# Problem Set 1 Solution

 $AAE~532:~Orbital~Mechanics\\ MWF:~11:30-12:30\\ Professor~Howell$ 

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#### Problem 1

- (a) Consider the candidate asteroid 101955 Bennu. Go to a NASA website and collect information about Bennu. Information from near Earth observations suggests that Bennu has an interesting shape as shown above. Find its size, dimensions, and density. Compare its size to a structure in your local community. Bennu is a C-type asteroid. What does it mean?
- (b) Bennu's surface is believed to be relatively rough and strewn with boulders. Can you find an image of the surface? Osiris-REx will touch down on the surface to collect a sample. Describe that plan in a short paragraph. Note the one of the concepts is reflected in this image. [For any image, please include the website and use only vetted sites. Wikipedia is not such a site.] What date is Osiris-REx scheduled to collect the sample?
- (c) Take a look at the orbit of 101955 Bennu. This site is the JPL Small-Body Database Browser. We entered '101955 Bennu' in the Search box; it switches to the page with the 'orbit diagram' and data. There is a link on the left that says [Show Orbit Diagram] that you can click if the orbit is not already in the view. Use the 'Play' button to observe the orbit evolution. There is a menu in the upper left to adjust the view. Use 'Default' for a standard view. You can also click 'Settings' to add additional planets to the view, for example. You can use the mouse buttons to zoom in-and-out as well as rotate for a 3D view. Note that the asteroid orbit is in two different colors; where the colors change indicates the location at which the asteroid crosses the orbital plane of the Earth about the Sun, i.e., the ecliptic plane. Locate each of the bodies in their heliocentric orbits at the dates specified below by clicking on the clock in the upper right and clicking a calendar 'Date'. The date in the bottom left of the diagram will change to reflect this selection.
  - (i) Save the image on the date that corresponds to YOUR birthday in 2021 in two different views: (1) one view straight down onto the ecliptic plane to assess the locations of some of the planets on the same date; (2) a view that is edge-on to the Earth's orbit to highlight the out-of-plane nature of the orbit.
  - (ii) On the applet, the distance of the asteroid relative to the Earth and the Sun are given in the bottom left in AU (1 AU = 1 Sun-Earth distance). On this date, how far is Bennu from the Earth? Is this distance to the Earth considered small?
  - (iii) Does the heliocentric orbit of Bennu cross the orbits of any of the planets? Does Bennu have a significant out-of-plane component relative to the Earth orbital plane? Where is the Sun relative to Bennu and the Earth?
  - (iv) Using the tables below the Java applet, find the inclination and period of Bennu. Note that the inclination is reported relative to the ecliptic plane. How do these orbital parameters compare to those of the Earth in its orbit about the Sun?
  - (v) Advance the date to August 1, 2023 using either the "Date" selection box or the time stepping arrows and step size options. Step through each day in the months of August and September 2023 using the time stepping arrows and determine the date on which Bennu is closest to the Earth during these two months. What is this date? What is the smallest distance between Earth and Bennu in AU? In km?

#### (c) Contunued

- (vi) During August/September 2023, does Bennu pass within the Moon's orbit about the Earth? (1 Earth-Moon distance 384,400 km) How do you know?
- (vii) On the date of Bennu's closest Earth approach, save two images using the same view as in part (i). Is Bennu significantly above, significantly below or close to the Earth's orbital plane on this date?
- (viii) Imagine that you were a mission designer working on the Osiris-REx mission concept several years ago and had to select an asteroid approach date for the robotic rendezvous to eventually collect the sample. The arrival phase started in April 2018 with 'rendezvous' in December 2018. View the relative positions of Earth and Bennu over those months. Based on the knowledge you have gained during this exercise about the orbit of Bennu, why might you have selected December 2018 for the approach date? How does it compare with the arrival video in the problem introduction? Could the spacecraft have arrived in November 2018?

#### **Problem 1 Solution**

(a)

Source: https://solarsystem.nasa.gov/asteroids-comets-and-meteors/asteroids/101955-bennu/overview/, https://solarstory.net/asteroids

Bennu's equatorial radius is 0.25km, which makes it 25484.0x smaller than Earth. Its volume is  $0.0623km^3$ . The density is  $1.26g/cm^3$ , while Earth has  $5.513g/cm^3$  as the density. C-type asteroids are of the most common type and have characteristics like low albedo due to their mostly carbon and mineral composition. These are chondrite asteroids consist mostly of clay and silicate rocks.

(b)

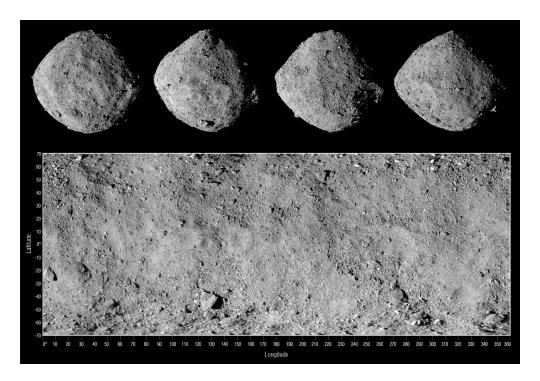
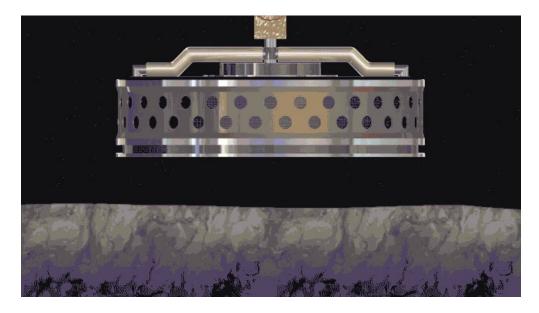


Figure 1: Surface of Bennu, looked from four different sides, Source: https://solarsystem.nasa.gov/asteroids-comets-and-meteors/asteroids/101955-bennu/galleries

OSIRIS-REx is equipped with TAGSAM (Touch-and-Go-Sample-Acquisition-Mechanism) instrument to collect a sample of regolith. At the end of the instrument is a round sampler head, which will be pushed against the surface of Bennu using the momentum of OSIRIS-REx. At the brief contact (5 seconds), nitrogen gas will blow onto the surface to roil up dust and small pebbles, which will be captured inside the TAGSAM head. The spacecraft has three nitrogen gas canisters on boards, allowing for three sampling attempts. The sample collection date is scheduled on October 20th, 2020. Source: http://www.spaceref.com/news/viewsr.html?pid=53681



 $\label{eq:source:figure} Figure \ 2: \quad TAGSAM \quad instrument \quad Source: \quad \ \ https://www.asteroidmission.org/asteroid-operations/$ 

(c)-(i)

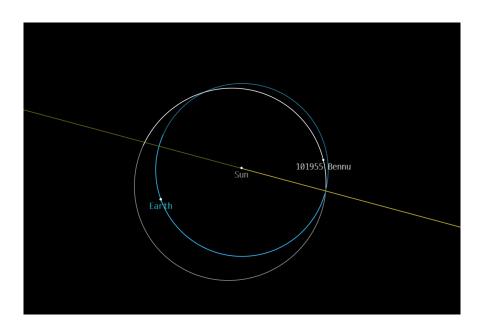


Figure 3: Bennu on 04/26/2021 from above

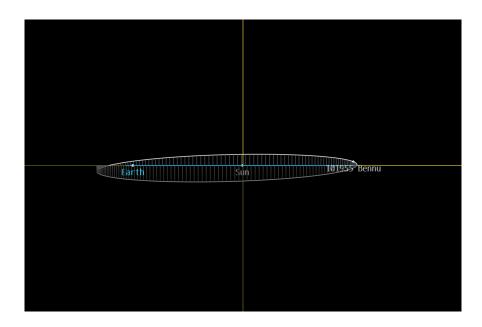


Figure 4: Bennu on 04/26/2021 edge-on view

(c)-(ii)

On 04/26/2021, the distance between Earth and Bennu is 1.938AU, or roughly 2.9e+8 km. This is relatively a big distance, since Earth and Bennu appear to be at the opposite sides when looked from above. For the following problems we will see much shorter distances than this.

(c)-(iii)

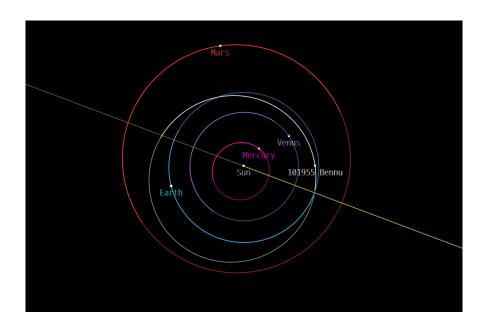


Figure 5: Bennu with other planets: above view

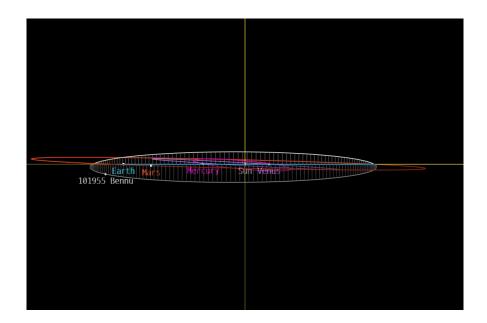


Figure 6: Bennu with other planets edge-on view

If we define "crossing" to have an intersection when looking at the ecliptic plane from above, Bennu only crosses the orbit of the Earth. Note however, that when these intersections happen, there is a difference in the out-of-plane component. Otherwise it would mean that Bennu collides with the Earth! When looking from the edge-on view, we can see that Bennu has significantly larger out-of-plane component compared to Mars. So we can consider it to have a significant out-of-plane component. Although all the bodies in the figure, including the Sun, planets and Bennu are

exerting gravitational force on each other, the biggest force on Earth and Bennu is from the Sun. As a result we have an elliptic orbits for both Earth and Bennu, and the Sun is located at one of th foci of these elliptic orbits.

## (c)-(iv)

• Inclination:  $6.0349^{\circ}$ 

• Period: 436.65 days

We know that the orbital period of the Earth is about 365.25 days, so Bennu takes about 71.4days more to complete one revolution around the Sun. The inclination is measured with respect to Earth's ecliptic plane at J2000. Although Earth's orbit also change over time, we can approximately say that Bennu's orbit is inclined by 6.0349° with respect to Earth's ecliptic plane, as of today.

#### (c)-(v)

From 08/25 to 08/30, The distance between the Earth and Bennu is the closest. The distance is approximately 0.456AU, or 6.8240e+7 km.

#### (c)-(vi)

Since the minimum distance is about 6.8240e + 7km > 384,400km, Bennu does not pass within the Moon's orbit radius.

## (c)-(vii)

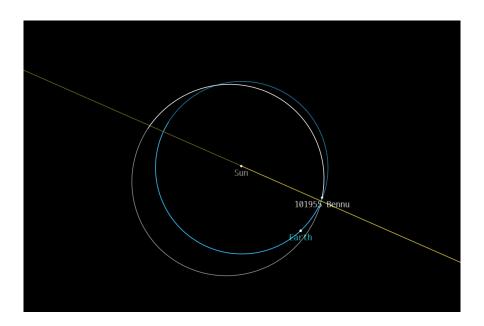


Figure 7: Bennu on 08/30/2023: above view

From the edge-on view, we can see that Bennu is really close to the Earth's orbital plane on the date(08/30/2023).

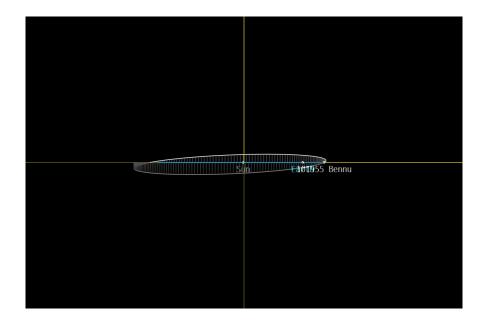


Figure 8: Bennu on 08/30/2023: edge-on view

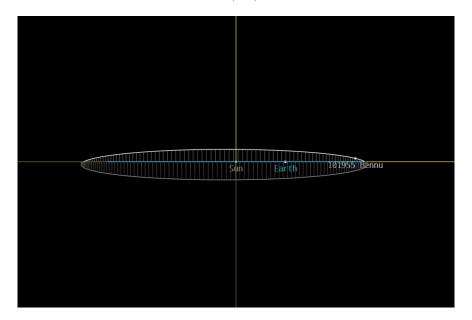


Figure 9: Bennu in December 2018

#### (c)-(viii)

We can see that November - December 2018 is when Bennu is close to the Earth orbit plane (ecliptic plane). The video shows that the spacecraft had Bennu in sight from the onboard camera and executed maneuvers to orbit around Bennu. The spacecraft probably could have arrived in November 2018 by executing more maneuvers for an earlier rendezvous, but it would have resulted in more propellent usage. Also note that OSIRIS-REx used Earth's gravity to perform the necessary inclination change, and Bennu was near the Earth's orbit plane at the time of the flyby of the spacecraft (September 2017).

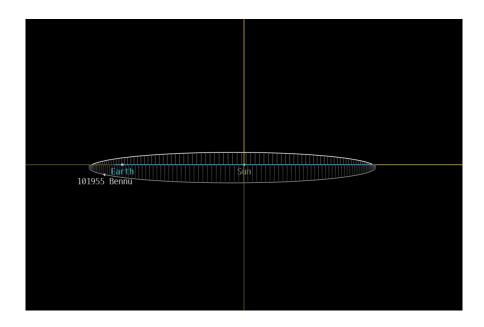


Figure 10: Bennu in April 2018

#### Problem 2

In 2006, the word 'planet' was officially given an updated scientific definition by the International Astronautical Federation; textbooks were quickly updated, and many objects were re-evaluated. Thus, the category 'dwarf planet' was also introduced. Following the new definitions, NASA relatively quickly acknowledged the first 5 dwarf planets and dwarf planet number 6 is currently on the 'watch list'. However, it is generally agreed that there may be 100 to 2000 objects that fit the definition.

- (a) Go to the NASA Solar System Exploration website and obtain the definition of a dwarf planet. Identify the five that are now generally accepted as dwarf planets plus on that might be on the watch list. Include an interesting fact about each one.
- (b) Using the website from Problem 1, just type the name of a dwarf planet in the 'Search' box. Then grab one image of the orbit for each of the first 5 dwarf planets. Use an image that highlights their out-of-plane motion. Find their inclinations and periods.
- (c) To reduce the propulsive requirement for a spacecraft, it is most efficient in terms of propellant to approach a dwarf planet when it is close to the ecliptic plane. There are currently some early proposals for a mission to Haumea. Haumea is a triple system with two moons, Hi'iaka and Namaka, so it is intriguing. Consider an edge-on view of the orbit of Haumea. Given its period, when is the earliest month/year that a mission to Haumea can arrive at the dwarf planet as it crosses the ecliptic plane? How long do we have to wait from today? Print the image for that arrival condition.
- (d) The spacecraft Dawn encountered Ceres during its recent mission. What date did Dawn arrive in the vicinity of Ceres? What date did it depart? Produce an image of the location of Ceres during Dawn's encounter. Are the encounter. Are the encounter conditions similar to those of Osiris-REx arriving at Bennu? Differences? Similarities?
- (e) Besides distance to Earth, also consider the line-of-sight(LOS) to Earth during the Bennu and Ceres arrival phases. Why might LOS be an important trajectory design condition? During the Bennu sample collection, is LOS to Earth 'clear'? What might impede LOS to Earth? When is LOS a challenge?

#### Problem 2 Solution

(a)

A dwarf planet is a planetary-mass object that is in direct orbit around Sun and is massive enough to compress itself into spheroid, but has not cleared the neighborhood of other materials(e.g. asteroids) around its orbit. Source: https://solarsystem.nasa.gov/.

- Pluto: Pluto's largest moon, Charon, is half the size of Pluto. This makes Charon the largest satellite relative to the planet in our solar system.
- Ceres: Ceres is the only dwarf planet located in the inner solar system. It comprises 25 percent of the asteroid belt, while this only makes it 1/14 of Pluto's total mass.
- Haumea: It only takes 4 hours for Haumea to complete its one rotation. It is one of the fastest rotating large objects in our solar system.
- Eris: Eris is about the same size as Pluto, which triggered a debate in scientific community that eventually led Pluto to be classified as dwarf planets.
- Makemake: Makemake is the second brightest object in Kuiper Belt (Pluto is the brightest.) It, along with Eris, triggered to reconsider the definition of a planet.
- 50000 Quaoar (on the watchlist): It is about half the size of Pluto. Signs of crystalline ice was found on it in 2004.

(b)

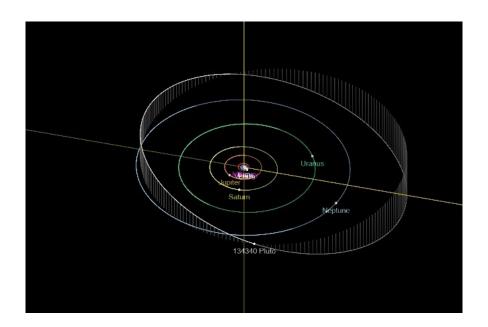


Figure 11: Pluto

- Pluto: 17.089°, 247.74 years (90487 days)
- Ceres: 10.594°, 4.60 years (1681 days)
- Haumea: 28.206°, 285.39 years (104240 days)
- Eris: 44.199°, 556.60 years (203297 days)
- $\bullet$  Makemake: 28.983°, 308.69 years (112750 days)

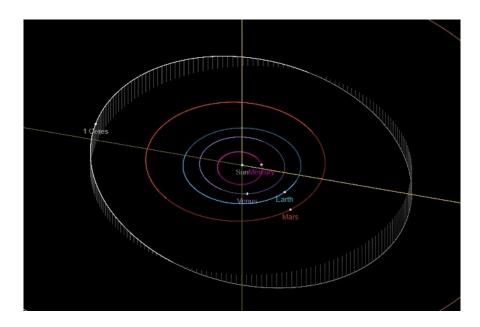


Figure 12: Ceres

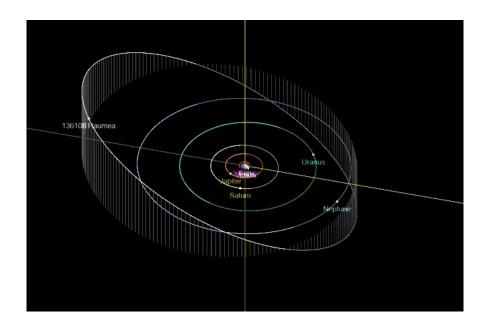


Figure 13: Haumea

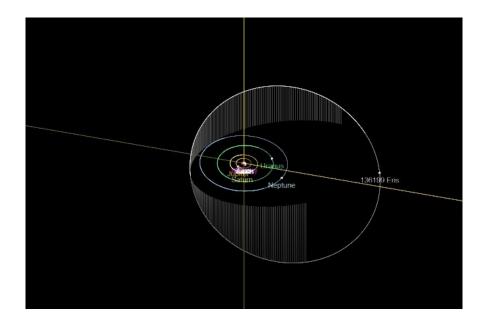


Figure 14: Eris

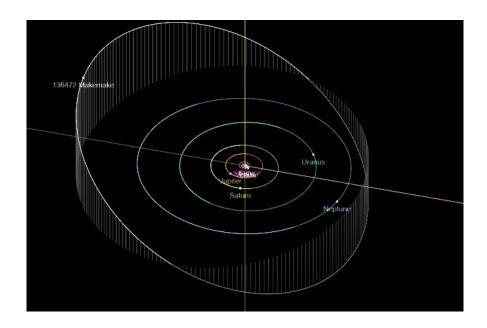


Figure 15: Makemake

(c)

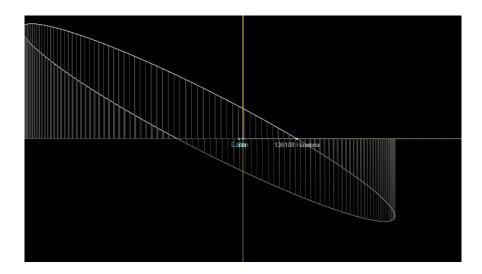


Figure 16: Haumea in May 2100: edge-on view

Haumea crosses the ecliptic plane in May 2100, we have to wait for 80 years for the configuration to happen!

(d)

Dawn arrived in the vicinity of Ceres on March 6th, 2015. Source: https://solarsystem.nasa.gov/missions/dawn/overview/ It did not leave Ceres, it stayed there after exhausting its fuel. As can be seen from the above figure, Ceres is close to the Earth ecliptic orbit at the encounter, which is similar to Bennu. The difference is that OSIRIS-REx's encounter with Bennu happened at the vicinity periapse of Bennu's orbit, while for Dawn's encounter with Ceres this was not the case.

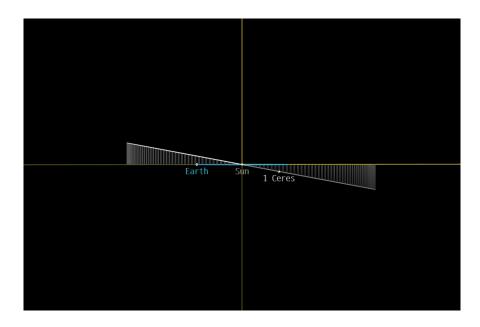


Figure 17: Ceres location at Dawn's encounter

(e)

Line-of-sight to Earth is needed for the data transmission, back and forth. Spacecraft should be able to take commands from the Earth, and it should also be able to transmit whatever data that it collected. For example, data from the navigation system, camera, system maintenance are all required to properly operate a spacecraft and thus securing the LOS is critical. For a spacecraft that orbits another body, the body will periodically block the LOS to the Earth. Also, in order to communicate, the antenna of the spacecraft should beface the Earth. This may be challenging when the spacecraft should be facing a certain direction in space. For example operating the solar panels to the maximum capacity, thrusting in a specific direction and taking a photo all put restrictions on the orientation of the spacecraft, making it challenging to secure the LOS to the Earth. Outside the orientation, the solar noise can hinder the communication when the Sun is within a few degrees from the LOS. In this aspect, Bennu has a 'clear' LOS during the sample collection as the Sun does not lie on the LOS to the Earth.