College of Engineering
School of Aeronautics and Astronautics

AAE 532
Orbital Mechanics

PS 5
Time Elapse of Orbits and 3D Orbits

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Problem 1: Return to the use of multiple propagators in GMAT. (Demonstrated in a previous GMAT Tip.) The propagator 'TwoBody' or 'EarthPointMass' is already available (under the name you have selected for previous assignments). Produce the new propagators: 'EarthMoon', 'EarthSun', 'EarthMoonSun'. Add the different forces to create the new propagators as described in the GMAT Tips document. Add the Moon's orbit (Luna) to the output image. Propagate for 60 days.

(a) Use a date October 2, 2020 16:00:00. Use the Earth J2000Eq coordinates throughout the simulations. In a Keplerian Coordinate Type, introduce initial conditions such that

$$r_p = 1.5R_\oplus, \qquad \Omega = 0$$

$$r_a = 200R_\oplus, \qquad \theta^* = 0$$

$$\omega = 0°, \qquad i = 30°$$

Explore the 4 propagators (use different color for each propagated path). Propagate all the trajectories for 60 days. [Sometimes it is convenient to use the 'Animation' button on the top bar if you have not already tried it! Watch each simulation evolve.]

Produce a plot with a view approximately down the Moon Orbit Normal with all four spacecraft. Add views on two other dates: October 7, 2020 and October 11, 2020 at the same time of day. Choose another date in October and add a figure.

These simulations all use the relative vector equation of motion for the spacecraft relative to the Earth from Notes Page D2; the perturbations on the right-hand side of the equation vary for each propagator.

Does the model make a difference? Is the two-body model adequate for this particular problem? Why or why not? For the trajectory in this analysis, which relative orbit model would you recommend: two-body, three-body, four-body? Why? Which bodies would you include?

What is the impact of the different epoch dates? Why is there such a difference in the paths?

From the given values of distance of periapsis and apoapsis we can calculate the semi-major axis and eccentricity

$$a = \frac{1}{2}(r_p + r_a) = \frac{1}{2}(1.5R_\oplus + 200R_\oplus)$$

Since, $R_\oplus = 6378.1\ km$

$$a \ = \ 6.4259e + 5 \ \ km \ .$$

Then,

$$r_p = a(1 - e)$$

$$e = 1 - \frac{r_p}{a}$$

$$e \ = \ 0.9851$$

Set this in the settings for “Sat1” in GMAT. Duplicate this for 3 more satellites.

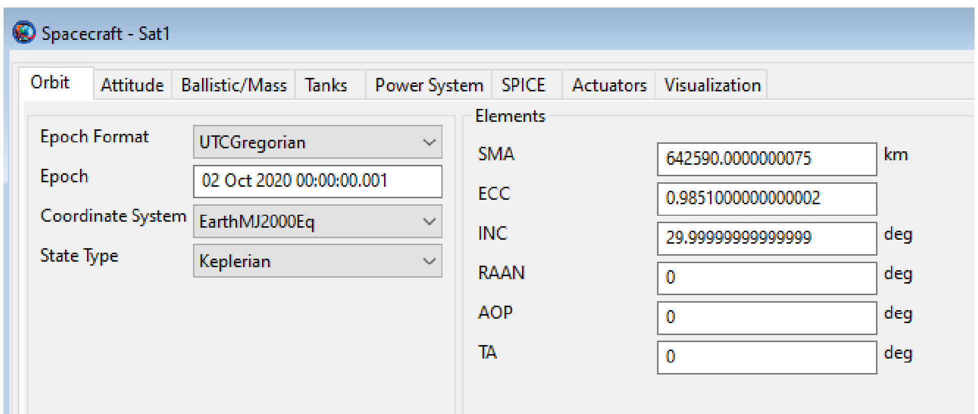


Figure 1: Sat1 settings

Define 4 missions, “Propagate1”, “Propagate2”, “Propagate3”, and “Propagate4”. Each propagator corresponds to the propagator “TwoBody”, “EarthMoon”, “EarthSun”, and “EarthMoonSun” respectively. For the “Stopping conditions” use the “ElapsedDays” and set that to 60 days.

On the next page you will see the plots for each satellite tracking each propagation defined. Each plot indicates the location of the satellite for the dates October 2, 2020 00:00:00:001, October 7, 2020 00:00:00:001, and October 11, 2020 00:00:00:001 respectively. Additionally, the date October 30, 2020 00:00:00:001 was selected, and the plot is shown as the fourth one.

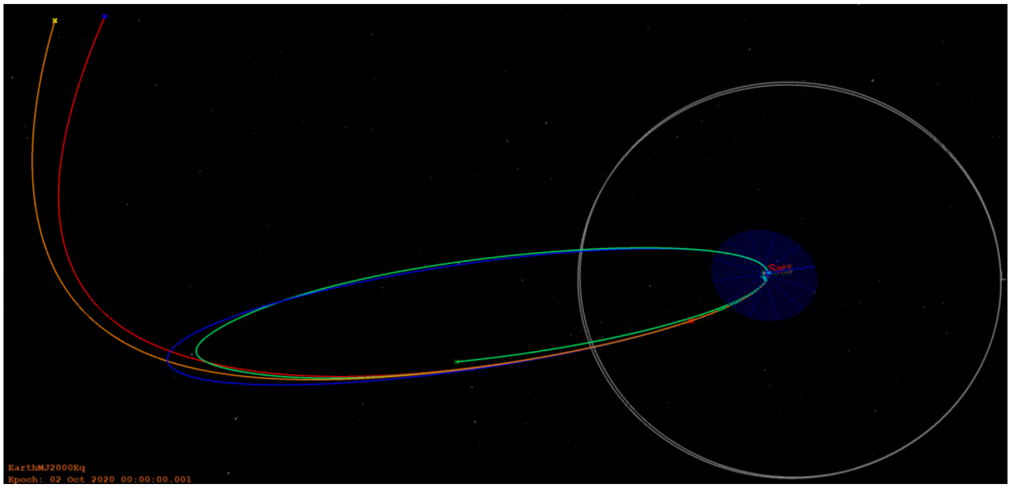


Figure 2: October 2 orbit

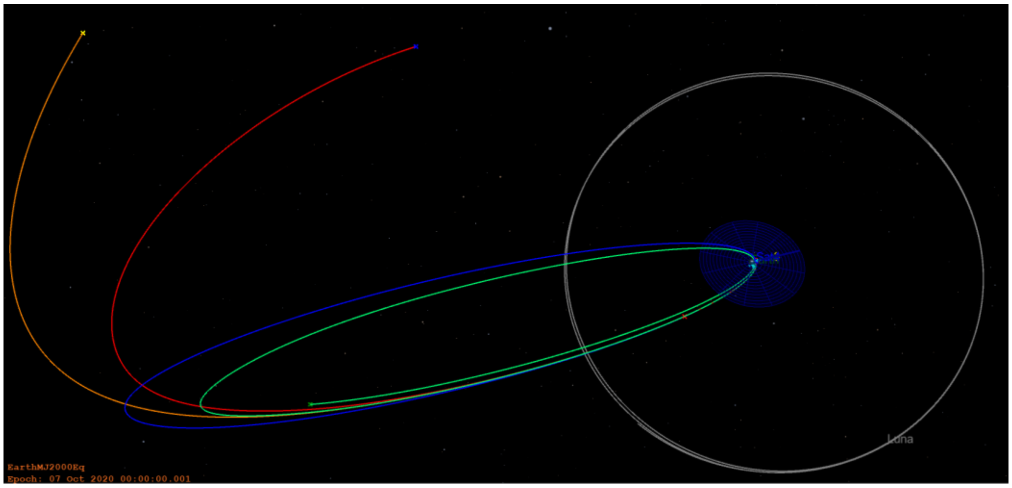


Figure 3: October 7 orbit

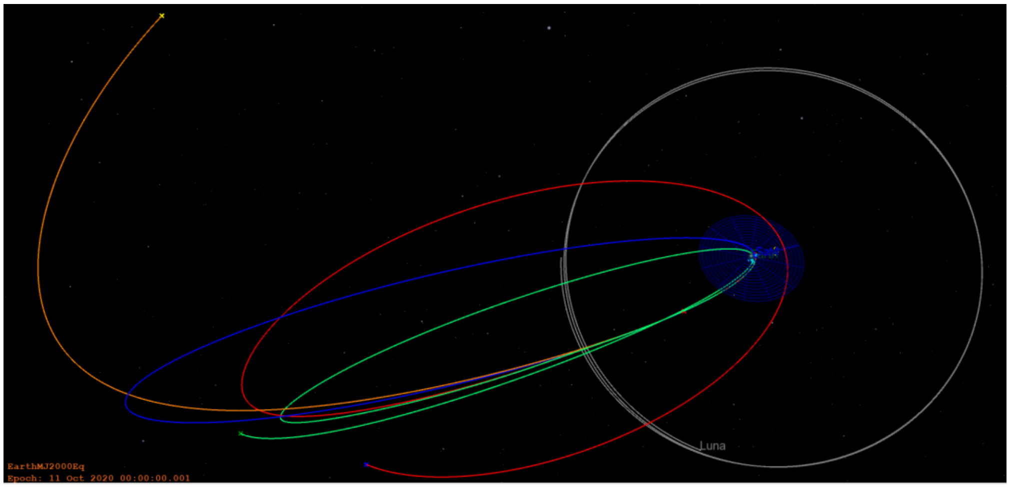


Figure 4: October 11 orbit

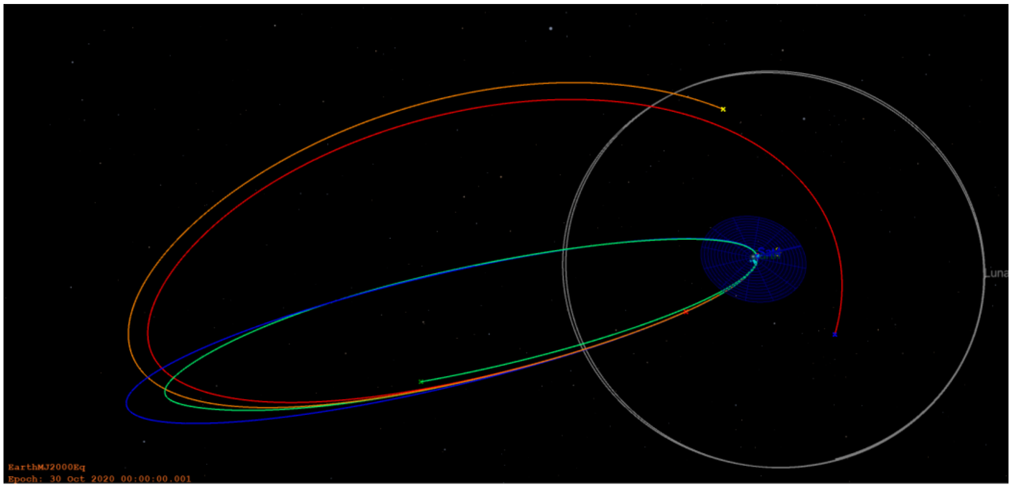


Figure 5: October 30 orbit

The model does make a difference. The point mass model of the Earth and the three-body problem of the Earth-Moon-spacecraft is almost alike when the Moon is relatively farther away from the spacecraft's orbit throughout the simulation time; however, when it is close to the spacecraft (figures 3 and 4) the orbits diverge largely. Also, when the Sun is incorporated into the model the orbit largely diverges from the elliptical orbit that the orbital models without the Sun travels. Thus, the two-body model is not adequate for this problem. Although, the three-body is not enough as well from what we have seen when the Moon is close to the spacecraft. That is the orbital behavior changes significantly when we disregard the moon while the Moon is close to the orbit of the spacecraft. Thus, we conclude that we must incorporate both the Moon and Sun in our model which makes the model a four-body model of Earth-Moon-Sun-spacecraft.

From the starting epoch difference the initial relative distance of the Moon to the spacecraft changes and the behavior of the orbit changes significantly on forward of the simulation. We can observe this from the figures 3 and 4. Just with a 4-day difference in the starting point of the simulation causes the "EarthSun" and "EarthMoonSun" propagation to have a large discrepancy.

(b) Output some information for each spacecraft at $t = t_f$, the end of the propagation. Determine the following information from the GMAT output: $a, e, r_p, \boldsymbol{\mathcal{E}}, h, r_f, v_f, \theta_f^*, \gamma_f$. Compare the closest approach altitude for all the spacecraft at the end of the simulation. Are any spacecraft in danger of Earth impact? Which perturbation reduced the r_p? Does it occur at all starting epochs? (Note that, if the model is not a true conic – as is the case for three of the four propagators – GMAT computes instantaneous values of these quantities. Hint: check the output at the end of the final propagate segment.)

The data from the reports of GMAT is processed using MATLAB and the following tables and graphs are created. $a, e, r_p, \mathcal{E}, h$ are all final values.

October 2, 2020 00:00:00:001	**Earth Point Mass**	**Earth Moon**	**Earth Sun**	**Earth Moon Sun**
$\boldsymbol{a}\ [\boldsymbol{km}]$	6.4259E+05	6.2375E+05	1.1981E+06	1.0653E+06
$\boldsymbol{e}$	0.9851	0.9841	0.3928	0.4497
$\boldsymbol{r_p}\ [\boldsymbol{km}]$	9.5746E+03	9.9413E+03	7.2752E+05	5.8628E+05
$\boldsymbol{\mathcal{E}}\ [\boldsymbol{km^2/s^2}]$	-0.3102	-0.3195	-0.1663	-0.1871
$\boldsymbol{h}\ [\boldsymbol{km^2/s}]$	8.7040E+04	8.8668E+04	6.3553E+05	5.8205E+05
$\boldsymbol{r_f}\ [\boldsymbol{km}]$	1.6695E+05	6.4202E+05	1.6609E+06	1.5433E+06
$\boldsymbol{v_f}\ [\boldsymbol{km/s}]$	2.0383	0.7763	0.3838	0.3774
$\boldsymbol{\theta_f^*}\ [\boldsymbol{deg}]$	154.1002	170.0558	173.1045	182.4081
$\boldsymbol{\gamma_f}\ [\boldsymbol{deg}]$	75.1805	79.7525	4.4200	-1.9649

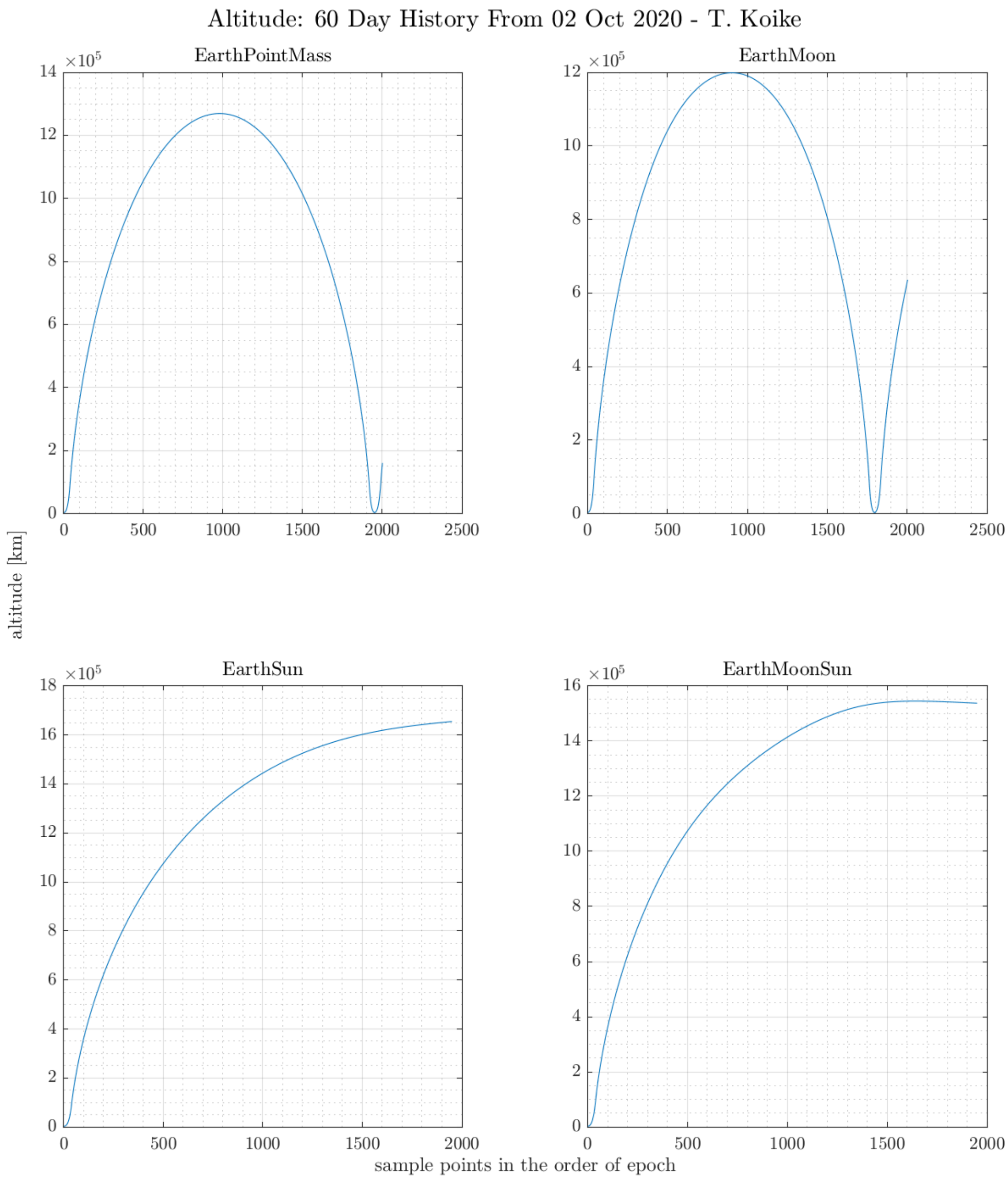
October 7, 2020 00:00:00:001	**Earth Point Mass**	**Earth Moon**	**Earth Sun**	**Earth Moon Sun**
$\boldsymbol{a}\ [\boldsymbol{km}]$	6.4259E+05	5.7788E+05	1.0950E+06	7.5403E+05
$\boldsymbol{e}$	0.9851	0.9830	0.3814	0.6218
$\boldsymbol{r_p}\ [\boldsymbol{km}]$	9.5746E+03	9.8391E+03	6.7738E+05	2.8519E+05
$\boldsymbol{\mathcal{E}}\ [\boldsymbol{km^2/s^2}]$	-0.3102	-0.3449	-0.1820	-0.2643
$\boldsymbol{h}\ [\boldsymbol{km^2/s}]$	8.7040E+04	8.8187E+04	6.1072E+05	4.2937E+05
$\boldsymbol{r_f}\ [\boldsymbol{km}]$	1.6695E+05	9.1054E+05	1.4911E+06	8.0809E+05
$\boldsymbol{v_f}\ [\boldsymbol{km/s}]$	2.0383	0.4310	0.4130	0.6767
$\boldsymbol{\theta_f^*}\ [\boldsymbol{deg}]$	154.1002	174.5760	192.3951	226.5451
$\boldsymbol{\gamma_f}\ [\boldsymbol{deg}]$	75.1805	77.0140	-7.4325	-38.2599

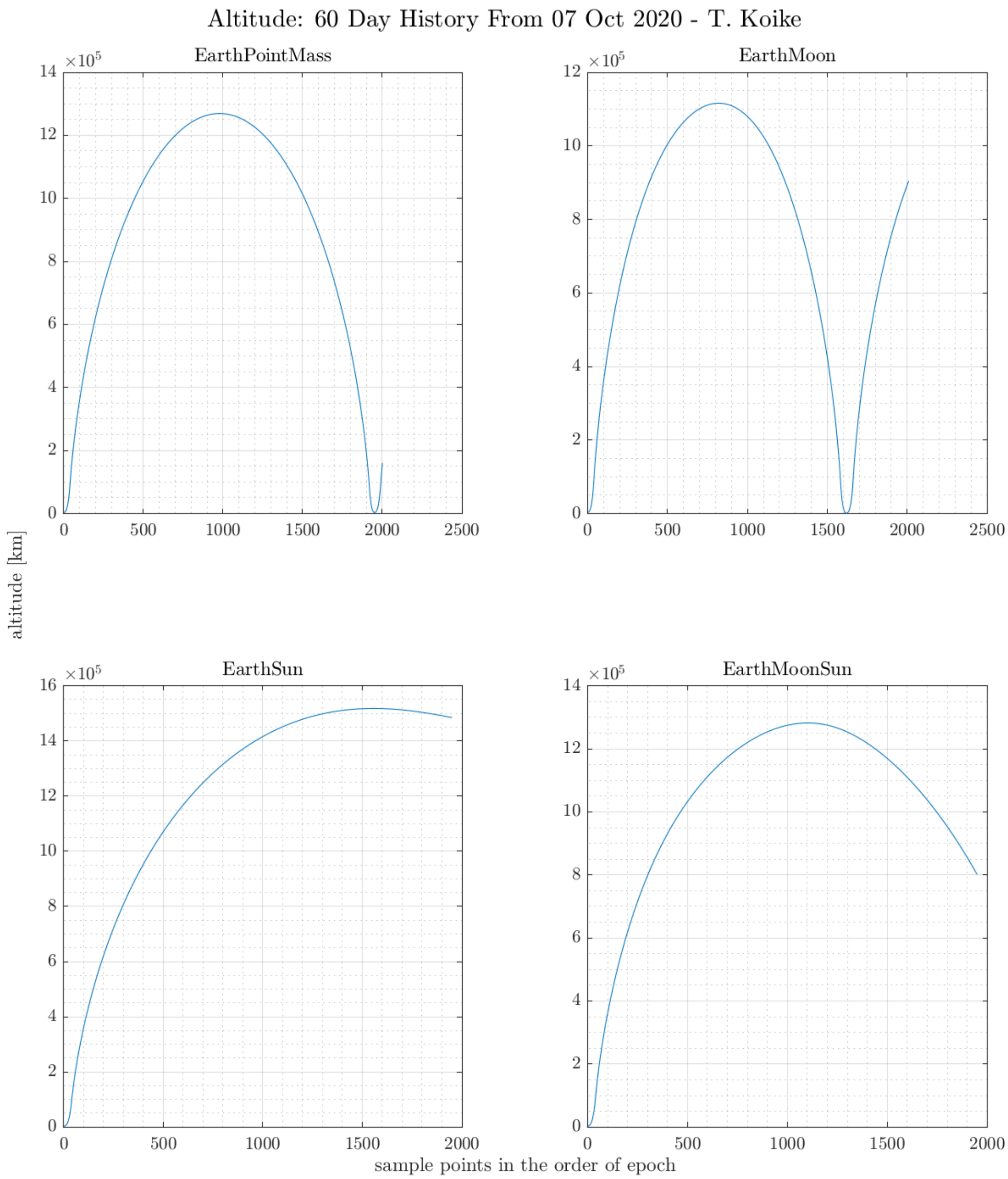
October 11, 2020 00:00:00:001	**Earth Point Mass**	**Earth Moon**	**Earth Sun**	**Earth Moon Sun**
$\boldsymbol{a}\ [\boldsymbol{km}]$	6.4259E+05	5.4642E+05	1.0218E+06	4.8835E+05
$\boldsymbol{e}$	0.9851	0.9832	0.4131	0.8699
$\boldsymbol{r_p}\ [\boldsymbol{km}]$	9.5746E+03	9.1542E+03	5.9965E+05	6.3535E+04
$\boldsymbol{\mathcal{E}}\ [\boldsymbol{km^2/s^2}]$	-0.3102	-0.3647	-0.1950	-0.4081
$\boldsymbol{h}\ [\boldsymbol{km^2/s}]$	8.7040E+04	8.5068E+04	5.8118E+05	2.1761E+05
$\boldsymbol{r_f}\ [\boldsymbol{km}]$	1.6695E+05	1.0734E+06	1.3361E+06	8.5524E+05
$\boldsymbol{v_f}\ [\boldsymbol{km/s}]$	2.0383	0.1148	0.4545	0.3405
$\boldsymbol{\theta_f^*}\ [\boldsymbol{deg}]$	154.1002	178.9663	207.7009	171.8384
$\boldsymbol{\gamma_f}\ [\boldsymbol{deg}]$	75.1805	46.3637	-16.8473	41.6375

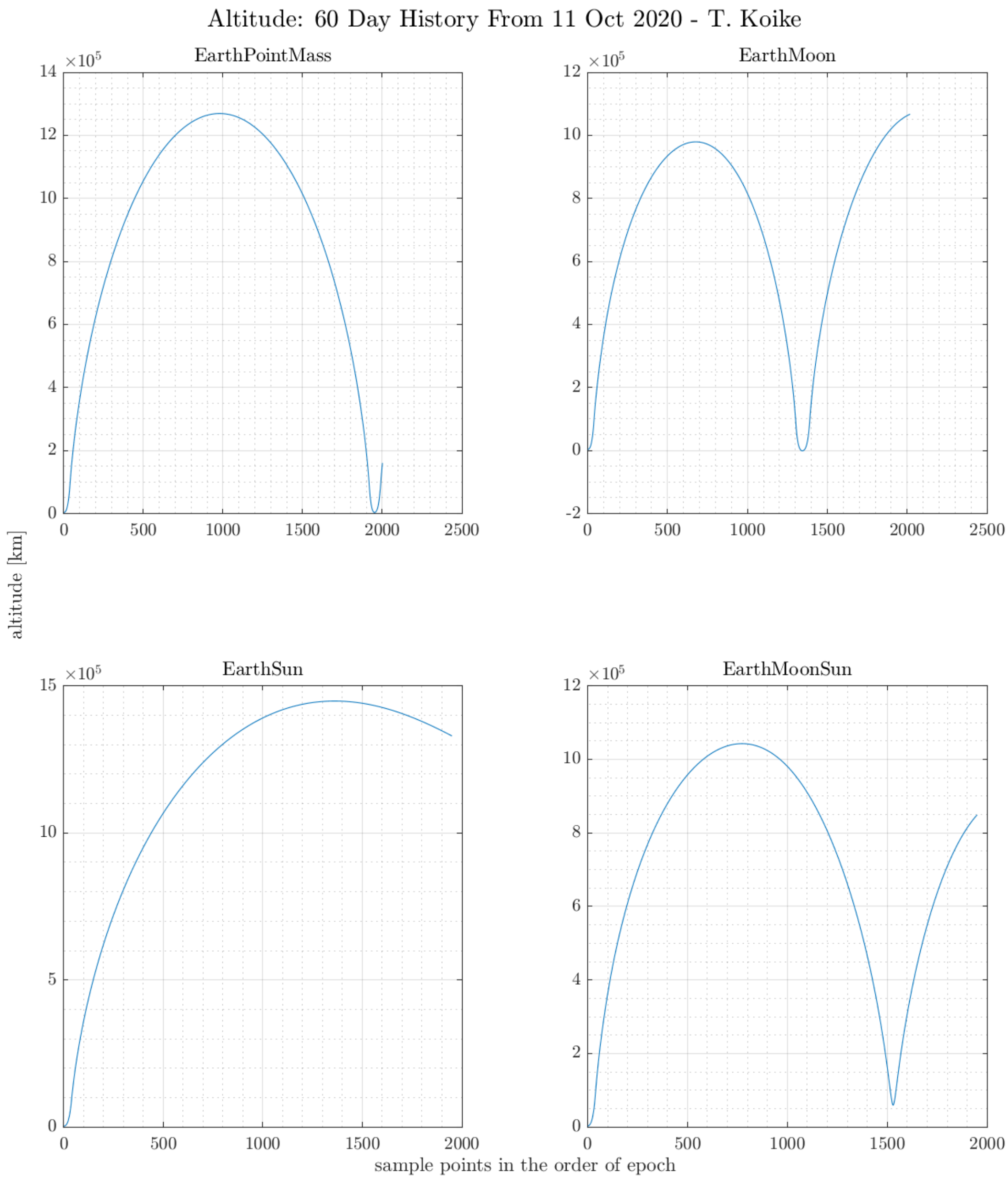
October 30, 2020 00:00:00:001	**Earth Point Mass**	**Earth Moon**	**Earth Sun**	**Earth Moon Sun**
$\boldsymbol{a}\ [\boldsymbol{km}]$	6.4259E+05	6.1896E+05	7.7179E+05	7.0913E+05
$\boldsymbol{e}$	0.9851	0.9839	0.7440	0.7720
$\boldsymbol{r_p}\ [\boldsymbol{km}]$	9.5746E+03	9.9520E+03	1.9759E+05	1.6168E+05
$\boldsymbol{\mathcal{E}}\ [\boldsymbol{km^2/s^2}]$	-0.3102	-0.3220	-0.2582	-0.2810
$\boldsymbol{h}\ [\boldsymbol{km^2/s}]$	8.7040E+04	8.8713E+04	3.7061E+05	3.3793E+05
$\boldsymbol{r_f}\ [\boldsymbol{km}]$	1.6695E+05	6.9249E+05	3.0171E+05	2.0831E+05
$\boldsymbol{v_f}\ [\boldsymbol{km/s}]$	2.0383	0.7122	1.4580	1.8069
$\boldsymbol{\theta_f^*}\ [\boldsymbol{deg}]$	154.1002	170.8820	281.0148	60.9116
$\boldsymbol{\gamma_f}\ [\boldsymbol{deg}]$	75.1805	79.6375	-32.5945	26.1293

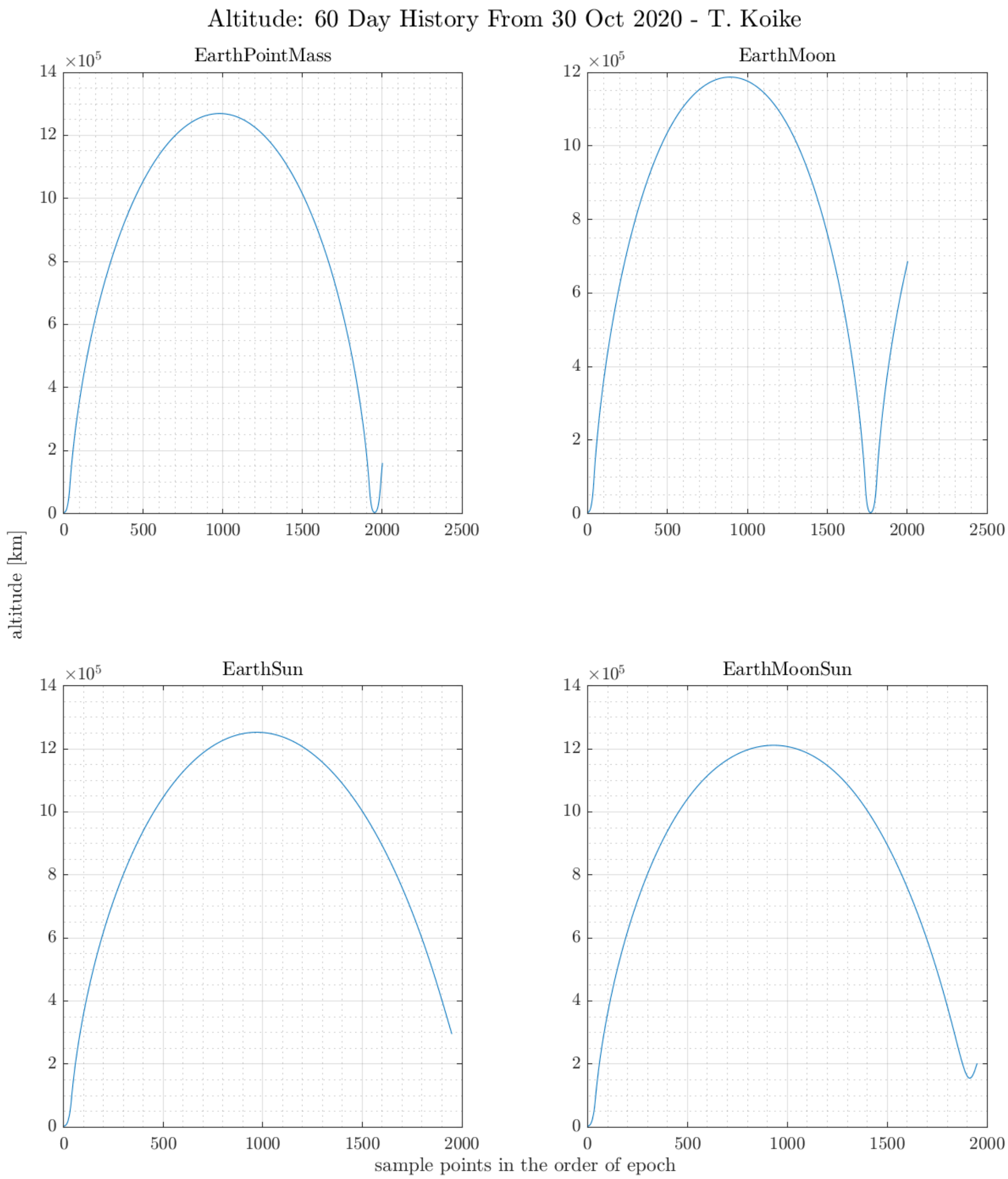
The raw data from the report is in the appendix.

Also, from GMAT the altitude data is retrieved and processed in MATLAB. The plot is on the following page.


Altitude: 60 Day History From 02 Oct 2020 - T. Koike
EarthPointMass
EarthMoon
EarthSun
EarthMoonSun
altitude [km]
sample points in the order of epoch
$\times 10^5$


Altitude: 60 Day History From 07 Oct 2020 - T. Koike
EarthPointMass
EarthMoon
EarthSun
EarthMoonSun
altitude [km]
sample points in the order of epoch
$\times10^5$


Altitude: 60 Day History From 11 Oct 2020 - T. Koike
EarthPointMass
EarthMoon
EarthSun
EarthMoonSun
altitude [km]
sample points in the order of epoch
×10^5


Altitude: 60 Day History From 30 Oct 2020 - T. Koike
EarthPointMass
EarthMoon
EarthSun
EarthMoonSun
×10^5
altitude [km]
sample points in the order of epoch

From the plots we can tell that the "EarthPointMass" and "EarthMoon" model spacecraft has a moment where the altitude approaches very close to 0. Thus, these 2 models are in danger of colliding into Earth.

From the tables, we can see that the model for "EarthMoon" with an epoch of "11 Oct 2020" has a reduced radius of periapsis. Mostly, the "EarthMoon" model has a radius of periapsis closest to the point mass model.

From the plots we can see that the starting epoch for all models have a local minimum of the altitude. However, still the altitudes are not 0 and 3000~ km. For the Earth point mass and Earth-Moon model there are points that have lower than the starting epoch such as "EarthMoon" with an epoch of "11 Oct 2020". Thus, not all models have the point of lowest altitude at the starting epoch.

Problem 2: A spacecraft is in orbit about Mars and is characterized such that $r_p = 1.5R_♂$ **and** $\boldsymbol{r_a = 6.5}R_♂$. The vehicle is currently located such that $M = -90°$.

(a) Determine the following orbit parameters and spacecraft state information.

$$a, e, p, h, \mathcal{P}, \boldsymbol{\mathcal{E}}, r, v, \theta^*, E, \gamma, (t - t_p)$$

From the radius of periapsis and apoapsis

$$a = 0.5(r_p + r_a) = 0.5(1.5R_♂ + 6.5R_♂) = 4R_♂ \ .$$

Since, $R_♂ = 3397\ km$, the semi-major axis becomes

$$a = 13588\ km \ .$$

Next, from the periapsis and apoapsis we can calculate the eccentricity

$$e = \frac{r_a - r_p}{r_a + r_p} = 0.6250 \ .$$

Then, the semi-latus rectum, p becomes

$$p = a(1 - e^2) = 8280.1875\ km \ .$$

When the gravitational parameter, $\mu \cong 42828\ km^3/s^2$, the specific angular momentum becomes

$$h = \sqrt{\mu p} = 18831.5287\ \frac{km^2}{s} \ .$$

The period of this orbit is

$$\mathcal{P} = 2\pi\sqrt{\frac{a^3}{\mu}} = 48089.2147\ s = 0.5566\ days \ .$$

The constant specific energy of this orbit is

$$\mathcal{E} = -\frac{\mu}{2a} = -1.5760\ \frac{km^2}{s^2} \ .$$

At the current location where the mean anomaly is $-90°$ we can find the eccentric anomaly. A MATLAB function solving the transcendental equation with the Bessel function was used to solve this. The code is shown below.

```
function E = M2E_anomaly(M, e, unit, tol)
 %{
       NAME:    M2E_anomaly
       AUTHOR:  TOMOKI KOIKE
       INPUTS:  (1) M:    MEAN ANOMALY
```

```
                 (2) e:     ECCENTRICITY
                 (3) unit: GRAVITATIONAL PARAMETER
                 (4) tol:   TOLERANCE
        OUTPUTS: (1) E:     ECCENTRIC ANOMALY
        DESCRIPTION: CALCULATES THE ECCENTRIC ANOMALY FROM THE MEAN ANOMALY
        USING THE BESSEL FUNCTIONS EXPLICIT APPROACH.
 %}

    %Checking for user inputed tolerance
    if nargin == 3
        %using default value
        tol = 10^-8;
    elseif nargin > 4
        error('Too many inputs.')
    elseif nargin < 3
        error('Too few inputs.')
    end

    % Check units
    if unit == "deg"
        M = deg2rad(M);
    end

    % Check tolerance
    if tol > 0.01
        error('Set a tolerance smaller than 0.001')
    end

    % Bessel function method
    del1 = 1; del2 = 1; del3 = 1;
    N = 1; ct = 1;
    Estore = [];
    while ~(del1 < tol && del2 < tol && del3 < tol && ct > 10)
        f = 0;
        for m = 1:N
            f = f + (1 / m) * besselj(m,m*e) * sin(m*M);
        end
        Estore = [Estore, M + 2*f];
        if ct > 4
            del1 = abs(Estore(ct) - Estore(ct-1));
            del2 = abs(Estore(ct-1) - Estore(ct-2));
            del3 = abs(Estore(ct-2) - Estore(ct-3));
        end
        N = N + 1; ct = ct + 1;
    end
    E = Estore(end);
end
```

This algorithm computes the eccentric anomaly to be

$$E = -120.7691^\circ \quad \text{equivalent to} \quad \mathrm{E} = 239.2309^\circ \;.$$

Then from the eccentric anomaly we can calculate the true anomaly. The following MATLAB function is used to do this computation

```
function theta_star = E2T_anomaly(e, E, unit)
%{
       NAME:    E2T_anomaly
       AUTHOR:  TOMOKI KOIKE
       INPUTS:  (1) e:    ECCENTRICITY
                (2) E:    ECCENTRIC ANOMALY
                (3) unit: DEGREES OR RADIANS
       OUTPUTS: (1) theta_star: TRUE ANOMALY
       DESCRIPTION: CALCULATES THE TRUE ANOMALY FROM THE ECCENTRIC ANOMALY.
 %}

    ee = sqrt((1 + e) / (1 - e));
    if unit == "deg"
        theta_star = 2*atand(ee * tand(E / 2));
        if theta_star < 0
            theta_star = 360 + theta_star;
        end
    else
        theta_star = 2*atan(ee * tan(E/ 2));
        if theta_star < 0
            theta_star = 2*pi + theta_star;
        end
    end
end
```

This gives the following result,

$$\theta^* = 210.5465° \ = \ -149.4535° \,.$$

Then,

$$r = \frac{p}{1 + e\cos(\theta^*)}$$

$$r \ = \ 17932.5912 \ \ km$$

The position gives us the velocity.

$$v = \sqrt{\mu\left(\frac{2}{r} - \frac{1}{a}\right)}$$

$$v = 1.2746 \ \ \frac{km}{s} \ .$$

Then, since the orbit is descending the flight path angle is

$$\gamma = -arccos\left(\frac{h}{rv}\right)$$

$$\gamma = -34.5256° \ .$$

Finally, from the mean anomaly, we can compute (M in radians and positive)

$$(t - t_p) = M\sqrt{\frac{a^3}{\mu}}$$

$$(t - t_p) = 36066.9110 \ \ s = 0.4174 \ \ day \ .$$

(b) Write $\overline{r_0}$ and $\overline{v_0}$ in terms of components in the directions $\hat{e}$ and $\hat{p}$.

From the true anomaly we can represent the position and velocity vectors as

$$\overline{r_0} = r(cos(\theta^*)\hat{e} + sin(\theta^*)\hat{p}) \ = \ (17932.5912 \ \ km)(cos(210.5465° \,)\hat{e} + sin(210.5465° \,)\hat{p})$$

$$\overline{r_0} \ = \ -15443.8560\hat{e} - 9114.0249\hat{p} \ .$$

$$\overline{v_0} = \ v(cos(\theta^* + 90° + |\gamma|)\hat{e} + sin(\theta^* + 90° + |\gamma|))$$

$$\overline{v_0} \ = \ \ 1.1559\hat{e} - 0.5372\hat{p} \ .$$

(c) Determine θ^* after a time equal to 50% of the period, i.e., $\Delta t = 0.5\mathcal{P}$. Use f and g relationships to write $\bar{r}, \bar{v}$ in terms of $\bar{r}_0, \bar{v}_0$. Prove that $f(\theta^* - \theta_0^*), g(\theta^* - \theta_0^*)$ produce the same results as $f(E - E_0), g(E - E_0)$.

When the time is equal to 50% of the period,

$$(t - t_0) = \pi\sqrt{\frac{a^3}{\mu}}$$

and t_0 is the time when $\theta_0^* = \theta^* = 210.5465°$ and $E_0 = E = 239.2309°$ in part (a).

Using the formula on notes **G7,** we can compute the eccentric anomaly for when half a period has passed from the this point where $E_0 = E = 239.2309°$.

$$\sqrt{\frac{\mu}{a^3}}(t_0 - t_p) = E_0 - esinE_0$$

$$\sqrt{\frac{\mu}{a^3}}(t - t_p) = E_f - esinE_f \ .$$

The subscript of “f” is the notation for the final point we are looking for. Now, subtract the first equation from the second one and we obtain

$$\sqrt{\frac{\mu}{a^3}}\left(t-t_p\right)-\sqrt{\frac{\mu}{a^3}}\left(t_0-t_p\right)=E_f-esinE_f-(E_0-esinE_0)$$

$$\sqrt{\frac{\mu}{a^3}}(t-t_0)=\left(E_f-E_0\right)-e\left(sinE_f-sinE_0\right)\ .$$

Then,

$$\because\ (t-t_0)=\pi\sqrt{\frac{a^3}{\mu}}$$

$$\sqrt{\frac{\mu}{a^3}}\left(\pi\sqrt{\frac{a^3}{\mu}}\right)=\left(E_f-E_0\right)-e\left(sinE_f-sinE_0\right)$$

$$\pi=\left(E_f-E_0\right)-e\left(sinE_f-sinE_0\right)$$

$$\therefore\ E_f-esinE_f=\pi+E_0-esinE_0$$

Since we know that in part (a) $M=-90°$

$$E_f-esinE_f=\pi+M_0=\pi-\frac{\pi}{2}=\frac{\pi}{2}\ .$$

$$\therefore\ M_f=\frac{\pi}{2}\ .$$

We solve this with the same algorithm we used in part (a) to compute the eccentric anomaly from the mean anomaly, and we get

$$E_f=120.7691°\ .$$

Now, compute the true anomaly from the eccentric anomaly

$$tan\frac{\theta_f^*}{2}=\left(\frac{1+e}{1-e}\right)^{1/2}tan\frac{E_f}{2}$$

$$\theta_f^*=149.4535°\ .$$

To find the final position and velocity vectors for an ellipse we use the following formula from notes H2

$$\bar{r}=\left(1-\frac{a}{r_0}\left(1-cos(E-E_0)\right)\right)\bar{r}_0+\left((t-t_0)+\left(\frac{sin(E-E_0)+(E-E_0)}{n}\right)\right)\bar{v}_0$$

$$\bar{v}=-\frac{na^2}{rr_0}sin(E-E_0)\bar{r}_0+\left(1-\frac{a}{r}\left(1-cos(E-E_0)\right)\right)\bar{v}_0$$

Here is a MATLAB function that computes the coefficients and the final position and velocity vectors

```
function [rvec, vvec, f, g, fdot, gdot] = FandG_elp(a, mu, dE, dt, r0vec, v0vec)
%{
       NAME:    rv_vecTconv_elp
       AUTHOR:  TOMOKI KOIKE
       INPUTS:  (1) a:      SEMI MAJOR AXIS
                (2) mu:     GRAVITATIONAL PARAMETER
                (3) dE:     DIFFERENCE OF ECCENTRIC ANOMALY
                (4) dt:     DIFFERENCE OF TIME
                (5) r0vec:  R-VECTOR AT INITIAL POINT
                (6) v0vec:  V-VECTOR AT INITIAL POINT
       OUTPUTS: (1) rvec:   R-VECTOR AT FINAL POINT
                (2) vvec:   V-VECTOR AT FINAL POINT
                (3) f:      THE F COEFFICIENT
                (4) g:      THE G COEFFICIENT
                (5) fdot:   THE FDOT COEFFICIENT
                (6) gdot:   THE GDOT COEFFICIENT
       DESCRIPTION: CALCULATES THE F AND G FOR POSITION VECTOR AND VELOCITY
       VECTOR AS WELL AS THE FINAL VECTORS FOR AN ELLIPSE.
 %}

    if (dE > 3.14 || -3.14 > dE) || dE > 6.28
        error("Enter the units in radius.")
    elseif ~(numel(r0vec) >= 2 && numel(v0vec) >= 2) || (numel(r0vec) ~=
numel(v0vec))
        error("The initial position and velocity vectors should have equal
dimensions larger than 2.")
    end

    % Compute F
    n = sqrt(mu / a^3);  % mean motion
    r0 = norm(r0vec);  % magnitude of initial position vector
    f = 1 - (a/r0) * (1 - cos(dE));
    % Compute G
    g = dt + (sin(dE) - dE) / n;
    %Compute rvec
    rvec = f * r0vec + g * v0vec;
    % Compute Fdot
    r = norm(rvec);  % magnitude of final position vector
    fdot = -(n * a^2 / r / r0) * sin(dE);
    % Compute Gdot
    gdot = 1 - (a / r) * (1 - cos(dE));
    % Compute vvec
    vvec = fdot * r0vec + gdot * v0vec;
end
```

Using this, we acquire the following results,

$$\begin{pmatrix} f(E-E_0) \\ g(E-E_0) \end{pmatrix} = \begin{pmatrix} -0.1188 \\ -14948.9295 \end{pmatrix}$$

$$\begin{pmatrix} \dot{f}(E-E_0) \\ \dot{g}(E-E_0) \end{pmatrix} = \begin{pmatrix} 6.550e-5 \\ -0.1188 \end{pmatrix}$$

Thus,

$$\bar{r} = (-15443.8460\bar{r}_0 + 9114.0249\bar{v}_0)\ km$$

$$\bar{v} = (-1.1559 - 0.5372\bar{v}_0)\ \ km/s\ .$$

Now, if we use the true anomaly, we use the following formula on notes H4

$$\bar{r} = \left(1 - \frac{r}{p}\left(1 - cos(\theta^* - \theta_0^*)\right)\right)\bar{r}_0 + \frac{rr_0}{\sqrt{\mu p}} sin(\theta^* - \theta_0^*)\bar{v}_0$$

$$\bar{v} = \left(\frac{\bar{r}_0 \cdot \bar{v}_0}{pr_0}\left(1 - cos(\theta^* - \theta_0^*)\right) - \frac{1}{r_0}\sqrt{\frac{\mu}{p}} sin(\theta^* - \theta_0^*)\right)\bar{r}_0 + \left(1 - \frac{r_0}{p}\left(1 - cos(\theta^* - \theta_0^*)\right)\right)\bar{v}_0$$

Use this MATLAB function,

```
function [rvec, vvec, f, g, fdot, gdot] = FandG_conic(p, mu, dTA, r0vec, v0vec,
rfmag)
%{
       NAME:    rv_vecTconv_conic
       AUTHOR:  TOMOKI KOIKE
       INPUTS:  (1) p:      SEMI LATUS RECTUM
                (2) mu:     GRAVITATIONAL PARAMETER
                (3) dTA:    DIFFERENCE OF TRUE ANOMALY
                (4) r0vec:  R-VECTOR AT INITIAL POINT
                (5) v0vec:  V-VECTOR AT INITIAL POINT
                (6) rfmag:  THE RADIAL DISTANCE AT THE FINAL POINT
       OUTPUTS: (1) rvec:   R-VECTOR AT FINAL POINT
                (2) vvec:   V-VECTOR AT FINAL POINT
                (3) f:      THE F COEFFICIENT
                (4) g:      THE G COEFFICIENT
                (5) fdot:   THE FDOT COEFFICIENT
                (6) gdot:   THE GDOT COEFFICIENT
       DESCRIPTION: CALCULATES THE F AND G FOR POSITION VECTOR AND VELOCITY
       VECTOR AS WELL AS THE FINAL VECTORS FOR AN ELLIPSE.
 %}

    d = abs(dTA / 2 / pi);
    if d > 6.28
        error("Enter the units in radius.")
```

```
    elseif ~(numel(r0vec) >= 2 && numel(v0vec) >= 2) || (numel(r0vec) ~=
numel(v0vec))
        error("The initial position and velocity vectors should have equal
dimensions larger than 2.")
    end

    % Compute F
    r0 = norm(r0vec);  % magnitude of initial position vector
    f = 1 - (rfmag/p) * (1 - cos(dTA));
    % Compute G
    g = (rfmag * r0 / sqrt(mu * p)) * sin(dTA);
    %Compute rvec
    rvec = f * r0vec + g * v0vec;
    % Compute Fdot
    fdot = (dot(r0vec, v0vec) / p / r0) * (1 - cos(dTA)) -
(1/r0)*sqrt(mu/p)*sin(dTA);
    % Compute Gdot
    gdot = 1 - (r0 / p) * (1 - cos(dTA));
    % Compute vvec
    vvec = fdot * r0vec + gdot * v0vec;
end
```

Computing the same values using this function we get the exact same values

$$\begin{pmatrix} f(\theta^* - \theta_0^*) \\ g(\theta^* - \theta_0^*) \end{pmatrix} = \begin{pmatrix} -0.1188 \\ -14948.9295 \end{pmatrix}$$

$$\begin{pmatrix} \dot{f}(\theta^* - \theta_0^*) \\ \dot{g}(\theta^* - \theta_0^*) \end{pmatrix} = \begin{pmatrix} 6.550e - 5 \\ -0.1188 \end{pmatrix}$$

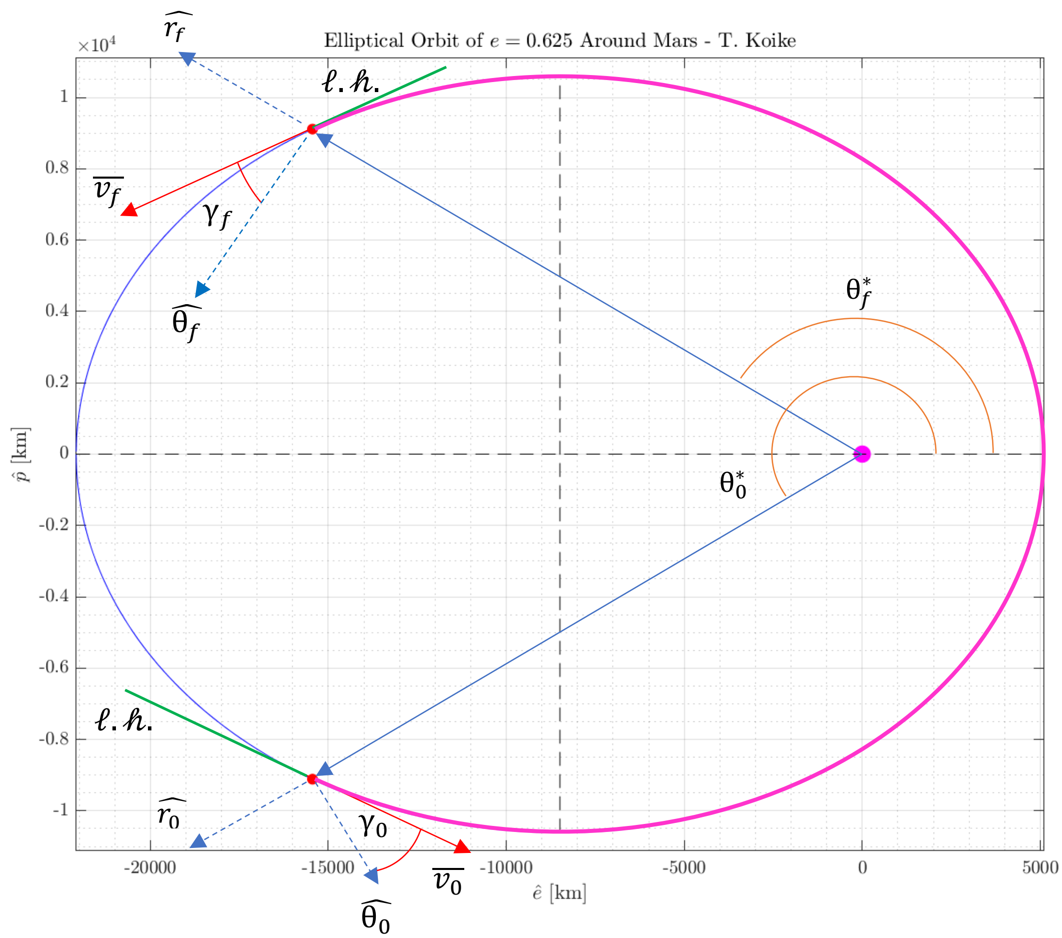
Thus,

$$\bar{r} = (-15443.8460\bar{r}_0 + 9114.0249\bar{v}_0)\ km$$

$$\bar{v} = (-1.1559 - 0.5372\bar{v}_0)\ \ km/s\ \ .$$

Hence, it is proven that the f, g function for both the elliptical and conical version agree.

(d) Plot the orbit with your MATLAB script. By hand, mark on the plot where the spacecraft is currently located by marking $\hat{r}$, $\hat{\theta}$, $\bar{r}_0$, θ_0^*; also sketch the local horizon, $\bar{v}_0$, and γ_0. Do the same at the second location. Identify the arc from t_0 to t.



The arc outlined in pink is the arc from t_0 to t. (The MATLAB code is in the appendix.)

Problem 3: Assume that a vehicle is currently moving in Earth orbit. Assume a two-body relative motion of the spacecraft with respect to Earth. The orbit is described with the following characteristics (relative to an Earth equatorial coordinate system) at the initial time t_i:

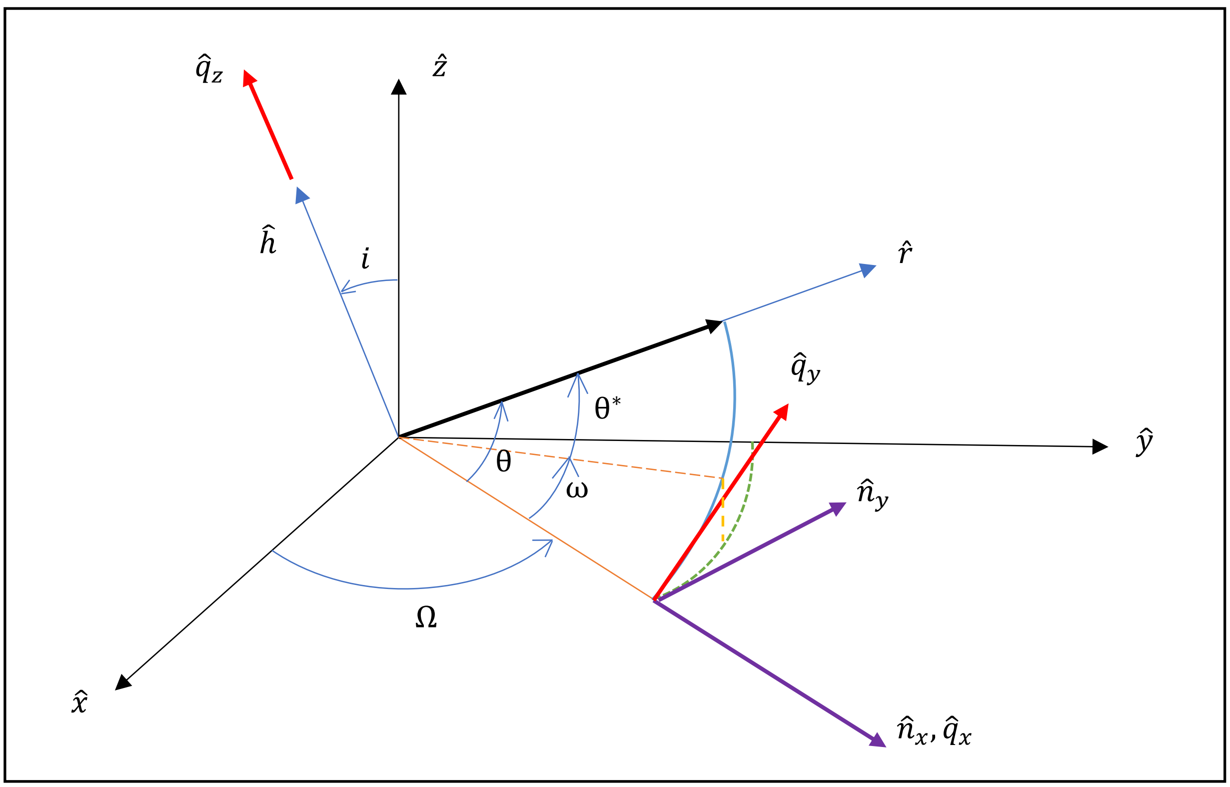
$$a = 20R_{\oplus}, \qquad \Omega = 45°$$

$$e = 0.6, \qquad \theta = 235°$$

$$\omega = 30°, \qquad i = 34°$$

(a) Determine the current state in terms of $\bar{r}, \bar{v}, r, v, \gamma, \theta^*, \mathcal{P}, M, E, (t - t_p)$; write $\bar{r}, \bar{v}$ in terms of both rotating orbit unit vectors $(\hat{r}, \hat{\theta}, \hat{h})$, unit vectors $(\hat{n}_x, \hat{n}_y, \hat{n}_z)$ as well as inertial unit vectors $(\hat{x}, \hat{y}, \hat{z})$.

3-1-3 (body-two) Euler Sequence:



$$(\hat{q}_x \quad \hat{q}_y \quad \hat{q}_z) = (\hat{r} \quad \hat{\theta} \quad \hat{h})\begin{pmatrix} c_\theta & s_\theta & 0 \\ -s_\theta & c_\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{1}$$

$$(\hat{n}_x \quad \hat{n}_y \quad \hat{n}_z) = (\hat{q}_x \quad \hat{q}_y \quad \hat{q}_z)\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_i & s_i \\ 0 & -s_i & c_i \end{pmatrix} \tag{2}$$

$$(\hat{x} \quad \hat{y} \quad \hat{z}) = (\hat{n}_x \quad \hat{n}_y \quad \hat{n}_z)\begin{pmatrix} c_\Omega & s_\Omega & 0 \\ -s_\Omega & c_\Omega & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{3}$$

From θ and ω, we know

$$\theta^* = \theta - \omega \; = \; 235 - 30 \; = \; 205° \; .$$

And, the semi latus rectum is

$$p \; = a(1 - e^2) = \; 81640.1446 \;\; km \; .$$

Then, we can find the magnitude of $\bar{r}$

$$r = \frac{p}{1 + ecos\theta^*} \; = \; 178950.9024 \;\; km \; .$$

Since, $\mu = 398600.4415 \;\; km^3/s^2$, the magnitude of the velocity vector is

$$v = \sqrt{\mu\left(\frac{2}{r} - \frac{1}{a}\right)} \; = \; 1.1533 \;\; km/s \; .$$

The specific angular momentum is

$$h = \sqrt{\mu p} = 180393.4525 \;\; km^2/s \; .$$

The flight path angle is

$$\gamma = arccos\left(\frac{h}{rv}\right) = -29.0659° \qquad (\because \theta^* > 180°) \; .$$

The period of this orbit is

$$\mathcal{P} = 2\pi\sqrt{\frac{a^3}{\mu}} \; = \; 453415.8190 \;\; s \; = \; 5.2479 \; days \; .$$

The eccentric anomaly can be calculated using,

$$tan\frac{\theta^*}{2} = \left(\frac{1+e}{1-e}\right)^{1/2} tan\frac{E}{2}$$

$$E \; = \; 227.8240° \; .$$

The mean anomaly becomes

$$M = E - esinE$$

$$M = 253.3006°$$

The time elapse from periapsis is

$$(t - t_p) = M\sqrt{\frac{a^3}{\mu}} = 319029.1896\ s = 3.6925\ days$$

Using the radial distance we can express the r vector in the orbital frame

$$\bar{r} = r\hat{r} = 178950.9024\ km\ \hat{r}\ .$$

Then, the velocity vector becomes

$$\bar{v} = v\left(sin(\gamma)\hat{r} + cos(\gamma)\hat{\theta}\right)$$

$$\bar{v} = \left(-0.5603\hat{r} + 1.0081\hat{\theta}\right)\ km/s\ .$$

To compute the frame transformation the following MATLAB function is used

```
function resvec = orbit_frame_transform(theta, i, Omega, vec, frame, unit)
%{
       NAME:    orbit_frame_transform
       AUTHOR:  TOMOKI KOIKE
       INPUTS:  (1) theta:  ARGUMENT OF LATITUDE
                (2) i:      INCILNATION
                (3) Omega:  RIGHT ASCENSION OF ASCENDING NODE
                (4) vec:    VECTOR TO TRANSFORM
                (5) frame:  THE STARTING FRAME (ORBITAL OR INERTIAL)
                (6) unit:   DEGREE OR RADIANS
       OUTPUTS: (1) resvec: RESULTING VECTOR STRUCTURE
       DESCRIPTION: TRANSFORMS THE ORBITAL VECTOR (POSITION OR VELOCITY)
       USING THE ORBITAL ANGLES.
 %}

    if unit == "radian"
        theta = rad2deg(theta);
        i = rad2deg(i);
        Omega = rad2deg(Omega);
    end

    % Direction cosine matrices
    % r,theta,h --> qx,qy,qz
    C_oq = [cosd(theta), sind(theta), 0; -sind(theta), cosd(theta), 0; 0,0,1];
    % qx,qy,qz --> nx,ny,nz
    C_qn = [1,0,0;0,cosd(i),sind(i);0,-sind(i),cosd(i)];
    % nx,ny,nz --> x,y,z
    C_ni = [cosd(Omega),sind(Omega),0;-sind(Omega),cosd(Omega),0;0,0,1];

    % For reverse
    C_in = C_ni'; C_nq = C_qn'; C_qo = C_oq';
```

```
    % Transform
    if frame == "orbital"
        % Orbital to q-frame
        resvec.q = vec * C_oq;
        % q-frame to n-frame
        resvec.n = resvec.q * C_qn;
        % n-frame to inertial
        resvec.i = resvec.n * C_ni;
    elseif frame == "inertial"
        % Inertial to n-frame
        resvec.n = vec * C_in;
        % n-frame to q-frame
        resvec.q = resvec.n * C_nq;
        % q-frame to orbital frame
        resvec.o = resvec.q * C_qo;
    else
        error("Enter a frame of either 'orbital' or 'inertial'.");
    end
end
```

From (1) and (2),

$$(\hat{n}_x \quad \hat{n}_y \quad \hat{n}_z) = (\hat{r} \quad \hat{\theta} \quad \hat{h})\begin{pmatrix} c_\theta & s_\theta & 0 \\ -s_\theta & c_\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_i & s_i \\ 0 & -s_i & c_i \end{pmatrix}$$

$$(\hat{n}_x \quad \hat{n}_y \quad \hat{n}_z) = (\hat{r} \quad \hat{\theta} \quad \hat{h})\begin{pmatrix} c_\theta & c_i s_\theta & s_i s_\theta \\ -s_\theta & c_i c_\theta & c_\theta s_i \\ 0 & -s_i & c_i \end{pmatrix}$$

Hence, to transform the vectors from the orbital frame ("o") to the n-frame ("n") is

$$\bar{r}^n = \bar{r}^o \begin{pmatrix} c_\theta & c_i s_\theta & s_i s_\theta \\ -s_\theta & c_i c_\theta & c_\theta s_i \\ 0 & -s_i & c_i \end{pmatrix}$$

$$\bar{v}^n = \bar{v}^o \begin{pmatrix} c_\theta & c_i s_\theta & s_i s_\theta \\ -s_\theta & c_i c_\theta & c_\theta s_i \\ 0 & -s_i & c_i \end{pmatrix}$$

Thus, in the n-frame the position vector and velocity vectors become

$$\bar{r}^n = \left(-102642.0209\hat{n}_x - 121526.9577\hat{n}_y - 81970.9680\hat{n}_z\right) \ km \ .$$

$$\bar{v}^n = \left(1.1471\hat{n}_x - 0.0988\hat{n}_y - 0.0667\hat{n}_z\right) \ km/s \ .$$

Then to transform it to the inertial frame ("i"), from (3)

$$\bar{r}^i = \bar{r}^n \begin{pmatrix} c_\Omega & s_\Omega & 0 \\ -s_\Omega & c_\Omega & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Compute this using MATLAB and we get,

$$\bar{\mathrm{r}}^{i} = (13353.6669\hat{x} - 158511.4049\hat{y} - 81970.9680\hat{z})\ km\ .$$

$$\bar{\mathrm{v}}^{i} = (0.8810\hat{x} + 0.7412\hat{y} - 0.0667\hat{z})\ \ km/s\ .$$

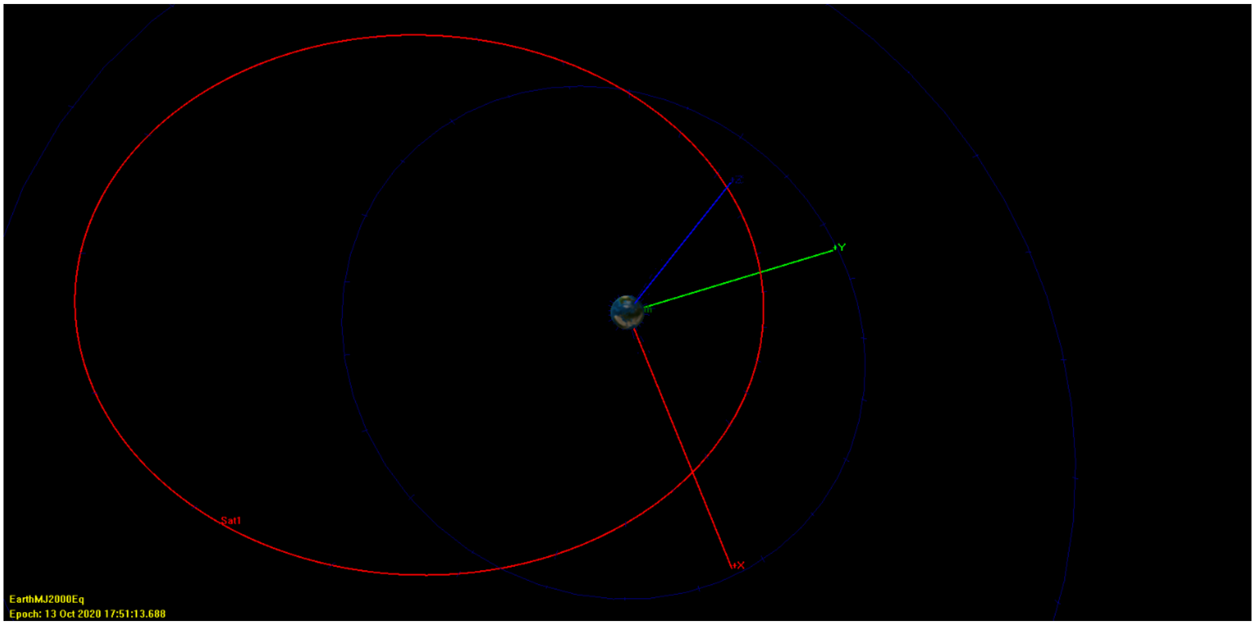
(b) Confirm the general results in GMAT with the conic propagator. Plot the GMAT image viewing down onto the orbit plane.

The GMAT results give us $\bar{r}, \bar{v}, r, v, \gamma, \theta^{*}, \mathcal{P}, M, E, (t - t_p)$

```
Propagate Command: Propagate1
Spacecraft       : Sat1
Coordinate System: EarthMJ2000Eq

Time System   Gregorian                     Modified Julian
----------------------------------------------------------------------
UTC Epoch:    13 Oct 2020 17:56:23.819      29136.2474979045
TAI Epoch:    13 Oct 2020 17:57:00.819      29136.2479261452
TT  Epoch:    13 Oct 2020 17:57:33.003      29136.2482986452
TDB Epoch:    13 Oct 2020 17:57:33.001      29136.2482986262

Cartesian State                       Keplerian State
---------------------------           --------------------------------
X  =   13353.666846499 km             SMA  =   127562.72599965 km
Y  =  -158511.40493158 km             ECC  =   0.5999999999984
Z  =  -81970.967996886 km             INC  =   34.000000000000 deg
VX =   0.8810382912704 km/sec         RAAN =   45.000000000000 deg
VY =   0.7412445371591 km/sec         AOP  =   29.999999999962 deg
VZ =  -0.0666745675888 km/sec         TA   =   204.99999999877 deg
                                      MA   =   253.30062046024 deg
                                      EA   =   227.82397421787 deg

Spherical State                       Other Orbit Data
---------------------------           --------------------------------
RMAG =   178950.90250115 km           Mean Motion        =   1.385744618e-05 deg/sec
RA   =  -85.184533212648 deg          Orbit Energy       =  -1.5623703490827 km^2/s^2
DEC  =  -27.262251997392 deg          C3                 =  -3.1247406981653 km^2/s^2
VMAG =   1.1533071717959 km/s         Semilatus Rectum   =   81640.144640011 km
AZI  =   111.15056655935 deg          Angular Momentum   =   180393.45247994 km^2/s
VFPA =   119.06592694950 deg          Beta Angle         =  -21.012833536281 deg
RAV  =   40.074930888723 deg          Periapsis Altitude =   44646.954100056 km
DECV =  -3.3142102788421 deg          VelPeriapsis       =   3.5353872196127 km/s
                                      VelApoapsis        =   0.8838468049075 km/s
                                      Orbit Period       =   453415.81894988 s

Planetodetic Properties
---------------------------
LST       =   275.14019961493 deg
MHA       =   291.85203753226 deg
Latitude  =  -27.258513764093 deg
Longitude =  -16.711837917326 deg
Altitude  =   172577.24511534 km


Spacecraft Properties
------------------------------
Cd                    =   2.200000
Drag area             =   15.00000 m^2
Cr                    =   1.800000
Reflective (SRP) area =   1.000000 m^2
```

```
      Dry mass              =   850.00000000000 kg
      Total mass            =   850.00000000000 kg
      SPADDragScaleFactor   =   1.000000
      SPADSRPScaleFactor    =   1.000000


  ========================================================================
```

The values in red verifies our answers. (The FPA is (90– VFPA) on the report above.)



(c) Use Kepler's equation and determine the values of $\bar{r}, \bar{v}, \theta^*, \gamma$ in exactly 3 days, i.e., time t_2. For this value of $(t_2 - t_1)$, what are the corresponding values of $(\theta_2^* - \theta_1^*)$, $(E_2 - E_1)$. Confirm the result in GMAT.

3 days is a time elapse of 259,200 seconds.

The mean anomaly becomes

$$M_2 - M_1 = \sqrt{\frac{\mu}{a^3}}(t_2 - t_1) = 259200\sqrt{\frac{\mu}{a^3}} = 205.7978°$$

$$M = 99.08984°$$

Using the MATLAB function in problem 2 (on page 13-14) we can find the eccentric anomaly at this second point to be

$$E_2 = 126.6716°$$

The true anomaly at this time is

$$\theta_2^* = 151.8108°$$

Thus, knowing that 3 days is less than one full orbit

$$\theta_2^* - \theta_1^* = 151.8108° - (205° - 360°) = 306.8108°$$

$$E_2 - E_1 = 126.6716° - (227.8240° - 360°) = 258.8477°$$

Using the f and g function we can compute the position and velocity vectors after this time.

$$\overline{r_2} = \left(1 - \frac{a}{r_1}\left(1 - cos(E_2 - E_1)\right)\right)\bar{r}_1 + \left((t_2 - t_1) + \left(\frac{sin(E_2 - E_1) + (E_2 - E_1)}{n}\right)\right)\bar{v}_1$$

$$\overline{v_2} = -\frac{na^2}{r_2 r_1} sin(E_2 - E_1)\bar{r}_1 + \left(1 - \frac{a}{r_2}\left(1 - cos(E_2 - E_1)\right)\right)\bar{v}_1$$

Using the MATLAB function we used in problem 2 (on page 18) we get the following results

$$\bar{r}_2{}^i = (-119251.6914\hat{x} - 125671.2021 - 3061.7827)\ \ km\ .$$

$$\bar{v}_2{}^i = (0.2022\hat{x} - 1.0410\hat{y} - 0.5929\hat{z})\ \ km/s\ .$$

Finally the flight path angle is

$$\gamma = arccos\left(\frac{h}{r_2 v_2}\right) = 31.0291°$$

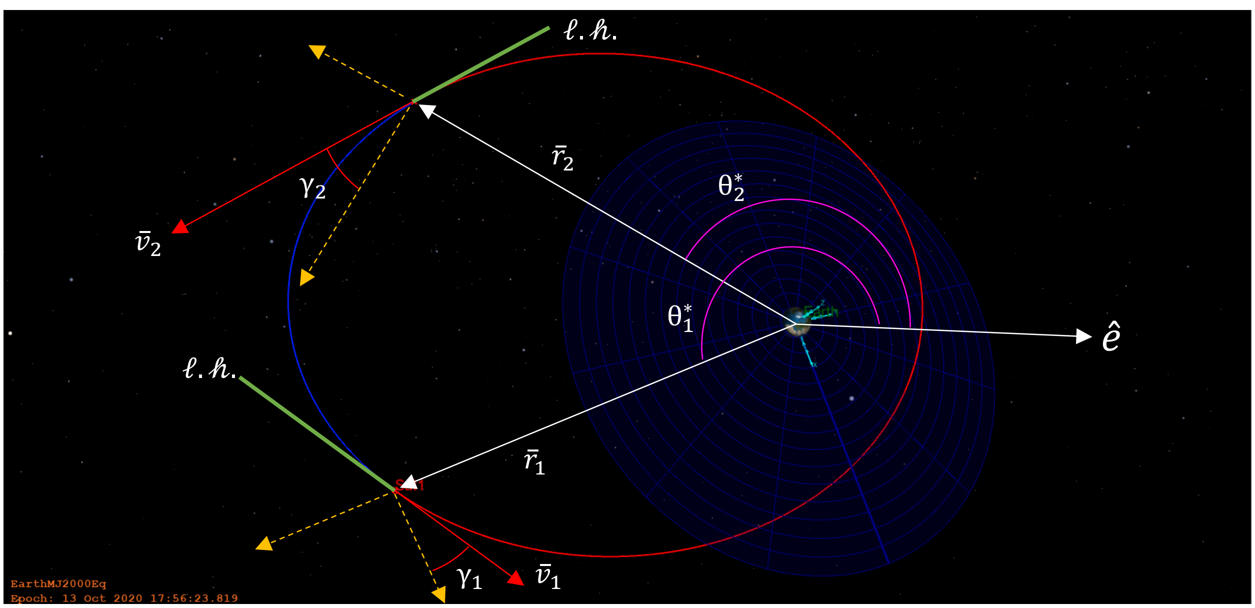
```
Propagate Command: Propagate1
Spacecraft       : Sat1
Coordinate System: EarthMJ2000Eq

Time System   Gregorian                     Modified Julian
----------------------------------------------------------------------
UTC Epoch:    11 Oct 2020 11:59:28.000      29133.9996296296
TAI Epoch:    11 Oct 2020 12:00:05.000      29134.0000578704
TT  Epoch:    11 Oct 2020 12:00:37.184      29134.0004303704
TDB Epoch:    11 Oct 2020 12:00:37.182      29134.0004303513

Cartesian State                       Keplerian State
---------------------------           --------------------------------
X  =  -119251.69147723 km             SMA  =   127562.72599965 km
Y  =  -125671.20223367 km             ECC  =   0.5999999999984
Z  =  -3061.7827422874 km             INC  =   34.000000000000 deg
VX =   0.2022456483717 km/sec         RAAN =   45.000000000000 deg
VY =  -1.0409621417831 km/sec         AOP  =   29.999999999962 deg
VZ =  -0.5929473913807 km/sec         TA   =   151.81082438204 deg
                                      MA   =   99.098468928579 deg
                                      EA   =   126.67164814234 deg

Spherical State                       Other Orbit Data
---------------------------           --------------------------------
RMAG =   173273.17018107 km           Mean Motion        =   1.385744618e-05 deg/sec
RA   =  -133.49860225159 deg          Orbit Energy       =  -1.5623703490826 km^2/s^2
DEC  =  -1.0124842709371 deg          C3                 =  -3.1247406981653 km^2/s^2
VMAG =   1.2149453040594 km/s         Semilatus Rectum   =   81640.144640011 km
AZI  =   123.98673315762 deg          Angular Momentum   =   180393.45247994 km^2/s
VFPA =   58.970842503039 deg          Beta Angle         =  -21.401642701711 deg
RAV  =  -79.005139108244 deg          Periapsis Altitude =   44646.954100056 km
DECV =  -29.212134042839 deg          VelPeriaspsis      =   3.5353872196127 km/s
```

```
                                              VelApoapsis        =   0.8838468049075 km/s
                                              Orbit Period       =   453415.81894988 s

        Planetodetic Properties
        ---------------------------
        LST       =   226.76913615495 deg
        MHA       =   200.40385314776 deg
        Latitude  =  -1.0911644074554 deg
        Longitude =   26.365283007198 deg
        Altitude  =   166895.04162097 km


        Spacecraft Properties
        ------------------------------
        Cd                    =   2.200000
        Drag area             =   15.00000 m^2
        Cr                    =   1.800000
        Reflective (SRP) area =   1.000000 m^2
        Dry mass              =   850.00000000000 kg
        Total mass            =   850.00000000000 kg
        SPADDragScaleFactor   =   1.000000
        SPADSRPScaleFactor    =   1.000000

   ========================================================================
```

The values in red match with our values. Thus, our results are verified. (The FPA is (VFPA - 90) on the report above.)

(d) Plot the orbit in MATLAB or GMAT. Mark $\bar{r}, \bar{v}$ at the two times; mark the usual quantities (vectors, local horizon, γ, θ^*) and highlight the arc between the two times.

The plot from GMAT is the following.



Problem 4: A vehicle is moving in some Earth orbit; assume a two-body model. At a certain time, the following information is given

$$\bar{r} = 0.15R_{\oplus}\hat{x} - 1.44R_{\oplus}\hat{y} - 0.65R_{\oplus}\hat{z}$$

$$\bar{v} = 6.62\hat{x} + 2.7\hat{y} - 1.56\hat{z}$$

(a) Determine $a, e, i, \omega, \Omega, \gamma, \theta^*, M, E, (t - t_p)$. Are you sure it is an ellipse? Why? What quantity do you check to assess the type of conic?

The magnitude of the position and the velocity are

$$r = |\bar{r}| = 10122\ km$$

$$v = |\bar{v}| = 7.3176\ \ km/s\ \ .$$

We know that

$$R_{\oplus} = 6378.1\quad km\quad and\quad \mu = 3.9860\mathrm{e}+5\quad km^3/s^2$$

Then,

$$\bar{h} = \bar{r} \times \bar{v} = (25521\hat{x} - 25953\hat{y} + 63385\hat{z})\quad km^2/s$$

and

$$\hat{h} = \frac{\bar{h}}{|\bar{h}|} = 0.34917\hat{x} - 0.35507\hat{y} + 0.86719\hat{z}\ \ .$$

Also, the unit r-vector is

$$\hat{r} = \frac{\bar{r}}{|\bar{r}|} = 0.094517\hat{x} - 0.90737\hat{y} - 0.40958\hat{z}\ \ .$$

The other unit vector can be found from the two unit vectors with the cross product

$$\hat{\theta} = \hat{h} \times \hat{r}\ = 0.93228\hat{x} + 0.22497\hat{y} - 0.28326\hat{z}\quad .$$

We check if this is an elliptical orbit

$$v = 7.3176\quad km/s < 8.8746\quad km/s = \sqrt{\frac{2\mu}{r}} = \sqrt{2}v_c\ \ .$$

Thus, this is an elliptical orbit.

Now, the semi major axis is

$$a = \frac{\frac{-\mu}{2}}{\left(\frac{v^2}{2} - \frac{\mu}{r}\right)} = 15811 \;\; km \;\; .$$

The eccentricity becomes

$$e = \sqrt{1 - \frac{h^2}{\mu a}} = 0.39026 \;\; .$$

The radius of periapsis and apoapsis are

$$r_p = a(1-e) = 9640.7 \;\; km$$

$$r_a = a(1+e) = 21982 \;\; km \;\; .$$

The semi latus rectum is

$$p = a(1-e^2) = 13403 \;\;\; km \;\; .$$

Since these three parameters are larger than the radius of the Earth, we can say that there is no risk of collision.

Now using the rotational transformation of frames

	$\hat{r}$	$\hat{\theta}$	$\hat{h}$
$\hat{x}$	$c_\Omega c_\theta - s_\Omega c_i s_\theta$	$-c_\Omega s_\theta - s_\Omega c_i s_\theta$	$s_\Omega s_i$
$\hat{y}$	$s_\Omega c_\theta + c_\Omega c_i s_\theta$	$-s_\Omega s_\theta + c_\Omega c_i s_\theta$	$-c_\Omega s_i$
$\hat{z}$	$s_i s_\theta$	$s_i c_\theta$	c_i

Plugging in the values that we have, we know that

	$\hat{r}$	$\hat{\theta}$	$\hat{h}$
$\hat{x}$	0.094517	0.93228	0.34917
$\hat{y}$	−0.90737	0.22497	−0.35507
$\hat{z}$	−0.40958	−0.28326	0.86719

Thus, the inclination can be found initially

$$i = arccos(0.86719) \;\; = \;\; \pm 29.867°$$

$$\because 0° \leq i \;\; \leq 180°$$

$$i \;\; = \;\; 29.867°$$

Then, from

$$s_\Omega s_i = 0.34917$$

$$\Omega_1 = 44.520°, 135.48°$$

$$-c_\Omega s_i = -0.35507$$

$$\Omega_2 = \pm 135.48$$

Thus, the common angle value is

$$\Omega = 135.48° \ .$$

Similarly

$$s_i s_\theta = -0.40958$$

$$\theta_1 = -55.332°, -124.67°$$

$$s_i c_\theta = -0.28326$$

$$\theta_2 = \pm 124.67°$$

The common angle of the two is

$$\theta = -124.67°$$

To figure out if this point is in ascending or descending, we do the following

$$\dot{r} = \bar{v} \cdot \hat{r} = (6.62\hat{x} + 2.7\hat{y} - 1.56\hat{z}) \cdot (0.094517\hat{x} - 0.90737\hat{y} - 0.40958\hat{z})$$

$$\dot{r} = -1.1852 < 0$$

It is smaller than 0, thus the position is descending in orbit.

The true anomaly becomes

$$\theta^* = \pm arccos\left(\frac{1}{e}\left(\frac{p}{r} - 1\right)\right) = \pm 33.843°$$

Since the orbit is descending,

$$\theta^* = -33.843°$$

Then the argument of periapsis becomes

$$\omega = \theta - \theta^* = -124.67° - (-33.843°) = -90.825°$$

The flight path angle is

$$\gamma = arccos\left(\frac{h}{rv}\right) = -9.3213°$$

The eccentric anomaly can be found using the following formula (solved by the MATLAB function below)

$$tan\frac{\theta^*}{2} = \left(\frac{1+e}{1-e}\right)^{1/2} tan\frac{E}{2}$$

```
function E = T2E_anomaly(e, theta_star, unit)
%{
       NAME:    T2E_anomaly
       AUTHOR:  TOMOKI KOIKE
       INPUTS:  (1) e:          ECCENTRICITY
                (2) theta_star: TRUE ANOMALY
                (3) unit:       DEGREES OR RADIANS
       OUTPUTS: (1) E:          ECCENTRIC ANOMALY
       DESCRIPTION: CALCULATES THE ECCENTRIC ANOMALY FROM THE TRUE ANOMALY.
 %}

    ee = sqrt((1 - e) / (1 + e));
    if unit == "deg"
        E = 2*atand(ee * tand(theta_star / 2));
    else
        E = 2*atan(ee * tan(theta_star / 2));
    end
end
```

$$E \;=\; -22.783°$$

Then the mean anomaly becomes

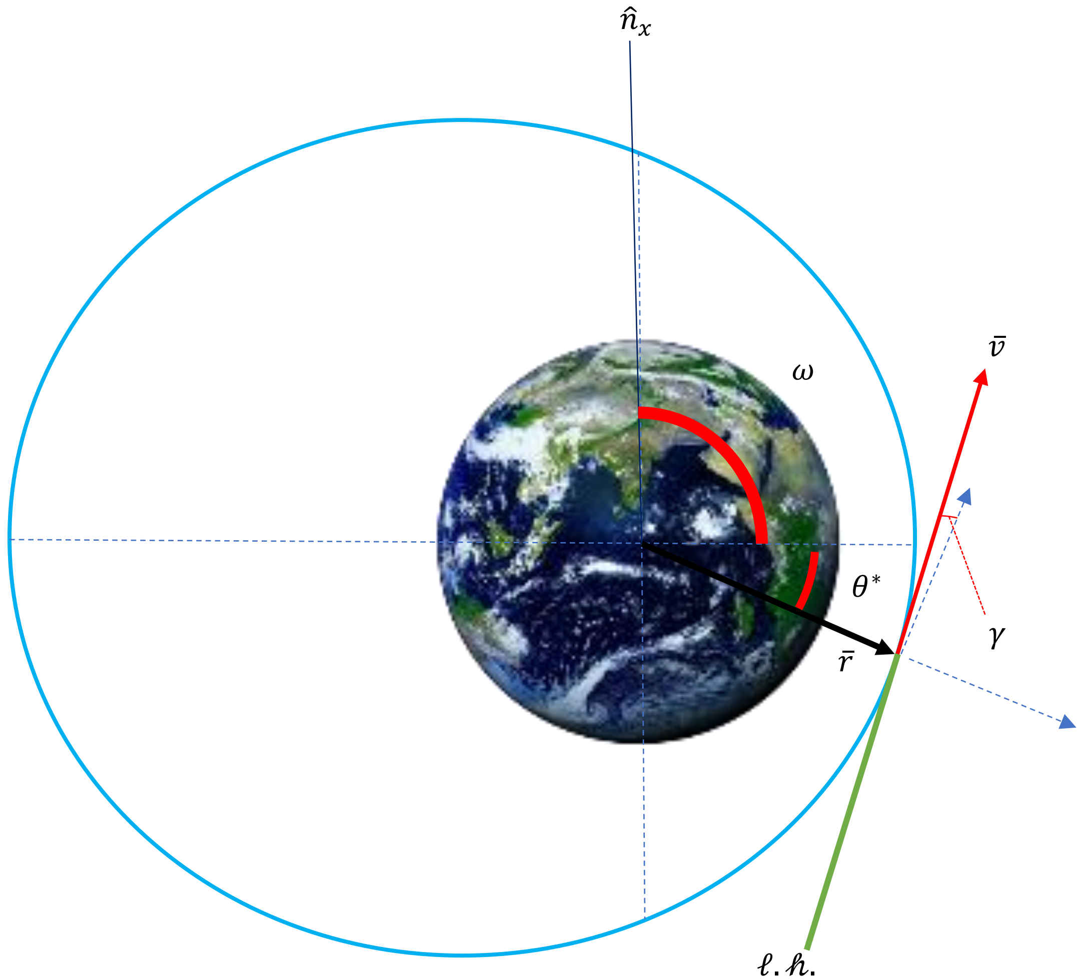
$$M \;=\; E \;-\; esinE = -14.124° \;=345.88°$$

And,

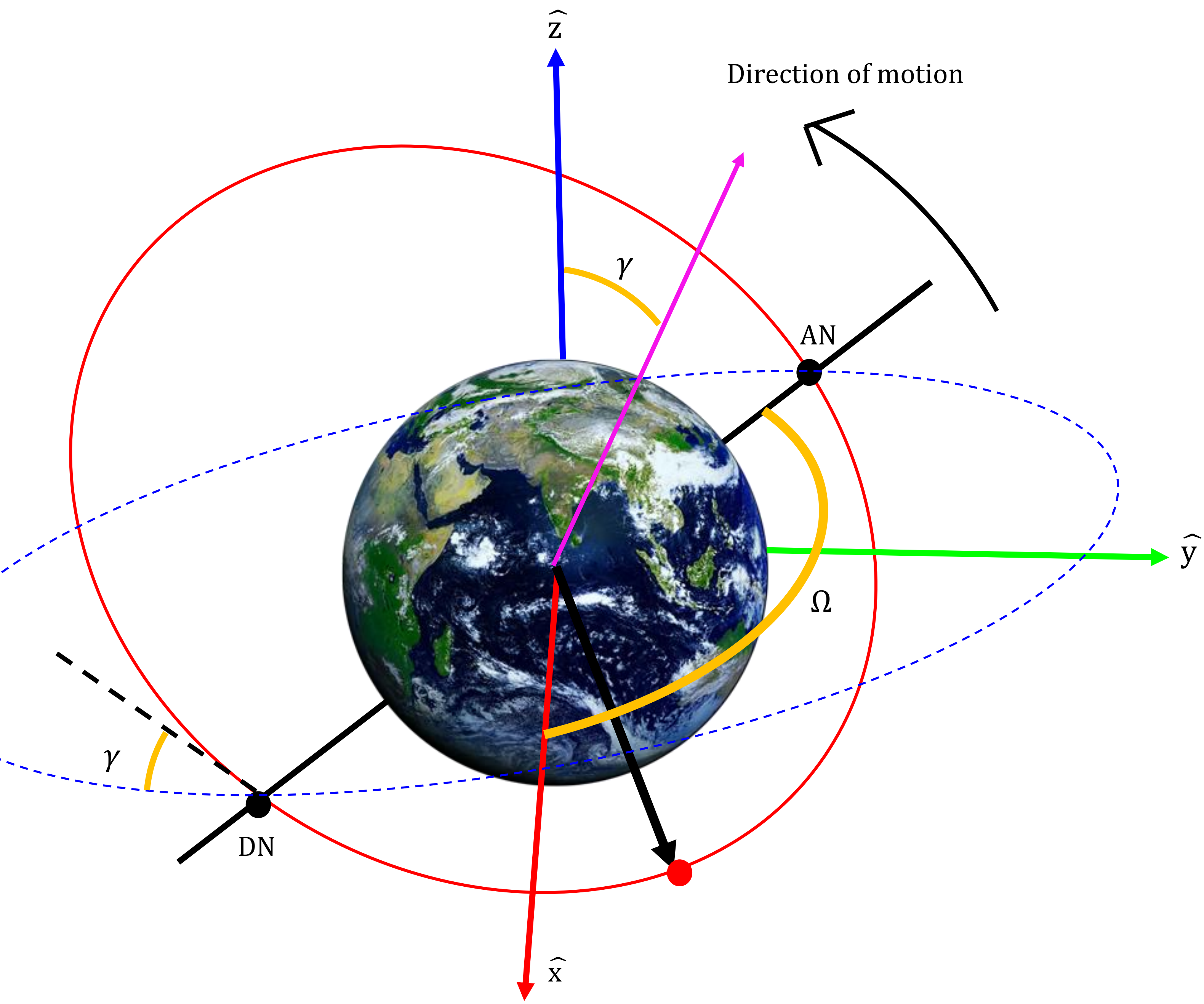
$$\left(t - t_p\right) = M\sqrt{\frac{a^3}{\mu}} = 19010\;s \;\;=\;\; 0.22002\;\; days \quad .$$

As discussed in the calculation process, we have concluded that the orbit was an ellipse by proving that the velocity and the point was smaller than the circular velocity multiplied by the square root of two. Also, assessing the eccentricity tells that the conic type of the orbit is an ellipse.

(b) Sketch the orbit in the orbit plane: add $r, v, \theta^*, \gamma, \ell.\hbar., \omega, \hat{n}_x$.



(c) Sketch the orbit in 3D (or a section of the orbit) to mark the following quantities: Ω, i, $\hat{h}$, AN (Ascending Node), DN (Descending Node), direction of motion. Is periapsis above or below the fundamental plane? How do you know? What is θ^* at the AN? DN?



The periapsis is below the fundamental plane. We can tell that from the signs of $\omega, \Omega, and\ \theta$.

At AN $\theta^* = 90.825°$, and at DN $\theta^* = 270.825°$.

Appendix

MATLAB CODE

```
%% AAE 532 HW 5 Problem 1
% Tomoki Koike
close all; clear all; clc;
fdir = 'C:\Users\Tomo\Desktop\studies\2020-Fall\AAE532\matlab\outputs\ps5';
set(groot, 'defaulttextinterpreter','latex');
set(groot, 'defaultAxesTickLabelInterpreter','latex');
set(groot, 'defaultLegendInterpreter','latex');
format longG;

% Load the GMAT data
warning('off','all');
reportDir = 'C:\Users\Tomo\Desktop\studies\2020-Fall\AAE532\gmat\ps5\reports';
for i = 0:3
    for j = 1:4
        if j == 1
            T(i+1).earthpointmass = readtable( ...
                fullfile(reportDir,"ps5_p1_"+num2str(i)+num2str(j)+".txt"));
        elseif j == 2
            T(i+1).earthmoon = readtable( ...
                fullfile(reportDir,"ps5_p1_"+num2str(i)+num2str(j)+".txt"));
        elseif j == 3
            T(i+1).earthsun = readtable( ...
                fullfile(reportDir,"ps5_p1_"+num2str(i)+num2str(j)+".txt"));
        else
            T(i+1).earthmoonsun = readtable( ...
                fullfile(reportDir,"ps5_p1_"+num2str(i)+num2str(j)+".txt"));
        end
    end
end

warning("on");

% Process the data to filter out the unnecessary values
for i = 1:4
    for j = 1:4
        if j == 1
           t =  T(i).earthpointmass.Sat1_Earth_RMAG;
           x1 = T(i).earthpointmass.Sat1_Earth_RMAG(1);
           x2 = T(i).earthpointmass.Sat1_Earth_RMAG(end);
           XX = find(t ~= x1 & t ~= x2);
        elseif j == 2
           t =  T(i).earthmoon.Sat2_Earth_RMAG;
           y1 = T(i).earthmoon.Sat2_Earth_RMAG(1);
           y2 = T(i).earthmoon.Sat2_Earth_RMAG(end);
           YY = find(t ~= y1 & t ~= y2);
        elseif j == 3
```

```
            t =  T(i).earthsun.Sat3_Earth_RMAG;
            z1 = T(i).earthsun.Sat3_Earth_RMAG(1);
            z2 = T(i).earthsun.Sat3_Earth_RMAG(end);
            ZZ = find(t ~= z1 & t ~= z2);
        else
            t =  T(i).earthmoonsun.Sat4_Earth_RMAG;
            w1 = T(i).earthmoonsun.Sat4_Earth_RMAG(1);
            w2 = T(i).earthmoonsun.Sat4_Earth_RMAG(end);
            WW = find(t ~= w1 & t ~= w2);
         end
    end
    T(i).earthpointmass = T(i).earthpointmass([XX(1)-1, XX', XX(end)+1], :);
    T(i).earthmoon      =      T(i).earthmoon([YY(1)-1, YY', YY(end)+1], :);
    T(i).earthsun       =       T(i).earthsun([ZZ(1)-1, ZZ', ZZ(end)+1], :);
    T(i).earthmoonsun   =   T(i).earthmoonsun([WW(1)-1, WW', WW(end)+1], :);

    % Compute the FPA
    d1 = T(i).earthpointmass.Sat1_Earth_HMAG;
    d2 = T(i).earthpointmass.Sat1_Earth_RMAG;
    d3 = T(i).earthpointmass.Sat1_EarthMJ2000Eq_VMAG;
    T(i).earthpointmass.Sat1_EarthMJ2000Eq_FPA = acosd(d1 ./ d2 ./ d3);
    dd = find(T(i).earthpointmass.Sat1_Earth_TA > 180);
    T(i).earthpointmass.Sat1_EarthMJ2000Eq_FPA(dd) = -
T(i).earthpointmass.Sat1_EarthMJ2000Eq_FPA(dd);

    d1 = T(i).earthmoon.Sat2_Earth_HMAG;
    d2 = T(i).earthmoon.Sat2_Earth_RMAG;
    d3 = T(i).earthmoon.Sat2_EarthMJ2000Eq_VMAG;
    T(i).earthmoon.Sat2_EarthMJ2000Eq_FPA = acosd(d1 ./ d2 ./ d3);
    dd = find(T(i).earthmoon.Sat2_Earth_TA > 180);
    T(i).earthmoon.Sat2_EarthMJ2000Eq_FPA(dd) = -
T(i).earthmoon.Sat2_EarthMJ2000Eq_FPA(dd);

    d1 = T(i).earthsun.Sat3_Earth_HMAG;
    d2 = T(i).earthsun.Sat3_Earth_RMAG;
    d3 = T(i).earthsun.Sat3_EarthMJ2000Eq_VMAG;
    T(i).earthsun.Sat3_EarthMJ2000Eq_FPA = acosd(d1 ./ d2 ./ d3);
    dd = find(T(i).earthsun.Sat3_Earth_TA > 180);
    T(i).earthsun.Sat3_EarthMJ2000Eq_FPA(dd) = -
T(i).earthsun.Sat3_EarthMJ2000Eq_FPA(dd);

    d1 = T(i).earthmoonsun.Sat4_Earth_HMAG;
    d2 = T(i).earthmoonsun.Sat4_Earth_RMAG;
    d3 = T(i).earthmoonsun.Sat4_EarthMJ2000Eq_VMAG;
    T(i).earthmoonsun.Sat4_EarthMJ2000Eq_FPA = acosd(d1 ./ d2 ./ d3);
    dd = find(T(i).earthmoonsun.Sat4_Earth_TA > 180);
    T(i).earthmoonsun.Sat4_EarthMJ2000Eq_FPA(dd) = -
T(i).earthmoonsun.Sat4_EarthMJ2000Eq_FPA(dd);
end

% a = []; e = []; rp = []; E = []; h = [];
% for i = 1:4
```

```
%     t1 = T(i).earthpointmass;
%     t2 = T(i).earthmoon;
%     t3 = T(i).earthsun;
%     t4 = T(i).earthmoonsun;
%     for j = 1:4
%         if j == 1
%             a(i,j)  = mean(t1.Sat1_Earth_SMA);
%             e(i,j)  = mean(t1.Sat1_Earth_ECC);
%             rp(i,j) = mean(t1.Sat1_Earth_RadPer);
%             En(i,j) = mean(t1.Sat1_Earth_Energy);
%             h(i,j)  = mean(t1.Sat1_Earth_HMAG);
%         elseif j == 2
%             a(i,j)  = mean(t2.Sat2_Earth_SMA);
%             e(i,j)  = mean(t2.Sat2_Earth_ECC);
%             rp(i,j) = mean(t2.Sat2_Earth_RadPer);
%             En(i,j) = mean(t2.Sat2_Earth_Energy);
%             h(i,j)  = mean(t2.Sat2_Earth_HMAG);
%         elseif j == 3
%             a(i,j)  = mean(t3.Sat3_Earth_SMA);
%             e(i,j)  = mean(t3.Sat3_Earth_ECC);
%             rp(i,j) = mean(t3.Sat3_Earth_RadPer);
%             En(i,j) = mean(t3.Sat3_Earth_Energy);
%             h(i,j)  = mean(t3.Sat3_Earth_HMAG);
%         else
%             a(i,j)  = mean(t4.Sat4_Earth_SMA);
%             e(i,j)  = mean(t4.Sat4_Earth_ECC);
%             rp(i,j) = mean(t4.Sat4_Earth_RadPer);
%             En(i,j) = mean(t4.Sat4_Earth_Energy);
%             h(i,j)  = mean(t4.Sat4_Earth_HMAG);
%         end
%     end
% end

% Get the last values for each column
a = []; e = []; rp = []; E = []; h = []; rf = []; vf = []; TA_f = []; FPA_f = [];
for i = 1:4
    t1 = T(i).earthpointmass;
    t2 = T(i).earthmoon;
    t3 = T(i).earthsun;
    t4 = T(i).earthmoonsun;
    for j = 1:4
        if j == 1
            a(i,j)     = t1.Sat1_Earth_SMA(end);
            e(i,j)     = t1.Sat1_Earth_ECC(end);
            rp(i,j)    = t1.Sat1_Earth_RadPer(end);
            En(i,j)    = t1.Sat1_Earth_Energy(end);
            h(i,j)     = t1.Sat1_Earth_HMAG(end);
            rf(i,j)    = t1.Sat1_Earth_RMAG(end);
            vf(i,j)    = t1.Sat1_EarthMJ2000Eq_VMAG(end);
            TA_f(i,j)  = t1.Sat1_Earth_TA(end);
            FPA_f(i,j) = t1.Sat1_EarthMJ2000Eq_FPA(end);
        elseif j == 2
```

```
            a(i,j)     = t2.Sat2_Earth_SMA(end);
            e(i,j)     = t2.Sat2_Earth_ECC(end);
            rp(i,j)    = t2.Sat2_Earth_RadPer(end);
            En(i,j)    = t2.Sat2_Earth_Energy(end);
            h(i,j)     = t2.Sat2_Earth_HMAG(end);
            rf(i,j)    = t2.Sat2_Earth_RMAG(end);
            vf(i,j)    = t2.Sat2_EarthMJ2000Eq_VMAG(end);
            TA_f(i,j)  = t2.Sat2_Earth_TA(end);
            FPA_f(i,j) = t2.Sat2_EarthMJ2000Eq_FPA(end);
        elseif j == 3
            a(i,j)     = t3.Sat3_Earth_SMA(end);
            e(i,j)     = t3.Sat3_Earth_ECC(end);
            rp(i,j)    = t3.Sat3_Earth_RadPer(end);
            En(i,j)    = t3.Sat3_Earth_Energy(end);
            h(i,j)     = t3.Sat3_Earth_HMAG(end);
            rf(i,j)    = t3.Sat3_Earth_RMAG(end);
            vf(i,j)    = t3.Sat3_EarthMJ2000Eq_VMAG(end);
            TA_f(i,j)  = t3.Sat3_Earth_TA(end);
            FPA_f(i,j) = t3.Sat3_EarthMJ2000Eq_FPA(end);
        else
            a(i,j)     = t4.Sat4_Earth_SMA(end);
            e(i,j)     = t4.Sat4_Earth_ECC(end);
            rp(i,j)    = t4.Sat4_Earth_RadPer(end);
            En(i,j)    = t4.Sat4_Earth_Energy(end);
            h(i,j)     = t4.Sat4_Earth_HMAG(end);
            rf(i,j)    = t4.Sat4_Earth_RMAG(end);
            vf(i,j)    = t4.Sat4_EarthMJ2000Eq_VMAG(end);
            TA_f(i,j)  = t4.Sat4_Earth_TA(end);
            FPA_f(i,j) = t4.Sat4_EarthMJ2000Eq_FPA(end);
        end
    end
end

% Get array for data
arr1 =
[a(1,:);e(1,:);rp(1,:);En(1,:);h(1,:);rf(1,:);vf(1,:);TA_f(1,:);FPA_f(1,:)];
arr2 =
[a(2,:);e(2,:);rp(2,:);En(2,:);h(2,:);rf(2,:);vf(2,:);TA_f(2,:);FPA_f(2,:)];
arr3 =
[a(3,:);e(3,:);rp(3,:);En(3,:);h(3,:);rf(3,:);vf(3,:);TA_f(3,:);FPA_f(3,:)];
arr4 =
[a(4,:);e(4,:);rp(4,:);En(4,:);h(4,:);rf(4,:);vf(4,:);TA_f(4,:);FPA_f(4,:)];

% Convert arrays to table
M1 = array2table(arr1);
M2 = array2table(arr2);
M3 = array2table(arr3);
M4 = array2table(arr4);

% Save table data as excel file
warning('off','MATLAB:xlswrite:AddSheet'); %optional
writetable(M1,fullfile(fdir, 'p1_data.xlsx'),'Sheet',1);
```

```
writetable(M2,fullfile(fdir, 'p1_data.xlsx'),'Sheet',2);
writetable(M3,fullfile(fdir, 'p1_data.xlsx'),'Sheet',3);
writetable(M4,fullfile(fdir, 'p1_data.xlsx'),'Sheet',4);

% Plotting

for i = 1:4
    t1 = T(i).earthpointmass;
    t2 = T(i).earthmoon;
    t3 = T(i).earthsun;
    t4 = T(i).earthmoonsun;

    fig = figure("Renderer","painters", "Position",[10 10 900 1000]);
    subplot(2,2,1)
    plot(t1.Sat1_Earth_Altitude)
    title('EarthPointMass')
    grid on; grid minor; box on;
    subplot(2,2,2)
    plot(t2.Sat2_Earth_Altitude)
    title('EarthMoon')
    grid on; grid minor; box on;
    subplot(2,2,3)
    plot(t3.Sat3_Earth_Altitude)
    title('EarthSun')
    grid on; grid minor; box on;
    subplot(2,2,4)
    plot(t4.Sat4_Earth_Altitude)
    title('EarthMoonSun')
    grid on; grid minor; box on;
    % Give common xlabel and ylabel to your figure
    han=axes(fig,'visible','off');
    han.XLabel.Visible='on';
    han.YLabel.Visible='on';
    xlabel(han,'sample points in the order of epoch');
    ylabel(han,'altitude [km]');

    ch_str = ["02 Oct 2020", "07 Oct 2020", "11 Oct 2020", "30 Oct 2020"];
    title_str = 'Altitude: 60 Day History From ' + ch_str(i) + ' - T. Koike';
    sgtitle(title_str)

    file_str = 'alt' + ch_str(i) + '.png';
    saveas(fig, fullfile(fdir, file_str));
end
```

```
%% AAE 532 HW 5 Problem 2
% Tomoki Koike
close all; clear all; clc;
fdir = 'C:\Users\Tomo\Desktop\studies\2020-Fall\AAE532\matlab\outputs\ps5';
set(groot, 'defaulttextinterpreter','latex');
set(groot, 'defaultAxesTickLabelInterpreter','latex');
set(groot, 'defaultLegendInterpreter','latex');
format longG;
```

```
% Set constants
planet_consts = setup_planetary_constants();  % Function that sets up all the
constants in the table
sun = planet_consts.sun;  % structure of sun
earth = planet_consts.earth;  % structure of earth
mars = planet_consts.mars;  % structure of mars
G = 6.6743015e-20;  % Gravitational constant [km^3/kg/s^2]

% (a) Calculate parameters
rp = 1.5*mars.mer;  % radius of periapsis
ra = 6.5*mars.mer;  % radius of apoapsis
M = -90;  % mean anomaly
a = 0.5*(rp + ra);  % semi major axis
e = (ra - rp) / (ra + rp);  % eccentricity
p = a * (1 - e^2);  % semi latus rectum
mu = mars.gp;  % gravitational parameter
h = sqrt(mu * p);  % specific angular momentum
IP = 2 * pi * sqrt(a^3 / mu);  % period
IP_days = IP / 60 / 60 / 24;  % period in days
En = -mu / 2 / a;  % specific energy
E = rad2deg(M2E_anomaly(M, e, "deg"))  % eccentric anomaly
E0 = E;
E = 360 + E
TA = E2T_anomaly(e,E,"deg")  % true anomaly
r = p / (1 + e * cosd(TA))  % position
v = vis_viva(r, a, mu)  % velocity
gamma = -acosd(h / r / v)  % flight path angle
del_t = deg2rad(M + 360) * sqrt(a^3 / mu)  % time elapse
del_t_day = del_t / 60 / 60 / 24

% (b)
r0vec = r * [cosd(TA), sind(TA)];
v0vec = v * [cosd(TA + 90 + abs(gamma)), sind(TA + 90 + abs(gamma))];

% (c)
Mf = 90;
Ef = rad2deg(M2E_anomaly(Mf, e, "deg"));
TAf = E2T_anomaly(e, Ef, "deg");
rfmag = p / (1 + e * cosd(TAf));

[rfvec, vfvec, f, g, fdot, gdot] = FandG_elp(a, mu, deg2rad(Ef-E0), 0.5*IP,
r0vec, v0vec);
[rfvec2, vfvec2, f2, g2, fdot2, gdot2] = FandG_conic(p, mu, deg2rad(TAf-(TA-
360)),r0vec,v0vec,rfmag);

% (d)
Th = -pi:0.01:pi;
R = p ./ (1 + e * cos(Th)); X = R .* cos(Th); Y = R .* sin(Th);
fig = figure("Renderer","painters", "Position",[10 10 900 700]);
    plot(X, Y, '-b')
    title('Elliptical Orbit of $e=0.625$ Around Mars - T. Koike')
```

```
    hold on; grid on; grid minor; box on; axis equal;
    plot(r*cosd(TA), r*sind(TA), '.r', 'MarkerSize', 18)
    plot(rfmag*cosd(TAf), rfmag*sind(TAf), '.r', 'MarkerSize', 18)
    % Mars
    plot(0, 0, '.m', 'MarkerSize', 30)
    % Axes
    nanikore = -ra:rp; korenani = -a*sqrt(1-e^2):a*sqrt(1-e^2);
    plot(nanikore, zeros(size(nanikore)), '--k')
    plot(-a*e*ones(size(korenani)), korenani, '--k')
    hold off
    xlabel('$\hat{e}$ [km]')
    ylabel('$\hat{p}$ [km]')
saveas(fig, fullfile(fdir, 'p2_orbit.png'));
```

```
% AAE 532 HW 5 Problem 3
% Tomoki Koike
close all; clear all; clc;
fdir = 'C:\Users\Tomo\Desktop\studies\2020-Fall\AAE532\matlab\outputs\ps5';
set(groot, 'defaulttextinterpreter','latex');
set(groot, 'defaultAxesTickLabelInterpreter','latex');
set(groot, 'defaultLegendInterpreter','latex');
format longG;

% Set constants
planet_consts = setup_planetary_constants();  % Function that sets up all the
constants in the table
earth = planet_consts.earth;  % structure of earth
G = 6.6743015e-20;  % Gravitational constant [km^3/kg/s^2]

Re = earth.mer;
a = 20*Re   % semi major axis
RA = 45;  % right ascension
e = 0.6;  % eccentricity
refA = 235;  % anomaly from reference line
AP = 30;  % argument of periapsis
i = 34;  % inclination
mu = earth.gp

C_pq = [cosd(refA), sind(refA), 0; -sind(refA), cosd(refA), 0; 0,0,1];
C_qn = [1,0,0;0,cosd(i),sind(i);0,-sind(i),cosd(i)];
C_ni = [cosd(RA),sind(RA),0;-sind(RA),cosd(RA),0;0,0,1];

% i: inertial frame, n: n-frame, q: q-frame, p: orbital frame (polar)
TA          = refA - AP                         % true anomaly
p           = a * (1 - e^2);                     % semi latus rectum
r1          = p / (1 + e * cosd(TA));
v1          = vis_viva(r1, a, mu);
h           = sqrt(mu * p);                      % specific angular momentum
FPA         = -acosd(h / r1 / v1);               % flight path angle
IP          = 2 * pi * sqrt(a^3 / mu);           % period
IP_day      = IP / 60 / 60 / 24;
E           = T2E_anomaly(e, TA, "deg");          % eccentric anomaly
```

```
M          = rad2deg(deg2rad(E) - e*sind(E));  % mean anomaly
dt_per     = deg2rad(M) * sqrt(a^3 / mu);       % time elapse from periapsis
dt_per_day = dt_per / 60 / 60 / 24;

r1vec_p = r1 * [1,0,0];                     % position vector in orbital frame
v1vec_p = v1 * [sind(FPA), cosd(FPA), 0];  % velocity vector in orbital frame
r1vec_n = r1vec_p * C_pq * C_qn;
v1vec_n = v1vec_p * C_pq * C_qn;
r1vec_i = r1vec_n * C_ni;
v1vec_i = v1vec_n * C_ni;

% syms theta i Omega
% assume([theta, i, Omega], 'real');
% Cm1 = [cos(theta), sin(theta), 0; -sin(theta), cos(theta), 0; 0,0,1];
% Cm2 = [1,0,0;0,cos(i),sin(i);0,-sin(i),cos(i)];
% Cm3 = [cos(Omega),sin(Omega),0;-sin(Omega),cos(Omega),0;0,0,1];
% Cm1*Cm2

% (c)

dt2 = 3*24*60*60;
M2 = mod((rad2deg(sqrt(mu / a^3) * dt2)+M),360)
E2 = rad2deg(M2E_anomaly(M2,e,"deg"))
TA2 = E2T_anomaly(e,E2,"deg")
dTA12 = TA2 - (TA-360)
dE12 = E2 - (E-360)

[rvec2, vvec2, f2, g2, fdot2, gdot2] =
FandG_elp(a,mu,deg2rad(dE12),dt2,r1vec_i,v1vec_i)

FPA2 = acosd(h / norm(rvec2) / norm(vvec2))
```

```
% AAE 532 HW 5 Problem 3
% Tomoki Koike
close all; clear all; clc;
fdir = 'C:\Users\Tomo\Desktop\studies\2020-Fall\AAE532\matlab\outputs\ps5';
set(groot, 'defaulttextinterpreter','latex');
set(groot, 'defaultAxesTickLabelInterpreter','latex');
set(groot, 'defaultLegendInterpreter','latex');
format shortG;

% Set constants
planet_consts = setup_planetary_constants();  % Function that sets up all the
constants in the table
earth = planet_consts.earth;  % structure of earth
G = 6.6743015e-20;  % Gravitational constant [km^3/kg/s^2]

% Setup
Re = earth.mer;
rvec_i = Re*[0.15, -1.44, -0.65];  % the position vector in inertial frame
vvec_i = [6.62, 2.7, -1.56];  % the velocity vector in inertial frame
```

```
mu = earth.gp;  % gravitational parameter

r = norm(rvec_i)  % magnitude of the position vector
v = norm(vvec_i)  % magnitude of the velocity vector
hvec_i = cross(rvec_i, vvec_i)  % the h vector perpendicular to the position and
velocity vectors
hhat_i = hvec_i / norm(hvec_i)  % the unit h vector in inertial frame
h = norm(hvec_i)  % magnitude of the h vector
rhat_i = rvec_i / r  % the unit r vector in inertial frame
thetahat_i = cross(hhat_i, rhat_i)  % the unit vector of theta direction

check = v < sqrt(2 * mu / r)  % check to see if this orbit is an ellipse

a = (-mu / 2) / (v^2/2 - mu/r)  % semi major axis
e = sqrt(1 - h^2 / mu / a)  % the eccentricity
rp = a*(1 - e)  % radius of periapsisc
ra = a*(1 + e)  % radius of apoapsis
p = a*(1 - e^2) % semi latus rectum

% Check that there are no collisions with the Earth
check = rp > Re
check = ra > Re

thetastar = acos_dbval(1/e * (p/r - 1), "deg")  % double valued true anomaly

i = acos_dbval(hhat_i(3), "deg")
i = i(0 <= i & i <= 180)  % inclination

Omega1 = asin_dbval(hhat_i(1) / sind(i), "deg")
Omega2 = -acos_dbval(hhat_i(2) / sind(i), "deg")
Omega = intersect(round(Omega1,5), round(Omega2,5))

theta1 = asin_dbval(rhat_i(3) / sind(i), "deg")
theta2 = acos_dbval(thetahat_i(3) / sind(i), "deg")
theta = intersect(round(theta1,5), round(theta2,5))

rdot = dot(vvec_i, rhat_i)
thetastar = thetastar(thetastar < 0)
omega = theta - thetastar
gamma = -acosd(h /r / v)

E = T2E_anomaly(e, thetastar, "deg")
M = rad2deg(deg2rad(E) - e*sind(E))
dtp = deg2rad(M+360)*sqrt(a^3 / mu)
dtp_day = dtp / 60 / 60/ 24

ang = [0:0.01:2*pi];
R = p./(1 + e*cos(ang));
X = R .* cos(ang);
Y = R .* sin(ang);
plot(X,Y)
axis equal
```

```
function res = asin_dbval(x, unit)
    if unit == "deg"
        ang1 = asind(x);
        if (0<=ang1 && ang1<=180)
            ang2 = 180 - ang1;
        elseif -90<=ang1 && ang1<0
            ang2 = -ang1 - 180;
        else
            ang2 = 540 - ang1;
        end
    else
        ang1 = asin(x);
        if (0<=ang1 && ang1<=pi)
            ang2 = pi - ang1;
        elseif -pi/2<=ang1 && ang1<0
            ang2 = -ang1 - pi;
        else
            ang2 = 3*pi - ang1;
        end
    end
    res = [ang1, ang2];
end

function res = acos_dbval(x, unit)
    if unit == "deg"
        ang1 = acosd(x);
        if (0<=ang1 && ang1<=180)
            ang2 = -ang1;
        else
            ang2 = 360 - ang1;
        end
    else
        ang1 = asin(x);
        if (0<=ang1 && ang1<pi)
            ang2 = -ang1;
        else
            ang2 = 2*pi - ang1;
        end
    end
    res = [ang1, ang2];
end
```