Name	Team Number
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AAE 251: Introduction to Aerospace Design

Assignment 2—Subsonic Wind Tunnels and Space Environment

Due Tuesday 29 January, 10:00 am on Blackboard

Instructions

Write or type your answers into the appropriate boxes. Make sure you submit a single PDF on Blackboard. Your homework will be a handy study guide.

Problem Number	Points Possible	Points Earned
Problem 1	8	
Problem 2	12	
Problem 3	10	
Problem 4	5	
Total	35	

Problem 1:

The Coefficient of Pressure is expressed as:

$$C_P = \frac{p - p_\infty}{\frac{1}{2}\rho_\infty V_\infty^2}$$

where p is the local pressure, p_∞ is the freestream pressure, and the denominator gives the free stream dynamic pressure. Suppose you own a low speed GA aircraft which measures airspeed using a pitot tube mounted at the leading edge of its wing. Find a reasonable maximum cruise speed for a typical GA aircraft and assume it is the same for your aircraft. Develop an expression to show how the pressure measured by the pitot tube varies with the aircraft's airspeed at sea level conditions. Plot your result for the velocity range you found ($0 < V < V_{max}$) using Matlab and paste your code and figure into your solution. Looking back at your solution and the expression given above, can you say what would be the C_p at the leading edge of your aircraft? How does it vary with airspeed?

Problem #1

A long distance commercial passenger aircraft has a typical crusing airspeed of **880-926 km/h** (475-500 knots; 547-575 mph). In m/s this is **244-257 m/s**.

Therefore, at sea level condition (from Appendix A of textbook)

$$\rho_\infty = 1.2250 \ \frac{\text{kg}}{m^3}$$

$$P_\infty = 1.01325 \times 10^5 \ \frac{N}{m^2}$$

$$0 \leq V \leq V_{\max} \implies 0 \leq V \leq 257 \ \frac{m}{s}$$

From Bernoulli's Equation

$$p = p_\infty + \frac{1}{2}\rho_\infty V_\infty^2$$

```
V_inf = 0:1:257;     % The cruising airspeed of the aircraft [m/s]
rho_inf = 1.2250;    % Freestream density at sea level [kg/m^3]
P_inf = 1.01325*10^5;    % Freestream pressure at sea level [N/m^3 or Pa]
```

Thus, using Bernoulli's Equation

```
% calculating the pressure meassured at the pitot tube (local pressure)
P_local = P_inf + 0.5 * rho_inf .* V_inf;
```

also because the pressure coefficient is

$$C_p = \frac{P - P_\infty}{\frac{1}{2}\rho_\infty V_\infty^2}$$

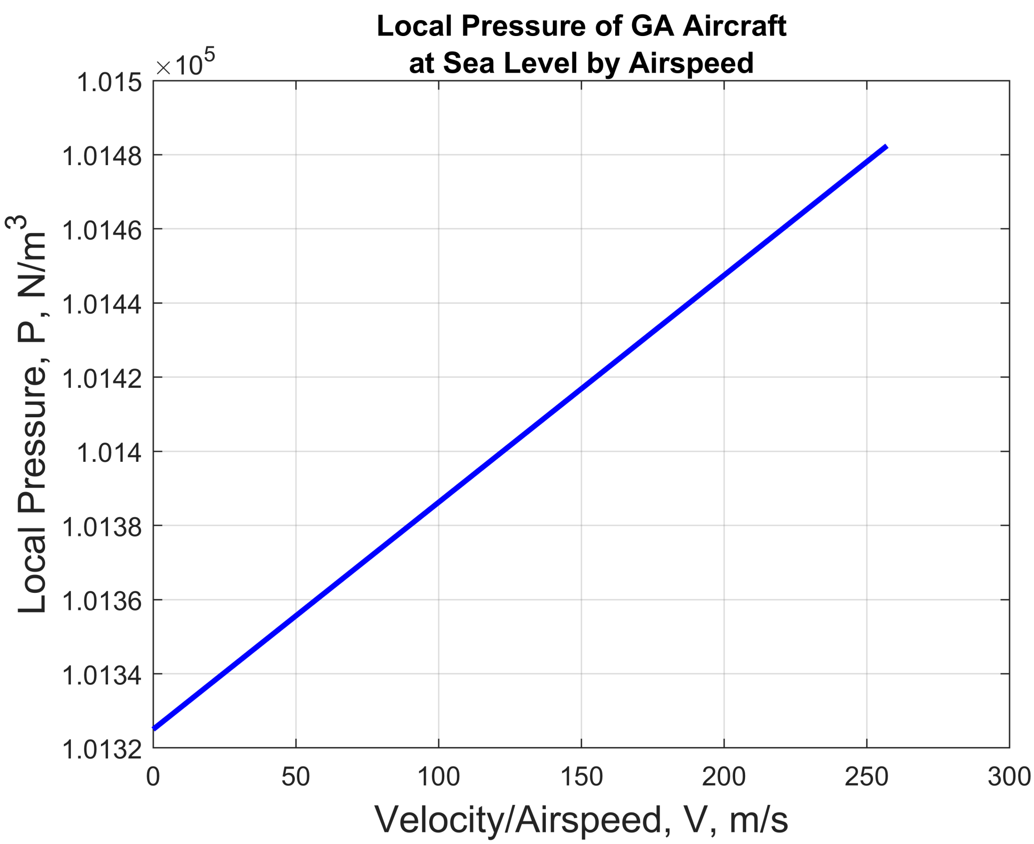
```
% Calculating the pressure coefficient depending on the airspeed
C_p = (P_local - P_inf) / 0.5 / rho_inf ./ V_inf;
```

Now, we plot a graph of local pressure by velocity/airspeed

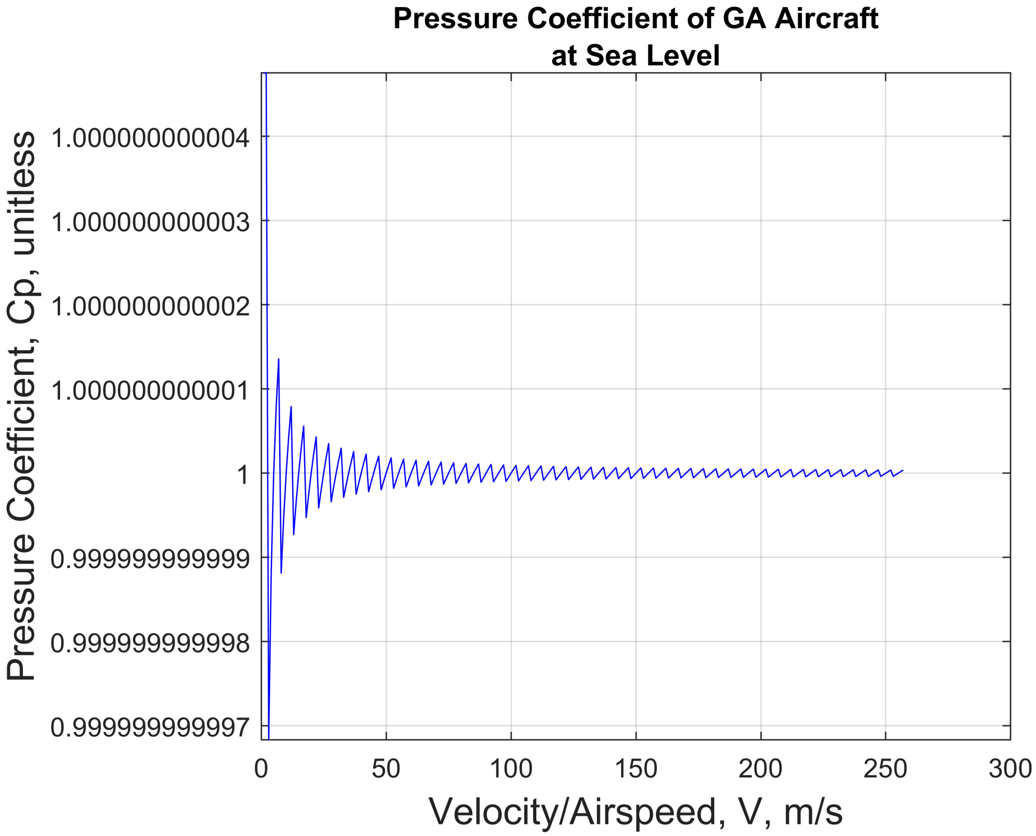
and also the Cp (pressure coefficient by velocity/airspeed

```
% Adjusting fontsize and linewidth
fontsize = 14;
linewidth = 2;

figure(1)
plot(V_inf, P_local, 'Color','b', 'LineStyle','-', 'LineWidth', linewidth)
```

```
title({'Local Pressure of GA Aircraft', 'at Sea Level by Airspeed'})
xlabel('Velocity/Airspeed, V, m/s', 'FontSize',fontsize)
ylabel('Local Pressure, P, N/m^3', 'FontSize',fontsize)
box on
grid on
```




```
figure(2)
plot(V_inf, C_p,'Color','b', 'LineStyle','-')
title({'Pressure Coefficient of GA Aircraft', 'at Sea Level'})
xlabel('Velocity/Airspeed, V, m/s','FontSize',fontsize)
ylabel('Pressure Coefficient, Cp, unitless','FontSize',fontsize)
box on
grid on
```



Analysis:

- What could the pressure coefficient be at the leading edge of the aircraft?

ANS: From the graph of the pressure coefficient, the Cp fluctuates with the value of 1 being the center. Therefore, the most plausible value for the coefficient is 1.

- How does it vary with airspeed?

ANS: As the velocity increases the pressure coefficient flucutuates while decreasing. And at maximum velocity it asymptotes to 1.

Problem 2:

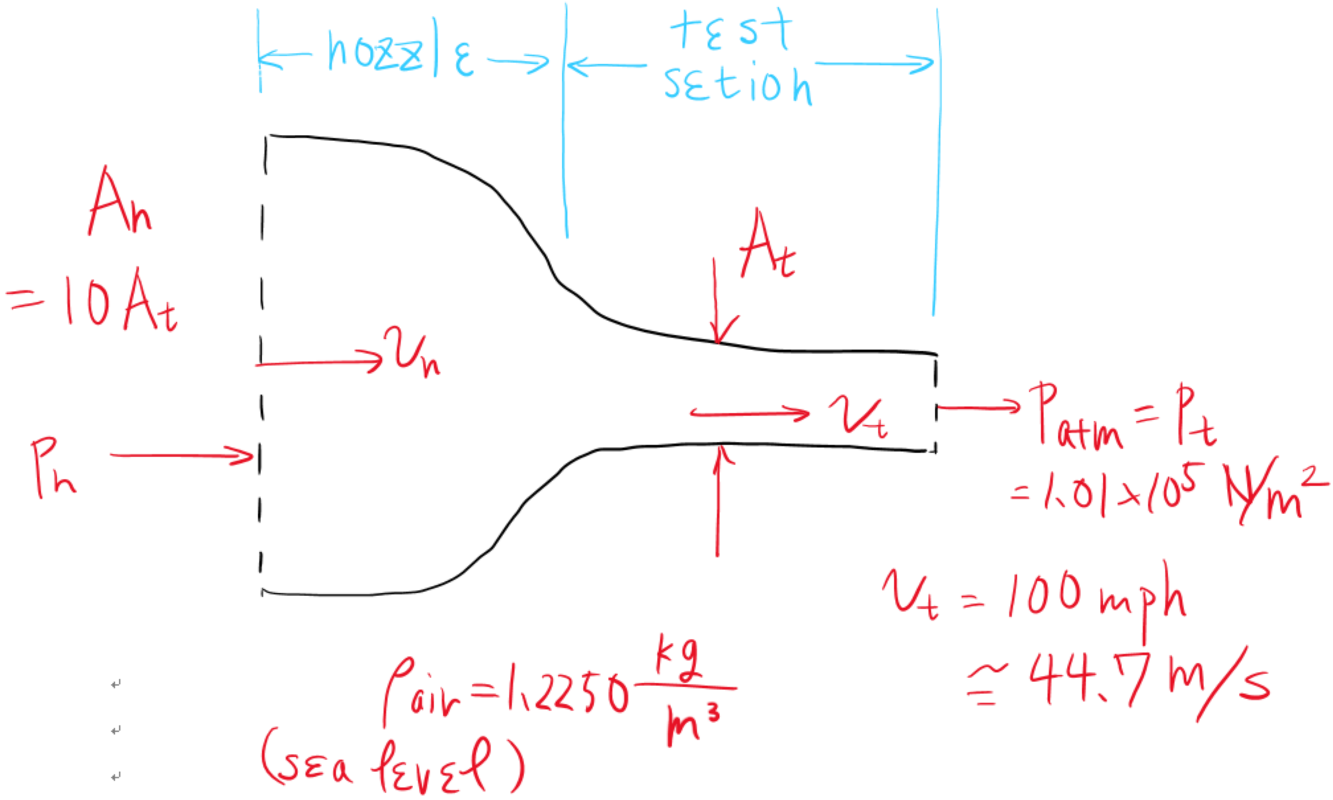
You want to operate a low-speed subsonic wind tunnel so that the flow in the test section has a velocity of 100 mph. You are given the following details about the wind tunnel:

- It is an arrangement of a nozzle and a constant area test section. The flow at the exit of the test section dumps out to the surrounding atmosphere, where atmospheric pressure is $1.01 \times 10^5 N/m^2$. A settling chamber or reservoir provides the inlet pressure to the nozzle.
- The contraction ratio of the nozzle is 10:1.

Assuming incompressible flow at standard sea-level density answer the following showing all the equations and steps required:

(a) Sketch the wind tunnel with the appropriate pressures, areas, and velocities indicated. Label the different parts of the wind tunnel.
(b) Calculate the pressure at the inlet of the nozzle.
(c) By how much should you increase this inlet pressure to achieve 200 mph in the test section of this wind tunnel?
(d) Comment on the magnitude of this increase in pressure relative to the increase in test-section velocity.

(a)



(b) from the continuity eqn

$$A_n v_n = A_t v_t$$

$$\therefore v_n = \frac{A_t}{A_n} v_t$$

since $A_n = 10A_t$

$$v_n = \frac{1}{10} v_t = \frac{1}{10}(44.7 m/s)$$

$$\therefore v_n = 4.47 \ m/s \ \bullet\bullet\bullet \ (1)$$

next from Bernoulli's Eqn

$$P_n + \frac{1}{2}\rho_{air} v_n^2 = P_t + \frac{1}{2}\rho_{air} v_t^2$$

$$P_n = P_t + \frac{1}{2}\rho_{air}(v_t^2 - v_n^2)$$

$$= (1.01\times10^5\ Pa) + \frac{1}{2}\left(1.2250\frac{kg}{m^3}\right)(44.7^2 - 4.47^2)\frac{m^2}{s^2}$$

$$\simeq 1.022\times10^5\ Pa$$

$$\boxed{P_n = 1.02\times10^5\ \frac{N}{m^2}}$$

(c)

using the same Eqn from above

$$P_n(v_t) = P_t + \frac{1}{2}\rho_{air}\left(v_t^2 - \frac{1}{100}v_t^2\right)$$

$$= P_t + \frac{\rho_{air}}{2}\cdot\frac{99}{100}v_t^2$$

$$= P_t + \frac{99\rho_{air}}{200}v_t^2 \qquad \text{say } q = \frac{99\rho_{air}}{200}$$

$$P_n + \Delta P_n = P_t + q(v_t + \Delta v_t)^2$$

$$P_n + \Delta P_n = P_t + q v_t^2 + 2q v_t \Delta v_t + q\Delta v_t^2$$

$$\Delta P_h = 2 q V_t \Delta V_t + q \Delta V_t^2$$

since $q = 0.6664$

$$\Delta V_t = 100\,\text{mph} \cong 44.7\ m/s$$

$$\Delta P_h = 2(0.6664)(44.7^2)\frac{m^2}{s^2} + (0.6664)(44.7^2)\frac{m^2}{s^2}$$

$$\cong 3635\ \text{Pa}$$

$$\boxed{\text{increase of } 3640\ \frac{N}{m^2}}$$

(d)

Compared to the original Pn the velocity in the test section can be increased with a relatively same change of the pressure because the original pressure of Pn is 10 to the power of 5 whereas the increase in the pressure at the nozzle to raise the velocity in the test section is merely 10 to the power of three which is 0.01 of the original.

Problem 3:

We are testing a 1:10 scale model of a wing in a subsonic tunnel. The actual wing spans 15 m and its average chord is 0.8 m. We are operating at sea level. We would like to estimate the drag at 160 km/hr, but our wind tunnel does not go that fast. We do have the following data available from a previous test.

Speed (m/s)	**Drag (N)**
14.9	0.31
18.1	0.44
21.6	0.61
25.2	0.8
29.3	1.08
32	1.3
34.2	1.46
36.9	1.7
39	1.9

Drag, as we will see in class, can be expressed as follows:

$$D = \frac{1}{2}\rho V_\infty^2 C_D S$$

where ρ is the air density, V_∞^2 is the free stream velocity, C_D is the drag coefficient, and S is the area. You will need to do two things to estimate the drag at 160 km/hr: (1) establish whether you have a reasonable basis for extrapolation (i.e., do we have *dynamic similarity*?), and (2) if extrapolation is indeed reasonable, solve for the drag at this velocity. Your answer must include a properly formatted Matlab code and plot, showing the variation in experimentally obtained drag coefficient with Reynolds number.

Problem #3

(1)

First we must go over the concepts behind the drag coefficient.

the drag coefficient, C_d is

$$C_D = C_{\text{do}} + C_{\text{di}} \quad \cdots (1)$$

where

C_{do} is the sum of the skin friction and form, and this is inversly proportional to the $\text{Re} \equiv$ Reynold's Number, which is,

$$\text{Re} = \rho \frac{\text{VL}}{\mu} \quad \cdots (2)$$

where ρ is the density of the fluid, $V$is the velocity, L is the length of the wing, and μ is the viscosity of the fluid.

And C_{di} is the induced drag which is derived as

$$C_{\text{di}} = \frac{C_l^2}{\pi(\text{AR})e} \quad \cdots (3)$$

C_l is the lift coefficient,

$\text{AR} \equiv \text{ Aspect Ratio} = \frac{b^2}{S} = \frac{(\text{wing span})^2}{(\text{wing area})}$, and e is the efficiency factor.

From (1), (2), and (3) we can see that there is a relation between the Reynolds Number and the drag coefficient,

and to verify that it is appropriate to extrapolate the velocity by drag force data given we will manipulate the given data and attempt to elucidate the relationship between the two values.

This will provide dynamic similarity in that the object and flow velocity combination with the same Reynold's Number exhibit the same flow characteristics.

```
% First we set up the given data of the velocity
% and the drag Forces
V_inf = [14.9, 18.1, 21.6, 25.2, 29.3, 32, 34.2, 36.9, 39]; % [m/s]
drag_F = [0.31, 0.44, 0.61, 0.8, 1.08, 1.3, 1.46, 1.7, 1.9]; % [N]

% And set-up other constants given at the condition of sea-level
dens = 1.2250; % [kg/m^3]
% At the sea-level temperature which is 288.16 K = 15.01 C, the viscosity is
visco = 17.89 * 10^(-6); % [N*s/m^2]
% The wing span and the average chord of the wing are also given (at the scale of 1:10)
span = 1.5; % [m]
avg_chord = 0.08; % [m]
% From the wing span and the average chord the area is
```

```
area = span * avg_chord; % [m^2]
```

Subsequently, using the drag force formula

$$D = \frac{1}{2}\rho V_\infty^2 C_D S$$

where ρ is density, V_∞ is the velocity, C_D is the drag coefficient, and S is the area of the wing.

We will solve this formula for C_D

$$C_D = \frac{2D}{\rho V_\infty^2 S}$$

```
% The drag coefficient (vector) becomes
drag_coeff = 2 * drag_F / dens ./ V_inf / area;
```

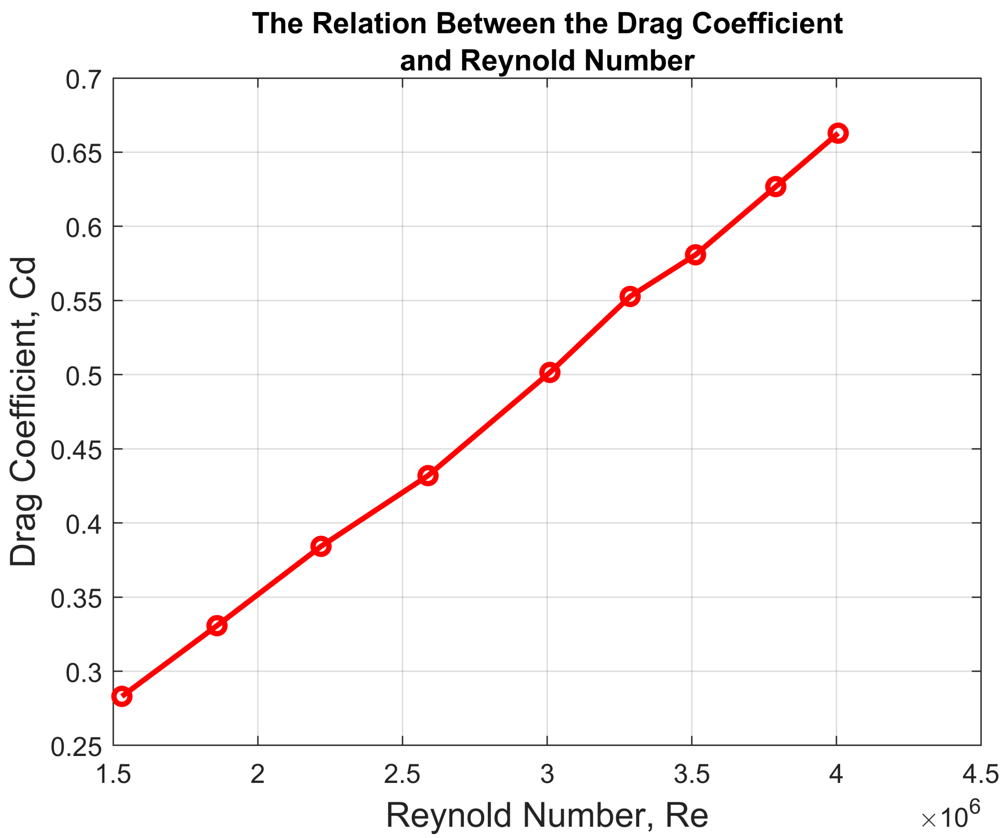
Also using

$$\mathrm{Re} = \rho \frac{\mathrm{VL}}{\mu}$$

```
% The Reynold's number (vector) becomes
reynold = dens * V_inf * span / visco;
```

Now we plot the drag coefficient (y-axis) by the Reynold's Number (x-axis) to find their relation

```
% Adjusting the fontsize and linewidth
fontsize = 13;
linewidth = 2;

% Plotting
figure(1)
plot(reynold, drag_coeff, 'Color','r','LineStyle','-','Marker','o','LineWidth',linewidth)
title({'The Relation Between the Drag Coefficient', 'and Reynold Number'})
xlabel('Reynold Number, Re','fontsize', fontsize)
ylabel('Drag Coefficient, Cd', "FontSize",fontsize)
box on
grid on
```



From this, we can say that the drag coefficient and the Reynold's Number has a linearly proportional relationship which approves of the dynamic similarity of the given data.

In conclusion, we are able to extrapolate the data in order to obtain the drag force for the velocity of 160 km/hr.

(2)

To extrapolate the drag force for velocity of the given data set we will use the method of linear regression

```
% Use polyfit to fit the data to the predicted linear regression
p = polyfit(V_inf, drag_F, 1);
% Call polyval to call the predicted drag force values for the
% obtained linear regression
drag_F_fit = polyval(p, V_inf);
% The SSE value (Sum of Squared Errors) is
sse = sum((drag_F - drag_F_fit).^2);
% The SST value (Sum of Squared Total) is
sst = sum((drag_F - mean(drag_F)).^2);
% Thus the coefficient of determination, Rsq is
Rsq = 1 - sse/sst;
```

The variance of the predicted linear regression is

```
display(Rsq);
```

```
Rsq = 0.9843
```

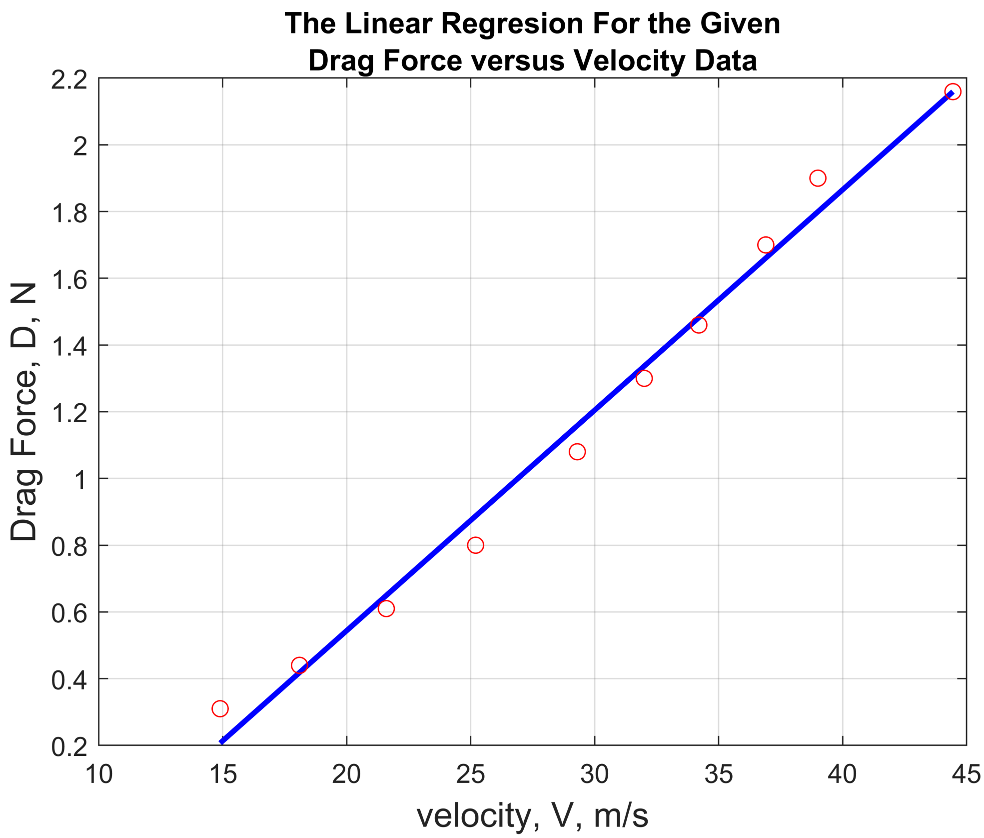
Which is very good. Hence we can use this reliable linear regression

to obtain the drag force at velocity 160 km/hr.

```
% The linear regression has
% A slope of
p(1);
% A y-intercept of
p(2);

% Convert 160 km/hr to m/s
V_find = 160 * 1000 / 3600; % [m/s]

% Finally the drag force is
drag_F160 = p(1) * V_find + p(2);
```

Additionally, the graph with the point of (v, D) = (V_find, drag_F160) (*which is the point for when the velocity is 160 km/hr) will be plotted for clarification

```
% Concatenate the data point for the velocity of 160 km/hr and the corresponding drag force
V_inf = [V_inf V_find];
drag_F = [drag_F drag_F160];
drag_F_fit = [drag_F_fit drag_F160];

% Plotting
figure(2)
plot(V_inf, drag_F_fit, 'color','b', 'LineStyle','-','LineWidth',linewidth)
title({'The Linear Regresion For the Given','Drag Force versus Velocity Data'})
xlabel('velocity, V, m/s','FontSize',fontsize)
ylabel('Drag Force, D, N', 'FontSize',fontsize)
box on
grid on
hold on
plot(V_inf, drag_F, 'color', 'r', 'Marker','o', 'LineStyle','none')
hold off
```



ANS:

```
fprintf('The drag force at 160 km/hr is %f N.',drag_F160);
```

```
The drag force at 160 km/hr is 2.159273 N.
```

Problem 4:

Complete your space hazards summary that you began in class on Tuesday and scan it in with your homework.

Source/Cause	Hazard	Parameters	Effect	Danger Zone	Mitigation
Earth's atmosphere	Drag	Air density	Shortens orbital lifetime	LEO	Go to higher orbit
		Ballistic coefficient			Periodically adjust orbit
	Atomic oxygen	mass	degrades spacecraft surface	LEO and beyond	atmospheric ozane $O+O \Rightarrow O_2$
Being in a vacuum	Out-gassing	molecule or material released	can cause electronics to malfunction	960 km	Bake materials
					ensure materials don't have trapped gasses
	Cold-welding	part material temperature	materials bind and cause failure	960 km	use lubricants avoid moving parts
	Inability to shed heat	temperature radiation	Things get not quirky	vacuum 960 km	big radiating surface areas
Past and present missions	Space debris	size	physical change to spacecraft	Above 65 km LEO	track debris
		speed			minimize debris
		collision (angle) point			use stronger material
		composition			carry replacement materials
Solar system	Micrometeoroids	size	shortens orbital lifespan	outside 65 km	strong materials
		speed			track micrometeroids
		collision point			
		composition			
The sun	Radiation	$10^1 - 10^2$ rad for biological matter above 10^2 for other	human injury	outside magnetosphere	materials stronger to radiation.
			part degradation		use magnetic or electrostatic sheilds
			over heating		
	Solar pressure	spacecraft surface area	orbital perturbations	outside magnetosphere	use solar pressure as motive force ⇒ Solar Sailing
Solar wind and flares	Charged particles	mass of particles	charging	outside mesosphere	replace with stronger material
Galactic cosmic rays		Temp. / Energy	sputtering		magnetic field or specific material layer
Van Allen radiation belts		wave length Plank's Constant	Single Event phenomenon		Some material or shield to prevent ultrafast electrons to cause electrical overload.
		Distance	Total dose effect		

aurora $\begin{cases} \text{borealis} \\ \text{austrialis} \end{cases}$