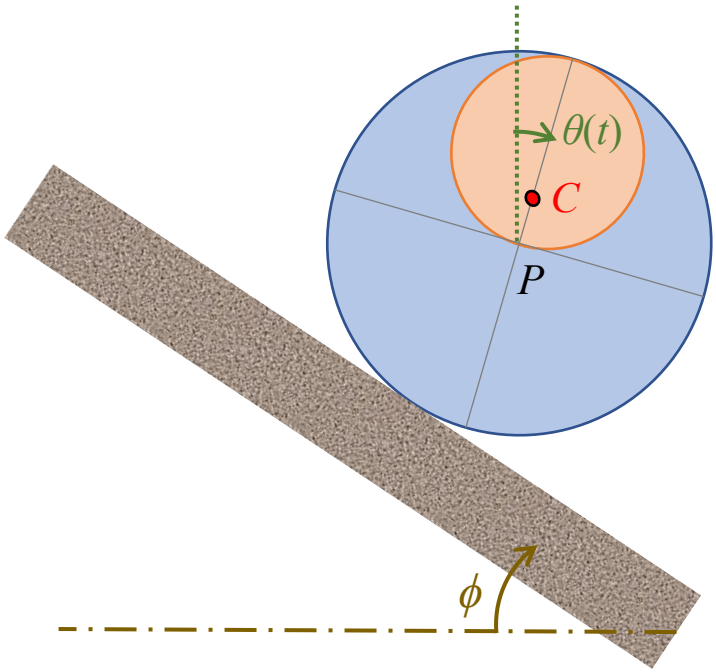
HW4: Rolling of Cylinder (with non-axisymmetric mass distribution)

Consider a circular cylinder of radius *R* made up of two materials – blue material and red material – on an inclined plane as shown below. The red cylinder has a radius of *R*/2 and the density of the red material is 5 times the density of the blue cylinder. The total mass of the cylinder is *m*.

- Calculate the location of the center of mass, the moment of inertia about the center of mass *C*, and the moment of inertia about the geometric center *P* in terms of *m* and *R*. Note: these are the only inertial properties that show up in the equations of motion.
- If the surfaces are frictionless, derive the equations of motion for the motion of the cylinder.
- Solve the equations for the motion of the cylinder starting from rest for $\theta(0) = 0$ and $\theta(0) = \phi$. Do the points *C* and/or *P* have a constant acceleration? Does the cylinder have a non-zero angular acceleration?
- Now assume that there is sufficient friction so that the cylinder is always rolling when it in contact with the surface. Derive the equations of motion for the motion of the cylinder. Make sure that the number of equations is equal to number of unknowns.



You can solve the problem using the Euler equations about *C* (as we have derived in class) or derive the equations about *P*. Feel free to do extra work ☺. For example:

- Simulate the equations of motion for rolling cylinder for $\theta(0) = 0$.
- Talk about the motion. Is it uniform?
- Do the equations indicate if the cylinder might lose contact with the inclined plane at any time (this may be indicated by a negative normal reaction in the solution)?
- Derive equations for a given coefficient of friction assuming that the cylinder is slipping.
- What if the inclined plane is curved?