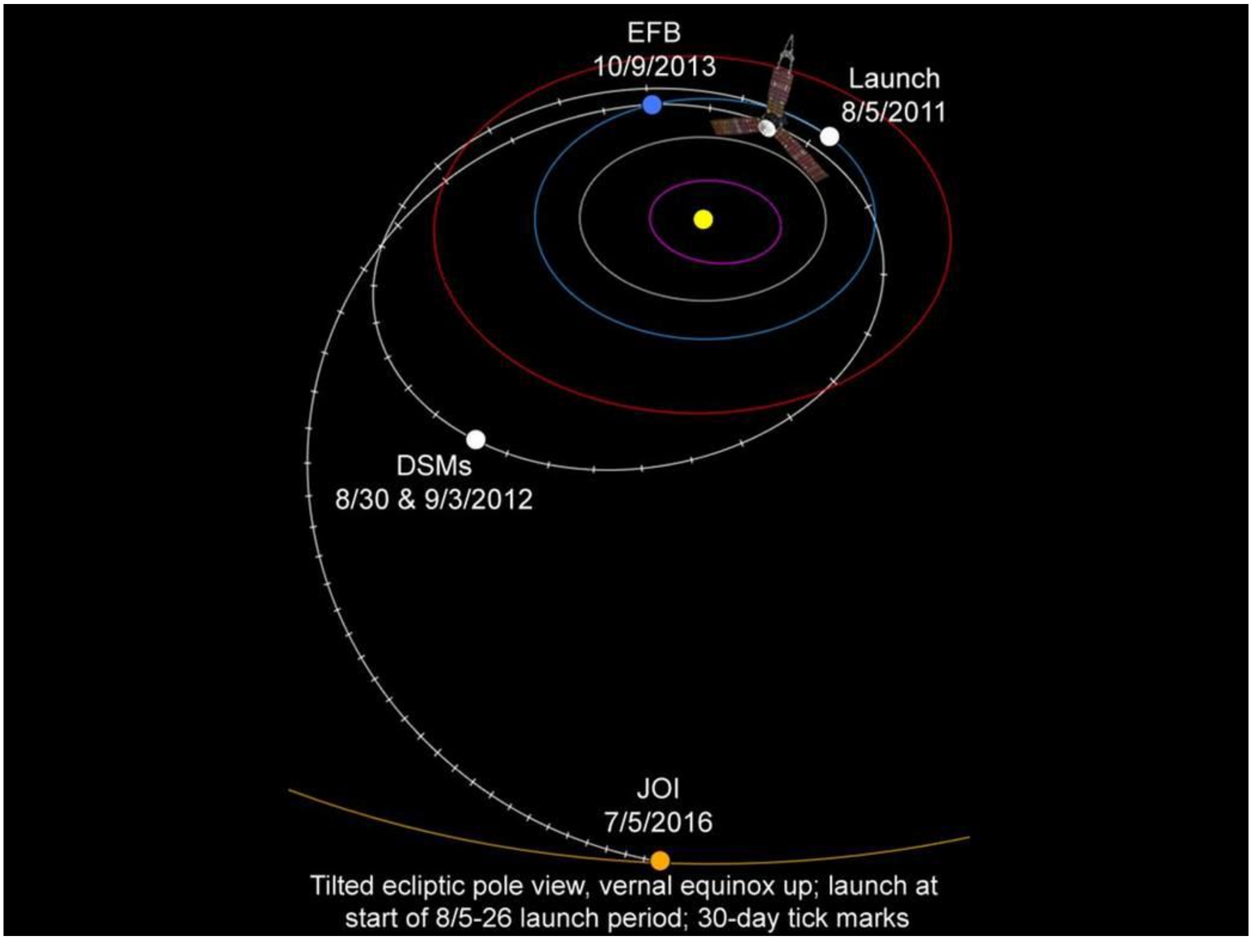
Author:
Tomoki Koike

Supervisor:
K. C. Howell

November 6th, 2020 Friday
Purdue University
West Lafayette, Indiana

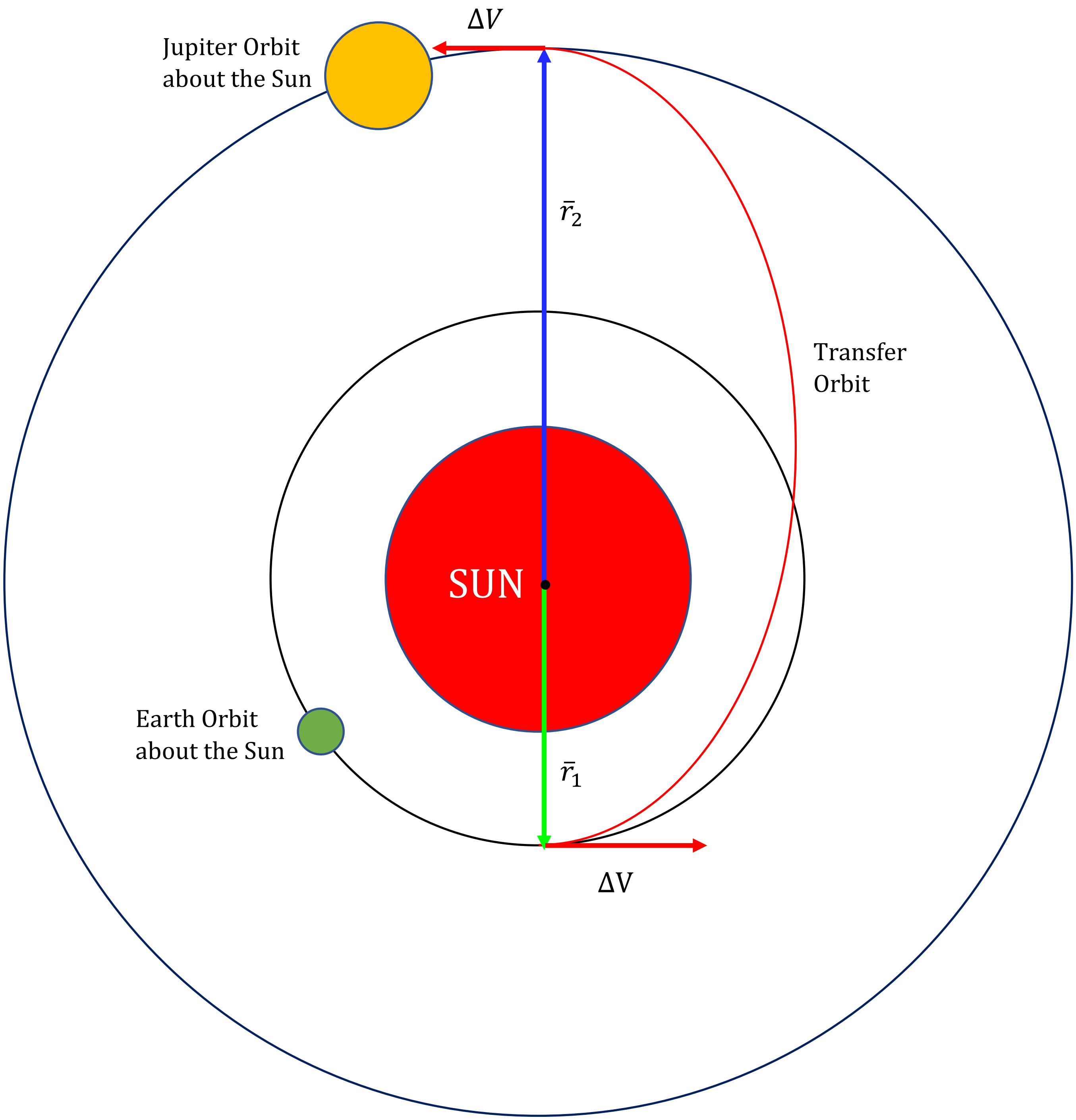
Problem 1:

Recall Problem 2 in PS7. You computed the cost $|\Delta\bar{v}|$ and TOF associated with a Hohmann transfer to Jupiter. But, in that preliminary analysis, you neglected the local gravity fields.



(a) Re-examine the Hohmann transfer but include the local fields. Assume the planetary orbits are circular; the Earth dark-side departure is from a 250 km altitude parking orbit. For the Juno spacecraft, the eventual science orbit at Jupiter was very close to the planet. So, assume that arrival at Jupiter occurs on the light side and the spacecraft is captured into a circular Jovian orbit of radius $2.8R_{jupiter}$. Of course, include all diagrams representing the local views. Compare the results with the cost Δv_{dep}, Δv_{arr}, and Δv_{total} as well as TOF in PS7. [The Earth departure maneuver is Δv_{dep}; then the maneuver to capture at Jupiter is Δv_{arr}.] Does adding the local fields impact the total $|\Delta\bar{v}|$? Does the inclusion of the local fields increase or decrease the cost? Does the maneuver cost increase or decrease at Earth? At Jupiter?

Hohmann Transfer from Earth to Jupiter:



Two-Body Problem #1 (near Earth):

The gravitational parameter and eccentricity for the Earth orbit (around the Sun) are

$$\mu_{\oplus} = 3.9860\mathrm{e}+5 \ \ \mathrm{km}^3/\mathrm{s}^2$$

$$e_{\oplus} = 0 \ .$$

The periapsis of the heliocentric transfer ellipse is

$$r_1 = (Earth\ Semi - Major\ Axis\ of\ Orbit) + R_{\oplus} + (250\mathrm{km}) = 1.4960e+8\ km \ .$$

Also, the apoapsis of the heliocentric transfer ellipse is

$$r_2 = (Jupiter\ Semi - Major\ Axis\ of\ Orbit) - 2.8R_{♃} \ = \ 7.7808e+8 \ \ km \ .$$

Then the semi-major axis of the transfer ellipse becomes

$$a_T = 0.5(r_1 + r_2) = 4.6384e+8 \ \ km \ .$$

The eccentricity of the transfer ellipse will be defined as

$$e_T = \frac{r_2 - r_1}{r_2 + r_1} = 0.6775.$$

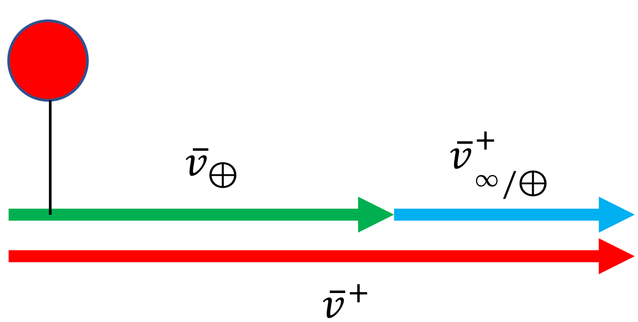
Then we know that the velocity we need to enter this transfer ellipse at the periapsis is

$$v^+ = \sqrt{\mu_{\odot}\left(\frac{2}{r_1} - \frac{1}{a_T}\right)} = \ 38.5754 \ \ km/s \ .$$

We also know the heliocentric velocity of the Earth with respect to the Sun, which is

$$v_{\oplus} = \sqrt{\frac{\mu_{\odot}}{(Earth\ Semi - Major\ Axis\ of\ Orbit)}} \ = \ 29.7847 \ \ km/s \ .$$

The vector diagram becomes is as follows.



$$\therefore v^+_{\infty/\oplus} = \mathrm{v}^+ - \mathrm{v}_{\oplus} \ = \ 8.7907 \ \ \mathrm{km/s} \ .$$

Next, we will compute the delta V required to place the s/c on the heliocentric ellipse with the required velocity for a Hohmann transfer at periapsis.

The circular velocity at the parking orbit is

$$v_c = \sqrt{\frac{\mu_\oplus}{250 + R_\oplus}} = 7.7548 \;\; km/s \; .$$

Then the delta V we are looking for becomes

$$\Delta v_i = \sqrt{\left(v^{+}_{\infty/\oplus}\right)^2 + \frac{2\mu_\oplus}{250 + R_\oplus}} - v_c = \color{red}{6.3005 \;\; km/s} \; .$$

Two-Body Problem #2 (influence of Sun):

Since the local fields of the Earth and Jupiter are turned off in during this trajectory the velocity at the end of the transfer ellipse at its apoapsis can be computed as

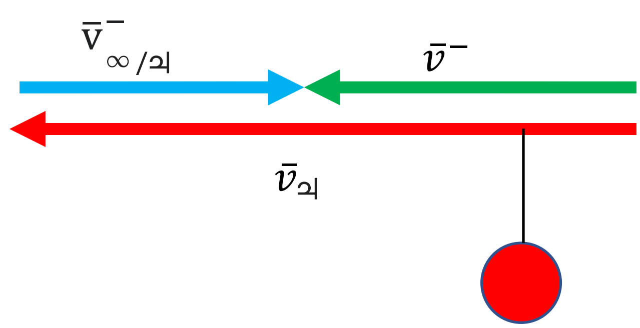
$$v^- = v_a = \sqrt{\mu_\odot\left(\frac{2}{r_2} - \frac{1}{a_T}\right)} = 7.4170 \;\; km/s \; .$$

Two-Body Problem #3 (near Jupiter):

The Heliocentric velocity of Jupiter with respect to the Sun is

$$v_{♃} = \sqrt{\frac{\mu_\odot}{(Jupiter\ Semi-Major\ Axis\ of\ Orbit)}} = 13.0583 \;\; km/s \; .$$

The vector diagram for the capture becomes



$$\therefore v^{-}_{\infty/♃} = |v^- - v_{♃}| = 5.6413 \;\; km/s \; .$$

Next, we will compute the delta V required to have the s/c be captured in a circular orbit around Jupiter from the Hohmann transfer ellipsis at its apoapsis.

The circular velocity at the capture orbit will be

$$v_{c♃} = \sqrt{\frac{\mu_{♃}}{2.8R_{♃}}} = 25.1595 \ km/s \ .$$

Then the delta V we are looking for becomes

$$\Delta v_f = \sqrt{\left(v^-_{\infty/♃}\right)^2 + \frac{2\mu_{♃}}{2.8R_{♃}}} - v_{c♃} = 10.8658 \ km/s \ .$$

Hence, the total delta V becomes

$$\Delta v_{total} = \Delta v_i + \Delta v_f = 17.1663 \ km/s \ .$$

To calculate the time of flight we have to calculate two durations, the hyperbolic trajectory around the Earth and the transfer ellipse.

For the hyperbola,

$$\xi = \frac{\left(v^+_{\infty/\oplus}\right)^2}{2} = 38.6383 \ km^2/s^2$$

$$a_H = -\frac{\mu_\oplus}{2\xi} = -5.1581e+3 \ km$$

$$e_H = 1 - \frac{r_{pH}}{a_H} = 2.2850 \qquad \because r_{pH} = 250 + R_\oplus$$

$$cos\theta^*_\infty = -\frac{1}{e_H} \quad \Rightarrow \quad \theta^*_\infty = 115.9533°$$

$$tan\frac{\theta^*_\infty}{2} = \left(\frac{e_H+1}{e_H-1}\right)^{0.5} tanh\frac{H_\infty}{2} \quad \Rightarrow \quad H_\infty = 90°$$

$$\therefore \mathrm{TOF_{hyperbola}} = \sqrt{\frac{|\mathrm{a_H}|^3}{\mu_\oplus}}\left(\mathrm{e_H sinH_\infty - H_\infty}\right) = 2.1638e+3 \ s \ .$$

The transfer ellipse is

$$TOF_T = \pi\sqrt{\frac{a_T^3}{\mu_\odot}} = 8.6149e+7 \ s \ .$$

Thus,

$$TOF_{total} = TOF_{hyperbola} + TOF_T = 8.6151e + 7\ s\ =\ 997.1180\ days\ =\ 2.7318\ years\ .$$

Now from our results we know that

	PS7 Case #1	*PS7 Case #2*	*PS8*
Δv_{dep} *[km/s]*	8.7925	8.6348	6.3005
Δv_{arr} *[km/s]*	5.6432	5.6257	10.8658
Δv_{total} *[km/s]*	14.4357	14.2605	17.1663
TOF [years]	2.7326	2.5675	2.7318

(*Case #1 → Jupiter orbit assumed to be circular. Case #2 → Jupiter orbit assumed to be eccentric)

From the table above we can see that the inclusion of the local fields changes the total delta V. Specifically, it increases the total delta V of the mission, which means that it increases the cost. However, the maneuver cost around the Earth decreased and the maneuver cost around Jupiter increased significantly.

(b) The cost will differ depending on the capture orbit at Jupiter. As an alternative, assume into a capture orbit that is similar to the insertion orbit actually used by Juno—an eccentric Jovian orbit. Let capture orbit characteristics be $r_p = 2.8R_{jupiter}$ and $e = 0.90$. Consider insertion into the capture orbit at perijove and compute the insertion cost, that is, the $|\Delta\bar{v}_{arr}|$.
Does the total cost improve in terms of Δv_{arr} and Δv_{total}? Why do you think this difference occurs?
The Juno spacecraft first entered this eccentric orbit at Jupiter, then used a series of maneuvers to reduce the size and eventually reach the science orbit. Discuss: why do you think the eccentric insertion orbit was used for Juno?

We will recompute the arrival delta V. First, we have to find the velocity of the s/c at the perijove of the elliptical capture orbit.

$$a_{cap} = \frac{r_{p,cap}}{1 - e_{cap}} = \frac{2.8R_{♃}}{1 - 0.90} = 2.0018e + 6\ km$$

$$v_{cap} = \sqrt{\mu_{♃}\left(\frac{2}{r_{p,cap}} - \frac{1}{a_{cap}}\right)} = 34.6800\ \ km/s\ \ .$$

Then the new delta V we are looking for becomes

$$\Delta v_{arr,new} = \Delta v_{f,new} = \sqrt{\left(v^{-}_{\infty/♃}\right)^2 + \frac{2\mu_{♃}}{2.8R_{♃}}} - v_{cap} = 1.3454 km/s.$$

Hence, the total delta V becomes

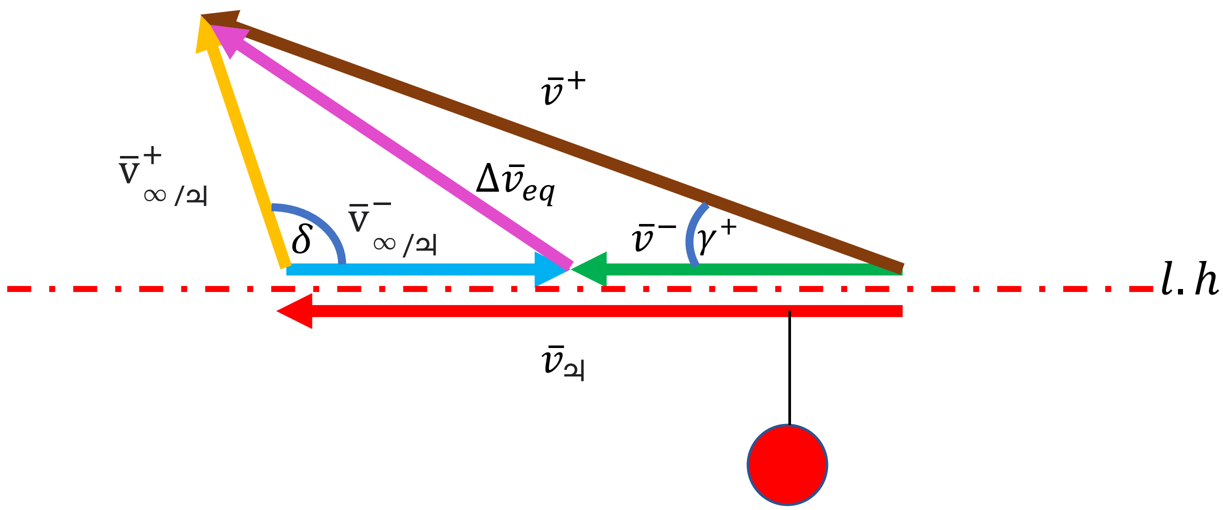
$$\Delta v_{total,new} = \Delta v_i + \Delta v_f = \ 7.6458\ \ km/s\ \ .$$

This is a very significant change in the cost around Jupiter. We can see that the chosen eccentricity of the capture orbit has a high value close to 1. This means that the transition from the hyperbola to this capture orbit has a very small change in energy compared to going from a hyperbola to circular orbit. Those the amount of delta V and propellant required for this near Jupiter transfer has a very small velocity change.

As we can see from our calculations the cost of using an eccentric capture orbit is significantly lower than a circular capture orbit. Also, how Juno reduced the size to the science orbit with a series of operations enable to lower the eccentricity which changes the energy levels gradually which is cost efficient and can leverage the orbital decay.

(c) Reconsider the Jupiter arrival. Assume that the vehicle arrives at Jupiter but does not capture. Instead, it is just a flyby. You should already have the arrival conditions in the heliocentric orbit: $r^-, v^-, \gamma^-, \theta^{*-}$. Compute the orbital characteristics of the new heliocentric orbit: $a, e, r_p, r_a, \mathbb{P}, \xi, \Delta\omega$. Did the spacecraft gain or lose energy?

The vector diagram for the flyby is as follows.



We know that

$$r^- = r_2 = 7.7808e+8 \ \ km \ .$$

$$v^- = 7.4170 \ \ km/s \ .$$

$$\gamma^- = 0° \ .$$

$$\theta^{*-} = 180° \ .$$

For the flyby hyperbola characteristics,

$$\xi_{fb} = \frac{\left(v^+_{\infty/♃}\right)^2}{2} = \frac{\left(v^-_{\infty/♃}\right)^2}{2} = 38.6383 km^2/s^2$$

$$a_{fb} = -\frac{\mu_\odot}{2\xi_{fb}} = -3.9817e+6 \ \ km$$

$$e_{fb} = 1 - \frac{2.8R_{♃}}{a_{fb}} = 1.0503$$

$$\delta = 2arcsin\left(\frac{1}{e_{fb}}\right) = 144.4008°$$

From cosine rule,

$$(v^+)^2 = \left(v^+_{\infty/\text{♃}}\right)^2 + v_{\text{♃}}^2 - 2\left(v^+_{\infty/\text{♃}}\right)(v_{\text{♃}})cos\delta \quad \Rightarrow \quad v^+ = 17.9483 km/s \ .$$

Then from the sine rule

$$\frac{v^+_{\infty/\text{♃}}}{sin\gamma^+} = \frac{v^+}{sin\delta} \ \Rightarrow \ \gamma^+ = 10.5423° \ .$$

The flight path angle is positive since the orbit is ascending at this point.

The new true anomaly becomes

$$tan\theta^{*+} = \frac{\left(\frac{r_2(v^+)^2}{\mu_\odot}\right)sin\gamma^+ cos\gamma^+}{\left(\frac{r_2(v^+)^2}{\mu_\odot}\right)cos^2\gamma^+ - 1} \quad \Rightarrow \ \theta^{*+} = 22.3701° \ .$$

Then,

$$\Delta\omega = \theta^{*-} - \theta^{*+} = 157.6299° \ .$$

$$a_N = -\frac{\mu_\odot}{2\left(\frac{(v^+)^2}{2} - \frac{\mu_\odot}{r_2}\right)} = 6.9897e+9 \ \ km$$

$$h_N = r_2 v^+ cos\gamma^+ = 1.3729e+10 \ \ km^2/s$$

$$p_N = \frac{h_N^2}{\mu_\odot} = 1.4204e+9 \ \ km$$

$$e_N = \sqrt{1 - \frac{p_N}{a_N}} = 0.8926$$

$$r_{aN} = a_N(1+e_N) = 1.3229e+10 \ \ km$$

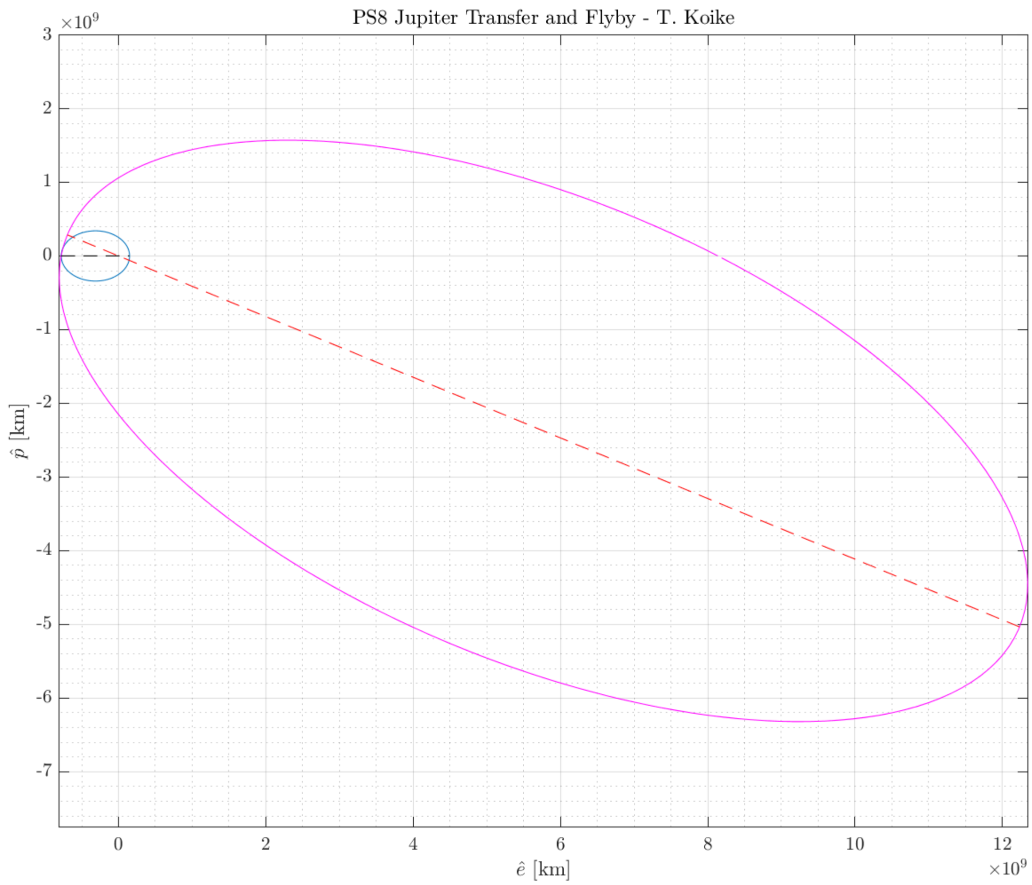
$$r_{pN} = a_N(1-e_N) = 7.5046e+8 \ \ km$$

$$\xi_N = -\frac{\mu_\odot}{2a_N} = -9.4935 \ \ km^2/s^2$$

$$\mathbb{P} = 2\pi\sqrt{\frac{a_N^3}{\mu_\odot}} = 1.0079e+10 \ \ s \ = \ 319.5963 \ \ years$$

The orbit gained energy since the ellipse became more eccentric.

(d) Plot the old and new heliocentric orbit of the spacecraft in Matlab. Compute the equivalent Δv_{eq} and α. Will the spacecraft reach the orbit of Saturn? Uranus? If timed correctly, could encounters of Saturn and, maybe, Uranus now occur?



From the vector diagram in part (c), we can compute

$$\Delta v_{eq} = 2v^{+}_{\infty/\text{♃}} sin\frac{\delta}{2} = 10.7425 \ \ km/s$$

$$\alpha = \frac{180° - \delta}{2} = 17.7996° \ .$$

The semi-major axis of Saturn and Uranus are

Saturn	1.4274e+9 km
Uranus	2.8705e+9 km

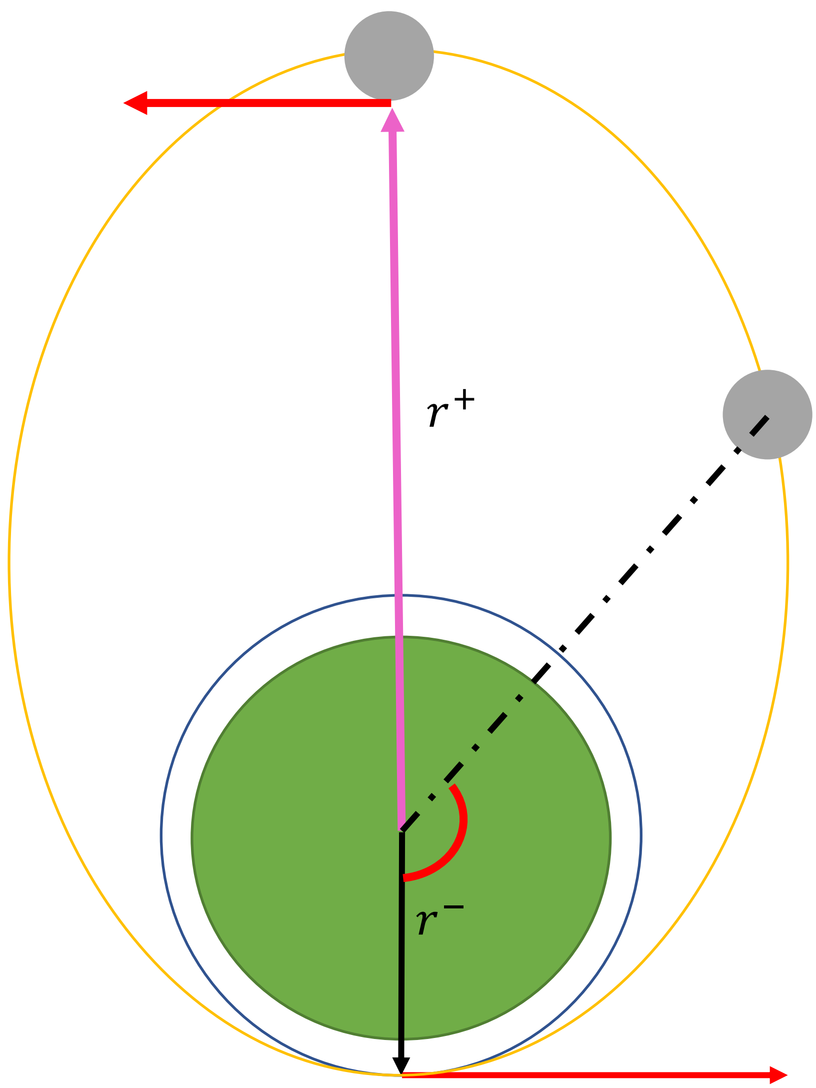
From the plot, we can see that the new orbit goes beyond 1e+10 km in distance and this means that it intersects with the orbit of Saturn and Uranus. Thus, the spacecraft is feasible to reach the orbit of the two planets, and if it is timed correctly to satisfy a precise phase angle, it is possible for encounters of Saturn and Uranus to occur.

Problem 2:

The US is currently planning for humans to reach the Moon in 2024. Consider a Hohmann transfer to the Moon. Assume departure from a 190 km altitude circular Earth parking orbit; include the local gravity field at the Moon.

(a) Determine the Δv and *TOF* for a Hohmann transfer to the Moon if the spacecraft drops into a circular orbit at the Moon with radius of altitude 200 km. Assume arrival on the near (light) side.
What are the transfer characteristics for the geocentric orbit: $r^-, v^-, \gamma^-, \theta^{*-}$ at lunar arrival, $a^-, e^-, r_p^-, r_a^+, \mathbb{P}^-, \xi^-$?
What is the phase angle at departure from the Earth parking orbit?

The orbit diagram:



We know the that,

$$\mu_{\oplus} = 3.9860e + 5 \;\; km^3/s^2$$

$$r^- = r_p^- = R_{\oplus} + 190 = 6.5681e + 3 \;\; km$$

$$r^+ = r_a^+ = (distance\ from\ Earth\ to\ Moon) - 200 - R_{\leftmoon} = 3.8246\text{e} + 5 \;\; km.$$

The semi-major axis of the (Hohmann) transfer ellipse is

$$a^- = a_T = 0.5(r^- + r^+) = 1.9451e + 5 \;\; km \;\; .$$

The eccentricity is

$$e^- = e_T = \frac{r^+ - r^-}{r^+ + r^-} = 0.9662 \;\; .$$

The period of this transfer ellipse is,

$$\mathbb{P}^- = 2\pi\sqrt{\frac{a_T^3}{\mu_{\oplus}}} \;=\; 9.8816\; days \;\; .$$

And the energy is

$$\xi^- = -\frac{\mu_{\oplus}}{2a_T} = -1.0246 \;\; km^2/s^2 \;\; .$$

The parking orbit of Earth is circular, so

$$v_{PO} = \sqrt{\frac{\mu_{\oplus}}{r^-}} \;=\; 7.7902 \;\; km/s \;\; .$$

And the velocity to escape into the transfer orbit is

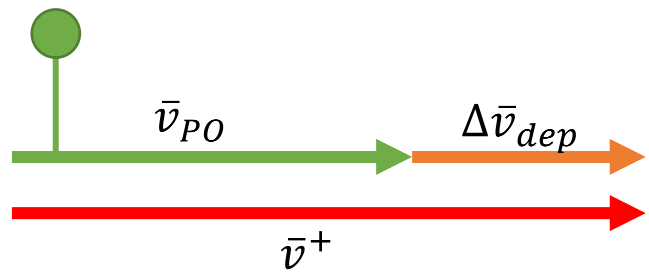
$$v^+ = \sqrt{\mu_{\oplus}\left(\frac{2}{r^-} - \frac{1}{a_T}\right)} \;=\; 10.9236 \;\; km/s \;\; .$$

The flight path angle at both departure and arrival are 0 because they occur at the periapsis and apoapsis, respectively.

$$\gamma^- = \gamma^+ = 0° \;\; .$$

The true anomaly at lunar arrival is

$$\theta^{*-} = 180° \;\; .$$



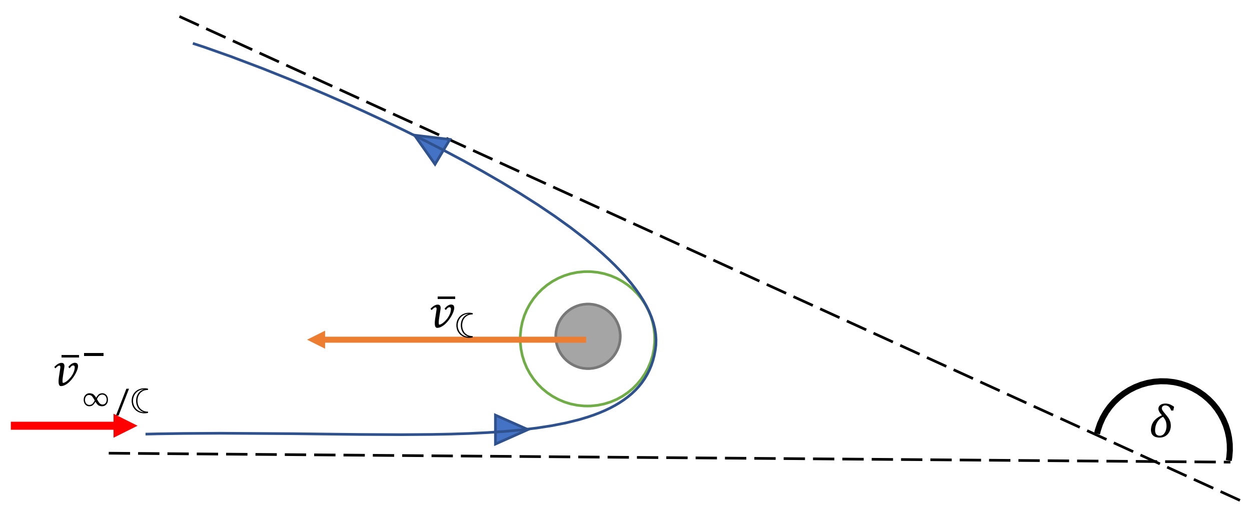
From the vector diagram, we can compute the delta V at departure,

$$\Delta v_{dep} = v^{+} - v_{PO} = 3.1334 \;\; km/s \;.$$

Then the velocity at the apoapsis of the transfer orbit is (at arrival)

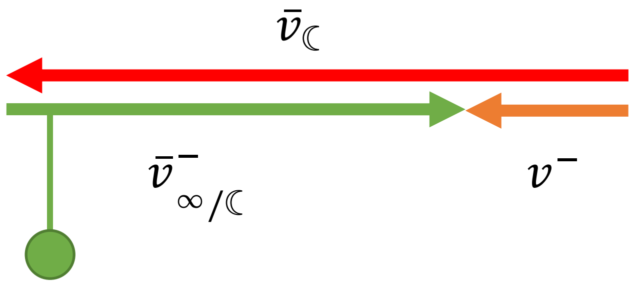
$$v^{-} = \sqrt{\mu_{\oplus}\left(\frac{2}{r^{+}} - \frac{1}{a_T}\right)} = 0.1876 \;\; km/s \;.$$

The arrival to Moon looks like the following diagram.



The geocentric velocity of the Moon is

$$v_{☾} = \sqrt{\frac{\mu_{\oplus}}{384400}} = 1.0183 \;\; km/s \;.$$



Then from the vector diagram above we can compute

$$v^{-}_{\infty/☾} = |v^{-} - v_{☾}| = 0.8307 \ km/s \ .$$

Now, the velocity in the circular orbit around the Moon is

$$v_{c☾} = \sqrt{\frac{\mu_{☾}}{R_{☾} + 200}} = 1.5905 \ km/s \ .$$

Thus,

$$\Delta v_{arr} = \sqrt{\left(v^{-}_{\infty/☾}\right)^2 + \frac{2\mu_{☾}}{R_{☾} + 200}} - v_{c☾} = 0.8073 \ km/s \ .$$

Hence, the total delta V is

$$\Delta v_{total} = \Delta v_{dep} + \Delta v_{arr} = 3.9407 \ km/s \ .$$

The time of flight is equivalent to half the period of the transfer orbit, which is

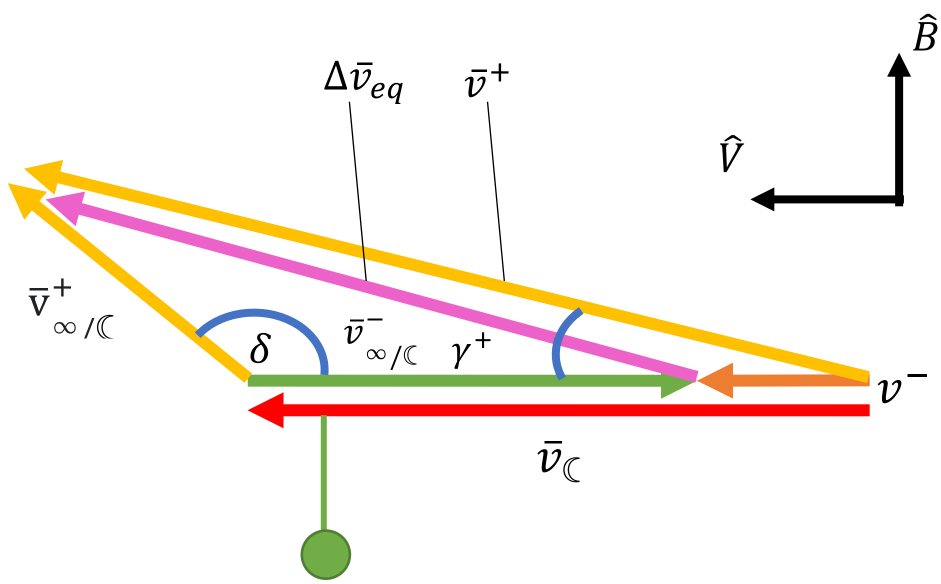
$$TOF = \mathbb{P}^{-}/2 = 4.9408 \ days \ .$$

Finally, the phase angle, ϕ is

$$\phi = \pi - \sqrt{\frac{\mu_{\oplus}}{384400^3}} \times TOF = 2.0107 \text{ rad} = 115.2072° \ .$$

(b) If the spacecraft does not execute the capture maneuver, determine the orbit of the vehicle relative to the Earth, that is, the new geocentric orbit. What turn angle δ is delivered by the Moon?
What are the orbital characteristics, relative the Earth, after the lunar encounter , i.e., $r^+, v^+, \gamma^+, \theta^{*+}$ at the Moon after encounter, $a^+, e^+, r_p^+, r_a^+, \mathbb{P}^+, \xi^+, \Delta\omega$ in the new orbit.
Does the spacecraft gain or lose energy? Why?
Will the new orbit close the Earth? If there is a crew onboard and the lunar capture maneuver is not successfully implemented, can the crew return to Earth?

If the spacecraft is not captured into the orbit of the Moon and does a flyby the vector diagram will be as follows.



We know that

$$v^- = 0.1876 \ km/s .$$

$$\gamma^- = 0°$$

$$\theta^{*-} = 180°$$

$$r^+ = 3.8246e + 5 \ km \ .$$

From the hyperbola characteristics

$$\xi_{fb} = \frac{\left(v^+_{\infty/☾}\right)^2}{2} = \frac{\left(v^-_{\infty/☾}\right)^2}{2} = 0.3450 \ km^2/s^2$$

$$a_{fb} = -\frac{\mu_\oplus}{2\xi_{fb}} = -5.7762e+5 \;\; km$$

$$e_{fb} = 1 - \frac{200 + R_☾}{a_{fb}} = 1.0034$$

$$\delta = 2arcsin\left(\frac{1}{e_{fb}}\right) = 170.6257°$$

From cosine rule,

$$(v^+)^2 = \left(v^+_{\infty/☾}\right)^2 + v_☾^2 - 2\left(v^+_{\infty/☾}\right)(v_☾)cos\delta \quad \Rightarrow \quad v^+ = 1.8429 \;\; km/s \;.$$

Then from the sine rule

$$\frac{v^+_{\infty/☾}}{sin\gamma^+} = \frac{v^+}{sin\delta} \;\; \Rightarrow \;\; \gamma^+ = 4.2106° \;.$$

The flight path angle is positive since the orbit is ascending at this point.

The new true anomaly becomes

$$tan\theta^{*+} = \frac{\left(\frac{r^+(v^+)^2}{\mu_\oplus}\right)sin\gamma^+cos\gamma^+}{\left(\frac{r^+(v^+)^2}{\mu_\oplus}\right)cos^2\gamma^+ - 1} \;\; \Rightarrow \;\; \theta^{*+} = 6.0774° \;.$$

Then,

$$\Delta\omega = \theta^{*-} - \theta^{*+} = 173.9226° \;.$$

$$a^+ = a_N = -\frac{\mu_\oplus}{2\left(\frac{(v^+)^2}{2} - \frac{\mu_\oplus}{r^+}\right)} = -3.0384e+5 \; km$$

$$h_N = r^+v^+cos\gamma^+ = 7.0293e+5 \;\; km^2/s$$

$$p_N = \frac{h_N^2}{\mu_\oplus} \; = \; 1.2396e+6 \;\;\; km$$

$$e^+ = e_N = \sqrt{1 - \frac{p_N}{a_N}} \; = \; 2.2538$$

$$r_a^+ = r_{aN} = \infty$$

$$r_p^+ = r_{pN} = |a_N|(e_N \; - \; 1) = 3.8097e+5 \; km$$

$$\xi^+ = \xi_N = -\frac{\mu_\oplus}{2a_N} = 0.6559 \;\; km^2/s^2$$

$$\mathbb{P}^+ = N.A.$$

Since the new orbit is a hyperbola it will NOT close Earth. If the maneuver is unsuccessful, they will NOT be able to return to Earth without extra maneuvers to transfer to an orbit back to Earth.

(c) What is the equivalent Delta-V, i.e., $|\Delta\bar{v}_{eq}|$ and the in-plane angle α, produced via the lunar flyby in (b)? Express the $\Delta\bar{v}_{eq}$ in terms of VNB coordinates.

From the vector diagram in part (b), we can compute

$$|\Delta\bar{v}_{eq}| = 2v^{+}_{\infty/☾} sin\frac{\delta}{2} = 1.6559\ \ km/s\ .$$

And α becomes

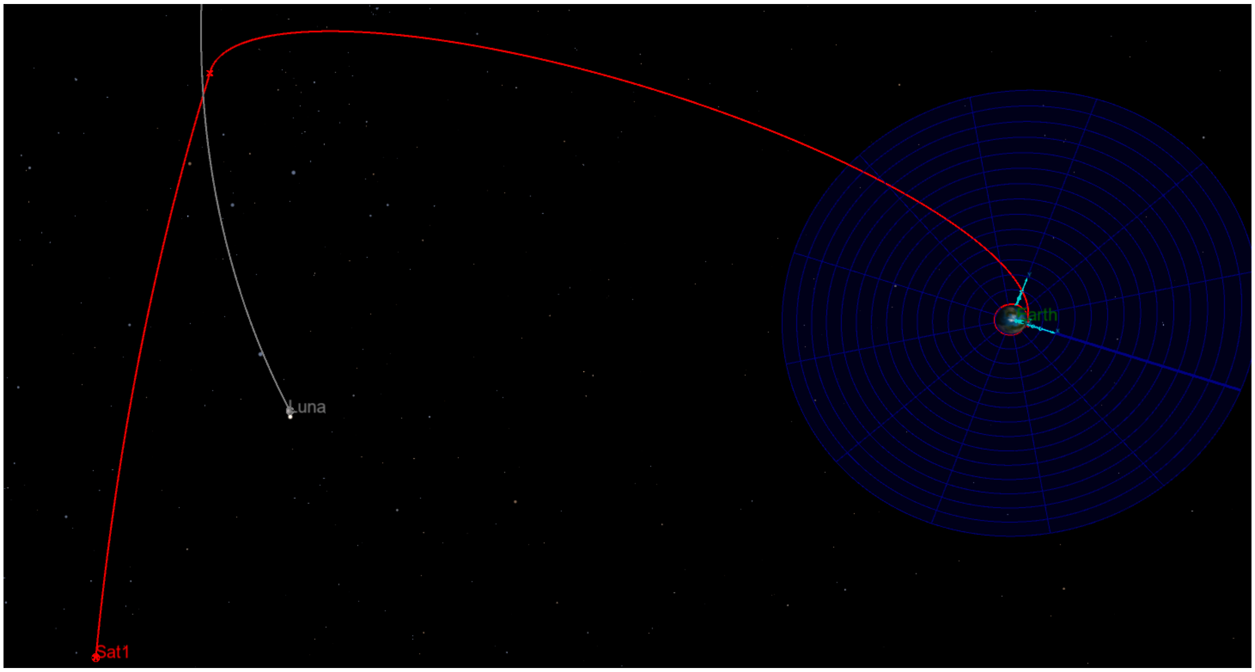
$$\alpha = \frac{180° - \delta}{2} = 4.6872°\ .$$

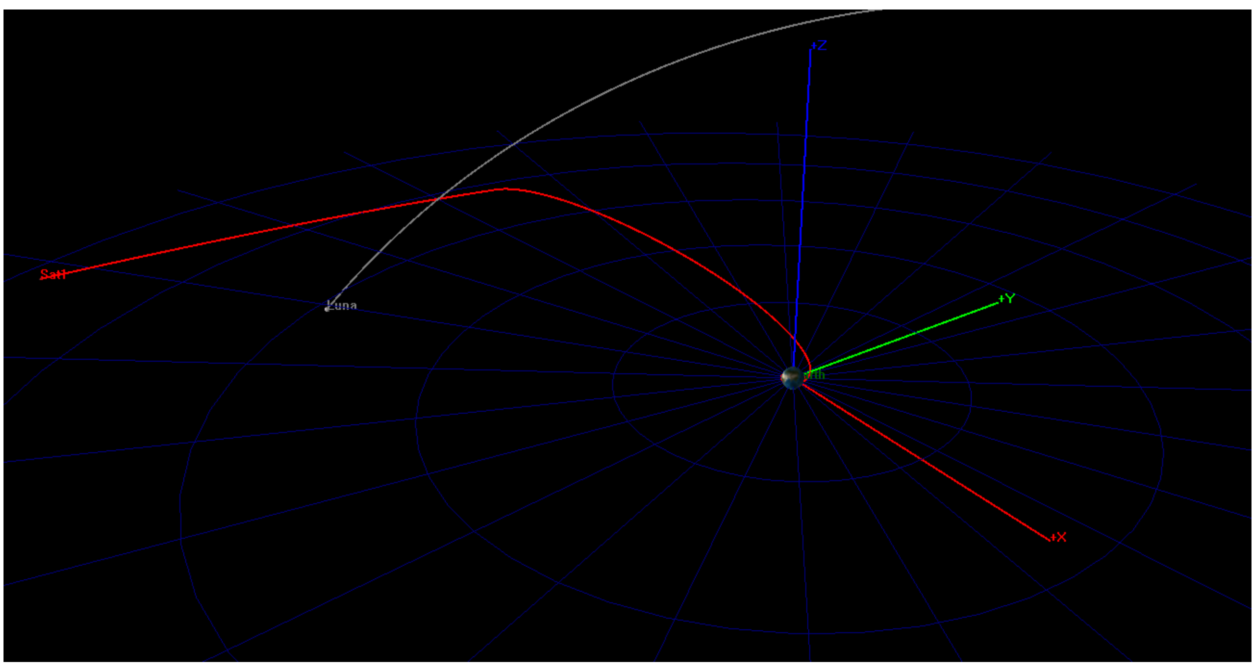
Thus,

$$\Delta\bar{v}_{eq} = |\Delta\bar{v}_{eq}|\left(cos\alpha\hat{V} + sin\alpha\hat{B}\right) = 1.6503\hat{V} + 0.1353\hat{B}\ \ km/s\ .$$

(d) Plot the initial Earth orbit in GMAT and use a two-body Earth propagator. Add the equivalent $\Delta\bar{v}_{eq}$ to reflect the impact of lunar gravity; the $\Delta\bar{v}_{eq}$ is added in terms of its VNB components. The new orbit should be the same as the outbound orbit in (b). Compare new orbital characteristics from GMAT output.

The GMAT results is as follows.





```
******  Changes made to the mission will not be reflected ******
******  in the data displayed until the mission is rerun  ******

        Propagate Command: Propagate3
        Spacecraft       : Sat1
        Coordinate System: EarthMJ2000Eq

        Time System   Gregorian                     Modified Julian
        ----------------------------------------------------------------------
        UTC Epoch:    13 Nov 2020 12:01:08.769      29167.0007959374
        TAI Epoch:    13 Nov 2020 12:01:45.769      29167.0012241781
        TT  Epoch:    13 Nov 2020 12:02:17.953      29167.0015966781
        TDB Epoch:    13 Nov 2020 12:02:17.952      29167.0015966632

        Cartesian State                       Keplerian State
        ---------------------------           --------------------------------
        X  =  -370635.05217044 km             SMA  =  -303907.45225599 km
        Y  =  -308936.23933987 km             ECC  =   2.2534204397077
        Z  =   0.0000000000000 km             INC  =   0.0000000000000 deg
        VX =   0.2278164808671 km/sec         RAAN =   0.0000000000000 deg
        VY =  -1.7064273251830 km/sec         AOP  =   173.92266568088 deg
        VZ =   0.0000000000000 km/sec         TA   =   45.889628200019 deg
                                              MA   =   42.050261345580 deg
                                              HA   =   30.833753670419 deg

        Spherical State                       Other Orbit Data
        ---------------------------           --------------------------------
        RMAG =   482505.89827985 km           Mean Motion        =   3.768398854e-06 deg/sec
        RA   =  -140.18770611910 deg          Orbit Energy       =   0.6557924765271 km^2/s^2
        DEC  =   0.0000000000000 deg          C3                 =   1.3115849530542 km^2/s^2
        VMAG =   1.7215674732888 km/s         Semilatus Rectum   =   1239305.3173545 km
        AZI  =   90.000000000000 deg          Angular Momentum   =   702842.54755300 km^2/s
        VFPA =   57.792009969965 deg          Beta Angle         =  -18.079706352799 deg
        RAV  =  -82.395696149132 deg          Periapsis Altitude =   374545.67613716 km
        DECV =   0.0000000000000 deg          VelPeriapsis       =   1.8451000557203 km/s

        Planetodetic Properties               Hyperbolic Parameters
        ---------------------------           --------------------------------
        LST       =   220.07973208488 deg     BdotT          =   613705.23996826 km
        MHA       =   233.35123676853 deg     BdotR          =   0.0000000000000 km
        Latitude  =  -0.0878419046300 deg     B Vector Angle =   0.0000000000000 deg
        Longitude =  -13.271504683650 deg     B Vector Mag   =   613705.23996826 km
        Altitude  =   476127.76203003 km      DLA            =   0.0000000000000 deg
                                              RLA            =  -69.732674799416 deg


        Spacecraft Properties
        ------------------------------
        Cd                    =   2.200000
        Drag area             =   15.00000 m^2
        Cr                    =   1.800000
        Reflective (SRP) area =   1.000000 m^2
        Dry mass              =   850.00000000000 kg
        Total mass            =   850.00000000000 kg
        SPADDragScaleFactor   =   1.000000
        SPADSRPScaleFactor    =   1.000000
```

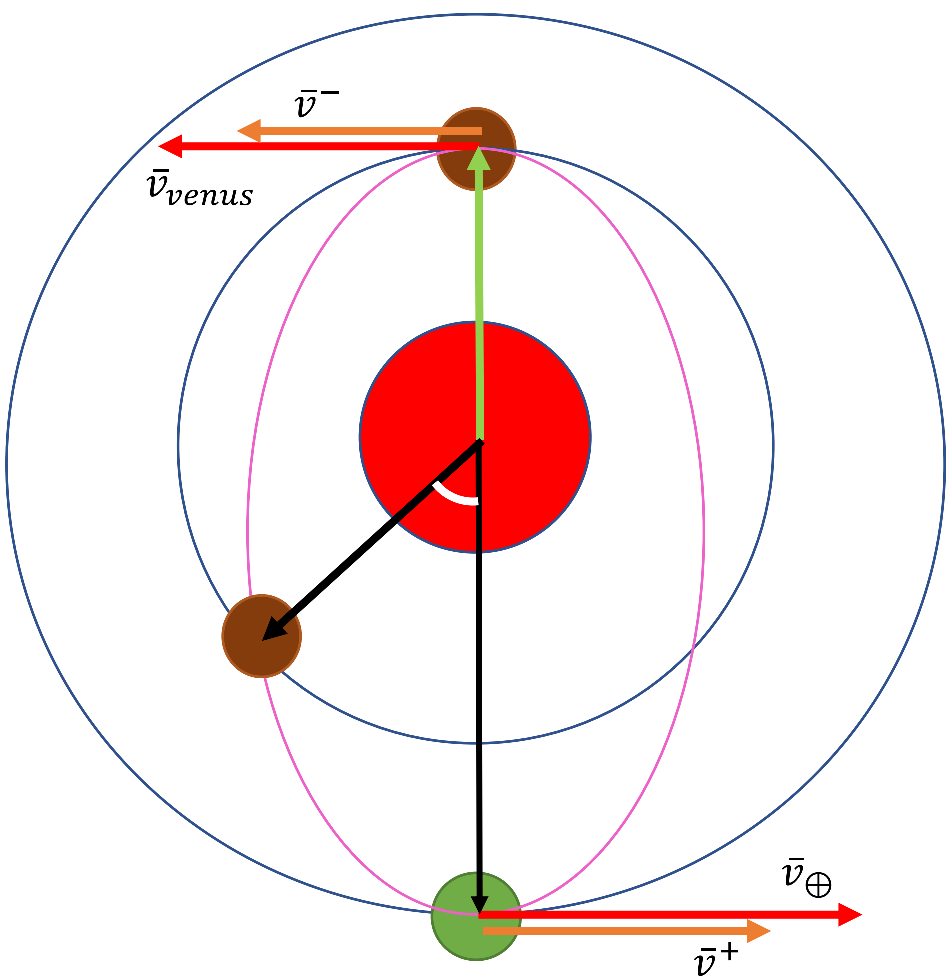
The values colored in red agree with our results in the previous part. Thus, we can verify our results for the new orbit.

Problem 3:

Some years ago, JAXA, the Japanese space agency, launched a Venus orbiter May 20, 2010 to study the planet's climate. The mission was known as Akatsuki. The spacecraft arrived at Venus December 7, 2010. There was a valve problem, however, and the capture maneuver failed. Nevertheless, assume that you are preparing a preliminary trajectory design for a mission design to Venus. Assume that Earth and Venus are in circular, coplanar orbits; include local gravity fields in the analysis.

(a) Start with a Hohmann transfer from Earth to Venus. What is the TOF? Determine the phase angle required to arrive at Venus.
Include a diagram of the heliocentric view; indicate the velocity vectors $\bar{v}^+, \bar{v}^-, \bar{v}_\oplus, \bar{v}_{venus}$. Locate Earth at departure and Venus at departure and arrival.

The orbit diagram is as follows.



We know the that,

$$\mu_{\oplus} = 3.9860e + 5 \ \ km^3/s^2$$

$$\mu_{\odot} = 1.3271e + 11 \ \ km^3/s^2$$

$$\mu_{venus} = 3.2486e + 5 \ \ km^3/s^2$$

$$r_1 = 1.4960e + 8 \ \ km$$

$$r_2 = 1.0821e + 8 \ \ km.$$

The semi-major axis of the (Hohmann) transfer ellipse is

$$a_T = 0.5(\mathrm{r\_1} + \mathrm{r\_2}) = 1.2890e + 8 \ \ km \ \ .$$

The eccentricity is

$$e_T = \frac{r_1 - r_2}{r_1 + r_2} = 0.1605 \ \ .$$

The velocity to enter the transfer ellipse is

$$v^+ = v_{T1} = \sqrt{\mu_{\odot}\left(\frac{2}{r_1} - \frac{1}{a_T}\right)} = 27.2892 \ \ km/s.$$

The velocity to exit the transfer ellipse is

$$v^- = v_{T2} = \sqrt{\mu_{\odot}\left(\frac{2}{r_2} - \frac{1}{a_T}\right)} = 37.7276 \ \ km/s.$$

The heliocentric velocity of Earth is

$$v_{\oplus} = \sqrt{\frac{\mu_{\odot}}{r_1}} \ = \ 29.7847 \ \ km/s \ \ .$$

The heliocentric velocity of Venus is

$$v_{venus} = \sqrt{\frac{\mu_{\odot}}{r_2}} \ = \ 35.0209 \ \ km/s \ \ .$$

The period of this transfer ellipse is,

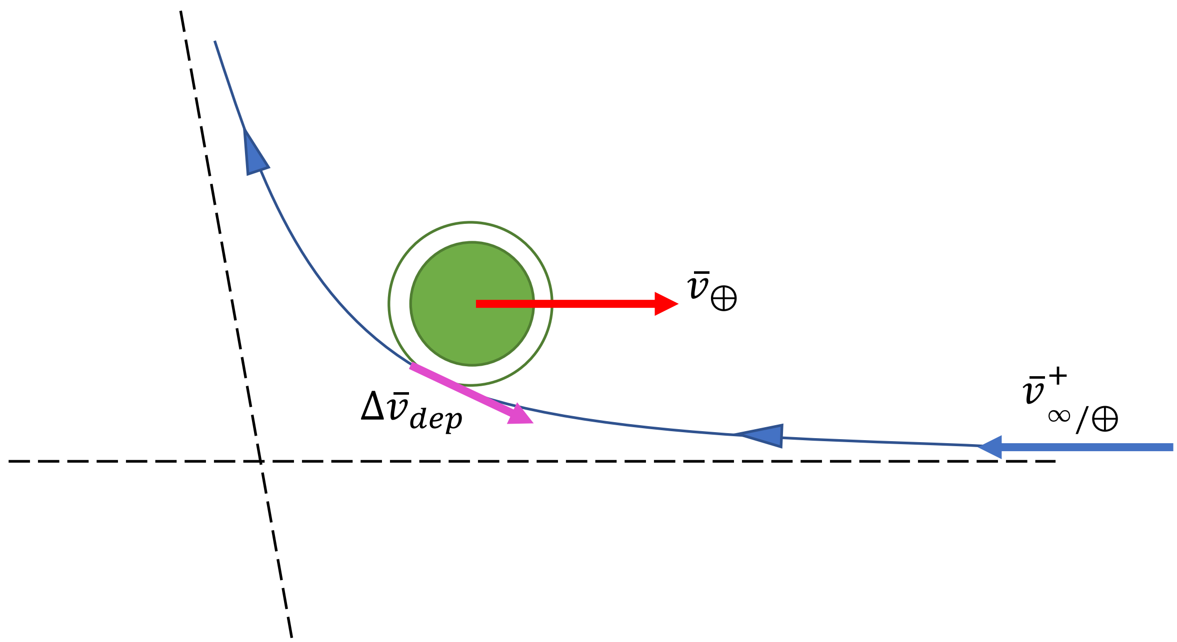
$$TOF = \pi\sqrt{\frac{a_T^3}{\mu_{\odot}}} \ = \ 0.4002 \text{ years} \ \ .$$

The phase angle, ϕ becomes

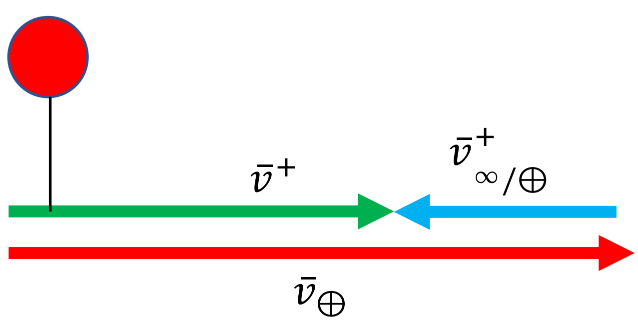
$$\phi = \pi - \sqrt{\frac{\mu_{\odot}}{{r_2}^3}} \times TOF \; = \; -0.9431 \; rad = -54.0347^{\circ} \; .$$

(b) Consider a spacecraft (e.g., Akatsuki) departure from Earth in a geocentric diagram. What is the $\bar{v}^{+}_{\infty/\oplus}$ that the spacecraft must possess to be on the correct heliocentric transfer orbit? Assuming a 210 km altitude Earth parking orbit, what $\Delta\bar{v}_{dep}$ will yield this $\bar{v}^{+}_{\infty/\oplus}$? In the diagram of the geocentric view; indicate the velocity vectors $\bar{v}^{+}_{\infty/\oplus}$, $\bar{v}_{\oplus}$, $\Delta\bar{v}_{dep}$.

Diagram of geocentric orbit:



The vector diagram becomes is as follows.



$$\therefore v^{+}_{\infty/\oplus} = |\mathrm{v}^{+} - \mathrm{v}_{\oplus}| = 2.4955 \ \text{km/s} \ .$$

$$\bar{v}^{+}_{\infty/\oplus} = -2.4955 km/s(totheleft) \quad (-\hat{V}) - dir$$

Next, we will compute the delta V required to place the s/c on the heliocentric ellipse with the required velocity for a Hohmann transfer at periapsis.

The circular velocity at the parking orbit is

$$v_{c\oplus} = \sqrt{\frac{\mu_{\oplus}}{210 + R_{\oplus}}} = 7.7784 \ km/s \ .$$

Then the delta V we are looking for becomes

$$\Delta v_{dep} = \sqrt{\left(v^{+}_{\infty/\oplus}\right)^2 + \frac{2\mu_{\oplus}}{210 + R_{\oplus}}} - v_{c\oplus} = 3.5014 \ km/s.$$

The $\Delta\bar{v}_{dep}$ is going to be dark side departure which is directed in the path which leads to the apoapsis of the transfer ellipse.

(c) The spacecraft arrives at Venus along a light side passage and enters a circular orbit at an altitude of 2000 km altitude. Determine arrival conditions: r_p^-, r_a^-, ξ^-, r^-, v^-, γ^-, θ^{*-}. What velocity $\bar{v}_{\infty/venus}^-$ results from the Hohmann transfer? What is the required orbit insertion $\Delta\bar{v}_{arr}$? Include the Venus-centered diagram. What is the Δv_{dep}, Δv_{arr}, and Δv_{total} for this transfer plan?

From what we know

$$r^- = r_p^- = r_2 - R_{venus} - 2000 = 1.0820e+8\ km$$

$$r_a^- = r_1 + R_\oplus + 210 = 1.4960e+8\ km$$

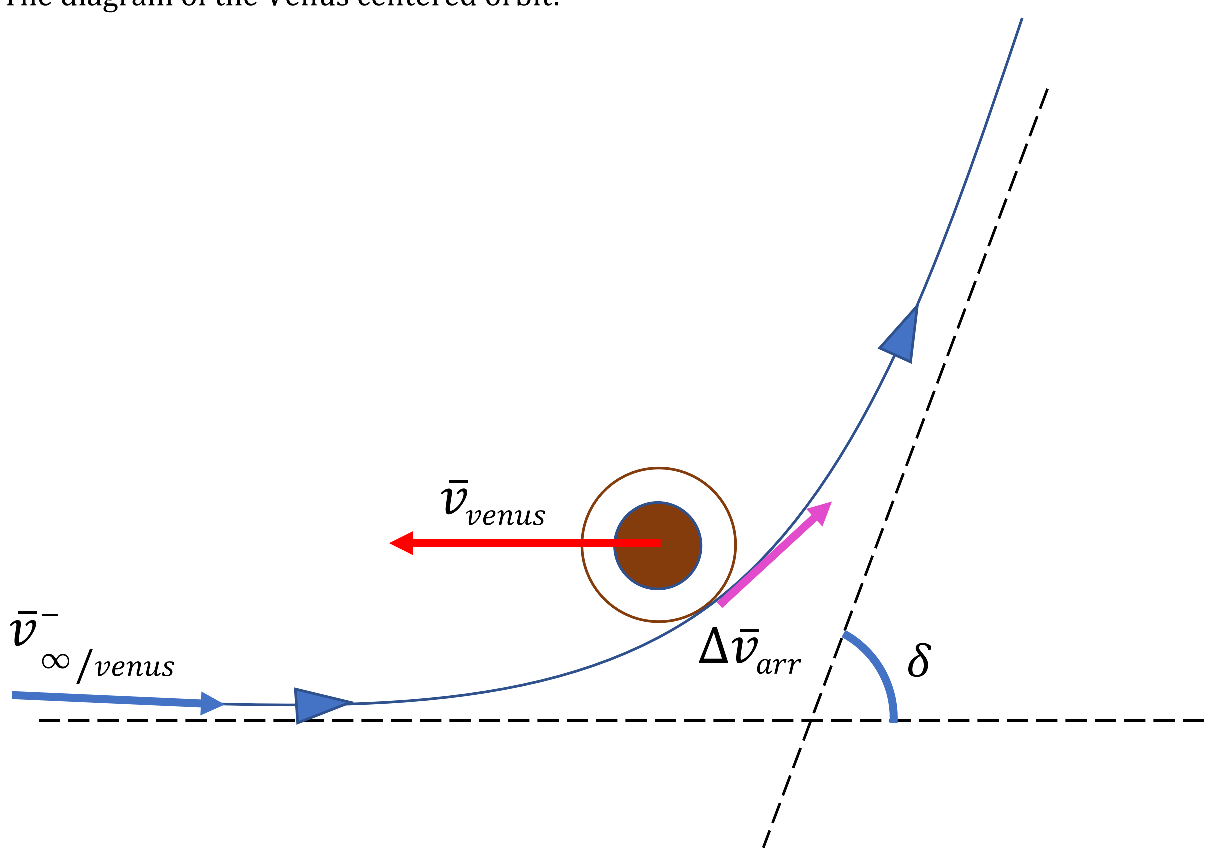
$$v^- = \sqrt{\mu_\odot\left(\frac{2}{r^-} - \frac{1}{a_T}\right)} = 37.7276\ km/s$$

$$\gamma^- = 0°$$

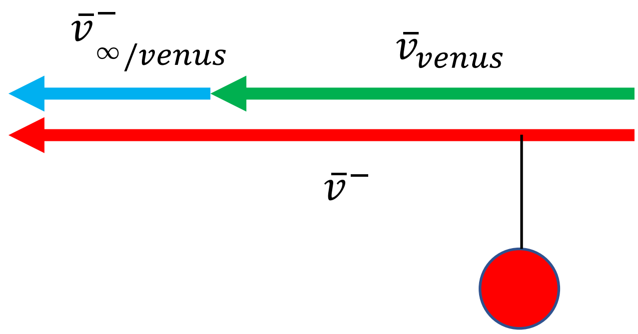
$$\xi^- = -\frac{\mu_\odot}{2a_T} = -514.7780\ km^2/s^2$$

$$\theta^{*-} = 180°\ .$$

The diagram of the Venus centered orbit:



The vector diagram at Venus arrival is



$$\therefore \bar{v}^-_{\infty/venus} = \bar{v}^- - \bar{v}_{venus} = 2.7067 \ km/s \ (to \ the \ left) \ \hat{V} - dir.$$

Next, we will compute the delta V required to have the s/c be captured in a circular orbit around Venus from the Hohmann transfer ellipsis at its periapsis.

The circular velocity at the capture orbit will be

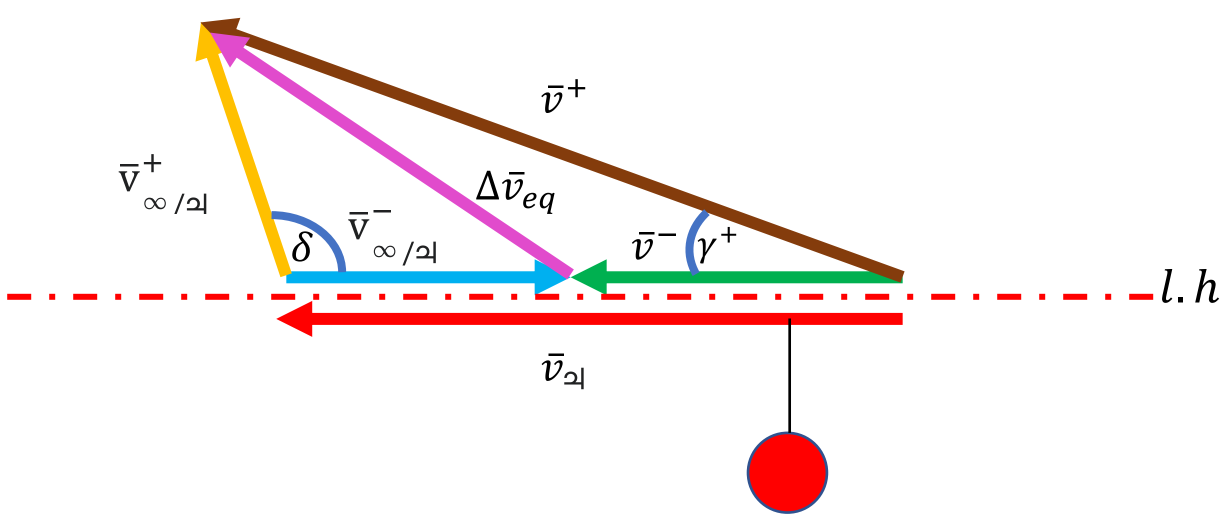
$$v_{c,\text{venus}} = \sqrt{\frac{\mu_{\text{venus}}}{2000 + R_{venus}}} = 6.3518 \ km/s \ .$$

Then the delta V we are looking for becomes

$$\Delta v_{arr} = \sqrt{\left(v^-_{\infty/\text{venus}}\right)^2 + \frac{2\mu_{\text{venus}}}{2000 + R_{venus}}} - v_{c,\text{venus}} = 3.0299 \ km/s \ \text{(to slow down)} \ .$$

Hence, the total delta V becomes

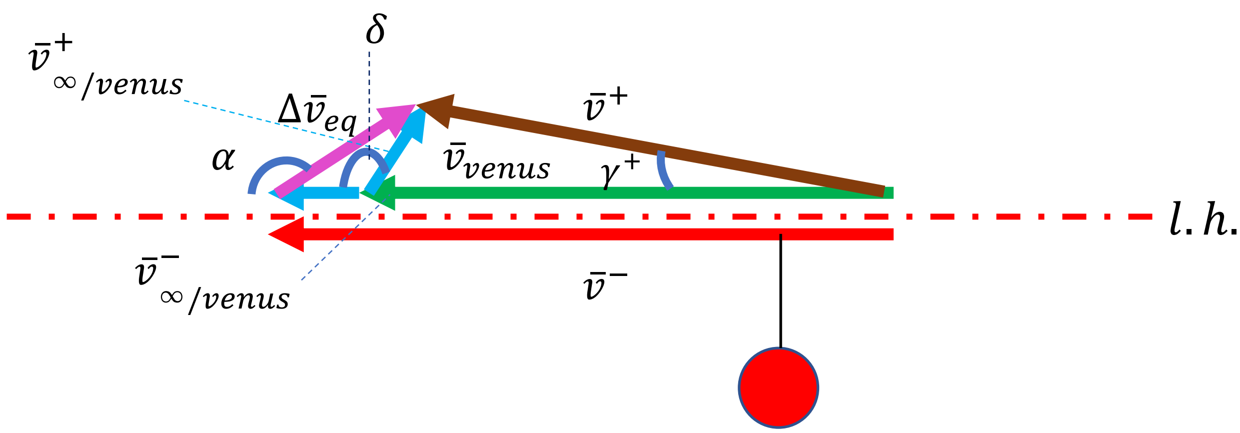
$$\Delta v_{total} = \Delta v_{dep} + \Delta v_{arr} = 6.5314 \ km/s \ .$$



(d) Consider the trajectory consequences if the Venus orbit insertion (VOI) maneuver implementation fails. Compute the turn angle δ and $v^+_{\infty/\text{venus}}$.

What is the new heliocentric velocity in terms of $r^+,\ v^+,\ \gamma^+,\ \theta^{*+}$? What is the equivalent $\Delta\bar{v}_{eq}$ due to the flyby, i.e. $|\Delta\bar{v}_{eq}|$, α? Has the spacecraft gained or lost energy? Compute the following characteristics of the new heliocentric orbit: $r_p^+, r_a^+, \Delta\omega, \xi^+$.

The diagram becomes as follows:



We know that

$$r^+ = r^- = r_2 = 1.0820e+8km.$$

$$v^- = 37.7276 \quad km/s \quad .$$

$$\gamma^- = 0° \ .$$

$$\theta^{*-} = 0° \text{ (for the transfer orbit)} .$$

For the flyby hyperbola characteristics,

$$\xi_{fb} = \frac{\left(v^+_{\infty/\text{venus}}\right)^2}{2} = \frac{\left(v^-_{\infty/\text{venus}}\right)^2}{2} = 3.6631 \ \ km^2/s^2$$

$$a_{fb} = -\frac{\mu_{\text{venus}}}{2\xi_{fb}} = -4.4342e+4 \ \ km$$

$$e_{fb} = 1 - \frac{2000 + R_{venus}}{a_{fb}} = 1.1816$$

$$\delta = 2arcsin\left(\frac{1}{e_{fb}}\right) = 115.6271°$$

$$v^+_{\infty/\text{venus}} = v^-_{\infty/\text{venus}} = 2.7067 \ \ km/s \ .$$

From cosine rule,

$$(v^+)^2 = \left(v^+_{\infty/\text{venus}}\right)^2 + v^2_{\text{venus}} - 2\left(v^+_{\infty/\text{venus}}\right)(v_{\text{venus}})cos\delta \quad \Rightarrow \quad v^+ = 36.2738\ km/s\ .$$

Then from the sine rule

$$\frac{v^+_{\infty/\text{venus}}}{sin\gamma^+} = \frac{v^+}{sin\delta} \ \Rightarrow\ \gamma^+ = 3.8577°\ .$$

The flight path angle is positive since the orbit is ascending at this point.

The new true anomaly becomes

$$tan\theta^{*+} = \frac{\left(\frac{r^+(v^+)^2}{\mu_\odot}\right)sin\gamma^+ cos\gamma^+}{\left(\frac{r^+(v^+)^2}{\mu_\odot}\right)cos^2\gamma^+ - 1} \quad \Rightarrow\ \theta^{*+} = 46.6849°\ .$$

Then,

$$\Delta\omega = \theta^{*-} - \theta^{*+} = 133.3151°\ .$$

$$a_N = -\frac{\mu_\odot}{2\left(\frac{(v^+)^2}{2} - \frac{\mu_\odot}{r^+}\right)} = 1.1671\text{e}+8\ \ km$$

$$h_N = r^+v^+cos\gamma^+ = 3.9159\text{e}+9\ \ km^2/s$$

$$p_N = \frac{h_N^2}{\mu_\odot} = 1.1555\text{e}+8\ \ km$$

$$e_N = \sqrt{1 - \frac{p_N}{a_N}} = 0.0998$$

$$r_a^+ = r_{aN} = a_N(1+e_N) = 1.2835e+8\ \ km$$

$$r_p^+ = r_{pN} = a_N(1-e_N) = 1.0506e+8\ \ km$$

$$\xi^+ = \xi_N = -\frac{\mu_\odot}{2a_N} = -568.5710\ \ km^2/s^2$$

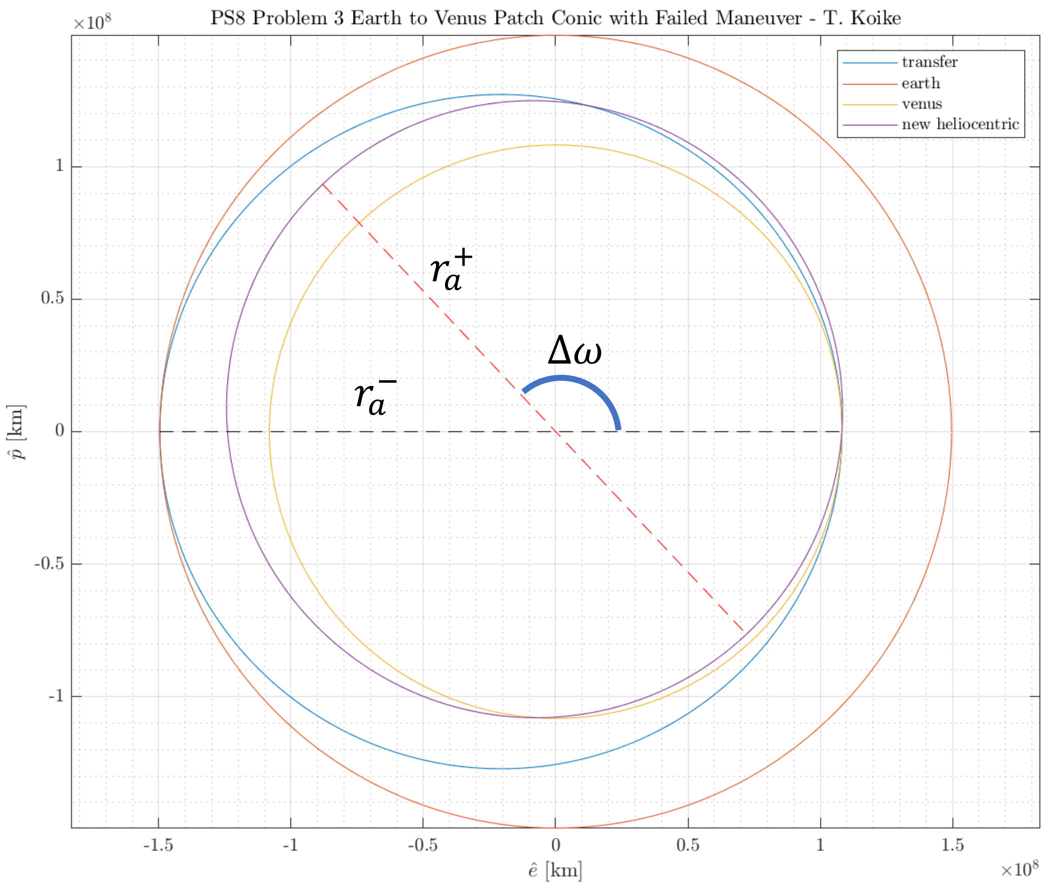
The orbit lost energy.

$$\Delta v_{eq} = 2v^+_{\infty/♃}sin\frac{\delta}{2} = 4.5815\ \ km/s$$

$$\alpha = 180° - \frac{180° - \delta}{2} = 147.8135°.$$

(e) Plots the orbits in MATLAB: Earth orbit, Venus orbit, transfer orbit, new heliocentric orbit that results post-encounter. Mark the $r_a^-, r_a^+, \Delta\omega$.
Is the spacecraft ascending or descending after the encounter? Will the spacecraft cross Earth's orbit again? Will the spacecraft reach the orbit of Mercury?
Discuss: How might the new heliocentric orbit change if the Venus encounter was a dark-side passage?

The MATLAB plot is the following,



From the plot we can see that the new heliocentric orbit is ascending after the encounter. It will not cross Earth's orbit again due to a small semi-major axis and eccentricity. This orbit will not cross the orbit of Mercury because it is too circular and has an orbit slightly larger than Venus but smaller than the Earth.

If the encounter was a dark side one, the line of apsides will be shifted by $(180° - \Delta\omega) \times 2$ (counter clockwise) and the periapsis and apoapsis of this new heliocentric orbit will be shifted by the same angle. This is because if the encounter is a dark side passage the spacecraft will go under Venus in the circumstance of the arrival maneuver being unsuccessful. Whereas, in this problem the spacecraft went above Venus because it had a light side passage.

Appendix

MATLAB Code

Problem 1

```
% AAE 532 HW 8 Problem 1
% Tomoki Koike
close all; clear all; clc;
fdir = 'C:\Users\Tomo\Desktop\studies\2020-Fall\AAE532\matlab\outputs\ps8';
set(groot, 'defaulttextinterpreter','latex');
set(groot, 'defaultAxesTickLabelInterpreter','latex');
set(groot, 'defaultLegendInterpreter','latex');

% Set constants
planet_consts = setup_planetary_constants();  % Function that sets up all the
constants in the table
sun = planet_consts.sun;  % structure of sun
earth = planet_consts.earth;  % structure of earth
mars = planet_consts.mars;  % structure of mars
jupiter = planet_consts.jupiter;  % structure of jupiter
G = 6.6743015e-20;  % Gravitational constant [km^3/kg/s^2]

% (a)
h_PO  =                        250;  % altitude of Earth Parking Orbit
h_JCO =           2.8 * jupiter.mer;  % altitude of Jovian capture orbit
r_1 = earth.smao + earth.mer + h_PO  % initial radial distance w.r.t Sun
r_2 =          jupiter.smao - h_JCO  % final radial distance w.r.t Sun into Jovian
capture orbit

% Gravitational parameters
mu_sun =         sun.gp;
mu_earth =     earth.gp;
mu_jupiter = jupiter.gp;

% Two-Body Problem #1 (near Earth)
a_T =         0.5 * (r_1 + r_2)
e_T = (r_2 - r_1) / (r_2 + r_1)
p_T = a_T * (1 - e_T^2);

% Velocity required to enter the transfer ellipse
v_plus = vis_viva(r_1, a_T, mu_sun)

% Heliocentric velocity of the Earth wrt the Sun
v_earth =  sqrt(mu_sun / earth.smao)

% Excess velocity
v_inf_earth = v_plus - v_earth

% Circular velocity at parking orbit
v_c = sqrt(mu_earth / (h_PO + earth.mer))

% Required delta V for the first burn
```

```
Dv_i = sqrt(v_inf_earth^2 + 2*mu_earth / (h_PO + earth.mer)) - v_c

% Two-Body Problem #2 (influence of Sun)
% Velocity at the apoapsis of the transfer ellipse
v_minus = vis_viva(r_2, a_T, mu_sun)

% Heliocentric velocity of Jupiter wrt Sun
v_jupiter = sqrt(mu_sun / jupiter.smao)

% Excess velocity for the capture
v_inf_jupiter = abs(v_minus - v_jupiter)

% The circular velocity in the capture orbit of Jupiter
v_c_jupiter = sqrt(mu_jupiter / h_JCO)

% Required delta V for the capture sequence
Dv_f = sqrt(v_inf_jupiter^2 + 2*mu_jupiter / h_JCO) - v_c_jupiter

% Total delta V
Dv_total = Dv_i + Dv_f

% TOF
% The hyperbola
xi = v_inf_earth^2 / 2
a_H = -mu_earth / 2 / xi
r_pH = h_PO + earth.mer;
e_H = 1 - r_pH / a_H
p_H = abs(a_H)*(e_H^2 - 1)
TA_infty = acosd(-1 / e_H)
HA_infty = T2H_anomaly(e_H, TA_infty, "deg")
TOF_hyperbola = sqrt(abs(a_H)^3 / mu_earth) * (e_H * sinh(deg2rad(HA_infty)) -
deg2rad(HA_infty))
TOF_hyperbola_hr = TOF_hyperbola / 60 / 60

% The transfer ellipse
TOF_T = pi * sqrt(a_T^3 / mu_sun)

% Total TOF
TOF_total = TOF_hyperbola + TOF_T
TOF_total_day = TOF_total / 60 / 60 / 24
TOF_total_year = TOF_total_day / 365

% (b)
r_p_cap = h_JCO;
e_cap = 0.90;
a_cap = r_p_cap / (1 - e_cap)
v_cap = vis_viva(r_p_cap, a_cap, mu_jupiter)

% The new required delta V for the capture sequence
Dv_f_new = sqrt(v_inf_jupiter^2 + 2*mu_jupiter / h_JCO) - v_cap

% The new total delta v
Dv_total_new = Dv_i + Dv_f_new

% (c)
```

```
xi_fb = v_inf_jupiter^2 / 2
a_fb = -mu_jupiter / 2 / xi_fb
e_fb = 1 - h_JCO / a_fb
delta = 2*asind(1 / e_fb)
v_plus = sqrt( v_inf_jupiter^2 + v_jupiter^2 -
2*v_inf_jupiter*v_jupiter*cosd(delta) )
FPA_plus = asind(sind(delta) * v_inf_jupiter / v_plus)

% True anomaly
temp = r_2 * v_plus^2 / mu_sun;
TA_plus = atand( temp*sind(FPA_plus)*cosd(FPA_plus) / ( temp*cosd(FPA_plus)^2 - 1 ))
Domega = 180 - TA_plus

% characterisitics
a_N = -mu_sun / 2 / (v_plus^2 / 2 - mu_sun / r_2)
h_N = r_2*v_plus*cosd(FPA_plus)
p_N = h_N^2 / mu_sun
e_N =  sqrt(1 - p_N / a_N)
r_aN = a_N * (1 + e_N)
r_pN = a_N * (1 - e_N)
xi_N = -mu_sun / 2 / a_N
IP_N = 2*pi * sqrt(a_N^3 / mu_sun)
IP_N_year = IP_N / 60 /60 / 24 / 365

% (d)
Dv_eq = 2*v_inf_jupiter*sind(delta / 2)
alpha = (180 - delta) / 2


% Plotting for visualization
% old orbit
angles = 0:0.01:2*pi;
RR = p_T ./ (1 + e_T*cos(angles)); XX = RR.*cos(angles); YY = RR.*sin(angles);

% new orbit
RR_new = p_N ./ (1 + e_N*cos(angles - deg2rad(Domega)));
XX_new = RR_new.*cos(angles);
YY_new = RR_new.*sin(angles);
rp_vec = r_pN*[cosd(Domega), sind(Domega)];
ra_vec = r_aN*[cosd(Domega+180), sind(Domega+180)];

Xsun = sun.mer*cos(angles); Ysun = sun.mer*sin(angles);
fig = figure("Renderer","painters","Position",[10, 10, 900, 700]);
    plot(XX,YY)
    hold on; grid on; grid minor; box on; axis equal;
    plot(Xsun, Ysun, '-r')
    plot(XX_new, YY_new, '-m')
    plot([-r_2, r_1], [0, 0], '--k')
    plot([ra_vec(1), rp_vec(1)],[ra_vec(2), rp_vec(2)], '--r')
    hold off
    title('PS8 Jupiter Transfer and Flyby - T. Koike')
    xlabel('$\hat{e}$ [km]')
    ylabel('$\hat{p}$ [km]')
saveas(fig, fullfile(fdir, "p1-tf_and_flyby.png"))
```

Problem 2

```
% AAE 532 HW 8 Problem 2
% Tomoki Koike
close all; clear all; clc;
fdir = 'C:\Users\Tomo\Desktop\studies\2020-Fall\AAE532\matlab\outputs\ps8';
set(groot, 'defaulttextinterpreter','latex');
set(groot, 'defaultAxesTickLabelInterpreter','latex');
set(groot, 'defaultLegendInterpreter','latex');

% Set constants
planet_consts = setup_planetary_constants();  % Function that sets up all the
constants in the table
sun = planet_consts.sun;  % structure of sun
earth = planet_consts.earth;  % structure of earth
moon = planet_consts.moon;  % structure of moon
G = 6.6743015e-20;  % Gravitational constant [km^3/kg/s^2]

% Set given constants
h_PO = 190;  % earth parking orbit altitude
h_cap = 200;  % moon capture orbit altitude

% (a)
r_minus = earth.mer + h_PO
r_plus = moon.smao - h_cap - moon.mer
a_T = 0.5*(r_minus+r_plus)
e_T = (r_plus - r_minus) / (r_plus + r_minus)
p_T = a_T * (1 - e_T^2)
IP_minus = 2*pi*sqrt(a_T^3 / earth.gp)
IP_minus_days = IP_minus / 60 / 60 / 24
xi_minus = -earth.gp / 2 / a_T

v_PO = sqrt(earth.gp / r_minus)
v_plus = vis_viva(r_minus, a_T, earth.gp)
Dv_dep = v_plus - v_PO

v_minus = vis_viva(r_plus, a_T, earth.gp)
v_moon = sqrt(earth.gp / moon.smao)
v_inf_moon = abs(v_minus - v_moon)
v_c_moon = sqrt(moon.gp / (moon.mer + h_cap))
Dv_arr = sqrt( v_inf_moon^2 + 2*moon.gp / (moon.mer + 200) ) - v_c_moon
Dv_total = Dv_dep + Dv_arr
TOF = IP_minus / 2
TOF_days = TOF / 60 / 60 / 24
phi = rad2deg(pi - sqrt(earth.gp / moon.smao^3)*TOF)

% (b)

xi_plus = v_inf_moon^2 / 2
a_plus = -earth.gp / 2 / xi_plus
e_plus = 1 - (h_cap + moon.mer) / a_plus
delta = 2*asind(1 / e_plus)
v_plus = sqrt( v_inf_moon^2 + v_moon^2 - 2*v_inf_moon*v_moon*cosd(delta) )

FPA_plus = asind( v_inf_moon / v_plus * sind(delta))
```

```
% True anomaly
temp = r_plus * v_plus^2 / earth.gp;
TA_plus = atand( temp*sind(FPA_plus)*cosd(FPA_plus) / ( temp*cosd(FPA_plus)^2 - 1 ))
Domega = 180 - TA_plus

% characterisitics
a_N = -earth.gp / 2 / (v_plus^2 / 2 - earth.gp / r_plus)
h_N = r_plus*v_plus*cosd(FPA_plus)
p_N = h_N^2 / earth.gp
e_N =  sqrt(1 - p_N / a_N)
r_aN = a_N * (1 + e_N)
r_pN = abs(a_N) * (e_N - 1)
xi_N = -earth.gp / 2 / a_N
IP_N = 2*pi * sqrt(a_N^3 / earth.gp)
IP_N_year = IP_N / 60 /60 / 24 / 365

% (c)
Dv_eq = 2 * v_inf_moon * sind(delta / 2)
alpha = (180 - delta) / 2
Dv_eq_vec = Dv_eq * [cosd(alpha), sind(alpha), 0]

% Plotting for visualization
% old orbit
angles = 0:0.01:pi;
RR = p_T ./ (1 + e_T*cos(angles)); XX = RR.*cos(angles); YY = RR.*sin(angles);

% new orbit
RR_new = p_N ./ (1 + e_N*cos(angles - deg2rad(Domega)));
XX_new = RR_new.*cos(angles);
YY_new = RR_new.*sin(angles);

new_angles = TA_plus:20;
Xsun = sun.mer*cos(new_angles); Ysun = sun.mer*sin(new_angles);
fig = figure("Renderer","painters","Position",[10, 10, 900, 700]);
    plot(XX,YY)
    hold on; grid on; grid minor; box on; axis equal;
    plot(XX_new, YY_new, '-m')
    hold off
    xlabel('$\hat{e}$ [km]')
    ylabel('$\hat{p}$ [km]')
```

Problem 3

```
% AAE 532 HW 8 Problem 3
% Tomoki Koike
close all; clear all; clc;
fdir = 'C:\Users\Tomo\Desktop\studies\2020-Fall\AAE532\matlab\outputs\ps8';
set(groot, 'defaulttextinterpreter','latex');
set(groot, 'defaultAxesTickLabelInterpreter','latex');
set(groot, 'defaultLegendInterpreter','latex');

% Set constants
planet_consts = setup_planetary_constants();  % Function that sets up all the
constants in the table
sun = planet_consts.sun;  % structure of sun
earth = planet_consts.earth;  % structure of earth
venus = planet_consts.venus;  % structure of venus
G = 6.6743015e-20;  % Gravitational constant [km^3/kg/s^2]

% (a)
r_1 = earth.smao
r_2 = venus.smao

a_T = mean([r_1, r_2])
e_T = (r_1 - r_2) / (r_2 + r_1)
p_T = a_T * (1 - e_T^2);
v_plus = vis_viva(r_1, a_T, sun.gp)
v_minus = vis_viva(r_2, a_T, sun.gp)
v_earth = sqrt(sun.gp / r_1)
v_venus = sqrt(sun.gp / r_2)
TOF = pi * sqrt(a_T^3 / sun.gp)
TOF_years = TOF / 60 / 60 / 24 / 365
phi = rad2deg(pi - sqrt(sun.gp / r_2^3) * TOF)

% (b)
v_inf_earth = abs(v_plus - v_earth)
v_c_earth = sqrt(earth.gp / (210 + earth.mer))
Dv_dep = sqrt(v_inf_earth^2 + 2*earth.gp/(210 + earth.mer)) - v_c_earth

% (c)
h_cap = 2000;
r_p_minus = r_2 - venus.mer - h_cap
r_a_minus = r_1 + earth.mer + 210
r_minus = r_p_minus
xi_minus = -sun.gp / 2 / a_T

v_inf_venus = v_minus - v_venus
v_c_venus = sqrt(venus.gp / (h_cap + venus.mer))
Dv_arr = v_c_venus - sqrt(v_inf_venus^2 + 2*venus.gp / (h_cap + venus.mer))
Dv_total = Dv_dep + Dv_arr

% (d)
r_plus = r_minus;
xi_fb = v_inf_venus^2 / 2
a_fb = -venus.gp / 2 / xi_fb
e_fb = 1 - (h_cap+venus.mer) / a_fb
```

```
delta = 2*asind(1 / e_fb)
v_plus = sqrt( v_inf_venus^2 + v_venus^2 - 2*v_inf_venus*v_venus*cosd(delta) )
FPA_plus = asind(sind(delta) * v_inf_venus / v_plus)

% True anomaly
temp = r_plus * v_plus^2 / sun.gp;
TA_plus = atand( temp*sind(FPA_plus)*cosd(FPA_plus) / ( temp*cosd(FPA_plus)^2 - 1 ))
Domega = 0 - TA_plus

% characterisitics
a_N = -sun.gp / 2 / (v_plus^2 / 2 - sun.gp / r_2)
h_N = r_plus*v_plus*cosd(FPA_plus)
p_N = h_N^2 / sun.gp
e_N =  sqrt(1 - p_N / a_N)
r_aN = a_N * (1 + e_N)
r_pN = a_N * (1 - e_N)
xi_N = -sun.gp / 2 / a_N
IP_N = 2*pi * sqrt(a_N^3 / sun.gp)
IP_N_year = IP_N / 60 /60 / 24 / 365

% (d)
Dv_eq = 2*v_inf_venus*sind((delta) / 2)
alpha = 180 - (180 - delta) / 2

% (e)

% Plotting for visualization
% transfer orbit
angles = 0:0.01:2*pi;
RR = p_T ./ (1 + e_T*cos(angles)); XX = RR.*cos(angles); YY = RR.*sin(angles);

% new orbit
RR_new = p_N ./ (1 + e_N*cos(angles - deg2rad(Domega)));
XX_new = RR_new.*cos(angles);
YY_new = RR_new.*sin(angles);
rp_vec = r_pN*[cosd(Domega), sind(Domega)];
ra_vec = r_aN*[cosd(Domega+180), sind(Domega+180)];

% Earth orbit
X_earth = earth.smao * cos(angles);
Y_earth = earth.smao * sin(angles);

% Venus orbit
X_venus = venus.smao * cos(angles);
Y_venus = venus.smao * sin(angles);

fig = figure("Renderer","painters","Position",[10, 10, 900, 700]);
    % old transfer orbit
    plot(XX,YY)
    hold on; grid on; grid minor; box on; axis equal;

    % earth
    plot(X_earth, Y_earth, '-')

    % venus
```

```
    plot(X_venus, Y_venus, '-')

    % new heliocentric orbit
    plot(XX_new, YY_new, '-')

    plot([-r_1, r_2], [0, 0], '--k')
    plot([ra_vec(1), rp_vec(1)],[ra_vec(2), rp_vec(2)], '--r')
    hold off
    legend('transfer', 'earth', 'venus', 'new heliocentric')
    title('PS8 Problem 3 Earth to Venus Patch Conic with Failed Maneuver - T. Koike')
    xlabel('$\hat{e}$ [km]')
    ylabel('$\hat{p}$ [km]')
saveas(fig, fullfile(fdir, "p3-patch-conic.png"))
```