Name	Team Number
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AAE 251: Introduction to Aerospace Design

Assignment 6—The Rocket Equation, Rocket Thrust and Staging

Due Tuesday March 5, 10:00 am on Blackboard

No 24hr extensions

Instructions

Write or type your answers into the appropriate boxes. ***Make sure you submit a single PDF on Blackboard.***

Make sure you keep a record of submission receipts or the confirmation emails after each submission as a proof that your submission was accepted.

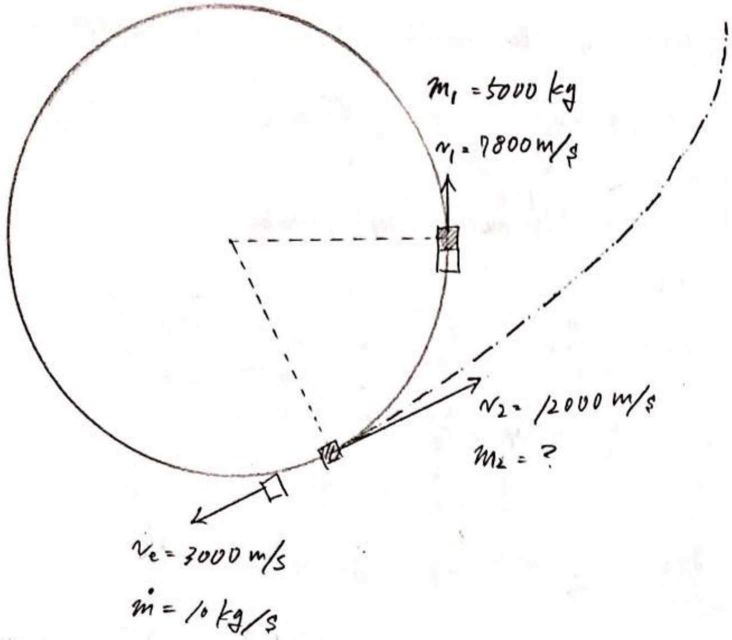
There is no 24hr extension on this homework. Any submission after March 5, 10:00 am will not be accepted.

	Score	Max
Question 1		**8**
Question 2		**9**
Question 3		**10**
Question 4		**10**
Question 5		**13**
Question 6		**25**
TOTAL		**75**

Question 1

A spacecraft weighing 5,000 kg is travelling in an orbit around Earth at a velocity of 7,800 m/s. Its engine places it on an escape trajectory by accelerating it to a velocity of 12,000 m/s. The engine expels mass at a rate of 10 kg/s at an effective exhaust velocity of 3,000 m/s. What is the duration of the burn?

Answer 1:



Total change in momentum $\equiv \Delta p = m_1 v_1 - m_2 v_2$

$$F_{thrust} = \frac{dm\, v_e}{dt} = \dot{m} v_e$$

and $(F_{thrust})\Delta t = \Delta p \iff \Delta t = \dfrac{\Delta p}{F_{thrust}} = \dfrac{m_1 v_1 - m_2 v_2}{F_{thrust}} \quad \cdots ①$

since we still do not know m_2

$$\because \quad dv = -v_e \frac{dm}{m} \qquad v_2 - v_1 = -v_e \ln \frac{m_2}{m_1}$$

$$\int_{v_1}^{v_2} dv = -v_e \int_{m_1}^{m_2} \frac{dm}{m} \qquad m_2 = m_1 \exp\left(-\frac{v_2 - v_1}{v_e}\right) \quad \cdots ②$$

$$\because ② \Rightarrow m_2 = (5000\ \text{kg}) \exp\left(-\frac{12000\ \text{m/s} - 7800\ \text{m/s}}{3000\ \text{m/s}}\right) \cong 944.378\ \text{kg}$$

$\because$ ① & ②

$$\Delta t = \frac{(5000\ \text{kg})(7800\ \text{m/s}) - (944.378\ \text{kg})(12000\ \text{m/s})}{(10\ \text{kg/s})(3000\ \text{m/s})} \cong \boxed{922.25\ \text{s}}$$

Question 2

A single-stage rocket is used to launch a satellite weighing 1000 kg into a circular orbit at an altitude of 200 km. The specific impulse of the rocket is 20,000 m/s. The structural mass of the rocket is 20% of the initial mass. Calculate the mass of propellant needed for this mission.

$GM = 3.986 \times 10^5\ km^3/s^2; \quad R_e = 6378\ km$

Answer 2:

this is mass based so v_e has unit m/s $v_e = 20000$ m/s

$GM = \mu = 3.986 \times 10^5\ km^3/s^2 = 3.986 \times 10^{14}\ m^3/s^2$

$R_e = 6378\ km = 6.378 \times 10^6\ m$

$R = 200\ km$, satellite mass $= m = 1000\ kg$

initial velocity $v_i = 0$

final velocity $v_f = \sqrt{\frac{\mu}{r}} = \sqrt{\frac{\mu}{R + R_e}} = \sqrt{\frac{3.986 \times 10^{14}\ m^3/s^2}{2.00 \times 10^5 + 6.378 \times 10^6}}$

$\cong 7784.34$ m/s

now using

$dv = -v_e \frac{dm}{m}$

$v_f - v_i = v_e \ln \frac{m_i}{m_f}$

$f_{inert} = \frac{0.2 m_i}{m_i - m_{pay}}$

thus

$m_{pay} = 1000\ kg$

$m_{inert} = 0.2 m_i$

$m_i = m_{prop} + m_{pay} + m_{inert} = m_{prop} + m_{pay} + 0.2 m_i$

$m_{prop} = 0.8 m_i - m_{pay}$ ... ①

$m_f = m_{pay} + m_{inert} = m_{pay} + 0.2 m_i$

$v_f - \cancel{v_i}^{0} = v_e \ln \frac{m_i}{m_f}$

$m_f = m_i \exp(-\frac{v_f}{v_e})$

$m_f \cong 0.67759\ m_i$

$0.67759 m_i = m_{pay} + 0.2 m_i$

$m_i = 2093.85\ kg$

$m_{prop} = ① = 675.08$

$\boxed{675.08\ kg}$

Question 3

The ARIANE 5 launch vehicle has two P230 solid propellant boosters and a main Vulcain engine that are ignited at lift-off. The two components of the launch vehicle have the following characteristics:

Effective exhaust velocities: $c_{Vulcain} = 3285$ m/s; $c_{P230} = 2355$ m/s

Mass flows: $\dot{m}_{Vulcain} = 255$ kg/s; $\dot{m}_{P230} = 1835$ kg/s

Calculate the average effective exhaust velocity and average specific impulse for the launch vehicle.

Answer 3:

$$\text{mean}(C_{eff}) = \frac{C_{vulcain}\dot{m}_{vulcain} + 2C_{p230}\dot{m}_{p230}}{\dot{m}_{vulcain} + 2\dot{m}_{p230}}$$

$$= \frac{(3285\ m/s)(255\ kg/s) + 2(2355\ m/s)(1835\ kg/s)}{255\ kg/s + 2(1835\ kg/s)}$$

$$\cong \boxed{2415.42\ m/s}$$

$$\text{mean}(I_{sp}) = \frac{\text{mean}(C_{eff})}{g_0} = \frac{2415.42\ m/s}{8.31\ m/s^2} \cong \boxed{290.66\ s}$$

Question 4

The mass of the payload of a spacecraft is 1000 kg, and the inert mass fraction is 0.10. If the velocity change, ΔV and the exit velocity of the propellant are 4860 m/s and 4267 m/s respectively,

a) What is the mass of the propellant on board the spacecraft?

b) What is the inert mass of the spacecraft?

c) What is the initial mass of the spacecraft?

Answer 4:

$$m_{pay} = 1000 \text{ kg}$$

$$f_{inert} = \beta = 0.10$$

$$\Delta V = 4860 \text{ m/s}$$

$$v_e = 4267 \text{ m/s}$$

(a) using the formula

$$m_{prop} = \frac{m_{pay}\left(e^{\frac{\Delta V}{v_e}} - 1\right)(1-\beta)}{1 - \beta e^{\frac{\Delta V}{v_e}}} = \frac{(1000 \text{ kg})\left(e^{\frac{4860}{4267}} - 1\right)(1-0.10)}{1-(0.10)e^{\frac{4860}{4267}}}$$

$$\approx \boxed{2779.35} \text{ kg}$$

(b)

$$\beta = \frac{m_{inert}}{m_{inert} + m_{prop}} \Leftrightarrow m_{inert} = \frac{\beta}{1-\beta} m_{prop} = \frac{0.10}{0.90}(2779.35 \text{ kg})$$

$$\approx \boxed{308.82 \text{ kg}}$$

(c)

$$m_i = m_{pay} + m_{inert} + m_{prop}$$

$$= 1000 \text{ kg} + 308.82 \text{ kg} + 2779.35 \text{ kg}$$

$$= \boxed{4088.17 \text{ kg}}$$

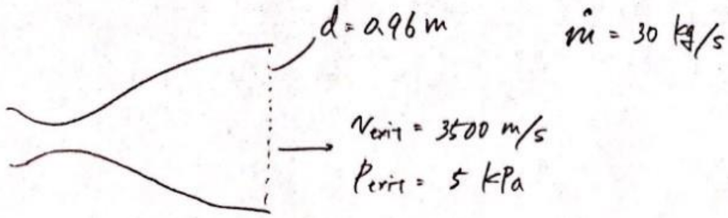
Question 5

A rocket's engine has an exit diameter of 0.96 m. It is ejecting mass at a rate of 30 kg/s with an exit velocity of 3,500 m/s. The pressure at the exit of the nozzle is 5 kPa. Calculate:

a) The thrust of the engine in vacuum
b) Final mass of the rocket if the initial mass was 1000 kg and the burn duration was 25s.
c) Specific impulse
d) Effective exhaust velocity

Answer 5:

Q5.



(a) at vacuum $P_{atm} = 0$

so

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.96\,m)^2 \cong 0.7238\ m^2$$

and

$$F_{THRUST} = \dot{m}[v_{exit} + (P_{exit} - \overset{0}{\cancel{P_{atm}}})A]$$
$$= (30\ kg/s)[3500\ m/s] + (5 \times 10^3\ Pa)(0.7238\ m^2)$$
$$= \boxed{108619\ N}$$

(b)

$$\frac{dm}{dt} = \dot{m}$$
$$\int_i^f dm = \int_0^{25} -\dot{m}\, dt$$

$$m_f - m_i = -\dot{m}\Delta t$$
$$m_f = -\dot{m}\Delta t + m_i$$
$$= -(30\ kg/s)(25\ s) + 1000\ kg = \boxed{250\ kg}$$

(c) specific impulse $\equiv I_{sp} = \frac{F_{THRUST}}{\dot{m} g_0}$

$$= \frac{108619\,N}{(30\ kg/s)(9.81\ m/s^2)} \cong \boxed{369.08\ s}$$

(d)

$$c = v_e = \frac{F_{THRUST}}{\dot{m}} = \boxed{3620.13\ m/s}$$

Question 6

The shape of NACA airfoils is described through their numerical identifier. We can input the parameters from the identifier into equations to precisely generate the cross-section of the airfoil. The four-digit series, in particular, defines the profile through three parameters. The first digit describes maximum camber as a percentage of the chord. The second digit provides the distance of the maximum camber from the airfoil leading edges in tens of percentages of the chord. The last two digits give us the maximum thickness of the airfoil as a percentage of the chord.

For example, the NACA 2215 airfoil has a maximum camber of 2%, located at 0.2 chords (20%) from the leading edge of the airfoil, with a maximum thickness of 15% of the chord. Since we can use equations to describe the shape of the airfoil, we can automate the process of drawing different airfoils!

Write a MATLAB script that takes a NACA 4-digit series code and the chord length as inputs and plots the shape of the airfoil. On your plot, include the chord and mean camber line. Plot three different airfoils to show that your algorithm works with various inputs. Include a symmetric airfoil, a cambered airfoil, and a third airfoil of your choice.

If you need inspiration when choosing NACA airfoils to plot, you can find airfoil designations for a variety of aircraft on "*The Incomplete Guide to Airfoil Usage*" at https://m-selig.ae.illinois.edu/ads/aircraft.html. On this website, you will notice that the Supermarine Spitfire has a different airfoil for the root and the tip of the wing. The Air Tractor, on the other hand, maintains the same airfoil from root to tip. Why do you think that is?

For this question, you will be graded on your MATLAB code, plot, and discussion on the last prompt. Make sure your code is well commented and describes the variables and equations you have used to generate the plots. Make is easy for a third person to read and understand.

Airfoil Plotter

- This program will plot the airfoil of a NACA 4-digit series aircraft.

Theory

First of all. the nomenclature of the 4-digit is the following

1. First digit: The **maximum camber** as percentage of chord
2. Second Digit: **Distance of the maximum camber** from the airfoil leading edge in tens of percents of chord
3. Last two digits: The **maximum thickness** of airfoil as percentage of chord

Then, the equations we will use to calculate the airfoil shape are shown below.

The equation for the airfoil is experessed as a pair of parameteric equations for X and Y using the parameter θ for $\theta = [0, 2\pi]$. The equations are

$$X(\theta) = 0.5 + 0.5\frac{|\cos(\theta)|^B}{\cos(\theta)} \quad \cdots (1)$$

$$Y(\theta) = \frac{T}{2}\frac{|\sin(\theta)|^B}{\sin(\theta)}(1 - X^P) + \mathrm{C}\sin(X^E\pi) + \mathrm{R}\sin(\mathrm{X}2\pi) \quad \cdots (2)$$

where

1. B: Describes the **Base shape coefficient.** This parameter determines mainly the shape of the leading edge. When B is closer to 2 the base shape of the airfoil becomes more ellliptical, whereas when it is more closer to 1 the shape becomes more rectangular.
2. T: Describes **thickness** as a fraction/percentage of the chord
3. P: Describes the **Taper Exponent**. The more P is closer to 1 the thickness tends to decrease more linearly when approaching 0, whereas the more P is a higher value the thickness decreases more suddenly
4. E: Describes the **Camber Exponent**. This defines the position of the maximum camber point on the chord. Where E = 1 indicates the camber point to be at the middle of the airfoil, that is 50% camber point. Smaller value of E shifts the camber point more toward the leading edge
5. R: Describes the **Reflex Parameter**. When this value is a positive value the trailing edge becomes reflexed, whereas when it is negative one emulates flaps

Next, the Equation to plot the mean chord line will be simply horizontal line connecting $x = [0, 1] \cdots (3)$.

Finally, the equation for the mean camber line will be

$$C_m(\theta) = \mathrm{C}\sin(X^E\pi) + \mathrm{R}\sin(\mathrm{X}2\pi) \quad \cdots (4)$$

Now the equations from (1) to (4) is the thoery of this function.

Function

```
function  airfoil_plotter_func(fourDigitCode, B_coeff, Taper_coeff, reflex_para)
```

>>INPUT

1. fourDigitCode: The NACA 4-digit code
2. B_coeff: Base shape coefficient
3. Taper_coeff: Taper exponent
4. reflex_para: Reflex parameter

>>OUTPUT

1. none (only plot)

>>SETUP

```
% Take out the parameters from the 4-digit code
% Hold the code
place_holder = fourDigitCode;
% Maximum camber
C_max = floor(fourDigitCode / 1000);
% Take the modulus of the four digit code for further manipulation
fourDigitCode = mod(fourDigitCode, 1000);
% Position of maximum camber
C_exp = floor(fourDigitCode / 100);
% Repeat modulus operation
fourDigitCode = mod(fourDigitCode, 100);
% Maximum thickness
Th_max = fourDigitCode;

% Default settings for the input parameters
% base shape coefficient
if B_coeff == 0
    B_coeff = 2;
end
% Taper coefficient
if Taper_coeff == 0
    Taper_coeff = 1;
end
% Reflex parameter
if reflex_para == 0
    reflex_para = 0;
end

% Assign simpler varibales to make the subsequent calculations easier
% while fixing the parameters from percent to decimal
C = C_max / 100;
B = B_coeff;
P = Taper_coeff;
E = C_exp / 10;
T = Th_max / 100;
R = reflex_para;
```

```
```

>>CALCULATIONS

```
% The main function will be a for loop to retreive all the points for the plots
% The stepsize/increment for each iteration will of the loop will be
stepsize = 2 * pi / 1000;

% Before the loop we must preallocate the vector including all the x- and y-values
% and point for mean camber line
X_theta = zeros(1000, 1);
Y_theta = zeros(1000, 1);
Cm_theta = zeros(1000,1);

% The loop
% Initialize index counter
n = 1;

for theta = 0:stepsize:2*pi
    % Calculating the x-value
    X = 0.5 + 0.5 * (abs(cos(theta)))^B / cos(theta);
    % Calculating the y-value
    Y = T * (abs(sin(theta)))^B * (1-X^P) / 2 / sin(theta) + C * sin(X^E * pi)...
        + R * sin(X * 2 * pi);
    % Calculating point for mean camber line
    Cm = C * sin(X^E * pi) + R * sin(X * 2 * pi);

    % Apending these values into the vector
    X_theta(n) = X;
    Y_theta(n) = Y;
    Cm_theta(n) = Cm;

    % Increment counter
    n = n + 1;
end
% The chord line will be
X_chord = [0,1];
Y_chord = [0,0];
```

>>PLOTTING

```
figure(1)
plot(X_theta, Y_theta,"-b",'Linewidth', 2.5)
title(['The Airfoil Geometry of NACA' num2str(place_holder)])
xlabel('0 to 1 from Leading Edge to Trailing Edge')
ylabel('Vertical Direction')
grid on
grid minor
ylim([-0.2, 0.2])
box on
hold on
```

```
plot(X_theta, Cm_theta,"-r", 'Linewidth', 1.5)
plot(X_chord, Y_chord, "--g")
hold off
```

Terminate Function

```
end
```

>>Execution

- Plotting 3 types of airfoils

#1. NACA 0015

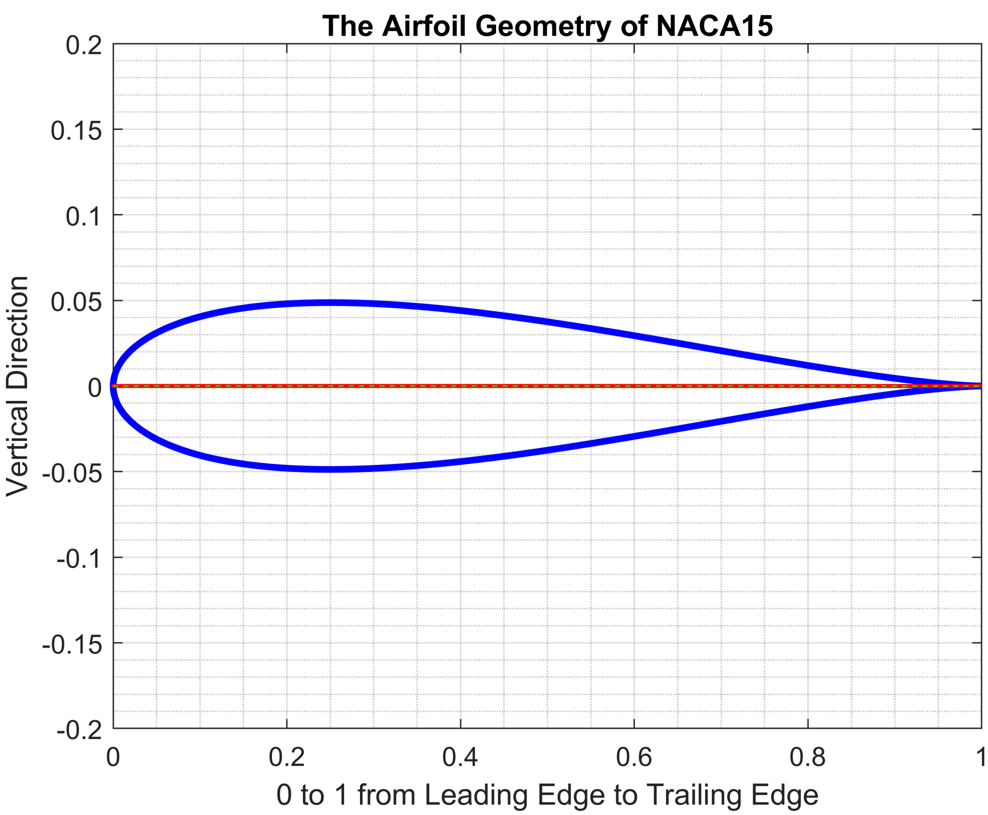
With

base shape coeffiecient = B = default (0)

taper exponent = P = default (0)

reflex parameter = R = default (0)

```
airfoil_plotter_func(0015,0,0,0);
```



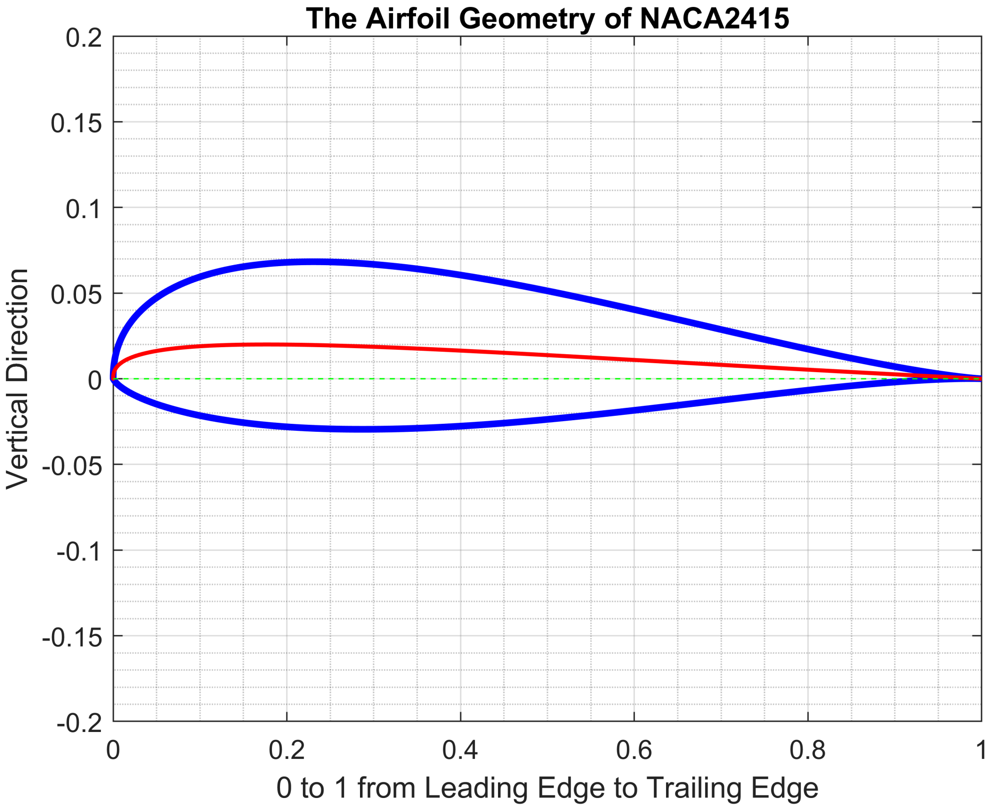
#2. NACA 2415

base shape coeffiecient = B = default (0)

taper exponent = P = default (0)

reflex parameter = R = default (0)

```
airfoil_plotter_func(2415,0,0,0);
```



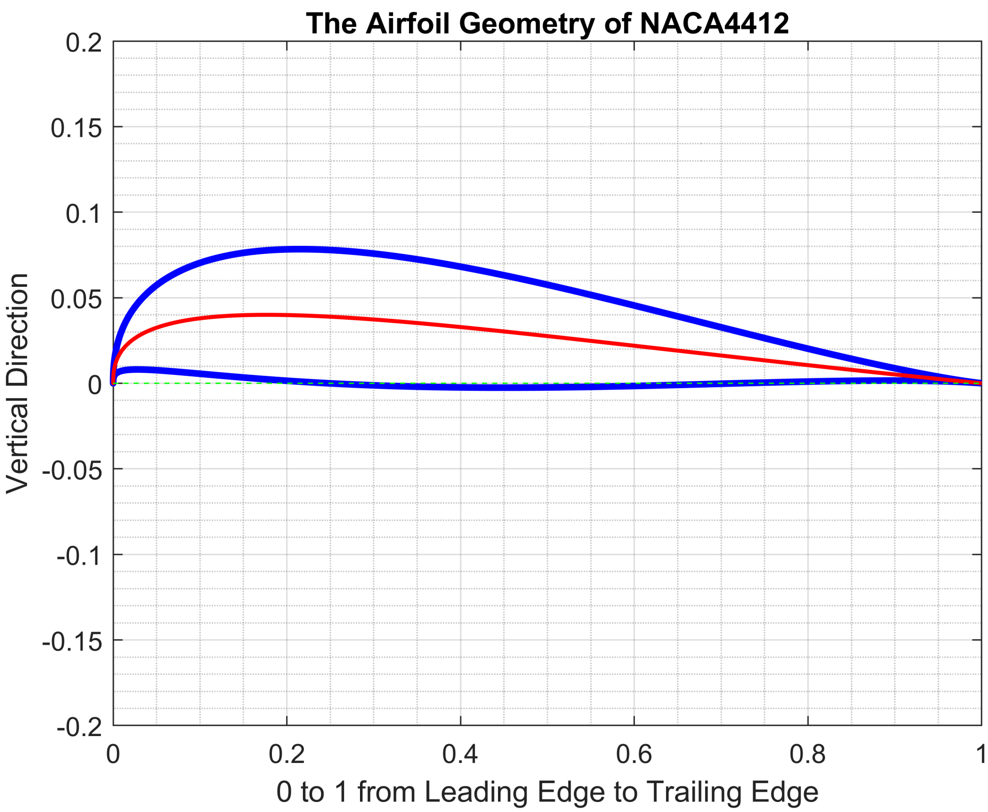
#3. NACA 4412

base shape coeffiecient = B = default (0)

taper exponent = P = default (0)

reflex parameter = R = default (0)

```
airfoil_plotter_func(4412,0,0,0);
```



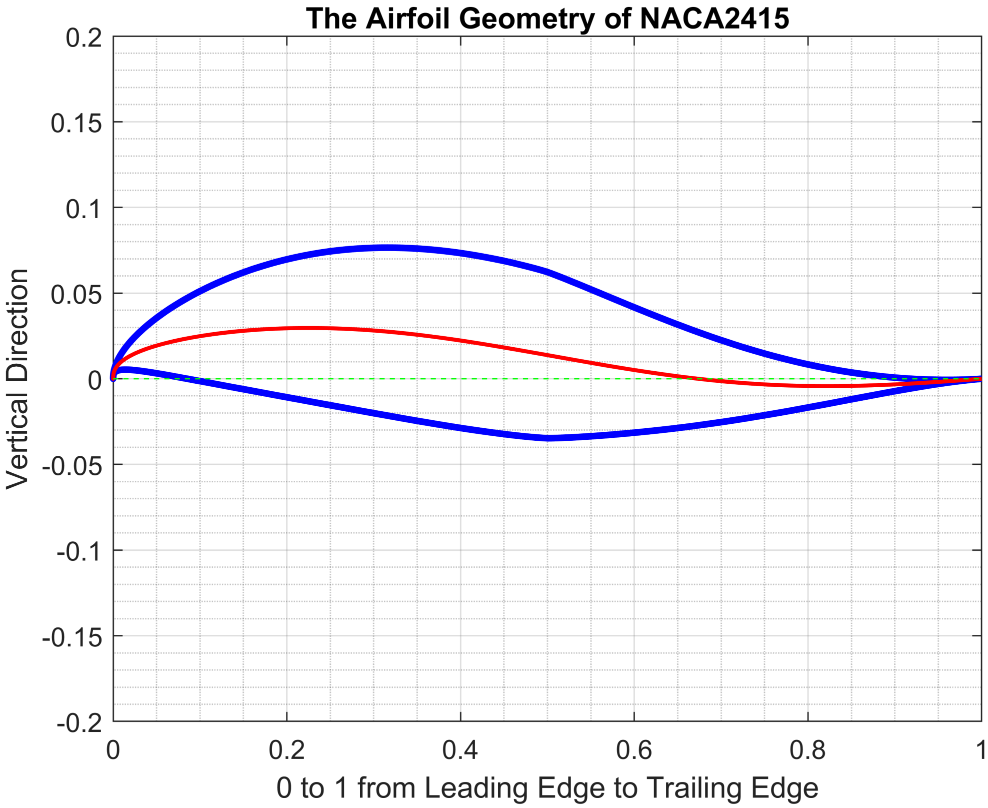
#4. NACA 2415 (NOT DEFAULT SETTINGS)

base shape coeffiecient = B = 2.5

taper exponent = P = default 1.8

reflex parameter = R = default 0.02

```
airfoil_plotter_func(2415,2.5,1.5,0.01);
```


The Airfoil Geometry of NACA2415
Vertical Direction
0 to 1 from Leading Edge to Trailing Edge
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
0
0.2
0.4
0.6
0.8
1

Analysis:

The Supermarine Spitfire has an unique wing configuration of having the root and tip of the wings be different airfoils. At the time of its design this was a new approach, and this was done to reduce the induced drag and improving performance. This is because the airfoil at the tip managed to make the lift coefficient smaller. Spitfire's main requirements was to gain victory in dogfights, and therefore, higher maneuverability was expected. In contrast, the Air Tractor is active in high production agriculture required to maintain a stable flight in low altitudes with slow speed. Due to its main usage of disseminating chemicals to grow vast fields of crops, it is rather ideal to maintain the same airfoil from root to tip to have a constant lift and drag.

REFERENCES

Ziemkiewicz, David. "*Simple Analytic Equation for Airfoil Shape Descritpion*." December2016, https://www.researchgate.net/publication/312222678_Simple_analytic_equation_for_airfoil_shape _description. Accessed on 1 March 2019.