September 18, 2020

AAE 564 Fall 2020

Homework Four

Due: Friday, September 25

Exercise 1 Determine (by hand) whether or not the following system of linear equations has a solution. If a solution exists, determine whether or not it is unique, and if not unique, obtain an expression for all solutions.

$$\begin{array}{rcrcrcrcr} x_1 & - & x_2 & + & 2x_3 & + & x_4 & = & 5 \\ x_1 & - & x_2 & + & x_3 & & & = & 3 \\ -2x_1 & + & 2x_2 & & & + & 2x_4 & = & -2 \\ 2x_1 & - & 2x_2 & - & x_3 & - & 3x_4 & = & 0 \end{array}$$

Exercise 2 We consider here systems of linear equations of the form

$$Ax = b$$

$$\text{with} \qquad x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}.$$

For each of the following cases, we present the reduced row echelon form of the augmented matrix $[A\,b]$. Determine (by hand) whether or not the corresponding system of linear equations has a solution. If a solution exists, determine whether or not it is unique; if not unique, obtain an expression for all solutions and give two solutions.

$$\text{(a)} \begin{pmatrix} 1 & 5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{(b)} \begin{pmatrix} 1 & -2 & 0 & 6 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{(c)} \begin{pmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{(d)} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Exercise 3 (By hand) Consider the three vectors

$$\mathbf{v_1} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \qquad \mathbf{v_2} = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} \qquad \mathbf{v_3} = \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix}.$$

(a) Express the vector

$$\mathbf{v} = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}$$

as a linear combination of $\mathbf{v_1}$, $\mathbf{v_2}$ and $\mathbf{v_3}$.

(b) Can every vector of the form

$$\mathbf{v} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

be expressed as a linear combination of $\mathbf{v_1}$, $\mathbf{v_2}$ and $\mathbf{v_3}$?

(c) Are the vectors $\mathbf{v_1}$, $\mathbf{v_2}$, $\mathbf{v_3}$ linearly independent?

Exercise 4 (By hand)

(a) Find a basis for the null space of the matrix,

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \end{pmatrix}.$$

(b) What is the nullity of A?

Check your answer using MATLAB.

Exercise 5 (By hand)

(a) Find a basis for the range of the matrix,

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \end{pmatrix}.$$

(b) What is the rank of A?

Check your answer using MATLAB.

Exercise 6 Consider a 5×5 matrix

$$A = vw^T$$

where v and w are 5×1 matrices.
(a) What is the rank of A?
(b) What is the nullity of A?

Exercise 7 Suppose $\mathcal{U}$ and $\mathcal{W}$ are subspaces of a vector space $\mathcal{V}$. Prove or disprove (by counterexample) the following statements.

(a) $\mathcal{U} \cap \mathcal{W}$ is a subspace of $\mathcal{V}$.

(b) $\mathcal{U} \cup \mathcal{W}$ is a subspace of $\mathcal{V}$.