AAE 666

Homework Nine

Exercise 1 Obtain the describing function of the nonlinear function

$$\phi(y) = y^5$$

Exercise 2 Determine whether or not the following Duffing system has a periodic solution. Determine the approximate amplitude and period of all periodic solutions.

$$\ddot{y} - y + y^3 = 0$$

Exercise 3 Determine whether or not the following system has a periodic solution. Determine the approximate amplitude and period of all periodic solutions.

$$\ddot{y} + \mu(\dot{y}^3/3 - \dot{y}) + y = 0$$

where μ is a positive real number.

Exercise 4 Use the describing function method to predict period solutions to

$$\dot{x}(t) = -x(t) - 2\,\mathrm{sgm}\,(x(t-h))$$

Illustrate your results with numerical simulations.

Exercise 5 Consider the double integrator

$$\ddot{q} = u$$

subject to a saturating PID controller

$$u = -k_P q - k_D \dot{q} - \mathrm{sat}(\tilde{u}) \qquad \text{where} \qquad \tilde{u} = k_I \int q$$

(a) For $k_P = 1$ and $k_D = 2$ determine the largest of $k_I \geq 0$ for which the closed loop system is asymptotically stable about $q(t) \equiv 0$.

(b) For $k_P = 1$ and $k_D = 2$, use the describing function method to determine the smallest value $k_I \geq 0$ for which the closed loop system has a periodic solution.