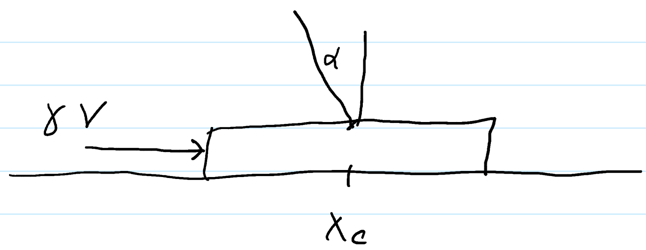
LAB 4



$u(t)$ is a square wave of period 20 seconds with Amplitude .2

LONG PENDULUM

$$\dot{x} = Ax + BV$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \qquad x_1 = x_c \; ; \; x_2 = \alpha$$

$$x_3 = \dot{x}_c \; , \; x_4 = \dot{\alpha}$$

$$A : \mathbb{C}^4 \longrightarrow \mathbb{C}^4 \qquad B : \mathbb{C} \longrightarrow \mathbb{C}^4$$

Integral controller

Add a new STATE x_5

$$\dot{x}_5 = u - x_1 = u - x_c$$

$$x_5 = \int (u - x_1)\, dt$$

$$x_c = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}$$

New STATE SPACE SYSTEM

$$\dot{x}_c = A_c x_c + B_c V + Du$$

$$\dot{x}_5 = u - x_1$$ IN STEADY STATE

$$\dot{x}_5 \to 0 \Rightarrow x_1 \to u$$

$$X_i = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} X \\ \hline X_5 \end{bmatrix}$$

$$\dot{X}_i = \begin{bmatrix} \dot{X} \\ \dot{X}_5 \end{bmatrix} = \begin{bmatrix} AX + BV \\ u - x_1 \end{bmatrix}$$

$$\dot{X}_i = \begin{bmatrix} \dot{X} \\ \dot{X}_5 \end{bmatrix} = \left[\begin{array}{c|c} A & 0 \\ \hline [-1\ 0\ 0\ 0] & 0 \end{array}\right] \begin{bmatrix} X \\ x_5 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u + \begin{bmatrix} B \\ \hline 0 \end{bmatrix} V$$

$$A_i = \left[\begin{array}{c|c} A & \vec{0} \\ \hline [-1\ 0\ 0\ 0] & 0 \end{array}\right]; \quad B_i = \begin{bmatrix} B \\ \hline 0 \end{bmatrix}, \quad D = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\boxed{\dot{X}_i = A_i X_i + B_i V + D u}$$

STATE FEED BACK

$$V = -K X_i = -k_1 x_1 - k_2 x_2 - k_3 x_3 - k_4 x_4 - k_5 x_5$$

$$V = -[k_1\ k_2\ k_3\ k_4\ k_5] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}$$

$$K = [k_1\ k_2\ k_3\ k_4\ k_5]$$

$V = -K X_i \Longrightarrow$ The FEED BACK SYSTEM

$$\dot{X}_i = (A_i - B_i K) X_i + D u$$

Need $A_i - B_i K$ STABLE

$$K = lqr(A_i, B_i, diag([q_1\ q_2\ q_3, q_4, q_5]), R)$$

$A_i - B_i K$ is STABLE