AE 6210: Advanced Dynamics HW5

Consider the double compound pendulum moving in the *x-y* plane as shown below. The rods are equal length, equal mass, and rigid. The hinges are frictionless. Derive the equations of motion using:

- 1) Lagrangian mechanics or Hamilton's principle or Lagrange's equations
- 2) Newton's laws or Newton-Euler equations

As you derive the equations you will have to:

- a) choose your generalized coordinates (can be different for each method if you prefer)
- b) choose your motion variables/generalized velocities if you like for Newton's laws

Show that both approaches give equivalent equations of motion. You can do this either (i) analytically (by deriving one set of equations from another – you may be lucky in some instances and derive exactly the same equations from two or more methods), or (ii) semi-analytically (by numerically calculating the accelerations using the different sets of equations for a given numerical value of generalized coordinates and velocities), or (iii) computationally (by simulating the system using the different sets of equations for a given initial condition).



Note: Kinetic energy of a rigid body (a scalar) is given by:

$$T = \frac{1}{2} m \, \vec{v}^{\,C} \cdot \vec{v}^{\,C} + \frac{1}{2} \vec{\omega}_B \cdot \vec{\vec{I}}^{\,C} \cdot \vec{\omega}_B$$

where, the velocity vector is of the CM, moment of inertia matrix is about the CM, and the angular velocity is of the body.

The above equation can be and should be simplified for the 2D problem before using it.