AAE 440: Spacecraft Attitude Dynamics

PS9*

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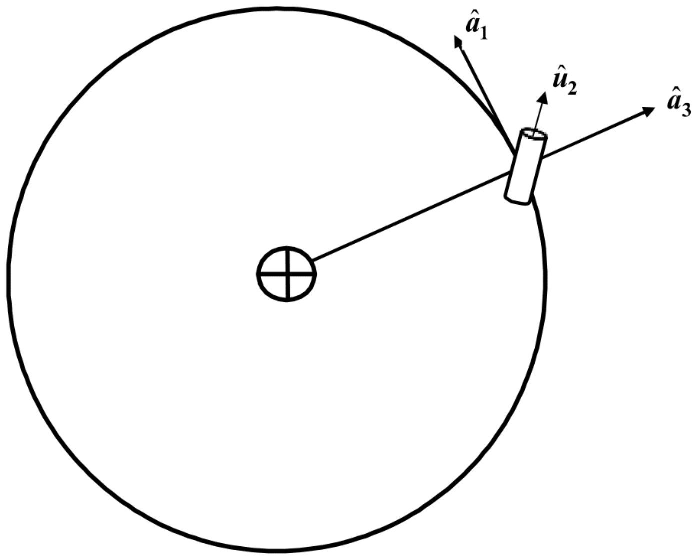
Spacecraft Attitude Dynamics
Problem Set 9*
Due: 4/10/20

Recall from PS8* that the axisymmetric rigid body U (spacecraft) can move in an inertial reference frame N in a circular orbit. Let $\hat{n}_i$ and $\hat{u}_i$ be unit vectors fixed in N and U, respectively. Assume again

$$\bar{\bar{I}}^{U/U^*} = 400\hat{u}_1\hat{u}_1 + 100\hat{u}_2\hat{u}_2 + 400\hat{u}_3\hat{u}_3 \ \text{ kg-met}^2$$

The mass of the vehicle is 200 kg. Assume that the spacecraft moves in a circular Earth orbit at a constant rate Ω with respect to N. Define an orbit-fixed frame A such that $\hat{a}_3$ is directed radially outward from the Earth toward U*, $\hat{a}_1$ is 90° from $\hat{a}_3$ and in the direction of motion. Then, $\hat{a}_2$ is parallel to orbital angular momentum and ${}^N\bar{\omega}^A = \Omega\,\hat{a}_2$.



Recall that we introduced an intermediate frame C is introduced such that $\hat{c}_2 = \hat{u}_2$ at all times. Define the measure numbers such that

$$\left|{}^C\bar{\omega}^U\right| = q \qquad \text{and} \qquad {}^N\bar{\omega}^U = \omega_i\hat{c}_i$$

Note that we will use the differential equations that you derived in PS8*. Check the solution and be sure that the equations that you derived are correct.

Problem 1: The nominal motion of interest is a constant spin of the spacecraft in N about an axis parallel to the orbit normal, i.e., $\left|{}^N\bar{\omega}^U\right| = k\Omega$ where k is a constant.

(a) If s is selected properly, the unit vectors $\hat{c}_i$ remain fixed relative to the orbit during the nominal motion. What choice of s is required? Why?
Given the proper choice of s, what is the resulting particular solution of the nonlinear differential equations that corresponds to the nominal motion?
Note that this is a constant solution. Demonstrate that all the differential equations are satisfied.

If the nominal motion of interest is a const. spin of the S/C in N about an axis parallel to the orbit normal

$$\omega_2 = k\Omega \quad , \quad \omega_1 = \omega_3 = 0$$

In the previous Problem Set, PS8* we have derived the following KDE (Kinematic Differential Equations)

$$2\dot{\varepsilon}_1 = \cancel{\omega_1\varepsilon_4} - (\omega_2 + \Omega - \dot{q})\varepsilon_3 + \cancel{\omega_3\varepsilon_2}$$
$$2\dot{\varepsilon}_2 = \cancel{\omega_1\varepsilon_3} + (\omega_2 - \Omega - \dot{q})\varepsilon_4 - \cancel{\omega_3\varepsilon_1}$$
$$2\dot{\varepsilon}_3 = -\cancel{\omega_1\varepsilon_2} + (\omega_2 + \Omega - \dot{q})\varepsilon_1 + \cancel{\omega_3\varepsilon_4}$$
$$2\dot{\varepsilon}_4 = -\cancel{\omega_1\varepsilon_1} - (\omega_2 - \Omega - \dot{q})\varepsilon_2 - \cancel{\omega_3\varepsilon_3}$$

$$\downarrow$$

$$2\dot{\varepsilon}_1 = -(\omega_2 + \Omega - \dot{q})\varepsilon_3$$
$$2\dot{\varepsilon}_2 = (\omega_2 - \Omega - \dot{q})\varepsilon_4$$
$$2\dot{\varepsilon}_3 = (\omega_2 + \Omega - \dot{q})\varepsilon_1$$
$$2\dot{\varepsilon}_4 = -(\omega_2 - \Omega - \dot{q})\varepsilon_2$$

for the KDEs to yield a const. soln. $\rightarrow$ $\dot{\varepsilon} = 0$ & $\dot{\omega} = 0$

since all $\varepsilon_i \neq 0$ $\because$ $\varepsilon_1^2 + \varepsilon_2^2 + \varepsilon_3^2 + \varepsilon_4^2 = 1$

thus, the probable condition is

some $\varepsilon_i = 0$ AND $\omega_2 + \Omega - \dot{q} = 0$ OR $\omega_2 - \Omega - \dot{q} = 0$

Option #1

requisite: $\omega_{20} + \Omega - q_1 = 0$

to satisfy this

$$q_1 = \omega_{20} + \Omega$$

Now, assuming $\hat{c}_2 = \hat{a}_2 = \hat{u}_2$

$$\begin{aligned}
{}^{N}\bar{\omega}^{U} &= {}^{N}\bar{\omega}^{A} + {}^{A}\bar{\omega}^{C} + {}^{C}\bar{\omega}^{U} \\
\omega_{20}\,\hat{a}_2 &= \Omega\hat{a}_2 + {}^{A}\bar{\omega}^{C} + q_1\hat{c}_2 \\
\omega_{20}\,\hat{a}_2 &= \Omega\hat{a}_2 + {}^{A}\bar{\omega}^{C} + (\omega_{20}+\Omega)\hat{a}_2 \\
{}^{A}\bar{\omega}^{C} &= -2\Omega\hat{a}_2
\end{aligned}$$

Then, the KDEs become

$$\begin{aligned}
2\dot{\varepsilon}_1 &= 0 \\
2\dot{\varepsilon}_2 &= \varepsilon_4(-2\Omega) \\
2\dot{\varepsilon}_3 &= 0 \\
2\dot{\varepsilon}_4 &= \varepsilon_2(2\Omega)
\end{aligned}$$

These can still be a particular soln. but the dependent variables are not const.

Option #2

requisite: $\omega_{20} - \Omega - q_1 = 0$

to satisfy this

$$q_1 = \omega_{20} - \Omega$$

Now, assuming $\hat{c}_2 = \hat{a}_2 = \hat{u}_2$

$$\begin{aligned}
{}^{N}\bar{\omega}^{U} &= {}^{N}\bar{\omega}^{A} + {}^{A}\bar{\omega}^{C} + {}^{C}\bar{\omega}^{U} \\
\omega_{20}\,\hat{a}_2 &= \Omega\hat{a}_2 + {}^{A}\bar{\omega}^{C} + q_1\hat{c}_2
\end{aligned}$$

$$\omega_{20}\hat{a}_2 = \Omega\hat{a}_2 + {}^{A}\bar{\omega}^{C} + (\omega_{20}-\Omega)\hat{a}_2$$
$${}^{A}\bar{\omega}^{C} = \bar{0}$$

OR

$$\hat{c}_i = \hat{a}_i \quad \text{where Euler angle} := \theta = 0$$

is also a possibility. Which means

$${}^{A}\bar{\varepsilon}^{C} = \hat{\lambda}\sin\frac{\theta}{2} = 0 \quad , \quad {}^{A}\varepsilon_4^{C} = \cos\frac{\theta}{2} = 1$$
$$\therefore \ \varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 0 \ \text{ AND } \ \varepsilon_4 = 1$$

Thus, option #2 is what we should choose

Whereby, the motion of interest satisfies

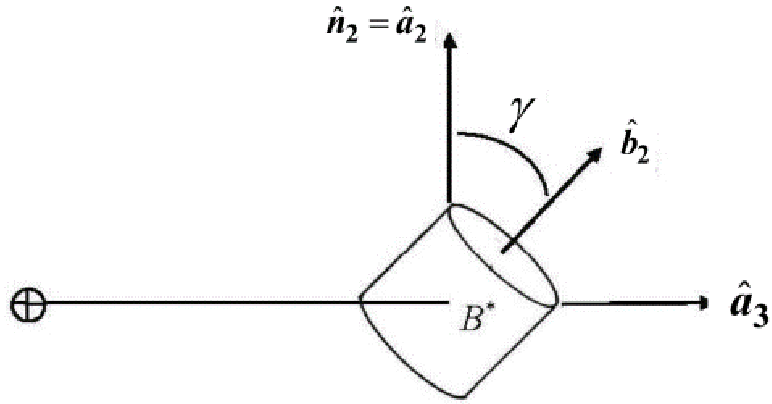
$$\boxed{\begin{array}{l} \bullet \ q = \omega_{20} - \Omega \\ \bullet \ \omega_1 = \omega_3 = 0 \ \text{ AND } \ \omega_2 = \omega_{20} \\ \bullet \ \varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 0 \ \text{ AND } \ \varepsilon_4 = 1 \end{array}}$$

Check the DDEs (Dynamic Differential Equations) AND KDEs

$$\dot{\omega}_1 = -K\omega_2\cancelto{0}{\omega_3} + q\cancelto{0}{\omega_3} + 6\Omega^2K(\cancelto{0}{\varepsilon_2\varepsilon_3} + \cancelto{0}{\varepsilon_1\varepsilon_4})(1 - \cancelto{0}{2\varepsilon_1^2} - \cancelto{0}{2\varepsilon_2^2})$$
$$\dot{\omega}_2 = 0$$
$$\dot{\omega}_3 = K\cancelto{0}{\omega_1}\omega_2 - q\cancelto{0}{\omega_1} - 12\Omega^2K(\cancelto{0}{\varepsilon_3\varepsilon_1} - \cancelto{0}{\varepsilon_2\varepsilon_4})(\cancelto{0}{\varepsilon_2\varepsilon_3} + \cancelto{0}{\varepsilon_1\varepsilon_4})$$
$$2\dot{\varepsilon}_1 = \cancelto{0}{\omega_1}\varepsilon_4 - (\omega_2+\Omega-q)\cancelto{0}{\varepsilon_3} + \cancelto{0}{\omega_3}\varepsilon_2$$
$$2\dot{\varepsilon}_2 = \cancelto{0}{\omega_1}\varepsilon_3 + \cancelto{0}{(\omega_2-\Omega-q)}\varepsilon_4 - \cancelto{0}{\omega_3\varepsilon_1}$$
$$2\dot{\varepsilon}_3 = -\cancelto{0}{\omega_1\varepsilon_2} + (\omega_2+\Omega-q)\cancelto{0}{\varepsilon_1} + \cancelto{0}{\omega_3\varepsilon_4}$$
$$2\dot{\varepsilon}_4 = -\cancelto{0}{\omega_1\varepsilon_1} - \cancelto{0}{(\omega_2-\Omega-q)}\varepsilon_2 - \cancelto{0}{\omega_3\varepsilon_3}$$

$$\boxed{\text{All } \dot{\varepsilon}_i = 0 \text{ AND } \dot{\omega}_i = 0 \text{ are sufficed, hence all DE are sufficed.}}$$

(b) Assume an initial perturbation in the orientation. There are no perturbations in the angular velocity. Model the initial orientation as follows.

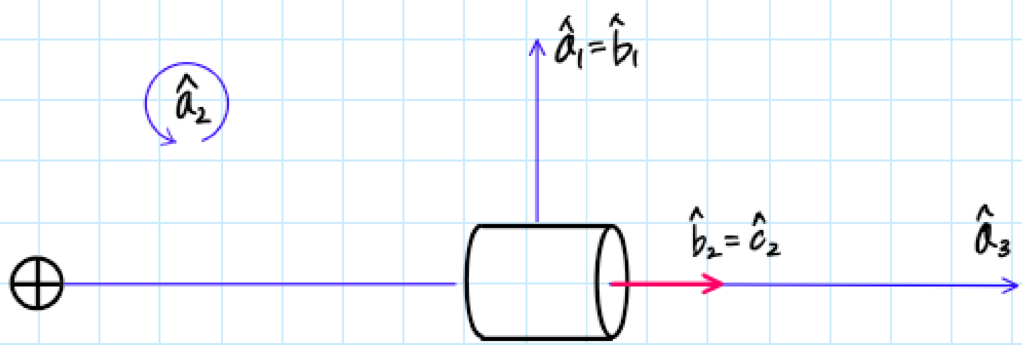


The differential equations are numerically integrated to produce a time history for the nutation angle γ. Why do you compute this angle using a body 2-1-2 angle sequence? Where are the unit vectors $\hat{a}_1, \hat{b}_1$ in the sketch? Use them to justify this choice of angle sequence.

Note : Body-frame = $\hat{b}_i$

We compute the nutation angle γ using a body-two 2-1-2 sequence because the rotations are about the following axes in the specific order

$$\hat{c}_2 \rightarrow \hat{c}_1 \rightarrow \hat{c}_2$$



(c) Now numerically integrate the non-dimensional differential equations. At each step in the integration process, include the Euler parameter constraint K, that is, evaluate the constant $K=\left[\varepsilon_1^2+\varepsilon_2^2+\varepsilon_3^2+\varepsilon_4^2\right]^{\frac{1}{2}}$. Integrate given $\gamma(0)=\gamma_o$; plot γ as a of ν (number of revs). Integrate for 4 revs and complete the following simulations:

(i) Confirm that if $\gamma(0)=0^{\circ}$, all the Euler parameters and $\gamma(t)$ are constant.

Integrate given $\gamma(0)=8^{\circ}$; for k = -1, 15. Plot the result $\Delta K = K - K(0)$ as a function of ν. What is the accuracy of the integration? Is it approximately the same for both values of k? Integrate again for $\gamma(0)=4^{\circ}$. For each value of k, plot both initial value curves on the same plot. Estimate the period (number of orbits) for one cycle of the angle γ. Does the general behavior change for different values of $\gamma(0)$?

Discuss your conclusions concerning 'stability' of the particular solution given these numerical results.

Simulation

∇ ICs

(1) nondimensional angular velocity measure numbers

$$w_0 = \begin{bmatrix} 0 & k & 0 \end{bmatrix}$$

where k := spin factor

(2) Euler parameters

$$\varepsilon_0 = \begin{bmatrix} \sin\frac{\gamma_0}{2} & 0 & 0 & \cos\frac{\gamma_0}{2} \end{bmatrix}$$

where γ_0 := initial perturbation

∇ Cases to Simulate

(1) $k=-1$, $\gamma_0 = 0^{\circ}, 8^{\circ}, 4^{\circ}$

(2) $k=15$, $\gamma_0 = 0^{\circ}, 8^{\circ}, 4^{\circ}$

∇ Nutation Angle, γ

In this body-two 2-1-2 sequence, from the DCM

Body-two: 2-1-2

	$\mathbf{b}_1$	$\mathbf{b}_2$	$\mathbf{b}_3$
$\mathbf{a}_1$	$-s_1c_2s_3 + c_3c_1$	s_1s_2	$s_1c_2c_3 + s_3c_1$
$\mathbf{a}_2$	s_2s_3	c_2	$-s_2c_3$
$\mathbf{a}_3$	$-c_1c_2s_3 - c_3s_1$	c_1s_2	$c_1c_2c_3 - s_3s_1$

from supplemental document

$$C_{22} = \cos\theta_2 \quad \text{where } \theta_2 = \gamma$$

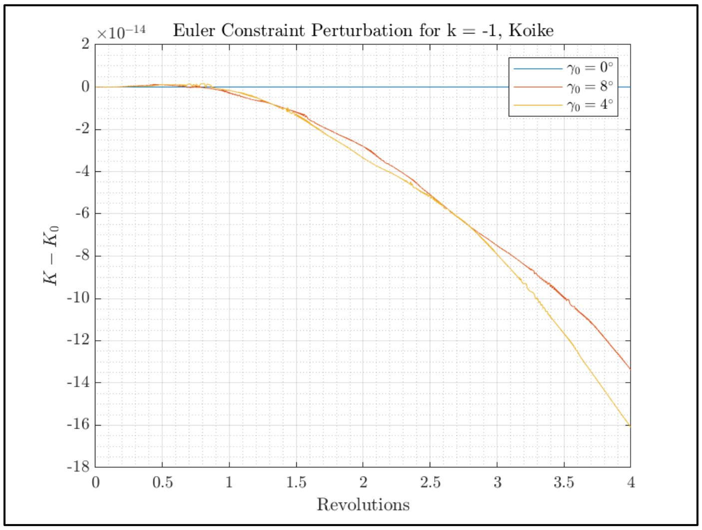
from Euler parameter relations C_{22} can be expressed as

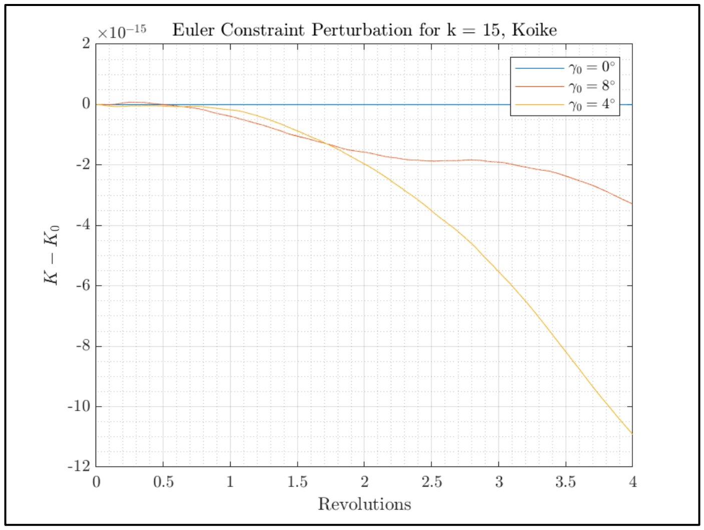
$$C_{22} = 1 - 2\varepsilon_3^2 - 2\varepsilon_1^2$$

thus, from the 2 eqns.

$$\gamma = \arccos\left(1 - 2\varepsilon_3^2 - 2\varepsilon_1^2\right)$$

Euler Constraint Plots

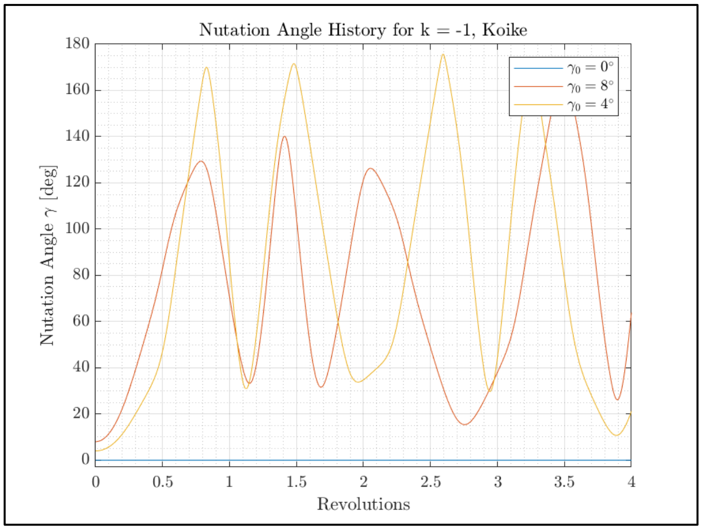


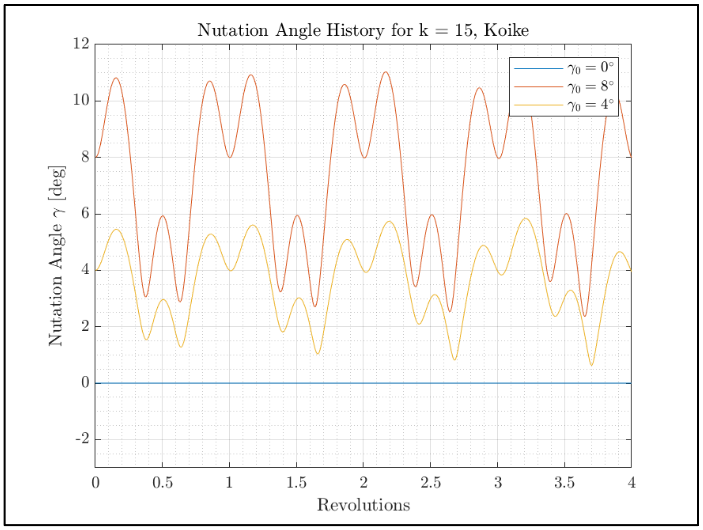


Analysis

1. Euler Constraint ΔK ranges in the order of 1e-15 ~ 1e-14 when the tolerance is set to 1e-13. The accuracy of the numerical integration is thereby proven.
2. The spin factor of k = 15 has a higher accuracy of 1e-15 than k = -1 which has an error of 1e-14. This stipulates that with a higher magnitude of spin factor k the more the error ΔK is improved. Which, in other words, means that k = -1 has more perturbation than k = 15.

Nutation Angle History Plots



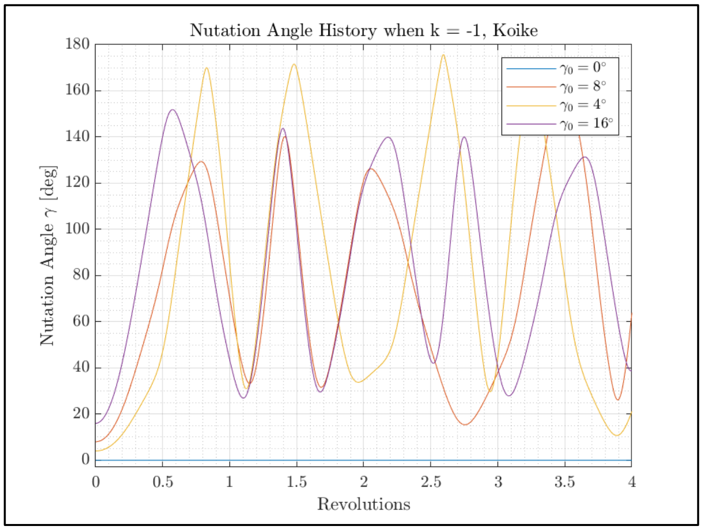


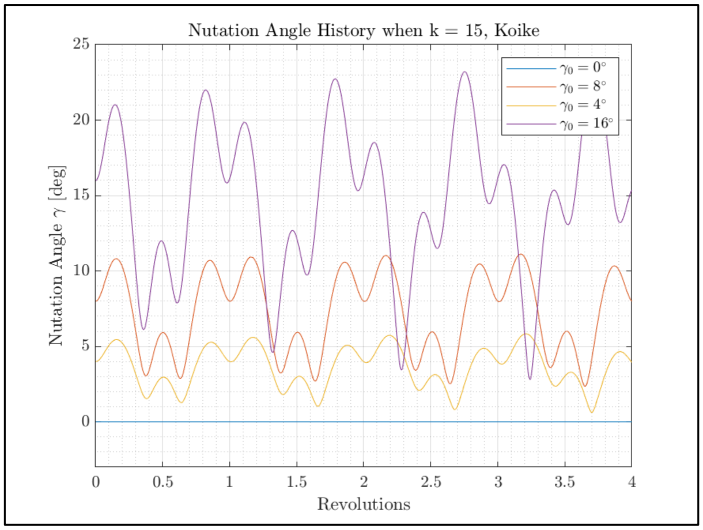
Analysis

1. The differential equations should all be a constant output when the $\Upsilon_0 = 0$ deg, and this can be observed from the plots above.
2. The plot with the spin factor k equal to -1 displays a result that is chaotic in that the plots do align with each other in the domain of revolutions from 1.7 to 4. Therefore, we can say that the motions for when $\Upsilon_0 = 4$ deg and 8 deg is different from each other.
3. The plot for k = -1 does not seem to have a period for each fluctuation pattern because the width of each wave seems to change. Whereas, the k = 15 seem to have a solid period which can be observed from the equally spaced patterns. These patterns seem to show that the k = 15 has a period that is approximately 1 revolution.
4. As aforementioned, the general behavior for when k = -1 differs for each initial nutation angle value. However, for k = 15 besides the magnitude of nutation angle Υ changing, the general behavior is same for all initial nutation angles.
5. In terms of stability, generally the nutation angle stabilizes with a higher magnitude value of the spin factor k which also means that there is a higher initial angular velocity ω_{20} applied to the body.
6. The plots for k = -1 have a larger deviation for each different initial nutation angle value (the y-tick values are evidently larger than that of k = 15), and therefore, we can say that the motion is unstable for when k = -1. On the contrary, when k = 15 the periodic trend and small deviation for each initial nutation angle shows that the motion is stable.

(ii) What happens in (i) if you double the initial angle $\gamma(0)=16^{\circ}$? Plot all three comparison solutions with the particular solution being tested. Are the results consistent with our previous plot? Does this new information change your conclusions?

Nutation Angle History Plots

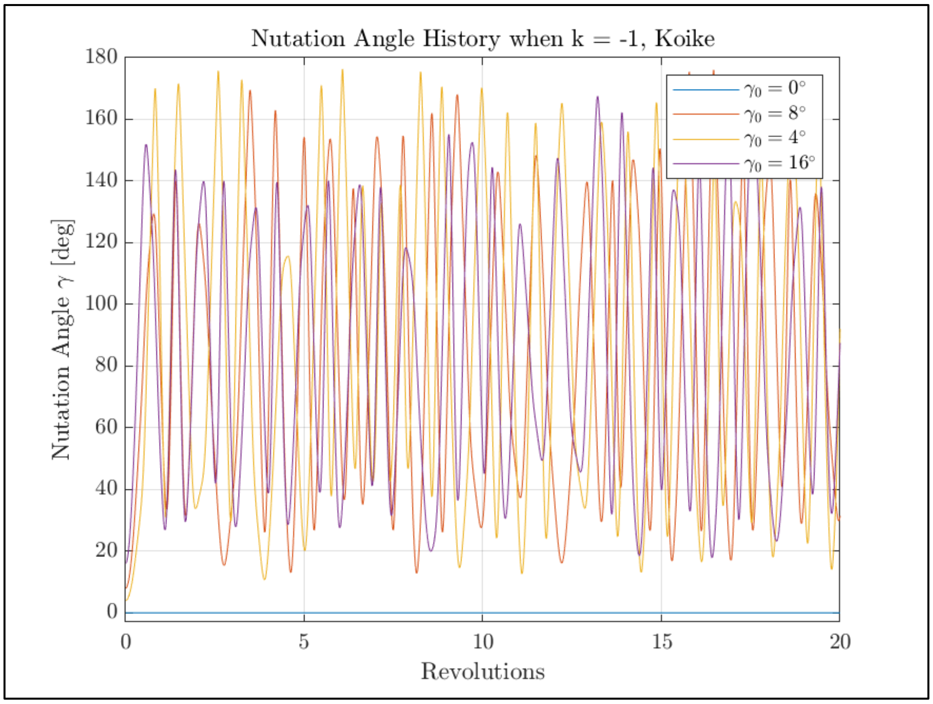


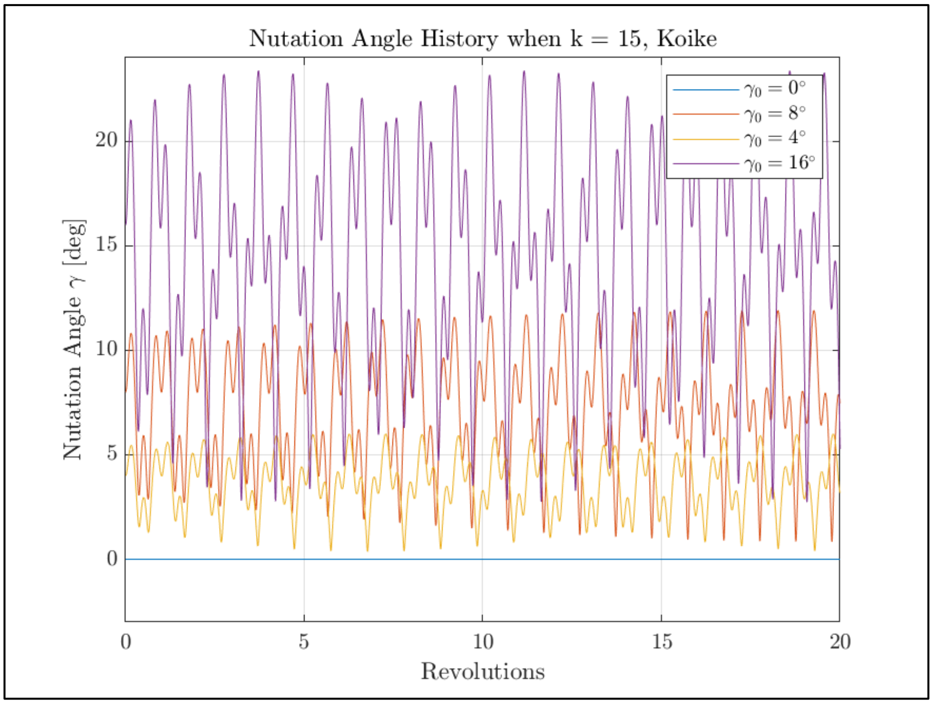


Analysis

1. For k = -1, the new initial nutation angle Υ_0 = 16 deg shows about the same kind of motion compared to the other initial nutation angles that we have discussed in the previous question. Thus, we can conclude that the plot remains consistent and does not change the conclusion of k = -1 showing a chaotic and unstable motion.
2. For k = 15, Υ_0 = 16 deg also shows a pattern which has a fixed period like the other initial nutation angles. Thus, the plot stays consistent with the conclusion we have drawn in the previous question (c)(i) as we have stated that the motion is in a orderly manner and is stable.
3. As a general behavior, we can observe that the higher the Υ_0 becomes the larger the amplitude of the wave (or the range of nutation angles) become. This leads to instability.

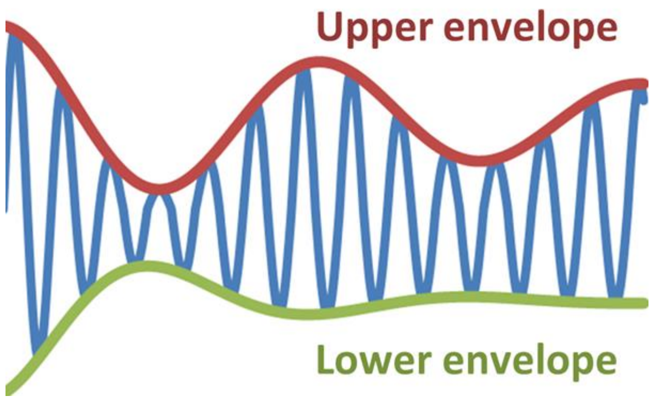
(d) Run some additional simulations on your own to discuss the following:
Does additional simulation time (maybe 20 revs) change your conclusions?
Consider k = -2 or k = 20. Can you predict the behavior?
Is the response symmetrical about $k = 0$? Should it be? (i.e., is the result the same if $k = \pm 2, \pm 20$?)





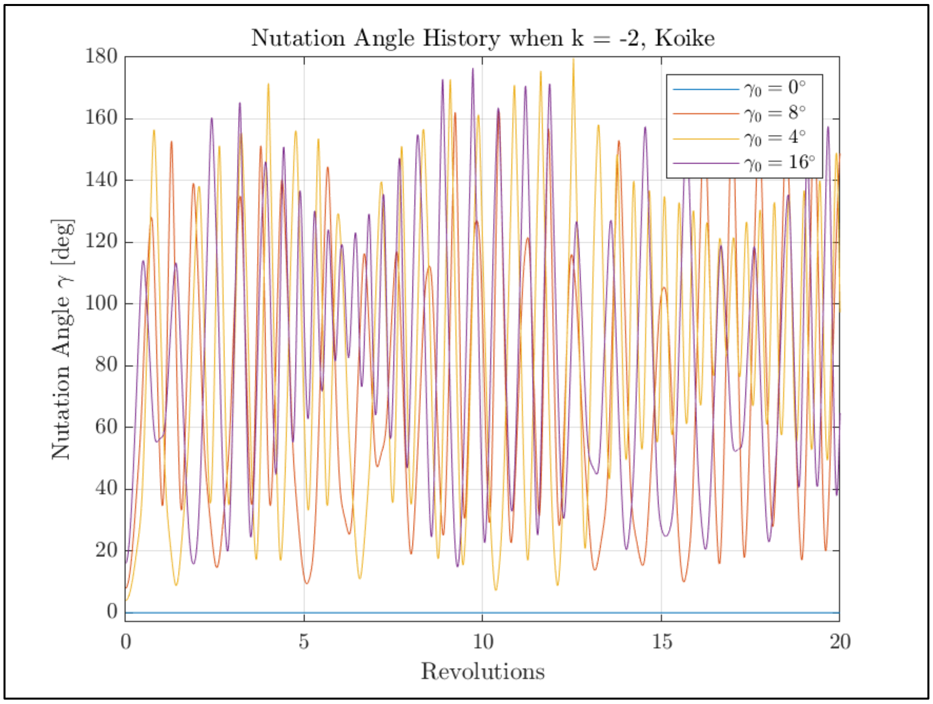
Analysis

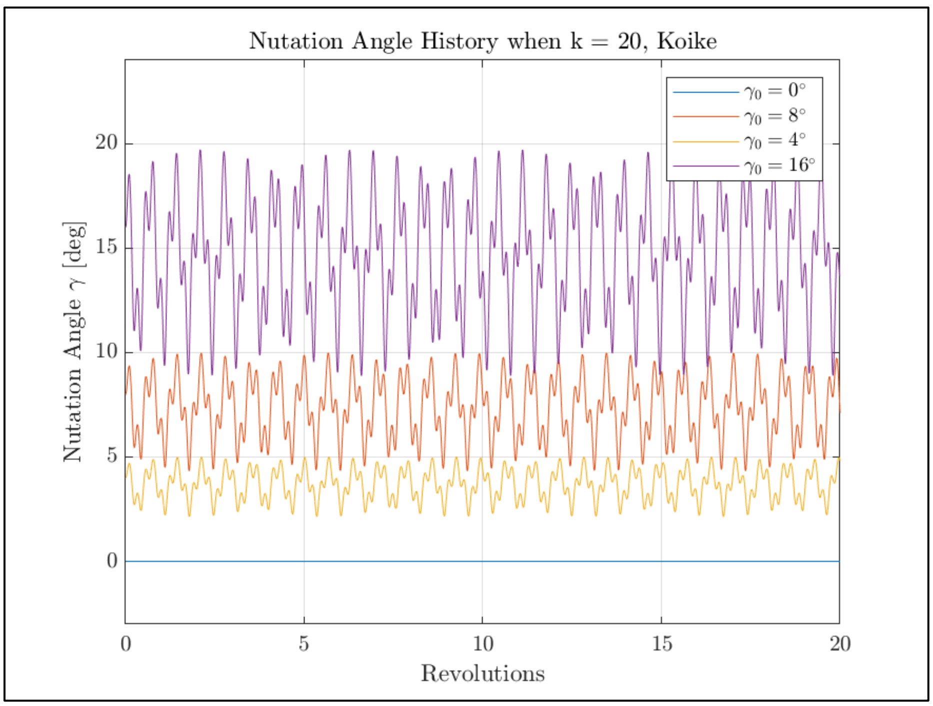
1. For k = -1, the motion's behavior remains chaotic and we are not able to see any kind of pattern or a certain period to characterize the motion. Thus, our conclusion is consistent with what we have for the previous two questions.
2. For k =15, by extending the simulation numbers of revolutions we are able to make a new observation that there seems to be a certain kind of pattern to the peaks of the nutation angle for $\Upsilon_0 = 16$. Namely, we can see an upper and lower envelope for the upper and lower peaks like the image below (*the image is only for visualization).
3. Nevertheless, we have a new observation for k = 15. The general behavior seems to nothing different from the conclusion we have drawn in the previous questions. Hence, the conclusion will not be changed for our simulations.

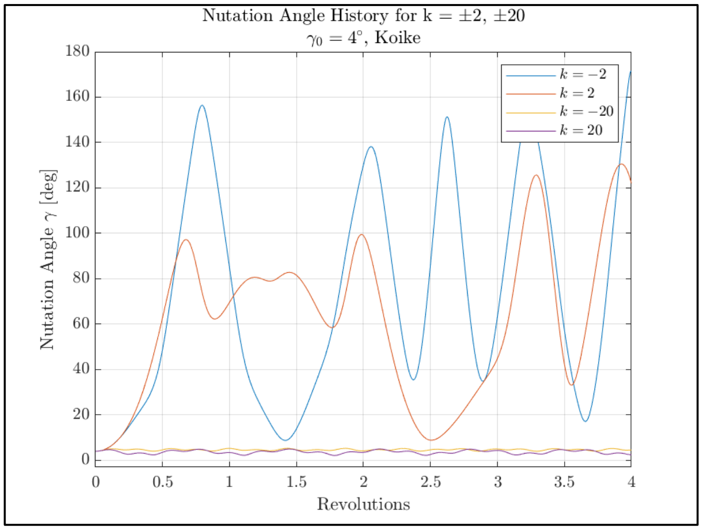


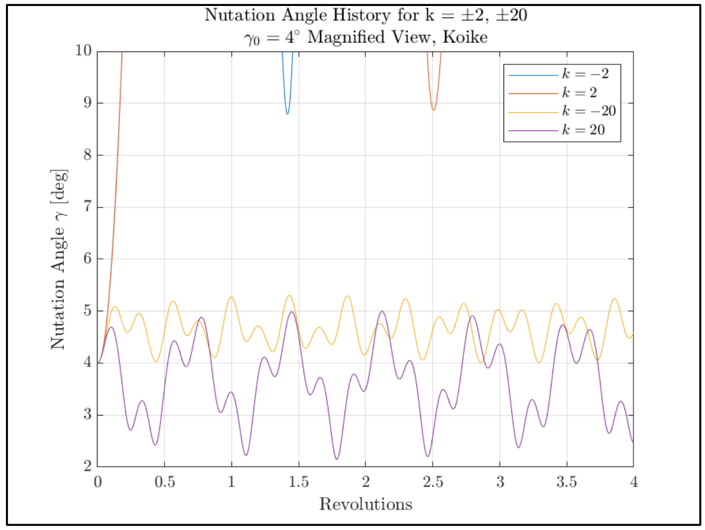
Predictions for k = -2 & k = 20

- Compared to k = -1 and k = 15, since we have discovered that with higher |k| the motion tends to become more stable, and therefore, for k = -2 and k = 20 the motion will become more stable compared to -1 and 15.
- k = -2 is still too low compared to 15 and will output a behavior that is chaotic and unstable with a wide range of nutation angle.
- Contrasted with k = 15, k = 20 will display a better motion, that is a more stable motion with a smaller range of nutation angles compared to small values of k.


Nutation Angle History when k = -2, Koike
Nutation Angle γ [deg]
Revolutions
$\gamma_0 = 0°$
$\gamma_0 = 8°$
$\gamma_0 = 4°$
$\gamma_0 = 16°$
0
20
40
60
80
100
120
140
160
180
0
5
10
15
20


Nutation Angle History when k = 20, Koike
Nutation Angle γ [deg]
Revolutions
$\gamma_0 = 0°$
$\gamma_0 = 8°$
$\gamma_0 = 4°$
$\gamma_0 = 16°$
0
5
10
15
20
0
5
10
15
20





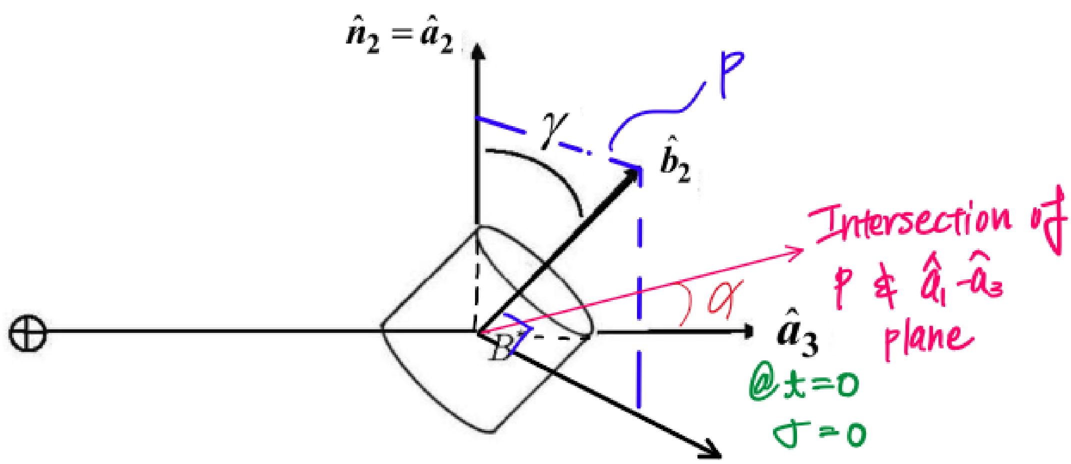
Analysis

1. The plots verify that our predictions were correct and that the stability for the nutation angles improve as the spin factor increases to k = -2 and 20. With the highest magnitude we have simulated being |k| = 20; from the plot, we can observe that the nutation only varies from approximately 2 ~ 5 for k = 20.
2. For the motions for when k = 2, 20 and k = -2, -20 are not symmetric. This is because nevertheless the spin rate is directed in the opposite direction the initial nutation angle remains in the same direction. Thus, we can see from the plot that they do not seem to be symmetric but behave in an unpredictable manner.

Problem 2: In Problem 1, you examined perturbations in the nutation angle and the influence of the rate k on the determination of instability in the nutation angle. Now consider the precession angles more carefully. Define P as the plane defined by $\hat{a}_2$ and $\hat{c}_2 = \hat{b}_2$. Also define:

γ	–	Angle between $\hat{a}_2$ and $\hat{b}_2$
σ	–	Angle between a fixed reference line and $\hat{a}_3$
α	–	Angle between $\hat{a}_3$ and plane P
β	–	Angle between plane P and the inertially fixed plane with which P coincides initially

(a) Let $\gamma(0) = 8^\circ$ and k = -15, 6, 12, 30, 80. Simulate for 2 revs. Make the following 4 plots: one for each angle; each plot includes five curves, one for each value of k. (Compute β as a continuous angle.)
Discuss the relationship between the angles γ, α, β. Which one is precession? Precession relative to which frame? How do you know?
Discuss how the curves change when k changes. What happens if k = +100?

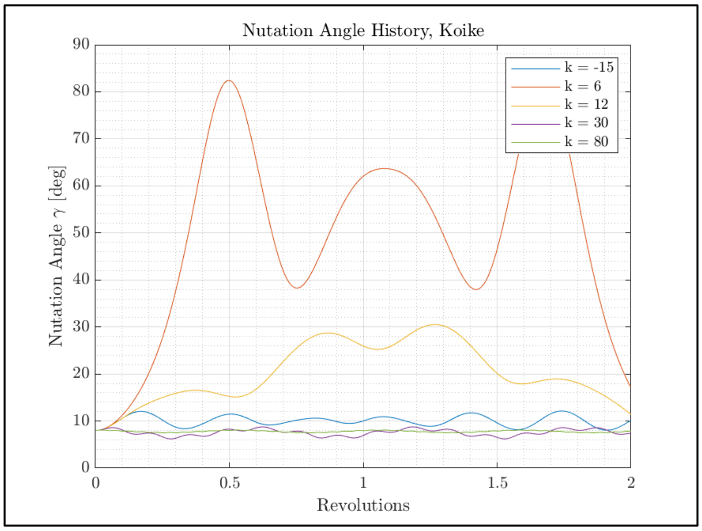


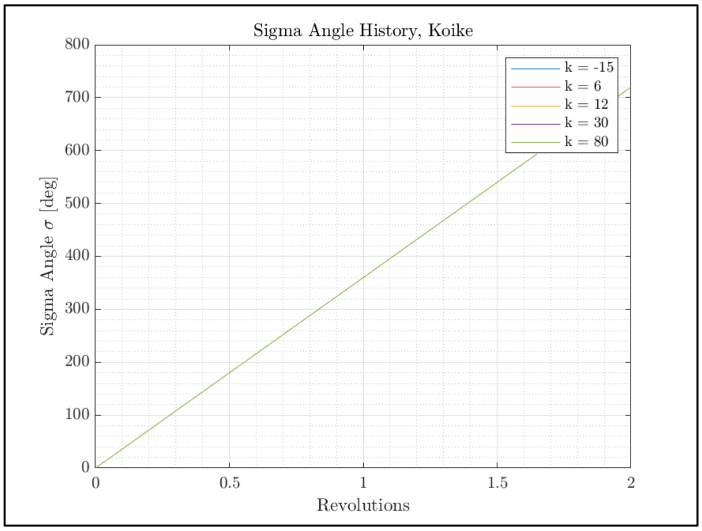
Simulation

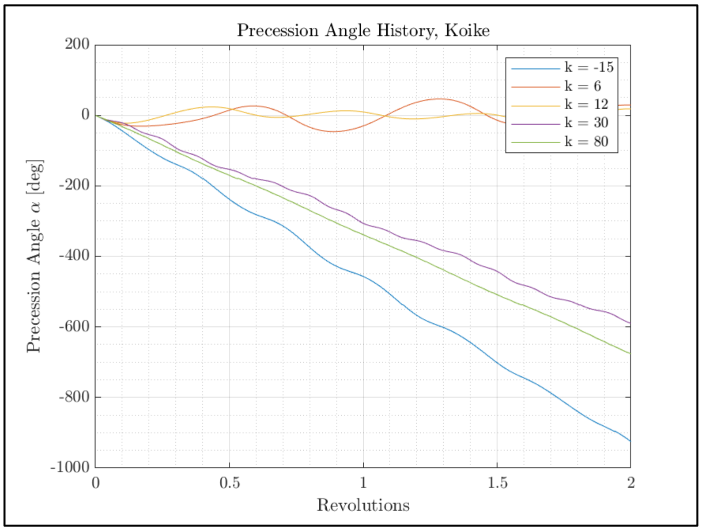
- Gamma, γ : Nutation angle of the body in $\hat{c}_i$ vector basis
- Sigma, σ : The angle between $\hat{n}_3$ and $\hat{a}_3$ when a fixed refence line $\hat{n}_3$ is assumed.

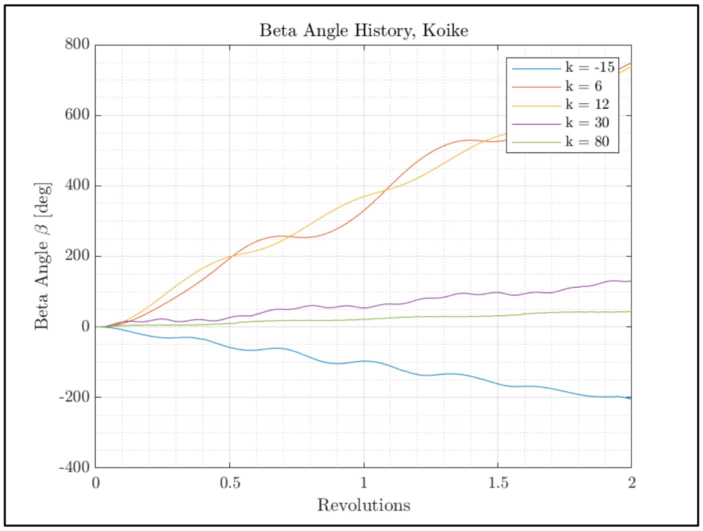
  $\sigma = \Omega t \longrightarrow \sigma = 2\pi\nu$
- Alpha, α : The angle between P and $\hat{a}_3$, which is the precession in $\hat{c}_i$ vector basis
- Beta, β : The angle between P and $\hat{n}_2$-$\hat{n}_3$ plane which is $\beta = \alpha + \sigma$

Angle History Plots





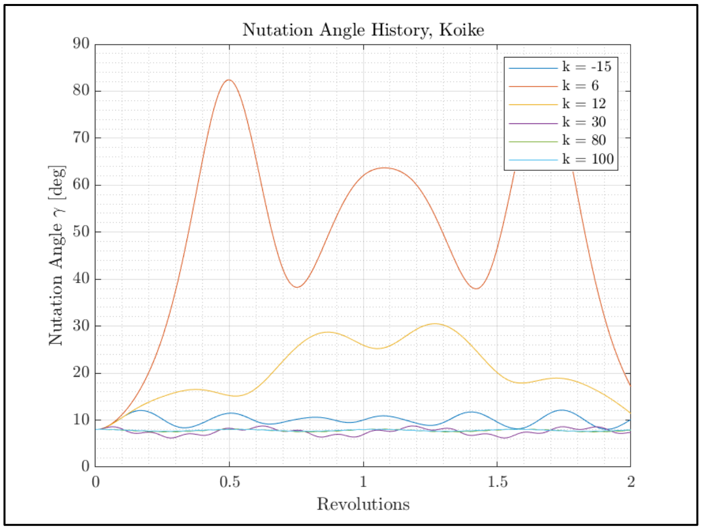


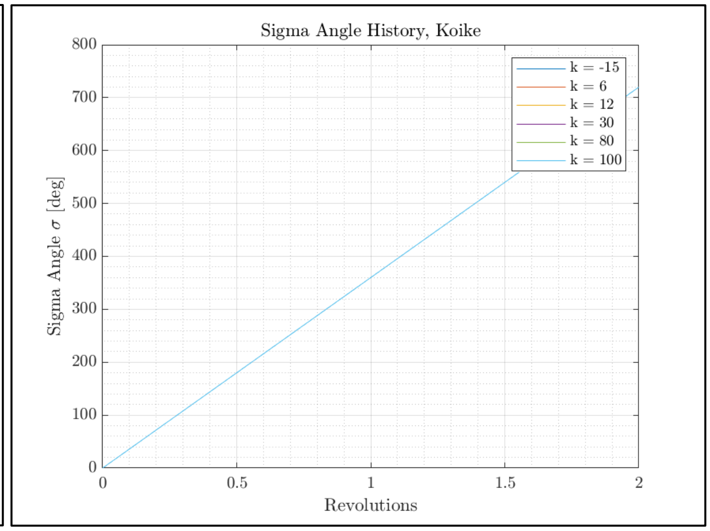


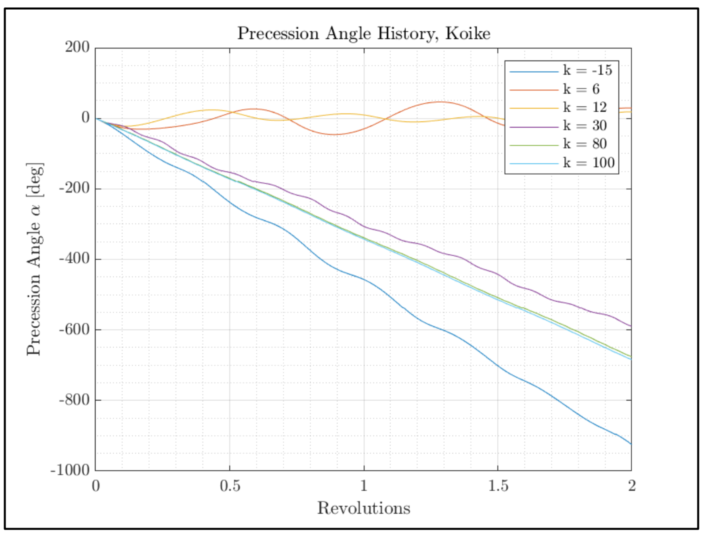
Analysis

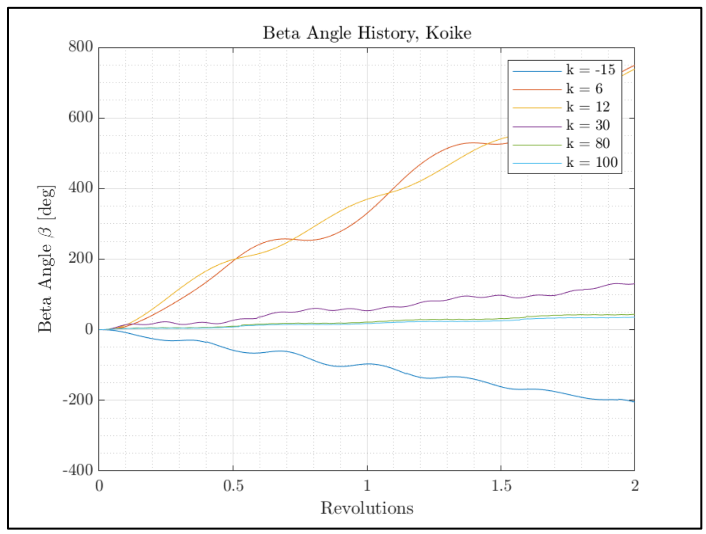
1. Υ is the nutation angle and α is the precession angle. The two angles are both defined in the body frame, and the angle β is defined as the sum of the two angles α and σ.
2. From the plots of the nutation angle and precession angle we can observe that with a spin factor k which has a small magnitude the nutation angle tends to have a larger range while when it is large the range becomes smaller; whereas, with the precession angle it is the opposite. That is when the magnitude of the spin factor k is smaller the range of the precession angle becomes smaller and when the k factor is large the angle keeps on rotating and diverges.
3. From the comparison of the nutation angle Υ and angle β we can observe a correlation between when the spin factor k has larger magnitude the angles seem to shrink down into a narrower range, which indicates more stability for the 2-revolution simulation.
4. The angle α is the precession angle in the body frame and β is the precession angle in the inertial frame. As discussed above, when the spin factor k has a relatively small magnitude like $k = 6, 12$ the angle seems to oscillate but with larger k-values it diverges. In contrast, the precession in the inertial frame, β regardless of the spin factor they all seem to diverge; however, when the spin factor has a larger magnitude the rate of divergence appears to be smaller then when the k-value is small in magnitude. This is because, when the magnitude of the spin factor becomes larger the angles α and σ interact in a way that they offset each other.
5. Overall, we can say that the magnitude of the spin factor k works proportionally to improve the stability in the inertially fixed frame.

Angle History Plots with k = 100 added









Analysis

1. We have predicted that with a larger magnitude of the spin factor k leads to more stability, and from the plots above, we can validate that our prediction was correct. This is because, we can observe that the precession angle ϒ varies with the smallest range so far, and the same can be said about the inertial precession angle β.

(b) Estimate the value of $\Delta\beta$ from the equation presented in lecture that is actually derived from perturbation theory. What does it tell you about α ? Add it to the β plot for the values $k =$ -15, 6, 12, 30, 80.
From your numerical results, estimate the maximum amplitude of the oscillation relative to the slope along each β curve. Does the amplitude increase or decrease as the input value $\left|{}^{N}\bar{\omega}^{U}\right| = k\,\Omega$ increases in magnitude?

From our lecture notes, we know that

$$\Delta\beta = 3\pi\left(\frac{\Omega}{\omega_{2_0}}\right)\left(\frac{I}{J}-1\right)\cos\gamma_0$$

$$\because\quad k = \frac{\omega_{2_0}}{\Omega}$$

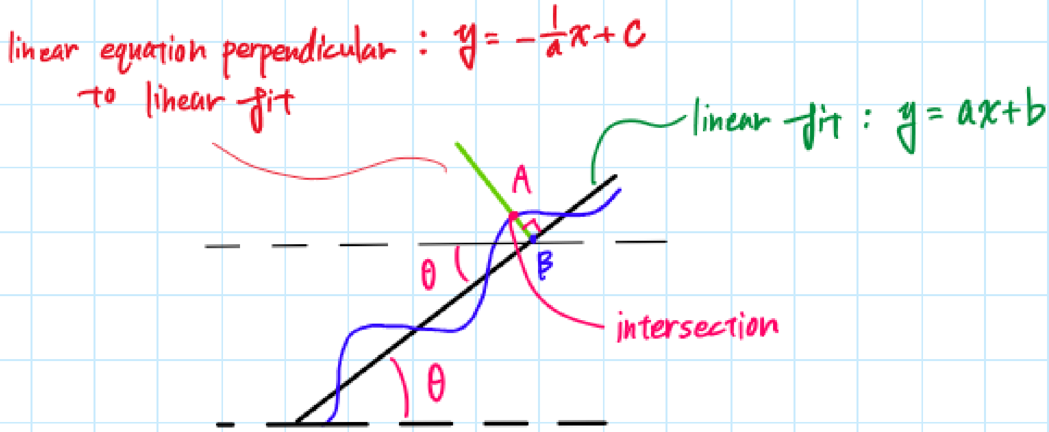
$$\therefore\quad \boxed{\Delta\beta = \frac{3\pi}{k}\left(\frac{I}{J}-1\right)\cos\gamma_0}$$

Since $I = 400\ \text{kg-m}^2$, $J = 100\ \text{kg-m}^2$, $\gamma_0 = 8\ \text{deg}$
$k = -15, 6, 12, 30, 80$

thus, for each k-value $\Delta\beta$ becomes

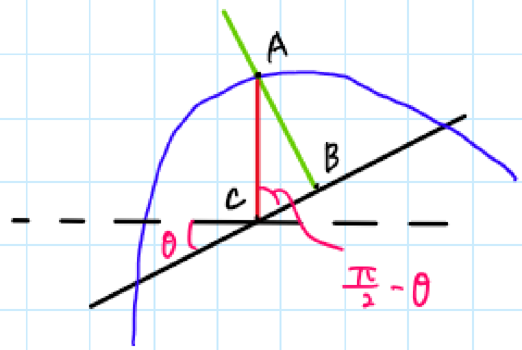
$$\boxed{\begin{aligned}
\Delta\beta\big|_{k=-15} &= -106.9490\ \text{deg/rev}\\
\Delta\beta\big|_{k=6} &= 267.3724\ \text{deg/rev}\\
\Delta\beta\big|_{k=12} &= 133.6862\ \text{deg/rev}\\
\Delta\beta\big|_{k=30} &= 53.4745\ \text{deg/rev}\\
\Delta\beta\big|_{k=80} &= 20.0529\ \text{deg/rev}
\end{aligned}}$$

Maximum Amplitude



closer view

pseudo code for MATLAB



$$\text{Amplitude} := |\overline{AB}| = |\overline{AC}| \sin\left(\frac{\pi}{2} - \theta\right) = |\overline{AC}| \cos\theta$$

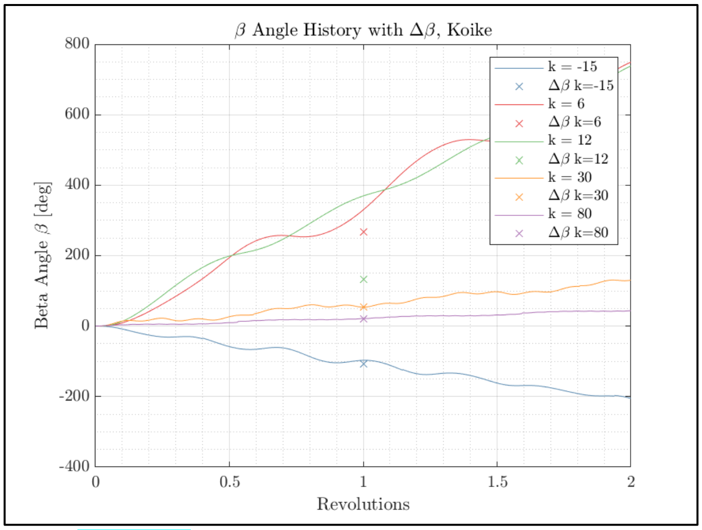
since $\quad \dfrac{dy}{dx} = a \quad$ (from linear fit)

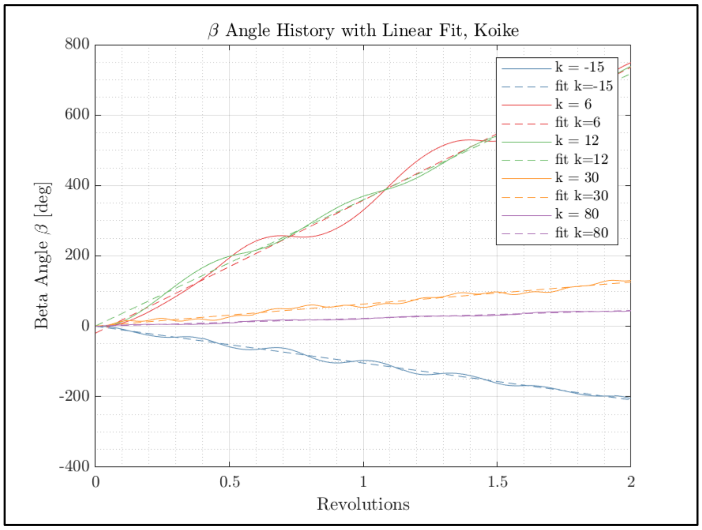
and $\quad \theta = \arctan\left(\dfrac{dy}{dx}\right) = \arctan(a)$

$$|\overline{AB}| = |\overline{AC}| \cos\left[\arctan(a)\right]$$

$$\Longrightarrow \boxed{\max\left(|\overline{AB}|\right) = \max\left(|\overline{AC}|\right) \cos\left[\arctan(a)\right]}$$

Angle β Plots





Analysis

1. The theoretical value of Δβ seem to only match for the simulations with a high magnitude spin factor value. As we can see that for k = 6, 12 the x-mark does not lie on the curve for β angle history.
2. From the previous analysis, we can generalize that the larger the magnitude of spin factor k is the smaller the Δβ becomes.
3. The maximum amplitude can be found by the method noted in the previous page, and by using this method we have computed the maximum amplitudes for each spin factor k. The results are tabulated below. From the tabulated results below, we can deduce that with a higher magnitude of spin factor k the larger the amplitude relative to the slope of the fitted linear regression becomes; however, the caveat is that the highest k-value does not necessarily mean that it has the highest amplitude.
4. Thus, as the input value of $k\Omega$ increases in magnitude it is likely that the amplitude increases.

Spin Factor, k	Maximum Amplitude [deg]
-15	0.1093
6	0.1146
12	0.0653
30	0.1782
80	0.1328

(c) Does your discussion/conclusions about stability in Problem 1 change now that you have information concerning more than one variable?

Do you think you could input sufficient spin $\left|{}^{N}\bar{\omega}^{U}\right|$ at a rate that could maintain a precession angle $\alpha = 0^{\circ}$? $\beta = 0^{\circ}$? Select another, higher value of k and try it. Discuss your answer.

Analysis

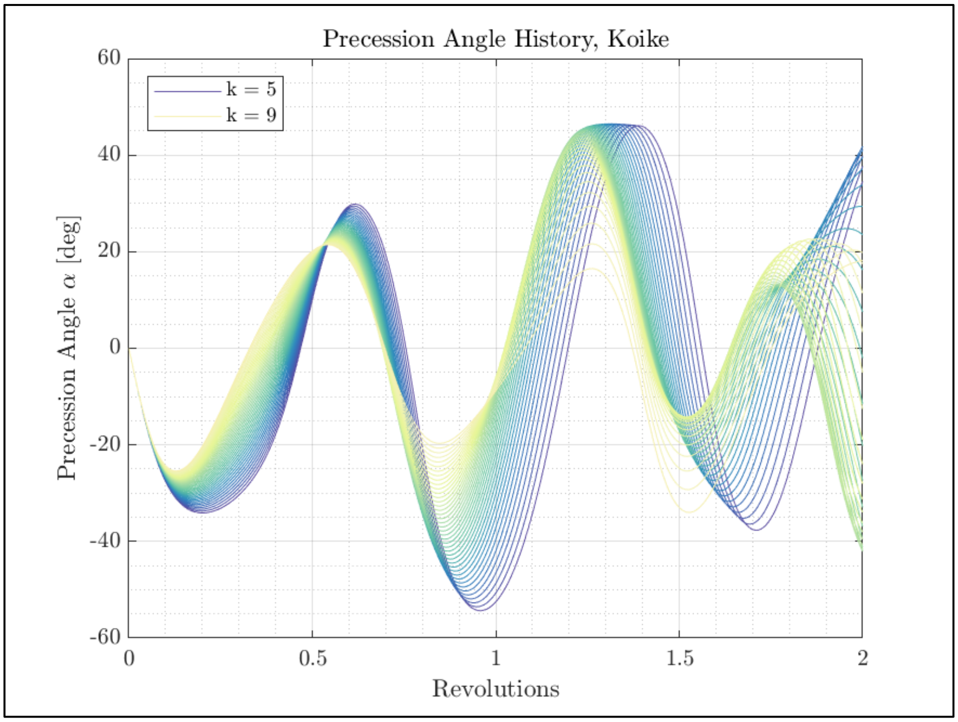
1. From the results posed in question (b), we can see that the results are consistent with what we have examining throughout this simulation. That is, for the precession angle, in question (a), we have analyzed that there is more stability observed when the magnitude of the k-value is smaller which is the opposite of properties such as the nutation angle in which higher spin factor k show improvement in stability. In the analysis of question (b), we have examined that the amplitudes correlate to the results of question (a) in that when the magnitude of k becomes larger the amplitude is more likely to become larger as well. Thus, introducing another variable turned out to reinforce our analysis.

Discussion of maintaining constant precession angle $\alpha = 0^{o}$ & constant $\beta = 0^{0}$

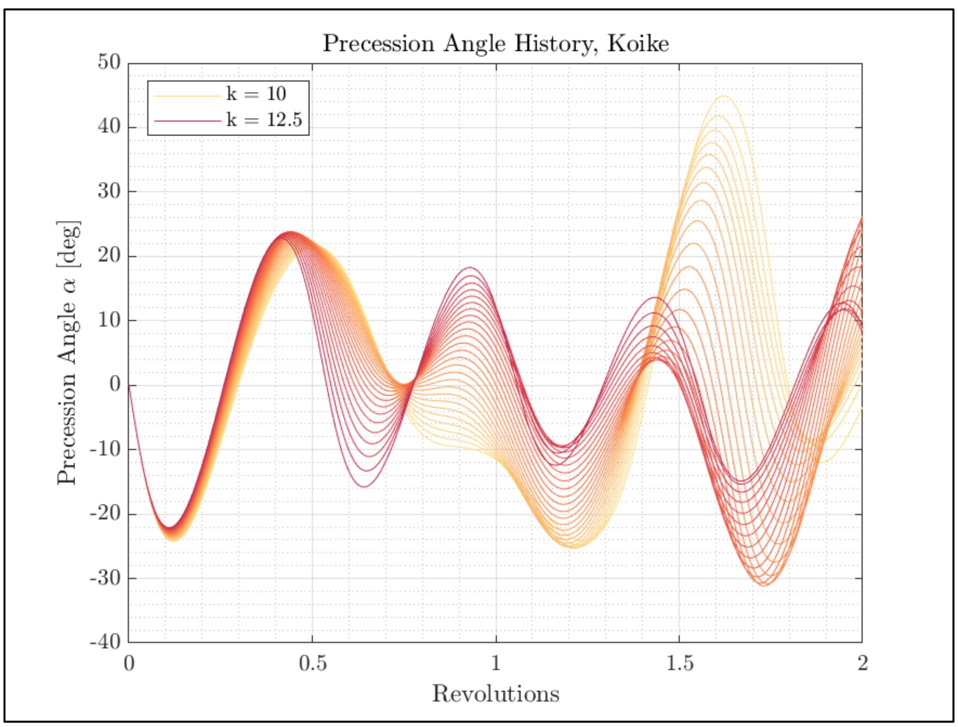
- In the previous question, we have seen that smallest maximum amplitude is output when k = 12.
- From this, we can compute the precession angle α and angle β using a k-value near 12 to find the most optimal value to suffice the condition of constant 0 degrees for the two angles
- From multiple simulations we found out that in the ranges shown below, the precession angle was maintained a small angle that could satisfy our requirements.

$$k \in [5, 9]\ AND\ [10, 12.5]$$

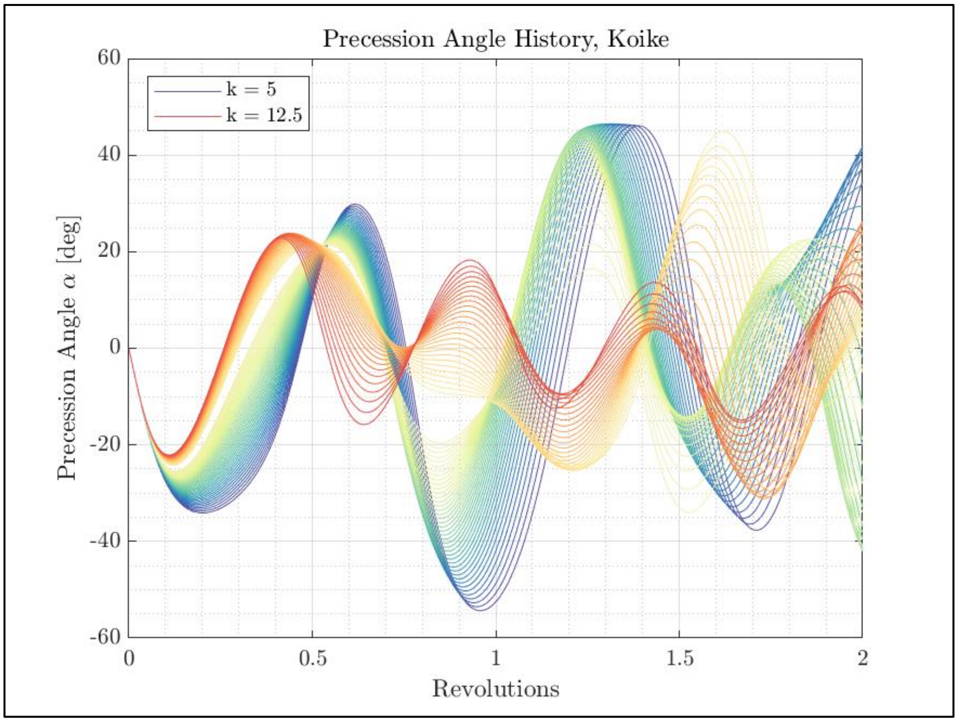
Plots for < k ∈ [5, 9] > with increments of 0.1



Plots for < k ∈ [10, 12.5] > with increments of 0.1



Plots for $< k \in \{[5, 9] \cup [10, 12.5]\} >$ with increments of 0.1



Analysis

1. From the plots above, we can observe that the angles of precession vary within the range of approximately -50 deg ≤ α ≤ 50 deg. This is relatively small but not small enough to say that it satisfies the requirement to make the angles α and β nearly zero.
2. This concludes that the precession angle is unpredictable and is nearly impossible to find a optimal solution for the spin factor k.

Optimization Analysis

For further analysis we have used the global optimization tool in MATLAB. This involves the implementation of the command "pattern search" which allows us to find the global minimum/maximum of a certain function that is dependent of a certain variable. Incorporating this will fortify our analysis by finding the most optimal spin factor k that produces the minimum value of the following relation.

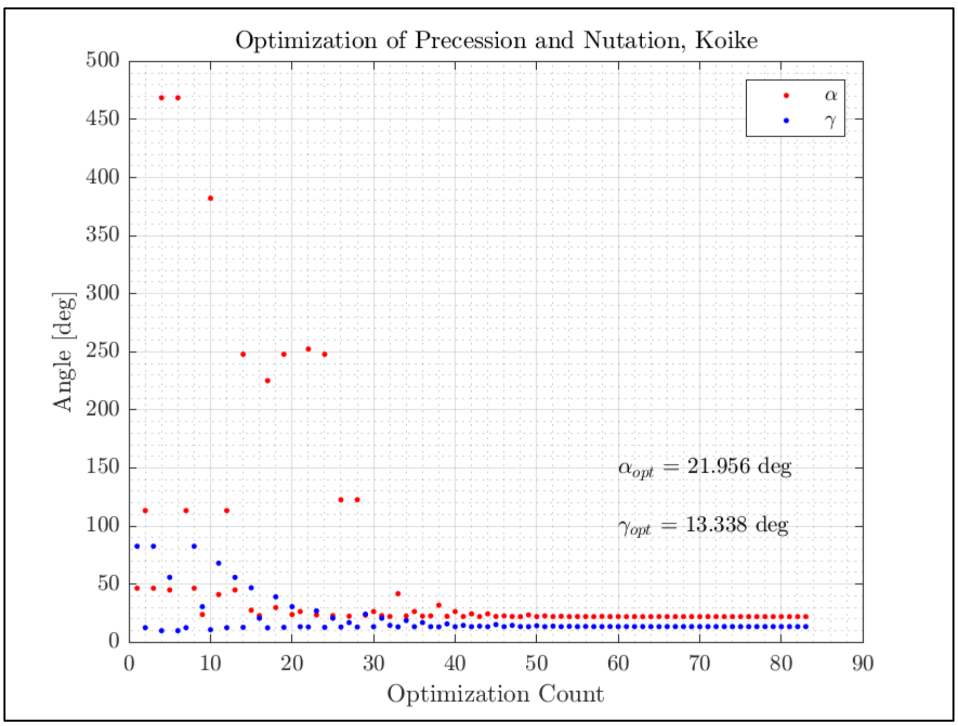
OBJ = (Maximum Amplitude of α) + (Maximum Amplitude of ϒ)

(*OBJ = objective to optimize)

We want to see if this optimal k-value falls within or close to the bounds we expect, that is $k \in \{[5, 9] \cup [10, 12.5]\}$. We ran the optimization in the bounds of $k \in [5, 20]$ and at a starting value of k = 6. Then MATLAB gave the following output.

Optimal Spin Factor k_{opt}	OBJ [deg]
12.7121	35.294

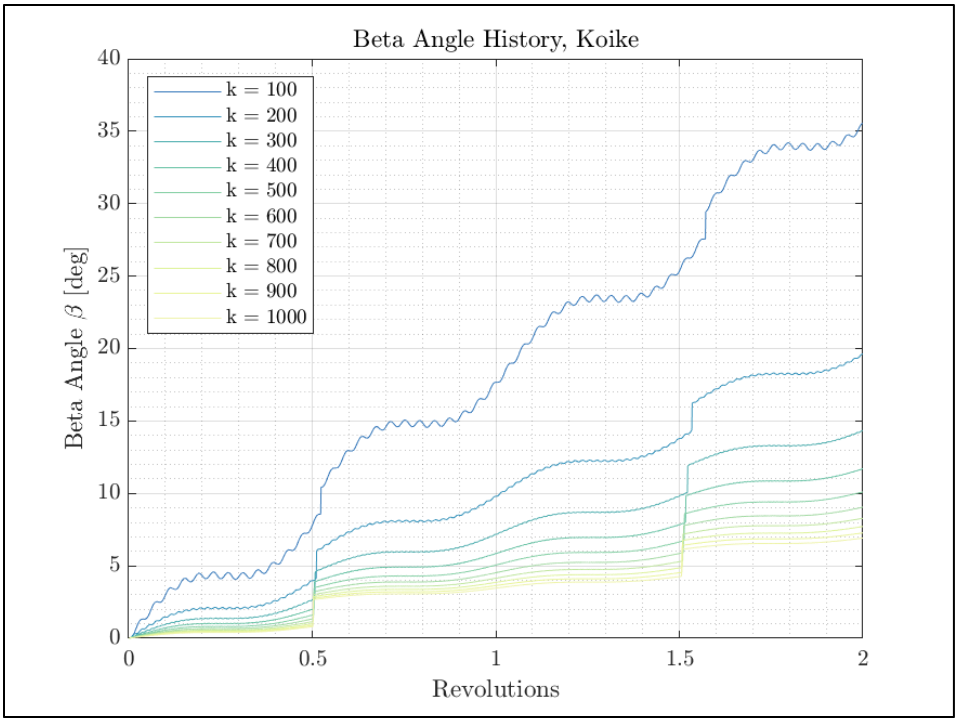
Additionally, the precession and nutation angles throughout the optimization is plotted as the following (*the optimal values for the two angles are also noted in the figure).

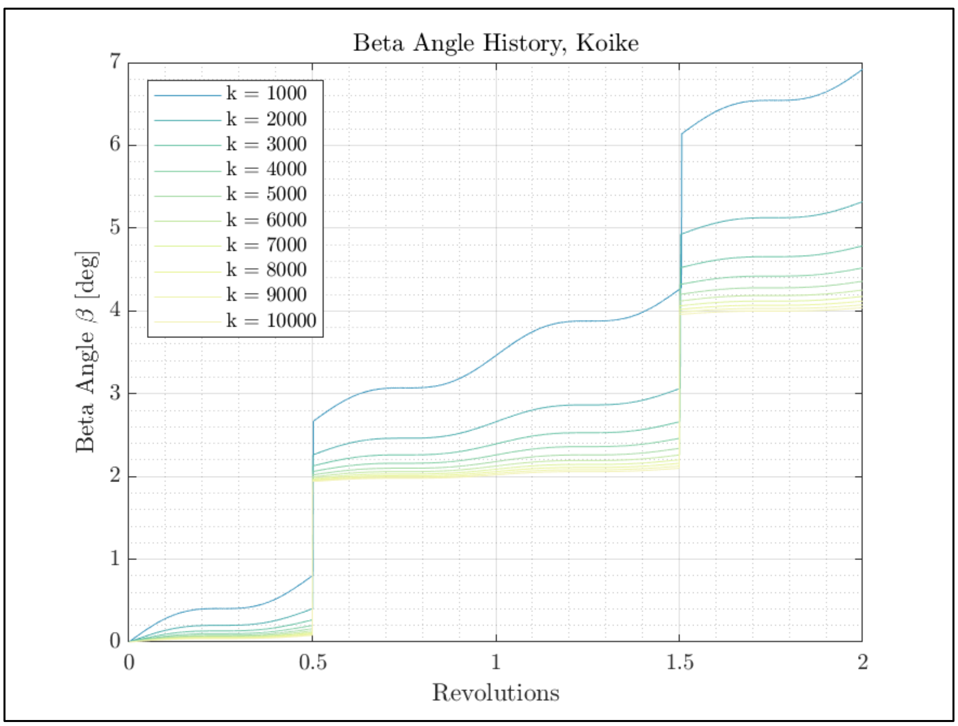


- From the results of the optimization we can tell that our analysis for k in range of 5 ~ 9 and 10 ~ 12.5 was accurate enough to observe a small angle for precession angle α ≈ 0° – 20°.

Inertial Precession β Plots

Since we have observed in the previous questions that with higher values of k we observe improvement in stability for inertial precession. Thus, we make the values of k significantly high to test extreme values and conduct analysis from the outputs.





Analysis

1. For each revolution the angle for β seem to step up a certain increment and as we can see that the higher the value of k becomes the smaller that increase in inertial precession becomes. The unique feature about the plot above is that as the value of k becomes very high the plot seems to flatten out and looks like a step response.
2. The jump in the inertial precession can be observed exactly at revolutions 0.5 and 1.5.
3. Since in this simulation of revolutions of within 0 to 2, we can see that the inertial precession continues to increase in a orderly manner; hence, we presumably cannot meet the requirement of achieving a constant inertial precession value of $\beta = 0^0$.

Problem 3: Complete a linear analysis of the particular solution to gain insight into the problem and consider what additional information is available.

(a) The nominal motion of interest is a constant spin of the spacecraft in N about an axis parallel to the orbit normal, i.e., $\left|{}^N\bar{\omega}^U\right| = k\Omega$ where k is a constant. Recall that the motion of interest can be modeled as C fixed in A. Again, what value of s corresponds to this solution?
What is the particular solution (in terms of the dependent variables) that corresponds to the nominal motion?

we know that $s = \omega_{20} - \Omega = q$

EOM

$$\dot{\omega}_1 = \omega_3 q + \frac{I-J}{I}\left[6\Omega^2(\varepsilon_2\varepsilon_3 + \varepsilon_1\varepsilon_4)(1-2\varepsilon_1^2-2\varepsilon_2^2) - \omega_2\omega_3\right]$$

$$\dot{\omega}_2 = 0$$

$$\dot{\omega}_3 = -\omega_1 q + \frac{J-I}{I}\left[12\Omega^2(\varepsilon_3\varepsilon_1 - \varepsilon_2\varepsilon_4)(\varepsilon_2\varepsilon_3 + \varepsilon_1\varepsilon_4) - \omega_1\omega_2\right]$$

$$2\dot{\varepsilon}_1 = \omega_3\varepsilon_2 - (\omega_2 - q + \Omega)\varepsilon_3 + \omega_1\varepsilon_4$$

$$2\dot{\varepsilon}_2 = -\omega_3\varepsilon_1 + \omega_1\varepsilon_3 + (\omega_2 - q - \Omega)\varepsilon_4$$

$$2\dot{\varepsilon}_3 = (\omega_2 - q + \Omega)\varepsilon_1 - \omega_1\varepsilon_2 + \omega_3\varepsilon_4$$

$$2\dot{\varepsilon}_4 = -\omega_1\varepsilon_1 - (\omega_2 - q - \Omega)\varepsilon_2 - \omega_3\varepsilon_3$$

The particular solutions for this nominal motion is

$$\boxed{\begin{array}{ll} \omega_1 = \omega_3 = 0, & \omega_2 = \omega_{20} = k \\ \varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 0, & \varepsilon_4 = 1 \end{array}}$$

$$\Downarrow$$

$$\boxed{\begin{array}{ll} \varepsilon_i = 0 + \tilde{\varepsilon}_i & (i = 1,2,3) \\ \varepsilon_4 = 1 + \tilde{\varepsilon}_4 & \\ \omega_j = 0 + \tilde{\omega}_j & (j = 1,3) \end{array}}$$

(b) Linearize relative to the particular solution and derive first-order, linear variational equations that govern the behavior near the reference solution.

Continuing on what we've obtained in (a)

plug-in

$$s = q = \omega_{20} - \Omega \ , \quad x = \frac{J}{I} - 1 \ , \quad y = \frac{\omega_{20}}{\Omega} - 1$$

HoT := High Order Term

$$2\dot{\tilde{\varepsilon}}_1 = \cancelto{HoT}{\tilde{\omega}_3\tilde{\varepsilon}_3} - [\cancel{\omega_2} - (\cancel{\omega_{20}} - \Omega) + \Omega]\tilde{\varepsilon}_3 + \tilde{\omega}_1(1 + \cancelto{HoT}{\tilde{\varepsilon}_4})$$

$$2\dot{\tilde{\varepsilon}}_1 = -2\Omega\tilde{\varepsilon}_3 + \tilde{\omega}_1$$

$$\therefore \ \dot{\tilde{\varepsilon}}_1 = -\Omega\tilde{\varepsilon}_3 + \frac{\tilde{\omega}_1}{2}$$

$$2\dot{\tilde{\varepsilon}}_2 = -\cancelto{HoT}{\tilde{\omega}_3\tilde{\varepsilon}_1} + \cancelto{HoT}{\tilde{\omega}_1\tilde{\varepsilon}_3} + [\cancelto{0}{\omega_2 - (\omega_{20} - \Omega) - \Omega}](1 + \tilde{\varepsilon}_4)$$

$$\therefore \ \dot{\tilde{\varepsilon}}_2 = 0$$

$$2\dot{\tilde{\varepsilon}}_3 = [\tilde{\omega}_2 - (\omega_{20} - \Omega) + \Omega]\tilde{\varepsilon}_1 - \cancelto{HoT}{\tilde{\omega}_1\tilde{\varepsilon}_2} + \tilde{\omega}_3(1 + \cancelto{HoT}{\tilde{\varepsilon}_4})$$

$$2\dot{\tilde{\varepsilon}}_3 = 2\Omega\tilde{\varepsilon}_1 + \tilde{\omega}_3$$

$$\therefore \ \dot{\tilde{\varepsilon}}_3 = \Omega\tilde{\varepsilon}_1 + \frac{\tilde{\omega}_3}{2}$$

$$2\dot{\tilde{\varepsilon}}_4 = -\cancelto{HoT}{\tilde{\omega}_1\tilde{\varepsilon}_1} - [\cancelto{0}{\omega_2 - (\omega_{20} - \Omega) - \Omega}]\tilde{\varepsilon}_2 - \cancelto{HoT}{\tilde{\omega}_3\tilde{\varepsilon}_3}$$

$$\therefore \ \dot{\tilde{\varepsilon}}_4 = 0$$

$$\dot{\omega}_1 = \omega_3 q_r + \frac{I-J}{I}\left[6\Omega^2(\varepsilon_2\varepsilon_3+\varepsilon_1\varepsilon_4)(1-2\varepsilon_1^2-2\varepsilon_2^2) - \omega_2\omega_3\right]$$

$$\rightarrow \dot{\tilde{\omega}}_1 = \tilde{\omega}_3\Omega y - x\left[6\Omega^2\left(\cancelto{\text{HoT}}{\tilde{\varepsilon}_2\tilde{\varepsilon}_3} + \tilde{\varepsilon}_1(1+\cancelto{\text{HoT}}{\tilde{\varepsilon}_4})\right)\left(1-\cancelto{\text{HoT}}{2\tilde{\varepsilon}_1^2}-\cancelto{\text{HoT}}{2\tilde{\varepsilon}_2^2}\right) - \omega_{20}\tilde{\omega}_3\right]$$

$$\dot{\tilde{\omega}}_1 = \tilde{\omega}_3\Omega y - (6\Omega^2\tilde{\varepsilon}_1 - \omega_{20}\tilde{\omega}_3)x$$

$$\dot{\tilde{\omega}}_1 = \left(y + x\frac{\omega_{20}}{\Omega}\right)\tilde{\omega}_3\Omega - 6x\Omega^2\tilde{\varepsilon}_1$$

$$\because \frac{\omega_{20}}{\Omega} = y+1$$

$$\dot{\tilde{\omega}}_1 = [y+(y+1)x]\tilde{\omega}_3\Omega - 6x\Omega^2\tilde{\varepsilon}_1$$

$$\because a = y + (1+y)x$$

$$\therefore \dot{\tilde{\omega}}_1 = a\tilde{\omega}_3\Omega - 6x\Omega^2\tilde{\varepsilon}_1$$

$$\dot{\omega}_3 = -\omega_1 q_r + \frac{J-I}{I}\left[12\Omega^2(\varepsilon_3\varepsilon_1-\varepsilon_2\varepsilon_4)(\varepsilon_2\varepsilon_3+\varepsilon_1\varepsilon_4) - \omega_1\omega_2\right]$$

$$\rightarrow \dot{\tilde{\omega}}_3 = -\tilde{\omega}_1\Omega y + x\left[12\Omega^2\left(\cancelto{\text{HoT}}{\tilde{\varepsilon}_3\tilde{\varepsilon}_1} - \tilde{\varepsilon}_2(1+\cancelto{\text{HoT}}{\tilde{\varepsilon}_4})\right)\left(\cancelto{\text{HoT}}{\tilde{\varepsilon}_2\tilde{\varepsilon}_3}+\tilde{\varepsilon}_1(1+\cancelto{\text{HoT}}{\tilde{\varepsilon}_4})\right) - \tilde{\omega}_1\omega_{20}\right]$$

$$\dot{\tilde{\omega}}_3 = -\tilde{\omega}_1\Omega y + x\left(-12\Omega^2\cancelto{\text{HoT}}{\tilde{\varepsilon}_2\cdot\tilde{\varepsilon}_1} - \tilde{\omega}_1\omega_{20}\right)$$

$$\dot{\tilde{\omega}}_3 = -\tilde{\omega}_1\Omega y - x\tilde{\omega}_1\omega_{20}$$

$$\dot{\tilde{\omega}}_3 = -\left(y + x\frac{\omega_{20}}{\Omega}\right)\tilde{\omega}_1\Omega$$

$$\because \frac{\omega_{20}}{\Omega} = y+1$$

$$\dot{\tilde{\omega}}_3 = -[y+x(y+1)]\tilde{\omega}_1\Omega$$

$$\because a = y+(y+1)x$$

$$\therefore \dot{\tilde{\omega}}_3 = -a\Omega\tilde{\omega}_1$$

Thus, the 1^{st} order Linear Variational Equations become

$$\boxed{\begin{aligned}
\dot{\tilde{\varepsilon}}_1 &= \frac{\tilde{\omega}_1}{2} - \Omega\tilde{\varepsilon}_3 \\
\dot{\tilde{\varepsilon}}_2 &= 0 \\
\dot{\tilde{\varepsilon}}_3 &= \frac{\tilde{\omega}_2}{2} + \Omega\tilde{\varepsilon}_1 \\
\dot{\tilde{\varepsilon}}_4 &= 0 \\
\dot{\tilde{\omega}}_1 &= Q\Omega\tilde{\omega}_3 - 6\gamma\Omega^2\tilde{\varepsilon}_1 \\
\dot{\tilde{\omega}}_3 &= -Q\Omega\tilde{\omega}_1
\end{aligned}}$$

(c) Derive the characteristic equation.

Define $z = [\tilde{\varepsilon}_1 \ \tilde{\varepsilon}_2 \ \tilde{\varepsilon}_3 \ \tilde{\varepsilon}_4 \ \tilde{\omega}_1 \ \tilde{\omega}_3]$

Then $\dot{z} = zA$ where

$$A = \begin{bmatrix} 0 & 0 & \Omega & 0 & -6x\Omega^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -\Omega & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & -2\Omega \\ 0 & 0 & \frac{1}{2} & 0 & 2\Omega & 0 \end{bmatrix}$$

→ Since $\dot{\varepsilon}_2 = \dot{\varepsilon}_4 = 0$ at least 2 eigenvalues are 0

so say $\lambda_1 = 0$ & $\lambda_2 = 0$

exclude the 0 terms

$$[\dot{\tilde{\varepsilon}}_1 \ \dot{\tilde{\varepsilon}}_3 \ \dot{\tilde{\omega}}_1 \ \dot{\tilde{\omega}}_3] = [\tilde{\varepsilon}_1 \ \tilde{\varepsilon}_3 \ \tilde{\omega}_1 \ \tilde{\omega}_3] \begin{bmatrix} 0 & \Omega & -6x\Omega^2 & 0 \\ -\Omega & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & -2\Omega \\ 0 & \frac{1}{2} & 2\Omega & 0 \end{bmatrix}$$

the CE is calculated as the following

$$|A-\lambda U| = \begin{vmatrix} -\lambda & \Omega & -6x\Omega^2 & 0 \\ -\Omega & -\lambda & 0 & 0 \\ \frac{1}{2} & 0 & -\lambda & -Q\Omega \\ 0 & \frac{1}{2} & Q\Omega & -\lambda \end{vmatrix} = 0$$

$$= -\lambda \begin{vmatrix} -\lambda & 0 & 0 \\ 0 & -\lambda & -Q\Omega \\ \frac{1}{2} & Q\Omega & -\lambda \end{vmatrix} - \Omega \begin{vmatrix} -\Omega & 0 & 0 \\ \frac{1}{2} & -\lambda & -Q\Omega \\ 0 & Q\Omega & -\lambda \end{vmatrix} - 6x\Omega^2 \begin{vmatrix} -\Omega & -\lambda & 0 \\ \frac{1}{2} & 0 & -Q\Omega \\ 0 & \frac{1}{2} & -\lambda \end{vmatrix}$$

$$= \lambda^2 \begin{vmatrix} -\lambda & -Q\Omega \\ Q\Omega & -\lambda \end{vmatrix} + \Omega^2 \begin{vmatrix} -\lambda & -Q\Omega \\ Q\Omega & -\lambda \end{vmatrix} - 6x\Omega^2 \left(-\Omega \begin{vmatrix} 0 & -Q\Omega \\ \frac{1}{2} & -\lambda \end{vmatrix} + \lambda \begin{vmatrix} \frac{1}{2} & -Q\Omega \\ 0 & -\lambda \end{vmatrix} \right)$$

$$= \lambda^2(\lambda^2 + Q^2\Omega^2) + \Omega^2(\lambda^2 + Q^2\Omega^2) - 6x\Omega^2\left[-\Omega\left(\frac{Q\Omega}{2}\right) + \lambda\left(-\frac{\lambda}{2}\right)\right]$$

$$= \lambda^4 + \lambda^2 Q^2\Omega^2 + \lambda^2\Omega^2 + Q^2\Omega^4 - 6x\Omega^2\left(-\frac{Q\Omega^2}{2} - \frac{\lambda^2}{2}\right)$$

$$= \lambda^4 + \lambda^2 Q^2\Omega^2 + \lambda^2\Omega^2 + Q^2\Omega^4 - 3xQ\Omega^4 + 3x\lambda^2\Omega^2$$

$$= \lambda^4 + (Q^2\Omega^2 + \Omega^2 + 3x\Omega^2)\lambda^2 + Q^2\Omega^4 - 3xQ\Omega^4$$

$$= \lambda^4 + (Q^2 + 3x + 1)\Omega^2\lambda^2 + (Q - 3x)Q\Omega^4 = 0$$

$$\boxed{\text{CE:} \quad \lambda^4 + (Q^2 + 3x + 1)\Omega^2\lambda^2 + (Q - 3x)Q\Omega^4 = 0}$$

(d) Determine the characteristic roots for k = -40, -15, 6, 12, 30, 80.
What predictions about stability are available from the roots? Are they consistent with your conclusions from the numerical results?
Determine the minimum positive and negative values of k that yield potential stability to three significant digits. Do the critical positive and negative values have the same magnitude? Is that reasonable?

Let $\lambda^2 = \Lambda$ (* capital lambda)

then

$$CE: \Lambda^2 + (Q^2+3x+1)\Omega^2\Lambda + (Q-3x)Q\Omega^4 = 0$$

solve this for Λ

$$\Lambda = \frac{1}{2}\left\{-(Q^2+3x+1)\Omega^2 \pm \sqrt{[(Q^2+3x+1)\Omega^2]^2 - 4(Q-3x)Q\Omega^4}\right\}$$

thus,

$$\lambda_i = \pm\sqrt{\frac{1}{2}\left\{-(Q^2+3x+1)\Omega^2 \pm \sqrt{[(Q^2+3x+1)\Omega^2]^2 - 4(Q-3x)Q\Omega^4}\right\}}$$

$(i = 3, 4, 5, 6)$

$$\lambda_i = \pm\sqrt{\frac{1}{2}\left\{-(Q^2+3x+1) \pm \sqrt{[(Q^2+3x+1)]^2 - 4(Q-3x)Q}\right\}}\;\Omega$$

$(i = 3, 4, 5, 6)$

since

$$Q = y + (y+1)x, \quad x = \frac{J}{I} - 1, \quad y = k - 1$$

Now, for

$$k = -40, -15, 6, 12, 30, 80$$

Obtain eigen values λ per Ω $\rightarrow$ $\frac{\lambda_i}{\Omega}$

Tabulated Results of Eigenvalues (calculated using MATLAB)

$\frac{\lambda_i}{\Omega}$	k = -40	k = -15	k = 6	k = 12	k = 30	k = 80
$i = 1$	0	0	0	0	0	0
$i = 2$	0	0	0	0	0	0
$i = 3$	1.1089i	1.3019i	1.2493	0.41372	0.82779i	0.94172i
$i = 4$	-1.1089i	-1.3019i	-1.2493	-0.41372	-0.82779i	-0.94172i
$i = 5$	10.887i	4.4292i	0.74877i	1.7091i	6.3494i	18.944i
$i = 6$	-10.887i	-4.4292i	-0.74877i	-1.7091i	-6.3494i	-18.944i

Analysis

k	Stability for Linear Systems	Stability for Non-Linear Systems
k = -40	Marginally stable	Inconclusive
k = -15	Marginally stable	Inconclusive
k = 6	Unstable	Unstable
k = 12	Unstable	Unstable
k = 30	Marginally stable	Inconclusive
k = 80	Marginally stable	Inconclusive

- Overall, our analysis in the numerical simulations are congruent with the results indicated by the eigenvalues. That is, when the |k| value is small the motion is inclined to become an unstable motion, whereas when |k| is large stability is improved. Whereby, it can be said that the increase of spin is a factor of stabilizing the system.

Bounds of Spin Factor k

Subsequently, we find the bounds for the spin factor k in which the system tends to instability. This is done using MATLAB.

Lower Bound of k	Upper Bound of k
-8.189	13.000

- The minimum positive k-value: 13.000
- The maximum negative k-value: -8.189
- The system is fixed in the orbit frame and exposed to gravity torque, and therefore, it is logical that the lower and upper bounds have different magnitudes. This explains why we are required to have more spin enable to shift the system to a stable state by counteracting the gravity torque imposed on the body in a certain direction.

(e) Select two more values of k that have not yet been examined. Select a value that linear analysis predicts as unstable; select a value that may have a chance for stability. Plot the time history for γ. Are the numerical results consistent with your predictions?
Plot the results for 10 revolutions. Any change in the conclusions?

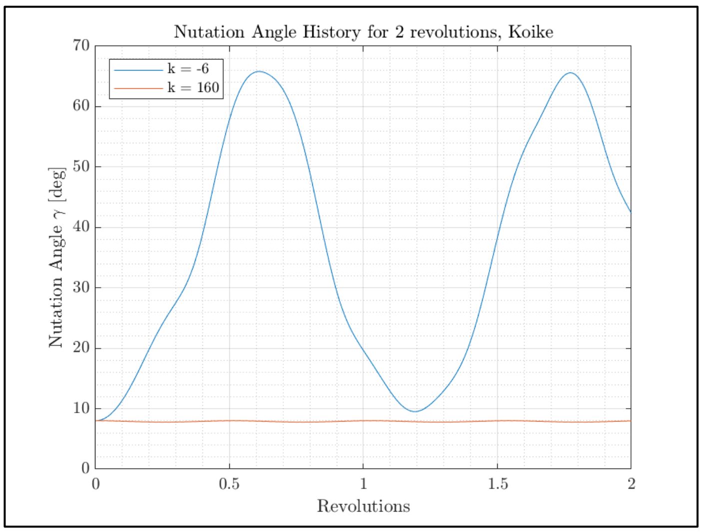
The spin factor k that has been selected for this analysis are the following.

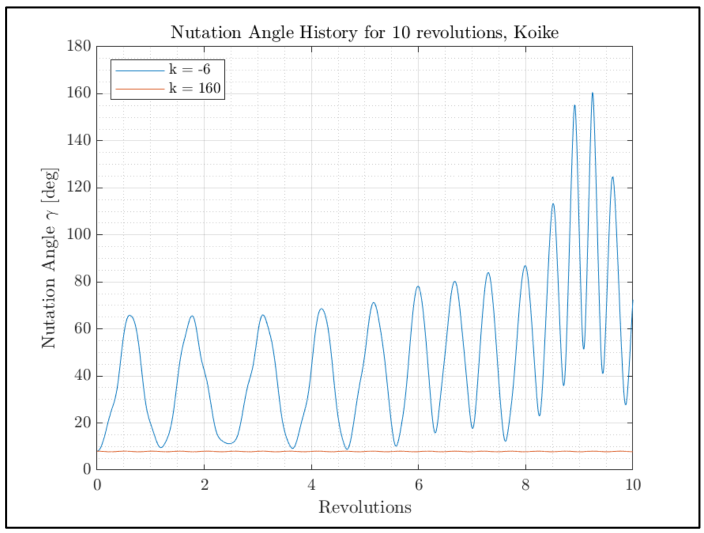
1. Predicted as unstable: -6
2. Chance of being stable: 160

For this simulation we will use the following initial conditions

1. $\Upsilon_0 = 8^o$
2. 2 revolutions
3. 10 revolutions for the nutation angle plot

Simulation Results





Analysis

1. The numerical integration illustrates results that are congruent with the simulations that we have done throughout PS9*. When the k-value was within the range of [-8.189, 13.000] the system became instable as we have predicted and when the it was outside the range the body was supplied with enough spin to overcome the factors that were causing instability.
2. From the plot with 10 revolutions, we can tell that by extending the simulation to 10 revolutions was not enough to alter our conclusion.

(f) Do the numerical results support conclusions from the analytical investigation of stability? Comment. Does the numerical study add any information that was unavailable from the analytical prediction alone?

Analysis

1. From all the simulations conducted, we can deduce that the numerical solutions do in fact support the conclusions from the analytical investigation of stability. This is because when the system is shown to unstable from our analytical analysis, our numerical results also display results that have unpredictable behaviors. Whereas, when we determine the system to be marginally stable from our analytical results, our simulations output plots which can be identified as stable.
2. The analytical solution only allows us to determine whether the system is stable, which in our case was indicated as marginally stable, or as unstable. However, the numerical solution provides us with the following additional information:
    a. If the motion is to be stable at what values of the dependent variables such as the time or number of revolutions is the motion apt to be unstable.
    b. The history of the angles ϒ, α, β, and σ which characterize the motion, and the comparisons of what kind of inclinations the angles show at when the system is stable or unstable.
    c. The specific bounds of the angles for which determines the values to be stable or unstable.

Appendix

AAE 440 PS9 Problem 1

```
clear all; close all; clc;
fdir = 'C:\Users\Tomo\Desktop\studies\2020-Spring\AAE440\MATLAB\outputs\HW9';
set(groot, 'defaulttextinterpreter','latex');
set(groot, 'defaultAxesTickLabelInterpreter','latex');
set(groot, 'defaultLegendInterpreter','latex');
```

(c)

```
% Defining System Properties
I       = 400;          % transverse moment of inertia [kg-m2]
J       = 100;          % axial moment of inertia      [kg-m2]
k       = [-1 15 -2 20];    % spin factor
gamma0  = [0 8 4 16];      % Initial perturbation  [deg]
rev     = 0:0.001:4;       % Integration revolutions

% Iterations for different spin factors
for i = 1:2
    % *Problem (c) part (i)
    % Iterations for different initial gammas
    for j = 1:3
        % Nutation History
        [rev, Nut, Prec, deltaK] = Nut_Prec_History(rev,k(i),gamma0(j),I,J);
        % Plot Nutation History vs Revolutions
        fig1 = figure(i);
            grid on; grid minor; box on; hold on;
            if i == 1
                ylim([-3 180]);
            else
                ylim([-3 12]);
            end
            plot(rev, Nut)
        % Plot Euler Constraint Perturbation
        fig2 = figure(i+2);
            grid on; grid minor; box on; hold on;
            plot(rev, deltaK)
    end
    figure(i)
        legend('$\gamma_0 = 0^\circ$', '$\gamma_0 = 8^\circ$', '$\gamma_0 = 4^\circ$')
        ylabel('Nutation Angle $\gamma$ [deg]')
        xlabel('Revolutions')
        title_str = sprintf('Nutation Angle History for k = %d, Koike',k(i));
        title(title_str)
    figure(i+2)
        legend('$\gamma_0 = 0^\circ$', '$\gamma_0 = 8^\circ$', '$\gamma_0 = 4^\circ$')
        ylabel('$K - K_0$')
        xlabel('Revolutions')
        title_str = sprintf('Euler Constraint Perturbation for k = %d, Koike',k(i));
        title(title_str)

    % Save Figures
    file_str = sprintf('P1c-i_Nutation_History_k=%d.png',k(i));
    saveas(fig1, fullfile(fdir, file_str));
    file_str = sprintf('P1c-i_Euler_Constraint_Perturbation_k=%d.png',k(i));
    saveas(fig2, fullfile(fdir, file_str));
```

```
  for j = 4:4
    % ** Problem (c) part (ii)
    % Nutation History
    [rev, Nut, Prec, deltaK] = Nut_Prec_History(rev,k(i),gamma0(j),I,J);

    % Plot Nutation History vs Revolutions
    fig1 = figure(i);
      hold on;
      if i == 1
        ylim([-3 180]);
      else
        ylim([-3 25]);
      end
      plot(rev, Nut)

    % Plot Euler Constraint Perturbation
    fig2 = figure(i+2);
      hold on;
      plot(rev, deltaK)
  end

    figure(i)
      legend('$\gamma_0 = 0^\circ$', '$\gamma_0 = 8^\circ$', ...
          '$\gamma_0 = 4^\circ$', '$\gamma_0 = 16^\circ$')
      ylabel('Nutation Angle $\gamma$ [deg]')
      xlabel('Revolutions')
      title_str = sprintf('Nutation Angle History when k = %d, Koike',k(i));
      title(title_str)

    figure(i+2)
      legend('$\gamma_0 = 0^\circ$', '$\gamma_0 = 8^\circ$', ...
          '$\gamma_0 = 4^\circ$', '$\gamma_0 = 16^\circ$')
      ylabel('$K - K_0$')
      xlabel('Revolutions')
      title_str = sprintf('Euler Constraint Perturbation when k = %d, Koike',k(i));
      title(title_str)

  % Save Figures
  file_str = sprintf('P1c-ii_Nutation_History_k=%d.png', k(i));
  saveas(fig1, fullfile(fdir, file_str));
  file_str = sprintf('P1c-ii_Euler_Constraint_Perturbation_k=%d.png', k(i));
  saveas(fig2, fullfile(fdir, file_str));
end
close all;
```

(d)

```
rev = 0:0.001:20; % Integration revolutions

% Iteration for different spin factors
for i = 1:4
  % *Problem (d) part (i)
  % Iteration for different initial gammas
  for j = 1:4

    % Nutation History
    [rev, Nut] = Nut_Prec_History(rev,k(i),gamma0(j),I,J);
```

```
    % Plot Nutation History vs Revolutions
    fig = figure(i);
      grid on; grid minor; box on; hold on;
      if i == 1 || i == 3
        ylim([-3 180]);
      else
        ylim([-3 24]);
      end
      plot(rev, Nut)
  end
  figure(i)
    legend('$\gamma_0 = 0^\circ$', '$\gamma_0 = 8^\circ$', ...
        '$\gamma_0 = 4^\circ$', '$\gamma_0 = 16^\circ$')
    ylabel('Nutation Angle $\gamma$ [deg]')
    xlabel('Revolutions')
    title_str = sprintf('Nutation Angle History when k = %d, Koike',k(i));
    title(title_str)

  % Saving Figures
  file_str = sprintf('P1d_Nutation_History_k=%d.png', k(i));
  saveas(fig, fullfile(fdir, file_str));

end

close all;

% Symmetry Test
for i = 1:4

  % Defining System Properties
  gamma0 = 4;             % Initial perturbation  [deg]
  rev    = 0:0.001:4;     % Integration Revolutions
  k      = [-2 2 -20 20]; % spin factor

  % Nutation History
  [rev, Nut] = Nut_Prec_History(rev,k(i),gamma0,I,J);

  % Plotting Nutation History vs Revolutions
  fig = figure(1);
    grid on; grid minor; box on; hold on;
    plot(rev, Nut)
end

for i = 1:2
  % Plotting Nutation Angle History vs Revolutions
  figure(1);
    legend('$k = -2$', '$k = 2$', '$k = -20$', '$k = 20$')
    ylabel('Nutation Angle $\gamma$ [deg]')
    xlabel('Revolutions')

  if i == 1
    ylim([-3 180]);
    title({'Nutation Angle History for k = $\pm$2, $\pm$20','$\gamma_0 = 4^\circ$, Koike'})
    saveas(fig, fullfile(fdir, 'P1d_symmetry_test.png'));
  else % Magnified View
    ylim([2 10]);
    title({'Nutation Angle History for k = $\pm$2, $\pm$20', '$\gamma_0 = 4^\circ$ Magnified View, Koike'})
    saveas(fig, fullfile(fdir, 'P1d_symmetry_test_close_up.png'));
```

```
    end
end
```

AAE 440 PS9 Problem 2

```
clear all; close all; clc;
fdir = 'C:\Users\Tomo\Desktop\studies\2020-Spring\AAE440\MATLAB\outputs\HW9';
set(groot, 'defaulttextinterpreter','latex');
set(groot, 'defaultAxesTickLabelInterpreter','latex');
set(groot, 'defaultLegendInterpreter','latex');
rgb = linspecer(70);
```

```
% Defining System Properties
I       = 400;          % transverse moment of inertia [kg m2]
J       = 100;          % axial moment of inertia      [kg m2]
gamma0  = 8;            % Initial perturbation  [deg]
rev     = 0:0.001:2;    % Integration revolutions

% spin factor
k0      = [-15 6 12 30 80];
k       = [-15 6 12 30 80 100];
```

(a)

```
% Iteration for different spin factors
for i = 1:length(k)-1
    % Nutation and Precession History
    [rev, Nut, Prec] = Nut_Prec_History(rev,k(i),gamma0,I,J);
    % Plot Gammas
    fig1 = figure(1);
        hold on;
        plot(rev, Nut, 'DisplayName',['k = ',num2str(k(i))])
    % Plot Sigmas
    sigma = 360*rev;
    fig2 = figure(2);
        hold on;
        plot(rev, sigma, 'DisplayName',['k = ',num2str(k(i))])
    % Plot Alphas
    fig3 = figure(3);
        hold on;
        plot(rev, Prec, 'DisplayName',['k = ',num2str(k(i))])
    % Plot Betas
    beta = Prec+sigma;
    fig4 = figure(4);
        hold on;
        plot(rev, beta, 'DisplayName',['k = ',num2str(k(i))])
end

% Saving Figures
for i = 1:4
    fig = figure(i);
        grid on; grid minor; box on; legend; xlabel('Revolutions');
        switch i
            case 1
                ylabel('Nutation Angle $\gamma$ [deg]')
                title({'Nutation Angle History, Koike'})
                saveas(fig, fullfile(fdir, 'P2a_Nutation_History.png'));
            case 2
                ylabel('Sigma Angle $\sigma$ [deg]')
                title({'Sigma Angle History, Koike'})
```

```
            saveas(fig, fullfile(fdir, 'P2a_Sigma_History.png'));
        case 3
            ylabel('Precession Angle $\alpha$ [deg]')
            title({'Precession Angle History, Koike'})
            saveas(fig, fullfile(fdir, 'P2a_Precession_History.png'));
        case 4
            ylabel('Beta Angle $\beta$ [deg]')
            title({'Beta Angle History, Koike'})
            saveas(fig, fullfile(fdir, 'P2a_Beta_History.png'));
    end
end

% Add k = 100 case to figures
% Nutation and Precession History
[rev, Nut, Prec] = Nut_Prec_History(rev,k(end),gamma0,I,J);
% Plotting Gammas
figure(1);
  hold on;
  plot(rev, Nut, 'DisplayName',['k = ',num2str(k(end))])
% Plotting Sigmas
sigma = 360*rev;
figure(2);
  hold on;
  plot(rev, sigma, 'DisplayName',['k = ',num2str(k(end))])
% Plotting Alphas (Precession)
figure(3);
  hold on;
  plot(rev, Prec, 'DisplayName',['k = ',num2str(k(end))])
% Plotting Betas (Inertial Precession)
beta = Prec+sigma;
figure(4);
  hold on;
  plot(rev, beta, 'DisplayName',['k = ',num2str(k(end))])

% Saving Figures
for i = 1:4
  fig = figure(i);
    switch i
        case 1
            saveas(fig, fullfile(fdir, 'P2a_Nutation_History_k=100.png'));
        case 2
            saveas(fig, fullfile(fdir, 'P2a_Sigma_History_k=100.png'));
        case 3
            saveas(fig, fullfile(fdir, 'P2a_Precession_History_k=100.png'));
        case 4
            saveas(fig, fullfile(fdir, 'P2a_Beta_History_k=100.png'));
    end
end
close all;
```

(b)

```
% Sample calculations for the delta_beta of perturbation theory
d_B = zeros([1 5]);
ct = 1;
for m = k0
  d_B(ct) = rad2deg(3*pi*(I/J - 1)*cosd(gamma0)/m);
  ct = ct + 1;
end
```

```
maxAmp = zeros([5,1]);
N = length(k)-1;
rgb_b = linspecer(N);
for j = 1:2
    ct = 1;
    % Iteration 1: beta with deltaBeta
    % Iteration 2: beta with linear fit beta
    for i = 1:length(k)-1
        % Nutation angle and precession angle history
        [rev, Nut, Prec] = Nut_Prec_History(rev,k(i),gamma0,I,J);
        % Plot Betas
        sigma = 360*rev;
        beta = Prec+sigma;
        switch j
            case 1
            % Compute deltaBeta
            deltaBeta = delta_inerPrec(I,J,k(i),gamma0);
            fig1 = figure(1);
                hold on;
                plot(rev, beta, '-', 'Color', rgb_b(ct,:), ...
                    'DisplayName',['k = ',num2str(k(i))])
                plot(1, deltaBeta,'x', 'Color', rgb_b(ct,:), ...
                    'DisplayName',['$\Delta\beta$ k=',num2str(k(i))])
            case 2
            % Compute Linear Fit and Maximum Amplitude
            p = polyfit(rev,beta,1);
            beta_fit = polyval(p,rev);
            AC = abs(beta-beta_fit);
            theta = atan(p(1));
            maxAmp(i) = max(AC)*cos(theta);
            fig2 = figure(2);
                hold on;
                plot(rev, beta, '-', 'Color', rgb_b(ct,:), ...
                    'DisplayName',['k = ',num2str(k(i))])
                plot(rev, beta_fit, '--', 'Color', rgb_b(ct,:), ...
                    'DisplayName',['fit k=',num2str(k(i))])
        end
        ct = ct + 1;
    end
    grid on; grid minor; box on; legend;
    xlabel('Revolutions'); ylabel('Beta Angle $\beta$ [deg]')
    switch j
        case 1
        title({'$\beta$ Angle History with $\Delta\beta$, Koike'})
        saveas(fig1, fullfile(fdir, 'P2b_Beta_History_with_deltaBeta.png'));
        case 2
        title({'$\beta$ Angle History with Linear Fit, Koike'})
        saveas(fig2, fullfile(fdir, 'P2b_Beta_History_with_linfit.png'));
    end
end
close all;
```

(c)

```
% Constant Precession
N = length([5:0.1:9,10:0.1:12.5,1:10]);
rgb_c = linspecer(N);
for j = [1 2 3]
```

```
    if j == 1
        k = 5:0.1:9;
        ct = 1;
    elseif j == 2
        k = 10.0:0.1:12.5;
        ct = length(5:0.1:9) + 6;
    else
        k = [5:0.1:9, 10:0.1:12.5];
        ct = 1;
    end
    for i = 1:length(k)
        % Nutation and Precession History
        [rev, Nut, Prec] = Nut_Prec_History(rev,k(i),gamma0,I,J);
        % Plot Alphas
        figure(j);
            hold on;
            if i == 1 || i == length(k)
                plot(rev, Prec, 'Color', rgb_c(ct,:), 'DisplayName',['k = ',num2str(k(i))])
            else
                plot(rev, Prec, 'Color', rgb_c(ct,:), 'HandleVisibility','off')
            end
        ct = ct + 1;
    end
    hold off;
    % Save Figures
    fig = figure(j);
        grid on; grid minor; box on; lgd = legend; xlabel('Revolutions');
        lgd.Location = "northwest";
        ylabel('Precession Angle $\alpha$ [deg]')
        title({'Precession Angle History, Koike'})
        file_str = sprintf('P2c_Precession_History_k=%.2f-%.2f.png', k(1),k(end));
        saveas(fig, fullfile(fdir, file_str));
end
```

```
% Optimization for k - value
% Find the optimal k where the sum of the maximum amplitude of both
% nutation and precession become the minimum
[k_opt,fval,exitflag,output] = HW9_P2_pattern_search_opt_k(6,5,20);
```

```
% Open created txt file with all the precession and nutation
opt_data = readmatrix("inputs\opt_prec_nut_data.txt");
prec_opt = opt_data(:,1);
nut_opt = opt_data(:,2);
ct = 1:numel(prec_opt);

% Plot the transition of the optimization for the precession and nutation
fig0 = figure("Renderer","painters");
    plot(ct,prec_opt,'.r',"MarkerSize",7)
    title("Optimization of Precession and Nutation, Koike")
    xlabel("Optimization Count")
    ylabel("Angle [deg]")
    hold on;
    plot(ct,nut_opt,'.b',"MarkerSize",7)
    hold off
    grid on; grid minor; box on;
    legend("$\alpha$","$\gamma$")
    t1 = sprintf("$\\alpha_{opt}$ = %.3f deg",prec_opt(end));
    t2 = sprintf("$\\gamma_{opt}$ = %.3f deg",nut_opt(end));
```

```
    text(60,150,t1);
    text(60,100,t2);
saveas(fig0,fullfile(fdir,"opt_prec_nut.png"));
```

```
% Constant Inertial Precession
for j = 1:2
    close all;
    switch j
        % spin factors
        case 1
            k = 1e2:1e2:1e3;
        case 2
            k = 1e3:1e3:1e4;
        case 3
            k = 1e4:1e4:1e5;
    end
    for i = 1:length(k)
        % Nutation and Precession History
        [rev, Nut, Prec] = Nut_Prec_History(rev,k(i),gamma0,I,J);
        % Calculating Inertial Precession
        sigma = 360*rev;
        beta = Prec+sigma;
        % Plot Betas
        figure(2);
            hold on;
            plot(rev, beta, 'Color', rgb((i+j)*3,:), 'DisplayName',['k = ',num2str(k(i))])
    end
    % Saving Figures
    fig = figure(2);
        grid on; grid minor; box on; lgd = legend; xlabel('Revolutions');
        lgd.Location = "northwest";
        ylabel('Beta Angle $\beta$ [deg]')
        title({'Beta Angle History, Koike'})
        file_str = sprintf('P2c_Inertial_Precession_History_k=%d-%d.png', k(1),k(end));
        saveas(fig, fullfile(fdir, file_str));
end
```

AAE 440 PS9 Problem 3

```
clear all; close all; clc;
fdir = 'C:\Users\Tomo\Desktop\studies\2020-Spring\AAE440\MATLAB\outputs\HW9';
set(groot, 'defaulttextinterpreter','latex');
set(groot, 'defaultAxesTickLabelInterpreter','latex');
set(groot, 'defaultLegendInterpreter','latex');
```

```
% Defining System Properties
I      = 400;          % transverse moment of inertia [kg m2]
J      = 100;          % axial moment of inertia      [kg m2]
gamma0  = 8;           % Initial perturbation  [deg]
rev     = 0:0.001:2;   % Integration revolutions

% spin factor
k      = [-40, -15, 6, 12, 30, 80];
```

(d)

```
% Characteristic Roots for different spin factor
lambda = zeros(6,6);
for i = 1:length(k)
```

```
    lambda(:,i) = eigenvalues(I,J,k(i));
end
res_array = [lambda(:,1) lambda(:,2) lambda(:,3) ...
            lambda(:,4) lambda(:,5) lambda(:,6)];
res_table = array2table(res_array, "VariableNames",...
            {'k = -40','k = -15','k = 6','k = 12','k = 30','k = 80'});
```

```
% Finding bounds for stable k
k_neg = -15:0.0001:0;
k_neg_bound = 0;
for i = 1:length(k_neg)
    lambda = eigenvalues(I,J,k_neg(i));
    for j = 1:6
        if real(lambda(j)) <= 0
            continue
        else
            k_neg_bound = k_neg(i);
            break
        end
    end
    if k_neg_bound == k_neg(i)
        break
    end
end
k_pos = 30:-0.0001:0;
k_pos_bound = 0;
for i = 1:length(k_pos)
    lambda = eigenvalues(I,J,k_pos(i));
    for j = 1:6
        if real(lambda(j)) <= 0
            continue
        else
            k_pos_bound = k_pos(i);
            break
        end
    end
    if k_pos_bound == k_pos(i)
        break
    end
end
k_bound = [k_neg_bound, k_pos_bound];
```

(e)

```
% Unstable and Potentially Stable k
k = [-6, 160];
for j = 1:2
    % Repeat for different number of revolutions
    switch j
        case 1
        rev = 0:0.001:2;
        case 2
        rev = 0:0.001:10;
    end
    for i = 1:length(k)
        % Numerical Integration
        [rev, Nut, Prec] = Nut_Prec_History(rev,k(i),gamma0,I,J);
        % Plot Nutation
        figure(j);
```

```
        hold on;
        plot(rev, Nut, 'DisplayName',['k = ',num2str(k(i))])
    end

    fig = figure(j);
        grid on; grid minor; box on; lgd = legend; xlabel('Revolutions');
        lgd.Location = 'northwest';
        ylabel('Nutation Angle $\gamma$ [deg]')
        str = sprintf('Nutation Angle History for %d revolutions, Koike',rev(end));
        title(str)
        str = sprintf('P3e_Nutation_History_rev=%d.png',rev(end));
    saveas(fig, fullfile(fdir, str));
end
hold off; close all;
```

Functions

```
function lambdas = eigenvalues(I,J,k)
    %{
    This function outputs EIGENVALUES PER OMEGA (orbital angular velocity)
    to the characteristic equation
    %}

    x = J/I - 1;        % shape factor
    y = k - 1;          % spin factor
    Q = y + (1 + y)*x;
    A = 3*x+Q^2+1;
    B = 3*x*Q+Q^2;
    % Eigenvalues
    lambdas = [ 0                        ;
                0                        ;
              sqrt((-A + sqrt(A^2-4*B))/2);
             -sqrt((-A + sqrt(A^2-4*B))/2);
              sqrt((-A - sqrt(A^2-4*B))/2);
             -sqrt((-A - sqrt(A^2-4*B))/2)];
end
```

```
function dy = nond_EOM(v,y,I,J,k)
    %{
     Function:    nond_EOM()
     Author:      Tomoki Koike
     Description: The nondimensional differential equations of dynamic and
                  kinematic equations for the numerical integration
                  process.
     >>Inputs
        v:  number of revolutions
        y:  nond angular velocities, euler parameters, K-K0
        I:  transverse moment of inertia
        J:  axial moment of inertia
        k:  spin factor
     Outputs<<
        dy: differential y
    %}

    q = k - 1;      % spin factor
    K = 1 - J/I;    % shape factor

    % nond Dynamic EOMs
```

```
    dy = zeros(8,1);
    dy(1) = 2*pi*(-K*y(2)*y(3) + q*y(3) +  6*K*(y(5)*y(6) + y(4)*y(7))*(1 - 2*y(4)^2 - 2*y(5)^2));
    dy(2) = 0;
    dy(3) = 2*pi*( K*y(1)*y(2) - q*y(1) - 12*K*(y(6)*y(4) - y(5)*y(7))*(y(5)*y(6) + y(4)*y(7)));

    % nond Kinematical EOMs
    dy(4) = pi*( y(1)*y(7) - (y(2) + 1 - q)*y(6) + y(3)*y(5) );
    dy(5) = pi*( y(1)*y(6) + (y(2) - 1 - q)*y(7) - y(3)*y(4) );
    dy(6) = pi*(-y(1)*y(5) + (y(2) + 1 - q)*y(4) + y(3)*y(7) );
    dy(7) = pi*(-y(1)*y(4) - (y(2) - 1 - q)*y(5) - y(3)*y(6) );

    % K - K0
    dy(8) = EulerConstraint(y(4),y(5),y(6),y(7));
end


function [rev, Nut, Prec, deltaK] = Nut_Prec_History(rev,k,gamma0,I,J)
    %{
     Function:    Nut_Prec_History()
     Author:      Tomoki Koike
     Description: Computes the ode45 for a spcecifc system for PS9 problem
                  1, and returns the history of the nutation, precession,
                  and Euler constraints for each revolution of the
                  simulation.
     >>Inputs
        rev:     number of revolutions
        k:       spin factor
        gamma0:  initial perturbation [deg]
        I:       transverse moment of inertia
        J:       axial moment of inertia
     Outputs<<
        rev:     revolutions
        Nut:     nutation angle history
        Prec:    precession angle history
        deltaK:  Euler Constraint Perturbation
    %}

    % Initial Conditions
    w0 = [0 k 0];
    e0 = [sind(gamma0/2) 0 0 cosd(gamma0/2)];
    y0 = [w0 e0 0];

    % Numerical Integration
    opt = odeset('RelTol', 1e-13, 'AbsTol', 1e-13);
    [rev, data] = ode45(@(v,y) nond_EOM(v,y,I,J,k),rev,y0,opt);

    % Calculate Nutation angle
    Nut = acosd(1-2*data(:,6).^2-2*data(:,4).^2);

    % Calculate Precession angle
    C12   = 2.*(data(:,4).*data(:,5)-data(:,6).*data(:,7));
    C32   = 2.*(data(:,5).*data(:,6)+data(:,4).*data(:,7));
    Prec  = zeros([length(Nut),1]);
    if gamma0 ~= 0
        for i = 1:length(Nut)
            Prec1 = round([     acosd(C32(i)/sind(Nut(i))), ...
                              -acosd(C32(i)/sind(Nut(i))), ...
                              -acosd(C32(i)/sind(Nut(i)))],3);
```

```
        Prec2 = round([    asind(C12(i)/sind(Nut(i))), ...
                     180-asind(C12(i)/sind(Nut(i))), ...
                    -180-asind(C12(i)/sind(Nut(i)))],3);
        Prec(i) = intersect(Prec1, Prec2);
      end
   end
   % Culmulative Precession
   Prec = unwrap(Prec);
   % Euler Constraint
   deltaK = data(:,8);
end
```

```
function delta_B = delta_inerPrec(I,J,k,gamma0)
   % Computing deltaBeta
   delta_B = rad2deg(3*pi/k*(I/J-1)*cosd(gamma0));
end
```

```
function obj = HW9_alphaANDgamma_opt_k(k)
   I      = 400;           % transverse moment of inertia [kg m2]
   J      = 100;           % axial moment of inertia      [kg m2]
   gamma0  = 8;            % Initial perturbation  [deg]
   rev     = 0:0.001:2;    % Integration revolutions
   % Nutation and Precession History
   [rev, Nut, Prec] = Nut_Prec_History(rev,k,gamma0,I,J);

   obj1 = max(abs(Prec));
   obj2 = max(abs(Nut));

   obj = obj1 + obj2;

   % Write to file
   afile = fopen("inputs/opt_prec_nut_data.txt",'a+');
   fprintf(afile,"%.8f %.8f\n",obj1,obj2);
   fclose(afile);
end
```

```
function [x,fval,exitflag,output] = HW9_P2_pattern_search_opt_k(x0,lb,ub)
   % Start with the default options
   options = optimoptions('patternsearch');
   % Modify options setting
   options = optimoptions(options,'Display', 'off');
   options = optimoptions(options,'PlotFcn', { @psplotbestx @psplotbestf @psplotfuncount });
   [x,fval,exitflag,output] = ...
   patternsearch(@HW9_alphaANDgamma_opt_k,x0,[],[],[],[],lb,ub,[],options);
end
```