Paper Review: Consensus and Cooperation in Networked Multi-Agent Systems

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***Abstract*—This paper has its purpose on reviewing the renown paper for the consensus theory of multi-agent system controls by R. Olafti-Saber, J. Alex Fax, and Richard M. Murray [1]. The specific paper will be dissected by identifying the thesis and it's motivation as well as evaluating the theories and methods elucidated. Furthermore, this paper will perform an analysis to recognize any points of improvement which can then be a foundation for another research topic in the field of multi-agent system controls. Namely, the use of non-cooperative/uncooperative agents will be featured since it was not spotlighted in the reviewed paper. Additionally, the theoretical robustness of the discrete-time algorithm which utilizes Perron matrices will be another focus as a probable betterment.**

***Index Terms*—consensus algorithm; gossip-based algorithms; graph Laplacians; graph theory; matrix theory; multi-agent systems; topology;**

I. Introduction and Motivation

The thesis or problem being addressed in this paper is rather vague due to the paper's nature of being a compilation of foundational theory for multi-agent systems and the network consensus theory. Providing the relatively young history of the research on multi-agent systems, the paper commits to congregate the theories formulated and teach them to the readers of the paper in an order in which the they can formulate a solution for current issues in the field.

The methods and techniques discussed in this paper begins with the summary of what consensus theory is about. The paper states that consensus is a state within a multi-agent system where the agents and neighbors are capable of exchanging information to update their states accordingly to reach a convergence of the all agents. The mathematical theories such as the Laplacian matrix and graph theory along with the relation of topology is explained thoroughly. Another major topic that the paper concentrates on is *cooperation*. Cooperation, as the word is defined literally, means that each agent adheres to a protocol that serves all the agents' objective without having any agent opposing or being an outlier. Focusing on networked consensus and cooperation, the paper provides an overview of applications of multi-agent system controls with consensus, information consensus with theoretical results, analysis of convergence and performance of time-varying multi-agent networks, and a theoretical framework to enable a cooperative control of multi-agent systems with consensus.

With the all posed theories and analyses, this paper will delve into the potential advantages and possible ideas of the use of uncooperative multi-agent networks which is not under consideration in the reviewed paper. Also it might be worthwhile to discuss the robustness of continuous-time and discrete-time algorithms for consensus in the subsequent sections of this paper.

This review is driven by the notion of comprehending the theories and practices that construct multi-agent system controls and cast a possible question that might be a possible topic of research.

II. Problem Formulation

In the first few sections of the reviewed paper, it clarifies the possible consensus network algorithms that were theorized at that time. The paper then utilizes that to perform a comparative simulation using those algorithms with the assumption of cooperation and connectivity with the neighbors that enable a relatively fast convergence. The dedication to describe the algorithms backup the reader with sufficient information before moving onto the applications and simulations. The paper lays out the focused algorithms such as the distributed consensus algorithm and average-consensus algorithm that serves as a algorithm to achieve global or non-global asymptotic convergence of multi-agent systems with graph Laplacian.

The discussion is then expanded to the the algebraic connectivity which is defined by the second smallest eigenvalue. This is brought up due to it's characteristic of illustrating the speed of the systems convergence. Since the adjacency or Laplacian matrix for the consensus network is possible to be very large in dimensions, it is important to make use of the Gershgorin Theorem which gives a rough estimate of eigenvalues. To be more precise it would be best to use the power method or shifted inverse power method to actually identify the eigenvalues computationally and analyze the convergence of the consensus algorithm for further investigation in terms of convergence and stability.

Along the lines of the discussion of convergence, the reviewed paper goes over the notion of strongly connected diagraphs which are also balanced. With this trait, the distributed consensus algorithm becomes asymptotically stable if the network is not dynamic, in other words, a switching topology.

The discussion so far was about consensus algorithms concerning static and continuous-time systems. But now the consensus of discrete-time and time varying systems are theoretically defined for multi-agent systems. For a discrete time system the matrix describing the communication between neighbors is indicated by the Perron matrix and not the Laplacian matrix like in a continuous-time system. The Perron matrix being a non-negative row stochastic matrix, the eigenvectors for the Perron matrix become identical with the Laplacian matrix. Whereby the Gershgorin theorem posits that the eigenvalues result in the seemingly same convergence with a continuous-time counterpart system. This relation is defined to be the Perron-Frobenius Theorem and applies to primitive non-negative matrices. Thus, the decision criteria of convergence apply to both continuous-time and discrete-time systems. However, in reality, the performance of the consensus algorithm for continuous-time and discrete-time is hypothetically not going to be as congruent with the theory. Therefore, it is possible to pose a question on whether this paper is sufficient enough to discuss more realistic demonstrations of the consensus theorem such as systems with disturbances and noise. This will be probed in the section IV which is about feasible reinforcement of the theory in the reviewed paper.

In the third section, the reviewed paper segues into the topic of switching networks which are dynamic diagraphs where the system involves topology that may change by time. The system can be modeled by implementing a switching signal that alters the connection between nodes which as a result changes the adjacency matrix for a continuous-time and the Perron matrix for the discrete-time system. It is concluded in this section that for either continuous or discrete the system is capable of converging asymptotically.

The theory setup in the reviewed paper is organized and thoroughly presented to formulate a common understanding for the simulations in the subsequent section.

III. Main Results

In the fourth section of the reviewed paper, the authors provide a system-theoretic framework to address the issues revolving around cooperative controls with multi-agent systems. The interaction of each agent updating the status of one another as well as the agent itself influencing its own states with the interaction with the environment which follow the rules of dynamics. Incorporating the realistic variations into the system introduce a higher dimension of complexity requiring higher orders of decision making for the control of the agents and the system of agents as a whole. To incorporate and control physical disruptions the paper suggests the use of a distributed controller where the feedback is a consensus feedback with the cooperation or communication between agents work as a system controller.

Assessing the multi-agent with the theories discussed in the previous sections and designing a optimal controller from control theory, simulations can be done to experiment the convergence of the multi-agent system. The paper performs a simulation with a 100 nodes/agents with an average of 300 links and connectivity of 6 neighbors for each. This is categorized as a small-world network topology and is capable of reaching a average-consensus that converges with a short period of time. This supports the effective control characteristic which the small-world network topology allows.

With all the theory stated in the paper, the amount of simulations presented seem to be very limited. Hence, it would be better to incorporate more simulations with different topological networks which provide more insight to how the actual controllers perform and follow the theoretical results.

IV. Your Ideas of Further Improvements

The authors of this paper engage with the other literature of the field very well. The amount of citations are fecund and this is due to how they compile the numerous amount of theories proposed by other experts and formulate a unified theory for the consensus and cooperation of multi-agent system controls. The literature does not contradict and does support the findings mentioned.

Next, the validity or reasonableness of the conclusion drawn based upon the analysis will be discussed. The simulations results does draw a relatively reliable conclusion for the authors in that it is sufficient enough to provide a theoretical framework for the analysis of consensus algorithms to the readers of the paper. The authors provide a strong basis fortified with theories and those theoretical frameworks are not psychoanalytical or developmental but well established. Since this paper aims to educate readers concerning consensus networks and their cooperation it is not proper to review this paper in terms of it using the best approach or not. Rather it would be plausible to consider whether it provided enough information so that the reader can spawn they're own ideas that solve or improve multi-agent systems. This is done successfully because from this review the following notions were developed after the reading: the affect of non-cooperate agents in the system and the analysis on robustness for continuous- and discrete-time systems.

In another reading by Yilun Shang, the topic of uncooperative neighbors is brought up. This reading was effective in that in the reviewed paper it only assumes a cooperative system and rules out non-cooperative instances. Cooperativeness intuitively seems likely to converge and satisfy the rule of consensus; however, with certain conditions the uncooperative agents might have an advantage. For example, path planning of a dynamic environment requires update of visual and dynamic information with high latency to agree on a optimal path for each time-frame. But if these specifications are not as promising the use of multi-agents might be a protocol. The agents could conduct a wider scan of the environment and compute an optimal path for each agent. This can be shared between connected neighbor. Although for path planning there exists a level of fuzziness to the data and with such ambiguity cooperation might be difficult. The use of uncooperative agents can design such complex decision-making cases. Shang posits the use of a resilient consensus in continuous-time systems where malicious agents can alter states arbitrarily and unconstrainedly [2]. A projection based resilient consensus algorithm was produced and successfully converged for examples of where the uncooperative agents updated states to something harmful to the system [2]. This does not precisely justify the thoughts mentioned above in this paragraph but does pose a possibility of utilizing uncooperative agents to design more complex multi-agent systems.

The second auxiliary literature studies the use of a fast consensus under bounded noise (FCBN) algorithm that allows a fast asymptotic convergence of multi-agent discrete-time systems with bounded noise [3]. This paper is presented to support or provide further insight on how actual computations on robotics process the updates of states to reach consensus of a multi-agent system. Since in a more realistic case the system is practically modeled as a discrete-time involving noise and disturbance the paper by *He et al.* is enlightening in that it demonstrates the feasibility of a fast convergence with discrete-time systems with external influence. The reviewed paper very lightly touched on the use of Kalman filters for the multi-agent system and the further reading encourages more investigation of methods and algorithms to deal with noise and disturbance for consensus algorithms.

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