AAE 564 Fall 2020

Homework Ten

Friday, November 6

Exercise 1 Determine (by hand) whether or not each of the following systems are controllable.

$$\begin{aligned}\dot{x}_1 &= -x_1 + u \\ \dot{x}_2 &= x_2 + u\end{aligned} \qquad \begin{aligned}\dot{x}_1 &= -x_1 \\ \dot{x}_2 &= x_2 + u\end{aligned} \qquad \begin{aligned}\dot{x}_1 &= x_1 + u \\ \dot{x}_2 &= x_2 + u\end{aligned}$$

Exercise 2 (By hand.) Determine whether or not the following system is controllable.

$$\begin{aligned}\dot{x}_1 &= 5x_1 + x_2 - x_3 + u_1 \\ \dot{x}_2 &= -x_1 + 3x_2 - x_3 + u_1 + u_2 \\ \dot{x}_3 &= -2x_1 - 2x_2 + 4x_3 + u_2\end{aligned}$$

If the system is uncontrollable, compute the uncontrollable eigenvalues.

Exercise 3 Carry out the following for linearizations L1, L3, L7 of the two pendulum cart system.

(a) Determine which linearizations are controllable?

(b) Compute the singular values of the controllability matrix.

(b) Determine the uncontrollable eigenvalues for the uncontrollable linearizations.

You may want to use MATLAB.

Exercise 4 (BB in Laundromat: external excitation.) Obtain a state space representation representation of the following system.

$$\begin{aligned}m\ddot{q}_1 - m\Omega^2 q_1 + k(q_1 - q_2) &= 0 \\ m\ddot{q}_2 - m\Omega^2 q_2 - k(q_1 - q_2) &= u\end{aligned}$$

Determine whether or not your state space representation system is controllable.

Exercise 5 (BB in Laundromat: self excited.) (By hand.) Obtain a state space representation of the following system.

$$\begin{aligned}m\ddot{\phi}_1 - m\Omega^2\phi_1 + k(\phi_1 - \phi_2) &= -u \\ m\ddot{\phi}_2 - m\Omega^2\phi_2 - k(\phi_1 - \phi_2) &= u \\ y &= \phi_1\end{aligned}$$

(a) Determine the uncontrollable eigenvalues. Consider $\omega := \sqrt{k/2m} > \Omega$.

(b) Obtain a basis for its controllable subspace.

(c) Obtain a reduced order controllable system which has the same input-output behavior as the original system when initial conditions are zero.

Exercise 6 (By hand.) Consider a system described by

$$\begin{aligned} \dot{x}_1 &= \lambda_1 x_1 + b_1 u \\ \dot{x}_2 &= \lambda_2 x_2 + b_2 u \\ &\vdots \\ \dot{x}_n &= \lambda_n x_n + b_n u \end{aligned}$$

where all quantities are scalar. Obtain conditions on the numbers $\lambda_1, \cdots, \lambda_n$ and $b_1, \cdots, b_n$ which are necessary and sufficient for the controllability of this system. (Hint: PBH time.)

Exercise 7 Consider the system described by

$$\begin{aligned} \dot{x} &= x_2 + u \\ \dot{x}_2 &= 4x_1 + 2u\,. \end{aligned}$$

Find (by hand) a non-zero vector w such for every input $u(\cdot)$, every solution $x(\cdot)$ of this system satisfies

$$w'x(t) = e^{-2t} w'x(0)\,.$$

Exercise 8 Suppose that λ is a uncontrollable complex eigenvalue of the system

$$\dot{x} = Ax + Bu$$

where x, A and B are real. Show that there are real vectors u and v such that for every initial conditon $x(0) = x_0$ and every $u(\cdot)$,

$$\begin{aligned} u'x(t) &= e^{\alpha t}(u\cos\omega t - v\sin\omega t)' x_0 \\ v'x(t) &= e^{\alpha t}(u\sin\omega t + v\cos\omega t)' x_0 \end{aligned}$$

where $\lambda = \alpha + \jmath\omega$.