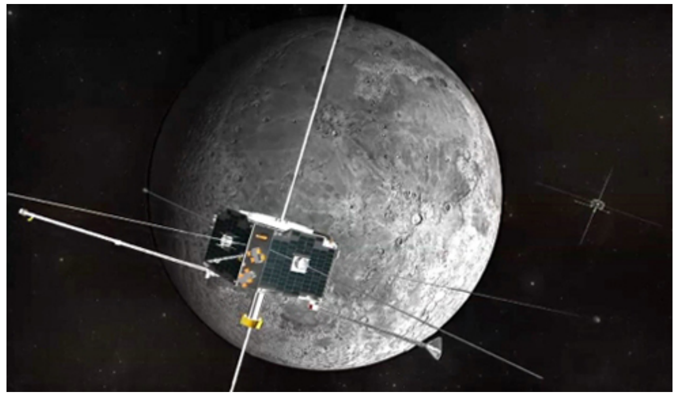
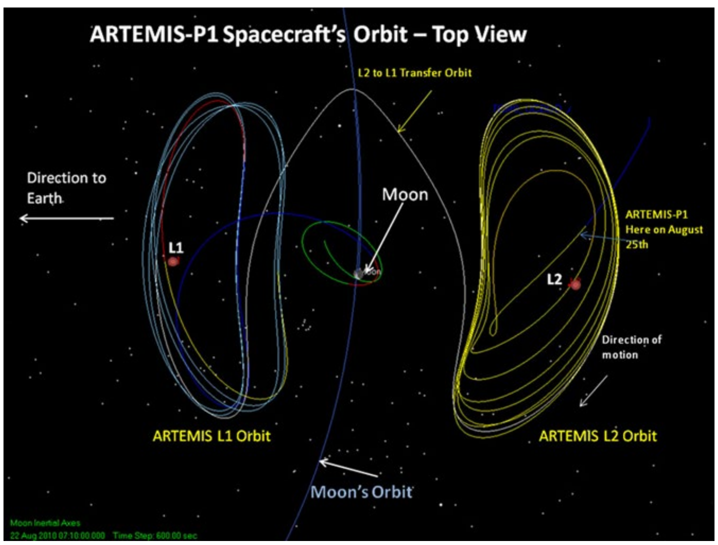
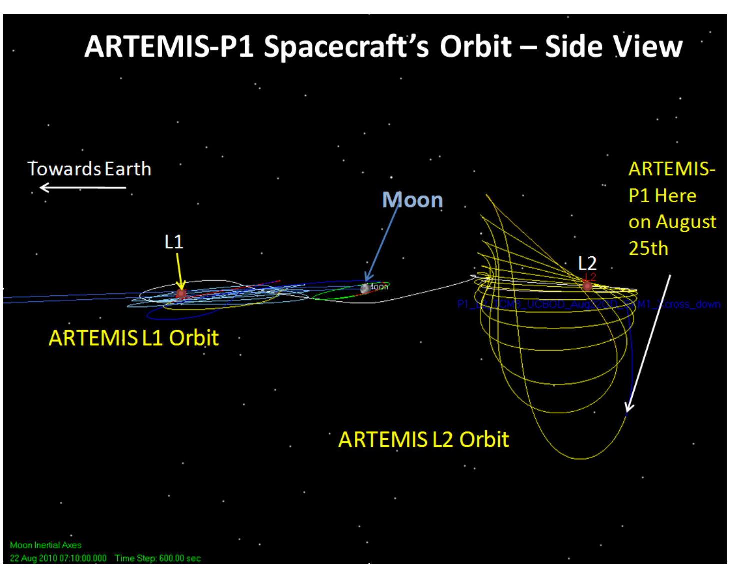
AAE 532 – Orbit Mechanics
Problem Set 2
Due: 9/11/20

ARTEMIS spacecraft



Problem 1: After a complex route to the vicinity of the Moon, the two identical—the original—ARTEMIS spacecraft (P1 and P2) arrived in the lunar vicinity on August 23 and October 22, 2010, respectively. The spacecraft eventually inserted into lunar orbits on June 27 and July 17, 2011. The trajectories in the lunar vicinity prior to lunar orbit insertion were influenced significantly by the gravity fields of other bodies, particularly the Earth and the Sun. The P1 path from arrival in the Moon vicinity to the lunar insertion point appears in the images below. Note that it is far from a familiar elliptical orbit.





Define a system that is comprised of five particles. The law of gravity between each pair is the familiar <u>inverse square law</u>. Obviously, the planets are not truly aligned simultaneously, but <u>assume</u> that the Sun, spacecraft (s/c), and other bodies are collinear and positioned as indicated below:

Earth — Moon — s/c — Sun — Jup

Assume that a single spacecraft is instantaneously located along the transfer path such that the distance between the s/c and the Moon is 140,000 km. The total mass of each ARTEMIS spacecraft is about 130 kg. The distances of the other planets from the Sun are assumed to be equal to the semi-major axis as listed in the Table of Constants located under Supplementary Documents on Brightspace.

(a) Locate the center of mass of the 5-particle system. Identify it on a sketch. Add unit vectors and appropriate position vectors.

(b) Write the vector differential equation for motion of the s/c with respect to the center of mass, i.e., $\bar{r}_i = \bar{r}_{s/c}$. You should obtain an expression for the accelerations on the s/c, i.e., $\ddot{\bar{r}}_{s/c} =$ (sum of 4 terms). Assuming the alignment above, compute the accelerations on the s/c due to each of the other bodies. Include the directions. Which body produces the largest acceleration on the s/c? smallest? What is the descending order? Net acceleration in km/sec^2?

[Did you use a consistent number of significant digits in your computations?]

(c) Compare the relative size of the acceleration terms (gravitational forces) and their directions on the s/c. Is the order of influence what you expected? Which gravity term dominates? Do the acceleration terms seem consistent with your expectations?

Problem 2: Return to the system in Problem 1.

(a) Write the expression for the acceleration of the <u>spacecraft relative to the Moon</u> due to the gravity of the Earth, Moon, Sun and Jupiter. Write this expression in the form $\ddot{\bar{r}}_{☾\to s/c} =$ (sum of terms). Label each of the following terms in this expression: dominant term, direct perturbing terms, indirect perturbing terms, net perturbing term.

(b) Compute the magnitude and direction of each of the terms in your expression for $\ddot{\bar{r}}_{☾\to s/c}$, as well as the net perturbing accelerations for each body and the total net acceleration. Include the directions.

Does the 'dominant' Moon term also possess the largest magnitude? If not, which body contributes the largest individual magnitude terms? Is the largest magnitude term a direct or indirect term?

Compare the magnitude and direction of the dominant acceleration and the <u>net</u> perturbing acceleration from each 'perturbing' body. Which 'perturbing' body has the largest impact?

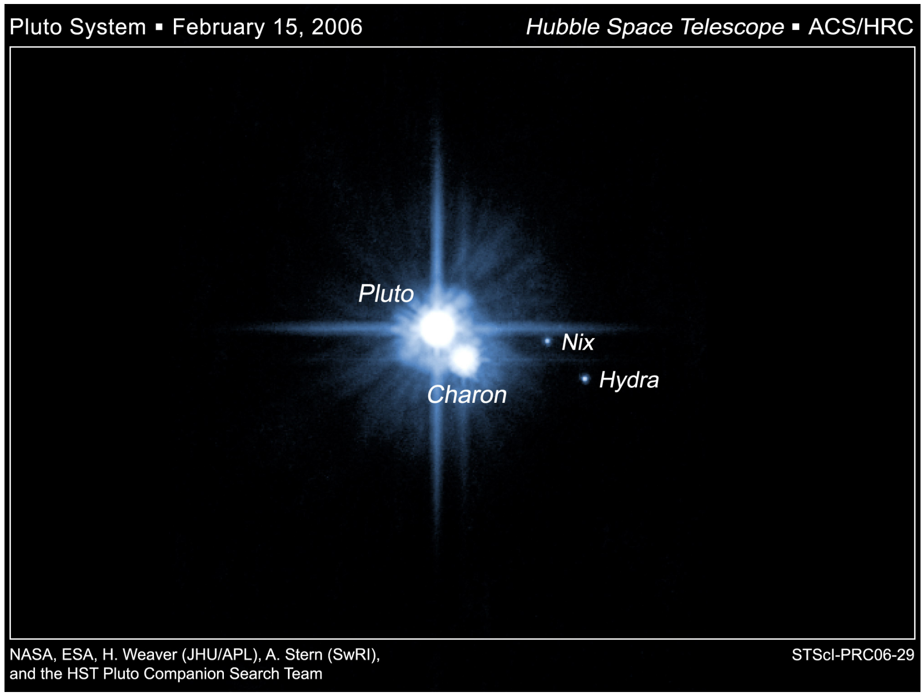
If the indirect perturbing acceleration terms are neglected, compare the magnitude of the dominant and the direct perturbing accelerations. Which body would have the largest impact?

Do the indirect perturbing term matter?

(c) Assume that all the perturbing bodies are neglected. Given the values of the acceleration terms at this instant along the path, is it reasonable to model the motion of the spacecraft using a two-body problem (i.e., only Moon and s/c)?
If a two-body model is not adequate, is there a three-body system that may provide a more reasonable model for the spacecraft motion, e.g., Moon/sc/Earth or Moon/sc/Sun?
Would you rather use a four-body model? Which bodies? Why?

(d) Recast the problem and write the expression for the acceleration of the <u>spacecraft relative to the Earth</u> due to the gravity of the Earth, Moon, Sun and Jupiter.
Now, evaluate the dominant acceleration and each perturbing acceleration term.
Is this model equally valid? Observing the terms, compare the values of the dominant acceleration and the net perturbing acceleration from each additional body. Given the assessment in (b), will any of your conclusions change with this formaution?
Which model for the spacecraft motion is correct (a) or (d)? Why?

Problem 3: The dwarf planet Pluto has 5 known moos. The largest moon, Charon, is nearly half the size of Pluto and is the largest known moon in comparison to its parent body. Assume the Pluto-Charon system is modeled as an isolated two-body problem for the motion of Charon relative to Pluto due to the mutual gravity. Ignore all other forces.



	Pluto	Charon*
Gm	981.601 km^3/s^2	119.480 km^3/s^2
Diameter	1162 km	606 km
Semi-major axis of orbit for Charon relative to Pluto = 19596 km		

*https://nssdc.gsfc.nasa.gov/planetary/factsheet/plutofact.html

(a) Sketch the system and define appropriate unit vectors; let $\hat{i}, \hat{j}, \hat{k}$ be an inertial set of unit vectors such that $\hat{i}$ is parallel to $\bar{r}_{PC}$ at the initial time. Locate the center of mass and define position vectors for each object with respect to the fixed center of mass. Is the cm outside the radius of Pluto?

(b) Let $\bar{r}_{PC} = \bar{r}$ be the relative position of Charon with respect to Pluto. Write the kinematic expressions for the relative position and velocity for the motion of Charon relative to Pluto, that is, $\bar{r}, \bar{v}$ in terms of rotating unit vectors $\hat{r}, \hat{\theta}$.

At $t = 0$, the inertial velocities are known such that $\dot{\bar{r}}_C = .211319 \text{km/s}\, \hat{j}$ and $\dot{\bar{r}}_P = -.025717 \text{km/s}\, \hat{j}$. Determine angular velocity for the motion of Charon relative to Pluto.

(c) Determine the system linear momentum; use this result to compute the velocity of the system center of mass. Does this result make sense?

(d) Determine the constant $\bar{C}_3$ for this system. What are the correct units?

(e) Evaluate the energy constant C_4; of course, include the units!