AAE 339: Aerospace Propulsion

HW 10: Gas Generator Cycles & Space Missions

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Problem 1

In this problem you will learn a bit about gas generator cycles, and use CEA to make performance calculations. We will use the RS-68, the core engine on the Delta IV as an example, using information from Sutton's textbook. For more information on the Delta IV, see the User's Manual (https://www.ulalaunch.com/docs/default-source/rockets/delta-iv-user%27s-guide.pdf). Since the available information is not comprehensive (and maybe not even accurate), we will have to make some assumptions. There are a number of sources of information on the RS-68 on the internet, please feel free to learn more about the RS-68 and its actual operation. This is a long problem statement – be sure to read through it completely and carefully before you begin.

The RS-68 is a gas generator cycle engine produced by Aerojet Rocketdyne that was notable during its development as being "designed to cost," e.g., see http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.457.7767&rep=rep1&type=pdf Information needed to complete this problem can be found in Figure 6-10 on page 28 of Lecture 24 and Table 11.2 below, both from Sutton. Shown on page 28 of the notes is a schematic of the the RS-68, from the paper cited above, that shows this engine uses a single gas generator to drive the ox and fuel turbopumps.

TABLE 11–2. Comparison of Rocket Engines Using Liquid Oxygen and Liquid Hydrogen Propellants[a]

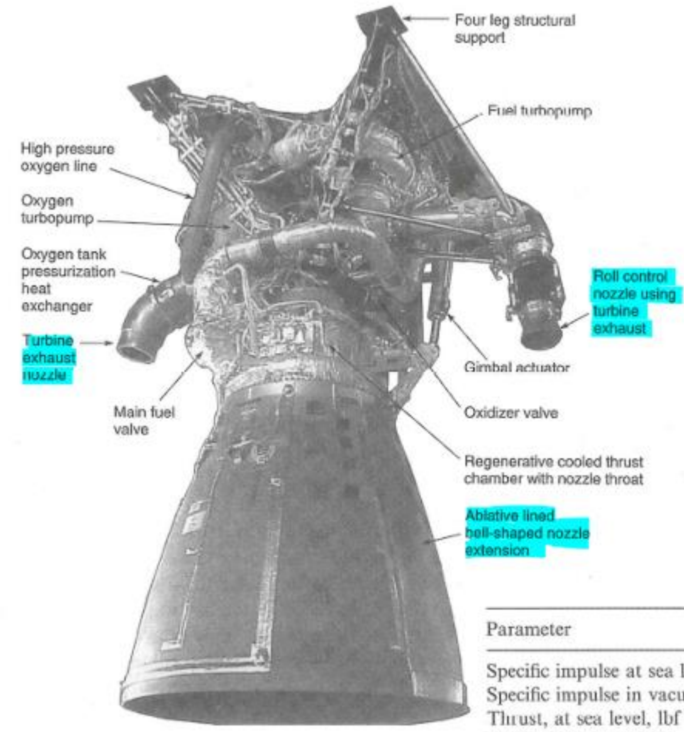
Engine Designation, Engine Cycle, Manuf. or Country (Year Qualified)	Vehicle	Thrust in Vacuum, kN (lbf)	Specific Impulse in Vacuum (sec)	Chamber Pressure, bar (psia)	Mixture Ratio	Nozzle Area Ratio	Engine Mass (dry), kg
SSME, staged combustion, Pratt & Whitney Rocketdyne (1998)	Space Shuttle (3 required)	2183 (490,850)	452.5	196 (2747)	6.0	68.8	3400
RS-68, gas generator, Pratt & Whitney Rocketdyne (2000)	Delta	3313 (745,000)	410	97.2 (1410)	6.0	21.5	6800
LE-5A, Expander bleed, MHI, Japan, (1991)	HII	121.5 (27,320)	452	37.2 (540)	5.0	130	255
LE-7, staged combustion, MHI, Japan (1992)	HII	1080 (242,800)	445.6	122 (1769)	6.0	52	1720
Vulcain, gas generator, SEP (circa 1996)	Ariane 5 2^{nd} stage	1120 (251,840)	433	112 (1624)	5.35	45	1585
HM-7, gas generator, SEP, France (1986)	Ariane 1,2,3,4 3^{rd} stage	62.7 (14,100)	444.2	36.2 (525)	5.1	45	155
RL 10-A3, expander, Pratt & Whitney Rocketdyne (1965)	Various upper stages	73.4 (16,500)	444.4	32.75 (475)	5.0	61	132
RL 10-B2, same as above (1998)	same as above	110 (24,750)	466.5	44.12 (640)	6.0	375	275
YF 73, China (circa 1981)	Long March 3^{rd} stage	44,147 (10,000)	420	26.28 (381)	5.0	40	236
YF 75 (2 required), China (circa 1991)	same	78.45 (17,600)	440	36.7 (532)	5.0	80	550

Propellants are LOX and LH2. In gas generator cycle engines, a few percent of the total propellant flows goes through a gas generator (a combustor that operates at high ϕ/low *O/F*) to produce warm gas that is used to do work on the turbine. The spinning turbine rotates a shaft to

drive pumps that do work on the propellants, pushing them to a high pressure so they can be injected into the main chamber.

You will see from the schematic and Figure 6-10 that there are three nozzles – one is used to exhaust products from the oxidizer turbopump assembly (TPA), one is used to exhaust products from the fuel TPA, and the big one is obviously the main chamber nozzle. Each one generates thrust. Figure 6-10 also indicates how much thrust is generated from the engine and the thrust chamber, and their respective values of specific impulse. The difference between engine thrust and the thrust chamber thrust is the amount of thrust that comes from the TPA exhaust.

a) assume the engine and thrust chamber operate at 100% efficiency and use the values for I_{sp} and thrust from Figure 6-10 in the notes to *calculate flowrate through the engine, through the thrust chamber, and through the gas generator for both sea level and vacuum conditions*. Using the vacuum condition numbers, use a mass balance to *calculate the O/F in the gas generator*.



Parameter	Thrust chamber	Engine
Specific impulse at sea level (max.), sec	363	357
Specific impulse in vacuum (max.), sec	415	409
Thrust, at sea level, lbf	640,700	656,000
Thrust in vacuum lbf	732,400	751,000
Mixture ratio	6.74	6.0

FIGURE 6–10. Simplified view of the RS-68 rocket engine with a gas generator cycle. For engine data see Table 10–3. (Courtesy of Pratt & Whitney, Rocketdyne.)

We are given $I_{sp,TC} = 363$ sec , $I_{sp,EG} = 357$ sec

$I_{vac,TC} = 415$ sec , $I_{vac,EG} = 409$ sec

TC := Thrust Chamber

EG := Engine

GG := Gas Generator

$F^T_{sl,TC} = 640,700 \text{ lbf} = 2.8500 \text{ MN}$

$F^T_{vac,TC} = 732,400 \text{ lbf} = 3.2579 \text{ MN}$

$F^T_{sl,EG} = 656,000 \text{ lbf} = 2.9180 \text{ MN}$

$F^T_{vac,EG} = 751,000 \text{ lbf} = 3.3406 \text{ MN}$

$\eta = 100\% = 1$

The mass flow rate, $\dot{m}$ is computed

$$\boxed{\text{Engine at Sea Level}: \dot{m}_1 = \frac{F^T_{sl,EG}}{\eta g I_{sp,EG}} = \frac{2.9180 \text{ MN}}{(9.81 \text{ m/s}^2)(357 \text{ s})} \quad \dot{m}_1 = 833.2069 \text{ kg/s}}$$

Similarly,

Engine at Vacuum : $\dot{m}_2 = 832.5952$ kg/s

Thrust Chamber at Sea Level : $\dot{m}_3 = 800.3231$ kg/s

Thrust Chamber at Vacuum : $\dot{m}_4 = 800.2349$ kg/s

Then,

Gas Generator at Sea level :

$$\dot{m}_5 = \dot{m}_1 - \dot{m}_3 = 32.8838 \text{ kg/s}$$

Gas Generator at Vacuum :

$$\dot{m}_6 = \dot{m}_2 - \dot{m}_4 = 32.3603 \text{ kg/s}$$

Since, we know $(O/F)_{EG} = 6.0$ & $(O/F)_{TC} = 6.74$
and that

$$O/F = \frac{\dot{m}_{ox}}{\dot{m}_f} \quad \text{for vacuum condition}$$

For $(O/F)_{EG} \Rightarrow 6.0 = \dfrac{\dot{m}_{ox,EG}}{\dot{m}_{f,EG}}$

$$\because \dot{m}_{ox,EG} + \dot{m}_{f,EG} = \dot{m}_2$$

$$6.0 = \frac{\dot{m}_2 - \dot{m}_{f,EG}}{\dot{m}_{f,EG}}$$

$$\dot{m}_{f,EG} = \frac{\dot{m}_2}{7} = 118.9422 \text{ kg/s}$$

$$\dot{m}_{ox,EG} = 713.6530 \text{ kg/s}$$

For $(O/F)_{TC} \Rightarrow 6.74 = \dfrac{\dot{m}_{ox,TC}}{\dot{m}_{f,TC}} = \dfrac{\dot{m}_4 - \dot{m}_{f,TC}}{\dot{m}_{f,TC}}$

$$\dot{m}_{f,TC} = \frac{\dot{m}_4}{7.74} = 103.3895 \text{ kg/s}$$

$$\dot{m}_{ox,TC} = 696.8454 \text{ kg/s}$$

Now, since
$$\begin{cases} \dot{m}_{ox,TC} + \dot{m}_{ox,GG} = \dot{m}_{ox,EG} \\ \dot{m}_{f,TC} + \dot{m}_{f,GG} = \dot{m}_{f,EG} \end{cases}$$

$$\Rightarrow \quad \dot{m}_{ox,GG} = 16.8076 \text{ kg/s}$$
$$\dot{m}_{f,GG} = 15.5526 \text{ kg/s}$$

Thus,
$$(O/F)_{GG} = \frac{\dot{m}_{ox,GG}}{\dot{m}_{f,GG}} = \frac{16.8076 \text{ kg/s}}{15.5526 \text{ kg/s}}$$

$$\boxed{(O/F)_{GG} = 1.0807}$$

Now, let's assume that the performance values on the figure are "delivered" values that take efficiency into account. Including Table 11-2 above, we have all the data necessary (O/F, p_{0t}, ε) to use CEA to calculate ideal (1-D equilibrium) performance *for the thrust chamber.*

b) *calculate ideal performance at sea level and vacuum conditions* for the thrust chamber, and *calculate an overall thrust efficiency* $\eta_{ERE} = I_{sp,del}/I_{sp,ideal}$. Since $I_{sp} = c^* c_f$, $\eta_{ERE} = \eta_{c^*}\,\eta_{cf}$. Assume a thrust coefficient efficiency $\eta_{cf} = 0.98$ and *calculate c* efficiency*. Taking these inefficiencies into account, *calculate the actual flow through the thrust chamber, the throat area of the thrust chamber, and the exit area of the thrust chamber nozzle.*

Calculate the required values using CEA with the following inputs

oxidizer : LOX O/F : 6.74 ε : 21.5

fuel : LH_2 P_{0t} : 97.2 bar

CEA gives the following results (full results in Appendix)

$$I_{sp,CEA} = 3982.6 \text{ m/s} \implies \boxed{I_{sp,ideal} = \frac{I_{sp,CEA}}{g} = 405.9735 \text{ s}}$$

$$I_{vac,CEA} = 4241.8 \text{ m/s} \implies \boxed{I_{vac,ideal} = \frac{I_{vac,CEA}}{g} = 432.3955 \text{ s}}$$

also from CEA

$$\boxed{\begin{aligned} C^*_{ideal} &= 2254.3 \text{ m/s} \\ C_{f,ideal} &= 1.7667 \end{aligned}}$$

Thus,

$$\eta_{ERE} = \frac{I_{sp,TC}}{I_{sp,ideal}}$$

$$\eta_{ERE} = \frac{363 \text{ s}}{405.9735 \text{ s}}$$

$$\boxed{\eta_{ERE} = 0.8941}$$

Then,

$$\eta_{ERE} = \eta_{c^*} \cdot \eta_{cf}$$

$$\because \eta_{cf} = 0.98$$

$$\eta_{c^*} = \frac{\eta_{ERE}}{\eta_{cf}} = \frac{0.8941}{0.98}$$

$$\boxed{\eta_{c^*} = 0.9124}$$

Now, taking these inefficiencies into account

$$C^*_{act} = \eta_{c^*} \cdot C^*_{ideal} = 2056.8 \text{ m/s}$$

$$C_{f,act} = \eta_{cf} \cdot C_{f,ideal} = 1.7314$$

Then,

$$\dot{m}_{act} = \frac{\frac{1}{3} F^T_{sl,TC} + \frac{2}{3} F^T_{vac,TC}}{C^*_{act} \cdot C_{f,act}}$$

$$\boxed{\dot{m}_{act} = 876.6713 \text{ kg/s}}$$

Next,

$$A_t = \frac{C^*_{act} \cdot \dot{m}_{act}}{P_{o,t}}$$

$$A_t = \frac{(2056.8 \text{ m/s})(876.6713 \text{ kg/s})}{(97.2 \text{ bar})}$$

$$\boxed{A_t = 0.1855 \text{ m}^2}$$

Finally,

$$\varepsilon = \frac{A_e}{A_t} \iff A_e = A_t \varepsilon$$

$$A_e = (0.1855 \text{ m}^2)(21.5)$$

$$\boxed{A_e = 3.9884 \text{ m}^2}$$

We want to know more about the turbomachinery. In part a, you calculated the thrust generated by the turbine exhaust, and the flowrates going through the gas generator that feeds each turbine. Now let's do a simplified analysis of the pump work using values from part a.

c) assume the gas generator operates at the same pressure as the thrust chamber, and that its c^* efficiency is 100%. Also, since the RS-68 uses the same propellants as the SSME, assume the RS-68 tank conditions are the same that were given in the notes for the SSME. Use CEA to *calculate the gas generator c* and the properties of the gas (T, MW, c_p, γ) that is used to drive the turbines.*

CEA analysis

$P_{0t} = 97.2$ bar

$\varepsilon = 21.5$

LOX & LH2

$(O/F)_{GG} = 1.0807 \rightarrow$ from part (a)

We get the following results

Gas Generator

$$c^* = 2130.4$$

$$T = \begin{cases} \text{Chamber} : 1051.6 \text{ K} \\ \text{Throat} : 891.23 \text{ K} \\ \text{Exit} : 290.67 \text{ K} \end{cases}$$

$$MW = 4.194$$

$$c_p = \begin{cases} \text{Chamber} : 7.6187 \text{ kJ/kg-K} \\ \text{Throat} : 7.444 \text{ kJ/kg-K} \\ \text{Exit} : 61.2218 \text{ kJ/kg-K} \end{cases}$$

$$\gamma = \begin{cases} \text{Chamber} : 1.3517 \\ \text{Throat} : 1.3631 \\ \text{Exit} : 1.1325 \end{cases}$$

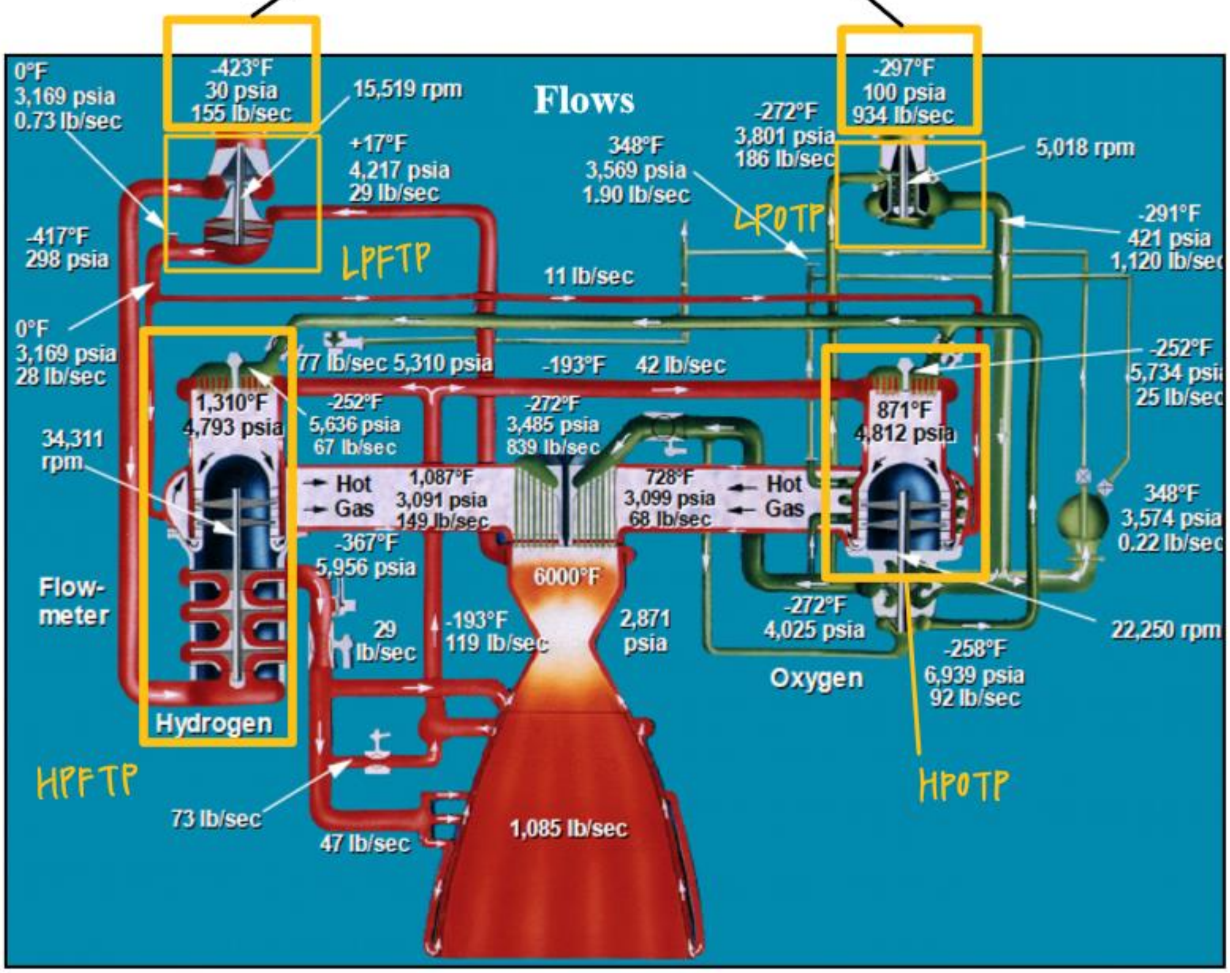
The R-68 has a single gas generator that provides warm gas to drive two separate turbines. Assume the flowrate to each turbine is proportional to the power requirement of the pump it drives. Refer to the slides on turbopumps in Lecture 25 for pump equations.

d) *Neglect pressure losses from the pump outlet to the thrust chamber, and calculate the amount of work required to pump each propellant from tank conditions to the TCA. Assume that the pump and turbine efficiencies are the same as those given for the high pressure SSME turbopumps on page 42 of Lecture 24. Calculate the pressure and temperature of the gas at the exit of each turbine. Neglect the tank head and calculate the net positive suction head (NPSH) for each pump.*

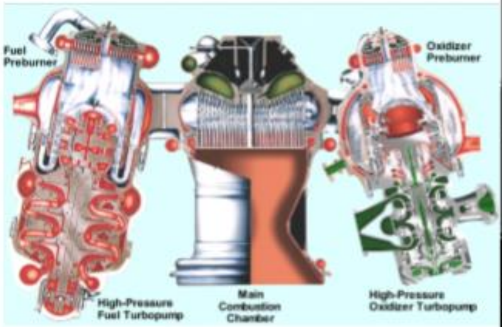
$$P_{F2} = P_{ox2} = P_{ot} = 97.2 \times 10^5 \ Pa$$

$$P_{F1} = 30 \text{ psi} = 2.06843 \text{ bar} \qquad P_{ox1} = 100 \text{ psi} = 6.89476 \text{ bar}$$

Tank Conditions



SSME Pumps



Turbopump Figure Reference	LPOTP 6-7	HPFTP 6-5	HPOTP Main 6-18	HPOTP Boost
Pump:				
Fluid	LOX	LH_2	LOX	LOX
Inlet density, lb/ft^3	71.03	4.32	70.28	70.28
Total inlet pressure, psia	100.0	250	374.8	4,031
Total discharge pressure, psia	408.8	6,024.8	4,129.8	6,952.2
Pump developed head, ft	625.9	168,920	7,591	5,934
Flow rate, lb/s	890.3	148.5	1,067	101.2
Volumetric flow at inlet, gpm	5,626	15,436	6,814	594.7
Shaft speed, rpm	4,961	33,974	27,039	27,039
Efficiency, %	68.6	77.3	67.3	82.5
Shaft power, bhp	1,476	58,970	21,882	1,330
Turbine[a]:				
Fluid	LOX	Hot gas	Hot gas	
Inlet total pressure, psia	3.961	4,933	4,924	
Discharge total pressure, psia	408.8	3,376	3,286	
Isentropic velocity ratio	0.465	0.356	0.286	
Pressure ratio, T-T	--	1.461	1.498	
Inlet temperature, °R	191.3	1835.9	1430	
Discharge temperature, °R	189.7	1698.3	1314.9	
Flow rate, lb/s	176.9	145.6	60.6	
Horsepower	1,476	58,972	23,212	
Shaft speed, rpm	4,961	33,974	27,039	
Efficiency, %	63.1	79.6	78.1	

Efficienies) Fuel Turbopump $\eta_{FTP} = 0.773$
Fuel Turbine $\eta_{FT} = 0.796$
Oxidizer Turbopump $\eta_{OTP} = 0.673$
Oxidizer Turbine $\eta_{OT} = 0.781$

Densities) LH2 $\rho_F = 4.32\ lb/ft^3 = 69.20\ kg/m^3$
LOX $\rho_{ox} = 70.28\ lb/ft^3 = 1125.78\ kg/m^3$

Work Required

Equation: $w_{req} = h_2 - h_1 = \dfrac{p_2 - p_1}{\rho \eta}$

Fuel Pump

$$w_{FP,req} = \frac{P_{F2} - P_{F1}}{\rho_F \eta_{FTP}}$$

$$w_{FP,req} = \frac{(97.2 - 2.06843)\times 10^5 \text{ Pa}}{(0.773)(69.20 \text{ kg/m}^3)}$$

$$\boxed{w_{FP,req} = 177.84 \text{ kJ/kg}}$$

Oxidizer Pump

$$w_{op,req} = \frac{(P_{ox2} - P_{ox1})}{\rho_{ox} \eta_{OTP}}$$

$$\boxed{w_{op,req} = 11.919 \text{ kJ/kg}}$$

Power of Pump

$$P_P = \underbrace{\dot{m}_{ox} \frac{\Delta P_{ox}}{\eta_{ox} \rho_{ox}}}_{P_{Pox}} + \underbrace{\dot{m}_f \frac{\Delta P_f}{\eta_f \rho_f}}_{P_{Pf}}$$

$$P_{Pox} = \dot{m}_{ox,EG} \frac{P_{ox2} - P_{ox1}}{\eta_{OTP} \rho_{ox}} = \dot{m}_{ox,EG}\, w_{op,req}$$

$$P_{Pox} = (713.6538 \text{ kg/s})\, w_{op,req}$$

$$\boxed{P_{Pox} = 8.5061 \text{ MW}}$$

$$P_{Pf} = \dot{m}_{f,EG}\, w_{FP,req}$$

$$P_{Pf} = (118.9422\ \mathrm{kg/s})\, w_{FP,req}$$

$$\boxed{P_{Pf} = 21.153\ \mathrm{MW}}$$

Mass flow for each turbine proportional to required power

$$\dot{m}_{GG,f} = \dot{m}_{GG}\left(\frac{P_{Pf}}{P_{Pf} + P_{Pox}}\right)$$

$\dot{m}_{GG}$ @ vacuum condition

$$\dot{m}_{GG,f} = (32.3603\ \mathrm{kg/s})\left(\frac{P_{Pf}}{P_{Pf} + P_{Pox}}\right)$$

$$\boxed{\dot{m}_{GG,f} = 23.0795\ \mathrm{kg/s}}$$

$$\dot{m}_{GG,ox} = \dot{m}_{GG} - \dot{m}_{GG,f}$$

$$\boxed{\dot{m}_{GG,ox} = 9.2807\ \mathrm{kg/s}}$$

Pressure and Temperatures at each exit of turbine

since $P_T = P_P$ (turbine Power) = (pump power)

$$P_{Pf} = \dot{m}_{GG,f}\, \eta_{FT}\, C_{p,GG}\, (T_{1f} - T_{2f})$$

where $C_p = 7.6187 \times 10^3\ \mathrm{J/kg\text{-}K}$ → from (C)

$T_{1f} = T_{GG} = 1051.6\ \mathrm{K}$

$$T_{2f} = T_{GG} - \frac{P_{Pf}}{\dot{m}_{GG,f}\, \eta_{FT}\, C_{p,GG}}$$

$$\boxed{T_{2f} = 900.4687\ \mathrm{K}}$$

Then from isentropic relations

$$P_{2f} = P_{1f}\left(\frac{T_{2f}}{T_{1f}}\right)^{\gamma_{GG}/\gamma_{GG}-1}$$

where $P_{1f} = P_{F2} = P_{0t} = 97.2 \times 10^5\ Pa$

$\gamma_{GG} = 1.3517 \quad \because (c)$

$$\boxed{P_{2f} = 5.3542\ MPa}$$

similarly,

$$P_{pox} = \dot{m}_{GG,ox}\,\eta_{oT}\,c_{p,GG}\left(T_{1ox} - T_{2ox}\right)$$

where $T_{1ox} = T_{1f} = T_{GG} = 1051.6\ K$

$$T_{2ox} = T_{1ox} - \frac{P_{pox}}{\dot{m}_{GG,ox}\,\eta_{oT}\,c_{p,GG}}$$

$$\boxed{T_{2ox} = 897.5660\ K}$$

$$P_{2ox} = P_{1ox}\left(\frac{T_{2ox}}{T_{1ox}}\right)^{\gamma_{GG}/\gamma_{GG}-1}$$

$$\boxed{P_{2ox} = 5.2882\ MPa}$$

Calculate the Net Positive Suction Head

$$NPSH = \frac{1}{g}\left(\frac{P_s}{\rho} + gz - \frac{P_{vap}}{\rho}\right)$$

where z = negligible

$P_{s,F} = P_{F1} = 2.06843 \times 10^5\ Pa$

$P_{s,ox} = P_{ox1} = 6.89476 \times 10^5\ Pa$

$P_{vap,f} = 1atm = 101325\ Pa$
$P_{vap,ox} = 1atm = 101325\ Pa$ } propellants are stored in normal boiling points

$\rho_F = 69.20\ kg/m^3$

$\rho_{ox} = 1125.78\ kg/m^3$

$$NPSH_F = \frac{1}{g}\left(\frac{P_{s,F}}{\rho_F} - \frac{P_{vap,f}}{\rho_F}\right)$$

$$= \frac{1}{9.81\ m/s^2}\ \frac{206843\ Pa - 101325\ Pa}{69.20\ kg/m^3}$$

$$= \boxed{155.4359\ m}$$

$$NPSH_{ox} = \frac{1}{g}\left(\frac{P_{s,ox}}{\rho_{ox}} - \frac{P_{vap,ox}}{\rho_{ox}}\right)$$

$$= \boxed{53.2557\ m}$$

Problem 2

Using a Hohmann transfer (two tangent burns), calculate the ΔV required to move a global positioning satellite (GPS) from a circular orbit that is 300 km above the Earth surface, to a coplanar circular orbit with an altitude of 20,000 km.

Given Properties

$R_\oplus = 6378.1 \text{ km}$

$G = 6.6740431 \times 10^{-11} \text{ m}^3 \text{kg}^{-1} \text{s}^{-2}$

$M_\oplus = 5.9724 \times 10^{24} \text{ kg}$

$\mu_\oplus = 3.986029 \times 10^{14} \text{ m}^3 \text{s}^{-2}$

$h_1 = 300 \text{ km}$

$h_2 = 20000 \text{ km}$

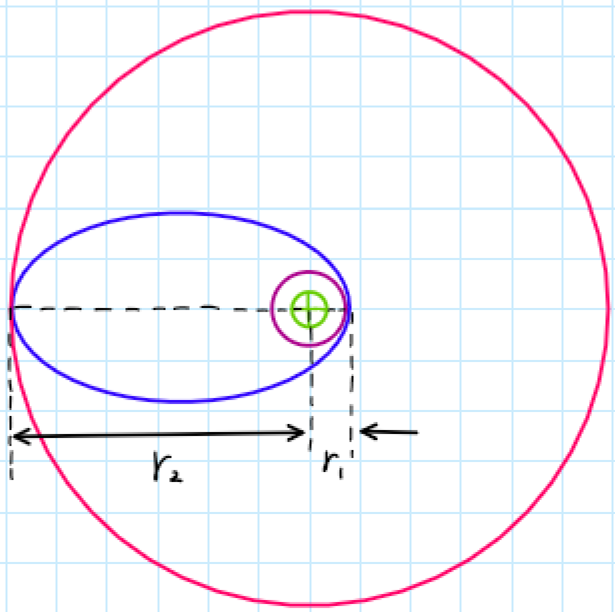
$r_1 = R_\oplus + h_1$

$r_2 = R_\oplus + h_2$

orbit 1

transfer orbit

orbit 2



semi-major axis

$$a = \frac{r_1 + r_2}{2} = 16528.100 \text{ km}$$

Orbit 1 → Transfer Orbit

Orbit 1: $V_{01} = \sqrt{\frac{\mu_\oplus}{r_1}} = \sqrt{\frac{\mu_\oplus}{R_\oplus + h_1}}$

$$V_{01} = \sqrt{\frac{3.986029 \times 10^{14}\ m^3 s^{-2}}{6378.1\ km + 300\ km}} = 7.7258 \times 10^3\ m/s$$

$$V_{01} = 7.7258\ km/s$$

Perigee of Transfer Orbit

$$V_{T1} = \sqrt{\mu_\oplus \left(\frac{2}{r_1} - \frac{1}{a}\right)}$$

$$V_{T1} = 9.7601\ km/s$$

$$\Delta V_1 = V_{T1} - V_{01} = 9.7601 - 7.7258 = 2.0343\ km/s$$

Transfer Orbit → Orbit 2

Apogee of Transfer Orbit

$$V_{T2} = \sqrt{\mu_\oplus \left(\frac{2}{r_2} - \frac{1}{a}\right)}$$

$$V_{T2} = 2.4709\ km/s$$

Orbit 2 $V_{02} = \sqrt{\frac{\mu_\oplus}{r_2}}$

$$V_{02} = 3.8873\ km/s$$

$$\Delta V_2 = V_{02} - V_{T2} = 1.4164\ km/s$$

$$\therefore \Delta V_{req} = \Delta V_1 + \Delta V_2$$

$$\Delta V_{req} = 2.0343 + 1.4164\ km/s$$

$$\boxed{\Delta V_{req} = 3.4506\ km/s}$$

Problem 3

In HW 9 Problem 2, we used the Atlas V users guide to determine the minimal Atlas V configuration that could be used to launch 4500 kg into a GTO. It was suggested that you should use the Atlas 401 for this mission, even though its use would require that the spacecraft supply an additional $\Delta V \sim 300$ m/s. I suggested the 401 because it does not use a solid rocket booster (SRB) and the calculations for mass properties and performance are relatively simple compared to the 411, for instance. In this problem the SRB is included in the calculations.

The users guide has limited data on the SRB. Mass and performance data can be found at the website https://www.spacelaunchreport.com/atlas5.html. Although solid motors can have a tailored thrust history, for this problem assume the *flowrate is constant*. Estimate the average specific impulse same as for the core booster, $I_{sp,av} = 1/3\ I_{sp,SL} + 2/3\ I_{sp,vac}$.

a) *Calculate the propellant mass fraction of the SRB, and compare it with your calculation of the core stage λ from Problem 2.*

The data sheet shows that the solid rockets have a burn time of 90 s, and are separated from the core while the core booster engine continues to provide thrust.

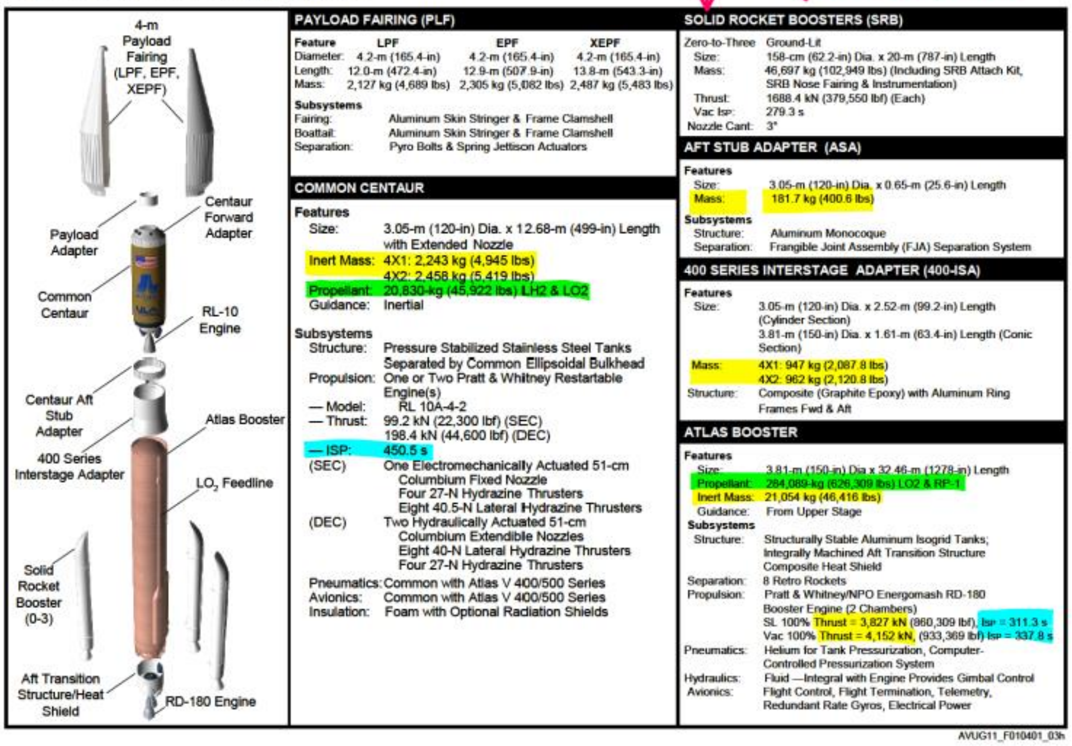
b) *Calculate the velocity of the Atlas V 411 at the end of the SRB burn (after it is burned out, but before it is jettisoned), the velocity of the rocket at the end of the core Stage 1 burn, GLOW, F, and F/W at take-off.*

Table 2.6-1: Atlas V 400/500 Series and HLV Performance Capabilities Summary

Orbit Type (ΔV to GSO)	400 Series				500 Series						HLV
	Number of Solid Rocket Boosters										
	0	1	2	3	0	1	2	3	4	5	N/A
	Payload Systems Weight (PSW), kg (lb)										
GTO (1804 m/s)	4,750 (10,470)	5,950 (13,110)	6,890 (15,180)	7,700 (16,970)	3,775 (8,320)	5,250 (11,570)	6,475 (14,270)	7,475 (16,470)	8,290 (18,270)	8,900 (19,620)	13,000 (28,660)
GTO (1500 m/s)	3,460 (7,620)	4,450 (9,810)	5,210 (11,480)	5,860 (12,910)	2,690 (5,930)	3,900 (8,590)	4,880 (10,750)	5,690 (12,540)	6,280 (13,840)	6,860 (15,120)	--
GSO	--	--	--	--	--	--	2,632 (5,802)	3,192 (7,037)	3,630 (8,003)	3,904 (8,608)	6,454 (14,229)
LEO I =28.5 deg	9,797* (21,598)	12,150* (26,787)	14,067* (31,012)	15,718* (34,653)	8,123 (17,908)	10,986 (24,221)	13,490 (29,741)	15,575 (34,337)	17,443 (38,456)	18,814 (41,478)	29,400* (64,816)*
LEO Sun-sync	7,724 (17,028)	8,905 (19,633)	10,290 * (22,687)	11,704 * (25,803)	6,424 (14,163)	8,719 (19,223)	10,758 (23,717)	12,473 (27,498)	14,019 (30,908)	15,179 (33,464)	--
Atlas V 400 Series • All Performance is SEC • Quoted Performance is with 4-m EPF					**Atlas V 500 Series and HLV** • All Performance is SEC • Quoted Performance is with 5-m Short PLF • HLV LEO Performance is DEC • HLV Quoted Performance is with 5-m Long PLF						
* For 400 series, PSW above 9,072 kg (20,000 lb) may require mission-unique accommodations. For 500 series and HLV, PSW above 19,051 kg (42,000 lb) may require mission-unique accommodations.											
Notes: GTO (1804 m/s): ≥185 x 35,786 km (≥ 100 x 19,323 nmi), Inclination = 27.0 deg, Argument of Perigee = 180 deg, CCAFS GTO (1500 m/s): Apogee Height = 35,786 km (19,323 nmi), Argument of Perigee = 180 deg, CCAFS GSO: 35,786 km Circular (19,323 nmi Circular), Inclination = 0 deg, CCAFS LEO 28.5 deg: 200 km (108 nmi) Circular, CCAFS LEO Sun-sync: 200 km (108 nmi) Circular, VAFB GCS: Guidance Commanded Shutdown, 2.33 sigma for CCAFS, and for VAFB											

Figure 1.4.1-3: Atlas V 400 Series Launch System

ignore this SRB data



From the provided website in the instructions we obtain the following data

Vehicle Components

	SRBs (Aerojet)
Diameter (m)	1.55 m
Length (m)	17.7 m
Propellant Mass (tons)	42.63 t
Total Mass (tons)	46.26 t
Engine	AJ-62
Engine Manufacturer	Aerojet
Fuel	HTPB
Oxidizer	HTPB
Thrust (sea level, tonnes)	172.2 t = 1532 kN
Thrust (vac (avg) tonnes)	126.98 t = 1130 kN
ISP (sea level, sec)	245 s
ISP (vac sec)	275 s
Burn Time (sec)	90 s
No. Engines	1

(a) Since $m_{p,SRB} = 42.63 \times 10^3$ kg

$m_{tot,SRB} = 46.26 \times 10^3$ kg

$$\lambda_{SRB} = \frac{m_{p,SRB}}{m_{tot,SRB}}$$

$$\boxed{\lambda_{SRB} = 0.9215}$$

The λs of the core stages computed in HW9 problem 2 are

$$\lambda_1 = 0.9310$$

$$\lambda_2 = 0.8159$$

$\boxed{\text{The SRB has a greater propellant mass fraction than the } 2^{nd} \text{ stage but is smaller than the } 1^{st} \text{ stage.}}$

(b) With the SRB the takeoff mass is increased by the amount of the SRB

$$\dot{m}_1 = \left(\frac{I_{sp1}}{F_1^T g}\right)_{SL} = \left(\frac{3827 \times 10^3 \text{ N}}{(311.3 \text{ s})(9.81 \text{ m/s}^2)}\right) \Rightarrow 1253.2 \text{ kg/s}$$

Initial mass (at takeoff)

$$m_{PL} = 4500 \text{ kg}$$

$$m_{p2} = 20830 \text{ kg}$$

$$m_{in2} = 2243 + 2458 = 4701 \text{ kg}$$

$$m_{f2} = 181.7 + 947 + 962 = 2090.7 \text{ kg}$$

$$m_{p1} = 284089 \text{ kg}$$

$$m_{in1} = 21054 \text{ kg}$$

$$m_{p,SRB} = 42.63 \times 10^3 \text{ kg}$$

$$m_{in,SRB} = m_{tot,SRB} - m_{p,SRB} = 3630 \text{ kg}$$

$$\Rightarrow m_i = (\text{sum of all masses above})$$

$$m_i = 383520 \text{ kg}$$

Mass after SRB burns out (but before jettisoned)

$$t_{SRB} = 90 \text{ s}$$

$$m_f = m_i - m_{p,SRB} - \dot{m}_1 t_{SRB}$$

$$\left(\dot{m}_{SRB} = \frac{m_{p,SRB}}{t_{SRB}} = \frac{42.63\times10^3 \text{ kg}}{90 \text{ s}} = 473.6667 \text{ kg/s} \right)$$

$$\therefore \; m_f = m_i - m_{p,SRB} - \dot{m}_1 t_{SRB}$$

$$m_f = 228110 \text{ kg}$$

Now, we compute the Isp from the thrust of the 1st stage booster and SRB

$$F^T_{sl1} = 3827 \text{ kN} \qquad F^T_{sl,SRB} = 1532 \text{ kN}$$

$$F^T_{vac1} = 4152 \text{ kN} \qquad F^T_{vac,SRB} = 1130 \text{ kN}$$

$$F^T_{avg1} = \frac{1}{3} F^T_{sl1} + \frac{2}{3} F^T_{vac1} = 4.0437 \text{ MN}$$

$$F^T_{avg,SRB} = 1.2640 \text{ MN}$$

Isp for parallel ignition (SRB & 1st stage)

$$\tilde{I}_{avg} = \frac{F^T_{avg1} + F^T_{avg,SRB}}{\dot{m}_1 + \dot{m}_{SRB}} \cdot \frac{1}{g}$$

$$\boxed{\tilde{I}_{avg} = 313.3164 \text{ s}}$$

Therefore

$$\Delta V_{SPB+1} = g \tilde{I}_{avg} \ln \frac{m_i}{m_f}$$

$$\boxed{\Delta V_{SPB+1} = 1.597 \quad km/s}$$

After, SPB is jettisoned the remaining propellant of stage 1 is

$$m_{p1}' = m_{p1} - \dot{m}_1 t_{SPB}$$

$$m_{p1}' = 171300 \text{ kg}$$

new initial mass is

$$m_i' = m_{PL} + m_{p2} + m_{in2} + m_{12} + m_{p1}' + m_{in1}$$

$$m_i' = 224480 \text{ kg}$$

$$m_f' = m_i' - m_{p1}' = 53176 \text{ kg}$$

$$\Delta V_{end1} = g_0 I_{avg1} \ln\left(\frac{m_i'}{m_f'}\right)$$

$$\because I_{avg1} = \frac{F^T_{avg1}}{\dot{m}_1 g}$$

$$\therefore \boxed{\Delta V_{end1} = 4.6471 \text{ km/s}}$$

$$m_{tot} = m_{p1} + m_{in1} + m_{p2} + m_{in2} + m_{12} + m_{PL} + m_{tot,SPB}$$

$$= m_i$$

$$\boxed{GLOW = m_i g_0 = 3.7624 \quad MN}$$

and

$$F = F^T_{avg,1} + F^T_{avg,SRB}$$

$$F = 4.0437\ MN + 1.2640\ MN$$

$$\boxed{F = 5.3077\ MN}$$

Thus

$$F/W = \frac{F}{GLOW}$$

$$F/W = \frac{5.3077\ MN}{3.7624\ MN}$$

$$\boxed{F/W = 1.4107}$$

Appendix

MATLAB CODE

AAE 339 HW 10

```
clear all; close all; clc;
% Global constants
R_e = 6378.1e3;  % [m]
G = 6.6740831e-11;  % [m3kg-1s-2]
M_e = 5.9724e24;  % [kg]
mu_e = G*M_e;
g = 9.81;
```

P1 (a)

```
% Specific impulse [s]
Isp_TC = 363;
Ivac_TC = 415;
Isp_EG = 357;
Ivac_EG = 409;

% Thrusts [N]
FT_sl_TC = lbf2newtons(640700);
FT_vac_TC = lbf2newtons(732400);
FT_sl_EG = lbf2newtons(656000);
FT_vac_EG = lbf2newtons(751000);

% Efficiency
eta = 1;

% Create array for convenience
arr = [Isp_EG, Ivac_EG, FT_sl_EG, FT_vac_EG;
       Isp_TC, Ivac_TC, FT_sl_TC, FT_vac_TC];
T = array2table(arr,"RowNames",{'EG',
'TC'},"VariableNames",{'Isp','Ivac','slThrust','vacThrust'});
T.slMdot = T.slThrust./T.Isp/g/eta;
T.vacMdot = T.vacThrust./T.Ivac/g/eta;
T.slMdot("GG") = T.slMdot(1) - T.slMdot(2);
T.vacMdot("GG") = T.vacMdot(1) - T.vacMdot(2);
T.slThrust("GG") = T.slThrust(1) - T.slThrust(2);
T.vacThrust("GG") = T.vacThrust(1) - T.vacThrust(2);

mdot1 = T.slMdot("EG");
mdot2 = T.vacMdot("EG");
mdot3 = T.slMdot("TC");
mdot4 = T.vacMdot("TC");
mdot5 = T.slMdot("GG");
mdot6 = T.vacMdot("GG");

% Mixture ratio
phi_EG = 6.0;
phi_TC = 6.74;
mdot_f_EG = mdot2/(1 + phi_EG);
```

```
mdot_ox_EG = mdot_f_EG*phi_EG;
mdot_f_TC = mdot4/(1 + phi_TC);
mdot_ox_TC = mdot_f_TC*phi_TC;
mdot_ox_GG = mdot_ox_EG - mdot_ox_TC;
mdot_f_GG = mdot_f_EG - mdot_f_TC;

phi_GG = mdot_ox_GG/mdot_f_GG;
```

(b)

```
% CEA results
Isp_CEA = 3982.6;
Isp_ideal = Isp_CEA/g;
Ivac_CEA = 4241.8;
Ivac_ideal = Ivac_CEA/g;
cstar_ideal = 2254.3;
cf_ideal = 1.7667;

% Overall effficiency
eta_ERE = Isp_TC/Isp_ideal;
eta_cf = 0.98;
eta_cstar = eta_ERE/eta_cf;

% act values
cstar_act = eta_cstar*cstar_ideal;
cf_act = eta_cf*cf_ideal;
FT_act = 1/3*FT_sl_TC + 2/3*FT_vac_TC;  % [N]
P0t = 97.2e5;  % [Pa]
epsilon = 21.5;
mdot_act = FT_act/cstar_act/cf_act;
At = cstar_act*mdot_act/P0t;
Ae = At*epsilon;
```

(c)

```
Cp_GG = 7.6187e3;  % [J/kg/K]
gamma_GG = 1.3517;
T_GG = 1051.6;  %[K]
Cctar_GG = 2130.4;
```

(d)

```
% Pressures
P_f2 = P0t;
P_ox2 = P0t;
P_f1 = 2.06843e5;
P_ox1 = 6.89476e5;

% Efficiencies
eta_FTP = 0.773;
eta_FT = 0.796;
eta_OTP = 0.673;
eta_OT = 0.781;

% Densities
```

```
rho_F = 69.20;
rho_ox = 1125.78;
```

```
% Work required
w_FP_req = (P_f2 - P_f1)/rho_F/eta_FTP;
w_OP_req = (P_ox2 - P_ox1)/rho_ox/eta_OTP;

% Pump power
P_p_ox = mdot_ox_EG*w_OP_req;
P_p_f = mdot_f_EG*w_FP_req;

% Mass flow for each turbine
mdot_GG = mdot6;
mdot_GG_f = mdot_GG*P_p_f/(P_p_f + P_p_ox);
mdot_GG_ox = mdot_GG*P_p_ox/(P_p_f + P_p_ox);
```

```
% Pressure and temperatures
T2f = T_GG - P_p_f/mdot_GG_f/eta_FT/Cp_GG;
P2f = P_from_isentropic_relation(P0t,T2f,T_GG,gamma_GG,"num");
T2ox = T_GG - P_p_ox/mdot_GG_ox/eta_OT/Cp_GG
P2ox = P_from_isentropic_relation(P0t,T2ox,T_GG,gamma_GG,"num")
```

```
% NPSH
Pvap_f = 101325;
Pvap_ox = Pvap_f;
% fuel
NPSH_F = (P_f1 - Pvap_f)/g/rho_F
% oxidizer
NPSH_ox = (P_ox1 - Pvap_ox)/g/rho_ox
```

P2

```
% Given properties
h1 = 300e3;
h2 = 20000e3;
r1 = R_e + h1;
r2 = R_e + h2;
a = mean([r1 r2]);

% Anonymous function for circular velocity
circ_v = @(r) sqrt(mu_e/r);
% Anonymous function for elliptic velocity
ellip_v = @(r,a) sqrt(mu_e*(2/r - 1/a));


%  Orbit 1 -> Transfer orbit
Vo1 = circ_v(r1);
Vt1 = ellip_v(r1,a);
dv1 = Vt1 - Vo1;

% Transfer orbit -> orbit 2
Vt2 = ellip_v(r2,a);
Vo2 = circ_v(r2);
```

```
dv2 = Vo2 - Vt2;

% Delta v required
dv_req = dv1 + dv2;
```

P3 (a)

```
% Given properties of the SRB
m_p_SRB = 42.63e3;  % [kg]
m_tot_SRB = 46.26e3;  % [kg]
m_in_SRB = m_tot_SRB - m_p_SRB;  % [kg]
FT_sl_SRB = 1532e3; % [N]
FT_vac_SRB = 1130e3; % [N]

% Propellant mass fraction of SRB
lambda_SRB = m_p_SRB/m_tot_SRB;
```

(b)

```
% Atlas V 401 masses [kg]
m_p_2   = 20830;
m_p_1   = 284089;
m_in_2  = 4701;
m_in_1  = 21054;
m_mid   = 2090.7;
m_pl    = 4500;

% Isp
% First stage
Isp1 = 311.3;
Ivac1 = 337.8;
FT_SL_1 = 3827e3;
FT_vac_1 = 4152e3;

% Time for SRB to burn out
t_SRB = 90;  % [s]

% Velocity after SRB burns out
% Mass and mass flow
m_i = m_pl + m_mid + m_in_1 + m_in_2 + m_p_1 + m_p_2 + m_tot_SRB
mdot_SRB = m_p_SRB/t_SRB;
mdot1 = FT_SL_1/Isp1/g
m_f = m_i - m_p_SRB - mdot1*t_SRB
% Isp
% First stage thrust
FT_sl_1 = 3827e3;
FT_vac_1 = 4152e3;
FT_avg_1 = 1/3*FT_sl_1 + 2/3*FT_vac_1

FT_avg_SRB = 1/3*FT_sl_SRB + 2/3*FT_vac_SRB
% Total thrust
FT_sl = FT_sl_1 + FT_sl_SRB
FT_vac = FT_vac_1 + FT_vac_SRB
```

```
Iavg_tilde = (FT_avg_1 + FT_avg_SRB)/(mdot1 + mdot_SRB)/g
dv_SRB_1 = rocket_eqn(Iavg_tilde,m_i,m_f)
```

```
m_p_1_new = m_p_1 - mdot1*t_SRB
m_i_new = m_pl + m_mid + m_in_1 + m_in_2 + m_p_1_new + m_p_2
m_f_new = m_i_new - m_p_1_new

Iavg1 = FT_avg_1/mdot1/g
dv_end1 = rocket_eqn(Iavg1,m_i_new,m_f_new)

GLOW = m_i*g
F = FT_avg_1 + FT_avg_SRB
F_W = F/GLOW
```

Functions

```
function dv = rocket_eqn(Isp,m1,m2)
    g = 9.81;
    dv = g*Isp*log(m1/m2);
end
```

```
function P2 = P_from_isentropic_relation(P1, T2, T1, gamma, type)
    % type => type you want to find
    if type == "num"
        P2 = P1 * (T2 / T1)^(gamma / (gamma - 1));
    elseif type == "den"
        P2 = P1 * (T2 / T1)^(-gamma / (gamma - 1));
    else
        disp("You can only enter num or den for type.")
    end
end
```

CEA RESULTS

Problem 1 (b)

```
*******************************************************************************
**

         NASA-GLENN CHEMICAL EQUILIBRIUM PROGRAM CEA2, FEBRUARY 5, 2004
                   BY  BONNIE MCBRIDE AND SANFORD GORDON
      REFS: NASA RP-1311, PART I, 1994 AND NASA RP-1311, PART II, 1996


*******************************************************************************
**



 ### CEA analysis performed on Wed 22-Apr-2020 22:15:56

 # Problem Type: "Rocket" (Infinite Area Combustor)

 prob case=1111___________1727 ro equilibrium

 # Pressure (1 value):
 p,bar= 97.2
 # Supersonic Area Ratio (1 value):
 supar= 21.5

 # Oxidizer/Fuel Wt. ratio (1 value):
 o/f= 6.74

 # You selected the following fuels and oxidizers:
 reac
 fuel H2(L)             mole=100.0000
 oxid O2(L)             mole=100.0000

 # You selected these options for output:
 # short version of output
 output short
 # Proportions of any products will be expressed as Mole Fractions.
 # Heat will be expressed as siunits
 output siunits

 # Input prepared by this script:prepareInputFile.cgi

 ### IMPORTANT:  The following line is the end of your CEA input file!
 end




              THEORETICAL ROCKET PERFORMANCE ASSUMING EQUILIBRIUM

           COMPOSITION DURING EXPANSION FROM INFINITE AREA COMBUSTOR

 Pin =  1409.8 PSIA
```

```
 CASE = 1111____________

             REACTANT                                  MOLES         ENERGY      TEMP
                                                                   KJ/KG-MOL      K
 FUEL        H2(L)                              100.0000000    -9012.000     20.270
 OXIDANT     O2(L)                              100.0000000   -12979.000     90.170

 O/F=    6.74000  %FUEL= 12.919897  R,EQ.RATIO= 1.177549  PHI,EQ.RATIO=
1.177549

                 CHAMBER   THROAT     EXIT
 Pinf/P            1.0000   1.7306   187.02
 P, BAR            97.200   56.165  0.51972
 T, K             3604.05  3418.64  1935.94
 RHO, KG/CU M    4.7378 0 2.9211 0 5.0356-2
 H, KJ/KG         -930.79 -2020.24 -8861.46
 U, KJ/KG        -2982.36 -3942.99 -9893.54
 G, KJ/KG        -61190.1 -59179.6 -41230.2
 S, KJ/(KG)(K)    16.7199  16.7199  16.7199

 M, (1/n)          14.606   14.783   15.596
 (dLV/dLP)t      -1.03360 -1.02815 -1.00022
 (dLV/dLT)p        1.5813   1.5142   1.0068
 Cp, KJ/(KG)(K)    9.3254   8.8498   3.2335
 GAMMAs            1.1351   1.1332   1.2003
 SON VEL,M/SEC     1526.0   1476.1   1113.0
 MACH NUMBER        0.000    1.000    3.578

 PERFORMANCE PARAMETERS

 Ae/At                      1.0000   21.500
 CSTAR, M/SEC               2254.3   2254.3
 CF                         0.6548   1.7667
 Ivac, M/SEC                2778.7   4241.8
 Isp, M/SEC                 1476.1   3982.6


 MOLE FRACTIONS

 *H               0.03218  0.02737  0.00056
 HO2              0.00009  0.00005  0.00000
 *H2              0.18209  0.17545  0.15060
 H2O              0.70545  0.73138  0.84853
 H2O2             0.00002  0.00001  0.00000
 *O               0.00617  0.00457  0.00000
 *OH              0.06486  0.05381  0.00032
 *O2              0.00914  0.00736  0.00000

  * THERMODYNAMIC PROPERTIES FITTED TO 20000.K
```

Problem 1 (c)

```
*******************************************************************************
**

         NASA-GLENN CHEMICAL EQUILIBRIUM PROGRAM CEA2, FEBRUARY 5, 2004
                   BY  BONNIE MCBRIDE AND SANFORD GORDON
      REFS: NASA RP-1311, PART I, 1994 AND NASA RP-1311, PART II, 1996


*******************************************************************************
**




 ### CEA analysis performed on Wed 22-Apr-2020 22:27:24

 # Problem Type: "Rocket" (Infinite Area Combustor)

 prob case=1111___________1727 ro equilibrium

 # Pressure (1 value):
 p,bar= 97.2
 # Supersonic Area Ratio (1 value):
 supar= 21.5

 # Oxidizer/Fuel Wt. ratio (1 value):
 o/f= 1.0807

 # You selected the following fuels and oxidizers:
 reac
 fuel H2(L)            mole=100.0000
 oxid O2(L)            mole=100.0000

 # You selected these options for output:
 # short version of output
 output short
 # Proportions of any products will be expressed as Mole Fractions.
 # Heat will be expressed as siunits
 output siunits

 # Input prepared by this script:prepareInputFile.cgi

 ### IMPORTANT:  The following line is the end of your CEA input file!
 end




             THEORETICAL ROCKET PERFORMANCE ASSUMING EQUILIBRIUM

          COMPOSITION DURING EXPANSION FROM INFINITE AREA COMBUSTOR

 Pin =  1409.8 PSIA
 CASE = 1111___________
```

```
            REACTANT                       MOLES         ENERGY      TEMP
                                                       KJ/KG-MOL       K
FUEL        H2(L)                      100.0000000     -9012.000     20.270
OXIDANT     O2(L)                      100.0000000    -12979.000     90.170

O/F=    1.08070  %FUEL= 48.060749  R,EQ.RATIO= 7.344020  PHI,EQ.RATIO=
7.344020

                CHAMBER   THROAT     EXIT
Pinf/P            1.0000   1.8713   290.67
P, BAR            97.200   51.944  0.33440
T, K             1051.16   891.23   295.75
RHO, KG/CU M    4.6648 0 2.9402 0 6.0615-2
H, KJ/KG        -2359.23 -3563.22 -8487.57
U, KJ/KG        -4442.91 -5329.88 -9039.25
G, KJ/KG        -37975.1 -33760.2 -18508.4
S, KJ/(KG)(K)    33.8824  33.8824  33.8824

M, (1/n)           4.194    4.194    4.457
MW, MOL WT         4.194    4.194    4.194
(dLV/dLP)t      -1.00000 -1.00000 -1.08934
(dLV/dLT)p        1.0000   1.0000   2.6024
Cp, KJ/(KG)(K)    7.6187   7.4421  61.2218
GAMMAs            1.3517   1.3631   1.1325
SON VEL,M/SEC     1678.2   1551.8    790.4
MACH NUMBER        0.000    1.000    4.429

PERFORMANCE PARAMETERS

Ae/At                      1.0000   21.500
CSTAR, M/SEC               2130.4   2130.4
CF                         0.7284   1.6433
Ivac, M/SEC                2690.2   3658.5
Isp, M/SEC                 1551.8   3501.0


MOLE FRACTIONS

*H2              0.86383  0.86383  0.86383
H2O              0.13617  0.13617  0.07718
H2O(L)           0.00000  0.00000  0.05899

 * THERMODYNAMIC PROPERTIES FITTED TO 20000.K
```