AAE 334: Aerodynamics

Homework 3: Flat Plate Theory and Effects of Flaps and Slats

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1. [15 pts] Starting from the definition of the moment about the leading edge:

$$M'_{LE} = -\rho_\infty V_\infty \int_0^c \xi\, \gamma(\xi) d\xi$$

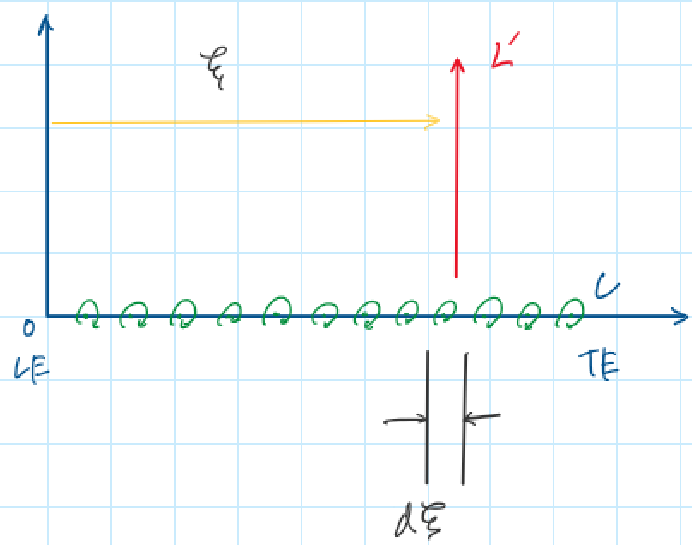
and the series solution for the circulation density:

$$\gamma(\theta) = 2V_\infty \left(A_0 \frac{1+\cos\theta}{\sin\theta} + \sum_{n=1}^{\infty} A_n \sin n\theta \right)$$

derive the equation for the moment coefficient about the leading edge:

$$c_{m,LE} = -\frac{\pi}{2}\left(A_0 + A_1 - \frac{A_2}{2}\right)$$

Recall that our standard change of variables is $\xi = \frac{c}{2}(1-\cos\theta)$.



from the Kutta-Jhukovsky law

$$dL' = \rho_\infty V_\infty \Gamma' = \rho_\infty V_\infty \gamma\, d\xi$$

then

$$M'_{LE} = -\int_0^c \xi\, dL' = -\rho_\infty V_\infty \int_0^c \xi\, \gamma(\xi)\, d\xi$$

plug in the substitutions

$$\begin{cases} \xi = \frac{c}{2}(1-\cos\theta) \implies d\xi = \frac{c}{2}\sin\theta\, d\theta \\ \gamma(\theta) = 2V_\infty\left(A_0 \frac{1+\cos\theta}{\sin\theta} + \sum_{n=1}^{\infty} A_n \sin n\theta\right) \end{cases}$$

$$M'_{LE} = -\rho_\infty V_\infty \int_0^c \frac{c}{2}(1-\cos\theta)\left[2V_\infty\left(A_0\frac{1+\cos\theta}{\sin\theta} + \sum_{n=1}^{\infty} A_n \sin n\theta\right)\right] d\xi$$

$$= -\rho_\infty V_\infty^2 c \int_0^\pi \left[A_0 \frac{(1-\cos^2\theta)}{\sin\theta} + (1-\cos\theta)\sum_{n=1}^{\infty} A_n \sin n\theta\right] \frac{c}{2}\sin\theta\, d\theta$$

$$= -\frac{\rho_\infty V_\infty^2 c^2}{2}\int_0^\pi \left[A_0 \sin\theta + (1-\cos\theta)\sum_{n=1}^{\infty} A_n \sin n\theta\right]\sin\theta\, d\theta$$

$$= -\frac{\rho_\infty V_\infty^2 c^2}{2}\int_0^\pi \left[A_0 \sin^2\theta + \sin\theta(1-\cos\theta)\sum_{n=1}^{\infty} A_n \sin n\theta\right] d\theta$$

$$= -\frac{\rho_\infty V_\infty^2 c^2}{2} A_0 \int_0^\pi \sin^2\theta\, d\theta - \frac{\rho_\infty V_\infty^2 c^2}{2}\sum_{n=1}^{\infty} A_n \int_0^\pi \sin\theta(1-\cos\theta)\sin n\theta\, d\theta$$

$$= -\frac{\rho_\infty V_\infty^2 c^2}{2}\left[A_0 \int_0^\pi \sin^2\theta\, d\theta + \sum_{n=1}^{\infty} A_n \int_0^\pi \sin\theta \sin n\theta\, d\theta - \sum_{n=1}^{\infty} A_n \int_0^\pi \sin\theta\cos\theta\sin n\theta\, d\theta\right]$$

$$= A_0 \int_0^{\pi} \sin^2\theta \, d\theta$$

$$= A_0 \int_0^{\pi} \frac{1}{2}(1 - \cos 2\theta)\, d\theta = \frac{A_0}{2}\left[\theta - \frac{1}{2}\sin 2\theta\right]_0^{\pi}$$

$$= \frac{A_0}{2}\pi$$

$$= \sum_{n=1}^{\infty} A_n \int_0^{\pi} \sin\theta \sin n\theta \, d\theta$$

here

$$\int_0^{\pi} \sin k\theta \sin m\theta = \begin{cases} \frac{\pi}{2} & k = m \\ 0 & k \neq m \end{cases}$$

$$= A_1 \cdot \frac{\pi}{2} = \frac{A_1}{2}\pi$$

this relation

$$= -\sum_{n=1}^{\infty} A_n \int_0^{\pi} \sin\theta \cos\theta \sin n\theta \, d\theta$$

$$= -\sum_{n=1}^{\infty} A_n \int_0^{\pi} \frac{1}{2} \sin 2\theta \sin n\theta \, d\theta$$

$$= -A_2 \int_0^{\pi} \frac{1}{2} \sin 2\theta \sin 2\theta \, d\theta$$

$$= -\frac{A_2}{4}\pi$$

Thus,

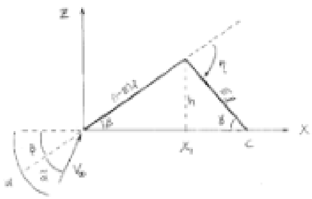
$$M'_{LE} = -\frac{\rho_\infty V_\infty^2 c^2}{2}\left(\frac{A_0}{2}\pi + \frac{A_1}{2}\pi - \frac{A_2}{4}\pi\right)$$

since

$$C_{m,LE} = \frac{M'_{LE}}{\frac{1}{2}\rho_\infty V_\infty^2 c^2}$$

$$\therefore \boxed{C_{m,LE} = -\frac{\pi}{2}\left(A_0 + A_1 - \frac{A_2}{2}\right)}$$

2. [35 pts] For the example in the class notes on thin airfoil theory applied to a flat plate airfoil with a flap, do the following:



(a) Determine the moment coefficient about the aerodynamic center, c_{mac}, as a function of the flap deflection angle, η, and the flap-chord ratio, E. As the flap is deflected downward, does the moment created about the aerodynamic center tend to pitch the airfoil up or down?

(a) first define $z(x)$

$$z(x) = \begin{cases} \frac{h}{x_1}x & x \in [0, x_1] \\ -\frac{h}{c-x_1}x + \frac{hc}{c-x_1} & x \in [x_1, c] \end{cases}$$

the

$$\frac{dz}{dx} = \begin{cases} \frac{h}{x_1} & x \in [0, x_1] \iff \theta \in [0, \theta_1] \\ -\frac{h}{c-x_1} & x \in [x_1, c] \iff \theta \in [\theta_1, \pi] \end{cases}$$

then because the eqn. for $C_{mac} = C_{m,c/4}$ is

$$\boxed{C_{mac} = -\frac{\pi}{4}(A_1 - A_2)}$$

we will find A_1 & A_2

$$A_1 = \frac{2}{\pi}\int_0^\pi \frac{dz}{dx}\cos\theta \, d\theta$$

$$= \frac{2}{\pi}\int_0^{\theta_1} \frac{h}{x_1}\cos\theta \, d\theta - \frac{2}{\pi}\int_{\theta_1}^{\pi} \frac{h}{c-x_1}\cos\theta \, d\theta$$

$$= \frac{2h}{\pi x_1}\left[\sin\theta\right]_0^{\theta_1} - \frac{2h}{\pi(c-x_1)}\left[\sin\theta\right]_{\theta_1}^{\pi}$$

$$= \frac{2h}{\pi x_1}\sin\theta_1 + \frac{2h}{\pi(c-x_1)}\sin\theta_1 = \frac{2h}{\pi}\sin\theta_1\left(\frac{1}{x_1} + \frac{1}{c-x_1}\right)$$

$$A_2 = \frac{2}{\pi}\int_0^{\pi} \frac{dz}{dx}\cos 2\theta\, d\theta$$

$$= \frac{2}{\pi}\int_0^{\theta_1} \frac{h}{x_1}\cos 2\theta\, d\theta - \frac{2}{\pi}\int_{\theta_1}^{\pi} \frac{h}{c-x_1}\cos 2\theta\, d\theta$$

$$= \frac{2h}{\pi x_1}\left[\frac{1}{2}\sin 2\theta\right]_0^{\theta_1} - \frac{2h}{\pi(c-x_1)}\left[\frac{1}{2}\sin 2\theta\right]_{\theta_1}^{\pi}$$

$$= \frac{h}{\pi x_1}\sin 2\theta_1 + \frac{h}{\pi(c-x_1)}\sin 2\theta_1 = \frac{h}{\pi}\sin 2\theta_1\left(\frac{1}{x_1} + \frac{1}{c-x_1}\right)$$

then

$$C_{m_{ac}} = -\frac{\pi}{4}\left[\frac{2h}{\pi}\sin\theta_1\left(\frac{1}{x_1} + \frac{1}{c-x_1}\right) - \frac{h}{\pi}\sin 2\theta_1\left(\frac{1}{x_1} + \frac{1}{c-x_1}\right)\right]$$

$$= -\frac{h}{4}\left(\frac{1}{x_1} + \frac{1}{c-x_1}\right)\left(2\sin\theta_1 - \sin 2\theta_1\right) = -\frac{h}{2}\left(\frac{1}{x_1} + \frac{1}{c-x_1}\right)\cdot \sin\theta_1\left(1-\cos\theta_1\right)$$

now we want to find θ_1 from the equ.

from flat plate assumption

$c \simeq l$

$x_1 \approx (1-E)c \Leftrightarrow \frac{x_1}{c} \approx 1-E$

$c - x_1 \approx Ec$

$$x = \frac{c}{2}(1-\cos\theta) \qquad \text{manipulate this}$$

$$\cos\theta_1 = 1 - 2\frac{x_1}{c} \simeq 1 - 2(1-E) = 2E - 1$$

then

$$\sin\theta_1 = \sqrt{1-\cos^2\theta_1} = \sqrt{1-(2E-1)^2}$$

$$= \sqrt{1-4E^2+4E-1} = 2\sqrt{E(1-E)}$$

$$\left.\begin{aligned} \tan\beta &\approx \frac{h}{x_1} \approx \beta \\ \tan\gamma &\approx \frac{h}{c-x_1} \approx \gamma \end{aligned}\right\} \text{small angle assumption}$$

now since $\eta = \beta + \gamma$

$$C_{mac} = -\frac{1}{2}\left(\frac{h}{x_1} + \frac{h}{c-x_1}\right)\sin\theta_1\left(1-\cos\theta_1\right)$$

$$= -\frac{1}{\cancel{2}}\left(\beta + \gamma\right)\cancel{2}\sqrt{E(1-E)}\left(1-(2E-1)\right)$$

$$= -\eta\sqrt{E(1-E)}\left(2-2E\right)$$

$$= -2\eta\sqrt{E(1-E)}\left(1-E\right)$$

$$\boxed{C_{mac} = -2\eta(1-E)\sqrt{E(1-E)}}$$

(b) Plot c_{mac}/η as a function of flap-chord ratio, E.

(c) Determine the value of E for which the magnitude $|c_{mac}/\eta|$ is a maximum. Typical flaps have $E = 0.20$. Is this value near where the maximum occurs? For an airplane taking off, flaps are needed to generate a large lift force at low velocity. Does the moment generated make it easy or difficult for the airplane to take off? (Hint: review your answer to part (a).) (This question is not for credit: can you think of a reason unrelated to aerodynamics for why a typical wing flap would have E = 0.20? Consider how a wing is constructed.)

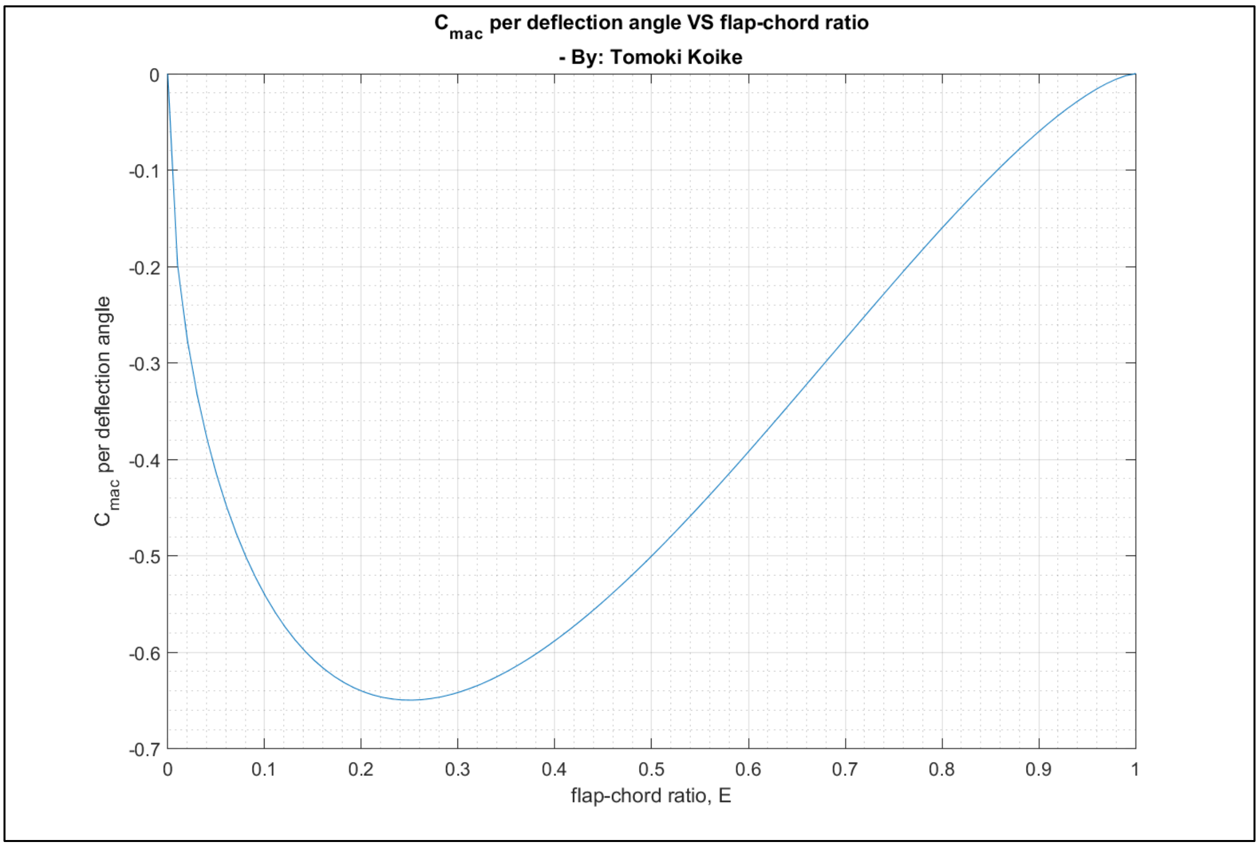
(d) [part (d) is no longer required. The ideal angle of attack is found by setting the Fourier coefficient $A_0 = 0$. For the geometry given, this results in

$$\alpha_{ideal} = -\eta\left(1 - E - \frac{\theta_1}{\pi}\right)$$

where $\theta_1 = \cos^{-1}(2E - 1)$. The ideal angle of attack relative to the main element of the airfoil is $\tilde{\alpha}_{ideal} = \alpha_{ideal} - \beta \approx \alpha_{ideal} - E\eta$. For E = 0.20 and $\eta = 20^{\circ} = 0.34907$ radians, $\alpha_{ideal} = -0.03322$ radians $= -1.90^{\circ}$ and $\tilde{\alpha}_{ideal} = -0.10303\ radians = -5.9^{\circ}$.]

~~For $E = 0.20$ and $\eta = 20^{\circ}$, determine the ideal (design) angle of attack and determine the lift coefficient at the ideal angle of attack.~~

(b)



(C) the equation is

$$\Phi = \frac{C_{mac}}{\eta} = -2(1-E)\sqrt{E(1-E)} = -2\left[E(1-E)^3\right]^{1/2}$$

$$= -2\left[E(1-E^3-3E+3E^2)\right]^{1/2} = -2\left[E-3E^2+3E^3-E^4\right]^{1/2}$$

Take the derivative

$$\frac{d\Phi}{dE} = -2\cdot\frac{1}{2}\left(E-3E^2+3E^3-E^4\right)^{-1/2}\left(1-6E+9E^2-4E^3\right)$$

$$= -\frac{1-6E+9E^2-4E^3}{\sqrt{E-3E^2+3E^3-E^4}}$$

$\because$ denominator $\neq 0$ only consider numerator

$$\frac{d\Phi}{dE} = 0 \iff 1-6E+9E^2-4E^3 = 0$$

$$E = 1,\ 0.25$$

$$\boxed{E = 0.25}$$

this is 25% difference from $E_{Typical} = 0.20$
this is somewhat close

how does moment make it easier to lift-off

$\boxed{\text{from the answers of (a) \& (b) we can tell that } C_m \text{ is negative making the plane to nose-down thus, makes it difficult to take off.}}$

(e) At the ideal angle of attack found in part (d), determine the vortex sheet strength distribution, $\gamma(x)/V_\infty$, and plot it. Do this by writing a Matlab program (or use another language if you wish, e.g., Python). Include enough Fourier coefficients in your computation that the results are accurate (i.e., if you use more Fourier coefficients, you do not make a significant difference in the results). Turn in the formulation of how $\gamma(x)/V_\infty$ is computed, a listing of you program, and the plot of the vortex sheet strength. Tell how many Fourier coefficients you used. The vortex sheet strength gives a picture of the local contribution to the lift. Discuss how the lift distribution along the chord of the airfoil is related to the geometry of the airfoil.

$$\alpha_{ideal} = -\eta\left(1 - E - \frac{\theta_1}{\pi}\right)$$

$$= -0.03322 = -1.90^\circ$$

from problem 1

$$\gamma(\theta) = 2V_\infty\left(A_0\frac{1+\cos\theta}{\sin\theta} + \sum_{n=1}^{\infty} A_n \sin n\theta\right)$$

$$\frac{\gamma(\theta)}{V_\infty} = 2A_0\frac{1+\cos\theta}{\sin\theta} + 2\sum_{n=1}^{\infty} A_n \sin n\theta$$

@ ideal α $\quad A_0 \to 0$

$$\frac{\gamma(\theta)}{V_\infty} = 2\sum_{n=1}^{\infty} A_n \sin n\theta$$

V_∞ $\quad n=1$

here

$$A_n = \frac{2}{\pi}\int_0^{\pi} \frac{\partial z}{\partial x}\cos n\theta \, d\theta$$

$$= \frac{2}{\pi}\int_0^{\theta_1} \frac{h}{x_1}\cos n\theta \, d\theta - \frac{2}{\pi}\int_{\theta}^{\pi} \frac{h}{c-x_1}\cos n d\theta$$

$$= \frac{2h}{\pi x_1}\left[\frac{1}{n}\sin n\theta\right]_0^{\theta_1} - \frac{2h}{\pi(c-x_1)}\left[\frac{1}{n}\sin n\theta\right]_{\theta_1}^{\pi}$$

$$= \frac{2}{n\pi}\,\frac{h}{x_1}\,\sin n\theta_1 + \frac{2}{n\pi}\,\frac{h}{c-x_1}\sin n\theta_1$$

$$= \frac{2}{n\pi}\sin n\theta_1\left(\frac{h}{x_1} + \frac{h}{c-x_1}\right)$$

$$= \frac{2}{n\pi}\eta \sin n\theta_1$$

$$\because \cos\theta_1 = 2E - 1 \iff \theta_1 = \arccos(2E-1)$$

$$\longrightarrow \quad A_n = \frac{2}{\pi}\eta\,\frac{\sin[n\arccos(2E-1)]}{n}$$

then

$$\frac{\gamma(\theta)}{V_\infty} = 2\sum_{n=1}^{\infty}\frac{2}{\pi}\eta\,\frac{\sin[n\arccos(2E-1)]}{n}\sin n\theta$$

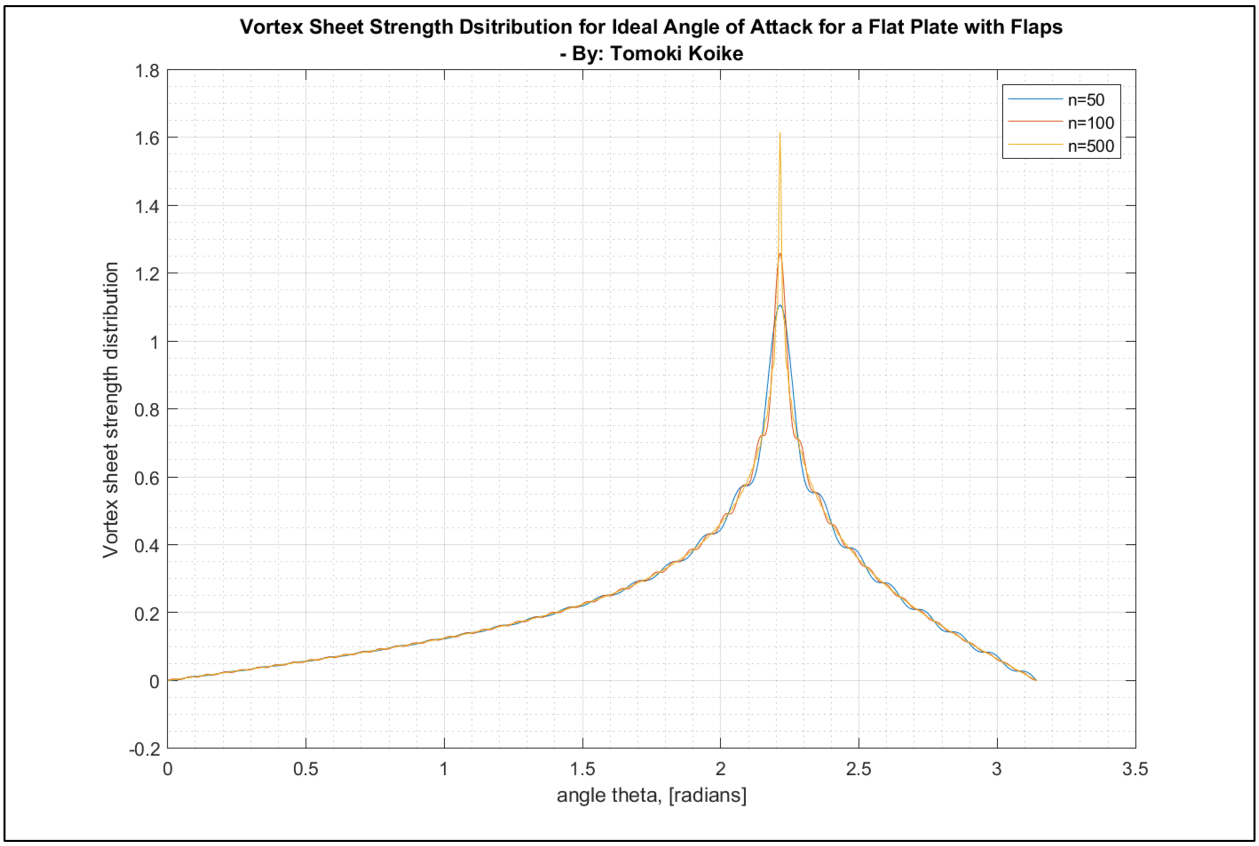
$$\boxed{\frac{\gamma(\theta)}{V_\infty} = \frac{4}{\pi}\eta\sum_{n=1}^{\infty}\frac{\sin[n\arccos(2E-1)]}{n}\sin n\theta}$$

if $E = 0.20 \quad \eta = 20^\circ = \frac{\pi}{9}$

$$\frac{\gamma(\theta)}{V_\infty} = \frac{4}{\pi} \cdot \frac{\pi}{9} \sum_{n=1}^{\infty} \frac{\sin n\left[\arccos(0.40-1)\right]}{n} \sin n\theta$$

$$\boxed{\frac{\gamma(\theta)}{V_\infty} = \frac{4}{9} \sum_{n=1}^{\infty} \frac{\sin\left[\arccos(-0.6)\, n\right] \sin n\theta}{n}}$$

now plot this in matlab

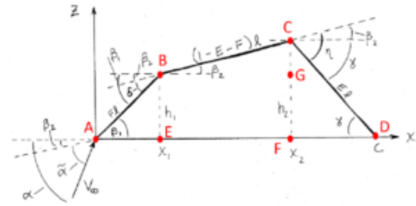


(code in Apendix, section "problem 2")

I used $n = 50$, 100, and 500

By the change of geometry the lift distribution also changes. For example, with a flap and slat the pressure distribution becomes stronger in that the lift overall increases. Likewise, cambered airfoils with optimal camber allows to increase the lift coefficient let alone the lift distribution. Besides that the thickness of the airfoil also changes the behavior of lift change after stall angle by preventing to fall abruptly.

3. [50 pts] Modify the example done in class by applying thin airfoil theory to a flat plate with a flap and a leading edge slat. The geometry is shown below. The flap deflection angle (measured from the line along the main element) is η and the fraction of chord made up by the flap is E, as in the example done in class. The slat deflection angle (measured from the line along the main element) is δ and the fraction of chord made up by the slat is F. Both angles η and δ are defined as positive for downward deflection and negative for upward deflection. Assume the angles η and δ are small so that small angle approximations can be used. The angle of attack, α, used in thin airfoil theory is defined relative to a chord line between the leading edge of the slat and the trailing edge of the flap, as in the class notes and as shown below. The geometric angle of attack relative to the wing (the main element of the airfoil) is $\tilde{\alpha}$, as in the class notes and as shown below. To help ensure we all use the same notation, define the lengths and angles as shown below.



(a) By applying thin airfoil theory the lift coefficient can be written as

$$c_\ell = 2\pi(\tilde{\alpha} - \alpha_{ZL}) .$$

Determine α_{ZL} in terms of E, F, η and δ.

[Hint: You should be able to show the following by using small angle approximations. For triangle ABE, $h_1 = F\ell \sin\beta_1 \approx Fc(\delta + \beta_2)$. For triangle CDF, $h_2 = E\ell \sin\gamma \approx Ec(\eta - \beta_2)$. For triangle BCG, $h_2 - h_1 = (1-E-F)\ell \sin\beta_2 \approx (1-E-F)c\beta_2$. Combine these results to show $\beta_2 = E\eta - F\delta$. You should then be able to do some further algebra and show that the slopes of the camber line are given by

$$\frac{dz}{dx} = \begin{cases} (1-F)\delta + E\eta & 0 \le x \le x_1 \\ E\eta - F\delta & x_1 \le x \le x_2 \\ -(1-E)\eta - F\delta & x_2 \le x \le c \end{cases}$$

Upon integrating the camber line slopes to find the Fourier coefficients, A_n, you should be able to show

$$\alpha_{ZL} = -\frac{1}{\pi}\left[-\cos^{-1}(1-2F) + 2\sqrt{F(1-F)}\right]\delta - \frac{1}{\pi}\left[\pi - \cos^{-1}(2E-1) + 2\sqrt{E(1-E)}\right]\eta \ .]$$

$$\frac{dz}{dx} = \begin{cases} (1-F)\delta + E\eta & x \in [0, x_1] \\ E\eta - F\delta & x \in [x_1, x_2] \\ -(1-E)\eta - F\delta & x \in [x_2, c] \end{cases}$$

$\Downarrow$

in terms of θ

$\theta \in [0, \theta_1]$

$\theta \in [\theta_1, \theta_2]$

$\theta \in [\theta_2, \pi]$

now

$$A_0 = \alpha - \frac{1}{\pi}\int_0^{\pi} \frac{dz}{dx} d\theta$$

$$= \alpha - \frac{1}{\pi}\int_0^{\theta_1} [(1-F)\delta + E\eta] d\theta - \frac{1}{\pi}\int_{\theta_1}^{\theta_2} (E\eta - F\delta) d\theta - \frac{1}{\pi}\int_{\theta_2}^{\pi} [-(1-E)\eta - F\delta] d\theta$$

$$= \alpha - \frac{1}{\pi}[(1-F)\delta + E\eta]\theta_1 - \frac{1}{\pi}[E\eta - F\delta](\theta_2 - \theta_1) - \frac{1}{\pi}[-(1-E)\eta - F\delta](\pi - \theta_2)$$

$$= \alpha - \frac{1}{\pi}\Big[\delta\theta_1 - \cancel{F\delta\theta_1} + \cancel{E\eta\theta_1} + E\eta\theta_2 - \cancel{E\eta\theta_1} - F\delta\theta_2 + \cancel{F\delta\theta_1} + (-\eta + E\eta - F\delta)(\pi - \theta_2)\Big]$$

$$= \alpha - \frac{1}{\pi}\Big[\delta\theta_1 + \cancel{E\eta\theta_2} - \cancel{F\delta\theta_2} - \pi\eta + \pi E\eta - \pi F\delta + \eta\theta_2 - \cancel{E\eta\theta_2} + \cancel{F\delta\theta_2}\Big]$$

$$= \alpha - \frac{1}{\pi}\Big[\delta\theta_1 - \pi\eta + \pi E\eta - \pi F\delta + \eta\theta_2\Big]$$

and

$$A_1 = \frac{2}{\pi}\int_0^{\pi} \frac{dz}{dx}\cos\theta\, d\theta$$

$$= \frac{2}{\pi}\Big[\int_0^{\theta_1} [(1-F)\delta + E\eta]\cos\theta\, d\theta + \int_{\theta_1}^{\theta_2} (E\eta - F\delta)\cos\theta\, d\theta + \int_{\theta_2}^{\pi} [-(1-E)\eta - F\delta]\cos\theta\, d\theta$$

$$= \frac{2}{\pi}\Big[(\delta - F\delta + E\eta)\sin\theta_1 + (E\eta - F\delta)(\sin\theta_2 - \sin\theta_1) + (-\eta + E\eta - F\delta)(-\sin\theta_2)\Big]$$

$$= \frac{2}{\pi}\Big(\delta\sin\theta_1 - \cancel{F\delta\sin\theta_1} + \cancel{E\eta\sin\theta_1} + \cancel{E\eta\sin\theta_2} - \cancel{E\eta\sin\theta_1} - F\cancel{\delta\sin\theta_2} + F\cancel{\delta\sin\theta_1} + \eta\sin\theta_2 - E\cancel{\eta\sin\theta_2} + F\cancel{\delta\sin\theta_2}\Big)$$

$$= \frac{2}{\pi}\left(\delta\sin\theta_1 + \eta\sin\theta_2\right)$$

now from the eqn.

$$\boxed{C_l = 2\pi\left(A_0 + \frac{1}{2}A_1\right)}$$

$$C_\ell = 2\pi\left(q - \frac{\delta\theta_1}{\pi} + \eta - E\eta + F\delta - \frac{\eta\theta_2}{\pi} + \frac{\delta\sin\theta_1}{\pi} + \frac{\eta\sin\theta_2}{\pi}\right)$$

from small angle assumption & flat plate assumption

$$\tan\beta_1 = \frac{h}{x_1} \approx \beta_1 \qquad x_1 \approx F\ell \approx Fc \iff \frac{x_1}{c} = F$$

$$\tan\beta_2 = \frac{h_2-h_1}{x_2-x_1} \approx \beta_2 \qquad x_2 - x_1 \approx (1-E-F)\ell \Rightarrow x_2 = (1-E-F)c + Fc = (1-E)c$$

$$c - x_2 \approx E\ell \qquad \Downarrow$$

$$\tan\gamma = \frac{h_2}{c-x_2} \approx \gamma \qquad c \approx \ell \qquad \frac{x_2}{c} \approx 1-E$$

and

$$x = \frac{c}{2}(1-\cos\theta) \iff \cos\theta_1 = 1 - \frac{2x_1}{c} = 1-2F$$

$$\sin\theta_1 = \sqrt{1-1+4F-4F^2} = 2\sqrt{F(1-F)}, \quad \theta_1 = \arccos(1-2F)$$

$$\cos\theta_2 = 1 - \frac{2x_2}{c} = 1-2(1-E) = 2E-1$$

$$\sin\theta_2 = \sqrt{1-(2E-1)^2} = \sqrt{1-4E^2+4E-1} = 2\sqrt{E(1-E)}, \quad \theta_2 = \arccos(2E-1)$$

thus, $\because q = \tilde{q} + \beta_2$

$$C_\ell = 2\pi\Big(\tilde{q} + \beta_2 - \frac{\delta}{\pi}\arccos(1-2F) + \eta - E\eta + F\delta - \frac{\eta}{\pi}\arccos(2E-1) + \frac{\delta}{\pi}2\sqrt{F(1-F)} + \frac{\eta}{\pi}2\sqrt{E(1-E)}\Big)$$

from the relations of $\triangle ABE$, $\triangle CDF$, and $\triangle BCG$

$$h_1 = F\ell\sin\beta_1 \simeq Fc(\delta+\beta_2)$$

$$h_2 = E\ell\sin\gamma \approx Ec(\eta-\beta_2)$$

$$h_2 - h_1 = (1-E-F)\ell\sin\beta_2 \approx (1-E-F)c\beta_2$$

now

$$h_2 - h_1 = C(1-E-F)c\beta_2 = Ec(\eta - \beta_2) - Fc(\delta + \beta_2)$$

$$c\beta_2 - \cancel{Ec\beta_2} - \cancel{Fc\beta_2} = Ec\eta - \cancel{Ec\beta_2} - Fc\delta - \cancel{Fc\beta_2}$$

$$\beta_2 = E\eta - F\delta$$

then

$$C_l = 2\pi\Big(\tilde{\alpha} + \cancel{E\eta} - \cancel{F\delta} - \frac{\delta}{\pi}\arccos(1-2F) + \eta - \cancel{E\eta} + \cancel{F\delta}$$
$$- \frac{\eta}{\pi}\arccos(2E-1) + \frac{\delta}{\pi}2\sqrt{F(1-F)} + \frac{\eta}{\pi}2\sqrt{E(1-E)}\Big)$$

$$= 2\pi\Big\{\tilde{\alpha} - \frac{1}{\pi}\left[\arccos(1-2F) - 2\sqrt{F(1-F)}\right]\delta$$
$$- \frac{1}{\pi}\left[-\pi + \arccos(2E-1) - 2\sqrt{E(1-E)}\right]\eta\Big\}$$

since $C_l = 2\pi(\tilde{\alpha} - \alpha_{\delta L})$

$-\alpha_{\delta L} =$

$$\therefore \quad \boxed{\alpha_{\delta L} = -\frac{1}{\pi}\left[-\arccos(1-2F) + 2\sqrt{F(1-F)}\right]\delta - \frac{1}{\pi}\left[\pi - \arccos(2E-1) + 2\sqrt{E(1-E)}\right]\eta}$$

(b) Assume $F = 0$ and show you recover the formula found for the example worked in class. Plot the flap effectiveness, $-\alpha_{ZL}/\eta$, as a function of E for $0 \le E \le 1$.

if $F=0$

$$q_{ZL} = -\frac{1}{\pi}\left[\cancelto{0}{-\arccos(1)} + 0\right]\delta - \frac{1}{\pi}\left[\pi - \arccos(2E-1) + 2\sqrt{E(1-E)}\right]\eta$$

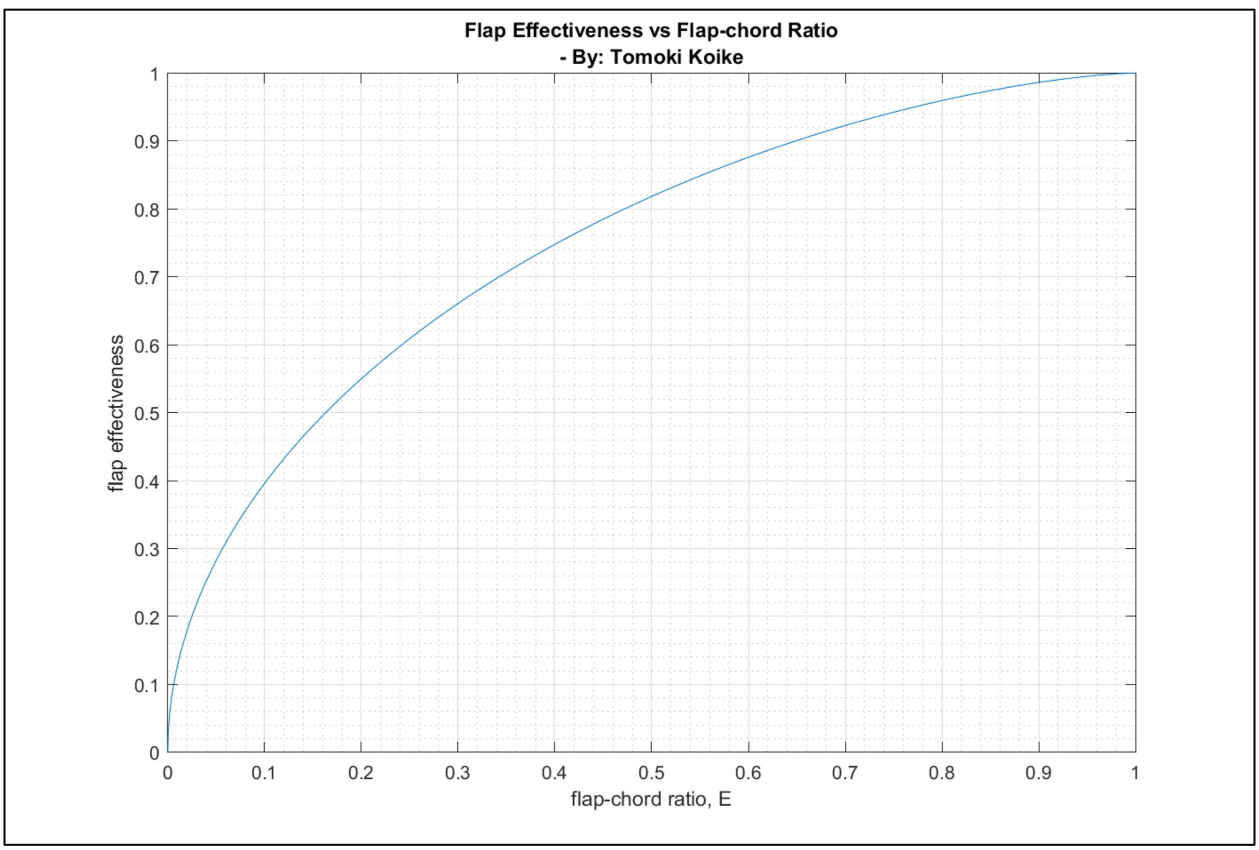
$$q_{ZL} = -\frac{1}{\pi}\left[\pi - \arccos(2E-1) + 2\sqrt{E(1-E)}\right]\eta$$

$$\alpha_{ZL} = -\eta - \frac{\eta}{\pi}\left(-\arccos(2E-1) + 2\sqrt{E(1-E)}\right)$$

agrees with equation on lecture notes

$$\boxed{-\frac{q_{ZL}}{\eta} = 1 + \frac{1}{\pi}\left(-\arccos(2E-1) + 2\sqrt{E(1-E)}\right)}$$

The plot is the following the Matlab code is in the appendix in section problem3 <i>



(c) Assume $E = 0$ so that the airfoil has a leading edge slat but no flap. Plot the slat effectiveness, $-\alpha_{ZL}/\delta$, as a function of F for $0 \le F \le 1$.

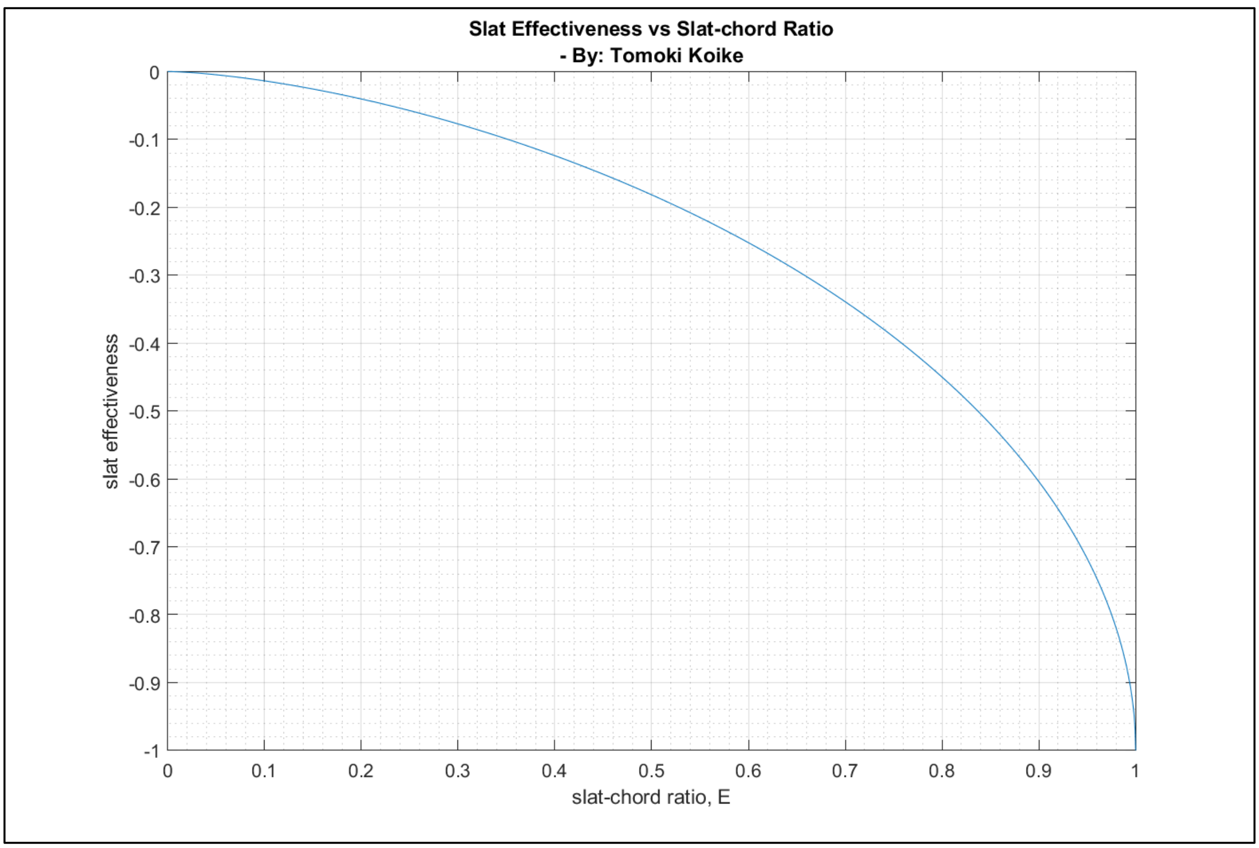
when $E = 0$

$$\alpha_{ZL} = -\frac{1}{\pi}\left[-\arccos(1-2F) + 2\sqrt{F(1-F)}\right]\delta$$

$$\cancel{-\frac{1}{\pi}(\pi - \pi + 0)\eta} \rightarrow 0$$

$$\boxed{-\frac{\alpha_{ZL}}{\delta} = \frac{1}{\pi}\left[-\arccos(1-2F) + 2\sqrt{F(1-F)}\right]}$$

The plot is the following.
The matlab code is in the Appendix in section problem 3 <ii>



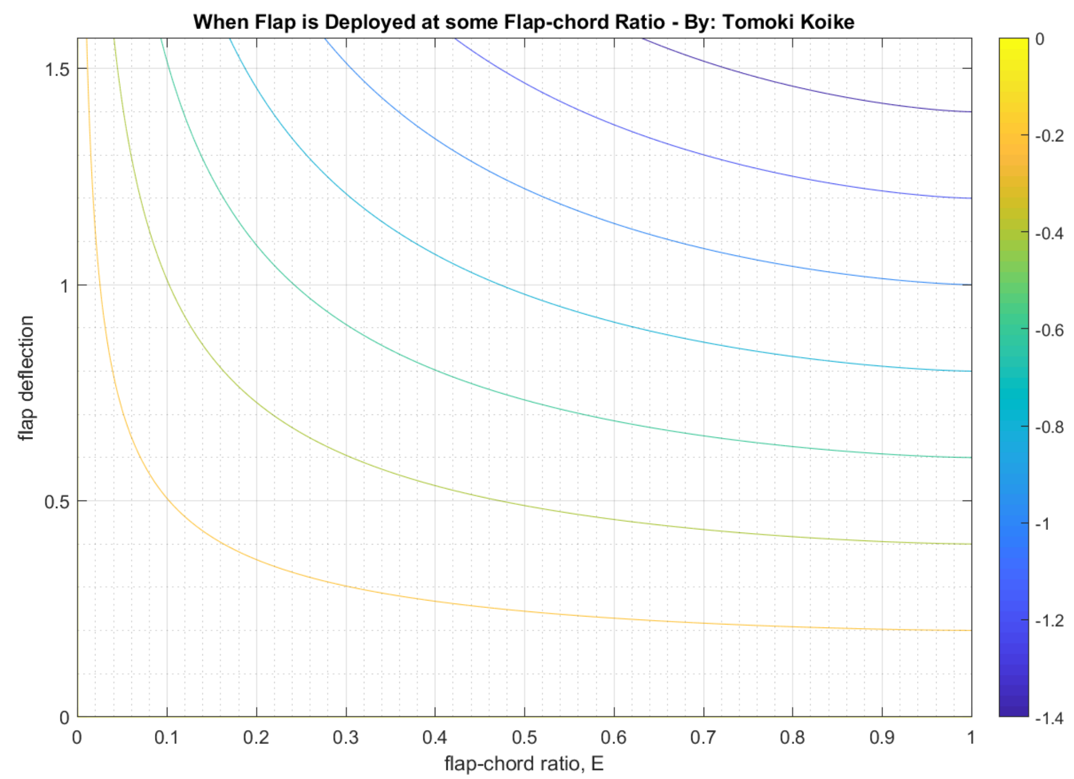
(d) As the flap is deployed (η increased from zero to positive values), what happens to α_{ZL}? As the slat is deployed (δ increased from zero to positive values), what happens to α_{ZL}? For a small slat or flap (E or $F \leq 0.25$) how do the relative magnitudes of the change in α_{ZL} compare for a flap and a slat? Discuss how your answers to these questions compare to the trends seen in figures 4.54 and 4.55 of the text. (Note: we cannot determine c_ℓ^{max} using an inviscid theory, but you can determine other aspects of the lift curve.)

The contour plot for $\alpha_{ZL} = -\eta - \frac{\eta}{\pi}\left(-\arccos(2E-1) + 2\sqrt{E(1-E)}\right)$

for $y = \eta$ $\quad \eta \in [0, \frac{\pi}{2}]$ $\quad (* \frac{\pi}{2} = 90°)$

$x = E$

is the following. $\longrightarrow$ Matlab code appendix <iii>



from this, we can say that for a fixed E value the larger the deflection when flap is deployed α_{ZL} moves to the left in the negative direction.

then

When slat is deployed what happens?

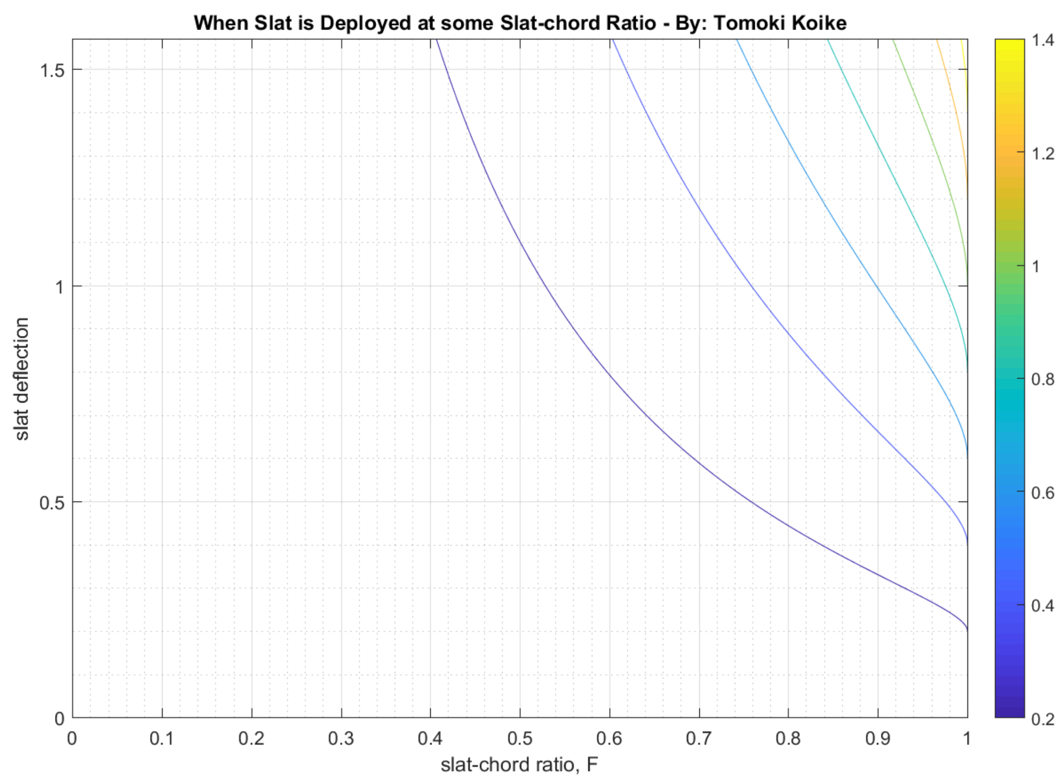
we can examine that from the next contour plot

where

$y = \delta \quad \delta \in [0, \frac{\pi}{2}] \quad (* \frac{\pi}{2} = 90°)$

$x = F$

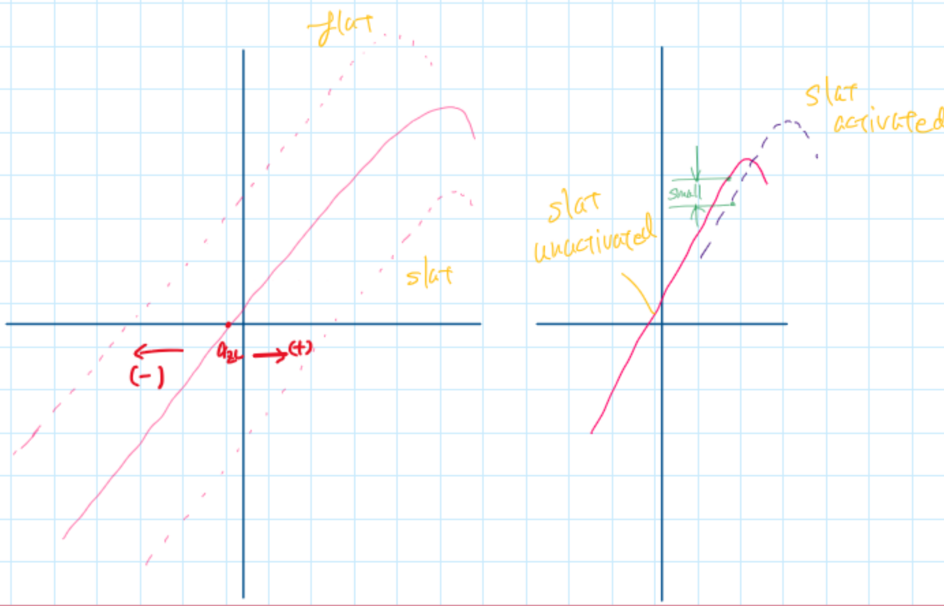
$$q_{\delta L} = -\frac{1}{\pi}\left[-\arccos(1-2F) + 2\sqrt{F(1-F)}\right]\delta$$



$\boxed{\text{the following shows that as flat is deployed } q_{\delta L} \text{ moves to right in the positive direction.}}$

for a small F or $F \leq 0.25$
the flap has more effect on the a_{zL}; that is,
makes $|a_{zL}|$ larger. Whereas F at small F does
not have observable effect.

These results agree with figure 4.54 & 4.55



Appendix

HW 3 - matlab code

problem 1

```
clear all; close all; clc


% Plotting the moment coefficient about the aerodynamic center
E = linspace(0,1,100);  % flap-chord ratio
C_mac_over_deflect = -2*(1-E).*sqrt(E.*(1-E));  % coefficient per deflection
angle
fig1 = figure('Renderer', 'painters', 'Position', [10 10 900 600]);
plot(E, C_mac_over_deflect)
xlabel('flap-chord ratio, E')
ylabel('C_m_a_c per deflection angle')
title({'C_m_a_c per deflection angle VS flap-chord ratio', ...
    '- By: Tomoki Koike'})
grid on
grid minor
box on
saveas(fig1, 'mmt_coeff.png')
```

problem 2

```
theta = linspace(0, pi, 2^10);  % Define the angle theta
gamma_over_Vinf_50 = 0;  % Initiate vortex sheet strength distribution
for n = 1:50
    A_n = sin(n*acos(-0.6)).*sin(n.*theta)/n;  % n-th Fourier coefficient
    gamma_over_Vinf_50 = gamma_over_Vinf_50 + A_n;  % Summation
end
gamma_over_Vinf_50 = 4/9 * gamma_over_Vinf_50;


gamma_over_Vinf_100 = 0;  % Initiate vortex sheet strength distribution
for n = 1:100
    A_n = sin(n*acos(-0.6)).*sin(n.*theta)/n;  % n-th Fourier coefficient
    gamma_over_Vinf_100 = gamma_over_Vinf_100 + A_n;  % Summation
end
gamma_over_Vinf_100 = 4/9 * gamma_over_Vinf_100;


gamma_over_Vinf_500 = 0;  % Initiate vortex sheet strength distribution
for n = 1:500
    A_n = sin(n*acos(-0.6)).*sin(n.*theta)/n;  % n-th Fourier coefficient
```

```
    gamma_over_Vinf_500 = gamma_over_Vinf_500 + A_n;  % Summation
end
gamma_over_Vinf_500 = 4/9 * gamma_over_Vinf_500;


% Plotting
fig2 = figure('Renderer', 'painters', 'Position', [10 10 900 600]);
plot(theta, gamma_over_Vinf_50)
xlabel('angle theta, [radians]')
ylabel('Vortex sheet strength distribution')
title({'Vortex Sheet Strength Dsitribution for Ideal Angle of Attack for a Flat
Plate with Flaps', ...
    '- By: Tomoki Koike'})
hold on
plot(theta, gamma_over_Vinf_100)
plot(theta, gamma_over_Vinf_500)
hold off
grid on
grid minor
box on
legend('n=50','n=100','n=500')
saveas(fig2, 'vortex_sheet_strength.png')
```

problem 3 <i>

```
E = linspace(0,1,2^11);
alpha_zl_E = 1 + (-acos(2.*E-1) + 2*sqrt(E.*(1-E)))/pi;


% Plotting
fig3 = figure('Renderer', 'painters', 'Position', [10 10 900 600]);
plot(E, alpha_zl_E)
xlabel('flap-chord ratio, E')
ylabel('flap effectiveness')
title({'Flap Effectiveness vs Flap-chord Ratio', ...
    '- By: Tomoki Koike'})
grid on
grid minor
box on
saveas(fig3, 'flap_effectiveness.png')
```

<ii>

```
F = linspace(0,1,2^11);
alpha_zl_F = (-acos(1-2.*F) + 2*sqrt(F.*(1-F)))/pi;


% Plotting
```

```
fig3 = figure('Renderer', 'painters', 'Position', [10 10 900 600]);
plot(F, alpha_zl_F)
xlabel('slat-chord ratio, E')
ylabel('slat effectiveness')
title({'Slat Effectiveness vs Slat-chord Ratio', ...
    '- By: Tomoki Koike'})
grid on
grid minor
box on
saveas(fig3, 'slat_effectiveness.png')
```

<iii>

```
eta = linspace(0,pi/2,2^8);
[X1, Y1] = meshgrid(E, eta);
alpha_zl_E = -Y1.*(1 + (-acos(2.*X1-1) + 2*sqrt(X1.*(1-X1)))/pi);
fig4 = figure('Renderer', 'painters', 'Position', [10 10 900 600]);
contour(X1,Y1, alpha_zl_E);
xlabel('flap-chord ratio, E')
ylabel('flap deflection')
title('When Flap is Deployed at some Flap-chord Ratio - By: Tomoki Koike')
colorbar
grid on
grid minor
box on
saveas(fig4, 'flap_deployed.png')
```

<iv>

```
delta = linspace(0,pi/2,2^8);
[X2, Y2] = meshgrid(F, delta);
alpha_zl_F = -Y2.*(-acos(1-2.*X2) + 2*sqrt(X2.*(1-X2)))/pi;
fig4 = figure('Renderer', 'painters', 'Position', [10 10 900 600]);
contour(X2,Y2, alpha_zl_F);
xlabel('slat-chord ratio, F')
ylabel('slat deflection')
title('When Slat is Deployed at some Slat-chord Ratio - By: Tomoki Koike')
colorbar
grid on
grid minor
box on
saveas(fig4, 'slat_deployed.png')
```