P3: S

Friday, April 3, 2020 06:30 PM

Investigation: Axisymmetric Body in Circular Orbit

some time history for dep vars

EOM: Particular Solution (motion of interest)

Nominal motion to be investigated MUST be a solution to the **nonlinear** differential equations

nonlinear soln → ω's, ε's as func of time ??

Easiest to obtain (and to investigate) a particular solution when the dependent variables are **constants**

⇨ To determine a particular solution, first consider the possibility that a constant solution even exists

$$\underbrace{\omega_j = \text{const.} \quad ; \quad \varepsilon_i = \text{const.}}_{\text{does this exist}}$$

yes if we get help.

S

not same as torque-free

Consider the following motion: symmetry axis remains normal to the orbit

$${}^N\bar{\omega}^B = \omega_3 \hat{a}_3 = \underset{\uparrow \text{ const.}}{\omega_3} \hat{b}_3 \qquad (\omega_1 = \omega_2 = 0, \ \underbrace{\varepsilon_i = \varepsilon_{i0}}_{??})$$

Check differential equations to see if such a motion can exist (Remove zero ω terms first)

$$\dot{\omega}_3 = 0 \qquad \rightarrow \omega_3 = \omega_{3_o} = \text{constant} \ \checkmark$$

$$\dot{\omega}_1 = \cancel{-s\omega_2} + \left(1 - \frac{J}{I}\right)\left[\cancel{\omega_2}\omega_3 - 12\Omega^2(\varepsilon_1\varepsilon_2 - \varepsilon_3\varepsilon_4)(\varepsilon_3\varepsilon_1 + \varepsilon_2\varepsilon_4)\right]$$

$$\dot{\omega}_2 = \cancel{s\omega_1} - \left(1 - \frac{J}{I}\right)\left[\cancel{\omega_1}\omega_3 - 6\Omega^2(\varepsilon_3\varepsilon_1 + \varepsilon_2\varepsilon_4)(1 - 2\varepsilon_2^2 - 2\varepsilon_3^2)\right]$$

$$2\dot{\varepsilon}_1 = \varepsilon_2(\omega_3 - s + \Omega) \cancel{- \varepsilon_3\omega_2} \cancel{+ \varepsilon_4\omega_1}$$

$$2\dot{\varepsilon}_2 = \cancel{\varepsilon_3\omega_1} \cancel{+ \varepsilon_4\omega_2} - \varepsilon_1(\omega_3 - s + \Omega)$$

$$2\dot{\varepsilon}_3 = \varepsilon_4(\omega_3 - s - \Omega) \cancel{+ \varepsilon_1\omega_2} \cancel{- \varepsilon_2\omega_1}$$

$$2\dot{\varepsilon}_4 = \cancel{-\varepsilon_1\omega_1} \cancel{- \varepsilon_2\omega_2} - \varepsilon_3(\omega_3 - s - \Omega)$$

start with these

if soln constant then ALL 1st deriv must equal $\emptyset$

What ε equations are left?

$$2\dot{\varepsilon}_1 = \varepsilon_2(\omega_3 - s + \Omega)$$

what constant ε

$$2\dot{\varepsilon}_1 = \varepsilon_2(\omega_3 - s + \Omega)$$

$$2\dot{\varepsilon}_2 = -\varepsilon_1(\omega_3 - s + \Omega)$$

$$2\dot{\varepsilon}_3 = \varepsilon_4(\omega_3 - s - \Omega)$$

$$2\dot{\varepsilon}_4 = -\varepsilon_3(\omega_3 - s - \Omega)$$

what constant ε be to make derivate

To obtain a "constant" solution, all $\dot{\varepsilon} = 0$, $\dot{\omega} = 0$

Either $\boxed{\text{ALL } \varepsilon_i = 0}$ ← not possible cuz does not satisfy constraint $\varepsilon_1^2 + \varepsilon_2^2 + \varepsilon_3^2 + \varepsilon_4^2 = 1$

OR

$\varepsilon_3 = \varepsilon_4 = 0$ $\qquad$ $\varepsilon_1 = \varepsilon_2 = 0$

some $\varepsilon_i = 0$ AND $\boxed{(\omega_{3_o} - s + \Omega) = 0}$ or $\boxed{(\omega_{3_o} - s - \Omega) = 0}$

option 2 $\qquad$ option 1

$\boxed{\text{Option \#1}}$ $\quad \varepsilon_1 = \varepsilon_2 = 0$

$(\omega_{3_o} - s - \Omega) = 0$ → true if $\boxed{s = \omega_{3_o} - \Omega}$

What does this mean for the values of ε_i?

$${}^{N}\underline{\omega}^{B} = {}^{N}\underline{\omega}^{A} + {}^{A}\underline{\omega}^{C} + {}^{C}\underline{\omega}^{B}$$

$\omega_{30}\hat{a}_3 = \Omega\hat{c}_3 \qquad {}^{A}\underline{\omega}^{C} \qquad s\hat{c}_3$

$(\omega_{30} = \Omega)$

assuming $\hat{c}_3 = \hat{b}_3 = \hat{a}_3$

⇨ if $s = \omega_{3_o} - \Omega$ then ${}^{A}\underline{\omega}^{C} = \underline{0}$ or $\hat{c} = \hat{a}$

$\Rightarrow$ if $s = \omega_{3_o} - \Omega$ then ${}^{A}\bar{\omega}^{C} = 0$ or $C = A$

$\Rightarrow$ ${}^{A}\varepsilon^{C} = \hat{\lambda}\sin\frac{\theta}{2}$ ${}^{A}\varepsilon_4^{C} = \cos\frac{\theta}{2}$

$$\boxed{\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 0 \qquad \varepsilon_4 = 1}$$

Option #2

$(\omega_{3_o} - s + \Omega) = 0$ → true if $\boxed{s = \omega_{3_o} + \Omega}$

What does this mean for the values of ε_i?

${}^{N}\bar{\omega}^{B} = {}^{N}\bar{\omega}^{A} + {}^{A}\bar{\omega}^{C} + {}^{C}\bar{\omega}^{B}$

$\omega_{3_0} + \Omega$

$\Rightarrow$ $\omega_{3_0}\hat{\lambda}_3 = \Omega\hat{a}_3 + {}^{A}\bar{\omega}^{C} + s\hat{c}_3$

$\hat{c}_3$

${}^{A}\bar{\omega}^{C} = -2\Omega\hat{a}_3$

differential equations for ε_i

$$\left.\begin{aligned}
2\dot{\varepsilon}_1 &= 0 \\
2\dot{\varepsilon}_2 &= 0 \\
2\dot{\varepsilon}_3 &= \varepsilon_4(-2\Omega) \\
2\dot{\varepsilon}_4 &= -\varepsilon_3(-2\Omega)
\end{aligned}\right\}$$

ε's not const. } can still be a particular solution

Certainly an acceptable nominal solution BUT the dependent variables are not constant!

$\dot{\varepsilon}_3 + \Omega\varepsilon_3 = 0$ → $\varepsilon_3 = \sin\Omega/2$

$\varepsilon_4 = \cos\Omega/2$

So……………………..use option #1

Motion of interest:

$s = \omega_{30} - \Omega$

particular solution

Use this particular solution for our investigation

$$s = \omega_{3_0} - \Omega$$

$$\omega_1 = \omega_2 = 0 \quad \omega_3 = \omega_{3_0}$$
$$\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 0 \quad \varepsilon_4 = 1$$

Use this particular solution for our investigation

Check final DE to be sure

$$\dot{\omega}_3 = 0 \qquad \rightarrow \omega_3 = \omega_{3_o} = \text{constant}$$

$$\dot{\omega}_1 = -s\omega_2 + \left(1 - \frac{J}{I}\right)\left[\omega_2\omega_3 - 12\Omega^2\left(\varepsilon_1\varepsilon_2 - \varepsilon_3\varepsilon_4\right)\left(\varepsilon_3\varepsilon_1 + \varepsilon_2\varepsilon_4\right)\right]$$

$$\dot{\omega}_2 = s\omega_1 - \left(1 - \frac{J}{I}\right)\left[\omega_1\omega_3 - 6\Omega^2\left(\varepsilon_3\varepsilon_1 + \varepsilon_2\varepsilon_4\right)\left(1 - 2\varepsilon_2^2 - 2\varepsilon_3^2\right)\right]$$

$$2\dot{\varepsilon}_1 = \varepsilon_2\left(\omega_3 - s + \Omega\right) - \varepsilon_3\omega_2 + \varepsilon_4\omega_1$$

$$2\dot{\varepsilon}_2 = \varepsilon_3\omega_1 + \varepsilon_4\omega_2 - \varepsilon_1\left(\omega_3 - s + \Omega\right)$$

$$2\dot{\varepsilon}_3 = \varepsilon_4\left(\omega_3 - s - \Omega\right) + \varepsilon_1\omega_2 - \varepsilon_2\omega_1$$

$$2\dot{\varepsilon}_4 = -\varepsilon_1\omega_1 - \varepsilon_2\omega_2 - \varepsilon_3\left(\omega_3 - s - \Omega\right)$$