AAE 364 HW #10 Fall 2019

B-7-3

(1) $G_1(s) = \frac{1+S}{1+2S}$ $G_2(s) = \frac{1-S}{1+2S}$

Solution:

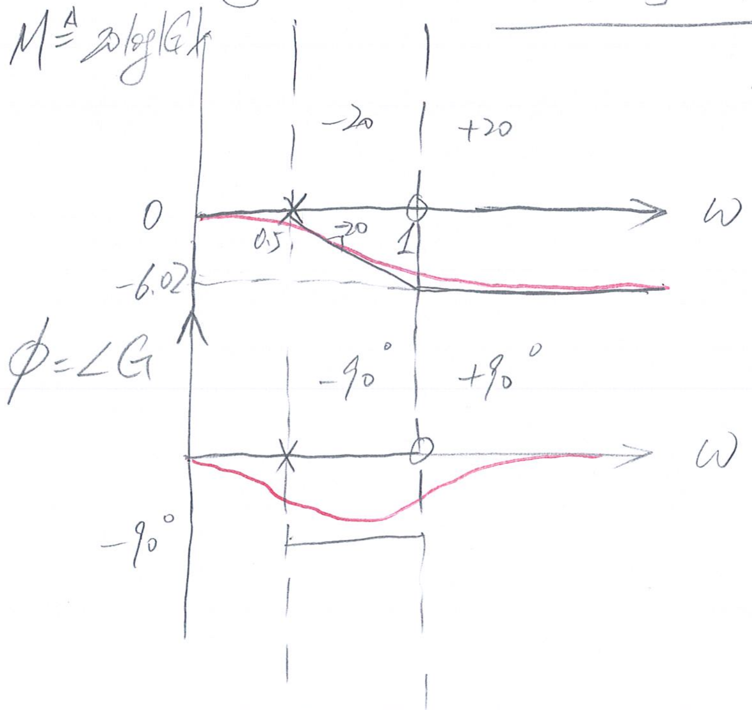
(1) For $G_1(j\omega)$, the corner frequencies include:

$\omega_c = 1$ ("stable zero), 0.5 ("stable pole")

Initial angle $\lim_{\omega \to 0} \angle G_1(j\omega) = 0°$

Initial magnitude $\lim_{\omega \to 0} |G_1(j\omega)| = 1 = 0\ dB.$

Also $\lim_{\omega \to \infty} |G(j\omega)| = 0.5 \cong -6.02\ dB.$



(2) Corner frequencies:

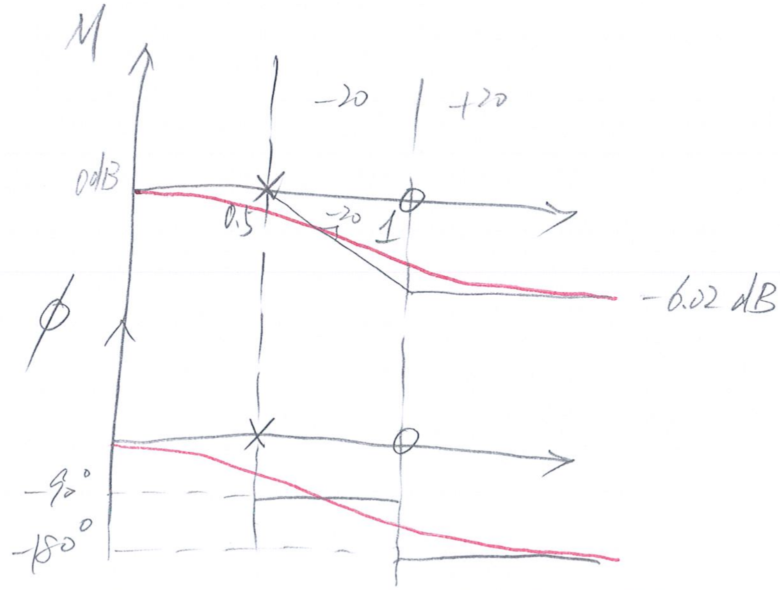
W_c : 1 ("unstable zero"), 0.5 ("stable pole")

$$\lim_{\omega \to 0} \angle G(j\omega) = 0^\circ$$

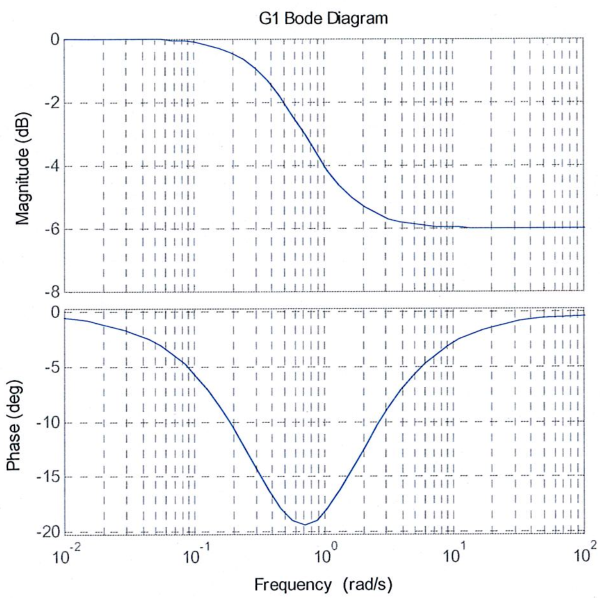
$$\lim_{\omega \to 0} |G(j\omega)| = 1$$

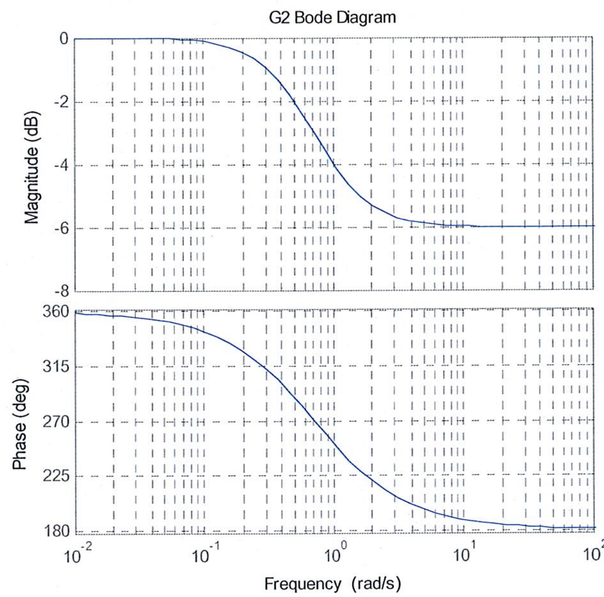
$$\lim_{\omega \to \infty} |G(j\omega)| = \frac{1}{2}$$

- Unstable zeroes result in a non-minimum phase behavior. The magnitude response corresponding to such a zero stays the same as for stable zeroes, but the phase angle at $\omega = \infty$ becomes -90°. [*Pg 415 in Section 7-2*]
- Unstable poles result in a non-minimum phase behavior. The magnitude response corresponding to such a pole is unchanged, but the phase angle at $\omega = \infty$ becomes +90°.



Problem B-7-3





[Matlab code]

```
clear all; close all; clc

num1=[1 1]; %numerator of G1(s)
den1=[2 1]; %denominator of G1(s)

num2=[-1 1]; %numerator of G2(s)
den2=[2 1];  %denominator of G2(s)

figure()
subplot(1,2,1)
bode(num1, den1)
grid on
title ('G1 Bode Diagram')

subplot(1,2,2)
bode(num2, den2)
grid on
title ('G2 Bode Diagram')
```

B-7-4

$$G(s) = \frac{10(s^2+0.4s+1)}{s(s^2+0.8s+9)}$$

Solution:

Corner frequencies: $1^{\times 2}$ ("stable zero")

$3^{\times 2}$ ("stable pole")

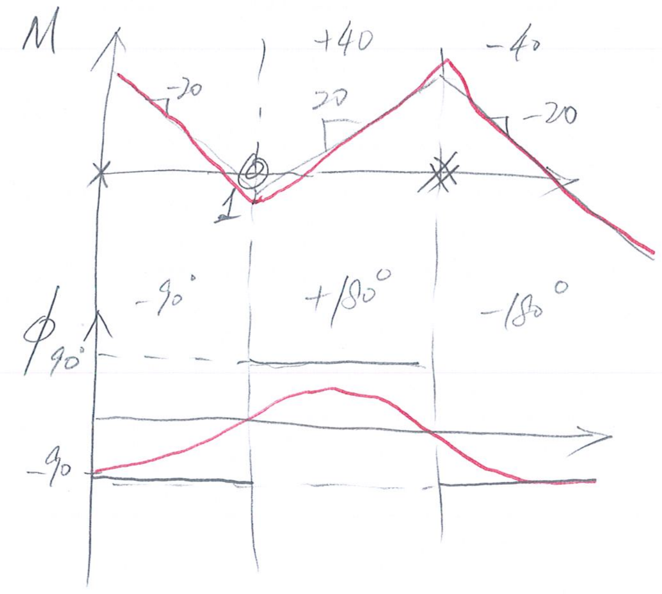
0.

$$\lim_{\omega\to 0^+} \angle G(j\omega) = \lim_{\omega\to 0} \angle \frac{10(-\omega^2+0.4j\omega+1)}{j\omega(-\omega^2+0.8j\omega+9)}$$

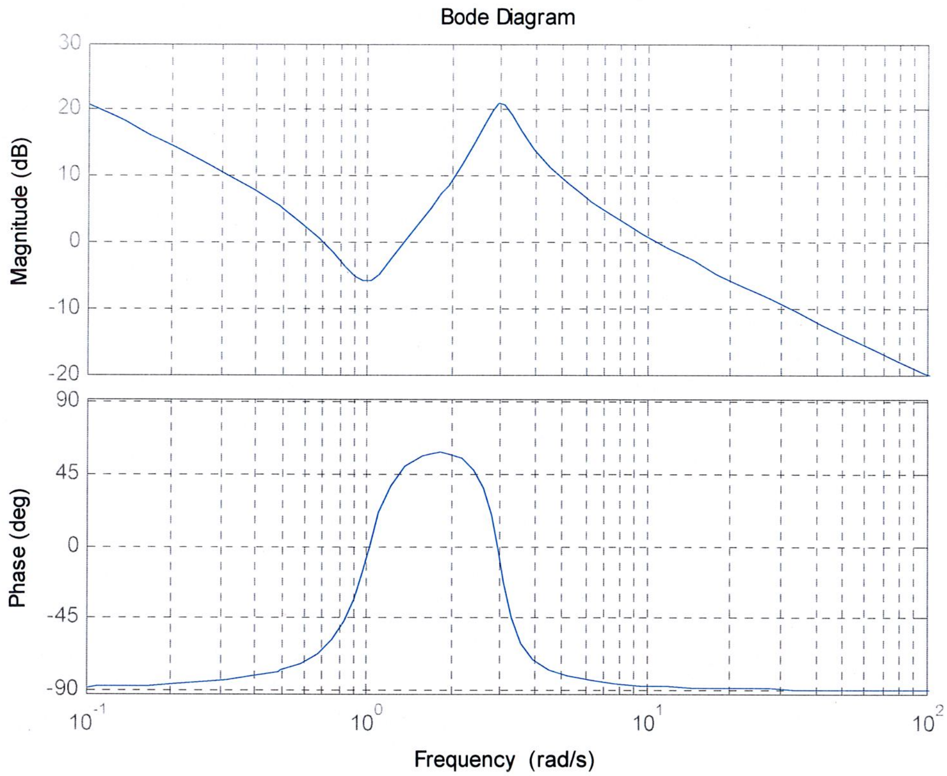
$$= \lim_{\omega\to 0}\left(\angle 10 + \angle(-\omega^2+0.4j\omega+1) - \angle j\omega - \angle(-\omega^2+0.8j\omega+9)\right)$$

$$= -90^\circ$$

(Crossover frequency: $|G(j\omega^*)| = 1$ or $0\,dB$)



B-7-4



[Matlab code]

```
sys=tf([10 4 10],[1 0.8 9 0])

figure()
bode(sys)
grid on
```

B-7-7

$$G(s) = K\frac{(T_a s+1)(T_b s+1)}{s^2(Ts+1)}$$

(a) $T_a > T_b > T$

(b) $T > T_a > T_b$

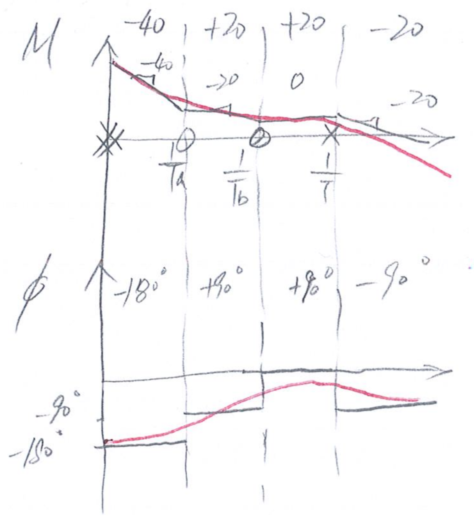
Assume $T_a > T_b$ WLOG

Solution:

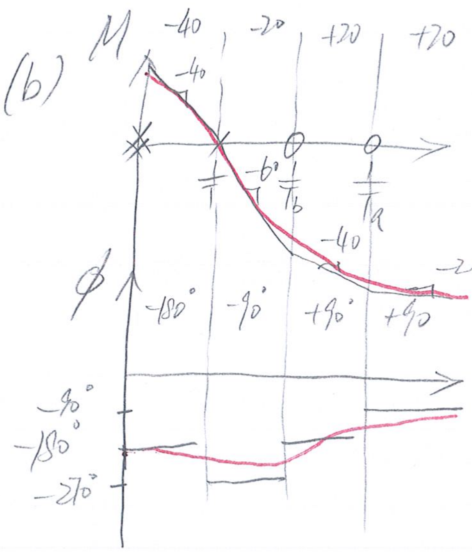
Corner frequencies: $\underbrace{\frac{1}{T_a}, \frac{1}{T_b}}_{\text{("stable zero")}}, \underbrace{\frac{1}{T}}_{\text{("stable pole")}}, 0^{\times 2}$

$$\lim_{\omega \to 0} \angle G(j\omega) = -180^\circ$$

(a)



(b)



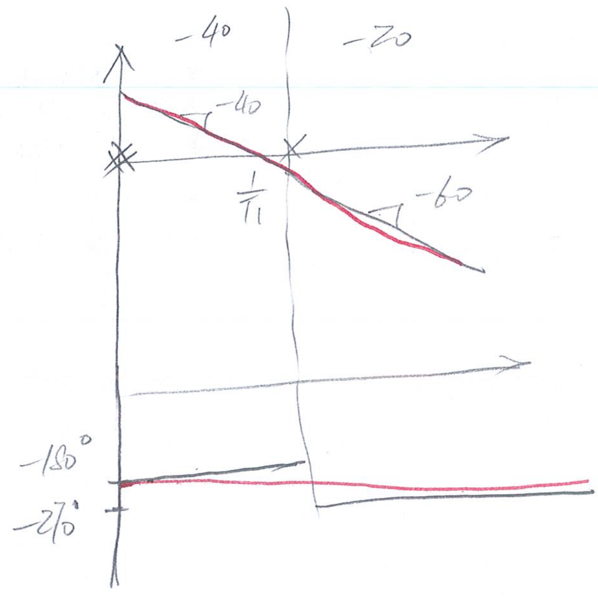
B-7-9.

$$G = \frac{K}{s^2(T_1 s + 1)}$$

Solution:

Corner frequencies: $0^{\times 2}$, $\underbrace{\frac{1}{T_1}}$
("stable pole")

As $\omega \to 0$, $G(j\omega) \to +\infty \angle -180°$



B-7-13

$$G = \frac{1}{s^3 + 0.2s^2 + s + 1}$$

Solution:

poles of G: -0.7246, $\underbrace{0.2623 \pm j1.1451}_{\text{"unstable"}}$

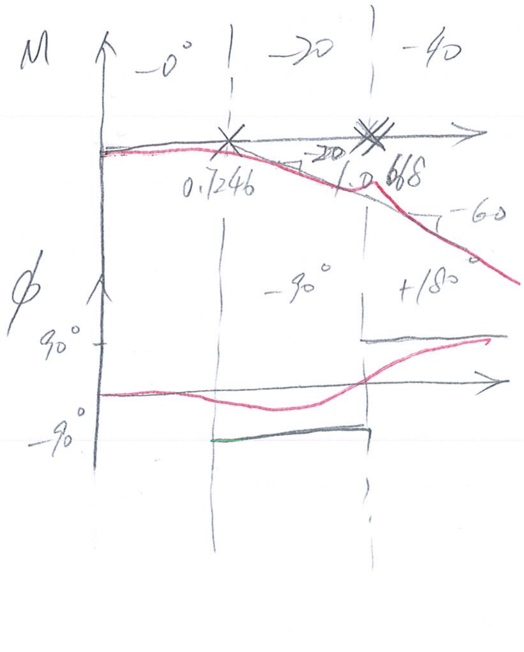
$\Downarrow$

$$G = \frac{1}{(s+0.7246)(s^2 - 0.5246s + 1.1381)}$$

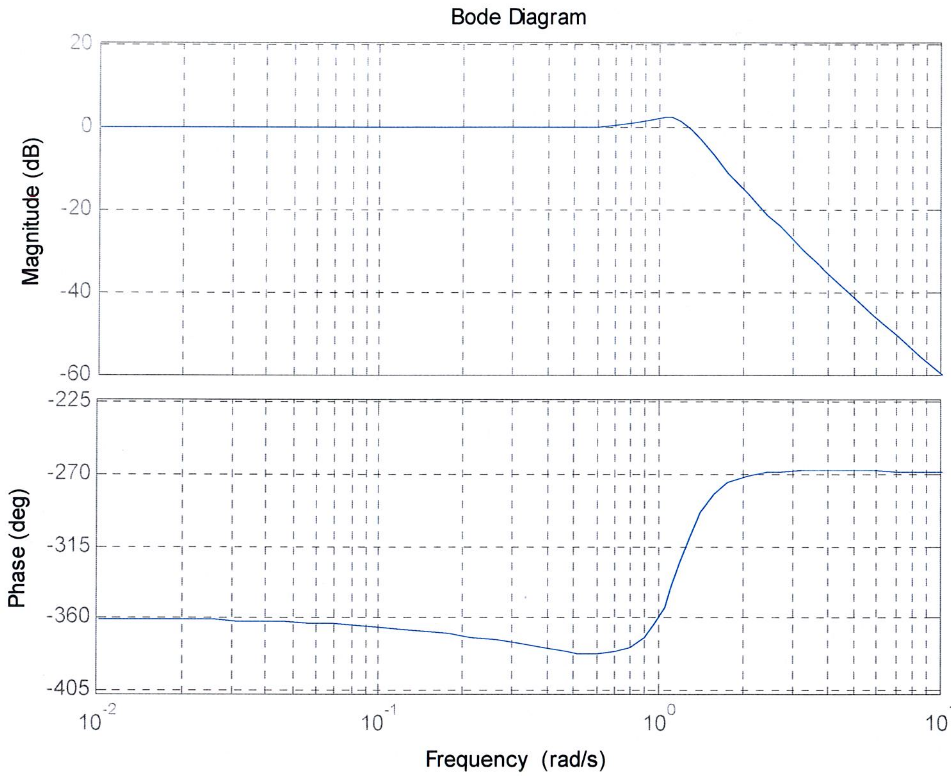
$$G(j\omega) = \frac{1}{0.7246 \times 1.1381 \left(\frac{j\omega}{0.7246} + 1\right)\left(1 - \frac{0.5246}{1.1381} j\omega + \frac{(j\omega)^2}{1.1381}\right)}$$

Corner frequencies: $+0.7246$, $\sqrt{1.1381}^{\times 2}$ ("unstable poles")

$\lim_{\omega \to 0} \angle G(j\omega) = 0°$ $\quad \lim_{\omega \to 0} |G(j\omega)| = 1 = 0\,dB$



B-7-13



[Matlab code]

```
sys=tf(1,[1 0.2 1 1])

figure()
bode(sys)
grid on
```

B-7-14.

$$G = \frac{s^2+2s+1}{s^3+0.2s^2+s+1}$$

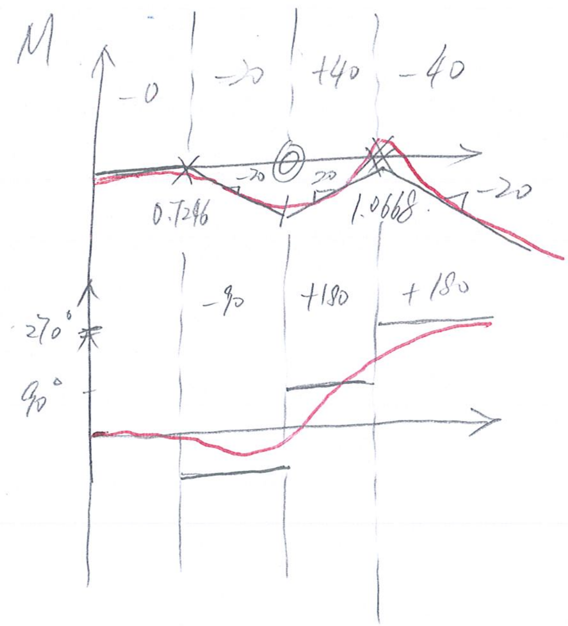
Solution:

From B-7-13, we know poles are -0.7246 and $0.2623 \pm j1.1451$.

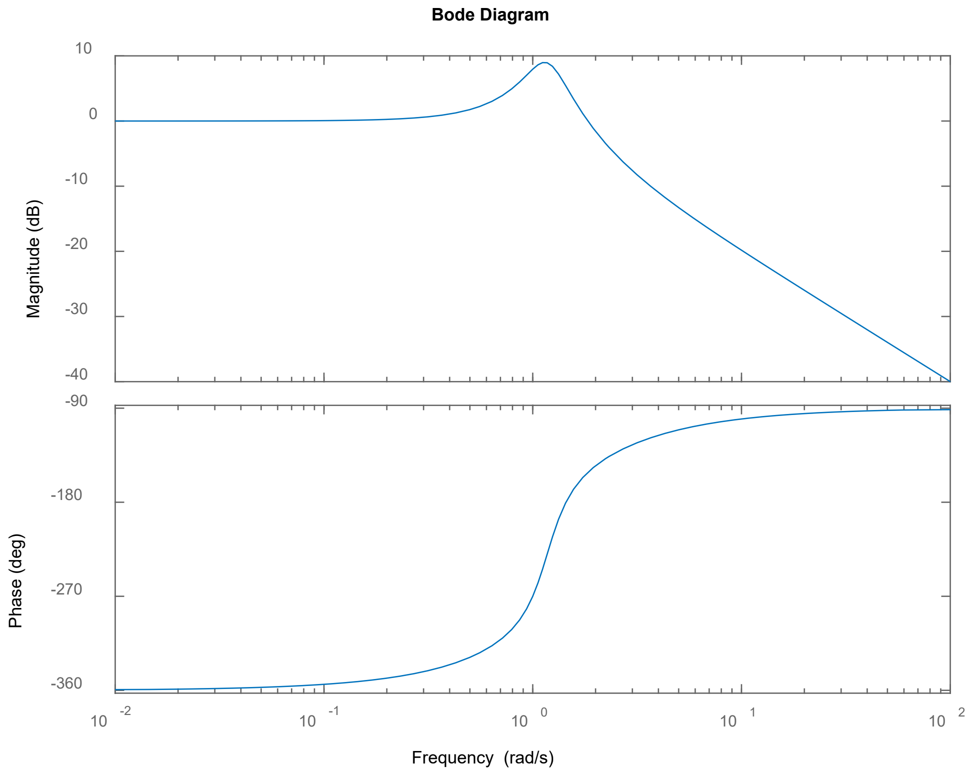
Corner frequencies: $\underbrace{0.7246}_{}$, $\underbrace{1}_{}^{\times 2}$, $1.0668^{\times 2}$

("stable pole") ("stable zeros") ("unstable poles")

As $\omega \to 0$, $G(j\omega) \to 1 \angle 0°$



B-7-14




```
G = tf([1 2 1],[1 0.2 1 1])

bode(G)
```