October 22, 2020

AAE 564 Fall 2020

Homework Nine

Due: Friday, October 30

Exercise 1 Determine (by hand) whether or not each of the following systems are observable.

$$
\begin{array}{rcl}
\dot{x}_1 &=& -x_1 \\
\dot{x}_2 &=& x_2 + u \\
y &=& x_1 + x_2
\end{array}
\qquad
\begin{array}{rcl}
\dot{x}_1 &=& -x_1 \\
\dot{x}_2 &=& x_2 + u \\
y &=& x_2
\end{array}
\qquad
\begin{array}{rcl}
\dot{x}_1 &=& x_1 \\
\dot{x}_2 &=& x_2 + u \\
y &=& x_1 + x_2
\end{array}
\qquad
\begin{array}{rcl}
\dot{x}_1 &=& x_2 \\
\dot{x}_2 &=& 4x_1 + u \\
y &=& -2x_1 + x_2
\end{array}
$$

Exercise 2 (BB in laundromat) Obtain a state space representation of the following system.

$$
\begin{aligned}
m\ddot{q}_1 - m\Omega^2 q_1 + k(q_1 - q_2) &= 0 \\
m\ddot{q}_2 - m\Omega^2 q_2 - k(q_1 - q_2) &= 0 \\
y &= q_1
\end{aligned}
$$

Determine whether or not your state space representation is observable.

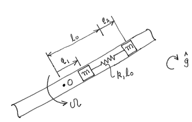

Figure 1: Beavis and Butthead in the laundromat

Exercise 3 For each system in Exercise 1 which is not observable, obtain a basis for the unobservable subspace.

Exercise 4 Determine the unobservable eigenvalues for each of the systems of Exercise 1.

Exercise 5 Determine (by hand) whether or not the following system is observable.

$$\begin{aligned}
\dot{x}_1 &= 5x_1 - x_2 - 2x_3 \\
\dot{x}_2 &= x_1 + 3x_2 - 2x_3 \\
\dot{x}_3 &= -x_1 - x_2 + 4x_3 \\
y_1 &= x_1 + x_2 \\
y_2 &= x_2 + x_3
\end{aligned}$$

If the system is unobservable, compute the unobservable eigenvalues.

Exercise 6 Consider a system described by

$$\begin{aligned}
\dot{x}_1 &= \lambda_1 x_1 + b_1 u \\
\dot{x}_2 &= \lambda_2 x_2 + b_2 u \\
&\vdots \\
\dot{x}_n &= \lambda_n x_n + b_n u \\
y &= c_1 x_1 + c_2 x_2 + \cdots + c_n x_n
\end{aligned}$$

where all quantities are scalar. Obtain conditions on the numbers $\lambda_1, \cdots, \lambda_n$ and $c_1, \cdots, c_n$ which are necessary and sufficient for the observability of this system. (Hint: PBH time.)

Exercise 7 Using MATLAB, carry out the following for linearizations L1, L3, L7 of the two pendulum cart system.

(a) Determine which linearizations are observable?

(b) Determine the unobservable eigenvalues for the unobservable linearizations.

Exercise 8 (BB in laundromat: mass center observations.) Obtain a state space representation of the following system.

$$\begin{aligned}
m\ddot{q}_1 - m\Omega^2 q_1 + k(q_1 - q_2) &= 0 \\
m\ddot{q}_2 - m\Omega^2 q_2 - k(q_1 - q_2) &= 0 \\
y &= \tfrac{1}{2}(q_1 + q_2)
\end{aligned}$$

(a) Obtain a basis for its unobservable subspace.

(b) Determine the unobservable eigenvalues. Consider $\omega := \sqrt{k/2m} > \Omega$.