AAE 564 Fall 2020

Homework Two

Due : Friday, September 11

Exercise 1 Obtain the A, B, C, D matrices for a state space representation of the following systems:

(a)

$$\begin{aligned} u &= a_0 q + a_1 \dot{q} + \ldots + a_{n-1} q^{(n-1)} + q^{(n)} \\ y &= \beta_0 q + \beta_1 \dot{q} + \ldots + \beta_{n-1} q^{(n-1)} + \gamma u \end{aligned}$$

where $u(t), y(t) \in \mathbb{R}$.

(b)

$$\begin{aligned} u(k) &= a_0 q(k) + a_1 q(k+1) + \ldots + a_{n-1} q(k+n-1) + q(k+n) \\ y(k) &= \beta_0 q(k) + \beta_1 q(k+1) + \ldots + \beta_{n-1} q(k+n-1) + \gamma u(k) \end{aligned}$$

where $y(k), u(k) \in \mathbb{R}$

(c) The 'simple structure' with outputs $y_1 = q_1$ and $y_2 = q_2$.

Exercise 2 Linearize (if possible) the following systems about *each* of their equilibrium states (if not possible, state why) and obtain the system A matrix for these linearized systems.

(a)

$$\dot{x} = x^3 - x$$

(b)

$$\dot{x} = \sqrt{|x|}$$

(c) The 'attitude dynamics' system. Consider non-symmetric case ($I_1 \neq I_2 \neq I_3$).

Exercise 3 (a) Obtain all equilibrium states of the following system:

$$\begin{aligned} \dot{x}_1 &= 2x_2(1 - x_1) - x_1 \\ \dot{x}_2 &= 3x_1(1 - x_2) - x_2 \end{aligned}$$

(b) Linearize the above system about the zero equilibrium state.

Exercise 4 For each of the following systems, linearize about each equilibrium solution and obtain the system A-matrix for a state space representation of these linearized systems.

(a)

$$\ddot{y} + (y^2 - 1)\dot{y} + y = 0\,.$$

where $y(t)$ is a scalar.

(b)

$$\ddot{y} + \dot{y} + y - y^3 = 0$$

where $y(t)$ is a scalar.

(c)

$$\begin{aligned}(M+m)\ddot{y} + ml\,\ddot{\theta}\cos\theta - ml\,\dot{\theta}^2\sin\theta + ky &= 0\\ ml\ddot{y}\cos\theta + ml^2\,\ddot{\theta} + mgl\,\sin\theta &= 0\end{aligned}$$

where $y(t)$ and $\theta(t)$ are scalars.

(d)

$$\ddot{y} + 0.5\dot{y}|\dot{y}| + y = 0\,.$$

where $y(t)$ is a scalar.

Exercise 5 Obtain the transfer function (matrix) of the following system

$$\begin{aligned}\ddot{y}_1 + \ddot{y}_2 + y_1 + y_2 &= u_1 + \dot{u}_2\\ 2\ddot{y}_1 + 3\ddot{y}_2 + y_1 - y_2 &= 0\end{aligned}$$

Exercise 6 Obtain the transfer function of the system with input u and output y described by

$$\begin{aligned}\ddot{q}_1 + 3\dot{q}_2 + \dot{q}_1 + 2q_2 &= \dot{u} + 4u\\ \ddot{q}_1 + 4\dot{q}_2 + 3q_2 &= u\\ y &= q_1 + q_2\end{aligned}$$

Exercise 7 Obtain a SIMULINK model of the following system.

$$\frac{d^3q}{dt^3} + 3\ddot{q} + \dot{q}\sin q + q^3 = 0\,.$$

Exercise 8 Obtain SIMULINK models of the following systems.

(a)

$$\begin{aligned}\ddot{q}_1 + 2\dot{q}_2 + q_1 &= 0\\ \ddot{q}_2 + \dot{q}_1 + 6q_2 &= 0\end{aligned}$$

(b)

$$
\begin{aligned}
\ddot{q}_1 + \ddot{q}_2 + 6q_2 &= 0 \\
\ddot{q}_1 - \ddot{q}_2 + 4q_1 &= 0
\end{aligned}
$$

(c)

$$
\begin{aligned}
\ddot{q}_1 + \dot{q}_1 + q_1 q_2 &= 0 \\
(1 + q_2^2)\dot{q}_2 + 2\dot{q}_1 q_2 &= 0
\end{aligned}
$$