September 24, 2020

AAE 564 Fall 2020

Homework Five

Due: Friday, October 2

Exercise 1 Compute the eigenvalues and eigenvectors of the matrix,

$$A = \begin{pmatrix} 2 & -3 & 0 \\ 2 & -3 & 0 \\ 3 & -5 & 1 \end{pmatrix}$$

.

Exercise 2 Compute the eigenvalues and eigenvectors of the following matrix.

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

Exercise 3 Determine whether or not the following matrix is nondefective.

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 1 & 1 \end{pmatrix}$$

Exercise 4 What is the companion matrix whose eigenvalues are -1, -2, and -3?

Exercise 5 What is the real 2×2 companion matrix with eigenvalues $1 + \jmath$, $1 - \jmath$?

Exercise 6 Suppose A is a real square matrix and the vectors

$$\begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix} \qquad \begin{pmatrix} 1 \\ 2 \\ 4 \\ 8 \end{pmatrix} \qquad \begin{pmatrix} 1 \\ \jmath \\ -1 \\ -\jmath \end{pmatrix}$$

are eigenvectors of A corresponding to eigenvalues -1 and 2 and $\jmath$, respectively. What is the response $x(t)$ of the system $\dot{x} = Ax$ to the following initial conditions.

$$(a) \quad x(0) = \begin{pmatrix} 2 \\ -2 \\ 2 \\ -2 \end{pmatrix} \qquad (b) \quad x(0) = \begin{pmatrix} -1 \\ -2 \\ -4 \\ -8 \end{pmatrix} \qquad (c) \quad x(0) = \begin{pmatrix} 0 \\ 3 \\ 3 \\ 9 \end{pmatrix} \qquad (d) \quad x(0) = \begin{pmatrix} 1 \\ 1 \\ -1 \\ -1 \end{pmatrix} ?$$

Exercise 7 Consider a discrete-time LTI system described by $x(k+1) = Ax(k)$.
(a) Suppose that the vectors

$$\begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

are eigenvectors of A corresponding to eigenvalues -2 and 3, respectively. What is the response $x(k)$ of the system to the initial condition

$$x(0) = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} ?$$

(b) Suppose A is a real matrix and the vector

$$\begin{pmatrix} 1 \\ \jmath \\ -1 \\ -\jmath \end{pmatrix}$$

is an eigenvector of A corresponding to the eigenvalue $2 + 3\imath$. What is the response $x(k)$ of the system to the initial condition

$$x(0) = \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} ?$$

Exercise 8 (You may use Matlab) Recall the 2 pendulum cart system. Consider the equilibrium configurations defined by

$$\begin{aligned} E1: \quad & (y^e, \theta_1^e, \theta_2^e) = (0, 0, 0) \\ E2: \quad & (y^e, \theta_1^e, \theta_2^e) = (0, \pi, \pi) \end{aligned}$$

Consider state space representations of the linearizations corresponding to the following combinations of parameters and equilibrium conditions:

L1	P1	E1
L2	P1	E2
L3	P2	E1
L4	P2	E2
L5	P3	E1
L6	P3	E2
L7	P4	E1
L8	P4	E2

(a) Determine the eigenvalues of all the linearized systems L1-L8.

(b) Compare the behavior of the nonlinear system with that of the linearized system for cases L7 and L8. Illustrate your results with time histories of y, θ_1 and θ_2.

Exercise 9 (You may use Matlab) This exercise refers to linearizations $L7$ and $L8$ of the two pendulum cart system.

(a) For $L7$ choose an initial state for the linearized system which results in a periodic solution for the linearized system.

(b) For $L8$ choose an initial state for the linearized system which results in a solution which asymptotically decays to zero for the linearized system.

(c) For $L8$ choose an initial state for the linearized system which results in a solution whose magnitude grows exponentially for the linearized system.

In each case, simulate both the linearized system and the nonlinear system with initial conditions corresponding to your chosen initial state for the linearized system.