AAE 532 – Orbit Mechanics
Problem Set 5
Due: 10/9/20

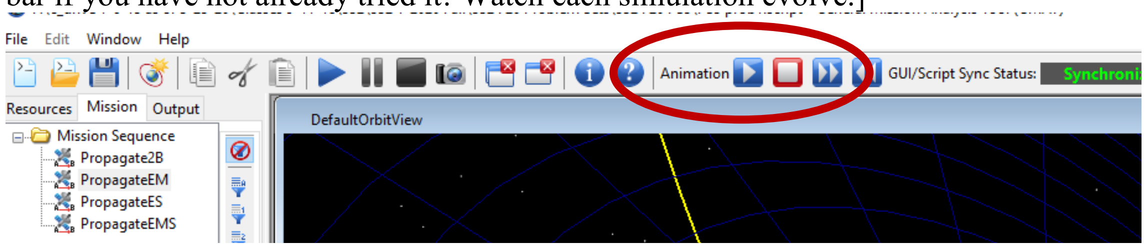
[Another short document is posted under GMAT Tips. This one is labelled ‘Propagation Relative to a Different Central Body’. We will use it in future problem sets!]

Problem 1: Return to the use of multiple propagators in GMAT. (Demonstrated in a previous GMAT Tip.) The propagator ‘TwoBody’ or ‘EarthPointMass’ is already available (under the name you have selected for previous assignments). Produce the new propagators: ‘EarthMoon’, ‘EarthSun’, ‘EarthMoonSun’. Add the different forces to create the new propagators as described in the GMAT Tips document. Add the Moon’s orbit (Luna) to the output image. Propagate for 60 days.

(a) Use a start date October 2, 2020 16:00:00. Use the Earth J2000Eq coordinates throughout the simulations. In a Keplerian Coordinate Type, introduce initial conditions such that

$$r_p = 1.5R_\oplus \qquad \Omega = 0^\circ$$
$$r_a = 200R_\oplus \qquad \theta^* = 0^\circ$$
$$\omega = 0^\circ$$
$$i = 30^\circ$$

Explore the 4 propagators (use a different color for each propagated path). Propagate all the trajectories for 60 days. [Sometimes it is convenient to use the ‘Animation’ button on the top bar if you have not already tried it! Watch each simulation evolve.]



Produce a plot with a view approximately down the Moon Orbit Normal with all four spacecraft. Add views on two other dates: October 7, 2020 and October 11, 2020 at the same time of day. Choose another date in October and add a figure.

These simulations all use the relative vector equation of motion for the spacecraft relative to the Earth from **Notes Page D2**; the perturbations on the right-hand side of the equation vary for each propagator.
Does the model make a difference? Is the two-body model adequate for this particular problem? Why or why not? For the trajectory in this analysis, which relative orbit model would you recommend: two-body, three-body, four-body? Why? Which bodies would you include?

What is the impact of the different epoch dates? Why is there such a difference in the paths?

(b) Output some information for each spacecraft at $t = t_f$, the end of the propagation. Determine the following information from the GMAT output: $a, e, r_p, \mathcal{E}, h; r_f, v_f, \theta_f^*, \gamma_f$. Compare the closest approach altitude for all the spacecraft at the end of the simulation. Are any spacecraft in danger of Earth impact? Which perturbation reduced the r_p? Does it occur at all starting epochs?
(Note that, if the model is not a true conic – as is the case for three of the four propagators – GMAT computes instantaneous values of these quantities. Hint: check the output at the end of the final Propagate segment.)

Problem 2: A spacecraft is in orbit about Mars and is characterized such that $r_p = 1.5R_{♂}$ and $r_a = 6.5R_{♂}$. The vehicle is currently located such that $M = -90^\circ$.

(a) Determine the following orbit parameters and spacecraft state information:

$$a,\ e,\ p,\ h,\ ,\ period,\ \mathcal{E};\ r,\ v,\ \theta^*, E,\ \gamma,\ (t - t_p)$$

(b) Write $\bar{r}_o$ and $\bar{v}_o$ in terms of components in the directions of $\hat{\boldsymbol{e}}$ and $\hat{\boldsymbol{p}}$.

(c) Determine θ^* after a time equal to 50% of the period, i.e., $\Delta t = 0.5IP$. Use f and g relationships to write $\bar{r}, \bar{v}$ in terms of $\bar{r}_o$, $\bar{v}_o$. Prove that $f(\theta^* - \theta_o^*), g(\theta^* - \theta_o^*)$ produce the same results as $f(E - E_o), g(E - E_o)$.

(d) Plot the orbit with your Matlab script. By hand, mark on the plot where the spacecraft is currently located by marking $\hat{r}, \hat{\theta}, \bar{r}_o, \theta_o^*$; also sketch the local horizon, $\bar{v}_o$, and γ_o. Do the same at the second location. Identify the arc from t_o to t?

Problem 3: Assume that a vehicle is currently moving in Earth orbit. Assume a two-body relative motion of the spacecraft with respect to Earth. The orbit is described with the following characteristics (relative to an Earth equatorial coordinate system) at the initial time t_1:

$$a = 20\ R_\oplus \qquad \Omega = 45^\circ$$
$$e = .6 \qquad \omega = 30^\circ$$
$$i = 34^\circ \qquad \theta = 235^\circ$$

(a) Determine the current state in terms of $\bar{r}, \bar{v}, r, v, \gamma, \theta^*, period, M, E, \left(t - t_p\right)$; write $\bar{r}, \bar{v}$ in terms both rotating orbit unit vectors ($\hat{r}, \hat{\theta}, \hat{h}$), unit vectors ($\hat{n}_x, \hat{n}_y, \hat{n}_z$) as well as inertial unit vectors ($\hat{x}, \hat{y}, \hat{z}$).

(b) Confirm the general results in GMAT with the conic propagator. Plot the GMAT image viewing down onto the orbit plane.

(c) Use Kepler's equation and determine the values of $\bar{r}, \bar{v}, \theta^*, \gamma$ in exactly 3 days, i.e., time t_2. For this value of ($t_2 - t_1$), what are the corresponding values of $(\theta_2^* - \theta_1^*)$, $(E_2 - E_1)$. Confirm the result in GMAT.

(d) Plot the orbit in Matlab or GMAT. Mark $\bar{r}, \bar{v}$ at the two times; mark the usual quantities (vectors, local horizon, γ, θ^*) and highlight the arc between the two times.

Problem 4: A vehicle is moving in some Earth orbit; assume a two-body model. At a certain time, the following information is given

$$\bar{r}_1 = 0.15\ R_\oplus \hat{x} - 1.44\ R_\oplus \hat{y} - 0.65\ R_\oplus \hat{z}$$
$$\bar{v}_1 = 6.62\hat{x} + 2.7\hat{y} - 1.56\hat{z} \text{ km/s}$$

(a) Determine $a, e, i, \omega, \Omega, \gamma, \theta^*, M, E, \left(t - t_p\right)$. Are you sure it is an ellipse? Why? What quantity do you check to assess the type of conic?

(b) Sketch the orbit in the orbit plane: add r, v, θ^*, λ, local horizon, $\omega, \hat{n}_x$.

(c) Sketch the orbit in 3D (or a section of the orbit) to mark the following quantities: $\Omega, i, \hat{h}$, AN (Ascending Node), DN (Descending Node), direction of motion.
Is periapsis above or below the fundamental plane? How do you know?
What is θ^* at the AN? DN?