Question 6

Span loading is defined as the ratio of aircraft's weight to its wing span. For an airplane in subsonic steady level flight, the drag force due to lift depends directly on the square of span loading through the relation

$$D_i = \frac{1}{\pi e q_\infty}\left(\frac{W}{b}\right)^2$$

where W is the weight of the airplane, b is the wing span, e is the Oswald efficiency factor, and q_∞ is the dynamic pressure.

a) Derive this relation.

b) Consider an aircraft with a wingspan of 37 ft and a gross weight of 10,000 lb. Assume that the Oswald efficiency factor is 0.8. The airplane is flying in steady, level flight at a velocity of 300 mi/h at a standard altitude of 5000 ft. Calculate the drag due to lift using the relation you derived.

c) What happens to the drag due to lift if the wing span is reduced to 34 ft? Why do you think the wingspan has an effect on the drag due to lift?

Q6

(a) SLUF

induced drag is $D_i = q_\infty S C_{Di}$

and $C_{Di} = \frac{C_L^2}{\pi e AR}$

and $C_L = \frac{W}{q_\infty S}$ and $AR = \frac{b^2}{S}$

thus $D_i = q_\infty S \left(\frac{S}{\pi e b^2}\right)\left(\frac{W^2}{q_\infty^2 S^2}\right)$

$= \boxed{\frac{1}{\pi e q_\infty}\left(\frac{W}{b}\right)^2}$

(b) $\rho|_{h=5000ft} = 2.0482 \times 10^{-3}$ $V = 440$ ft/s

$q_\infty = \frac{1}{2}(2.0482 \times 10^{-3})(440)^2 = 198.27$ lb/ft^2

$D_i = \frac{1}{\pi(0.8)(198.27)}\left(\frac{10000}{37}\right)^2 \approx \boxed{146.59 \text{ lb}}$

(c) if $b = 34$ ft

$D_i' \approx 173.60$ lb it increases!

It has effect on the induced drag because the turbulence produced by air flowing on the wing is distributed on the length of the wing span.