Exam #1

AE 6511 Optimal Guidance and Control

October 7, 2021

Instructions

1. This is a **open-book, open-notes** exam.
2. You will need to upload the exam to Canvas by 9:30pm ET. The system will close down automatically at that time.
3. Any scratch notes that you may want to include with the solution have to be uploaded at the same time as the exam.
4. You may use both sides of the sheets. Make sure that you scan and upload all sheets.
5. **No collaboration of any kind** between students is allowed.
6. Include all intermediate steps for full credit. Box your answer and **state the solution clearly.**
7. **Points will be subtracted for sloppiness**
8. Total number of points is 100.

Good luck!

Student Agreement

I certify that I have read and understand the above ground rules for the exam. I also understand that any violations of these rules or those of the Georgia Tech Honor Code will be treated as a violation of the Honor Code.

Name and Signature ______________________________

1. (**25pts**) Use the method of Lagrange multipliers to find the greatest and least distances from the point $(2, 1, -2)$ to the sphere with equation $x^2 + y^2 + z^2 = 1$.

2. (**25pts**) Consider the following problem

$$\min \mathcal{J} = \tfrac{1}{2} \int_0^T \left((\dot{x} - x)^2 - \alpha x^2 \right) \mathrm{d}t$$

subject to $x(0) = x(T) = 0$.

(a) Find the extremals for this problem for $\alpha > 1$.

(b) Is the Legendre condition satisfied?

(c) Investigate the existence (and location) of conjugate points for $\alpha = 2$ and $T = \pi$.

3. (**25pts**) Analyze the following problem

$$\min \int_0^1 (\dot{y}(t) - y(t))^2 \, \mathrm{d}t$$

subject to $y(0) = 0$ and $y(1) = (e - e^{-1})$.

4. (**25pts**) Suppose we want to solve the following equality maximization problem

$$\max f(x,y) \qquad \text{subject to} \quad g(x,y)=0$$

where $f : \mathbb{R}^{n+m} \to \mathbb{R}$ and $g : \mathbb{R}^{n+m} \to \mathbb{R}^n$, and $x \in \mathbb{R}^n$ $y \in \mathbb{R}^m$. Show that the second order necessary condition is equivalent to the statement that the matrix

$$L_{yy} - L_{yx} g_x^{-1} g_y - g_y^\intercal g_x^{-\intercal} L_{xy} + g_y^\intercal g_x^{-\intercal} L_{xx} g_x^{-1} g_y$$

evaluated at the candidate maximizer (x^*, y^*) is negative semi-definite, where $L(x,y,\lambda) = f(x,y) + \lambda^\intercal g(x,y)$.