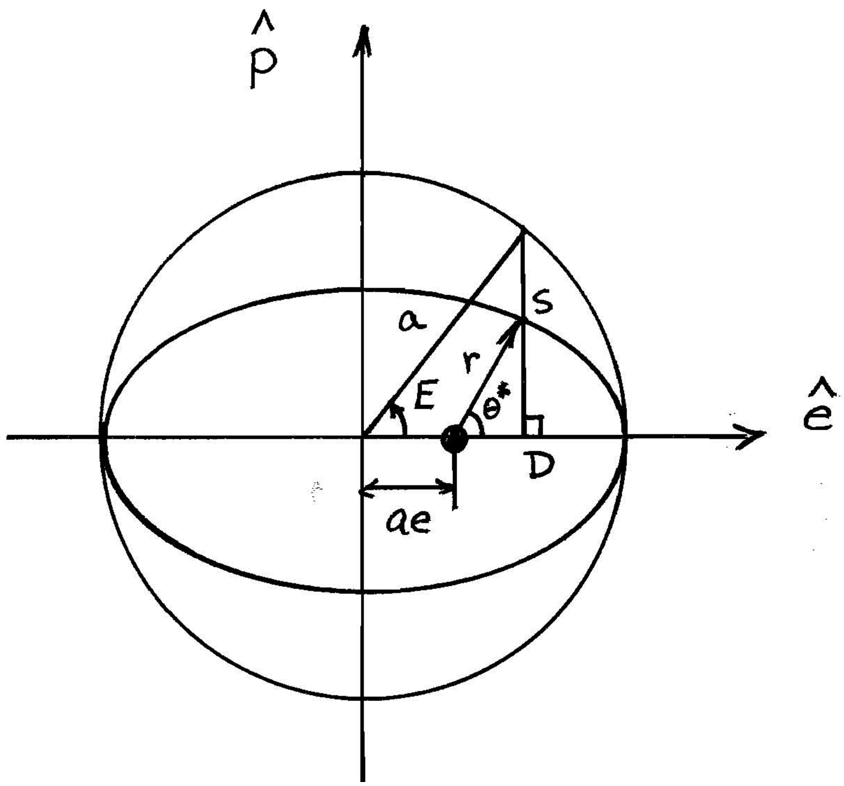
f and g Functions

E and H are most definitely useful for "time" relationships
But they are also useful in other ways.

$\longrightarrow$ new expressions for $\bar{r}, \bar{v}$

Begin with the elliptic case



$$\bar{r} = a\left(\cos E - e\right)\hat{e} + \overbrace{b\sin E}^{\text{SD}}\ \hat{p}$$

$$\frac{d}{dt}\left(M = nt = E - e\sin E\right) \rightarrow$$

$$\overline{v} = -\frac{a^2 n}{r}\sin E\ \hat{e} + \frac{abn}{r}\cos E\ \hat{p}$$

Evaluate $\hat{e}$, $\hat{p}$ at $t = t_0$ (i.e., $\overline{r}_0, \overline{v}_0$)

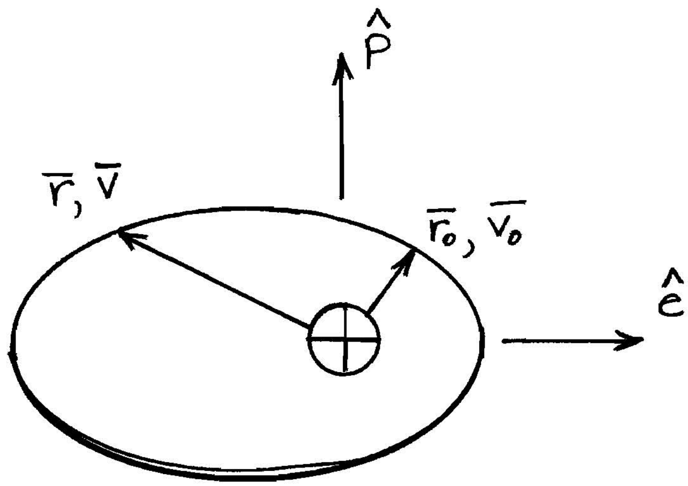
$$\hat{e} = \frac{1}{a\left(\cos E_0 - e\right)}\,\overline{r}_0 - \frac{b\sin E_0}{a\left(\cos E_0 - e\right)}\,\hat{p}$$

Substitute into $\overline{v}$ equation

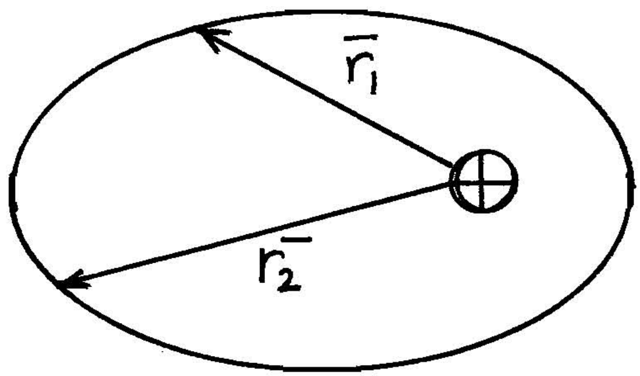
Substitute $\hat{e}$, $\hat{p}$ into original expressions

$$\overline{r} = \left\{1 - \frac{a}{r_0}\left[1 - \cos\left(E - E_0\right)\right]\right\}\overline{r}_0 + \left\{\left(t - t_0\right) + \left[\frac{\sin\left(E - E_0\right) - \left(E - E_0\right)}{n}\right]\right\}\overline{v}_0$$

$$\overline{v} = -\frac{na^2}{rr_0}\sin\left(E - E_0\right)\overline{r}_0 + \left\{1 - \frac{a}{r}\left[1 - \cos\left(E - E_0\right)\right]\right\}\overline{v}_0$$



Example:



$\bar{r}_2 = f\ \bar{r}_1 + g\ \bar{v}_1$

Do same in terms of θ^*;
Do same in hyperbolic orbits

f and g Relationships

Any conic

$$\bar{r} = \left\{1 - \frac{r}{p}\left[1 - \cos\left(\theta^* - \theta_0^*\right)\right]\right\}\bar{r}_0 + \frac{r\,r_0}{\sqrt{\mu p}}\sin\left(\theta^* - \theta_0^*\right)\bar{v}_0$$

$$\bar{v} = \left\{\frac{\bar{r}_0 \cdot \bar{v}_0}{p\,r_0}\left[1 - \cos\left(\theta^* - \theta_0^*\right)\right] - \frac{1}{r_0}\sqrt{\frac{\mu}{p}}\sin\left(\theta^* - \theta_0^*\right)\right\}\bar{r}_0 + \left\{1 - \frac{r_0}{p}\left[1 - \cos(\theta^* - \theta_0^*)\right]\right\}\bar{v}_0$$

Elliptic Orbits

$$\bar{r} = \left\{1 - \frac{a}{r_0}\left[1 - \cos\left(E - E_0\right)\right]\right\}\bar{r}_0 + \left\{\left(t - t_0\right) - \sqrt{\frac{a^3}{\mu}}\left[\left(E - E_0\right) - \sin\left(E - E_0\right)\right]\right\}\bar{v}_0$$

$$\bar{v} = -\frac{\sqrt{\mu a}}{r r_0}\sin\left(E - E_0\right)\bar{r}_0 + \left\{1 - \frac{a}{r}\left[1 - \cos\left(E - E_0\right)\right]\right\}\bar{v}_0$$

Hyperbolic Orbits

$$\bar{r} = \left\{1 - \frac{|a|}{r_0}\left[\cosh\left(H - H_0\right) - 1\right]\right\}\bar{r}_0 + \left\{\left(t - t_0\right) - \sqrt{\frac{|a|^3}{\mu}}\left[\sinh\left(H - H_0\right) - \left(H - H_0\right)\right]\right\}\bar{v}_0$$

$$\bar{v} = -\frac{\sqrt{\mu|a|}}{r r_0}\sinh\left(H - H_0\right)\bar{r}_0 + \left\{1 - \frac{|a|}{r}\left[\cosh\left(H - H_0\right) - 1\right]\right\}\bar{v}_0$$