Student Name____________________________________

Student ID______________________________________

Orbit Mechanics
11/20/20

Exam 3

Please read the problems carefully.

Write clearly and use diagrams when necessary.

Use the following constant values when appropriate

Body	GM (km^3/s^2)	Radius (km)
Earth	4.0000×10^5	6400
Sun	1.300×10^{11}	7.0000×10^5
Jupiter	1.300×10^8	7.0000×10^4
Sun-Earth Semimajor Axis 1.5000×10^8 km		

Purdue Honor Pledge "As a Boilermaker pursuing academic excellence, I pledge to be honest and true in all that I do. Accountable together—We Are Purdue."

For this exam, I understand it is a take-home exam with the following requirements:

1. I can use my own class notes and my own previously completed assignments.
2. I am not allowed to search for any resources online.
3. I can use my own calculator. I cannot use Matlab or other commercial software.
4. I am expected to work the exam on my own. I am not allowed to work with another person. I am not allowed to contact another person for help while completing the exam.
5. If I have any questions during the exam period, I will email Prof Howell (howell@purdue.edu) AND the TAs Beom Park (park1103@purdue.edu) + Nadia Numa (nnuma@purdue.edu). Given this exam period is 24 hours, we will answer as soon as possible.

Signature ____

(35 Points)

Problem 1: For motion throughout the solar system, it is reasonable to model the Sun-planet and the Sun-comet orbits as relative two-body problems. Thus, assume that a short period comet moves in a heliocentric orbit with characteristics such that:

$$a_{comet} = 8AU \quad e_{comet} = 0.8$$

When the comet is ***inbound***, it encounters Jupiter. At encounter, the Jovian orbit parameters are

$$r_{Jup} = 5.2AU \qquad \gamma_{Jup} = 5^\circ$$
$$v_{Jup} = 13.0 \text{ km/s} \quad \theta^*_{Jup} = 90^\circ$$

The comet passes Jupiter at $25R_{Jup}$ and the comet's heliocentric energy **increases**.
Assuming Jupiter and comet orbits are coplanar; orbital angular momentums are aligned.

(a) Determine the comet arrival conditions including θ^{*-}_{comet}. Sketch the heliocentric orbits of Jupiter and the comet.

(b) Sketch the heliocentric vector diagram for the encounter.

In a sketch of the Jovian-centered view, include the aimpoint, semiminor axis, asymptotes, and turn angle.

Evaluate the semimajor axis and the semiminor axis—express in terms of R_{Jup}; compute the turn angle.

During the encounter with Jupiter, will the passage be "ahead" or "behind"? How do you know?

(c) Determine the comet's new heliocentric velocity v^+_{comet} and γ^+_{comet} immediately after the maneuver.

(d) For the encounter, determine the $|\Delta\bar{v}_{eq}|$, α for the equivalent Delta-V.

(35 points)

Problem 2: A vehicle is currently in Earth orbit with characteristics as follows:

$$a = 5R_{\oplus} \qquad \Omega = 0^{\circ}$$
$$e = 0.4 \qquad \omega = 30^{\circ}$$
$$i = 30^{\circ}$$

A maneuver will be implemented at a pre-determined location such that $\theta^{*} = 60^{\circ}$.

(a) Sketch the 3D orbit as viewed in terms of the inertial coordinates. Identify the maneuver location and add the orbital angular momentum vector.
Without any calculations, orbital angular momentum is in what inertial plane? How do you know?

(b) The planned maneuver is given as

$$\Delta\bar{v} = -\frac{1}{2}\hat{y} \text{ km/s}$$

Express this $\Delta\bar{v}$ in terms of VNB coordinates.

(c) The maneuver can also be expressed in terms of a magnitude and two orientation angles. Recall that the in-plane angle α is in the orbital plane and the out-of-plane angle is β. There are two possible α, β combinations that describe this maneuver. Determine these two ways of representing the maneuver.

(d) The $\Delta\bar{v}$ vector is comprised of an in-plane component ($\Delta\bar{v}_{VB}$) and an out-of-pane component ($\Delta\bar{v}_{N}$). Use some sketches (3D or 2D) of these components to explain the α, β options.

(30 Points)

Problem 3: Assume that a two-body model is appropriate to examine a transfer between two Earth orbits. Consider a spacecraft Omega that is currently in a circular orbit of radius $3R_{\oplus}$ about the Earth. The spacecraft Omega will depart its current orbit and rendezvous with Space Station Alpha. Alpha moves in an eccentric orbit but rendezvous occurs when Alpha is located at a distance $4R_{\oplus}$. The orbits of Omega and Alpha are coplanar with parallel orbital angular momentum vectors (out of the page).

Given: Omega is at a departure distance $3R_{\oplus}$ and rendezvous with the Space Station occurs when Alpha is located at an arrival distance $4R_{\oplus}$. For various reasons the transfer angle is specified as 90°.

(a) Note the space triangle on the next page. Determine the chord.

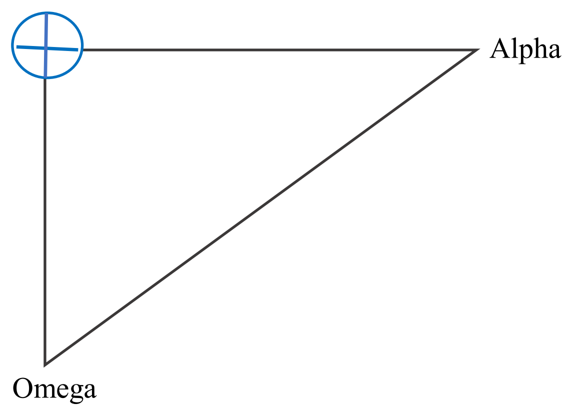
(b) There exists an elliptical transfer arc that satisfies these conditions. What is the minimum value for the semimajor axis that connects the two locations?

For a transfer with this minimum value of semimajor axis, what is the corresponding value of energy?

(c) Prove that the appropriate value of the semi-latus rectum for this transfer orbit is $2.4R_{\oplus}$. Compute the values for eccentricity and the semiminor axis.

(d) Determine the following quantities that characterize the transfer:
$r_p, r_a; r_{dep}, v_{dep}, \gamma_{dep}, E_{dep}, \theta^*_{dep}, \theta^*_{arr}$.

(e) Sketch the transfer orbit (reasonably accurately) over the space triangle. Identify departure/arrival locations, the line of apsides, the minor axis, ellipse center C and the two foci, r_p, r_a,, latus rectum, $r_{dep}, r_{arr}, \gamma_{dep}, \theta^*_{dep}, \theta^*_{arr}$. Mark the transfer arc.


Alpha
Omega