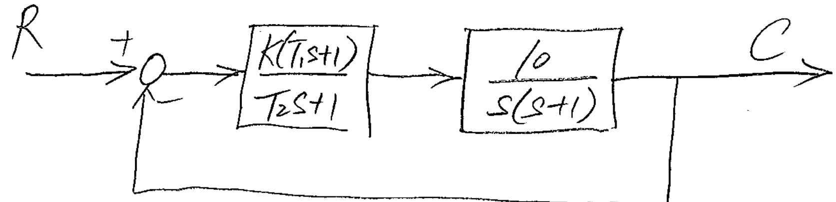
AAE 364 HW #8 Fall 2019

B-6-15

Given:



$\zeta^* = 0.5 \qquad W_n^* = 3$ rad/sec

Find: $K,\ T_1,\ T_2$.

Solution:

$$S^d_{1,2} = -\zeta W_n \pm j\sqrt{1-\zeta^2}\, W_n$$
$$= -1.5 \pm j\, 2.5981$$

$$CE: \quad 1 + \underbrace{\left(K\frac{T_1}{T_2}\right)}_{\bar{K}} \cdot \underbrace{\frac{\left(s+\frac{1}{T_1}\right)}{\left(s+\frac{1}{T_2}\right)} \cdot \frac{10}{s\,(s+1)}}_{L(s)} = 0$$

$$\text{or} \quad 1 + \bar{K} L(s) = 0.$$

By the angle condition,

$$\angle L(s^d_1) = -180^\circ \quad \left(\text{or } \angle L(s^d_2) = -180^\circ\right)$$

$$\angle L(s^d_1) = \angle\left(s^d_1 + \frac{1}{T_1}\right) - \angle\left(s^d_1 + \frac{1}{T_2}\right) - \angle s^d_1 - \angle\left(s^d_1 + 1\right)$$

Note that

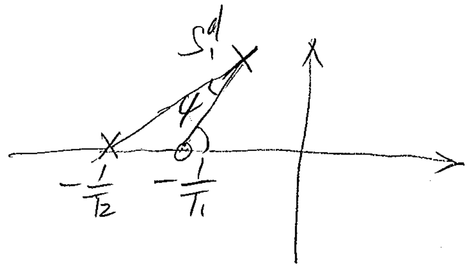
$$\angle(s_1^d) = 180^\circ - \tan^{-1}\left(\frac{2.5981}{1.5}\right)$$
$$\simeq 119.9998$$
$$\angle(s_1^d+1) = 180^\circ - \tan^{-1}\left(\frac{2.5981}{1.5-1}\right)$$
$$\simeq 100.8933$$

Then

Angle deficiency

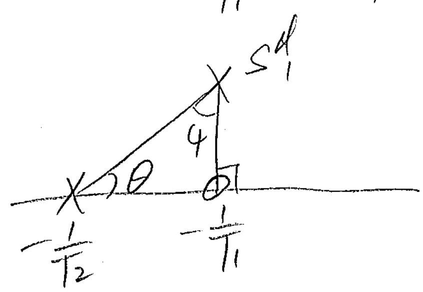
$$\varphi = \angle\left(s_1^d + \frac{1}{T_1}\right) - \angle\left(s_1^d + \frac{1}{T_2}\right) = \boxed{40.8931^\circ}$$

※ ($\varphi > 0 \Rightarrow$ $\boxed{K\frac{T_1 s+1}{T_2 s+1}}$ is a lead-compensator)



※ Solution for T_1 and T_2 is not unique.

Here we set $-\frac{1}{T_1} = -1.5$, then



$$\theta = \tan^{-1}\left(\frac{2.5981}{\frac{1}{T_2}-1.5}\right) = 90^\circ - 40.8931^\circ$$

$$\frac{1}{T_2} = 3.75$$

For the magnitude condition:

$$\left| \bar{K} L(s_1^d) \right| = 1$$

$$\left| L(s_1^d) \right| = \frac{10 \times |s_1^d + 1.5|}{|s_1^d| \cdot |s_1^d + 3.75| \cdot |s_1^d + 1|}$$

$$= 0.9524$$

$$\bar{K} = 1.05$$

$$K = \frac{T_2}{T_1} \bar{K} = \bar{K} \times \frac{\frac{1}{T_1}}{\frac{1}{T_2}}$$

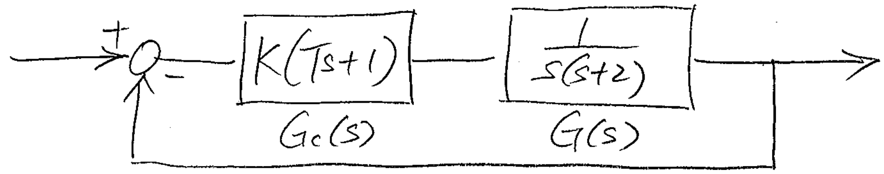
$$= 1.05 \times 1.5 / 3.75$$

$$= 0.42$$

$$\boxed{K = 0.42, \quad T_1 = 0.6667, \quad T_2 = 0.2667}$$

B-6-16

Given:



$s^d_{1,2} = -2 \pm j2$

Find: K, T.

Solution:

CE: $1 + G_c(s)G(s) = 0$

Angle condition: $\angle G_c(s^d_1) + \angle G(s^d_1) = -180°$

$\angle G(s^d_1) = -\angle s^d_1 - \angle(s^d_1 + 2)$

$= -\{180° - \tan^{-1}(\frac{2}{2})\} - 90°$

$= -225°$

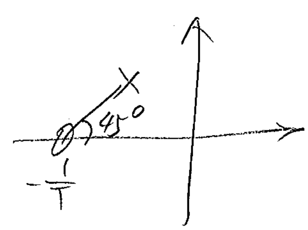
Angle defiency: $\varphi = -180° - \angle G(s^d_1)$

$= 45°$

$\angle G_c(s^d_1) = \varphi$

$\Downarrow$

$\angle(s^d_1 + \frac{1}{T}) = 45°$



$$-\frac{1}{T} = -4 \Rightarrow \boxed{T = 0.25}$$

Magnitude condition:

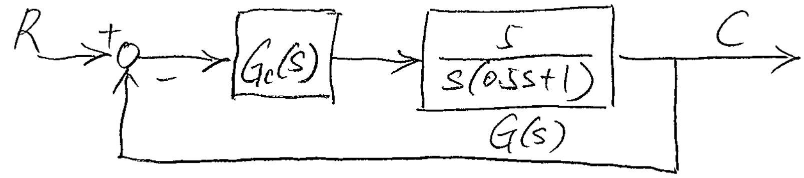
$$\left| G_c(s_1^d)\, G(s_1^d) \right| = 1$$

$$K \frac{|(0.25 \times s_1^d + 1)|}{|s_1^d| \cdot |s_1^d + 2|} = 1$$

$$\boxed{K = 8}$$

B-6-17

Given:



$$S^d_{1,2} = -2 \pm j2\sqrt{3}$$

Required: (1) $G_c(s)$; (2) Plot the unit-step response.
A Lead-compensator

Solution:

CE: $1 + G_c(s)\,G(s) = 0$.

$$\angle G(S^d_1) = -\angle S^d_1 - \angle(S^d+2)$$
$$= -120^\circ - 90^\circ$$
$$= -210^\circ$$

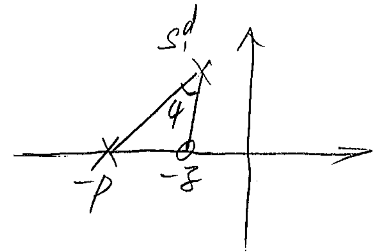
Angle deficiency: $\boxed{\begin{aligned}\phi &= -180^\circ - \angle G(S^d_1)\\ &= 30^\circ\end{aligned}}$

$\phi > 0 \Rightarrow$ Need a PD-controller or a lead-compensator. (The problem ask for a compensator)

※ Solution is not unique.

Consider a PD-controller: $G_c(s) = K\frac{(s+z)}{(s+p)}$

$$\angle G(s_i^d) = 4$$



Let $z = 2$

Then $\angle(s_i^d + p) = 60° \Rightarrow p = 4$

To determine K, we consider the magnitude condition:

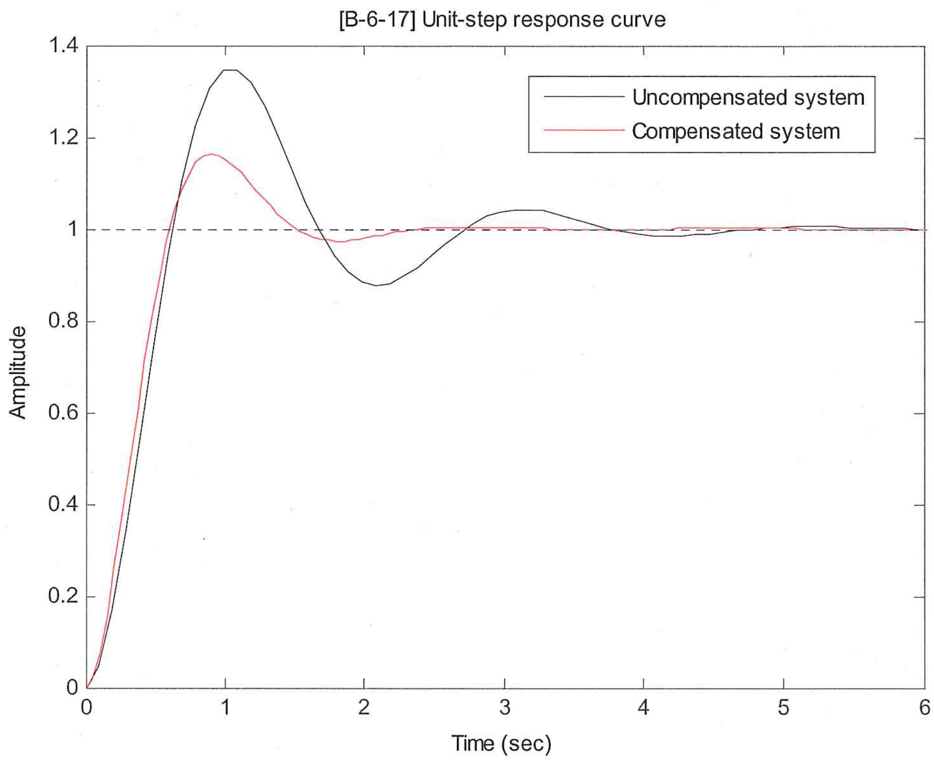
$$|G_c(s_i^d)\ G(s_i^d)| = 1$$

$$K \frac{|s_i^d + 2|}{|s_i^d + 4|} \cdot \frac{5}{|s_i^d| \cdot |s_i^d \times 0.5 + 1|} = 1$$

$$K = 1.6$$

$$\boxed{\text{Therefore, } G_c(s) = 1.6\,\frac{(s+2)}{(s+4)}}$$

Problem B-6-17

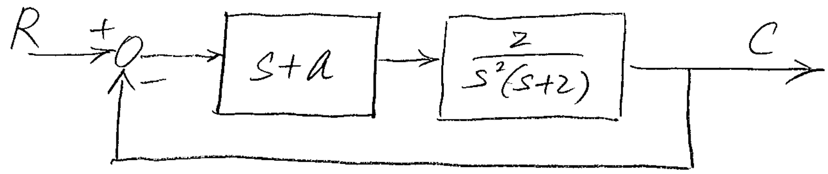


[Matlab code]

```
clear all; close all; clc

G=tf(5,[0.5 1 0]);
Gc=zpk(-2, -4, 1.6);

UncompensatedSYS=feedback(G,1);
CompensatedSYS=feedback(Gc*G,1);

figure()
step(UncompensatedSYS, 'k')
hold on
step(CompensatedSYS, 'r')
title ('[B-6-17] Unit-step response curve')
legend ('Uncompensated system', 'Compensated system')
```

B-6-26

Given:



$a \in (0, +\infty)$

$\zeta^d = 0.5$

Required: 1) Root loci ; 2) a s.t $\zeta^d = 0.5$

Solution:

1) CE : $1 + \frac{2(s+a)}{s^2(s+2)} = 0$

Rewrite it :

$$(s^3+2s^2+2s)+2a = 0$$

or

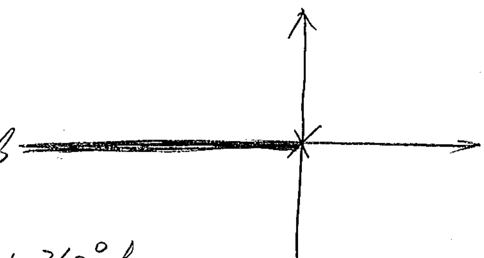
$$1 + a\underbrace{\frac{2}{s^3+2s^2+2s}}_{L(s)} = 0$$

$$L(s) = \frac{2}{s(s^2+2s+2)}$$

① $\begin{cases} \text{poles: } 0,\ -1 \pm j & n=3 \\ \text{zeros: } \phi & m=0 \end{cases}$

② Symmetry

③ R.L on real-axis



④ $\theta_a = \dfrac{180° + 360° l}{\cancel{n-m}\ 3}$, $l = 0, 1, 2$

$= 60°,\ 180°,\ 300°$

$\sigma_a = \dfrac{\sum \text{poles} - \sum \text{zeros}}{n-m} = \dfrac{-2}{3}$

⑤ $\dfrac{d}{ds}\left(\dfrac{1}{L(s)}\right) = 0$ for break-in/away points.

$\Downarrow$

$3s^2 + 4s + 2 = 0$ No real roots.

⑥ $-\angle(-1+j) - \theta_d - 90° = -180° \Rightarrow \theta_d = -45°$

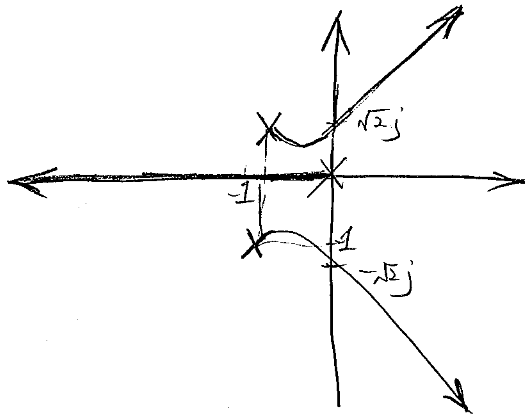
↑ Angle of departure.

⑦ R.L $\cap$ imaginary axis

$1 + a_0 L(j\omega) = 0$

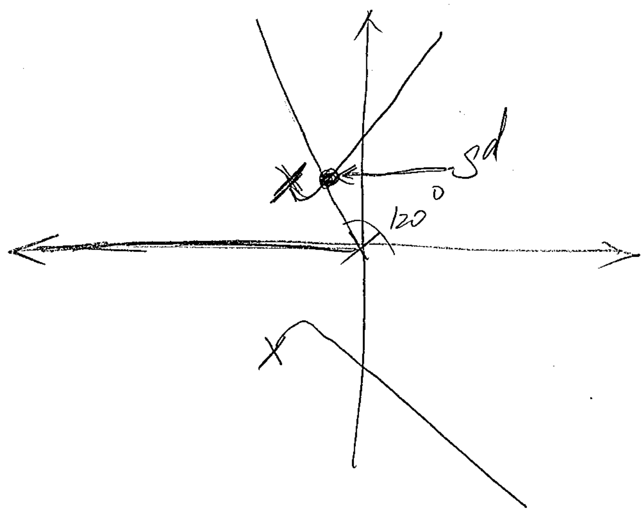
$\Rightarrow \begin{cases} 2a - 2\omega^2 = 0 \\ -\omega^3 + 2\omega = 0 \end{cases} \Rightarrow \begin{cases} a = 0 \\ \omega = 0 \end{cases}$ or $\begin{cases} a = 2 \\ \omega = \pm\sqrt{2} \end{cases}$

Not applicable



2) If dominant poles have damping ratio 0.5,

$$\angle s^d = 180^\circ - \cos^{-1}(\zeta) = 120^\circ$$



To analytically find the intersection, recall that

$$CE: \; s^3 + 2s^2 + 2s + 2a = 0.$$

From the root loci, we know that there are always two complex poles and a real pole. Then

$$s^3 + 2s^2 + 2s + 2a$$

can be written as

$$(s+p)(s^2 + 2\zeta w_n s + w_n^2) = 0$$

$$\Downarrow$$

$$s^3 + 2\zeta w_n s^2 + w_n^2 s + ps^2 + 2\zeta w_n ps + w_n^2 p$$

or

$$s^3 + (2\zeta w_n + p)s^2 + (w_n^2 + 2\zeta w_n p)s + w_n^2 p$$

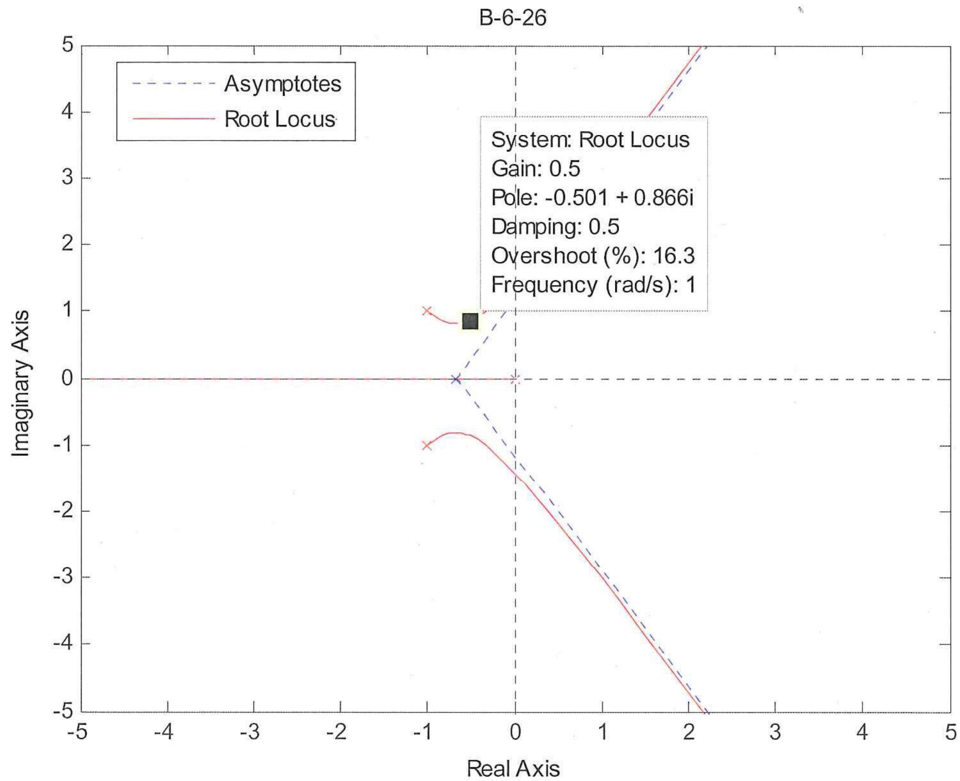
Therefore

$$\begin{cases} 2\zeta w_n + p = 2 & ① \\ w_n^2 + 2\zeta w_n p = 2 & ② \quad (\text{given } \zeta = 0.5) \\ 2a = w_n^2 p & ③ \end{cases}$$

$$\begin{matrix} ① \\ ② \end{matrix} \Rightarrow \begin{cases} w_n + p = 2 \\ w_n(w_n + p) = 2 \end{cases} \Rightarrow \begin{cases} w_n = 1 \\ p = 1 \end{cases}$$

$$\boxed{\text{Thus } a = \frac{1}{2}}$$

Problem B-6-26



[Matlab code]

```
clear all; close all; clc

num=2;                                  %numerator of L(s)
den=conv([1 0],[1 2 2]);                %denominator of L(s)
Asym=zpk([],[-2/3 -2/3 -2/3],1);

figure()
rlocus(Asym,':b')
hold on
rlocus(num,den,'r')
title ('B-6-26')
axis([-5 5 -5 5])
legend ('Asymptotes','Root Locus')
```

Problem 2

$$G(s) = \frac{5}{s^2+2} \quad , \text{ unity-feedback system}$$

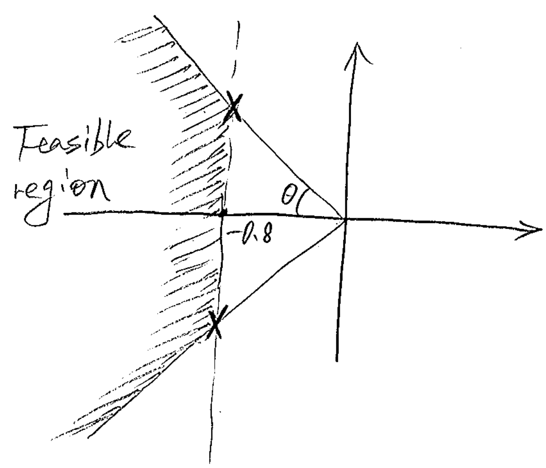
$M_p \leq 40\%$

$t_s \leq 5 \text{ sec} \qquad (2\% \text{ criterion})$

Solution:

Find the desired locations for dominant poles:

$$\begin{cases} M_p \leq 0.4 \Rightarrow e^{-\zeta\pi/\sqrt{1-\zeta^2}} < 0.4 \Rightarrow \zeta \geq 0.28 \\ t_s < 5 \Rightarrow \frac{4}{\sigma} \leq 5 \qquad \Rightarrow \sigma \geq 0.8 \end{cases}$$



$\theta = \cos^{-1}(0.28)$

We simply take:

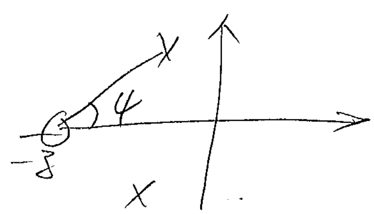
$$s_{1,2}^d = -2 \pm j\,2\sqrt{3}$$

Angle deficiency:

$$\phi = -180^\circ - \angle G(s_1^d)$$
$$= -180^\circ + \angle(s_1^d + \sqrt{2}j) + \angle(s_1^d - \sqrt{2}j)$$
$$= -180^\circ + \left\{ +180^\circ - \tan^{-1}\left(\frac{2\sqrt{3}-\sqrt{2}}{2}\right)\right\}$$
$$+ \left\{ +180^\circ - \tan^{-1}\left(\frac{2\sqrt{3}+\sqrt{2}}{2}\right)\right\}$$
$$= 66.5868^\circ > 0$$

Let's design a $\overbrace{\text{PD-controller}}^{K(s+z)}$ to meet the angle condition:

$$\angle(s_1^d + z) = \phi$$



$$\tan^{-1}\left(\frac{2\sqrt{3}}{z-2}\right) = 66.5868^\circ$$

$$\underline{z = 3.5}$$

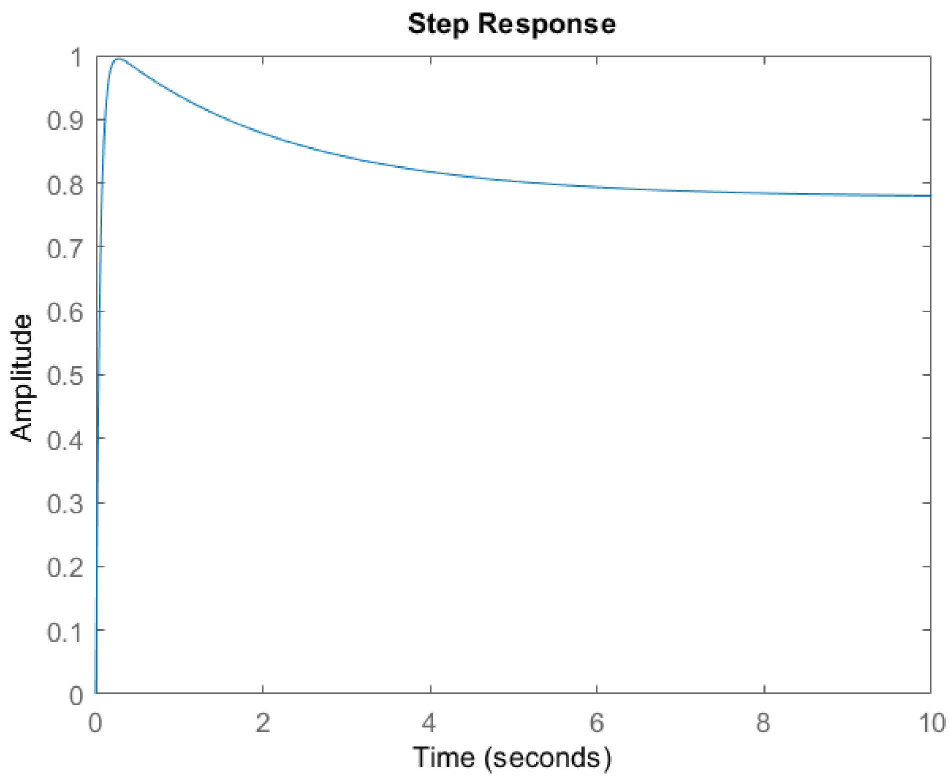
For K, we use the magnitude condition:

$$K \frac{|s_1^d + 3.5| \cdot 5}{|s_1^{d\,2} + 2|} = 1$$

$$\underline{K = 4}$$

Therefore $\underline{K(s) = 4(s+3.5)}$

Problem 2




```
%P2
G = tf([5],[1 0 2]);
Gc = zpk([-0.35],[],4);
Gcl = Gc*G/(1+Gc*G);
figure(1)
step(Gcl)
```

Problem 3

$$G(s) = \frac{1}{s^2}$$

$$M_p \le 4.6\% \quad , \quad t_s \le 8.16 \text{ sec.}$$

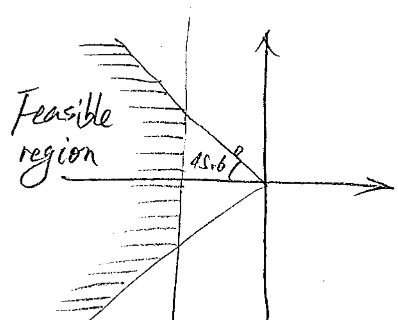
Solution:

Find the desired locations for dominant poles:

$$\begin{cases} e^{-\pi\zeta/\sqrt{1-\zeta^2}} \le 0.046 \\ \frac{4}{\sigma} \le 8.16 \end{cases}$$

$$\Downarrow$$

$$\begin{cases} \zeta \geq 0.7 \\ \sigma \geq 0.4902 \end{cases}$$



Without loss of generality, we take:

$$s^d_{1,2} = -1 \pm j$$

Angle deficiency: $\psi = -180° + \angle s_1^d + \angle s_1^d$
$= 90°$

$\psi > 0 \Rightarrow$ Need a PD-controller or a lead compensator.

Consider a PD-Controller: $G_c(s) = K(s+z)$

Then $\angle S_1^d + \delta = \psi = 90^\circ$

$\delta = +1$

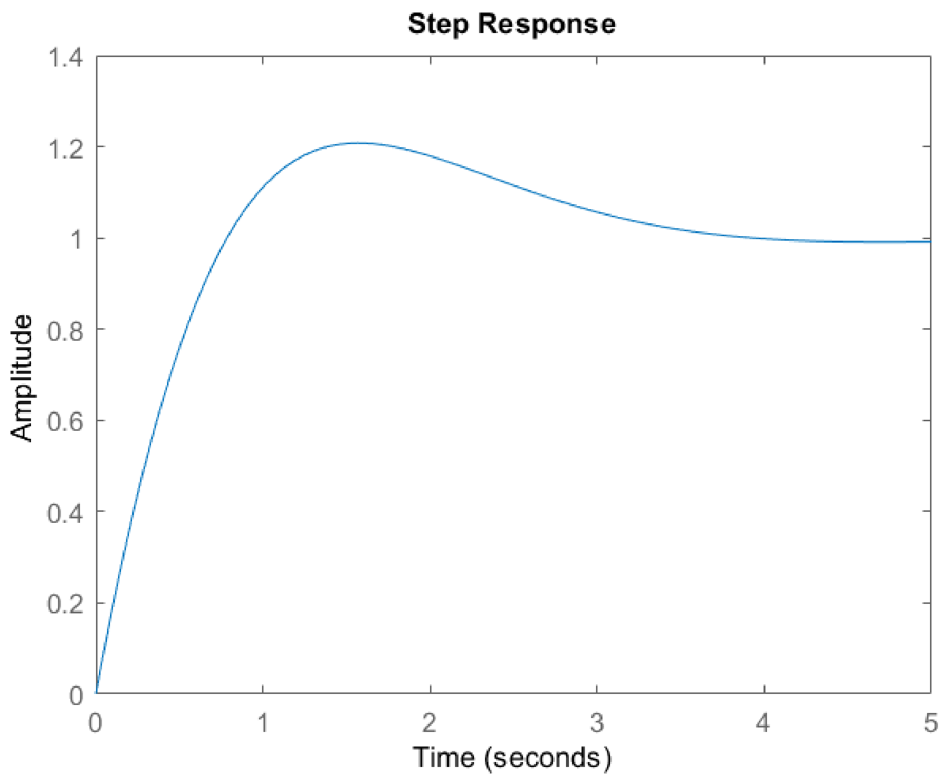
To find K:

$$K \cdot \frac{|S_1^d + 1|}{|S_1^{d\,2}|} = 1$$

$K = 2$

Therefore $K(s) = 2(s+1)$

Problem 3




```
%P3
G = tf([1],[1 0 0]);
Gc = zpk(-1,[],2);
Gcl = Gc*G/(1+Gc*G);
figure(2)
step(Gcl)
```