Student Name___________________________________

Student ID____________________________________

Orbit Mechanics
9/25/20

Exam 1

Please read the problems <u>carefully</u>.

Write clearly and use diagrams when necessary.

Use the following constant values when appropriate

Body	GM (km^3/s^2)	Radius (km)
Earth	4.0000×10^{5}	6400.0
Mars	4.2000×10^{4}	3400.0

Purdue Honor Pledge "As a Boilermaker pursuing academic excellence, I pledge to be honest and true in all that I do. Accountable together—We Are Purdue."

For this exam, I understand it is a take-home exam with the following requirements:

1. I can use my own class notes and my own previously completed assignments.
2. I am not allowed to search for any resources online.
3. I can use my own calculator. I cannot use Matlab or other commercial software.
4. I am expected to work the exam on my own. I am not allowed to work with another person. I am not allowed to contact another person for help while completing the exam.
5. If I have any questions during the exam period, I will email Prof Howell (howell@purdue.edu) AND the TAs Beom Park (park1103@purdue.edu) + Nadia Numa (nnuma@purdue.edu). Given this exam period is 24 hours, we will answer as soon as possible.

(35 Points)

Problem 1: NASA's SETI Program (Search for ExtraTerrestial Intelligence) sought out signs of intelligent life in the universe by scanning the sky. Assume that we have information concerning a distant system of three bodies. At a certain instant, the three bodies are known to be described with the following characteristics:

Body*	GM (km^3/s^2)	Distance (km)
A	2.0000×10^8	$r_{AB} = 8.0000\times 10^8$
B	5.0000×10^8	$r_{BC} = 4\sqrt{3}\times 10^8$
C	1.0000×10^8	$r_{AC} = 4.0000\times 10^8$

*Assume that the bodies are spherically symmetric

(a) What is the significance of the assumption that bodies are spherically symmetric?

Sketch (by hand) the three-body system.
Locate the system center of mass; add it to your sketch.

(b) The inverse square law of gravitation still applies! To consider the future locations of each body, you need the equations of motion.

(i) Since GM for body C is the smallest mass, write the differential equations for the motion of body C relative to body A or body B.
Which one is correct? Why?

(ii) Write the governing vector differential equation for the relative motion that you selected.

(iii) Identify the independent variable and the set of scalar dependent variables.

(c) For the vector differential equation in part (b), consider the acceleration on body C.

(i) Compute the dominant acceleration, the direct acceleration, the indirect acceleration and the net perturbing acceleration, each as a magnitude and direction.

(ii) Compare the magnitudes of each of the quantities in part (c)(i).

(iii) Discuss the dominant term in comparison to the perturbing term. For the two bodies involved in the dominant term, is the perturbing term – at this instant – tending to increase or decrease the distance between them? Why?

(30 points)

Problem 2: In the relative two-body problem, one type of solution is elliptical. In an ellipse, there are certain locations that are of particular interest: periapsis, apoapsis, ends of the latus rectum and ends of the minor axis. Compare the characteristics at these locations.

(a) Sketch an ellipse (by hand). Identify and mark the points of interest.

(b) The scalar distance *r* at each location can be written in terms of semimajor axis and eccentricity, i.e., only *a* and *e*. Derive these four expressions, given the following relationships

$$r = \frac{p}{1 + e\cos\theta^*} \quad , \qquad r = a(1 - e\cos E)$$

(c) The scalar magnitude of the relative velocity, *v*, at each location can be written in term of only the circular velocity and eccentricity, i.e., v_c and *e*. Given the constant $\mathcal{E}$, derive these four expressions.

(d) If $e = 0.5$, compare the radii and speeds at these four location, i.e., evaluate *r* in terms of *a*; *v* in terms of v_c.

(e) For a spacecraft moving relative to Earth, let $e = 0.5000$ and $a = 4.0000R_\oplus$ and write the vectors $\bar{r}$ and $\bar{v}$ in terms of unit vectors $\hat{e}$, $\hat{p}$ and units km and km/s for one location at the end of the minor axis and descending

(35 points)

Problem 3: The year is 20?? and a spacecraft is in orbit about Mars. Assume that the problem is modeled in terms of the relative two-body model. The orbit is characterized with the following quantities:

Spacecraft Mass = 8000.0 kg (<<< mass Mars!)
System Linear Momentum $\overline{C}_1 = \overline{0}$
Specific Angular Momentum $\left|\overline{h}\right| = 2.5000 \times 10^4 \,\text{km}^2/\text{s}$
Specific Energy $\left|\mathcal{E}\right| = 1.2351 \,\text{km}^2/\text{s}^2$

(a) System linear momentum is zero. What does that information tell you?

(b) Determine the following information about the orbit if $\mathcal{E} < 0$:
a, e, p, b, r_p, r_a, period (express distance in terms of $R_{♂}$)

(c) At the location where $\theta^* = -110^\circ$, determine r, v, γ

Sketch the orbit; mark $\overline{r}, \overline{v}, \gamma, a, b$, local horizon, center

(d) Make one change to the orbital characteristics: $\mathcal{E} > 0$. Repeat (b) and (c).
How does the orbit change?

(e) Compute one additional quantity for each orbit; discuss the significance of this additional number.