F BP

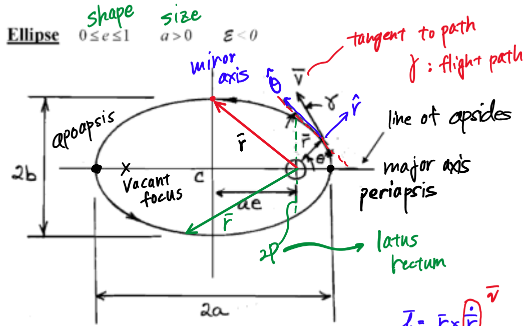
Monday, September 14, 2020 01:39 PM

$\hat{\theta}$: local horizon

$\hat{b}$ semiminor axis

Conic Sections

Ellipse shape $0 \le e \le 1$ size $a > 0$ $\varepsilon < 0$



$$r = \frac{p}{1 + e\cos\theta^*}$$

$\gamma > 0$ ascending

$\gamma < 0$ descending

$\bar{h} = \bar{r} \times \dot{\bar{r}}$ $\bar{v}$

$\hat{h} = \frac{\bar{h}}{h}$

$\hat{e}, \hat{p}, \hat{h}$

$\hat{h} = \hat{e} \times \hat{p}$

periapsis $\theta^* = 0$ $\quad r_p = \frac{p}{1+e} = a(1-e)$

apoapsis $\theta^* = 180^\circ$ $\quad r_a = \frac{p}{1-e} = a(1+e)$

$\to p = \frac{h^2}{\mu}$

$p = a(1-e^2)$

$a = \frac{1}{2}(r_p + r_a)$ also known as mean distance

Circle (special case; $e = 0$)

$a = r = p$ $\qquad \varepsilon = -\frac{\mu}{2a} = \frac{v^2}{2} - \frac{\mu}{r}$

$v_c = \sqrt{\frac{\mu}{r}}$

$\bar{v} = v_r \hat{r} + v_\theta \hat{\theta}$

top half orbit

γ is always above L.H. (local horizon)

positive γ → distance between 2 particles are growing

→ ascending

true anomaly $0^\circ \sim 180^\circ$ → v_r is positive

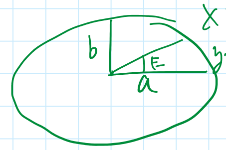
bottom half orbit

γ is always below L.H.

negative γ → distance is decreasing

→ descending

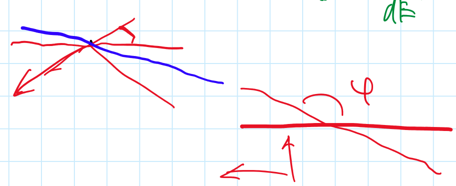
true anomally $180^\circ \sim 360^\circ$ (or $-180^\circ \sim 0^\circ$) → v_r is negative



$x = a\cos E$

$y = b\sin E$

$$\frac{dy}{dx} = \frac{\frac{dy}{dE}}{\frac{dx}{dE}} = \frac{b\cos E}{-a\sin E}$$



$$\varphi = \arctan\left(-\frac{b\cos E}{a\sin E}\right)$$

General Ellipse:

$$\varepsilon = \frac{v^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2a}$$

$v_c = \sqrt{\frac{\mu}{r}} \Rightarrow v_c^2 = \frac{\mu}{r}$ $\quad \varepsilon = \frac{-\mu}{2a}$ negative

$$v^2 = \frac{2\mu}{r} - \frac{\mu}{a} = 2v_c^2 - \underbrace{\frac{\mu}{a}}_{\text{positive by definition}}$$

$\frac{\mu}{a} > 0$

$v^2 = 2v_c^2 -$ stuff $\quad \boxed{v < \sqrt{2}\, v_c}$ → always!!

$$\frac{dA}{dt} = \frac{h}{2} \quad \to \quad dt = \frac{2}{h} dA$$

period: $IP = \frac{2}{h}(\pi ab)$ ← area of ellipse $\quad b = a\sqrt{1-e^2}$

$= \frac{2}{h}\pi a\left(a\sqrt{1-e^2}\right)$

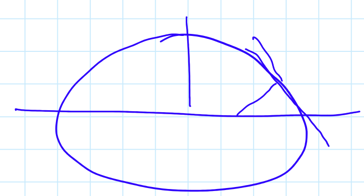
$= \frac{2\pi a^2\sqrt{1-e^2}}{\sqrt{\mu p}}$ $\quad \because p = a(1-e^2)$

$$\mathbb{P} = 2\pi\sqrt{\frac{a^3}{\mu}}$$

$\hat{r} = (\cos\theta^*, \sin\theta^*)$

$$\hat{\theta} = \hat{r}\begin{pmatrix} \cos 90^\circ & -\sin 90^\circ \\ \sin 90^\circ & \cos 90^\circ \end{pmatrix}$$

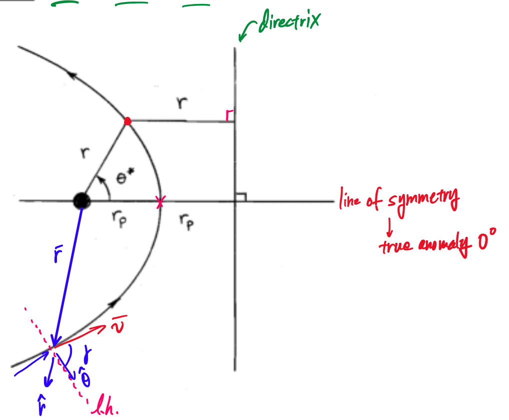
$\theta^* + 90^\circ$



F3

Parabola $e = 1$ $a = \infty$ $\mathcal{E} = 0$

$\mathcal{E} = \frac{v^2}{2} - \frac{\mu}{r} = -\cancel{\frac{\mu}{2a}}^{0}$ $a \to \infty$



Orbit NOT closed; particle leaves vicinity of attracting body

$\frac{v^2}{2} - \frac{\mu}{r} = 0 \rightarrow v^2 = \frac{2\mu}{r} = 2v_c^2 \Rightarrow v = \underbrace{\sqrt{2}\, v_c}_{\text{escape speed at distance } r}$

$\frac{v_\infty^2}{2} - \cancel{\frac{\mu}{r_\infty}}^{0} = 0 \rightarrow v_\infty = 0$

can just barely escape grav influence of attracting particle

relative velocity → ∅

F4

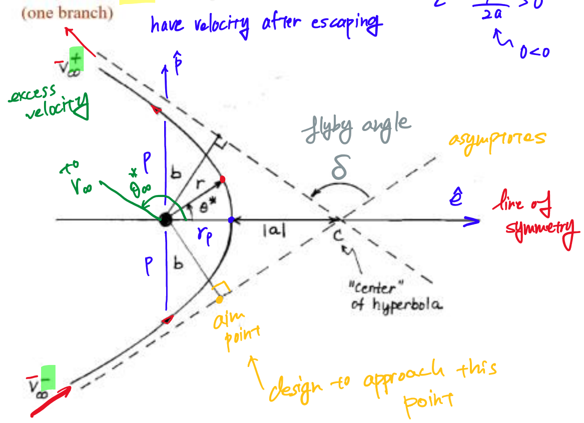
Hyperbola $e > 1$ $a < 0$ (by convention) $\mathcal{E} > 0$

(one branch)

$\mathcal{E} = \frac{-\mu}{2a} > 0$

$a < 0$

have velocity after escaping



rel vel

$\mathcal{E} = \frac{v^2}{2} - \frac{\mu}{r}$ const.

$\mathcal{E} = -\frac{\mu}{2a} = +\frac{\mu}{2|a|} = \frac{v_\infty^2}{2} - \cancel{\frac{\mu}{r_\infty}}^{0} = \frac{v_\infty^2}{2}$ $\quad \mathcal{E} > 0 \quad v_\infty \neq 0$

$r_p = a(1-e) = |a|(e-1)$ $\quad e > 1 \quad a < 0 \Rightarrow \oplus$

$p = a(1-e^2) = |a|(e^2-1)$ $\quad e > 1 \quad a < 0 \rightarrow \oplus$

$$r = \frac{p}{1+e\cos\theta^*} = \frac{p}{1+e\,c_{\theta^*}}$$

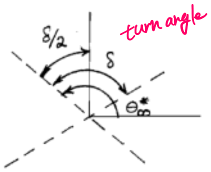
F5

parallel to asymptote $$r_\infty = \frac{|a|\left(e^2-1\right)}{1+e\cos\theta^*_\infty}$$

OR

$$1+e\cos\theta^*_\infty = \cancelto{\text{zero}}{\frac{|a|\left(e^2-1\right)}{r_\infty}}$$

$$\longrightarrow \cos\theta^*_\infty = -\frac{1}{e}$$



$$\theta^*_\infty = \frac{\delta}{2} + 90^\circ$$

$$\cos\theta^*_\infty = -\sin\frac{\delta}{2}$$

$$\sin\frac{\delta}{2} = \frac{1}{e}$$

$$\mathcal{E} = \frac{+\mu}{2|a|} = \frac{v^2}{2} - \frac{\mu}{r}$$

$$v^2 = \underbrace{\frac{2\mu}{r}}_{v_e^2} + \underbrace{\frac{\mu}{|a|}}_{\text{positive}} \Longrightarrow v > \sqrt{2}\,v_c \quad \text{always}$$