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Gyrostats

Assume that we want the advantages of spin stabilization without spinning the entire vehicle

Simplest way: add a spinning part to the spacecraft

$\Downarrow$

simplest spinner is an axisymmetric body, i.e., a **<u>rotor</u>**

Fundamental concept associated with most (non-mass expulsive) devices used for stabilization:

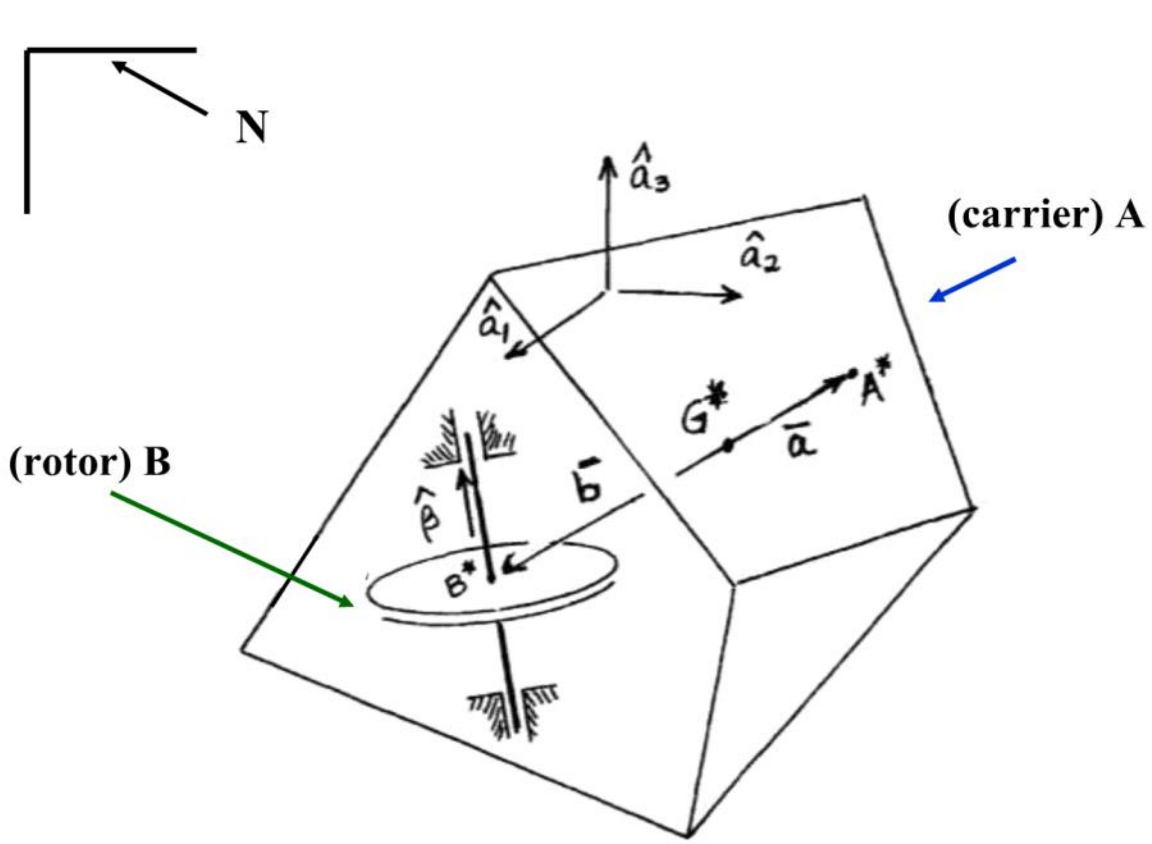
Reaction wheels
Momentum wheels
Control moment gyros

Create a new "system"

- ✓ at least two connected rigid bodies
- ✓ move relative to each other
- ✓ also standard to use 3 rotors (at least one per axis) for three-axis stabilization

Consider the simplest form to illustrate the advantages:

Gyrostat = $\underbrace{\text{rigid body (generally unsymmetric) + axisymmetric rotor}}_{\textbf{set up problem and re-derive EOMS}}$



A rigid body (generally unsymmetric)

B axisymmetric rotor
(assume B^* and axis of rotation fixed in A)

G gyrostat = A + B

$\hat{\beta}$ parallel to rotor axis (fixed in A)

$\hat{a}_i$ parallel to central principal axes of **G**

$$\overline{\overline{I}}^{G/G^*} = I_1 \hat{a}_1 \hat{a}_1 + I_2 \hat{a}_2 \hat{a}_2 + I_3 \hat{a}_3 \hat{a}_3$$

$$\overline{\overline{I}}^{B/B^*} = K\left(\hat{\delta}\hat{\delta} + \hat{\gamma}\hat{\gamma}\right) + J\hat{\beta}\hat{\beta}$$

$$\hat{\beta} = \beta_i \hat{a}_i$$

$${}^N\overline{\omega}^A = \omega_i \hat{a}_i$$

$${}^A\overline{\omega}^B = {}^A\omega^B \ \hat{\beta}$$

Gyrostats -- Equations of Motion

$$\bar{M}^{G^*} = \frac{{}^N d\, {}^N \bar{H}^{G/G^*}}{dt}$$

Kinematics:

$${}^N \bar{H}^{G/G^*} = \bar{\bar{I}}_G \bullet {}^N \bar{\omega}^A + \bar{\bar{I}}^{B/B^*} \bullet {}^A \bar{\omega}^B$$

$$\bar{\bar{I}}_G = I_1 \hat{a}_1 \hat{a}_1 + I_2 \hat{a}_2 \hat{a}_2 + I_3 \hat{a}_3 \hat{a}_3$$

$$\bar{H}_R = I_1 \omega_1 \hat{a}_1 + I_2 \omega_2 \hat{a}_2 + I_3 \omega_3 \hat{a}_3$$

$$\bar{H}_I = J\, {}^A \omega^B \hat{\beta}$$

$$\begin{aligned} {}^N \bar{H}^{G/G^*} = & \left(I_1 \omega_1 + J\, {}^A \omega^B \beta_1 \right) \hat{a}_1 \\ & + \left(I_2 \omega_2 + J\, {}^A \omega^B \beta_2 \right) \hat{a}_2 \\ & + \left(I_3 \omega_3 + J\, {}^A \omega^B \beta_3 \right) \hat{a}_3 \end{aligned}$$

$$\frac{{}^N d\, {}^N \bar{H}^{G/G^*}}{dt} = \frac{{}^A d\, {}^N \bar{H}^{G/G^*}}{dt} + {}^N \bar{\omega}^A \times {}^N \bar{H}^{G/G^*}$$

$$\frac{{}^N d\, {}^N \bar{H}^{G/G^*}}{dt} = \Big[I_1 \dot{\omega}_1 + J\, {}^A\dot{\omega}^B \beta_1 + (I_3 - I_2)\,\omega_2\omega_3 + J\, {}^A\omega^B (\beta_3\omega_2 - \beta_2\omega_3) \Big] \hat{a}_1$$
$$+ \Big[I_2 \dot{\omega}_2 + J\, {}^A\dot{\omega}^B \beta_2 + (I_1 - I_3)\,\omega_2\omega_1 + J\, {}^A\omega^B (\beta_1\omega_3 - \beta_3\omega_1) \Big] \hat{a}_2$$
$$+ \Big[I_3 \dot{\omega}_3 + J\, {}^A\dot{\omega}^B \beta_3 + (I_2 - I_1)\,\omega_1\omega_2 + J\, {}^A\omega^B (\beta_2\omega_1 - \beta_1\omega_2) \Big] \hat{a}_3$$

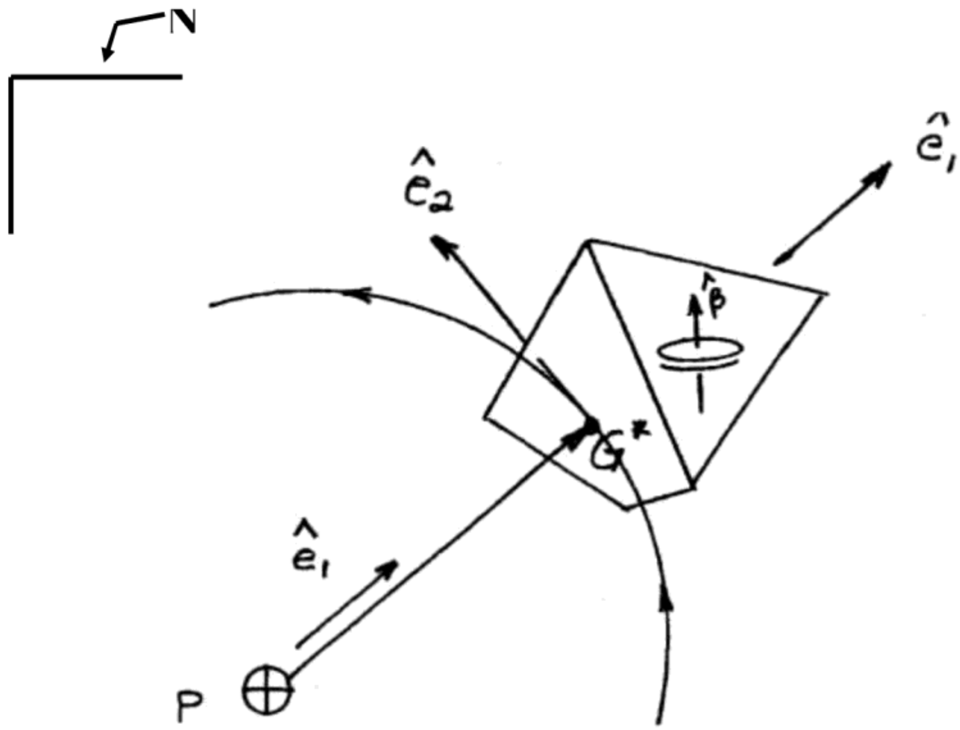
$$\bar{M}^{G^*} = M_i\, \hat{a}_i$$

Place gyrostat in orbit

Dynamic DE

$$\boxed{\begin{aligned}
\dot{\omega}_1 &= K_1\omega_1\omega_3 - \frac{J}{I_1}\Big[{}^A\dot{\omega}^B \beta_1 - {}^A\omega^B (\beta_2\omega_3 - \beta_3\omega_2) \Big] + \frac{M_1}{I_1} \\
\dot{\omega}_2 &= K_2\omega_3\omega_1 - \frac{J}{I_2}\Big[{}^A\dot{\omega}^B \beta_2 - {}^A\omega^B (\beta_3\omega_1 - \beta_1\omega_3) \Big] + \frac{M_2}{I_2} \\
\dot{\omega}_3 &= K_3\omega_1\omega_2 - \frac{J}{I_3}\Big[{}^A\dot{\omega}^B \beta_3 - {}^A\omega^B (\beta_1\omega_2 - \beta_2\omega_1) \Big] + \frac{M_3}{I_3}
\end{aligned}}$$

$$\boxed{K_1 = \frac{I_2 - I_3}{I_1} \qquad K_2 = \frac{I_3 - I_1}{I_2} \qquad K_3 = \frac{I_1 - I_2}{I_3}}$$



Let $\hat{e}_i$ be orbit-fixed unit vectors

Force/Moment Models

$$\bar{F} = -\frac{\mu m}{R^2}\hat{e}_1 \qquad (m = m_G)$$

(assume a circular orbit)

$$\bar{M}^{G^*} = -\frac{3\mu}{R^3}\hat{e}_1 \times \bar{\bar{I}}^{G/G^*} \bullet \hat{e}_1$$

Translational motion does NOT depend on orientation; Rotational motion IS influenced by the orbit

$$\Omega = \sqrt{\frac{\mu}{R^3}}$$ as usual for a circular orbit

Nominal motion: A fixed in E
(want to use spin to help stabilize – assume $\hat{\beta} = \hat{a}_3$)

So $\hat{\beta} = \hat{a}_3$ $\beta_1 = \beta_2 = 0$ $\beta_3 = 1$

This simplifies the gyrostat model

$$\bar{M}^{G^*} = -3\Omega^2 I_1 K_1 C_{12} C_{13} \hat{a}_1 - 3\Omega^2 I_2 K_2 C_{13} C_{11} \hat{a}_2 - 3\Omega^2 I_3 K_3 C_{11} C_{12} \hat{a}_3$$

which $C's$ are these?

$$\boxed{\begin{aligned} \dot{\omega}_1 &= K_1 \omega_1 \omega_3 - \frac{J}{I_1} {}^A\omega^B \omega_2 - 3\Omega^2 K_1 C_{12} C_{13} \\ \dot{\omega}_2 &= K_2 \omega_3 \omega_1 - \frac{J}{I_2} {}^A\omega^B \omega_1 - 3\Omega^2 K_2 C_{13} C_{11} \\ \dot{\omega}_3 &= K_3 \omega_1 \omega_2 - \frac{J}{I_3} {}^A\dot{\omega}^B - 3\Omega^2 K_3 C_{11} C_{12} \end{aligned}}$$

What else do we need?

3 equations, 7 unknowns $\left(\omega_1, \omega_2, \omega_3, {}^A\omega^B, C_{11}, C_{12}, C_{13}\right)$

Collect more differential equations

Kinematic Equations:

orbit $\rightarrow$ ${}^EC^A$ $\leftarrow$ Basis in which $\bar{\bar{I}}$ expressed

$${}^E\dot{C}^A = {}^EC^A\ {}^E\tilde{\omega}^A \qquad {}^E\tilde{\omega}^A = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}$$

$${}^N\bar{\omega}^A = {}^N\bar{\omega}^E + {}^E\bar{\omega}^A$$

$$\Rightarrow \quad {}^E\bar{\omega}^A = \underbrace{{}^N\bar{\omega}^A}_{\omega_i\hat{a}_i} - \underbrace{{}^N\bar{\omega}^E}_{-\Omega\,\hat{e}_3}$$

need ${}^EC^A$

$${}^E\bar{\omega}^A = \underbrace{\left({}^N\omega_1^A - \Omega\,{}^EC_{31}^A\right)}_{{}^A\omega_1^B}\hat{a}_1 + \underbrace{\left({}^N\omega_2^A - \Omega\,{}^EC_{32}^A\right)}_{{}^A\omega_2^B}\hat{a}_2 + \underbrace{\left({}^N\omega_3^A - \Omega\,{}^EC_{33}^A\right)}_{{}^A\omega_3^B}\hat{a}_3$$

Kinematic DE

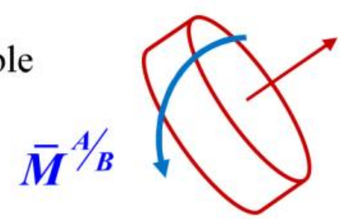
$$\boxed{\begin{aligned}
\dot{C}_{11} &= C_{12}\omega_3 - C_{13}\omega_2 + \Omega\left(C_{13}C_{32} - C_{12}C_{33}\right)\\
\dot{C}_{12} &= C_{13}\omega_1 - C_{11}\omega_3 + \Omega\left(C_{11}C_{33} - C_{13}C_{31}\right)\\
\dot{C}_{13} &= C_{11}\omega_2 - C_{12}\omega_1 + \Omega\left(C_{12}C_{31} - C_{11}C_{32}\right)\\
\dot{C}_{31} &= C_{32}\omega_3 - C_{33}\omega_2\\
\dot{C}_{32} &= C_{33}\omega_1 - C_{31}\omega_3\\
\dot{C}_{33} &= C_{31}\omega_2 - C_{32}\omega_1
\end{aligned}}$$

9 nonlinear, coupled DE for 10 variables
$\left(\omega_1, \omega_2, \omega_3, {}^{A}\omega^{B}, C_{11}, C_{12}, C_{13}, C_{31}, C_{32}, C_{33}\right)$

What to do with ${}^{A}\omega^{B}$?

1. Keep constant
2. Determine diff equation that governs ${}^{A}\omega^{B}$

FBD of rotor will introduce new variable



By assuming ${}^{A}\omega^{B} = {}^{A}\omega_o^{B}$ <u>constant</u>

Now 9 simultaneous equations in 9 unknowns
(rotor speed now an input parameter)

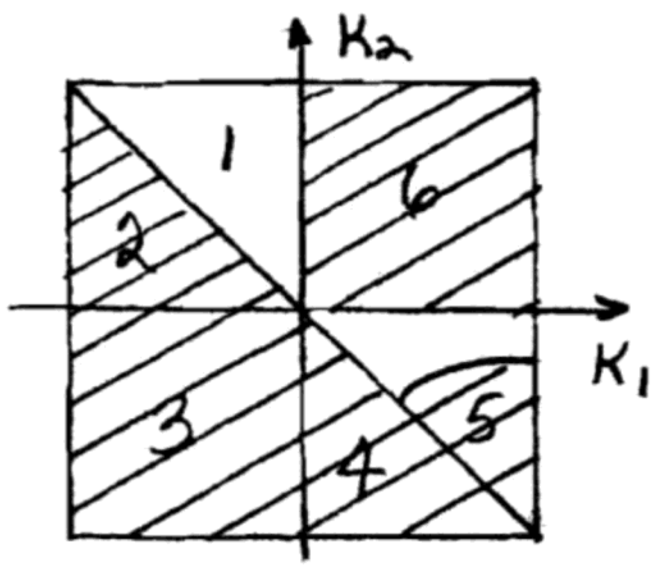
If ${}^{A}\omega^{B} = 0$, reproduces equations for single unsymmetric rigid body

Recall problem:
We had single, unsymmetric (in general) rigid body that we wished to keep fixed in orbit frame
Nominal motion $\longrightarrow$ fixed in E
(only possible if principal axis of carrier aligned with $\hat{e}'s$ orbit axes)

Effect of gravity torque: sometimes helpful /sometimes not helpful
Even so, stability "tenuous"

Did a linear stability analysis help us understand stability characteristics?

Produce instability chart



Much of chart known to be unstable / "unknown" regions may be in trouble, too

How can we help?
How can addition of a rotor be helpful?

Take a case KNOWN to be unstable and stabilize (numerically) with a rotor – can you?