AE 6230 – HW4: Mode Shapes and Responses of 1-D Continuous Systems

Out: November 15, 2022; **Due:** November 23, 2022 by 11:59 PM ET in Canvas

Guidelines

- Read each question carefully before doing any work;
- If you find yourself doing pages of math, pause and consider if there is an easier approach;
- You can consult any relevant materials;
- You can discuss solution approaches with others, but your submission must be your own work;
- If you have doubts, please ask questions in class, during office hours, and/or Piazza (no questions via email);
- The solution to each question should concisely and clearly show the steps;
- Simplify your results as much as possible;
- Box the final answer for each question;
- Submit any code with the solution (but remember to also submit all relevant plots).

Problem 1 – 30 points

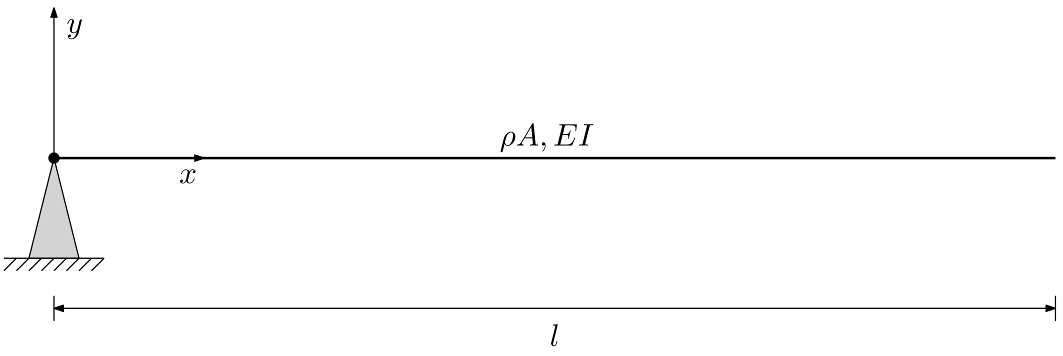

Figure 1: Schematic of a pinned-free uniform beam undergoing out-of-plane bending.

Table 1: Parameter values for Problem 1.

Parameter	Symbol	Value
Length	l	1 m
Bending stiffness	EI	5 Nm^2
Mass per unit length	ρA	0.5 kg/m

Figure 1 shows a simplified model for the anti-symmetric out-of-plane bending vibrations of an aircraft. Using the anti-symmetry condition, the model considers one half wing represented as a pinned-free uniform beam. Considering the parameters in Table 1, answer the following questions:

1. Verify that the characteristic equation is given by

$$\tanh \alpha l - \tan \alpha l = 0 \tag{1}$$

2. Evaluate the first four eigenvalues α_i;

3. Evaluate the natural frequencies ω_i;

4. Determine the analytical expressions of the eigenfunctions $X_i(x)$;

5. Plot the mode shapes $\phi_i(x)$ obtained by normalizing the eigenfunctions to have unit maximum displacement;

6. Verify (mathematically) that there is one rigid-body eigenfunction.

Problem 2 – 15 points

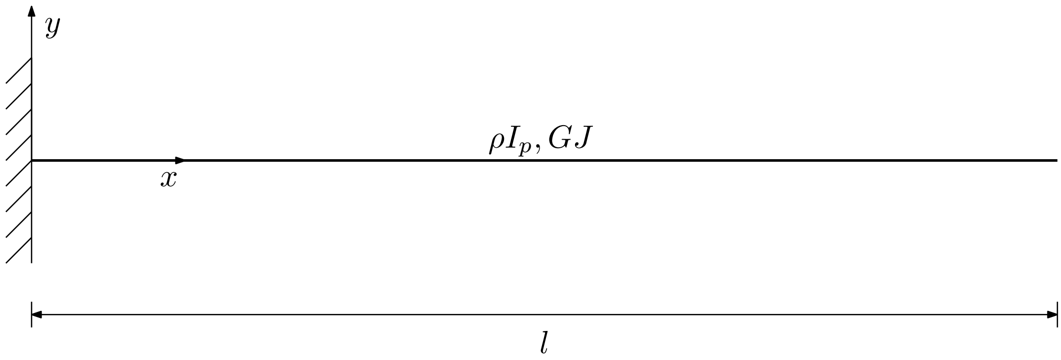

Figure 2: Schematic of a clamped-free uniform beam undergoing torsion.

Figure 2 shows a clamped-free uniform beam undergoing torsion. The beam is subject to the initial conditions

$$\theta(x,0) = \theta_0(x) = \frac{\bar{\theta}x}{l} \qquad \dot{\theta}(x,0) = \dot{\theta}_0(x) = 0 \tag{2}$$

where $\bar{\theta}$ is the tip twist angle at the initial time. Answer the following questions:

1. Determine the analytical expressions of the modal initial conditions $\eta_{0_i}, \dot{\eta}_{0_i}$;
2. Write the undamped free response in the form

$$\theta(x,t) = \sum_{i=1}^{\infty} \phi_i(x)\eta_i(t) \tag{3}$$

   showing the appropriate expressions of the mode shapes, natural frequencies, and modal coordinates;
3. Verify that the response from Question 2 satisfies the initial conditions.

Problem 3 – 30 points

Table 2: Parameter values for Problem 3.

Parameter	Symbol	Value
Length	l	1.0 m
Torsional stiffness	GJ	5.0 Nm^2
Moment of inertia per unit length	ρI_p	0.005 kg·m
Modal viscous damping factor	ζ_i	0.02
Excitation amplitude	r_0	1 N
Excitation frequency	ω_0	125 rad/s

Consider the beam of Problem 2 but now subject to the distributed moment

$$r(x,t) = r_0 \sin \omega_0 t \tag{4}$$

Considering the parameters in Table 2, answer the following question:

1. Determine the analytical expressions of the modal forces $N_i(t)$;
2. Write the damped steady-state response in the form

$$\theta(x,t) = \sum_{i=1}^{\infty} \phi_i(x)\eta_i(t) \tag{5}$$

   showing the appropriate expressions of the modal coordinates;
3. Tabulate the quantities needed to evaluate Eq. (5) considering the first four modes[1];
4. Plot the tip twist angle for $0 \le t \le 0.2$ s considering an increasing number of modes $N = 1, 2, 3, 4$;
5. Explain the trend in the results for increasing N;
6. How should the distributed moment $r(x,t)$ be shaped to avoid exciting the first torsional vibration mode?

[1]MATLAB prinouts are acceptable.

Problem 4 – 25 points

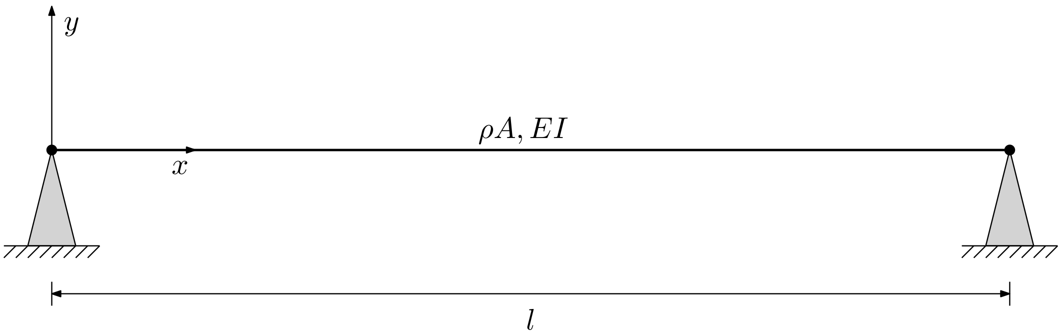

Figure 3: Schematic of a pinned-pinned uniform beam undergoing out-of-plane bending.

Table 3: Parameter values for Problem 4.

Parameter	Symbol	Value
Length	l	1 m
Bending stiffness	EI	50 Nm^2
Mass per unit length	ρA	0.25 kg/m
Excitation amplitude	F_0	1 N
Excitation application point	x_0	$l/2$

Figure 3 shows a pinned-pinned uniform beam in bending. The beam is at rest when it experiences the excitation

$$F(t) = F_0\,\delta(t) \quad \text{at} \quad x = x_0 \tag{6}$$

where F_0 is the amplitude, x_0 the application point, and $\delta(t)$ the unit impulse function. The eigenvalues and mode shapes (normalized to have unit maximum displacement) are the same as for a uniform string

$$\alpha_i = \frac{i\pi}{l} \qquad \phi_i(x) = \sin\left(\frac{i\pi x}{l}\right) \tag{7}$$

Considering the parameters in Table 3, answer the following questions:

1. Determine the analytical expressions of the modal forces $N_i(t)$;
2. Write the undamped forced response in the form

   $$v(x,t) = \sum_{i=1}^{\infty} \phi_i(x)\eta_i(t) \tag{8}$$

   showing the appropriate expressions of the natural frequencies and modal coordinates;
3. Tabulate the quantities needed to evaluate Eq. (7) considering the first eight modes[2];
4. Plot the midpoint displacement for $0 \le t \le 0.1$ s considering an increasing number of modes $N = 1, \ldots, 8$;
5. Explain the trend in the results for increasing N.

[2]MATLAB prinouts are acceptable.