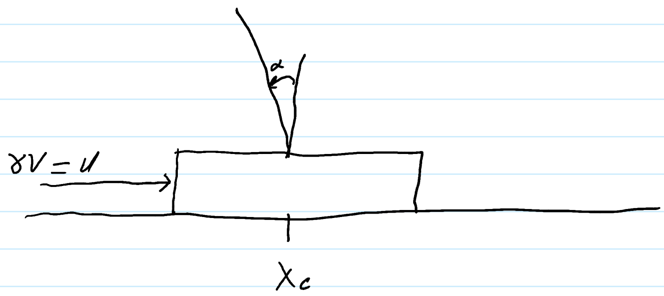
CONTROL of the Inverted pendulum



$$\dot{X} = AX + BV \qquad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \qquad x_1 = x_c \quad x_2 = \alpha \qquad x_3 = \dot{x}_c \quad x_4 = \dot{\alpha}$$

STATE FEEDBACK

$$V = r(t) - KX = r(t) - k_1 x_1 - k_2 x_2 - k_3 x_3 - k_4 x_4$$

$$K = [k_1 \ k_2 \ k_3 \ k_4] : \mathbb{C}^4 \longrightarrow \mathbb{C}$$

$$A : \mathbb{C}^4 \longrightarrow \mathbb{C}^4 \quad \text{and} \quad B : \mathbb{C} \longrightarrow \mathbb{C}^4$$

$$V = r(t) - KX = r(t) - [k_1 \ k_2 \ k_3 \ k_4] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$$\dot{X} = AX + BV = AX + B(r(t) - KX)$$

FEEDBACK SYSTEM

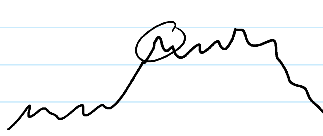
$$\dot{X} = (A - BK)X + B\,r(t)$$

Choose K TO CONTROL the pendulum

A is unstable

Choose K To make $(A-BK)$ STABLE

$$x_1 = x_c \qquad x_3 = \dot{x}_c$$



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Pole placement

Assume $\{A, B\}$ is controllable

$\text{rank}([B, AB, A^2B, A^3B]) = 4$

$K = \text{place}(A, B, [\lambda_1, \lambda_2, \lambda_3, \lambda_4])$

$\text{eig}(A - BK) = \underline{\{\lambda_1, \lambda_2, \lambda_3, \lambda_4\}}$

$\dot{x} = (A - BK)x + Br$

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$$\min \int_0^\infty (q_1 x_1^2 + q_2 x_2^2 + q_3 x_3^2 + q_4 x_4^2 + R V^2)\, dt$$

$q_1 > 0$, $q_2 > 0$, $q_3 > 0$, $q_4 > 0$ $R > 0$

Choose R large "feedback tends to be slow"

Choose R small "feedback tend to go faster"

small ~ large K

If $\{A, B\}$ is controllable $\Longrightarrow$

$\underline{A} - \underline{B}K$ is STABLE

The feedback System is stable

Pole placement MAY NOT BE ROBUST

$\boxed{\text{LQR IS ROBUST}}$

IN MATLAB

$K = \text{lqr}(A, B, \text{diag}([q_1, q_2, q_3, q_4]), R)$