HW #3   ME 6444 Nonlinear Systems
Spring 2021

Due Date: Tuesday, 5 October

1. (20 points) Hamilton's Principle - Nonlinear String

   a. Derive the equations of motion for a spatial string (i.e., string with motions in the *x*, *y*, and *z* directions) using both the full NL strain expression and a NL material law of the form $\sigma = E_1\varepsilon + E_2\varepsilon^2$. Do not reduce the theory using a quasistatic stretching assumption.
   b. Introduce a linear damping (i.e. Kelvin-Voigt) such that the material law takes the form $\sigma = E_1\varepsilon + E_2\varepsilon^2 + \alpha\dot{\varepsilon}$ and re-derive the equations of motion. **Hint:** The problem is now nonconservative and it may be easier to calculate the virtual work done by internal forces rather than use the concept of strain energy. Please see Lecture 11 for more information.

2. (30 points) Galerkin's Method – Nonlinear String

   Continuing from 1b., study only the in-plane vibration (i.e., set *u(x,t)* = *w(x,t)* =0) and only nonlinear terms due to damping (i.e., the only nonlinear terms to keep in the model are those dependent on the damping parameter α).

   a. Determine the single PDE governing free motions. Clearly identify the terms arising from Kelvin-Voigt damping. Remark on the character of the damping – would the linear system have a contribution due to damping?
   b. Study free motions of a pinned-pinned string using the first two mode shapes of the corresponding linear system. Use Galerkin's method to obtain two nonlinear ODEs.
   c. For a parameter set of your choosing, look at pseudo-phase planes in which you plot the free response due to a given set of initial modal displacements and speeds. You should generate two pseudo-phase planes: one corresponding to modal displacement and speed of the first mode, and the second corresponding to the second mode. This can be accomplished using the 'scene = [x (t),y(t)]' option in Maple with the DEplot function. Verify that the response for each mode is in fact damped.