College of Engineering
School of Aeronautics and Astronautics

AAE 532
Orbital Mechanics

PS 4
Conical Orbits and Kepler's Equation

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Problem 1: At some initial time, t_0, a spacecraft is in orbit about the Earth. Its orbit is characterized by $e = 0.6$ and $p = 6R_\oplus$. It is currently located at the point in the orbit such that $\theta_0^* = 90°$.

(a) Determine the following orbit parameters and spacecraft state information: $a, r_p, r_a, \mathcal{P}, \mathcal{E}, ; r_0, v_0, E_0^*, \gamma_0$. [Always list all angles in degrees.] Compare v at this location with $\sqrt{2}v_c$. Should $v < \; or \; v > \sqrt{2}v_c$? Can your v value be correct? How do you know?

*All calculations are handled using MATLAB (code is in appendix).

$$R_\oplus = 6.3781\text{e+3 km}$$

$$a = \frac{p}{1-e^2} = \frac{6R_\oplus}{1-0.6^2} = \frac{6(6.3781\text{e+3 km})}{1-0.6^2}$$

$$\boxed{a = 5.9795\text{e+4 km}}$$

$$r_p = a(1-e) = (5.9795\text{e+4 km})(1-0.6)$$

$$\boxed{r_p = 2.3918\text{e+4 km}}$$

$$r_a = a(1+e) = (5.9795\text{e+4 km})(1+0.6)$$

$$\boxed{r_a = 9.5672\text{e+4 km}}$$

$$\mu = G(m_\oplus + m_{s/c}) \approx Gm_\oplus \quad \because m_{s/c} \ll m_\oplus$$

$$\mu = 3.9860\text{e+5 km}^3/\text{s}^2$$

$$\mathbb{P} = 2\pi\sqrt{\frac{a^3}{\mu}} = 2\pi\sqrt{\frac{(5.9795\text{e+4 km})^3}{3.9860\text{e+5 km}^3/\text{s}^2}}$$

$$\boxed{\mathbb{P} = 1.4552\text{e+5 s}}$$

$$\mathcal{E} = -\frac{\mu}{2a} = -\frac{3.9860e+5 \ km^3/s^2}{2(5.9795e+4 \ km)}$$

$$\boxed{\mathcal{E} = -3.3331 \ km^2/s^2}$$

$$r_0 = \frac{p}{1+e\cos\theta^*} = \frac{6(6.3781e+3 \ km)}{1+0.6\cos 90^\circ}$$

$$\boxed{r_0 = 3.8269e+4 \ km}$$

from $\mathcal{E} = \frac{v_0^2}{2} - \frac{\mu}{r_0} \rightarrow v_0^2 = \mu\left(\frac{2}{r_0} - \frac{1}{a}\right)$

$$v_0 = \sqrt{\mu\left(\frac{2}{r_0} - \frac{1}{a}\right)}$$

$$\boxed{v_0 = 3.7637 \ km/s}$$

from $\tan\frac{\theta^*}{2} = \left(\frac{1+e}{1-e}\right)^{1/2}\tan\frac{E}{2}$

$$E = 2\arctan\left[\left(\frac{1-e}{1+e}\right)^{1/2}\tan\frac{\theta^*}{2}\right]$$

$$\boxed{E_0^* = 53.1301 \ deg}$$

$$h = \sqrt{\mu p} = 1.2351e+5 \ km^2/s$$

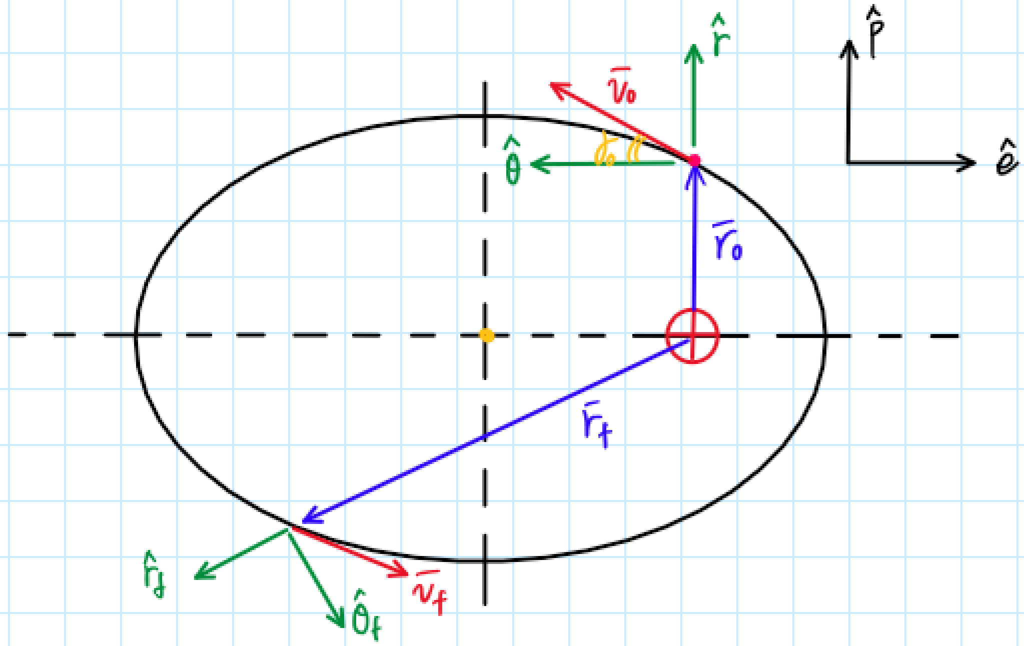
$$\gamma_0 = \arccos\left(\frac{h}{r_0 v_0}\right)$$

$$\boxed{\gamma_0 = 30.9638 \ deg}$$

$$v_0 = 3.7637 \text{ km/s} > 3.6513 \text{ km/s} = \sqrt{2}\, v_c = \sqrt{\frac{2\mu}{a}}$$

This not correct since for a general ellipse the velocity magnitude is always smaller than $\sqrt{2} v_c$

(b) Write $\overline{r_0}$ and $\overline{v_0}$ in terms of components in the directions of $\hat{e}$ and $\hat{p}$. At some later time, t_f, determine $r_f, v_f, \theta_f^*, \gamma_f$ when $E_f^* = 225$. Write $\bar{r}_f$ and $\bar{v}_f$ in terms of components in the directions of $\hat{e}$ and $\hat{p}$.



$$\boxed{\bar{r}_0 = r_0\hat{p} = 3.8269e{+}4 \text{ km } \hat{p}}$$

$$\bar{v}_0 = v_0(\cos\gamma_0\hat{\theta} + \sin\gamma_0\hat{r})$$

$$\bar{v}_0 = v_0(\cos\gamma_0(-\hat{e}) + \sin\gamma_0\hat{p})$$

$$\boxed{\bar{v}_0 = -3.2274 \text{ km/s } \hat{e} + 1.9364 \text{ km/s } \hat{p}}$$

from $r = a(1 - e\cos E)$

$$r_f = a(1 - e\cos 225°)$$

$$\boxed{r_f = 8.5164e{+}4 \text{ km}}$$

$$v_f = \sqrt{\mu\left(\frac{2}{r_f} - \frac{1}{a}\right)}$$

$$\boxed{v_f = 1.6415 \text{ km/s}}$$

$$\theta_f^* = 2\arctan\left[\left(\frac{1+e}{1-e}\right)^{1/2} \tan\frac{E_f^*}{2}\right]$$

$$\boxed{\theta_f^* = 203.4018 \text{ deg}}$$

$$\gamma_f = -\arccos\left(\frac{h}{r_f v_f}\right)$$

$$\boxed{\gamma_f = -27.9384 \text{ deg}}$$

$$\bar{r}_f = r_f\left(\cos\theta_f^*\,\hat{e} + \sin\theta_f^*\,\hat{p}\right)$$

$$\boxed{\bar{r}_f = -7.8159\text{e+4 km}\,\hat{e} - 3.3825\text{e+4 km}\,\hat{p}}$$

$$\bar{v}_f = v_f\left[\cos(\theta_f^* + 90° + |\gamma_f|)\,\hat{e} + \sin(\theta_f^* + 90° + |\gamma_f|)\,\hat{p}\right]$$

$$\boxed{\bar{v}_f = 1.2818 \text{ km/s}\,\hat{e} - 1.0255 \text{ km/s}\,\hat{p}}$$

(c) Determine the time t_0 relative to periapsis, i.e., at $\theta_0^* = 90°$; also determine the final time t_f relative to periapsis, i.e., when $E_f^* = 225°$. What is the time-of-flight (TOF), i.e., $(t_f - t_0)$ as well as $\Delta\theta^*$ and ΔE?

from the Notes G7

$$\sqrt{\frac{\mu}{a^3}}(t_0 - t_p) = E_0^* - e\sin E_0^*$$

let $t_p = 0$

$$t_0 = \sqrt{\frac{a^3}{\mu}}\left(E_0^* - e\sin E_0^*\right)$$

from previous questions

$$\theta_0^* = 90° \longrightarrow E_0^* = 53.1301°$$

*plug in E_0^* as radians

$$\boxed{t_0 = 1.0359e+4 \text{ s}}$$

Then

$$t_f = \sqrt{\frac{a^3}{\mu}}\left(E_f^* - e\sin E_f^*\right)$$

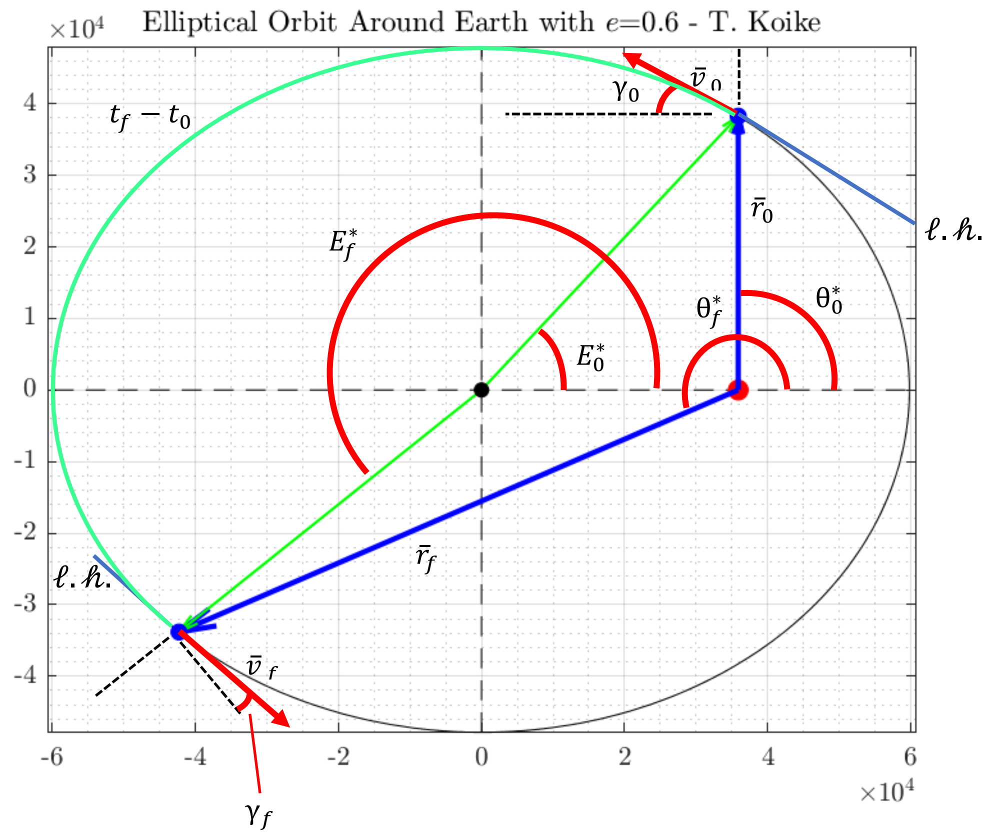
$$\boxed{t_f = 1.0077e+5 \text{ s}}$$

$$t_f - t_0 = 9.0414e+4 \text{ s}$$

$$\Delta\theta = \theta_f^* - \theta_0^* = 113.4018 \text{ deg}$$

$$\Delta E = E_f^* - E_0^* = 171.8699 \text{ deg}$$

(d) PLOT the entire orbit in MATLAB. (Do not use polar plots; compute $\hat{e}$ and $\hat{p}$ components along the path.) By hand, on the plot, mark the location of the satellite at t_0 and t_f; at each location, indicate $r, \theta^*, \bar{v}, E, \gamma$; also, sketch the local horizon. Indicate the arc used between t_0 and t_f.

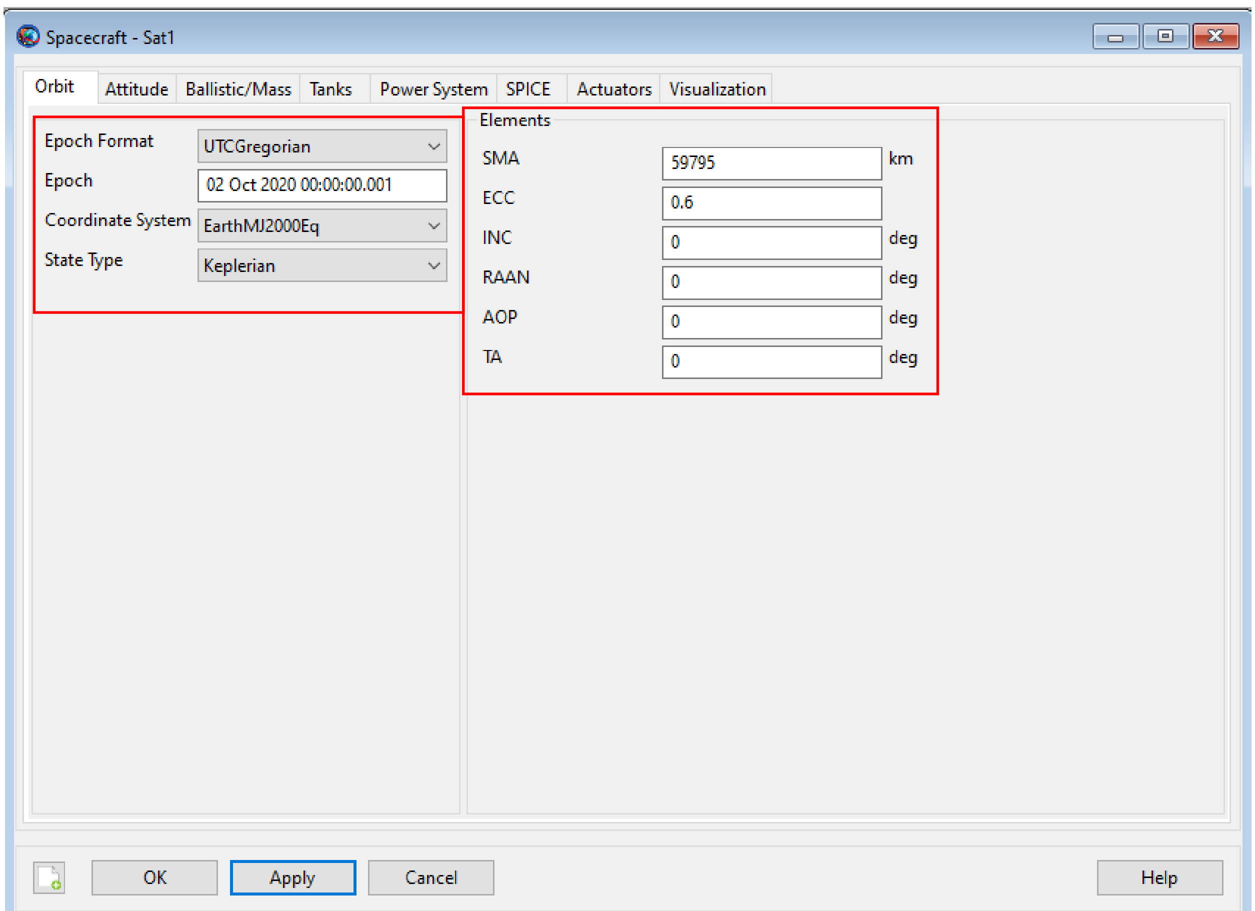


Problem 2: Return to Problem 1 and confirm your results in GMAT. Use October 2, 2020 as the start date.

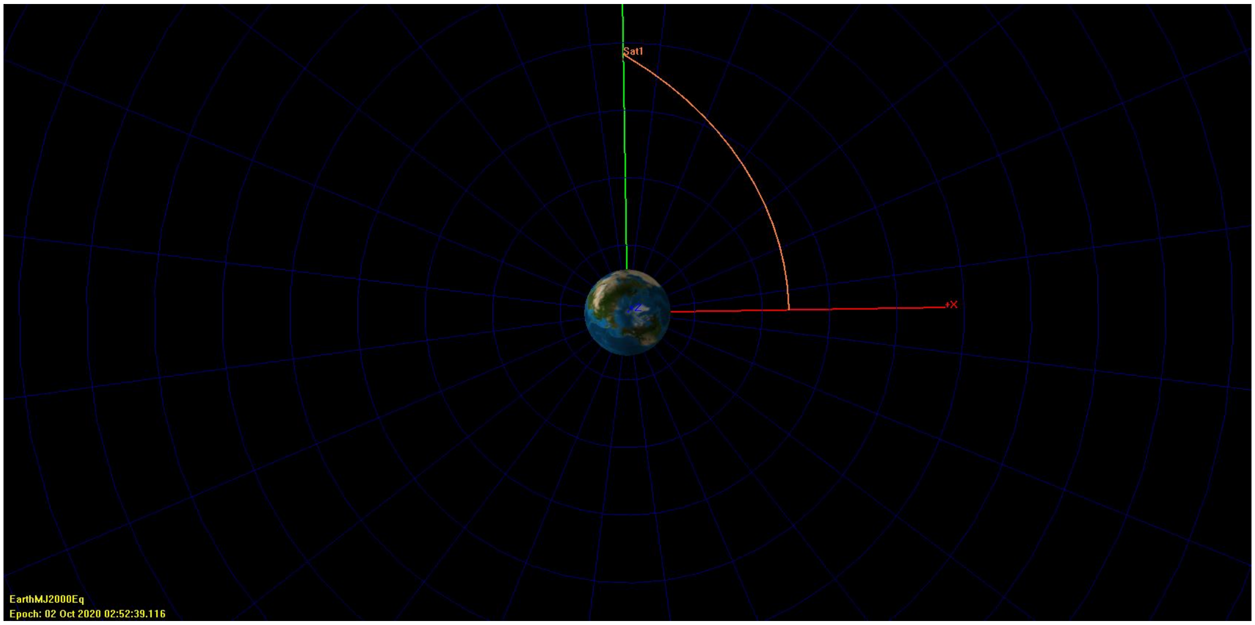
(a) What initial state can be input to GMAT for Sat1? Can you locate the rest of the quantities that were requested in Problem 1(a) and 1(b); do they confirm that your computations are correct? Can you determine the time difference $(t_f - t_0)$? Compare you MATLAB plot and the GMAT plot. Is your GMAT plot consistent with your MATLAB plot?

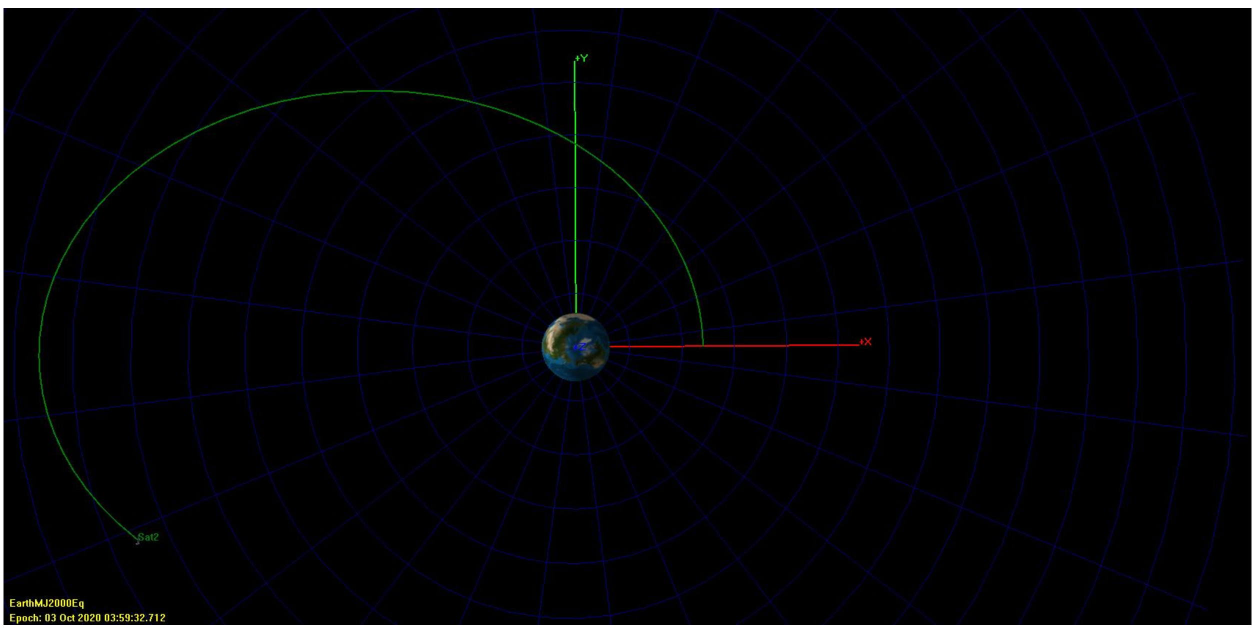
The initial conditions to Sat1 will be set as

- Epoch: UTCGregorian - 02 Oct 2020 00:00:00.001
- State Type: Keplerian
- SMA: 59795 km
- ECC: 0.6



The propagation for the 2 eccentric anomalies E_0^* and E_f^* are the following





From the report generated by GMAT we are able to confirm the quantities $a, r_p, r_a, \mathcal{P}, \mathcal{E}, ; r_0, v_0, E_0^*, \gamma_0$ and $r_f, v_f, \theta_f^*, \gamma_f$ when $E_f^* = 225$. We

For the orbit up to E_0^*

periapsis [km]	23918
apoapsis [km]	95672
energy [kJ]	-3.333058
semi-major axis [km]	59795
semi-latus rectum [km]	38269
period	145515
$\boldsymbol{X}$ **[km]**	0.0020326
$\boldsymbol{Y}$**[km]**	38269
$\boldsymbol{Z}$ **[km]**	0
$\boldsymbol{r_0} = \sqrt{(\boldsymbol{X^2+Y^2+Z^2})}$ **[km]**	38269
$\boldsymbol{V_x}$**[km/s]**	-3.2274
$\boldsymbol{V_y}$**[km/s]**	1.9364
$\boldsymbol{V_z}$**[km/s]**	0
$\boldsymbol{v_0} = \sqrt{\boldsymbol{V_x^2+V_y^2+V_z^2}}$ **[km/s]**	3.7637
θ* [deg]	90
time-elapsed $\boldsymbol{t_0}$	10359

For the orbit up to E_f^*

$\boldsymbol{X}$ **[km]**	**-78158**
$\boldsymbol{Y}$**[km]**	-33825
$\boldsymbol{Z}$ **[km]**	0
$\boldsymbol{r_0} = \sqrt{(\boldsymbol{X^2+Y^2+Z^2})}$ **[km]**	85163
$\boldsymbol{V_x}$**[km/s]**	1.2818
$\boldsymbol{V_y}$**[km/s]**	-1.0255
$\boldsymbol{V_z}$**[km/s]**	0
$\boldsymbol{v_0} = \sqrt{\boldsymbol{V_x^2+V_y^2+V_z^2}}$ **[km/s]**	1.6415
θ* [deg]	203.40
time-elapsed $\boldsymbol{t_0}$	100773

The table above all agree with the results in Problem 1. Therefore, we can say that the GMAT result confirms our answers.

From the time elapse data,

$$\boxed{t_f - f_0 = 100773 - 10359 = 90414 \ s}$$

This agrees with our calculations in Problem 1.

(b) Also print out the data from GMAT. (You can submit output from the file generated in the propagate window under the Mission Sequence. Cut-and-paste the sections with the required data into a Word document. Highlight the requested quantities. You can also create a Report file; you may not want to include the entire file but, again cut-and-paste.)

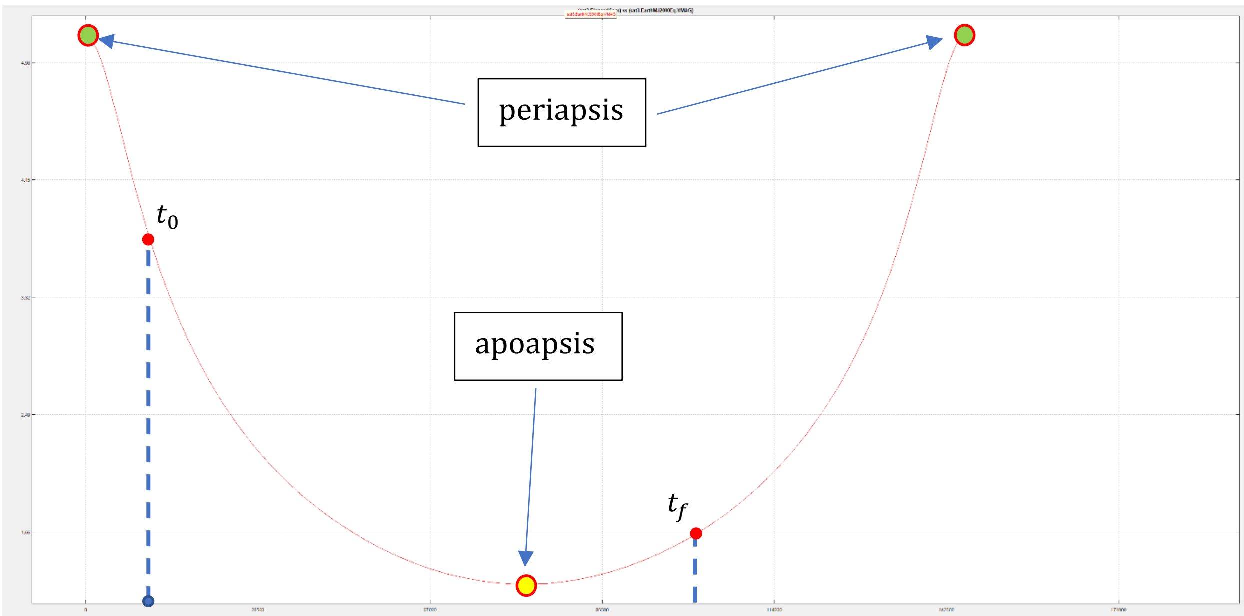
The constants from the report

Sat1.Earth.RadApo	Sat1.Earth.RadPer	Sat1.Earth.SMA	Sat1.Earth.SemilatusRectum	Sat1.Earth.Energy	Sat1.Earth.OrbitPeriod
95672	23918	59795	38268.8	-3.333058295	145515.175
95672	23918	59795	38268.8	-3.333058295	145515.175
95672	23918	59795	38268.8	-3.333058295	145515.175
95672	23918	59795	38268.8	-3.333058295	145515.175
95672	23918	59795	38268.8	-3.333058295	145515.175
95672	23918	59795	38268.8	-3.333058295	145515.175

On the next, page there is a table for the velocity, position, and elapse-time. The specific time for t_0 and t_f are highlighted.

Sat1. VX	Sat1. VY	Sat1.VZ	Sat1.X	Sat1. Y	Sat1.Z	Sat1.ElapsedSecs
0	5.163764747	0	23918	0	0	0
-0.04180402	5.163493991	0	23916.74585	309.8204696	0	59.99999988
-0.162518739	5.159670198	0	23899.02547	1205.004061	0	233.4193771
-0.512376379	5.122832596	0	23727.50067	3815.381452	0	740.8400658
-0.897720259	5.036396539	0	23319.18205	6752.969377	0	1318.718584
...	...	...	...	...	...	...
-3.163624424	2.574602722	0	6764.817645	33534.38749	0	8249.895158
-3.209227423	2.277976586	0	3808.322783	35781.71315	0	9176.954888
-3.226977383	1.985647356	0	578.5224334	37917.2734	0	10179.8525
-3.227352967	1.936411951	0	0.002032598	38268.79878	0	10359.11481
...	...	...	...	...	...	...
0.625639348	-1.229718757	0	-91260.32255	-18033.3843	0	86948.02516
0.748572524	-1.202926741	0	-89405.56726	-21318.67931	0	89648.02516
0.873611317	-1.17045327	0	-87216.13658	-24524.07901	0	92348.02516
1.001153939	-1.131730649	0	-84685.81353	-27633.50716	0	95048.02516
1.131626053	-1.086042462	0	-81807.26968	-30629.16103	0	97748.02516
1.265483661	-1.032486715	0	-78571.98686	-33491.06266	0	100448.0252
1.281831493	-1.025465192	0	-78158.44993	-33825.16003	0	100772.7109

(c) Add an X-Y plot to the output. Plot speed as a function of elapsed time in seconds. Print the plot. Mark your time that you computed in Problem 1. Does the max velocity location in your plot correlate to the periapsis time in the GMAT plot?



From this plot we can tell that the periapsis which is the closest to the Earth has the largest velocity magnitude. Thus, we can say that the GMAT plot agrees with the theoretical results from Kepler's equations.

Problem 3: To investigate the requirements for departure, assume that a spacecraft is departing the vicinity of the Earth along a parabolic path. Consider the spacecraft to be located at perigee on the parabola.

(a) A circular parking orbit about the Earth may be defined at 225 km altitude. At a perigee altitude of 225 km on the parabola, compare the escape velocity on the parabola with the relative velocity in a circular orbit with the same altitude. To shift from the circular orbit to the escape trajectory, what % increase in velocity is required.

For the circular orbit

$$r_c = R_\oplus + h \quad \text{where } R_\oplus = 6378.1 \text{ km}, \quad h = 225 \text{ km}$$

$$v_c = \sqrt{\frac{\mu_\oplus}{r_c}} = \sqrt{\frac{398.6004e{+}3 \text{ km}^3/\text{s}^2}{6603.1 \text{ km}}}$$

$$\boxed{v_c = 7.7695 \text{ km/s}}$$

The escape speed of the parabola is

$$\boxed{v_{esc} = \sqrt{2}\, v_c = 10.9877 \text{ km/s}}$$

The percent of velocity change

$$\boxed{\frac{\sqrt{2}\, v_c - v_c}{v_c} \times 100\% = 41.4214\%}$$

(b) Compute the velocity along the parabola as it departs the vicinity of the Earth, that is, at the following distances: $r = 2R_\oplus, 10R_\oplus, 75R_\oplus, 200R_\oplus, 800R_\oplus$ one additional distance of your choice $\rightarrow 2000R_\oplus$. Determine the true anomaly θ* that corresponds to each distance. Also include the time since passing the periapsis at each distance (in days).

$r_p = 225$ km

since, $r_p = \frac{p}{1+e} = \frac{p}{2}$ (for parabola)

$$p = 2r_p = 450 \text{ km}$$

For each velocity

$$v = \sqrt{\frac{2\mu}{r}}$$

and for each $r = 2R_\oplus, 10R_\oplus, \cdots, 2000R_\oplus$
compute v with MATLAB

and for each θ^*, using

$$r = \frac{p}{1+\cos\theta^*}$$

$$\cos\theta^* = \frac{p}{r} - 1$$

$$\theta^* = \arccos\left(\frac{p}{r} - 1\right)$$

The θ^* will also be computed with MATLAB

From Barker's Eqn

$$6\sqrt{\frac{\mu}{p^3}}(t - \cancel{t_p}^{\,0}) = \tan^3\frac{\theta^*}{2} + 3\tan\frac{\theta^*}{2}$$

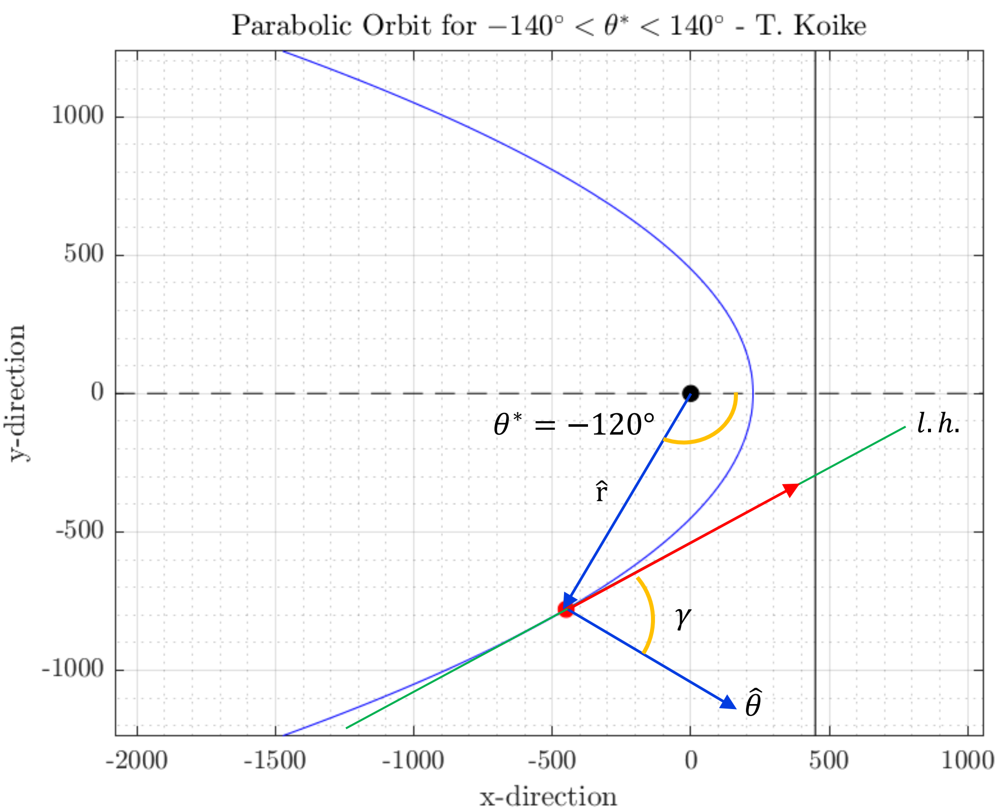
$$t = \frac{1}{6}\sqrt{\frac{p^3}{\mu}}\left(\tan^3\frac{\theta^*}{2} + 3\tan\frac{\theta^*}{2}\right)$$

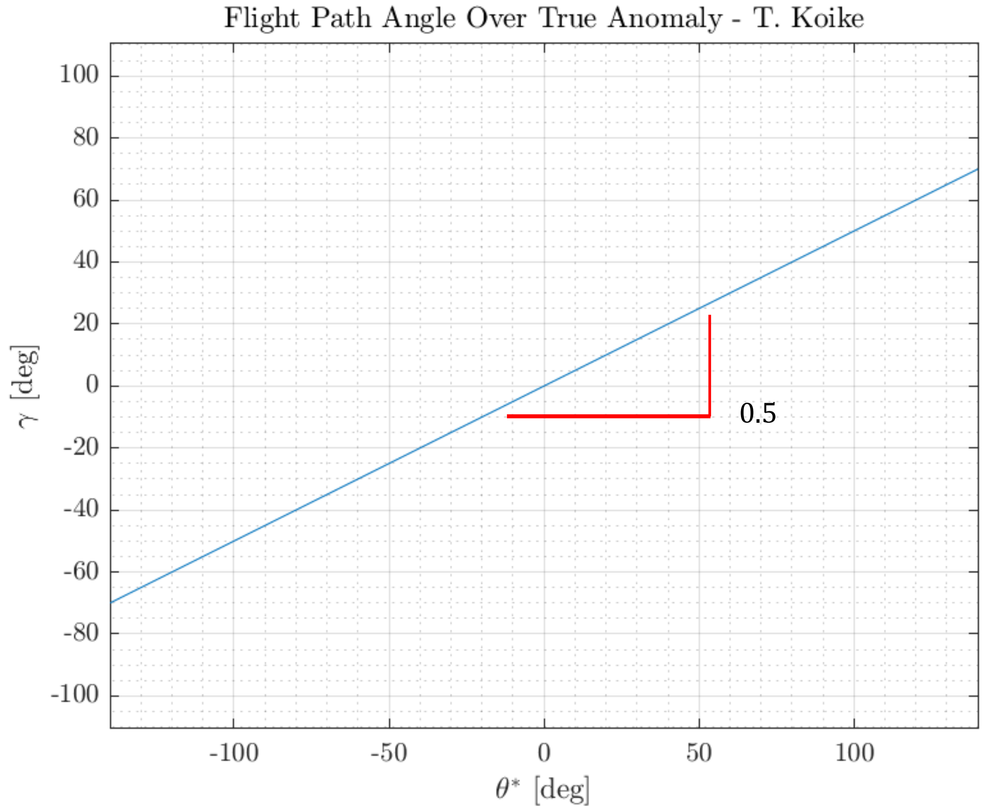
plug the computed true anomalies to find the time.

The results computed from MATLAB are tabulated below.

Distances	*Velocity [km/s]*	*True Anomaly [deg]*	*Time since* r_p *[days]*
$2R_{\oplus}$	7.9054	164.7360	0.012776
$10R_{\oplus}$	3.5354	173.1899	0.13994
$75R_{\oplus}$	1.2909	177.5146	2.8612
$200R_{\oplus}$	0.79054	178.4781	12.4541
$800R_{\oplus}$	0.39527	179.2391	99.6129
$2000R_{\oplus}$	0.24999	179.5187	393.7389

(c) In the MATLAB script from the first problem, plot the parabola corresponding for altitude 225 km between $-140° < \theta^* < 140°$. Mark on the plot, r, v, γ at $\theta^* = -120°$; also, sketch the l.h. (local horizon). Also add the directrix. Compare θ^* and γ is there a pattern?





From the graph above, we can see the relationship between the flight path angle and the true anomaly for a parabola is

$$\boxed{\theta^* = 2\gamma}$$

(d) At $r = 75R_\oplus$, is it reasonable to model the problem as a two-body problem (Earth and spacecraft)?

Since the distance of the Moon from the Earth is 384400 km $\cong 60R_\oplus$, we can say that we cannot model this as a two-body problem. The spacecraft is farther away from than the Moon and is possible for the spacecraft to be near the Moon. This puts the spacecraft under the influence of the Moon's gravitation, and therefore, we must model the problem as a three-body problem rather than a two-body problem.

Problem 4: As part of the new lunar initiative, an unmanned probe is approaching the Moon on a hyperbola. The hyperbola is defined such that $|a| = 7050\ km$ and the passage altitude is 800 km altitude. At the "current" time, the probe is located at $\theta^* = -60°$.

(a) Determine the following additional orbital characteristics: $r_p, v_p, b, h, \delta, v_\infty$, $\mathcal{E}$.

Determine the following quantities at the current time: r, v, γ, H, time till perilune.

*All calculations are done in MATLAB (code in appendix).

The passage altitude is equivalent to the periapsis

$$\boxed{r_p = 800\ km}$$

Since, $|a| = 7050\ km$

$$\boxed{e = \frac{r_p}{|a|} + 1 = 1.1135}$$

Then,

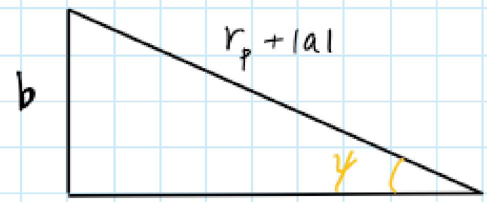
$$\boxed{p = |a|(e-1) = 1690.8\ km}$$

$$\boxed{b = |a|\sqrt{e^2-1} = 3452.5\ km}$$

Since,

$$\mu = \mu_D = GM_D = 4.9028\ e{+}3\ km^3/s^2$$

$$\boxed{h = \sqrt{\mu p} = 2.8792\ e{+}3\ km^2/s}$$



$$\psi = \frac{180^\circ - \delta}{2}$$

Thus,

$$\sin\left(\frac{180^\circ - \delta}{2}\right) = \frac{b}{r_p + |a|}$$

$$\frac{180^\circ - \delta}{2} = \arcsin\left(\frac{b}{r_p + |a|}\right)$$

$$\delta = 180^\circ - 2\arcsin\left(\frac{b}{r_p + |a|}\right)$$

$$\boxed{\delta = 127.8160 \text{ deg}}$$

Next,

$$v_\infty = \sqrt{\frac{\mu}{|a|}}$$

$$\boxed{v_\infty = 0.83393 \text{ km/s}}$$

$$\mathcal{E} = -\frac{\mu}{2a}$$

$$\boxed{\mathcal{E} = 0.34772 \text{ km}^2/\text{s}^2}$$

When $\theta^* = -60^\circ$

$$r = \frac{|a|(e^2 - 1)}{1 + e\cos\theta^*}$$

$$\boxed{r = 1086.1 \text{ km}}$$

$$v = \sqrt{\mu\left(\frac{2}{r} - \frac{1}{a}\right)}$$

$$\boxed{v = 3.1183 \text{ km/s}}$$

$$\gamma = -\arccos\left(\frac{h}{rv}\right)$$

$$\boxed{\gamma = -31.7755 \text{ deg}}$$

Finally,

$$\frac{|a| + r}{e} = |a| \cosh H$$

$$\cosh H = \frac{|a| + r}{|a| e}$$

$$H = \operatorname{arccosh}\left(\frac{|a| + r}{|a| e}\right)$$

$$\boxed{H = \begin{cases} 0.26917 \text{ rad} \\ 15.4225 \text{ deg} \end{cases}}$$

(b) Use your MATLAB script and plot the hyperbola between $\theta^* = \pm 100°$. Mark the probe at $\theta^* = -60°$ and label b, aim point, $\frac{\delta}{2}, v, \gamma, r, \theta^*$. (Always include the local horizon!)

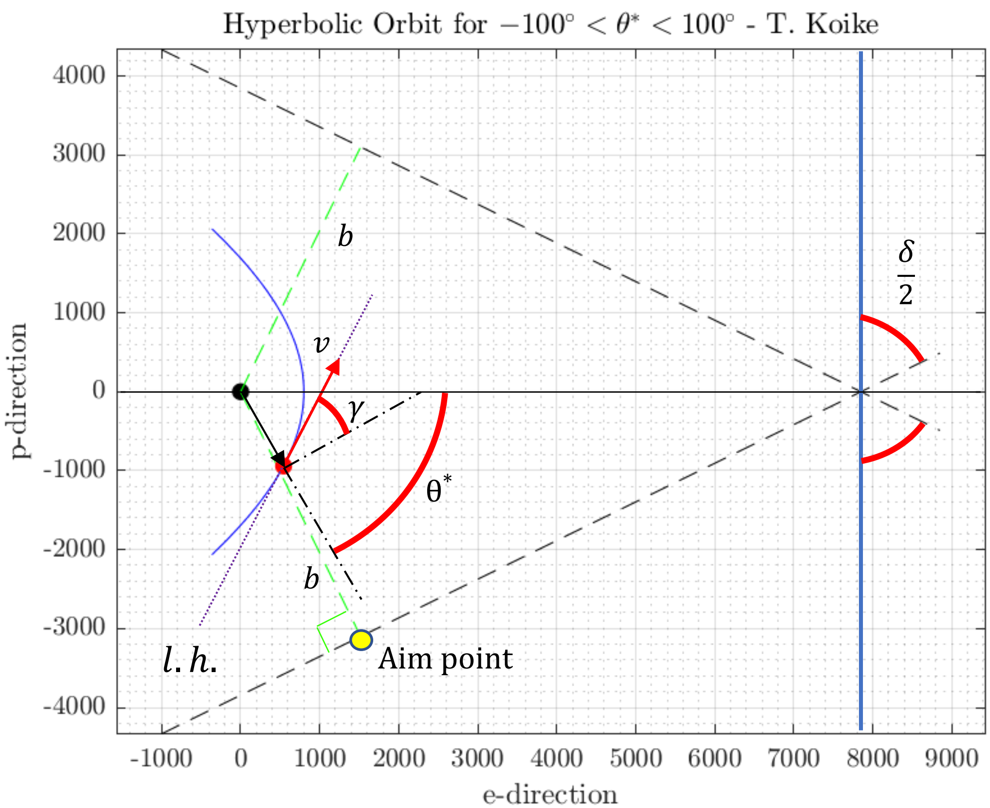
Since the hyperbola equation is defined as

$$\frac{x^2}{|a|^2} - \frac{y^2}{|a|^2(e^2 - 1)} = 1$$

The asymptotes are expressed as

$$y = \pm \frac{|a|\sqrt{e^2 - 1}}{|a|} x = \pm \sqrt{e^2 - 1} x$$

Keeping this in mind we plot the following graph



The red dot indicated in the plot represents the point at $\theta^* = -60°$.

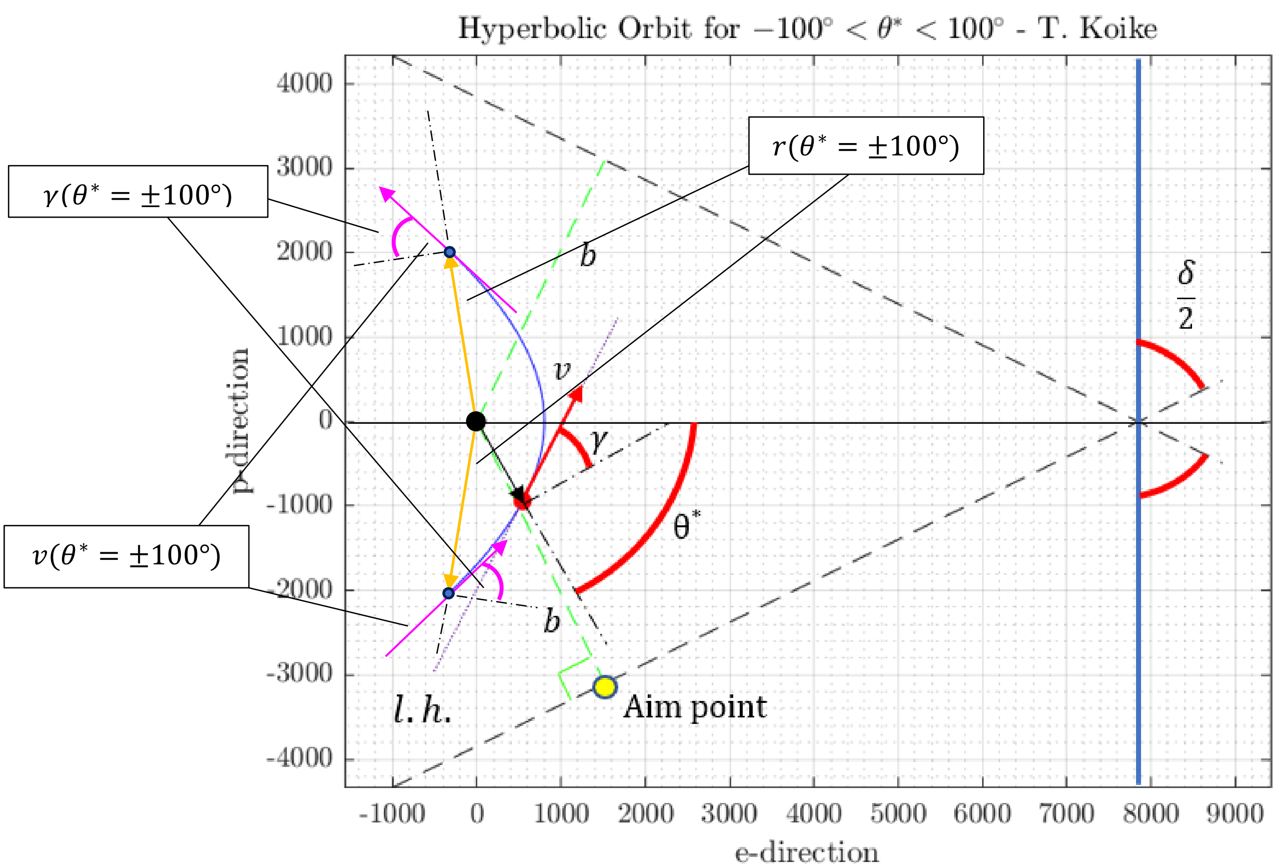
(c) Determine r, v, γ at $\theta^* = \pm 100°$; add this information to the plot.

when $\theta^* = \pm 100°$

$$r = \frac{p}{1 + e\cos 100°} = 2096.1 \text{ km}$$

$$v = \sqrt{\mu\left(\frac{2}{r} - \frac{1}{a}\right)} = 2.3181 \text{ km/s}$$

$$\gamma = \arccos\left(\frac{h}{rv}\right) = \pm 53.6612 \text{ deg}$$



Appendix

MATLAB Code

Problem 1

```
%% AAE 532 HW 4 Problem 1
% Tomoki Koike
close all; clear all; clc;
fdir = 'C:\Users\Tomo\Desktop\studies\2020-Fall\AAE532\matlab\outputs\ps4';
set(groot, 'defaulttextinterpreter','latex');
set(groot, 'defaultAxesTickLabelInterpreter','latex');
set(groot, 'defaultLegendInterpreter','latex');
format shortEng;

% Set constants
planet_consts = setup_planetary_constants();  % Function that sets up all the
constants in the table
earth = planet_consts.earth;  % structure of earth
G = 6.6743015e-20;  % Gravitational constant [km^3/kg/s^2]

% (a)
r_E = earth.mer;  % radius of earth [km]
e = 0.6;  % eccentricity
theta_star = 90;  % true anomaly [deg]
p = 6*r_E;  % latus rectum [km]
a = p/(1 - e^2);  % semi mahor axis [km]
b = a * sqrt(1 - e^2);
rp = a*(1 - e);  % periapsis [km]
ra = a*(1 + e);  % apoapsis [km]
mu = earth.gp;  % gravitational parameter [km^3/s^2]
IP = 2*pi*sqrt(a^3 / mu);  % period [s]
En = -mu / 2 / a;  % specific energy [km^2/s^2]
r0 = p;
v0 = sqrt(mu * (2/r0 - 1/a));
E0 = trueAnomaly2EccAnomaly(e, theta_star, "deg");  % eccentric anomaly [deg]
h = sqrt(mu * p);  % specific angular momentum [km^2/s]
gamma0 = acosd(h / r0 / v0);  % flight path angle [deg]

vc = sqrt(mu / a )

% (b)
r0_vec = r0 * [0, 1, 0];
v0_vec = v0 * [-cosd(gamma0), sind(gamma0), 0];
Ef = 225;  % eccentric anomaly at a certain time tf [deg]
rf = a * (1 - e*cosd(Ef));
vf = vis_viva(rf, a, mu);
theta_star_f = eccAnomaly2trueAnomaly(e, Ef, "deg");
gamma_f = acosd(h / rf / vf);

rf_hat = [cosd(theta_star_f), sind(theta_star_f), 0];
```

```
vf_hat = [cosd(theta_star_f+90+gamma_f), sind(theta_star_f+90+gamma_f), 0];
rf_vec = rf * rf_hat;
vf_vec = vf * vf_hat;
```

```
% (c)
t0 = ellipse_time_KeplerEqn(0, E0, "deg", mu, a, e);
tf = ellipse_time_KeplerEqn(0, Ef, "deg", mu, a, e);
Dt = tf - t0;
Dtheta = theta_star_f - theta_star;
DE = Ef - E0;
```

```
% (d)

% Arrow drawing function
drawArrow = @(x,y,varargin) quiver( x(1),y(1),x(2)-x(1),y(2)-y(1),0,
varargin{:} );

% Plotting
fig1 = figure("Renderer","painters");
hold on; grid on; grid minor; box on; axis equal;
% ylim([-5.5e+4, 5.5e+4]); xlim([-7e+4, 7e+4])
tt = 0:0.01:2*pi;
X = a * cos(tt); Y = b * sin(tt);

% Ellipse
plot(X, Y, '-k')

% Axes
x_axis = linspace(-a, a, 2^9);
y_axis = linspace(-b, b, 2^9);
plot(x_axis, zeros(size(x_axis)), '--k')
plot(zeros(size(y_axis)), y_axis, '--k')

% Center
plot(a*e, 0, '.r', 'MarkerSize', 25)

% t0 point
rx_t0 = [a*e, a * cosd(E0)]; ry_t0 = [0, b * sind(E0)];
drawArrow(rx_t0, ry_t0, 'linewidth',2,'Color',[0 0 1]);
rx_t0E = [0, a * cosd(E0)]; ry_t0E = [0, b * sind(E0)];
drawArrow(rx_t0E, ry_t0E, 'linewidth',1,'Color',[0 1 0]);
plot(a * cosd(E0), b * sind(E0), '.b', 'MarkerSize', 20)

% tf point
rx_tf = [a*e, a * cosd(Ef)]; ry_tf = [0, b * sind(Ef)];
drawArrow(rx_tf, ry_tf, 'linewidth',2,'Color',[0 0 1]);
rx_tfE = [0, a * cosd(Ef)]; ry_tfE = [0, b * sind(Ef)];
drawArrow(rx_tfE, ry_tfE, 'linewidth',1,'Color',[0 1 0]);
plot(a * cosd(Ef), b * sind(Ef), '.b', 'MarkerSize', 20)

% Origin
plot(0, 0, '.k', 'MarkerSize', 18)
```

```
hold off

title('Elliptical Orbit Around Earth with $e$=0.6 - T. Koike')
saveas(fig1, fullfile(fdir, 'p1_orbitplot.png'));
```

Problem 3

```
%% AAE 532 HW 4 Problem 3
% Tomoki Koike
close all; clear all; clc;
fdir = 'C:\Users\Tomo\Desktop\studies\2020-Fall\AAE532\matlab\outputs\ps4';
set(groot, 'defaulttextinterpreter','latex');
set(groot, 'defaultAxesTickLabelInterpreter','latex');
set(groot, 'defaultLegendInterpreter','latex');
format shortEng;
```

```
% Set constants
planet_consts = setup_planetary_constants();  % Function that sets up all the
constants in the table
earth = planet_consts.earth;  % structure of earth
G = 6.6743015e-20;  % Gravitational constant [km^3/kg/s^2]
```

```
% (a)
h = 225;  % altitude [km]
r_E = earth.mer;  % radius of earth [km]
mu = earth.gp;
r_c = r_E + h;
v_c = sqrt(mu / r_c);
v_esc = sqrt(2) * v_c;
```

```
% (b)
r_p = h;
p = 2 * r_p
dist = r_E * [2, 10, 75, 200, 800, 2000];
v_para = []; TA = []; t_rp = [];
for ri = dist
    v_para = [v_para, parabola_for_r(ri, mu)];
    temp = true_anomaly_calc(p, ri);
    TA = [TA, temp];
    t_rp = [t_rp, backer_eqn_time(p, mu, temp)];
end
t_rp = t_rp / 60 / 60/ 24;
```

```
% (c)

% Plotting
TA = -140:0.01:140;
R = p ./ (1 + cosd(TA));
X = R .* cosd(TA);
Y = R .* sind(TA);

% Find point for -120 degrees
R_n120= R(TA == -120);
```

```
fig1 = figure("Renderer","painters");
title('Parabolic Orbit for $-140^{\circ}<\theta^*<140^{\circ}$ - T. Koike')
xlabel('x-direction')
ylabel('y-direction')
plot(X, Y, '-b')
grid on; grid minor; box on; axis equal;
hold on;
plot(R_n120*cosd(-120), R_n120*sind(-120), '.r', 'MarkerSize', 20)

% Directrix
direx_y = fig1.CurrentAxes.YLim(1):fig1.CurrentAxes.YLim(2);
direx_x = ones(size(direx_y))*2*r_p;
plot(direx_x, direx_y, '-k')

% x-axis
xaxis_x = fig1.CurrentAxes.XLim(1):fig1.CurrentAxes.XLim(2);
xaxis_y = zeros(size(xaxis_x));
plot(xaxis_x, xaxis_y, '--k')

% Center
plot(0, 0, '.k', 'MarkerSize', 20)
hold off
saveas(fig1, fullfile(fdir, 'p3_plot.png'))
```

```
% Plotting gamma with TA

% gamma
rs = calc_parabola_r(p, TA);
vs = parabola_for_r(rs, mu);
gammas = acosd(sqrt(mu*p) ./ rs ./ vs);
idx1 = find(TA == -140);
idx2 = find(TA==0);
gammas(idx1:idx2) = -1*gammas(idx1:idx2);

fig2 = figure("Renderer","painters");
plot(TA, gammas)
grid on; grid minor; box on; axis equal;
title('Flight Path Angle Over True Anomaly - T. Koike')
xlabel('$\theta^*$ [deg]')
ylabel('$\gamma$ [deg]')
saveas(fig2, fullfile(fdir, 'p3_gamma_theta.png'))
```

```
function v = parabola_for_r(r,mu)
    v = sqrt(2 * mu ./ r);
end

function r = calc_parabola_r(p, TA)
    r = p ./ (1 + cosd(TA));
end

function TA = true_anomaly_calc(p, r)
```

```
    TA = acosd(p/r - 1);
end

function et = backer_eqn_time(p, mu, TA)
    et = sqrt(p^3 / mu) / 6 * (tand(TA/2)^3 + 3*tand(TA/2));
end
```

Problem 4

```
%% AAE 532 HW 4 Problem 4
% Tomoki Koike
close all; clear all; clc;
fdir = 'C:\Users\Tomo\Desktop\studies\2020-Fall\AAE532\matlab\outputs\ps4';
set(groot, 'defaulttextinterpreter','latex');
set(groot, 'defaultAxesTickLabelInterpreter','latex');
set(groot, 'defaultLegendInterpreter','latex');
format shortEng;
```

```
% Set constants
planet_consts = setup_planetary_constants();  % Function that sets up all the
constants in the table
moon = planet_consts.moon;  % structure of moon
G = 6.6743015e-20;  % Gravitational constant [km^3/kg/s^2]
```

```
% Find characterisitics
mu = moon.gp;  % gravitational parameter
a = 7050;  % semi major axis [km]

r_p = 800;
e = r_p/a + 1;
p = a * (e^2 - 1)
b = a * sqrt(e^2 - 1)
h = sqrt(mu * p)
FBA = 180 - 2*asind(b / (r_p + a))
v_inf = sqrt(mu / a)
En = mu / 2 / a
```

```
% at TA = -60 deg current time
TA_curr = -60;
r_curr = p / (1 + e*cosd(TA_curr));
v_curr = vis_viva(r_curr, -a, mu);
FPA_curr = -acosd(h / r_curr / v_curr);
H_curr = acosh((a + r_curr) / a / e)
```

```
% (b) Plotting
% Hyperbola
TA = -100:1:100;
R = p ./ (1 + e*cosd(TA));
X = R .* cosd(TA);
Y = R .* sind(TA);

fig1 = figure("Renderer","painters");
plot(X, Y, '-b')
```

```
title('Hyperbolic Orbit for $-100^{\circ}<\theta^*<100^{\circ}$ - T. Koike')
xlabel('e-direction')
ylabel('p-direction')
grid on; grid minor; box on; axis equal; hold on;

% Asymptotes
x_asymp = -1000:(a*e+1000);
y_asymp_p = sqrt(e^2 - 1)*(x_asymp - a*e);
y_asymp_n = -sqrt(e^2 - 1)*(x_asymp - a*e);

plot(x_asymp, y_asymp_p, '--k')
plot(x_asymp, y_asymp_n, '--k')

% Center
plot(0, 0, '.k', 'MarkerSize', 20)

% b line
x_b = 0:1513;
y_bp = 1/sqrt(e^2-1) .* x_b;
y_bn = -1/sqrt(e^2-1) .* x_b;

plot(x_b, y_bp, '--g')
plot(x_b, y_bn, '--g')

% Plot TA=-60
Xi = X(TA==-60);
Yi = Y(TA==-60);
plot(Xi, Yi, '.r', 'MarkerSize', 20)
hold off
saveas(fig1, fullfile(fdir, 'p4_hyperbola.png'))
```

```
% at TA = 100 deg current time
TA_curr = 100;
r_curr = p / (1 + e*cosd(TA_curr));
v_curr = vis_viva(r_curr, -a, mu);
FPA_curr = acosd(h / r_curr / v_curr);
```