JS BP

Saturday, October 10, 2020 11:47 PM

Orbital Maneuvers

relative 2BP

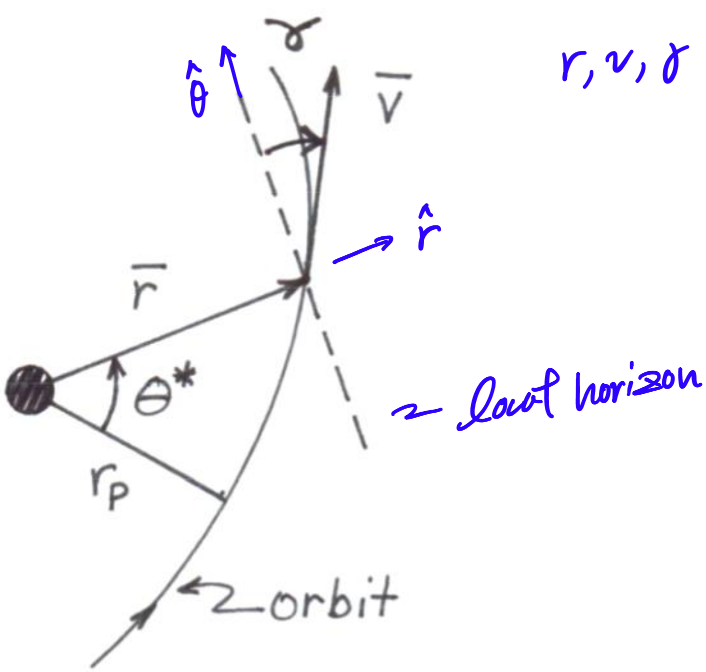
Thus far, only consider orbit <u>characteristics</u>

But, artificial satellites change orbits ➡ consider maneuvers and estimate velocity changes required for particular mission objectives

Discussion:
1. Orbit establishment ✓
2. Single impulse adjustments ✓
3. Transfers ✓

1. Orbit Establishment

Relating position/velocity to orbit characteristics



r, v, γ

$$\bar{v} = \underbrace{v s_\gamma}_{v_r} \hat{r} + \underbrace{v c_\gamma}_{v_\theta} \hat{\theta}$$

r, v, γ characterize orbit (in orbit plane)

Known: $\bar{h} = \bar{r} \times \bar{v} \Rightarrow h = r v_\theta$ $\qquad h = r\overbrace{v c_\gamma}^{v_\theta}$

$$\boxed{v_\theta = \frac{h}{r} = \frac{\mu}{h}\left(1 + e\cos\theta^*\right)} \tag{JS.1}$$

$$v_r = \dot{r} = \frac{dr}{d\theta}\dot{\theta} = \left(\frac{dr}{d\theta}\right)\frac{h}{r^2} \qquad v_r = v s_\gamma$$

$$= \frac{d}{d\theta}\left(\frac{h^2/\mu}{1 + e\cos(\theta - \omega)}\right)\frac{h}{r^2}$$

$$\boxed{v_r = \frac{\mu e}{h}\sin\theta^*} \tag{JS.2}$$

Rearrange (JS.1) and (JS.2)

$$e\cos\theta^* = \frac{h v_\theta}{\mu} - 1$$

$$e\sin\theta^* = \frac{h v_r}{\mu} = \frac{r v_\theta v_r}{\mu}$$

$$e^2 = \left[\left(e\cos\theta *\right)^2 + \left(e\sin\theta *\right)^2\right]$$

$$= \left(\frac{hv\cos\gamma}{\mu} - 1\right)^2 + \frac{r^2 v^2 (\cos^2\gamma) v^2 (\sin^2\gamma)^2}{\mu^2} \qquad \boxed{1 - \cos^2\gamma}$$

$$= \frac{r^2 v^4 (\cos^4\gamma)}{\mu^2} - \frac{2rv^2\cos^2\gamma}{\mu} + 1 + \frac{r^2 v^4 (\cos^2\gamma)(\sin^2\gamma)^2}{\mu^2}$$

$$\boxed{\sin^2\gamma + \cos^2\gamma}$$

no change in h

no change in r

change $v \longrightarrow$ how to keep e a constant

$$\boxed{e^2 = \left(\frac{rv^2}{\mu} - 1\right)^2 \cos^2\gamma + \sin^2\gamma}$$ (JS.3)

$$\tan\theta^* = \frac{rv_\theta v_r/\mu}{\frac{hv_\theta}{\mu} - 1} = \frac{rv^2 \sin\gamma\cos\gamma}{\mu\left(\frac{rv^2\cos^2}{\mu} - 1\right)}$$

$$\tan\theta^* = \frac{\sin\theta^*}{\cos\theta^*}$$

(JS.4)

$$\boxed{\tan\theta^* = \frac{\left(\frac{rv^2}{\mu}\right)\cos\gamma\sin\gamma}{\left(\frac{rv^2}{\mu}\right)\cos^2\gamma - 1}}$$

$r, v, \gamma \longrightarrow$ define other orbital parameters

2. Single Impulse Adjustments

Use a single impulse to adjust / change an orbit:

- Eliminate launch errors
- Bring s/c to a more desirable orbit → limited
- Planned correction maneuvers

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.

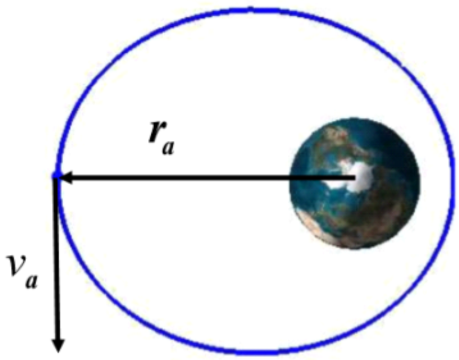
Note: Transfer to a new orbit with a single impulse is not possible unless the new orbit intersects the original orbit → must intersect!

Assume: only in plane changes
impulsive thrust applications

thrust possible in any directions

Example 1

Satellite in an established Earth orbit: $a = 3R_\oplus \quad e = .5 \quad \left(r_p = 1.5R_\oplus\right)$



Goal to change orbit subject to:
- e constant
- $a_N = 4R_\oplus$
- Δv (thrust) must be applied at apogee

Determine magnitude and direction of $\Delta\bar{v}$ to accomplish goal

Solution
- (a) Current orbit already established a, e in plane $\longrightarrow$ r, v, γ
- (b) Conditions at thrust point <u>before</u> maneuver/thrust

$$r_a = a(1+e) = 4.5R_\oplus$$

$$\frac{v_a^2}{2} = \frac{\mu_\oplus}{r_a} - \frac{\mu_\oplus}{2a} \quad \Rightarrow \quad v_a = 2.64 \text{ km/s}$$

$\gamma = 0°$

$\theta^* = 180°$

Note: to increase a, likely requires increase in v

$$\frac{v^2}{2} = \cancel{\frac{\mu}{r}} - \frac{\mu}{2a}$$

same

$r^- = r^+$

$a \uparrow$ term $-\frac{\mu}{2a}$ becomes less negative

RHS $\uparrow$

If increase v and maintain same r, does e change?

For same e, higher v must be in a different part of new orbit

Does θ^* change? yes!

Does γ change? yes!

γ is going to change $e(r, v, \gamma)$

(c) Determine (if possible) conditions at thrust point after maneuver

$$\overset{r^+}{r_N} = \overset{r^-}{r_a} = 4.5 R_\oplus$$ (cannot change r during maneuver)

$$a^+ = a_N = 4R_\oplus \qquad \bar{e} = .5 \qquad \text{given}$$

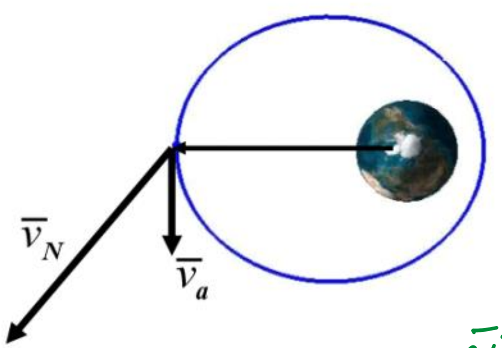
$$\frac{v_N^2}{2} = \frac{\mu_\oplus}{r_N} - \frac{\mu_\oplus}{2a_N} \quad \Rightarrow \qquad v^+ = v_N = 3.49 \text{ km/s}$$

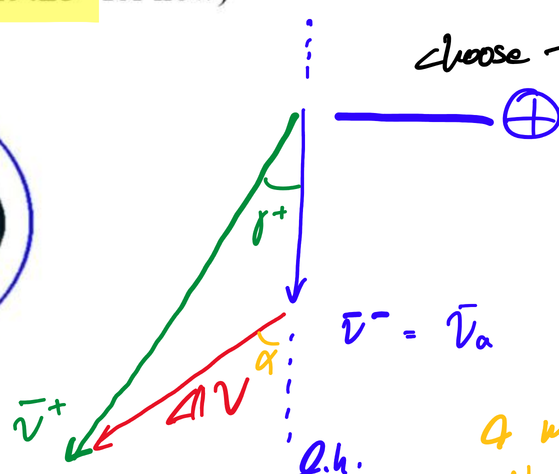
$$h = rv\cos\gamma$$

$$\sqrt{\mu_\oplus \underset{p^+}{p_N}} = r_N v_N \cos\gamma_N \quad \Rightarrow \quad \cos\gamma_N = \frac{\sqrt{\mu p^+}}{r^+ v^+} \Rightarrow \gamma_N = \gamma^+ = \pm 29.23^\circ$$

(d) Sketch a vector diagram of the situation

(Choose $\Delta\gamma = +29.23^\circ$ for now)





$\Delta V = \bar{v}^+ - \bar{v}^-$

α measured w.r.t the original vel vector ($\bar{v}^-$)

away from Earth

α is −

away from Earth

q is $+$

q is $-$

Cosine Law

$$\Delta v^2 = v_N^2 + v_a^2 - 2v_N v_a \cos\Delta\gamma_N$$

OR

$$\Delta v = \left[v_N^2 + v_a^2 - 2v_N v_a \cos\Delta\gamma_N\right]^{1/2}$$

$\boxed{\Delta v = 1.75 \text{ km/s}}$

At angle α wrt initial velocity

Sine law or geometry

$\boxed{\alpha = 76^\circ}$

$r^+ = r^-$

v^+, γ^+

Know r_N, v_N, γ_N $\longrightarrow$ $\theta_N^* = \tan^{-1}\left\{\dfrac{\left(\dfrac{rv^2}{\mu}\right)\cos\gamma\sin\gamma}{\left(\dfrac{rv^2}{\mu}\right)\cos^2\gamma - 1}\right\}$

$\theta^{*+} = \theta_N^* = -48.4^\circ$ OR $+131.6^\circ$

which one?

$\gamma^+ > 0 \Rightarrow \theta_N^* = +131.6^\circ$

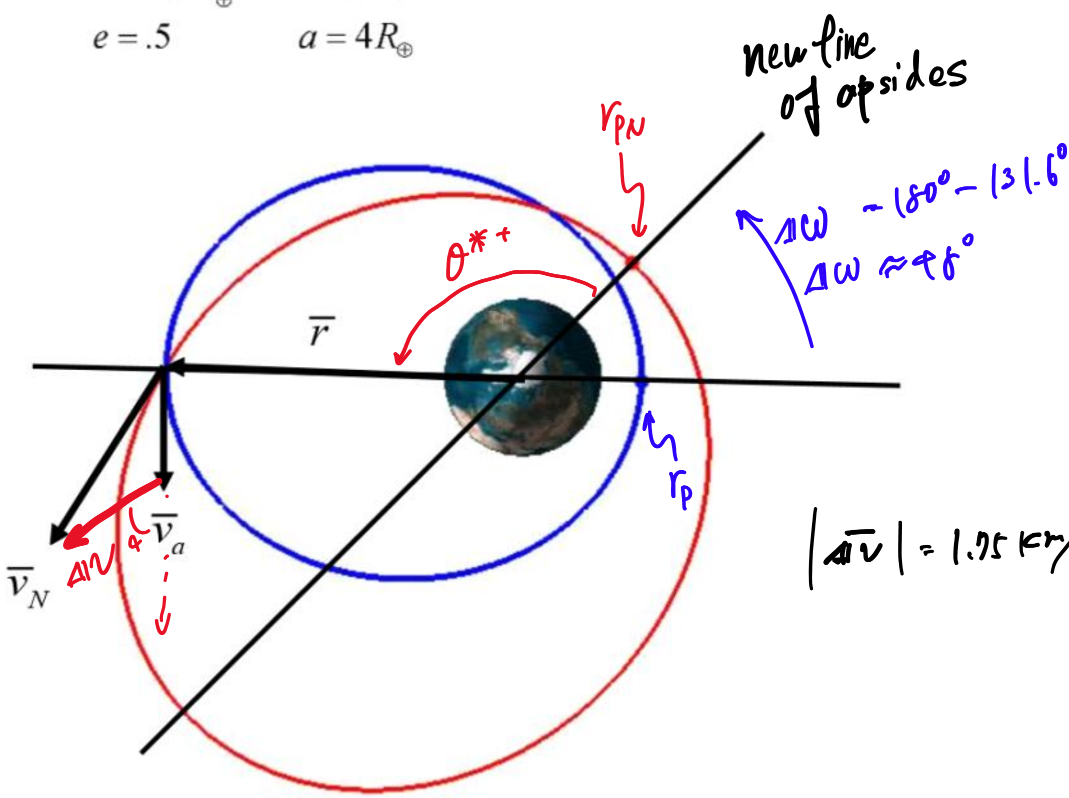
How do I know the quadrant??

Originally $\theta^* = 180^\circ$ → Now $\theta^* = 131.6^\circ$

New orbit:

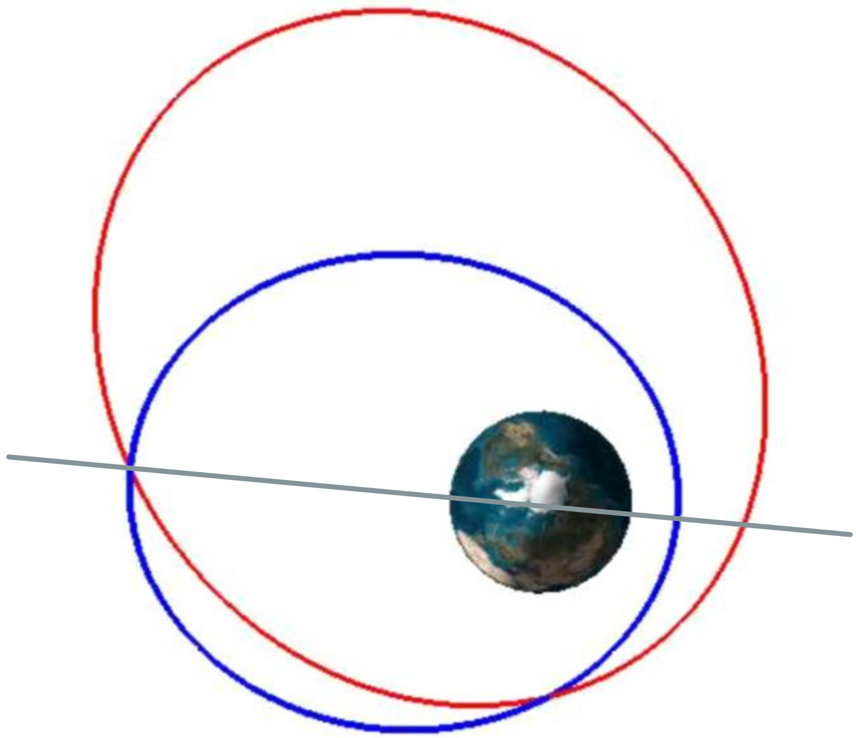
$r = 4.5R_\oplus$ $v = 3.49$ km/s

$e = .5$ $a = 4R_\oplus$



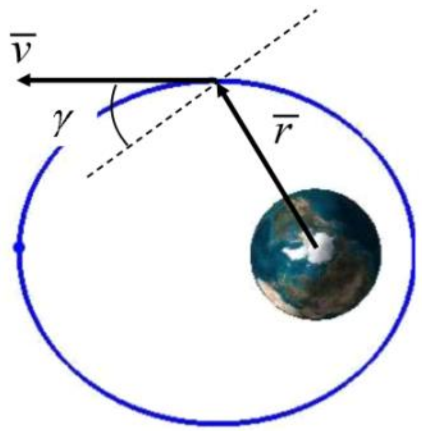
$|\Delta \bar{v}| = 1.75$ km/s

If $\Delta\gamma = -29.23^{\circ}$ ← enter new orbit in descending part of the orbit.



Can same be accomplished for lower cost?
Put maneuver in a different location?

Currently: $a = 3R_{\oplus}$ $\quad e = .5 \quad (r_p = 1.5R_{\oplus})$



Maneuver at $\theta^* = 120^\circ$
$e_N = e = .5$ constant
$a_N = 4R_{\oplus}$

Solution
(a) Current orbit already established
(b) Conditions at thrust point <u>before</u> maneuver/thrust

$r = 3R_{\oplus} \longrightarrow r = a \Rightarrow$ end of minor axis

$v^- = 4.5642$ km/s

$\gamma^- = {}^+30^\circ$ asending

Consider how to accomplish objective –
Increase/decrease velocity?

$$\frac{v^2}{2} = \cancel{\frac{\mu}{r}} - \frac{\mu}{2a} \quad \Rightarrow \quad a\uparrow \; v\uparrow$$

0

Is a tangential Δv possible?

$$e^2 = \left(\frac{rv^2}{\mu} - 1\right)^2 \cos^2\gamma + \sin^2\gamma \Rightarrow$$ cannot keep e const. unless change γ

no change

(c) Desired conditions after maneuver

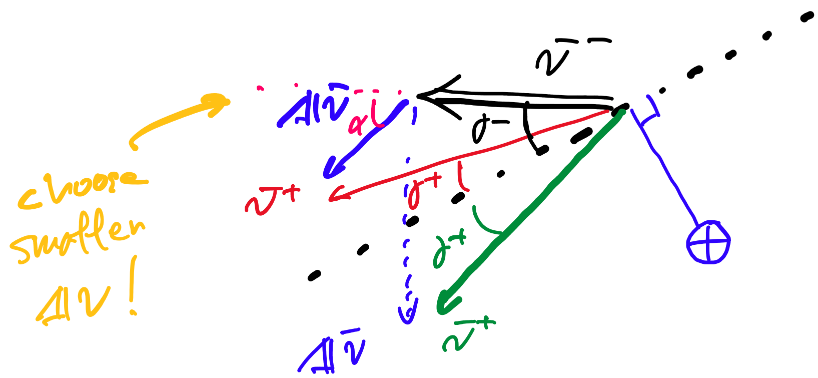
$r_N = r_a = 3R_\oplus$

$v^+ = v_N =$ 5.103 km/s $\qquad h = r v \cos\gamma$ ±

$a_N = 4R_\oplus \qquad e = .5$

(d) Vector diagram

Sketch a vector diagram of the situation



Cosine Law

$\Delta v^2 = v_N^2 + v_a^2 - 2 v_N v_a \cos \Delta\gamma_N$

$\boxed{\Delta v = 0.6115 \text{ km/s}}$

Sine Law

$$\frac{\Delta v}{\sin \Delta\gamma} = \frac{v_N}{\sin\beta} \quad \Rightarrow \beta = 150^\circ$$

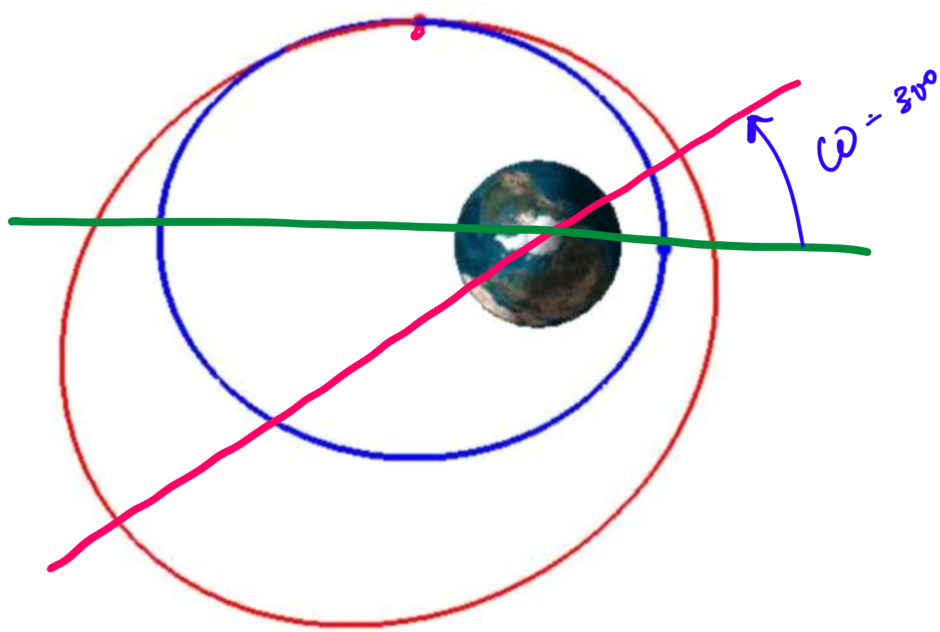
$\boxed{\alpha = -30^\circ}$

$r^+, v^+, \gamma^+ \Longrightarrow \theta^* = \pm 90^\circ$ (?)

b/c $\gamma^+ > 0$ θ^{*+} ascending

$120^\circ - 90^\circ$

$$\Delta\omega = \theta_o^* - \theta_N^* = +30^\circ$$



<u>Example</u>

At a certain instant, an Earth observing satellite is described in terms of the following state

$$r = 1.65R_{\oplus}$$
$$v = 5.7\,\text{km/s}$$
$$\gamma = -10.2^{\circ}$$

At this point, a maneuver such that $\Delta v = 1.2\,\text{km/s}$ and $\alpha = +25^{\circ}$. Determine the final orbit.

Solution:

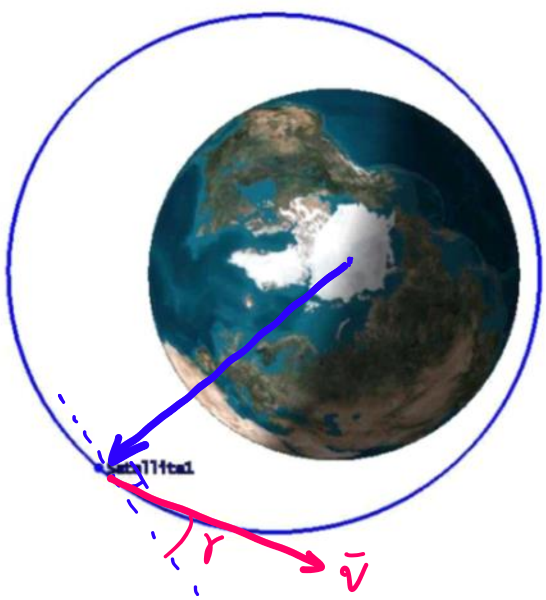
(a) Establish current orbit / current location

$$a = 1.4446R_{\oplus} \quad E = -129.05^{\circ}$$
$$e = .22571 \quad \theta^{*} = -138.52^{\circ}$$
$$r_p = 1.1185R_{\oplus} \quad IP = 2.4449\text{hr} = .10187\text{da}$$
$$r_a = 1.7710R_{\oplus} \quad (t - t_p) = -.80823\text{hr}$$

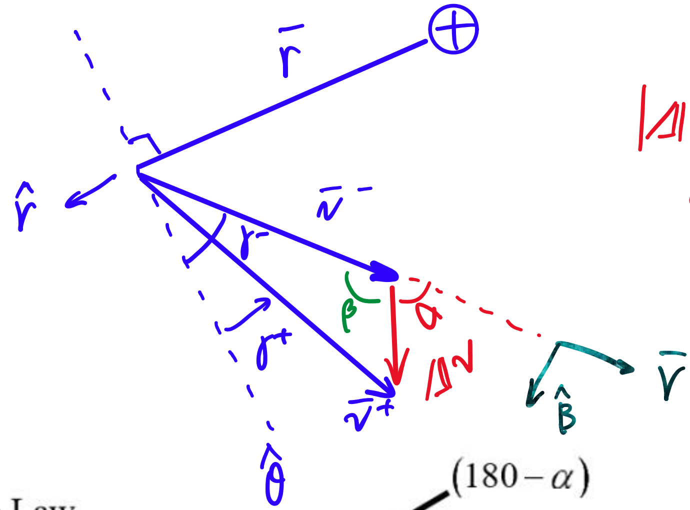
(b) Conditions immediately prior to maneuver are given

Add $\Delta\bar{v}$:

$$|\Delta\bar{v}| = 1.2\,\text{km/s}$$
$$\alpha = +25^{\circ}$$



(c) Conditions immediately <u>after</u> the impulse



$|\Delta\bar{v}| = 1.2$ km/s

$\alpha = +25°$

$\bar{v}^+ = \bar{v}^- + \Delta\bar{v}$

$\hat{V} = \dfrac{\bar{V}}{|\bar{V}|}$

$\hat{N} = \hat{h}$ normal

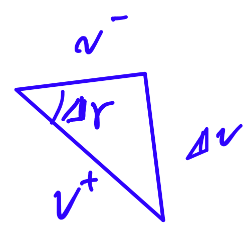
$\hat{B} = \hat{V} \times \hat{N}$ binormal

$(180 - \alpha)$

Cosine Law

$$v^{+2} = v_N^2 = v^2 + \Delta v^2 - 2v\Delta v\cos\beta$$

$\Longrightarrow$ $v^+ = v_N = 6.806$ km/s



$$\cos\Delta\gamma = \frac{\Delta v^2 - v_N^2 - v^2}{-2v_N v}$$

$\Longrightarrow$ $\Delta\gamma = 4.273°$

$\Longrightarrow$ $\gamma^+ = -5.927°$

(d) Establish the new orbit

$a^+ = a_N = 2.124\, R_\oplus$

$e^+ = e_N = 0.2448$

$r_p^+ = r_{pN} = 1.6446\, R_\oplus$

$E^+ = E_N = -24.76°$

$\theta^* = \theta_N^* = -30.874°$