AAE340 HW#4

<1b> & <1c>

Numerically integrate

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = \frac{f_o}{m}\cos(\omega t)$$

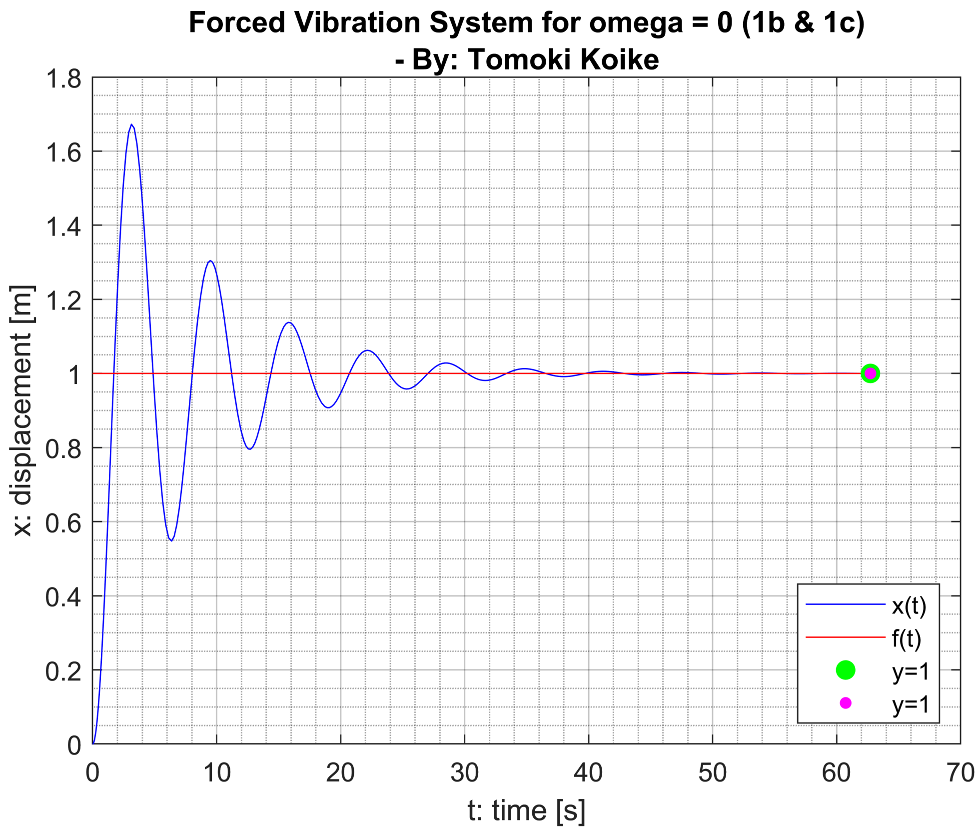
```
% Solving with ode45
zeta = 1/8; % Setting the zeta variable
omega_n = 1; % Setting the omega variable
f_over_m = 1; % Setting the f over m variable
```

CASE 1: omega = 0

```
omega = 0;
tspan = [0 10*2*pi/omega_n]; % Defining the interval of t
x0 = [0 0]; % Defining the initial conditions
[t, x] = ode45(@(t,x) fcn(t,x,zeta,omega_n,f_over_m,omega), tspan, x0);
% Solving the ode with ode45
x0_numer = x(:,1); % Assigning the x_numerical values to a variable
f_t = f_over_m * cos(omega .* t);
```

Plotting the result of the solve differential equation

```
figure(1)
plot(t, x0_numer, '-b')
ylabel('x: displacement [m]')
xlabel('t: time [s]')
title({'Forced Vibration System for omega = 0 (1b & 1c)','- By: Tomoki Koike'})
grid on
grid minor
box on
hold on
plot(t, f_t, '-r')
plot(62.7214, 1, '.g','MarkerSize',25)
plot(62.7214, 1, '.m','MarkerSize',15)
hold off
legend('x(t)', 'f(t)', 'y=1', 'y=1', 'Location','southeast')
```

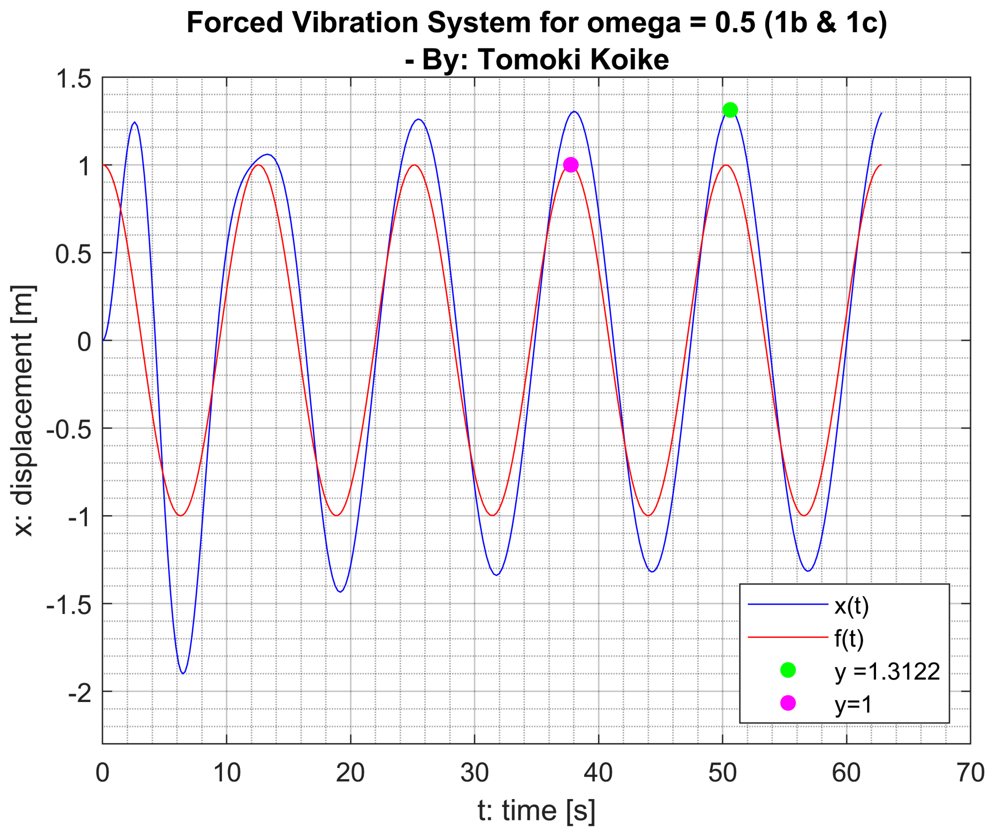


CASE 2: omega = 0.5

```
omega = 0.5;
tspan = [0 10*2*pi/omega_n]; % Defining the interval of t
x0 = [0 0]; % Defining the initial conditions
[t, x] = ode45(@(t,x) fcn(t,x,zeta,omega_n,f_over_m,omega), tspan, x0);
% Solving the ode with ode45
x05_numer = x(:,1); % Assigning the x_numerical values to a variable
f_t = f_over_m * cos(omega .* t);
```

Plotting the result of the solve differential equation

```
figure(1)
plot(t, x05_numer, '-b')
ylabel('x: displacement [m]')
xlabel('t: time [s]')
title({'Forced Vibration System for omega = 0.5 (1b & 1c)','- By: Tomoki Koike'})
grid on
grid minor
box on
ylim([-2.3 1.5])
hold on
plot(t, f_t, '-r')
plot(50.6157, 1.3122, '.g','MarkerSize',20)
plot(37.7581, 1, '.m','MarkerSize',20)
hold off
legend('x(t)', 'f(t)', 'y =1.3122', 'y=1', 'Location','southeast')
```

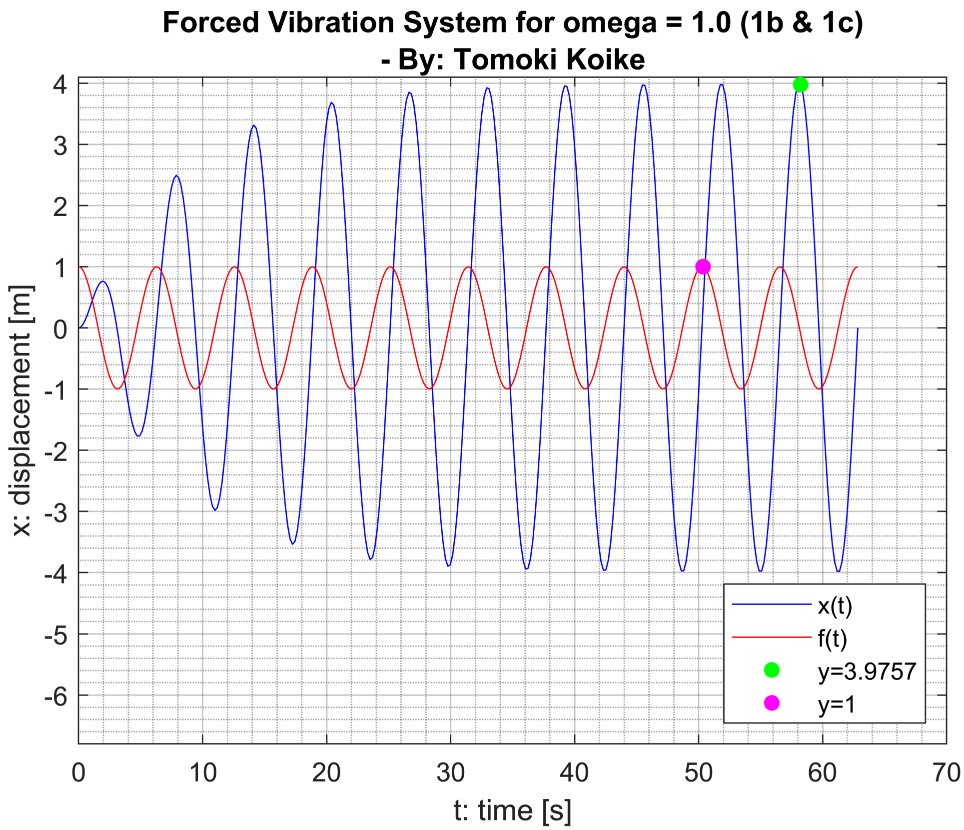


CASE 3: omega = 1.0

```
omega = 1.0;
tspan = [0 10*2*pi/omega_n]; % Defining the interval of t
x0 = [0 0]; % Defining the initial conditions
[t, x] = ode45(@(t,x) fcn(t,x,zeta,omega_n,f_over_m,omega), tspan, x0);
% Solving the ode with ode45
x10_numer = x(:,1); % Assigning the x_numerical values to a variable
f_t = f_over_m * cos(omega .* t);
```

Plotting the result of the solve differential equation

```
figure(1)
plot(t, x10_numer, '-b')
ylabel('x: displacement [m]')
xlabel('t: time [s]')
title({'Forced Vibration System for omega = 1.0 (1b & 1c)','- By: Tomoki Koike'})
grid on
grid minor
box on
ylim([-6.8 4.1])
hold on
plot(t, f_t, '-r')
plot(58.2175, 3.9757, '.g','MarkerSize',20)
plot(50.3516, 1, '.m','MarkerSize',20)
hold off
legend('x(t)', 'f(t)', 'y=3.9757', 'y=1','Location','southeast')
```

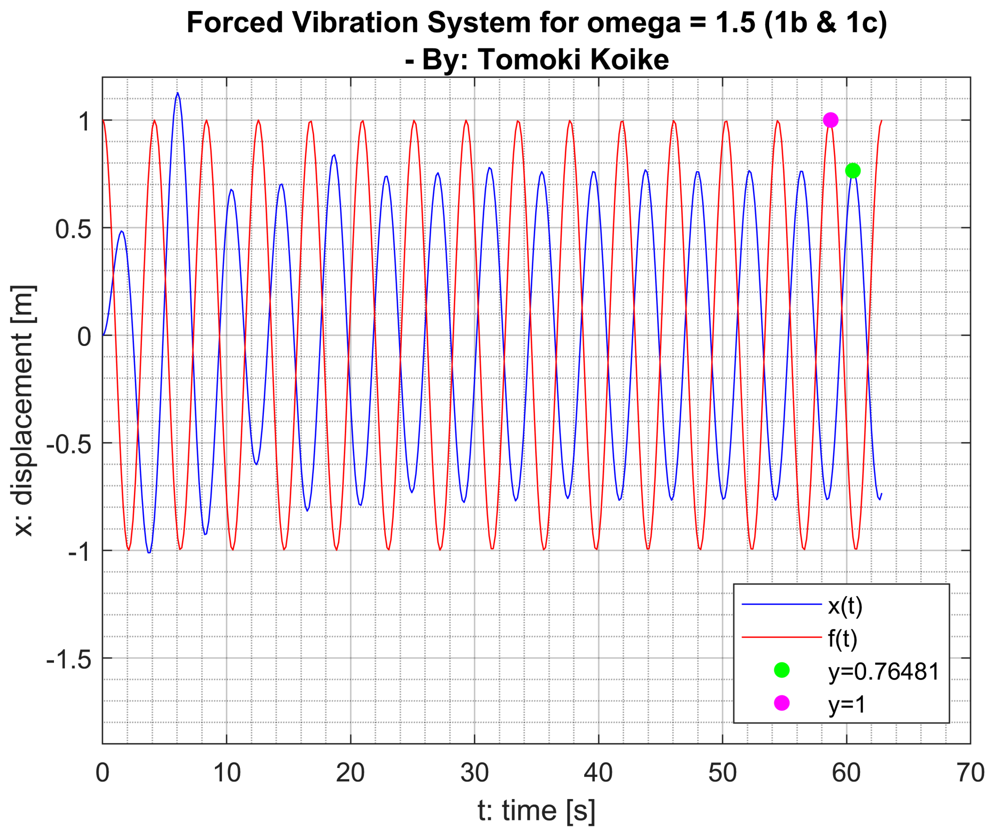


CASE 4: omega = 1.5

```
omega = 1.5;
tspan = [0 10*2*pi/omega_n]; % Defining the interval of t
x0 = [0 0]; % Defining the initial conditions
[t, x] = ode45(@(t,x) fcn(t,x,zeta,omega_n,f_over_m,omega), tspan, x0);
% Solving the ode with ode45
x15_numer = x(:,1); % Assigning the x_numerical values to a variable
f_t = f_over_m * cos(omega .* t);
```

Plotting the result of the solve differential equation

```
figure(1)
plot(t, x15_numer, '-b')
ylabel('x: displacement [m]')
xlabel('t: time [s]')
title({'Forced Vibration System for omega = 1.5 (1b & 1c)','- By: Tomoki Koike'})
grid on
grid minor
box on
ylim([-1.9 1.2])
hold on
plot(t, f_t, '-r')
plot(60.5038, 0.76481, '.g','MarkerSize',20)
plot(58.7128, 1, '.m','MarkerSize',20)
hold off
legend('x(t)', 'f(t)', 'y=0.76481', 'y=1','Location','southeast')
```

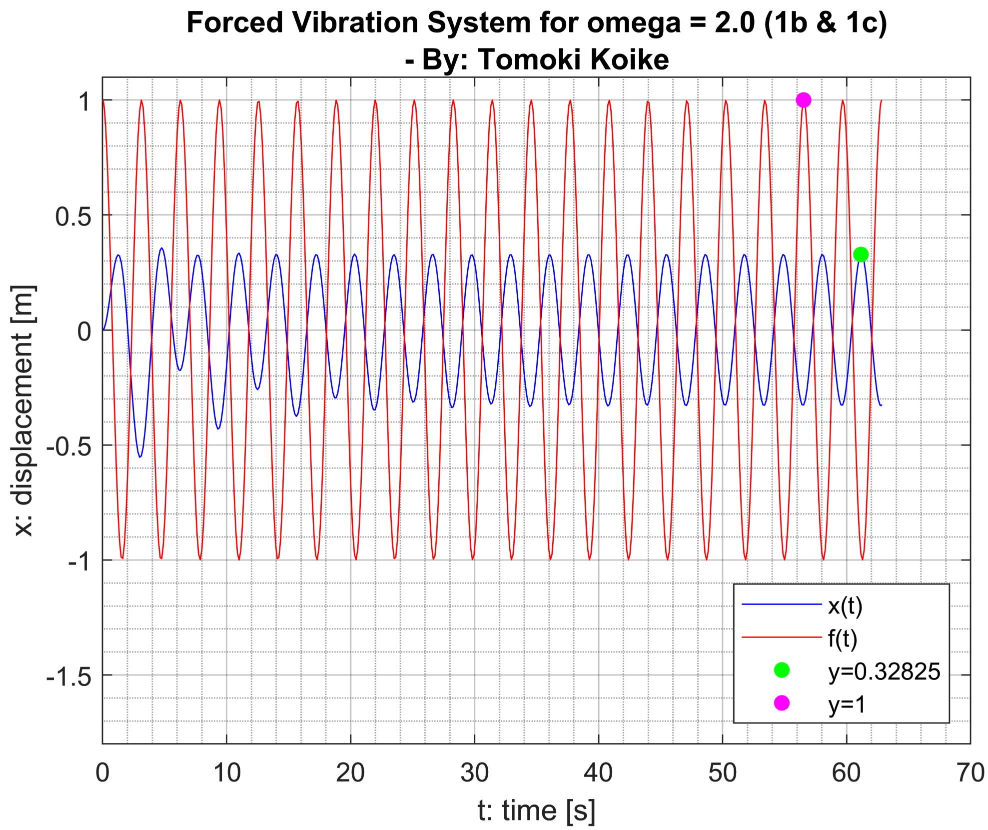


CASE 5: omega = 2.0

```
omega = 2.0;
tspan = [0 10*2*pi/omega_n]; % Defining the interval of t
x0 = [0 0]; % Defining the initial conditions
[t, x] = ode45(@(t,x) fcn(t,x,zeta,omega_n,f_over_m,omega), tspan, x0);
% Solving the ode with ode45
x20_numer = x(:,1); % Assigning the x_numerical values to a variable
f_t = f_over_m * cos(omega .* t);
```

Plotting the result of the solve differential equation

```
figure(1)
plot(t, x20_numer, '-b')
ylabel('x: displacement [m]')
xlabel('t: time [s]')
title({'Forced Vibration System for omega = 2.0 (1b & 1c)','- By: Tomoki Koike'})
grid on
grid minor
box on
ylim([-1.8 1.1])
hold on
plot(t, f_t, '-r')
plot(61.1502, 0.32825, '.g','MarkerSize',20)
plot(56.5255, 1, '.m','MarkerSize',20)
hold off
legend('x(t)', 'f(t)', 'y=0.32825', 'y=1','Location','southeast')
```



<1d> & <1e>

The measurement for each cases are on the plots as green markers for x(t) amplitude and

magenta for f/m*cos(omega*t) amplitude.

The measurements are organized as the following

```
omegas = [0; 0.5; 1.0; 1.5; 2.0];
Amp_ft = [1; 1; 1; 1; 1];
Amp_xt = [1; 1.3122; 3.9757; 0.76481; 0.32825];
measured_ratios = [1; 1.3122; 3.9757; 0.76481; 0.32825];
T = table(omegas, Amp_ft, Amp_xt, ratios);
disp(T);
```

omegas	Amp_ft	Amp_xt	ratios
0	1	1	1
0.5	1	1.3122	1.3122
1	1	3.9757	3.9757
1.5	1	0.76481	0.76481
2	1	0.32825	0.32825

The theoretical values are the same as the AF

```
AF = zeros(1, 5); % Allocate the AF vector
for i = 0:4
    AF(i+1) = 1 / sqrt((1-(i*0.5/omega_n)^2)^2+(2*zeta*i*0.5/omega_n)^2);
```

```
end
```

Update the table to

```
AF = AF';
T = table(omegas, Amp_ft, Amp_xt, measured_ratios, AF);
disp(T);
```

omegas	Amp_ft	Amp_xt	measured_ratios	AF
0	1	1	1	1
0.5	1	1.3122	1.3122	1.3152
1	1	3.9757	3.9757	4
1.5	1	0.76481	0.76481	0.76626
2	1	0.32825	0.32825	0.3288

AF agree with the measured values.

<1f>

```
errs = zeros(1, 5); % Allocate the error vector
for i = 1:5
    errs(i) = abs(measured_ratios(i)-AF(i))/AF(i)*100;
end
```

Update the tables to include the percent errors (%)

```
errors = errs';
T = table(omegas, Amp_ft, Amp_xt, measured_ratios, AF, errors);
disp(T);
```

omegas	Amp_ft	Amp_xt	measured_ratios	AF	errors
0	1	1	1	1	0
0.5	1	1.3122	1.3122	1.3152	0.22749
1	1	3.9757	3.9757	4	0.6075
1.5	1	0.76481	0.76481	0.76626	0.18936
2	1	0.32825	0.32825	0.3288	0.16666

<ANALYSIS>

The errors are consistent in that taking 10 periods are enough to find the steady state amplitude for the Forced Vibration Problem so that they agree with the theoretical values. That being said, the very small errors are cohesive with the other results obtained.

Creating a function for the differential equation (1a)

```
function dxdt = fcn(t,x,zeta,omega_n,f_over_m,omega)
dxdt = zeros(2,1);  % Defining a zero vector to store the dxdt terms
dxdt(1) = x(2); % Derivative of x1 = x2
dxdt(2) = f_over_m*cos(omega*t)-omega_n^2*x(1)-2*zeta*omega_n*x(2);
% Derivative of x2 = -2*zeta*omega*x2
```

```
end
```