HW # 5 ME 6444 Nonlinear Systems
Fall 2021

Due Date: Tuesday, 23 November

1. Averaging – Autonomous System
   Consider the nonlinear system $\ddot{x} + \varepsilon(x^2 + \dot{x}^2 - 4)\dot{x} + x = 0$.

   a. Use the method of averaging to find a periodic solution (i.e., limit cycle) for this system. Report the amplitude and phase of the limit cycle you find – expect dependence on θ in the amplitude. You can assume an amplitude a_1 at a solution phase corresponding to $\theta = 0$.
   b. Find the period of the limit cycle.
   c. Generate a phase plane (using Maple, Mathematica, Matlab, etc.) to verify the limit cycle's existence.

2. Lindstedt-Poincaré and Multiple Scales – Autonomous System
   Consider Rayleigh's equation: $\ddot{x} + \varepsilon\left(\frac{1}{3}\dot{x}^3 - \dot{x}\right) + x = 0$ with initial conditions $x(0) = a$ and $\dot{x}(0) = 0$. Carry-out a **first-order** approximation as follows:

   a. Use Lindstedt-Poincaré's method to find an approximate solution for *x(t)*.
   b. Use the Multiple Scales approach to find an approximate solution for *x(t)*.
   c. Generate a phase plane (using Maple, Mathematica, Matlab, etc.) to verify the limit cycle's existence.