Problem 1

Equations of Motion and Demonstration that dH/dt=0

The mass has two velocities --- L/2*diff(sin(theta),t) in the plane, and L/2*sin(theta)*Omega out of the plane

Each spring compresses by an amount L/2*(1-cos(theta))

```
> T := 1/2*m*( (l/2*diff(sin(theta(t)),t))^2 + (l/2*sin(theta(t))*Omega)^2);
  U := k*(l/2*(1-cos(theta(t))))^2;
```

$$T := \frac{1}{2}\, m \left(\frac{1}{4}\, l^2 \cos(\theta(t))^2 \left(\frac{d}{dt}\,\theta(t)\right)^2 + \frac{1}{4}\, l^2 \sin(\theta(t))^2\,\Omega^2\right)$$

$$U := \frac{1}{4}\, k\, l^2\, (1-\cos(\theta(t)))^2 \tag{1.1}$$

```
> L := T-U;
```

$$L := \frac{1}{2}\, m \left(\frac{1}{4}\, l^2 \cos(\theta(t))^2 \left(\frac{d}{dt}\,\theta(t)\right)^2 + \frac{1}{4}\, l^2 \sin(\theta(t))^2\,\Omega^2\right) - \frac{1}{4}\, k\, l^2\, (1 - \cos(\theta(t)))^2 \tag{1.2}$$

Maple doesn't like to take the derivative of a function of time, so I use x to do it with an inverse substitution

```
> dLdthetaDot := subs(x=diff(theta(t),t),diff(subs(diff(theta(t),t)=x,L),x));
  dLdtheta := subs(x=theta(t),diff(subs(theta(t)=x,L),x));
```

$$dLdthetaDot := \frac{1}{4}\, m\, l^2 \cos(\theta(t))^2 \left(\frac{d}{dt}\,\theta(t)\right)$$

$$dLdtheta := \frac{1}{2}\, m \left(-\frac{1}{2}\, l^2 \cos(\theta(t)) \left(\frac{d}{dt}\,\theta(t)\right)^2 \sin(\theta(t)) + \frac{1}{2}\, l^2 \sin(\theta(t))\,\Omega^2 \cos(\theta(t))\right) - \frac{1}{2}\, k\, l^2\, (1 - \cos(\theta(t)))\sin(\theta(t)) \tag{1.3}$$

Equation of motion resulting from Lagrange's equations

```
> EOM := diff(dLdthetaDot,t)-dLdtheta = 0;
```

$$EOM := -\frac{1}{2}\, m\, l^2 \cos(\theta(t)) \left(\frac{d}{dt}\,\theta(t)\right)^2 \sin(\theta(t)) + \frac{1}{4}\, m\, l^2 \cos(\theta(t))^2 \left(\frac{d^2}{dt^2}\,\theta(t)\right) - \frac{1}{2}\, m \left(-\frac{1}{2}\, l^2 \cos(\theta(t)) \left(\frac{d}{dt}\,\theta(t)\right)^2 \sin(\theta(t)) + \frac{1}{2}\, l^2 \sin(\theta(t))\,\Omega^2 \cos(\theta(t))\right) + \frac{1}{2}\, k\, l^2\, (1 - \cos(\theta(t)))\sin(\theta(t)) = 0 \tag{1.4}$$

Hamiltonian

```
> H := L - dLdthetaDot*diff(theta(t),t);
```

$$H := \frac{1}{2}\, m \left(\frac{1}{4}\, l^2 \cos(\theta(t))^2 \left(\frac{d}{dt}\,\theta(t)\right)^2 + \frac{1}{4}\, l^2 \sin(\theta(t))^2\,\Omega^2\right) - \frac{1}{4}\, k\, l^2\, (1 \tag{1.5}$$

$$-\cos(\theta(t)))^2 - \frac{1}{4} m l^2 \cos(\theta(t))^2 \left(\frac{d}{dt}\theta(t)\right)^2$$

Look at dH/dt to see if it is zero - i.e., H is constant

```
> Hdot := subs(x=theta(t),diff(subs(theta(t)=x,H),x))*diff
  (theta(t),t) + subs(x=diff(theta(t),t),diff(subs(diff(theta
  (t),t)=x,H),x))*diff(theta(t),t,t);
```

$$Hdot := \left(\frac{1}{2} m \left(-\frac{1}{2} l^2 \cos(\theta(t)) \left(\frac{d}{dt}\theta(t)\right)^2 \sin(\theta(t)) + \frac{1}{2} l^2 \sin(\theta(t))\,\Omega^2 \cos(\theta(t))\right) - \frac{1}{2} k l^2 (1-\cos(\theta(t)))\sin(\theta(t)) + \frac{1}{2} m l^2 \cos(\theta(t)) \left(\frac{d}{dt}\theta(t)\right)^2 \sin(\theta(t))\right)\left(\frac{d}{dt}\theta(t)\right) - \frac{1}{4} m l^2 \cos(\theta(t))^2 \left(\frac{d}{dt}\theta(t)\right)\left(\frac{d^2}{dt^2}\theta(t)\right) \quad (1.6)$$

```
> simplify(Hdot);
```

$$\frac{1}{4}\left(\frac{d}{dt}\theta(t)\right) l^2 \left(m\cos(\theta(t))\left(\frac{d}{dt}\theta(t)\right)^2 \sin(\theta(t)) + m\sin(\theta(t))\,\Omega^2\cos(\theta(t)) - 2k\sin(\theta(t)) + 2k\sin(\theta(t))\cos(\theta(t)) - m\cos(\theta(t))^2\left(\frac{d^2}{dt^2}\theta(t)\right)\right) \quad (1.7)$$

We can solve for thetaDotDot using the EOM, and then subsitute that back into Hdot to show it is zero

```
> thetaDotDot := solve(EOM,diff(theta(t),t,t));
```

$$thetaDotDot := \frac{\sin(\theta(t))\left(m\cos(\theta(t))\left(\frac{d}{dt}\theta(t)\right)^2 + m\,\Omega^2\cos(\theta(t)) - 2k + 2k\cos(\theta(t))\right)}{m\cos(\theta(t))^2} \quad (1.8)$$

Do the subsitution

```
> simplify(subs(diff(theta(t),t,t)=thetaDotDot,Hdot));
```

$$0 \quad (1.9)$$

Fixed Points and Their Stability

Put in first order form and then find the fixed points - equivalent to setting time derivatives of theta to zero in EOM

```
> fixedPointEq := simplify(eval(4/l^2*subs({diff(theta(t),t)=0}
  ,EOM)));
```

$$fixedPointEq := -\sin(\theta(t))\left(m\,\Omega^2\cos(\theta(t)) - 2k + 2k\cos(\theta(t))\right) = 0 \quad (2.1)$$

```
> solve(fixedPointEq,theta(t));
```

$$0, \arccos\left(\frac{2k}{m\,\Omega^2 + 2k}\right) \quad (2.2)$$

First-order form of equations

```
> f1 := y;
  f2 := m*y^2*tan(x)+m*Omega^2*tan(x)-2*k*(1-cos(x))*tan(x)/cos
  (x);
```

$$f1 := y$$

$$f2 := m\,y^2 \tan(x) + m\,\Omega^2 \tan(x) - \frac{2\,k\,(1-\cos(x))\,\tan(x)}{\cos(x)} \tag{2.3}$$

```
> with(linalg):
> A:=matrix(2,2,[diff(f1,x),diff(f1,y),diff(f2,x),diff(f2,y)]);
```

$$A := \left[\left[0, 1 \right], \tag{2.4}$$

$$\left[m\,y^2 \left(1+\tan(x)^2\right) + m\,\Omega^2 \left(1+\tan(x)^2\right) - \frac{2\,k \sin(x)\,\tan(x)}{\cos(x)} - \frac{2\,k\,(1-\cos(x))\left(1+\tan(x)^2\right)}{\cos(x)} - \frac{2\,k\,(1-\cos(x))\,\tan(x)\,\sin(x)}{\cos(x)^2}, 2\,m\,y\tan(x) \right] \right]$$

```
> simplify(A[2,1]);
```

$$\frac{2\,k\cos(x)^2 + m\,y^2\cos(x) + m\,\Omega^2\cos(x) + 2\,k\cos(x) - 4\,k}{\cos(x)^3} \tag{2.5}$$

Matrices holding A evaluated at each fixed point

```
> A_atFP1 := matrix(2,2,[]);
  A_atFP2 := matrix(2,2,[]);
```

$$A\_atFP1 := array(1..2, 1..2, [\ ])$$

$$A\_atFP2 := array(1..2, 1..2, [\ ]) \tag{2.6}$$

```
> A_atFP1[1,1] := eval(subs({x=0,y=0},A[1,1]));
  A_atFP1[1,2] := eval(subs({x=0,y=0},A[1,2]));
  A_atFP1[2,1] := eval(subs({x=0,y=0},A[2,1]));
  A_atFP1[2,2] := eval(subs({x=0,y=0},A[2,2]));
```

$$A\_atFP1_{1,1} := 0$$

$$A\_atFP1_{1,2} := 1$$

$$A\_atFP1_{2,1} := m\,\Omega^2$$

$$A\_atFP1_{2,2} := 0 \tag{2.7}$$

```
> A_atFP2[1,1] := simplify(eval(subs({x=arccos(2*k/(m*
  Omega^2+2*k)),y=0},A[1,1])));
  A_atFP2[1,2] := simplify(eval(subs({x=arccos(2*k/(m*Omega^2+2*
  k)),y=0},A[1,2])));
  A_atFP2[2,1] := simplify(eval(subs({x=arccos(2*k/(m*Omega^2+2*
  k)),y=0},A[2,1])));
  A_atFP2[2,2] := simplify(eval(subs({x=arccos(2*k/(m*Omega^2+2*
  k)),y=0},A[2,2])));
```

$$A\_atFP2_{1,1} := 0$$

$$A\_atFP2_{1,2} := 1$$

$$A\_atFP2_{2,1} := -\frac{1}{4}\,\frac{(m\,\Omega^2+2\,k)\,m\,\Omega^2\,(m\,\Omega^2+4\,k)}{k^2}$$

$$A\_atFP2_{2,2} := 0 \tag{2.8}$$

```
> eigsFP1 := eigenvals(A_atFP1);
```

$$eigsFP1 := \sqrt{m}\,\Omega,\ -\sqrt{m}\,\Omega \tag{2.9}$$

← fixed point @ (0,0) is a saddle

```
> eigsFP2 := eigenvals(A_atFP2);
```

$$eigsFP2 := \frac{1}{2}\,\frac{\sqrt{-m^3\,\Omega^4-6\,m^2\,k\,\Omega^2-8\,m\,k^2}\,\Omega}{k},\ -\frac{1}{2}\,\frac{\sqrt{-m^3\,\Omega^4-6\,m^2\,k\,\Omega^2-8\,m\,k^2}\,\Omega}{k} \tag{2.10}$$

Here is the term inside of the radical - it determines stability

```
> insideRadical := -m^3*Omega^4-6*m^2*k*Omega^2-8*m*k^2;
```

$$insideRadical := -m^3\,\Omega^4-6\,m^2\,k\,\Omega^2-8\,m\,k^2 \tag{2.11}$$

The above is a quadratic in Omega^2

```
> quadraticEqnForOmegaSquared := subs({Omega^4=z^2,Omega^2=z},
  insideRadical);
```

$$quadraticEqnForOmegaSquared := -z^2\,m^3-6\,z\,m^2\,k-8\,m\,k^2 \tag{2.12}$$

Use the solutions to factor the term inside the radical

```
> solve(quadraticEqnForOmegaSquared,z);
```

$$-\frac{4\,k}{m},\ -\frac{2\,k}{m} \tag{2.13}$$

Or, another approach is to use Maple's factor command

```
> factor(insideRadical);
```

$$-m\,(m\,\Omega^2+4\,k)\,(m\,\Omega^2+2\,k) \tag{2.14}$$

From the last line, we see that no matter the choice of Omega (positive), the eigenvalues are imaginary and thus the fixed points associated with the arccos() are centers. Using H, we can also show this and say it <u>conclusively</u>

```
>
```

Problem 2 - Part(a) - Fixed Points: (0,0) ~~(1,0) (-1,0)~~

> $x := 0 : y := 0 :$

> $a := 0 : b := 1 : c := -1 - 3 \cdot x^2 : d := -2 \cdot \mu :$

> $q := a \cdot d - b \cdot c; p := a + d; \Delta := p^2 - 4 \cdot q;$

$$q := 1$$

$$p := -2\,\mu$$

$$\Delta := 4\,\mu^2 - 4 \qquad (1)$$

Good

$\mu = 1$ degenerate stable node (shown)

$\mu < 1$ stable spiral

$\mu > 1$ stable node

> $x := 1 : y := 0 :$

> $a := 0 : b := 1 : c := -1 - 3 \cdot x^2 : d := -2 \cdot \mu :$

> $q := a \cdot d - b \cdot c; p := a + d; \Delta := p^2 - 4 \cdot q;$

$$q := -2$$

$$p := -2\,\mu$$

$$\Delta := 4\,\mu^2 + 8 \qquad (2)$$

Non-Real

> $x := -1 : y := 0 :$

> $a := 0 : b := 1 : c := -1 - 3 \cdot x^2 : d := -2 \cdot \mu :$

> $q := a \cdot d - b \cdot c; p := a + d; \Delta := p^2 - 4 \cdot q;$

$$q := -2$$

$$p := -2\,\mu$$

$$\Delta := 4\,\mu^2 + 8 \qquad (3)$$

Non-Real

Part(b) - Fixed Points: (0,0) (1,0) (-1,0)

> $x := 0 : y := 0 :$

> $a := 0 : b := 1 : c := -1 + 3 \cdot x^2 : d := -2 \cdot \mu :$

> $q := a \cdot d - b \cdot c; p := a + d; \Delta := p^2 - 4 \cdot q;$

$$q := 1$$

$$p := -2\,\mu$$

$$\Delta := 4\,\mu^2 - 4 \qquad (4)$$

$\mu = 1$ degenerate stable node (shown)

$\mu < 1$ stable spiral

$\mu > 1$ stable node

> $x := 1 : y := 0 :$

> $a := 0 : b := 1 : c := -1 + 3 \cdot x^2 : d := -2 \cdot \mu :$

> $q := a \cdot d - b \cdot c; p := a + d; \Delta := p^2 - 4 \cdot q;$

$$q := -2$$

$$p := -2\,\mu$$

$$\Delta := 4\,\mu^2 + 8 \qquad (5)$$

Saddle (shown)

> $x := -1 : y := 0 :$

> $a := 0 : b := 1 : c := -1 + 3 \cdot x^2 : d := -2 \cdot \mu :$

> $q := a \cdot d - b \cdot c; p := a + d; \Delta := p^2 - 4 \cdot q;$

$$q := -2$$

$$p := -2\,\mu$$

$$\Delta := 4\,\mu^2 + 8 \qquad (6)$$

Saddle (shown)

Part(c) - Fixed Points: (0,0) (1,0) (-1,0)

> $x := 0 : y := 0 :$

> $a := 0 : b := 1 : c := 1 - 3 \cdot x^2 : d := -2 \cdot \mu :$

> $q := a \cdot d - b \cdot c; p := a + d; \Delta := p^2 - 4 \cdot q;$

$$q := -1$$
$$p := -2\,\mu$$
$$\Delta := 4\,\mu^2 + 4 \quad (7)$$

saddle (shown)

> $x := 1 : y := 0 :$

> $a := 0 : b := 1 : c := 1 - 3 \cdot x^2 : d := -2 \cdot \mu :$

> $q := a \cdot d - b \cdot c; p := a + d; \Delta := p^2 - 4 \cdot q;$

$$q := 2$$
$$p := -2\,\mu$$
$$\Delta := 4\,\mu^2 - 8 \quad (8)$$

$\mu = \sqrt{2}$ deg. stable node

$\mu > \sqrt{2}$ stable node

$\mu < \sqrt{2}$ stable spiral (shown)

> $x := -1 : y := 0 :$

> $a := 0 : b := 1 : c := 1 - 3 \cdot x^2 : d := -2 \cdot \mu :$

> $q := a \cdot d - b \cdot c; p := a + d; \Delta := p^2 - 4 \cdot q;$

$$q := 2$$
$$p := -2\,\mu$$
$$\Delta := 4\,\mu^2 - 8 \quad (9)$$

see above

Part(d) - Fixed Points: (0,0) ~~(1,0) (-1,0)~~

> $x := 0 : y := 0 :$

> $a := 0 : b := 1 : c := 1 + 3 \cdot x^2 : d := -2 \cdot \mu :$

> $q := a \cdot d - b \cdot c; p := a + d; \Delta := p^2 - 4 \cdot q;$

$$q := -1$$
$$p := -2\,\mu$$
$$\Delta := 4\,\mu^2 + 4 \quad (10)$$

Saddle (shown)

> $x := I : y := 0 :$

> $a := 0 : b := 1 : c := 1 + 3 \cdot x^2 : d := -2 \cdot \mu :$

> $q := a \cdot d - b \cdot c; p := a + d; \Delta := p^2 - 4 \cdot q;$

$$q := 2$$
$$p := -2\,\mu$$
$$\Delta := 4\,\mu^2 - 8 \quad (11)$$

Non-Real

> $x := -I : y := 0 :$

> $a := 0 : b := 1 : c := 1 + 3 \cdot x^2 : d := -2 \cdot \mu :$

> $q := a \cdot d - b \cdot c; p := a + d; \Delta := p^2 - 4 \cdot q;$

$$q := 2$$
$$p := -2\,\mu$$
$$\Delta := 4\,\mu^2 - 8 \quad (12)$$

Non-Real

Problem 3

```
> restart
> with(plots) : with(linalg) : with(DEtools) :
> interface(Typesetting='extended') :
> with(Typesetting) :
> Settings(dot = t, usedot = true, prime = x, useprime = true) :
> Settings(functionassign = true) :
```

Part(a) - Fixed Points: (0,0)

> μ := 1

$$\mu := 1 \tag{1}$$

> EQ1 := y(t) = diff(x(t), t)

$$EQ1 := y(t) = \dot{x}(t) \tag{2}$$

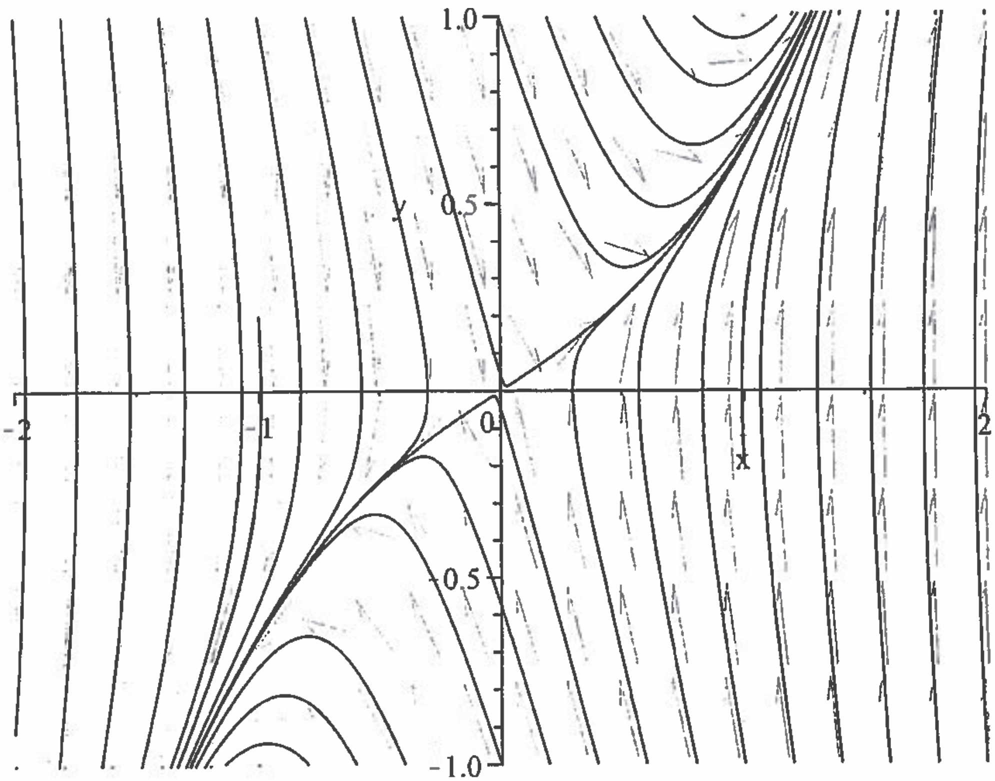
> EQ2 := diff(y(t), t) = -2·μ·y(t) − x(t) − x(t)^3

$$EQ2 := \dot{y}(t) = -2\,y(t) - x(t) - x(t)^3 \tag{3}$$

> p1 := DEplot([EQ1, EQ2], [x(t), y(t)], t=0..10, x=-2..2, [[x(0) = -2, y(0) = 1], [x(0) = -1.8, y(0) = 1], [x(0) = -1.6, y(0) = 1], [x(0) = -1.4, y(0) = 1], [x(0) = -1.2, y(0) = 1], [x(0) = -1, y(0) = 1], [x(0) = -0.8, y(0) = 1], [x(0) = -0.6, y(0) = 1], [x(0) = -0.4, y(0) = 1], [x(0) = -2, y(0) = 1], [x(0) = 0, y(0) = 1], [x(0) = 0.2, y(0) = 1], [x(0) = 0.4, y(0) = 1], [x(0) = 0.6, y(0) = 1], [x(0) = 0.8, y(0) = 1], [x(0) = 1.0, y(0) = 1], [x(0) = 1.2, y(0) = 1], [x(0) = 1.4, y(0) = 1], [x(0) = 1.6, y(0) = 1], [x(0) = 1.8, y(0) = 1], [x(0) = 2.0, y(0) = 1], [x(0) = -2, y(0) = -1], [x(0) = -1.8, y(0) = -1], [x(0) = -1.6, y(0) = -1], [x(0) = -1.4, y(0) = -1], [x(0) = -1.2, y(0) = -1], [x(0) = -1, y(0) = -1], [x(0) = -0.8, y(0) = -1], [x(0) = -0.6, y(0) = -1], [x(0) = -0.4, y(0) = -1], [x(0) = -2, y(0) = -1], [x(0) = 0, y(0) = -1], [x(0) = 0.2, y(0) = -1], [x(0) = 0.4, y(0) = -1], [x(0) = 0.6, y(0) = -1], [x(0) = 0.8, y(0) = -1], [x(0) = 1.0, y(0) = -1], [x(0) = 1.2, y(0) = -1], [x(0) = 1.4, y(0) = -1], [x(0) = 1.6, y(0) = -1], [x(0) = 1.8, y(0) = -1], [x(0) = 2.0, y(0) = -1], [x(0) = -1, y(0) = -0.2], [x(0) = -1, y(0) = 0.2], [x(0) = 1, y(0) = -0.2], [x(0) = 1, y(0) = 0.2],], linecolor = black, stepsize = 0.01, thickness = 1)

[Length of output exceeds limit of 1000000] (4)

> display(p1, view = [-2..2, -1..1])



Part(b) - Fixed Points: (0,0) (1,0) (-1,0)

> $\mu := 1$

$$\mu := 1 \qquad (5)$$

> $EQ1 := y(t) = diff(x(t), t)$

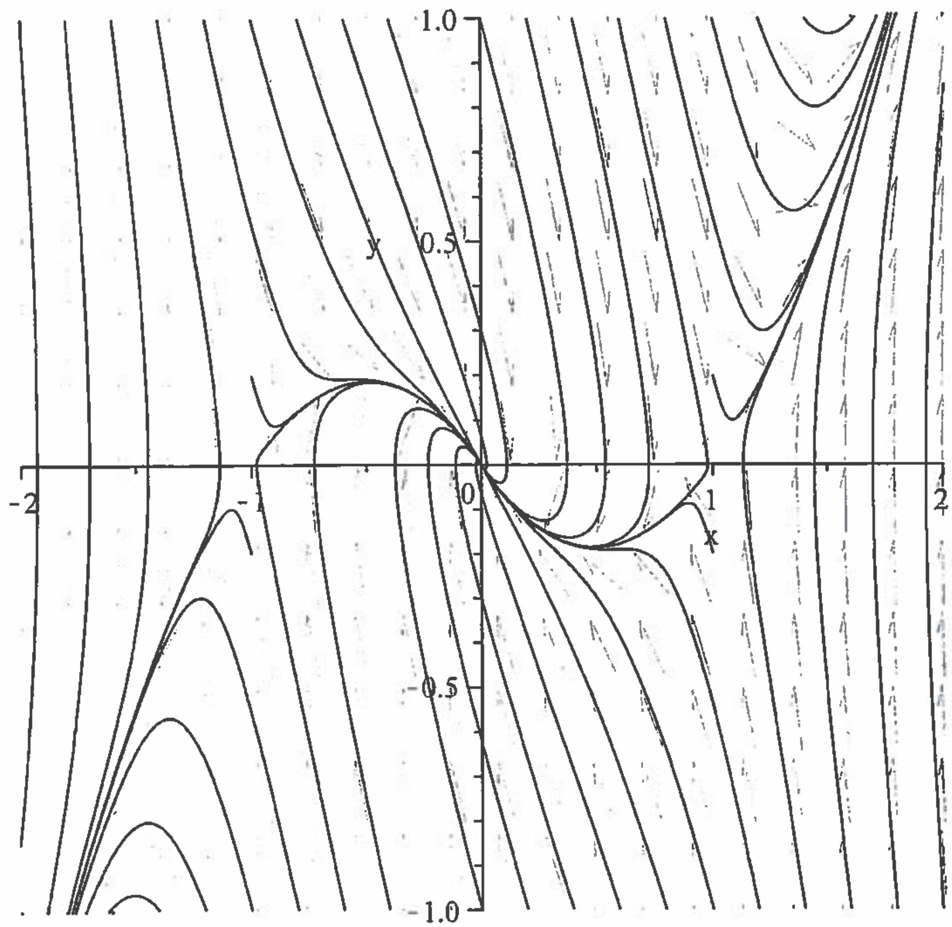
$$EQ1 := y(t) = \dot{x}(t) \qquad (6)$$

> $EQ2 := diff(y(t), t) = -2 \cdot \mu \cdot y(t) - x(t) + x(t)^3$

$$EQ2 := \dot{y}(t) = -2\,y(t) - x(t) + x(t)^3 \qquad (7)$$

> *p1* := *DEplot*([*EQ1*, *EQ2*], [x(t), y(t)], t = 0 ..10, x = -2 ..2, [[x(0) = -2, y(0) = 1], [x(0) = -1.8, y(0) = 1], [x(0) = -1.6, y(0) = 1], [x(0) = -1.4, y(0) = 1], [x(0) = -1.2, y(0) = 1], [x(0) = -1, y(0) = 1], [x(0) = -0.8, y(0) = 1], [x(0) = -0.6, y(0) = 1], [x(0) = -0.4, y(0) = 1], [x(0) = -2, y(0) = 1], [x(0) = 0, y(0) = 1], [x(0) = 0.2, y(0) = 1], [x(0) = 0.4, y(0) = 1], [x(0) = 0.6, y(0) = 1], [x(0) = 0.8, y(0) = 1], [x(0) = 1.0, y(0) = 1], [x(0) = 1.2, y(0) = 1], [x(0) = 1.4, y(0) = 1], [x(0) = 1.6, y(0) = 1], [x(0) = 1.8, y(0) = 1], [x(0) = 2.0, y(0) = 1], [x(0) = -2, y(0) = -1], [x(0) = -1.8, y(0) = -1], [x(0) = -1.6, y(0) = -1], [x(0) = -1.4, y(0) = -1], [x(0) = -1.2, y(0) = -1], [x(0) = -1, y(0) = -1], [x(0) = -0.8, y(0) = -1], [x(0) = -0.6, y(0) = -1], [x(0) = -0.4, y(0) = -1], [x(0) = -2, y(0) = -1], [x(0) = 0, y(0) = -1], [x(0) = 0.2, y(0) = -1], [x(0) = 0.4, y(0) = -1], [x(0) = 0.6, y(0) = -1], [x(0) = 0.8, y(0) = -1], [x(0) = 1.0, y(0) = -1], [x(0) = 1.2, y(0) = -1], [x(0) = 1.4, y(0) = -1], [x(0) = 1.6, y(0) = -1], [x(0) = 1.8, y(0) = -1], [x(0) = 2.0, y(0) = -1], [x(0) = -1, y(0) = -0.2], [x(0) = -1, y(0) = 0.2], [x(0) = 1, y(0) = -0.2], [x(0) = 1, y(0) = 0.2],], *linecolor* = *black*, *stepsize* = 0.01, *thickness* = 1) :

> *display*(*p1*, *view* = [-2 ..2, -1 ..1])



Part(c) - Fixed Points: (0,0) (1,0) (-1,0)

> $\mu := 1$

$$\mu := 1 \tag{8}$$

> $EQ1 := y(t) = diff(x(t), t)$

$$EQ1 := y(t) = \dot{x}(t) \tag{9}$$

> $EQ2 := diff(y(t), t) = -2 \cdot \mu \cdot y(t) + x(t) - x(t)^3$

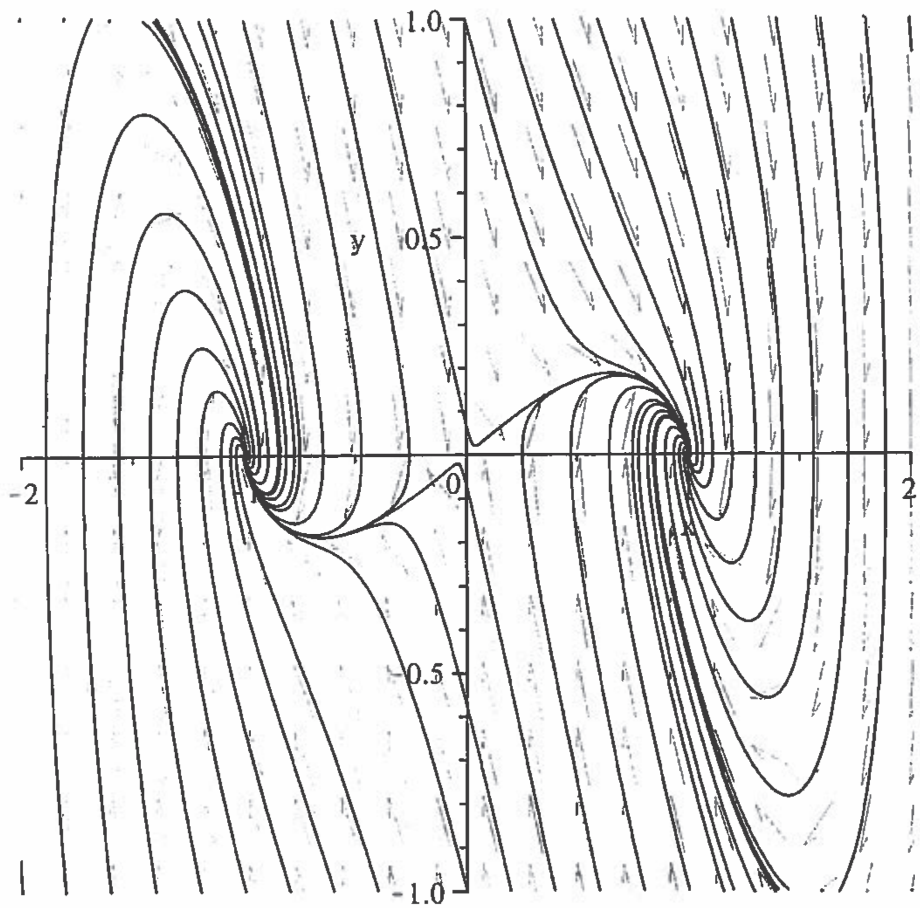
$$EQ2 := \dot{y}(t) = -2\, y(t) + x(t) - x(t)^3 \tag{10}$$

> $pl := DEplot([EQ1, EQ2], [x(t), y(t)], t = 0..10, x = -2..2, [[x(0) = -2, y(0) = 1], [x(0) = -1.8, y(0) = 1], [x(0) = -1.6, y(0) = 1], [x(0) = -1.4, y(0) = 1], [x(0) = -1.2, y(0) = 1], [x(0) = -1, y(0) = 1], [x(0) = -0.8, y(0) = 1], [x(0) = -0.6, y(0) = 1], [x(0) = -0.4, y(0) = 1], [x(0) = -2, y(0) = 1], [x(0) = 0, y(0) = 1], [x(0) = 0.2, y(0) = 1], [x(0) = 0.4, y(0) = 1], [x(0) = 0.6, y(0) = 1], [x(0) = 0.8, y(0) = 1], [x(0) = 1.0, y(0) = 1], [x(0) = 1.2, y(0) = 1], [x(0) = 1.4, y(0) = 1], [x(0) = 1.6, y(0) = 1], [x(0) = 1.8, y(0) = 1], [x(0) = 2.0, y(0) = 1], [x(0) = -2, y(0) = -1], [x(0) = -1.8, y(0) = -1], [x(0) = -1.6, y(0) = -1], [x(0) = -1.4, y(0) = -1], [x(0) = -1.2, y(0) = -1], [x(0) = -1, y(0) = -1], [x(0) = -0.8, y(0) = -1], [x(0) = -0.6, y(0) = -1], [x(0) = -0.4, y(0) = -1], [x(0) = -2, y(0) = -1], [x(0) = 0, y(0) = -1], [x(0) = 0.2, y(0) = -1], [x(0) = 0.4, y(0) = -1], [x(0)$

$= 0.6, y(0) = -1], [x(0) = 0.8, y(0) = -1], [x(0) = 1.0, y(0) = -1], [x(0) = 1.2, y(0) = -1], [x(0) = 1.4, y(0) = -1], [x(0) = 1.6, y(0) = -1], [x(0) = 1.8, y(0) = -1], [x(0) = 2.0, y(0) = -1], [x(0) = -1, y(0) = -0.2], [x(0) = -1, y(0) = 0.2], [x(0) = 1, y(0) = -0.2], [x(0) = 1, y(0) = 0.2],], linecolor = black, stepsize = 0.01, thickness = 1)$

[Length of output exceeds limit of 1000000] **(11)**

> *display(p1, view = [-2..2, -1..1])*



Part(d) - Fixed Points: (0,0)

> $\mu := 1$

$$\mu := 1 \qquad (12)$$

> $EQ1 := y(t) = diff(x(t), t)$

$$EQ1 := y(t) = \dot{x}(t) \qquad (13)$$

> $EQ2 := diff(y(t), t) = -2\cdot\mu\cdot y(t) + x(t) + x(t)^3$

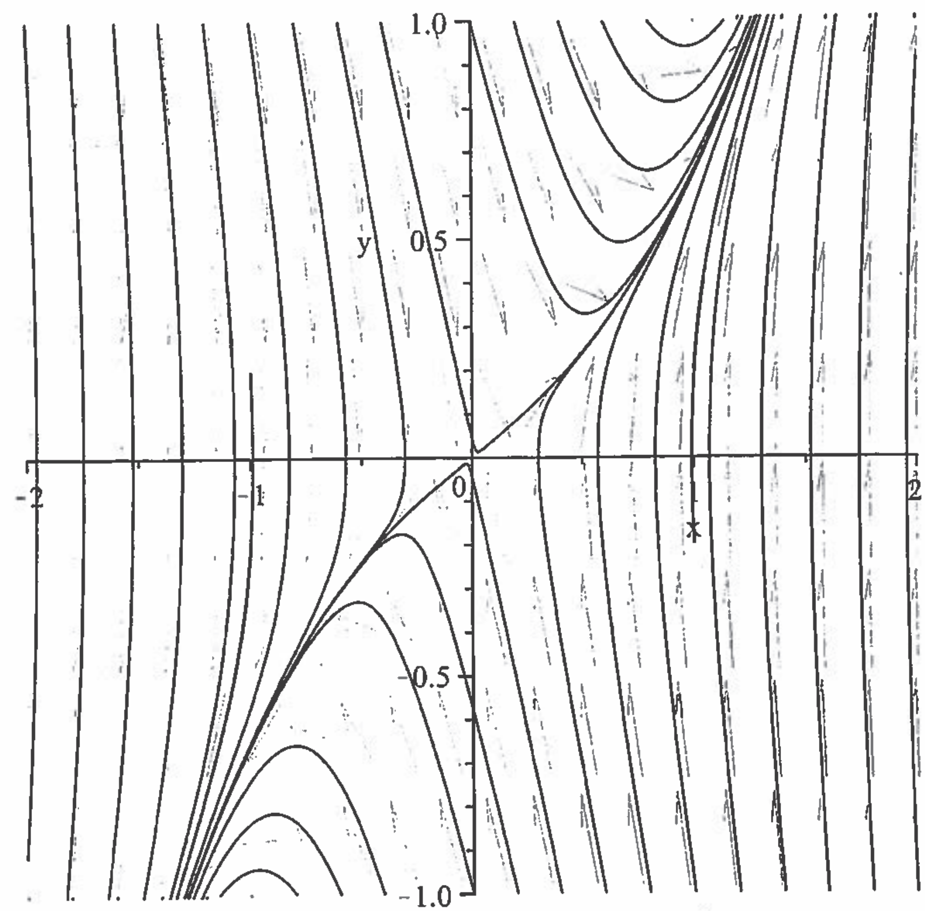
$$EQ2 := \dot{y}(t) = -2\,y(t) + x(t) + x(t)^3 \qquad (14)$$

> $p1 := DEplot([EQ1, EQ2], [x(t), y(t)], t = 0..10, x = -2..2, [[x(0) = -2, y(0) = 1], [x(0) = -1.8, y(0) = 1], [x(0) = -1.6, y(0) = 1], [x(0) = -1.4, y(0) = 1], [x(0) = -1.2, y(0) = 1], [x(0) = -1, y(0) = 1], [x(0) = -0.8, y(0) = 1], [x(0) = -0.6, y(0) = 1], [x(0) = -0.4, y(0) = 1], [x(0) = -2, y(0) = 1], [x(0) = 0, y(0) = 1], [x(0) = 0.2, y(0) = 1], [x(0) = 0.4, y(0) = 1], [x(0) = 0.6, y(0) = 1], [x(0) = 0.8, y(0) = 1], [x(0) = 1.0, y(0) = 1], [x(0)$

$= 1.2, y(0) = 1], [x(0) = 1.4, y(0) = 1], [x(0) = 1.6, y(0) = 1], [x(0) = 1.8, y(0) = 1], [x(0) = 2.0, y(0) = 1], [x(0) = -2, y(0) = -1], [x(0) = -1.8, y(0) = -1], [x(0) = -1.6, y(0) = -1], [x(0) = -1.4, y(0) = -1], [x(0) = -1.2, y(0) = -1], [x(0) = -1, y(0) = -1], [x(0) = -0.8, y(0) = -1], [x(0) = -0.6, y(0) = -1], [x(0) = -0.4, y(0) = -1], [x(0) = -2, y(0) = -1], [x(0) = 0, y(0) = -1], [x(0) = 0.2, y(0) = -1], [x(0) = 0.4, y(0) = -1], [x(0) = 0.6, y(0) = -1], [x(0) = 0.8, y(0) = -1], [x(0) = 1.0, y(0) = -1], [x(0) = 1.2, y(0) = -1], [x(0) = 1.4, y(0) = -1], [x(0) = 1.6, y(0) = -1], [x(0) = 1.8, y(0) = -1], [x(0) = 2.0, y(0) = -1], [x(0) = -1, y(0) = -0.2], [x(0) = -1, y(0) = 0.2], [x(0) = 1, y(0) = -0.2], [x(0) = 1, y(0) = 0.2],], linecolor = black, stepsize = 0.01, thickness = 1)$

$$pl := PLOT(...) \qquad (15)$$

> $display(pl, view = [-2..2, -1..1])$



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