AAE 364: Control Systems Analysis

HW 10: Bode & Nyquist Plots

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B–7–3. Using MATLAB, plot Bode diagrams of $G_1(s)$ and $G_2(s)$ given below.

$$G_1(s) = \frac{1+s}{1+2s}$$

$$G_2(s) = \frac{1-s}{1+2s}$$

$G_1(s)$ is a minimum-phase system and $G_2(s)$ is a nonminimum-phase system.

Minimum Phase Sys.

$$G_1(s) = \frac{1+s}{1+2s} \Rightarrow G_1(j\omega) = \frac{1+j\omega}{1+j\omega 2}$$

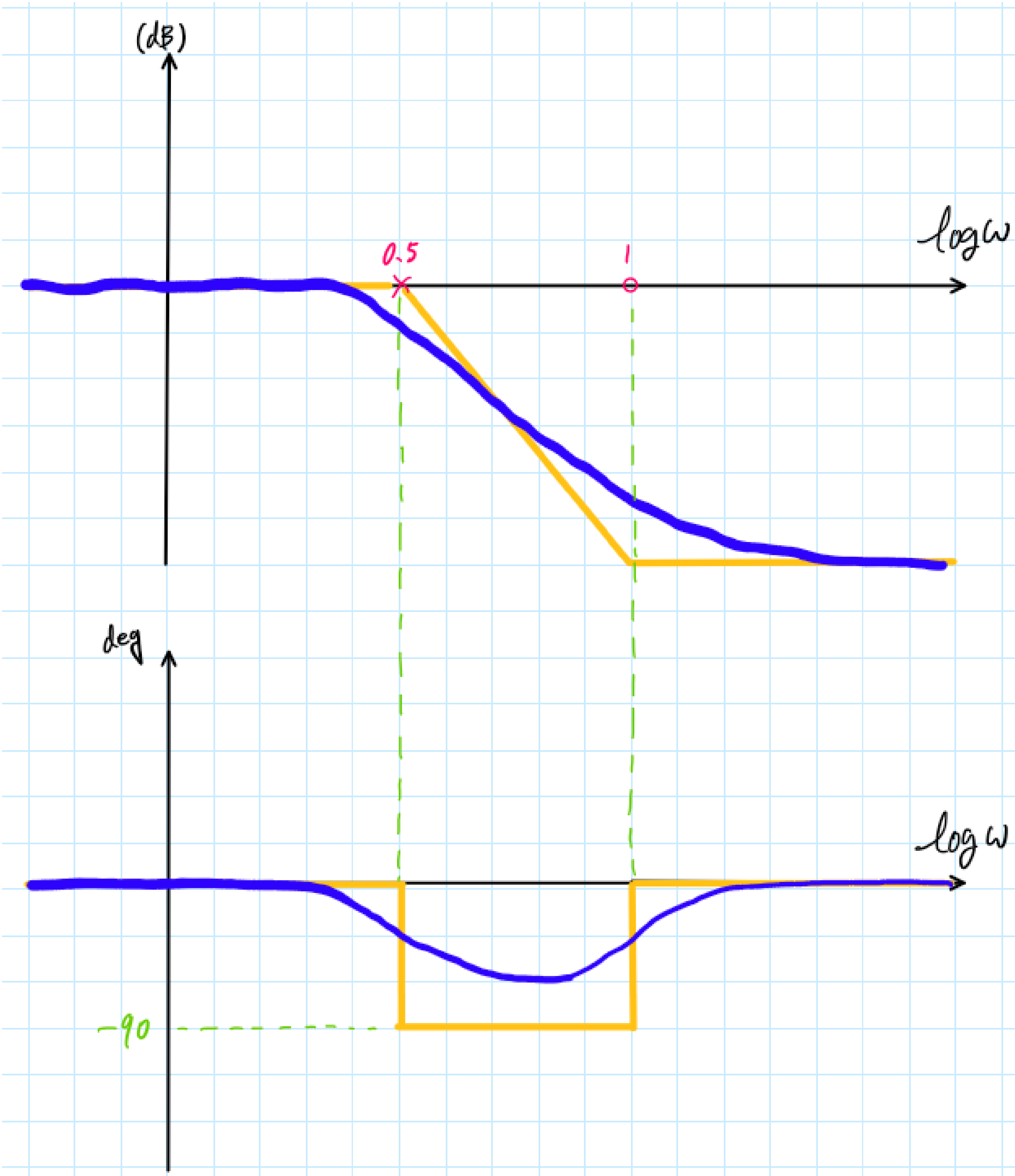
corner frequencies : $\omega_1 = 1$, $\omega_2 = \frac{1}{2} = 0.5$

- Magnitude

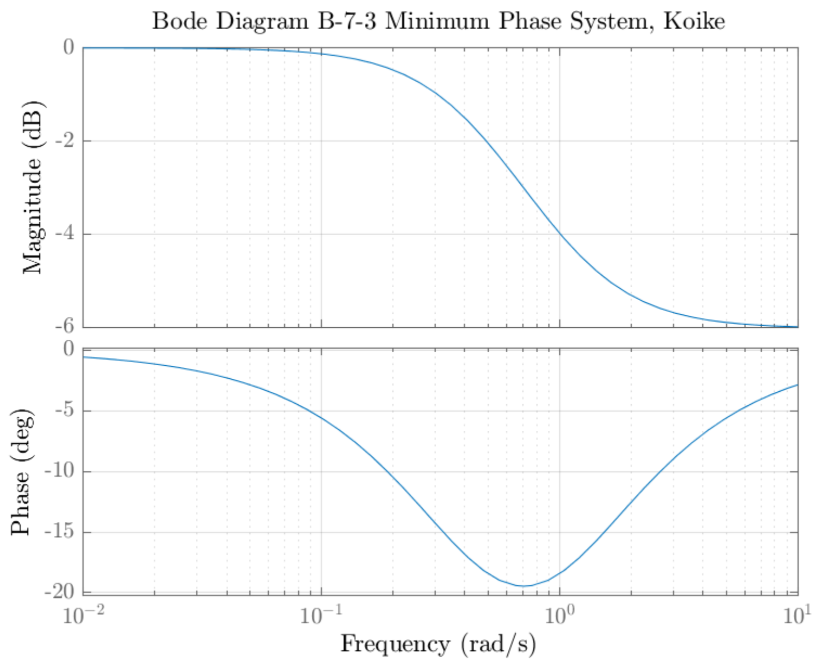
$$20\log|G(j\omega)| = 20\log|1+j\omega| - 20\log|1+j\omega 2|$$

- Phase

$$\arg[G(j\omega)] = \arg(1+j\omega) - \arg(1+j\omega 2)$$



(dB)
log ω
0.5
1
deg
log ω
−90



Non-Minimum Phase Sys.

$$G_1(s) = \frac{1-s}{1+2s} \Rightarrow G_1(j\omega) = \frac{1-j\omega}{1+j\omega 2}$$

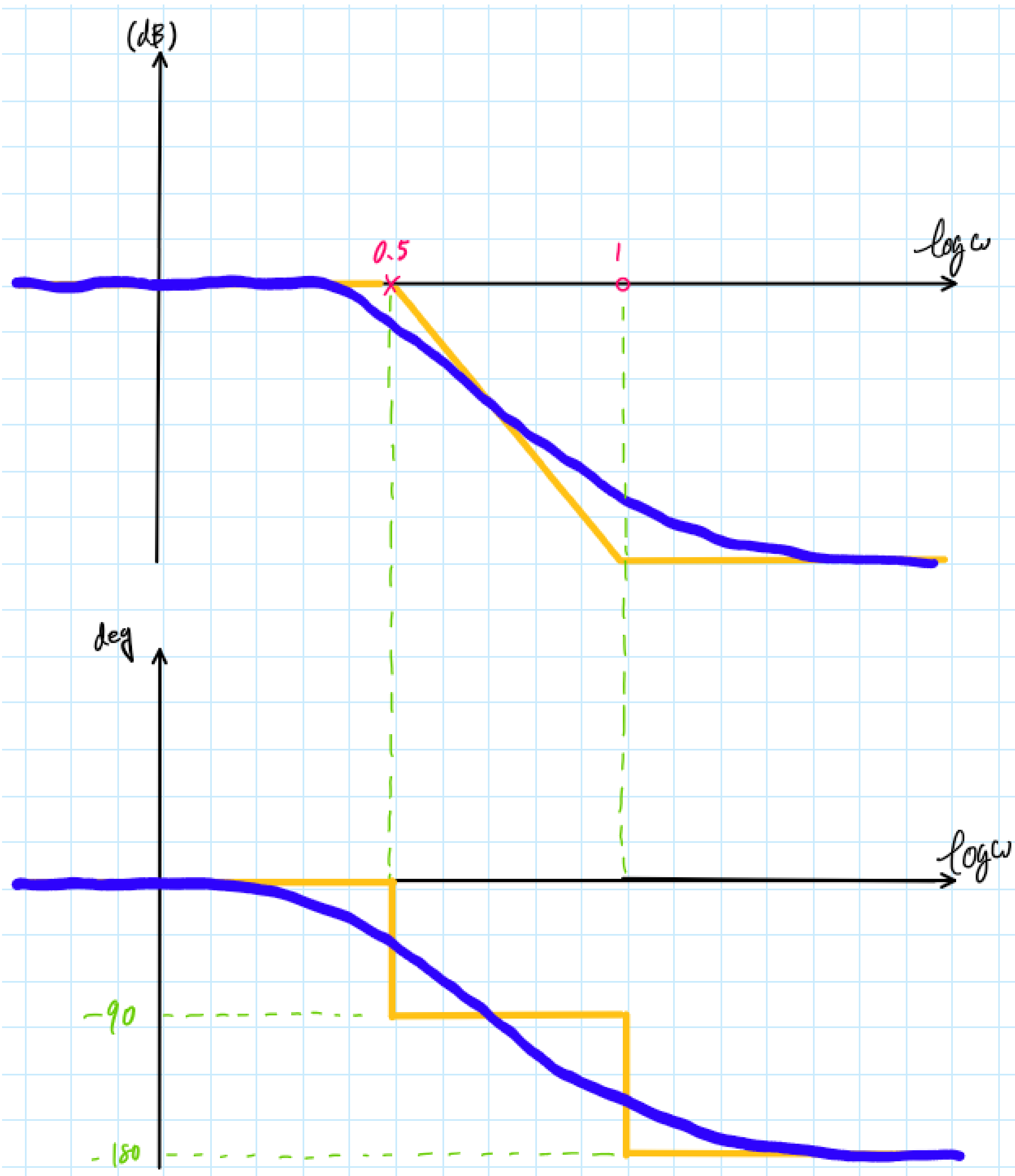
corner frequencies : $\omega_1 = 1$, $\omega_2 = \frac{1}{2} = 0.5$

- Magnitude

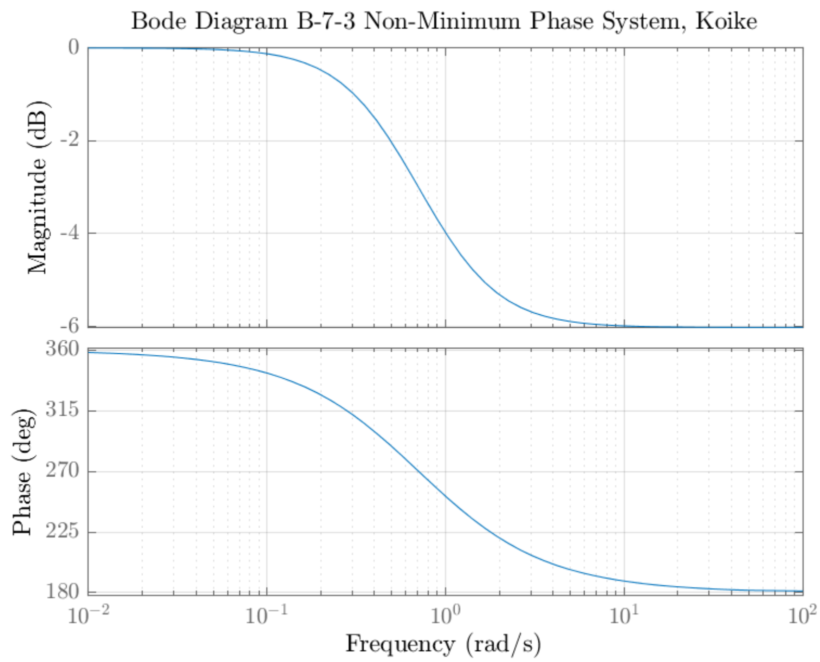
$$20\log|G(j\omega)| = 20\log|1-j\omega| - 20\log|1+j\omega 2|$$

- Phase

$$\arg[G(j\omega)] = \arg(1-j\omega) - \arg(1+j\omega 2)$$



(dB)
0.5
1
log ω
deg
log ω
-90
-180


Bode Diagram B-7-3 Non-Minimum Phase System, Koike
Magnitude (dB)
0
-2
-4
-6
Phase (deg)
360
315
270
225
180
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
Frequency (rad/s)

B–7–8. Draw a Nyquist locus for the unity-feedback control system with the open-loop transfer function

$$G(s) = \frac{K(1-s)}{s+1}$$

Using the Nyquist stability criterion, determine the stability of the closed-loop system.

Bode Plot

$$G(j\omega) = \frac{K(1-j\omega)}{j\omega+1}$$

Magnitude

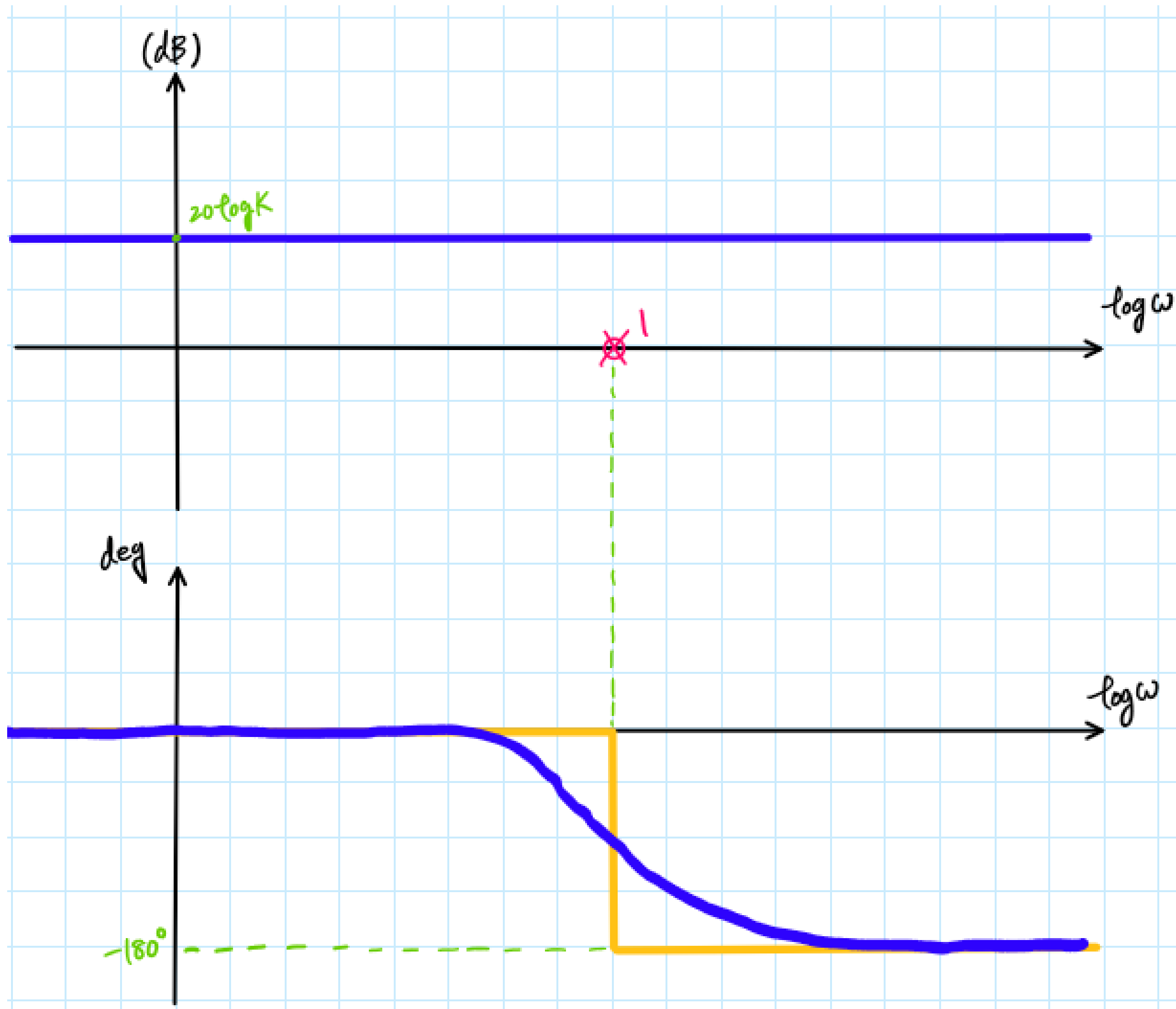
$$20\log[G(j\omega)] = 20\log K + \cancel{20\log|1-j\omega|} - \cancel{20\log|j\omega+1|}$$

Phase

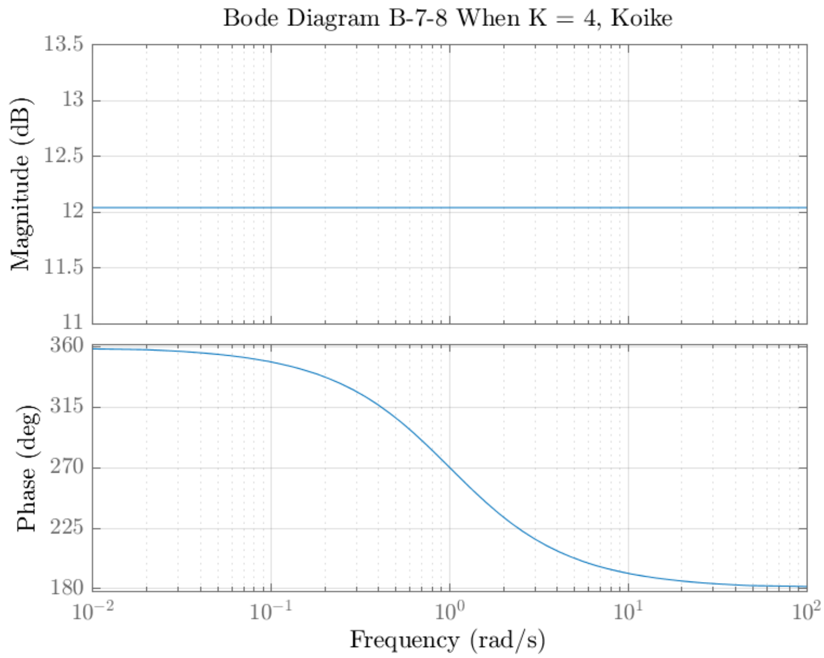
$$\arg[G(j\omega)] = \cancelto{0}{\arg(K)} + \arg(1-j\omega) - \arg(j\omega+1)$$

Corner freq.

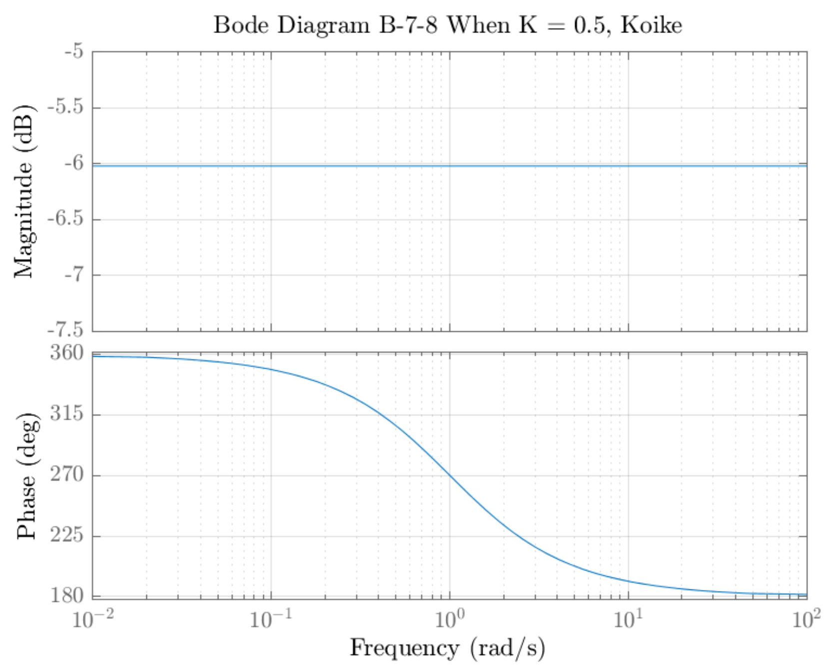
$$\omega_1 = 1$$


(dB)
20logK
log ω
1
deg
log ω
-180°

|K| > 1

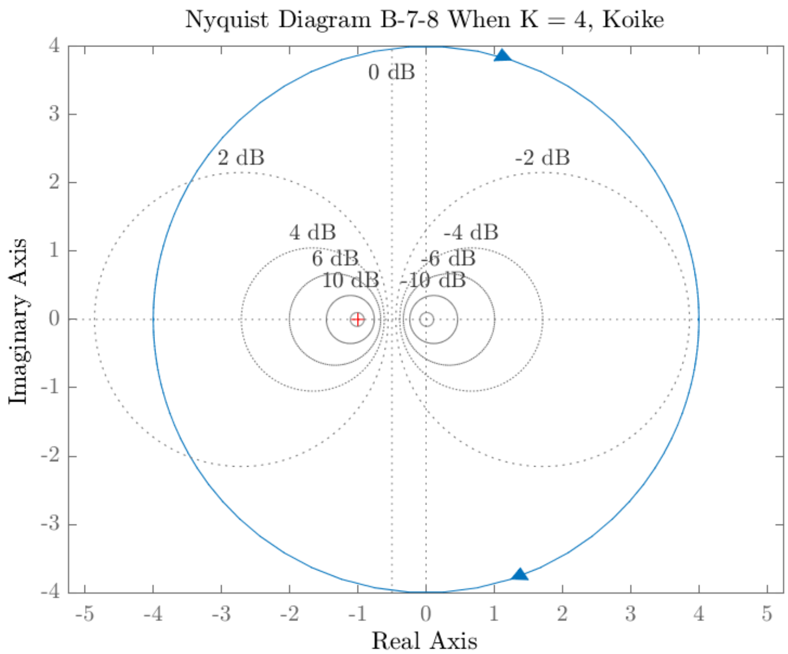

Bode Diagram B-7-8 When K = 4, Koike
Magnitude (dB)
Phase (deg)
Frequency (rad/s)

|K| ≤ 1

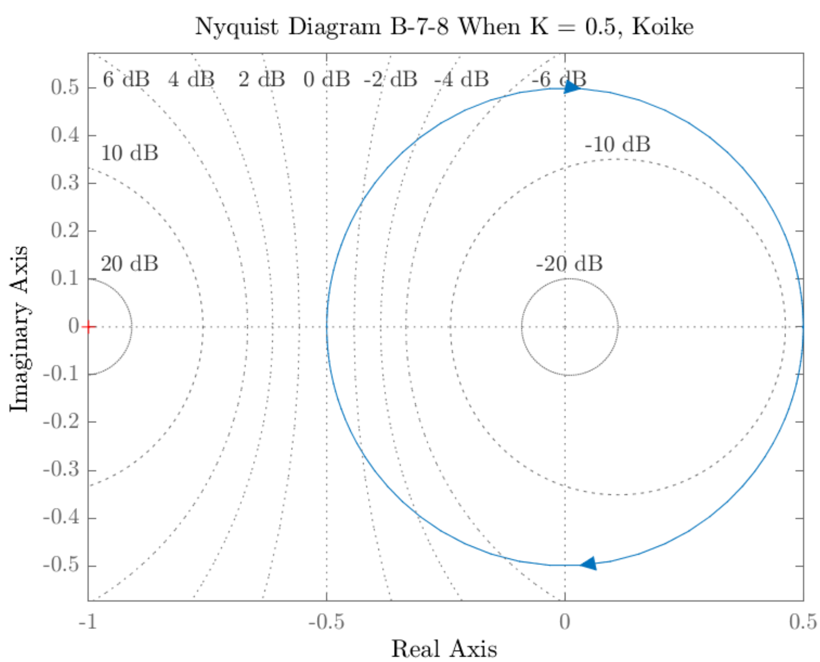

Bode Diagram B-7-8 When K = 0.5, Koike
Magnitude (dB)
Phase (deg)
Frequency (rad/s)

Nyquist Plot

|K| > 1



|K| ≤ 1



Nyquist Stability Criterion

P : the # of OL poles in RHP
N : the # of clockwise encirclements about -1
Z : the # of CL poles in RHP $\Rightarrow Z = N + P$

$|K| > 1$

$P = 0, \ N = 1 \Rightarrow Z = 1$

$|K| \leq 1$

$P = 0, \ N = 0 \Rightarrow Z = 0$

The system is
- unstable if $|K| > 1$
- stable if $|K| \leq 1$

B–7–10. Consider the closed-loop system with the following open-loop transfer function:

$$G(s)H(s) = \frac{10K(s + 0.5)}{s^2(s + 2)(s + 10)}$$

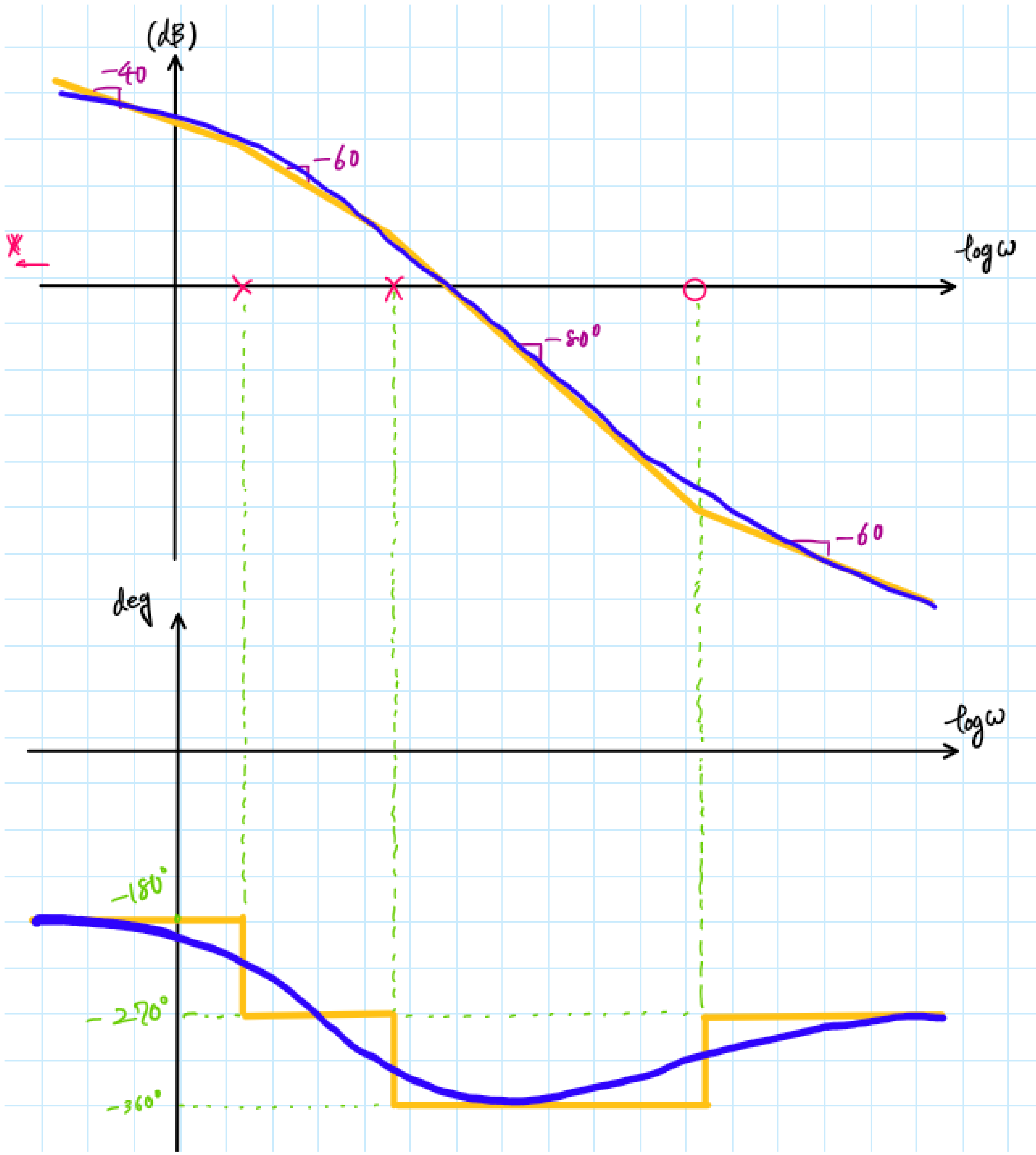
Plot both the direct and inverse polar plots of $G(s)H(s)$ with $K = 1$ and $K = 10$. Apply the Nyquist stability criterion to the plots, and determine the stability of the system with these values of K.

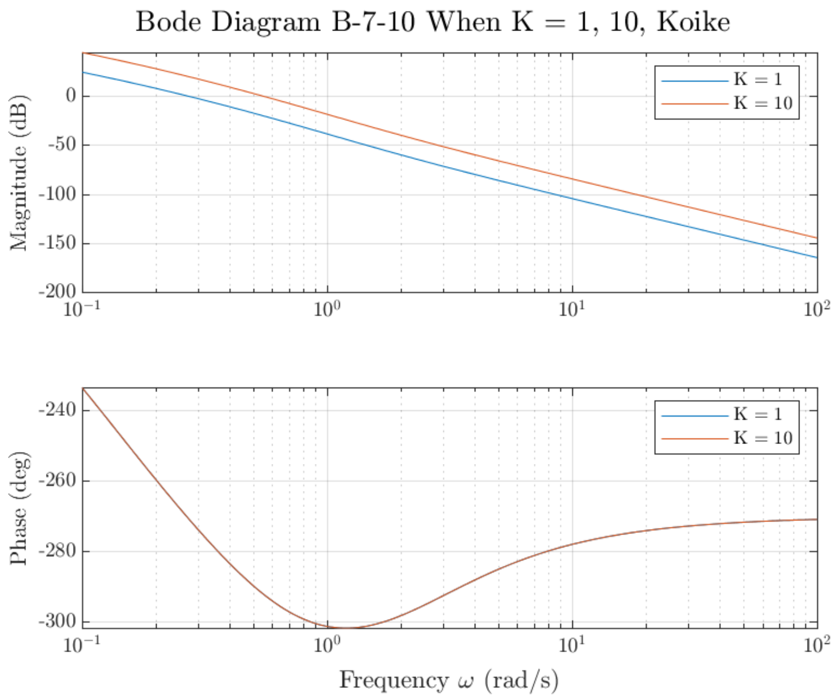
Bode Plot

$$G(j\omega) = \frac{10K(j\omega + 0.5)}{-\omega^2(j\omega + 2)(j\omega + 10)} = \frac{5K(0.5j\omega + 1)}{-20\omega^2(2j\omega+1)(10j\omega+1)}$$

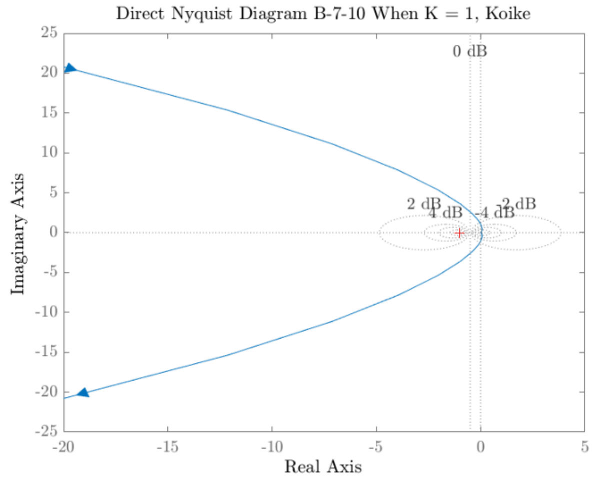
$$\omega \to 0: \quad G \approx -\frac{5K}{20\omega^2} \longrightarrow \infty \angle -180^\circ$$

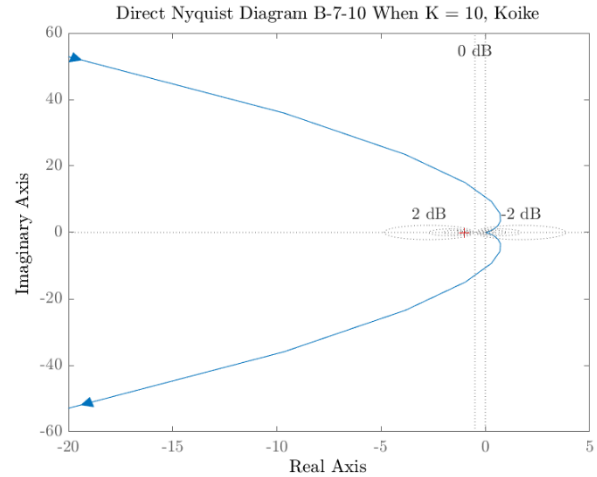
$$\omega \to \infty: \quad G \approx \frac{5K(0.5j\omega)}{-20\omega^2(2j\omega)(10j\omega)} = \frac{K}{160\omega^3} j \longrightarrow 0 \angle 90^\circ$$

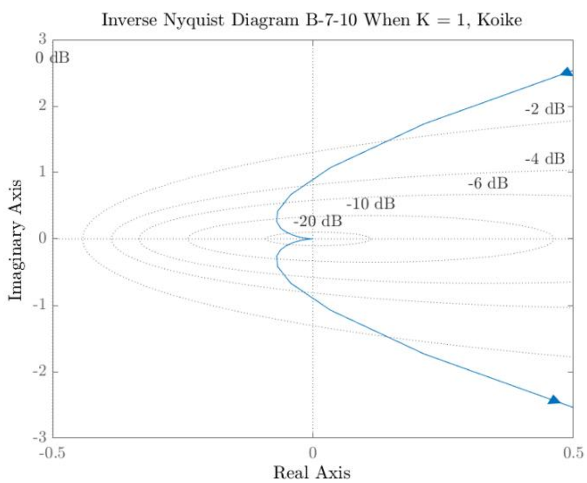

(dB)
-40
-60
log ω
-80°
-60
deg
log ω
-180°
-270°
-360°

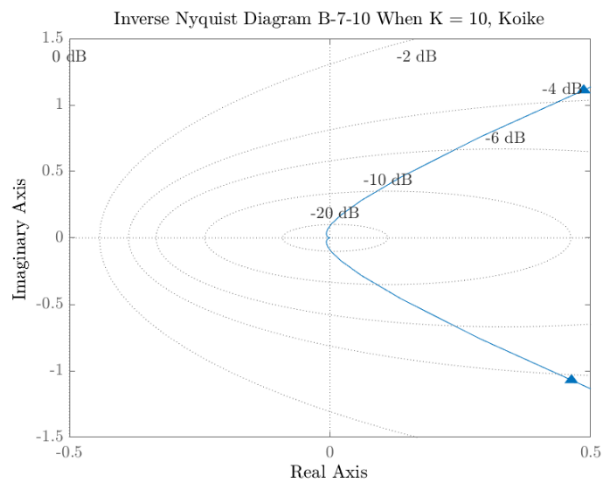

Bode Diagram B-7-10 When K = 1, 10, Koike
Magnitude (dB)
0
-50
-100
-150
-200
K = 1
K = 10
Phase (deg)
-240
-260
-280
-300
10^{-1}
10^{0}
10^{1}
10^{2}
Frequency ω (rad/s)

Nyquist Plots









Nyquist Stability Criterion

P : the # of OL poles in RHP
N : the # of clockwise encirclements about -1
Z : the # of CL poles in RHP $\Rightarrow Z = N + P$

$K = 1$

$P = 0$, $N = 0$ $\Rightarrow Z = 0$ for both direct and inverse

$K = 10$

$P = 0$, $N = 0$ $\Rightarrow Z = 0$ for both direct and inverse

The system is
- stable at $K = 1$
- stable at $K = 10$

B–7–13. Consider a unity-feedback control system with the following open-loop transfer function:

$$G(s) = \frac{1}{s^3 + 0.2s^2 + s + 1}$$

Draw a Nyquist plot of $G(s)$ and examine the stability of the system.

Bode Plot

$$G(s) = \frac{1}{s^3+0.2s^2+s+1} \longrightarrow G(j\omega) = \frac{1}{-j\omega^3 - 0.2\omega^2 + j\omega + 1}$$

let poles be p_i

$$p_1 = -0.7246, \quad p_2 = 0.2623 + j1.1451, \quad p_3 = 0.2623 - j1.1451$$

let $q_i = -p_i$

$$G(s) = \frac{1}{(s+q_1)[s^2 + (q_2+q_3)s + q_2 q_3]}$$

$$G(s) = \frac{1}{(s+0.7246)(s^2+0.5246s+1.3801)}$$

$$G(s) = \frac{1}{0.7246 \cdot 1.3801 \left(\frac{s}{0.7246}+1\right)\left(\frac{s^2}{1.3801} + 0.3801 s + 1\right)}$$

$$G(j\omega) = \frac{1}{\left(\frac{j\omega}{0.7246}+1\right)\left[\frac{(j\omega)^2}{1.3801} + 0.3801(j\omega) + 1\right]}$$

Magnitude

$$20\log[G(j\omega)] = \cancelto{0}{20\log|1|} - 20\log\left|\frac{j\omega}{0.7246}+1\right| - 20\log\left|\frac{(j\omega)^2}{1.3801} + 0.3801(j\omega) + 1\right|$$

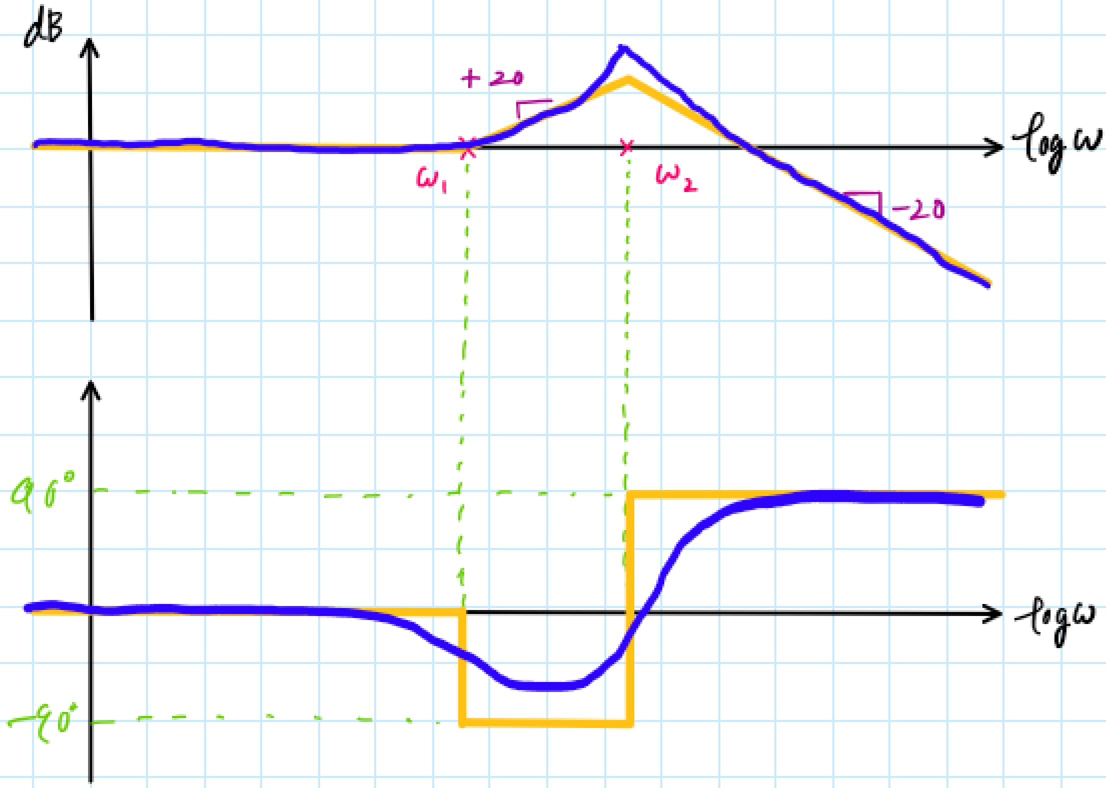
Phase

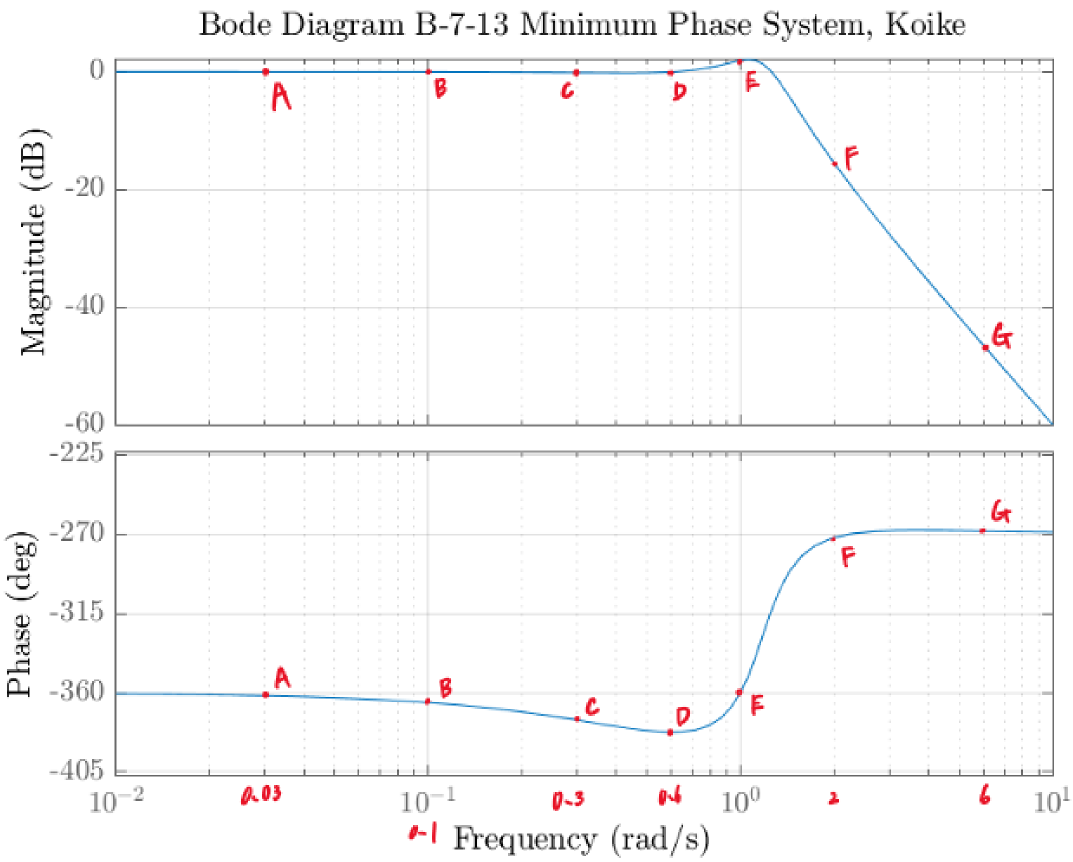
$$\arg[G(j\omega)] = \cancel{\arg(1)}^{\,0} - \arg\left(\frac{j\omega}{0.7246}+1\right) - \underbrace{\arg\left[\frac{(j\omega)^2}{1.3801} + 0.3801(j\omega) + 1\right]}_{+180^\circ \text{ since roots are in RHP}}$$

Corner freq.

$\omega_1 = 0.7246 \implies 0 < \omega_1 < 1$ slope $-(-20) = +20$

$\omega_2 = \sqrt{1.3801} = 1.1748$

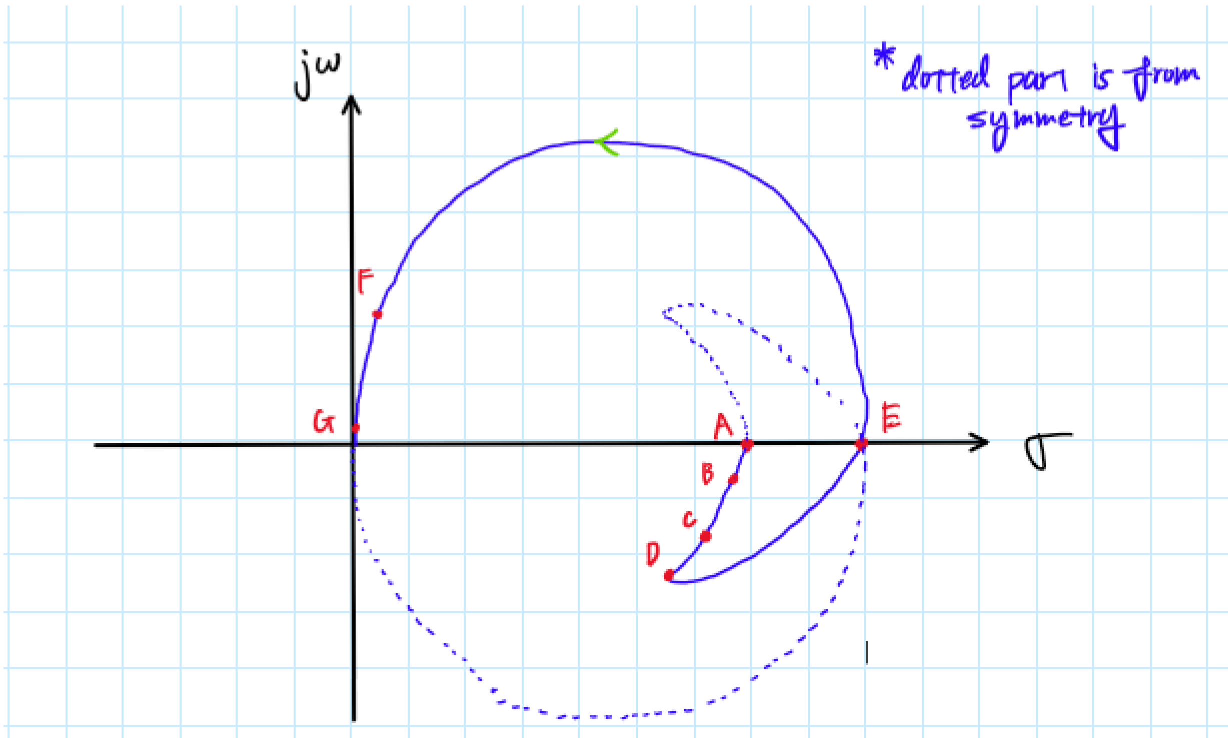


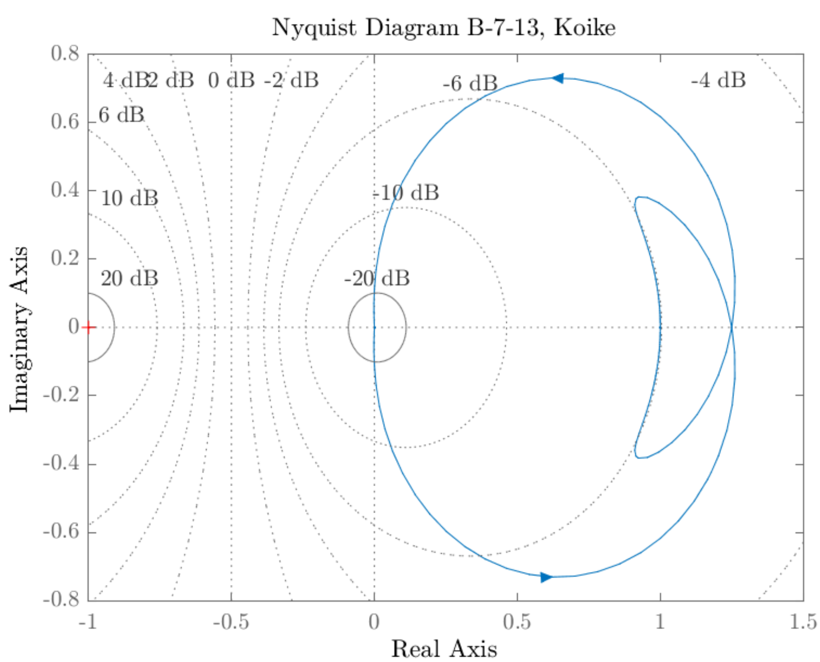


Nyquist Plot

approx.

Point	ω	$\angle G$	$20\log\lvert G\rvert$	$\lvert G\rvert$
A	0.03	$-360°$	0	1
B	0.1	$-364°$	0	1
C	0.3	$-375°$	0	1
D	0.6	$-382°$	0	1
E	1	$-360°$	2	1.2589
F	2	$-271°$	-18	0.1259
G	6	$-270°$	-46	0.00501


jω
*dotted part is from symmetry
F
G
A
B
C
D
E
σ


Nyquist Diagram B-7-13, Koike
Imaginary Axis
Real Axis
4 dB
2 dB
0 dB
-2 dB
6 dB
10 dB
20 dB
-6 dB
-4 dB
-10 dB
-20 dB
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-0.5
0
0.5
1
1.5

Nyquist Stability Criterion

P : the # of OL poles in RHP
N : the # of clockwise encirclements about -1
Z : the # of CL poles in RHP $\Rightarrow Z = N + P$

since $P = 2$, $N = 0$

$Z = 2 \Longrightarrow$ system is $\boxed{\text{unstable}}$

B–7–15. Consider the unity-feedback system with the following $G(s)$:

$$G(s) = \frac{1}{s(s-1)}$$

Suppose that we choose the Nyquist path as shown in Figure 7–156. Draw the corresponding $G(j\omega)$ locus in the $G(s)$ plane. Using the Nyquist stability criterion, determine the stability of the system.

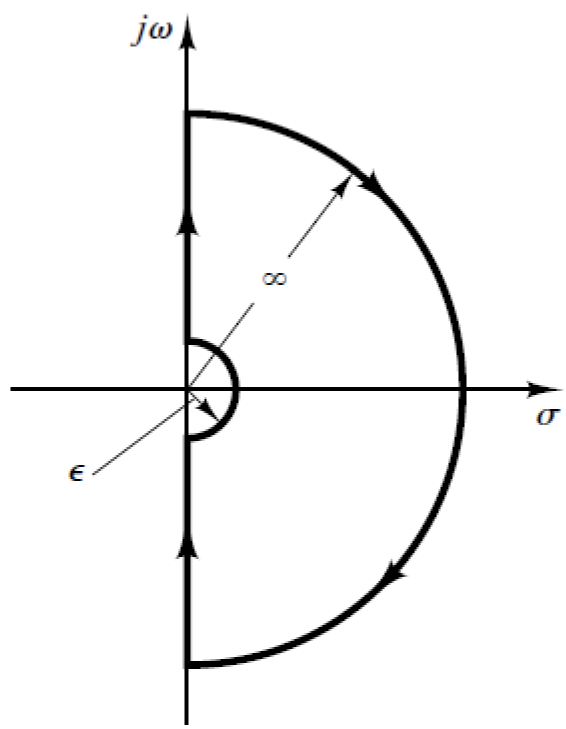


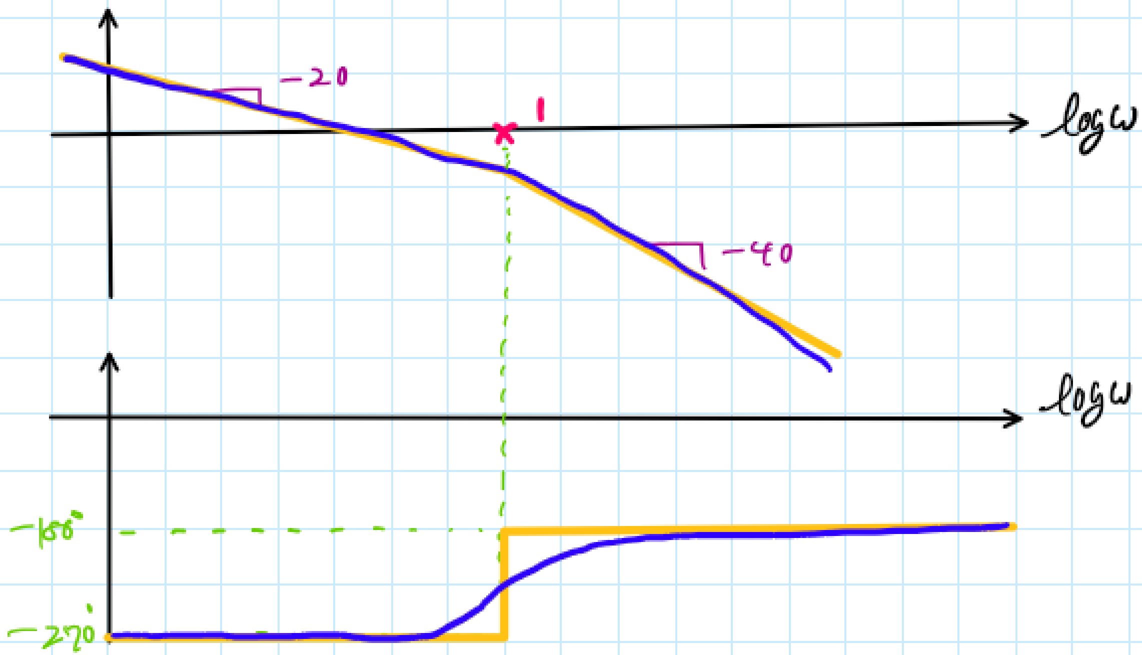
Figure 7–156
Nyquist path.

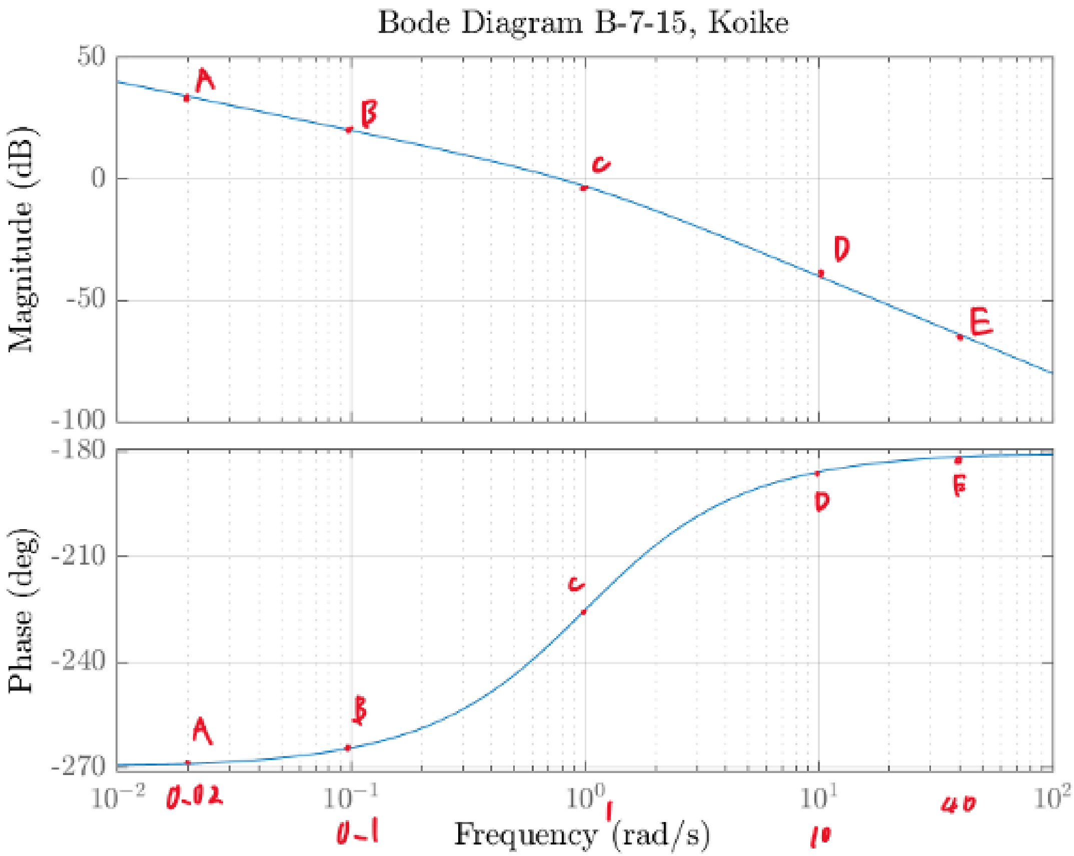
$$G(j\omega) = \frac{1}{j\omega(j\omega+1)}$$

$$20\log|G(j\omega)| = \cancel{20\log|1|}^{\,0} - 20\log|j\omega| - 20\log|j\omega - 1|$$

$$\arg[G(j\omega)] = \cancel{\arg(1)}^{\,0} - \arg(j\omega) - \arg(j\omega - 1)$$

Bode Plot

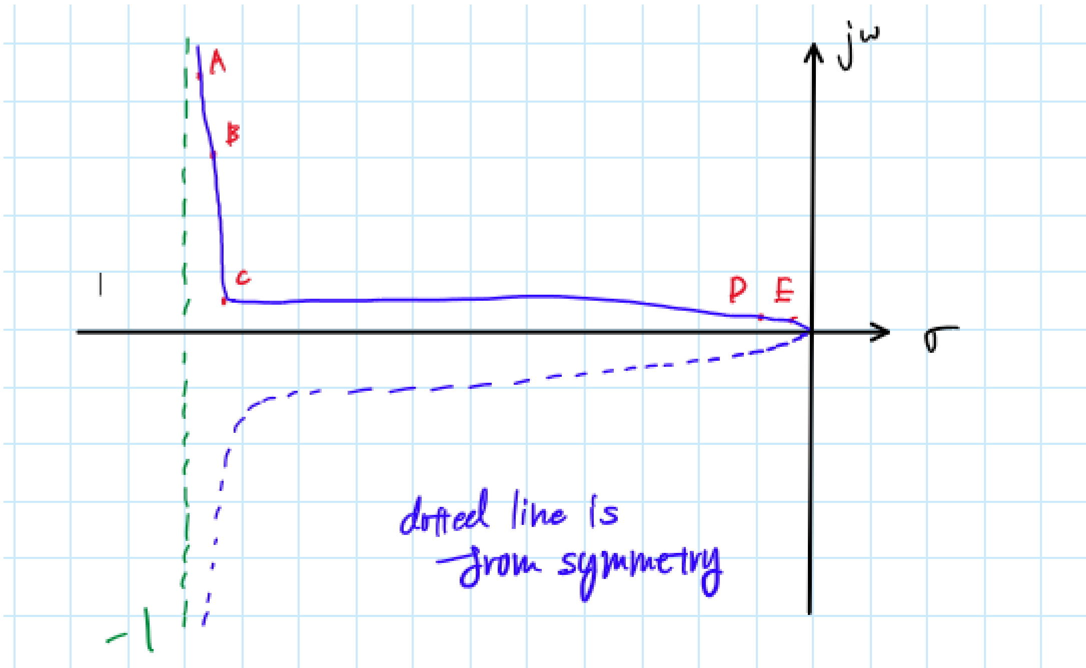




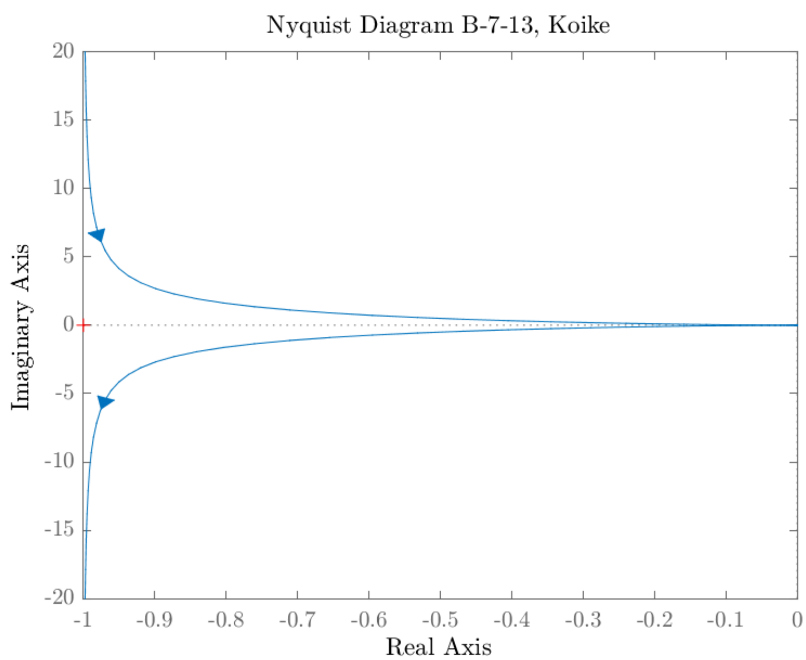
Nyquist Plot

approx.

Points	ω	$\angle G$	$20\log\lvert G\rvert$	$\lvert G\rvert$
A	0.02	-270	30	31.6228
B	0.1	-265	20	10
C	1	-225	-3	0.7079
D	10	-185	-46	0.0050
E	40	-182	-65	0.0006



jω
A
B
C
D
E
σ
-1
dotted line is
from symmetry



Nyquist Diagram B-7-13, Koike
Imaginary Axis
Real Axis
20
15
10
5
0
-5
-10
-15
-20
-1
-0.9
-0.8
-0.7
-0.6
-0.5
-0.4
-0.3
-0.2
-0.1
0

Nyquist Stability Criterion

P : the # of OL poles in RHP
N : the # of clockwise encirclements about -1
Z : the # of CL poles in RHP $\Rightarrow Z = N + P$

$P = 1$ & $N = 0$

thus, $Z = 1 \Longrightarrow$ $\boxed{\text{unstable}}$

Problem 2

1. Sketch the Bode plots of the following three systems:

   (a) $G(s) = \frac{T_1 s+1}{T_2 s+1}$, $(T_1 > T_2 > 0)$

   (b) $G(s) = \frac{T_1 s-1}{T_2 s+1}$, $(T_1 > T_2 > 0)$

   (c) $G(s) = \frac{-T_1 s+1}{T_2 s+1}$, $(T_1 > T_2 > 0)$

(a)

$$G(j\omega) = \frac{j\omega T_1 + 1}{j\omega T_2 + 1} \quad (T_1 > T_2 > 0)$$

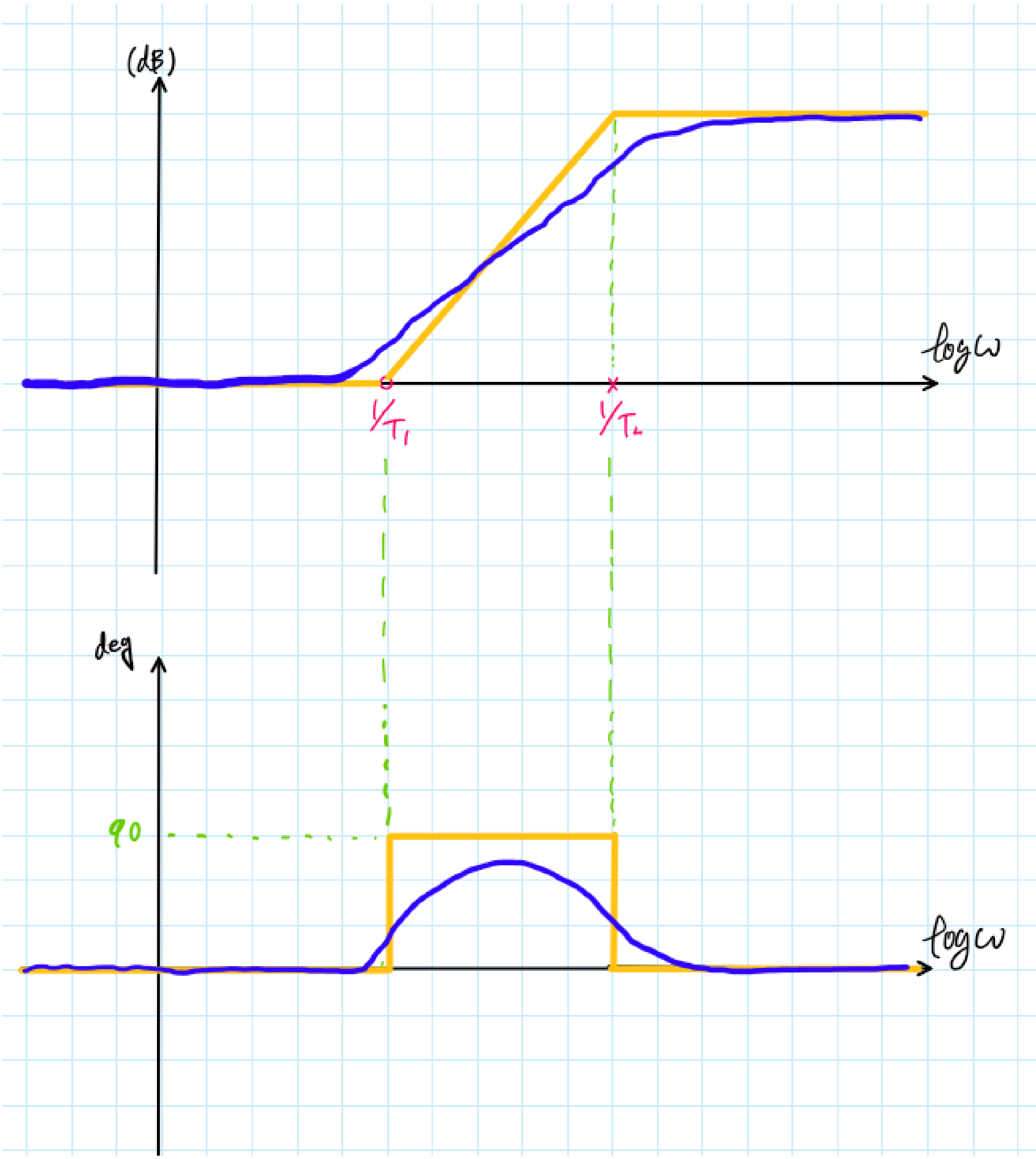
corner frequency : $\omega_1 = \frac{1}{T_1}$ $\omega_2 = \frac{1}{T_2}$ $\frac{1}{T_1} < \frac{1}{T_2}$

magnitude

$$20\log|G(j\omega)| = 20\log|j\omega T_1 + 1| - 20\log|j\omega T_2 + 1|$$

phase

$$\arg[G(j\omega)] = \arg(j\omega T_1 + 1) - \arg(j\omega T_2 + 1)$$


(dB)
log ω
1/T1
1/T2
deg
90
log ω

(b)

$$G(j\omega) = \frac{j\omega T_1 - 1}{j\omega T_2 + 1} \quad (T_1 > T_2 > 0)$$

corner frequency : $\omega_1 = \frac{1}{T_1}$ $\omega_2 = \frac{1}{T_2}$ $\frac{1}{T_1} < \frac{1}{T_2}$

magnitude

$$20\log|G(j\omega)| = 20\log|j\omega T_1 - 1| - 20\log|j\omega T_2 + 1|$$ same as (a)

phase

$$\arg[G(j\omega)] = \arg(j\omega T_1 - 1) - \arg(j\omega T_2 + 1)$$

since $T_1 > T_2 > 0$

$$\arg[G(j\omega)] = 180° + \arctan(-\omega T_1) - \arctan(\omega T_2)$$

$$\Downarrow$$

if $\omega \to 0$

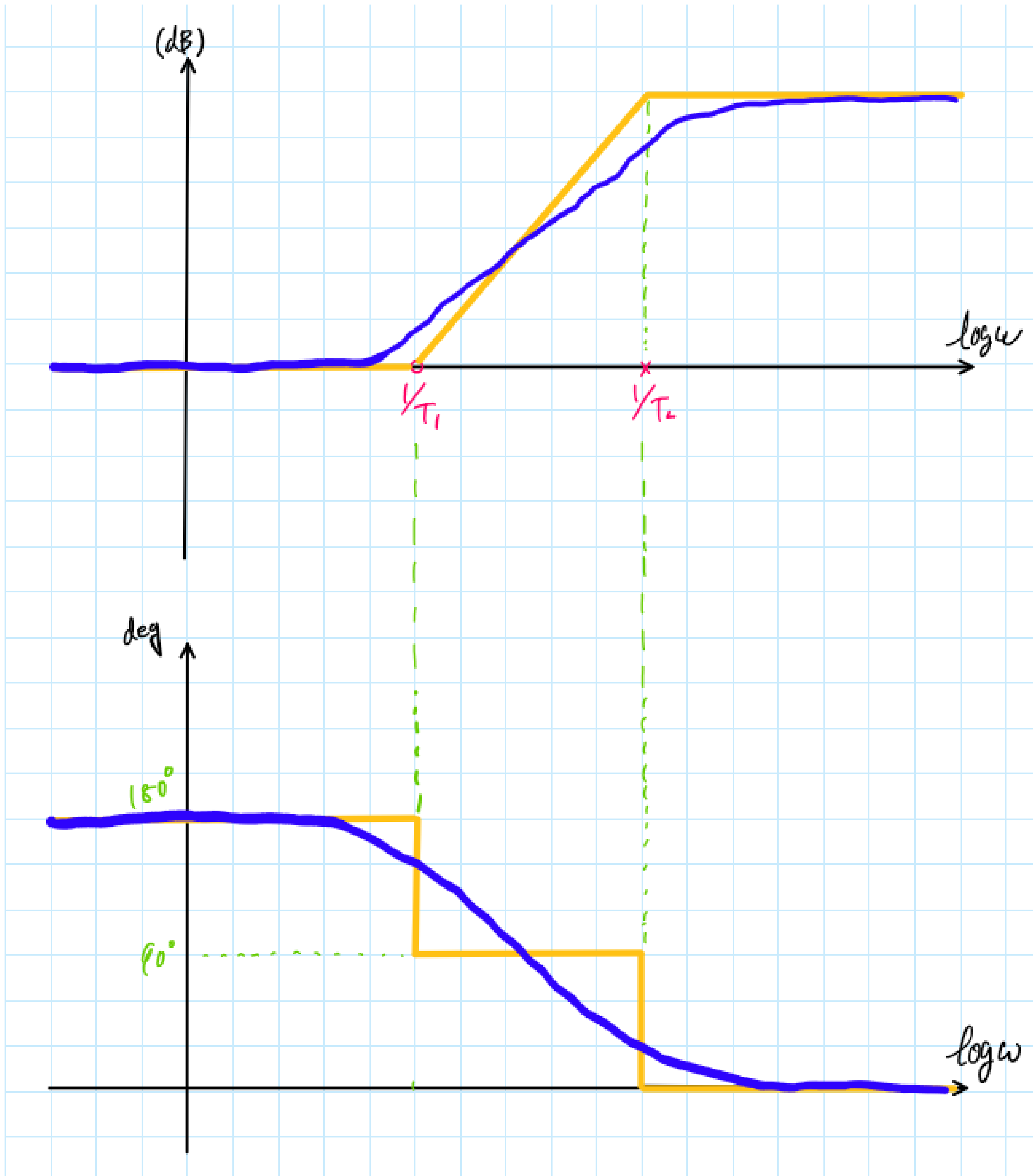
$$\varphi_0 = 180° + 0 - 0 = 180°$$

if $\omega \to 1/T_1$

$$\varphi_1 = \varphi_0 - 90° = 90°$$

if $\omega \to 1/T_2$

$$\varphi_2 = \varphi_1 - 90° = 0$$


(dB)
logω
1/T1
1/T2
deg
180°
90°
logω

(c)

$$G(j\omega) = \frac{-j\omega T_1 + 1}{j\omega T_2 + 1} \quad (T_1 > T_2 > 0)$$

corner frequency : $\omega_1 = \frac{1}{T_1}$ $\omega_2 = \frac{1}{T_2}$ $\frac{1}{T_1} < \frac{1}{T_2}$

magnitude

$$20\log|G(j\omega)| = 20\log|-j\omega T_1 + 1| - 20\log|j\omega T_2 + 1|$$ same as (a)

phase

$$\arg[G(j\omega)] = \arg(-j\omega T_1 + 1) - \arg(j\omega T_2 + 1)$$

since $T_1 > T_2 > 0$

$$\arg[G(j\omega)] = -\arctan(\omega T_1) - \arctan(\omega T_2)$$

$$\Downarrow$$

if $\omega \to 0$

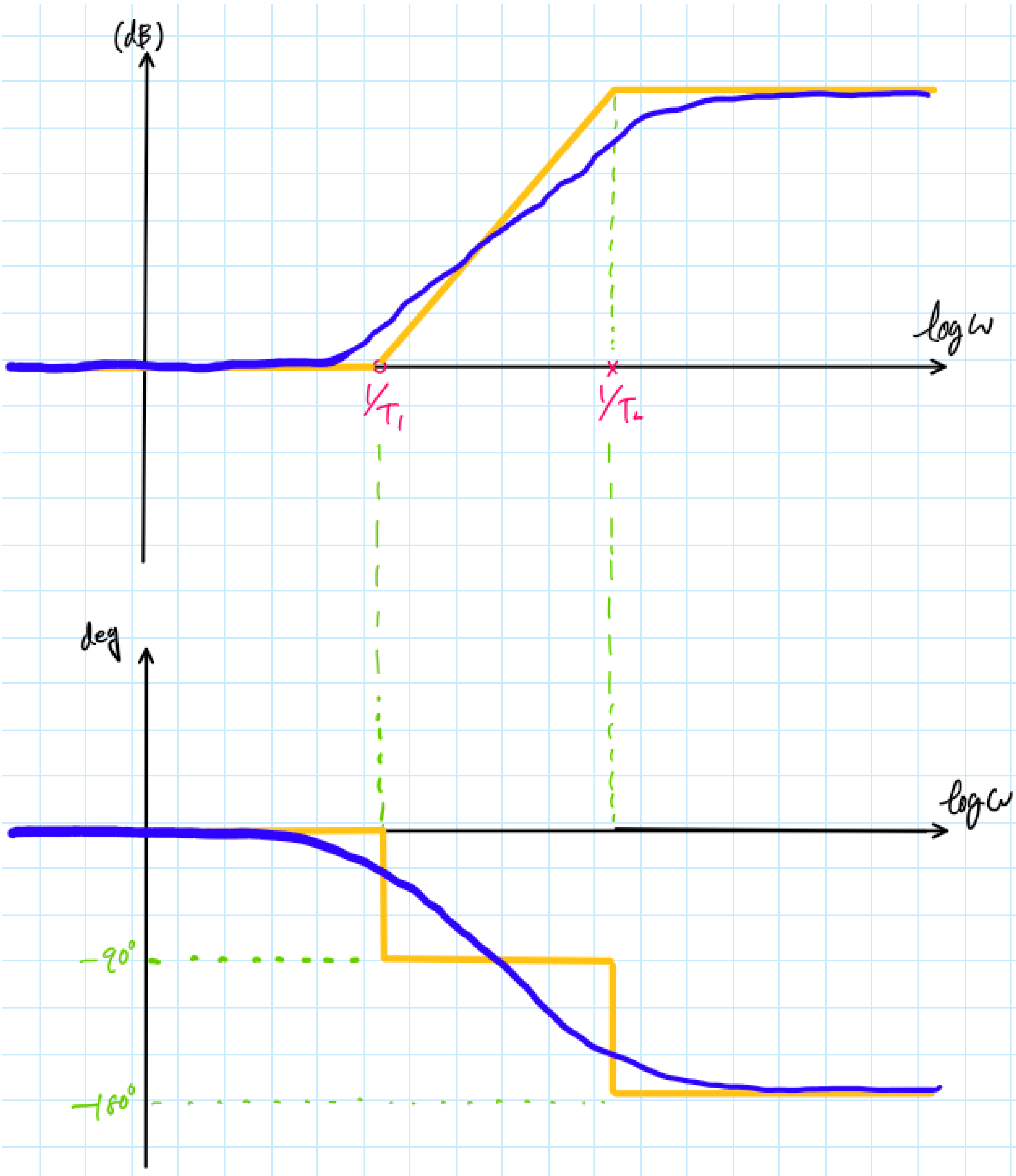
$$\varphi_0 = 0^\circ$$

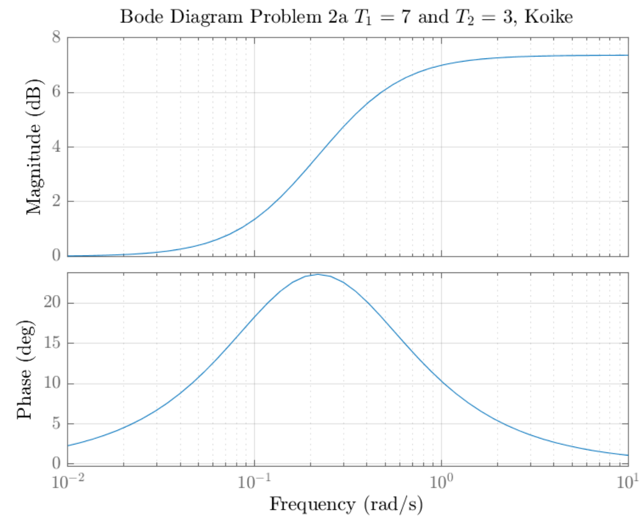
if $\omega \to 1/T_1$

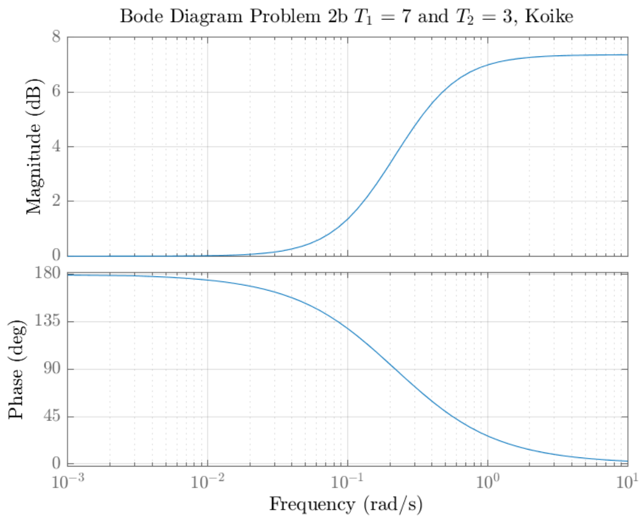
$$\varphi_1 = \varphi_0 - 90^\circ = -90^\circ$$

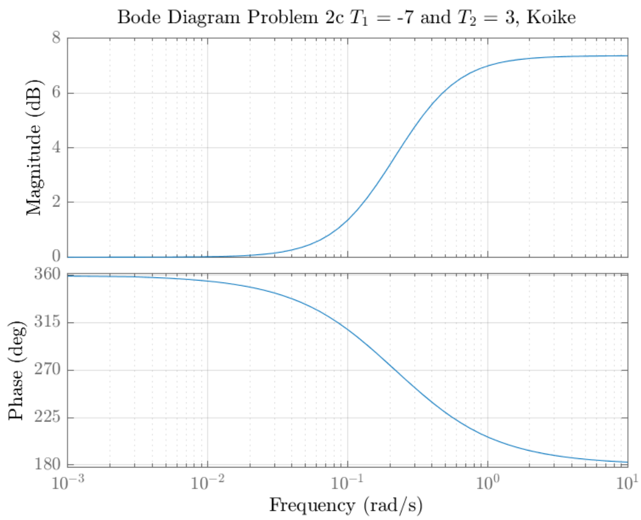
if $\omega \to 1/T_2$

$$\varphi_2 = \varphi_1 - 90^\circ = -180^\circ$$


(dB)
log ω
1/T1
1/T2
deg
log ω
-90°
-180°


Bode Diagram Problem 2a $T_1 = 7$ and $T_2 = 3$, Koike
Magnitude (dB)
Phase (deg)
Frequency (rad/s)


Bode Diagram Problem 2b $T_1 = 7$ and $T_2 = 3$, Koike
Magnitude (dB)
Phase (deg)
Frequency (rad/s)


Bode Diagram Problem 2c $T_1 = $ -7 and $T_2 = 3$, Koike
Magnitude (dB)
Phase (deg)
Frequency (rad/s)

Appendix

AAE364 HW10

```
clear all; close all; clc;
fdir = 'C:\Users\Tomo\Desktop\studies\2020-Spring\AAE364\matlab\matlab_output\hw10';
set(groot, 'defaulttextinterpreter',"latex");
set(groot, 'defaultAxesTickLabelInterpreter',"latex");
set(groot, 'defaultLegendInterpreter',"latex");
```

```
% Bode plot options
opts_bd = bodeoptions('cstprefs');
opts_bd.Title.Interpreter = "latex";
opts_bd.XLabel.Interpreter = "Latex";
opts_bd.YLabel.Interpreter = "Latex";
opts_bd.Grid = 'on';
% Nyquist plot options
opts_nq = nyquistoptions("cstprefs");
opts_nq.Title.Interpreter = 'latex';
opts_nq.XLabel.Interpreter = "Latex";
opts_nq.YLabel.Interpreter = "Latex";
opts_nq.Grid = 'on';
```

B-7-3

```
% Minimum Phase System
num = [1 1];
den = [2 1];
G = tf(num,den);
fig = figure("Renderer","painters");
    opts_bd.Title.String = "Bode Diagram B-7-3 Minimum Phase System, Koike";
    bodeplot(G,opts_bd);
saveas(fig,fullfile(fdir,"B-7-3_min_bode.png"));

% Non-minimum Phase System
num = [-1 1];
den = [2 1];
G = tf(num,den);
fig = figure("Renderer","painters");
    opts_bd.Title.String = "Bode Diagram B-7-3 Non-Minimum Phase System, Koike";
    bodeplot(G,opts_bd);
saveas(fig,fullfile(fdir,"B-7-3_nonmin_bode.png"));
```

B-7-8

```
% |K| > 1
% Draw the Bode plot
K = 4;
num = K*[-1 1];
den = [1 1];
G = tf(num,den);
fig = figure("Renderer","painters");
```

```
    opts_bd.Title.String = "Bode Diagram B-7-8 When K = 4, Koike";
    bodeplot(G,opts_bd);
saveas(fig,fullfile(fdir,"B-7-8_bode_K=4.png"));

% Nyquist plot
fig = figure("Renderer","painters");
    opts_nq.Title.String = "Nyquist Diagram B-7-8 When K = 4, Koike";
    nyquistplot(G,opts_nq);
    axis equal;
saveas(fig,fullfile(fdir,"B-7-8_nyquist_K=4.png"));

% |K| <= 1
% Draw the Bode plot
K = 0.5;
num = K*[-1 1];
den = [1 1];
G = tf(num,den);
fig = figure("Renderer","painters");
    opts_bd.Title.String = "Bode Diagram B-7-8 When K = 0.5, Koike";
    bodeplot(G,opts_bd);
saveas(fig,fullfile(fdir,"B-7-8_bode_K=0.5.png"));

% Nyquist plot
fig = figure("Renderer","painters");
    opts_nq.Title.String = "Nyquist Diagram B-7-8 When K = 0.5, Koike";
    nyquistplot(G,opts_nq);
    axis equal;
saveas(fig,fullfile(fdir,"B-7-8_nyquist_K=0.5.png"));
```

B-7-10

```
close all;
% Define the OL transfer function
num = 5*[0.5 1];
den = conv([2 1],[10 1]);
den = conv(den,20*[1 0 0]);
w = logspace(-1,2,200);
for i = 0:1
    switch i
        case 0
            K = 1; G = tf(K*num,den);
            [mag,phase,w] = bode(G,w);
            mag1dB = 20*log10(mag(:)); phase1 = phase(:);
        case 1
            K = 10; G = tf(K*num,den);
            [mag,phase,w] = bode(G,w);
            mag2dB = 20*log10(mag(:)); phase2 = phase(:);
    end

    % Direct Nyquist plot
    fig1 = figure(1+i);
        title_txt = sprintf("Direct Nyquist Diagram B-7-10 When K = %d,
Koike",K);
```

```
        opts_nq.Title.String = title_txt;
        nyquistplot(G,opts_nq);
        xlim([-20 5])
        file_txt = sprintf("B-7-10_dir_nyquist_K=%d.png",K);
    saveas(fig1,fullfile(fdir,file_txt));

    % Inverse Nyquist plot
    fig2 = figure(3+i);
        title_txt = sprintf("Inverse Nyquist Diagram B-7-10 When K = %d,
Koike",K);
        opts_nq.Title.String = title_txt;
        nyquistplot(inv(G),opts_nq);
        xlim([-0.5 0.5])
        file_txt = sprintf("B-7-10_inv_nyquist_K=%d.png",K);
    saveas(fig2,fullfile(fdir,file_txt));
end

% Bode Plot
fig = figure("Renderer","painters");
    subplot(2,1,1);
        semilogx(w,mag1dB); ylabel('Magnitude (dB)');
        grid on; hold on;
        semilogx(w,mag2dB); hold off; legend('K = 1','K = 10');
    subplot(2,1,2);
        semilogx(w,phase1); ylabel('Phase (deg)');
        grid on; hold on;
        semilogx(w,phase2); hold off; legend('K = 1','K = 10');
    % Give common xlabel, ylabel and title to your figure
    han = axes(fig,'visible','off');
    han.XLabel.Visible = 'on';
    xlabel(han,'Frequency $\omega$ (rad/s)');
    title_txt = sprintf("Bode Diagram B-7-10 When K = %d, %d, Koike",1,10);
    sgtitle(title_txt);
    file_txt = sprintf("B-7-10_bode_K=%d.png",K);
saveas(fig,fullfile(fdir,file_txt));
```

B-7-13

```
% Define the transfer function
num = [0 1];
den = [1 0.2 1 1];
p = roots(den);
a = -p(3)
b = p(1) + p(2)
c = p(1)*p(2)
b_c = b/c
Q = 1/a/c
c_sqrt = sqrt(c)
G = tf(num,den);
```

```
fig = figure("Renderer","painters");
    opts_bd.Title.String = "Bode Diagram B-7-13 Minimum Phase System, Koike";
    bodeplot(G,opts_bd);
```

```
saveas(fig,fullfile(fdir,"B-7-13_bode.png"));
```

```
% Nyquist Plot
fig = figure("Renderer","painters");
    opts_nq.Title.String = "Nyquist Diagram B-7-13, Koike";
    nyquistplot(G,opts_nq);
saveas(fig,fullfile(fdir,"B-7-13_nyquist.png"));
```

B-7-15

```
num = [0 1];
den = conv([1 0],[1 -1]);
G = tf(num,den);
% Bode plot
fig = figure("Renderer","painters");
    opts_bd.Title.String = "Bode Diagram B-7-15, Koike";
    bodeplot(G,opts_bd);
saveas(fig,fullfile(fdir,"B-7-15_bode.png"));
```

```
% some calculations
arr_log = [30 20 -3 -46 -65];
arr = 10.^(arr_log/20);
```

```
% Nyquist Plot
fig = figure("Renderer","painters");
    opts_nq.Title.String = "Nyquist Diagram B-7-13, Koike";
    opts_nq.Grid = 'off';
    nyquistplot(G,opts_nq);
saveas(fig,fullfile(fdir,"B-7-15_nyquist.png"));
```

P2

```
% (a)
num = [7 1];
den = [3 1];
G = tf(num,den);
fig = figure("Renderer","painters");
    opts_bd.Title.String = "Bode Diagram Problem 2a $T_1$ = 7 and $T_2$ = 3,
Koike";
    bodeplot(G,opts_bd);
saveas(fig,fullfile(fdir,"P2-a_bode.png"));

% (b)
num = [7 -1];
den = [3 1];
G = tf(num,den);
fig = figure("Renderer","painters");
    opts_bd.Title.String = "Bode Diagram Problem 2b $T_1$ = 7 and $T_2$ = 3,
Koike";
    bodeplot(G,opts_bd);
saveas(fig,fullfile(fdir,"P2-b_bode.png"));
```

```
% (c)
num = [-7 1];
```

```
den = [3 1];
G = tf(num,den)
fig = figure("Renderer","painters");
    opts_bd.Title.String = "Bode Diagram Problem 2c $T_1$ = -7 and $T_2$ = 3,
Koike";
    bodeplot(G,opts_bd);
saveas(fig,fullfile(fdir,"P2-c_bode.png"));
```