College of Engineering
School of Aeronautics and Astronautics

AAE 532
Orbital Mechanics

PS 9
Transfers with Return Orbits

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Problem 1:

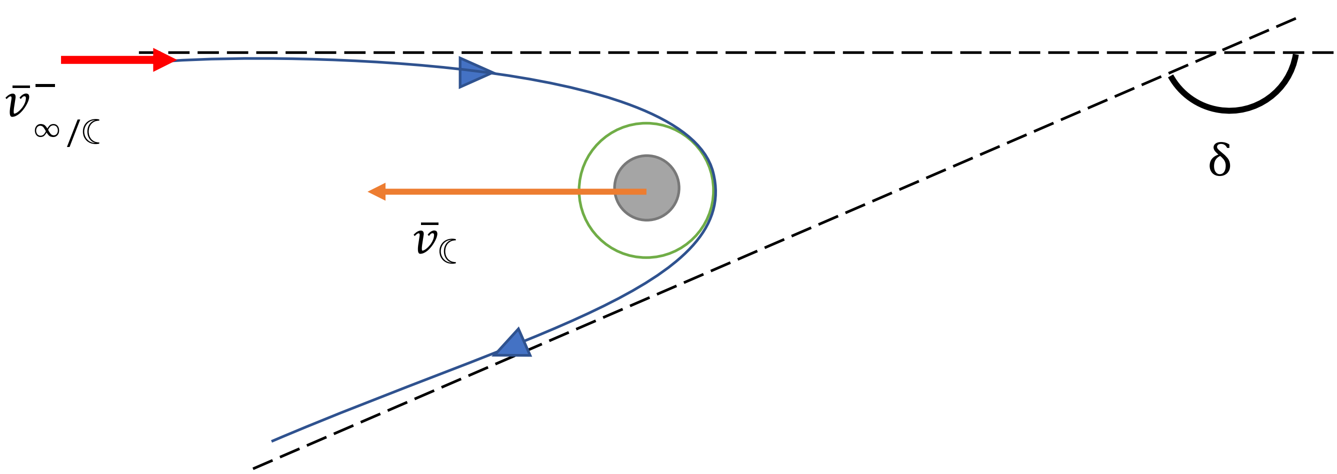
As noted last week in Problem 2 in PS 8, the US is currently planning for humans to reach the Moon in 2024. Return to consideration of a trajectory to the Moon and its return. Assume departure from a 190 km altitude circular Earth parking orbit; include the local gravity field at the Moon.

(a) The path to the Moon last week was planned as a Hohmann transfer so the outbound leg was a 180° transfer. But, the passage by the Moon modified the orbit relative to the Earth. Recall the conditions immediately after the lunar encounter, i.e., $r^+, v^+, \gamma^+, \theta^{*+}$.

Assume that the goal is to return to the Earth orbit. Should the vehicle pass on the light side or the dark side? Was the lunar passage a light side or dark side pass in PS8? If the goal is to return to the Earth, which pass is better, light-side or dark-side? Why?

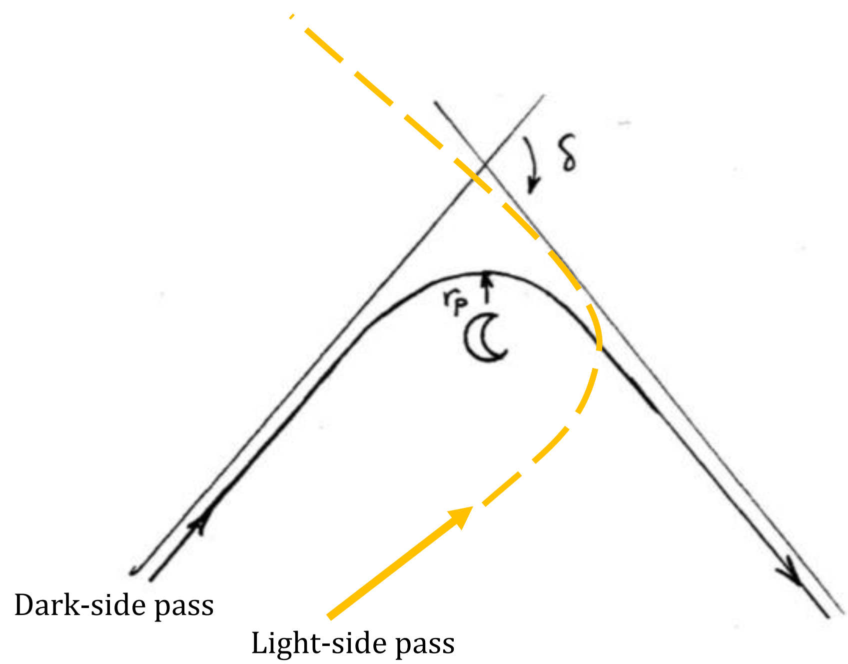
Assume that the pass occurs with a plan to return to Earth; what are the post-encounter conditions? To immediately, return to Earth orbit, assume that a maneuver is implemented to offset the lunar gravity and to return the crew to the second half of the Hohmann transfer path and to the original Earth orbit. Determine the maneuver $|\Delta\bar{v}|$ and α that would be required to immediately return the vehicle to the Hohmann transfer path for the return/inbound arc back to the Earth parking orbit.

Does this maneuver seem reasonable? Recall the analysis from last week, if this return maneuver is missed, can the crew return?

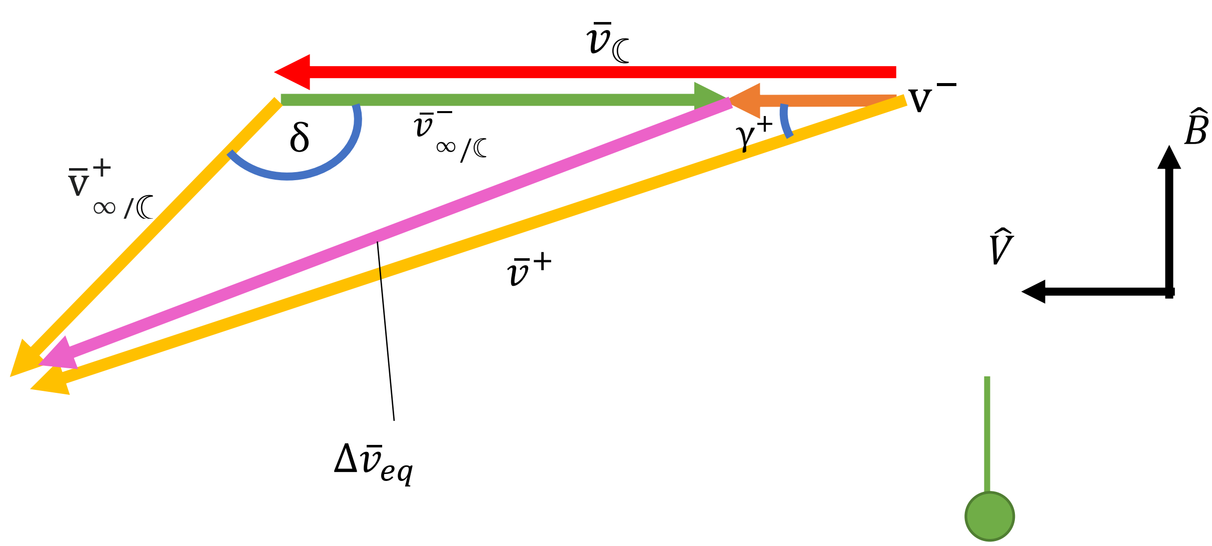


In the diagram above, the Earth is positioned downwards, and as we can see from it, when the goal is to return to the Earth's orbit, it is important to have the vehicle pass on the dark-side so that the vehicle travels under the Earth with a new velocity that is directed towards the Earth. However, even if you pass the light-side you can theoretically have it enter a large orbit centered by the Earth. Then, if the fuel is sufficient, by conducting several maneuvers we can shrink the orbit while maintaining the Earth as the center of the orbit. Thus, if the goal is to return to an Earth's orbit it could be both sides.

In problem 2 of PS8, the passage was a light-side pass. Now, if the goal is to return to the Earth, like in notes K-FR 3 (diagram below) we can see that it is important to have the vehicle pass the dark-side to have the spacecraft go below the Moon and on an orbit that heads back to the Earth.



Since we are assuming dark-side pass to return to the transfer ellipse, the vector diagram for pre and post encounter will look as follows.



From problem 2 of PS8 we have the values

$$v^-_{\infty/\mathbb{C}} = 0.8307 \ \ km/s$$

$$v^- = 0.1876 \ \ km/s$$

$$r^+ = 3.8246e + 5 \ \ km$$

We also know from PS8 that the hyperbola characteristics for the flyby are

$$\xi_{\text{fb}} = \frac{\left(v^+_{\infty/\mathbb{C}}\right)^2}{2} = \frac{\left(v^-_{\infty/\mathbb{C}}\right)^2}{2} = 0.3450 \ \ km^2/s^2$$

$$a_{\text{fb}} = -\frac{\mu_\oplus}{2\xi_{\text{fb}}} = -7.1047e + 3 \ \ km$$

$$e_{fb} = 1 - \frac{200 + R_{\mathbb{C}}}{a_{fb}} = 1.2728$$

$$\delta = 2arcsin\left(\frac{1}{e_{fb}}\right) = 103.5647°$$

From cosine rule,

$$(v^+)^2 = \left(v^+_{\infty/\mathbb{C}}\right)^2 + v_{\mathbb{C}}^2 - 2\left(v^+_{\infty/\mathbb{C}}\right)(v_{\mathbb{C}})cos\delta \quad \Rightarrow \quad \text{v}^+ = 1.4573 \text{ km/s} \ .$$

Then from the sine rule

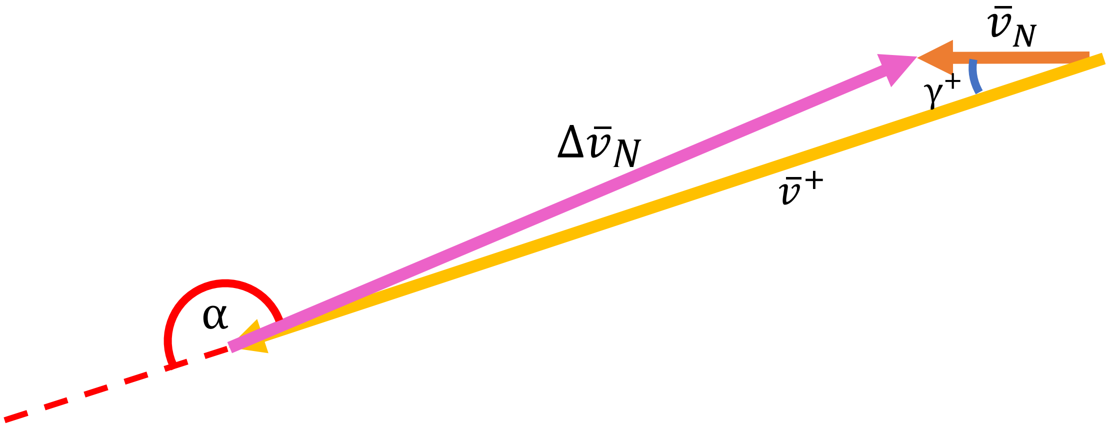
$$\frac{v^+_{\infty/\mathbb{C}}}{sin\gamma^+} = \frac{v^+}{sin\delta} \ \Rightarrow \ \gamma^+ = 33.6501° \ .$$

The flight path angle is positive since the orbit is ascending at this point.

The new true anomaly becomes

$$tan\theta^{*+} = \frac{\left(\frac{r^+(v^+)^2}{\mu_\oplus}\right)sin\gamma^+cos\gamma^+}{\left(\frac{r^+(v^+)^2}{\mu_\oplus}\right)cos^2\gamma^+ - 1} \ \Rightarrow \ \theta^{*+} = 66.3258° \ .$$

Now if we want to return immediately to the original transfer ellipse from the flyby the spacecraft will have to undergo a maneuver depicted as the following vector diagram. Note that the new velocity $\bar{v}_N$ has an equal magnitude and direction to $\bar{v}^-$.



This $\Delta\bar{v}$ has an equivalent magnitude as $\Delta\bar{v}_{eq}$ and opposite direction as it. From PS8 we know that

$$\Delta v_N = |\Delta\bar{v}_{eq}| = 2v^+_{\infty/☾} sin\frac{\delta}{2} = 1.3053\ km/s.$$

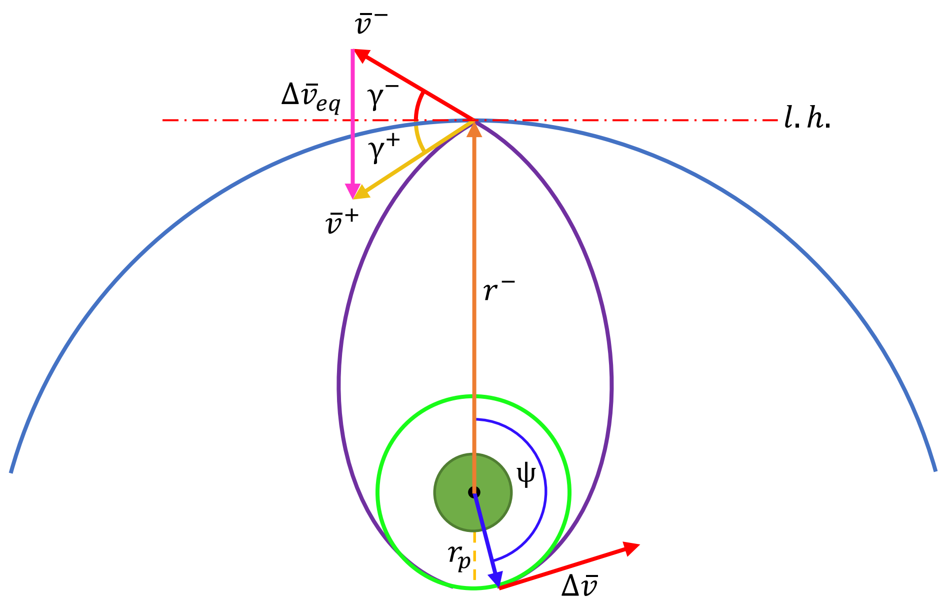
And α becomes

$$\alpha = \gamma^+ + \frac{180° + \delta}{2} = 175.4324°\ .$$

This maneuver seems very counterproductive in that it requires a large magnitude in the opposite direction of the $\bar{v}^+$ to decrease the velocity to the velocity at the apoapsis of the transfer orbit. Thus, it is rather unreasonable.

Recalling problem 2 of PS8, we know that the spacecraft misses the maneuver the spacecraft will enter a hyperbolic orbit meaning that the crew on board will not be able to return to Earth.

(b) To avoid reliance on a return maneuver, reconsider the transfer as a free-return. Assume a circular, coplanar lunar orbit. Assume that the vehicle departs the same 190-km Earth parking orbit. However, rather than a Hohmann transfer, design a free-return trajectory such that the transfer angle is 173.8° . Determine the transfer orbit characteristics: $a, e, r_p, r_a, \mathbb{P}, \mathcal{E}$.
The Earth →Moon TOF on the outbound leg?
Phase angle at departure from the parking orbit?



We know from the altitude of the Earth parking orbit, $h_\oplus = 190km$, that

$$r_p = R_\oplus + h_\oplus = 6.5681e + 3 \ \ km \ .$$

From the diagram, we can tell that

$$r^- = 384400 \ km \ .$$

The transfer angle, $\psi = \ 173.8°$ implies that $\theta^* = \psi$, so

$$r^- = \frac{r_p(1+e)}{1 + ecos\psi} \quad \Rightarrow \quad e = 0.9720$$

$$a = \frac{r_p}{1-e} = 2.3450e + 5 \ km$$

$$r_a = a(1+e) = 4.6242e + 5 \ km$$

$$\mathbb{P} = 2\pi\sqrt{\frac{a^3}{\mu_\oplus}} = 1.1301e + 6 \ s \ = \ 13.0797 \ days$$

$$\mathcal{E} = -\frac{\mu_\oplus}{2a} = -0.8499\ km^2/s^2\ .$$

To find the *TOF* at the arrival point we first find the eccentric anomaly at the point using the following relation

$$\tan\frac{\theta^*}{2} = \left(\frac{1+e}{1-e}\right)^{1/2} \tan\frac{E}{2}$$

$$\Rightarrow\ E\ =\ 131.1236°$$

Thus, the *TOF* of the outbound leg from the Earth to Moon can be computed as

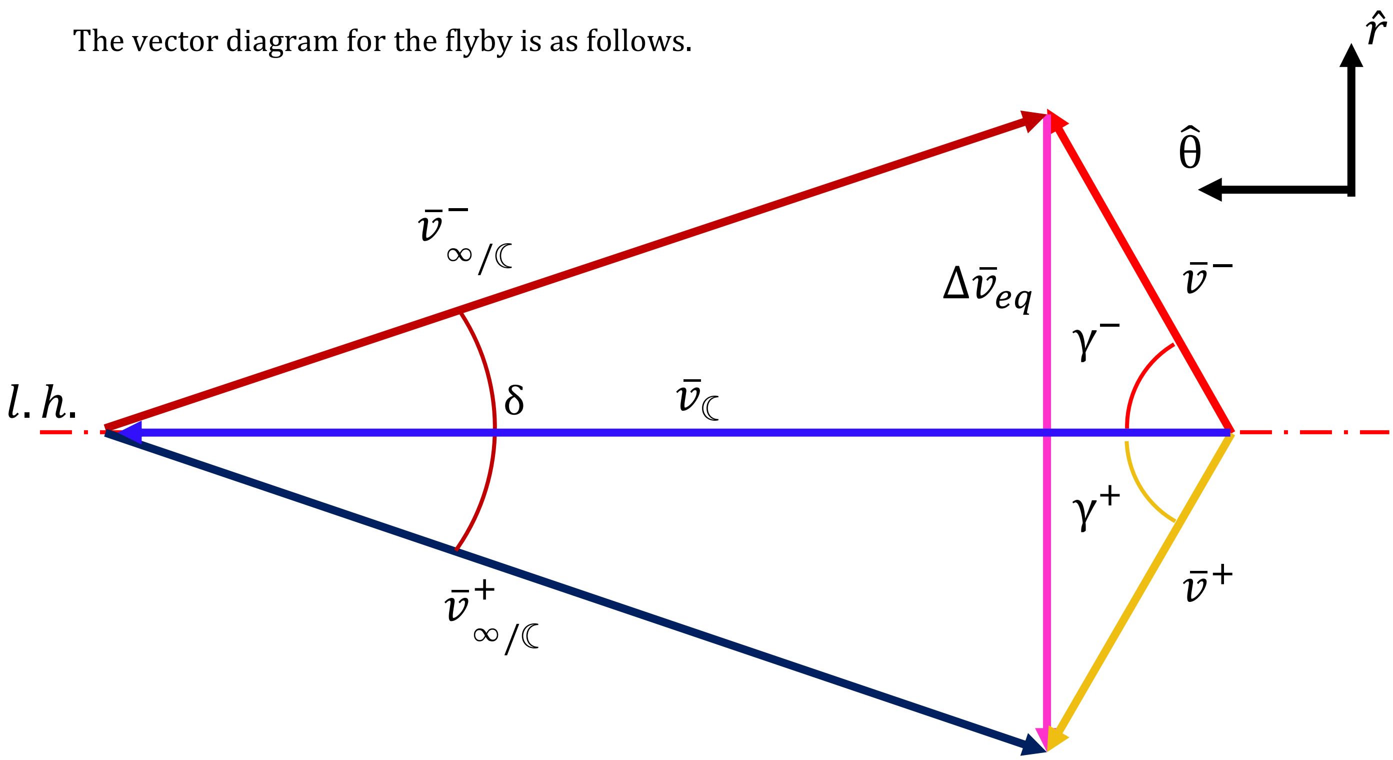
$$\text{TOF} = \sqrt{\frac{a^3}{\mu_\oplus}}(E - esinE)\ =\ 3.2399\ days\ .$$

Then the phase angle, ϕ from the departure of parking orbit becomes

$$\sqrt{\frac{\mu_\oplus}{a^3}}(TOF) = \psi - \phi\ \ \Rightarrow\ \ \phi = 131.3131°\ \ .$$

(c) What is the pass distance r_p at the Moon to ensure a free return? Altitude? Is this altitude reasonable?
What are the orbital characteristics, relative to the Earth, after the lunar encounter, i.e., $r^+, v^+, \gamma^+, \theta^{*+}, a,^+, e^+, r_p^+, r_a^+, \mathbb{P}^+, \mathcal{E}^+, \Delta\omega$ in the new orbit?
What is the $|\Delta\bar{v}_{eq}|$, α that corresponds to the free return?

The vector diagram for the flyby is as follows.



We first find the pre and post encounter velocities.

$$v^- = \sqrt{\mu_\oplus\left(\frac{2}{r^-} - \frac{1}{a}\right)} = 0.6116\ km/s\ .$$

$$\therefore v^+ = v^- = 0.6116\ km/s\ .$$

The flight path angle is

$$\gamma^- = arccos\left(\frac{\sqrt{\mu_\oplus a(1-e^2)}}{r^- v^-}\right) = 72.2043°$$

$$\therefore \gamma^+ = 72.2043°\ .$$

From the properties of an isosceles triangle we know that

$$|\Delta\bar{v}_{eq}| = 2v^- sin\gamma^- = 1.1647\ km/s\ .$$

The geocentric velocity of the Moon is

$$v_{☾} = \sqrt{\frac{\mu_{\oplus}}{r^-}} = 1.0183\ km/s\ .$$

Next the v-infinity can be calculated using vector calculation

$$\bar{v}^- = v^-\left(sin(\gamma^-)\hat{r} + cos(\gamma^-)\hat{\theta}\right) = 0.5823\hat{r} + 0.1869\hat{\theta}\ km/s$$

$$\bar{v}_{☾} = 1.0183\hat{\theta}\ km/s$$

$$\therefore \bar{v}^-_{\infty/☾} = \bar{v}^- - \bar{v}_{☾} = 0.5823\hat{r} - 0.8314\hat{\theta}\ km/s\ .$$

The magnitude of v-infinity is

$$v^-_{\infty/☾} = v^+_{\infty/☾} = \left|\bar{v}^-_{\infty/☾}\right| = 1.0150\ km/s\ .$$

Then

$$\delta = 2arcsin\left(\frac{\left|\Delta\bar{v}_{eq}\right|}{2v^-_{\infty/☾}}\right) = 70.0187°\ .$$

Then we can find the eccentricity of the flyby hyperbola

$$e_{fb} = \frac{1}{sin\left(\frac{\delta}{2}\right)} = 1.7430\ .$$

Also,

$$\mathcal{E}_{fb} = \frac{\left(v^-_{\infty/☾}\right)^2}{2} = 0.5152 km^2/s^2$$

$$a_{fb} = -\frac{\mu_{☾}}{2\mathcal{E}_{fb}} = -4.7585e + 3\ km\ .$$

Then, finally we can compute the periapsis of the flyby hyperbola

$$r_{p,fb} = a_{fb}\left(1 - e_{fb}\right) = 3.5358e + 3\ km\ .$$

This periapsis of the flyby hyperbola is about twice of the mean radius of the Moon. This is somewhat large but is a feasible passage. Thus, it is reasonable.

After the lunar encounter the characteristics of the orbit are going to be

$$r^+ = r^- = 384400\ km$$

$$v^+ = 0.6116\ km/s$$

$$\gamma^+ = \gamma^- = 72.2043°$$

$$tan\theta^{*+} = \frac{\left(\frac{r^+(v^+)^2}{\mu_\oplus}\right) sin\gamma^+ cos\gamma^+}{\left(\frac{r^+(v^+)^2}{\mu_\oplus}\right) cos^2\gamma^+ - 1} \Rightarrow \theta^{*+} = -173.8°$$

$$a^+ = \frac{\frac{-\mu_\oplus}{2}}{\left(\frac{(v^+)^2}{2} - \frac{\mu_\oplus}{r^+}\right)} = 2.3450e + 5\ km$$

$$h^+ = r^+ v^+ cos\gamma^+ = 7.1853e + 4\ km^2/s$$

$$p^+ = \frac{(h^+)^2}{\mu_\oplus} = 1.2952e + 4\ km$$

$$e^+ = e_N = \sqrt{1 - \frac{p_N}{a_N}} = 0.9720$$

$$r_p^+ = a^+(1 - e^+) = 6.6581e + 3\ km$$

$$r_a^+ = a^+(1 + e^+) = 4.6242e + 5\ km$$

$$\mathbb{P}^+ = \mathbb{P} = 13.0797\ days$$

$$\mathcal{E}^+ = -0.8499\ km^2/s^2\ .$$

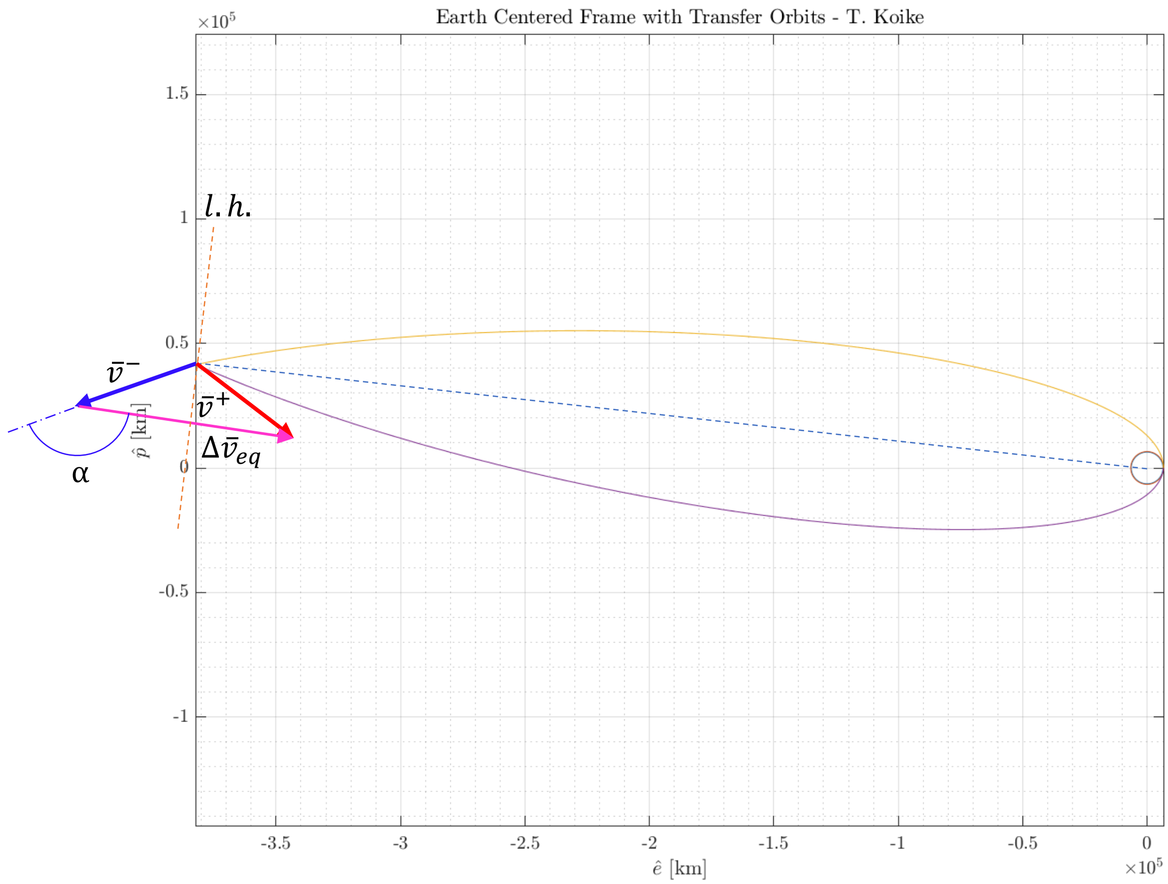
$$\Delta\omega = \theta^{*-} - \theta^{*+} = -12.4°$$

Finally,

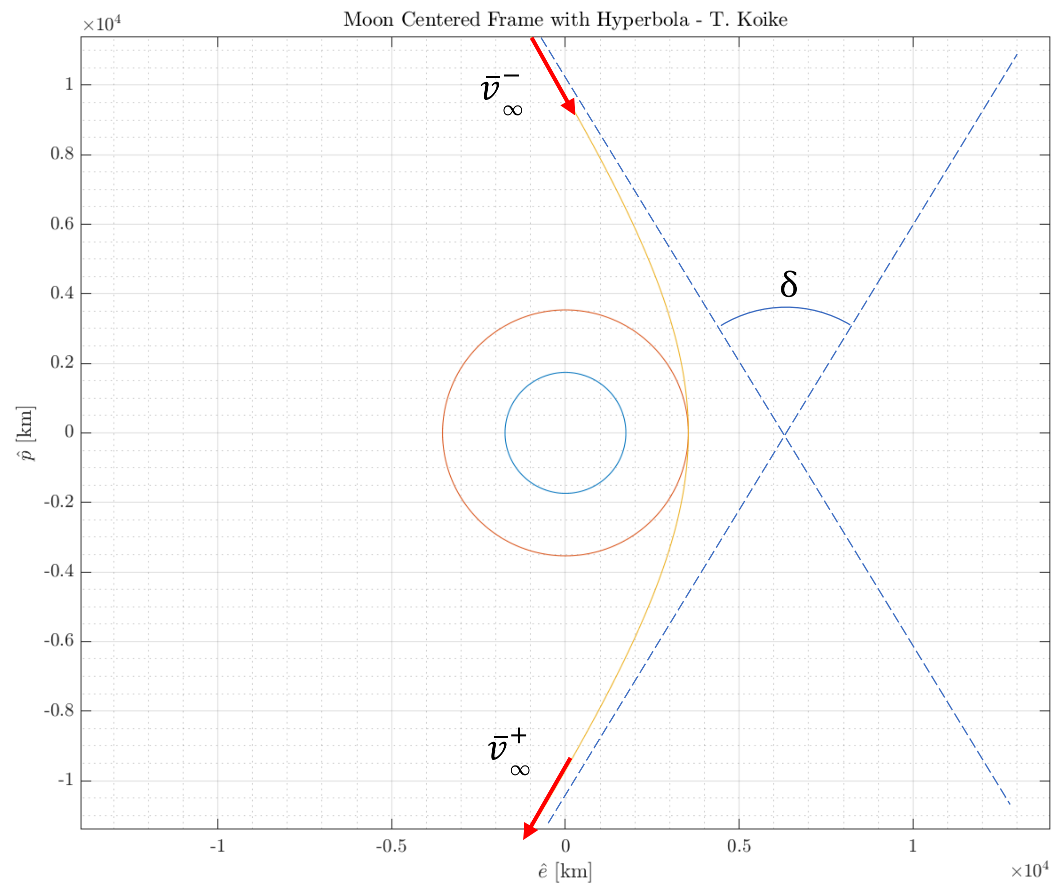
$$\alpha = 90° + \gamma^- = 162.2043°\,.$$

(d) For the free-return, plot the orbit in Matlab: (i) in the Earth centered frame, plot the parking orbit, then the outbound and the return arcs <u>only</u>. On the plot, add $\bar{v}^-, \bar{v}^+, l.h., \Delta\bar{v}_{eq}, \alpha$. (ii) in the Moon centered frame, plot the hyperbola. Add the asymptotes, the vectors, $\bar{v}_\infty^-, \bar{v}_\infty^+, \delta$.

(i)



(ii)


Moon Centered Frame with Hyperbola - T. Koike
$\times 10^4$
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
$\hat{p}$ [km]
-1
-0.5
0
0.5
1
$\hat{e}$ [km]
$\times 10^4$
$\bar{v}_\infty^-$
$\bar{v}_\infty^+$
δ

MATLAB

```
% AAE 532 HW 9 Problem 1
% Tomoki Koike
close all; clear all; clc;
fdir = 'C:\Users\Tomo\Desktop\studies\2020-Fall\AAE532\matlab\outputs\ps9';
set(groot, 'defaulttextinterpreter','latex');
set(groot, 'defaultAxesTickLabelInterpreter','latex');
set(groot, 'defaultLegendInterpreter','latex');

% Set constants
planet_consts = setup_planetary_constants();  % Function that sets up all the
constants in the table
sun = planet_consts.sun;  % structure of sun
earth = planet_consts.earth;  % structure of earth
moon = planet_consts.moon;  % structure of moon
G = 6.6743015e-20;  % Gravitational constant [km^3/kg/s^2]

%% From Problem 2 of PS8
% Set given constants
h_PO = 190;  % earth parking orbit altitude
h_cap = 200;  % moon capture orbit altitude

% (a)
r_minus = earth.mer + h_PO;
r_plus = moon.smao - h_cap - moon.mer ;
a_T = 0.5*(r_minus+r_plus);
e_T = (r_plus - r_minus) / (r_plus + r_minus);
p_T = a_T * (1 - e_T^2);
IP_minus = 2*pi*sqrt(a_T^3 / earth.gp);
IP_minus_days = IP_minus / 60 / 60 / 24;
xi_minus = -earth.gp / 2 / a_T;

v_PO = sqrt(earth.gp / r_minus);
v_plus = vis_viva(r_minus, a_T, earth.gp);
Dv_dep = v_plus - v_PO;
v_minus = vis_viva(r_plus, a_T, earth.gp);
v_moon = sqrt(earth.gp / moon.smao);
v_inf_moon = abs(v_minus - v_moon);
v_c_moon = sqrt(moon.gp / (moon.mer + h_cap));
Dv_arr = sqrt( v_inf_moon^2 + 2*moon.gp / (moon.mer + 200) ) - v_c_moon;
Dv_total = Dv_dep + Dv_arr;
TOF = IP_minus / 2 ;
TOF_days = TOF / 60 / 60 / 24;
phi = rad2deg(pi - sqrt(earth.gp / moon.smao^3)*TOF);
% (b)

xi_plus = v_inf_moon^2 / 2
a_plus = -moon.gp / 2 / xi_plus
e_plus = 1 - (h_cap + moon.mer) / a_plus
delta = 2*asind(1 / e_plus)
v_plus = sqrt( v_inf_moon^2 + v_moon^2 - 2*v_inf_moon*v_moon*cosd(delta) )
FPA_plus = asind( v_inf_moon / v_plus * sind(delta))

% True anomaly
```

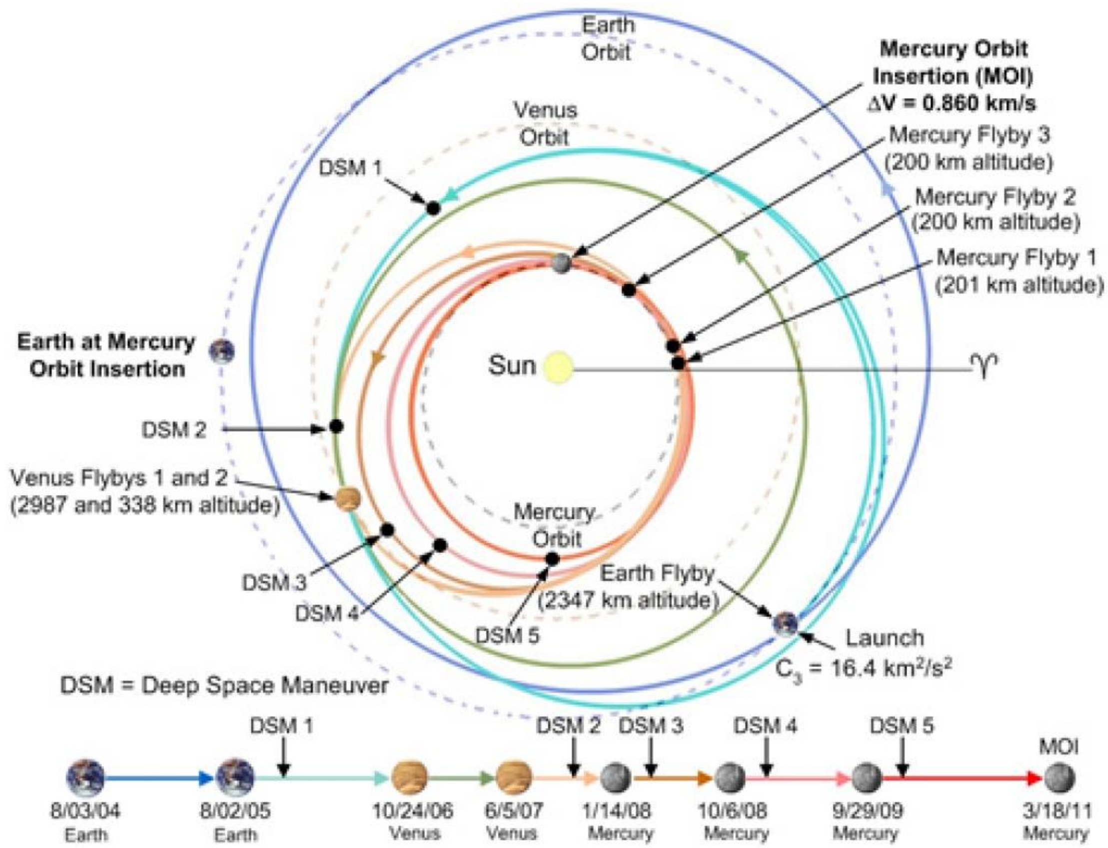
```
temp = r_plus * v_plus^2 / earth.gp
TA_plus = atand( temp*sind(FPA_plus)*cosd(FPA_plus) / ( temp*cosd(FPA_plus)^2 - 1 ))
Domega = 180 - TA_plus
% characterisitic
a_N = -earth.gp / 2 / (v_plus^2 / 2 - earth.gp / r_plus)
h_N = r_plus*v_plus*cosd(FPA_plus)
p_N = h_N^2 / earth.gp
e_N =  sqrt(1 - p_N / a_N)
r_aN = a_N * (1 + e_N)
r_pN = abs(a_N) * (e_N - 1)
xi_N = -earth.gp / 2 / a_N
IP_N = 2*pi * sqrt(a_N^3 / earth.gp)
IP_N_year = IP_N / 60 /60 / 24 / 365
% (c);
Dv_eq = 2 * v_inf_moon * sind(delta / 2)
alpha = (180 - delta) / 2
Dv_eq_vec = Dv_eq * [cosd(alpha), sind(alpha), 0]

% (a)

Dv_N = Dv_eq
alpha = FPA_plus + (180 + delta) / 2

% (b)

% Find the transfer orbit characteristics
psi = 173.8;  % transfer angle
h_earth = 190;  % Altitude of parking orbit around the Earth
r_p = h_earth + earth.mer  % periapsis of transfer orbit
r_minus = moon.smao
e = (r_minus/r_p - 1) / (1 - r_minus/r_p * cosd(psi))  % eccentricity
a = r_p / (1 - e)  % semi-major axis
r_a = a*(1 + e)  % apoapsis
IP = 2*pi*sqrt(a^3 / earth.gp)  % period in seconds
IP_day = IP / 60 / 60 / 24  % period in days
En = -earth.gp / 2 / a  % specific energy

% Find the TOF at the arrival point
EA = T2E_anomaly(e, psi, "deg")  % the eccentric anomaly at the arrival point
TOF = sqrt(a^3 / earth.gp) * (deg2rad(EA) - e*sind(EA))  % the tof at the arrival
point
TOF_day = TOF / 60 / 60 / 24

% Find the phase angle for this departure
phi = rad2deg(deg2rad(psi) - sqrt(earth.gp / moon.smao^3) * TOF )

% (c)

% Find the periapsis distance for the lunar flyby
v_minus = vis_viva(r_minus, a, earth.gp)  % velocity pre encounter
p_minus = a*(1 - e^2);
FPA_minus = acosd(sqrt(earth.gp*p_minus) / r_minus / v_minus)  % flight path angle
Dv_eq = 2*v_minus*sind(FPA_minus)  % equivalent delta-V
v_moon = sqrt(earth.gp / r_minus)  % the geocentric velocity of the moon
v_minus_vec = v_minus * [sind(FPA_minus), cosd(FPA_minus), 0]
```

```
v_moon_vec = v_moon * [sind(0), cosd(0), 0]
v_inf_moon_vec = v_minus_vec - v_moon_vec
v_inf_moon = norm(v_inf_moon_vec)
delta = 2*asind(Dv_eq / 2 / v_inf_moon)

% characteristics of the hyperbolic flyby
e_fb = 1 / sind(delta / 2)
En_fb = v_inf_moon^2 / 2
a_fb = -moon.gp / 2 / En_fb
r_p_fb = a_fb * (1 - e_fb)

r_plus = r_minus;
v_plus = v_minus;
FPA_plus = -FPA_minus;
% True anomaly
temp = r_plus * v_plus^2 / earth.gp;
TA_plus = atan_dbval( temp*sind(FPA_plus)*cosd(FPA_plus) / ( temp*cosd(FPA_plus)^2 -
1 ), "deg")
TA_plus = TA_plus(TA_plus < 0);
TA_plus = TA_plus + 360;

a_plus = (-earth.gp / 2) / (v_plus^2 / 2 - earth.gp / r_plus)
h_plus = r_plus*v_plus*cosd(FPA_plus)
p_plus = h_plus^2 / earth.gp
e_plus = sqrt(1 - p_plus / a_plus)
r_p_plus = a_plus*(1 - e_plus)
r_a_plus = a_plus * ( 1 + e_plus)
Domega = psi - TA_plus
alpha = 90 + FPA_minus

% (d)

% plotting
% (i)
angles = 0:0.001:2*pi;

% Earth
Xearth = earth.mer * cos(angles); Yearth = earth.mer * sin(angles);

% Earth parking orbit
Xearth_po = r_p * cos(angles); Yearth_po = r_p * sin(angles);

% Outbound
angles = 0:0.001:deg2rad(psi);
Rout = p_minus ./ (1 + e*cos(angles)); Xout = Rout.*cos(angles); Yout =
Rout.*sin(angles);

% return
angles = psi:0.1:360;
Rre = p_plus ./ (1 + e_plus*cosd(angles - (Domega)));
Xre = Rre .* cosd(angles); Yre = Rre .* sind(angles);


fig = figure("Renderer","painters","Position",[10, 10, 900, 700]);
    plot(Xearth,Yearth)
```

```
    hold on; grid on; grid minor; box on; axis equal;
    plot(Xearth_po, Yearth_po)
    plot(Xout, Yout)
    plot(Xre, Yre)
    hold off
    title('Earth Centered Frame with Transfer Orbits - T. Koike')
    xlabel('$\hat{e}$ [km]')
    ylabel('$\hat{p}$ [km]')
saveas(fig, fullfile(fdir, "p1_earthCenter.png"))

% (ii)
% moon
angles = 0:0.1:360;
Xmoon = moon.mer * cosd(angles); Ymoon = moon.mer * sind(angles);

% moon parking orbit
Xmoon_po = (r_p_fb) * cosd(angles); Ymoon_po = r_p_fb * sind(angles);

% hyperbola
angles = -95:0.1:95;
Rhyp = a_fb*(1 - e_fb^2) ./ (1 + e_fb * cosd(angles));
Xhyp = Rhyp .* cosd(angles);  Yhyp = Rhyp .* sind(angles);

fig = figure("Renderer","painters","Position",[10, 10, 900, 700]);
    plot(Xmoon,Ymoon)
    hold on; grid on; grid minor; box on; axis equal;
    plot(Xmoon_po, Ymoon_po)
    plot(Xhyp, Yhyp)
    hold off
    xlabel('$\hat{e}$ [km]')
    ylabel('$\hat{p}$ [km]')
    title("Moon Centered Frame with Hyperbola - T. Koike")
saveas(fig, fullfile(fdir, "p1_moonCenter.png"));
```

Problem 2:

The Messenger spacecraft offered exploration of the planet Mercury after launch in 2004 with Mercury orbit insertion in March 2011. The transfer to Mercury employed more than one Venus gravity assist.



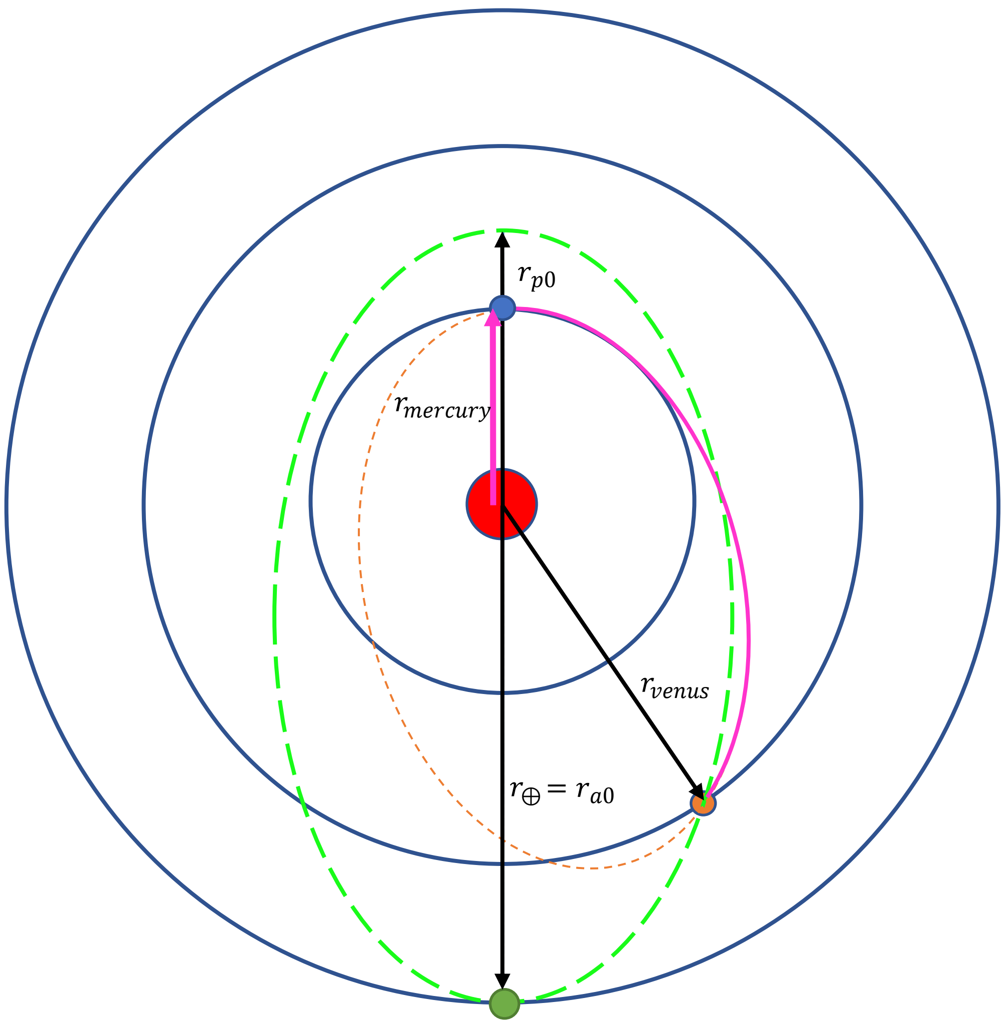
(a) Determine the actual values for *TOF* for the following:
  (i) Earth launch to the first Venus flyby
  (ii) Earth launch to first Mercury flyby
  (iii) Earth launch to MOI (Mercury Orbit Insertion)

	Start	***End***	**TOF *[days]***
(i)	08/03/2004	10/24/2006	812
(ii)	08/03/2004	01/14/2008	1259
(iii)	08/03/2004	03/18/2011	2418

(b) Assume that you are completing a preliminary analysis for such a Mercury mission but assume that all planetary orbits are coplanar and circular and use patch conics. A Venus gravity assist will aid in reducing the launch maneuver. So, to assess the possible gravity assist, consider a Hohmann transfer; but let the transfer path possess a perihelion distance equal to 0.50 AU. As a result, the transfer path does not reach Mercury; however, along the path the spacecraft encounters Venus. Examine a Venus gravity assist and explore whether Venus can deliver the spacecraft to Mercury.

Sketch the heliocentric view to describe the path and identify the location for the Venus encounter. Compute the θ* along the heliocentric path at which the Venus encounter occurs. *TOF* from Earth to Venus?

The sketch of the orbit is as follows.



The original transfer orbit has the characteristics of

$$r_{p0} = 0.5\ AU = 74798949\ km$$

$$r_{a0} = 1\ AU = 149597898\ km$$

$$a_0 = 0.5(r_{p0} + r_{a0}) = 1.1220e + 8\ \ km$$

$$e = \frac{r_{a0} - r_{p0}}{r_{a0} + r_{p0}} = 0.3333$$

$$p_0 = a_0(1 - e_0^2) = \ 9.9732e + 7\ km\ \ .$$

Then the true anomaly at the Venus encounter can be computed as

$$r_{venus} = \frac{p_0}{1 + e_0 cos\theta^*_{venus}} \quad \because r_{venus} = 1.0821e + 8\ km$$

$$\Rightarrow \theta^*_{venus} = arccos\left(\frac{1}{e_0}\left(\frac{p_0}{r_{venus}} - 1\right)\right) = -103.5902° \ \ .$$

This true anomaly is in the descending orbit.

The eccentric anomaly corresponding to this true anomaly is

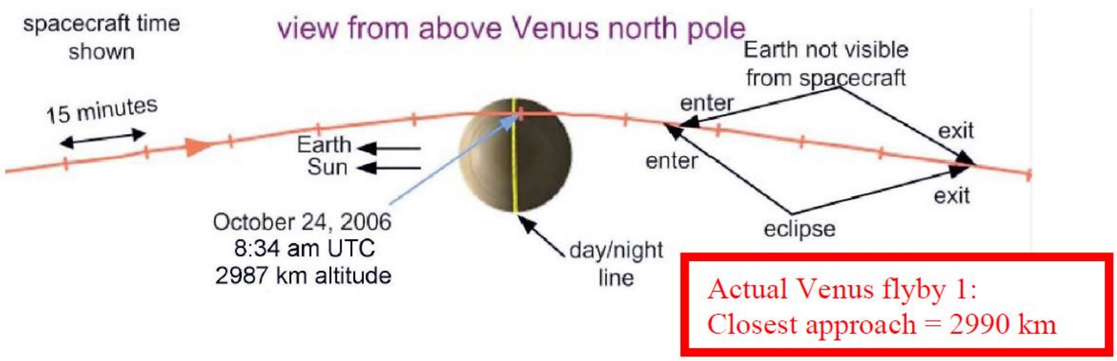
$$\tan\frac{\theta^*_{venus}}{2} = \left(\frac{1 + e_0}{1 - e_0}\right)^{1/2} \tan\frac{E_{venus}}{2}$$

$$\Rightarrow E_{venus} = -83.8739°.$$

Thus, the time of flight becomes

$$\mathrm{TOF_{venus}} = \pi\sqrt{\frac{a^3}{\mu_\odot}} - \sqrt{\frac{a^3}{\mu_\odot}}\left(|E_{venus}| - e_0 sin(|E_{venus}|)\right) = 75.8616\ days.$$

(c) For the actual Venus Flyby 1, the flyby distance was 2990 km altitude. How many Venus radii was the actual encounter? Assume that your Venus encounter uses the same pass distance.
To continue down to Mercury most efficiently, is it desirable to gain or lose energy? Should the spacecraft pass 'ahead' or 'behind' Venus? Determine the following quantities for the post-encounter heliocentric orbit: $a, e, r, v, \gamma, \theta^*, r_p, r_a, \mathbb{P}, \Delta\omega$. {Don't forget the Venus-centered vector diagram!]



Determine the equivalent $\Delta\bar{v}_{eq}$ due to the Venus encounter. What is the magnitude and direction, i.e. $|\Delta\bar{v}_{eq}|$ and α?

The flyby altitude of 2990 km is $0.4941R_{venus}$, and the actual distance from the center of Venus is $1.4941R_{venus}$.

$$r_{p.fb} = 1.4941R_{venus} = 9.0419e + 6\ km\ .$$

Since, we want to shorten the perihelion distance of the original transfer orbit, it is desirable to lose energy whilst the flyby. That means that the spacecraft should pass ahead of Venus.

The heliocentric velocity of Venus is

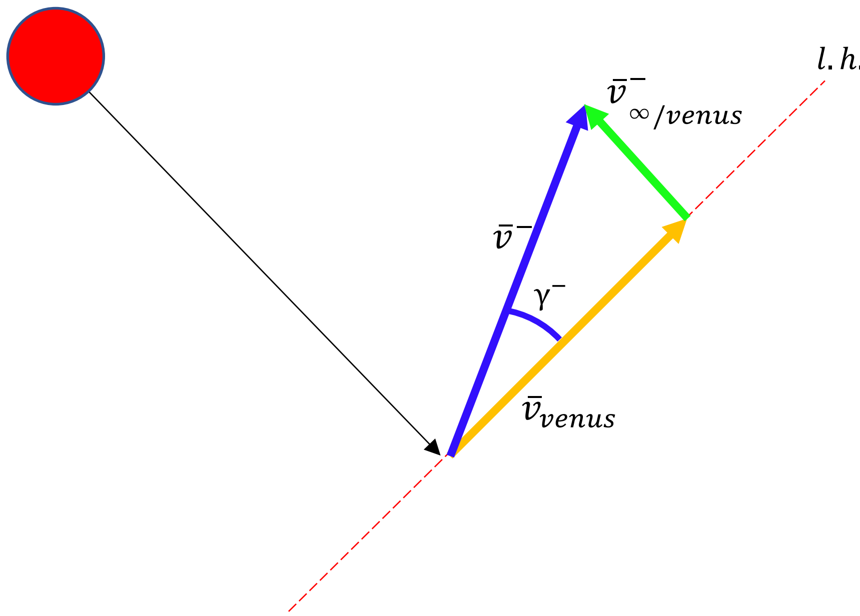
$$v_{venus} = \sqrt{\frac{\mu_\odot}{r_{venus}}} = 35.0209\ \ km/s\ .$$

The pre-encounter conditions are

$$v^- = \sqrt{\mu_\odot\left(\frac{2}{r_{venus}} - \frac{1}{a_0}\right)} = 35.6384\ km/s\ .$$

$$\gamma^- = arccos\left(\frac{\sqrt{\mu_\odot p_0}}{r_{venus}v^-}\right) = -19.3683°.$$

From the vector diagram



$$v^-_{\infty/venus} = \sqrt{v_{venus}^2 + (v^-)^2 - 2v_{venus}v^- cos\gamma^-} = 11.9017\ km/s\ .$$

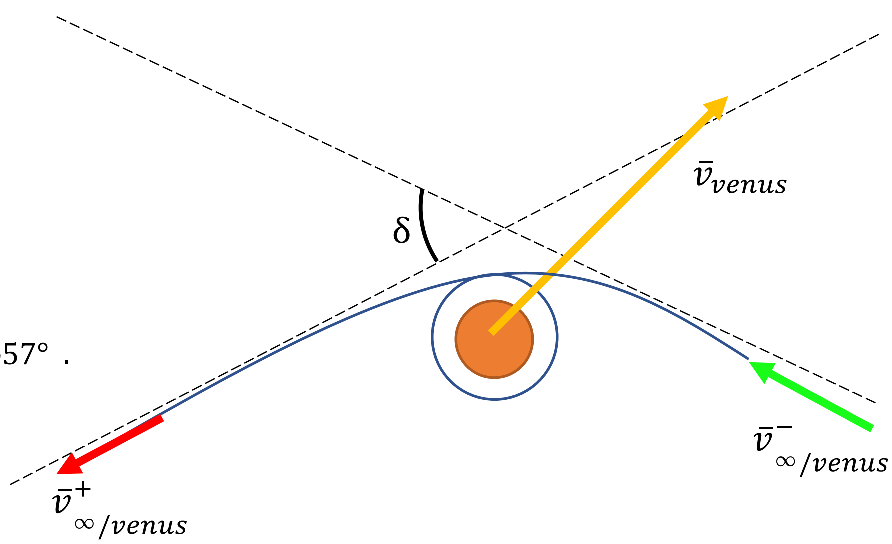
The hyperbola has the characteristics of

$\mathcal{E} = v_\infty^2/2 = -\mu_{\text{venus}}/2a_{fb}$

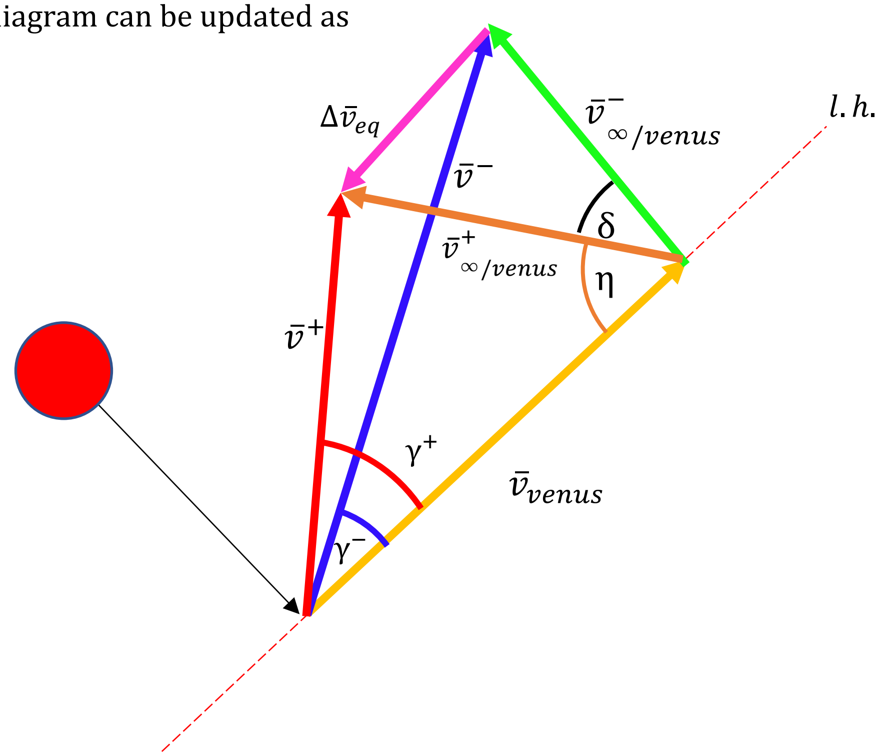
$\Rightarrow a_{fb} = -2.2934e+3\ km$

$e_{fb} = 1 - r_{p,fb}/a_{fb} = 4.9426$

$sin(\delta/2) = 1/e_{fb} \Rightarrow \delta = 23.3457°$.



Then the diagram can be updated as



From the sine rule

$$\frac{v^{-}}{sin(\delta+\eta)} = \frac{v^{-}_{\infty/venus}}{sin\gamma^{-}} \quad\Rightarrow\quad \eta = 59.9015°$$

From cosine rule

$$v^{+} = \sqrt{\left(v^{+}_{\infty/venus}\right)^2 + v_{venus}^2 - 2\left(v^{-}_{\infty/venus}\right)v_{venus}cos\eta} \;=\; 30.8231\ km/s\ .$$

At the position of

$$r^{+} = r_{venus} = 1.0821e+8\ km\ .$$

Then from the sine rule the flight path angles becomes (note that it is descending).

$$\frac{v^{+}_{\infty/venus}}{sin\gamma^{+}} = \frac{v^{+}}{sin\eta} \quad\Rightarrow\quad \gamma^{+} = -19.5156°\ .$$

Then

$$tan\theta^{*+} = \frac{\left(\frac{r^{+}(v^{+})^2}{\mu_{\odot}}\right)sin\gamma^{+}cos\gamma^{+}}{\left(\frac{r^{+}(v^{+})^2}{\mu_{\odot}}\right)cos^2\gamma^{+} - 1} \quad\Rightarrow\quad \theta^{*+} = 38.0341°, -141.9659°\ .$$

Since it is in the descending orbit $\theta^{*+} = -141.9659°$.

Now we can compute the post-encounter characteristics

$$h^+ = r^+ v^+ cos\gamma^+ = 3.1437e+9\ km^2/s$$

$$p^+ = \frac{(h^+)^2}{\mu_\odot} = 7.4467e+7\ km\ .$$

$$e^+ = \frac{1}{cos\theta^{*+}}\left(\frac{p^+}{r^+} - 1\right) = 0.3959$$

$$a^+ = \frac{p^+}{1-(e^+)^2} = 8.8306e+7\ km\,.$$

$$r_p^+ = a^+(1-e^+) = 5.3348e+7\ km\,.$$

$$r_a^+ = a^+(1+e^+) = 1.2326e+8\ km\,.$$

$$\mathbb{P}^+ = 2\pi\sqrt{\frac{(a^+)^3}{\mu_\odot}} = 1.4312e+7\ \ s\ = 165.6527\ \ days\ .$$

$$\Delta\omega = \theta^{*-} - \theta^{*+} = 38.3758°\ .$$

The new orbit reaches mercury at a true anomaly of

$$\theta^*_{mercury} = arccos\left(\frac{1}{e^+}\left(\frac{p^+}{r_{mercury}} - 1\right)\right) = 43.7576°\ .$$

Finally,

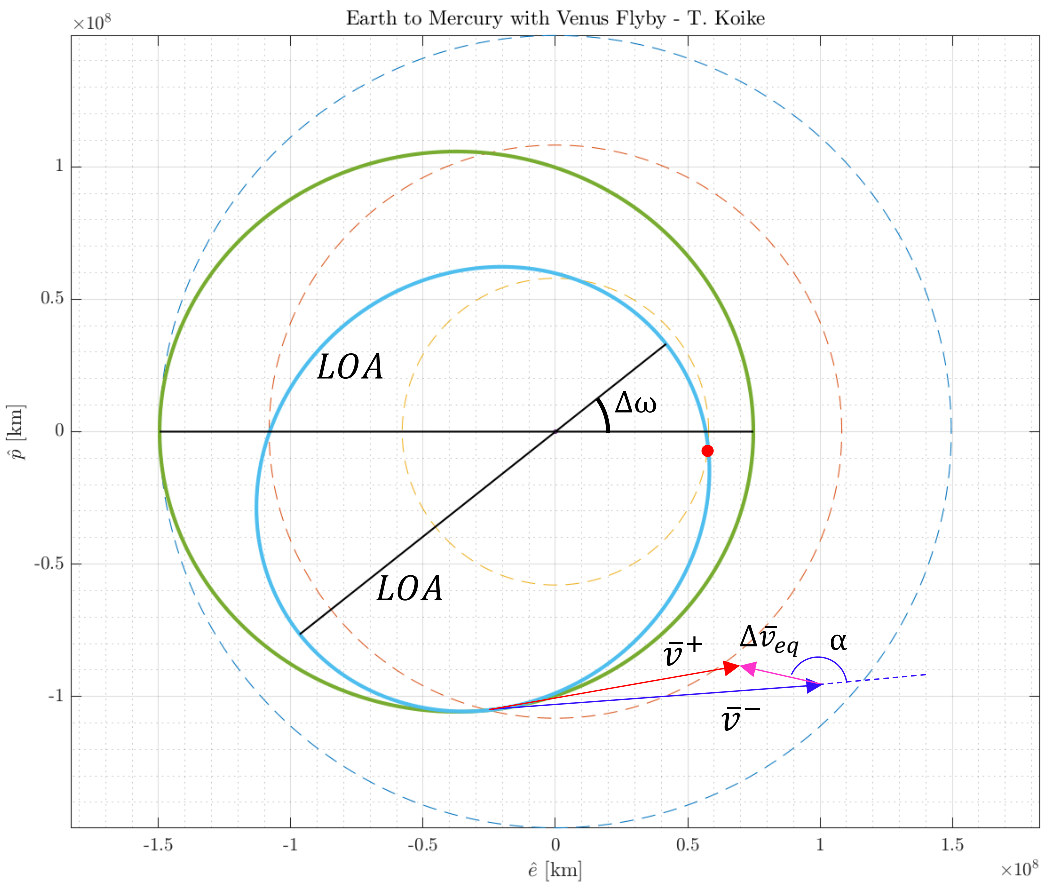
$$\Delta v_{eq} = 2v^-_{\infty/venus} sin\frac{\delta}{2} = 4.8160\ km/s\ .$$

And

$$\alpha = (|\gamma^+| - |\gamma^-|) + \frac{180° - \delta}{2} + (180° - \eta - |\gamma^+|) = 179.0573°$$

(d) Plot the old and the new orbits. (Use either Matlab or GMAT.) identify $\bar{v}^-, \bar{v}^+, \Delta\bar{v}_{eq}, \alpha$ line of apsides, $\Delta\omega$. Add Mercury's orbit to the plot.
Does the s/c now reach the orbit of Mercury? If it does, mark the crossing.

If the orbit does cross Mercury's orbit, you could further reduce the launch cost by launching into a smaller heliocentric orbit (selecting a larger value of the perihelion in part (b)). If your resulting orbit does NOT reach Mercury, you will need to increase the launch maneuver cost by selecting a smaller perihelion value. Discuss: if you try a new initial heliocentric orbit for the transfer, what perihelion distance will you try? Why? [Note: no more calculations! Just discuss what you might select and why.]



From the plot we can see that the new orbit does cross Mercury's orbit, and the point is indicated with the red dot.

To reduce the cost of the mission it would be preferrable to select a larger distance for the perihelion of the first orbit because we can see from the plot that we can still largen the new orbit by a small amount and still have the new orbit intersect with Mercury's orbit.

MATLAB

```
% AAE 532 HW 9 Problem 2
% Tomoki Koike
close all; clear all; clc;
fdir = 'C:\Users\Tomo\Desktop\studies\2020-Fall\AAE532\matlab\outputs\ps9';
set(groot, 'defaulttextinterpreter','latex');
set(groot, 'defaultAxesTickLabelInterpreter','latex');
set(groot, 'defaultLegendInterpreter','latex');

% Set constants
planet_consts = setup_planetary_constants();  % Function that sets up all the
constants in the table
sun = planet_consts.sun;  % structure of sun
earth = planet_consts.earth;  % structure of earth
mercury = planet_consts.mercury;  % structure of mercury
venus = planet_consts.venus;  % structure of venus
G = 6.6743015e-20;  % Gravitational constant [km^3/kg/s^2]

% (a)

% Original Transfer Orbit
mu_sun = sun.gp;
mu_earth = earth.gp;
mu_venus = venus.gp;
mu_mercury = mercury.gp;
r_p0 = 0.5 * earth.smao
r_a0 = earth.smao
a_0 = 0.5*(r_p0 + r_a0)
e_0 = (r_a0 - r_p0) / (r_a0 + r_p0)
p_0 = a_0 * (1 - e_0^2)
r_venus = venus.smao
TA_venus = acos_dbval(1 / e_0 * (p_0 / r_venus - 1), "deg")
TA_venus = TA_venus(TA_venus < 0)

E_venus = T2E_anomaly(e_0, TA_venus, "deg")
MM_0 = sqrt(a_0^3 / mu_sun);
TOF_venus = pi*MM_0 - MM_0 * (deg2rad(-E_venus) - e_0 * sind(-E_venus))
TOF_venus_days = TOF_venus / 60 / 60 / 24

% (b)

% What is the actual venus radii of the encounter
h_venus = 2990;
r_p_fb = h_venus + venus.mer

% Find the pre-encounter conditions
v_venus = sqrt(mu_sun / r_venus)
v_minus = vis_viva(r_venus, a_0, mu_sun)
FPA_minus = -acosd(sqrt(mu_sun*p_0) / r_venus / v_minus)

% Find the v-infinity
v_inf_venus = sqrt(v_venus^2 + v_minus^2 - 2*v_venus*v_minus*cosd(FPA_minus))

% Find hyperbola characteristics
```

```
En_fb = v_inf_venus^2 / 2
a_fb = -mu_venus / 2 / En_fb
e_fb = 1 - r_p_fb / a_fb
delta = 2*asin_dbval(1 / e_fb, "deg")
delta = delta(1)

% New orbit
eta = asind(v_minus / v_inf_venus * sind(-FPA_minus)) - abs(delta)
v_plus = sqrt(v_inf_venus^2 + v_venus^2 - 2*v_inf_venus*v_venus*cosd(eta))
FPA_plus = -asind(v_inf_venus / v_plus * sind(eta))

% True anomaly
r_plus = r_venus;
temp = r_plus * v_plus^2 / mu_sun;
TA_plus = atan_dbval( temp*sind(FPA_plus)*cosd(FPA_plus) / ( temp*cosd(FPA_plus)^2 -
1 ), "deg")
TA_plus = TA_plus(TA_plus == min(TA_plus))

h_plus = r_plus*v_plus*cosd(FPA_plus)
p_plus = h_plus^2 / mu_sun
e_plus = 1/cosd(TA_plus) * (p_plus/r_plus - 1)
a_plus = p_plus / (1 - e_plus^2)
r_p_plus = a_plus * (1 - e_plus)
r_a_plus = a_plus * (1 + e_plus)
IP_plus = 2*pi * sqrt(a_plus^3 / mu_sun)
IP_plus_years = IP_plus / 60 / 60 / 24
Domega = TA_venus - TA_plus

r_mercury = mercury.smao;
TA_mercury = acosd(1/e_plus * (p_plus/r_mercury - 1))

Dv_eq = 2*v_inf_venus*sind(delta / 2)
alpha = (abs(FPA_plus)-abs(FPA_minus)) + (180 - delta)/2 + (180 - eta -
abs(FPA_plus))

% (d)

% plotting
angles = 0:0.001:2*pi;

% Earth orbit
Xearth = earth.smao * cos(angles); Yearth = earth.smao * sin(angles);

% Venus orbit
Xvenus = venus.smao * cos(angles); Yvenus = venus.smao * sin(angles);

% Mercury orbit
Xmercury = mercury.smao * cos(angles); Ymercury = mercury.smao * sin(angles);

% Sun
Xsun = sun.mer * cos(angles); Ysun = sun.mer * sin(angles);

% Earth to Venus
angles = 0:0.1:360;
```

```
R_E2V = p_0 ./ (1 + e_0*cosd(angles)); X_E2V = R_E2V.*cosd(angles); Y_E2V =
R_E2V.*sind(angles);

% Venus to Mercury
R_V2M = p_plus ./ (1 + e_plus*cosd(angles - Domega));
X_V2M = R_V2M .* cosd(angles); Y_V2M = R_V2M .* sind(angles);


fig = figure("Renderer","painters","Position",[10, 10, 900, 700]);
    plot(Xearth,Yearth, '--')
    hold on; grid on; grid minor; box on; axis equal;
    plot(Xvenus, Yvenus, '--')
    plot(Xmercury, Ymercury, '--')
    plot(Xsun, Ysun)
    plot(X_E2V, Y_E2V, "LineWidth", 2)
    plot(X_V2M, Y_V2M, "LineWidth", 2)

    plot([-r_a0, r_p0], [0, 0], '-k', 'LineWidth', 1)

    r_a_plus_vec = r_a_plus * [-cosd(Domega), -sind(Domega)];
    r_p_plus_vec = r_p_plus * [cosd(Domega), sind(Domega)];
    temp = vertcat(r_a_plus_vec, r_p_plus_vec);
    plot(temp(:, 1), temp(:, 2), '-k', "LineWidth", 1)

    hold off
    title('Earth to Mercury with Venus Flyby - T. Koike')
    xlabel('$\hat{e}$ [km]')
    ylabel('$\hat{p}$ [km]')
saveas(fig, fullfile(fdir, "p2_earth2mercury.png"))
```

Problem 3:

The Juno spacecraft remains in orbit about Jupiter until at least 2021; now consider a follow-up mission to the Jovian system. Currently, 79 moons are orbiting Jupiter and the number is increasing as sky searches continue. Assume there exists a new Jovian moon (Remus) with the following characteristics:

$$a_R = 15R_{jup}, \qquad R_R = 3000km$$

$$e_R = 0.25, \qquad \mu_R = 1 \times 10^5\ km^3/s^2$$

The spacecraft orbit is in the same plane as Remus with

$$r_p = 7.5R_{Jup}$$

$$e = 0.5$$

(a) The spacecraft encounters the moon when Remus is at the end of the minor axis and is ascending. The spacecraft is outbound in its orbit.
Compare the orientation of the s/c orbit line of apsides with that of Remus prior to the encounter. Sketch the orbits. What is the angle between the lines of apsides?

For the Remus orbit

$$p_R = a_R(1 - e_R^2) = 1.0054e + 6\ km\ .$$

For the spacecraft orbit

$$a_0 = \frac{r_{p0}}{(1 - e_0)} = 1.0724e + 6\ km \qquad \because r_{p0} = r_p,\ \ e_0 = e$$

$$p_0 = a_0(1 - e_0^2) = 8.0429e + 5\ \ km\ .$$

The true anomaly of the Remus at the end of the semi-minor axis in the ascending orbit will be

$$\theta^*_{R,b} = arccos\left(\frac{1}{e_R}\left(\frac{p_R}{a_R} - 1\right)\right) = 104.4775°\ .$$

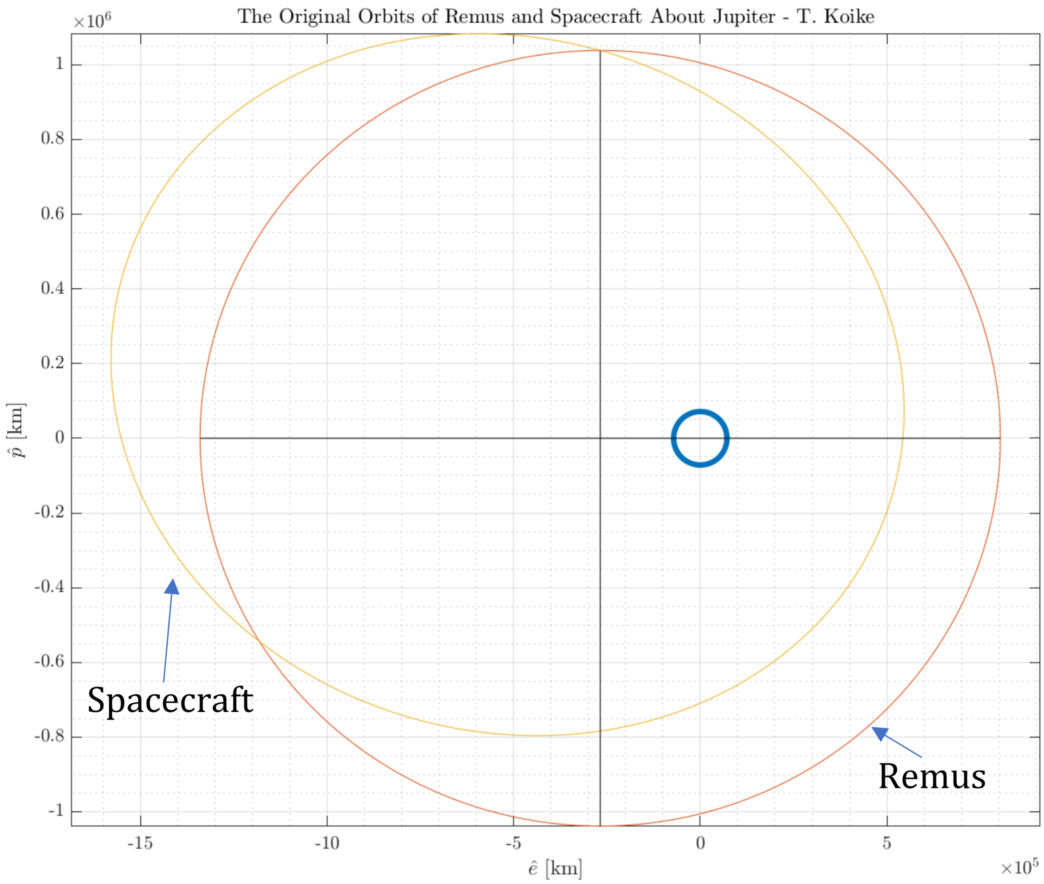
Similarly, for the spacecraft

$$\theta^*_{0,\mathrm{b}} = arccos\left(\frac{1}{e_0}\left(\frac{p_0}{a_R} - 1\right)\right) = 120.0000°.$$

Then

$$\Delta\omega_0 = \theta^*_{R,b} - \theta^*_{0,\mathrm{b}} = -15.5225°\ .$$

Thus, the sketch or MATLAB plot is as follows.



(b) For the spacecraft, determine $r^{-}, v^{-}, \gamma^{-}, \theta^{*-}$.

Since the location of encounter is at the end of the semi-minor axis of Remus's orbit, the distance of the spacecraft from Jupiter becomes

$$r^{-} = a_R = 1.0724e + 6\ km\ .$$

Then the velocity can be computed as

$$v^{-} = \sqrt{\mu_{Jup}\left(\frac{2}{r^{-}} - \frac{1}{a_0}\right)} = 10.8702\ km/s\ .$$

Then the flight path angle can be found by first calculating the specific angular momentum

$$h^{-} = \sqrt{\mu_{Jup} p_0} = 1.0095e + 7\ km^2/s\,.$$

$$\gamma^{-} = arccos\left(\frac{h^{-}}{r^{-}v^{-}}\right) = 30.0000°\ .$$

And, from part (a),

$$\theta^{*-} = \theta^{*}_{0,\mathrm{b}} = arccos\left(\frac{1}{e_0}\left(\frac{p_0}{a_R} - 1\right)\right) = 120.0000°\ .$$

(c) Sketch the vector diagram for the encounter and the appropriate Remus-centered trajectory.
The Remus gravity assist will be used to change the spacecraft orbit and the goal is to decrease the s/c orbital energy. Should the spacecraft pass "ahead" or "behind" the moon? Why?

The velocity of Remus at this point is

$$v_R = \sqrt{\mu_{Jup}\left(\frac{2}{r^-} - \frac{1}{a_R}\right)} = 10.8702\ km/s\ .$$

The flight path angle is

$$\gamma_R = arccos\left(\frac{\sqrt{\mu_{Jup} p_R}}{r^- v_R}\right) = 14.4775°\ .$$

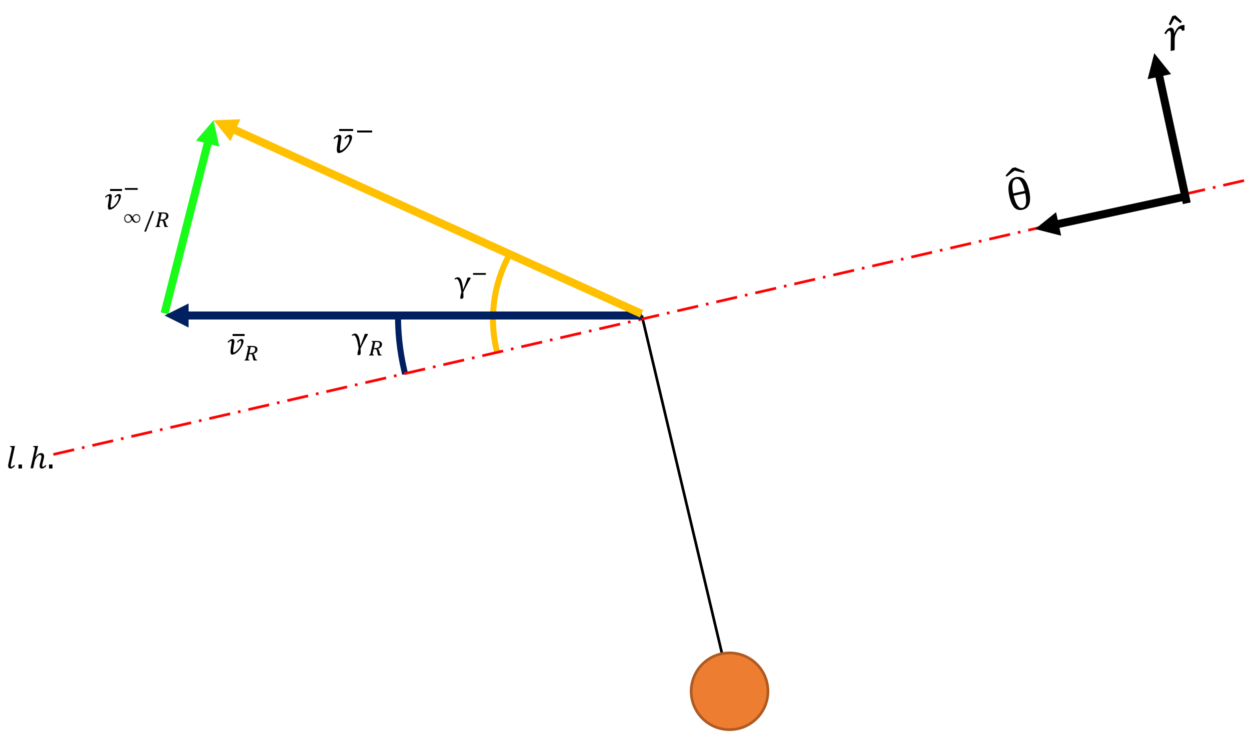
The velocity vectors we need to know are

$$\bar{v}^- = v^-\left(sin\gamma^-\hat{r} + cos\gamma^-\hat{\theta}\right) = 5.4351\hat{r} + 9.4138\hat{\theta}\ \ km/s$$

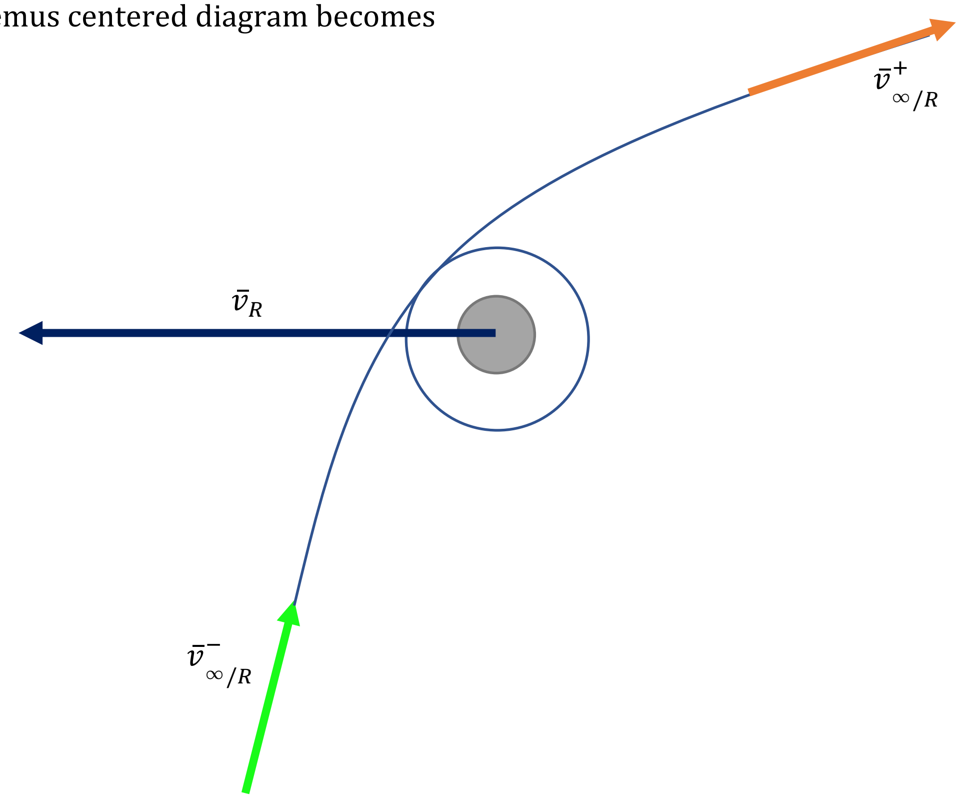
$$\bar{v}_R = v_R\left(sin\gamma_R\hat{r} + cos\gamma_R\hat{\theta}\right) = 2.7175\hat{r} + 10.5250\hat{\theta}\ km/s$$

$$\bar{v}^-_{\infty/R} = \bar{v}^- - \bar{v}_R = 2.7175\hat{r} - 1.1112\hat{\theta}\ km/s$$

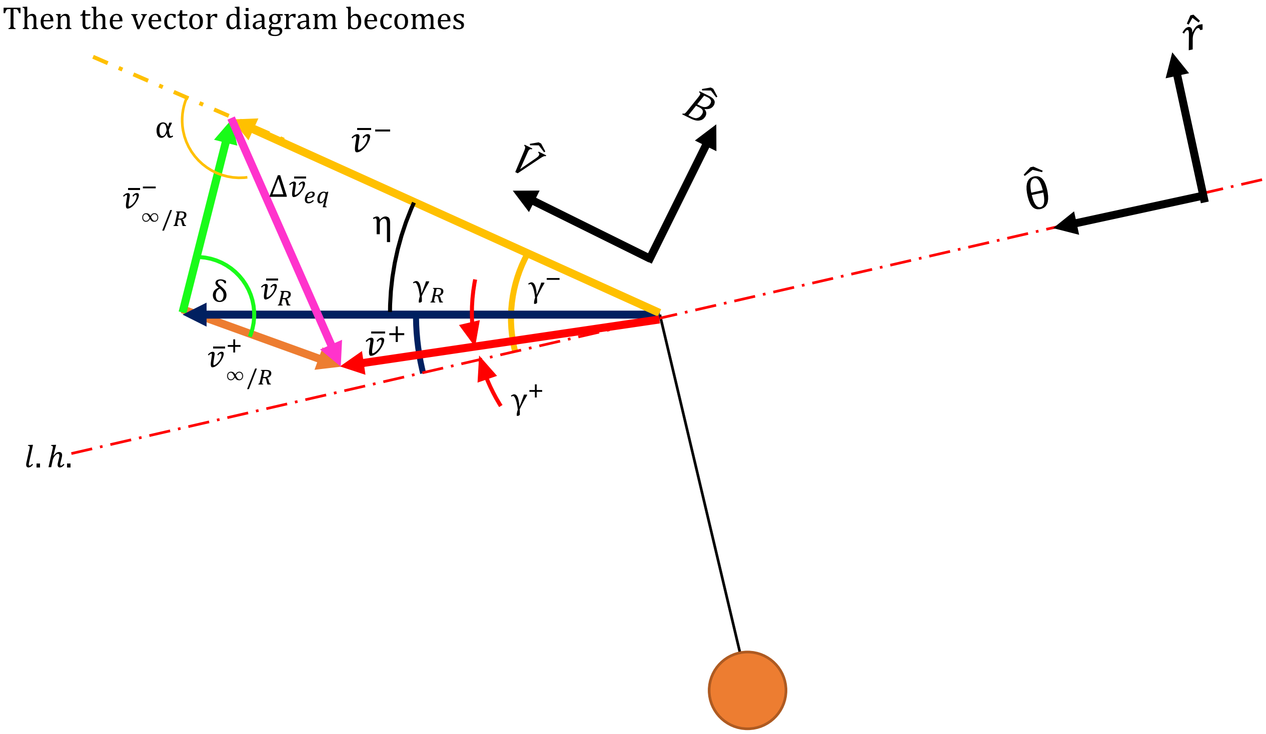
$$v^-_{\infty/R} = 2.9359\ km/s\ .$$



The Remus centered diagram becomes



Then the vector diagram becomes



We can see from the diagrams that when v^+ decreases, the flyby should occur ahead of Remus. When v^+ decreases the energy is decreased, and thus it is reasonable.

(d) The closest approach during the encounter with Remus is 1500 km altitude. Compute the new spacecraft orbit relative to Jupiter, i.e., determine $r^+, v^+, \gamma^+, \theta^{*+}$.
Also determine the new orbital characteristics: $a, e, r_p, r_a, \mathbb{P}, \mathcal{E}, \Delta\omega$.
Use your vector diagram and determine the equivalent $\Delta\bar{v}_{eq}$ with its magnitude and direction and α. Express $\Delta\bar{v}_{eq}$ in *VNB* coordinates.

First, we look at the flyby hyperbola characteristics

$$\mathcal{E}_{fb} = \frac{\left(v_{\infty/R}^-\right)^2}{2} = -\frac{\mu_R}{2a_{fb}} \;\Rightarrow\; a_{fb} = -1.1601e+4 \;\; km$$

$$\because r_{p,fb} = 1500 + R_R = 4500 \;\; km$$

$$e_{fb} = 1 - \frac{r_{p,fb}}{a_{fb}} = 1.3879$$

$$sin\frac{\delta}{2} = \frac{1}{e_{fb}} \;\Rightarrow\; \delta = 92.1949°.$$

From the vector diagram

$$\bar{v}_{\infty/R}^+ = \bar{v}_{\infty/R}^- \begin{pmatrix} cos(-\delta) & sin(-\delta) \\ -sin(-\delta) & cos(-\delta) \end{pmatrix} = (2.7175 \quad -1.1112) \begin{pmatrix} cos(-\delta) & sin(-\delta) \\ -sin(-\delta) & cos(-\delta) \end{pmatrix}$$

$$\bar{v}_{\infty/R}^+ = -1.2144\hat{r} - 2.6730\hat{\theta} \text{ km/s}$$

Then

$$\bar{v}^+ = \bar{v}_R + \bar{v}_{\infty/R}^+ = (-1.2144\hat{r} - 2.6730\hat{\theta} \text{ km/s}) + (2.7175\hat{r} + 10.5250\hat{\theta} \; km/s)$$

$$\bar{v}^+ = 1.5031\hat{r} + 7.8520\hat{\theta} \; km/s \;.$$

$$\therefore v^+ = 7.9946 \; km/s$$

At the position of

$$r^+ = r^- = 1.0724e+6 \; km \,.$$

Then from the vector of $\bar{v}^+$

$$\gamma^+ = arctan\left(\frac{1.5031}{7.8520}\right) = 10.8371° \;.$$

Then

$$tan\theta^{*+} = \frac{\left(\frac{r^+(v^+)^2}{\mu_{Jup}}\right)sin\gamma^+ cos\gamma^+}{\left(\frac{r^+(v^+)^2}{\mu_{Jup}}\right)cos^2\gamma^+ - 1} \quad \Rightarrow \quad \theta^{*+} = -11.7978°, 168.2022° \ .$$

Since it is in the ascending orbit $\Rightarrow \theta^{*+} = 168.2022°$.

Now we can compute the post-encounter characteristics

$$h^+ = r^+ v^+ cos\gamma^+ = 8.4203e + 6 \ km^2/s$$

$$p^+ = \frac{(h^+)^2}{\mu_{Jup}} = 5.5955e + 5 \ km \ .$$

$$e^+ = \frac{1}{cos\theta^{*+}}\left(\frac{p^+}{r^+} - 1\right) = 0.4885$$

$$a^+ = \frac{p^+}{1-(e^+)^2} = 7.3496e + 5 \ km \ .$$

$$r_p^+ = a^+(1-e^+) = 3.7590e + 5 \ km \ .$$

$$r_a^+ = a^+(1+e^+) = 1.0940e + 6 \ km \ .$$

$$\mathbb{P}^+ = 2\pi\sqrt{\frac{(a^+)^3}{\mu_{Jup}}} = 3.5170e + 5 \ \ s \ = 4.0705 \ \ days \ .$$

$$\Delta\omega = \theta^{*-} - \theta^{*+} = -48.2022° \ .$$

Finally,

$$\Delta\bar{v}_{eq} = \bar{v}^+ - \bar{v}^- = (1.5031\hat{r} + 7.8520\hat{\theta} km/s) - (5.4351\hat{r} + 9.4138\hat{\theta} km/s)$$

$$\Delta\bar{v}_{eq} = -3.3920\hat{r} - 1.5618\hat{\theta} \ km/s$$

$$\Delta v_{eq} = 4.2308 \ km/s$$

$$\alpha = \arccos\left(\frac{\bar{v}^- \cdot \Delta\bar{v}_{eq}}{|\bar{v}^-||\Delta\bar{v}_{eq}|}\right) = 141.6638° \ .$$

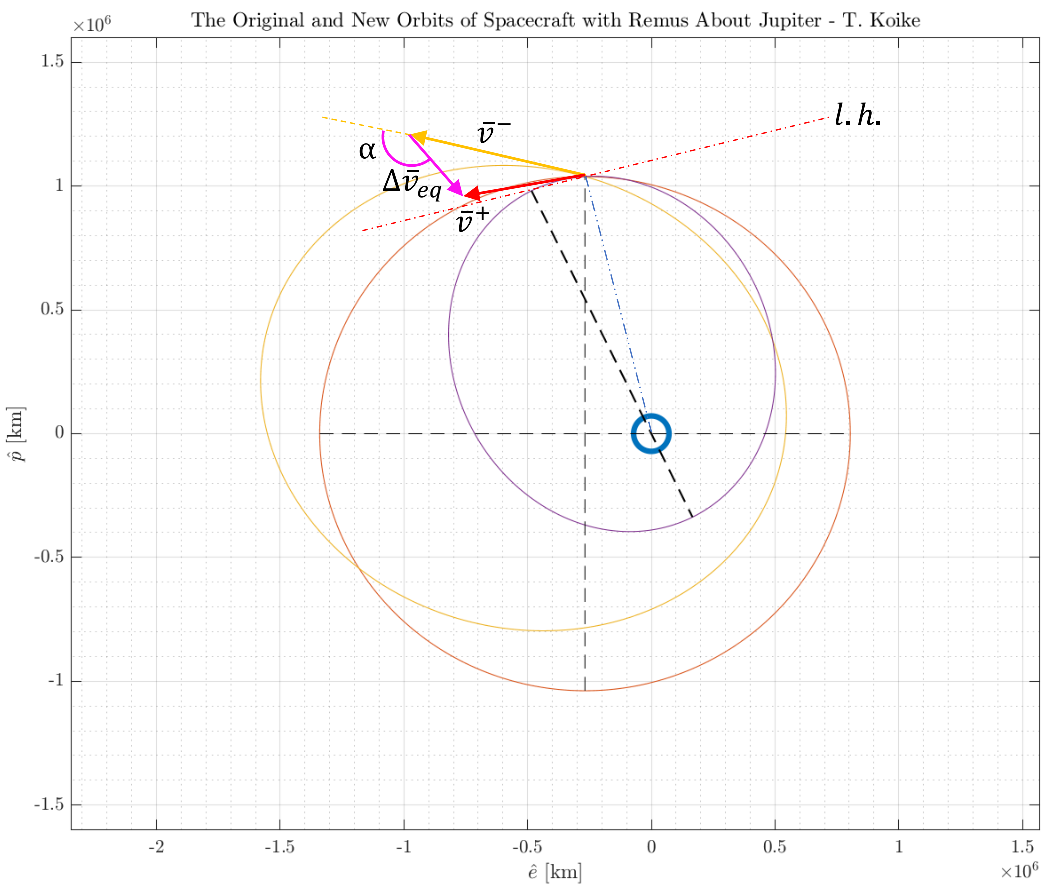
In the *VNB* coordinate

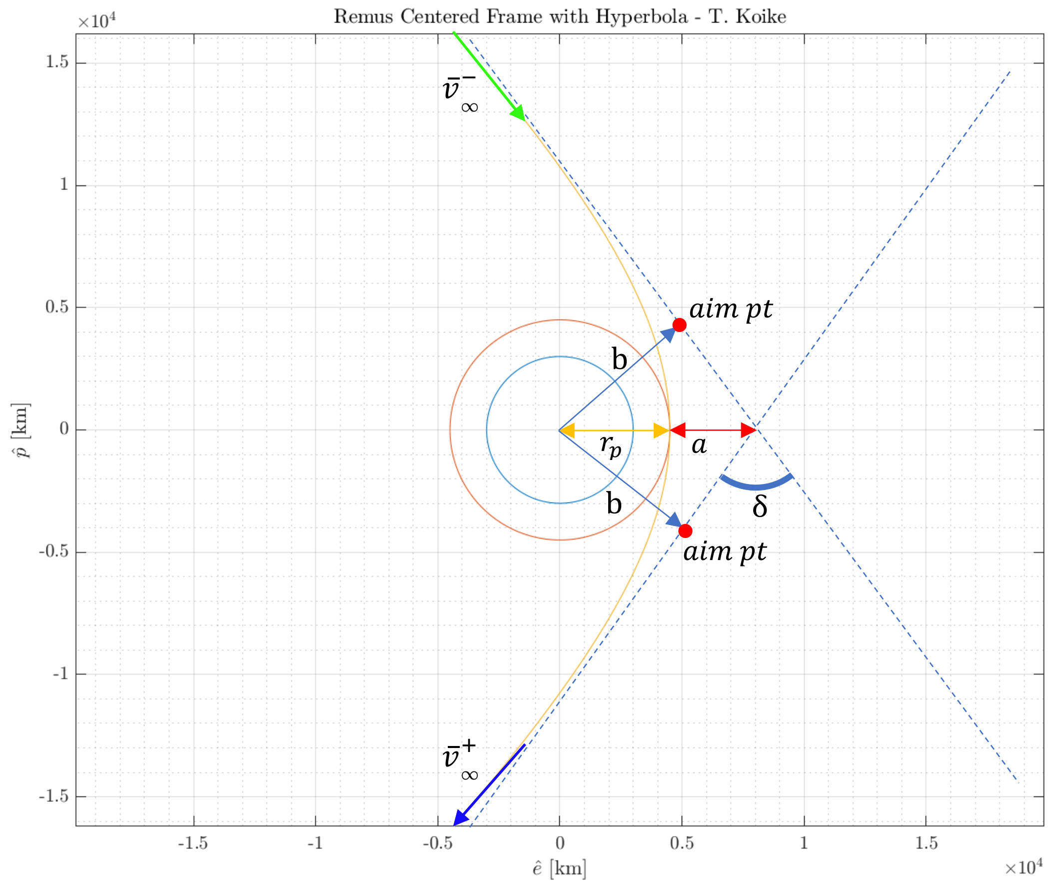
$$\Delta\bar{v}_{eq} = \Delta v_{eq}\left(cos\alpha\hat{V} + sin\alpha(-\hat{B})\right) = -3.3186\hat{V} - 2.6243\hat{B} \ \ km/s \ .$$

(e) Plot the old and new spacecraft orbits:

(i) in the Jupiter centered frame, plot old and new spacecraft orbits. On the plot, add $\bar{v}^-, \bar{v}^+, l.h., \Delta\bar{v}_{eq}, \alpha$.

(ii) in the Remus centered frame, plot the hyperbola. Add the asymptotes, the vectors , $\bar{v}_\infty^-, \bar{v}_\infty^+$ as well as a, b, r_p the aim point, and the flyby angle δ.





Remus Centered Frame with Hyperbola - T. Koike
$\times 10^4$
1.5
1
0.5
0
-0.5
-1
-1.5
$\hat{p}$ [km]
-1.5
-1
-0.5
0
0.5
1
1.5
$\hat{e}$ [km]
$\times 10^4$
$\bar{v}_\infty^-$
$\bar{v}_\infty^+$
aim pt
aim pt
b
b
r_p
a
δ

MATLAB

```
% AAE 532 HW 9 Problem 3
% Tomoki Koike
close all; clear all; clc;
fdir = 'C:\Users\Tomo\Desktop\studies\2020-Fall\AAE532\matlab\outputs\ps9';
set(groot, 'defaulttextinterpreter','latex');
set(groot, 'defaultAxesTickLabelInterpreter','latex');
set(groot, 'defaultLegendInterpreter','latex');

% Set constants
planet_consts = setup_planetary_constants();  % Function that sets up all the
constants in the table
sun = planet_consts.sun;  % structure of sun
earth = planet_consts.earth;  % structure of earth
mercury = planet_consts.mercury;  % structure of mercury
venus = planet_consts.venus;  % structure of venus
jupiter = planet_consts.jupiter;  % structure of Juipter
G = 6.6743015e-20;  % Gravitational constant [km^3/kg/s^2]

% (a)

mu_jup = jupiter.gp;

% Remus
a_R = 15 * jupiter.mer
R_R = 3000;
e_R = 0.25;
mu_R = 1e5;
p_R = a_R * (1 - e_R^2)
b_R = a_R * sqrt(1 - e_R^2)
rp_R = a_R * (1 - e_R)
ra_R = a_R * (1 + e_R)

% Spacecraft
r_p = 7.5 * jupiter.mer
e_0 = 0.5;
a_0 = r_p / (1 - e_0)
p_0 = a_0 * (1 - e_0^2)

% True anomaly at the semi-minor axis of Remus
TA_Rb = acosd(1 / e_R * (p_R / a_R - 1))
TA_0b = acosd(1 / e_0 * (p_0 / a_R - 1))
Domega_0 = -TA_0b + TA_Rb

% % Plot
% % Jupiter
% angles = 0:0.1:360;
% Xj = jupiter.mer * cosd(angles); Yj = jupiter.mer * sind(angles);
% % Remus
% Rrem = p_R ./ (1 + e_R * cosd(angles));
% Xrem = Rrem .* cosd(angles); Yrem = Rrem .* sind(angles);
% % spacecraft
% Rsc = p_0 ./ (1 + e_0 * cosd(angles - Domega_0));
% Xsc = Rsc .* cosd(angles); Ysc = Rsc .* sind(angles);
```

```
%
% fig = figure("Renderer","painters","Position",[10, 10, 900, 700]);
%     plot(Xj,Yj, '-', 'LineWidth', 3)
%     hold on; grid on; grid minor; box on; axis equal;
%     plot(Xrem, Yrem)
%     plot(Xsc, Ysc)
%
%     plot([-ra_R, rp_R], [0, 0], '-k')
%     plot([-a_R*e_R, -a_R*e_R], [-b_R, b_R], '-k')
%     hold off
%     title('The Original Orbits of Remus and Spacecraft About Jupiter  - T. Koike')
%     xlabel('$\hat{e}$ [km]')
%     ylabel('$\hat{p}$ [km]')
% saveas(fig, fullfile(fdir, "p3_remus_sc_original.png"))

% (b)
% Spacecraft
r_minus = a_R;
v_minus = vis_viva(r_minus, a_0, mu_jup)
h_minus = sqrt(mu_jup * p_0)
FPA_minus = acosd(h_minus / r_minus / v_minus)
TA_minus = TA_0b;

% Remus
v_R = vis_viva(r_minus, a_R, mu_jup)
FPA_R = acosd(sqrt(mu_jup * p_R) / r_minus / v_R)

% (c)

v_minus_vec = v_minus * [sind(FPA_minus), cosd(FPA_minus)]
v_R_vec = v_R * [sind(FPA_R), cosd(FPA_R)]
v_inf_R_vec = v_minus_vec - v_R_vec
v_inf_R = norm(v_inf_R_vec)

% (d)

% Hyperbola
E_fb = v_inf_R^2 / 2;
a_fb = -mu_R / 2 / E_fb
rp_fb = R_R + 1500
e_fb = 1 - rp_fb / a_fb
delta = 2 * asind(1 / e_fb)

v_inf_R_plus_vec = v_inf_R_vec * [cosd(-delta), sind(-delta); -sind(-delta), cosd(-
delta)]
v_plus_vec = v_R_vec + v_inf_R_plus_vec
v_plus = norm(v_plus_vec)
r_plus = r_minus;
FPA_plus = atand(v_plus_vec(1) / v_plus_vec(2))
% True anomaly
temp = r_plus * v_plus^2 / mu_jup;
TA_plus = atan_dbval( temp*sind(FPA_plus)*cosd(FPA_plus) / ( temp*cosd(FPA_plus)^2 -
1 ), "deg")
TA_plus = TA_plus(TA_plus == max(TA_plus))
```

```
h_plus = r_plus*v_plus*cosd(FPA_plus)
p_plus = h_plus^2 / mu_jup
e_plus = 1/cosd(TA_plus) * (p_plus/r_plus - 1)
a_plus = p_plus / (1 - e_plus^2)
r_p_plus = a_plus * (1 - e_plus)
r_a_plus = a_plus * (1 + e_plus)
IP_plus = 2*pi * sqrt(a_plus^3 / mu_jup)
IP_plus_days = IP_plus / 60 / 60 / 24
Domega = TA_minus - TA_plus

Dv_eq_vec = v_plus_vec - v_minus_vec
Dv_eq = norm(Dv_eq_vec)
alpha = acosd(dot(v_minus_vec, Dv_eq_vec) / Dv_eq / v_minus)
Dv_eq_vec_VBN = Dv_eq * [cosd(alpha), -sind(alpha)]

% Plot

% (i) Jupiter
angles = 0:0.1:360;
Xj = jupiter.mer * cosd(angles); Yj = jupiter.mer * sind(angles);
% Remus
Rrem = p_R ./ (1 + e_R * cosd(angles));
Xrem = Rrem .* cosd(angles); Yrem = Rrem .* sind(angles);
% spacecraft
Rsc = p_0 ./ (1 + e_0 * cosd(angles - Domega_0));
Xsc = Rsc .* cosd(angles); Ysc = Rsc .* sind(angles);

% new orbit
Rnew = p_plus ./ (1 + e_plus*cosd(angles - (Domega + Domega_0)));
Xnew = Rnew .* cosd(angles); Ynew = Rnew .* sind(angles);

fig = figure("Renderer","painters","Position",[10, 10, 900, 700]);
    plot(Xj,Yj, '-', 'LineWidth', 3)
    hold on; grid on; grid minor; box on; axis equal;
    plot(Xrem, Yrem)
    plot(Xsc, Ysc)
    plot(Xnew, Ynew)
    plot([-ra_R, rp_R], [0, 0], '--k')
    plot([-a_R*e_R, -a_R*e_R], [-b_R, b_R], '--k')

    r_a_plus_vec = r_a_plus * [-cosd(Domega + Domega_0), -sind(Domega + Domega_0)];
    r_p_plus_vec = r_p_plus * [cosd(Domega + Domega_0), sind(Domega + Domega_0)];
    temp = vertcat(r_a_plus_vec, r_p_plus_vec);
    plot(temp(:, 1), temp(:, 2), '--k', "LineWidth", 1)

    hold off
    ylim([-1.6e6, 1.6e6])
    title('The Original and New Orbits of Spacecraft with Remus About Jupiter  - T.
Koike')
    xlabel('$\hat{e}$ [km]')
    ylabel('$\hat{p}$ [km]')
saveas(fig, fullfile(fdir, "p3_jupCenter.png"))

% (ii)
% moon
```

```
angles = 0:0.1:360;
XR = R_R * cosd(angles); YR = R_R * sind(angles);

% moon parking orbit
Xpo = (rp_fb) * cosd(angles); Ypo = rp_fb * sind(angles);

% hyperbola
angles = -105:0.1:105;
Rhyp = a_fb*(1 - e_fb^2) ./ (1 + e_fb * cosd(angles));
Xhyp = Rhyp .* cosd(angles);  Yhyp = Rhyp .* sind(angles);

fig = figure("Renderer","painters","Position",[10, 10, 900, 700]);
    plot(XR,YR)
    hold on; grid on; grid minor; box on; axis equal;
    plot(Xpo, Ypo)
    plot(Xhyp, Yhyp)
    hold off
    xlabel('$\hat{e}$ [km]')
    ylabel('$\hat{p}$ [km]')
    title("Remus Centered Frame with Hyperbola - T. Koike")
saveas(fig, fullfile(fdir, "p3_remusCenter.png"));
```