Problem #4

This program aims to calculate the optimal initial mass for the first stage of a two-stage rocket. A plot of initial mass of 1st stage vs velocity ratio 1 will be manipulated.

>> Given Data

- required total velocity $\Delta V = 12\ \frac{\mathrm{km}}{s} = 12000\ \frac{m}{s}$
- payload mass $m_{\text{pay}} = 80\,\mathrm{kg}$
- inert mass ratio $f_{\text{inert}} = \frac{1}{12} \approx\ 0.0833$
- specific impulse for 1st stage $I_{\text{sp1}} = 280\,s$
- specific impulse for 2nd stage $I_{\text{sp2}} = 350\,s$

>> Assumptions

- ideal conditions
- no gravity loss
- perfectly expanded nozzle

>> Equations Used

- $\Delta V = \Delta V_1 + \Delta V_2 \qquad \cdots (1)$
- $\Delta V_1 = f_1 \cdot \Delta V, \qquad \Delta V_2 = f_2 \cdot \Delta V$
- $f_1 + f_2 = 1 \qquad \cdots (2)$
- $$m_{\text{initial,stage1}} = m_{\text{pay}} \cdot (1 - f_{\text{inert}})^n \cdot \prod \left[\frac{e^{\frac{\Delta V_n}{C_n}}}{1 - f_{\text{inert}} \cdot e^{\frac{\Delta V_n}{C_n}}} \right] \cdots (3)$$

$$\text{mass of propellent} = m_{\text{prop}} = m_{\text{pay}} \cdot (1 - f_{\text{inert}})^n \cdot \prod \left[\frac{\left(e^{\frac{\Delta V_n}{C_n}} - 1 \right)}{1 - f_{\text{inert}} \cdot e^{\frac{\Delta V_n}{C_n}}} \right] \cdots (4)$$

where $\quad c = \text{effective exhaust velocity, and } n = \#\text{ of stages}$

or $\quad m_{\text{prop}} = (1 - f_{\text{inert}})(m_{\text{initial}} - m_{\text{pay}}) \qquad \cdots (5)$

from equations (1) - (3) we obtain for a two-stage rocket

$$m_{\text{initial,stage1}}(f_1) = \frac{m_{\text{pay}} \cdot e^{\left(\frac{f_1 \Delta V}{C_1} + \frac{(1-f_1)\Delta V}{C_2}\right)} (1 - f_{\text{inert}})^2}{\left(1 - f_{\text{inert}} \cdot e^{\frac{f_1 \Delta V}{C_1}}\right)\left(1 - f_{\text{inert}} \cdot e^{\frac{(1-f_1)\Delta V}{C_2}}\right)}$$

$$m_{\text{prop,stage1}}(f_1) = \frac{m_{\text{pay}} \cdot \left(1 - e^{\frac{f_1 \Delta V}{C_1}}\right)\left(1 - e^{\frac{(1-f_1)\Delta V}{C_2}}\right)(1 - f_{\text{inert}})^2}{\left(1 - f_{\text{inert}} \cdot e^{\frac{f_1 \Delta V}{C_1}}\right)\left(1 - f_{\text{inert}} \cdot e^{\frac{(1-f_1)\Delta V}{C_2}}\right)}$$

and $c = g_0 \cdot I_{\text{sp}}$ where $g_0 = \text{gravitational accleration} = 9.81\ \frac{m}{s^2}$

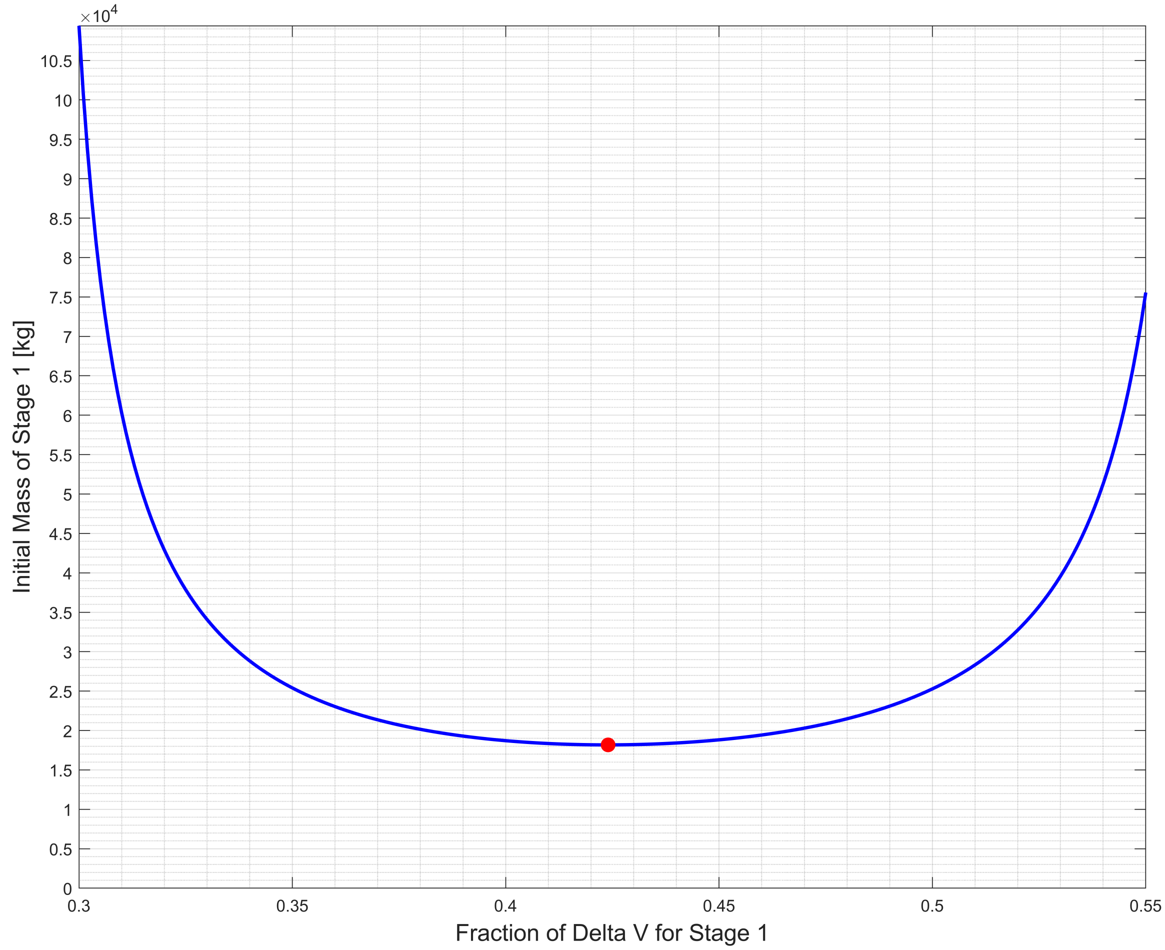
Algorithm

```
% Assigning variables to given values
% Required total velocity [m/s]
V_tot = 12000;
% Payload mass [kg]
m_pay = 80;
% Inert mass ratio
f_inert = 0.0833;
% Specific impulse for stage 1 [s]
Isp1 = 280;
% Spedific impulse for stage 2 [s]
Isp2 = 350;
% Gravitational acceleration [m/s^2]
g_o = 9.81;

% Since f1 has an optimum value interval of [0.3, 0.55] create the array for f1 values as
f1 = 0.3:0.001:0.55;
% Break down the equation to calculate the initial mass of stage 1
% Component 1
comp1 = exp(f1 * V_tot / g_o / Isp1);
% Component 2
comp2 = exp((1 - f1) * V_tot / g_o / Isp2);
% Thus, the initial mass of stage 1 becomes
m_stage1 = m_pay .* comp1 .* comp2 * (1 - f_inert)^2 ./ (1 - f_inert.*comp1)...
    ./ (1 - f_inert.*comp2);

% Finding the minimum mass for stage 1
m_min = min(m_stage1);
% The index of this minimum value
idx = find(m_stage1 == m_min);
% The corresponding f1 value
f1_min = f1(idx);
% Find f2 corresponding to this f1 value
f2_min = 1 - f1_min;
% Find the corresponding mass of propellent using equation (4)
% First manipulate the components to match the f1 with f1_min
comp1_min = exp(f1_min * V_tot / g_o / Isp1);  % Component 1
comp2_min = exp((1 - f1_min) * V_tot / g_o / Isp2); % Component 2
% Mass of propllent is
```

```
m_prop_min =  m_pay * (1 - comp1_min) * (1 - comp2_min) * (1 - f_inert)^2 ...
    / (1 - f_inert*comp1_min) / (1 - f_inert*comp2_min);
```

Plotting

```
% Adjusting fontsize and linewidth
fontsize = 14;
linewidth = 2;

% Plotting commands
figure(1)
plot(f1, m_stage1,'-b','Linewidth',linewidth)
axis([0.3 0.55 0 inf])
yticks(0:5000:110000)
xlabel('Fraction of Delta V for Stage 1', 'FontSize',fontsize)
ylabel('Initial Mass of Stage 1 [kg]', 'FontSize',fontsize)
grid on
grid minor
box on
hold on
plot(f1_min, m_min, '.r', 'MarkerSize',30)
% Control where plot is positioned
set(gcf,'PaperPositionMode','auto','Position',[0 0 1100 850])
```



Results

```
fprintf(fid,['The initial mass of stage 1 is at minimum when f1 = %.3f',...
```

```
The initial mass of stage 1 is at minimum when f1 = 0.424 with a value of 18176.37 kg.
```

```
    'with a value of %.2f kg.'], f1_min, m_min);
fprintf(fid,['At this minimum value f2 = %.3f',...
```

```
At this minimum value f2 = 0.576 and the mass of the propellent is 13278.03 kg.
```

```
    'and the mass of the propellent is %.2f kg.'], f2_min, m_prop_min);
```