AAE 532 – Orbit Mechanics
Problem Set 7
Due: JD 2459153.1875

Problem 1: As part of an interplanetary mission, a spacecraft is in the following orbit around Mars (relative to a Mars centered equatorial J2000 coordinate frame):

$$a = 6R_{♂} \qquad \Omega = 45^\circ$$
$$e = .5 \qquad \omega = 30^\circ$$
$$i = 30^\circ \qquad \theta^* = -165^\circ$$

When $\theta^* = 150^\circ$, the following maneuver is implemented:

$$\Delta\bar{v} = .1\hat{x} + .25\hat{y} + .35\hat{z} \text{ km/s}$$

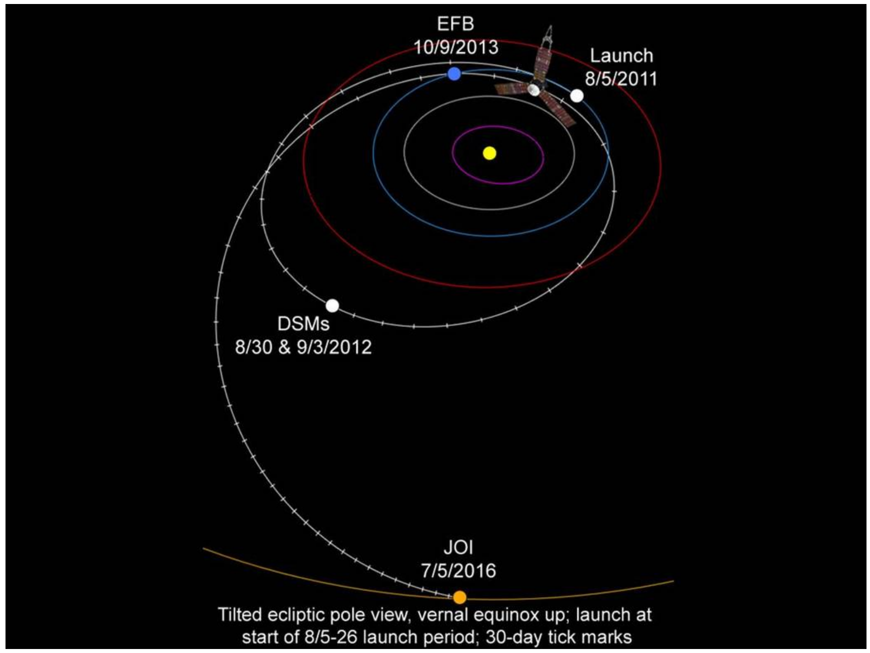
(a) Transform $\Delta\bar{v}$ into $\hat{r}, \hat{\theta}, \hat{h}$ and *VNB* components corresponding to the original orbit. How much of the $\Delta\bar{v}$ is out-of-plane? What is this value as a % of the total $|\Delta\bar{v}|$? (Define this out-of-plane component as $\Delta\bar{v}_h$.)
Define $\Delta\bar{v}_{r\theta}$ as the projection of $\Delta\bar{v}$ in the orbital plane. Determine $\Delta v_{r\theta}, \beta, \phi$.
Define $\Delta\bar{v}_{BV}$; is it equal to $\Delta\bar{v}_{r\theta}$? Determine α between the velocity vector in the original orbit and $\Delta\bar{v}_{BV}$.

(b) To apply the maneuver, all positions, velocities, and $\Delta\bar{v}$'s must be written in terms of the same set of unit vectors, such as the inertial unit vectors, $\hat{x}, \hat{y}, \hat{z}$. Determine the new $\bar{r}^+, \bar{v}^+$ immediately after the maneuver.

(c) Determine the orbital elements of the new orbit, i.e., $a^+, e^+, i^+, \Omega^+, \omega^+, \theta^{*+}$

(d) Insert the maneuver into GMAT; which GMAT components are the *VNB* directions? Use the scroll to verify your results.
Plot the new and the old orbits in Matlab in terms of $\hat{x}, \hat{y}, \hat{z}$ coordinates. (You already have a Matlab script that plots the orbit in the orbit plane in terms of $\hat{e}, \hat{p}, \hat{h}$; how can you use the transformation matrix to produce the $\hat{x}, \hat{y}, \hat{z}$ coordinates.) Add the following elements to the Matlab plot: Mars equator, Line of Nodes, Orbital Angular Momentum Vector, and Periapsis Vector. Also note the maneuver location.

Problem 2: The Juno spacecraft launched August 5, 2011 and arrived in the Jovian system July 5, 2016; it remains in operation till July 2021. For preliminary analysis, obtain some information for such a mission. Ignore local gravity fields.



(a) Clearly, the actual transfer to Jupiter is not a Hohmann. But, determine the corresponding Julian Dates and the actual TOF in days and **years**. (Use noon to represent a given Julian date.)

(b) Begin by examining at transfer from $\oplus$ to Jupiter. Let the Earth and Jupiter planetary orbits be assumed as coplanar and circular.

(c) Compute the total $|\Delta\bar{v}|$ and TOF (time of flight in **years**) for a Hohmann transfer from Earth to Jupiter. Ignore local fields. . [Do not forget vector diagrams! Each planar $\Delta\bar{v}$ still requires $|\Delta\bar{v}|$, α .]

Comment on the required $|\Delta\bar{v}|$ — is this value large/small? Easily delivered by a launch vehicle? How does the TOF for the Hohmann transfer compare to the actual TOF?

(d) Assuming the Hohmann transfer path is employed, what is the required phase angle at Earth departure for arrival at Jupiter? For missions to Jupiter, what is the synodic period?

(e) Jupiter's orbit is actually eccentric. Assume arrival at Jupiter perihelion and re-compute the Hohmann TOF and required $|\Delta\bar{v}|$. Compare these results to the Hohmann in (b). Does perihelion arrival same time or $|\Delta\bar{v}|$?

(f) Discuss: Do you think that arrival at perihelion affects the synodic period? Does it matter? [Return to the JPL small body database from PS1. View Jupiter's orbit. It may assist in the discussion.]

Problem 3: Consider transfers about the Earth that include a plane change. Assume that a vehicle is in a circular Earth equatorial orbit of altitude 200 km and the vehicle is to be shifted to an orbit with the same altitude but an inclination of 57° (consistent with the Kodiak launch site in Alaska).

(a) Consider a single maneuver to accomplish the plane change. Include the vector diagram and determine the appropriate values of $|\Delta\bar{v}|$, α, β.

(b) Use a bi-elliptic transfer and an intermediate radius of $55R_{\oplus}$. The cost includes all the required maneuvers; include vector diagrams and appropriate values of $|\Delta\bar{v}|$, α, β for each maneuver. Of course, the plane change occurs at the second maneuver.

(c) What is the total cost and the TOF for the bi-elliptic option? What is the $|\Delta\bar{v}|$ cost savings? Time penalty?

(d) Reproduce the bi-elliptic option in GMAT and confirm your results. Add the Moon's orbit to the 3D window. How close is the intermediate orbit to the lunar radius? Is this close enough to introduce a concern about lunar gravity?

(e) After thinking about the potential impact of lunar gravity in (d), add the Earth+Moon propagator. That is, use the same three maneuvers but propagate forward with the E+M propagator.
Is the result impacted significantly by the lunar gravity (for an intermediate radius of $55R_{\oplus}$)? Does the date that you use to initiate the propagation matter? Why?