P1: E BP

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Successive Rotations

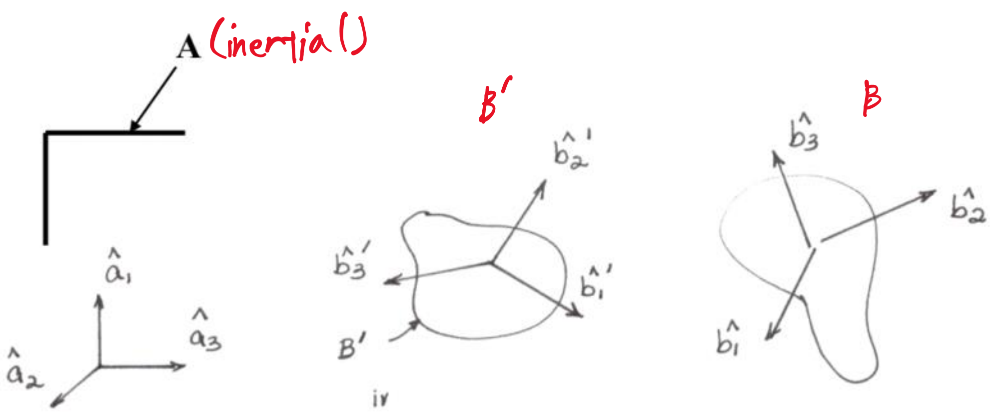
All changes in orientation CAN be described in terms of a single simple rotation

→ Not always most convenient
Generally difficult to visualize
Maybe not physically possible

Decompose into a sequence of rotations:

1) easier to visualize
2) may be a better model for analysis

So, consider methods of analysis for successive rotations:
Assume rigid body B (s/c) subject to two successive rotations —
analysis in terms of any variable set
Convenient to introduce notation for an intermediate frame B'



Initial

$\hat{a}_i = \hat{b}'_i = \hat{b}_i$

@ $t = t_0$

$$A \xrightarrow[{}^{A}C^{B'}]{} B' \xrightarrow[{}^{B'}C^{B}]{} B$$

$$\Downarrow$$

$${}^{A}C^{B} \; ?$$

Each rotation, as well as the equivalent single rotation, can be described in terms of direction cosines, Euler parameters, Euler axis/Euler angle, Each rotation, as well as the equivalent single rotation, can be described in terms of direction cosines, Euler parameters, Euler axis/Euler angle,

1. Direction Cosines

$$ {}^{A}C^{B'},\ {}^{B'}C^{B},\ {}^{A}C^{B} $$

$$ \left[\hat{b}_1'\ \hat{b}_2'\ \hat{b}_3'\right] = \left[\hat{a}_1\ \hat{a}_2\ \hat{a}_3\right]{}^{A}C^{B'} $$

$$ \left[\hat{b}_1\ \hat{b}_2\ \hat{b}_3\right] = \left[\hat{b}_1'\ \hat{b}_2'\ \hat{b}_3'\right]{}^{B'}C^{B} $$

produces

$$ \left[\hat{b}_1\ \hat{b}_2\ \hat{b}_3\right] = \left[\hat{a}_1\ \hat{a}_2\ \hat{a}_3\right]{}^{A}C^{B} $$

$$ \boxed{{}^{A}C^{B} = {}^{A}C^{B'}\ {}^{B'}C^{B}} $$

${}^{A}C^{B}$ is the relationship we seek

Can easily be extended

$$ \boxed{{}^{A}C^{B} = {}^{A}C^{B'}\ {}^{B'}C^{B''}\ {}^{B''}C^{B}} $$

→ direction cosine rule for successive rotation

2. Euler Parameters $A - B' - B$

$\boxed{{}^{A}\bar{\varepsilon}^{B'},\ {}^{A}\varepsilon_4^{B'},}$ $\boxed{{}^{B'}\bar{\varepsilon}^{B},\ {}^{B'}\varepsilon_4^{B},}$ $\boxed{{}^{A}\bar{\varepsilon}^{B}\ {}^{A}\varepsilon_4^{B}}$

Use Rodrigues version with relationships for ρ_i and ε_i, i.e.,

$$
{}^{A}\bar{\rho}^{B'} = \frac{{}^{A}\bar{\varepsilon}^{B'}}{{}^{A}\varepsilon_4^{B'}} \qquad {}^{B'}\bar{\rho}^{B} = \frac{{}^{B'}\bar{\varepsilon}^{B}}{{}^{B'}\varepsilon_4^{B}} \qquad {}^{A}\bar{\rho}^{B} = \frac{{}^{A}\bar{\varepsilon}^{B}}{{}^{A}\varepsilon_4^{B}}
$$

$$
\boxed{\begin{aligned}
{}^{A}\bar{\varepsilon}^{B} &= {}^{A}\bar{\varepsilon}^{B'}\ {}^{B'}\varepsilon_4^{B} + {}^{B'}\bar{\varepsilon}^{B}\ {}^{A}\varepsilon_4^{B'} + {}^{B'}\bar{\varepsilon}^{B'} \times {}^{A}\bar{\varepsilon}^{B'} \longrightarrow \text{vector} \\
{}^{A}\varepsilon_4^{B} &= {}^{A}\varepsilon_4^{B'}\ {}^{B'}\varepsilon_4^{B} - {}^{A}\bar{\varepsilon}^{B'} \cdot {}^{B'}\bar{\varepsilon}^{B} \longrightarrow \text{scalar}
\end{aligned}}
$$

1) order matters

2) for 3 or more rotations, must combine in sets of 2.

$A - B' - B'' - B''' - B$

$\downarrow$

not easy to do ∇

measure #5 $\rightarrow$ ${}^{A}\bar{\varepsilon}^{B} = {}^{B'}\bar{\varepsilon}^{B}$ (vector basis must be consistent)

Notes concerning these results

(a) $\bar{\varepsilon}$, ε_4 involves a **VECTOR** relationship

Be aware of vector basis in use

Write ε relationships in a matrix format

$$\begin{bmatrix} {}^A\varepsilon_1^B \\ {}^A\varepsilon_2^B \\ {}^A\varepsilon_3^B \\ {}^A\varepsilon_4^B \end{bmatrix} = \begin{bmatrix} {}^A\varepsilon_4^{B'} & -\varepsilon_3 & +\varepsilon_2 & +\varepsilon_1 \\ +\varepsilon_3 & +\varepsilon_4 & -\varepsilon_1 & +\varepsilon_2 \\ -\varepsilon_2 & +\varepsilon_1 & +\varepsilon_4 & +\varepsilon_3 \\ -\varepsilon_1 & -\varepsilon_2 & -\varepsilon_3 & +\varepsilon_4 \end{bmatrix} \begin{bmatrix} {}^{B'}\varepsilon_1^B \\ {}^{B'}\varepsilon_2^B \\ {}^{B'}\varepsilon_3^B \\ {}^{B'}\varepsilon_4^B \end{bmatrix} \tag{E.1}$$

vector basis is important

WARNING: use at your own risk

Go through the vector version; make sure that you know specifically the vector basis in which each element of equation (E.1) is expressed

Can you tell the vector basis in which the answer is expressed? ${}^A\bar{\varepsilon}^B$

(NEVER use an equation when you are not aware and thoughtful about the assumptions used in the derivation.)

(b) All equations make it apparent that order is important. Performing rotations in different order MAY alter the results

Whether it happens depends heavily on how the rotations are described (body-fixed axes or inertial axes)

Summary to date:

- We have so far described 3 sets of kinematic variables to be used this semester to consider s/c attitude (there are others)

  Hopefully come to better understanding of each by working with them; advantages and disadvantages

- Developed relationships between variable sets

- All change in orientation defined in terms of a simple rotation

- Simple rotation easy in concept but
  - Simple rotation can't always be accomplished
  - Analysis in terms of SR not always available or understandable
  - Necessary to analyze in terms of successive rotations

Still may need more help to understand what is going on { in each set of variables relationships to analyze final orientation in terms of any number of intermediate rotations

- Have not forgotten usefulness of **orientation angles** best physical insight

  In undergraduate mechanics, begin orientation discussions with angles as variables because of physical insight and analytical convenience

☆ good in terms of physical & analytical reasons

☆ infinite variety of angle combinations can be used

☆ relationships b/w all variable sets

$$A \longrightarrow B' \longrightarrow B'' \longrightarrow B$$

1. $\left.\begin{array}{l} {}^{A-B''}\underline{\varepsilon} \Rightarrow \text{vector} \\ {}^{A-B''}\varepsilon_4 \Rightarrow \text{scalar} \end{array}\right\}$ B' vector basis

2. Transform ${}^{A-B''}\underline{\varepsilon}$ from B' to either $\underline{\underline{A}}, \underline{\underline{B''}}$ [C]

3. $\left.\begin{array}{l} {}^{A-B}\underline{\varepsilon} = f\left({}^{A-B''}\underline{\varepsilon}, {}^{B''-B}\underline{\varepsilon}\right) \\ {}^{A}\varepsilon_4^{B} = f\left({}^{A-B''}\underline{\varepsilon}, {}^{B''-B}\underline{\varepsilon}\right) \end{array}\right\}$ B'' vector basis