AAE 440: Spacecraft Attitude Dynamics

PS7

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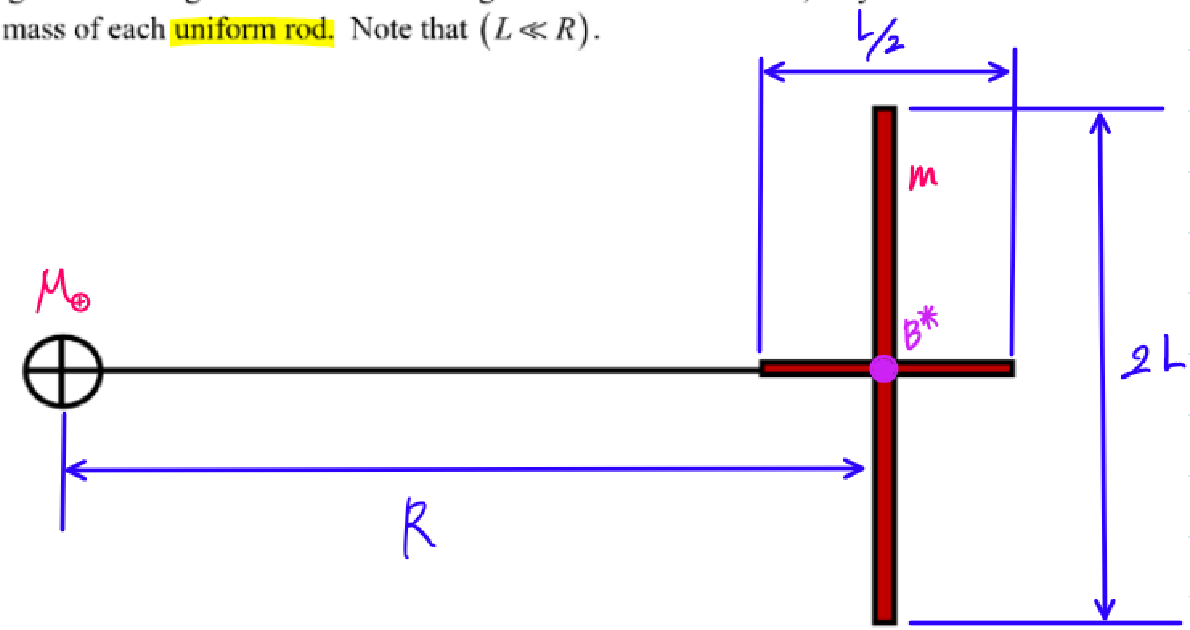
Problem 1: During assembly of a space station, the partially finished structure ($m \ll M_{\oplus}$) has a shape that resembles two thin rods oriented as indicated below. The long rod is of length $2L$ and the total length of the short rod is $L/2$; they cross at the center of mass of each uniform rod. Note that $(L \ll R)$.



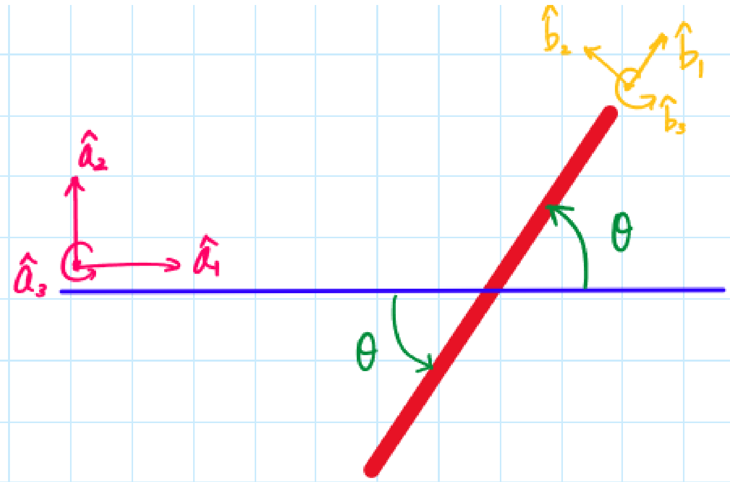
(a) Let R (Earth $\rightarrow$ B*) = 6488 km, that is, an orbit altitude of 110 km. Also, assume that L = 50 km. (Large!) Determine the resultant gravity force and locate the c.g. What is the distance between the c.m. and the c.g.?
On the sketch, indicate the relative positions of the c.m. and the c.g.
Do you think the attitude is "stable"? Why or why not?

Given properties

$R = R_e + h = 6378 \text{ km} + 110 \text{ km} = 6488 \text{ km}$ $\quad G = 6.6743 \times 10^{-11} \frac{m}{kg\text{-}s^2}$

$L = 50 \text{ km}$

First we will find the $\bar{\bar{I}}^{B/B^*}$ of this structure, but since 2 rods are passing through its CM we can consider the rods separately and compute each moment of inertia and sum them up to get the answer



A horizontal rod with a length of ℓ, has an inertia dyad

$$\bar{\bar{I}}^{B/B^*} = \frac{m\ell^2}{12}\left(\hat{b}_2\hat{b}_2 + \hat{b}_3\hat{b}_3\right)$$

from this we can find the inertia dyad for the horizontal & vertical rod of this problem

horizontal

$$\bar{\bar{I}}_h^{B/B^*} = \frac{m}{12}\left(\frac{L}{2}\right)^2\left(\hat{b}_2\hat{b}_2 + \hat{b}_3\hat{b}_3\right) = \frac{mL^2}{48}\left(\hat{b}_2\hat{b}_2 + \hat{b}_3\hat{b}_3\right) = \frac{mL^2}{48}\left(\hat{a}_2\hat{a}_2 + \hat{a}_3\hat{a}_3\right)$$

vertical

and using the DCM

${}^{A}C^{B}$	$\hat{b}_1$	$\hat{b}_2$	$\hat{b}_3$
$\hat{a}_1$	C_θ	$-S_\theta$	0
$\hat{a}_2$	S_θ	C_θ	0
$\hat{a}_3$	0	0	1

$$\bar{\bar{I}}^{B/B^*} = \frac{m(2L)^2}{12}\left(\hat{b}_2\hat{b}_2 + \hat{b}_3\hat{b}_3\right)$$
$$= \frac{mL^2}{3}\left(\hat{b}_2\hat{b}_2 + \hat{b}_3\hat{b}_3\right)$$

$$\bar{\bar{I}}_V^{B/B^*}\Big|_{\hat{a}} = {}^{A}C^{B}\ \bar{\bar{I}}^{B/B^*}\Big|_{\hat{b}}\ \left({}^{A}C^{B}\right)^T$$
$$= \begin{bmatrix} C_\theta & -S_\theta & 0 \\ S_\theta & C_\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{mL^2}{3} & 0 \\ 0 & 0 & \frac{mL^2}{3} \end{bmatrix} \begin{bmatrix} C_\theta & S_\theta & 0 \\ -S_\theta & C_\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{mL^2\sin^2\theta}{3} & -\frac{mL^2\cos\theta\sin\theta}{3} & 0 \\ -\frac{mL^2\cos\theta\sin\theta}{3} & \frac{mL^2\cos^2\theta}{3} & 0 \\ 0 & 0 & \frac{mL^2}{3} \end{bmatrix}$$

thus, @ $\theta = 90° = \frac{\pi}{2}$

$$\bar{\bar{I}}_V^{B/B^*} = \begin{bmatrix} \frac{mL^2}{3} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{mL^2}{3} \end{bmatrix}, \quad \bar{\bar{I}}^{B/B^*} = \frac{mL^2}{3}\left(\hat{a}_1\hat{a}_1 + \hat{a}_3\hat{a}_3\right)$$

thus,

$$\bar{\bar{I}}^{B/B^*} = \bar{\bar{I}}_h^{B/B^*} + \bar{\bar{I}}_v^{B/B^*} = \frac{mL^2}{48}(\hat{a}_2\hat{a}_2 + \hat{a}_3\hat{a}_3) + \frac{mL^2}{3}(\hat{a}_1\hat{a}_1 + \hat{a}_3\hat{a}_3)$$

$$\bar{\bar{I}}^{B/B^*} = mL^2\left(\frac{1}{3}\hat{a}_1\hat{a}_1 + \frac{1}{48}\hat{a}_2\hat{a}_2 + \frac{17}{48}\hat{a}_3\hat{a}_3\right)$$

next, since

$$\bar{F} = -\frac{Gmm'}{R^2}\left(\hat{a}_1 + \sum_{i=2}^{\infty}\bar{f}^{(i)}\right)$$

$$-\frac{Gmm'}{R^2}\hat{a}_1 = -\frac{\left(6.67408\times10^{-11}\,\frac{m^3}{kg\cdot s^2}\right)\mu_{\oplus}\, m}{(6488\times10^3\,m)^2}\,\hat{a}_1$$

and

$$\bar{f}^{(2)} = \frac{1}{mR_e^2}\left\{\frac{3}{2}\underbrace{\left[tr\left(\bar{\bar{I}}^{B/B^*}\right) - 5\,\hat{a}_1\cdot\bar{\bar{I}}^{B/B^*}\cdot\hat{a}_1\right]\hat{a}_1 + 3\,\bar{\bar{I}}^{B/B^*}\cdot\hat{a}_1}\right\}$$

$$= \frac{3}{2}\left[\frac{mL^2}{3} + \frac{mL^2}{48} + \frac{17}{48}mL^2 - 5\cdot\frac{mL^2}{3}\right]\hat{a}_1 + 3\cdot\frac{mL^2}{3}\hat{a}_1$$

$$= \frac{3}{2}mL^2\left[\frac{16}{48} + \frac{1}{48} + \frac{17}{48} - \frac{80}{48} + \frac{32}{48}\right]\hat{a}_1$$

$$= \frac{3}{2}mL^2\left[-\frac{14}{48}\right]\hat{a}_1 = \frac{3}{2}mL^2\left(-\frac{7}{24}\right)\hat{a}_1$$

$$= -\frac{7}{16}mL^2\,\hat{a}_1$$

neglect the higher order terms of $\bar{f}^{(i)}$ $(i>2)$

thus,

$$\bar{F} = -\frac{GM_{\oplus}m}{R^2}\hat{a}_1 - \frac{GM_{\oplus}m}{R^2}\cdot\frac{1}{mR^2}\left(-\frac{7}{16}mL^2\right)\hat{a}_1$$

$$= -\frac{GM_{\oplus}m}{R^2}\left(1-\frac{7mL^2}{16mR^2}\right)\hat{a}_1$$

since $m \ll M_{\oplus}$ $m \approx 1$ [kg]

$L \ll R_e$ $L \approx 50\times10^3$ [m]

$$\therefore \bar{F} = \boxed{(-9.4692\,\hat{a}_1)\ N}$$

then

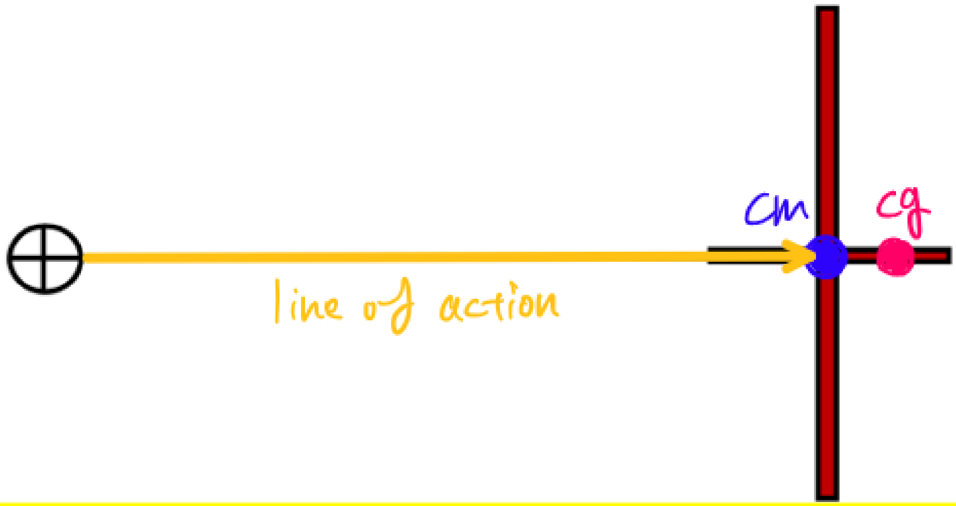
$$R^{cg} = \left[\frac{GM_{\oplus}m}{|\bar{F}|}\right]^{\frac{1}{2}}$$

$$= \left[\frac{\left(6.67408\times10^{-11}\frac{m^3}{kg\text{-}s^2}\right)\left(5.9723\times10^{24}\,kg\right)(1\,kg)}{(9.4692\ N)}\right]^{\frac{1}{2}}$$

$$= 6488084.29\ m$$

$$= \boxed{6488.08429\ km}$$

thus, $R^{cm} - R^{cg} = \boxed{84.29 \text{ m}}$



Unstable

because the moment will move the structure away from the initial cm.

(b) Is the body centrobaric? How do you know?

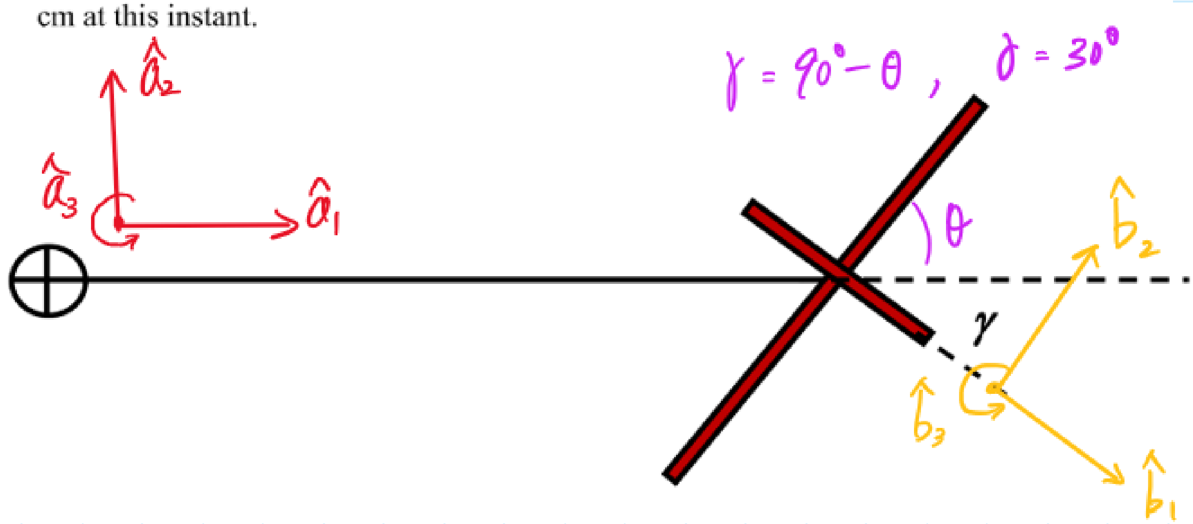
The body is **not** centrobaric because:

(1) $\bar{J}^{(2)} \neq 0$

(2) $cg \neq cm$

(c) Now assume that the space station is in a different orientation such that the vehicle is reoriented relative to the orbit as observed on the next page. However, $\hat{a}_1$ still passes through the center of mass. The angle γ measures the orientation between $\hat{a}_1$ and one of the principal directions. Let $\gamma = 30^\circ$. Now compare the principal directions $\hat{b}_i$ with the orbit-fixed directions $\hat{a}_i$.

Compute the approximate gravity force at this instant. Locate the cg relative to the cm at this instant.



$$\bar{\bar{I}}^{B/B^*} = mL^2\left(\frac{1}{3}\hat{a}_1\hat{a}_1 + \frac{1}{48}\hat{a}_2\hat{a}_2 + \frac{17}{48}\hat{a}_3\hat{a}_3\right) \quad \text{from (a)}$$

define the DCM

${}^A C^B$	$\hat{b}_1$	$\hat{b}_2$	$\hat{b}_3$
$\hat{a}_1$	C_γ	S_γ	0
$\hat{a}_2$	$-S_\gamma$	C_γ	0
$\hat{a}_3$	0	0	1

Using

$$\bar{f}^{(2)} = \frac{3}{mR^2}\Big\{\frac{1}{2}\left[I_1(1-3C_{11}^2) + I_2(1-3C_{12}^2) + I_3(1-3\cancel{C_{13}^2}^{\,0})\right]\hat{a}_1$$

$$+ \left[I_1 C_{21} C_{11} + I_2 C_{22} C_{12} + I_3 \cancel{C_{23} C_{13}}^{\,0}\right]\hat{a}_2$$

$$+ \left[I_1 \cancel{C_{31} C_{11}}^{\,0} + I_2 \cancel{C_{32} C_{12}}^{\,0} + I \cancel{C_{33} C_{13}}^{\,0}\right]\hat{a}_3\Big\}$$

$$= \frac{3}{mR^2}\Big\{\frac{1}{2}\left[I_1(1-3C_{11}^2) + I_2(1-3C_{12}^2) + I_3\right]\hat{a}_1$$

$$+ \left[I_1 C_{21} C_{11} + I_2 C_{22} C_{12}\right]\hat{a}_2\Big\}$$

since C_{ij} equals the corresponding DCM elements above, and also using the corresponding elements from $\underline{\underline{I}}^{B/B*}$

(compute using MATLAB)

$$\bar{f}^{(2)} = \left(-5.28146\times 10^{-6}\,\hat{a}_1 - 2.49484\times 10^{-5}\,\hat{a}_2\right)N$$

thus, (using MATLAB) we compute

$$\bar{F} = -\frac{GmM_e}{R_e}\left(\hat{a}_1 + \bar{f}^{(2)}\right)$$

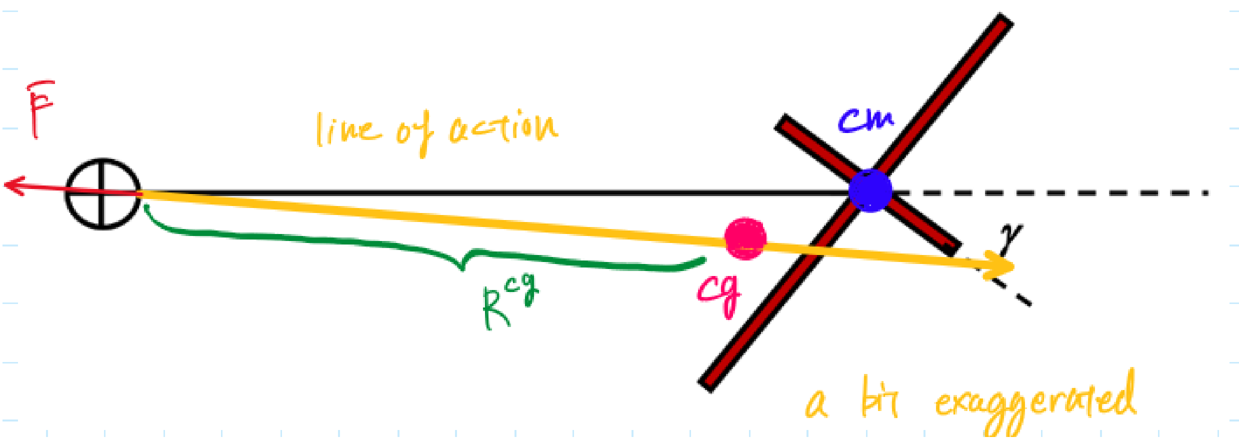
$$\bar{F} = \boxed{\left(-9.79893\,\hat{a}_1 + 2.49484 \times 10^{-5}\,\hat{a}_2\right) N}$$

then,

$$R^{cg} = \left[\frac{G m \mu_e}{|F|}\right]^{\frac{1}{2}}$$

$$= 6487998.25 \text{ m}$$

$$= \boxed{6487.998 \text{ km}}$$



(d) Compute the gravity moment about B^* for $R = 6488$ km.
The moment will cause the structure to rotate in which direction?
Is this direction consistent with the previous class discussion of "stability"?

say the origin is the center of the Earth

$$\bar{r}^{cg} = R^{cg}\, -\frac{\bar{F}}{|\bar{F}|}$$

$$= \frac{-(6487.998\ \text{km})}{\left[(-9.79893)^2 + (+2.49484\times10^{-5})^2\right]^{1/2}}\,\bar{F}$$

$$= (6487998.251\,\hat{a}_1 - 16.518666\,\hat{a}_2)\ \text{m}$$

$$\bar{r}^{cm} = R_e\,\hat{a}_1 = (6488000\,\hat{a}_1)\ \text{m}$$

$$\bar{r}^{cm\to cg} = \bar{r}^{cg} - \bar{r}^{cm}$$

$$= (-1.74849\times10^{5}\,\hat{a}_1 - 16.51866\,\hat{a}_2)\ \text{m}$$

now compute using MATLAB

$$\therefore\ \bar{M}^{cg} = \bar{r}^{cm\to cg}\times\bar{F}$$

$$\boxed{\bar{M}^{cg} = (-1.564232\times10^{2}\,\hat{a}_3)\ \text{N-m}}$$

out of the page clockwise

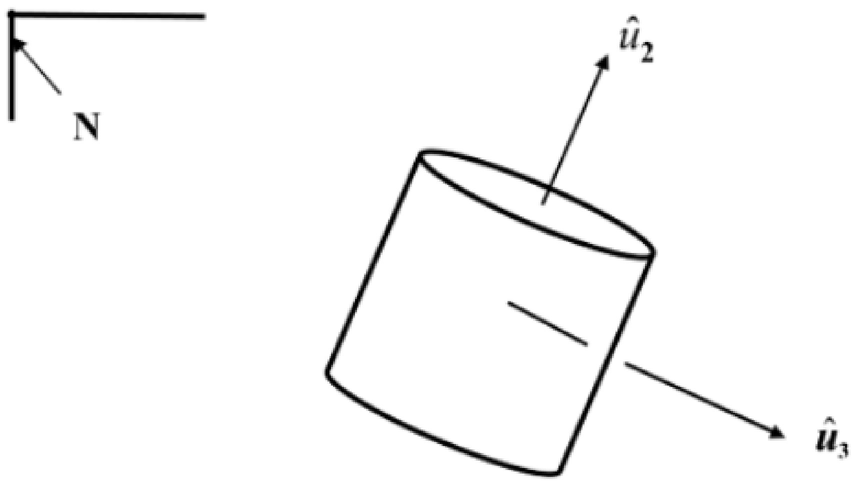
Discussion

The direction is congruent with the fact that the lower half which is closer to Earth is where the cg is located. This is where the stability is the most reinforced. However, with torque the structure deviates from the initial CM so it is unstable.

Problem 2: Recall that in PS5, it was assumed that a rigid body B can move in an inertial torque-free environment N.

$$\overline{\overline{I}}^{S/S^*} = 400\hat{u}_1\hat{u}_1 + 100\hat{u}_2\hat{u}_2 + 400\hat{u}_3\hat{u}_3 \quad \text{kg-met}^2$$

Let $\hat{n}_i$ be fixed in the inertial frame N and $\hat{u}_i$ define body-fixed unit vectors parallel to central principal axes of inertia.



(a) For this vehicle, compute the semi-diameters of the corresponding inertia ellipsoid. Plot three planar projections of the energy ellipsoid: $\hat{u}_1-\hat{u}_3$, $\hat{u}_1-\hat{u}_2$, $\hat{u}_2-\hat{u}_3$. (Use the same scale for each.) Can you plot a 3D image in Matlab? One of the projections is circular. What does that tell you? Is this body more "rod-like" or "disk-like"?

It is given that $\overline{\overline{I}}^{S/S^*} = 400\hat{u}_1\hat{u}_1 + 100\hat{u}_2\hat{u}_2 + 400\hat{u}_3\hat{u}_3$ kg-m^2

$$= I_1\hat{u}_1\hat{u}_1 + I_2\hat{u}_2\hat{u}_2 + I_3\hat{u}_3\hat{u}_3$$

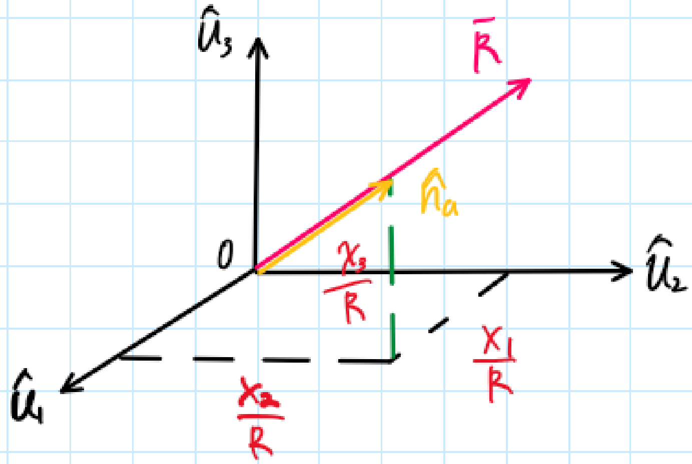
the DCM is

${}^N C^U$	$\hat{u}_1$	$\hat{u}_2$	$\hat{u}_3$
$\hat{n}_a$	C_{a1}	C_{a2}	C_{a3}
$\hat{n}_b$	C_{b1}	C_{b2}	C_{b3}
$\hat{n}_c$	C_{c1}	C_{c2}	C_{c3}

$$I_{aa} = \hat{n}_a \cdot \bar{\bar{I}}^{s/s^*} \cdot \hat{n}_a$$

$$I_{aa} = I_1 C_{a1}^2 + I_2 C_{a2}^2 + I_3 C_{a3}^2$$

(* I_{aa} can stand for all possible $\hat{n}_a$ directions)



$$\bar{R} = R\hat{n}_a = x_1 \hat{u}_1 + x_2 \hat{u}_2 + x_3 \hat{u}_3$$

$$\hat{n}_a = \frac{x_1}{R}\hat{u}_1 + \frac{x_2}{R}\hat{u}_2 + \frac{x_3}{R}\hat{u}_3$$

since

$$\hat{n}_a = C_{a1}\hat{u}_1 + C_{a2}\hat{u}_2 + C_{a3}\hat{u}_3$$

$$C_{a1} = \frac{x_1}{R} \qquad C_{a2} = \frac{x_2}{R} \qquad C_{a3} = \frac{x_3}{R}$$

then,

$$I_{aa} = I_1 \frac{x_1^2}{R^2} + I_2 \frac{x_2^2}{R^2} + I_3 \frac{x_3^2}{R^2}$$

$$I_{aa} = \frac{x_1^2}{(R\, I_1^{-\frac{1}{2}})^2} + \frac{x_2^2}{(R\, I_2^{-\frac{1}{2}})^2} + \frac{x_3^2}{(R\, I_3^{-\frac{1}{2}})^2}$$

$$\because \quad R = k I_{aa}^{-\frac{1}{2}}$$

$$I_{aa} = \frac{x_1^2}{\left(k I_{aa}^{-\frac{1}{2}} I_1^{-\frac{1}{2}}\right)^2} + \frac{x_2^2}{\left(k I_{aa}^{-\frac{1}{2}} I_2^{-\frac{1}{2}}\right)^2} + \frac{x_3^2}{\left(k I_{aa}^{-\frac{1}{2}} I_3^{-\frac{1}{2}}\right)^2}$$

$$\Longleftrightarrow \quad 1 = \frac{x_1^2}{\left(k I_1^{-\frac{1}{2}}\right)^2} + \frac{x_2^2}{\left(k I_2^{-\frac{1}{2}}\right)^2} + \frac{x_3^2}{\left(k I_3^{-\frac{1}{2}}\right)^2}$$

then the semi-diameters are

$$\begin{cases} d_1 = k I_1^{-\frac{1}{2}} \\ d_2 = k I_2^{-\frac{1}{2}} \\ d_3 = k I_3^{-\frac{1}{2}} \end{cases}$$

say $k = 1$ then

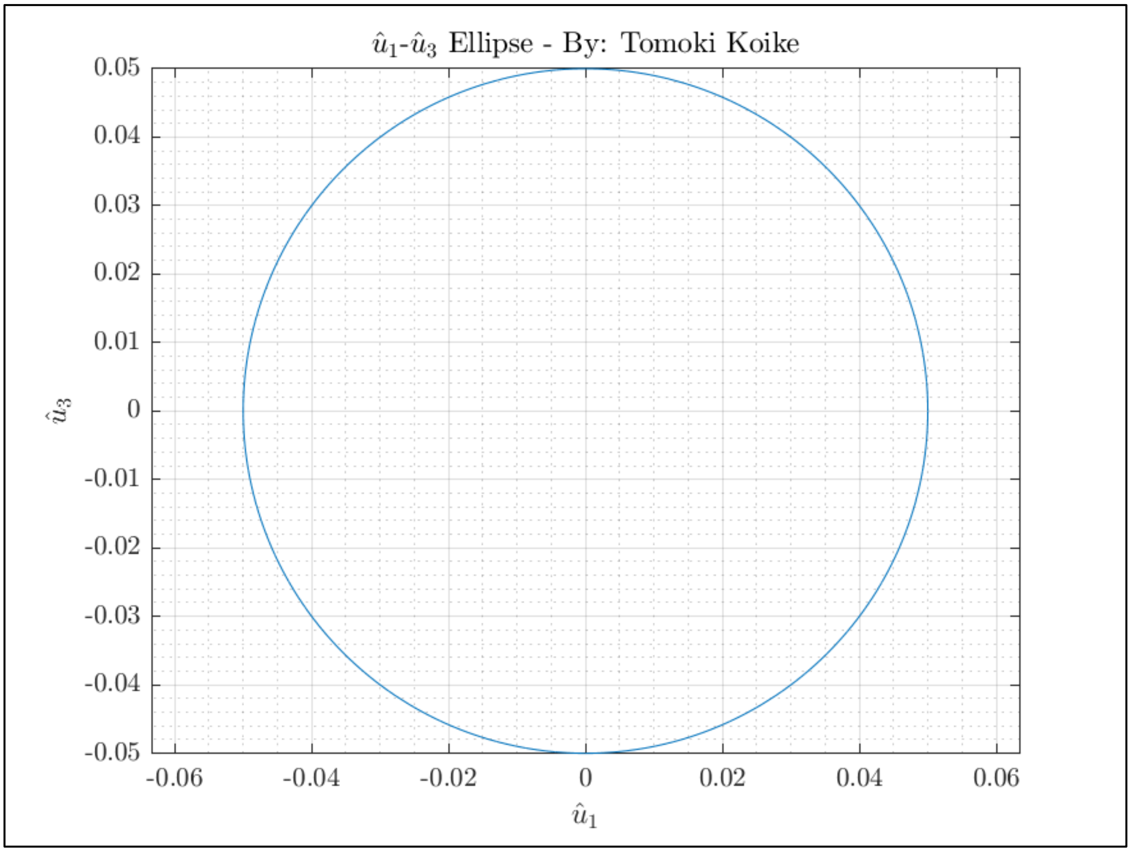
$$\boxed{\begin{aligned} d_1 &= 0.05 \\ d_2 &= 0.1 \\ d_3 &= 0.05 \end{aligned}}$$

the ellipsoid becomes

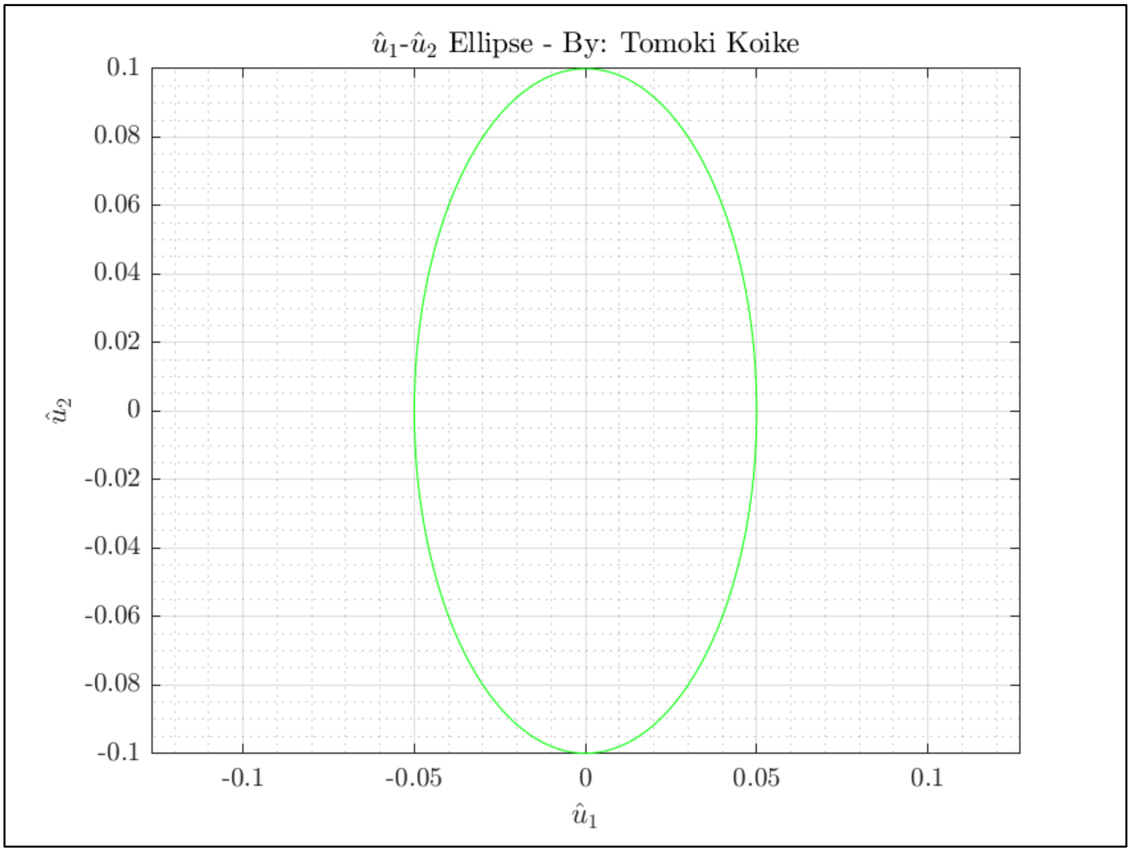
$$1 = \frac{x_1^2}{0.05^2} + \frac{x_2^2}{0.1^2} + \frac{x_3^2}{0.05^2}$$

$\hat{u}_1 - \hat{u}_3$

$$\frac{x_1^2}{0.05^2} + \frac{x_3^2}{0.05^2} = 1$$

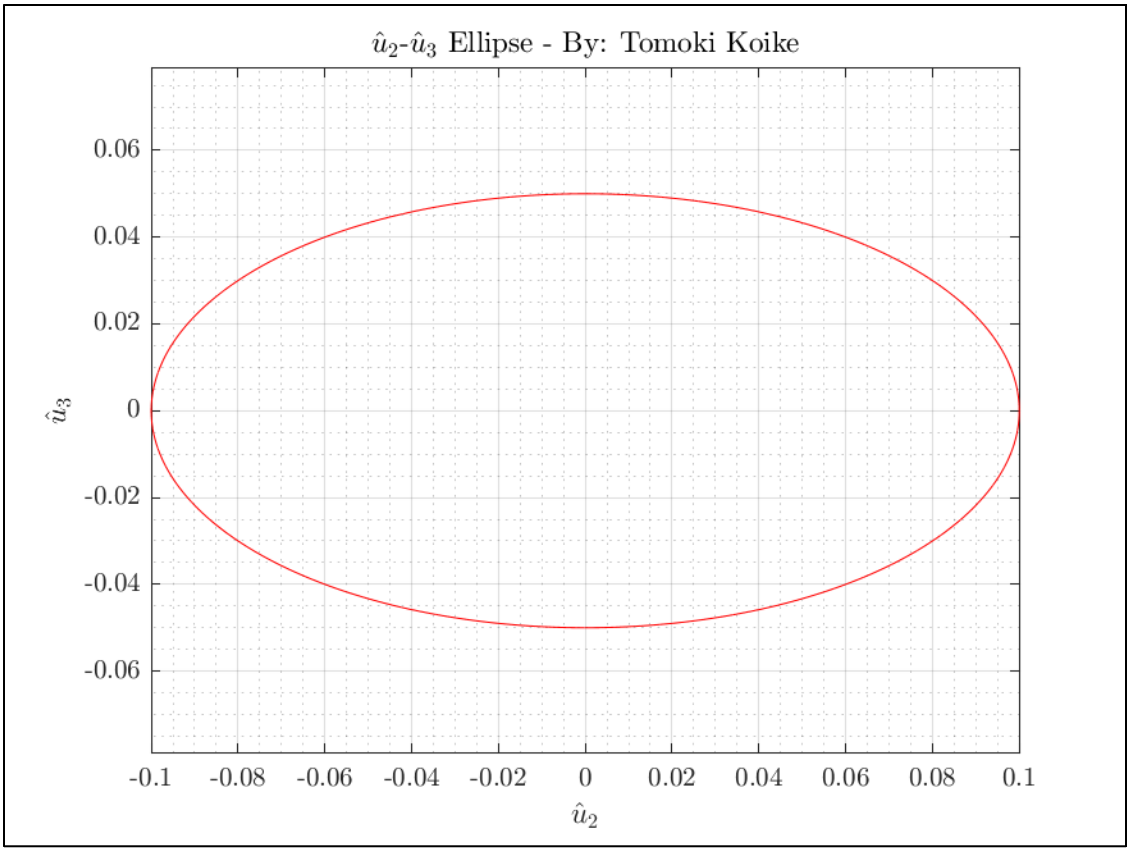


$\hat{u}_1 - \hat{u}_2$ $$\frac{x_1^2}{0.05^2} + \frac{x_2^2}{0.1^2} = 1$$

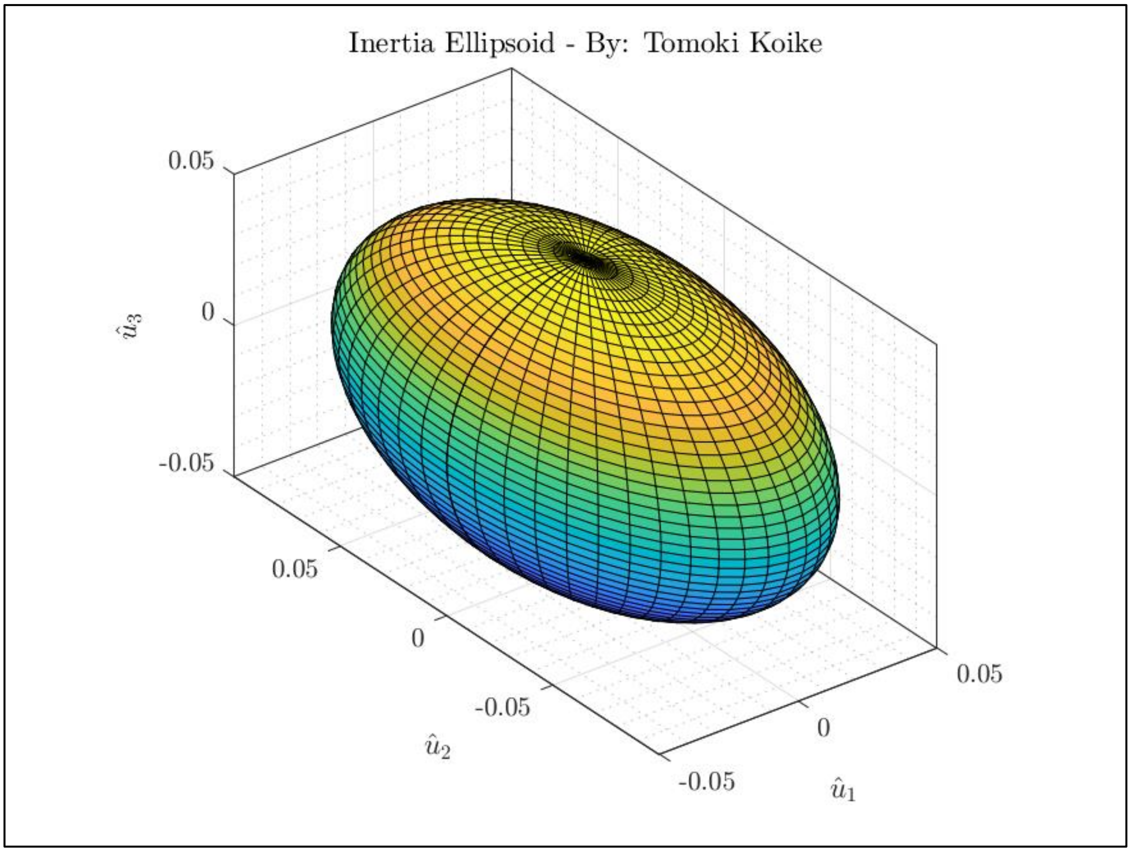


$\hat{u}_2 - \hat{u}_3$

$$\frac{x_2^2}{0.1^2} + \frac{x_3^2}{0.05^2} = 1$$



ellipsoid



Discussion

Because $\hat{u}_1-\hat{u}_3$ is a circle we can tell that the mass distribution on the $\hat{u}_1-\hat{u}_3$ plane is equally distributed. Also because $400 = I > J = 100$ this is a rod-like structure.

(b) Assume that the vehicle inertia characteristics are modified to be

$$\bar{\bar{I}}^{S/S^*} = 200\hat{u}_1\hat{u}_1 + 600\hat{u}_2\hat{u}_2 + 300\hat{u}_3\hat{u}_3 \quad \text{kg-met}^2$$

Now compute the semi-diameters of the corresponding inertia ellipsoid for these new vehicle characteristics. Again plot three planar projections of the energy ellipsoid: $\hat{u}_1 - \hat{u}_3$, $\hat{u}_1 - \hat{u}_2$, $\hat{u}_2 - \hat{u}_3$. (Use the same scale.) Can you plot a 3D image in Matlab?
Is the again a circular projection? Why or why not? What does that tell you?
Is this body more "rod-like" or "disk-like"?

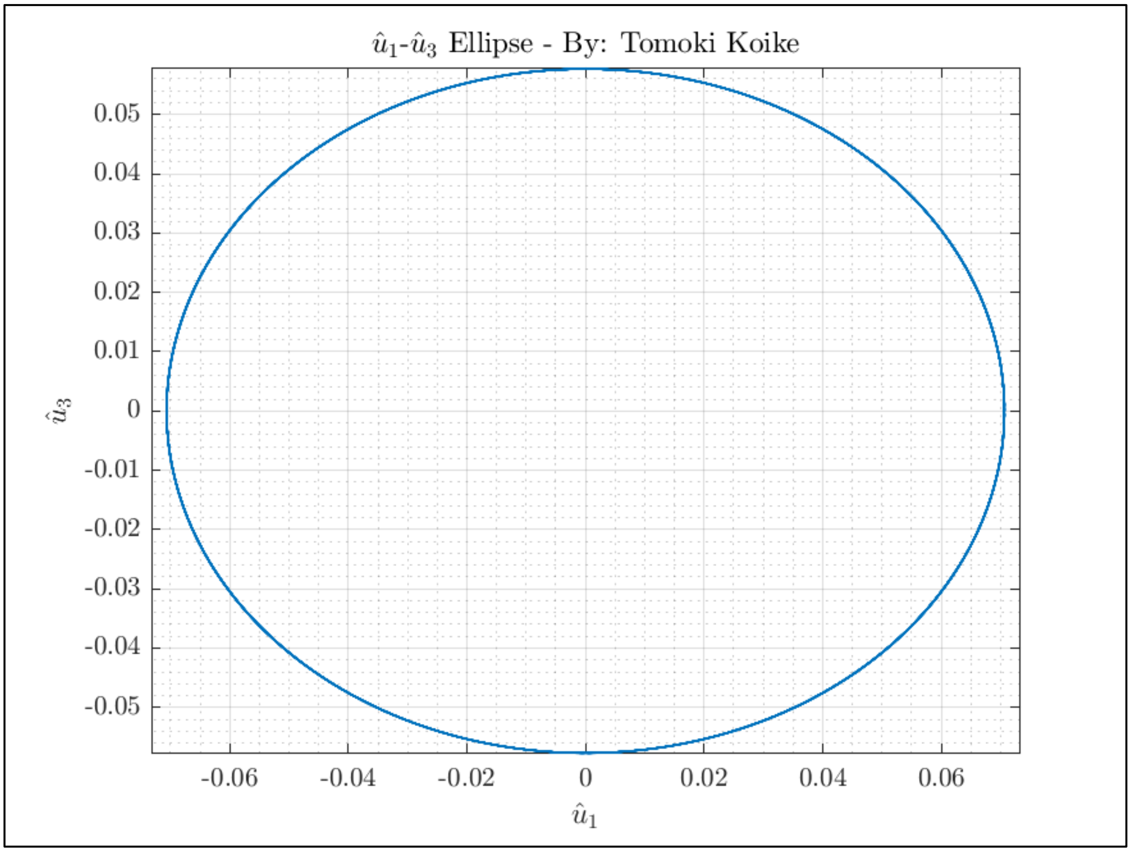
$$\begin{cases} d_1 = k I_1^{-\frac{1}{2}} \\ d_2 = k I_2^{-\frac{1}{2}} \\ d_3 = k I_3^{-\frac{1}{2}} \end{cases}$$

similar to (a)

$$k = 1 \Rightarrow \boxed{\begin{aligned} d_1 &= 0.0707 \\ d_2 &= 0.0408 \\ d_3 &= 0.0577 \end{aligned}}$$

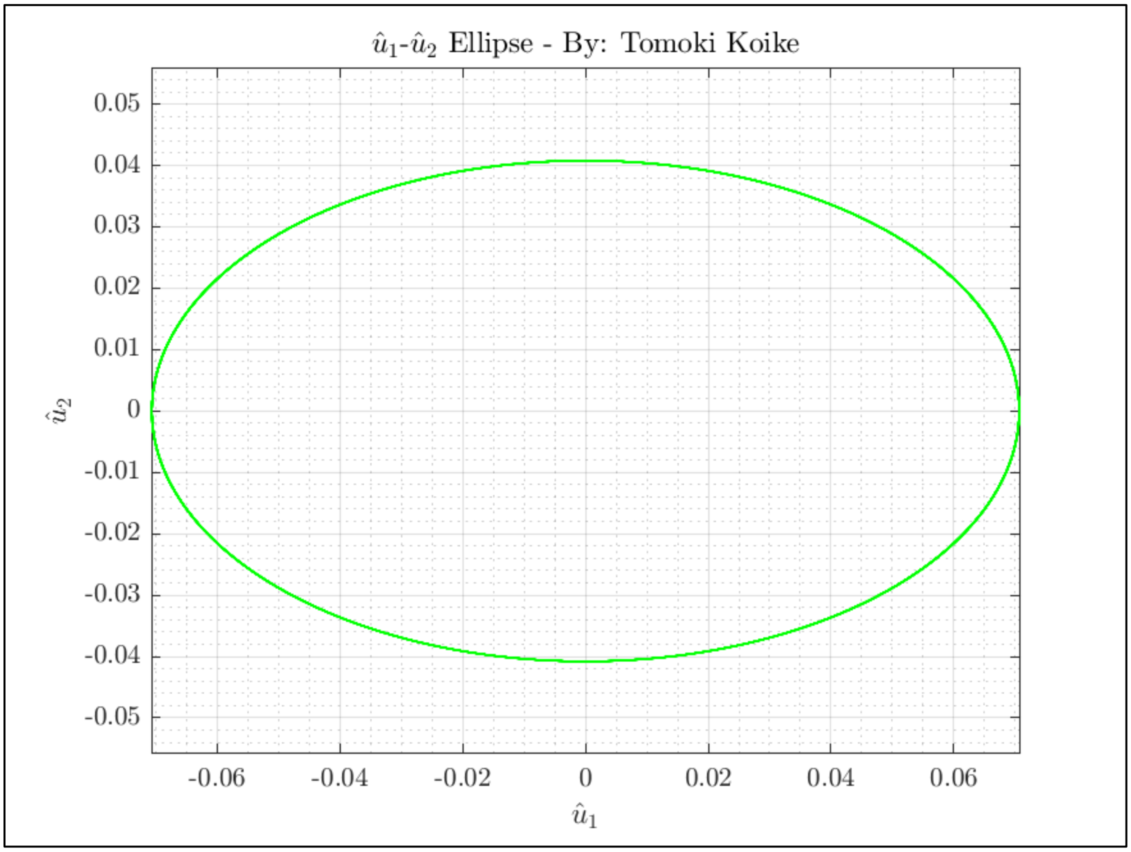
$$1 = \frac{x_1^2}{0.0707^2} + \frac{x_2^2}{0.0408^2} + \frac{x_3^2}{0.0577^2}$$

$\hat{u}_1 - \hat{u}_3$



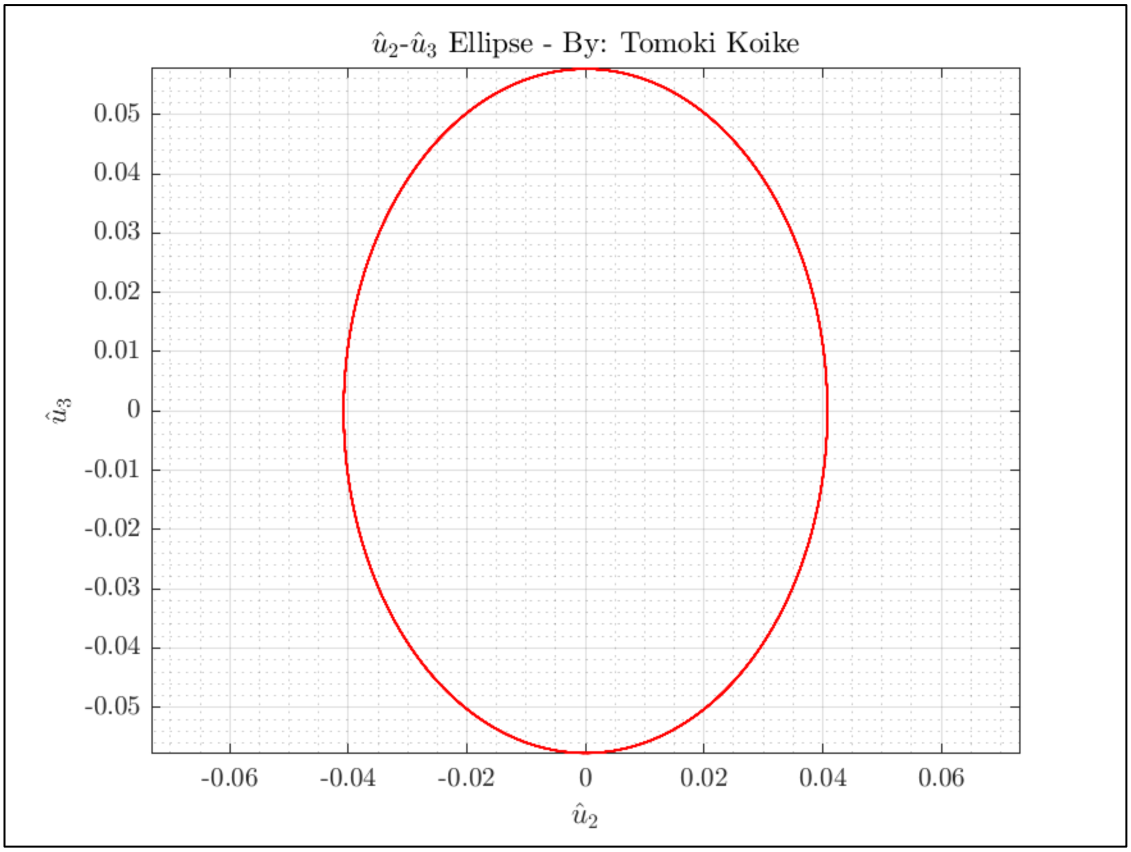
$\hat{u}_1$-$\hat{u}_3$ Ellipse - By: Tomoki Koike
0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
-0.05
$\hat{u}_3$
-0.06
-0.04
-0.02
0
0.02
0.04
0.06
$\hat{u}_1$

$\hat{u}_1 - \hat{u}_2$



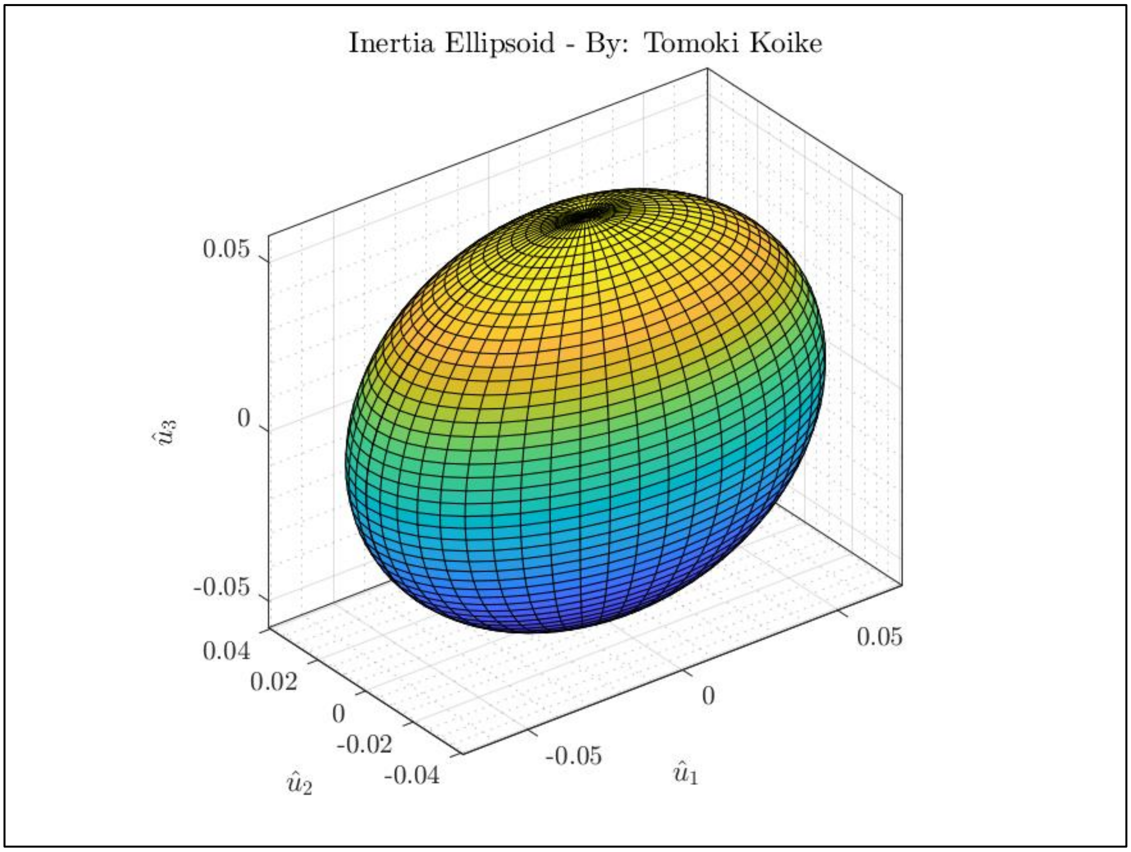
$\hat{u}_1$-$\hat{u}_2$ Ellipse - By: Tomoki Koike
0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
-0.05
$\hat{u}_2$
-0.06
-0.04
-0.02
0
0.02
0.04
0.06
$\hat{u}_1$

$\hat{u}_2 - \hat{u}_3$



$\hat{u}_2$-$\hat{u}_3$ Ellipse - By: Tomoki Koike
0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
-0.05
$\hat{u}_3$
-0.06
-0.04
-0.02
0
0.02
0.04
0.06
$\hat{u}_2$

ellipsoid



Inertia Ellipsoid - By: Tomoki Koike
0.05
0
-0.05
$\hat{u}_3$
0.04
0.02
0
-0.02
-0.04
$\hat{u}_2$
-0.05
0
0.05
$\hat{u}_1$

Discussion

There is no circular projection for this because $I_1 \neq I_2 \neq I_3$ and this indicates that no plane has mass distributed equally. Moreover, since

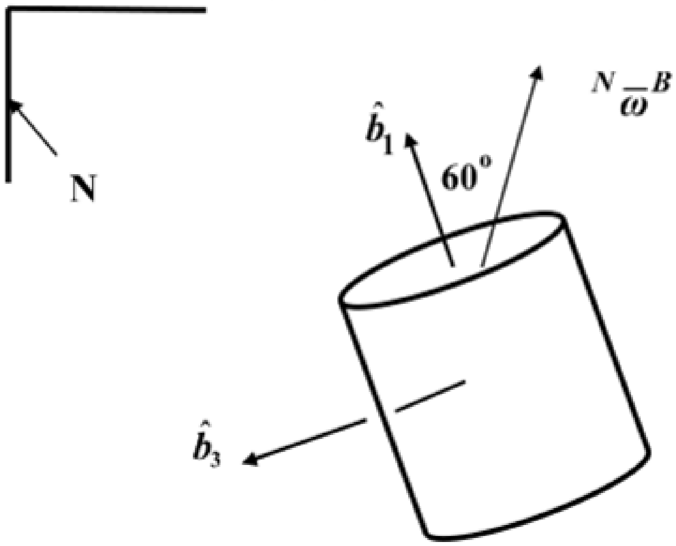
$600 = J > I_1 = 200$ and $600 = J > I_3 = 300$

this is a disk-like structure.

Problem 3: Assume that a rigid body B can move in an inertial torque-free environment N. Define some inertia characteristics that might be similar to a spacecraft like Cassini:

$$\bar{\bar{I}}^{B/B^*} = 4000\hat{b}_1\hat{b}_1 + 9000\hat{b}_2\hat{b}_2 + 9000\hat{b}_3\hat{b}_3 \text{ kg-met}^2$$

Let $\hat{n}_i$ be fixed in the inertial frame N and $\hat{b}_i$ define body-fixed unit vectors parallel to central principal axes of inertia. At the initial time ($t = 0$), $|{}^N\bar{\omega}^B| = 3$ rad/s and ${}^N\bar{\omega}^B$ is directed as shown, in the $\hat{b}_1 - \hat{b}_3$ plane.



(c) For this vehicle, compute the semi-diameters of the corresponding inertia ellipsoid. Plot three planar projections of the energy ellipsoid: $\hat{b}_1 - \hat{b}_3$, $\hat{b}_1 - \hat{b}_2$, $\hat{b}_2 - \hat{b}_3$.
One of the plots is circular. What does that tell you?
Is this body more "rod-like" or "disk-like"?

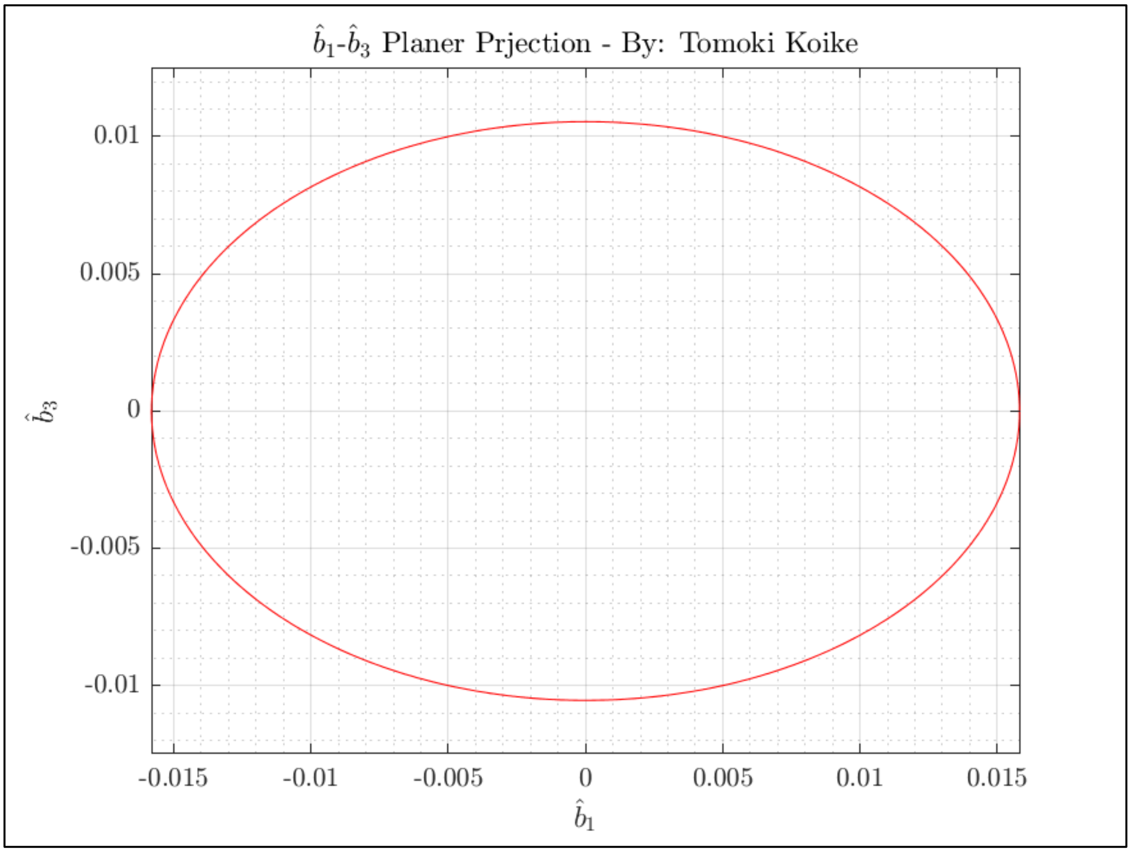
moment inertia : $\bar{\bar{I}}^{B/B^*} = 4000\,\hat{b}_1\hat{b}_1 + 9000\,\hat{b}_2\hat{b}_2 + 9000\,\hat{b}_3\hat{b}_3$

semi-diameters

$$\boxed{\begin{aligned} d_1 &= kI_1^{-\frac{1}{2}} = 0.0158\,k \\ d_2 &= kI_2^{-\frac{1}{2}} = 0.0105\,k \\ d_3 &= kI_3^{-\frac{1}{2}} = 0.0105\,k \end{aligned}}$$

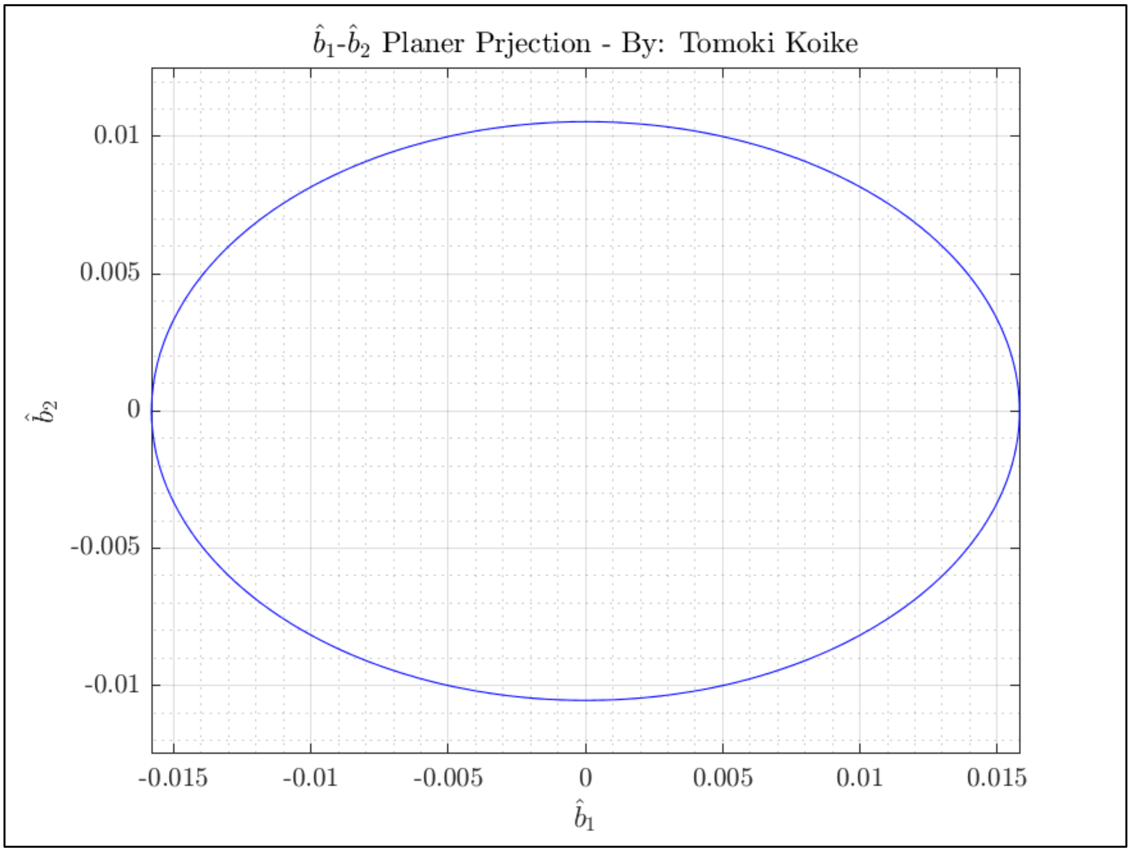
where k is an arbitrary scalar

$\hat{b}_1 - \hat{b}_3$



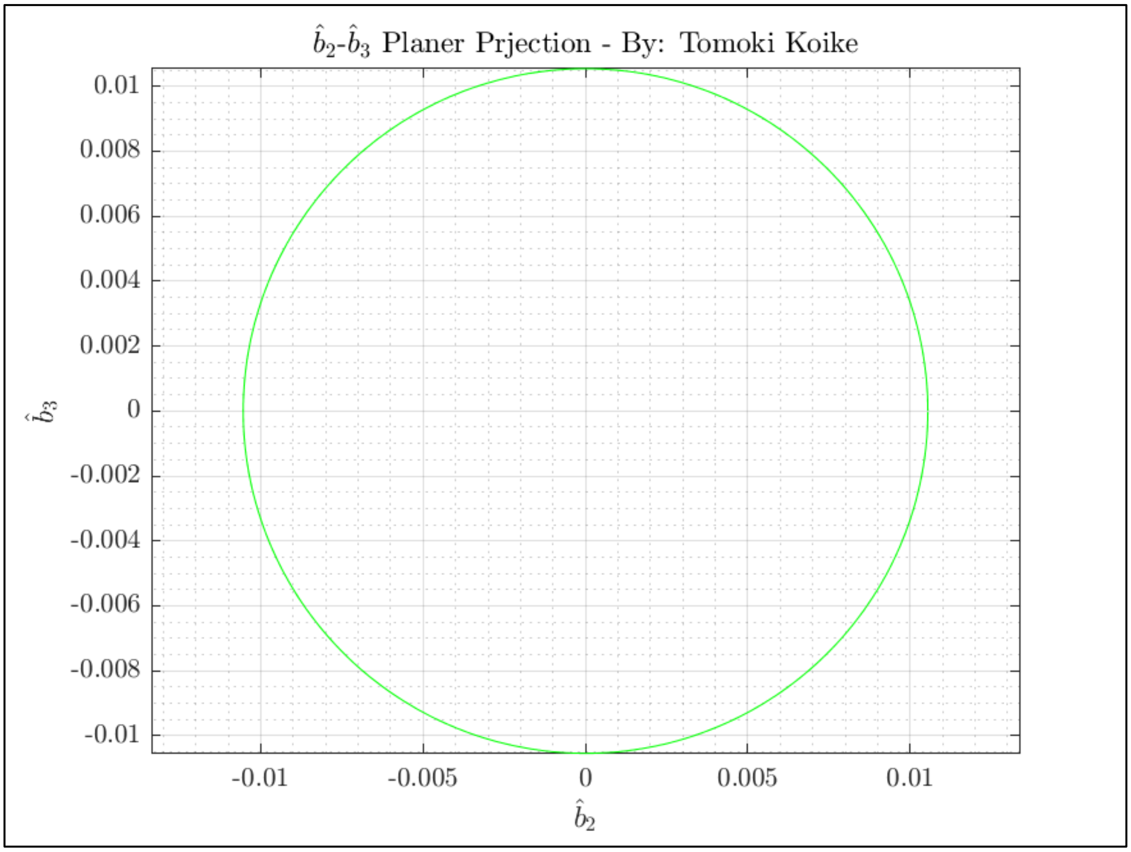
$\hat{b}_1$-$\hat{b}_3$ Planer Prjection - By: Tomoki Koike
0.01
0.005
0
-0.005
-0.01
$\hat{b}_3$
-0.015
-0.01
-0.005
0
0.005
0.01
0.015
$\hat{b}_1$

$\hat{b}_1 - \hat{b}_2$



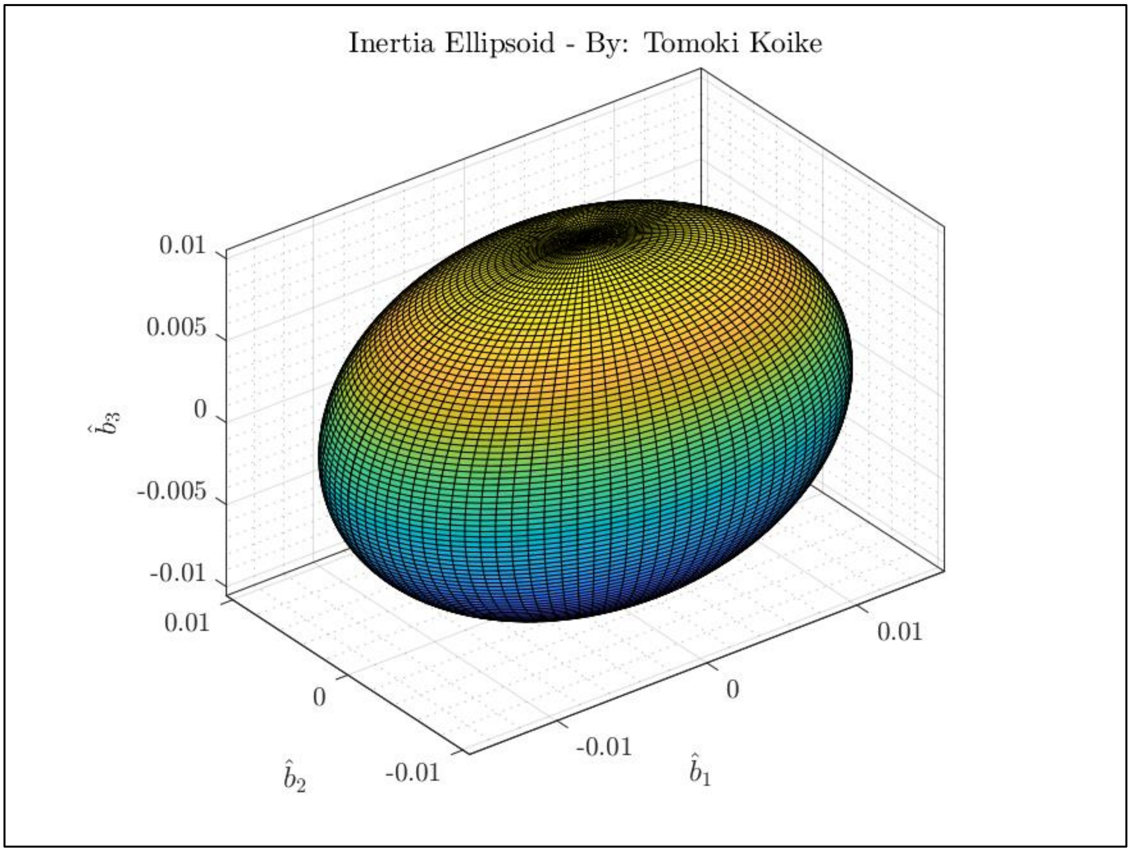
$\hat{b}_1$-$\hat{b}_2$ Planer Prjection - By: Tomoki Koike
0.01
0.005
0
-0.005
-0.01
$\hat{b}_2$
-0.015
-0.01
-0.005
0
0.005
0.01
0.015
$\hat{b}_1$

$\hat{b}_2 - \hat{b}_3$



$\hat{b}_2$-$\hat{b}_3$ Planer Prjection - By: Tomoki Koike
0.01
0.008
0.006
0.004
0.002
0
-0.002
-0.004
-0.006
-0.008
-0.01
$\hat{b}_3$
-0.01
-0.005
0
0.005
0.01
$\hat{b}_2$

3D ellipsoid



Discussion

- → the $\hat{b}_2 - \hat{b}_3$ cross-section is circular.
  This implies that the body is a cylinder with an equal mass distribution about $\hat{b}_1$ while $\hat{b}_1$ is the axis of rotation.

- → since $9000 = I > J = 4000$ the body is considered as rod-like

(d) Use the plot of the inertia ellipsoid in the $\hat{b}_1 - \hat{b}_3$ projection, but note that it proportionally represents the energy ellipsoid. Add the following vectors and other quantities to the plot:

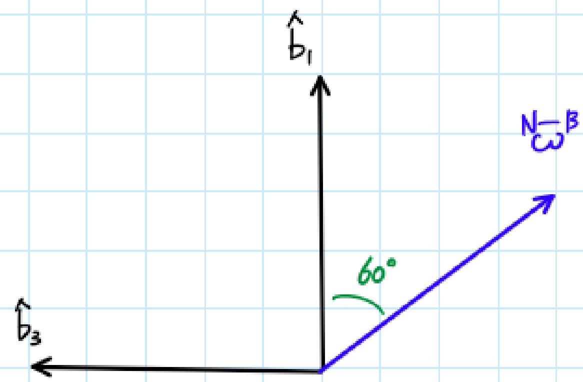
$^N\bar{H}^{B/B^*}$

$^N\bar{\omega}^B$

invariable plane π

nutation angle

Angular velocity



$$^N\bar{\omega}^B = |^N\bar{\omega}^B|(\cos 60^\circ \hat{b}_1 - \sin 60^\circ \hat{b}_2)$$

since $|^N\bar{\omega}^B| = 3$ rad/s @ $t = 0$

$$\Rightarrow \quad = 3\left(\frac{1}{2}\hat{b}_1 - \frac{\sqrt{3}}{2}\hat{b}_2\right)$$

$$\boxed{^N\bar{\omega}^B = \frac{3}{2}\hat{b}_1 - \frac{3\sqrt{3}}{2}\hat{b}_2}$$

Angular Momentum

$$^N\bar{H}^{B/B^*} = \bar{\bar{I}}^{B/B^*} \cdot {}^N\bar{\omega}^B$$

$$= (4000\hat{b}_1\hat{b}_1 + 9000\hat{b}_2\hat{b}_2 + 9000\hat{b}_3\hat{b}_3) \cdot \left(\frac{3}{2}\hat{b}_1 - \frac{3\sqrt{3}}{2}\hat{b}_3\right)$$

$$= 6000\hat{b}_1 - 13500\sqrt{3}\,\hat{b}_3$$

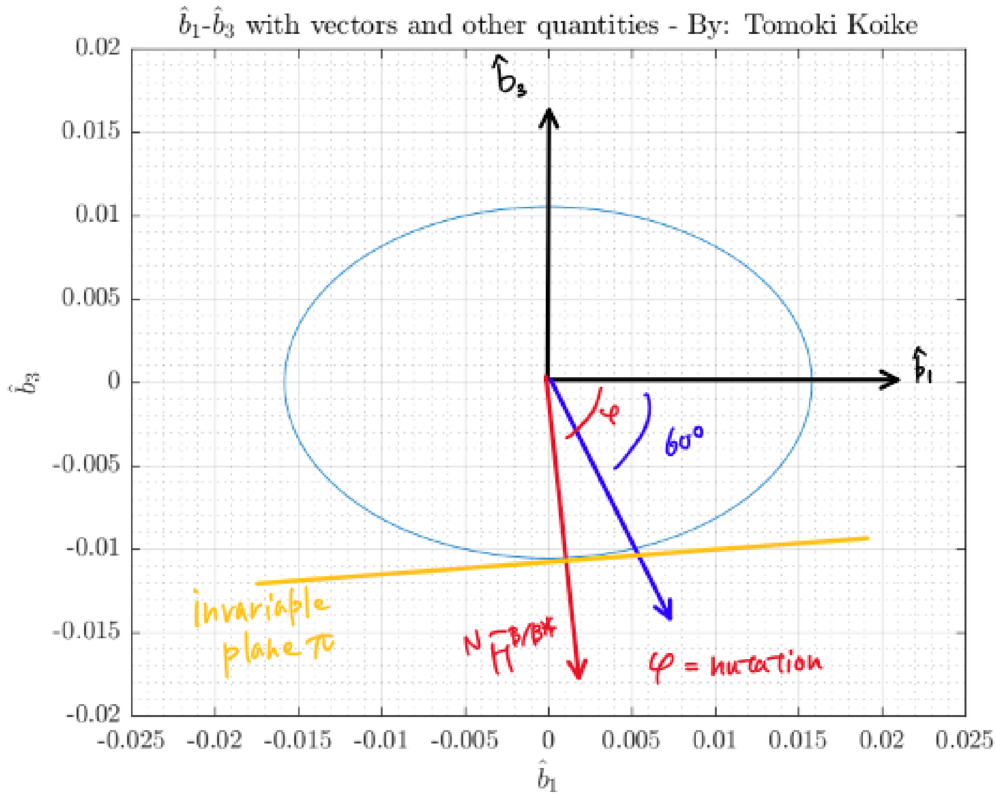
$$\boxed{^N\bar{H}^{B/B^*} = (6000\hat{b}_1 - 13500\sqrt{3}\,\hat{b}_3) \text{ kg-m}^2/\text{s}}$$

Invariable plane π

this plane is perpendicular to $^N\bar{H}^{B/B^*}$ and tangent to the energy ellipsoid.

Nutation angle

The nutation angle is the angle between the rotational axis $\hat{b}_1$ and the angular momentum vector ${}^{N}\bar{H}^{B/B*}$.

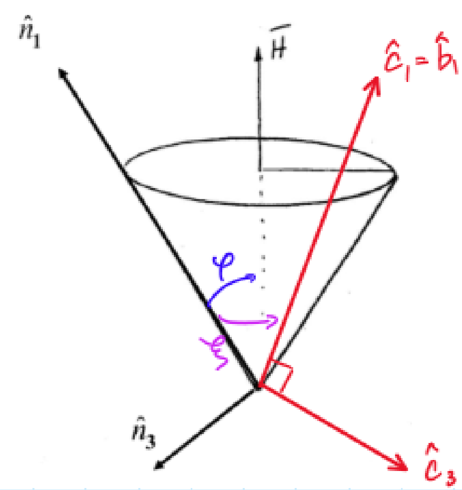


(c) Given a figure similar to the one on the next page, sketch the orientation of the unit vectors $\hat{c}_1$ and $\hat{c}_3$ with respect to $\hat{n}_1$ and $\hat{n}_3$ at an arbitrary time. Define ξ as the angle the angle between between $\hat{n}_1$ and $\hat{b}_1$. Where is ξ in the sketch?

Determine the following quantities at $t = 0.4$ sec; 3.0 sec, 6.0 sec:

precession, nutation, spin angles

precession, nutation, and spin rates



when $t = 1$

Precession rate

$$p = \frac{|{}^N\bar{H}^{B/B^*}|}{I} = \frac{\sqrt{6000^2 + (3500\sqrt{3})^2}}{9000} = \boxed{\begin{array}{l} 2.6822 \text{ rad/s} \\ 143.2394 \text{ deg/s} \end{array}}$$

Spin rate

$$s = \frac{I-J}{I}\omega_1 = \frac{9000-4000}{9000}\cdot\frac{3}{2} = \boxed{\begin{array}{l} 0.8333 \text{ rad/s} \\ 286.4789 \text{ deg/s} \end{array}}$$

Nutation rate

$$\because \varphi = \text{const} \qquad \boxed{\therefore \dot{\varphi} = 0}$$

since rates are indep. of time they are constant throughout $t = 0.4$, 3.0, 6.0

Nutation

$$\varphi = \arccos\left[\frac{J}{|\bar{H}|}\omega_1\right]$$

$$= \arccos\left[\frac{4000}{\sqrt{6000^2 + (3500\sqrt{3})^2}}\cdot\frac{3}{2}\right]$$

$$= \boxed{1.3196 \text{ rad} = 19.0986 \text{ deg/s}}$$

	0.4	3.0	6.0	Time [s]
NUTATION	75.6084	75.6084	75.6084	
PRECESSSION	61.4726	101.0442	202.0883	
SPIN	19.0986	143.2394	286.4789	
[deg]				

(d) What are the Euler parameters ${}^N\bar{\varepsilon}^B$, ${}^N\varepsilon_4^B$ that correspond to these orientations at the specified times?
Write the Euler vector in terms of unit vectors $\hat{c}$ as well as body-fixed unit vectors $\hat{b}$.

using $\quad {}^N\bar{\varepsilon}^B = {}^N\bar{\varepsilon}^C\,{}^C\varepsilon_4^B + {}^C\bar{\varepsilon}^B\,{}^N\varepsilon_4^C + {}^C\bar{\varepsilon}^B \times {}^N\bar{\varepsilon}^C$

$${}^N\bar{\varepsilon}^B = \sin\left(\frac{pt}{2}\right)\hat{h}\cos\left(\frac{st}{2}\right) + \sin\left(\frac{st}{2}\right)\hat{c}_1\cos\left(\frac{pt}{2}\right) + (\hat{c}_1\times\hat{h})\sin\left(\frac{st}{2}\right)\sin\left(\frac{pt}{2}\right)$$

where $\quad \hat{h} = \cos\varphi\,\hat{c}_1 - \sin\varphi\,\hat{c}_3$

$\quad\quad\quad \hat{c}_1\times\hat{h}_1 = -\sin\varphi\,\hat{c}_2$

$$\Rightarrow \sin\left(\frac{pt}{2}\right)(\cos\varphi\,\hat{c}_1 - \sin\varphi\,\hat{c}_3)\cos\left(\frac{st}{2}\right) + \sin\left(\frac{st}{2}\right)\hat{c}_1\cos\left(\frac{pt}{2}\right) - \sin\varphi\,\hat{c}_2\sin\left(\frac{st}{2}\right)\sin\left(\frac{pt}{2}\right)$$

$$\boxed{\begin{aligned} = &\left[\sin\left(\frac{pt}{2}\right)\cos\left(\frac{st}{2}\right)\cos\varphi + \sin\left(\frac{st}{2}\right)\cos\left(\frac{pt}{2}\right)\right]\hat{c}_1 \\ &- \sin\varphi\sin\left(\frac{st}{2}\right)\sin\left(\frac{pt}{2}\right)\hat{c}_2 \\ &- \sin\left(\frac{pt}{2}\right)\cos\left(\frac{st}{2}\right)\sin\varphi\,\hat{c}_3 \end{aligned}}$$

$$ {}^{N}\varepsilon_4^{B} = {}^{N}\varepsilon_4^{C}\,{}^{C}\varepsilon_4^{B} - {}^{N}\bar{\varepsilon}^{C} \cdot {}^{C}\bar{\varepsilon}^{B} $$

$$ = \cos\left(\frac{pt}{2}\right)\cos\left(\frac{st}{2}\right) - \sin\left(\frac{pt}{2}\right)\sin\left(\frac{st}{2}\right)\hat{h}\cdot\hat{c}_1 $$

$$ \because \hat{h}\cdot\hat{c}_1 = \cos\varphi $$

$$ = \boxed{\cos\left(\frac{pt}{2}\right)\cos\left(\frac{st}{2}\right) - \sin\left(\frac{pt}{2}\right)\sin\left(\frac{st}{2}\right)\cos\varphi} $$

plug in the values for p, s, φ, t using MATLAB

@ $t = 0.4$ s

$$ {}^{N}\bar{\varepsilon}^{B} = 0.2679\,\hat{c}_1 - 0.0821\,\hat{c}_2 - 0.4881\,\hat{c}_3 $$

$$ {}^{N}\varepsilon_4^{B} = 0.8265 $$

@ $t = 3.0$ s

$$ {}^{N}\bar{\varepsilon}^{B} = -0.1638\,\hat{c}_1 + 0.7095\,\hat{c}_2 + 0.2858\,\hat{c}_3 $$

$$ {}^{N}\varepsilon_4^{B} = -0.0184 $$

@ $t = 6.0$ s

$$ {}^{N}\bar{\varepsilon}^{B} = -0.3101\,\hat{c}_1 - 0.5690\,\hat{c}_2 + 0.7616\,\hat{c}_3 $$

$$ {}^{N}\varepsilon_4^{B} = 0.0075 $$

Use DCM to convert to $\hat{b}$-frame

$$
{}^{C}C^{B} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(5t) & -\sin(5t) \\ 0 & \sin(5t) & \cos(5t) \end{bmatrix}
$$

$$
{}^{N}\bar{\varepsilon}^{B}\Big|_{\hat{b}} = {}^{N}\bar{\varepsilon}^{B}\Big|_{\hat{c}} \; {}^{C}C^{B}
$$

@ $t = 0.4$

$$
{}^{N}\bar{\varepsilon}^{B} = 0.2679\,\hat{c}_1 - 0.2373\,\hat{c}_2 - 0.4344\,\hat{c}_3
$$

@ $t = 3.0$

$$
{}^{N}\bar{\varepsilon}^{B} = -0.6638\,\hat{c}_1 - 0.4273\,\hat{c}_2 - 0.6135\,\hat{c}_3
$$

@ $t = 6.0$

$$
{}^{N}\bar{\varepsilon}^{B} = -0.3101\,\hat{c}_1 - 0.8917\,\hat{c}_2 - 0.3295\,\hat{c}_3
$$

$$
{}^{N}\varepsilon^{B}_{4} = \text{const.}
$$

(e) What is the maximum value of ξ? Can you determine the time when the max value first occurs? What are the Euler parameters at that time?

$$\mathrm{Max}(\xi) = 2\varphi = 2 \times 1.3196 \text{ rad} = \boxed{2.6392 \text{ rad}}$$

$p = 2.6822$ rad/s

so $$t_{max} = \frac{\pi}{p} = \boxed{1.1713 \text{ s}}$$

The euler parameters are using same method in (d)

$$\boxed{\begin{aligned} {}^{N}\bar{\varepsilon}^{B} &= 0.2195\hat{e}_1 - 0.4542\hat{e}_2 - 0.8555\hat{e}_3 \\ {}^{N}\varepsilon_4^{B} &= -0.1165 \end{aligned}}$$

(f) Assume that you want to know the Space 3-1-2 angles at time $t = 6.0$ sec. Compute them.

@ $t = 6.0$ s

nutation $\longrightarrow$ $\theta_2 = 75.6084°$

precession $\longrightarrow$ $\theta_1 = 202.0883°$

spin $\longrightarrow$ $\theta_3 = 286.4784°$

$\Downarrow$

Body-two 1-2-1 plug-in θs

Body-two: 1-2-1

	$\mathbf{b}_1$	$\mathbf{b}_2$	$\mathbf{b}_3$
$\mathbf{a}_1$	c_2	$s_2 s_3$	$s_2 c_3$
$\mathbf{a}_2$	$s_1 s_2$	$-s_1 c_2 s_3 + c_3 c_1$	$-s_1 c_2 c_3 - s_3 c_1$
$\mathbf{a}_3$	$-c_1 s_2$	$c_1 c_2 s_3 + c_3 s_1$	$c_1 c_2 c_3 - s_3 s_1$

$$\hookrightarrow \quad {}^{N}C^{B} = \begin{bmatrix} 0.2485 & -0.9288 & 0.2748 \\ -0.3642 & -0.3525 & -0.8620 \\ 0.8975 & 0.1142 & -0.4259 \end{bmatrix}$$

Space-three: 3-1-2

	$\mathbf{b}_1$	$\mathbf{b}_2$	$\mathbf{b}_3$
$\mathbf{a}_1$	$s_1 s_2 s_3 + c_3 c_1$	$c_1 s_2 s_3 - c_3 s_1$	$c_2 s_3$
$\mathbf{a}_2$	$s_1 c_2$	$c_1 c_2$	$-s_2$
$\mathbf{a}_3$	$s_1 s_2 c_3 - s_3 c_1$	$c_1 s_2 c_3 + s_3 s_1$	$c_2 c_3$

$$-s_2 = -0.8620$$

$$\boxed{\theta_2 = 59.5456^\circ}$$

$$c_1 c_2 = -0.3525$$

$$\theta_1 = \arccos\left(\frac{-0.3525}{\cos\theta_2}\right)$$

$$\theta_1 = -134.0593$$

$$\theta_1 = \pi - \arcsin\left(\frac{-0.3642}{\cos\theta_2}\right)$$

$$\boxed{\theta_1 = -134.0593^\circ}$$

$$c_2 c_3 = -0.4254$$

$$\theta_3 = \arccos\left(\frac{-0.4254}{\cos\theta_2}\right)$$

$$= 147.1739^\circ$$

$$\theta_3 = \pi - \arcsin\left(\frac{0.2748}{\cos\theta_2}\right)$$

$$= \boxed{147.1739^\circ}$$

Appendix

AAE440 PS7 Problem 1

```
clear all; close all; clc;
format long e
digits(100)
```

```
syms m L theta
I_rod = [0 0 0; 0 m*L^2/3 0; 0 0 m*L^2/3];
C_AB = [cos(theta) -sin(theta) 0; sin(theta) cos(theta) 0; 0 0 1];
I_h = C_AB*I_rod*transpose(C_AB)
I_h = subs(I_h, theta, pi/2)
I_v = subs(I_rod, theta, pi/2)
clear all;
```

(a) & (b)

```
% Define given properties
M_e = 5.9723e24;  % [kg]
R_e = 6378e3;  % [m]
R = R_e + 110e3
G = 6.6743015e-11;  % [m3kg-1s-2]
m = 1;
L = 50e3;


% Resultant gravity force
m_horz = m;
m_vert = m;
l_horz = L/2;
l_vert = 2*L;
I_horz = m_horz*l_horz^2/12*[0 0 0; 0 1 0; 0 0 1];
I_vert = m_vert*l_vert^2/12*[1 0 0; 0 0 0; 0 0 1];
I_body = (I_horz + I_vert);
F_g = resultant_Gforce(M_e,m,R,I_body)
R_cg = distance2cg(M_e,m,F_g)
delta_cm_cg = R - R_cg
```

(c)

```
gamma = -30;  % [deg]
C_AB = [ cosd(gamma) sind(gamma) 0;
        -sind(gamma) cosd(gamma) 0;
                   0           0 1];
F_g_new = resultant_Gforce_with_DCM(M_e,m,R,I_body,C_AB)
R_cg_new = distance2cg(M_e,m,F_g_new)
```

(d)

```
r_cg = -F_g_new/norm(F_g_new)*R_cg_new
```

```
r_cm = [R 0 0]
r_cm_cg = r_cg - r_cm
Mmt_cg = cross(r_cm_cg,F_g_new)
```

AAE440 PS7 Problem 2

```
clear all; close all; clc;
```

```
fdir = 'C:\Users\Tomo\Desktop\studies\2020-Spring\AAE440\MATLAB\outputs\HW7';
set(groot, 'defaulttextinterpreter',"latex");
set(groot, 'defaultAxesTickLabelInterpreter',"latex");
set(groot, 'defaultLegendInterpreter',"latex");
```

```
% ellipsoid
I_body = [400 0 0; 0 100 0; 0 0 400];
k = 1;
d1 = k*I_body(1,1)^(-0.5)
d2 = k*I_body(2,2)^(-0.5)
d3 = k*I_body(3,3)^(-0.5)
```

```
% Plotting
theta = 0:0.1:360;
theta = deg2rad(theta);
% u1-u3
fig1 = figure("Renderer","painters");
plot(d1*cos(theta), d3*sin(theta))
title('$\hat{u}_1$-$\hat{u}_3$ Ellipse - By: Tomoki Koike')
xlabel('$\hat{u}_1$')
ylabel('$\hat{u}_3$')
grid on; grid minor; box on; axis equal;
saveas(fig1, fullfile(fdir,'u1_u3_ellipse.png'));


% u1-u2
fig2 = figure("Renderer","painters");
plot(d1*cos(theta), d2*sin(theta), 'g')
title('$\hat{u}_1$-$\hat{u}_2$ Ellipse - By: Tomoki Koike')
xlabel('$\hat{u}_1$')
ylabel('$\hat{u}_2$')
grid on; grid minor; box on; axis equal;
saveas(fig2, fullfile(fdir,'u1_u2_ellipse.png'));


% u1-u2
fig3 = figure("Renderer","painters");
plot(d2*cos(theta), d3*sin(theta), 'r')
title('$\hat{u}_2$-$\hat{u}_3$ Ellipse - By: Tomoki Koike')
xlabel('$\hat{u}_2$')
```

```
ylabel('$\hat{u}_3$')
grid on; grid minor; box on; axis equal;
saveas(fig3, fullfile(fdir,'u2_u3_ellipse.png'));
```

```
% Ellipsoid
fig4 = figure("Renderer","painters");
ellipsoid(0,0,0,d1,d2,d3,50)
title('Inertia Ellipsoid - By: Tomoki Koike')
xlabel('$\hat{u}_1$')
ylabel('$\hat{u}_2$')
zlabel('$\hat{u}_3$')
grid on; grid minor; box on; axis equal;
saveas(fig4, fullfile(fdir,'ellipsoid.png'));
```

```
clear all; close all;
```

```
fdir = 'C:\Users\Tomo\Desktop\studies\2020-Spring\AAE440\MATLAB\outputs\HW7';
set(groot, 'defaulttextinterpreter',"latex");
set(groot, 'defaultAxesTickLabelInterpreter',"latex");
set(groot, 'defaultLegendInterpreter',"latex");
% ellipsoid
I_body = [200 0 0; 0 600 0; 0 0 300];
k = 1;
d1 = k*I_body(1,1)^(-0.5)
d2 = k*I_body(2,2)^(-0.5)
d3 = k*I_body(3,3)^(-0.5)
```

```
% Plotting
theta = 0:0.1:360;
% u1-u3
fig5 = figure("Renderer","painters");
plot(d1*cos(theta), d3*sin(theta))
title('$\hat{u}_1$-$\hat{u}_3$ Ellipse - By: Tomoki Koike')
xlabel('$\hat{u}_1$')
ylabel('$\hat{u}_3$')
grid on; grid minor; box on; axis equal;
saveas(fig5, fullfile(fdir,'u1_u3_ellipse2.png'));


% u1-u2
fig6 = figure("Renderer","painters");
plot(d1*cos(theta), d2*sin(theta), 'g')
title('$\hat{u}_1$-$\hat{u}_2$ Ellipse - By: Tomoki Koike')
xlabel('$\hat{u}_1$')
ylabel('$\hat{u}_2$')
grid on; grid minor; box on; axis equal;
```

```
saveas(fig6, fullfile(fdir,'u1_u2_ellipse2.png'));


% u1-u2
fig7 = figure("Renderer","painters");
plot(d2*cos(theta), d3*sin(theta), 'r')
title('$\hat{u}_2$-$\hat{u}_3$ Ellipse - By: Tomoki Koike')
xlabel('$\hat{u}_2$')
ylabel('$\hat{u}_3$')
grid on; grid minor; box on; axis equal;
saveas(fig7, fullfile(fdir,'u2_u3_ellipse2.png'));
```

```
% Ellipsoid
fig8 = figure("Renderer","painters");
ellipsoid(0,0,0,d1,d2,d3,50)
title('Inertia Ellipsoid - By: Tomoki Koike')
xlabel('$\hat{u}_1$')
ylabel('$\hat{u}_2$')
zlabel('$\hat{u}_3$')
grid on; grid minor; box on; axis equal;
saveas(fig8, fullfile(fdir,'ellipsoid2.png'));
```

AAE 440 PS7 p3

```
clear all; close all; clc;
```

```
fdir = 'C:\Users\Tomo\Desktop\studies\2020-Spring\AAE440\MATLAB\outputs\HW7';
set(groot, 'defaulttextinterpreter',"latex");
set(groot, 'defaultAxesTickLabelInterpreter',"latex");
set(groot, 'defaultLegendInterpreter',"latex");
```

```
% Draw Arrow Function
drawArrow = @(x,y,varargin) quiver( x(1),y(1),x(2)-x(1),y(2)-y(1),0,
varargin{:});
```

```
% Setting given properties
I_body = [4000 0 0; 0 9000 0; 0 0 9000];
k = 1;
d_1 = k*I_body(1,1)^(-0.5);
d_2 = k*I_body(2,2)^(-0.5);
d_3 = k*I_body(3,3)^(-0.5);
theta = 0:0.01:2*pi;
```

<a>

```
% Plotting
% b1-b3
```

```
fig1 = figure("Renderer","painters");
    plot(d_1*cos(theta), d_3*sin(theta), 'r')
    title('$\hat{b}_1$-$\hat{b}_3$ Planer Prjection - By: Tomoki Koike')
    xlabel('$\hat{b}_1$')
    ylabel('$\hat{b}_3$')
    grid on; grid minor; box on; axis equal;
saveas(fig1, fullfile(fdir,'b1_b3_ellipse3.png'));


% b1-b2
fig2 = figure("Renderer","painters");
    plot(d_1*cos(theta), d_2*sin(theta), 'b')
    title('$\hat{b}_1$-$\hat{b}_2$ Planer Prjection - By: Tomoki Koike')
    xlabel('$\hat{b}_1$')
    ylabel('$\hat{b}_2$')
    grid on; grid minor; box on; axis equal;
saveas(fig2, fullfile(fdir,'b1_b2_ellipse3.png'));


% b2-b3
fig3 = figure("Renderer","painters");
    plot(d_2*cos(theta), d_3*sin(theta), 'g')
    title('$\hat{b}_2$-$\hat{b}_3$ Planer Prjection - By: Tomoki Koike')
    xlabel('$\hat{b}_2$')
    ylabel('$\hat{b}_3$')
    grid on; grid minor; box on; axis equal;
saveas(fig3, fullfile(fdir,'b2_b3_ellipse3.png'));


% Ellipsoid
fig4 = figure("Renderer","painters");
    ellipsoid(0,0,0,d_1,d_2,d_3,100)
    title('Inertia Ellipsoid - By: Tomoki Koike')
    xlabel('$\hat{b}_1$')
    ylabel('$\hat{b}_2$')
    zlabel('$\hat{b}_3$')
    grid on; grid minor; box on; axis equal;
saveas(fig4, fullfile(fdir,'ellipsoid3.png'));
```

<b>

```
% Defining properties
T = 0;  % Torque  [N-m]
I_cm = [4000 0 0; 0 9000 0; 0 0 9000];   % Inertia Dyadic  [kg-m2]
I = 9000;
J = 4000;


% Defining the angular velocity
w_mag = 3; % Magnitude of angular velocity
ang = deg2rad(60); % [rad]
w_NB = [w_mag*cos(ang) 0 -w_mag*sin(ang)];
```

```
% Angular Momentum
H_NB = I_cm*w_NB';




% Replotting b1-b3 and adding vectors and quantities
fig5 = figure("Renderer","painters");
    plot(d_1*cos(theta), d_3*sin(theta))
    title('$\hat{b}_1$-$\hat{b}_3$ with vectors and other quantities - By: Tomoki Koike')
    xlabel('$\hat{b}_1$')
    ylabel('$\hat{b}_3$')
    hold on; grid on; grid minor; box on; axis equal;
    xlim([-0.025, 0.025]); ylim([-0.020, 0.020]);
saveas(fig5, fullfile(fdir,'b1_b3_ellipse_new.png'));
```

<c>

```
% angles @ t = 1
p = norm(H_NB)/I; % precession rate
s = (I-J)/I*w_NB(1); % spin rate
phi = acos(J/norm(H_NB)*w_NB(1)); % nutation angle = constant


% t = 0.4, 3.0, 6.0
t = [0.4 3.0 6.0];
p_rad = p.*t;
s_rad = s.*t;
phi_deg = rad2deg(phi)
p_deg = rad2deg(p_rad)
s_deg = rad2deg(s_rad)
```

<d>

```
% Euler Parameters
e_04_c = EulerPara_from_OrientAngs(0.4,p,s,phi);
e_30_c = EulerPara_from_OrientAngs(3.0,p,s,phi);
e_60_c = EulerPara_from_OrientAngs(6.0,p,s,phi);


% t = 0.4s
t = 0.4;
e_04_c = [e_04_c(1) e_04_c(2) e_04_c(3)];
C_CB_04 = [1 0 0; 0 cos(s*t) -sin(s*t); 0 sin(s*t) cos(s*t)];
e_04_b = e_04_c*C_CB_04


% t = 3.0s
t = 3.0;
e_30_c = [e_30_c(1) e_30_c(2) e_30_c(3)];
C_CB_30 = [1 0 0; 0 cos(s*t) -sin(s*t); 0 sin(s*t) cos(s*t)];
e_30_b = e_30_c*C_CB_30
```

```
% t = 6.0s
t = 6.0;
e_60_c = [e_60_c(1) e_60_c(2) e_60_c(3)];
C_CB_60 = [1 0 0; 0 cos(s*t) -sin(s*t); 0 sin(s*t) cos(s*t)];
e_60_b = e_60_c*C_CB_60
```

<e>

```
% maximum of zeta
zeta = 2*phi;

% time corresponding to the maximum zeta
t_maxzeta = pi/p;


% Euler parameters corresponding
e_maxzeta = EulerPara_from_OrientAngs(t_maxzeta,p,s,phi)
```

<f>

```
% t = 6.0 s
C = DCM_Body(1, 2, 1, p_rad(3), phi, s_rad(3));
theta2 = asin(-C(2,3))
theta1 = -acos(C(2,2)/cos(theta2))
theta1 = -pi-asin(C(2,1)/cos(theta2))
theta3 = pi-asin(C(1,3)/cos(theta2))
theta3 = acos(C(3,3)/cos(theta2))

theta1 = rad2deg(theta1)
theta2 = rad2deg(theta2)
theta3 = rad2deg(theta3)


function epsilons = EulerPara_from_OrientAngs(t,p,s,phi)
    e1 = cos(phi)*sin(p*t/2)*cos(s*t/2)+cos(p*t/2)*sin(s*t/2);
    e2 = -sin(phi)*sin(s*t/2)*sin(p*t/2);
    e3 = -sin(phi)*sin(p*t/2)*cos(s*t/2);
    e4 = cos(p*t/2)*cos(s*t/2)-cos(phi)*sin(p*t/2)*sin(s*t/2);
    epsilons = [e1 e2 e3 e4];
end
```