Homework #2 Due: September 30, 2021

1. Solve the following equality minimization problem

$$\min f(x) = x$$

subject to

$$g(x,y) = y^2 + x^4 - x^3 = 0$$

2. Solve the following optimization problem:

$$\min \;\; x_1^2 - x_2$$

subject to

$$x_1^2 + x_2^2 \leq 1, \qquad x_2 \leq 2, \qquad x_1^3 + x_2 = 1$$

3. Solve the problem

$$\begin{array}{ll} \text{Minimize} & x_1 + x_2^2 + x_2 x_3 + 2x_3^2 \\ \text{subject to} & \frac{1}{2}(x_1^2 + x_2^2 + x_3^2) = 1 \end{array}$$

4. Solve the following problem

$$\max \;\; x_1$$

subject to

$$\begin{aligned} x_2 - (1 - x_1)^3 &\leq 0, \\ x_2 &\geq 0 \end{aligned}$$

Plot the feasible region for this problem, along with the optimal point. Draw the gradient of the constraints and the gradient of the function to be minimized. What do you observe?

5. Minimize

$$f(x_1, x_2) = -5x_1 - x_2$$

subject to the inequalities

$$\begin{aligned} g_1(x_1,x_2) &= -x_1 \leq 0 \\ g_2(x_1,x_2) &= 3x_1 + x_2 - 11 \leq 0 \\ g_3(x_1,x_2) &= x_1 - 2x_2 - 2 \leq 0 \end{aligned}$$

Sketch the region of feasible points in x_1, x_2 space. Check the Kuhn-Tucker necessary condition at the point which furnishes the minimum. Verify your answer using the `fmincon` command of MATLAB.

6. The previous problem is an example of a *Linear Programming* problem. The general linear programming (LP) problem has the form

$$\min c^{\mathsf{T}} x$$

subject to

$$Ax \leq b$$

where $A \in \mathbb{R}^{m\times n}$, $b \in \mathbb{R}^m$ and $c \in \mathbb{R}^n$. The MATLAB command `linprog` can be used to solve LP problems.

Consider the following transportation planning problem: A car dealer has purchased 500 cars in Seattle and 400 cars in Chicago. He then sells 200 cars to a customer in Denver, 360 to a customer in Miami and the remaining 340 cars to a customer in New York city. He wishes to determine the shipping schedule to deliver all these vehicles that will incur the minimum cost, given the freight rates (in dollars per vehicle) shown in the table below:

Seattle	\$400	\$550	\$600
Chicago	\$360	\$470	\$500
	Denver	Miami	New York

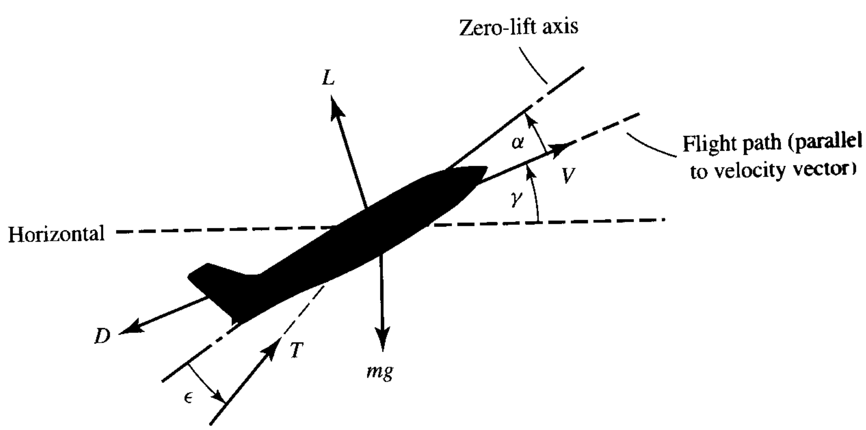
(a) Formulate the previous transportation problem as an LP problem. (Hint: Let x_1 be the number of cars to be shipped from Seattle to Denver, and let x_2 be the number of cars to be shipped from Seattle to Miami. Then $200-x_1$ will be the number of cars to be shipped from Chicago to Denver, $360-x_2$ will be the number of cars to be shipped from Chicago to Miami, etc).

(b) Use the `linprog` command of MATLAB to find the minimum-cost shipping schedule.

7. The net force on an aircraft maintaining a steady rate of climb must be zero. Choosing force components parallel and normal to the flight path (see figure) one obtains the equations

$$T(V)\cos(\alpha+\varepsilon) - D(V,\alpha) - mg\sin\gamma = 0$$
$$T(V)\sin(\alpha+\varepsilon) + L(V,\alpha) - mg\cos\gamma = 0$$

where V is the aircraft velocity, γ the flight path angle, α the angle of attack, m the mass of the aircraft, ε the angle between the thrust axis and the zero-lift axis (constant), L the lift, D the drag, and T the engine thrust. We want to find (α, V, γ) to maximize the rate of climb $f(V,\gamma) = V\sin\gamma$.



(a) Write down the necessary condition for maximizing the rate of climb for an aircraft while maintaining a steady rate of climb.

(b) Using the MATLAB function `fmicon` show that for a Boeing 727 aircraft the maximum rate of climb is 37.6 ft/sec and occurs at $V = 342$ ft/sec, $\alpha = 6.39\,\text{deg}$ and $\gamma = 6.31\,\text{deg}$.

The thrust, drag and lift in units of a/c weight W(=180,000 lb) for a B723 aircraft at take-off (maximum rate of climb) are given approximately by

$$
\begin{aligned}
T &= 0.2476 - 0.04312\,V + 0.008392\,V^2 \\
D &= (0.07351 - 0.08617\,\alpha + 1.996\,\alpha^2)\,V^2 \\
L &= (0.1667 + 6.231\,\alpha - 21.65\,[\max(0, \alpha - 0.2094)]^2)\,V^2
\end{aligned}
$$

where V is the velocity in units of $\sqrt{g\ell}$, $\ell = 2W/(\rho g S)$, $\rho = 0.002203$ slugs/ft^3 the density of air at sea level, and $S = 1560$ ft^2 the wing span. The angle of attack α is in radians and $\varepsilon = 0.0349$ rad.