AAE 421 Homework 1: Solutions

Fall 2020

Problem 1 (20pts)

The rotorcraft shown below is flying in an inertial reference frame O (x_f, y_f, z_f). The center of the rotor blade system is located at $10i$ ft and the blades rotate (non-deformed) in a plane perpendicular to the rotor axis with a rotational rate of $300rpm$, counter clockwise looking from left to right. The rotor blades have a radius of 20 feet. At a given instant in time the rotorcraft has the following motion state:

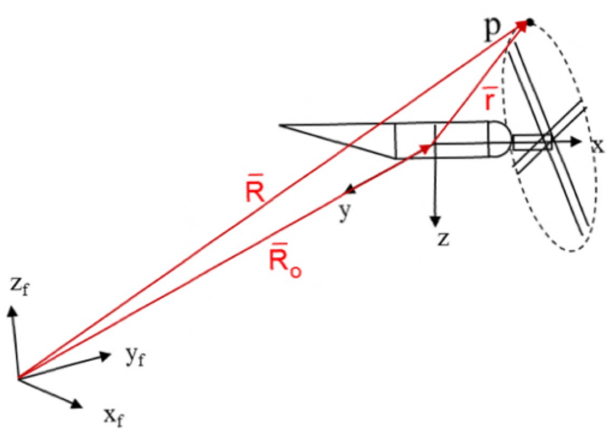
$$U = 80fps \qquad V = 20fps \qquad W = 30fps$$

$$P = 0.1rad/sec \qquad Q = 0.15rad/sec \qquad R = 0.05rad/sec$$

(a)

Sketch the traditional vectors R_0, r, and R used to define the location of the point.

sol.



(b)

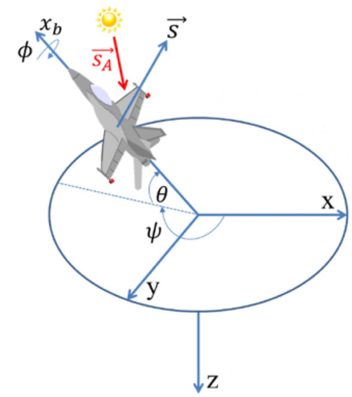
Determine the inertial velocity of point P at the instant in time where the point moves through its most top position with respect to the moving frame.

sol.

$$V_p = \frac{d\bar{R}}{dt} = V_0 + \frac{d\bar{r}}{dt} + (\omega \times \bar{r}) = 77\hat{\imath} + 605.8\hat{\jmath} + 28.5\hat{k} \quad ft/sec$$

Problem 2 (20 pts)

An aircraft made a sequence of 3-2-1 rotations with Euler angles $\Phi = 120^o$,$\Theta = 30^o$ and $\Phi = 10^o$. The vector $\bar{S}$ is along the z body axis of the aircraft. Vector $\bar{S}_A$ represents the sun light direction. Find the relative angle between $\bar{S}$ and $\bar{S}_A$ when the Sun has azimuth angle $a = 30^o$ and elevation angle $e = 45^o$.



sol.

$$Relative\,angle \;= \frac{\bar{S}_{A_{body}}.\bar{S}}{|\bar{S}_{A_{body}}||\bar{S}|} = 1.899\;rad = 108.82^o$$

Problem 3 (20 pts)

Suppose that for an ariplane the slope of the C_m vs C_L curve is -0.15 and $C_m = 0.08$ at $C_L = 0$.

(a)

Determine the trim lifting coefficient

sol.

$$C_{L_{trim}} = 0.53$$

(b)

If $x_{cg} = 0.3\bar{c}$ determine the neutral point location:

sol.

$$\frac{x_{np}}{\bar{c}} = h_n = 0.15$$

Problem 4 (20 pts)

(a)

List all of the equations for a reference condition under level, steady, symmetric, no rotational velocities flight

sol.

$$\begin{aligned}
X_0 - mg\sin(\theta_0) &= 0 \\
Y_0 &= 0 \\
Z_0 + mg\cos(\theta_0) &= 0 \\
L_0 = M_0 = N_0 &= 0 \\
P_0 = Q_0 = R_0 &= 0 \\
V_0 = W_0 &= 0 \\
\Phi_0 &= 0
\end{aligned}$$

(b)

Based on the above reference condition, derived the linear perturbation flight dynamics equations for Y froce, $m(\dot{v} + u_0 r) = \Delta Y + mg\cos(\theta_0)\phi$.

sol.

$$\dot{v} = \frac{1}{m}\Delta Y + g\cos(\theta_0)\phi - ru_0$$

Problem 5 (20 pts)

An airplane is in a constant-speed, constant-altitude (u_0=constant, h_0=constant, $w_0 = v_0 = q_0 = 0$) steady turn with a constant roll angle $f_0 < 90^o$ and constant turning rate W. The figure above shows the front view of the trim flight. Ignore any effects of rotors internal to the airplane

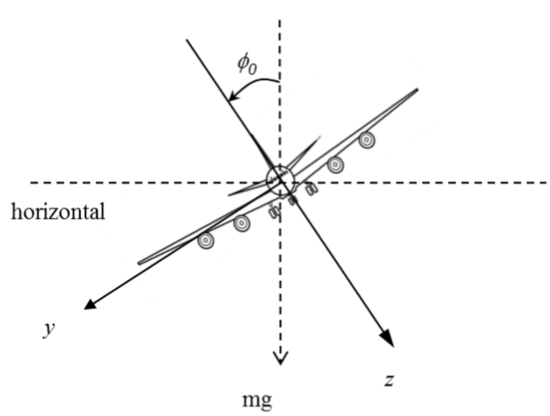


Figure 1: Caption

(a)

Show that the trim values of the angular rates are

$$q_0 = \Omega \sin \phi_0$$
$$r_0 = \Omega \cos \phi_0$$
$$p_0 = 0$$

sol.

We have,

$$\begin{aligned}
q_0 &= \dot{\theta}_0 \cos \phi + \dot{\phi}_0 \cos \theta_0 \sin \phi_0 \\
r_0 &= \dot{\theta}_0 \sin \phi + \dot{\psi}_0 \cos \theta_0 \cos \phi_0 \\
p_0 &= \dot{\phi}_0 - \dot{\psi}_0 \sin \theta_0
\end{aligned}$$

We get the result by substituting the following in the above equations:

$$\dot{\phi}_0 = 0 \qquad \dot{\theta}_0 = 0 \qquad \dot{\psi}_0 = \Omega \qquad \theta_0 = 0$$

(b)

Determine the expressions for the trim values of Y_0, L_0, M_0, N_0.

sol.

$$\begin{aligned}
Y_0 &= -mg \sin \phi_0 + mU_0 \Omega \cos \phi_0 \\
L_0 &= (I_z - I_y)\Omega^2 \sin \phi_0 \cos \phi_0 \\
M_0 &= -I_{xz}\Omega^2 \cos^2 \phi_0 \\
N_0 &= I_{xz}\Omega^2 \sin \phi_0 \cos \phi_0
\end{aligned}$$

Bonus

For problem 5, obtain the linearized y-force equation using the above trim condition. Show your work and clearly write down the result. Are the longitudinal and lateral linearized dynamics of the airplane in this case decoupled?

sol.

$$\dot{V} = \frac{1}{m}\Delta Y + g\phi \cos \phi_0 - rU_0 + u\Omega \cos \phi_0$$

Longitudinal and lateral linearized dynamics of the airplane are not decoupled.