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AAE 251: Introduction to Aerospace Design

Assignment 5—Lift and Drag

Due Tuesday, February 26, 10:00 am on Blackboard

Instructions

*Write or type your answers into the appropriate boxes. **Make sure you submit a single PDF on Blackboard.***

Make sure you keep a record of submission receipts or the confirmation emails after each submission as a proof that your submission was accepted.

	Score	Max
Question 1		25
Question 2		4
Question 3		20
Question 4		4
Question 5		6
Question 6		16
TOTAL		75

Question 1

Impressed by your aerodynamics skills, Beechcraft hired you as a summer intern. At the first day of your internship, they ask you to analyze some airfoils for them. You search through their databases and find that the Beechcraft Baron 58 uses the 23012 airfoil, and, conveniently enough, the airfoil is the same from root to tip (many aircraft don't have the same airfoil cross section across the entire wingspan). You make a reasonable assumption of 4 ft for the chord length and you assume that if a prototype is made and tested in a wind tunnel, it will be mounted such that the edges of the wing touch the wind tunnel walls. Calculate the lift, drag, and moment about the quarter-chord per unit span ($span = 1ft$) at standard sea-level conditions for the following airflow velocities, if the angle of attack is 7 degrees. Specify all your answers in Imperial units. Refer to the 'Understanding Imperial Units' handout posted on Blackboard.

a) 100 ft/s

b) 200 ft/s

For this case, what can you say about lift as the velocity is doubled? Can you say the same about drag? If yes, why and if no, why not?

Answer:

GIVEN 23012 airfoil, chord length $\equiv c = 4$ ft
wings span to edge of wind tunnel.
angle of attack $= \alpha = 7^\circ$

FIND

lift per unit span $\equiv l$, drag per unit span $\equiv d$

Moment about quarter-chord per unit span, $\equiv m_{c/4}$

@ standard sea level conditions $\longrightarrow \rho = 0.002377 \text{ slug/ft}^3$
for $v_\infty = 100 \text{ ft/s}$ & $v_\infty = 200 \text{ ft/s}$ $\mu = 3.737 \times 10^{-7} \text{ slug/ft}\cdot\text{s}$

SOLN

(a) when $v_\infty = 100 \text{ ft/s}$

From table on Anderson for 23012 airfoil with $\alpha = 7^\circ$
because the Reynolds # $= Re$ is

$$Re = \frac{\rho v_\infty c}{\mu} = \frac{(0.002377 \text{ slug/ft}^3)(100 \text{ ft/s})(4 \text{ ft})}{(3.737 \times 10^{-7} \text{ slug/ft}\cdot\text{s})} \approx 2.54 \times 10^6$$

the coefficients

$$C_{m_{c/4}} = -0.0122, C_l = 0.89, C_d = 0.082$$

$$\text{dynamic pressure} \equiv q_\infty = \frac{1}{2} \rho v^2 = (0.5)(0.002377 \text{ slug/ft}^3)(100 \text{ ft/s})^2 \\ \approx 11.89 \text{ lb/ft}^2$$

$$l = q_\infty C_l = (11.89 \text{ lb/ft}^2)(4 \text{ ft})(0.89) \approx \boxed{42.33 \text{ lb/ft}}$$

$$d = q_\infty C_d = (11.89 \text{ lb/ft}^2)(4 \text{ ft})(0.082) \approx \boxed{3.900 \text{ lb/ft}}$$

$$m_{c/4} = q_\infty c^2 C_{m_{c/4}} = (11.89 \text{ lb/ft}^2)(4 \text{ ft})^2(-0.0122) \approx \boxed{-2.321 \text{ lb}\cdot\text{ft}^2/\text{ft}}$$

Answer:

(b) when $V_{\infty} = 200 \text{ ft/s}$

similar to (a)

$$Re \approx 5.089 \times 10^6$$

the coefficients become

$$C_{m_{\alpha}} = -0.0122, \quad C_L = 0.86, \quad C_D = 0.07$$

$$q_{\infty} = 47.54 \text{ lb/ft}^2$$

$$L = q_{\infty} C_L = 113.54 \text{ lb/ft}$$

$$D = q_{\infty} C_D = 13.311 \text{ lb/ft}$$

$$m_{\alpha} = q_{\infty} c^2 C_{m_{\alpha}} = -9.280 \text{ lb-ft/ft}$$

Analysis.

When the V_{∞} doubled the lift increased.

The drag increased as well.

this is because from the table the coefficients are rather more influenced by angle of attack than the velocity. Thus, even though the velocity was doubled this did not affect the coefficients much.

Whereas, the dynamic pressure is proportional to the square of velocity which increases the lift and drag.

Question 2

A famous aerial photographer is hoping to buy an aircraft with low stall speed, so she can fly slow enough to take good pictures. One of her options is an aircraft such as the Piper Warrior. Your boss asks you find the stall speed of the Piper Warrior at the maximum gross weight of 9600 N. The wing area is 16 m^2 and the maximum lift coefficient is 1.9 with flaps down. Your boss thinks you can change the design of the Piper to achieve a lower stall speed. What wing area would your aircraft need to reduce the stall speed to 60 km/h ? Assume the new design has the same gross weight and maximum lift coefficient as the Piper Warrior.

Answer:

GIVEN

$$W_{\text{gross}} = 9600 \text{ N}, \text{ wing area} = S = 16 \text{ m}^2, C_{l, \text{max}} = 1.9$$

FIND

① stall speed at initial condition

② wing area S' to make stall speed 60 km/h

SOLN

①

Since

$$V_{\text{stall}} = \sqrt{\frac{2 W_{\text{gross}}}{\rho_{\infty} S C_{l, \text{max}}}}$$

where, $L = W_{\text{gross}}$ & $\rho_{\infty} = 1.23 \text{ kg/m}^3$ standard sea level

$$V_{\text{stall}} = \sqrt{\frac{2 (9600 \text{ N})}{(1.23 \text{ kg/m}^3) (16 \text{ m}^2) (1.9)}}$$

$$\begin{aligned} &\approx 22.66 \text{ m/s} \\ &\approx 81.58 \text{ km/h} \end{aligned}$$

$$\textcircled{2} \quad V_{\text{stall}}' = 60 \text{ km/h} = \left(\frac{60 \text{ km}}{\text{h}} \right) \left(\frac{1000 \text{ m}}{1 \text{ km}} \right) \left(\frac{1 \text{ h}}{3600 \text{ s}} \right) \approx 16.67 \text{ m/s}$$

$$\begin{aligned} S &= \frac{2 W_{\text{gross}}}{\rho_{\infty} V_{\text{stall}}^2 C_{l, \text{max}}} \\ &= \frac{2 (9600 \text{ N})}{(1.23 \text{ kg/m}^3) (16.67 \text{ m/s})^2 (1.9)} \\ &\approx 29.56 \text{ m}^2 \end{aligned}$$

Question 3

The maximum lift-to-drag ratio, (L/D) is an important parameter in airfoil performance. It is a direct measure of aerodynamic efficiency. You want to estimate the maximum L/D of an airfoil that uses a NACA 2412 design at 8 different angles of attack, as shown in the first column of the table. The chord length of the airfoil is 1m.

Fill out the table with the values you obtain from the airfoil data charts.

Plot the variation in L/D with angle of attack. Assume the air pressure is $1.01 \times 10^5 \text{ Pa}$, the temperature is 30 degrees Celsius, and the velocity in the test section is 45 m/s.

Angle of Attack	Coefficient of lift	Coefficient of drag	L/D
-8	-0.6	0.009	-66.67
-4	-0.18	0.0072	-25.00
0	0.25	0.0054	46.30
4	0.65	0.0067	97.01
8	1.08	0.011	98.18

Is there a difference between the values of L/D you calculated and that for a real airplane? Justify your answer.

Answer:

Given

- NACA 2412
- Air pressure = $P = 1.01 \times 10^5$ Pa
- Temperature = 30C
- Free stream velocity = $V = 45$ m/s

Solution

Setup

```
Pres = 1.01*10^5; % [Pa]
Temp = 30; % [C]
Vel = 45; % [m/s]
Chord = 1; % [m]
R_air = 287.05; % [J/KgK] gas constant of air
Visc = 1.789 * 10^(-5); % [Pa-s]

% At this temperature from equation of state
% density
rho = Pres / R_air / (Temp + 273.15);

% The Reynold's # becomes
Re = rho * Vel * Chord / Visc;
```

Re = 2.9195E+06

Tabulated Data

```
% With the Reynold's number and Anderson's table
alpha = [-8 -4 0 4 8]; % Angle of attacks [degree]
l_coeff = [-0.6 -0.18 0.25 0.65 1.08]; % lift coefficients
d_coeff = [0.009 0.0072 0.0054 0.0067 0.011]; % drag coefficients
```

Because

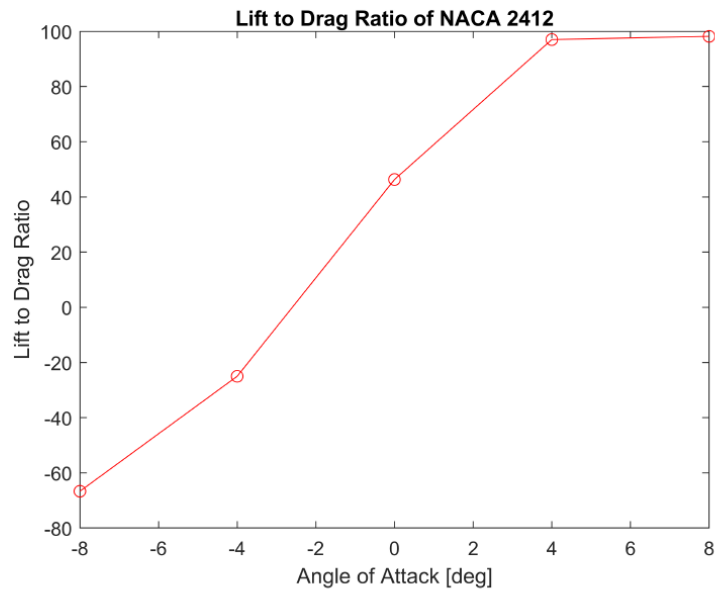
$$\frac{L}{D} = \frac{C_L}{C_D}$$

```
L_D = l_coeff ./ d_coeff; % lift drag ratio
```

Answer:

Plot

```
figure(1)
plot(alpha, L_D, "-or")
title('Lift to Drag Ratio of NACA 2412')
xlabel('Angle of Attack [deg]')
ylabel('Lift to Drag Ratio')
```



For the real airplane the wings are going to become finite wings which will alter the drag. The drag when it is a finite wing will be separated into two parts the parasitic and induced drag which in total make the total drag of the aircraft. Since this occurs the lift to drag ratio will differ from the graph that I have plotted above with the assumption of the wing being a infinite wing in a wind tunnel.

Question 4

Consider a NACA 1412 airfoil with a 2m chord in an airstream with a velocity of 50m/s at standard sea-level conditions. If the lift per unit span is 1353 N, what is the angle of attack?

Answer:

GIVEN

NACA 1412 airfoil, chord = $c = 2\text{m}$

$V = 50\text{ m/s}$ lift per unit span = $l = 1353\text{ N/m}$
sea level cond

FIND

α

SOLN

$$\rho_{\infty} = 1.225\text{ kg/m}^3$$

$$\mu = 1.789 \times 10^{-5}\text{ Pa}\cdot\text{s}$$

$$\text{Reynold's } \# = Re = \frac{\rho_{\infty} V c}{\mu} = \frac{(1.225\text{ kg/m}^3)(50\text{ m/s})(2\text{ m})}{(1.789 \times 10^{-5}\text{ Pa}\cdot\text{s})}$$
$$\approx 6.85 \times 10^6$$

$$l = \frac{1}{2} \rho_{\infty} V^2 c C_L$$

$$\therefore \text{lift coefficient} = C_L = \frac{2l}{\rho_{\infty} V^2 c} = \frac{2(1353\text{ N/m})}{(1.225\text{ kg/m}^3)(50\text{ m/s})^2(2\text{ m})}$$
$$\approx 0.442$$

Now from Anderson's table for NACA 1412

approximately

$$\alpha = 2.5\text{ degrees}$$

Question 5

The sizing expert in your team estimates that your revolutionary aircraft will need a wing area of 15 m^2 to handle an expected loading of 9500 N. The aspect ratio will be 7.2, and the span efficiency factor 0.63. You, the aerodynamic expert, are asked to calculate the induced drag if the aircraft flies at standard sea-level conditions with a velocity of 250 km/h. Assume it is in level flight.

Answer:

GIVEN

$$\text{wing area} = S = 15 \text{ m}^2, W_{\text{load}} = 9500 \text{ N}$$

$$AR = 7.2, \text{ span efficiency factor} = e = 0.63$$

$$\text{sea level cond velocity} = V = 250 \text{ km/h} \approx 69.44 \text{ m/s}$$

level flight

FIND

induced drag, P_i

SOLN

$$\rho_0 = 1.225 \text{ kg/m}^3$$

$$\text{at level flight } L = W_{\text{load}}$$

so

$$W_{\text{load}} = \frac{1}{2} \rho_0 V^2 S C_L$$

$$C_L = \frac{2 W_{\text{load}}}{\rho_0 V^2 S} = \frac{2(9500 \text{ N})}{(1.225 \text{ kg/m}^3)(69.44 \text{ m/s})^2(15 \text{ m}^2)} \approx 0.214$$

now that we have the lift coefficient find induced drag coefficient

$$C_{D,i} = \frac{C_L^2}{\pi e AR} = \frac{(0.214)^2}{\pi(0.63)(7.2)} \approx 3.214 \times 10^{-3}$$

$$P_i = \frac{1}{2} \rho_0 V^2 S C_{D,i} = \frac{1}{2} (1.225 \text{ kg/m}^3)(69.44 \text{ m/s})^2 (15 \text{ m}^2)(3.214 \times 10^{-3})$$

$$\approx 142.4 \text{ N}$$

$$P_i = 142.4 \text{ N}$$

Question 6

You have a finite rectangular wing with a wing span of 10m and chord length of 2m. The airfoil is a NACA 2415 airfoil section. The span efficiency factor is 0.85. The angle of attack is 6 degrees, and the airflow velocity is 40m/s. Assume sea level conditions.

Your goal is to calculate the total drag on the airfoil. To do so, calculate

- a) Lift curve slope for the infinite wing
- b) Parasitic drag coefficient (c_d)
- c) Lift curve slope for the finite wing
- d) Coefficient of lift for the finite wing at the given angle of attack
- e) Total drag coefficient (C_D)
- f) Total drag acting on the airfoil

Show your work for all the subparts of the question.

Answer:

GIVEN finite rectangular wing (NACA 2415)

\gg wing span $= b = 10 \text{ m}$

\gg chord length $= c = 2 \text{ m}$

\gg span efficiency factor $= e = 0.85$

\gg angle of attack $= \alpha = 6^\circ$

\gg air flow velocity $= V_\infty = 40 \text{ m/s}$

\gg sea level

$\rho = 1.225 \text{ kg/m}^3$

$\mu = 1.789 \times 10^{-5} \text{ Pa}\cdot\text{s}$

FIND Total drag

from Anderson's table

when Reynolds $Re = \frac{\rho V_\infty c}{\mu} = \frac{(1.225 \text{ kg/m}^3)(40 \text{ m/s})(2 \text{ m})}{(1.789 \times 10^{-5} \text{ Pa}\cdot\text{s})}$

$= 5.48 \times 10^6$

aspect ratio $= AR = \frac{b^2}{bc} = \frac{(10 \text{ m})^2}{(10 \text{ m})(2 \text{ m})} = 5$

(a) lift curve slope for infinite wing, a_0

from table at $Re = 5.48 \times 10^6$

$C_L = 0.20$ at $\alpha_{crit} = 0^\circ$, $C_L = 1.20$ at $\alpha_{crit} = 10^\circ$

$a_0 = \frac{dC_L}{d\alpha} = \frac{1.20 - 0.20}{10 - 0} = 0.10 \text{ per degree}$

(b) from Anderson's table at $\alpha = 6^\circ$

drag coefficient $= C_d = 0.008$

Answer:

(c) lift curve slope for finite wing, a

$$a = \frac{a_0}{1 + \frac{a_0}{\pi e AR}} = \frac{0.101}{1 + \frac{0.101}{\pi(0.85)(5)}} \approx 0.09926 \text{ per degree}$$

(d) coefficient of lift of finite wing C_L

from table: $\alpha_{L=0} = -2.0^\circ$ @ $\alpha = 6.0^\circ$

$$\therefore C_L = a(\alpha - \alpha_{L=0}) = 0.09926[6.0 - (-2.0)] = 0.7941$$

(e) Total drag coefficient, C_D

$$C_D = C_{D0} + \frac{C_L^2}{\pi e AR}$$
$$= 0.008 + \frac{0.7941^2}{\pi(0.85)(5)}$$

$$= 0.05523$$

(f) Total

$$D_{\text{total}} = \frac{1}{2} \rho V_\infty^2 S C_D$$
$$= \frac{1}{2} (1.225 \text{ kg/m}^3) (40 \text{ m/s})^2 (20 \text{ m}^2) (0.05523)$$
$$= 1082.508 \text{ N}$$

$$D_{\text{total}} = 1.083 \times 10^3 \text{ N}$$