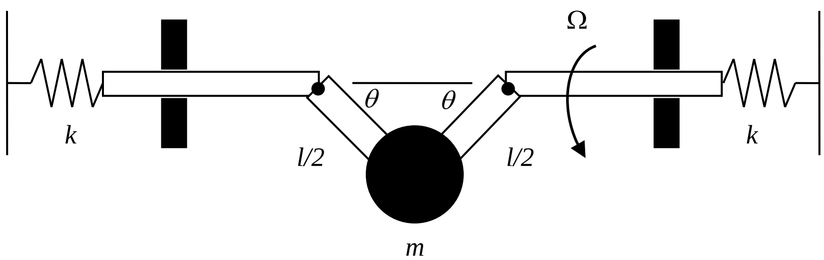
HW # 4 ME 6444 Nonlinear Systems
Fall 2021

Due Date: Thursday, 21 October

1. (30 points) Discrete NL Modeling, Hamiltonian, and Phase Plane Analysis

    a. Use Lagrange's eqs. to verify that the spinning shaft pictured is governed by,
    $$\frac{1}{4}m\ddot{\theta}-\frac{1}{4}m\dot{\theta}^2\tan\theta-\frac{1}{4}m\Omega^2\tan\theta+\frac{1}{2}k\frac{(1-\cos\theta)}{\cos\theta}\tan\theta=0 \;,$$
    where Ω is an imposed constant rotational spin. Assume massless links and an unstretched spring when θ is zero. Neglect gravitational potential energy (small compared to stored elastic energy).



    b. Sketch the local trajectories in phase plane about each fixed point assuming $\Omega > 0$.
    c. Show that the total energy $E = T + V$ is not conserved, and instead that the Hamiltonian $H = L - \frac{\partial L}{\partial \dot{\theta}}\dot{\theta}$ is conserved.
    d. Using H, determine the stability of the fixed points.

2. (20 points) Phase Plane with Stability Analysis

    Determine the character (type of fixed point and stability) of all fixed points associated with each equation of motion below. Plot the phase plane for each.

    a. $\ddot{u}+2\mu\dot{u}+u+u^3=0$
    b. $\ddot{u}+2\mu\dot{u}+u-u^3=0$
    c. $\ddot{u}+2\mu\dot{u}-u+u^3=0$
    d. $\ddot{u}+2\mu\dot{u}-u-u^3=0$

    In all cases treat the damping coefficient μ as greater than zero.