P2: M BP

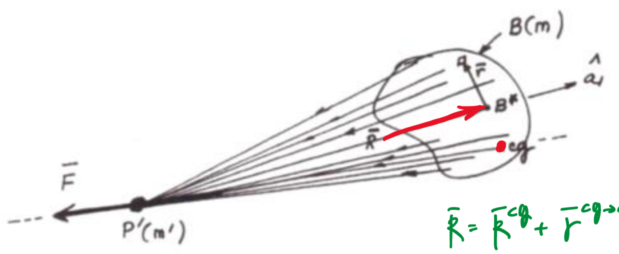
2020年3月2日 月曜日　　午後0:31

Gravitational Moment

$\bar{\mu}^{B^*}$ = moment arm

$= \bar{r}^{B^* \to pcm} \times \bar{F}$

Moment exerted on a body by a particle



$\bar{R} = \bar{R}^{cg} + \bar{r}^{cg \to cm}$

Resultant force $\bar{F}$ at P' represents force model on the system

We want to create an equivalent system at B^* $\left(B^{cm}\right)$ resultant $\bar{F}$

$\bar{M} \equiv$ moment about B^* of all forces exerted on B by P'

$$\bar{\mu}^{B^*} = -\bar{R} \times \bar{F}$$

$$= -\left(\bar{R}^{cg} + \bar{r}^{cg \to cm}\right) \times \bar{F}$$

$$= \cancel{-\bar{R}^{cg} \times \bar{F}}_{\to \emptyset} - \bar{r}^{cg \to cm} \times \bar{F}$$

$$\boxed{\bar{\mu}^{B^*} = -\bar{R} \times \bar{F} = \bar{r}^{cg \to cm} \times \bar{F}}$$

→ if cg ≠ cm Mom always exists

1. Inertia Matrices
   - ALWAYS symmetric
   - moments NEVER negative
   - changing unit vectors
2. DCM → orthogonalities & constraints

general $\bar{F}$ is nontrivial

$\bar{\mu}^{B^*} = -\bar{R}^{P' \to cm} \times \bar{F}$

$$\bar{M} = -R\,\hat{a}_1 \times \left[-\frac{Gm'm}{R^2} \left(\hat{a}_1 + \sum_{i=2}^{\infty} \bar{f}^{(i)} \right) \right]$$

$$\bar{\mu}^{B^*} = \frac{Gm'm}{R}\,\hat{a}_1 \times \sum_{i=2}^{\infty} \bar{f}^{(i)}$$

$$= \frac{Gm'm}{R} \left[\hat{a}_1 \times \bar{f}^{(2)} + \underbrace{\sum_{i=3}^{\infty} \hat{a}_1 \times \bar{f}^{(i)}}_{\bar{m}^{(i)}} \right]$$

because 'x' involved moment term with $\bar{f}^{(2)}$ may not always be largest

$$\bar{f}^{(2)} = \frac{1}{mR^2} \left\{ \frac{3}{2} \left[tr\left(\bar{\bar{I}}\right) - 5\,\hat{a}_1 \cdot \bar{\bar{I}} \cdot \hat{a}_1 \right] \hat{a}_1 + 3\bar{\bar{I}} \cdot \hat{a}_1 \right\}$$

$$\bar{M} = \frac{3Gm'}{R^3}\,\hat{a}_1 \times \bar{\bar{I}} \cdot \hat{a}_1 + \frac{Gm'm}{R} \sum_{i=3}^{\infty} \hat{a}_1 \times \bar{f}^{(i)}$$

$$\boxed{\bar{M}^{\beta^*} = \frac{3Gm'}{R^3}\hat{a}_1 \times \bar{\bar{I}}^{B/\beta^*} \times \hat{a}_1 + \frac{Gmm'}{R}\sum_{i=3}^{\infty}\bar{m}^{(i)}}$$

depends on orientation wrt $\hat{a}$'s ↑ orbit frame

collection of terms of degree i in $\frac{r}{R}$

Useful Approximation

$$\boxed{\tilde{M}^{\beta^*} = \frac{3Gm'}{R^3}\hat{a}_1 \times \bar{\bar{I}}^{B/\beta^*} \cdot \hat{a}_1} \qquad \text{due only to } \boxed{\bar{f}^{(2)}}$$

Convenient if it is available in component form

1. Vector basis $\hat{a}_i$ ← fixed in orbit

But orientation continually changing update I_{21}/I_{31} at every step

Let $I_{jk} = \hat{a}_j \cdot \bar{\bar{I}} \cdot \hat{a}_k$

$$\tilde{M} = \frac{3Gm'}{R^3}(\underbrace{I_{21}}\hat{a}_3 - \underbrace{I_{31}}_{\text{changing}}\hat{a}_2)$$

2. Vector basis $\hat{b}_i$ ← body fixed (central, principal) → principle for cm

Let $I_j = \hat{b}_j \cdot \bar{\bar{I}} \cdot \hat{b}_j$ $\qquad \bar{\bar{I}}^{B/B^*} = I_{11}\hat{b}_1\hat{b}_1 + I_{22}\hat{b}_2\hat{b}_2 + I_{33}\hat{b}_3\hat{b}_3$

$$\tilde{M} = \frac{3Gm'}{R^3}\hat{a}_1 \times \left(\hat{b}_1 I_1 \hat{b}_1 + \hat{b}_2 I_2 \hat{b}_2 + \hat{b}_3 I_3 \hat{b}_3\right) \cdot \hat{a}_1$$

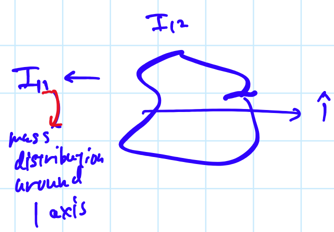
A = orbit
B = body

$$\tilde{M} = \frac{3Gm'}{R^3}\left[(I_3 - I_2)\,{}^{A}C^{B}_{12}C_{13}\,\hat{b}_1 + (I_1 - I_3)C_{13}C_{11}\,\hat{b}_2 + (I_2 - I_1)C_{11}C_{12}\,\hat{b}_3\right]$$

principal

{Torque : rotational behavior
inertia : resistance to rot.

$$\bar{\bar{I}} = \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{21} & I_{22} & I_{23} \\ I_{31} & I_{32} & I_{33} \end{bmatrix}$$



-1 absement
0 pos
1 vel
2 accel
3 jerk
4 snap/jounce
5 crackle
6 pop
7 lock
8 drop

Careful when using the approximation

$\tilde{M}$ may vanish when $\bar{M} \neq \bar{0}$:

1. $\tilde{M} = 0$ spherical central inertia ellipsoid

$I_{11} = I_{22} = I_{33}$ principal

due to $\bar{f}^{(2)}$

$I_{11} = I_{22} = I_{33}$

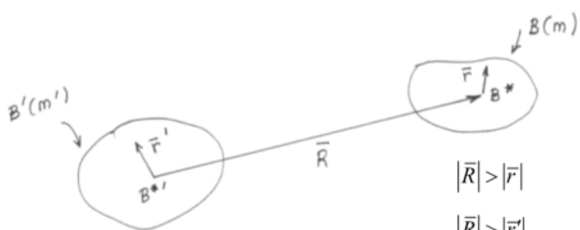
due to $\bar{f}^{(2)}$

2. $\hat{a}_i$ parallel to principal directions for $\hat{b}_i$

$\tilde{M} = 0$

M5

Moment exerted on a small body by a small body



$|\bar{R}| > |\bar{r}|$

$|\bar{R}| > |\bar{r}'|$

Assume ⊕ centrobaric

degree $\left(\frac{r}{R}\right)^i$

$$\boxed{\bar{M}^{B^*} = \frac{3Gm'}{R^3}\hat{a}_1 \times \bar{\bar{I}}^{B/B^*} \cdot \hat{a}_1 + \frac{Gmm'}{R}\sum_{i=3}^{\infty}\bar{m}^{(i)} + \frac{Gmm'}{R}\sum_{i=2}^{\infty}\sum_{j=3}^{\infty}\bar{m}^{(ij)}}$$

degree $\left(\frac{r}{R}\right)^i\left(\frac{r'}{R}\right)^j$

Moment about B^* of all forces exerted on B by B^*

M6

Moment about B^* of all forces exerted by B' does NOT equal moment about $\left(B^*\right)'$ of all forces exerted by B by B'

(equivalent system at different point)