LA BP

Thursday, November 12, 2020 02:42 PM

Transfer Orbits: Lambert Arcs

Two approaches to mission planning:

(a) Given the transfer orbit → initial and final positions are specified; relate to the time of flight

(b) Given the initial (departure) and final (target) points → determine the orbit that passes through the points

Transfer Orbit Design
(special class of boundary value problem)

1. **Geometrical relationships**

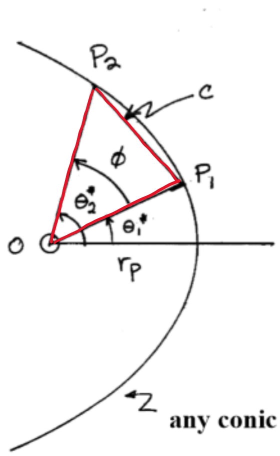
   Conic paths connecting two points that are fixed in space with focus at the attracting center

2. **Analytical Relationships**

3. **Lambert's Theorem**

Analytical Relationships

Objective: expression for *p*; *e*



$$r = \frac{p}{1 + e\cos\theta^*}$$

$$e\cos\theta_1^* = \frac{p}{r_1} - 1$$

$$e\cos\underbrace{\theta_2^*}_{} = e\cos(\theta_1^* + \varphi) = \frac{p}{r_2} - 1$$

Also known:

$$a\,e^2 = a - p$$

cosine law

$$c^2 = r_1^2 + r_2^2 - 2r_1 r_2\cos\phi$$

Given the following trig identity

$$\cos^2(\theta_1^* + \varphi) - 2\cos(\theta_1^* + \varphi)\cos\theta_1^*\cos\varphi + \cos^2\theta_1^* - \sin^2\varphi = 0$$

Sub above 5 expressions into trig identity and produce a quadratic in ***p***

only unknown : a, p

$$\boxed{a c^2 p^2 + r_1 r_2 (1 - \cos\phi)\left[-2a(r_1 + r_2) + r_1 r_2 (1 + \cos\phi)\right] p + a r_1^2 r_2^2 (-1 + \cos\phi)^2 = 0} \qquad \mathbf{I}$$

modify to more convenient form

definition of semi-perimeters of space triangle

Use $2s = r_1 + r_2 + c$ to rewrite term in brackets

$$\boxed{\left[-2a(r_1+r_2)+r_1 r_2(1+\cos\phi)\right]=2s(s-c-2a)+2ac} \qquad \textbf{A}$$

Also the last term

$$\boxed{r_1 r_2(1-\cos\phi)=2(s-r_1)(s-r_2)} \qquad \textbf{B}$$

AND add some new definitions:

IF transfer is elliptic arc

$$\boxed{\begin{aligned} \alpha &= 2\arcsin\sqrt{\frac{s}{2a}} \\ \beta &= 2\arcsin\sqrt{\frac{s-c}{2a}} \end{aligned}}$$

$$\left.\begin{aligned} s-c-2a &= -2a\cos^2\left(\frac{\beta}{2}\right) \\ s &= 2a\sin^2\left(\frac{\alpha}{2}\right) \\ c &= 2a\left[\sin^2\left(\frac{\alpha}{2}\right)-\sin^2\left(\frac{\beta}{2}\right)\right] \end{aligned}\right\} \quad \textbf{C}$$

Sub A, B, C into I

Quadratic for p

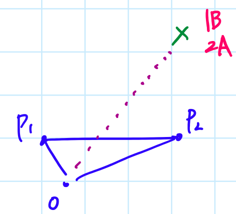
$$c^4 p^2 - 4a(s-r_1)(s-r_2)\left[\sin^2\left(\frac{\alpha+\beta}{2}\right)+\sin^2\left(\frac{\alpha-\beta}{2}\right)\right]c^2 p$$

$$+4a^2(s-r_1)^2(s-r_2)^2\sin^2\left(\frac{\alpha+\beta}{2}\right)\sin^2\left(\frac{\alpha-\beta}{2}\right)=0$$

Roots

$$\boxed{p = \frac{4a(s-r_1)(s-r_2)}{c^2}\sin^2\left(\frac{\alpha\pm\beta}{2}\right)}$$

larger $a e$ $|\overline{OF}|$
larger e
$\downarrow$
smaller p



If know a, produces two possible paths;
Each path possesses different values of p and e

@ $a = a_{\min}$

$2a_{\min} = s \quad\Rightarrow\quad \alpha = \pi$

$$p = \frac{4a(s-r_1)(s-r_2)}{c^2}\sin^2\left(\frac{\pi\pm\beta}{2}\right) \Rightarrow \text{one } p$$

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IF transfer is hyperbolic arc

$$\boxed{\begin{aligned}\alpha' &= 2\,\text{arcsinh}\sqrt{\frac{s}{2|a|}}\\ \beta' &= 2\,\text{arcsinh}\sqrt{\frac{s-c}{2|a|}}\end{aligned}}$$

$$s-c-2a = 2|a|\cosh^2\left(\frac{\beta'}{2}\right)$$

$$s = 2|a|\sinh^2\left(\frac{\alpha'}{2}\right)$$

$$c = 2|a|\left[\sinh^2\left(\frac{\alpha'}{2}\right) - \sinh^2\left(\frac{\beta'}{2}\right)\right]$$

C

Given $r_1\ r_2,\ \theta,\ c$

know 'a' or select 'a' — how to select "a" ?

$\downarrow$

α, β

$\downarrow$

p quad eqn $\Rightarrow$ roots choose p by geometry

1. pick some 'a' → try it
2. select another quantity

$$s=2|a|\sinh^2\left(\frac{\quad}{2}\right)$$

$$\left.\begin{array}{l} \\ c=2|a|\left[\sinh^2\left(\frac{\alpha'}{2}\right)-\sinh^2\left(\frac{\beta'}{2}\right)\right]\end{array}\right\} \quad \mathbf{C}$$

Using this C in I

Roots

$$\boxed{p=\frac{4|a|(s-r_1)(s-r_2)}{c^2}\sinh^2\left(\frac{\alpha'\pm\beta'}{2}\right)}$$

If known $|a|$, produces two possible hyperbolic paths

2. select another quantity

TOF

$\varepsilon=-\frac{\mu}{2a}$

$\underbrace{\text{related to 'a'}}_{\text{lambert}}$