AAE440: Space Attitude Dynamics

HW4: Kinematical Differentials & Orientations

Dr. Howell

Tomoki Koike

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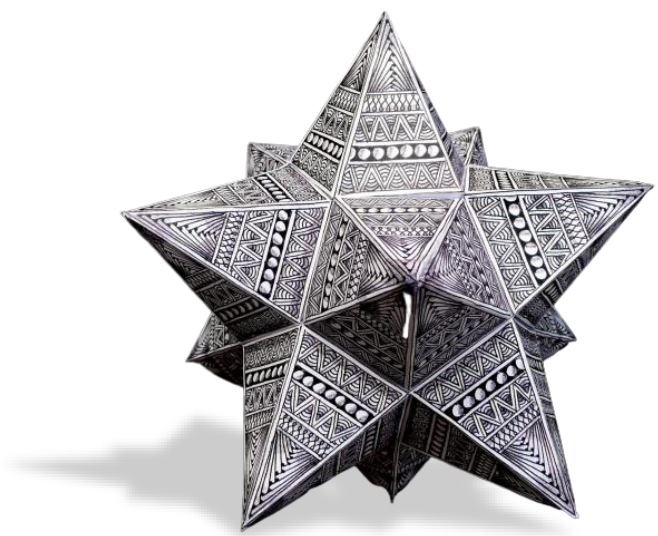
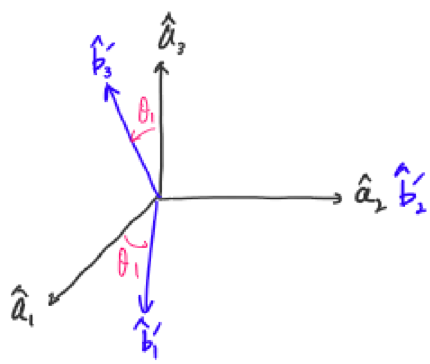


Problem 1: (a) Derive the final form for the direction cosine matrix for the following types of successive rotation sequences:

Space-two 2 – 3 – 2
Body-two 2– 3 – 2

(1) for space-two 2-3-2

ROT1 : ${}^{A}\hat{\lambda}^{B'} = \hat{a}_1 = \hat{b}'_2$



${}^{A}C^{B'}$	$\hat{b}'_1$	$\hat{b}'_2$	$\hat{b}'_3$
$\hat{a}_1$	c_1	0	s_1
$\hat{a}_2$	0	1	0
$\hat{a}_3$	$-s_1$	0	c_1

ROT2 : ${}^{B'}\hat{\lambda}^{B''} = \hat{a}_3 = -s_1\hat{b}_1 + c_1\hat{b}_3$

$$\Rightarrow \lambda_1 = -s_1 \ , \ \lambda_2 = 0 \ , \ \lambda_3 = c_1 \qquad \theta \Rightarrow \theta_2$$

thus,

$$
{}^{B'}C^{B''} = \begin{cases}
C_{11} = \cos\theta + \lambda_1^2(1-\cos\theta) & = c_2 + s_1^2(1-c_2) \\
C_{12} = -\lambda_3\sin\theta + \cancel{\lambda_1\lambda_2(1-\cos\theta)} & = -c_1 s_2 \\
C_{13} = \cancel{\lambda_2\sin\theta} + \lambda_3\lambda_1(1-\cos\theta) & = -c_1 s_1(1-c_2) \\
C_{21} = \lambda_3\sin\theta + \cancel{\lambda_1\lambda_2(1-\cos\theta)} & = c_1 s_2 \\
C_{22} = \cos\theta + \cancel{\lambda_2^2(1-\cos\theta)} & = c_2 \\
C_{23} = -\lambda_1\sin\theta + \cancel{\lambda_2\lambda_3(1-\cos\theta)} & = s_1 s_2 \\
C_{31} = -\cancel{\lambda_2\sin\theta} + \lambda_3\lambda_1(1-\cos\theta) & = -c_1 s_1(1-c_2) \\
C_{32} = \lambda_1\sin\theta + \cancel{\lambda_2\lambda_3(1-\cos\theta)} & = -s_1 s_2 \\
C_{33} = \cos\theta + \lambda_3^2(1-\cos\theta) & = c_2 + c_1^2(1-c_2)
\end{cases}
$$

$$ {}^{\beta'}C^{\beta''} = \begin{bmatrix} c_2 + s_1^2(1-c_2) & -c_1 s_2 & -c_1 s_1(1-c_2) \\ c_1 s_2 & c_2 & s_1 s_2 \\ -c_1 s_1(1-c_2) & -s_1 s_2 & c_2 + c_1^2(1-c_2) \end{bmatrix} $$

Rot 3 : $ {}^{\beta''}\hat{\lambda} = {}^{\beta''}\lambda_1 \hat{b}_1'' + {}^{\beta''}\lambda_2 \hat{b}_2'' + {}^{\beta''}\lambda_3 \hat{b}_3'' = {}^{A}\lambda_1 \hat{a}_1 + {}^{A}\lambda_2 \hat{a}_2 + {}^{A}\lambda_3 \hat{a}_3 $

$$ \begin{aligned} [\,{}^{\beta''}\lambda_1 \quad {}^{\beta''}\lambda_2 \quad {}^{\beta''}\lambda_3\,] &= [\,{}^{A}\lambda_1 \quad {}^{A}\lambda_2 \quad {}^{A}\lambda_3\,]\, {}^{A}C^{\beta''} \\ &= [\,0 \quad 1 \quad 0\,]\, {}^{A}C^{\beta'}\, {}^{\beta'}C^{\beta''} \\ &= [\,0 \quad 1 \quad 0\,] \begin{bmatrix} c_1 & 0 & s_1 \\ 0 & 1 & 0 \\ -s_1 & 0 & c_1 \end{bmatrix} {}^{\beta'}C^{\beta''} \\ &= [\,0 \quad 1 \quad 0\,] \begin{bmatrix} c_2 + s_1^2(1-c_2) & -c_1 s_2 & -c_1 s_1(1-c_2) \\ c_1 s_2 & c_2 & s_1 s_2 \\ -c_1 s_1(1-c_2) & -s_1 s_2 & c_2 + c_1^2(1-c_2) \end{bmatrix} \\ &= [\,c_1 s_2 \quad c_2 \quad s_1 s_2\,] \end{aligned} $$

this becomes

$$ {}^{\beta''}\lambda^{\beta} = \hat{a}_2 = c_1 s_2 \hat{b}_1'' + c_2 \hat{b}_2'' + s_1 s_2 \hat{b}_3'' $$

$$ {}^{\beta''}\hat{\theta}^{B} = \theta_3 $$

$$
{}^{B''}C^{B}\quad
\begin{cases}
C_{11} = \cos\theta + \lambda_1^2(1-\cos\theta) & = C_3 + C_1^2 S_2^2(1-C_3) \\
C_{12} = -\lambda_3\sin\theta + \lambda_1\lambda_2(1-\cos\theta) & = -S_1 S_2 S_3 + C_1 C_2 C_3(1-C_3) \\
C_{13} = \lambda_2\sin\theta + \lambda_3\lambda_1(1-\cos\theta) & = C_2 S_3 + S_1 S_2^2 C_1(1-C_3) \\
C_{21} = \lambda_3\sin\theta + \lambda_1\lambda_2(1-\cos\theta) & = S_1 S_2 S_3 + C_1 S_2 C_2(1-C_3) \\
C_{22} = \cos\theta + \lambda_2^2(1-\cos\theta) & = C_3 + C_2^2(1-C_3) \\
C_{23} = -\lambda_1\sin\theta + \lambda_2\lambda_3(1-\cos\theta) & = -C_1 S_2 S_3 + C_2 S_1 S_2(1-C_3) \\
C_{31} = -\lambda_2\sin\theta + \lambda_3\lambda_1(1-\cos\theta) & = -C_2 S_3 + S_1 S_2^2 C_1(1-C_3) \\
C_{32} = \lambda_1\sin\theta + \lambda_2\lambda_3(1-\cos\theta) & = C_1 S_2 S_3 + C_2 S_1 S_2(1-C_3) \\
C_{33} = \cos\theta + \lambda_3^2(1-\cos\theta) & = C_3 + S_1^2 S_2^2(1-C_3)
\end{cases}
$$

$$
{}^{B''}C^{B} = \begin{bmatrix}
C_3 + C_1^2 S_2^2(1-C_3) & -S_1 S_2 S_3 + C_1 S_2 C_2(1-C_3) & C_2 S_3 + S_1 S_2^2 C_1(1-C_3) \\
S_1 S_2 S_3 + C_1 S_2 C_2(1-C_3) & C_3 + C_2^2(1-C_3) & -C_1 S_2 S_3 + C_2 S_1 S_2(1-C_3) \\
-C_2 S_3 + S_1 S_2^2 C_1(1-C_3) & C_1 S_2 S_3 + C_2 S_1 S_2(1-C_3) & C_3 + S_1^2 S_2^2(1-C_3)
\end{bmatrix}
$$

$$
{}^{A}C^{B} = {}^{A}C^{B'}\,{}^{B'}C^{B''}\,{}^{B''}C^{B}
$$

$$
= \begin{bmatrix} C_1 & 0 & S_1 \\ 0 & 1 & 0 \\ -S_1 & 0 & C_1 \end{bmatrix}
\begin{bmatrix}
C_2 + S_1^2(1-C_2) & -C_1 S_2 & -C_1 S_1(1-C_2) \\
C_1 S_2 & C_2 & S_1 S_2 \\
-C_1 S_1(1-C_2) & -S_1 S_2 & C_2 + C_1^2(1-C_2)
\end{bmatrix} {}^{B''}C^{B}
$$

$$
= \begin{bmatrix}
C_1 C_2 + C_1 S_1^2(1-C_2) - C_1 S_1^2(1-C_2) & -C_1^2 S_2 - S_1^2 S_2 & -C_1^2 S_1(1-C_2) + S_1 C_2 + S_1 C_1^2(1-C_2) \\
C_1 S_2 & C_2 & S_1 S_2 \\
C_1 S_1^2(1-C_2) + C_1 C_2 + C_1^3(1-C_2) & C_1 S_1 S_2 - S_1 S_2 C_1 & C_1 S_1^2(1-C_2) + C_1 C_2 + C_1^2(1-C_2)
\end{bmatrix} {}^{B''}C^{B}
$$

$$= \begin{bmatrix} C_1C_2 & -S_2 & S_1C_2 \\ C_1S_2 & C_2 & S_1S_2 \\ -S_1 & 0 & C_1 \end{bmatrix} \begin{bmatrix} C_3+C_1^2S_2^2(1-C_3) & -S_1S_2S_3+C_1S_2C_2(1-C_3) & C_2S_3+S_1S_2^2C_1(1-C_3) \\ S_1S_2S_3+C_1S_2C_2(1-C_3) & C_3+C_2^2(1-C_3) & -C_1S_2S_3+C_2S_1S_2(1-C_3) \\ -C_2S_3+S_1S_2^2C_1(1-C_3) & C_1S_2S_3+C_2S_1S_2(1-C_3) & C_3+S_1^2S_2^2(1-C_3) \end{bmatrix}$$

$$= \left[\begin{array}{l} C_1C_2C_3+C_1^3C_2S_2^2(1-C_3)-S_1S_2^2S_3-C_1S_2^2C_2(1-C_3)-S_1C_2^2S_3+S_1^2S_2^2C_1C_2(1-C_3) \\ C_1S_2C_3+C_1^3S_2^3(1-C_3)+S_1S_2S_3C_2+C_1S_2C_2^2(1-C_3)-S_1S_2S_3C_2+S_1^2S_2^3C_1(1-C_3) \\ -C_3S_1-S_1C_1^2S_2^2(1-C_3)-C_1C_2S_3+S_1S_2^2C_1^2(1-C_3) \end{array}\right.$$

$$\begin{array}{l} -C_1C_2S_1S_2S_3+C_1^2S_2C_2^2(1-C_3)-S_2C_3-S_1C_2^2(1-C_3)+C_1C_2S_1S_2S_3+S_1^2C_2^2S_2(1-C_3) \\ \longrightarrow \quad -C_1S_1S_2^2S_3+C_1^2S_2^2C_2(1-C_3)+C_2C_3+C_1^3(1-C_3)+C_1S_1S_2^2S_3+C_1S_1^2S_2^2(1-C_3) \\ S_1^2S_2S_3-S_1C_1S_2C_2(1-C_3)+C_1^2S_2S_3+C_1C_2S_1S_2(1-C_3) \end{array}$$

$$\left.\begin{array}{l} C_1C_2^2S_3+C_1^2S_1S_2^2C_2(1-C_3)+C_1S_2^2S_3-C_2S_1S_2^2(1-C_3)+S_1C_2C_3+S_1^3S_2^2C_2(1-C_3) \\ \longrightarrow \quad C_1C_2S_2S_3+S_1S_2^3C_1^2(1-C_3)-C_1C_2S_2S_3+C_2^2S_1S_2(1-C_3)+S_1S_2C_3+S_1^3S_2^3(1-C_3) \\ -S_1C_2S_3-S_1^3S_2^2C_1(1-C_3)+C_1C_3+C_1S_1^2S_2^2(1-C_3) \end{array}\right]$$

$$\begin{aligned} C_{11} &= C_1C_2C_3+C_1^3C_2S_2^2(1-C_3)-S_1S_2^2S_3-C_1S_2^2C_2(1-C_3)-S_1C_2^2S_3+S_1^2S_2^2C_1C_2(1-C_3) \\ &= C_1C_2C_3+(1-C_3)(C_1^3C_2S_2^2-C_1S_2^2C_2+S_1^2S_2^2C_1C_2)-S_1S_2^2S_3-S_1C_2^2S_3 \\ &= C_1C_2C_3-S_1S_3(S_2^2+C_2^2) \\ &= \underline{C_1C_2C_3-S_1S_3} \end{aligned}$$

$$C_{12} = -c_1c_2s_1s_2s_3 + c_1^2 s_2 c_2^2 (1-c_3) - s_2c_3 - s_1 c_2^2 (1-c_3) + c_1c_2s_1s_2s_3 + s_1^2 c_2^2 s_2 (1-c_3)$$

$$= (1-c_3)(c_1^2 s_1 c_2^2 - s_2 c_2^2 + s_1^2 c_2^2 s_2) - s_2 c_3$$

$$= -s_2 c_3$$

$$C_{13} = c_1 c_2^2 s_3 + c_1^2 s_1 s_2^2 c_2 (1-c_3) + c_1 s_2^2 s_3 - c_2 s_1 s_2^2 (1-c_2) + s_1 c_2 c_3 + s_1^3 s_2^2 c_2 (1-c_3)$$

$$= c_1 s_3 + s_1 c_2 c_3 + (1+c_3) s_1 c_2 (c_1^2 s_2^2 - s_2^2 + s_1^2 s_2^2)$$

$$= c_1 s_3 + s_1 c_2 c_3 + (1+c_3)(s_1 c_2)(s_1^2 - s_2^2)$$

$$= c_1 s_3 + s_1 c_2 c_3$$

$$C_{21} = c_1 s_2 c_3 + c_1^3 s_2^3 (1-c_3) + s_1 s_2 s_3 c_2 + c_1 s_2 c_2^2 (1-c_3) - s_1 s_2 s_3 c_2 + s_1^2 s_2^3 c_1 (1-c_3)$$

$$= c_1 s_2 c_3 + (1-c_3) c_1 s_2 (c_1^2 s_2^2 + c_2^2 + s_1^2 s_2^2)$$

$$= c_1 s_2 c_3 + (1-c_3)(c_1 s_2)(c_2^2 + s_2^2)$$

$$= c_1 s_2$$

$$C_{22} = -c_1 s_1 s_2^2 s_3 + c_1^2 s_2^2 c_2 (1-c_3) + c_2 c_3 + c_2^3 (1-c_3) + c_1 s_1 s_2^2 s_3 + c_2 s_1^2 s_2^2 (1-c_3)$$

$$= c_2 c_3 + (1-c_3)(c_1^2 s_2^2 c_2 + c_2^3 + c_2 s_1^2 s_2^2)$$

$$= c_2 c_3 + (1-c_3) c_2 (c_1^2 s_2^2 + c_2^2 + s_1^2 s_2^2)$$

$$= c_2 c_3 + c_2 - c_2 c_3 (s_2^2 + c_2^2)$$

$$= c_2$$

$$C_{23} = -c_1 s_1 s_2^2 s_3 + c_1^2 s_2^2 c_2 (1-c_3) + c_2 c_3 + c_2^3 (1-c_3) + c_1 s_1 s_2^2 s_3 + c_2 s_1^2 s_2^2 (1-c_3)$$

$$= s_1 s_2 c_3 + (1-c_3)(s_1 s_2^3 c_1^2 + c_2^2 s_1 s_2 + s_1^3 s_2^3)$$

$$= s_1 s_2 c_3 + (1-c_3) s_1 s_2 (c_1^2 s_2^2 + c_2^2 + s_1^2 s_3^2)$$

$$= s_1 s_2$$

$$C_{31} = -C_3S_1 - S_1C_1^2S_2^2(1-C_3) - C_1C_2S_3 + S_1S_2^2C_1^2(1-C_3)$$

$$= \underline{-C_3S_1 - C_1C_2S_3}$$

$$C_{32} = S_1^2S_2S_3 - S_1C_1S_2C_2(1-C_3) + C_1^2S_2S_3 + C_1C_2S_1S_2(1-C_3)$$

$$= S_1^2S_2S_3 + C_1^2S_2S_3$$

$$= \underline{S_2S_3}$$

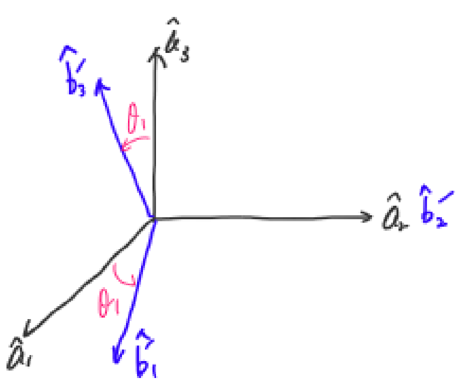
$$C_{33} = -S_1C_2S_3 - S_1^2S_2^2C_1(1-C_3) + C_1C_3 + C_1S_1^2S_2^2(1-C_3)$$

$$= \underline{-S_1C_2S_3 + C_1C_3}$$

$$\boxed{{}^{A}C^{B} = \begin{bmatrix} C_1C_2C_3 - S_3S_1 & -S_2C_3 & S_1C_2C_3 + S_3C_1 \\ C_1S_2 & C_2 & S_1S_2 \\ -C_1C_2S_3 - C_3S_1 & S_2S_3 & -S_1C_2S_3 + C_3C_1 \end{bmatrix}}$$

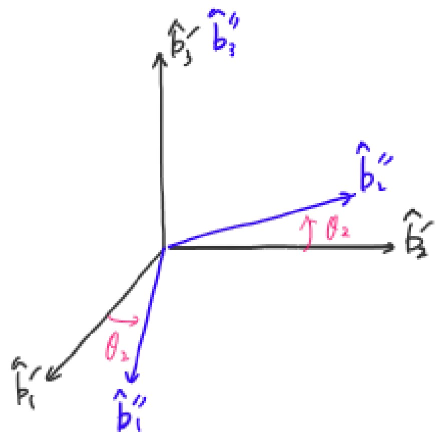
(2) Body-two 2-3-2

Rot 1: ${}^{A}\hat{\lambda}^{B'} = \hat{a}_2 = \hat{b}_1$



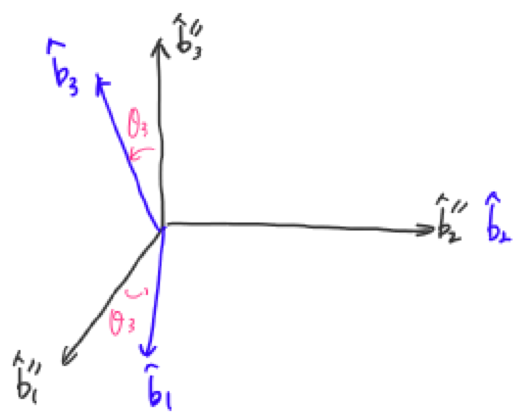
${}^{A}C^{B'}$	$\hat{b}_1'$	$\hat{b}_2'$	$\hat{b}_3'$
$\hat{a}_1$	C_1	0	S_1
$\hat{a}_2$	0	1	0
$\hat{a}_3$	$-S_1$	0	C_1

Rot 2: ${}^{B'}\hat{\lambda}^{B''} = \hat{b}'_3 = \hat{b}''_3$



${}^{B'}C^{B''}$	$\hat{b}''_1$	$\hat{b}''_2$	$\hat{b}''_3$
$\hat{b}'_1$	c_2	$-s_2$	0
$\hat{b}'_2$	s_2	c_2	0
$\hat{b}'_3$	0	0	1

Rot 3: ${}^{B''}\hat{\lambda}^{B} = \hat{b}''_2 = \hat{b}_2$



${}^{B''}C^{B}$	$\hat{b}_1$	$\hat{b}_2$	$\hat{b}_3$
$\hat{b}''$	c_3	0	s_3
$\hat{b}''_2$	0	1	0
$\hat{b}''_3$	$-s_3$	0	c_3

$${}^{A}C^{B} = {}^{A}C^{B'}\,{}^{B'}C^{B''}\,{}^{B''}C^{B}$$

$$= \begin{bmatrix} c_1 & 0 & s_1 \\ 0 & 1 & 0 \\ -s_1 & 0 & c_1 \end{bmatrix} \begin{bmatrix} c_2 & -s_2 & 0 \\ s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_3 & 0 & s_3 \\ 0 & 1 & 0 \\ -s_3 & 0 & c_3 \end{bmatrix}$$

$$= \begin{bmatrix} C_1C_2 & -C_1S_2 & S_1 \\ S_2 & C_2 & 0 \\ -S_1C_2 & S_1S_2 & C_1 \end{bmatrix} \begin{bmatrix} C_3 & 0 & S_2 \\ 0 & 1 & 0 \\ -S_3 & 0 & C_3 \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} C_1C_2C_3 - S_3S_1 & -C_1S_2 & C_1C_2C_3 + C_3S_1 \\ S_2C_3 & C_2 & S_2S_3 \\ -S_1C_2C_3 - S_3C_1 & S_1S_2 & -S_1C_2S_3 + C_3C_1 \end{bmatrix}}$$

(b) Assume that data is delivered and that the body-three 1-2-3 angles are known and given As $\phi_1 = +60^\circ$, $\phi_2 = -210^\circ$, $\phi_3 = +165^\circ$. Determine the equivalent space-two 2-3-2 angles and the equivalent body-two 2-3-2 angles.
Should the two sets of 2-3-2 angles be the same? Why or why not?

$$\begin{bmatrix} C_2C_3 & -C_2S_3 & S_2 \\ S_1S_2C_3+S_3C_1 & -S_1S_2S_3+C_3C_1 & -S_1C_2 \\ -C_1S_2C_3+S_3S_1 & C_1S_2S_3+C_3S_1 & C_1C_2 \end{bmatrix} \longrightarrow \begin{matrix} \varphi_1 = 60^\circ \\ \varphi_2 = -210^\circ \\ \varphi_3 = 165^\circ \end{matrix}$$

use MATLAB to calculate

$$ {}^{A}C^{B} = \begin{bmatrix} 0.8365 & 0.2241 & 0.5000 \\ -0.2888 & -0.5950 & 0.7500 \\ 0.4656 & -0.7718 & -0.4330 \end{bmatrix}$$

(1)

$$ {}^{A}C^{B} = \begin{bmatrix} C_1C_2C_3-S_3S_1 & -S_2C_3 & S_1C_2C_3+S_3C_1 \\ C_1S_2 & C_2 & S_1S_2 \\ -C_1C_2S_3-C_3S_1 & S_2S_3 & -S_1C_2S_3+C_3C_1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.8365 & 0.2241 & 0.5000 \\ -0.2888 & -0.5950 & 0.7500 \\ 0.4656 & -0.7718 & -0.4330 \end{bmatrix}$$

$$\cos\theta_2 = -0.5950 \leftrightarrow \theta_2 = 126.5151^\circ,\ -126.5151^\circ$$

when $\theta_2 = 126.5151°$

$\sin\theta_2 = 0.8037$

$\sin\theta_1 \sin\theta_2 = C_{23}$

$\sin\theta_1 = \frac{0.75}{\sin\theta_2}$

$\theta_1 = \begin{cases} 68.9367° \\ 111.0633° \end{cases}$

$\cos\theta_1 \sin\theta_2 = C_{21}$

$\cos\theta_1 = -\frac{0.2888}{\sin\theta_2}$

$\theta_1 = \begin{cases} 111.0633° \\ -111.0633° \end{cases}$

$-\sin\theta_2 \cos\theta_3 = C_{12}$

$\cos\theta_3 = -\frac{0.2241}{\sin\theta_2}$

$\theta_3 = \begin{cases} 106.1940° \\ -106.1940° \end{cases}$

$\sin\theta_2 \sin\theta_3 = C_{32}$

$\sin\theta_3 = -\frac{0.7718}{\sin\theta_2}$

$\theta_3 = \begin{cases} -73.8060° \\ -106.1940° \end{cases}$

when $\theta_2 = -126.5151°$

$\sin\theta_2 = -0.8037$

$\sin\theta_1 \sin\theta_2 = C_{23}$

$\sin\theta_1 = \frac{0.75}{\sin\theta_2}$

$\theta_1 = \begin{cases} -68.9367° \\ -111.0633° \end{cases}$

$\cos\theta_1 \sin\theta_2 = C_{21}$

$\cos\theta_1 = -\frac{0.2888}{\sin\theta_2}$

$\theta_1 = \begin{cases} 68.9367° \\ -68.9367° \end{cases}$

$-\sin\theta_2 \cos\theta_3 = C_{12}$

$\cos\theta_3 = -\frac{0.2241}{\sin\theta_2}$

$\theta_3 = \begin{cases} 106.1940° \\ -106.1940° \end{cases}$

$\sin\theta_2 \sin\theta_3 = C_{32}$

$\sin\theta_3 = -\frac{0.7718}{\sin\theta_2}$

$\theta_3 = \begin{cases} -73.8060° \\ -106.1940° \end{cases}$

$$\boxed{\begin{array}{lll} \hat{a}_2 : & \theta_1 = 111.0633° , & -68.9367° \\ \hat{a}_3 : & \theta_2 = 126.5151° , & -126.5151° \\ \hat{a}_2 : & \theta_3 = -106.1940° , & -106.1940° \end{array}}$$

Body-two 2-3-2

$$\begin{bmatrix} c_1c_2c_3 - s_3s_1 & -c_1s_2 & c_1c_2c_3+c_3s_1 \\ s_2c_3 & c_2 & s_2s_3 \\ -s_1c_2c_3 - s_3c_1 & s_1s_2 & -s_1c_2s_3+c_3c_1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.8365 & 0.2241 & 0.5000 \\ -0.2888 & -0.5950 & 0.7500 \\ 0.4656 & -0.7718 & -0.4330 \end{bmatrix}$$

$$\cos\theta_2 = -0.5950$$
$$\theta_2 = 126.5151°, -126.5151°$$

when $\theta_2 = 126.5151°$

$\sin\theta_2 = 0.8037$

$\sin\theta_1 \sin\theta_2 = C_{32}$

$$\sin\theta_1 = -\frac{0.7718}{\sin\theta_2}$$

$$\theta_1 = \begin{cases} -73.8060° \\ -106.1940 \end{cases}$$

$-\cos\theta_1 \sin\theta_2 = C_{12}$

$$\cos\theta_1 = -\frac{0.2241}{\sin\theta_2}$$

$$\theta_1 = \begin{cases} 106.1940° \\ -106.1940° \end{cases}$$

$\sin\theta_2 \cos\theta_3 = C_{21}$

$$\cos\theta_3 = -\frac{0.2888}{\sin\theta_2}$$

$$\theta_3 = \begin{cases} 111.0633° \\ -111.0633° \end{cases}$$

$\sin\theta_2 \sin\theta_3 = C_{23}$

$$\sin\theta_3 = \frac{0.75}{\sin\theta_2}$$

$$\theta_3 = \begin{cases} 68.9367° \\ 111.0633° \end{cases}$$

when $\theta_2 = -126.5151^\circ$

$$\sin\theta_2 = -0.8037$$

$$\sin\theta_1 \sin\theta_2 = C_{32}$$

$$\sin\theta_1 = -\frac{0.7718}{\sin\theta_2}$$

$$\theta_1 = \begin{cases} 73.8060^\circ \\ 106.1940^\circ \end{cases}$$

$$\cos\theta_1 \sin\theta_2 = C_{12}$$

$$\cos\theta_1 = -\frac{0.2241}{\sin\theta_2}$$

$$\theta_1 = \begin{cases} -73.8060 \\ 73.8060 \end{cases}$$

$$\sin\theta_2 \sin\theta_3 = C_{23}$$

$$\theta_3 = \begin{cases} 68.9367^\circ \\ 111.0633^\circ \end{cases}$$

$$-\sin\theta_2 \cos\theta_3 = C_{21}$$

$$\theta_3 = \begin{cases} 111.0633 \\ -111.0633 \end{cases}$$

$$\boxed{\begin{array}{lll} \hat{a}_2 & \theta_1 = -106.1940^\circ, & 73.8060^\circ \\ \hat{b}_3 & \theta_2 = 126.5151^\circ, & -126.5151^\circ \\ \hat{b}_2 & \theta_3 = 111.0633^\circ, & 111.0633^\circ \end{array}}$$

Body-two 2-3-2	Space-two 2-3-2
① $\theta_1 = -106.1940^\circ$	① $\theta_1 = 111.0633^\circ$
$\theta_2 = 126.5151^\circ$	$\theta_2 = 126.5151^\circ$
$\theta_3 = 111.0633^\circ$	$\theta_3 = -106.1940^\circ$
② $\theta_1 = 73.8060^\circ$	② $\theta_1 = -68.9367^\circ$
$\theta_2 = -126.5151^\circ$	$\theta_2 = -126.5151^\circ$
$\theta_3 = 111.0633^\circ$	$\theta_3 = -106.1940^\circ$

Discussions

☆ for solution ① the angles are the same but in different order

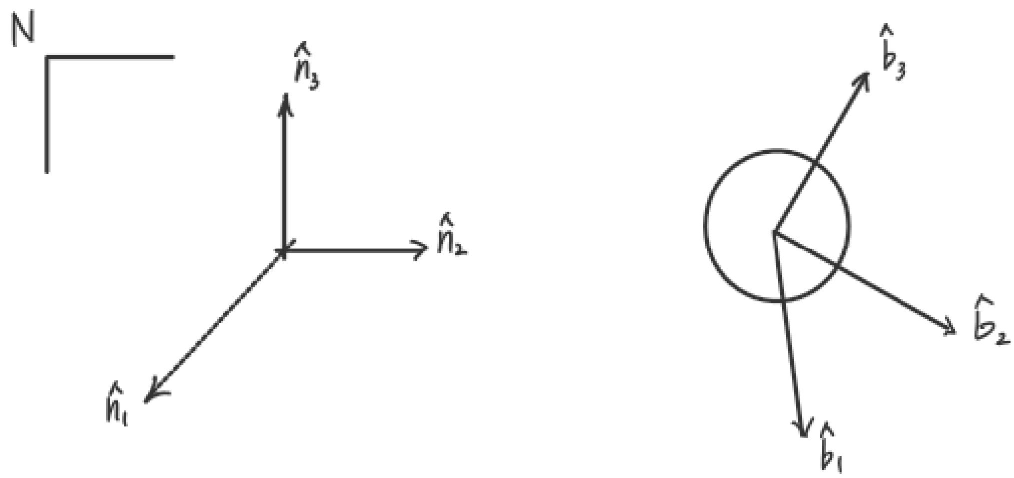
☆ The two did not match perfectly and differed slightly because they have different axis of rotations.

☆ for solution ② only θ_2 match.

Problem 2: A dextral set of orthogonal unit vectors $\hat{n}_i$ is fixed in a reference frame N; a set $\hat{b}_i$ is fixed in body B that can move with respect to N. At any instant, the angular velocity can be written in the form ${}^N\bar{\omega}^B = \omega_i \hat{b}_i$
(orientation and rate of change of orientation) is given as follows

$$ {}^N L^B = 2\hat{n}_1 - \hat{n}_2 + 3\hat{n}_3 \qquad {}^N\theta^B = +260^\circ $$
$$ {}^N\bar{\omega}^B = 1.0\hat{n}_1 - 3.0\hat{n}_2 - 1.5\hat{n}_3 \ \text{rad/s} $$

(a) Determine the rates of change of the Euler parameters, ${}^N\dot{\bar{\varepsilon}}^B$, ${}^N\dot{\varepsilon}_4^B$.
(b) Determine the rates of change of the direction cosines ${}^N\dot{C}^B$
(c) Determine the corresponding angles $\kappa_1, \kappa_2, \kappa_3$ and their rates $\dot{\kappa}_1, \dot{\kappa}_2, \dot{\kappa}_3$ are space-two 1-2-1 angles.



(a) from given ${}^N\bar{L}^B = 2\hat{n}_1 - \hat{n}_2 + 3\hat{n}_3 \quad {}^N\theta^B = +260^\circ$

$$ \lambda_1 = \frac{2}{\sqrt{2^2 + (-1)^2 + 3^2}} = \frac{2}{\sqrt{14}} = 0.53452 $$

$$ \lambda_2 = \frac{-1}{\sqrt{14}} = -0.26726 $$

$$ \lambda_3 = \frac{3}{\sqrt{14}} = 0.80178 $$

thus, ${}^N\hat{\lambda}^B = 0.5345\ \hat{b}_1 - 0.2673\ \hat{b}_2 + 0.8018\ \hat{b}_3$

$\hat{\lambda}$ is fixed in both N, B frame now

$$ {}^{N}\bar{\varepsilon}^{B} = {}^{N}\hat{\lambda}^{B} \sin\frac{\theta}{2} = {}^{N}\hat{\lambda}^{B} \sin 130^{\circ} $$

$$ \boxed{ {}^{N}\bar{\varepsilon}^{B} = 0.4095\ \hat{b}_1 - 0.2047\ \hat{b}_2 + 0.6142\ \hat{b}_3 } $$

and

$$ \boxed{ {}^{N}\varepsilon_4^{B} = \cos 130^{\circ} = -0.6428 } $$

from $\quad {}^{N}\bar{\omega}^{B} = 1.0\hat{n}_1 - 3.0\hat{n}_2 -$ [illegible] $\hat{n}_3$

also from ${}^{N}\hat{\lambda}^{B}$ & θ the DCM becomes

$$ C_{11} = \cos\theta + \lambda_1^2(1-\cos\theta) = 0.1617 $$
$$ C_{12} = -\lambda_3\sin\theta + \lambda_1\lambda_2(1-\cos\theta) = 0.6219 $$
$$ C_{13} = \lambda_2\sin\theta + \lambda_3\lambda_1(1-\cos\theta) = 0.7662 $$
$$ C_{21} = \lambda_3\sin\theta + \lambda_1\lambda_2(1-\cos\theta) = -0.9573 $$
$$ C_{22} = \cos\theta + \lambda_2^2(1-\cos\theta) = -0.0898 $$
$$ C_{23} = -\lambda_1\sin\theta + \lambda_2\lambda_3(1-\cos\theta) = 0.2749 $$
$$ C_{31} = -\lambda_2\sin\theta + \lambda_3\lambda_1(1-\cos\theta) = 0.2398 $$
$$ C_{32} = \lambda_1\sin\theta + \lambda_2\lambda_3(1-\cos\theta) = -0.7779 $$
$$ C_{33} = \cos\theta + \lambda_3^2(1-\cos\theta) = 0.5808 $$

(calculated w/ MATLAB)

$$ {}^{N}C^{B} = \begin{bmatrix} 0.1617 & 0.6219 & 0.7662 \\ -0.9573 & -0.0898 & 0.2749 \\ 0.2398 & -0.7779 & 0.5808 \end{bmatrix} $$

then $\quad \hat{b}_i = \hat{n}_i\, {}^{N}C^{B}$

so change ${}^{N}\omega^{B}$ to $\hat{b}$ - basis

$$ ({}^{N}\omega^{B})_{\hat{b}} = ({}^{N}\omega^{B})_{\hat{n}}\, {}^{N}C^{B} $$

$$ \therefore \boxed{ {}^{N}\omega^{B} = 2.6738\,\hat{b}_1 + 2.0582\,\hat{b}_2 - 0.9298\,\hat{b}_3 } $$

then using MATLAB (code in appendix) calculate

$$\dot{\varepsilon} = \frac{1}{2} \omega E^T$$

where

$$E = \begin{bmatrix} \varepsilon_4 & -\varepsilon_3 & \varepsilon_2 & \varepsilon_1 \\ \varepsilon_3 & \varepsilon_4 & -\varepsilon_1 & \varepsilon_2 \\ -\varepsilon_2 & \varepsilon_1 & \varepsilon_4 & \varepsilon_3 \\ -\varepsilon_1 & -\varepsilon_2 & -\varepsilon_3 & \varepsilon_4 \end{bmatrix}$$

thus,

$${}^N\dot{\varepsilon}^B = [\,-1.3962 \quad 0.3500 \quad 0.9939 \quad -0.0512\,]$$

$$\boxed{{}^N\dot{\underline{\varepsilon}}^B = -1.3962\,\hat{b}_1 + 0.3500\,\hat{b}_2 + 0.9939\,\hat{b}_3}$$

$$\boxed{{}^N\dot{\varepsilon}_4^B = -0.0512}$$

(b) To calculate ${}^N\dot{C}^B$

$${}^N\dot{C}^B = {}^NC^B \; {}^N\tilde{\omega}^B$$

where

$${}^N\tilde{\omega}^B = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}$$

using MATLAB for the computation, we obtain

$$
{}^{N}\dot{C}^{B} = \begin{bmatrix} 0.1617 & 0.6219 & 0.7662 \\ -0.9573 & -0.0898 & 0.2749 \\ 0.2398 & -0.7779 & 0.5808 \end{bmatrix} \begin{bmatrix} 0 & 0.9298 & 2.0582 \\ -0.9298 & 0 & -2.6738 \\ -2.0582 & 2.6738 & 0 \end{bmatrix}
$$

$$
{}^{N}\dot{C}^{B} = \boxed{\begin{bmatrix} -2.1553 & 2.1990 & -1.3302 \\ -0.4823 & -0.1550 & -1.7301 \\ -0.4722 & 1.7760 & 2.5735 \end{bmatrix}}
$$

(c) from the relationship

$$
{}^{N}\bar{\omega}^{B} = {}^{N}\omega_1^{B}\,\hat{b}_1 + {}^{N}\omega_2^{B}\,\hat{b}_2 + {}^{N}\omega_3^{B}\,\hat{b}_3
$$

$$
{}^{N}\bar{\omega}^{B} = \dot{k}_1\,{}^{N}\hat{\lambda}_1^{B}\,\hat{b}_1 + \dot{k}_2\,{}^{N}\hat{\lambda}_2^{B}\,\hat{b}_2 + \dot{k}_3\,{}^{N}\hat{\lambda}_3^{B}\,\hat{b}_3
$$

and from the supplementary document we know that for space-two 1-2-1 the DCM is

Space-two: 1-2-1

	b_1	b_2	b_3
a_1	c_2	$s_1 s_2$	$c_1 s_2$
a_2	$s_2 s_3$	$-s_1 c_2 s_3 + c_3 c_1$	$-c_1 c_2 s_3 - c_3 s_1$
a_3	$-s_2 c_3$	$s_1 c_2 c_3 + s_3 c_1$	$c_1 c_2 c_3 - s_3 s_1$

$$
\begin{bmatrix} c_2 & s_1 s_2 & c_1 s_2 \\ s_2 s_3 & -s_1 c_2 s_3 + c_3 c_1 & -c_1 c_2 s_3 - c_3 s_1 \\ -s_2 c_3 & s_1 c_2 c_3 + s_3 c_1 & c_1 c_2 c_3 - s_3 s_1 \end{bmatrix}
$$

$$= \begin{bmatrix} 0.1617 & 0.6219 & 0.7662 \\ -0.9573 & -0.0898 & 0.2749 \\ 0.2398 & -0.7779 & 0.5808 \end{bmatrix}$$

using MATLAB solve this system equation

$$\boxed{\begin{aligned} K_1 &= 0.6819 = 39.0671 \\ K_2 &= 1.4084 = 80.6956 \\ K_3 &= -1.8162 = -104.0630 \end{aligned}}$$

then finally also from the supplemental document.

$$\dot{K}_1 = \omega_1 - \frac{(\omega_2 \sin K_1 + \omega_3 \cos K_1)\cos K_2}{\sin K_2}$$

$$\dot{K}_2 = \omega_2 \cos K_1 - \omega_3 \sin K_1$$

$$\dot{K}_3 = \frac{\omega_2 \sin K_1 + \omega_3 \cos K_1}{\sin K_2}$$

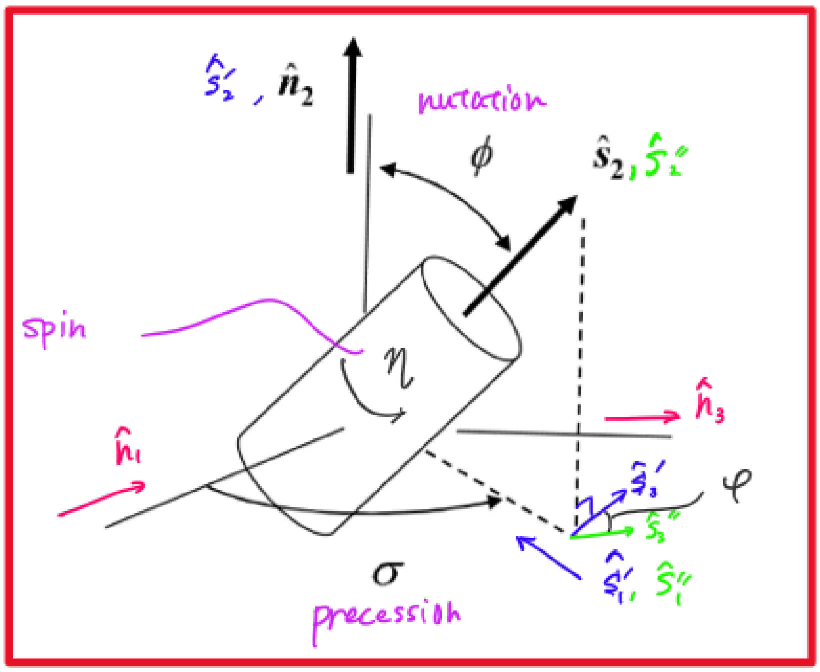
use MATLAB to calculate this we get

$$\boxed{\begin{aligned} \dot{K}_1 &= 2.5795 \\ \dot{K}_2 &= 2.1840 \\ \dot{K}_3 &= 0.5829 \end{aligned}}$$

Problem 3: A weather satellite has apparently malfunctioned. You need to produce some understanding of its current motion and compare it to some pre-determined specifications. The satellite moves in frame N and unit vectors $\hat{s}_i$ are fixed in the satellite. Determine the precession, nutation, and spin angles (σ, ϕ, η
compute the satellite angular velocity in terms of precession rate ($\dot{\sigma}$), nutation rate ($\dot{\phi}$), and spin rate ($\dot{\eta}$). However, the kind of data that you are currently receiving from the satellite is based on the space-three 2-3-1 angles $\beta_1, \beta_2, \beta_3$
the values

$$\beta_1 = 30^\circ \qquad \dot{\beta}_1 = 2 \text{ rad/s}$$
$$\beta_2 = -50^\circ \qquad \dot{\beta}_2 = -1.5 \text{ rad/s}$$
$$\beta_3 = 160^\circ \qquad \dot{\beta}_3 = 1 \text{ rad/s}$$

Ans.



(a) Add unit vectors $\hat{n}_1, \hat{n}_3, \hat{s}_1', \hat{s}_3', \hat{s}_1'', \hat{s}_3''$ such that σ, ϕ, η **body-two 2-1-2** angles; add η to the figure. Identify the angles as precession, nutation, or spin.

(b) Determine the direction cosine matrix ${}^N C^S$

for space-three 2-3-1, from supplementary document

$$
{}^{N}C^{S} = \begin{bmatrix} C_1C_2 & -S_2 & S_1C_2 \\ C_1S_2C_3+S_3S_1 & C_2C_3 & S_1S_2C_3-S_3C_1 \\ C_1S_2S_3-C_3S_1 & C_2S_3 & S_1S_2S_3+C_3C_1 \end{bmatrix}
$$

using MATLAB calculate w/ angles β_i:

$$
= \boxed{\begin{bmatrix} 0.5567 & 0.7660 & 0.3214 \\ 0.7944 & -0.6040 & 0.0637 \\ 0.2429 & 0.2198 & -0.9448 \end{bmatrix}}
$$

(c) Determine precession, nutation, and spin angles as well as the precession rate, nutation rate, and spin rate. i.e. $\dot{\sigma}, \dot{\phi}, \dot{\eta}$.

find σ, φ, η for body-two 2-1-2

from supplemental document. the DCM is

$$
\begin{bmatrix} -S_1C_2S_3+C_3C_1 & S_1S_2 & S_1C_2C_3+S_3C_1 \\ S_2S_3 & C_2 & -S_2C_3 \\ -C_1C_2S_3-C_3S_1 & C_1S_2 & C_1C_2C_3-S_3S_1 \end{bmatrix}
$$

$$
= \begin{bmatrix} 0.5567 & 0.7660 & 0.3214 \\ 0.7944 & -0.6040 & 0.0637 \\ 0.2429 & 0.2198 & -0.9448 \end{bmatrix}
$$

$$\cos\theta_2 = -0.6040$$

$$\Longleftrightarrow \quad \theta_2 = \boxed{127.1586^\circ}, \quad -127.1586^\circ$$

since we want $0^\circ < \theta_2 < 180^\circ$

$$s_2 = 0.7970$$

then $s_2 s_3 = 0.7944$

$$s_3 = \frac{0.7944}{s_2}$$

$$\theta_3 = \arcsin\left(\frac{0.7944}{s_2}\right)$$

$$-s_2 c_3 = 0.0637$$

$$c_3 = -\frac{0.0637}{s_2}$$

$$\theta_3 = \arccos\left(-\frac{0.0637}{s_2}\right)$$

$$\boxed{\theta_3 = 94.5862^\circ}$$

also $s_1 s_2 = 0.7660$

$$\theta_1 = \arcsin\left(\frac{0.7660}{s_2}\right)$$

$$c_1 s_2 = 0.2198$$

$$\theta_1 = \arccos\left(\frac{0.2198}{s_2}\right)$$

$$\boxed{\theta_1 = -73.9871^\circ}$$

hence

$$\boxed{\begin{aligned} \delta &= \theta_1 = -73.9871^\circ \\ \varphi &= \theta_2 = 127.1586^\circ \\ \eta &= \theta_3 = 94.5862^\circ \end{aligned}}$$

Find $\dot{\delta}, \dot{\varphi}, \dot{\eta}$

from $\dot{\beta}$ we can find $\dot{\delta}, \dot{\varphi}, \dot{\eta}$

the below is from the supplemental document

Space-three: 2-3-1

$\omega_1 = -\dot{\theta}_2 s_1 + \dot{\theta}_3 c_1 c_2$	$\dot{\theta}_1 = (\omega_1 c_1 + \omega_3 s_1) s_2 / c_2 + \omega_2$
$\omega_2 = \dot{\theta}_1 - \dot{\theta}_3 s_2$	$\dot{\theta}_2 = -\omega_1 s_1 + \omega_3 c_1$
$\omega_3 = \dot{\theta}_2 c_1 + \dot{\theta}_3 s_1 c_2$	$\dot{\theta}_3 = (\omega_1 c_1 + \omega_3 s_1)/c_2$

Calculated with MATLAB

since $[\dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3] = [2 \quad -1.5 \quad 1]$

$$\omega_1 = -\dot{\theta}_2 s_1 + \dot{\theta}_3 c_1 c_2 = 1.3067 \text{ rad/s}$$

$$\omega_2 = \dot{\theta}_1 - \dot{\theta}_3 s_2 = 2.7660 \text{ rad/s}$$

$$\omega_3 = \dot{\theta}_2 c_1 + \dot{\theta}_3 s_1 c_2 = -0.9776 \text{ rad/s}$$

since being fixed in $\hat{s}$-frame

use Body-two 2-1-2

Body-two: 2-1-2

$\omega_1 = \dot\theta_1 s_2 s_3 + \dot\theta_2 c_3$	$\dot\theta_1 = (\omega_1 s_3 - \omega_3 c_3)/s_2$
$\omega_2 = \dot\theta_1 c_2 + \dot\theta_3$	$\dot\theta_2 = \omega_1 c_3 + \omega_3 s_3$
$\omega_3 = -\dot\theta_1 s_2 c_3 + \dot\theta_2 s_3$	$\dot\theta_3 = (-\omega_1 s_3 + \omega_3 c_3)c_2/s_2 + \omega_2$

from this calculate with MATLAB

$$\dot\delta = \dot\theta_1 = (\omega_1 s_3 - \omega_3 c_3)/s_2 = 1.5362 \text{ rad/s}$$

$$\dot\varphi = \dot\theta_2 = \omega_1 c_3 + \omega_3 s_3 = -1.0790 \text{ rad/s}$$

$$\dot\eta = \dot\theta_3 = (-\omega_1 s_3 + \omega_3 c_3)c_2/s_2 + \omega_2 = 3.6940 \text{ rad/s}$$

(d) Correspondingly, find the associated ${}^N\bar{\varepsilon}^S$, ${}^N\varepsilon_4^S$

$${}^N\varepsilon_4^S = \frac{1}{2}\sqrt{1 + C_{11} + C_{22} + C_{33}} = 0.0443$$

$${}^N\varepsilon_1^S = \frac{C_{32} - C_{23}}{4\varepsilon_4} = 0.8811$$

$${}^N\varepsilon_2^S = \frac{C_{13} - C_{31}}{4\varepsilon_4} = 0.4427$$

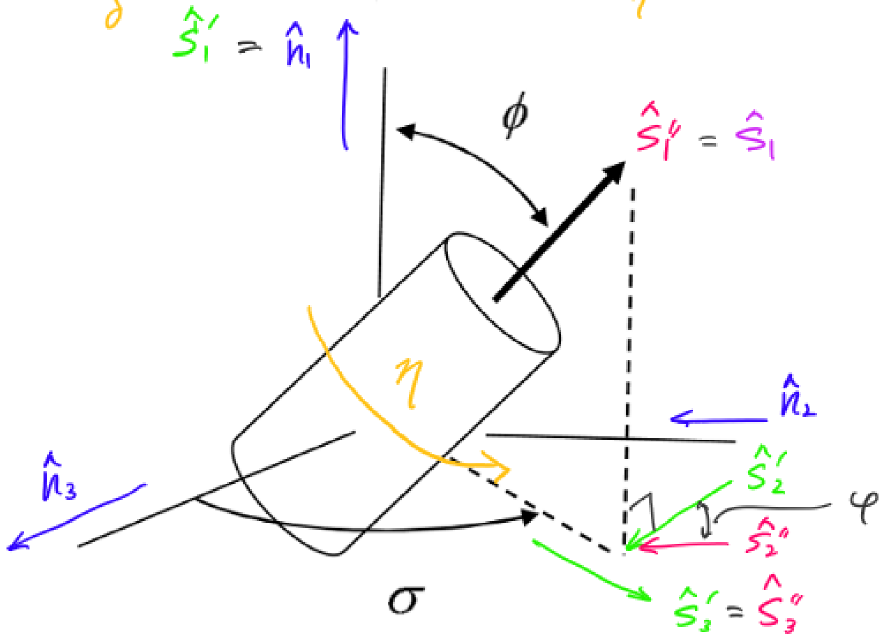
$${}^N\varepsilon_3^S = \frac{C_{21} - C_{12}}{4\varepsilon_4} = 0.1601$$

using MATLAB

Optional: Re-define the unit vectors for a sequence that reflects a body-two 1-3-1 set of angles for precession, nutation, and spin.

Body two 1-3-1

$$\underset{\delta}{\hat{n}_1 = \hat{s}_1'} \longrightarrow \overset{\varphi}{\hat{s}_3' = \hat{s}_3''} \longrightarrow \underset{\eta}{\hat{s}_1'' = \hat{s}_1}$$



Appendix

AAE440 HW4 MATLAB CODE problem 1

```
clear all; close all; clc;
% Defining body-three 1-2-3 system
theta_d = [60 -210 165];
theta = deg2rad(theta_d);  % convert the thetas to radians

% Function that calculates DCM in Body System from angles and rotational axes
C_body_123 = DCM_Body(1, 2, 3, theta(1), theta(2), theta(3));
```

```
function C_body = DCM_Body(Rot1, Rot2, Rot3, theta1, theta2, theta3)

    Rot = [Rot1, Rot2, Rot3];
    theta = [theta1, theta2, theta3]; % radians
    C_body = zeros([3,3]);
    C = zeros([3,3,3]);

    for i=1:3
        if Rot(i) == 1
            % DCM for rotation about axis 1
            C(:,:,i) = [ 1              0               0;
                         0  cos(theta(i))  -sin(theta(i));
                         0  sin(theta(i))   cos(theta(i))];
        elseif Rot(i) == 2
            % DCM for rotation about axis 2
            C(:,:,i) = [ cos(theta(i))  0   sin(theta(i));
                                     0  1               0;
                        -sin(theta(i))  0   cos(theta(i))];
        else
            % DCM for rotation about axis 3
            C(:,:,i) = [ cos(theta(i))  -sin(theta(i))  0;
                         sin(theta(i))   cos(theta(i))  0;
                                     0               0  1];
        end
    end
    C_body  =  C(:,:,1)*C(:,:,2)*C(:,:,3);

end
```

AAE440 HW4 MATLAB CODE problem 2

```
clear all; close all; clc;
```

<a>

```
% Given
L_NB_vec = [2 -1 3];  % n-hat
theta = 260/180*pi;
% Calculate lambda
lambda_NB = L_NB_vec/sqrt(sum(L_NB_vec.^2)); % b-hat
```

```
% Calculate epsilons
e_NB_vec = lambda_NB*sin(theta/2);  % b-hat
e_NB_4 = cos(theta/2);
% Angular velocity
omega_NB_n = [1 -3 -1.5];  % n-hat
% Find the DCM
C_NB = DCM_from_EulerAxisAng(lambda_NB,theta);
% Change basis of omega from n-hat to b-hat
omega_NB_b = omega_NB_n*C_NB;
e_NB_dot = EulerParaDot_from_AngVel([e_NB_vec, e_NB_4], [omega_NB_b, 0]);
```

<b>

```
% Calculating the rate of change of DCM
C_dot = DCM_Dot_from_PoissonKineEq(C_NB, omega_NB_b);
```

<c>

```
% Angles
kappa2 = acos(C_NB(1,1)); % Neglecting the second solution (Class Convention)
kappa1 = asin(C_NB(1,2)/sin(kappa2));
kappa1 = acos(C_NB(1,3)/sin(kappa2));   % Check if it satisfies condition
kappa3 = -(pi+asin(C_NB(2,1)/sin(kappa2)));
kappa3 = -acos(-C_NB(3,1)/sin(kappa2));

% Angles in degrees
kappa1_d = rad2deg(kappa1);
kappa2_d = rad2deg(kappa2);
kappa3_d = rad2deg(kappa3);

% Angular velocities
omega = omega_NB_b;
kappa1_dot = omega(1) - (omega(2)*sin(kappa1) +
omega(3)*cos(kappa1))*cos(kappa2)/sin(kappa2);
kappa2_dot = omega(2)*cos(kappa1) - omega(3)*sin(kappa1);
kappa3_dot = (omega(2)*sin(kappa1) + omega(3)*cos(kappa1))/sin(kappa2);

function C_mat = DCM_from_EulerAxisAng(lambdas, theta)

    % Euler Axis
    lambda1 = lambdas(1);
    lambda2 = lambdas(2);
    lambda3 = lambdas(3);

    % Calculating DCM from Euler Axis and Euler Angle
    C11 = cos(theta) + lambda1^2*(1-cos(theta));
    C12 = -lambda3*sin(theta) + lambda1*lambda2*(1-cos(theta));
    C13 = lambda2*sin(theta) + lambda3*lambda1*(1-cos(theta));
    C21 = lambda3*sin(theta) + lambda1*lambda2*(1-cos(theta));
    C22 = cos(theta) + lambda2^2*(1-cos(theta));
    C23 = -lambda1*sin(theta) + lambda2*lambda3*(1-cos(theta));
    C31 = -lambda2*sin(theta) + lambda3*lambda1*(1-cos(theta));
    C32 = lambda1*sin(theta) + lambda2*lambda3*(1-cos(theta));
    C33 = cos(theta) + lambda3^2*(1-cos(theta));

    C_mat = [C11 C12 C13; C21 C22 C23; C31 C32 C33];
```

```
end
function e_dot = EulerParaDot_from_AngVel(epsilons, omega)
    %{
    epsilons: 1x4 row vector with 3 vector elements and the 4th being a
    scalar element
    omega: 1x4 row vector with 4th element equal to 0
    %}
    e1 = epsilons(1);
    e2 = epsilons(2);
    e3 = epsilons(3);
    e4 = epsilons(4);

    % E matrix
    E = [e4 -e3 e2 e1; e3 e4 -e1 e2; -e2 e1 e4 e3; -e1 -e2 -e3 e4];
    % Output
    e_dot = 0.5*omega*E.';
end
function C_dot = DCM_Dot_from_PoissonKineEq(C_mat, omegas)
    omega1 = omegas(1);
    omega2 = omegas(2);
    omega3 = omegas(3);
    omega_wave = [0 -omega3 omega2; omega3 0 -omega1; -omega2 omega1 0];
    C_dot = C_mat*omega_wave;
end
```

AAE440 HW4 MATLAB CODE problem 3

```
clear all; close all; clc;
% Angles and angular velocities
Beta = deg2rad([30 -50 160]); % rad
Beta_dot = [2 -1.5 1];      % rad s-1
% Creating DCM for space three 2-3-1 (N_C_S)
C_NS_space_231 = DCM_Space(2, 3, 1, Beta(1), Beta(2), Beta(3));

delta = 1.291318577654501;   % Precession
phi = 2.219335441889180;  % Nutation
eta = 1.650841195002049;    % Spin

theta1 = delta;
theta2 = phi;
theta3 = eta;

% Calculating omega
omega1 = -Beta_dot(2)*sin(Beta(1)) + Beta_dot(3)*cos(Beta(1))*cos(Beta(2));
omega2 = Beta_dot(1) - Beta_dot(3)*sin(Beta(2));
omega3 = Beta_dot(2)*cos(Beta(1)) + Beta_dot(3)*sin(Beta(1))*cos(Beta(2));

% Rates
delta_dot = (omega1*sin(theta3) - omega3*cos(theta3))/sin(theta2);
phi_dot = omega1*cos(theta3) + omega3*sin(theta3);
```

```
eta_dot = (-omega1*sin(theta3) + omega3*cos(theta3))*cos(theta2)/sin(theta2) +
omega2;

% Calcuate the euler parameters
epsilons = EulerPara_from_DCM(C_NS_space_231);
```

```
function epsilons = EulerPara_from_DCM(C_mat)

    epsilon4 = 0.5*sqrt(1+C_mat(1,1)+C_mat(2,2)+C_mat(3,3));
    epsilon1 = (C_mat(3,2)-C_mat(2,3))/4/epsilon4;
    epsilon2 = (C_mat(1,3)-C_mat(3,1))/4/epsilon4;
    epsilon3 = (C_mat(2,1)-C_mat(1,2))/4/epsilon4;
    epsilons = [epsilon1 epsilon2 epsilon3 epsilon4];

end
```