August 27, 2020

AAE 564 Fall 2020

Homework One

Friday, September 4

Exercise 1 Consider a system described by a single n^{th}-order linear differential equation of the form

$$q^{(n)} + a_{n-1}q^{(n-1)} + \ldots a_1\dot{q} + a_0 q = u$$

where $q(t) \in \mathbb{R}$ and $q^{(n)} := \frac{d^n q}{dt^n}$. By appropriate definition of state variables, obtain a first order state space description of this system.

Exercise 2 By appropriate definition of state variables, obtain a first order state space description of the following systems where q_1 and q_2 are real scalars.

(i)

$$\begin{aligned} 2\ddot{q}_1 + \ddot{q}_2 + \sin q_1 &= 0 \\ \ddot{q}_1 + 2\ddot{q}_2 + \sin q_2 &= 0 \end{aligned}$$

(ii)

$$\begin{aligned} \ddot{q}_1 + \dot{q}_2 + q_1^3 &= 0 \\ \dot{q}_1 + \dot{q}_2 + q_2^3 &= 0 \end{aligned}$$

Exercise 3 Obtain a state-space description of the following system.

$$\begin{aligned} \ddot{q}_1 + \dot{q}_2 + \sin q_1 &= u \\ \dot{q}_2 + q_1 + q_2 &= 0 \\ y &= q_1 + q_2 \end{aligned}$$

Exercise 4 Consider the discrete-time system described by

$$q(k+3) + 7q(k+2) + q(k+1) + 6q(k) + 7u(k) = 0$$

Obtain a state space description of this system.

Exercise 5 Obtain a state space representation of the following system:

$$q(k+n) + a_{n-1}q(k+n-1) + \ldots + a_1 q(k+1) + a_0 q(k) = 0$$

where $q(k) \in \mathbb{R}$.

Exercise 6 Obtain a state description of the following system:

$$
\begin{aligned}
&q_1(k+2) + q_2(k+1) + q_1(k) = u(k) \\
&q_1(k+2) - q_2(k+1) + q_2(k) = 0 \\
&y(k) = q_1(k+1) + q_2(k)
\end{aligned}
$$

Exercise 7 Recall the two pendulum cart example in the notes. Consider the following parameter sets

	m_0	m_1	m_2	l_1	l_2	g
P1	2	1	1	1	1	1
P2	2	1	1	1	0.99	1
P3	2	1	0.5	1	1	1
P4	2	1	1	1	0.5	1

and initial conditions,

	y	θ_1	θ_2	$\dot{y}$	$\dot{\theta}_1$	$\dot{\theta}_2$
IC1	0	$-10°$	$10°$	0	0	0
IC2	0	$10°$	$10°$	0	0	0
IC3	0	$-90°$	$90°$	0	0	0
IC4	0	$-90.01°$	$90°$	0	0	0
IC5	0	$100°$	$100°$	0	0	0
IC6	0	$100.01°$	$100°$	0	0	0
IC7	0	$179.99°$	$0°$	0	0	0

Simulate the system with $u = 0$ using the following combinations:

$$
\begin{aligned}
P1: &\qquad IC1, IC2, IC3, IC7 \\
P4: &\qquad IC1, IC2, IC3, IC4
\end{aligned}
$$