K BP

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Passage through "Local" Gravity Fields

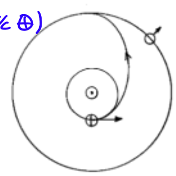
To examine interplanetary transfers completely, it would be necessary to consider all gravitational influences at all times. However, that is an inconvenient approach and can be solved only numerically. But, it is possible to obtain a pretty good approximation to the $\Delta\bar{v}$ requirements by considering the transfer in three phases, each of which involves only a two-body problem, for which there are a large number of analysis techniques.

For an example, consider that you are planning an Earth-to-Mars mission. Assume that the planets are in circular orbits about the Sun and move in the same plane. To determine the $\Delta\bar{v}$, examine the mission as 3 two-body problems:

I. Short time near departure planet (S/C ⊕)

II. Long time under solar influence (S/C ⊙)

III. short time near arrival planet (S/C ♂)



Denoted the "patched conic" approach.
Approximate but yields a good guess of the requirements. Consider each phase separately.

Two-Body Problem #1 (near ⊕)

Assume that the vehicle is originally in a circular "parking" orbit around the Earth. Motion is influenced solely by the ⊕, totally neglecting the Sun since, at that close range, the Earth is certainly dominant. The vehicle will transfer "instantaneously" from the influence of the ⊕ to the influence of the ⊙. To escape the ⊕, the spacecraft must be on a parabolic or hyperbolic orbit with respect

Observations:

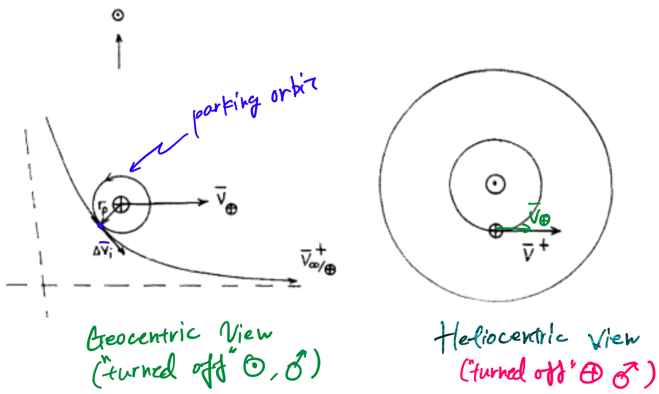
(1) $\Delta\bar{V} = \bar{v}_2 - \bar{v}_1 \longrightarrow$ vector eqn

(2) E eccentric anomaly
how is it measured?

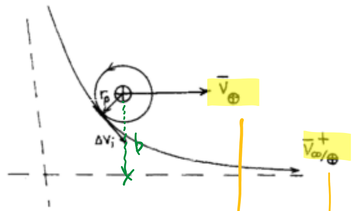
(3) Diagrams larger

(4) $\hat{B}$ unit vector CK how defined?!

to the Earth. Once "escaped", the vehicle must be moving on the correct transfer orbit about the ⊙ to enable arrival at ♂. Consider two views of the situation.



- Circular parking at ⊕ (could be any orbit actually)
- No effect of ⊙
- Transfer "instantaneously" from influence of ⊕ to influence of ⊙
- To escape ⊕, must depart on parabola or hyperbola
- Once escaped, possess exactly correct velocity for transfer orbit about ⊙
- For trip to ♂, s/c velocity wrt ⊙ must be > velocity of Earth wrt ⊙ (gain energy) ⟶



assuming $\Delta\overline{V}$
periapsis of $\oplus$
hyperbola
tangential

Geocentric: maneuver $\Delta\overline{v}_i$ (tangential) so spacecraft jumps from circular $\oplus$ orbit to hyperbolic orbit to escape $\oplus$. After escape, velocity is $\overline{v}^{+}_{\infty/\oplus}$. Note that this is a vector quantity. To determine s/c velocity with respect to $\odot$, relate vectors

$$\overline{v}^{+} = \overline{v}^{+}_{\infty/\oplus} + \overline{v}_{\oplus}$$

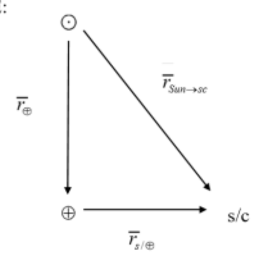
heliocentric velocity of s/c on ellipse wrt

excess velocity at $\oplus$ escape

heliocentric velocity of $\oplus$ wrt $\odot$

V_∞ velocity along the hyperbola

NOTE:



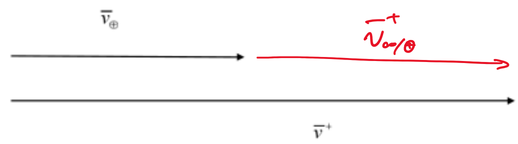
/ ← relative "slash"

$$\boxed{\begin{aligned} \overline{r}_s &= \overline{r}_{s/\oplus} + \overline{r}_{\oplus} \\ \overline{v}_s &= \underbrace{\overline{v}_{s/\oplus}}_{\frac{{}^{i}d\overline{r}_{s/\oplus}}{dt}} + \overline{v}_{\oplus} \\ \overline{v}^{+} &= \overline{v}^{+}_{\infty/\oplus} + \overline{v}_{\oplus} \end{aligned}}$$

VECTOR Equation → requires vector diagram

a tong depart
for Hohmann transfer

$$\bar{V}^{+} = \bar{V}_{\oplus} + \underbrace{\bar{V}^{+}_{\infty/\oplus}}$$



Assuming knowledge of the required $\bar{v}^{+}$ to transfer to ♂, solve for the $\bar{v}^{+}_{\infty/\oplus}$ vector required to escape

Use excess velocity to compute the exact vector $\Delta\bar{v}_i$ to jump from the parking orbit to the required hyperbola.

Jump at hyperbolic perigee most effective

required change in velocity

At perigee, velocity on hyperbola: $v_c + \Delta v_i$

circular velocity in PO (parking orbit)

tangential maneuver

$\boxed{\text{energy } \varepsilon \text{ on hyperbolic orbit}}$ → $$\varepsilon = \frac{v_\infty^2}{2} = \frac{(v_c + \Delta v_i)^2}{2} - \frac{\mu_\oplus}{r_p}$$

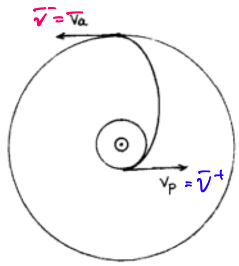
$$\boxed{\Delta v_i = \sqrt{v_\infty^2 + \frac{2\mu_\oplus}{r_p}} - v_c}$$

← magnitude of Δv_i to depart ⊕ orbit

Δv_i: initial burn required to place s/c on heliocentric ellipse with correct v^{+}

Two-Body Problem #2 (influence of ⊙)

- assume: leaving and approaching massless planets
- $v^{+} = v_p$ i.e., perihelion on Hohmann transfer ellipse
- easily compute velocity wrt ⊙ at ♂ arrival, i.e., the apohelion velocity on Hohmann transfer ellipse
- $v_a = v^{-}$ i.e., heliocentric velocity for ♂ approach



Note: departure from ⊕ phased correctly so Mars is in proper position when s/c rendezvous

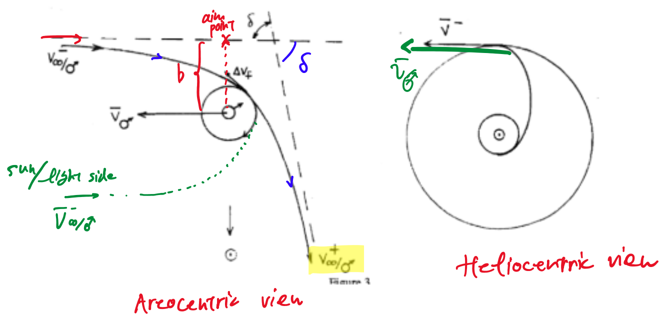
Two-Body Problem #3 (near ♂)

- Assume that the goal is capture into a circular orbit about ♂ at a given radius r_f; also consider the circular orbit must be defined in a particular direction
- Since s/c at apohelion on transfer ellipse, s/c will be moving

- s/c will enter Martian vicinity on hyperbola and – at appropriate time – add Δv_f to slow s/c for capture

- final velocity

dark side arrival

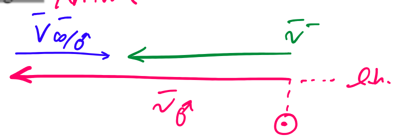


Use heliocentric velocity to determine the velocity on the hyperbola:

$$\bar{v}^-_{\infty/♂} = \bar{v}^- - \bar{v}_♂$$

$\bar{V}^- = \bar{V}_♂ + \bar{V}^-_{\infty/♂}$

Vector Diagram Arrival



CAPTURE

Solve the vector equation for $\bar{v}^-_{\infty/♂}$ and determine Δv_f

$\bar{V}_c = \bar{V}_{ph} - \Delta$

$$\boxed{\frac{V_\infty^{2}/♂}{2} = \frac{(V_c + \Delta V_f)^2}{2} - \frac{\mu_♂}{r_f}}$$

Total: $\Delta v_{total} = \Delta v_i + \Delta v_f$

SWINGBY

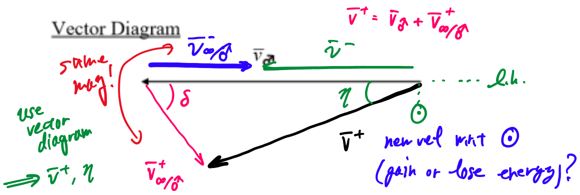
Rather than capture into ♂ orbit, assume pass by of planet at a closest approach of r_f

⇒ same approach hyperbola, but continues past planet on outbound leg of hyperbola

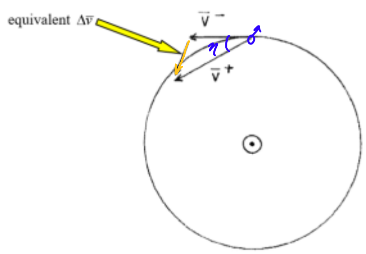
Below: passes on the "dark" side i.e., "behind" Mars

Use vector relationships to determine impact of passage on spacecraft heliocentric velocity

Vector Diagram



passage through the local gravity field of Mars instantaneously changes the **magnitude** and **direction** of the spacecraft heliocentric velocity



$\underbrace{\bar{v}^+ - \bar{v}^-}_{\text{instantaneous } \Delta v} = \Delta\bar{v}_{eq}$

Obtained a change in heliocentric velocity for "free" !!!
Can calculate new r, v, γ for elliptical heliocentric orbit

Consider the following:

How can you calculate $|\bar{v}^+|$ and α? from vector diagram
Does the spacecraft gain or lose energy via the Mars passage?
How would $|\bar{v}^+|$ and α change if the spacecraft passed on the "light" side of the planet? $|\bar{v}^+| > |\bar{v}^-|$
Recall that arrival occurred tangentially. What if arrival occurred at an arbitrary point on the transfer ellipse?
How can the departure from Earth be timed such that Mars is in the assigned position at the proper time?

$$\left.\begin{aligned} \mathcal{E}^- &= \frac{v^{-2}}{2} - \frac{\mu_\odot}{r^-} \\ \mathcal{E}^+ &= \frac{v^{+2}}{2} - \frac{\mu_\odot}{r^+} \end{aligned}\right\} \quad r^- = r^+ \text{ in heliocentric view}$$

Amount of "gravity assist" generated depends on closest pass to Mars (or planet)

Example: $v_\oplus = 29.45$ km/s

$v^- = 21.48$ km/s $\qquad v_{♂} = 24.13$ km/s

$$\boxed{\begin{aligned} a &= \frac{1}{2}(r_\oplus + r_{♂}) = 1.888\times10^8 \text{ km } (1.26\text{AU}) \\ \mathcal{E} &= \frac{v_\infty^2}{2} - \frac{\mu_\odot}{r_{♂}} \stackrel{\odot}{=} -\frac{\mu}{2a} \Rightarrow v_a = 21.48 \text{ km/s} \end{aligned}}$$

Hohmann on apoapsis

$v^-_{\infty/♂} =$ 2.65 km/s

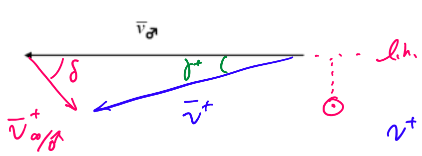
Assume closest approach to ♂

$r_f = R_{♂} + 1500\text{km} \quad (4.91\times10^3 \text{ km})$ r_p on hyperbola

$\delta =$ 67.36°

$$\boxed{\begin{aligned} \mathcal{E} &= \frac{v_\infty^2}{2} = \frac{\mu_{♂}}{2|a|} \Rightarrow |a| = 6.1132\times10^3 \text{ km} \\ e &= \frac{r_p}{|a|} + 1 \\ \sin\frac{\delta}{2} &= \frac{1}{e} \end{aligned}}$$

hyperbola



cosine law

$v^+ = 23.2$ km/s

$\gamma^+ = -1.65°$ → below l.h.

$\bar{v}^+_{\infty/♂}$ $\bar{v}^+$ ⊙

cosine law

$v^+ = 23.2$ km/s

$\gamma^+ = -1.65°$ → below l.h.

sin law

K10

Alternatively:

$r_f = R_{♂} + 200\text{km}$ $\quad (3.61 \times 10^3 \text{ km})$

$\delta = 77.9°$

$v^+ = 23.7$ km/s

$\gamma^+ = -6.3°$

gain energy

- Of course, a sunside passage will yield same v^+ but now γ^+ positive → changes orbit relative to Sun
  But $v^+ > v^-$ → gain energy (free Δv)

- Project Galileo: at one time, plans included a flyby of ♂ on way to Jupiter

  Hohmann to ♂ — 8.5 months
  actual TOF ~ 3 months
  poss distance (r_p on hyp) ~275 km altitude

- Patched-conic method for calculating trajectories (or Δv estimates) yields pretty accurate thrust requirements (surprisingly) → pretty accurate nominal trajectories [some error in transfer time because only time along actual Hohmann is incorporated]

  hyperbola

- Greatest difficulty in practice is <u>implementation</u>
  Hyperbolic excess speed, v_∞, is extremely sensitive to small errors in injection velocity

  Example: ⊕ departure

circ park → $\bar{v}_c + \Delta\bar{v} = \bar{v}_p$ ← Velocity required at periapsis of hyperbola

added thrust

from energy $\mathcal{E}$: energy on hyperbola for ⊕ departure

$v_\infty^2 = v_p^2 - \frac{2\mu}{r_p}$ first-order difference eqn for small errors in v_p

$$2 v_\infty \delta v_\infty = 2 v_p \delta v_p$$

$$\frac{\delta v_\infty}{v_\infty} = \left(\frac{v_p}{v_\infty}\right)^2 \frac{\delta v_p}{v_p}$$

Hohmann to Mars → $v_p \cong 11.5\,\mathrm{km/s}$ $v_\infty = 3\,\mathrm{km/s}$

1% error in v_p

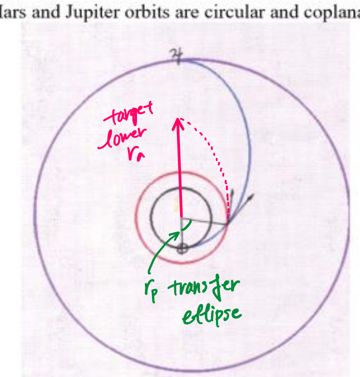
⟹ 15% error in v_∞

assume perfect implementation

Example: Swingby when Arrival not Tangential

Assume Hohmann transfer to Jupiter and intercept Mars
How will Mars swingby affect orbit?

Assume Mars and Jupiter orbits are circular and coplanar



→ smaller r_a
ΔV lower than necessary at ⊕
⇓
more efficient with fly-by
complications:
- shift in line-of-apsides
- 2 phase angle
- navigations
- untangential arrival

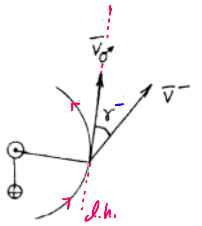
Determine conditions as Mars arrival

$$a = \frac{1}{2}\left(r_\oplus + r_{Jup}\right) = 3.1\,\text{AU}$$

$$r_\oplus = r_p = a(1-e) \Rightarrow e = .677 \qquad (p = 1.68\,\text{AU})$$

♂ radius $r_{♂} = \dfrac{a\left(1-e^2\right)}{1+e\cos\theta^*} \Rightarrow$ $\theta^* = 81.3°$

Assume: Mars is in circular orbit : $v_{♂}$ circular vel along l.h.



$$\boxed{\bar{v}_{♂} = 24.187\,\text{km/s}}$$

$$\frac{(v^-)^2}{2} - \frac{\mu_\odot}{r_{♂}} = -\frac{\mu_\odot}{2a_T}$$ } for transfer path

$$\longrightarrow \boxed{v^- = 29.7\,\text{km/s}}$$

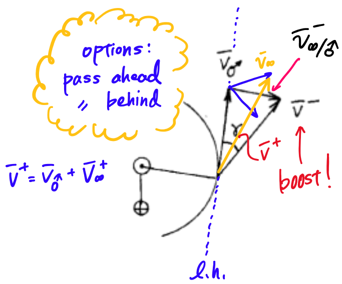
variety of ways to determine γ^-

$$\frac{v_p^2}{2} - \frac{\mu_\odot}{r_p} = -\frac{\mu_\odot}{2a} \longrightarrow v_p = 38.66\,\text{km/s}$$

$$h = \sqrt{\mu_\odot p} = r_{♂} v^- \cos\gamma \longrightarrow \gamma = 31.3^\circ \quad \text{(why +?)}$$

$\gamma = \pm 31.3^\circ$

↑ ascending on transfer orbit
$0^\circ < \theta^* < 180^\circ$



known known

$$\bar{v}^- = \bar{v}_{♂} + \bar{v}^-_{\infty/♂}$$ Vector eqn. ← use to solve for v_∞

$$\bar{v}^-_{\infty/♂} = \bar{v}^- - \bar{v}_{♂}$$

Solve for $v_{\infty/♂}$ from cosine law

$$v^2_{\infty/♂} = v^2_{♂} + (v^-)^2 - 2v_{♂}v^-\cos\gamma$$ $\measuredangle \gamma$

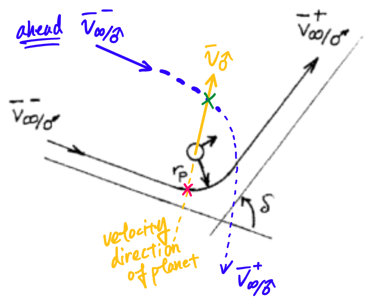
$$\boxed{v^-_{\infty/♂} = 15.476\ \text{km/s}}$$

That is fast!

get a "boost" to outer solar system by Mars encounter

which way to pass to increase or decrease energy?

Mars is moving away in this instance



"leading" or "trailing"

"ahead" or "behind"

GAIN energy?
Pass behind

$r_f = R_♂ + 300\,\text{km}$ altitude

choose a close pass of ♂

$r_{pass} = r_p$ of hyp.

Hyperbolic orbit wrt ♂

$\rightarrow |\bar{V}^-_{\infty/♂}| = |\bar{V}^+_{\infty/♂}|$

known

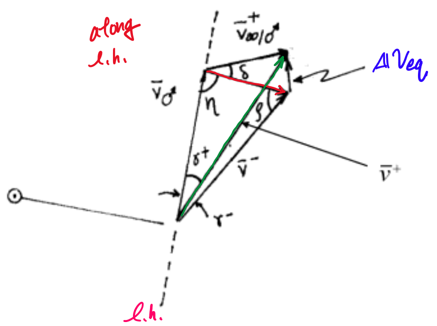
$$\mathcal{E} = \frac{v_\infty^2}{2} = \frac{\mu_\odot}{2|a|} \longrightarrow |a| = 179.536\,\text{km}$$

$$e = \frac{r_p}{|a|} + 1 = 21.55$$

$$\sin\frac{\delta}{2} = \frac{1}{e} \longrightarrow \boxed{\delta = 5.3^\circ}$$ passing a small planet very fast

what impact

$\rightarrow \bar{V}^{-}_{\infty/\sigma}$



$$\frac{v^{-}}{\sin\eta}=\frac{v^{-}_{\infty/\sigma}}{\sin\gamma^{-}}$$ $\eta =$ 85.57° or 94.43° how to check?

$$\frac{v_{\sigma}}{\sin\varsigma_{\rho}}=\frac{v^{-}_{\infty/\sigma}}{\sin\gamma^{-}}$$ ρ $\varsigma =$ 54.3° or 125.7°

$$\left(v^{-}\right)^{2}=\underbrace{\left(v^{-}_{\infty/\sigma}\right)^{2}}+v_{\sigma}^{2}-2v^{-}_{\infty/\sigma}v_{\sigma}\cos\eta \rightarrow \cos\eta \text{ negative}$$

Need to add to this term

$v^{-} = 29.7$ km/s

$$\left(v^{+}\right)^{2}=v_{\sigma}^{2}+\left(v^{+}_{\infty/\sigma}\right)^{2}-2v_{\sigma}v^{+}_{\infty/\sigma}\cos(\eta+\delta)$$ $\boxed{v^{+} = 30.84 \text{ km/s}}$

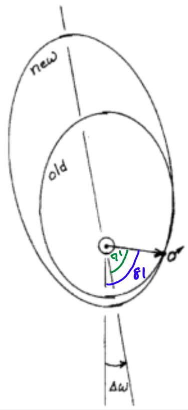
Increased velocity $\rightarrow$ higher energy orbit (pass ahead, drop vel?)

change $v \sim 1$ km/s

$$\frac{v^{+}_{\infty/\sigma}}{\sin\gamma^{+}}=\frac{v^{+}}{\sin(\eta+\delta)} \rightarrow \boxed{\gamma^{+}=29.63^{\circ}}$$

$\theta^{*-} = 81.3^\circ$

$$\tan\theta^* = \frac{\left(\frac{r v^2}{\mu}\right)\sin\gamma\cos\gamma}{\left(\frac{r v^2}{\mu}\right)\cos^2\gamma - 1} \longrightarrow \theta^{*+} = 71.9^\circ \text{ or } 252^\circ \quad (\gamma > 0)$$



$\Delta\varpi = 9.4^\circ$

perihelion advances

	before encounter	after encounter
r	1.52 AU	1.52 AU
v	29.7 km/s	30.84 km/s
γ	31.3°	29.63°
e	.677	.7348
θ^*	81.3°	71.9°
a	3.07 AU	4.02 AU
r_p	1 AU	1.066 AU
r_a	5.2 AU	6.97 AU