I BP

Sunday, October 4, 2020 12:15 AM

Orbits in Three Dimensions

Previously, we considered everything in 2D
Now, try 3D problems ← some background necessary to define an orbit in space

First, define coordinate systems (3D) to help

many available

Two basic types for us to use:

$x_\varepsilon, y_\varepsilon$

(1) **Ecliptic System** – fundamental plane is the plane of the ⊕'s orbit about the Sun (latitude, longitude)

x, y

(2) **Equatorial System** – Fundamental plane is the plane of the body's equator (right ascension, declination)

Obliquity of ecliptic (ε) – inclination of ecliptic with respect to equator

To effectively use a coordinate system, reference directions must be known and understood; we need a fixed reference direction in the fundamental plane from which measurements are made

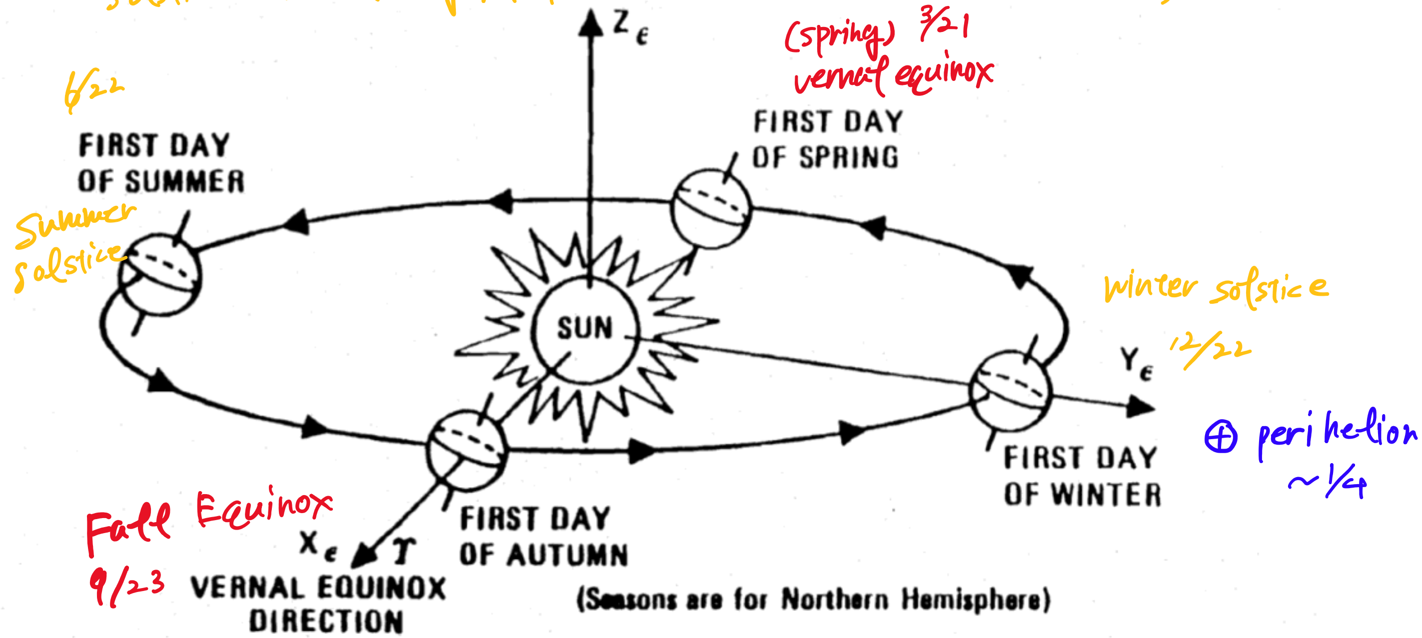
→ vernal equinox

$x_\varepsilon = x$ intersection of ecliptic and Earth equitorial plane

Earth orbit almost circular ; axis of rotation tilted wrt ecliptic $\cong 23.5°$

equinox — when length of night/day equal

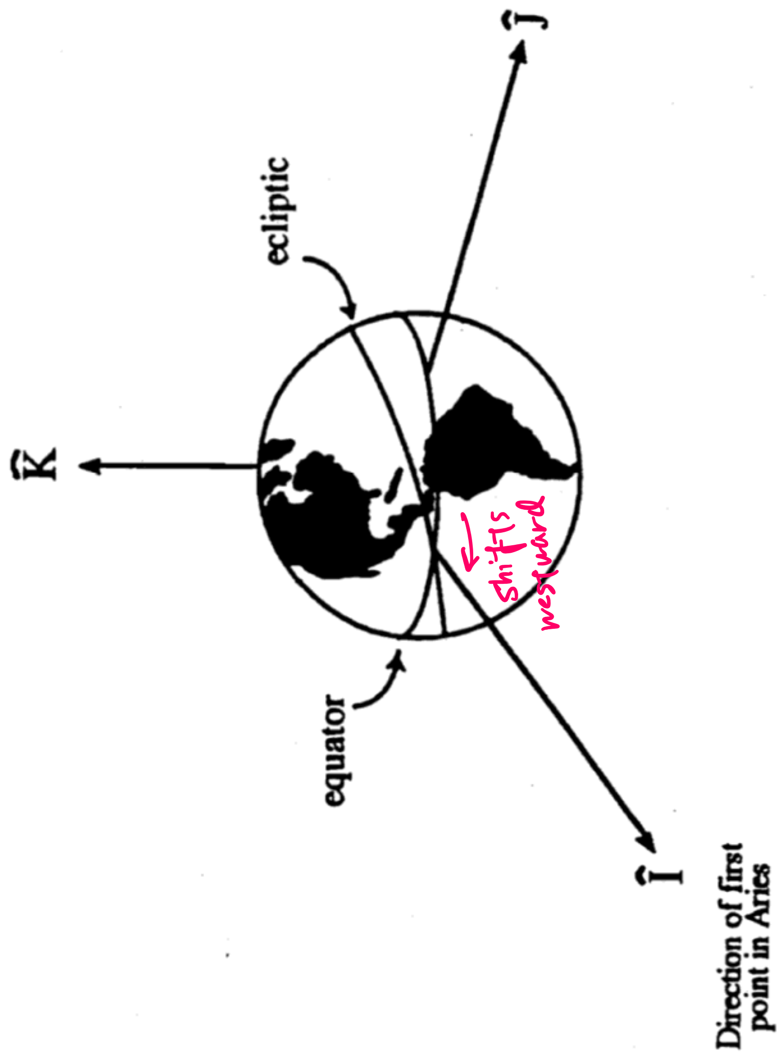
solstice — when axis of rot & dir to Sun make largest/smallest $\measuredangle$ angle

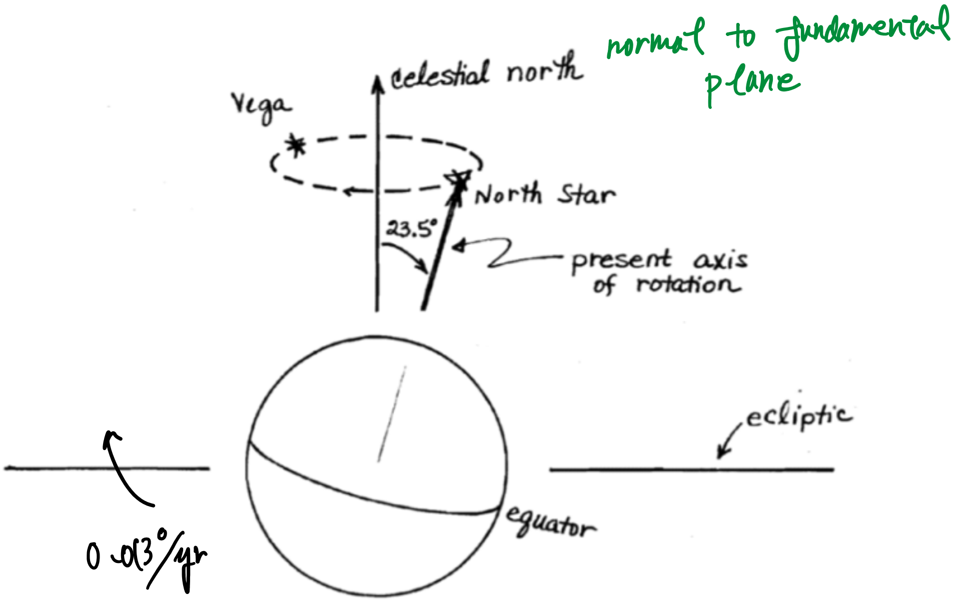


♈ first point in Aries ← the ram

~2000 yrs since vernal equinox pointing in dir of constellation Aries

(currently direction is toward Pisces) ← ~70 B.C.


ecliptic
$\hat{K}$
$\hat{J}$
shifts westward
equator
$\hat{I}$
Direction of first point in Aries



"precession of the equinoxes" – change in direction of Earth spin axis

Caused by perturbing forces on its attitude, i.e., ☉ and ☾ gravity forces
These apply a precessing motion (same as the precessing motion of a spinning top or a torque-free rigid body)

Known as early as 2nd century BC to Greek astronomer Hipparchus

Time for complete precession is 26,000 years

shift / swing thru $\frac{360°}{26000\text{yrs}} = \frac{4}{5}\ \frac{\text{arcmin}}{\text{yr}}$

$= 0.13°/\text{yr}$

Consequence

1. Cataloging of celestial objects must refer to a specific date → epoch → currently 0.0 hrs
2. We will assume ϒ fixed; reasonable over the relatively short intervals of interest

eclip
equatorial

Reference System

$\hat{x}$ direction of the vernal equinox
$\hat{z}$ normal to fundamental plane; + north
$\hat{y}$ $= \hat{z} \times \hat{x}$

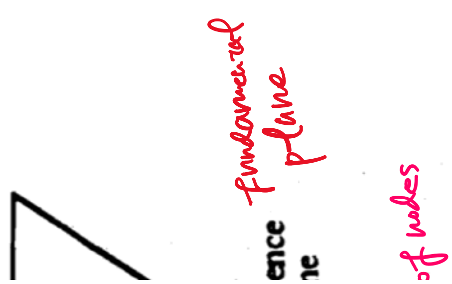
So, to locate s/c in space :

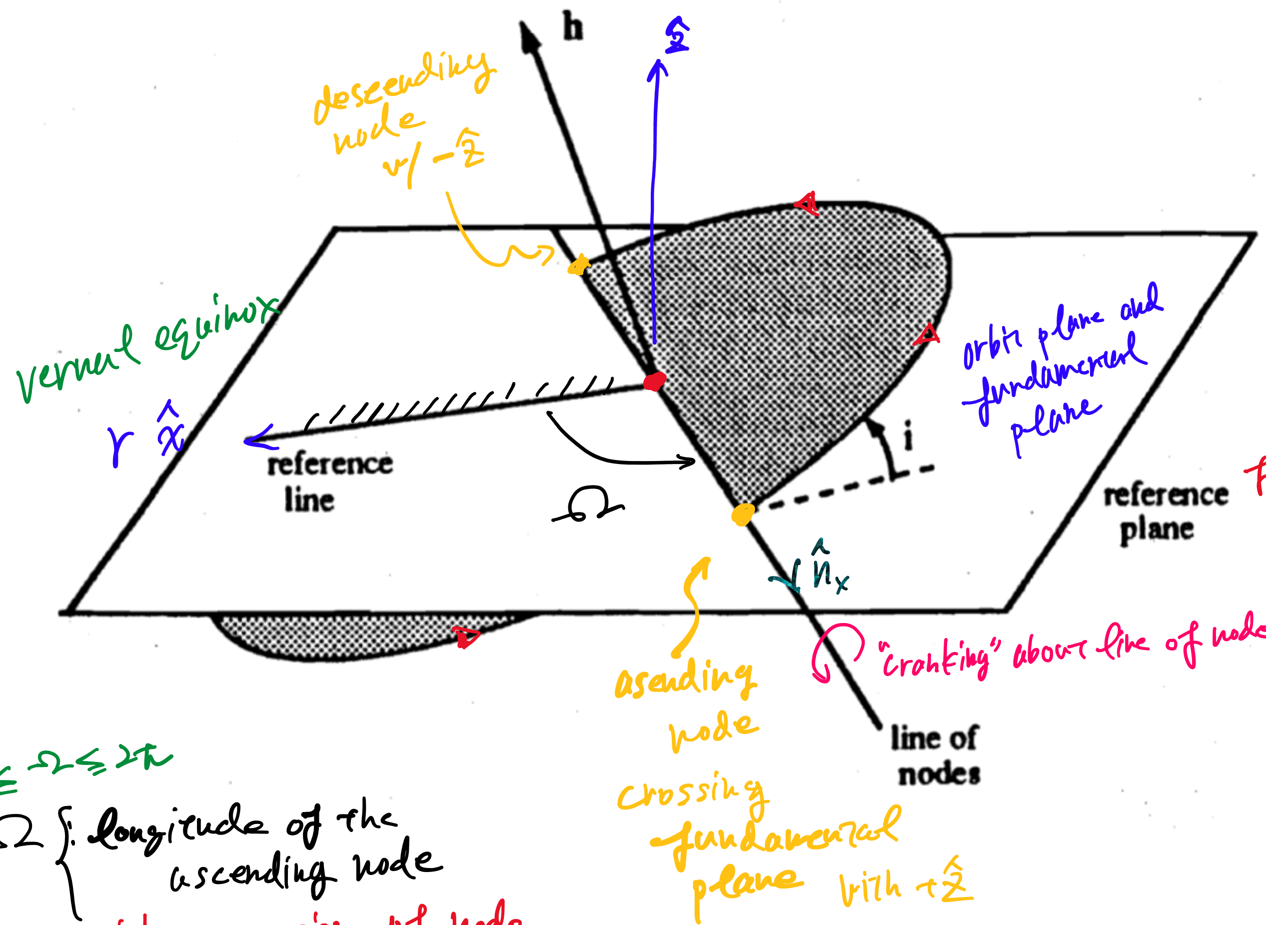
time

Argument of periapsis

(1) locate s/c in orbit (θ^*, E, M)
(2) identify orientation of orbit within orbit plane (ω); size and shape of orbit (a, e)
(3) identify orientation of orbit plane in space (Ω, i)

w/ reference to some fundamental plane





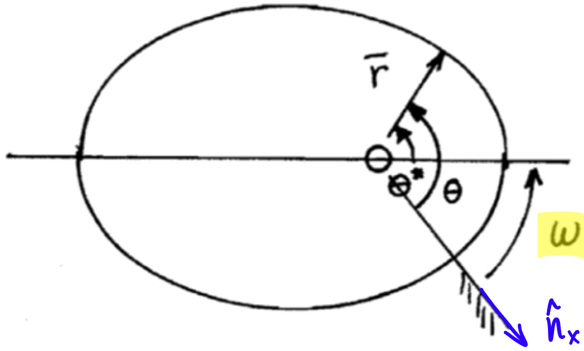
$0 \leq \Omega \leq 2\pi$

Ω : longitude of the ascending node

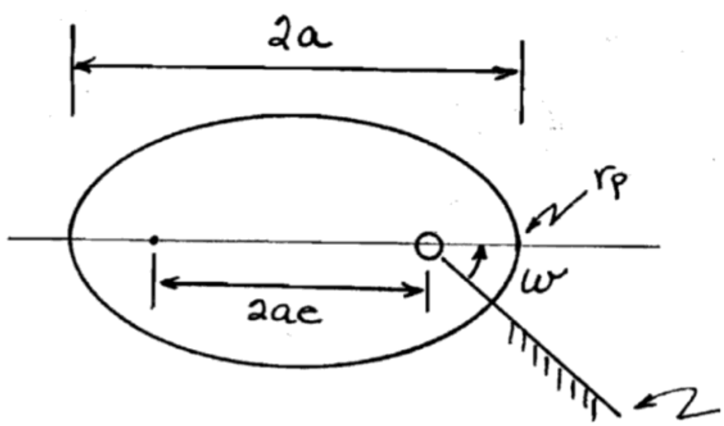
right ascension of node

i : inclination $\quad 0 \leq i \leq \pi$

(1) Locate s/c in orbit: time ← $\overset{\text{time}}{\downarrow}$ M, E, θ^* → one of these



(2) Within orbit plane : orbit size and shape (a, e)
orbit orientation within plane (ω)



(3) orientation of orbit plane : Ω, i

orbital elements : $a, e, i, \Omega, \omega, M\ (\tau_p)$
$\uparrow$ E, θ^*

write $\bar{r}, \bar{v} \longrightarrow \hat{r}, \hat{\theta} / \hat{e}, \hat{p}$

$\hat{\theta} = \hat{h} \times \hat{r}$

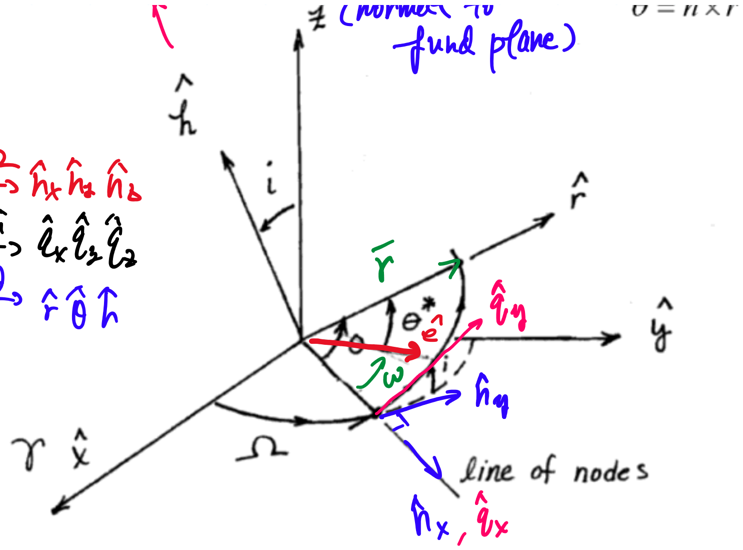
$\hat{x}\,\hat{y}\,\hat{z} \xrightarrow{\Omega} \hat{n}_x\,\hat{n}_y\,\hat{n}_z$

$\hat{n}_x\,\hat{n}_y\,\hat{n}_z \xrightarrow{i} \hat{l}_x\,\hat{l}_y\,\hat{l}_z$

$\hat{l}_x\,\hat{l}_y\,\hat{l}_z \xrightarrow{\theta} \hat{r}\,\hat{\theta}\,\hat{h}$

plus

$\hat{l}_x\,\hat{l}_y\,\hat{l}_z \xrightarrow{\omega} \hat{e}\,\hat{p}\,\hat{h}$



$\hat{q}_x, \hat{q}_y, \hat{q}_z \xrightarrow{\omega} \hat{e}, \hat{p}, \hat{h}$

ω { argument of periapsis $0 \leq \omega \leq 2\pi$

$\Omega + \omega = \varpi$ { longitude of periapsis

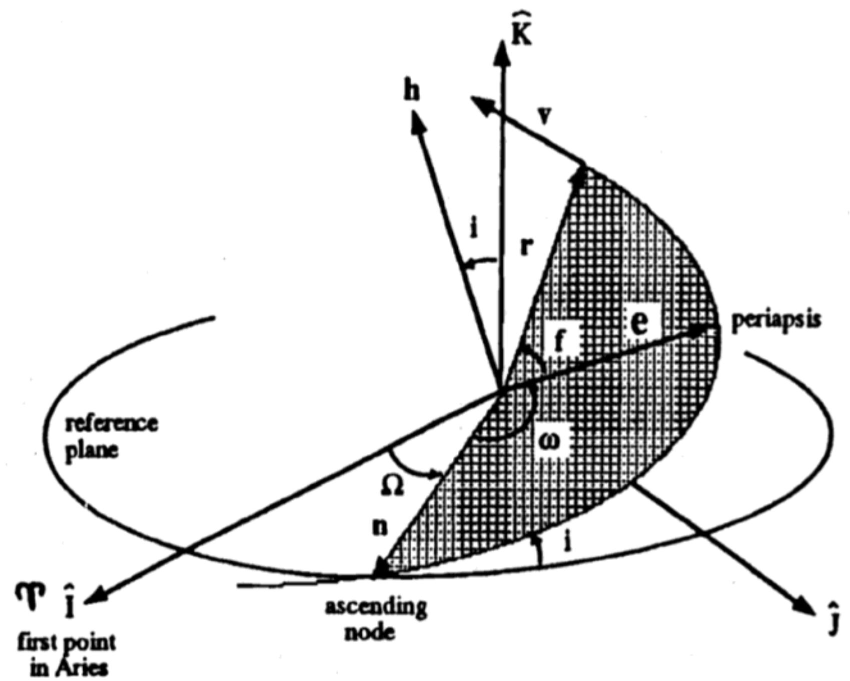
$\varpi + \theta^* = L$ { true longitude at epoch $L = \Omega + \omega + \theta^*$

$\Omega \; i \; \theta$

3-1-3 (body-two) Euler sequence $\Longrightarrow$ transformation matrix

$\hat{z} - \hat{n}_x - \hat{h}$

	$\hat{r}$	$\hat{\theta}$	$\hat{h}$
$\hat{x}$	$c_\Omega c_\theta - s_\Omega c_i s_\theta$	$-c_\Omega s_\theta - s_\Omega c_i c_\theta$	$s_\Omega s_i$
$\hat{y}$	$s_\Omega c_\theta + c_\Omega c_i s_\theta$	$-s_\Omega s_\theta + c_\Omega c_i c_\theta$	$-c_\Omega s_i$
$\hat{z}$	$s_i s_\theta$	$s_i c_\theta$	c_i



Ω : longitude of the ascending node

ω : argument of periapsis

$\tilde{\omega} = \Omega + \omega$: longitude of periapsis

f : true anomaly

$L = \tilde{\omega} + f$: true longitude