AAE 440: Spacecraft Attitude Dynamics

PS10

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Problem 1: In PS9, you examined perturbations in the nutation angle and the influence of the spin rate $k\Omega$ on the motion from numerical simulations as represented in terms of both the nutation angle and precession. In PS9, Problem 3, you also completed a linear analysis. One conclusion is that neither approach offers all the insight and additional information is gained from each approach.

(a) Consider the meaning of the rates for $\omega_{2_o} = k\Omega$. The orbital angular velocity in a circular orbit, Ω, is equal to the mean motion. For the Earth, $\Omega = \sqrt{\dfrac{3.986\times10^5 km^3/s^2}{(R_\oplus + h)^3}}$ where radius of the Earth $R_\oplus = 6378km$ and h is the orbit altitude. Choose altitudes $h = 200$, 800, 2000, 25000 km.

(i) What is the orbital rate in each case? If $k = 2$ or 100, what is the angular velocity $\omega_{2_o} = k\Omega$ in rad/s in each case?

By plugging in the altitude values h = [200, 800, 2000, 25000] into the equation

$$\Omega = \sqrt{\frac{\mu_\oplus}{(R_\oplus + h)^3}} \quad where\ \mu_\oplus = 3.986 \times 10^5\ km^3 s^{-2}\ \ and\ R_\oplus = 6378\ \ km$$

And then, using the output Ω we can compute the orbital rate from the following equation

$$orbital\ rate := T = \frac{2\pi}{\Omega}$$

Next, we take the product of the spin factor k and Ω to find the angular velocity.

$$\omega_{2o} = k\Omega \quad for \quad k = [2, 100]$$

The calculations for when h = 200 km becomes,

(a)<i>

When $h_1 = 200$

$$\Omega_1 = \sqrt{\frac{\mu_\oplus}{(R_\oplus + h)^3}} = \sqrt{\frac{3.986\times10^5\ km^3 s^{-2}}{[(6378 + 200)\ km]^3}}$$

$$\boxed{\Omega_1 = 0.011834\ rad/s}$$

$$orbital\ rate := T_1 = \frac{2\pi}{\Omega_1} = \frac{2\pi}{0.011834\ rad/s}$$

$$\boxed{T_1 = 5309.4}$$

then for $k = 2, 100$

$$\boxed{\begin{aligned} k=2: &\quad \omega_{20} = k\Omega_1 = 0.0023168\ rad/s \\ k=100: &\quad \omega_{20} = k\Omega_1 = 0.11834\ rad/s \end{aligned}}$$

With the same procedure we can obtain the rest. Then, we get the following tabulated results.

h_i [km]	Ω_i [rad/s]	Orbital Rate T_i [s]	ω_{2o} $[rad/s]$ $k=2$	ω_{2o} $[rad/s]$ $k=100$
200	0.0011834	5309.4	0.0023668	0.11834
800	0.0010382	6052.2	0.0020763	0.10382
2000	0.00082330	7631.7	0.0016466	0.082330
25000	0.00011359	55315.8	0.00022718	0.011359

(ii) The spin rate for the MAP spacecraft was 0.464 rpm. What k values would that number represent?

If

$$k\Omega = 0.464 \text{ rpm}$$
$$k\Omega = (0.464rpm)\left(\frac{2\pi\ rad}{1\ rev}\right)\left(\frac{1min}{60sec}\right)$$
$$k_j = \frac{0.04859\,\frac{rad}{s}}{\Omega_j}\quad where\ \ \Omega_j = \Omega_i$$

Example, calculations for the first altitude values becomes,

$$k_1 = \frac{174.9239\ \frac{rad}{s}}{\Omega_1} = \frac{174.9239\ \frac{rad}{s}}{0.0011834}$$
$$\boxed{k_1 = 147816}$$

Thus, by plugging in the numbers we get the values for spin factor k_j

Ω_i [rad/s]	$SpinFactor k_j$
0.0011834	41.0600
0.0010382	46.8040
0.00082330	59.0185
0.00011359	427.7754

(iii) Find a spacecraft that is spin-stabilized and its spin rate. (Be sure to include your source.)

One example, of a spin stabilized spacecraft is the Lunar Prospector.
On NASA's archive it is noted that the nominal spin rate of the Lunar Prospector is 12 rpm.
(NASA).

(b) With all the results PS9, discuss the impact of the gravity torque on an axisymmetric vehicle in circular orbit from this analytical and numerical analysis.
Is spin effective in compensating for the gravity torque? Why? Does the direction of spin matter? Why might spin be preferred over thrusters?
Reaction wheels, momentum wheels, and control moment gyros are all control devices for spacecraft that are based on the use of spin for stabilization. Do you think they are also useful for other particular solutions (nominal motions)? Why or why not?

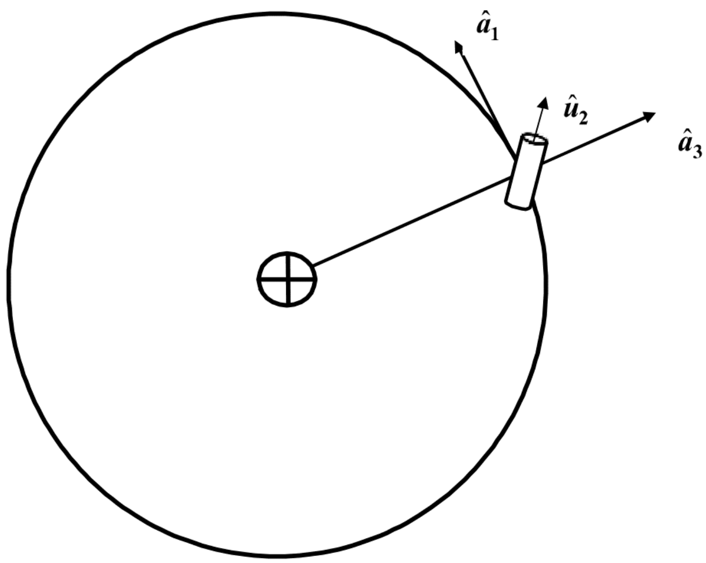
Discussion

- Spin enables the spacecraft to resist perturbations such as gravity force, and therefore, is effective.
- The direction of the spin matters depending on the magnitude of the spin rate. With a higher spin rate, the direction does not matter as much, whereas for low spin rates the perturbation acts more strongly on the spacecraft and the effect can be mitigated as well as exacerbated based on the direction of rotation. This has been observed from our analysis in PS9*.
- Spin stabilization plays a pivotal role in satellites that are expected to remain in a stable inertially fixed direction. Reaction wheels, momentum wheels, gyros, etc. enable for example weather satellites or surveillance satellites to tidally lock itself and improve data acquisition. Other methods such as thrusters are implemented as well to cope with perturbation.
- As we have analyzed in PS9* problem 3 by analytically assessing the stability by linearizing the Euler equation based on a nominal motion. From that we were able to correlate the stability and the spin factor clearly, which shows that spin stabilization and control methods are very important for other particular solutions.

Problem 2: In PS8 and PS9, the focus was an axisymmetric rigid body U (spacecraft) that can move in an inertial reference frame N in a circular orbit. For $\hat{n}_i$ and $\hat{u}_i$ as unit vectors fixed in N and U, respectively, the inertia characteristics are

$$\bar{\bar{I}}^{U/U^*} = 400\hat{u}_1\hat{u}_1 + 100\hat{u}_2\hat{u}_2 + 400\hat{u}_3\hat{u}_3 \quad \text{kg-met}^2$$

The nominal motion of interest is a constant spin of the spacecraft in N about an axis parallel to the orbit normal, i.e., $\left|{}^N\bar{\omega}^U\right| = k\Omega$ where k is a constant.



For this same problem, now explore the impact of the **shape** on the response. Recall that the spin factor is $y = \frac{\omega_{2_o}}{\Omega} - 1$ and the shape factor is $x = \frac{J}{I} - 1$. Assume the the focus is on only two spin rates: $y = 1$ and $y = -1$

(a) What spin rates $\left|{}^N\bar{\omega}^U\right| = k\Omega$ correspond to the two y values?

Explore four shape factors: $x = -0.75, -0.4, +0.2, +0.8$; what x value have you been working with thus far? What are the corresponding values of I and J for your previous analysis?

Assume that I remains the same; what is the axial inertia J for each new x value? Describe the difference in the shape of the spacecraft for each x. Add sketches to illustrate the differences.

Compute k

$$\boxed{\begin{aligned} y = 1 &\longrightarrow \frac{\omega_{20}}{\Omega} = 2 \longrightarrow k = 2 \\ y = -1 &\longrightarrow \frac{\omega_{20}}{\Omega} = 0 \longrightarrow k = 0 \end{aligned}}$$

Compute J

Using the relation

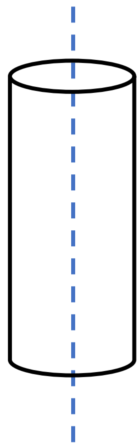
$$x = \frac{J}{I} - 1 \quad \Leftrightarrow \quad J = I(1 + \mathrm{x})\mathrm{where I} = 400\ \ \mathrm{kg} - \mathrm{m}^2$$

We can figure out the x, I, J, and the shape of the spacecraft. The tabulated results are the following.

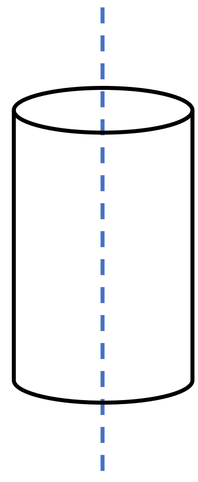
x	I [kg-m^2]	J [kg-m^2]	Shape of Spacecraft	
-0.75	400	100	I > J	Rod-like
-0.4	400	240	I > J	Rod-like
0.2	400	480	J > I	Disk-like
0.8	400	720	J > I	Disk-like

Illustrations

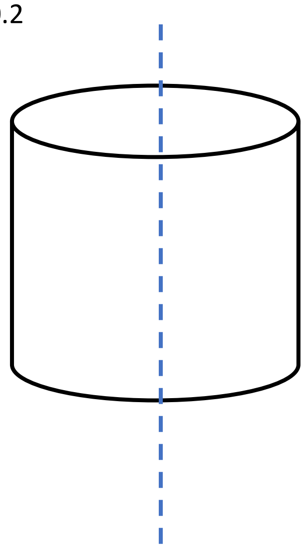
x = -0.75



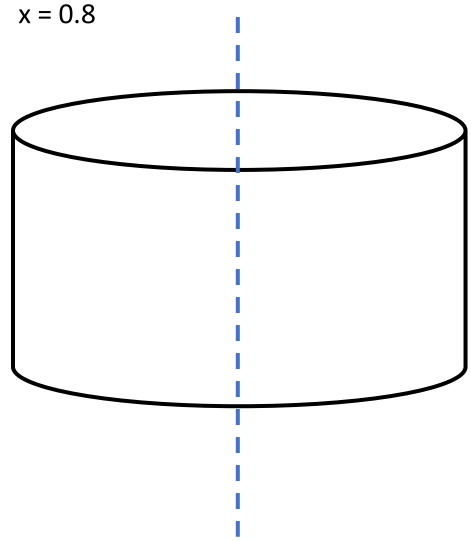
x = -0.4



x = 0.2



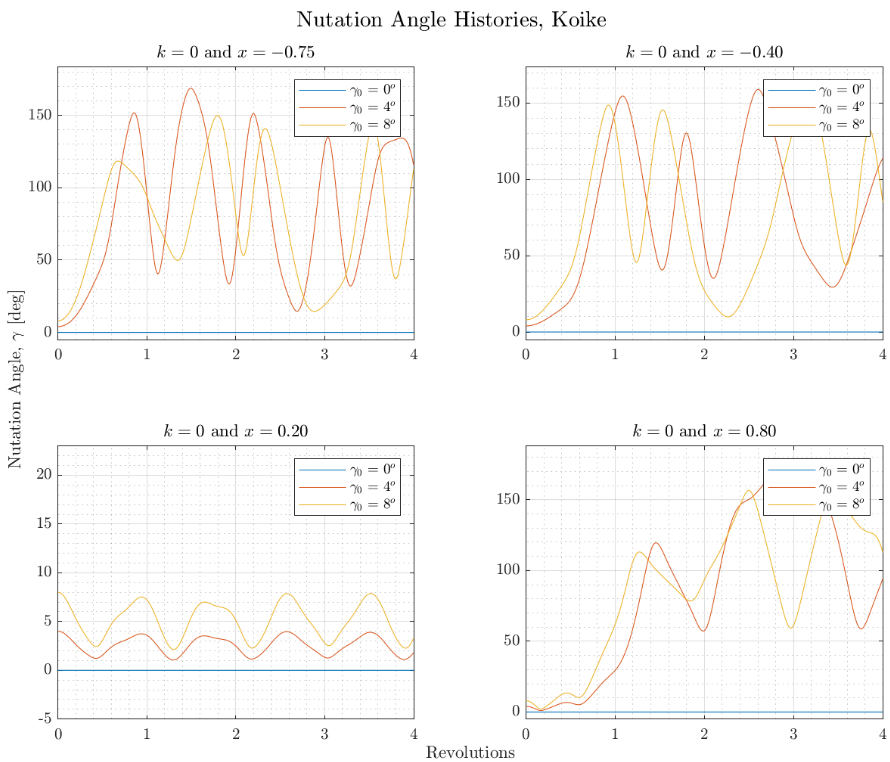
x = 0.8

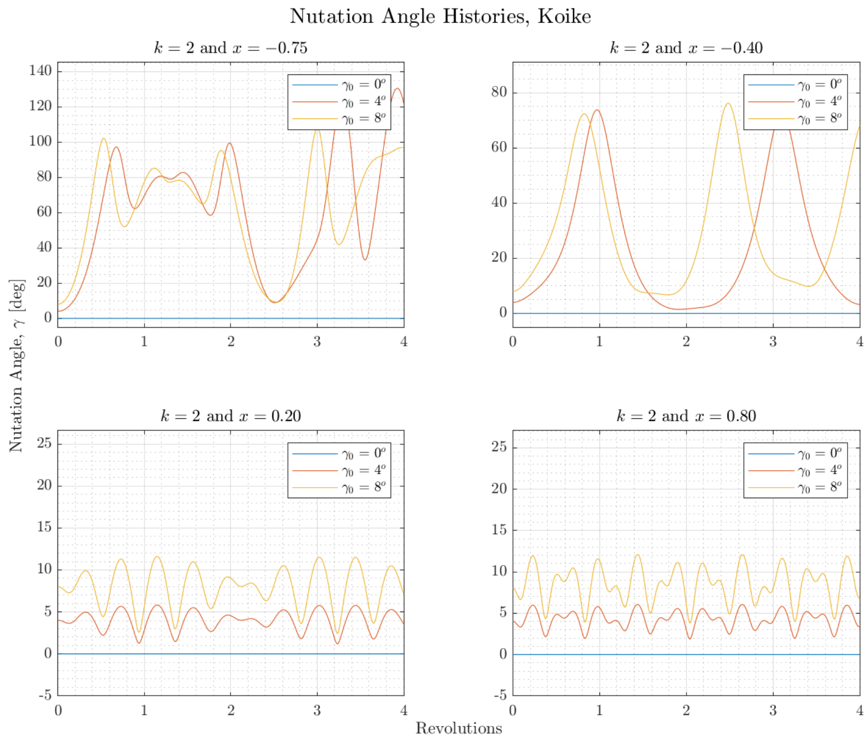


(b) The nominal motion is the same as in PS9. For the same comparison solutions that you examined previously, i.e., $\gamma(0) = 4^\circ, 8^\circ$ examine output of the nutation angle after 4 revs for each y and each x value. You should have 8 plots with 3 curves on each (the particular solution and two comparison solutions).

Comment on the impact of shape. Does a rod-shape or disk-shape matter?

Simulation Plots





Analysis

- Observing the case of k = 0 and y = -1, we can see that it shows instabilities for any x-value besides the value of x =0.2 This shows that the shape of the body does not effect the motion, but for only for small positive values of x.
- Next, observing the case of k=2 and y=1, only for these specific can we tell that the shape of the motion matters for the stability. For example, when the shape is disk-like the motion is stable, whereas when the shape is rod-like the motion is unstable. Additionally, the plot tells us that as the period of the patterns increase the x-value decreases.

(c) In the linear analysis, compute the eigenvalues for each x, y combination. What stability properties do the eigenvalues predict? Are the numerical results for nutation consistent with the analytical predictions? Which x- y combinations should you run for more revs? Why?

Run at least two combinations (more if necessary) for more revs. How many revs are sufficient? 20 revs? 40 revs? What do you conclude?

Eigenvalues

From non-dimensional characteristic equation in PS9* problem 3 the characteristic equ becomes

$$\tilde{\lambda}^2 \left[\tilde{\lambda}^4 + (12\pi^2 x + 4\pi^2)\tilde{\lambda}^2 + (48\pi^4 x Q + 16\pi^4 Q^2) \right] = 0$$

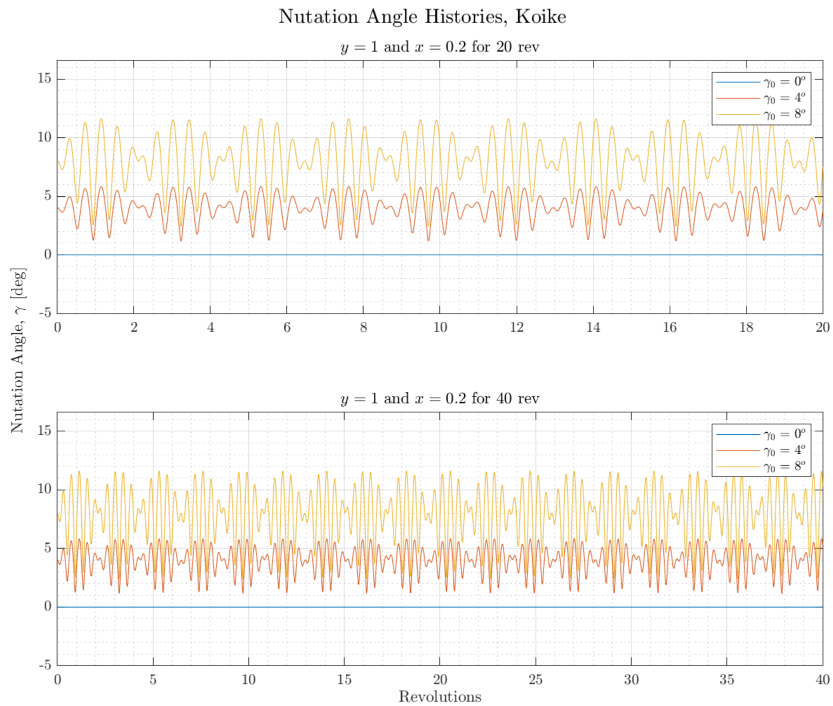
$$\because \quad \tilde{\lambda} = \frac{\lambda}{\Omega} \quad \& \quad Q = y + x(y+1)$$

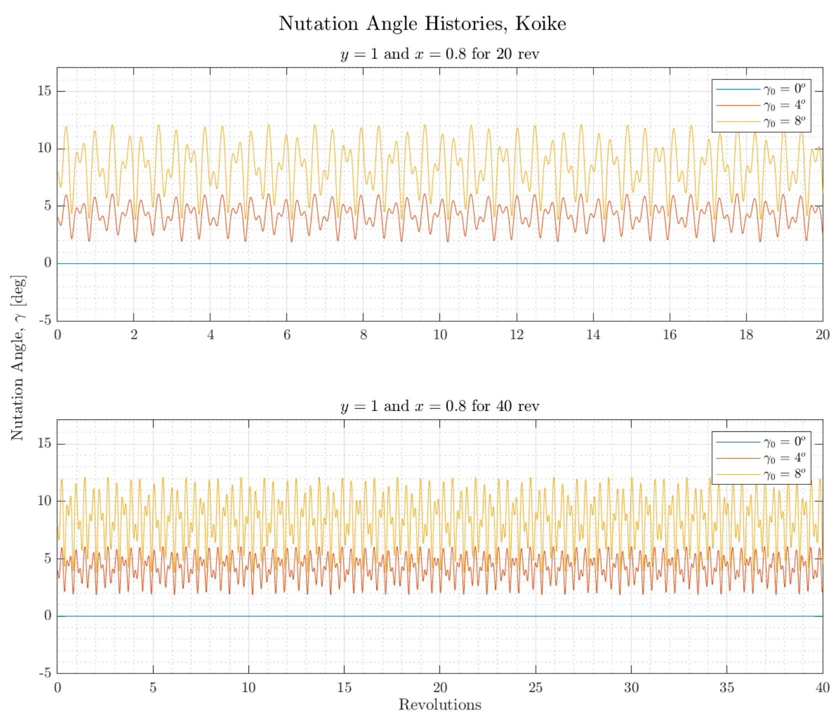
Now, solving $\tilde{\lambda}$ using MATLAB (code in appendix) by plugging in the provided values, we get the tabulated results on the next page.

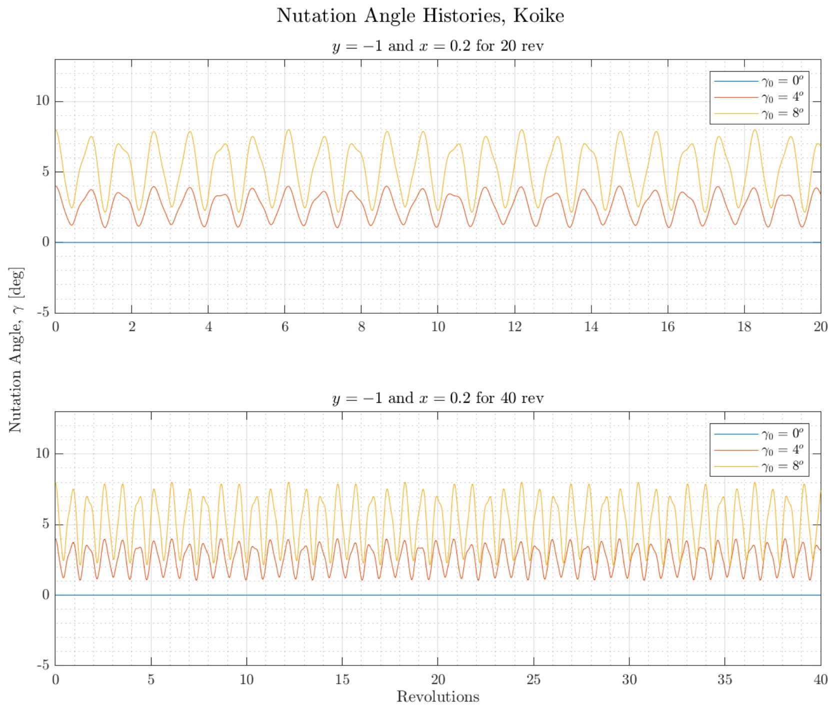
Tabulated Results

y	x	$\hat{\lambda}_{3,4,5,6}$	Prediction	Numerical Results	Linear Analytical Analysis	Congruency
1	-0.75	+2.9426i -2.9426i +1.2253i -1.2253i	Inconclusive	Probably Unstable	Marginally Stable	Yes
	-0.4	0 0 0 0	Inconclusive	Probably Unstable	Marginally Stable	Yes
	0.2	0 0 0 0	Inconclusive	Probably Stable	Marginally Stable	Yes
	0.8	0 0 0 0	Inconclusive	Probably Stable	Marginally Stable	Yes
-1	-0.75	+2.9426i -2.9426i +1.2253i -1.2253i	Inconclusive	Probably Unstable	Marginally Stable	Yes
	-0.4	+0.54589 -0.54589 +2.1675i -2.1675i	Unstable	Probably Unstable	Unstable	Yes
	0.2	0 0 0 0	Inconclusive	Probably Stable	Marginally Stable	Yes
	0.8	0 0 0 0	Inconclusive	Probably Unstable	Marginally Stable	Yes

Simulation Plots







Analysis

- Every plot shows no large change in the pattern of the nutation angles for both 20 revolutions and 40 revolutions. The angles for both simulated revolutions, the nutation angle amplitudes seem to remain within the range of which can be identified as stable.
- Though, we cannot conclude the system to be stable for the lack of information; however, we can say that the system is possibly/potentially stable.

Problem 3: Assume that a rigid body B can move in a torque-free environment but has a mass distribution such that the body is unsymmetric and

$$\bar{\bar{I}}^{B/B^*} = 400mL^2\hat{b}_1\hat{b}_1 + 100mL^2\hat{b}_2\hat{b}_2 + 800mL^2\hat{b}_3\hat{b}_3$$

Let $\hat{n}$ be inertial and $\hat{b}$ are fixed in the body; $\hat{n}_i = \hat{b}_i$ initially.

(a) Derive the differential equations that govern the torque-free response. (Recall that we are using vector basis $\hat{b}$ in the analysis for the unsymmetric body since frame $\hat{c}$ now offers no advantages.)

(a)

Deriving the DE for torque-free motion of unsymmetric rigid body.

Since $\bar{\bar{I}}^{B/B^*} = 400mL^2\hat{b}_1\hat{b}_1 + 100mL^2\hat{b}_2\hat{b}_2 + 800mL^2\hat{b}_3\hat{b}_3$

Let $I_1 = 400mL^2$, $I_2 = 100mL^2$, $I_3 = 800mL^2$

From Euler's EOM

$$\bar{M}^{B^*} = \frac{{}^N d\,{}^N\bar{H}^{B^*}}{dt} = \bar{0} \quad \cdots \text{①}$$

Where, ${}^N\bar{H}^{B^*} = \bar{\bar{I}}^{B/B^*} \cdot {}^N\bar{\omega}^B$

Since, ${}^N\bar{\omega}^B = \omega_i \hat{b}_i$

$${}^N\bar{H}^{B^*} = I_1\omega_1\hat{b}_1 + I_2\omega_2\hat{b}_2 + I_3\omega_3\hat{b}_3$$

Then,

$$\frac{{}^N d\,{}^N\bar{H}^{B^*}}{dt} \overset{BKE}{=} \frac{{}^B d\,{}^N\bar{H}^{B^*}}{dt} + {}^N\bar{\omega}^B \times {}^N\bar{H}^{B^*}$$

$$= I_1\dot{\omega}_1\hat{b}_1 + I_2\dot{\omega}_2\hat{b}_2 + I_3\dot{\omega}_3\hat{b}_3$$
$$+ (\omega_1\hat{b}_1 + \omega_2\hat{b}_2 + \omega_3\hat{b}_3) \times (I_1\omega_1\hat{b}_1 + I_2\omega_2\hat{b}_2 + I_3\omega_3\hat{b}_3)$$

$$= I_1\dot{\omega}_1\hat{b}_1 + I_2\dot{\omega}_2\hat{b}_2 + I_3\dot{\omega}_3\hat{b}_3$$
$$+ I_2\omega_1\omega_2\hat{b}_3 - I_3\omega_1\omega_3\hat{b}_2$$
$$- I_1\omega_1\omega_2\hat{b}_3 + I_3\omega_2\omega_3\hat{b}_1$$
$$+ I_1\omega_1\omega_3\hat{b}_2 - I_2\omega_2\omega_3\hat{b}_1$$

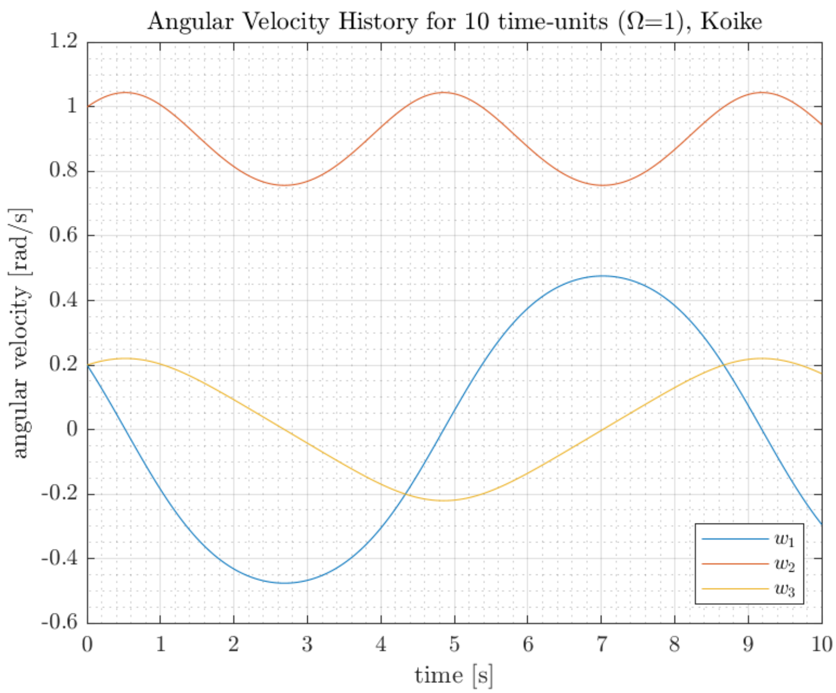
$$= \left[I_1\dot{\omega}_1 + (I_3 - I_2)\omega_2\omega_3\right]\hat{b}_1$$
$$+ \left[I_2\dot{\omega}_2 + (I_1 - I_3)\omega_1\omega_3\right]\hat{b}_2$$
$$+ \left[I_3\dot{\omega}_3 + (I_2 - I_1)\omega_1\omega_2\right]\hat{b}_3$$

since ①, the DEs become

$$I_1\dot{\omega}_1 + (I_3 - I_2)\omega_2\omega_3 = 0$$
$$I_2\dot{\omega}_2 + (I_1 - I_3)\omega_1\omega_3 = 0$$
$$I_3\dot{\omega}_3 + (I_2 - I_1)\omega_1\omega_2 = 0$$

$$\Rightarrow \boxed{\begin{aligned} \dot{\omega}_1 &= \frac{I_2 - I_3}{I_1}\omega_2\omega_3 \\ \dot{\omega}_2 &= \frac{I_3 - I_1}{I_2}\omega_1\omega_3 \\ \dot{\omega}_3 &= \frac{I_1 - I_2}{I_3}\omega_1\omega_2 \end{aligned}}$$

(b) The initial conditions are given such that ${}^N\bar{\omega}^B = 0.2\Omega\hat{b}_1 + 1.0\Omega\hat{b}_2 + .2\Omega\hat{b}_3$, where Ω is a constant. Numerically integrate the dynamic differential equations for the angular velocities for 10 time-units and plot the angular velocity components ω_j in the same figure. Can you identify the curves as elliptic functions? Which curves reflect the sn, cn, and dn functions?
From the curves, determine the maximum amplitudes and the period corresponding to each angular velocity component.



Analysis

- From this plot, we can identify the angular velocities as a Jacobi elliptic functions because the plots are periodic and are evidently sinusoidal.
- By drawing a vertical line at the point (0.5, 0), we can see that ω_1 decreases right after the initial point. This means that ω_1 is the $cn(x, k)$ elliptical function. Then because ω_3 increases right after the initial point, this means that ω_3 is sn(x, k). Thus, the remaining ω_2 which is somewhat distant from the other two becomes dn(x, k).
- Using MATLAB (code in Appendix) we can calculate the maximum amplitude by taking the difference of the maximum value and minimum value of each angular velocity waves and dividing that by 2. The results are tabulated below.

	Maximum Amplitude [rad/s]
ω_1	0.4761
ω_2	0.1438
ω_3	0.2204

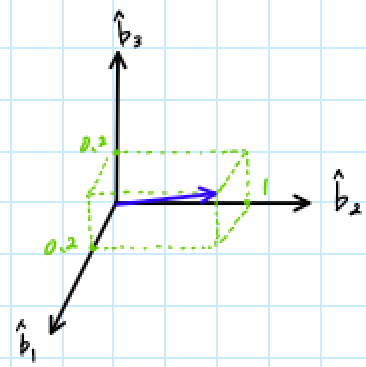
(c) Determine the following angles at the initial time:

(i) ξ — angle between ${}^N\bar{\omega}^B$ and $\hat{b}_2$

(ii) φ — angle between ${}^N\bar{H}^{B/B^*}$ and $\hat{b}_2$

(i)

From the figure in the right



$$\cos\xi = {}^N\hat{\omega}^B \cdot \hat{b}_2$$

where,

$$
\begin{aligned}
{}^N\hat{\omega}^B &= \frac{{}^N\bar{\omega}^B}{|{}^N\bar{\omega}^B|} \\
&= 0.1925\,\hat{b}_1 + 0.9623\,\hat{b}_2 + 0.1925\,\hat{b}_3
\end{aligned}
$$

$$\therefore \boxed{\xi = \arccos(0.9623) = 15.7932^\circ}$$

(ii)

Similarly, angle between ${}^N\bar{H}^{B/B^*}$ and $\hat{b}_2$ becomes

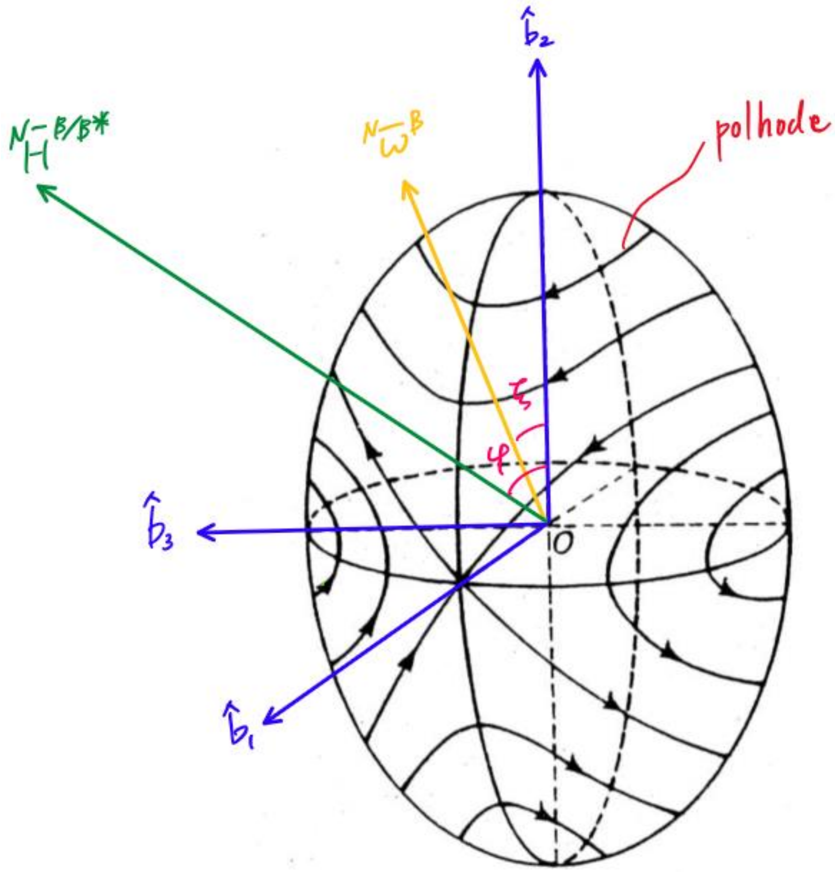
$$\arccos\left({}^N\hat{H}^{B^*} \cdot \hat{b}_2\right)$$

where

$$
\begin{aligned}
{}^N\hat{H}^{B^*} &= \frac{{}^N\bar{H}^{B^*}}{|{}^N\bar{H}^{B^*}|} = \frac{\bar{\bar{I}}^{B/B^*} \cdot {}^N\bar{\omega}^B}{|\bar{\bar{I}}^{B/B^*} \cdot {}^N\bar{\omega}^B|} \\
&= \frac{80\,\hat{b}_1 + 100\,\hat{b}_2 + 160\,\hat{b}_3}{204.9390} \\
&= 0.3904\,\hat{b}_1 + 0.4880\,\hat{b}_2 + 0.7807\,\hat{b}_3
\end{aligned}
$$

$$\therefore \boxed{\varphi = \arccos(0.4880) = 60.7941^\circ}$$

(d) Representative polhode curves from the Notes V appear on the last page of this document as discussed in class. Add vectors $\hat{b}_j$, ${}^N\bar{\omega}^B$, ${}^N\bar{H}^{B/B^*}$ to the sketch. Identify the polhode curve that is most likely to represent the motion in this problem. Why do you select it?

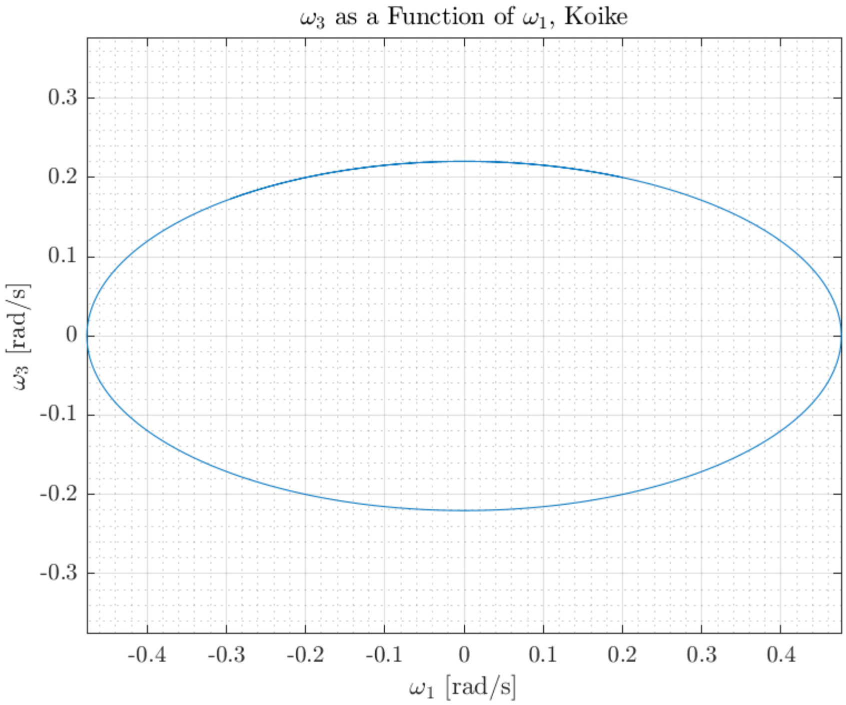


$$\arccos\left({}^N\hat{H}^{B*}\cdot\hat{b}_1\right) = \arccos(0.3904) = 67.02^\circ$$
$$\arccos\left({}^N\hat{H}^{B*}\cdot\hat{b}_3\right) = \arccos(0.7807) = 38.68^\circ$$

Discussion

- The specific polhode was selected because it is the only one that intersects with the angular velocity vector that we have sketched on the ellipsoid diagram. We also know that the movement is confined to one curve because it depends on the initial conditions of the angular velocity and since the largest angular velocity is the $\hat{b}_2$ component, and therefore the encirclement will tend to be near and around the $\hat{b}_2$-axis.

(e) Plot ω_3 as a function of ω_1. Given the polhode curves, discuss your conclusions regarding 'stability'.

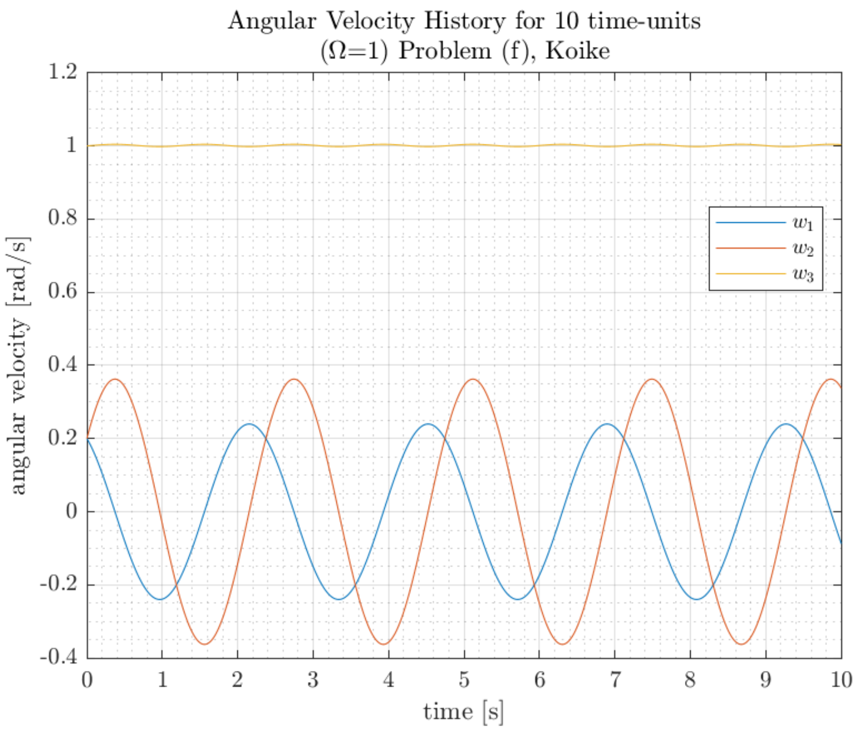


Analysis

- This system is marginally stable by our definition because the rotational axis is near the minimum moment of inertia which allows the rotational axis to remain close to the principle direction with the minimum moment of inertia. The closer the angular velocity vector is to the min/max moment of inertia the smaller the deviation in angle for each cycle.

(f) Integrate for a different set of initial angular velocities, i.e., ${}^N\bar{\omega}^B = 0.2\Omega\hat{b}_1 + 0.2\Omega\hat{b}_2 + 1.0\Omega\hat{b}_3$. Plot time histories for ω_j. What is the representative polhode? Do the polhodes differ from the response to the previous initial conditions?

Plot

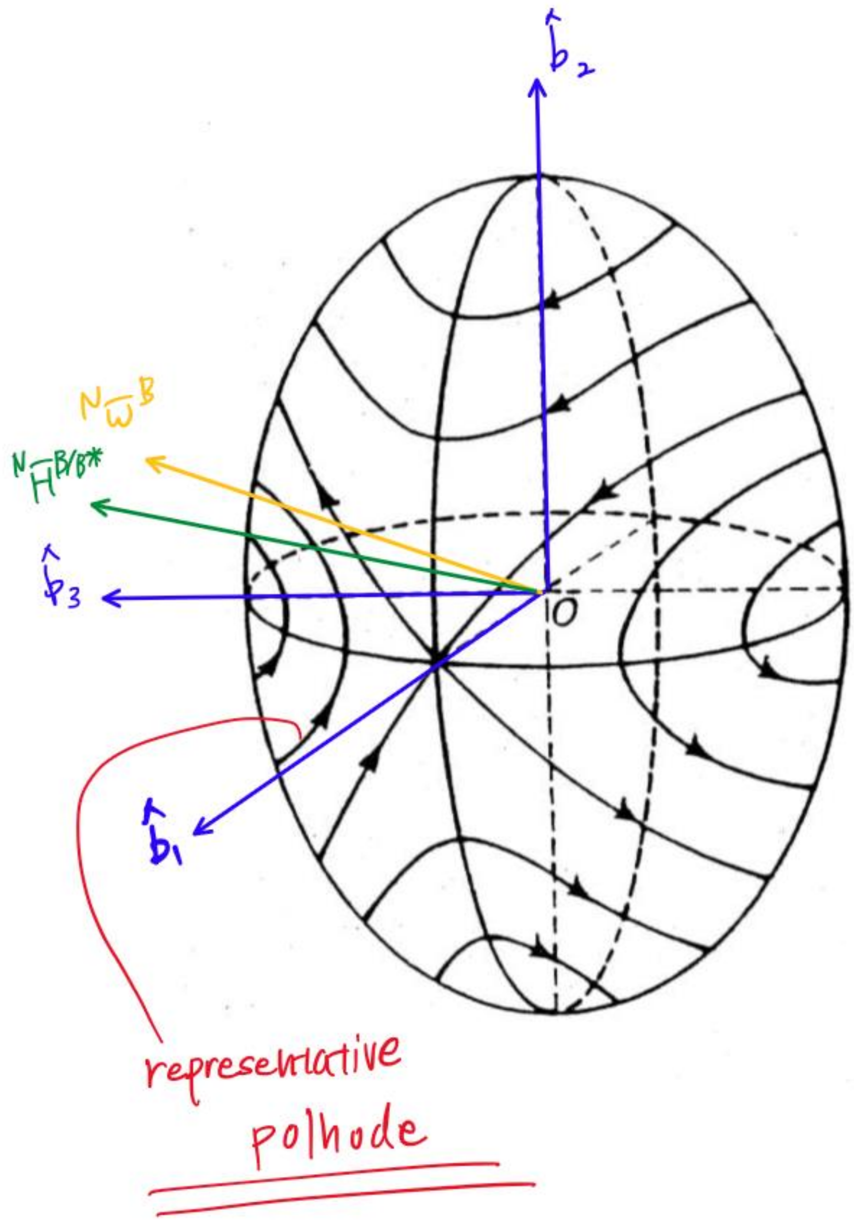


With the same procedure in problem (d) we find out that the angles between ${}^N\bar{\omega}^{\boldsymbol{B}}$ and each axis $\hat{b}_1$, $\hat{b}_2$, and $\hat{b}_3$ as well as ${}^N\bar{H}^{B/B*}$ and each axis $\hat{b}_1$, $\hat{b}_2$, and $\hat{b}_3$ are the following,

${}^N\bar{\omega}^{\boldsymbol{B}}$			${}^N\bar{H}^{B/B*}$		
$\hat{b}_1$,	$\hat{b}_2$,	$\hat{b}_3$	$\hat{b}_1$,	$\hat{b}_2$,	$\hat{b}_3$
78.9402°	78.9402°	15.7932°	84.2912°	88.5750°	5.8851°

Based on these angles we can sketch ${}^N\bar{\omega}^{\boldsymbol{B}}$ and ${}^N\bar{H}^{B/B*}$ onto the ellipsoid like how we did in problem (d).

Representative Polhode



Analysis

- The polhodes differ from what we had for the first initial conditions. This is because for the second initial conditions the angular velocity was the closest to $\hat{b}_3$-axis due to its largest component being the $\hat{b}_3$ component. Thus, the rotation will tend to trace a path near the $\hat{b}_3$-axis.

Appendix

AAE 440 PS10 Problem 1

```
clear all; close all; clc;
fdir = 'C:\Users\Tomo\Desktop\studies\2020-Spring\AAE440\MATLAB\outputs\HW10';
set(groot, 'defaulttextinterpreter','latex');
set(groot, 'defaultAxesTickLabelInterpreter','latex');
set(groot, 'defaultLegendInterpreter','latex');
```

(a)

(i)

```
% Given properties
Re = 6378;  % radius of the Earth [km]
mu_e = 3.986e5;  % [m3s-2]
h = [200 800 2000 25000];  % altitudes [km]
k = [2; 100];  % spin factor

% Anonymous function for mean motion Omega
Omega = @(h) sqrt(mu_e./(Re + h).^3);

% Orbital rates
OMG_i = Omega(h)
T_i = 2*pi./OMG_i

% Angular velocities
w_2o = k.*OMG_i
```

(ii)

```
w_2o = 0.464*2*pi/60;
k_j = w_2o./OMG_i
```

AAE 440 PS10 Problem 2-2

```
clear all; close all; clc;
fdir = 'C:\Users\Tomo\Desktop\studies\2020-Spring\AAE440\MATLAB\outputs\HW10';
set(groot, 'defaulttextinterpreter','latex');
set(groot, 'defaultAxesTickLabelInterpreter','latex');
set(groot, 'defaultLegendInterpreter','latex');
```

```
% Defining System Properties
I       = 400;                  % transverse moment of inertia [kg m2]
rev     = 0:0.001:4;            % Integration revolutions
x       = [-0.75 -0.4 0.2 0.8]; % shape factor
% x       = [-0.01 0.01 0.1 0.2];
y       = [1 -1];
```

(a)

```
% spin factor
k       = y + 1;
% axial moment of inertia [kg m2]
J       = I.*(1+x);
```

(b)
```
% Initial perturbation  [deg]
gamma0  = [0 4 8];
peak    = zeros(1,3);
for i = 1:length(k) % spin factors
    fig = figure("Renderer","painters","Position",[10 10 900 700]);
    fig.NumberTitle = i;
    for j = 1:length(J) % shape factors
        for n = 1:length(gamma0) % initial perturbations
            % Nutation History
            [rev, Nut] = Nut_History(rev,k(i),gamma0(n),I,J(j));
            peak(n) = max(Nut);

            % Plotting Gammas
            subplot(2,2,j);
            hold on;
            plot(rev, Nut, 'DisplayName',['$\gamma_0$ = ',num2str(gamma0(n)),
'$^o$'])
        end
        grid on; grid minor; box on; ylim([-5 max(peak)+15]);
        legend;
        str = sprintf('$k=%d$ and $x=%.2f$',k(i),x(j));
        title(str)
    end
    % Give common xlabel, ylabel and title to figure
    han = axes(fig,'Visible','off');
    han.XLabel.Visible = 'on';
    han.YLabel.Visible = 'on';
    han.Title.Visible = 'on';
    xlabel(han,'Revolutions');
    ylabel(han,'Nutation Angle, $\gamma$ [deg]');
    sgt = sgtitle("Nutation Angle Histories, Koike");
    sgt.Visible = 'on';
    str = sprintf('2b_Sub_Nut_History_k=%d.png',k(i));
    saveas(fig, fullfile(fdir, str));
    close all;
end
```

(c)
```
% More Revolutions
revN = [20, 40];
gamma0  = [0 4 8];
for i = 1:3
    % Combinations to be run for more revolutions
    % [y, x] = [1, 0.2], [1, 0.8], [-1, 0.2]
    fig = figure("Renderer","painters","Position",[10 10 900 700]);
    fig.NumberTitle = i;
    switch i
        case 1
            y = 1; x = 0.2;
```

```
        case 2
            y = 1; x = 0.8;
        case 3
            y = -1; x = 0.2;
    end

        k = y + 1;   % spin factor
        J = I*(1+x); % axial moment of inertia [kg m2]

        for j = 1:2
            rev = 0:0.001:revN(j);
            for n = 1:length(gamma0) % initial perturbations

                % Nutation History
                [rev, Nut] = Nut_History(rev,k,gamma0(n),I,J);
                peak(n) = max(Nut); % max nutation
                subplot(2,1,j);
                hold on;
                plot(rev, Nut, 'DisplayName', ...
                    ['$\gamma_0$ = ',num2str(gamma0(n)), '$^o$'])
            end
            grid on; grid minor; box on; legend; ylim([-5 max(peak)+5])
            str = sprintf('$y=%d$ and $x=%.1f$ for %d rev',y,x,rev(end));
            title(str)
        end
    % Give common xlabel, ylabel and title to figure
    han = axes(fig,'visible','off');
    han.XLabel.Visible = 'on';
    han.YLabel.Visible = 'on';
    han.Title.Visible = 'on';
    xlabel(han,'Revolutions');
    ylabel(han,'Nutation Angle, $\gamma$ [deg]');
    sgt = sgtitle("Nutation Angle Histories, Koike");
    sgt.Visible = 'on';
    str = sprintf('2.(c)Sub_Nut_History_y=%d_x=%.1f.png',y,x);
    saveas(fig, fullfile(fdir, str));
    close all;
end
```

AAE 440 PS10 Problem 3

```
clear all; close all; clc;
fdir = 'C:\Users\Tomo\Desktop\studies\2020-Spring\AAE440\MATLAB\outputs\HW10';
set(groot, 'defaulttextinterpreter','latex');
set(groot, 'defaultAxesTickLabelInterpreter','latex');
set(groot, 'defaultLegendInterpreter','latex');
```

(b)

```
% Defining System Properties
I = [400 100 800];          % moment of inertia [kg-m2]
Omega = 1;
w0 = Omega*[0.2 1 0.2];     % initial angular velocity [rad/s]
t_span = 0:0.001:10;       % 10 time-units [s]
```

```
% Numerical Integration
opt = odeset('RelTol', 1e-13, 'AbsTol', 1e-13);
[t, data] = ode45(@(t,w) Tfree_unsymm_EOM(t,w,I),t_span,w0,opt);

% Assign variables from the output of the numerical integration
w1 = data(:,1);
w2 = data(:,2);
w3 = data(:,3);

% Plot the angular velocities
fig1 = figure("Renderer","painters");
    plot(t,w1)
    title({'Angular Velocity History for 10 time-units ($\Omega$=1), Koike'})
    xlabel('time [s]')
    ylabel('angular velocity [rad/s]')
    hold on;
    plot(t,w2); plot(t,w3); hold off;
    grid on; grid minor; box on;
    legend('$w_1$','$w_2$','$w_3$',"Location",'southeast')
saveas(fig1,fullfile(fdir,'P3-b_angVel.png'));

% Find amplitude for each angular velocity
amp_w1 = (max(w1)-min(w1))/2
amp_w2 = (max(w2)-min(w2))/2
amp_w3 = (max(w3)-min(w3))/2
```

(c)

```
% Angle between omegaNB and b2 hat
b2_hat = [0 1 0];
w0_hat = w0/norm(w0)
zeta = acosd(dot(w0_hat,b2_hat))
```

```
% Angle between H_NB and b2 hat
H_NB = I.*w0
H_NB_hat = H_NB/norm(H_NB)
psi = acosd(dot(H_NB_hat,b2_hat))
```

(d)

```
% Plotting w3 as a function w1
fig2 = figure("Renderer","painters");
    plot(w1,w3)
    title({'$\omega_3$ as a Function of $\omega_1$, Koike'})
    xlabel('$\omega_1$ [rad/s]'); ylabel('$\omega_3$ [rad/s]')
    grid on; grid minor; box on; axis equal;
saveas(fig2,fullfile(fdir,'P3-d_w1_vs_w3.png'));
```

(f)

```
w0 = Omega*[0.2 0.2 1];     % initial angular velocity [rad/s]

% Numerical Integration
opt = odeset('RelTol', 1e-13, 'AbsTol', 1e-13);
```

```
[t, data] = ode45(@(t,w) Tfree_unsymm_EOM(t,w,I),t_span,w0,opt);

% Assign variables from the output of the numerical integration
w1 = data(:,1);
w2 = data(:,2);
w3 = data(:,3);

% Plot the angular velocities
fig3 = figure("Renderer","painters");
    plot(t,w1)
    title({'Angular Velocity History for 10 time-units', '($\Omega$=1) Problem
(f), Koike'})
    xlabel('time [s]')
    ylabel('angular velocity [rad/s]')
    hold on;
    plot(t,w2); plot(t,w3); hold off;
    grid on; grid minor; box on;
    legend('$w_1$','$w_2$','$w_3$',"Location",'best')
saveas(fig3,fullfile(fdir,'P3-e_angVel.png'));

% Angle between omegaNB and b2 hat
b2_hat = [0 1 0];
w0_hat = w0/norm(w0)
zeta = acosd(dot(w0_hat,b2_hat))
acosd(dot(w0_hat,[1 0 0]))
acosd(dot(w0_hat,[0 0 1]))

% Angle between H_NB and b2 hat
H_NB = I.*w0
H_NB_hat = H_NB/norm(H_NB)
psi = acosd(dot(H_NB_hat,b2_hat))
acosd(dot(H_NB_hat,[1 0 0]))
acosd(dot(H_NB_hat,[0 0 1]))
```

```
function lambdas = eigenvalues(I,J,k,type)
    %{
      Function:    eigenvalues
      Author:      Tomoki Koike
      Description: Eigenvalues per omega will be computed.
      >>Inputs
          I:    transverse moment of inertia
          J:    rotational moment of inertia
          k:    spin factor k
          type: 'd' = dimensional
                'nd' = non-dimensional
      Outputs<<
          lambdas: eigenvalues
    %}

    x = J/I - 1;        % shape factor
    y = k - 1;          % spin factor
    Q = y + (1 + y)*x;
```

```
    % Dimensional or Non_dimensional
    if type == 'd'
        A = 3*x+Q^2+1;
        B = 3*x*Q+Q^2;
    else % type == 'nd'
        A = (3*x+Q^2+1)*4*pi^2;
        B = (3*x*Q+Q^2)*16*pi^4;
    end

    % Eigenvalues
    lambdas = [ 0                           ;
                0                           ;
                sqrt((-A + sqrt(A^2-4*B))/2);
               -sqrt((-A + sqrt(A^2-4*B))/2);
                sqrt((-A - sqrt(A^2-4*B))/2);
               -sqrt((-A - sqrt(A^2-4*B))/2)];
end
```

```
function [rev, Nut, deltaK] = Nut_History(rev,k,gamma0,I,J)
    %{
      Function:    Nut_History
      Author:      Tomoki Koike
      Description: This function computes the nutation angle history from
                   the ode45 simulation for a rigid body motion.
      >>Inputs
          rev:    number of revolutions
          k:      spin factor
          gamma0: initial perturbation [deg]
          I:      transverse moment of inertia
          J:      axial moment of inertia
      Outputs<<
          rev: revolutions
          Nut: Nutation angle history
          deltaK: Euler Constraint Perturbation
    %}

    % Initial Conditions
    w0 = [0 k 0];
    e0 = [sind(gamma0/2) 0 0 cosd(gamma0/2)];
    y0 = [w0 e0 0];

    % Numerical Integration
    opt = odeset('RelTol', 1e-13, 'AbsTol', 1e-13);
    [rev, data] = ode45(@(v,y) nond_EOM(v,y,I,J,k),rev,y0,opt);

    % Calculating Nutation angle
    Nut = acosd(1-2*data(:,6).^2-2*data(:,4).^2);

    % Euler Constraint
    deltaK = data(:,8);
end
```

```
function dwdt = Tfree_unsymm_EOM(t,w,I)
    %{
      Function:    unsymm_Tfree_EOM
      Author:      Tomoki Koike
      Description: The dynamic differential equation of a torque free
                   motion for a unsymmetric rigid body.
      >>Inputs
          t:  time span to analyze
          w:  angular velocities
          I:  moment of inertia
      Outputs<<
          dwdt: differential w
    %}
    dwdt = zeros([3 1]);
    % Dynamic differential equations
    dwdt(1) = (I(2)-I(3))/I(1)*w(2)*w(3);
    dwdt(2) = (I(3)-I(1))/I(2)*w(1)*w(3);
    dwdt(3) = (I(1)-I(2))/I(3)*w(1)*w(2);
end
```

References

Williams, David R. "Lunar Prospector." *NASA Space Science Data Coordinated Archive*, NASA, 1998, nssdc.gsfc.nasa.gov/nmc/spacecraft/display.action?id=1998-001A.