Paper Review: Fast Linear Iterations for Distributed Averaging

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***Abstract*—We will analyze and dissect a paper by Lin Xiao and Stephen Boyd [1]. This paper investigates the use of a fast converging linear iteration for distributed averaging consensus of multi-agent systems. This algorithm can be solved globally because the with the assumption of the iteration being symmetric, we can determine that the iteration becomes semidefinite. With this approach a network with a large number of nodes and edges will be solved for consensus.**

***Index Terms*—Distributed consensus; Linear system; Spectral radius; Graph Laplacian; Semidefinite programming; Fastest distributed linear averaging (FDLA); Semidefinite program (SDP)**

I. Introduction and Motivation

Considering a network constructed of a set of nodes and its neighbors, the initial values of this system is possible to reach an average through consensus or agreement using a distributed algorithm. The thesis or problem being addressed in this paper is to probe the FDLA problem with the expectations of minimizing the time for it to reach an asymptotic convergence to some consensus. To accomplish this, the research focuses on experimenting the weight matrix W for the distributed linear iteration as follows.

$$x_i(t+1) = W_{ii}x_i(t) + \sum_{j\in\mathcal{N}_i} W_{ij}x_j(t), \qquad i = 1, ..., n$$

which can be rewritten in the following vector form

$$x(t+1) = Wx(t).$$

With the applications such as formation flights of unmanned air vehicles, clustered satellites, or other vehicles, the motivation of this research owes itself to the optimization of such systems to improve their performances in speed and accuracy[1].

II. Problem Formulation

First the conditions for convergence is discussed. From the given theorem, the convergence relies on the spectral radius and the spectral norm of

$$W - \mathbb{1}\mathbb{1}^T/n$$

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where the former is the radius of the maximum eigenvalue of the matrix and the latter is the maximum singular value which can be found by conducting the singular value decomposition [1]. Subsequently they discuss how the Laplacian matrix is defined for the network using the degree of each node and the edge connecting nodes. For the edges two methods are introduced to find its weight. One is the maximum degree method and another is to use the maximum degree of two incident nodes which is called the local-degree weight. The problem formulation becomes how to find the optimal symmetric weights for each edge and node which gives the minimum asymptotic convergence factor.

To accomplish this computationally, the paper proposes two methods: the interior-point method and subgradient method. The performance will be discussed in the main results section.

III. Main Results

For the interior point method a primal barrier method is introduced as a convex optimization to identify the weights by solving the SDP. This took roughly 20 to 80 steps with a cost of $(10/3)n^3 + (1/3)m^3$ flops per step cost.

The subgradient method is capable of solving FDLA problems for large-scale networks, however, it is slow in its computation compared the interior point method. It also does not have a termination criterion that allows the output to have a certain level of optimality in a confidence range. This algorithm is demonstrated on a network with 10000 nodes and 100000 edges. The results show a convergence factor of 0.473 which is significantly smaller than that of the local-degrees weight which is 0.730.

IV. Your Ideas of Further Improvements

For a nonlinear dynamical multiagent system, it would be difficult identify the communication network between each node and neighbor. Furthermore, with external factors the weight determination for each node and edge becomes difficult. Thus, it would be rather effective to adopt a system where followers work towards a consensus of a leader in the system. For this system, it could be possible to implement a distributed neural-network based adaptive control scheme to solve the FDLA problem.

REFERENCES

[1] Lin Xiao and Stephen Boyd. Fast linear iterations for distributed averaging. *Systems and Control Letters*, 53(1):65–78, 2004.