

AAE 666 Spring 2021

HOMEWORK ONE

Friday, January 29

Exercise 1 By appropriate definition of state variables, obtain a first order state space description of the following system where q_1 and q_2 are real scalars.

$$\begin{aligned} 2\ddot{q}_1 + \ddot{q}_2 + \sin q_1 &= 0 \\ \ddot{q}_1 + 2\ddot{q}_2 + \sin q_2 &= 0 \end{aligned}$$

Exercise 2 By appropriate definition of state variables, obtain a first order state space description of the following system where q_1 and q_2 are real scalars.

$$\begin{aligned} \ddot{q}_1 + \dot{q}_2 + q_1^3 &= 0 \\ \dot{q}_1 + \dot{q}_2 + q_2^3 &= 0 \end{aligned}$$

Exercise 3 By appropriate definition of state variables, obtain a first order state space description of the following system where q_1 and q_2 are real scalars.

$$\begin{aligned} \ddot{q}_1 + q_1 + 2\dot{q}_2 &= 0 \\ \ddot{q}_1 + \dot{q}_2 + q_2 &= 0 \end{aligned}$$

Exercise 4 By appropriate definition of state variables, obtain a first order state space description of the following system where q_1 and q_2 are real scalars.

$$\begin{aligned} q_1(k+2) + q_1(k) + 2q_2(k+1) &= 0 \\ q_1(k+2) + q_1(k+1) + q_2(k) &= 0 \end{aligned}$$

Exercise 5 Show that x^e is an equilibrium state of the system

$$x(k+1) = x(k) - \frac{g(x(k))}{g'(x(k))}$$

if and only if $g(x^e) = 0$.

Exercise 6 Draw the state portrait of the first nonlinear system.

Exercise 7 Draw the state portrait for

$$\dot{x} = x^4 - x^2.$$

Exercise 8 Obtain an explicit expression for all solutions of

$$\dot{x} = -x^3.$$

Exercise 9 Consider the Lorenz system described by

$$\begin{aligned}\dot{x}_1 &= \sigma(x_2 - x_1) \\ \dot{x}_2 &= rx_1 - x_2 - x_1x_3 \\ \dot{x}_3 &= -bx_3 + x_1x_2\end{aligned}$$

with $\sigma = 10$, $b = \frac{8}{3}$, and $r = 28$. Simulate this system with initial states

$$\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 0 \\ 1 + eps \\ 0 \end{pmatrix}$$

where eps is the floating point relative accuracy in MATLAB. Comment on your results for the integration interval $[0 \ 60]$.