October 27, 2020

AAE 564 Fall 2020

Test One

Problem 1 Obtain a state space description of the following system.

$$\begin{aligned} \dot{q}_1 + \ddot{q}_2 - q_2^3 &= u \\ \ddot{q}_2 + \dot{q}_3 - q_1^3 &= u \\ \dot{q}_3 - q_2 &= 0 \\ y &= q_1 + \ddot{q}_3 \end{aligned}$$

Problem 2 Obtain a state space realization of the transfer function,

$$\hat{G}(s) = \begin{pmatrix} \dfrac{s^2}{s^2-4} & \dfrac{s}{s-2} \\ \dfrac{1}{s+2} & -\dfrac{1}{s} \end{pmatrix}.$$

Problem 3 Obtain a state space description of the following single-input single-output system with input u and output y.

$$2\dot{q}_1 - \dot{q}_2 - 6q_2 = 3u \tag{1}$$

$$-\dot{q}_1 + 2\dot{q}_2 - 9q_1 = 6\dot{u} \tag{2}$$

$$y = q_1 \tag{3}$$

Problem 4 (a) Obtain the eigenvalues and eigenvectors of the following matrix.

$$A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & -2 \\ -1 & 0 & -2 \end{pmatrix}$$

(b) Is the matrix defective or non-defective? Justify your answer.

Problem 5 Two of the eigenvalues of a real matrix A are 1 and $\jmath$ with corresponding eigenvectors

$$\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \qquad \text{and} \qquad \begin{pmatrix} 1 \\ \jmath \\ 0 \end{pmatrix}$$

What is the solution to $\dot{x} = Ax$ with

$$(a) \quad x(0) = \begin{pmatrix} -2 \\ 0 \\ -2 \end{pmatrix}, \qquad (b) \quad x(0) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \qquad (c) \quad x(0) = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}?$$

Express your solutions in real form.

Problem 6 Compute e^{At} for the matrix

$$A = \begin{pmatrix} -2 & 0 \\ 1 & -1 \end{pmatrix}.$$

Problem 7 Consider an LTI system described by

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -4x_1 - 4x_2 + u \\ y &= -8x_1 - 4x_2 + u \end{aligned}$$

Is their a persistent input (does not go to zero) u for which the corresponding output always goes to zero regardless of initial conditions? If answer is yes, provide an example.

Problem 8 If possible, use linearization to determine the stability properties of each of the following systems about the zero solution.

(a)

$$\begin{aligned} \dot{x}_1 &= x_2^2 x_1 + (\cos x_1) x_2 \\ \dot{x}_2 &= (1 + \sin x_2) x_1 - x_1^2 x_2 \end{aligned}$$

(b)

$$\ddot{y} + \dot{y}^3 + y^5 = 0$$

(c)

$$\frac{d^3 y}{dt^3} - (\cos y)\dot{y} = 0$$