AAE 42100 **Exam 2** (Open-Book Open-Notes) **Name**______________________

Due 12am November 21st, 2020 (Print Your Name)

I certify that I have neither given help to, nor received help from, any individual in matters relating to this examination.

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Signature

Problem 1. (25 pts)

Given the following set of nonlinear differential equations:

$$\dot{V} = (T - D - mgsin\gamma)/m$$
$$\dot{\gamma} = (L - mgcos\gamma)/(mV)$$
$$\dot{h} = Vsin\gamma$$

where

$$L = \frac{1}{2}\rho V^2 S C_L \quad and\ C_L = C_{L_\alpha}\alpha$$
$$D = \frac{1}{2}\rho V^2 S\left(C_{D_0} + \epsilon C_L^2\right)$$

The parameters are set as $\epsilon = 0.9$, $AR = 0.86$, $m = 0.003kg$, $S = 0.017m^2$, $C_{d_0} = 0.02$, $\rho = 0.41405kg/m^3$at h=10,000m, $C_{L_\alpha} = 1.2936$.

(a) Find the trim condition for a leveled flight at altitude of $h = 10{,}000m$ flying at a speed of V_0 =3.7 m/s, where the state $\bar{x} = \begin{bmatrix} V \\ \gamma \\ h \end{bmatrix}$ and the control is denoted by $\bar{u} = \begin{bmatrix} \alpha \\ T \end{bmatrix}$. (Hint: to maintain a trim condition at level flight, we much have $\dot{V} = \dot{\gamma} = \dot{h} = 0$ and $\gamma = 0$)

(b) Develop a Simulink model to simulate the system state response for $t = 10$ sec with the initial condition set as the trim condition in Part (a) and input $\bar{u} = \begin{bmatrix} \alpha_{trim} \\ T_{trim} \end{bmatrix} * 105\%$.

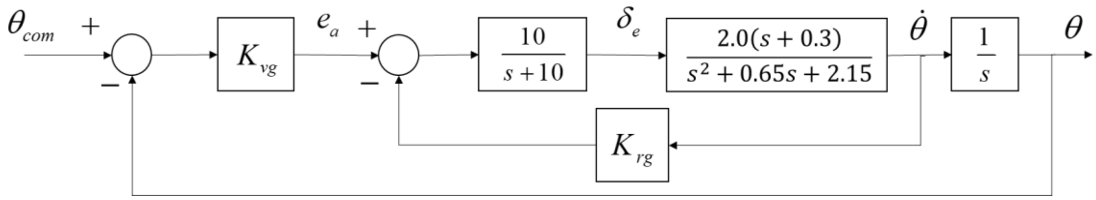
(c) Make use of the Matlab command 'linmod.m' to find the linearized state space model about the trim condition found in part (a), assuming the output $\bar{y} = \bar{x}$.

Problem 2. (25 pts)

Using the Dutch roll approximation, determine the state feedback gains so that the damping ratio and frequency of the Dutch roll are 0.3 and 1.0 rad/s, respectively. Assume the airplane has the following characteristics: $Y_\beta = -19.5\ ft/s^2$, $Y_r = 1.3\ ft/s$, $N_\beta = 1.5\ s^{-2}$, and $N_r = -0.21\ s^{-1}$, $U_0 = 400\ ft/s$.

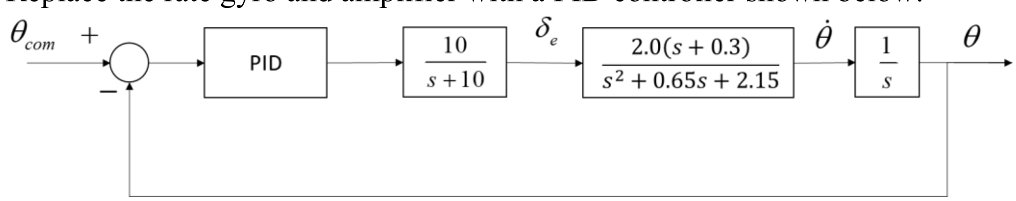
Problem 3. (25 pts)

For the pitch displacement autopilot system shown below,



(a) Determine the gain necessary to improve the system characteristics so that the control system has the following performance: $\zeta = 0.3$, $\omega_n = 2.0$ rad/s. Verify your solution by providing the root locus plot of the overall system and plot of system response to a 5^o step change in the commanded pitch attitude.

(b) Replace the rate gyro and amplifier with a PID controller shown below:



Design the PID gains using Matlab Control System Tuner. Compare the design results with part (a) by providing the plot of system response to a 5^o step change in the commanded pitch attitude.

Problem 4. (25 pts)

The equations of motion governing the aircraft's motion are

$$\Delta\dot{\alpha} = \frac{Z_\alpha}{U_0}\Delta\alpha - \Delta q$$

$$\Delta\dot{q} = M_\alpha\Delta\alpha + M_q\Delta q + M_\delta\Delta\delta_e$$

The performance index is set as

$$\mathrm{J} = \int_0^\infty \left[\left(\frac{\alpha}{\alpha_{max}}\right)^2 + \left(\frac{\delta_e}{\delta_{e_{max}}}\right)^2 + \left(\frac{q}{q_{max}}\right)^2\right] \mathrm{dt}$$

where α_{max}, $\delta_{e_{max}}$, q_{max} are given parameters. Derive the nonlinear algebraic equations to satisfy the algebraic Riccati equation that is required to be solved to design an LQR controller.

Bonus (5 pts)

Use state feedback to design an altitude hold control system. Assume the forward speed is held constant and the longitudinal equation can be modeled using the short-period approximation. The short-period equations are

$$\begin{bmatrix} \Delta\dot{\alpha} \\ \Delta\dot{q} \\ \Delta\dot{\theta} \end{bmatrix} = \begin{bmatrix} -1.5 & 1 & 0 \\ -4.0 & -1.0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta\alpha \\ \Delta q \\ \Delta\theta \end{bmatrix} + \begin{bmatrix} -0.2 \\ -8.0 \\ 0 \end{bmatrix} \Delta\delta_e$$

Assume the $\Delta\dot{h} = u_0(\Delta\theta - \Delta\alpha)$ where $u_0 = 200$ ft/s. Determine the state feedback gain if the closed-loop eigenvalues are located at

$$\lambda = -1.5 \pm 2.5\mathrm{i}$$

$$\lambda = -0.75 \pm 1.0\mathrm{i}$$