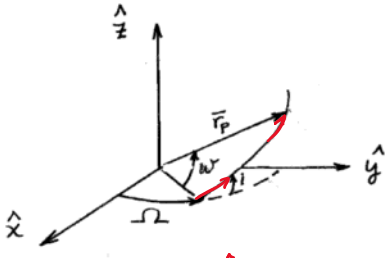
JS_3Dex 1

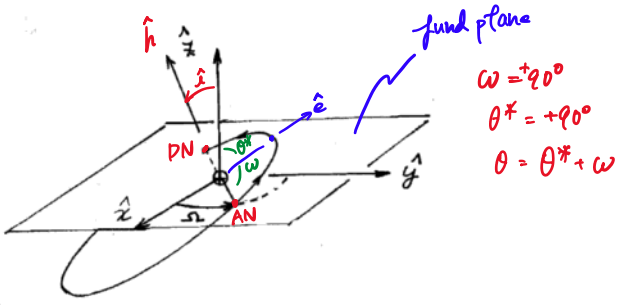
3D Example

Assume s/c moving in orbit about the Earth

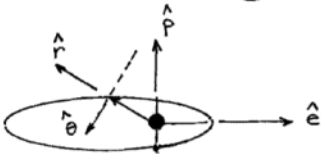
$a = 8R_\oplus$
$e = .7$
$i = 30^\circ$
$\Omega = 60^\circ$
$\omega = 90^\circ$

wrt "Earth centered Mean J2000 coordinates"



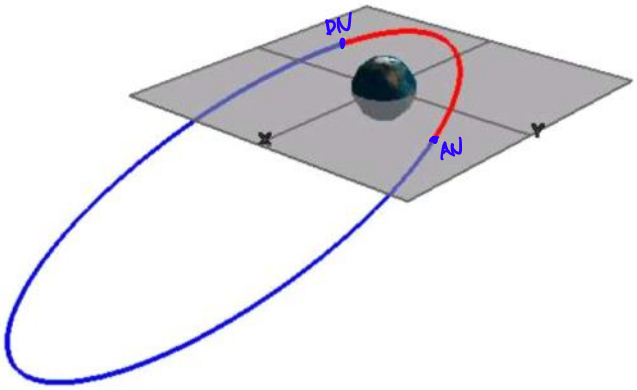


$\omega = +90^\circ$
$\theta^* = +90^\circ$
$\theta = \theta^* + \omega$



maneuver @ $\theta^* = 90^\circ$ (DN)

$r = 26022.8$ km
$V = 4.7777$ km/s
$\gamma = +34.99^\circ$ ascending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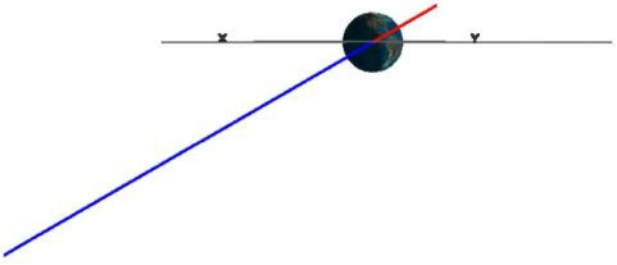
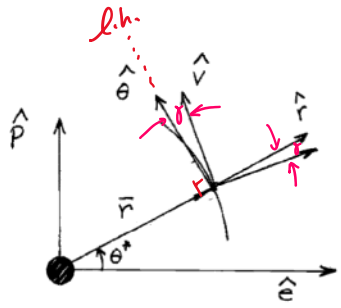


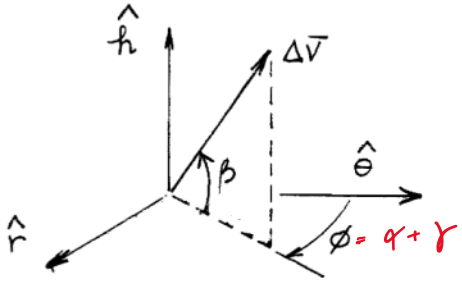


VNB coordinate frame
(useful for describing Δv's and maneuvers)

$\hat{V}$ parallel to velocity; tangent to path
$\hat{N}$ normal; out-of-plane
$\hat{B}$ bi-normal to curve; in plane of motion

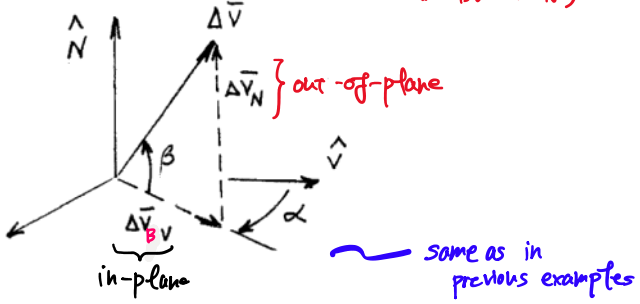
$$\hat{V} = \frac{\bar{v}}{|\bar{v}|}$$
$$\hat{N} = \hat{h} = \frac{\bar{r}\times\bar{v}}{|\bar{r}\times\bar{v}|}$$
$$\hat{B} = \hat{V}\times\hat{N}$$

} representation of maneuver



$$\Delta\bar{v} = \Delta v\,(\cos\beta\cos\varphi\,\hat{\theta} + \cos\beta\sin\varphi\,\hat{r} + \sin\beta\,\hat{h})$$

$$(\Delta v)^2 = (\Delta v_{B\bar{V}}^2 + \Delta v_N^2)$$



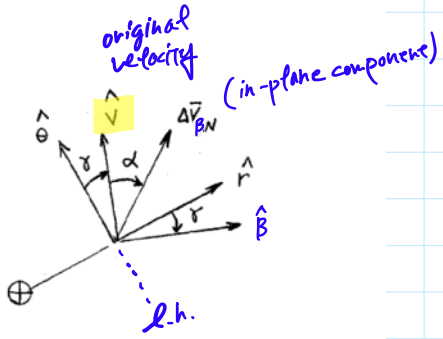
$$\Delta\bar{v} = \Delta v(\cos\beta\cos\varphi\,\hat{V} + \cos\beta\sin\varphi\,\hat{B} + \sin\beta\,\hat{N})$$

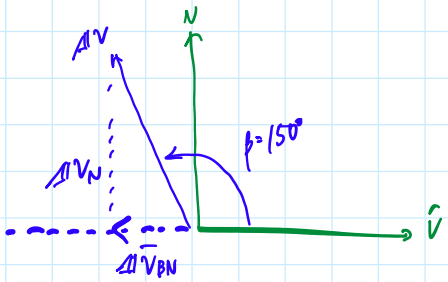
Assume a maneuver such that:

$\Delta v = 2\,\text{km/s} \qquad \alpha = 0^\circ \qquad \beta = 150^\circ$

$\Delta\bar{v} = -1.732\hat{V} + 1.0\hat{N}\ \text{km/s}$

$\varphi = \alpha + \gamma$





	$\hat{r}$	$\hat{\theta}$	$\hat{h}$
$\hat{x}$	$c_\Omega c_\theta - s_\Omega c_i s_\theta$	$-c_\Omega s_\theta - s_\Omega c_i c_\theta$	$s_\Omega s_i$
$\hat{y}$	$s_\Omega c_\theta + c_\Omega c_i s_\theta$	$-s_\Omega s_\theta + c_\Omega c_i c_\theta$	$-c_\Omega s_i$
$\hat{z}$	$s_i s_\theta$	$s_i c_\theta$	c_i

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$i = 30^\circ$
$\Omega = 60^\circ$
$\theta = \omega + \theta^* = 180^\circ$

	$\hat{r}$	$\hat{\theta}$	$\hat{h}$
$\hat{x}$	−.5	.75	.433
$\hat{y}$	−.866	−.433	−.25
$\hat{z}$	0	−.5	.866

$\bar{r} = 26022.80\hat{r}\ \text{km}$

$\bar{r} = -13011.40\hat{x} - 22536.40\hat{y}\ \text{km/s}$ ← Does it make sense (this is inconsistent $\Omega = 0$)

$v = 4.777328\,\text{km/s} \qquad \gamma = 34.992^\circ$

$\bar{v} = 2.739616\hat{r} + 3.91374\hat{\theta}\ \text{km/s}$

$= (-1.36981\hat{x} - 2.37258\hat{y}\ \text{km/s})$

$+ (2.93573\hat{x} - 1.69465\hat{y} - 1.95687\hat{z}\ \text{km/s})$

$\bar{v} = 1.56550\hat{x} - 4.06728\hat{y} - 1.95687\hat{z}\ \text{km/s}$

$\Delta\bar{v}$ ← also in inertial coordinates

$\Delta\bar{v} = -.134551\hat{x} + 1.22457\hat{y} + 1.57548\hat{z}\ \text{km/s}$

$\bar{v}_{new} = \bar{v}_{old} + \Delta\bar{v}$ $\quad \bar{V}^+ = \bar{V}^- + \Delta\bar{V}$ (all vectors in same basis)

$\bar{v}_{new} = 1.4309\hat{x} - 2.8427\hat{y} - 0.3114\hat{z}\ \text{km/s}$

$\bar{r}_{new} = \bar{r}_{old}$ $\quad \bar{r}^+ = \bar{r}^-$

Characteristics of new orbit?

$\hat{h}_{new} = \dfrac{\bar{r}^+ \times \bar{v}^+}{|\bar{r}^+ \times \bar{v}^+|} = 0.12299\hat{x} - 0.7101\hat{y} + 0.8858\hat{z}$

$$\hat{h}_{new} = \frac{\bar{r}^+ \times \bar{v}^+}{|\bar{r}^+ \times \bar{v}^+|} = 0.122989\hat{x} - 0.71101\hat{y} + 0.9848\hat{z}$$

$$\hat{h}^+ = \sin\Omega \sin i\,\hat{x} - \cos\Omega \sin i\,\hat{y} + \cos i\,\hat{z}$$

from where?

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$\cos i = .989864407 \longrightarrow$ $i = 8.158°$

$$\left.\begin{array}{l} \sin\Omega\sin i = .122989 \\ -\cos\Omega\sin i = -.071008 \end{array}\right\} \quad \begin{array}{l} \Omega = 60°, 120° \\ \Omega = \pm 60° \end{array}$$

$\Omega = 60°$ — same?

$|\bar{v}| = 3.206887\ \text{km/s}$

$$\frac{v^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2a} \longrightarrow a^+ = 19576.962\ \text{km}$$

$\bar{r} \bullet \bar{v} = +45476\ \text{km}^2/\text{s} \longrightarrow$ sign on γ is positive

$r\dot{r} = rv\sin\gamma$ $\quad \dot{r} > 0$ $\quad \gamma^+ = +33.014°$

$$e^2 = \left(\frac{rv^2}{\mu} - 1\right)^2 \cos^2\gamma + \sin^2\gamma \longrightarrow e^+ = .610802$$

$$\tan\theta^* = \frac{\left(\frac{rv^2}{\mu}\right)\sin\gamma\cos\gamma}{\left(\frac{rv^2}{\mu}\right)\cos^2\gamma - 1} \longrightarrow \theta^{*+} = -30.14346°,\ 149.88708°$$

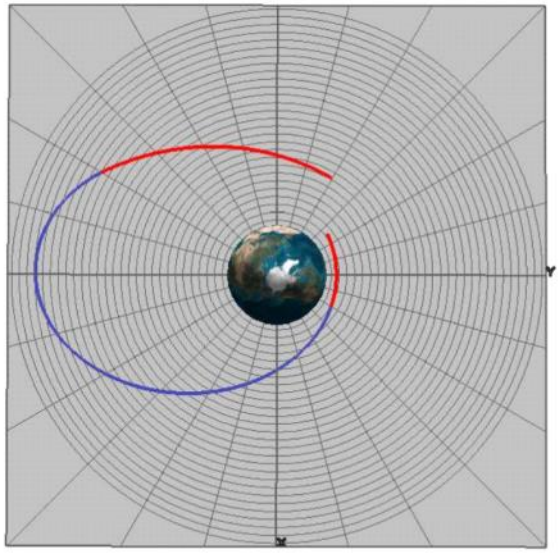
$r > 0$ $\dot{r} > 0$ $\quad \theta^* = 90°$

$\hat{r} \bullet \hat{z} = \sin i \sin\theta = 0 \longrightarrow \theta = 0°, 180°$ — still on node

$\hat{r} \bullet \hat{x} = \cos\Omega\cos\theta - \sin\Omega\cos i\sin\theta$ $\quad \theta = 180°$ still on DN

$\omega_{new} = \theta_{new} - \theta^*_{new} \longrightarrow \omega_{new} = 30.11292°$

$180° - 150°$


Y
Z