College of Engineering
School of Aeronautics and Astronautics

AAE 36401 Lab
Control Systems Lab

Lab 5 Report
The Control of Inverted Pendulum to Balance with Sine Wave Input

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Introduction

Objective

The object of this lab is to control an inverted pendulum on a cart which is place on a track. Alike the previous lab, the pendulum will be controlled in its upright orientation; however, in this experiment the pendulum will maintain the orientation while having the cart move sideways based on a sine wave input. This is almost identical to the last experiment where the input was a square wave. The objective is the input gains into the feedback system to maintain a stable upright orientation with a more complicated sine wave input.

Method

To accomplish the objective, we have to come up with the gains of the feedback control system. For this experiment, the gains will be computed by the LQR method and the pole placement method. Then the gains will be input to the Simulink model that is synced to the experimental setup. The cart will move based on the square wave input and the data is obtain from the response.

Results

Part (i)

The first gains used were the gains obtained from the prelab

The gains using LQR:

Table 1: gains of LQR from prelab

K_1	K_2	K_3	K_4	K_5
-49.9747	156.3945	-45.5208	36.5653	31.6228

The diagonal matrix, Q used for the linear quadratic regulator is the following

$$Q = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 10 \end{pmatrix}$$

The weight, R is

$$R = 0.01$$

The poles for LQR:

Table 2: poles of LQR from prelab

λ_1	λ_2	λ_3	λ_4	λ_5
-64.3972	-2.0915 + 1.3698i	-2.0915 - 1.3698i	-1.2125 + 1.1168i	-1.2125 - 1.1168i

The gains using Pole Placement:

Table 3: gains of pole placement from prelab

K_1	K_2	K_3	K_4	K_5
-76.5544	91.0760	-37.9289	18.5217	94.4953

The corresponding poles were

Table 4: poles of pole placement from prelab

λ_1	λ_2	λ_3	λ_4	λ_5
-5+3.5i	-5-3.5i	-3+4.5i	-3-4.5i	-3

The gains were tuned because of excessive oscillation. The tuned gains are tabulated below.

The tuned gains for LQR:

Table 5: the tuned gains of LQR

K_1	K_2	K_3	K_4	K_5
-57.6834	136.7788	-47.5748	27.8695	31.6228

The diagonal matrix, Q used for the linear quadratic regulator is the following

$$Q = \begin{pmatrix} 8 & 0 & 0 & 0 & 0 \\ 0 & 6 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 10 \end{pmatrix}$$

The weight, R is

$$R = 0.01$$

The poles for this was,

Table 6: the tuned gains of LQR

$\boldsymbol{\lambda_1}$	$\boldsymbol{\lambda_2}$	$\boldsymbol{\lambda_3}$	$\boldsymbol{\lambda_4}$	$\boldsymbol{\lambda_5}$
-24.3929	-6.9848	-3.0906	-1.3639+0.4658i	-1.3639-0.4658i

The tuned gains for Pole Placement:

Table 7: the tuned gains of pole placement

$\boldsymbol{K_1}$	$\boldsymbol{K_2}$	$\boldsymbol{K_3}$	$\boldsymbol{K_4}$	$\boldsymbol{K_5}$
-88.2269	114.1343	-46.3868	23.6105	88.4623

The corresponding poles were

Table 8: the tuned poles of pole placement

$\boldsymbol{\lambda_1}$	$\boldsymbol{\lambda_2}$	$\boldsymbol{\lambda_3}$	$\boldsymbol{\lambda_4}$	$\boldsymbol{\lambda_5}$
-5+3i	-5-3i	-5	-6	-3

Analysis & Discussions

Part (i)

The time histories for LQR and Pole Placement are graphed as the following.

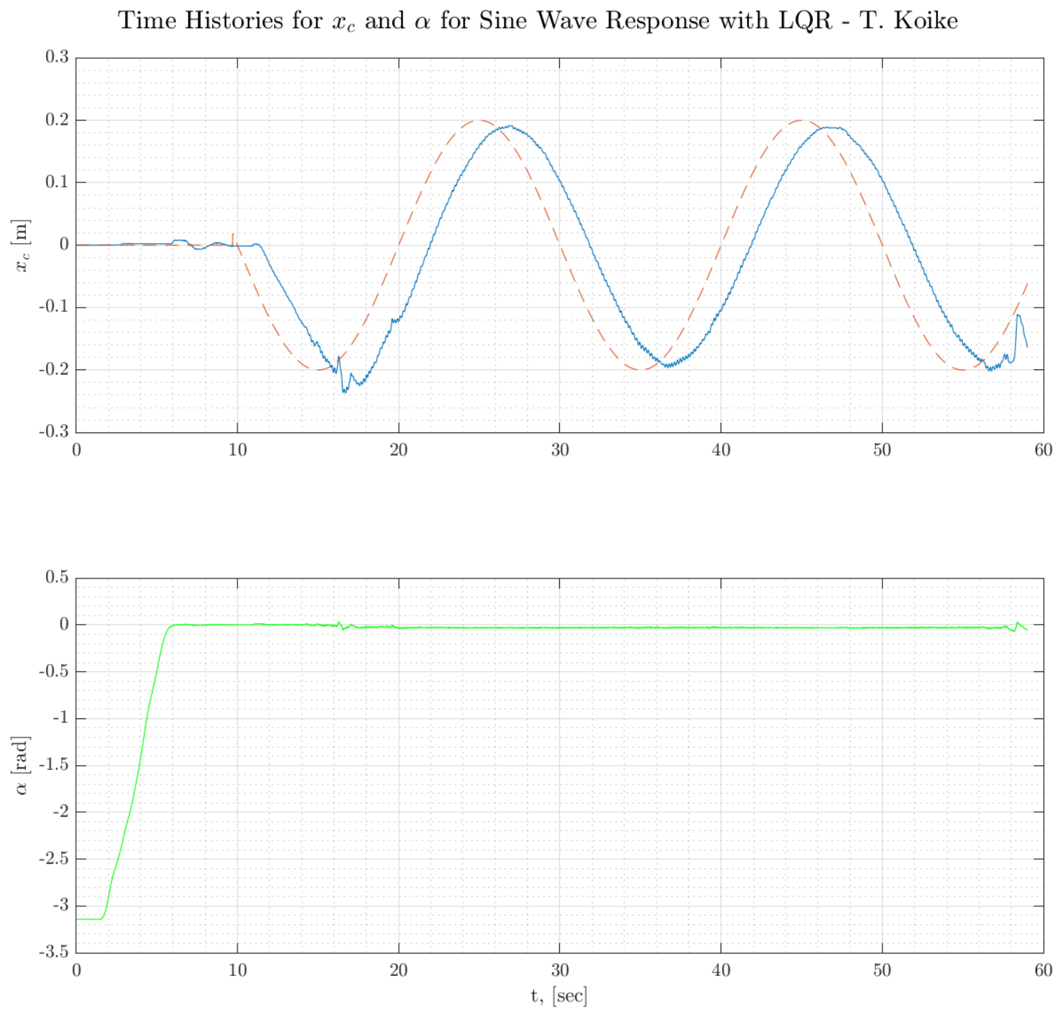


Figure 1: time history for tuned gains of LQR system

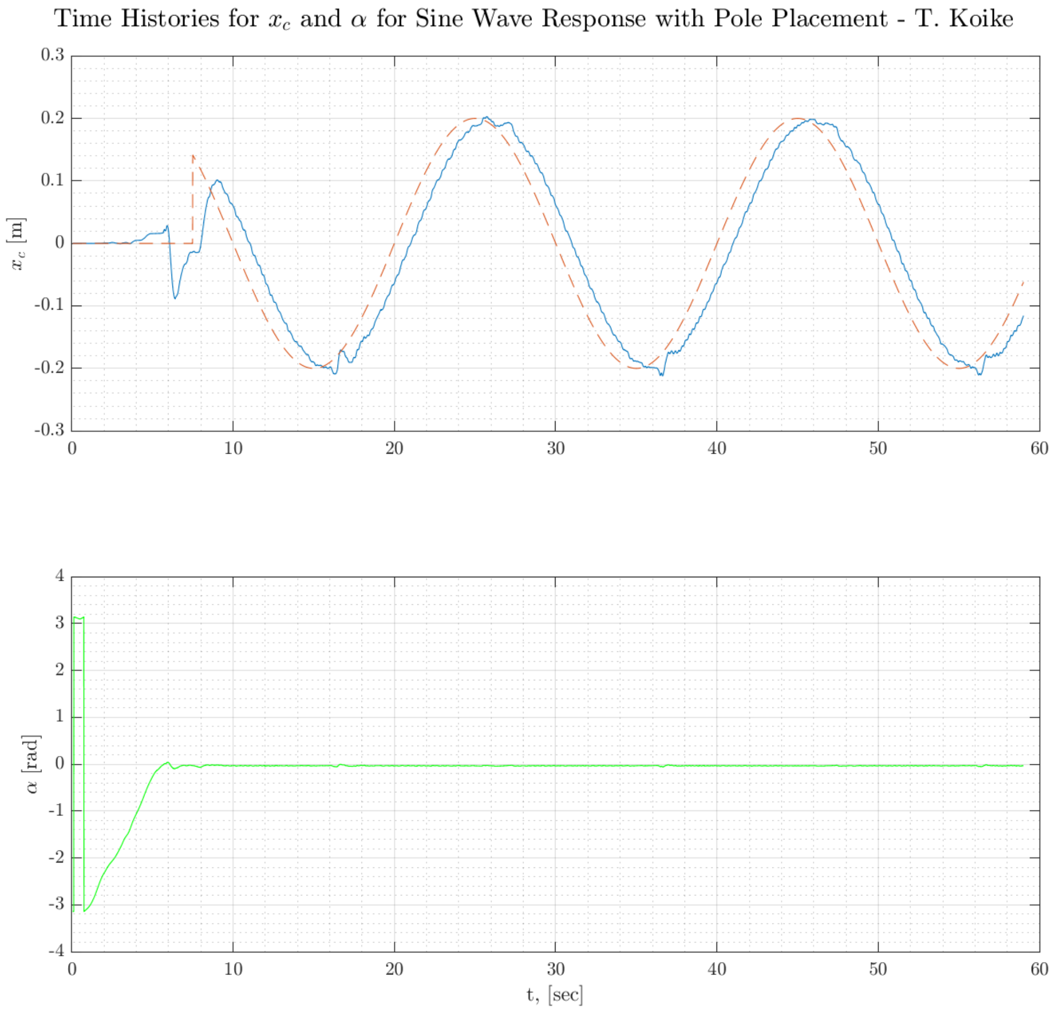


Figure 2: time history for tuned gains of pole placement system

Because the gains from the prelab had 2 complex pole pairs the response resulted in a highly shaky response. That is because when there are 2 complex pole pairs there is going to oscillation in the response. By fixing this, we were able to obtain a response that is not oscillating and follows the sine wave fairy well. As we can observe from the angle plot, the angle after the pendulum is lifted up slowly to an upright orientation is able to maintain the angle with a very small error with minimal perturbations.

However, looking closely we can see that when the amplitude is at the point of absolute magnitude the cart tends to shake to stabilize itself. We can observe more of this shake in the pole placement method than the LQR method.

Conclusion & Recommendation

Main Points

From the results, we can observe that the experiment is successful with a system that maintains a pendulum at an upright position while the cart has a dynamic motion. The angle of the pendulum is maintaining 0 degrees with a significantly small error while the cart follows a somewhat complicated sine wave input.

Theoretical/Experimental Limitations

The motion of the cart is one limitation that we can see from the plots. The motion that causes a subtle shake of the cart at turning points.

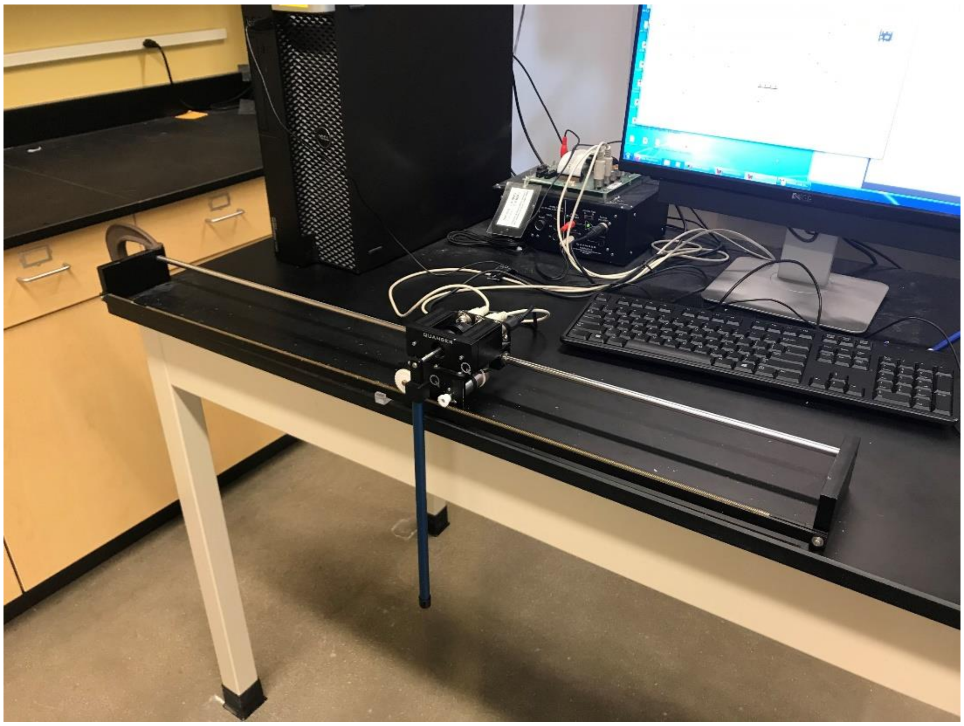
Another limitation common to all the other labs performed is the voltage limit that prevents us to have a gain larger than 200.

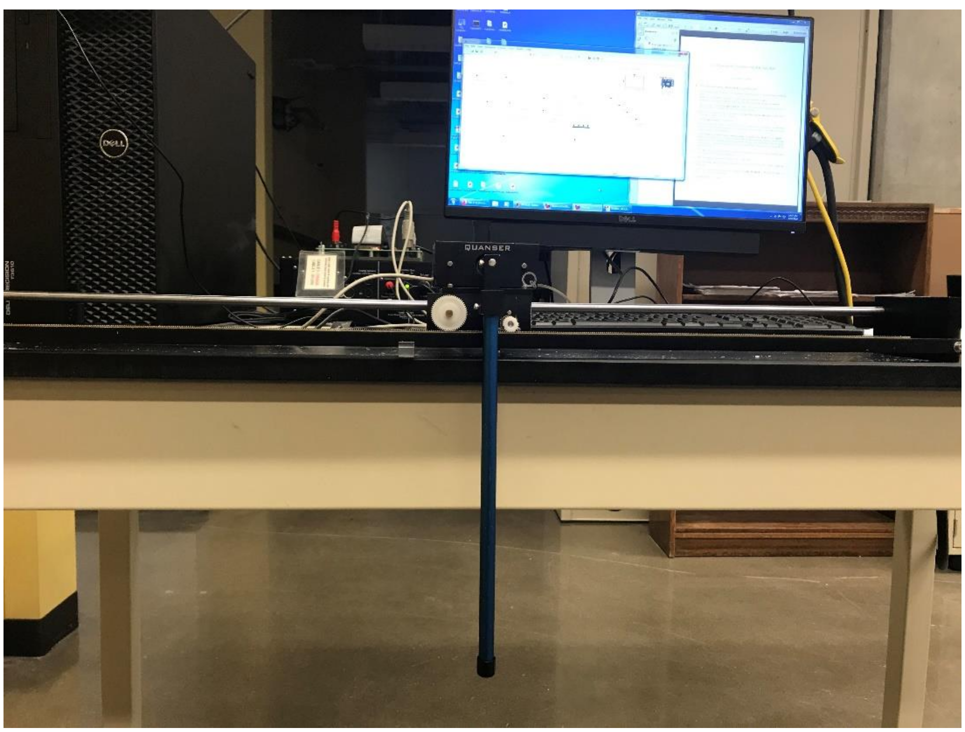
Lessons Learned & Suggestions for Improvement

The utmost lesson from this lab is the implementation of linear quadratic regulators to control complex tasks. It is rather better to use LQR for a more stable response compared to the pole placement method. We can consider evaluating the poles by conducting a Root Locus or Nyquist Analysis on the poles of the control system.

Appendix

Experiment Setup





Notations for Variables

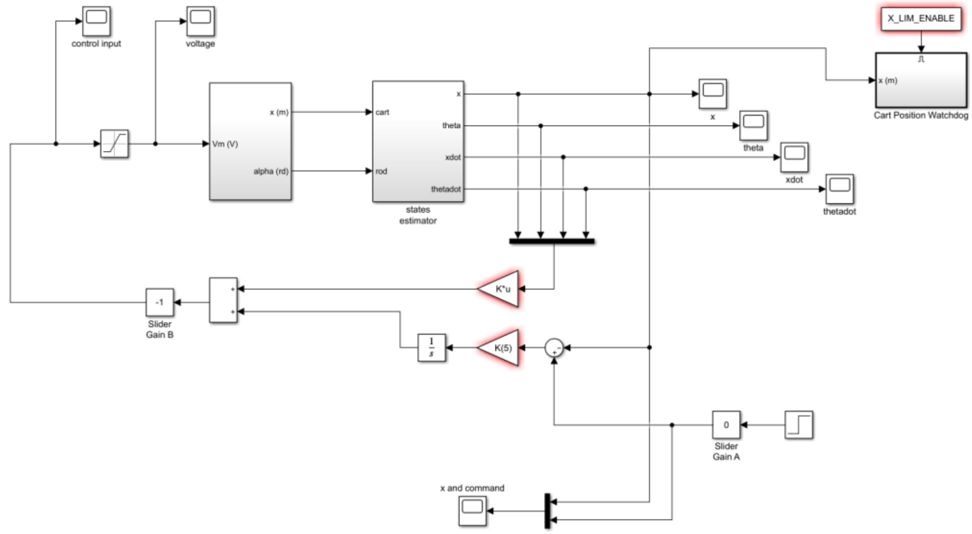
Symbol	Description	Value	Unit
R_m	motor armature resistance	2.6	Ω
L_m	motor armature inductance	0.18	mH
K_t	motor torque constant	0.00767	$N.m/A$
η_m	motor efficiency	100%	%
K_m	back-electromotive-force(EMF) constant	0.00767	$V.s/rad$
J_m	rotor moment of inertia	3.9×10^{-7}	$kg.m^2$
K_g	planetary gearbox ratio	3.71	
η_g	planetary gearbox efficiency	100%	%
M_{c2}	cart mass	0.57	kg
M_w	cart weight mass	0.37	kg
M_c	total cart weight mass including motor inertia	1.0731	kg
B_{eq}	viscous damping at motor pinion	5.4000	$N.s/m$
L_t	track length	0.990	m
T_c	cart travel	0.814	m
P_r	rack pitch	1.664×10^{-3}	$m/tooth$
r_{mp}	motor pinion radius	6.35×10^{-3}	m
N_{mp}	motor pinion number of teeth	24	
r_{pp}	position pinion radius	0.01482975	m
N_{pp}	position pinion number of teeth	56	
K_{EP}	cart encoder resolution	2.275×10^{-5}	$m/count$
M_p	long pendulum mass with T-fitting	0.230	kg
M_{pm}	medium pendulum mass with T-fitting	0.127	kg
L_p	long pendulum length from pivot to tip	0.6413	m
L_{pm}	medium pendulum length from pivot to tip	0.3365	m
l_p	long pendulum length: pivot to center of mass	0.3302	m
l_{pm}	medium pendulum length: pivot to center of mass	0.1778	m
J_p	long pendulum moment of inertia ⟳ center of mass	7.88×10^{-3}	$kg.m^2$
J_{pm}	medium pendulum moment of inertia ⟳ center of mass	1.20×10^{-3}	$kg.m^2$
B_p	viscous damping at pendulum axis	0.0024	$N.m.s/rad$
g	gravitational constant	9.81	m/s^2
v	voltage of servo motor	variable	V

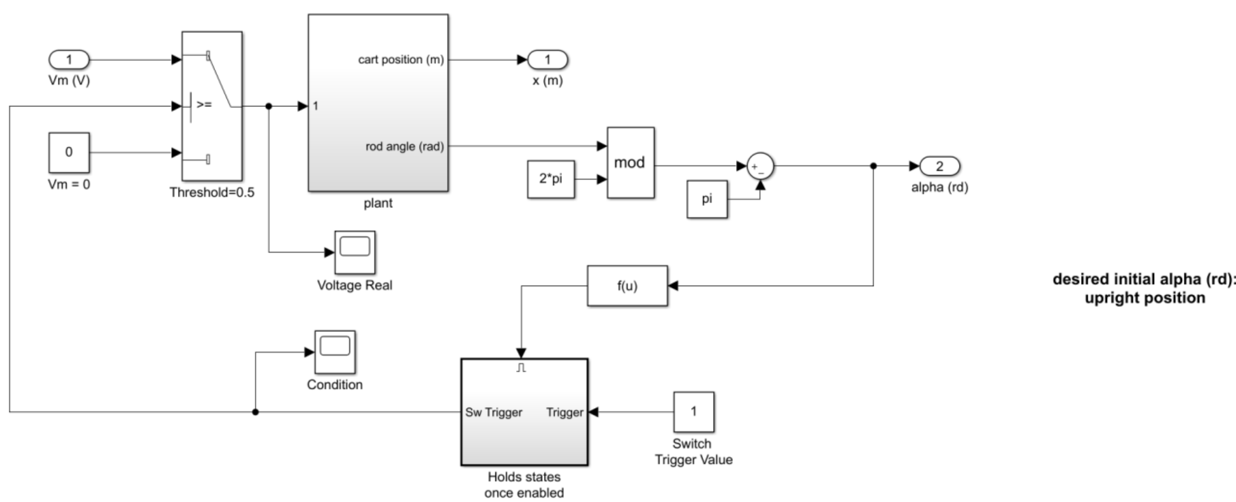
MATLAB Code

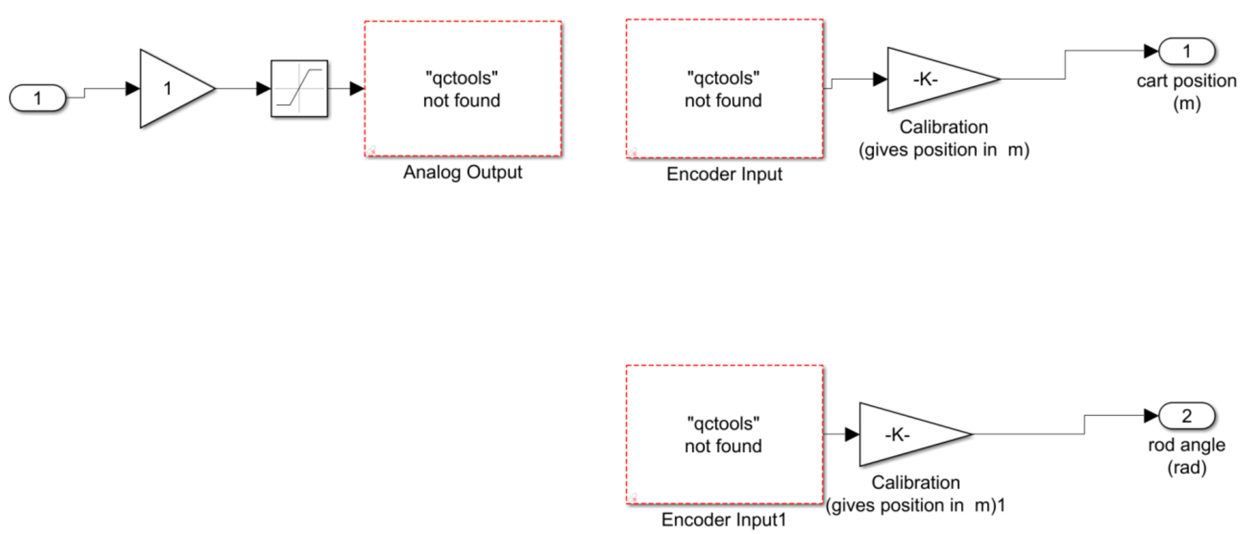
```
% AAE 364L Lab5 MATLAB Code
% TOMOKI KOIKE
clear all; close all; clc;
set(groot, 'defaulttextinterpreter','latex');
set(groot, 'defaultAxesTickLabelInterpreter','latex');
set(groot, 'defaultLegendInterpreter','latex');

lqr_xc_data = load("koike_lab5_lqr_xc.mat");
lqr_ang_data = load("koike_lab5_lqr_ang.mat");
pp_xc_data = load("koike_lab5_pp_xc.mat");
pp_ang_data = load("koike_lab5_pp_ang.mat");

% LQR
t = lqr_xc_data.x_and_command_part3.time;
xc = lqr_xc_data.x_and_command_part3.signals.values(:,1);
command = lqr_xc_data.x_and_command_part3.signals.values(:,2);
theta = lqr_ang_data.theta_part3.signals.values;

% Plotting
fig = figure('Renderer',"painters", 'Position', [10 10 900 800]);
subplot(2,1,1)
plot(t,xc)
grid on; grid minor; box on; hold on;
plot(t, command, '--')
hold off
ylabel('$x_c$ [m]')
subplot(2,1,2)
plot(t,theta, '-g')
grid on; grid minor; box on;
ylabel('$\alpha$ [rad]')
xlabel('t, [sec]')
sgtitle('Time Histories for $x_c$ and $\alpha$ for Sine Wave Response with LQR - T. Koike')
saveas(fig, 'response_lqr.png')

% PP
t = pp_xc_data.x_and_command_part3.time;
xc = pp_xc_data.x_and_command_part3.signals.values(:,1);
command = pp_xc_data.x_and_command_part3.signals.values(:,2);
theta = pp_ang_data.theta_part3.signals.values;

% Plotting
fig = figure('Renderer',"painters", 'Position', [10 10 900 800]);
subplot(2,1,1)
plot(t,xc)
grid on; grid minor; box on; hold on;
plot(t, command, '--')
hold off
ylabel('$x_c$ [m]')
```

```
subplot(2,1,2)
plot(t,theta, '-g')
grid on; grid minor; box on;
ylabel('$\alpha$ [rad]')
xlabel('t, [sec]')
sgtitle('Time Histories for $x_c$ and $\alpha$ for Sine Wave Response with Pole
Placement - T. Koike')
saveas(fig, 'response_pp.png')
```

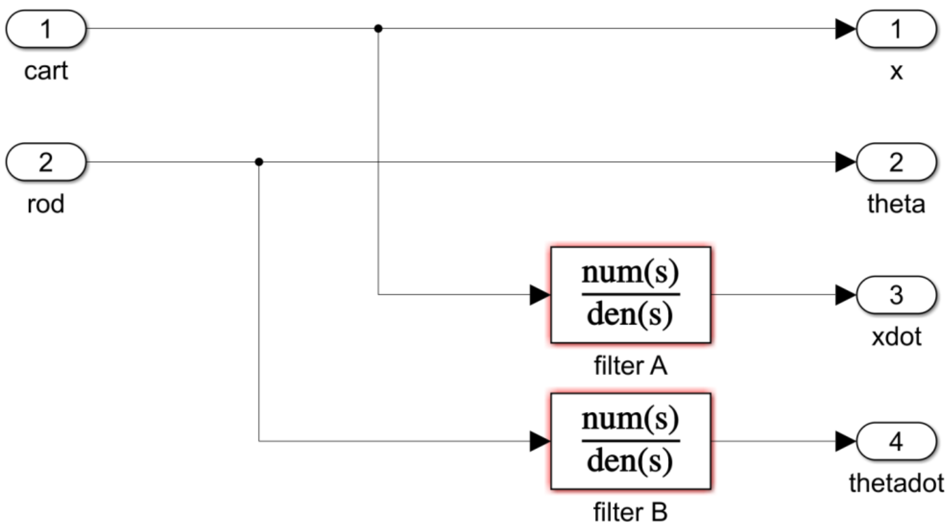
Simulink Models

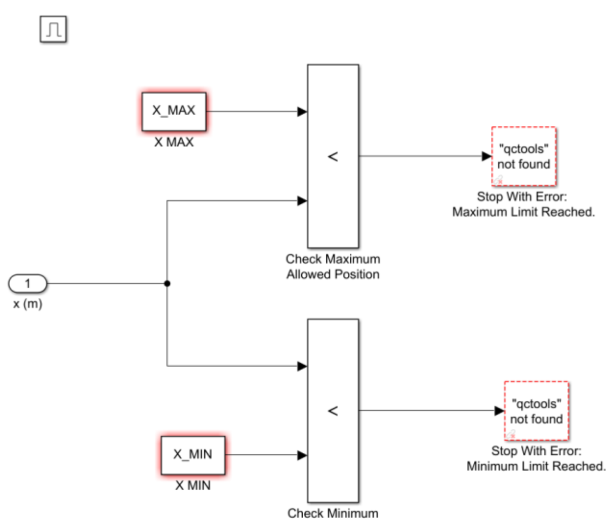







1
Trigger
1
Sw Trigger


1
cart
2
rod
1
x
2
theta
num(s)
den(s)
filter A
3
xdot
num(s)
den(s)
filter B
4
thetadot


X_MAX
X MAX
<
Check Maximum
Allowed Position
"qctools"
not found
Stop With Error:
Maximum Limit Reached.
1
x (m)
<
Check Minimum
Allowed Position
X_MIN
X MIN
"qctools"
not found
Stop With Error:
Minimum Limit Reached.