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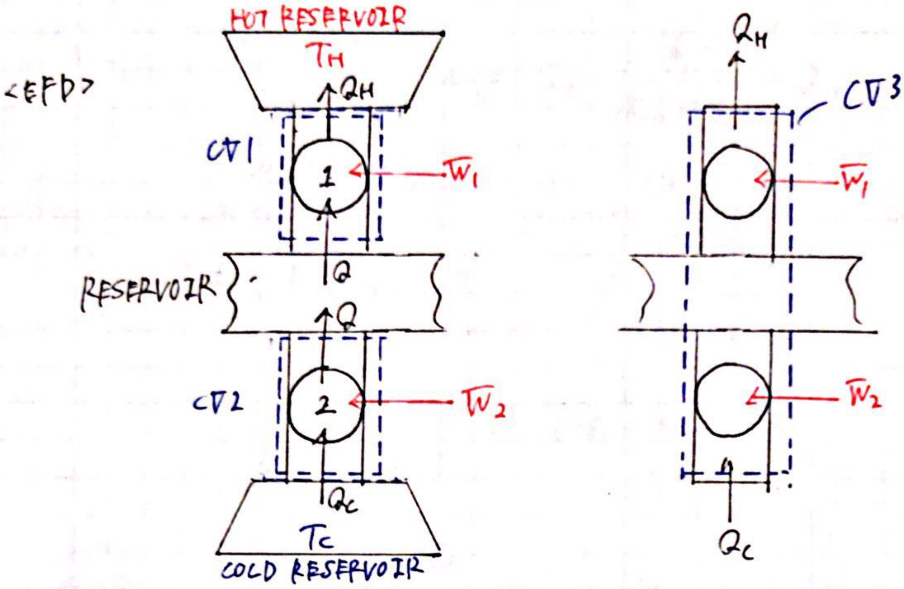
GIVEN

Two reversible refrigeration/heat pump cycles

<EFD>



FIND COP_R and COP_H (refrigeration & heat pump)

ASSUMP open sys, quasi-equilibrium, reversible (energy conserved)

EQN $\cancel{\frac{dE}{dt}}\Big|_{sys}^{0} = \dot{Q} - \dot{W}$, $\cancel{\Delta U}^{0} = Q - W$, $COP_R = \beta = \frac{Q_C}{W_{cyc}}$, $COP_H = \gamma = \frac{Q_H}{W_{cyc}}$

SOLN

(i) refrigeration

<CV1>

$$COP_{R1} = \frac{Q}{W_{cyc}} = \frac{Q}{Q_H - Q} \quad \cdots ①$$

$$0 = Q_H - Q - (-W_1) \iff W_1 = Q - Q_H$$

<CV2>

$$COP_{R2} = \frac{Q_C}{W_{cyc}} = \frac{Q_C}{Q - Q_C} \quad \cdots ②$$

$$0 = Q - Q_C - (-W_2) \iff W_2 = Q_C - Q$$

<CV3>

$$COP_R = \frac{Q_C}{Q_H - Q_C}$$

now say $COP_{R1} = \beta_1$, $COP_{R2} = \beta_2$, $COP_R = \beta$

from ① $Q_H \beta_1 = Q\beta_1 + Q \Rightarrow Q = \frac{\beta_1}{\beta_1 + 1} Q_H \quad \cdots ③$

from ② $Q\beta_2 - Q_c\beta_2 = Q_c \iff Q = \frac{\beta_2+1}{\beta_2}Q_c$ ... ④

equate ③ & ④ ③ = ④

$$\frac{\beta_1}{\beta_1+1}Q_H = \frac{\beta_2+1}{\beta_2}Q_c \iff Q_c = \frac{\beta_1\beta_2}{(\beta_1+1)(\beta_2+1)}Q_H$$

then

$$\beta = \frac{Q_c}{Q_H - Q_c} = \frac{\frac{\beta_1\beta_2}{(\beta_1+1)(\beta_2+1)}Q_H}{Q_H - \frac{\beta_1\beta_2}{(\beta_1+1)(\beta_2+1)}Q_H} = \frac{\frac{\beta_1\beta_2}{(\beta_1+1)(\beta_2+1)}}{\frac{\beta_1+\beta_2+1}{(\beta_1+1)(\beta_2+1)}} = \frac{\beta_1\beta_2}{1+\beta_1+\beta_2}$$

$$\therefore \boxed{COP_R = \beta = \frac{\beta_1\beta_2}{\beta_1+\beta_2+1}}$$

ii, heat pump

<CT1>

$COP_1 = \gamma_1 = \frac{Q_H}{Q_H - Q}$ ... Ⓐ

<CT2>

$COP_2 = \gamma_2 = \frac{Q}{Q - Q_c}$ ... Ⓑ

<CT3>

$COP = \gamma = \frac{Q_H}{Q_H - Q_c}$ ... Ⓒ

from Ⓐ $-\gamma_1 Q + \gamma_1 Q_H = Q_H \iff Q = \frac{\gamma_1 - 1}{\gamma_1}Q_H$ ... Ⓓ

from Ⓑ $\gamma_2 Q - Q_c\gamma_2 = Q \iff Q = \frac{\gamma_2}{\gamma_2 - 1}Q_c$ ... Ⓔ

equate Ⓓ & Ⓔ Ⓓ = Ⓔ

$$\frac{\gamma_1 - 1}{\gamma_1}Q_H = \frac{\gamma_2}{\gamma_2 - 1}Q_c \iff Q_c = \frac{(\gamma_1-1)(\gamma_2-1)}{\gamma_1\gamma_2}Q_H$$

then

$$\gamma = \frac{Q_H}{Q_H - Q_c} = \frac{Q_H}{Q_H - \frac{\gamma_1\gamma_2 - \gamma_1 - \gamma_2 + 1}{\gamma_1\gamma_2}Q_H} = \frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2-1}$$

$$\therefore \boxed{COP_H = \gamma = \frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2-1}}$$