D BP

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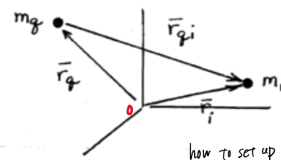
n – Body Problem

Problem: only 10 integrals of motion
12 are required to solve the 2BP

Observation: Do we really care about



We really want to know how it moves relative to some other point such as
⊕ ; ⊙

$\Longrightarrow$ $\bar{r}_{qi}$ more useful !

how to set up a math model that is valid ?

Redo the problem:

relative

Newton's 2nd Law

How to get the EOM governing $\bar{r}_{qi}$? ($\ddot{\bar{r}}_{qi}$)

For any acceleration still necessary to consider $\left\{ \begin{array}{l} \text{position vector } \cancel{\bar{r}_{qi}} \\ \text{frame of differentiation inertial} \end{array} \right.$

To apply Newton's Law of Motion MUST differentiate in inertial frame and base point of the position vector must be fixed in that frame

→ CANNOT use $\bar{F} = m\,\bar{A}$ directly with $\bar{r}_{qi}$ ← not valid for $\bar{r}_{qi}$

But $\bar{r}_{qi}$ can be written in terms of appropriate vectors

$$\boxed{\bar{r}_i = \bar{r}_{qi} + \bar{r}_q}$$
$$\ddot{\bar{r}}_i = \ddot{\bar{r}}_{qi} + \ddot{\bar{r}}_q$$

Apply Newton's law of motion validly to $\bar{r}_i$ and $\bar{r}_q$

$$\ddot{\bar{r}}_i = -G\sum_{\substack{j=1\\ j\neq i}}^{n}\frac{m_j}{r_{ji}^3}\bar{r}_{ji}$$

$$\ddot{\bar{r}}_q = -G\sum_{\substack{j=1\\ j\neq q}}^{n}\frac{m_j}{r_{jq}^3}\bar{r}_{jq}$$

what we want

Sub into $\ddot{\bar{r}}_{qi} + \ddot{\bar{r}}_q = \ddot{\bar{r}}_i$

↑ ↑

$$\ddot{\bar{r}}_{qi} - \underbrace{G\sum_{\substack{j=1\\ j\neq q}}^{n}\frac{m_j}{r_{jq}^3}\bar{r}_{jq}}_{\text{remove } i \text{ term}} = \underbrace{-G\sum_{\substack{j=1\\ j\neq i}}^{n}\frac{m_j}{r_{ji}^3}\bar{r}_{ji}}_{\text{remove } q \text{ term}}$$

← Note: only relative positions appear

$$\ddot{\bar{r}}_{qi} - G\frac{m_i}{r_{iq}^3}\bar{r}_{iq} \underbrace{-G\sum_{\substack{j=1\\ j\neq i,q}}^{n}\frac{m_j}{r_{jq}^3}\bar{r}_{jq}}_{\text{move to right}} = \underbrace{-G\frac{m_q}{r_{qi}^3}\bar{r}_{qi}}_{\text{move to left}} - G\sum_{\substack{j=1\\ j\neq i,q}}^{n}\frac{m_j}{r_{ji}^3}\bar{r}_{ji}$$

Equation for motion of m_i relative to m_q:

$$\boxed{\ddot{\bar{r}}_{qi} + \underbrace{\frac{G(m_i + m_j)}{r_{qi}^3}}_{\text{dominant acceleration}}\bar{r}_{qi} = \underbrace{+G\sum_{\substack{j=1\\ j\neq i,q}}^{n} m_j\left(\frac{\bar{r}_{ij}}{r_{ij}^3} - \frac{\bar{r}_{qj}}{r_{qj}^3}\right)}_{\text{perturbing acceleration}}}$$

dominant acceleration

perturbing acceleration

direct term

indirect term

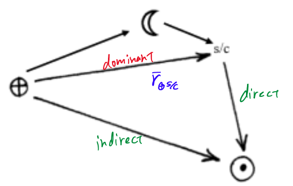
3 body 16 $\longrightarrow$ 6

loss of information

D3

Example: ⊕ ⊙ ☾ s/c

How does s/c move relative to ⊕ ?



Motion of mass 2 relative to mass 1; perturbed by masses 3,4

$\therefore\; q = \overset{1}{\oplus} \qquad i = \overset{2}{s/c} \qquad j = \overset{3}{\odot} \text{ or } \overset{4}{☽}$

$$\ddot{\bar{r}}_{12} + \underbrace{G\frac{(m_1+m_2)}{r_{12}^3}\bar{r}_{12}}_{} = G\,m_3\left(\underbrace{\frac{\bar{r}_{23}}{r_{23}^3}}_{} - \underbrace{\frac{\bar{r}_{13}}{r_{13}^3}}_{}\right) + G\,m_4\left(\frac{\bar{r}_{24}}{r_{24}^3} - \frac{\bar{r}_{14}}{r_{14}^3}\right)$$

direct effect of ☽ on s/c

indirect effect of ☽ on ⊕

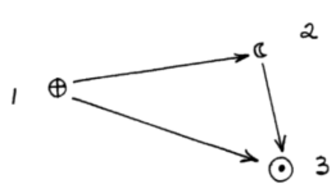
$$\ddot{\bar{r}}_{\oplus s/c} + \underbrace{G\frac{(m_\oplus+m_{s/c})}{r_{\oplus s/c}^3}\bar{r}_{\oplus s/c}}_{\text{dominant}} = \underbrace{G\,m_\odot\left(\frac{\bar{r}_{s/c\odot}}{r_{s/c\odot}^3} - \frac{\bar{r}_{\oplus\odot}}{r_{\oplus\odot}^3}\right)}_{\text{pert acceleration due to } \odot} + \underbrace{G\,m_{☾}\left(\frac{\bar{r}_{s/c☾}}{r_{s/c☾}^3} - \frac{\bar{r}_{\oplus☾}}{r_{\oplus☾}^3}\right)}_{\text{pert acc due to } ☽}$$

direct effect of ⊙ on s/c

indirect effect of ⊙ on ⊕

D3

Example: ⊕ ⊙ ☾



How does ☾ move relative to ⊕ ?

Motion of mass 2 relative to mass 1; perturbed by mass 3

$\rightarrow \ddot{\bar{r}}_{12}$

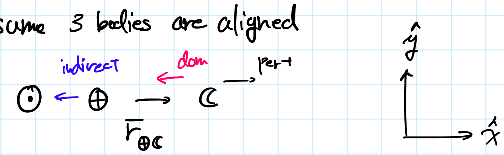
$\therefore\; q = 1 \qquad i = 2 \qquad j = 3$

$$\ddot{\bar{r}}_{12} + \underbrace{G\frac{(m_1+m_2)}{r_{12}^3}\bar{r}_{12}}_{} = G\,m_3\left(\underbrace{\frac{\bar{r}_{23}}{r_{23}^3}}_{} - \underbrace{\frac{\bar{r}_{13}}{r_{13}^3}}_{}\right)$$

$$\ddot{\bar{r}}_{\oplus☾} + G\frac{(m_\oplus+m_☾)}{r_{\oplus☾}^3}\bar{r}_{\oplus☾} = G\,m_\odot\left(\frac{\bar{r}_{☾\odot}}{r_{☾\odot}^3} - \frac{\bar{r}_{\oplus\odot}}{r_{\oplus\odot}^3}\right)$$

Careful – indirect effects frequently forgotten but can be significant!!!

assume 3 bodies are aligned



$$\ddot{\bar{r}}_{\oplus☾} = -\frac{G(m_\oplus+m_☾)}{r_{\oplus☾}^3}\bar{r}_{\oplus☾} + Gm_\odot\left(\frac{\bar{r}_{☾\odot}}{r_{☾\odot}^3} - \frac{\bar{r}_{\oplus\odot}}{r_{\oplus\odot}^3}\right)$$

dom $= 2.731\times10^{-6}\ \frac{km}{s^2}$

direct $= 5.8997\times10^{-6}\ km/s^2$

indirect $= 5.930\times10^{-6}\ km/s^2$

} perturbation accel $-30.36\times10^{-9}\ km/s^2$

$r_{\oplus☾} \qquad \left(r_{☾\odot} \qquad r_{\oplus\odot} \right)$

Careful – indirect effects frequently forgotten but can be significant!!!

D4

Given the equation of motion, do we now have an equation that we can solve?

⇒ Still can't solve if $n \geq 3$

$n = 3$: requires position of $\odot$ relative to $\oplus$ or ☾ ; an additional vector EOM is necessary

↓

to solve two 2nd-order vector DE requires 12 integrals of the motion; we only know 10 !!

$n = 2$: no m_j perturbing bodies
a 2nd-order vector DE in only one unknown position vector!

↓

6 scalar EOMs ; 6 dependent scalar variables ; requires only 6 integrals of motion (we have 10 !!)

→ Relative motion of two bodies solvable

two particles

$$\ddot{\bar{r}}_{12} + G\frac{(m_1 + m_2)}{r_{12}^3}\bar{r}_{12} = \bar{0}$$ solvable but nontrivial

OR

$$\ddot{\bar{r}} + \frac{\mu}{r^3}\bar{r} = \bar{0} \qquad \text{where} \quad \mu = G(m_1 + m_2)$$