College of Engineering
School of Aeronautics and Astronautics

AAE 36401 Lab
Control Systems Lab

Lab 4 Report
The Control of Inverted Pendulum to Balance

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West Lafayette, Indiana

Introduction

Objective

The object of this lab is to control an inverted pendulum on a cart which is place on a track. Alike the previous lab, the pendulum will be controlled in its upright orientation; however, in this experiment the pendulum will maintain the orientation while having the cart move sideways based on a square wave input. The objective is the input gains into the feedback system to maintain a stable upright orientation.

Method

To accomplish the objective, we have to come up with the gains of the feedback control system. For this experiment, the gains will be computed by the LQR method. Then the gains will be input to the Simulink model that is synced to the experimental setup. The cart will move based on the square wave input and the data is obtain from the response.

Results

Part (i)

The gains used for each method are the following

The gains using LQR:

Table 1: gains for linear quadratic regulator

$\boldsymbol{K_1}$	$\boldsymbol{K_2}$	$\boldsymbol{K_3}$	$\boldsymbol{K_4}$	$\boldsymbol{K_5}$
-68.8145	156.0625	-51.6130	33.2344	44.7214

The diagonal matrix, Q used for the linear quadratic regulator is the following

$$Q = \begin{pmatrix} 8 & 0 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 20 \end{pmatrix}$$

The weight, R is

$$R = 0.01$$

The poles for LQR:

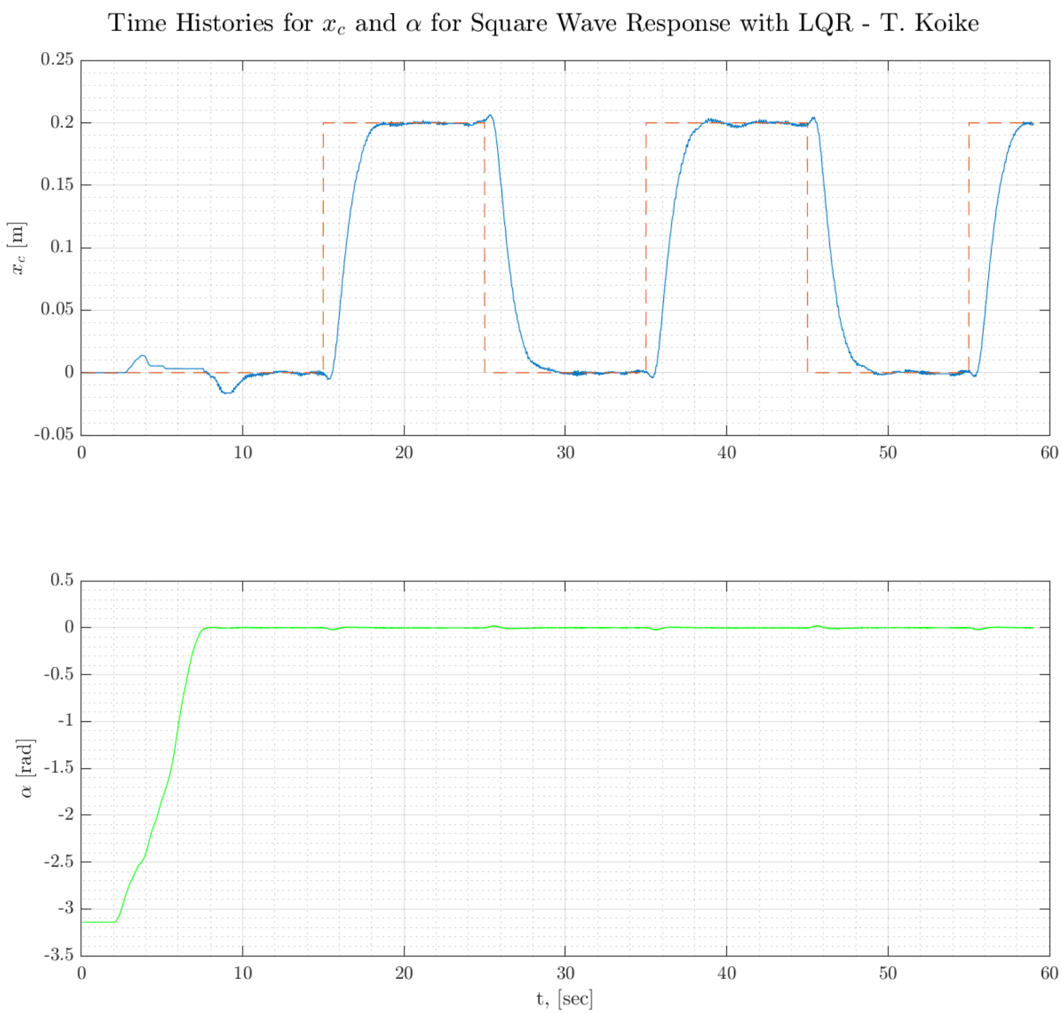
Table 2: poles for linear quadratic regulator

λ_1	λ_2	λ_3	λ_4	λ_5
-40.5785	-2.6398+0.8961i	-2.6398-0.8961i	-2.0395+0.8637i	-2.0395-0.8637i

Analysis & Discussions

Part (i)

The time history is graphed as the following.



Initially the angle of the pendulum is $-180°$, but its lifted up slowly be $0°$. From the results we can see that we have a system that satisfies the requirements of maintaining the upright orientation of the pendulum while the cart is moving sideways with a square wave input.

We can see that the position values of the cart have some degree of discrepancy. Namely, there is some steady state error at the end of each square wave but overall it is smooth and coherent with the input.

Conclusion & Recommendation

Main Points

From the results, we can observe that the experiment is successful with a system that maintains a pendulum at a upright position while the cart has a dynamic motion. The angle of the pendulum is maintaining 0 degrees with a significantly small error.

Theoretical/Experimental Limitations

The motion of the cart is one limitation that we can see from the plots.

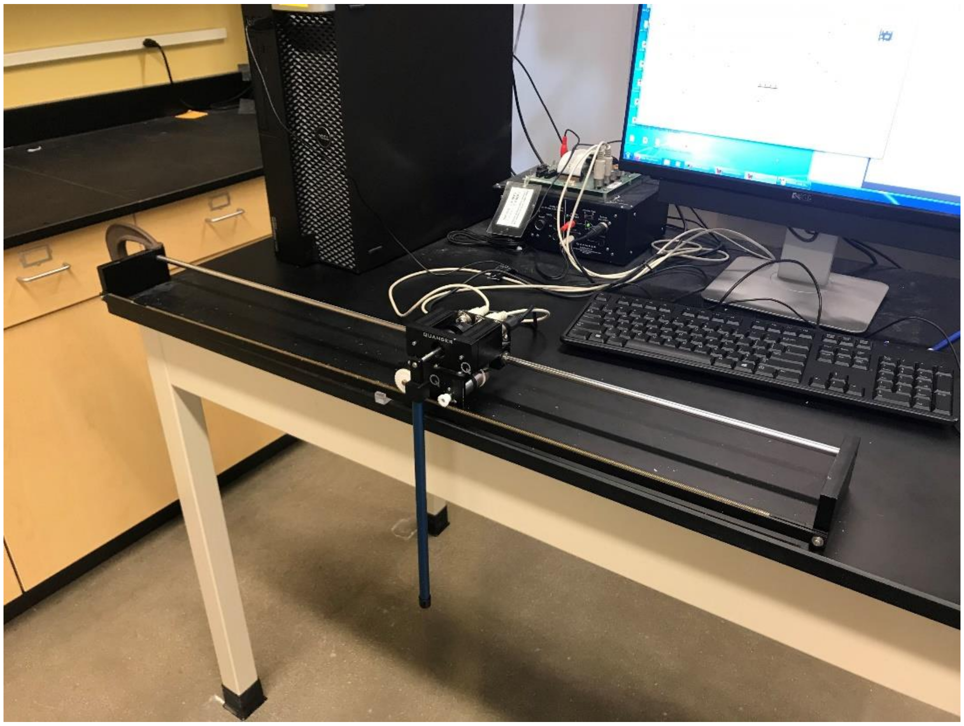
Another limitation common to all the other labs performed is the voltage limit that prevents us to have a gain larger than 200.

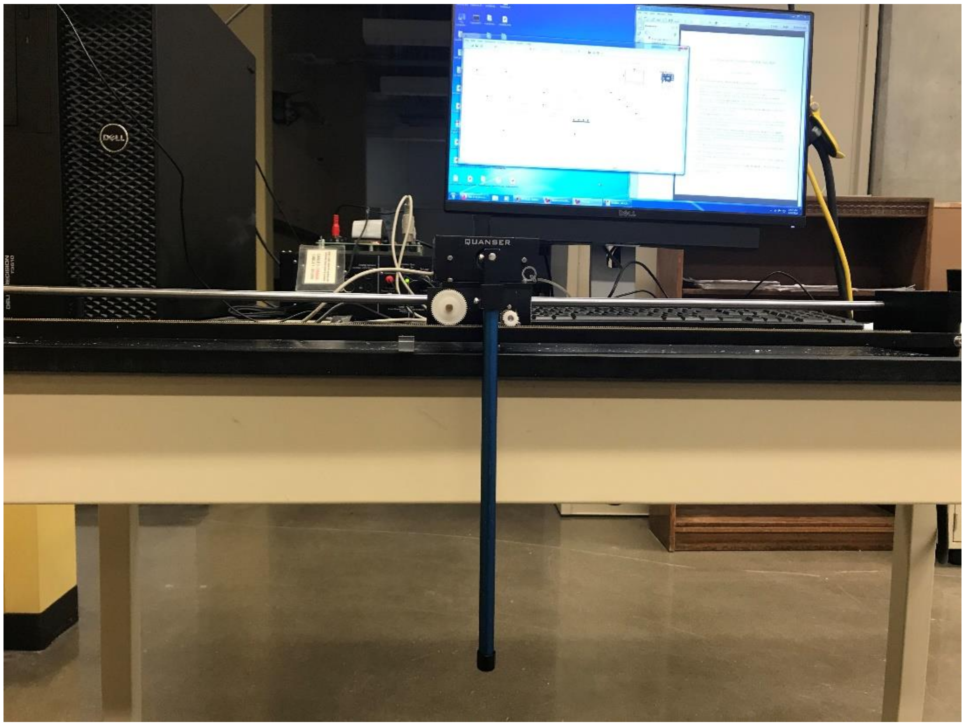
Lessons Learned & Suggestions for Improvement

The utmost lesson from this lab is the implementation of linear quadratic regulators to control complex tasks.

Appendix

Experiment Setup





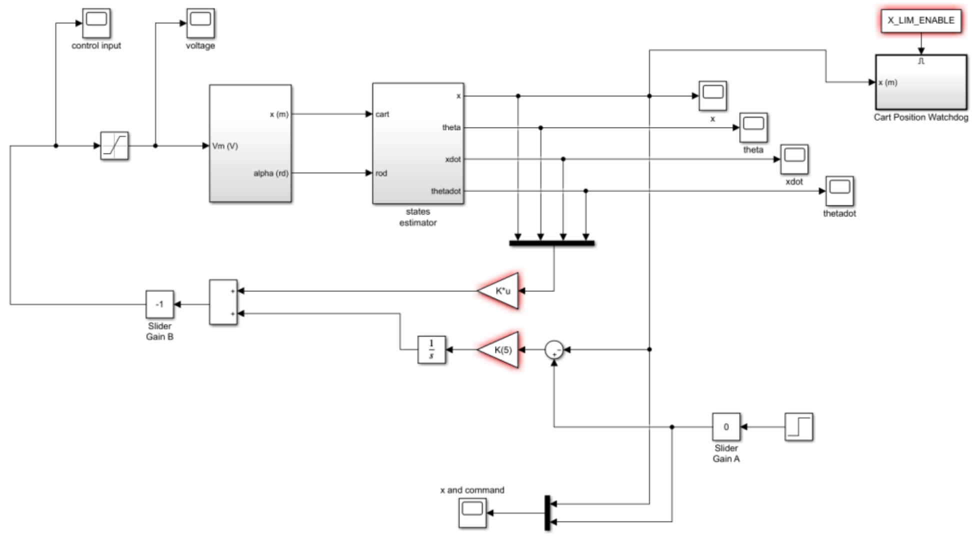
Notations for Variables

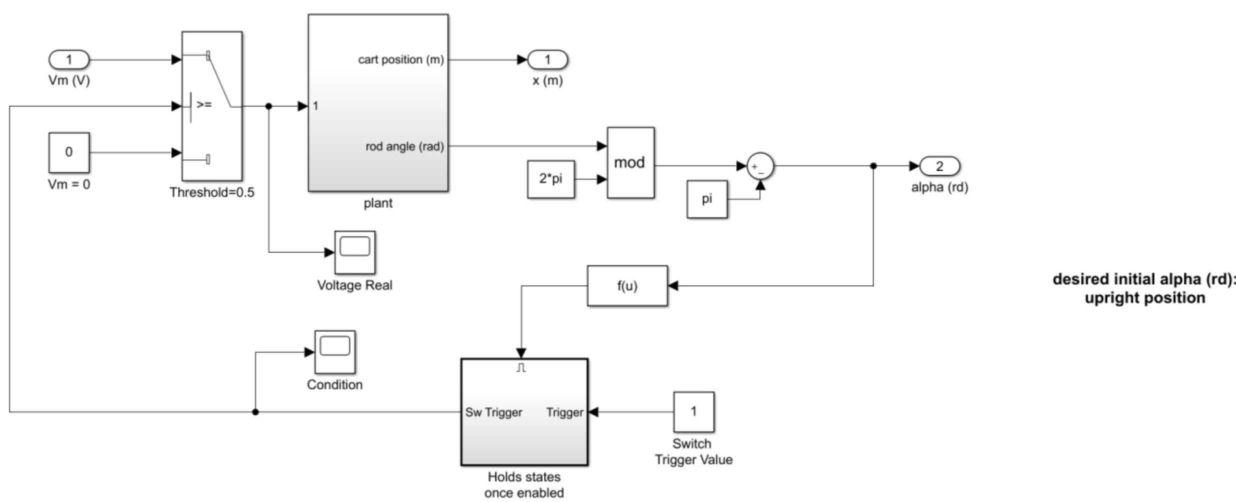
Symbol	Description	Value	Unit
R_m	motor armature resistance	2.6	Ω
L_m	motor armature inductance	0.18	mH
K_t	motor torque constant	0.00767	$N.m/A$
η_m	motor efficiency	100%	%
K_m	back-electromotive-force(EMF) constant	0.00767	$V.s/rad$
J_m	rotor moment of inertia	3.9×10^{-7}	$kg.m^2$
K_g	planetary gearbox ratio	3.71	
η_g	planetary gearbox efficiency	100%	%
M_{c2}	cart mass	0.57	kg
M_w	cart weight mass	0.37	kg
M_c	total cart weight mass including motor inertia	1.0731	kg
B_{eq}	viscous damping at motor pinion	5.4000	$N.s/m$
L_t	track length	0.990	m
T_c	cart travel	0.814	m
P_r	rack pitch	1.664×10^{-3}	$m/tooth$
r_{mp}	motor pinion radius	6.35×10^{-3}	m
N_{mp}	motor pinion number of teeth	24	
r_{pp}	position pinion radius	0.01482975	m
N_{pp}	position pinion number of teeth	56	
K_{EP}	cart encoder resolution	2.275×10^{-5}	$m/count$
M_p	long pendulum mass with T-fitting	0.230	kg
M_{pm}	medium pendulum mass with T-fitting	0.127	kg
L_p	long pendulum length from pivot to tip	0.6413	m
L_{pm}	medium pendulum length from pivot to tip	0.3365	m
l_p	long pendulum length: pivot to center of mass	0.3302	m
l_{pm}	medium pendulum length: pivot to center of mass	0.1778	m
J_p	long pendulum moment of inertia ⟳ center of mass	7.88×10^{-3}	$kg.m^2$
J_{pm}	medium pendulum moment of inertia ⟳ center of mass	1.20×10^{-3}	$kg.m^2$
B_p	viscous damping at pendulum axis	0.0024	$N.m.s/rad$
g	gravitational constant	9.81	m/s^2
v	voltage of servo motor	variable	V

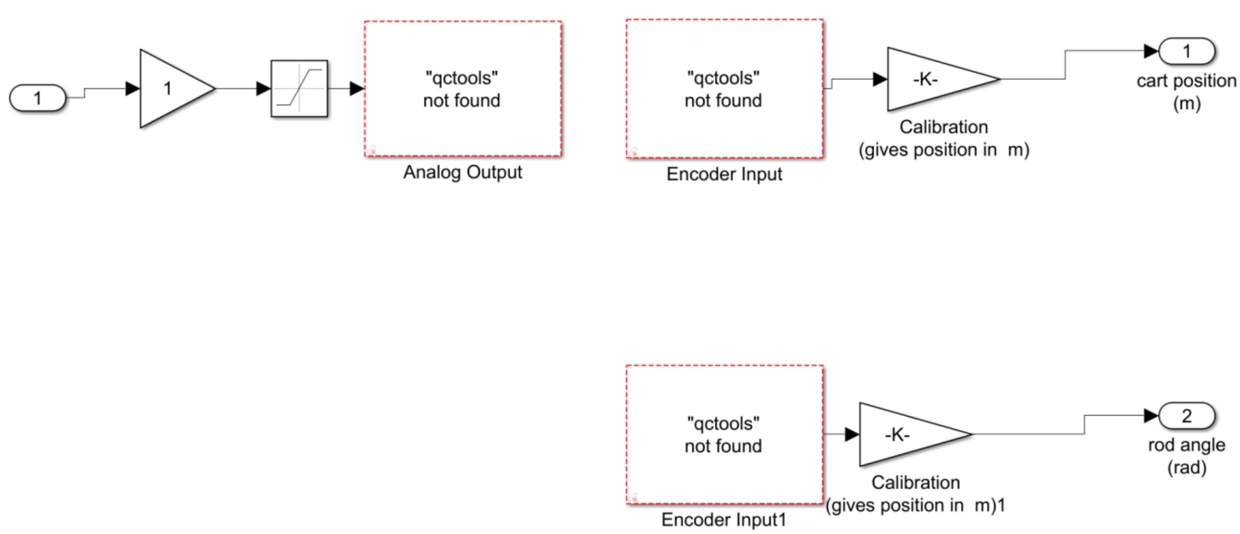
MATLAB Code

```
% AAE 364L LAB4 MATLAB CODE
% TOMOKI KOIKE
clear all; close all; clc;
set(groot, 'defaulttextinterpreter','latex');
set(groot, 'defaultAxesTickLabelInterpreter','latex');
set(groot, 'defaultLegendInterpreter','latex');

% Load result data

pos = load("koike_lab4\koike_lab4_xc.mat");
angle = load("koike_lab4\koike_lab4_theta.mat");

% PP

t = pos.x_and_coimmand_part3.time;
xc = pos.x_and_coimmand_part3.signals.values(:,1);
command = pos.x_and_coimmand_part3.signals.values(:,2);
theta = angle.theta_part3.signals.values;

% Plotting
fig = figure('Renderer',"painters", 'Position', [10 10 900 800]);
subplot(2,1,1)
plot(t,xc)
grid on; grid minor; box on; hold on;
plot(t, command, '--')
hold off
ylabel('$x_c$ [m]')
subplot(2,1,2)
plot(t,theta, '-g')
grid on; grid minor; box on;
ylabel('$\alpha$ [rad]')
xlabel('t, [sec]')
sgtitle('Time Histories for $x_c$ and $\alpha$ for Square Wave Response with LQR
- T. Koike')
saveas(fig, 'response.png')
```

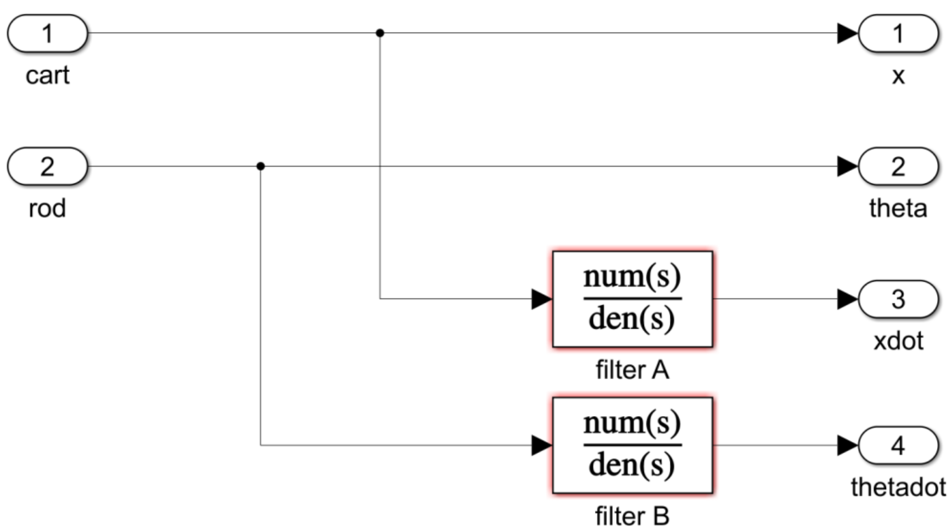
Simulink Models

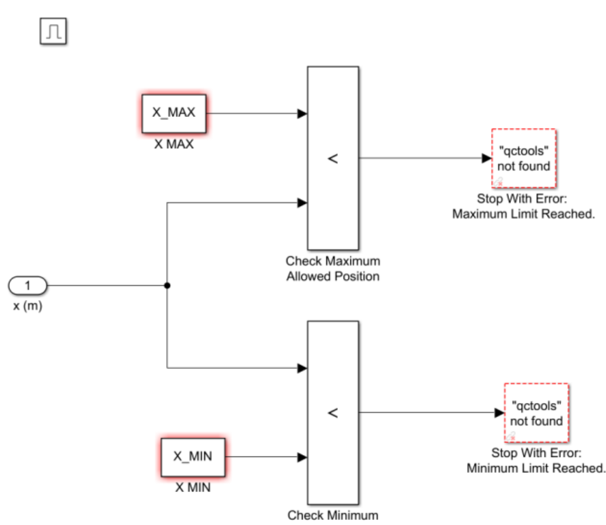







1
Trigger
1
Sw Trigger


1
cart
2
rod
num(s)
den(s)
filter A
num(s)
den(s)
filter B
1
x
2
theta
3
xdot
4
thetadot


X_MAX
X MAX
<
Check Maximum
Allowed Position
"qctools"
not found
Stop With Error:
Maximum Limit Reached.
1
x (m)
X_MIN
X MIN
<
Check Minimum
Allowed Position
"qctools"
not found
Stop With Error:
Minimum Limit Reached.