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AAE 251: Introduction to Aerospace Design

Assignment 7—Rocket Design

Due Tuesday March 26, 10:00 am on Blackboard

Instructions

*Write or type your answers into the appropriate boxes. **Make sure you submit a single PDF on Blackboard.***

Make sure you keep a record of submission receipts or the confirmation emails after each submission as a proof that your submission was accepted.

	Score	Max
Question 1		8
Question 2		10
Question 3		33
Question 4		10
Question 5		12
Question 6		12
TOTAL		85

Question 1

Four rocket teams from AAE251 join together to launch a four stage rocket with a small payload to the Moon. The rocket needs to reach the Earth escape velocity of $11,176 \text{ m/s}$. The rocket will launch the vehicle eastward from Kennedy Space Center where the speed of rotation of the Earth is 427 m/s . Assume gravitational losses of about 1500 m/s and aerodynamic velocity losses of 600 m/s . To keep cost down, four stages have the same effective exhaust velocity C and inert-mass fraction. Each stage burns kerosene and oxygen producing a specific impulse of 330 s . The inert-mass fraction of each stage is 0.1 , and they each deliver the same ΔV . What is the $m_{\text{payload}}/m_{\text{initial}}$ that is, the ratio of mass of payload to the initial mass on the launch pad?

Answer 1:

Q1.

GIVEN:

4-stage rocket to moon. Needs Earth escape velocity $\Delta V_{esp} = 11176 \text{ m/s}$

Launched from Kennedy Space Center where Earth rotation speed

$$\Delta V_{rot} = 427 \text{ m/s}.$$

Gravitational loss $\Delta V_g = 1500 \text{ m/s}$

Aerodynamic velocity loss $\Delta V_A = 600 \text{ m/s}$

All stages have same v_c (or C) and $f_{int} = 0.1$

$$I_{sp} = 330 \text{ s}$$

Each stage has same ΔV

FIND

$$\frac{m_{\text{payload}}}{m_{\text{initial}}}$$

SOLN

The equation for the required velocity will be

$$\Delta V = \Delta V_{esp} + \Delta V_g + \Delta V_A - \Delta V_{rot}$$

$$\text{Since, } \Delta V_1 = \Delta V_2 = \Delta V_3 = \Delta V_4 = \Delta V_{\text{stages}}$$

$$\Rightarrow \Delta V = (11176 + 1500 + 600 - 427) \text{ m/s} = 12849 \text{ m/s}$$

$$\Delta V_{\text{stage}} = \frac{\Delta V}{4}$$

$$\Delta V_{\text{stage}} = \frac{12849 \text{ m/s}}{4}$$

$$\Delta V_{\text{stage}} \approx 3212.25 \text{ m/s}$$

Answer 1:

$$v_e = g_0 I_{sp} = (9.81 \text{ m/s}^2)(330 \text{ s}) = 2943 \text{ m/s}$$

now if # of stages is n

the initial mass can be expressed as

$$m_{\text{initial}} = m_{\text{payload}} \left[\frac{\exp\left(\frac{\Delta v_{\text{stage}}}{n v_e}\right) (1 - f_{\text{iner}})}{1 - f_{\text{iner}} \exp\left(\frac{\Delta v_{\text{stage}}}{n v_e}\right)} \right]^n$$

thus for a 4-stage

$$m_{\text{initial}} = m_{\text{payload}} \left[\frac{\exp\left(\frac{3212.25}{4 \cdot 2943}\right) (1 - 0.1)}{1 - (0.1) \exp\left(\frac{3212.25}{4 \cdot 2943}\right)} \right]^4$$

$$m_{\text{initial}} \approx 3.4329 m_{\text{payload}}$$

$$\therefore \frac{m_{\text{payload}}}{m_{\text{initial}}} = \frac{m_{\text{payload}}}{3.4329 m_{\text{payload}}} = \frac{1}{3.4329}$$

$$\approx 0.2913$$

$$\boxed{0.291}$$

Question 2

In lecture 15, we presented the following equation:

$$m_{prop} = \frac{m_{pay} [e^{\frac{\Delta V}{c}} - 1] (1 - f_{inert})}{1 - f_{inert} e^{\frac{\Delta V}{c}}}$$

Using the four input equations (equations 1 through 4) given in the notes, derive this expression.

Answer 2:

GIVEN:

$$\Delta V = g_0 I_{sp} t_h \ln \frac{m_i}{m_f} \quad \dots \textcircled{1}$$

$$m_i = m_f + m_{prop} \quad \dots \textcircled{2}$$

$$m_f = m_{pay} + m_{inert} \quad \dots \textcircled{3}$$

$$f_{inert} = \frac{m_{inert}}{m_{inert} + m_{prop}} \quad \dots \textcircled{4}$$

FIND:

$$\text{Derive } m_{prop} = \frac{m_{pay} \left[\exp\left(\frac{\Delta V}{c}\right) - 1 \right] (1 - f_{inert})}{1 - f_{inert} \exp\left(\frac{\Delta V}{c}\right)}$$

SOLN:

$$\text{Using } \textcircled{4} \quad m_{inert} = \frac{f_{inert}}{1 - f_{inert}} m_{prop} \quad \dots \textcircled{5}$$

plug $\textcircled{5}$ into $\textcircled{3}$

$$m_f = m_{pay} + \frac{f_{inert}}{1 - f_{inert}} m_{prop} \quad \dots \textcircled{6}$$

plug $\textcircled{6}$ into $\textcircled{2}$

$$m_i = m_{pay} + \frac{f_{inert}}{1 - f_{inert}} m_{prop} + m_{prop}$$

$$m_i = m_{pay} + \frac{m_{prop}}{1 - f_{inert}} \quad \dots \textcircled{7}$$

from $\textcircled{1}$

$$\exp\left(\ln \frac{m_i}{m_f}\right) = \exp\left(\frac{\Delta V}{c}\right)$$

$$\frac{m_i}{m_f} = \exp\left(\frac{\Delta V}{c}\right) \quad \dots \textcircled{8}$$

Answer 2:

using ③ ⑦ ⑧

$$\frac{m_2}{m_1} = \frac{m_t}{m_1} + \frac{m_{prop}}{m_1}$$

$$1 = \frac{1}{\exp\left(\frac{\Delta v}{c}\right)} + \frac{m_{prop}}{m_{pay} + \frac{m_{prop}}{1 - f_{inert}}}$$

$$m_{pay} \exp\left(\frac{\Delta v}{c}\right) + \frac{\exp\left(\frac{\Delta v}{c}\right)}{1 - f_{inert}} m_{prop} = m_{pay} + \frac{m_{prop}}{1 - f_{inert}} + m_{prop} \exp\left(\frac{\Delta v}{c}\right)$$

$$\left[\frac{1}{1 - f_{inert}} + \exp\left(\frac{\Delta v}{c}\right) - \frac{\exp\left(\frac{\Delta v}{c}\right)}{1 - f_{inert}} \right] m_{prop} = m_{pay} [\exp\left(\frac{\Delta v}{c}\right) - 1]$$

$$\frac{1 + \exp\left(\frac{\Delta v}{c}\right) - f_{inert} \exp\left(\frac{\Delta v}{c}\right) - \exp\left(\frac{\Delta v}{c}\right)}{1 - f_{inert}} m_{prop} = m_{pay} [\exp\left(\frac{\Delta v}{c}\right) - 1]$$

$$\therefore m_{prop} = \frac{m_{pay} [\exp\left(\frac{\Delta v}{c}\right) - 1] (1 - f_{inert})}{1 - f_{inert} \exp\left(\frac{\Delta v}{c}\right)}$$

Question 3

We want to design a rocket system to take a payload of 6000 kg to a lunar orbit and from there observe the lunar surface using various instruments. We will achieve this mission with a three-stage mission using a Hohmann transfer: (1) launch to LEO, (2) transfer orbit to the moon, and (3) transfer to lunar orbit (see your slides for thoughts on ΔV required).

- a) What are the minimum number of burns we need for this entire mission?
- b) In general, when designing a space mission, what else might you want to know about the payload in addition to its mass? Give at least three factors.

Now, let's consider the transfer from LEO to the lunar orbit.

Transfer to the Moon:

Assume that a transfer from LEO to the lunar orbit will require about 4500 m/s of total ΔV and we will need 75% of the ΔV to enter the transfer orbit, 25% to enter lunar orbit. Let's consider two options:

Option 1:

- Two solid rocket stages, one for each ΔV
- $I_{sp} = 280$ s
- Assume $f_{inert} = 0.1$

Option 2: One liquid stage ignited twice

- N_2O_4/N_2H_4 (toxic but storable)
- $I_{sp} = 360$ s
- Assume $f_{inert} = 0.18$

- c) Calculate the masses for each option. (m_{prop} , m_{inert} , m_i , and m_f)
- d) Which option would you pick, and why?
- e) Discuss qualitatively how your design would change if we wanted a sample-return mission.

Answer 3:

GIVEN: Lunar orbit mission (3-stage rocket)

$m_{\text{pay}} = 6000 \text{ kg} \rightarrow$ Hohmann transfer

(1) To LEO \rightarrow (2) transfer orbit \rightarrow (3) lunar orbit

SOLN:

(a) Min # of burns required?

2 burns

(b) Besides mass what do you want to know about the payload?

volume, material, human/non-human

Assume that

LEO \rightarrow Lunar orbit requires $\Delta V_{\text{Hohmann}} = 4500 \text{ m/s}$

75% of this to enter transfer orbit $\Delta V_{T1} = 3375 \text{ m/s}$

25% to enter lunar orbit. $\Delta V_{T2} = 1125 \text{ m/s}$

(c)

<option 1>

$$m_{\text{prop}_2} = \frac{m_{\text{pay}} \left[\exp\left(\frac{\Delta V_{T2}}{g_0 I_{sp}}\right) - 1 \right] (1 - f_{\text{inert}})}{1 - f_{\text{inert}} \exp\left(\frac{\Delta V_{T2}}{g_0 I_{sp}}\right)} \quad (2^{\text{nd}} \text{ stage})$$

$$\text{Since } \begin{cases} \exp\left(\frac{\Delta V_{T1}}{g_0 I_{sp}}\right) = \exp\left(\frac{3375}{9.81 \cdot 280}\right) \approx 3.4168 \\ \exp\left(\frac{\Delta V_{T2}}{g_0 I_{sp}}\right) = \exp\left(\frac{1125}{9.81 \cdot 280}\right) \approx 1.5062 \end{cases}$$

Answer 3:

$$\therefore m_{prop2} = \frac{(6000 \text{ kg})(1.5062 - 1)(1 - 0.1)}{(1 - 0.1 \cdot 1.5062)} \approx 3218.21 \text{ kg}$$

$$m_{inert2} = \frac{f_{inert}}{1 - f_{inert}} m_{prop2} = \frac{1}{9} (3218.21 \text{ kg}) \approx 357.58 \text{ kg}$$

$$m_{f2} = m_{p2} + m_{inert2} = 6000 \text{ kg} + 357.58 \text{ kg} = 6357.58 \text{ kg}$$

$$m_{22} = m_{prop2} + m_{f2} = 3218.21 \text{ kg} + 6357.58 \text{ kg} = 9575.79 \text{ kg}$$

same for stage 1 $\rightarrow m_{p1} = m_{22} = 9576 \text{ kg}$

$$m_{prop1} = \frac{(9576 \text{ kg})(3.4168 - 1)(1 - 0.1)}{1 - 0.1 \cdot 3.4168} \approx 31639.55 \approx 3.16 \times 10^4 \text{ kg}$$

$$m_{inert1} = \frac{1}{9} (31639.55 \text{ kg}) \approx 3515.51 \text{ kg} \approx 3.52 \times 10^3 \text{ kg}$$

$$m_{f1} = 9576 \text{ kg} + 3515.51 \text{ kg} = 13091.51 \text{ kg} \approx 1.31 \times 10^4 \text{ kg}$$

$$m_{21} = 31639.55 \text{ kg} + 13091.51 \text{ kg} = 44731.06 \text{ kg} \approx 4.47 \times 10^4 \text{ kg}$$

Answer 3:

<option 2>

same to option 1

$$\text{since } \exp\left(\frac{\Delta V_{\text{mission}}}{g_0 I_{sp}}\right) = \exp\left(\frac{4500}{9.81 \cdot 360}\right) = 3.5759$$

$$m_{\text{prop}} = \frac{(6000 \text{ kg})(3.5759 - 1)(1 - 0.18)}{1 - 0.18 \cdot 3.5759} \approx 35565.75 \text{ kg} \approx \boxed{3.56 \times 10^4 \text{ kg}}$$

$$m_{\text{inert}} = \frac{0.18}{0.82} (35565.75) \approx 7807.12 \text{ kg} \approx \boxed{7.81 \times 10^3 \text{ kg}}$$

$$m_f = 6000 \text{ kg} + 7807.12 \text{ kg} = 13807.12 \text{ kg} \approx \boxed{1.38 \times 10^4 \text{ kg}}$$

$$m_i = 35565.75 \text{ kg} + 13807.12 \text{ kg} = 49372.87 \text{ kg} \approx \boxed{4.94 \times 10^4 \text{ kg}}$$

(d) The solid propellant because the initial mass of the rocket can be reduced significantly compared to the liquid propellant.

(e) To return from lunar orbit to LEO we must have more fuel to decelerate the rocket by producing a reverse thrust. This requires a larger tank (more inert mass) and an overall larger rocket design.

Question 4

NASA is designing a two-stage rocket with a required ΔV of 12 km/s and payload capacity of 80 kg to a Highly Elliptical Orbit (HEO) around Earth.

The two stages contribute ΔV_1 and ΔV_2 respectively to the total ΔV via the fractions f_1 and f_2 such that:

$$\Delta V = \Delta V_1 + \Delta V_2 = f_1 \Delta V + f_2 \Delta V, \text{ and}$$

$$f_1 + f_2 = 1$$

Assume that for both stages the mass ratio, R i.e. the inverse of f_{inert} is 12. The I_{sp} for stage 1 is 280 s, and for stage 2 is 350 s. Assume ideal conditions, i.e. no gravity loss or drag, perfectly expanded nozzle.

NASA wants to design the two-stage rocket with as minimum initial mass of stage 1 as possible. Write a MATLAB code to plot initial mass of stage 1 vs f_1 . On your plot indicate the point where the initial mass of stage 1 is the minimum. Find the corresponding f_1 value. Then calculate the values of f_2 , and the total mass of the propellant required.

Make sure to paste your MATLAB code and plot.

Hint: The optimum value of f_1 lies somewhere between 0.3 and 0.55

Problem #4

This program aims to calculate the optimal initial mass for the first stage of a two-stage rocket. A plot of initial mass of 1st stage vs velocity ratio 1 will be manipulated.

>> Given Data

- required total velocity $\Delta V = 12 \frac{\text{km}}{\text{s}} = 12000 \frac{\text{m}}{\text{s}}$
- payload mass $m_{\text{pay}} = 80 \text{ kg}$
- inert mass ratio $f_{\text{inert}} = \frac{1}{12} \approx 0.0833$
- specific impulse for 1st stage $I_{\text{sp1}} = 280 \text{ s}$
- specific impulse for 2nd stage $I_{\text{sp2}} = 350 \text{ s}$

>> Assumptions

- ideal conditions
- no gravity loss
- perfectly expanded nozzle

>> Equations Used

- $\Delta V = \Delta V_1 + \Delta V_2 \quad \dots (1)$
- $\Delta V_1 = f_1 \cdot \Delta V, \quad \Delta V_2 = f_2 \cdot \Delta V$
- $f_1 + f_2 = 1 \quad \dots (2)$

•

$$m_{\text{initial, stage1}} = m_{\text{pay}} \cdot (1 - f_{\text{inert}})^n \cdot \prod \left[\frac{e^{\frac{\Delta V_n}{C_n}}}{1 - f_{\text{inert}} \cdot e^{\frac{\Delta V_n}{C_n}}} \right] \dots (3)$$

$$\text{mass of propellant} = m_{\text{prop}} = m_{\text{pay}} \cdot (1 - f_{\text{inert}})^n \cdot \prod \left[\frac{\left(e^{\frac{\Delta V_n}{C_n}} - 1 \right)}{1 - f_{\text{inert}} \cdot e^{\frac{\Delta V_n}{C_n}}} \right] \dots (4)$$

where c = effective exhaust velocity, and n = # of stages

$$\text{or } m_{\text{prop}} = (1 - f_{\text{inert}})(m_{\text{initial}} - m_{\text{pay}}) \quad \dots (5)$$

from equations (1) - (3) we obtain for a two-stage rocket

$$m_{\text{initial, stage1}}(f_1) = \frac{m_{\text{pay}} \cdot e^{\left(\frac{f_1 \Delta V}{C_1} + \frac{(1-f_1) \Delta V}{C_2} \right)} (1 - f_{\text{inert}})^2}{\left(1 - f_{\text{inert}} \cdot e^{\frac{f_1 \Delta V}{C_1}} \right) \left(1 - f_{\text{inert}} \cdot e^{\frac{(1-f_1) \Delta V}{C_2}} \right)}$$

$$m_{\text{prop,stage1}}(f_1) = \frac{m_{\text{pay}} \cdot \left(1 - e^{-\frac{f_1 \Delta V}{c_1}}\right) \left(1 - e^{-\frac{(1-f_1) \Delta V}{c_2}}\right) (1 - f_{\text{inert}})^2}{\left(1 - f_{\text{inert}} \cdot e^{-\frac{f_1 \Delta V}{c_1}}\right) \left(1 - f_{\text{inert}} \cdot e^{-\frac{(1-f_1) \Delta V}{c_2}}\right)}$$

and $c = g_0 \cdot I_{\text{sp}}$ where $g_0 = \text{gravitational acceleration} = 9.81 \frac{m}{s^2}$

Algorithm

```
% Assigning variables to given values
% Required total velocity [m/s]
V_tot = 12000;
% Payload mass [kg]
m_pay = 80;
% Inert mass ratio
f_inert = 0.0833;
% Specific impulse for stage 1 [s]
Isp1 = 280;
% Specific impulse for stage 2 [s]
Isp2 = 350;
% Gravitational acceleration [m/s^2]
g_o = 9.81;

% Since f1 has an optimum value interval of [0.3, 0.55] create the array for f1 values as
f1 = 0.3:0.001:0.55;
% Break down the equation to calculate the initial mass of stage 1
% Component 1
comp1 = exp(f1 * V_tot / g_o / Isp1);
% Component 2
comp2 = exp((1 - f1) * V_tot / g_o / Isp2);
% Thus, the initial mass of stage 1 becomes
m_stage1 = m_pay .* comp1 .* comp2 * (1 - f_inert)^2 ./ (1 - f_inert.*comp1)...
    ./ (1 - f_inert.*comp2);

% Finding the minimum mass for stage 1
m_min = min(m_stage1);
% The index of this minimum value
idx = find(m_stage1 == m_min);
% The corresponding f1 value
f1_min = f1(idx);
% Find f2 corresponding to this f1 value
f2_min = 1 - f1_min;
% Find the corresponding mass of propellant using equation (4)
% First manipulate the components to match the f1 with f1_min
comp1_min = exp(f1_min * V_tot / g_o / Isp1); % Component 1
comp2_min = exp((1 - f1_min) * V_tot / g_o / Isp2); % Component 2
% Mass of propellant is
```

```

m_prop_min = m_pay * (1 - comp1_min) * (1 - comp2_min) * (1 - f_inert)^2 ...
/ (1 - f_inert*comp1_min) / (1 - f_inert*comp2_min);

```

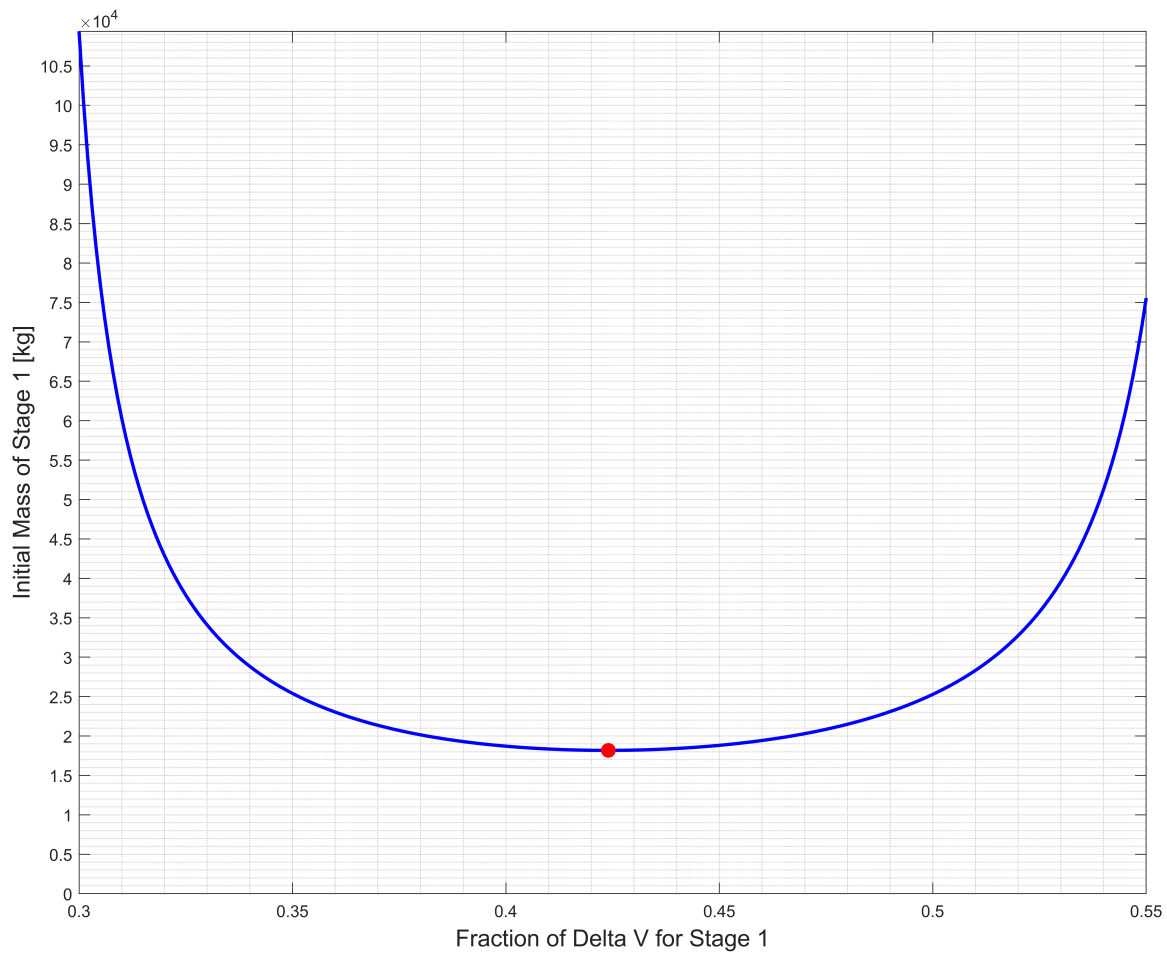
Plotting

```

% Adjusting fontsize and linewidth
fontsize = 14;
linewidth = 2;

% Plotting commands
figure(1)
plot(f1, m_stage1, '-b', 'Linewidth', linewidth)
axis([0.3 0.55 0 inf])
yticks(0:5000:110000)
xlabel('Fraction of Delta V for Stage 1', 'FontSize', fontsize)
ylabel('Initial Mass of Stage 1 [kg]', 'FontSize', fontsize)
grid on
grid minor
box on
hold on
plot(f1_min, m_min, '.r', 'MarkerSize', 30)
% Control where plot is positioned
set(gcf, 'PaperPositionMode', 'auto', 'Position', [0 0 1100 850])

```

Results

```
fprintf(fid,['The initial mass of stage 1 is at minimum when f1 = %.3f',...
```

The initial mass of stage 1 is at minimum when f1 = 0.424 with a value of 18176.37 kg.

```
    'with a value of %.2f kg.'], f1_min, m_min);
fprintf(fid,['At this minimum value f2 = %.3f',...
```

At this minimum value f2 = 0.576 and the mass of the propellant is 13278.03 kg.

```
    'and the mass of the propellant is %.2f kg.'], f2_min, m_prop_min);
```

Question 5:

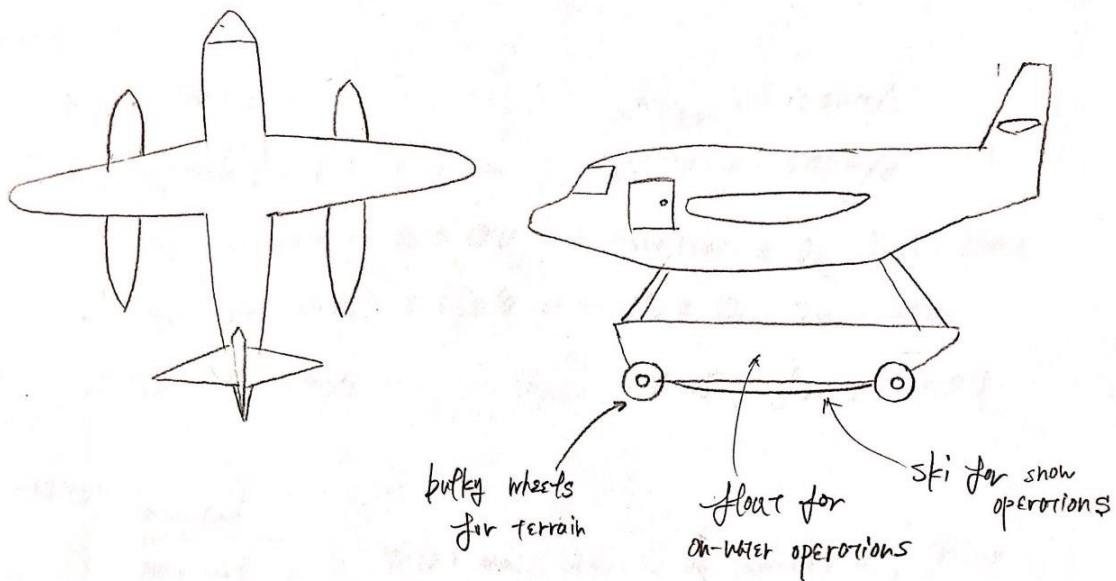
The following set of design requirements statements for an aircraft is written poorly. Describe a concept that would satisfy all the requirements and yet fail its mission. Point out the problems with the set of requirements. You may help illustrate your solution with a sketch.

Mission: An all-terrain airborne vehicle that can be used for fishing in all seasons in Alaska.

Requirements:

- a) The aircraft has a large wing that produces a lot of lift, so that it can take off in short distances.
- b) The aircraft can carry 100 fish, which is the average number of fish a fisher can catch in a workday.
- c) The aircraft can land on a lake to provide access to fishing opportunities.
- d) The aircraft has skis for winter operation.
- e) The aircraft should be equipped with fishing equipment.
- f) The aircraft has two occupants.

Answer 5:



For this concept of the aircraft it satisfies the given requirements; however, does not provide the crew a comfortable experiment of fishing because even if you try to fish from the entrance the aircraft it will be almost impossible to cast the fishing rod. The wings and floats are not suitable for a place to stand on or sit on to fish either. Thus, this concept does not satisfy the main mission of providing the crew a safe and stable station to fish.

Problems for each requirement:

- a. There are no specific metrics for (e.g, estimated AR, wing span, etc.)
- b. This requirement is not verifiable in that the stated metric varies depending on the fisher, weather, and other factors
- c. This should be more specific by stating that the aircraft should have a float to be able to provide access to fishing opportunities

- d. This is an independent statement that does not relate to the provided mission of fishing. Nevertheless, having all-terrain wheels on the floats is sufficient for the aircraft to operate in snow
- e. This is not verifiable. Question of need for this requirement: "Why cannot the crews just bring fishing equipment by themselves?"
- f. It does not state the reasoning for the number of crews

Question 6:

A two stage rocket is to be used to deliver a payload of 1000 kg to a circular low-Earth orbit altitude 1200 km; 40% of the delta-V will be delivered by the first stage, and 60% by the second stage. The vehicle will be launched Eastward from Kennedy Space Center where the speed of rotation of the Earth is 427 m/ sec. Assume gravitational velocity losses of about 1200 m/ sec and aerodynamic velocity losses of 500 m/ sec. The first stage burns kerosene and oxygen producing a mean specific impulse of 320 sec averaged over the flight, while the upper stage burns hydrogen and oxygen with an average specific impulse of 450 sec. The inert fraction of the first stage is 0.05 and that of the second is 0.07. Determine $m_{\text{payload}}/m_{\text{initial}}$ (the ratio of mass of payload to the initial mass), and the total mass of the vehicle on the launchpad. Assume perfectly expanded nozzles.

Suppose the same vehicle is to be used to launch a satellite into a north-south orbit from a launch complex on Kodiak island in Alaska. How does the mass of the payload change?

Answer 6:

GIVEN:

$$m_{\text{pay}} = 1000 \text{ kg}$$

$$\Delta V_{\text{grav}} = 1200 \text{ m/s}$$

$$\text{LFO (altitude)} \equiv h = 1200 \text{ km}$$

$$\Delta V_{\text{aero}} = 500 \text{ m/s}$$

$$1^{\text{st}} \text{ stage} \rightarrow \Delta V_1 = 0.4 \Delta V \rightarrow \text{kerogene \& O}_2 \quad I_{sp1} = 320 \text{ s}$$

$$2^{\text{nd}} \text{ stage} \rightarrow \Delta V_2 = 0.6 \Delta V \rightarrow \text{H}_2 \& \text{O}_2 \quad I_{sp2} = 450 \text{ s}$$

$$\Delta V_{\text{rot}} = 427 \text{ m/s}$$

$$f_{\text{iner1}} = 0.05, \quad f_{\text{iner2}} = 0.07$$

FIND:

$$\frac{m_{\text{payload}}}{m_{\text{initial}}} \quad \& \quad \text{total mass vehicle on launchpad, } m_{\text{tot}}$$

SIN:

$$\Delta V_{\text{LEO}} = \sqrt{\frac{\mu}{h+R_e}} = \sqrt{\frac{3.986 \times 10^{14} \text{ m}^3/\text{s}^2}{1200 \times 10^3 \text{ m} + 6378 \times 10^3 \text{ m}}} \approx 7252.56 \text{ m/s}$$

thus,

$$\begin{aligned} \Delta V &= \Delta V_{\text{LEO}} + \Delta V_{\text{grav}} + \Delta V_{\text{aero}} - \Delta V_{\text{rot}} \\ &= (7252.56 + 1200 + 500 - 427) \text{ m/s} \\ &= 8525.56 \text{ m/s} \end{aligned}$$

and

$$\Delta V_1 = 0.4 \Delta V \approx 3410.22 \text{ m/s}$$

$$\Delta V_2 = 0.6 \Delta V \approx 5115.34 \text{ m/s}$$

then

$$z_1 = \exp\left(\frac{\Delta V_1}{g_0 I_{sp1}}\right) = \exp\left(\frac{3410.22}{9.81 \cdot 320}\right) \approx 2.9634$$

$$z_2 = \exp\left(\frac{\Delta V_2}{g_0 I_{sp2}}\right) = \exp\left(\frac{5115.34}{9.81 \cdot 450}\right) \approx 3.1860$$

Answer 6:

$$\begin{aligned} \alpha = \frac{m_{\text{payload}}}{m_{\text{initial}}} &= \left(\frac{z_1 (1 - f_{\text{inert}1})}{1 - f_{\text{inert}1} \cdot z_1} \cdot \frac{z_2 (1 - f_{\text{inert}2})}{1 - f_{\text{inert}2} \cdot z_2} \right)^{-1} \\ &= \left(\frac{2.9634 (1 - 0.05)}{1 - 0.05 \cdot 2.9634} \cdot \frac{3.1860 (1 - 0.07)}{1 - 0.07 \cdot 3.1860} \right)^{-1} \\ &\approx \boxed{0.07935} \end{aligned}$$

$$\begin{aligned} m_{\text{initial}} &= \frac{m_{\text{payload}}}{\alpha} = \frac{1000 \text{ kg}}{0.07935} \approx 12602.39 \text{ kg} \\ &\approx \boxed{1.26 \times 10^4 \text{ kg}} \end{aligned}$$

Analysis:

Because Alaska is at a higher latitude than KSC the ΔV_{rot} (or ΔV_{FH}) will be smaller. This means that ΔV becomes smaller and $\Delta V_1, \Delta V_2$ will also become smaller correspondingly. This implies that $\frac{m_{\text{payload}}}{m_{\text{initial}}}$ becomes larger and if m_{initial} is fixed the mass of payload will increase.