P1: F BP

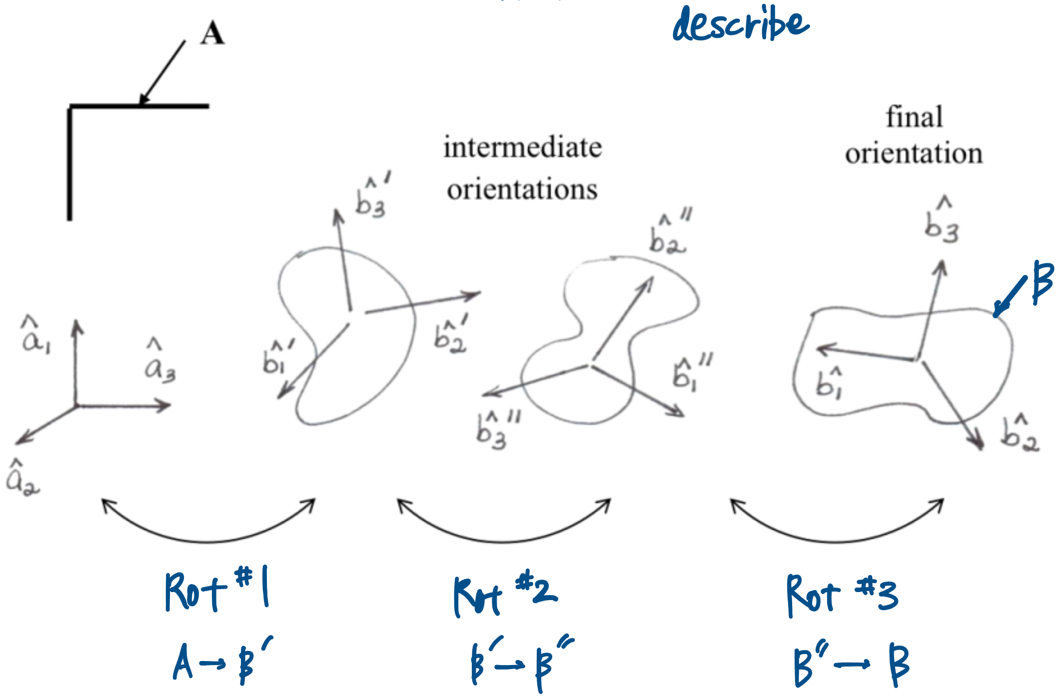
2020年1月31日 金曜日　　午後0:36

Orientation Angles

More familiar way to visualize and describe changes in orientation through successive rotations

Describe spacecraft final orientation relative to A; choose a sequence of three rotations

3 RDOF → 3 successive rot.s to describe



Infinite variety of ways that it can be decomposed into 3 rotations

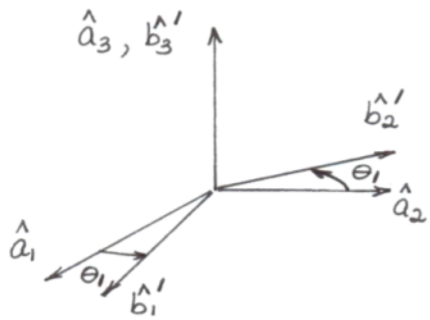
Choose one way and review the derivation

F2

body fixed $\hat{b}_3'$ $\hat{b}_2'$

Example: Body – three 3-2-1

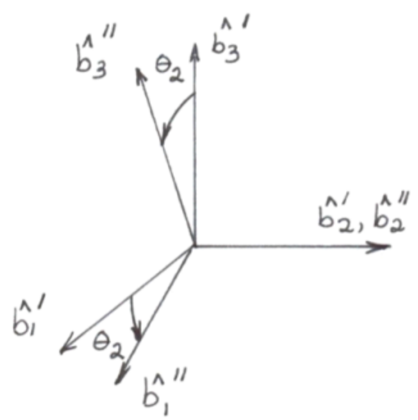
Rot #1 about $\hat{a}_3 = \hat{b}_3'$ $\theta = \theta_1$ ${}^{A}\hat{\lambda}^{B'} = \hat{a}_3 = \hat{b}_3'$



${}^{A}C^{B'}$	$\hat{b}_1'$	$\hat{b}_2'$	$\hat{b}_3'$
$\hat{a}_1$	$\cos\theta_1$	$-\sin\theta_1$	0
$\hat{a}_2$	$\sin\theta_1$	$\cos\theta_1$	0
$\hat{a}_3$	0	0	1

Visually OR relationships on pg. C3

Rot #2 about $\hat{b}_2' = \hat{b}_2''$ ${}^{B'}\hat{\lambda}^{B''} = \hat{b}_2' = \hat{b}_2''$ $\theta = \theta_2$



${}^{B'}C^{B''}$	$\hat{b}_1''$	$\hat{b}_2''$	$\hat{b}_3''$
$\hat{b}_1'$	$\cos\theta_2$	0	$\sin\theta_2$
$\hat{b}_2'$	0	1	0
$\hat{b}_3'$	$-\sin\theta_2$	0	$\cos\theta_2$

$${}^{A}C^{B''} = {}^{A}C^{B'}\,{}^{B'}C^{B''}$$

F3

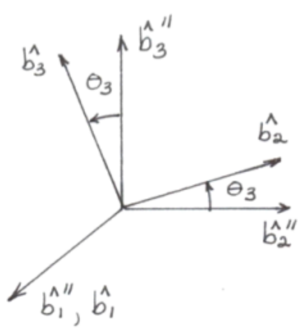
$${}^{A}C^{B''} = {}^{A}C^{B'}\;{}^{B'}C^{B''} = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_2 & 0 & \sin\theta_2 \\ 0 & 1 & 0 \\ -\sin\theta_2 & 0 & \cos\theta_2 \end{bmatrix}$$

$$
{}^{A}C^{B''} = {}^{A}C^{B'}\,{}^{B'}C^{B''} = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_2 & 0 & \sin\theta_2 \\ 0 & 1 & 0 \\ -\sin\theta_2 & 0 & \cos\theta_2 \end{bmatrix}
$$

$$
{}^{A}C^{B''} = \begin{bmatrix} \cos\theta_1\cos\theta_2 & -\sin\theta_1 & \cos\theta_1\sin\theta_2 \\ \sin\theta_1\cos\theta_2 & \cos\theta_1 & \sin\theta_1\sin\theta_2 \\ -\sin\theta_2 & 0 & \cos\theta_2 \end{bmatrix}
$$

$$
{}^{A}C^{B''} = \begin{bmatrix} C_1C_2 & -S_1 & C_1S_2 \\ S_1C_2 & C_1 & S_1S_2 \\ -S_2 & 0 & C_2 \end{bmatrix}
$$

Rot #3 about $\hat{b}_1'' = \hat{b}_1$ $\qquad$ ${}^{B''}\hat{\lambda}^{B} = \hat{b}_1'' = \hat{b}_1$



${}^{B''}C^{B}$	$\hat{b}_1$	$\hat{b}_2$	$\hat{b}_3$
$\hat{b}_1''$	1	0	0
$\hat{b}_2''$	0	$\cos\theta_3$	$-\sin\theta_3$
$\hat{b}_3''$	0	$\sin\theta_3$	$\cos\theta_3$

$$
{}^{A}C^{B} = {}^{A}C^{B''}\,{}^{B''}C^{B}
$$

$$ {}^{A}C^{B} = {}^{A}C^{B''}\ {}^{B''}C^{B} = \begin{bmatrix} \cos\theta_1\cos\theta_2 & -\sin\theta_1 & \cos\theta_1\sin\theta_2 \\ \sin\theta_1\cos\theta_2 & \cos\theta_1 & \sin\theta_1\sin\theta_2 \\ -\sin\theta_2 & 0 & \cos\theta_2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_3 & -\sin\theta_3 \\ 0 & \sin\theta_3 & \cos\theta_3 \end{bmatrix} $$

$$ {}^{A}C^{B} = \begin{bmatrix} c_1 c_2 & -s_1 c_3 + c_1 s_2 s_3 & s_1 s_3 + c_1 s_2 c_3 \\ s_1 c_2 & c_1 c_3 + s_1 s_2 s_3 & -c_1 s_3 + s_1 s_2 c_3 \\ -s_2 & c_2 s_3 & c_2 c_3 \end{bmatrix} $$

Body-two set
3-2-3

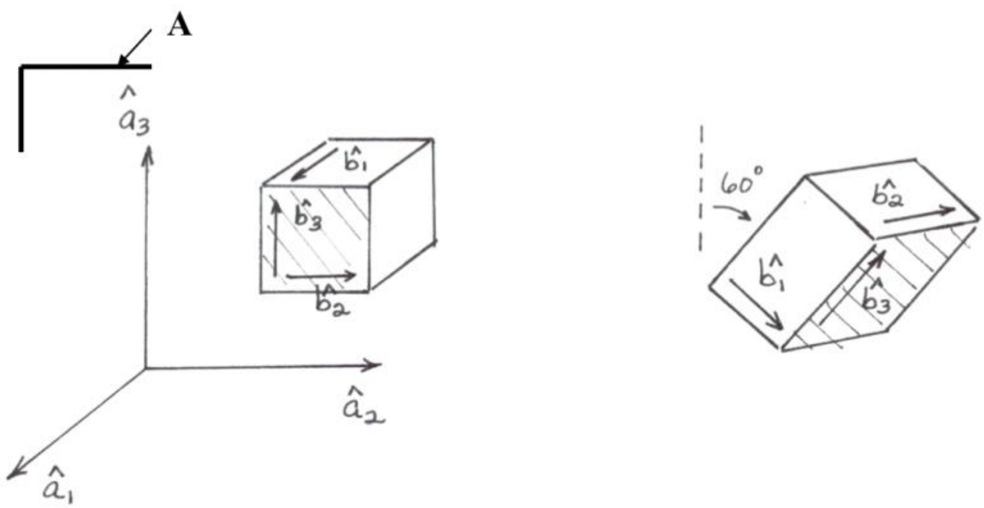
Result: final orientation relative to A $\left({}^{A}C^{B}\right)$
expressed in terms of body-three 3-2-1 angles

Notation —

$$\begin{array}{ccc} \theta_1\hat{a}_3 & & \theta_1\hat{b}'_3 \\ \theta_2\hat{b}'_2 & \text{OR} & \theta_2\hat{b}''_2 \\ \theta_3\hat{b}''_1 & & \theta_3\hat{b}_1 \end{array}$$

For **ANY** change in orientation: initial → final, can determine the set of three angles that are necessary to produce it

Previous Example



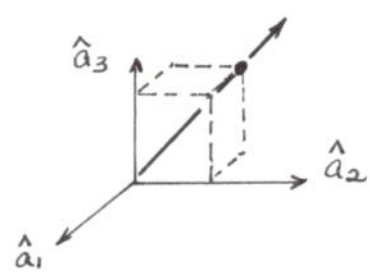
By inspection

${}^AC^B$	$\hat{b}_1$	$\hat{b}_2$	$\hat{b}_3$
$\hat{a}_1$	0	-1	0
$\hat{a}_2$	$\cos 60^\circ$	0	$\sin 60^\circ$
$\hat{a}_3$	$-\sin 60^\circ$	0	$\cos 60^\circ$

Equivalent single rotation to accomplish this change in orientation?

$$\hat{\lambda} = -\frac{1}{\sqrt{5}}\hat{a}_1 + \frac{1}{\sqrt{5}}\hat{a}_2 + \sqrt{\frac{3}{5}}\hat{a}_3$$

$$\theta = 104.5^\circ$$



What set of body-three 3-2-1 angles would produce the same result?

${}^AC^B$	$\hat{b}_1$	$\hat{b}_2$	$\hat{b}_3$
$\hat{a}_1$	0	–1	0
$\hat{a}_2$	$\cos 60^\circ$	0	$\sin 60^\circ$
$\hat{a}_3$	$-\sin 60^\circ$	0	$\cos 60^\circ$

$$= \begin{matrix} 0 & -1 & 0 \\ 0.5 & 0 & 0.866 \\ -0.866 & 0 & 0.5 \end{matrix}$$

$${}^AC^B = \begin{bmatrix} c_1 c_2 & -s_1 c_3 + c_1 s_2 s_3 & s_1 s_3 + c_1 s_2 c_3 \\ s_1 c_2 & c_1 c_3 + s_1 s_2 s_3 & -c_1 s_3 + s_1 s_2 c_3 \\ -s_2 & c_2 s_3 & c_2 c_3 \end{bmatrix}$$

Body three 3-2-1

$$\left.\begin{matrix} \text{Rot \#1} & \theta_1 & \hat{a}_3 \\ \text{Rot \#2} & \theta_2 & \hat{b}_2' \\ \text{Rot \#3} & \theta_3 & \hat{b}_1 \end{matrix}\right\}$$

we think these should be

$-s_2 = -s_{60^\circ} \Longrightarrow$ $-S_2 = -0.866$

$\theta_2 = 60^\circ, 120^\circ$

$\underline{\text{if } \theta_2 = 60^\circ}$

$c_1c_2 = 0 \quad \rightarrow$ $\theta_1 = 90^\circ, 270^\circ$

$s_1c_2 = c_{60^\circ} \quad \rightarrow$ $\theta_1 = 90^\circ$ $\Big\}$ $\theta_1 = 90^\circ$

$c_2s_3 = 0 \quad \rightarrow$ $\theta_3 = 0^\circ, 180^\circ$

$c_2c_3 = c_{60^\circ} \quad \rightarrow$ $\theta_3 = 0^\circ$ $\Big\}$ $\theta_3 = 0^\circ$

$\underline{\text{if } \theta_2 = 120^\circ}$

$c_1c_2 = 0 \quad \rightarrow \quad \theta_1 = 90^\circ, 270^\circ$

$s_1c_2 = c_{60^\circ} \quad \rightarrow \quad \theta_1 = 270^\circ$ $\Big\}$ $\theta_1 = 270^\circ$

$c_2s_3 = 0 \quad \rightarrow \quad \theta_3 = 0^\circ, 180^\circ$

$c_2c_3 = c_{60^\circ} \quad \rightarrow \quad \theta_3 = 180^\circ$ $\Big\}$ $\theta_3 = 180^\circ$

Possibilities

$\theta_1 = 90^\circ$	$\hat{a}_3$	$\theta_1 = 270^\circ$
$\theta_2 = 60^\circ$	$\hat{b}_2'$	$\theta_2 = 120^\circ$
$\theta_3 = 0^\circ$	$\hat{b}_1$	$\theta_3 = 180^\circ$

Always two possibilities — choose a convention to restrict the class to the same case (<u>not</u> necessary but convenient)

For first angle selected (θ_2)

If sin fcn → $-90^\circ < \theta_2 < 90^\circ$

If cos fcn → $0^\circ < \theta_2 < 180^\circ$

occur around a-axis (fixed)

Of course, a different set of angles can be used if the appropriate form of the direction cosine matrix is determined

Example: space-three 2-1-3 sequence

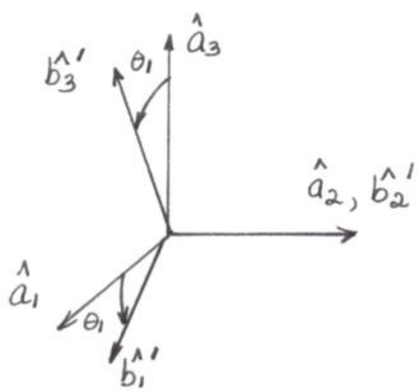
$A \to B'$ Rot #1 θ_1 $\hat{a}_2$

$B' \to B''$ Rot #2 θ_2 $\hat{a}_1$ } three successive rotations

$B'' \to B$ Rot #3 θ_3 $\hat{a}_3$

Rot #1 $A \to B'$

$${}^{A}\hat{\lambda}^{B'} = \hat{a}_2 = \hat{b}_2'$$



${}^{A}C^{B'}$	$\hat{b}_1'$	$\hat{b}_2'$	$\hat{b}_3'$
$\hat{a}_1$	c_1	0	s_1
$\hat{a}_2$	0	1	0
$\hat{a}_3$	$-s_1$	0	c_1

Rot #2 $B' \to B''$ but ${}^{B'}\hat{\lambda}^{B''} = \hat{a}_1 = c_1 \hat{b}_1' + s_1 \hat{b}_3'$

Rotate about $\hat{a}_1$ -- imagine rotating the $\hat{b}'$ above rotating about $\hat{a}_1$
It is tough to sketch;
Visual inspection is not useful to write the C matrix

${}^{B'}\hat{\lambda}^{B''} = \hat{a}_1$ Cannot use equations on page C3 directly; assumption in the derivation is violated!!

Use ${}^{A}C^{B'}$ to transform ${}^{B'}\hat{\lambda}^{B''} = \hat{a}_1$ into the appropriate unit vectors for use with the $\hat{\lambda}, \theta$ equations on page C3

$${}^{B'}\hat{\lambda}^{B''} = \hat{a}_1$$
$${}^{B'}\hat{\lambda}^{B''} = c_1 \hat{b}_1' + s_1 \hat{b}_3'$$

from pg C3

$$\left.\begin{aligned} \lambda_1 &= c_1 \\ \lambda_2 &= 0 \\ \lambda_3 &= s_1 \\ &\text{rotation angle } \theta_2 \end{aligned}\right\} \quad \begin{gathered}\hat{\lambda}, \theta \leftrightarrow C \\ \text{relationships}\end{gathered}$$

$${}^{B'}C^{B''} = \begin{matrix} c_2 + c_1^2(1-c_2) & -s_1 s_2 & s_1 c_1(1-c_2) \\ s_1 s_2 & c_2 & -c_1 s_2 \\ s_1 c_1(1-c_2) & c_1 s_2 & c_2 + s_1^2(1-c_2) \end{matrix}$$

Rot #3 $\quad B'' \rightarrow B$ but ${}^{B''}\hat{\lambda}^{B} = \hat{a}_3 \qquad \theta = \theta_3$

Rotate about $\hat{a}_3$ -- I cannot sketch this convincingly at all!!
Direction cosine matrix not available through visual inspection

Must use the derived relationships for $\hat{\lambda}, \theta$

$\hat{\lambda}, \theta$ equations on page C3 assume ${}^{B''}\hat{\lambda}^{B}$ is expressed in terms of $\hat{b}''$ or $\hat{b}$

$$
\begin{aligned}
{}^{B''}\hat{\lambda}^{B} &= {}^{B''}\lambda_1 \hat{b}_1'' + {}^{B''}\lambda_2 \hat{b}_2'' + {}^{B''}\lambda_3 \hat{b}_3'' \\
&= {}^{A}\lambda_1 \hat{a}_1 + {}^{A}\lambda_2 \hat{a}_2 + {}^{A}\lambda_3 \hat{a}_3
\end{aligned}
$$

$$\left[{}^{B''}\lambda_1 \quad {}^{B''}\lambda_2 \quad {}^{B''}\lambda_3\right] = \left[{}^{A}\lambda_1 \quad {}^{A}\lambda_2 \quad {}^{A}\lambda_3\right] {}^{A}C^{B''}$$

$$\left[{}^{B''}\lambda_1 \quad {}^{B''}\lambda_2 \quad {}^{B''}\lambda_3\right] = \left[{}^{A}\lambda_1 \quad {}^{A}\lambda_2 \quad {}^{A}\lambda_3\right] {}^{A}C^{B'} \, {}^{B'}C^{B''}$$

$$\left[{}^{B''}\lambda_1 \quad {}^{B''}\lambda_2 \quad {}^{B''}\lambda_3\right] =$$

$$
{}^{A}C^{B'} = \begin{bmatrix} c_1 & 0 & s_1 \\ 0 & 1 & 0 \\ -s_1 & 0 & c_1 \end{bmatrix}
$$

$\hat{b}_1'$

$$
{}^{B'}C^{B''} = \begin{bmatrix} c_2 + c_1^2(1 - c_2) & -s_1 s_2 & s_1 c_1 (1 - c_2) \\ s_1 s_2 & c_2 & -c_1 s_2 \\ s_1 c_1 (1 - c_2) & c_1 s_2 & c_2 + s_1^2 (1 - c_2) \end{bmatrix}
$$

$\hat{b}_3'$

$$\left[{}^{B''}\lambda_1 \quad {}^{B''}\lambda_2 \quad {}^{B''}\lambda_3\right] =$$

$\theta = \theta_3$

$${}^{B''}\hat{\lambda}^{B} = \hat{a}_3 = -s_1 c_2 \hat{b}_1'' + s_2 \hat{b}_2'' + c_1 c_2 \hat{b}_3'' \quad \Longrightarrow \quad \lambda_1 = -s_1 c_2$$

$\lambda_2 = s_2$

$\lambda_3 = c_1 c_2$

$$
{}^{B''}C^{B} = \begin{bmatrix} c_3 + s_1^2 c_2^2 (1-c_3) & -c_1 c_2 s_3 - s_1 c_2 s_2 (1-c_3) & s_2 s_3 - s_1 c_1 c_2^2 (1-c_3) \\ c_1 c_2 s_3 - s_1 c_2 s_2 (1-c_3) & c_3 + s_2^2 (1-c_3) & s_1 c_2 s_3 + s_2 c_1 c_2 (1-c_3) \\ -s_2 s_3 - s_1 c_1 c_2^2 (1-c_3) & -s_1 c_2 s_3 + s_1 c_2 c_2 (1-c_3) & c_3 + c_1^2 c_2^2 (1-c_3) \end{bmatrix}
$$

So $\quad {}^{A}C^{B} = {}^{A}C^{B'}\ {}^{B'}C^{B''}\ {}^{B''}C^{B}$

$$
{}^{A}C^{B} = \begin{bmatrix} -s_1 s_2 s_3 + c_3 c_1 & -c_2 s_3 & c_1 s_2 s_3 + c_3 s_1 \\ s_1 s_2 c_3 + s_3 c_1 & c_2 c_3 & -c_1 s_2 c_3 + s_3 s_1 \\ -s_1 c_2 & s_2 & c_1 c_2 \end{bmatrix}
$$

space three 2-1-3

What set of space-three 2-1-3 angles could be used in the example?

$$
{}^{A}C^{B} = \begin{bmatrix} 0 & -1 & 0 \\ c_{60^\circ} & 0 & s_{60^\circ} \\ -s_{60^\circ} & 0 & c_{60^\circ} \end{bmatrix}
$$

$s_2 = 0 \iff \theta_2 = 0,\ 180^\circ$

$$-s_2 = 0 \quad \Longrightarrow \quad \theta_2 = 0^\circ, 180^\circ$$

$$\left(\text{Choose } -\frac{\pi}{2} \le \theta_2 \le \frac{\pi}{2}\right)$$

$\theta_2 = 0^\circ$

$$-s_1 c_2 = -s_{60^\circ} \rightarrow \theta_1 = 60^\circ, 120^\circ$$
$$c_1 c_2 = c_{60^\circ} \rightarrow \theta_1 = \pm 60^\circ$$

$\theta_1 = 60^\circ$

$$-c_2 s_3 = -1 \rightarrow \theta_3 = 90^\circ, -270^\circ$$
$$c_2 c_3 = 0 \rightarrow \theta_3 = \pm 90^\circ$$

$\theta_3 = 90^\circ$

Possible Space 2-1-3 sequence:

$\theta_1 = 60^\circ$ about $\hat{a}_2$ → -120

$\theta_2 = 0^\circ$ about $\hat{a}_1$ → 180°

$\theta_3 = 90^\circ$ about $\hat{a}_3$ → -90°