October 8, 2020

AAE 564 Fall 2020

Homework Seven

Due: Friday, November 16

Exercise 1 Consider the differential equation

$$\dot{x} = Ax \tag{1}$$

where A is a square matrix. Show that if A has $\jmath 2\pi$ as an eigenvalue, then there is an nonzero initial state x_0 such that (1) has a solution x which satisfies $x(1) = x(0) = x_0$.

Exercise 2 Compute e^{At} at $t = \ln(2)$ for

$$A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

using *all* four methods mentioned in the notes.

Exercise 3 Compute e^{At} for the matrix

$$A = \begin{pmatrix} 3 & -5 \\ -5 & 3 \end{pmatrix}$$

by:

(a) using the eigenvalues and eigenvectors of A;

(b) using the Laplace transform.

Exercise 4 Compute e^{At} at $t = \ln(2)$ for

$$A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

Exercise 5 Obtain (by hand) the state response $x(\cdot)$ of each of the following systems due to a unit impulse input and the zero initial conditions. For each case, determine whether the response contains all the system modes.

a)

$$\begin{aligned} \dot{x}_1 &= -5x_1 + 2x_2 + u \\ \dot{x}_2 &= -12x_1 + 5x_2 + u \end{aligned}$$

b)

$$\begin{aligned}\dot{x}_1 &= -5x_1 + 2x_2 + u\\ \dot{x}_2 &= -12x_1 + 5x_2 + 2u\end{aligned}$$

c)

$$\begin{aligned}\dot{x}_1 &= -5x_1 + 2x_2 + u\\ \dot{x}_2 &= -12x_1 + 5x_2 + 3u\end{aligned}$$

Exercise 6 Consider the system with input u output y and state variables $x_1, \cdots, x_4$ described by

$$\begin{aligned}\dot{x}_1 &= -x_1\\ \dot{x}_2 &= x_1 - 2x_2\\ \dot{x}_3 &= x_1 - 3x_3 + u\\ \dot{x}_4 &= x_1 + x_2 + x_3 + x_4\\ y &= x_3\end{aligned}$$

Obtain (by hand) an expression for the impulse response of this system. Does it contain all the state space modes?

Exercise 7 Consider an LTI system described by

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= -2x_1 - 3x_2 + u\\ y &= 3x_1 - x_2\end{aligned}$$

Is their a persistent input (does not go to zero) u for which the corresponding output always going to zero regardless of initial conditions? If answer is yes provide an example.

Exercise 8 Consider an LTI system described by

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= -2x_1 - 3x_2 + u\\ y &= -x_1 - 3x_2 + u\end{aligned}$$

Is their a persistent input (does not go to zero) u for which the corresponding output always goes to zero regardless of initial conditions? If answer is yes provide an example.

Exercise 9 Disturbing the cart. Consider the pendulum cart system with parameter set $P4$. We will subject it to passive stabilization and a disturbance input w, that is, we let

$$u = -ky - c\dot{y} + w$$

where $k > 0$ and $c > 0$. Regard the resulting system as an input-output system with input w and output y and answer the following questions.

(a) Using MATLAB, obtain the poles and zeros of the system linearized about $E1$.

(b) Consider a sinusoidal disturbance input of the form

$$w(t) = a \sin(\omega t)$$

Choose ω so that the steady state response of the linearized system to this disturbance is zero. Simulate both the nonlinear system and the linearized system with zero initial conditions and this disturbance.