

# AAE 339

## HW7: Compressor Stage Analysis

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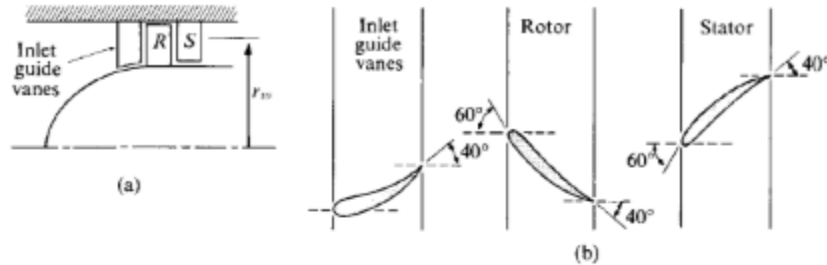
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## Problem 7.2

2. Estimate the power required to drive a single-stage compressor shown schematically in parts (a) and (b) of the figure.



### PROBLEM 2

At the mean radius ( $r_m = 30$  cm), the blade configuration is as shown in part (b). For simplicity, assume that the air angles and blade angles are identical.

The overall adiabatic efficiency of the stage is 90%. The hub-tip radius ratio is 0.8, high enough so that conditions at the mean radius are a good average of conditions from root to tip.

The axial velocity component at design flow rate is uniformly 125 m/s, and the inlet air is at 1 atm and 20°C. What should the shaft speed be under these conditions?

(\* Computations done in MATLAB)

Shaft speed,  $\omega_s$

@ the inlet of this compressor

$$p_1 = 1 \text{ atm} = 101325 \text{ Pa}$$

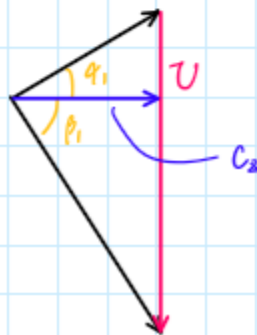
$$T_1 = 20^\circ\text{C} = 293.15 \text{ K}$$

$$C_{a1} = 125 \text{ m/s}$$

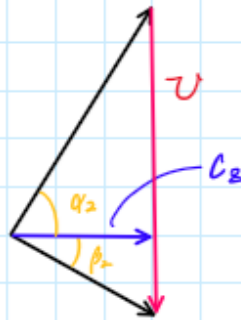
$$\tau = 0.8$$

draw the velocity triangles at mean radius

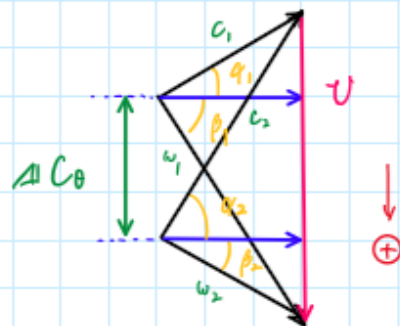
IGVs out  $\alpha_1 = 40^\circ \rightarrow$  Rotor in  $\beta_1 = 60^\circ$



Rotor out  $\beta_2 = 40^\circ \rightarrow$  Stator in  $\alpha_2 = 60^\circ$



this becomes



from this diagram

$$\begin{aligned} \Delta C_0 &= C_2 \tan \beta_1 - C_1 \tan \beta_2 \\ &= C_2 \tan \alpha_1 - C_1 \tan \alpha_2 \\ &= C_2 (\tan \beta_1 - \tan \beta_2) \\ &= (125 \text{ m/s}) (\tan 60^\circ - \tan 40^\circ) \\ &= \underline{111.6189 \text{ m/s}} \end{aligned}$$

$$\begin{aligned} \text{and } u &= C_2 \tan \alpha_1 + C_1 \tan \beta_1 \\ &= C_2 (\tan \alpha_1 + \tan \beta_1) \\ &= \underline{321.3938 \text{ m/s}} \end{aligned}$$

thus, the shaft speed (angular velocity) is

$$\omega_s = \frac{u}{r_m} = \left( \frac{321.3928 \text{ m/s}}{0.30 \text{ m}} \right) = \boxed{1071.3 \text{ rad/s}}$$

power required,  $P_s$

$$= \boxed{10230 \text{ rpm}}$$

using the values  $\Delta C_0$ ,  $u$  we know that

$$\begin{aligned} h_{02} - h_{01} &= u \Delta C_0 \\ C_p (T_{02} - T_{01}) &= u \Delta C_0 \end{aligned}$$

$\therefore$  no work is done in stator (assume)

$$C_p (T_{03} - T_{01}) = u \Delta C_0$$

now assuming isentropic,  $\gamma = 1.4$ ,  $R = 287.05 \frac{\text{J}}{\text{kg K}}$

$$\rho_1 = \frac{p_1}{RT_1} = 1.2041 \frac{\text{kg}}{\text{m}^3}$$

where  $A = \pi(r_t^2 - r_h^2)$   $\therefore r_t = \frac{2}{1.8} r_m$   $r_h = \frac{2}{1.8} \cdot 0.8 r_m$   
then

$$\dot{m} = \rho_1 C_d A = (1.2041 \frac{\text{kg}}{\text{m}^3})(125 \text{ m/s})(0.1257 \text{ m}^2)$$

$$\dot{m} = \underline{18.9142 \text{ kg/s}}$$

then,

$$\begin{aligned} \text{torque} := \tau_r &= \dot{m} r_m A C_\theta \\ &= (18.9142 \frac{\text{kg}}{\text{s}})(0.3 \text{ m})(111.6169 \text{ m/s}) \\ &= 633.3562 \text{ N-m} \end{aligned}$$

ultimately, the power required  $P_s$  becomes

$$P_s = \tau_s \omega_s$$

$$P_s = (633.3562 \text{ N-m})(1071.3 \frac{\text{rad}}{\text{s}})$$

$$P_s = 678521 \text{ W}$$

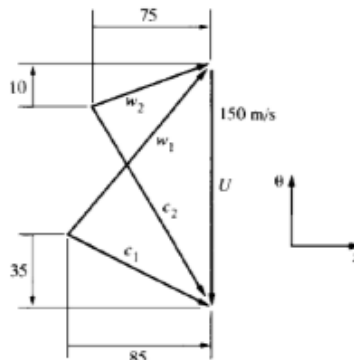
### Problem 7.3

3. At a certain operating condition the mid-radius velocity triangles for an axial compressor stage are as shown in the figure. Here subscripts 1 and 2 denote entrance to rotor and stator, respectively. The stagnation temperature and pressure at entrance to the rotor are 340 K and 185 kPa.

Neglecting frictional effects, determine:

- The specific work kJ/kg;
- The stagnation and static temperatures between rotor and stator;
- The stagnation and static pressures between rotor and stator;
- The mid-radius pressure coefficient

$$C_p = (p_2 - p_1) / \frac{1}{2} \rho_1 w_1^2.$$

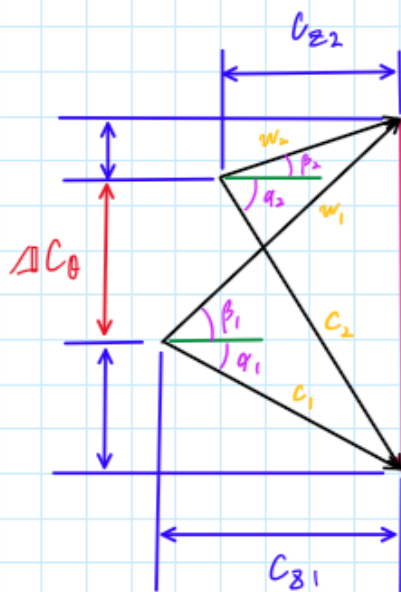


PROBLEM 3

Given properties,

$$P_{01} = 185 \times 10^3 \text{ Pa} \quad T_{01} = 340 \text{ K}$$

Draw the velocity triangles



$$C_{B1} = 85 \text{ m/s}$$

$$C_{B2} = 75 \text{ m/s}$$

and

$$\Delta C_\theta = 150 \text{ m/s} - 10 \text{ m/s} - 35 \text{ m/s} = 105 \text{ m/s}$$

(a) Specific work

since

$$\begin{aligned} h_{02} - h_{01} &= h_{03} - h_{01} = U \Delta C_0 \\ &= (150 \text{ m/s})(105 \text{ m/s}) = 15750 \frac{\text{J}}{\text{kg}} = \boxed{15.75 \frac{\text{kJ}}{\text{kg}}} \end{aligned}$$

(b) Stagnation & static temperatures between rotor and stator,  $T_{02}$  &  $T_2$

since

$$h_{02} - h_{01} = C_p (T_{02} - T_{01}) = U \Delta C_0$$

$$T_{02} = T_{01} + \frac{U \Delta C_0}{C_p}$$

$$\text{where } C_p = \frac{\gamma}{\gamma - 1} R = 1.0047 \times 10^3 \frac{\text{J}}{\text{kg} \cdot \text{K}} = 1.0047 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$\therefore T_{02} = 340 \text{ K} + \frac{15.75 \frac{\text{kJ}}{\text{kg}}}{1.0047 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}}$$

$$T_{02} = \boxed{355.68 \text{ K}}$$

then, using isentropic relations

$$\frac{T_{02}}{T_2} = \left[ 1 + \frac{\gamma - 1}{2} M_2^2 \right]$$

where

$$M_2 = \frac{C_{22}}{\sqrt{\gamma R T_2}}$$

$$\Leftrightarrow T_{02} = T_2 + T_2 \cdot \frac{\gamma - 1}{2} \cdot \frac{C_{22}^2}{\gamma R T_2}$$

$$T_{02} = T_2 + \frac{\gamma - 1}{2 R} C_{22}^2$$

$$T_2 = T_{02} - \frac{C_{22}^2}{2 C_p}$$

$$= 355.68 - \frac{(75 \text{ m/s})^2}{2 (1.0047 \times 10^3 \frac{\text{J}}{\text{kg} \cdot \text{K}})}$$

$$= \boxed{352.88 \text{ K}}$$

(c) The stagnation and static pressure between rotor and stator,  $P_{02}$  &  $P_2$

from isentropic relations

$$\frac{P_{02}}{P_{01}} = \left( \frac{T_{02}}{T_{01}} \right)^{\frac{\gamma}{\gamma-1}}$$

$$P_{02} = (85000 \text{ Pa}) \left( \frac{355.68 \text{ K}}{340 \text{ K}} \right)^{\frac{1.4}{0.4}}$$

$$P_{02} = 216620 \text{ Pa}$$

similarly

$$\frac{P_{02}}{P_2} = \left( \frac{T_{02}}{T_2} \right)^{\frac{\gamma}{\gamma-1}}$$

$$P_2 = P_{02} \left( \frac{T_{02}}{T_2} \right)^{\frac{-\gamma}{\gamma-1}}$$

$$P_2 = (216620 \text{ Pa}) \left( \frac{355.68 \text{ K}}{352.88 \text{ K}} \right)^{\frac{-1.4}{0.4}}$$

$$P_2 = 210710 \text{ Pa}$$

(d) The mid radius pressure coefficient

$$C_{p, \text{midR}} = \frac{(P_2 - P_1)}{\frac{1}{2} \rho_1 w_1^2}$$

from  $C_{21}$  we can compute  $T_1$

$$T_1 = T_{01} - \frac{C_{21}^2}{2C_p}$$

$$= 340 \text{ K} - \frac{(85 \text{ m/s})^2}{2 (1.0047 \times 10^3 \frac{\text{J}}{\text{kg} \cdot \text{K}})} = 336.40 \text{ K}$$

then from isentropic relations

$$\frac{P_{01}}{P_1} = \left( \frac{T_{01}}{T_1} \right)^{\frac{\gamma}{\gamma-1}}$$

$$P_1 = P_{01} \left( \frac{T_{01}}{T_1} \right)^{\frac{-\gamma}{\gamma-1}} = 178240 \text{ Pa}$$

and

$$\rho_1 = \frac{P_1}{RT_1} = 1.8458 \frac{\text{kg}}{\text{m}^3}$$

and

$$\begin{aligned}w_1 &= \sqrt{(10 \text{ m/s} + 11 C_0)^2 + C_{31}^2} \\&= \sqrt{(10 \text{ m/s} + 105 \text{ m/s})^2 + (85 \text{ m/s})^2} \\&= \underline{143.0035 \text{ m/s}}\end{aligned}$$

thus

$$C_{p, \text{midp}} = \frac{210710 \text{ Pa} - 178240 \text{ Pa}}{\frac{1}{2}(1.2458 \text{ kg/m}^3)(143.0035 \text{ m/s})^2} = \boxed{0.43}$$



### Problem 3

Analyze an aft stage (rotor plus stator) of a high-pressure compressor. It rotates at 8000 rpm and compresses 125 kg/s of air. The inlet pressure and temperature are  $p_1 = 2.0$  MPa and  $T_1 = 720$  K. The average radius of the rotor blades  $r_m = 0.35$  m, and their average height ( $r_o - r_i$ , or  $r_t - r_h$  where subscript  $t$  refers to tip and subscript  $h$  refers to hub) is 0.03 m. The angle of the flow entering the rotor in the stationary frame of reference is the same as the stator exit flow angle ( $\alpha_1 = \alpha_3 = 15^\circ$ ). The turning angle of the flow through the rotor is  $\beta_2 - \beta_1 = 25^\circ$ . The stage was designed for a constant axial velocity  $c_z$ . The efficiency of the stage is 0.90. Use  $c_p = 1.1$  kJ/kg-K,  $\gamma = 1.35$  based on the average static temperature of the stage ( $T_2$ ). Using the station definitions that 1 is the entry to the rotor, 2 is the exit of the rotor/inlet to the stator, and 3 is the exit of the stator, determine  $p_{01}$ ,  $T_{01}$ ,  $p_2$ ,  $T_2$ ,  $p_{02}$ ,  $T_{02}$ ,  $p_3$ ,  $T_3$ ,  $p_{03}$ , and  $T_{03}$ . Also determine the stator turning angle ( $\alpha_3 - \alpha_2$ ), the Mach number at the rotor and stator exits, the required power for the stage, and the percent reaction  $R$  for the stage. In performing the analysis, draw scaled velocity triangles at stations 1, 2, and 3.

First, make the preliminary calculations:  $U$  at meanline radius  $r_m$ , axial velocity  $c_{z1}$ , flow area  $A_1$ , and speed of sound  $a_1$ . Remember based on conservation of mass,  $A_1$  should be based on the axial velocity.

#### Rotor inlet

Use the calculated values of  $U$  and  $c_{z1}$ , and the given value of  $\alpha_1$  to make a scaled drawing of the velocity triangle at the rotor inlet. Calculate  $\beta_1$  and other velocities ( $w_1$ ,  $c_1$ ,  $c_{01}$ ,  $w_{01}$ ) and label them on the VT.

Calculate the Mach number in the rotating frame,  $\text{abs}(w_1)/a_1$ , and the Mach number in the absolute frame,  $\text{abs}(c_1)/a_1$ , to check compressibility and to determine whether shocks may occur.

Calculate the stagnation temperature and pressure using the isentropic flow equations.

#### Rotor exit-stator inlet

Find the blade exit angle  $\beta_2$  from  $\beta_1$  and the turning angle given in the problem statement. Remember that angles are defined with respect to the flow axis, and that angles turning in the direction of  $U$  are positive and angles turning opposite to  $U$  are negative. Realizing that the axial velocity  $c_z$  is constant, make a neat and scaled sketch of the velocity triangle. Label all angles and velocities. Use Euler's work equation to calculate the rise in stagnation enthalpy and stagnation temperature, and calculate the stage pressure ratio  $p_{02}/p_{01}$  using  $\eta_{st} = 0.90$ . Note that this assumes all the losses of the stage occur in the rotor - this is not necessarily true, there will be frictional losses in the stator as well.

Calculate  $T_2$  from the 1<sup>st</sup> law  $h_0 = h + u^2/2$ , using the absolute velocity  $c_2$  for  $u$ . Calculate the speed of sound using  $T_2$ .

Again, calculate the Mach number in the rotating frame,  $\text{abs}(w_2)/a_2$ , and the Mach number in the absolute frame,  $\text{abs}(c_2)/a_2$ , to check compressibility and the possibility of shocks occurring. Setting  $c_z$  and  $r_m$  to be constant, calculate  $A_2$  and the height of the flow annulus at station 2.

Calculate the power input to the rotor stage, and use the equations from the notes to calculate the degree of reaction  $R$ .

Superimpose the velocity triangles at stations 1 and 2 and comment on the symmetry and the effect of the degree of reaction on this symmetry.

#### Stator Exit

Letting  $c_z$  be constant, and using the value given for the absolute exit angle  $\alpha_3$ , calculate  $c_3$ . Calculate the turning angle of the stator.

Using the stagnation enthalpy form of the energy equation as above, calculate  $T_3$ , the speed of sound,  $a_3$ , and the stationary reference frame  $M_3$  based on  $\text{abs}(c_3)$ .

Assuming no frictional loss in the stator ( $p_{03} = p_{02}$ ), calculate the annulus height necessary to keep  $c_z$  constant at station 3.

\* Calculations are done using MATLAB

### Given properties

» Rotor and Stator compressor aft stage

- $\omega = 8000 \text{ rpm} = 837.758 \text{ rad/s}$
- $\dot{m} = 125 \text{ kg/s}$
- $p_1 = 2.0 \text{ MPa}$ ,  $T_1 = 720 \text{ K}$
- rotor entry  $\alpha_1 = 15^\circ$
- stator exit  $\alpha_3 = 15^\circ$
- turning angle  $\beta_2 - \beta_1 = 25^\circ$
- $r_m = 0.35 \text{ m}$
- $r_t - r_h = h = 0.03$
- $C_z = \text{const.}$
- $\eta_{st} = 0.90$
- $C_p = 1.1 \text{ kJ/kg-K}$
- $\gamma = 1.35$
- $R = 287.05 \text{ J/kg-K}$

## Preliminary calculations

first

$$v = r_m \omega = (0.35 \text{ m})(837.758 \text{ rad/s}) = 293.22 \text{ m/s}$$

since  $\begin{cases} r_t + r_h = 2r_m = 0.70 \text{ m} \\ r_t - r_h = 0.03 \text{ m} \end{cases}$

$$A_1 = \pi(r_+^2 - r_h^2) = \pi(r_+ + r_h)(r_+ - r_h) = \pi(0.70\text{m})(0.03\text{m}) = 0.0660\text{m}^2$$

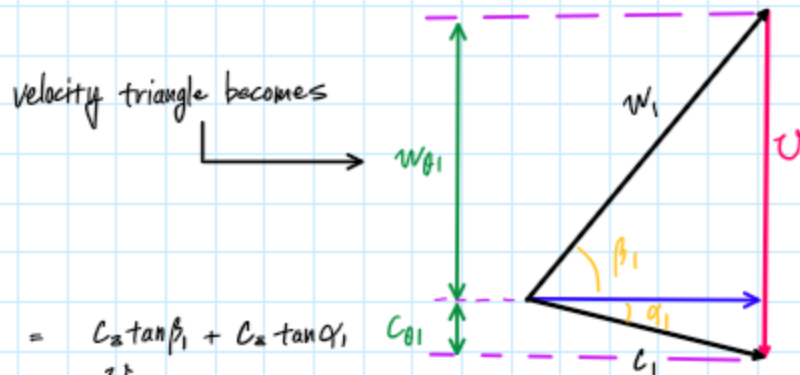
from  $\dot{m} = \rho_1 C_{B1} A_1$

$$C_{21} = \frac{\dot{m}}{\rho_1 A_1} = \frac{\dot{m}}{\frac{P_1}{RT_1} A_1} = 195.79 \text{ m/s}$$

$$a_1 = \sqrt{\gamma R T_1} = 528.22 \text{ m/s}$$

Rotor inlet

velocity triangle becomes



since  $U = C_2 \tan \beta_1 + C_2 \tan \alpha_1$

$$\tan \beta_1 = \frac{v}{c_z} - \tan \alpha_1$$

$$\beta_1 = \arctan\left(\frac{V}{C_2} - \tan \alpha_1\right) = \arctan\left(\frac{293.22 \text{ m/s}}{195.79 \text{ m/s}} - \tan(15^\circ)\right)$$

$$\beta_1 < 0 \rightarrow \beta_1 = -50.88^\circ$$

$$\text{then } w_1 = C_2 / \cos \beta_1 = 310.32 \text{ m/s}$$

$$c_1 = C_2 / \cos \alpha_1 = 202.70 \text{ m/s}$$

$$C_{\theta 1} = C_2 \tan \alpha_1 = 52.463 \text{ m/s}$$

$$w_{\theta 1} = C_2 \tan \beta_1 = 240.75 \text{ m/s} \quad (\text{negative direction})$$

$$\text{next } M_{w1} = \frac{\text{abs}(w_1)}{a_1} = 0.5875$$

$$M_{c1} = \frac{\text{abs}(c_1)}{a_1} = 0.3837$$

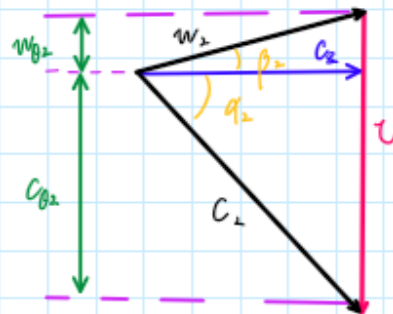
lastly from isentropic relations

$$T_{01} = T_1 \left[1 + \frac{\gamma-1}{2} M_{c1}^2\right] = 738.55 \text{ K}$$

$$P_{01} = P_1 \left[1 + \frac{\gamma-1}{2} M_{c1}^2\right]^{\gamma/(\gamma-1)} = 2.2062 \text{ MPa}$$

Rotor exit-stator inlet

velocity triangle becomes



$$\text{first } \beta_2 = 25^\circ + \beta_1 = -25.88^\circ$$

$$\text{then } w_2 = C_2 / \cos \beta_2 = 217.62 \text{ m/s}$$

$$w_{\theta 2} = C_2 \tan \beta_2 = 94.99 \text{ m/s} \quad (\text{negative direction})$$

$$\text{next } w_{\theta 2} - w_{\theta 1} = C_{\theta 2} - C_{\theta 1}$$

$$C_{\theta 2} = w_{\theta 2} - w_{\theta 1} + C_{\theta 1} = 198.23 \text{ m/s}$$

and

$$C_{\theta 2} = C_2 \tan \alpha_2$$

$$\alpha_2 = \arctan\left(\frac{C_{\theta 2}}{C_2}\right) = 45.35^\circ$$

then  $C_2 = C_8 / \cos 92 = 278.62 \text{ m/s}$

from  $h_{02} - h_{01} = C_p(T_{02} - T_{01}) = U \Delta C_\theta$

where  $\Delta C_\theta = C_{\theta 2} - C_{\theta 1}$

then  $T_{02} = T_{01} + \frac{U(C_{\theta 2} - C_{\theta 1})}{C_p} = 777.41 \text{ K}$

$$U \Delta C_\theta = 4.2740 \times 10^4 \text{ J/kg}$$

using isentropic relations ( $P_{03} = P_{02}$  all loss in rotor)

$$\frac{P_{02}}{P_{01}} = \left(1 + \eta_{st} \frac{U \Delta C_\theta}{C_p T_{01}}\right)^{\gamma/\gamma-1} = 1.1953 \quad P_{02} = 2.6372 \text{ MPa}$$

from  $h_0 = h + \frac{u^2}{2} \Leftrightarrow T_{02} = T_2 + \frac{C_8^2}{2C_p}$

then  $T_2 = T_{02} - \frac{C_8^2}{2C_p} = 759.98 \text{ K}$

so  $a_2 = \sqrt{\gamma R T_2} = 542.69 \text{ m/s}$

$$M_{w2} = \frac{\text{abs}(w_2)}{a_2} = 0.4010$$

$$M_{c2} = \frac{\text{abs}(C_2)}{a_2} = 0.5134$$

from isentropic relations

$$\frac{P_{02}}{P_2} = \left(\frac{T_{02}}{T_2}\right)^{\gamma/\gamma-1} \Leftrightarrow P_2 = P_{02} \left(\frac{T_{02}}{T_2}\right)^{-\gamma/\gamma-1} = 2.4164 \text{ MPa}$$

then  $A_2 = \frac{\dot{m}}{C_8 \rho_2} = \frac{\dot{m}}{C_8 \frac{P_2}{R T_2}} = 0.0576 \text{ m}^2$

thus

$$A_2 = \pi(r_+^2 - r_-^2) = \pi(r_+ - r_-)(r_+ + r_-)$$

$$A_2 = \pi h(2r_m)$$

$$h = \frac{A_2}{2\pi r_m} = 0.0262 \text{ m}$$

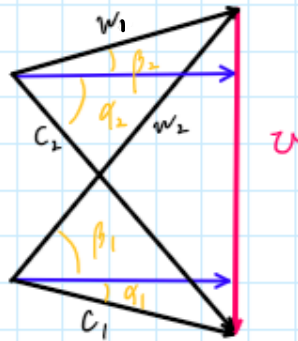
power input is

$$P_c = \dot{m} U \Delta C_\theta = 5.3426 \text{ MW}$$

degree of reaction  $R$  becomes

$$R_{rxn} = -\frac{C_2}{2u} \left( \frac{\tan \beta_1 + \tan \beta_2}{2} \right)$$

$$R_{rxn} = \boxed{0.5725}$$



#### Discussion

- since  $R_{rxn} = 57.25\%$   
thus the velocity triangle is close but not symmetric.
- $\alpha_1 \neq -\beta_2$ ,  $\alpha_2 \neq -\beta_1$

#### Stator Exit

first  $C_3 = C_2 / \cos \alpha_3 = \boxed{202.70 \text{ m/s}}$

then turning angle of stator

$$\alpha_3 - \alpha_2 = \boxed{-30.35^\circ}$$

since  $T_{03} = T_{02}$

$$= \boxed{777.41 \text{ K}}$$

$$T_2 = T_{03} - \frac{C_2^2}{2C_p} = \boxed{759.98 \text{ K}}$$

$$a_3 = \sqrt{\gamma R T_3} = \boxed{542.64 \text{ m/s}}$$

$$M_{C3} = \frac{\text{abs}(C_3)}{a_3} = \boxed{0.3735}$$

also  $P_3 = P_{03} \left( \frac{T_{03}}{T_3} \right)^{\frac{\gamma}{\gamma-1}} = \underline{2.4164 \text{ MPa}}$

$$^{\circ} P_{03} = P_{02}$$

the area  $A_3$  is

$$A_3 = \frac{\dot{m}}{\rho_3 C_3} = \frac{\dot{m}}{\frac{P_3}{RT_3} C_3}$$

$$A_3 = \underline{0.0576 \text{ m}^2}$$

now

$$A_3 = \pi h_3 \cdot 2r_m$$

$$h_3 = \frac{A_3}{2\pi r_m} = \boxed{0.0262 \text{ m}}$$

## Appendix

### AAE 339 HW 7 MATLAB CODE

```
close all; clear all; clc;
```

#### p1.

```
% Defining given properties
P1 = 101325; % [Pa]
T1 = 293.15; % [K]
c_z = 125; % [m/s]
r_m = 0.3; % [m]
zeta = 0.8;
alpha1 = 40; % [deg]
alpha2 = 60; % [deg]
beta1 = 60; % [deg]
beta2 = 40; % [deg]
gamma = 1.4;
R = 287.05;

delta_c_theta = c_z*(tand(beta1) - tand(beta2))
U = c_z*(tand(alpha1) + tand(beta1))
w_s = U/r_m
w_c = U*delta_c_theta
rho1 = P1/R/T1
% Calculate m_dot
r_t = 2/1.8*r_m;
r_h = r_t*0.8;
A = pi*(r_t^2 - r_h^2)
m_dot = rho1*A*c_z
tau_s = m_dot*r_m*delta_c_theta
Pow_s = tau_s*w_s
```

#### p2.

```
clear all; close all; clc;

% <a>
P01 = 185000; % [Pa]
T01 = 340; % [K]
delta_c_theta = 105; % [m/s]
U = 150; % [m/s]
c_z1 = 85; % [m/s]
c_z2 = 75; % [m/s]
Pow_s = U*delta_c_theta
% <b>
gamma = 1.4;
R = 287.05;
```

```

c_p = gamma/(gamma - 1)*R;
T02 = T01 + Pow_s/c_p
T2 = T02 - c_z2^2/c_p/2
% <c>
P02 = P_from_isentropic_relation(P01, T02, T01, gamma, "1")
P2 = P_from_isentropic_relation(P02, T02, T2, gamma, "2")
% <d>
T1 = T01 - c_z1^2/2/c_p
P1 = P_from_isentropic_relation(P01, T01, T1, gamma, "2")
rho1 = P1/R/T1
w1 = sqrt((10 + delta_c_theta)^2 + c_z1^2)
C_P_midR = (P2 - P1)/2/rho1/w1^2

```

### p3.

```

clear all; close all; clc;

omega = 8000*2*pi/60; % [rad/s]
m_dot = 125; % [kg/s]
P1 = 2e6; % [Pa]
T1 = 720; % [K]
    alpha1 = deg2rad(15);
alpha3 = alpha1;
turn_ang = deg2rad(25);
r_m = 0.35; % [m]
h = 0.03; % r_t - r_h
eta = 0.90;
cp = 1.1e3; % [kg/kg/K]
gamma = 1.35;
R = 287.05;

```

### PRELIMINARY CALCULATIONS

```

U = r_m*omega
A1 = pi*2*r_m*h
c_z1 = m_dot/(P1/R/T1)/A1
a1 = sqrt(gamma*R*T1)

```

### ROTOR INLET

```

c_z = c_z1
beta1 = -atan(U/c_z - tan(alpha1))
beta1_deg = rad2deg(beta1)
w1 = c_z/cos(beta1)
c1 = c_z/cos(alpha1)
c_theta1 = c_z*tan(alpha1)
w_theta1 = c_z*tan(beta1)
M_w1 = abs(w1)/a1
M_c1 = abs(c1)/a1
T01 = T_from_M_and_gamma(T1,M_c1,gamma,"stagnation")
P01 = p_from_M_and_gamma(P1,M_c1,gamma,"stagnation")

```



Rotor exit-stator inlet

```

beta2 = beta1 + turn_ang;
beta2_deg = rad2deg(beta2)
w2 = c_z/cos(beta2)
w_theta2 = c_z*tan(beta2)
c_theta2 = w_theta2 - w_theta1 + c_theta1
alpha2 = atan(c_theta2/c_z)
alpha2_deg = rad2deg(alpha2)
c2 = c_z/cos(alpha2)
T02 = T01 + U*(c_theta2-c_theta1)/cp
rise = U*(c_theta2-c_theta1)
P02_P01 = (1 + eta*rise/cp/T01)^(gamma/(gamma - 1))
P02 = P02_P01*P01
T2 = T02 - c_z^2/2/cp
a2 = sqrt(gamma*R*T2)
M_w2 = abs(w2)/a2
M_c2 = abs(c2)/a2
P2 = P02*(T02/T2)^(-gamma/(gamma - 1))
A2 = m_dot/c_z/(P2/R/T2)
h2 = A2/2/pi/r_m
Pc = m_dot*U*(c_theta2-c_theta1)
deg_rxn = -c_z/U*(tan(beta1) + tan(beta2))/2

```

Stator exit

```

c3 = c_z/cos(alpha3)
turning_stator = alpha3 - alpha2
turning_stator_deg = rad2deg(turning_stator)
T03 = T02
T3 = T03 - c_z^2/2/cp
a3 = sqrt(gamma*R*T3)
M_c3 = abs(c3)/a3
P03 = P02
P3 = P03*(T03/T3)^(-gamma/(gamma - 1))
A3 = m_dot/(P3/R/T3)/c_z
h3 = A3/2/pi/r_m

function P2 = P_from_isentropic_relation(P1, T2, T1, gamma, type)
    if type == "1"
        P2 = P1 * (T2 / T1)^(gamma / (gamma - 1));
    elseif type == "2"
        P2 = P1 * (T2 / T1)^(-gamma / (gamma - 1));
    else
        disp("You can only enter 1 or 2 for type.")
    end
end

function T2 = T_from_M_and_gamma(T1, M, gamma, type)
    if type == "stagnation"
        T2 = T1 * (1 + (gamma - 1) / 2 * M^2);
    elseif type == "static"

```

```

        T2 = T1 / (1 + (gamma - 1) / 2 * M^2);
    else
        disp("Error. Incorrect type. Type can only be 'stagnation' or 'static'.")
    end
end

function p2 = p_from_M_and_gamma(p1, M, gamma, type)
    if type == "stagnation"
        p2 = p1 * (1 + (gamma - 1) / 2 * M^2)^(gamma/(gamma - 1));
    elseif type == "static"
        p2 = p1 / (1 + (gamma - 1) / 2 * M^2)^(gamma/(gamma - 1));
    else
        disp("Error. Incorrect type. Type can only be 'stagnation' or 'static'.")
    end
end
end

```