AAE 532 – Orbit Mechanics
Problem Set 4
Due: 10/2/20

[Note that a new document is available under GMAT Tips on Brightspace: "Defining Multiple Spacecraft and Propagators." It may be useful in the future!]

Problem 1: At some initial time, t_o, a spacecraft is in orbit about the Earth. Its orbit is characterized by $e = 0.6$ and $p = 6\,\mathrm{R}_\oplus$. It is currently located at the point in the orbit such that $\theta_o^* = 90^\circ$.

(a) Determine the following orbit parameters and spacecraft state information:
a, r_p, r_a, *period*, $\mathcal{E}$; $r_o, v_o, E_o^*, \gamma_o$ [Always list all angles in degrees.]
Compare v at this location with $\sqrt{2}\,v_c$. Should $v >$ or $v > \sqrt{2}\,v_c$? Can your v value be correct? How do you know?

(b) Write $\bar{r}_o$ and $\bar{v}_o$ in terms of components in the directions of $\hat{\boldsymbol{e}}$ and $\hat{\boldsymbol{p}}$.
At some later time, t_f, determine $r_f, v_f, \theta_f^*, \gamma_f$ when $E_f^* = 225^\circ$. Write $\bar{r}_f$ and $\bar{v}_f$ in terms of components in the directions of $\hat{\boldsymbol{e}}$ and $\hat{\boldsymbol{p}}$.

(c) Determine the time t_o relative to periapsis, i.e., at $\theta_o^* = 90^\circ$; also determine the final time t_f relative to periapsis, i.e., when $E_f^* = 225^\circ$. What is the time-of-flight (TOF), i.e., $\left(t_f - t_o\right)$ as well as $\Delta\theta^*$ and ΔE?

(d) PLOT the entire orbit in MATLAB. (Do not use polar plots; compute $\hat{e}$ and $\hat{p}$ components along the path.)
By hand, on the plot, mark the location of the satellite at t_o and t_f; at each location, indicate $r, \theta^*, \bar{v}, E, \gamma$; also, sketch the local horizon. Indicate the arc used between t_o and t_f.

Problem 2: Return to Problem 1 and confirm your results in GMAT. Use October 2, 2020 as the start date.

(a) What initial state can be input to GMAT for Sat1? Can you locate the rest of the quantities that were requested in Problem 1(a) and 1(b); do they confirm that your computations are correct? Can you determine the time difference $\left(t_f - t_o\right)$?
Compare your Matlab plot and the GMAT plot. Is your GMAT plot consistent with your MATLAB plot?

(b) Also print out the data from GMAT. (You can submit output from the file generated in the propagate window under the Mission Sequence. Cut-and-paste the sections with the required data into a Word document. Highlight the requested quantities. You can also create a Report file; you may not want to include the entire file but, again cut-and-paste.)

(c) Add an X-Y plot to the output. Plot speed as a function of elapsed time in seconds. Print the plot. Mark your time that you computed in Problem 1. Does the max velocity location in your plot correlate to the periapsis time in the GMAT plot?

Problem 3: To investigate the requirements for departure, assume that a spacecraft is departing the vicinity of the Earth along a parabolic path. Consider the spacecraft to be located at perigee on the parabola.

(a) A circular parking orbit about the Earth may be defined at 225 km altitude. At a perigee altitude of 225 km on the parabola, compare the escape velocity on the parabola with the relative velocity in a circular orbit with the same altitude.
To shift from the circular orbit to the escape trajectory, what % increase in velocity is required?

(b) Compute the velocity along the parabola as it departs the vicinity of the Earth, that is, at the following distances: $r = 2R_{\oplus}, 10R_{\oplus}, 75R_{\oplus}, 200R_{\oplus}, 800R_{\oplus}$ one additional distance of your choice.
Determine the true anomaly θ^* that corresponds to each distance. Also include the time since periapsis at each distance (in days).

(c) In the MATLAB script from the first problem, plot the parabola corresponding for altitude 225 km between $-140^\circ < \theta^* < 140^\circ$. Mark on the plot, r, v, γ at $\theta^* = -120^\circ$; also, sketch the l.h. (local horizon). Also add the directrix.
Compare θ^* and γ is there a pattern?

(d) At $r = 75R_{\oplus}$, is it reasonable to model the problem as a two-body (Earth and spacecraft)?

Problem 4: As part of the new lunar initiative, an unmanned probe is approaching the Moon on a hyperbola. The hyperbola is defined such that $|a| = 7050$ km and the passage altitude is 800 km altitude. At the "current" time, the probe is located at $\theta^* = -60^\circ$.

(a) Determine the following additional orbital characteristics: $r_p, v_p, b, h, \delta, v_\infty, \boldsymbol{\mathcal{E}}$. Determine the following quantities at the current time: r, v, γ, H , time till perilune.

(b) Use your Matlab script and plot the hyperbola between $\theta^* = \pm 100^\circ$. Mark the probe at $\theta^* = -60^\circ$ and label b, aim point, $\frac{\delta}{2}$, v, γ, r, θ^*. (Always include the local horizon!)

(c) Determine r, v, γ at $\theta^* = +100^\circ$; add this information to the plot.