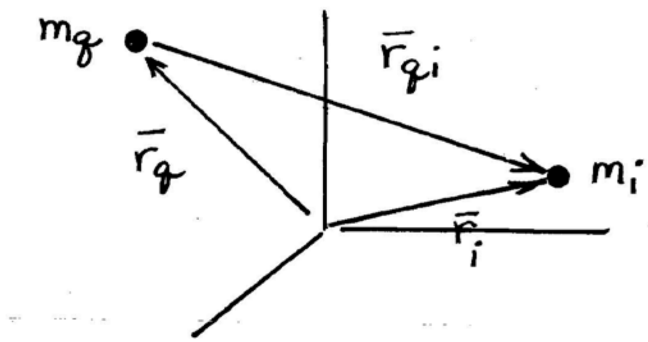
n – Body Problem

Problem: only 10 integrals of motion
12 are required to solve the 2BP

Observation: Do we really care about



We really want to know how it moves relative to some other point such as
⊕ ; ⊙

⟹

Redo the problem:

How to get the EOM governing $\bar{r}_{qi}$? ($\ddot{\bar{r}}_{qi}$)

For any acceleration
still necessary to consider $\Bigg\{$

To apply Newton's Law of Motion MUST differentiate in inertial frame and base point of the position vector must be fixed in that frame

→ CANNOT use $\bar{F} = m\,\bar{A}$ directly with $\bar{r}_{qi}$

But $\bar{r}_{qi}$ can be written <u>in terms of</u> appropriate vectors

$$\bar{r}_i = \bar{r}_{qi} + \bar{r}_q$$
$$\ddot{\bar{r}}_i = \ddot{\bar{r}}_{qi} + \ddot{\bar{r}}_q$$

Apply Newton's law of motion validly to $\bar{r}_i$ and $\bar{r}_q$

$$\ddot{\bar{r}}_i = -G\sum_{\substack{j=1\\ j\neq i}}^{n} \frac{m_j}{r_{ji}^3}\bar{r}_{ji}$$

$$\ddot{\bar{r}}_q = -G\sum_{\substack{j=1\\ j\neq q}}^{n} \frac{m_j}{r_{jq}^3}\bar{r}_{jq}$$

Sub into $\ddot{\bar{r}}_{qi} + \ddot{\bar{r}}_q = \ddot{\bar{r}}_i$

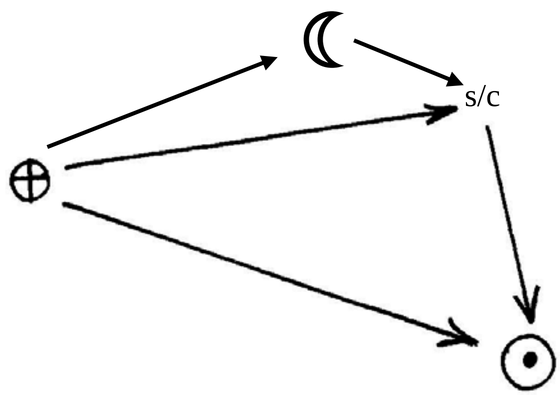
$$\ddot{\bar{r}}_{qi} - \underbrace{G\sum_{\substack{j=1\\ j\neq q}}^{n} \frac{m_j}{r_{jq}^3}\bar{r}_{jq}}_{\text{remove } i \text{ term}} = \underbrace{-G\sum_{\substack{j=1\\ j\neq i}}^{n} \frac{m_j}{r_{ji}^3}\bar{r}_{ji}}_{\text{remove } q \text{ term}}$$

Note: only relative positions appear

$$\ddot{\bar{r}}_{qi} - G\frac{m_i}{r_{iq}^3}\bar{r}_{iq} \underbrace{-G\sum_{\substack{j=1\\ j\neq i,q}}^{n} \frac{m_j}{r_{jq}^3}\bar{r}_{jq}}_{\text{move to right}} = \underbrace{-G\frac{m_q}{r_{qi}^3}\bar{r}_{qi}}_{\text{move to left}} - G\sum_{\substack{j=1\\ j\neq i,q}}^{n} \frac{m_j}{r_{ji}^3}\bar{r}_{ji}$$

Equation for motion of m_i relative to m_q:

Example: $\oplus$ $\odot$ ☾ s/c



How does s/c move relative to $\oplus$?

Motion of mass 2 relative to mass 1; perturbed by masses 3,4

$\therefore\ q =$ $\qquad\qquad$ $i =$ $\qquad\qquad$ $j =$

$$\ddot{\bar{r}}_{12} + \underbrace{G\frac{\left(m_1+m_2\right)}{r_{12}^3}}\,\bar{r}_{12} = G\,m_3\left(\underbrace{\frac{\bar{r}_{23}}{r_{23}^3}} - \underbrace{\frac{\bar{r}_{13}}{r_{13}^3}}\right) + G\,m_4\left(\frac{\bar{r}_{24}}{r_{24}^3} - \frac{\bar{r}_{14}}{r_{14}^3}\right)$$

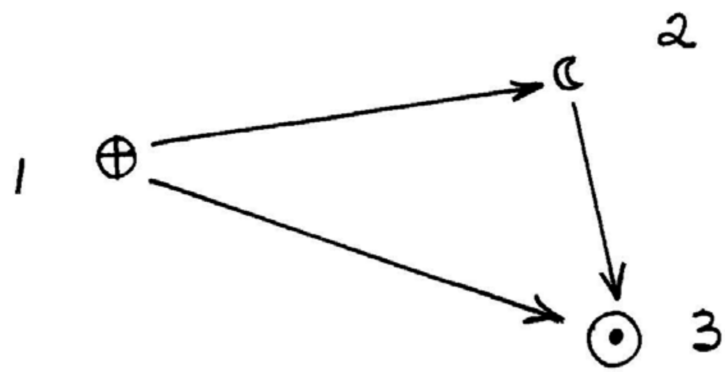
$$\ddot{\bar{r}}_{\oplus s/c} + G\frac{\left(m_\oplus+m_{s/c}\right)}{r_{\oplus s/c}^3}\,\bar{r}_{\oplus s/c} = G\,m_\odot\left(\frac{\bar{r}_{s/c\odot}}{r_{s/c\odot}^3} - \frac{\bar{r}_{\oplus\odot}}{r_{\oplus\odot}^3}\right) + G\,m_{\text{☾}}\left(\frac{\bar{r}_{s/c\text{☾}}}{r_{s/c}^3} - \frac{\bar{r}_{\oplus\text{☾}}}{r_{\oplus\text{☾}}^3}\right)$$

Example: ⊕ ⊙ ☾

How does ☾ move relative to ⊕ ?

Motion of mass 2 relative to mass 1;
perturbed by mass 3

➔ $\ddot{\bar{r}}_{12}$



$\therefore\ q = 1 \qquad i = 2 \qquad j = 3$

$$\ddot{\bar{r}}_{12} \quad + \quad \underbrace{G\frac{(m_1 + m_2)}{r_{12}^3}\bar{r}_{12}} \quad = \quad G\, m_3 \left(\underbrace{\frac{\bar{r}_{23}}{r_{23}^3}} \quad - \quad \underbrace{\frac{\bar{r}_{13}}{r_{13}^3}} \right)$$

$$\ddot{\bar{r}}_{\oplus\leftmoon} \quad + \quad G\frac{(m_\oplus + m_\leftmoon)}{r_{\oplus\leftmoon}^3}\bar{r}_{\oplus\leftmoon} \quad = \quad G\, m_\odot \left(\frac{\bar{r}_{\leftmoon\odot}}{r_{\leftmoon\odot}^3} \quad - \quad \frac{\bar{r}_{\oplus\odot}}{r_{\oplus\odot}^3} \right)$$

Careful – indirect effects frequently forgotten but can be significant!!!

Given the equation of motion, do we now have an equation that we can solve?

⇒ Still can't solve if $n \geq 3$

$n = 3$ **:** requires position of ⊙ relative to ⊕ or ☾ **;** an additional vector EOM is necessary

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to solve two 2nd-order vector DE requires 12 integrals of the motion**;** we only know 10 !!

$n = 2$ **:** no m_j perturbing bodies
a 2nd-order vector DE in only one unknown position vector!

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6 scalar EOMs **;** 6 dependent scalar variables **;** requires only 6 integrals of motion (we have 10 !!)

→ <u>Relative</u> motion of <u>two</u> bodies solvable

$$\ddot{\bar{r}}_{12} + G\frac{(m_1 + m_2)}{r_{12}^3}\bar{r}_{12} = \bar{0}$$ solvable but nontrivial

OR

$$\ddot{\bar{r}} + \frac{\mu}{r^3}\bar{r} = \bar{0}$$ where $\mu = G(m_1 + m_2)$