$$F^* = M\,{}^A\alpha^{C^*} = M(a_1\hat{b}_1 + a_2\hat{b}_2 + a_3\hat{b}_3)$$

$$a_1 = -Ru_2t_2 + \dot{u}_4 a_1 + \dot{u}_5 s_1 + Ru_1u_2(1+se_2^2)$$

$$a_2 = -\dot{u}_4 s_1 s_2 + \dot{u}_5 c_1 s_2 - Ru_1^2 - Ru_2^2 t_2^2$$

$$a_3 = R\dot{u}_1 + \dot{u}_4 s_1 c_2 - \dot{u}_5 c_1 c_2 + Ru_2^2 t_2$$

$$T^* = -I^B\cdot{}^A\alpha^C - {}^A\omega^C \times (I^B\cdot{}^A\omega^C)$$

$$= \frac{MR^2}{4}(\hat{b}_1\hat{b}_1 + \hat{b}_2\hat{b}_2 + 2\hat{b}_3\hat{b}_3)\cdot\{[\dot{u}_1 + u_2(u_3 - u_2t_2)]\hat{b}_1 + [\dot{u}_2 - u_1(u_3 - u_2t_2)]\hat{b}_2 + \dot{u}_3\hat{b}_3\}$$
$$- (u_1\hat{b}_1 + u_2\hat{b}_2 + u_3\hat{b}_3)\times\left[\frac{MR^2}{4}(\hat{b}_1\hat{b}_1 + \hat{b}_2\hat{b}_2 + \hat{b}_3\hat{b}_3)\cdot(u_1\hat{b}_1 + u_2\hat{b}_2 + u_3\hat{b}_3)\right]$$

$$= \frac{MR^2}{4}[\dot{u}_1 + u_2(u_3 - u_2t_2)]\hat{b}_1 + \frac{MR^2}{4}[\dot{u}_2 - u_1(u_3 - u_2t_2)]\hat{b}_2 + \frac{MR^2}{2}\dot{u}_3\hat{b}_3$$
$$- (u_1\hat{b}_1 + u_2\hat{b}_2 + u_3\hat{b}_3)\times\frac{MR^2}{4}(u_1\hat{b}_1 + u_2\hat{b}_2 + 2u_3\hat{b}_3)$$

$$= (\ \text{〃}\ ) - \frac{MR^2}{4}(u_1u_2\hat{b}_3 - 2u_1u_3\hat{b}_2 - u_1u_2\hat{b}_3 + 2u_2u_3\hat{b}_1 + u_1u_3\hat{b}_2 - u_2u_3\hat{b}_1)$$

$$= (\ \text{〃}\ ) - \frac{MR^2}{4}(u_2u_3\hat{b}_1 - u_1u_3\hat{b}_2)$$

$$= \frac{MR^2}{4}(\dot{u}_1 - u_2^2t_2)\hat{b}_1 + \frac{MR^2}{4}(\dot{u}_2 + u_1u_2t_2)\hat{b}_2 + \frac{MR^2}{2}\dot{u}_3\hat{b}_3$$

$$F_1^* = {}^A\omega_1^C\cdot T^* + {}^Av_1^{C^*}\cdot F^*$$

$$= \frac{MR^2}{4}(\dot{u}_1 - u_2^2t_2) + MR\underbrace{(R\dot{u}_1 + \dot{u}_4 s_1 c_2 - \dot{u}_5 c_1 c_2 + Ru_2^2t_2)}_{a_3}$$

$$F_2^* = {}^A\omega_2^C\cdot T^* + {}^Av_2^{C^*}\cdot F^*$$

$$= \frac{MR^2}{4}(\dot{u}_2 + u_1u_2t_2) - MRt_2\underbrace{[-Ru_2t_2 + \dot{u}_4 c_1 + \dot{u}_5 s_1 + Ru_1u_2(1+se_2^2)]}_{a_1}$$

$$F_3^* = {}^A\omega_3^C\cdot T^* + {}^Av_3^{C^*}\cdot F^*$$

$$= \frac{MR^2}{2}\dot{u}_3$$

$$F_4^* = {}^A\omega_4^C \cdot T^* + {}^A v_4^{C^*} \cdot F^*$$

$$= 0 + (c_1\hat{b}_1 - s_1 s_2 \hat{b}_2 + s_1 c_2 \hat{b}_3) \cdot M(a_1\hat{b}_1 + a_2\hat{b}_2 - a_3\hat{b}_3)$$

$$= Mc_1 a_1 - Ms_1 s_2 a_2 + Ms_1 c_2 a_3$$

$$F_5^* = (s_1\hat{b}_1 + c_1 s_2 \hat{b}_2 - c_1 c_2 \hat{b}_3) \cdot M(a_1\hat{b}_1 + a_2\hat{b}_2 + a_3\hat{b}_3)$$

$$= M(s_1 a_1 + c_1 s_2 a_2 - c_1 c_2 a_3)$$

$$\cancel{F_r}^{\,0} + F_r^* = 0 \qquad r = 1, 2, \ldots, 5$$

① $$R\dot{u}_1 - Ru_2^2 t_2 + 4R\dot{u}_1 + 4s_1 c_2 \dot{u}_4 - 4c_1 c_2 \dot{u}_5 + 4Ru_2^2 t_2 = 0$$

$$5R\dot{u}_1 + 4s_1 c_2 \dot{u}_4 - 4c_1 c_2 \dot{u}_5 = -3Rt_2 u_2^2 \quad \cdots ①$$

② $$R\dot{u}_2 + Ru_1 u_2 t_2 - 4t_2\left[-R\dot{u}_2 t_2 + \dot{u}_4 c_1 + \dot{u}_5 s_1 + Ru_1 u_2 (2 + t_2^2)\right] = 0$$

$$- (-4Rt_2^2 \dot{u}_2 + 4c_1 t_2 \dot{u}_4 + 4s_1 t_2 \dot{u}_5 + 8Ru_1 u_2 t_2 + 4Ru_1 u_2 t_2^3) = 0$$

$$R(1 + 4t_2^2)\dot{u}_2 - 4c_1 t_2 \dot{u}_4 - 4s_1 t_2 \dot{u}_5 = 7Rt_2 u_1 u_2 + 4Rt_2^3 u_1 u_2$$

$$= Rt_2(7 + 4t_2^2) u_1 u_2 \quad \cdots ②$$

③ $$\dot{u}_3 = 0 \quad \cdots ③$$

④ $$-Rc_1 t_2 \dot{u}_2 + c_1^2 \dot{u}_4 + s_1 c_1 \dot{u}_5 + Rc_1(2 + t_2^2) u_1 u_2$$

$$+ s_1^2 s_2^2 \dot{u}_4 - c_1 s_1 s_2^2 \dot{u}_5 + Rs_1 s_2 u_1^2 + Rs_1 s_2 t_2^2 u_2^2$$

$$+ Rs_1 c_2 \dot{u}_1 + s_1^2 c_2^2 \dot{u}_4 - c_1 s_1 c_2^2 \dot{u}_5 + Rs_1 s_2 u_2^2 = 0$$

$$Rs_1 c_2 \dot{u}_1 - Rc_1 t_2 \dot{u}_2 + \dot{u}_4 + Rs_1 s_2 u_1^2 + Rc_1(2 + t_2^2) u_1 u_2 + Rs_1 s_2 (1 + t_2^2) u_2^2 = 0$$

$$Rs_1 c_2 \dot{u}_1 - Rc_1 t_2 \dot{u}_2 + \dot{u}_4 = -Rs_1 s_2 u_1^2 - Rs_1 s_2 (1 + t_2^2) u_2^2 - Rc_1(2 + t_2^2) u_1 u_2$$

$$\cdots ④$$

⑤ $s_1 a_1 + c_1 s_2 a_2 - c_1 c_2 a_3 = 0$

$$-Rs_1 t_2 \dot{u}_2 + s_1 c_1 \dot{u}_4 + s_1^2 \dot{u}_5 + Rs_1(2+t_2^2) u_1 u_2$$
$$- s_1 c_1 s_2^2 \dot{u}_4 + c_1^2 s_2^2 \dot{u}_5 - Rc_1 s_2 u_1^2 - Rc_1 s_2 t_2^2 u_2^2$$
$$- Rc_1 c_2 \dot{u}_1 - s_1 c_1 c_2^2 \dot{u}_4 + c_1^2 c_2^2 \dot{u}_5 - Rc_1 s_2 u_2^2 = 0$$

$$-Rc_1 c_2 \dot{u}_1 - Rs_1 t_2 \dot{u}_2 + \dot{u}_5 - Rc_1 s_2 u_1^2 - Rc_1 s_2 (1+t_2^2) u_2^2 + Rs_1(2+t_2^2) u_1 u_2 = 0$$

$$-Rc_1 c_2 \dot{u}_1 - Rs_1 t_2 \dot{u}_2 + \dot{u}_5 = 2Rc_1 s_2 u_1^2 + Rc_1 s_2 (1+t_2^2) u_2^2 - Rs_1(2+t_2^2) u_1 u_2$$

. . . ⑤

$$\begin{bmatrix} 5R & 0 & 0 & 4s_1 c_2 & -4c_1 c_2 \\ 0 & R(1+4t_2^2) & 0 & -4c_1 t_2 & -4s_1 t_2 \\ 0 & 0 & 1 & 0 & 0 \\ Rs_1 c_2 & -Rc_1 t_2 & 0 & 1 & 0 \\ -Rc_1 c_2 & -2Rs_1 t_2 & 0 & 0 & 1 \end{bmatrix}$$