P3: U

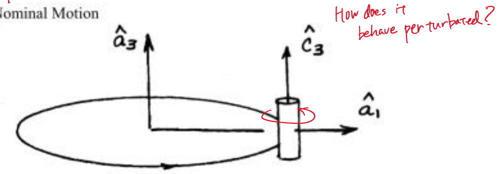
2020年3月9日 月曜日　　午後0:32

Numerical Investigation

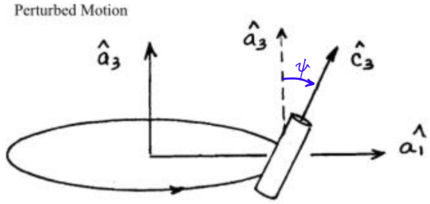
particular soln

1. Nominal Motion

How does it behave perturbated?



Perturbed Motion



Further narrow scope of numerical study: assume initial ω values correct for nominal motion; initial orientation includes perturbation

2. Variable of interest? ψ pointing angle

   a. no torque $\Rightarrow$ constant
   b. gravity torque? $\Rightarrow$ numerically integrate to see impact

3. Set up equations

rotation rate for particular orbit

nondimensionalize

$$2\dot{\varepsilon}_1 = \varepsilon_2\left(\omega_3 - s + \Omega\right) - \varepsilon_3\omega_2 + \varepsilon_4\omega_1$$
$$2\dot{\varepsilon}_2 = \varepsilon_3\omega_1 + \varepsilon_4\omega_2 - \varepsilon_1\left(\omega_3 - s + \Omega\right)$$
$$2\dot{\varepsilon}_3 = \varepsilon_4\left(\omega_3 - s - \Omega\right) + \varepsilon_1\omega_2 - \varepsilon_2\omega_1$$
$$2\dot{\varepsilon}_4 = -\varepsilon_1\omega_1 - \varepsilon_2\omega_2 - \varepsilon_3\left(\omega_3 - s - \Omega\right)$$
$$\dot{\omega}_1 = -s\omega_2 - x\left[\omega_2\omega_3 - 12\Omega^2\left(\varepsilon_1\varepsilon_2 - \varepsilon_3\varepsilon_4\right)\left(\varepsilon_3\varepsilon_1 + \varepsilon_2\varepsilon_4\right)\right]$$
$$\dot{\omega}_2 = s\omega_1 + x\left[\omega_1\omega_3 - 6\Omega^2\left(\varepsilon_3\varepsilon_1 + \varepsilon_2\varepsilon_4\right)\left(1 - 2\varepsilon_2^2 - 2\varepsilon_3^2\right)\right]$$

Use the following nondimensional parameters

No. of orbits $\quad \nu = \dfrac{\Omega t}{2\pi}$

Shape Factor $\quad x = \dfrac{J}{I} - 1$ $\quad \begin{cases} \text{rod-shape} & x < 0 \\ \text{disk-shape} & x > 0 \end{cases}$

Spin factor $\quad \gamma = \dfrac{\omega_{3_0}}{\Omega} - 1$

non-dim s

particular soln

$\dfrac{s}{\Omega} = \dfrac{\omega_{30}}{\Omega} - \dfrac{\Omega}{\Omega}$

if $\dfrac{\omega_{30}}{\Omega} = 1$

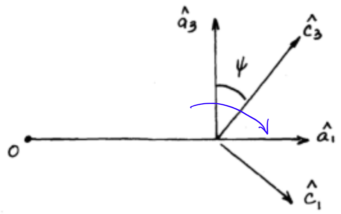
dim $\omega_0 = \Omega$

$n = \sqrt{\dfrac{\mu}{a^3}}$

Note: $\dfrac{df}{dt} = \dfrac{\Omega}{2\pi}\dfrac{df}{d\nu}$

Consider: under what conditions do these reduce to the torque-free equations?

4. Initial Conditions



$@t=0$

$\psi = \psi_o$

$\hat{c}'s$ oriented above (perhaps an orbit insertion error)

${}^N\bar{\omega}^B = \omega_{3_o}\hat{c}_3$ (ω_{3_o} constant)

$\omega_1 = \omega_2 = 0$ (nominal values)

${}^A\bar{\varepsilon}^C = \hat{\lambda}\sin\left(\frac{\theta}{2}\right) = \hat{c}_2 \sin\frac{\theta}{2} = \hat{c}_2 \sin\frac{\psi}{2}$

$\Longrightarrow$ $\varepsilon_1 = 0$ $\quad$ $\varepsilon_3 = 0$

$\varepsilon_2 = \sin\frac{\psi}{2}$ $\quad$ $\varepsilon_4 = \cos\frac{\psi}{2}$

How to interpret results

With torque, let $\psi = .1$ rad (5.7°) or .05 rad

Determine effect on ψ if change body shape; spin rate

5. Integrate – start with a couple of revs

6. Interpret Results – check ψ

Nominal motion	Perturbed Motion
No torque	no torque
Gravity torque	gravity torque

Stability?

(How do we evaluate stability in a numerical procedure?)

U5

Definitions

Independent variable $\{$ t or ν

Dependent variables z_i $\left\{ \begin{array}{l} \omega \leftarrow \quad W = \frac{\omega}{\Omega} \\ \varepsilon \leftarrow \end{array} \right.$

Then $z_i' = \dfrac{dz_i}{d\nu}$ $\qquad i = 1, \dots, n$

Diff Eqns $\quad z_1' = f_1$
$\qquad\qquad z_2' = f_2$
$\qquad\qquad \vdots$

(1) Particular solution of differential equations ← for dep var
p ← part. soln.

Known solution to NLDE
Solution whose stability is being tested

$\boxed{\text{ICs}}$ $\left. \begin{array}{l} z_1|_{\nu=0} = p_1(0) \\ z_2|_{\nu=0} = p_2(0) \\ \vdots \end{array} \right\}$ symmetry axis normal to the orbit

soln to test

(2) Comparison solution ← dep var ← when perturbated
q ← comparison solns

Known solution to NLDE
Solution whose stability is being tested

$\boxed{\text{ICs}}$ $\begin{array}{l} z_1|_{\nu=0} = q_1(0) \\ z_2|_{\nu=0} = q_2(0) \\ \vdots \end{array}$

U6

(I) If for every $\Delta > 0$ there exists a $\delta > 0$ such that

$$|p_i(0) - q_i(0)| < \delta \qquad (i = 1, \dots, n)$$

implies, for all $\nu \geq 0$

$$|p_i - q_i| < \Delta \qquad (i = 1, \dots, n)$$

then the particular solution $p_1, p_2, \dots p_n$ is called stable.

(II) If for every $\Delta > 0$ there exists a $\delta > 0$ such that

$$|p_i(0) - q_i(0)| < \delta \qquad (i = 1, \dots, n)$$

implies, for all $\nu \geq 0$

$$|p_i - q_i| < \Delta \qquad (i = 1, \dots, n)$$

AND

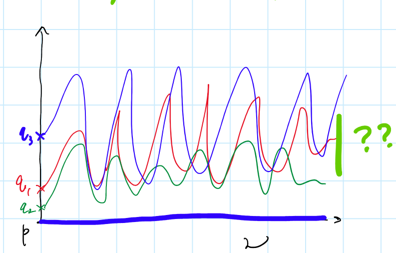
$$\lim_{\nu \to \infty} |p_i - q_i| = 0 \qquad (i = 1, \dots, n)$$

then the particular solution $p_1, p_2, \dots p_n$ is called asymptotically stable.

(III) A stable solution which is not asymptotically stable is called marginally stable.

(IV) A solution which is not stable is called unstable.

Always at least run 2 comparison solns



part. soln.

marginally stable for ν = # simulation

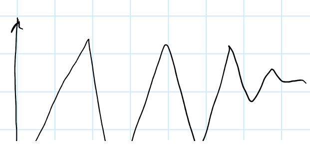
as $p(0)$ and $q(0)$ get closer, responses do get closer

U7

Run simulations ⟶ check output

(Two observations based on limitations that are true in general for numerical studies.)

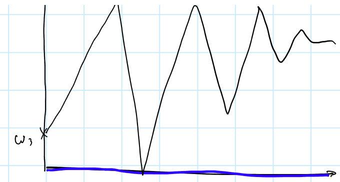
1. Possible to attempt only a finite number of initial conditions.
   Never state that you are 100% of any conclusion – impossible
2. Can only run a simulation for a finite time
   Never draw conclusions (in a nonlinear

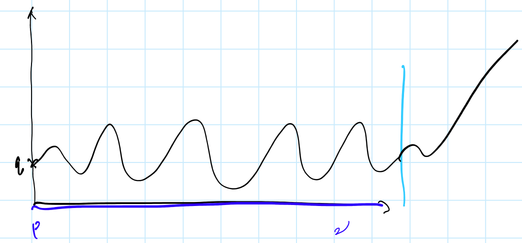


any conclusion ~ impossible

2. Can only run a simulation for a finite time

Never draw conclusions (in a nonlinear system) about results beyond this time
if you speculate likely to be wrong!





input $\frac{\omega_{30}}{\Omega}$

↓

simulation for a perturbation $\psi = 0, \underset{\sim}{6°}$ perturbation

⇓

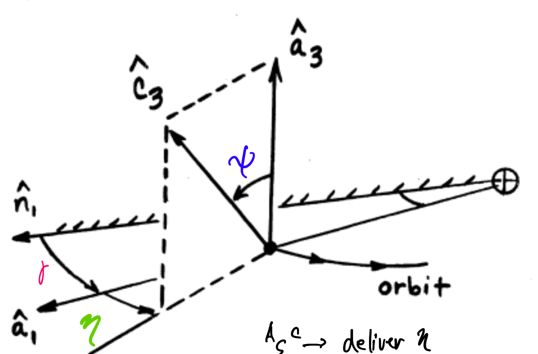
time (revs) histories for ψ

⇓

interpret $\underbrace{\text{"stability"}}_{\text{def.}}$ from $\underbrace{\text{plots}}_{???}$

numerical stability → length of sim
ICs
variable of interest

$$\Delta\lambda = 3\pi\left(\frac{\Omega}{\omega_{30}}\right)\left(\frac{I}{J} - 1\right)\cos\psi_0$$



A_{Σ}^{c} → deliver η

prec

$\lambda = \gamma + \eta$