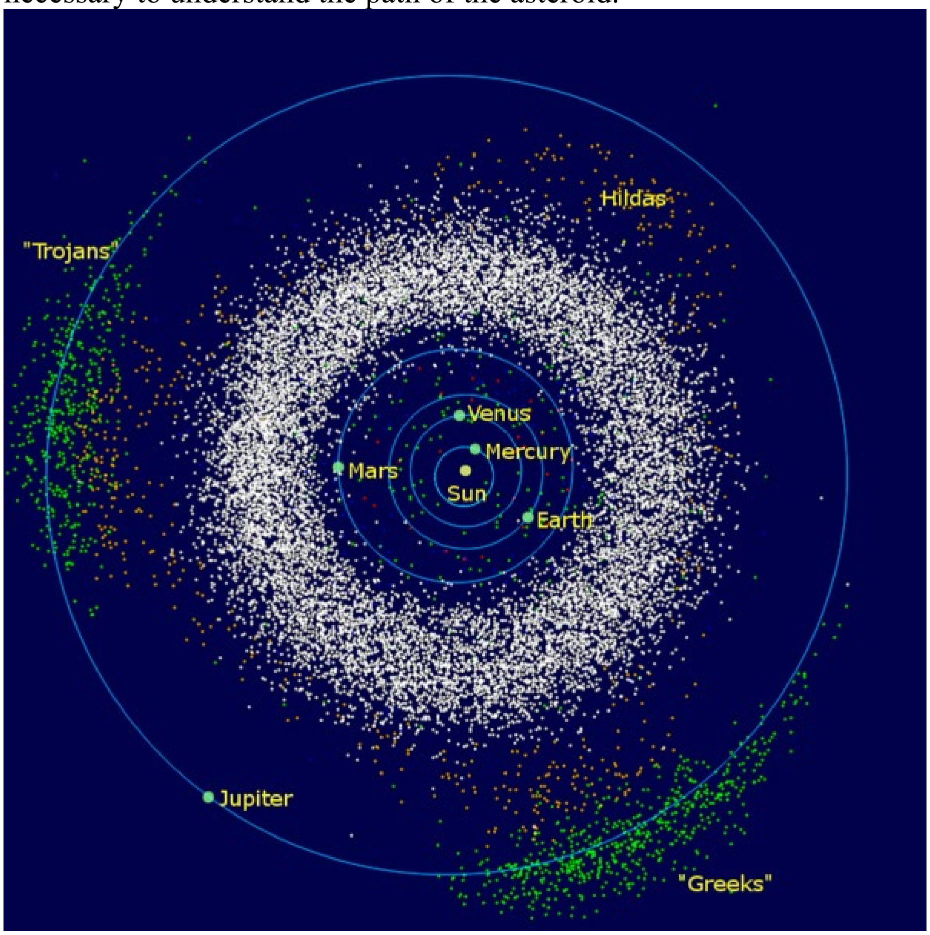
AAE 532 – Orbit Mechanics
Problem Set 3
Due: 9/18/20

Problem 1: A mission concept was proposed recently to deliver a spacecraft to one or more of the Trojan asteroids, e.g., 8317 Eurysaces. Of course, as seen in the figure, there are many, many asteroids. As they move along their orbital path, the 'green' Trojans remain in the same relative locations with respect to the Sun and Jupiter. To develop a trajectory for the mission, it is also necessary to understand the path of the asteroid.



The asteroids of the inner solar system and Jupiter

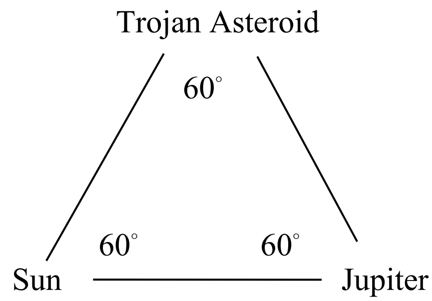
- ***Jupiter trojans***
- Asteroid belt
- Hilda asteroids
- Orbits of planets

Note that the Jupiter Trojans are divided into two groups: The Greek camp (in front of Jupiter) and the Trojan camp (trailing behind Jupiter).

(a) Return to the small body database and check the orbit of the asteroid Eurysaces. Use a view to "Look at Sun" and "Look from Above". Is Eurysaces in the Greek camp or the Trojan camp?
The database lists the orbital period of Eurysaces in years. Compare it to the period of Jupiter in its orbit relative to the Sun.

Take 3 images as Jupiter moves through its orbit relative to the Sun. Use a start date of 9/18/20 and select 3 other dates along Jupiter's orbit. From the images, measure approximately the angle between the lines Sun-Jupiter and Sun-Eurysaces. How much does the angle change over your three dates?

(b) For a preliminary assessment, the positions of a sample Trojan asteroid (e.g. Eurysaces), the Sun, and Jupiter can be modeled as located at the vertices of an equilateral triangle as envisioned below.



Assume that the distance between the Sun and Jupiter is equal to the semi-major axis of Jupiter's orbit. Let the mass of the asteroid be assumed as $\mu = 75 \text{ km}^3/\text{sec}^2$. Consider the net acceleration on the asteroid.

(i) Write the expression for the acceleration of the <u>asteroid relative to the Sun</u> where Jupiter is a perturbing body. [Note that the definition of a set of unit vectors is necessary.] Write this expression in the form $\ddot{\bar{r}}_{\odot \rightarrow asteroid} =$ (sum of terms). Label each of the following terms in this expression: dominant term, direct perturbing terms, indirect perturbing terms.
Determine the magnitude and direction of each of the terms in the expression, as well as the net perturbing accelerations for each body and the total net acceleration.

(ii) Re-formulate the problem and write the expression for the acceleration of the <u>asteroid relative to Jupiter.</u> Again, determine the magnitude and direction of each of the terms in the expression, as well as the net perturbing accelerations for each body and the total net acceleration.

(iii) Which term is the largest in each formulation? How does the magnitude of the dominant term in each formulation compare?

Determine the net perturbing acceleration in each case? Which has the largest impact in each formulation, Sun or Jupiter?

Is the net total acceleration on the asteroid the same in each formulation? Should it be the same? Why or why not?

Which formulation is correct? Why?

From the results here, is it reasonable to model the motion of the asteroid as a two-body problem, i.e., Sun-asteroid or Jupiter-asteroid? Why?

Problem 2: An Introductory Manual (Intro_Manual_F20 GMATR2020a with OpenFrames) for the General Mission Analysis Tool (GMAT) software is posted under GMAT on Brightspace. GMAT is open source and is easily downloaded.
To obtain some practice using GMAT, step through the manual carefully. Complete all the steps and view the final orbits.
(a) Now use an Epoch of 18 Sept 2020. Produce a satellite orbit with a 'semi-major axis' of 60,000 km, 'eccentricity' of 0.7, and an 'inclination' of 45°. Note that you are using an Earth point mass model.
(In the Resource tree, for the 'LowEarthProp' propagator, replace Gravity Model with 'JGM-2' but set degree and order to zero to render a point mass model. Under 'DefaultOtbitView' it will be a cleaner image if you do NOT enable the constellations.)
Plot images from GMAT. Also print the summary of the orbit details from the "Report" option. Under the Resources tree, right click on 'Output'. You are offered the opportunity to add a report file with numerical data from the simulation. The report will appear in the Output tree.
From the output data, determine

(i) radius at closest approach or Rad. Peri.
(ii) radius at farthest excursion or Rad. Apo.
(iii) energy
(iv) semi-major axis
(v) semi-latus rectum
(vi) angular momentum
(v) the Cartesian components of position and velocity at the initial time.
With what reference frame are these associated?

(Do not submit the entire output report; cut-and-paste the sections with the required data into a Word document. Highlight the requested quantities.)

(b) Given the orbit in the scenario in part (a), set the inclination to zero. Add a second spacecraft with an orbit of a different color. With the same eccentricity and zero inclination, try a different semi-major axis, i.e., 40,000 km. Add a third satellite with a = 75,000 km.
Repeat the exercise for a = 60,000 km and three eccentricities, i.e., e = 0.2, 0.65, 0.88 In each case, hold the inclination fixed at 45°. [Note that some combinations go below the radius of the Earth…they can be excluded.]
Repeat the exercise for e = 0.65 and three semi-major axes, i.e., a = 20,000 km, 35,000 km, 75,000 km.

You should have three plots from GMAT; use a view that is looking down on the orbit plane for variations in semi-major axis and eccentricity.

Problem 3: Consider only the relative two-body problem (Earth and spacecraft). An Earth-orbiting vehicle is tracked from ground stations; the spacecraft mass is 600 kg. At a certain instant (t_o), the following position and velocity information is obtained relative to an inertial observer:

Altitude = 8560 km
Radial component of relative velocity = +2.11 km/s
Transverse component of relative velocity = 4.89 km/s
(Our tracking system is perfect so these states are completely without error!!)

(a) Compute the total system angular momentum $\bar{\boldsymbol{C}}_3$, specific angular momentum, total kinetic energy for the system, total energy C_4, specific energy, areal velocity.
(b) What is the value of the coefficient by which to multiply C_4 to obtain specific energy?
(c) Within the context of the relative two-body problem, determine the following orbital characteristics: $p, e, a,$ period, γ, θ^*. Write the position vector in terms of the inertial unit vectors $\hat{e}$ and $\hat{p}$.
(d) Compare this relative velocity to the circular relative velocity at this altitude.