AAE 334: Aerodynamics

HW11

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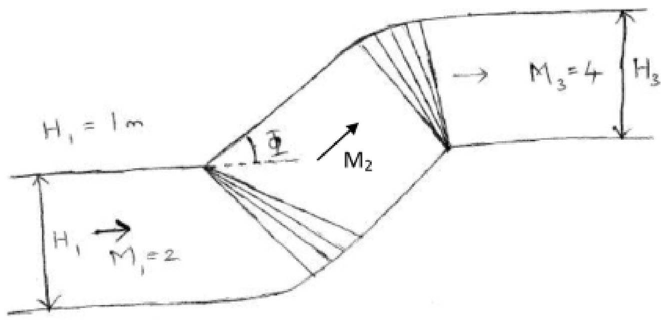
Friday April 24th, 2020



1.) [20 pts] Consider an airflow at Mach number M_1=2 through a duct. Assume that the flow is two- dimensional (no variations in the direction perpendicular to the plane of the paper). The upper wall is deflected upward through an angle Φ. The lower wall is curved in such a way that the expansion waves do not reflect from the lower wall. The flow then goes through a second expansion corner, as shown in the figure, so that it returns to its original direction. The flow after the two turns is at Mach number M_3=4.

(a) Determine the angle Φ.

(b) Determine the Mach number M_2 needed to match the specified conditions.

(c) If the height of the first duct is H_1=1 m, determine the height of the final duct, H_3.

(a)

1st Expansion fan

Using Prandtl-Meyer Function

$$\nu(M) = \sqrt{\frac{\gamma+1}{\gamma-1}} \arctan\left[\sqrt{\frac{\gamma-1}{\gamma+1}(M^2-1)}\right] - \arctan\sqrt{M^2-1} \quad \cdots (i)$$

$$\Rightarrow \quad \Phi = \nu(M_2) - \nu(M_1) \quad \cdots ①$$

2nd Expansion fan

Similarly,

$$\Phi = \nu(M_3) - \nu(M_2) \quad \cdots ②$$

from, ① & ②

$$\nu(M_2) - \nu(M_1) = \nu(M_3) - \nu(M_2)$$

$$\nu(M_2) = \frac{1}{2}\left[\nu(M_3) + \nu(M_1)\right]$$

Using eqn (i) and computing using MATLAB (code in Appendix) we obtain

$$\nu(M_1) = 26.3798^\circ$$
$$\nu(M_3) = 65.7848^\circ$$
$$\nu(M_2) = 46.0823^\circ$$

Next, plug in values to ① & ②, and we get

$$\Phi = 65.7848^\circ - 46.0823^\circ$$

$$\boxed{\Phi = 19.7025^\circ}$$

(b)

Now, since we have the value of $\nu(M_2)$ we can solve the Prandtl-Meyer function for M_2 using MATLAB (code in Appendix)

$$46.0823^\circ \cdot \frac{\pi}{180^\circ} = \sqrt{\frac{2.4}{0.4}}\arctan\left[\sqrt{\frac{0.4}{2.4}(M_2^2-1)}\right] - \arctan\sqrt{M_2^2-1}$$

$$\boxed{M_2 = 2.8162}$$

(c)

From isentropic relations we can obtain the area ratio for the region of M_1 & M_3.

$$\frac{A_3}{A_1} = \frac{M_1}{M_3}\sqrt{\left[\frac{1+\frac{\gamma-1}{2}M_3^2}{1+\frac{\gamma-1}{2}M_1^2}\right]^{\frac{\gamma+1}{\gamma-1}}}$$

$$\frac{A_3}{A_1} = 6.3519$$

$$A_3 = 6.3519 A_1$$

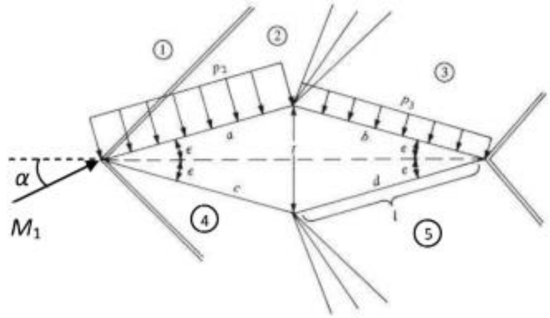
since $A_1 = \pi\left(\frac{h_1}{2}\right)^2 = 0.7854\ m^2$

$$\therefore A_3 = (6.3519)(0.7854\ m^2)$$

$$A_3 = 4.9887\ m^2$$

$$\pi\left(\frac{h_3}{2}\right)^2 = 4.9887\ m^2$$

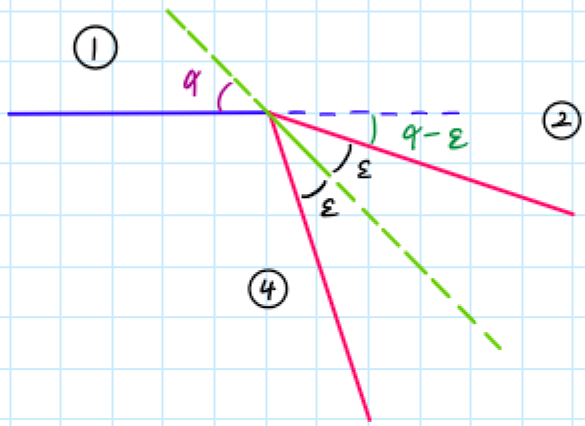
$$\boxed{h_3 = 2.5203\ m}$$



2.) [50 pts] Consider a symmetric diamond airfoil as in the figure above. The half-angle is $\epsilon = 4°$, the angle-of-attack is $\alpha = 5°$ and the freestream Mach number is $M_1 = 3.2$.

(a) Using shock-expansion theory, compute the pressure coefficient for each surface: C_{p2}, C_{p3}, C_{p4}, C_{p5}. Also calculate the lift coefficient c_l and the drag coefficient c_d.

1 → 2 Expansion Fan



Using Prandtl-Meyer Function

$$\nu(M) = \sqrt{\frac{\gamma+1}{\gamma-1}}\arctan\left[\sqrt{\frac{\gamma-1}{\gamma+1}(M^2-1)}\right] - \arctan\sqrt{M^2-1} \quad \cdots (1)$$

$$\Rightarrow \quad \alpha - \varepsilon = \nu(M_2) - \nu(M_1)$$

Using MATLAB compute $\nu(M_1)$ (code in Appendix)

$$\nu(M_1) = 53.4703°$$

thus,

$$5° - 4° = \nu(M_2) - 53.4703°$$

$$\nu(M_2) = 54.4703°$$

Using eqn (1) calculate M_2 from $\nu(M_2)$

$$M_2 = 3.2566$$

Then, from isentropic relations

$$\frac{P_2}{P_1} = \frac{P_0/P_1}{P_0/P_2} = \left(\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}\right)^{\gamma/\gamma-1} = \frac{49.4370}{53.7133}$$

$$\frac{P_2}{P_1} = 0.9204$$

Hence using the function

$$C_p = \frac{2}{\gamma M_\infty^2}\left(\frac{P}{P_\infty} - 1\right) \quad \cdots \text{(ii)}$$

$$C_{p2} = \frac{2}{\gamma M_1^2}\left(\frac{P_2}{P_1} - 1\right)$$

$$\boxed{C_{p2} = -0.0111}$$

① → ④ Oblique Shock

Looking at the sketch above we can tell that an oblique shock occurs. Then from the following function representing $\theta-\beta-M$ relations we can obtain β.

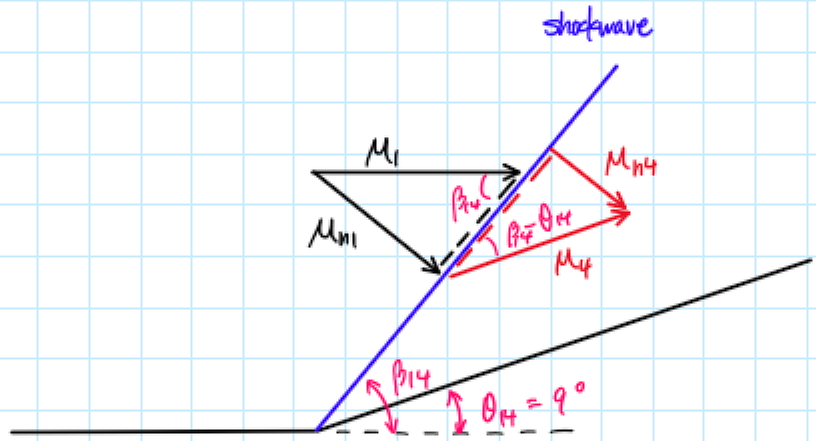
Note that $\theta_{14} = (9-\varepsilon) + 2\varepsilon = 9+\varepsilon = 5^\circ + 4^\circ = 9^\circ$

$$\tan\theta = 2\cot\beta \frac{M_1^2\sin^2\beta - 1}{M_1^2(\gamma + \cos 2\beta) + 2} \quad \cdots \text{(iii)}$$

Using MATLAB we can compute β (code in Appendix)

$$\theta = \theta_{14} = 9^\circ \quad \& \quad M_1 = 3.2$$

$$\beta_{14} = 25.1598^\circ$$



then, since $M_{n1} = M_1 \sin\beta_{14} = (3.2)\sin(25.1598°)$

$M_{n1} = 1.3605$

Next, using normal shock relations

$$M_2^2 = \frac{1 + \frac{\gamma-1}{2}M_1^2}{\gamma M_1^2 - \frac{\gamma-1}{2}} \quad \cdots \text{(iv)}$$

Plugging in $M_1 = M_{n1}$ we get $M_{n4} = M_2$

$$M_{n4} = 0.7570$$

Thus,

$$M_4 = \frac{M_{n4}}{\sin(\beta_{14} - \theta_{14})} = \frac{0.7570}{\sin(25.1598° - 9°)}$$

$$M_4 = 2.7198$$

Finally, using eqn (ii) we can compute C_{p4} since

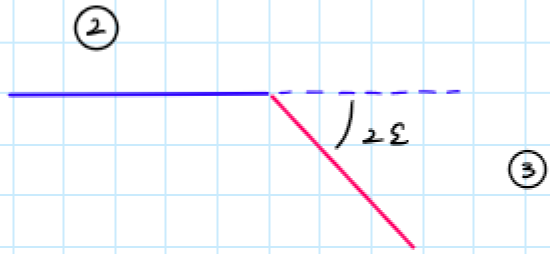
$$\frac{p_0}{p_4} = \left(1 + \frac{\gamma-1}{2}M_4^2\right)^{\gamma/\gamma-1} = 24.0026$$

$$\frac{p_4}{p_1} = \frac{p_0/p_1}{p_0/p_4} = \frac{49.4370}{24.0026} = 2.0597$$

$$C_{p4} = \frac{2}{\gamma M_1^2}\left(\frac{p_4}{p_1} - 1\right)$$

$$\boxed{C_{p4} = 0.1478}$$

② → ③ Expansion Fan



Using eqn (i)

$$\theta_{23} = 2\varepsilon = \nu(M_3) - \nu(M_2)$$

since $2\varepsilon = 8^\circ$ & $\nu(M_2) = 54.47030^\circ$

$$\nu(M_3) = 62.4703^\circ$$

Then solve eqn (i) for M_3, and we obtain with MATLAB

$$M_3 = 3.7600$$

Next,

$$\frac{P_0}{P_3} = \left(1 + \frac{\gamma-1}{2}M_3^2\right)^{\gamma/\gamma-1} = 109.6956$$

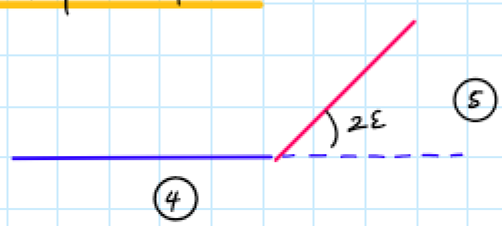
$$\frac{P_3}{P_1} = \frac{P_0/P_1}{P_0/P_3} = \frac{49.4370}{109.6956} = 0.4507$$

Hence,

$$C_{p3} = \frac{2}{\gamma M_1^2}\left(\frac{P_3}{P_1} - 1\right)$$

$$\boxed{C_{p3} = -0.0766}$$

④ → ⑤ Expansion Fan



Using eqn (i) $\nu(M_4) = 44.0488^\circ$

$$\theta_{45} = 2\varepsilon = \nu(M_5) - \nu(M_4)$$

since $\theta_{45} = 2\varepsilon = 8^\circ$

$$\nu(M_5) = 52.0488^\circ$$

Then solve eqn (i) for M_3, and we obtain with MATLAB

$$M_5 = 3.1216$$

Next,

$$\frac{P_0}{P_5} = \left(1+\frac{\gamma-1}{2}M_5^2\right)^{\gamma/\gamma-1} = 44.0344$$

$$\frac{P_5}{P_1} = \frac{P_0/P_1}{P_0/P_5} = \frac{49.4370}{44.0344} = 1.1227$$

Hence,

$$C_{p5} = \frac{2}{\gamma M_1^2}\left(\frac{P_5}{P_1}-1\right)$$

$$\boxed{C_{p5} = 0.0171}$$

Then,

$$C_n = \frac{1}{2}\left(C_{p4}+C_{p5}-C_{p2}-C_{p3}\right)$$

$$C_n = \frac{1}{2}\left(0.1478+0.0171+0.0111+0.0766\right)$$

$$C_n = 0.1263$$

$$C_a = \frac{1}{2}\left(C_{p2}+C_{p4}-C_{p3}-C_{p5}\right)\tan\varepsilon$$

$$C_a = \frac{1}{2}\left(-0.0111+0.1478+0.0766-0.0171\right)\tan(4^\circ)$$

$$C_a = 0.0069$$

Thus,

$$\begin{bmatrix} C_l \\ C_d \end{bmatrix} = \begin{bmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{bmatrix}\begin{bmatrix} C_n \\ C_a \end{bmatrix}$$

$$\boxed{\begin{aligned} C_l &= 0.1253 \\ C_d &= 0.0178 \end{aligned}}$$

(b) Compute the same quantities using linearized supersonic flow theory. For pressure coefficient, see Lecture 32; for the lift and drag coefficients, see Lecture 33, slide 22. (Remember that the turning angle in the linearized theory is relative to the freestream direction.) Comment on the accuracy of the linear theory for these conditions.

From the supersonic linearized theory we have the 2 eqns

$$C_{pu} = \frac{2\theta}{\sqrt{M_1^2 - 1}}, \quad C_{pl} = -\frac{2\theta}{\sqrt{M_1^2 - 1}}$$

Since, we know that

$$\theta_{12} = \alpha - \varepsilon = 1^\circ$$
$$\theta_{13} = \theta_{12} + \theta_{23} = 1^\circ + 8^\circ = 9^\circ$$
$$\theta_{14} = 9^\circ$$
$$\theta_{15} = \theta_{14} - \theta_{45} = 9^\circ - 8^\circ = 1^\circ$$

Thus,

$$\boxed{\begin{aligned} C_{p2} &= \frac{2\theta_{12}}{\sqrt{M_1^2-1}} = -0.0115 \\ C_{p3} &= \frac{2\theta_{13}}{\sqrt{M_1^2-1}} = -0.1034 \\ C_{p4} &= \frac{2\theta_{14}}{\sqrt{M_1^2-1}} = 0.1034 \\ C_{p5} &= \frac{2\theta_{15}}{\sqrt{M_1^2-1}} = 0.0115 \end{aligned}}$$

and

$$\boxed{\begin{aligned} C_l &= \frac{4\alpha}{\sqrt{M_1^2-1}} \quad \because \alpha = 5^\circ \\ C_l &= 0.1148 \\ C_d &= \frac{4(\alpha^2 + \tan^2\varepsilon)}{\sqrt{M_1^2-1}} \\ C_d &= 0.0165 \end{aligned}}$$

Table of Comparison

	Angle Relative to Freestream [deg]	Shockwave and Expansion Fan Analysis (I)	Supersonic Linearized Theory (II)	Difference (I) – (II)
C_{p2}	-1	-0.0111	-0.0115	0.0004
C_{p3}	-9	-0.0766	-0.1034	0.0267
C_{p4}	9	0.1478	0.1034	0.0445
C_{p5}	-1	0.0171	0.0115	0.0056
C_{l}	NaN	0.1253	0.1148	0.0104
C_{d}	NaN	0.0178	0.0165	0.0014

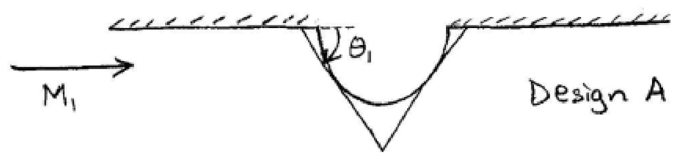
Analysis

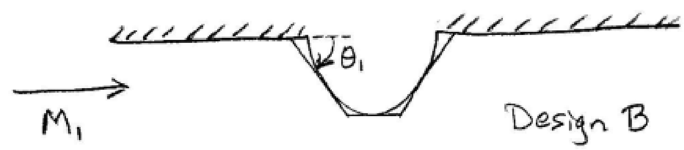
- From the tabulated results we can see that for small angles the error of the pressure coefficients computed using the supersonic linearized theory is small.
- The drag coefficient has a smaller error compared to the lift coefficient. This is probably due to the large deviations of the pressure coefficients with large angles relative to the freestream.

3.) [30 pts] A semicircular pod of radius R = 0.2 m housing an antenna array is attached to the bottom of a supersonic vehicle, as shown below. Treat this problem as two-dimensional. The airstream along the bottom of the vehicle has a Mach number M_1 = 6.0, static pressure p_1 = 5, 500 Pa, and static temperature T_1 = 216.7 K. In order to avoid the large drag associated with having a strong bow shock in front of the antenna pod, an aerodynamic shell is to be placed over the pod. Design A uses a symmetric ramp with $\theta_1 = 40^\circ$ (see design A below). The ramp is tangent to the semicircle making up the antenna housing. Someone suggests that the wave drag can be reduced by truncating the apex of the aerodynamic shell to be flush with the bottom of the semicircle (see design B below).

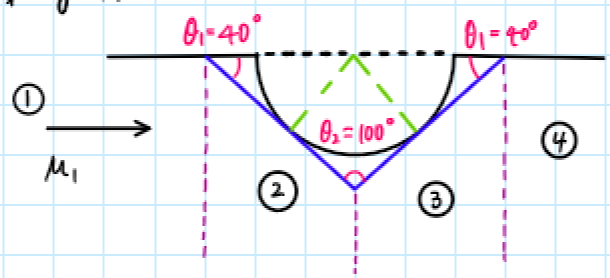
(a) Evaluate the wave drag associated with design A.

(b) Evaluate the wave drag associated with design B and compare to the result for design A.





(a) Design A



① → ② Oblique Shock

Using the $\theta-\beta-M$ relation we can find the shock-wave angle β_{12} from the following eqn.

$$\tan\theta = 2\cot\beta \frac{M_1^2 \sin^2\beta - 1}{M_1^2(\gamma + \cos 2\beta) + 2} \quad \cdots (i)$$

MATLAB computes β_{12} to be (code in Appendix)

$$\beta_{12} = 57.1883°$$

Then, $M_{n1} = M_1 \sin\beta_{12} = 5.0427$

From normal shock relations

$$M_2^2 = \frac{1 + \frac{\gamma-1}{2}M_1^2}{\gamma M_1^2 - \frac{\gamma-1}{2}} \quad \cdots (ii)$$

$$\Rightarrow \quad M_{n2} = 0.4146$$

Then, $M_2 = M_{n2} \csc(\beta_{12} - \theta_1)$

$$M_2 = 1.4031$$

Now, $$P_2 = P_1 \frac{P_0/P_1}{P_0/P_2} = P_1 \left(\frac{1 + \frac{\gamma-1}{2}M_1^2}{1 + \frac{\gamma-1}{2}M_2^2} \right)^{\gamma/\gamma-1}$$

$$P_2 = (5500\ Pa)\frac{1578.9}{3.1960}$$

$$\boxed{P_2 = 2.7171\ MPa}$$

② → ③ Expansion Fan

Using Prandtl Meyer Function

$$\nu(M) = \sqrt{\frac{\gamma+1}{\gamma-1}}\arctan\left[\sqrt{\frac{\gamma-1}{\gamma+1}(M^2-1)}\right] - \arctan\sqrt{M^2-1} \quad \cdots \text{(iii)}$$

MATLAB computes $\nu(M_2) = 9.0754$

$$\Rightarrow 180° - \theta_2 = \nu(M_3) - \nu(M_2)$$

$$\nu(M_3) = 89.0754°$$

Solve for M_3 using eqn (iii) and from MATLAB we obtain

$$M_3 = 6.6546$$

Then,

$$P_3 = P_1 \frac{P_0/P_1}{P_0/P_3} = P_1 \left(\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_3^2}\right)^{\gamma/\gamma-1}$$

$$\boxed{P_3 = 2.8883 \text{ kPa}}$$

③ → ④ Oblique Shock

Using the θ-β-M relation we can find the shockwave angle β_{12} from eqn (i)

MATLAB computes β_{34} to be (code in Appendix)

$$\beta_{34} = 56.1045°$$

Then,

$$M_{n3} = M_3 \sin\beta_{34} = 5.5237$$

From normal shock relations

$$M_2^2 = \frac{1+\frac{\gamma-1}{2}M_1^2}{\gamma M_1^2 - \frac{\gamma-1}{2}} \quad \cdots \text{(ii)}$$

$$\Rightarrow M_{n4} = 0.4087$$

Then,

$$M_4 = M_{n4}\csc(\beta_{34} - \theta_1)$$

$$M_4 = 1.4734$$

Now,

$$P_4 = P_1 \frac{P_0/P_1}{P_0/P_4} = P\left(\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_4^2}\right)^{\gamma/\gamma-1}$$

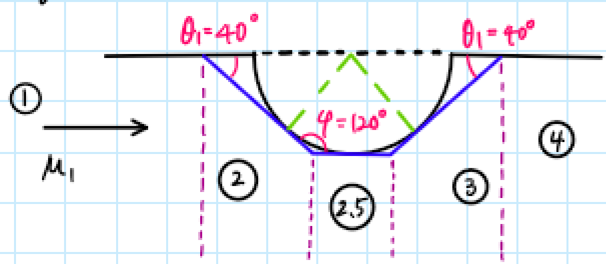
$$\boxed{P_4 = 2.4579 \text{ MPa}}$$

Therefore, Wave Drag for this design A becomes

$$W_{D,A} = P_2 - P_3 = 2.7171 \text{ MPa} - 2.8883 \text{ kPa}$$

$$\boxed{W_{D,A} = 2.7142 \text{ MPa}}$$

(b) Design B



For design B, regions ① & ② are the same as design A

② → ②.5 Expansion Fan

Using eqn (iii) we can find $M_{2.5}$

We already know that $\nu(M_2) = 9.0754$

Then,

$$\Rightarrow \quad \theta_1 = \nu(M_{2.5}) - \nu(M_2)$$

$$\nu(M_{2.5}) = 49.0754^\circ$$

Solve for $M_{2.5}$ using eqn (iii) and from MATLAB we obtain

$$M_{2.5} = 2.9649$$

Then,

$$P_{2.5} = P_1 \frac{P_0/P_1}{P_0/P_{2.5}} = P_1 \left(\frac{1 + \frac{\gamma-1}{2} M_1^2}{1 + \frac{\gamma-1}{2} M_{2.5}^2} \right)^{\gamma/\gamma-1}$$

$$\boxed{P_{2.5} = 0.24921 \text{ MPa}}$$

(2.5) → (3) Expansion Fan

Using eqn (iii) we can find M_3

Then,
$$\Rightarrow \quad \theta_1 = \nu(M_3) - \nu(M_{2.5})$$
$$\nu(M_3) = 89.0754°$$

Solve for $M_{2.5}$ using eqn (iii) and from MATLAB we obtain
$$M_3 = 6.6546$$

Then,
$$P_3 = P_1 \frac{P_0/P_1}{P_0/P_3} = P_1 \left(\frac{1 + \frac{\gamma-1}{2} M_1^2}{1 + \frac{\gamma-1}{2} M_3^2} \right)^{\gamma/\gamma-1}$$

$$\boxed{P_3 = 2.8883 \text{ kPa}}$$

Therefore, the Wave Drag for Design B becomes
$$W_{D,B} = P_2 - P_{2.5}$$
$$W_{D,B} = 2.7171 \text{ MPa} - 0.24921 \text{ MPa}$$

$$\boxed{W_{D,B} = 2.4678 \text{ MPa}}$$

For the 2 designs, design B reduces 0.24632 MPa worth of wave drag. Thus, design B would be a better option.

Appendix

AAE 334 HW11

```
clear all; close all; clc;
% Global constants
gamma = 1.4;
R = 287.05;
```

P1 (a)

```
% Given properties
M1 = 2;
M3 = 4;
h1 = 1;  % [m]

% Apply Prantl-Meyer
nu_M1 = Prandtl_Meyer_Expansion(M1,gamma);
nu_M3 = Prandtl_Meyer_Expansion(M3,gamma);
nu_M2 = 0.5*(nu_M1 + nu_M3);
Phi = nu_M3 - nu_M2;
```

(b)

```
% Find M2
M2 = calc_M_from_PrantlMeyer(nu_M2,gamma);
```

(c)

```
% Obtain the area ratio from isentropic relations
A3_A1 = areaRatio_from_isentropic_relation(M1,M3,gamma);
A1 = pi*(h1/2)^2;
A3 = A3_A1*A1;
h3 = 2*sqrt(A3/pi);
```

P2 (a)

```
% Given properties
alpha = 5;  % [deg]
epsilon = 4;  % [deg]
M1 = 3.2;
gamma = 1.4;

% 1 -> 2
nu_M1 = Prandtl_Meyer_Expansion(M1,gamma);
theta12 = alpha - epsilon;
nu_M2 = theta12 + nu_M1;
M2 = calc_M_from_PrantlMeyer(nu_M2,gamma);
P0_P1 = isentropic_relation_P_ratio(M1,gamma);
P0_P2 = isentropic_relation_P_ratio(M2,gamma);
P2_P1 = P0_P1/P0_P2;
Cp2 = calc_pressure_coeff(P2_P1,M1,gamma);
```

```
% 1 -> 4
theta14 = alpha + epsilon;
```

```
beta14 = theta_beta_M_relation(theta14,M1,gamma);
Mn1 = M1*sind(beta14);
Mn4 = normalShock_jump_M(Mn1,gamma);
M4 = Mn4/sind(beta14-theta14);
P0_P4 = isentropic_relation_P_ratio(M4,gamma);
P4_P1 = P0_P1/P0_P4;
Cp4 = calc_pressure_coeff(P4_P1,M1,gamma);
```

```
% 2 -> 3
theta23 = 2*epsilon;
nu_M3 = theta23 + nu_M2;
M3 = calc_M_from_PrantlMeyer(nu_M3,gamma);
P0_P3 = isentropic_relation_P_ratio(M3,gamma);
P3_P1 = P0_P1/P0_P3;
Cp3 = calc_pressure_coeff(P3_P1,M1,gamma);
```

```
% 4 -> 5
theta45 = 2*epsilon;
nu_M4 = Prandtl_Meyer_Expansion(M4,gamma);
nu_M5 = theta45 + nu_M4;
M5 = calc_M_from_PrantlMeyer(nu_M5,gamma);
P0_P5 = isentropic_relation_P_ratio(M5,gamma);
P5_P1 = P0_P1/P0_P5;
Cp5 = calc_pressure_coeff(P5_P1,M1,gamma);
```

```
% Lift and drag coefficients
Cn = 0.5*(Cp4 + Cp5 - Cp2 - Cp3);
Ca = 0.5*(Cp2 + Cp4 - Cp3 - Cp5)*tand(epsilon);
DCM = [cosd(alpha) -sind(alpha);
       sind(alpha)  cosd(alpha)];
res = DCM*[Cn; Ca];
Cl = res(1);
Cd = res(2);
```

(b)

```
Cp2_ssl = supersonic_linear_theory_Cp(-theta12,M1,"upper");
theta13 = theta12 + theta23;
Cp3_ssl = supersonic_linear_theory_Cp(-theta13,M1,"upper");
Cp4_ssl = supersonic_linear_theory_Cp(-theta14,M1,"lower");
theta15 = theta14 - theta45;
Cp5_ssl = supersonic_linear_theory_Cp(-theta15,M1,"lower");
Cl_ssl = 4*deg2rad(alpha)/sqrt(M1^2 - 1);
Cd_ssl = 4*((deg2rad(alpha))^2 + (tand(epsilon))^2)/sqrt(M1^2 - 1);

% Percent Errors
C1 = [Cp2 Cp3 Cp4 Cp5 Cl Cd];
C2 = [Cp2_ssl Cp3_ssl Cp4_ssl Cp5_ssl Cl_ssl Cd_ssl];
diff = C1 - C2;
```

P3 (a)

```
% Given properties
M1 = 6;
```

```
T1 = 216.7;  % [K]
P1 = 5500;  % [Pa]
theta1 = 40;  % [deg]
gamma = 1.4;
theta2 = 180 - 2*theta1;

% 1 -> 2
beta12 = theta_beta_M_relation(theta1,M1,gamma);
Mn1 = M1*sind(beta12);
Mn2 = normalShock_jump_M(Mn1,gamma);
M2 = Mn2*cscd(beta12 - theta1);
P0_P1 = isentropic_relation_P_ratio(M1,gamma);
P0_P2 = isentropic_relation_P_ratio(M2,gamma);
P2 = P1*P0_P1/P0_P2;
```

```
% 2 -> 3
nu_M2 = Prandtl_Meyer_Expansion(M2,gamma);
nu_M3 = 180 - theta2 + nu_M2;
M3 = calc_M_from_PrantlMeyer(nu_M3,gamma);
P0_P3 = isentropic_relation_P_ratio(M3,gamma);
P3 = P1*P0_P1/P0_P3;
```

```
% 3 -> 4
beta34 = theta_beta_M_relation(theta1,M3,gamma);
Mn3 = M3*sind(beta34);
Mn4 = normalShock_jump_M(Mn3,gamma);
M4 = Mn4*cscd(beta34 - theta1);
P0_P4 = isentropic_relation_P_ratio(M4,gamma);
P4 = P1*P0_P1/P0_P4;

% Wave drag
WD_A = P2 - P3;
```

(b)

```
phi = 180 - theta1;

% 2 -> 2.5
nu_M25 = theta1 + nu_M2;
M25 = calc_M_from_PrantlMeyer(nu_M25,gamma);
P0_P25 = isentropic_relation_P_ratio(M25,gamma);
P25 = P1*P0_P1/P0_P25;
```

```
% 2.5 - > 3
nu_M3 = theta1 + nu_M25;
M3 = calc_M_from_PrantlMeyer(nu_M3,gamma);
P0_P3 = isentropic_relation_P_ratio(M3,gamma);
P3 = P1*P0_P1/P0_P3;

% Wave Drag
WD_B = P2 - P25;

% Compare
```

```
WD_diff = WD_A - WD_B
```

Functions

```
function M = calc_M_from_PrantlMeyer(nu,gamma)
    M = sym('M');
    assume(M,["real","positive"]);
    a1 = sqrt((gamma + 1)/(gamma - 1));
    a2 = atand(a1^(-1)*sqrt(M^2 - 1));
    a3 = atand(sqrt(M^2 - 1));
    eqn = nu == a1*a2 - a3;
    M = double(vpasolve(eqn,M));
    if M < 0
        M = -M;
    end
end

function A2_A1 = areaRatio_from_isentropic_relation(M1,M2,gamma)
    % Calculate the Mach number at the inlet
    a1 = 1 + (gamma - 1)/2*M2^2;
    a2 = 1 + (gamma - 1)/2*M1^2;
    a3 = (gamma + 1)/2/(gamma - 1);
    A2_A1 = M1/M2 * (a1/a2)^(a3);
end

function P_rat = isentropic_relation_P_ratio(M,gamma)
    P_rat = (1 + (gamma - 1)/2*M^2)^(gamma/(gamma - 1));
end
```

```
function Cp = calc_pressure_coeff(P_rat,M,gamma)
    Cp = 2/gamma/M^2*(P_rat - 1);
end

function Cp = supersonic_linear_theory_Cp(theta,M_inf,type)
    theta = deg2rad(theta);
    if type == "upper"
        Cp = 2*theta/sqrt(M_inf^2 - 1);
    elseif type == "lower"
        Cp = -2*theta/sqrt(M_inf^2 - 1);
    end
end
```

```
function nu = Prandtl_Meyer_Expansion(M,gamma)
    %{
      Function:    Prandtl_Meyer_Expansion
      Author:      Tomoki Koike
      Description: This function calculates the Prandtl-Meyer function results 
for
                   a given flow with given Mach number to find the
                   expansion fan relations
      >>Inputs
          M1:    Mach number before expansion fan
          gamma: specific hear ratio
```

```
      Outputs<<
          nu:    Prandtl-Meyer function result [deg]
    %}
    a1 = sqrt((gamma + 1)/(gamma - 1));
    a2 = atand(a1^(-1)*sqrt(M^2 - 1));
    a3 = atand(sqrt(M^2 - 1));
    nu = a1*a2 - a3;
end
```

```
function M2 = normalShock_jump_M(M1,gamma)
    %{
      Function:    normalShock_jump_M
      Author:      Tomoki Koike
      Description: This function calculates the Mach number jump after a normal
shockwave.
      >>Inputs
          M1:    Mach number
          gamma: specific hear ratio
      Outputs<<
          M2: Mach number after shock
    %}
    a1 = (gamma - 1)/2;
    M2 = sqrt((1 + a1*M1^2)/(gamma*M1^2 - a1));
end
```