AAE 364 Control Systems Analysis
Problem Set 4

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Reading Assignment:

Sections 1,2 and 7 in Chapter 2 and Sections 1 and 2 in Chapter 3.

Problems

Solve B-2-1, B-2-3, B-2-4, and B-2-6 in Chapter 2 and B-3-6 in Chapter 3.

Problem 2: Aircraft Control

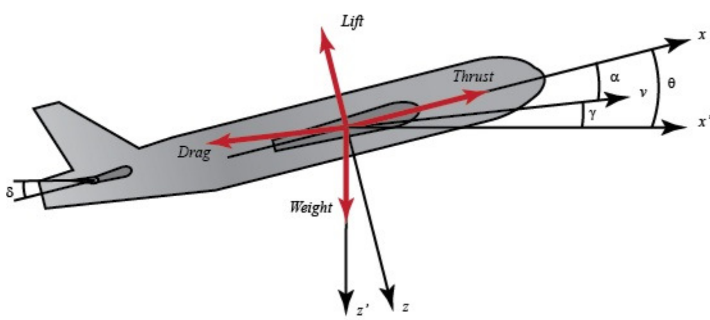

Figure 1: Forces acting on an aircraft in the Longitudinal plane.

Figure 1 shows the coordinate axes and forces acting on the aircraft in the longitudinal plane of motion. Assuming that the aircraft is cruising at constant velocity and altitude along with other simplifying assumptions, the equations of motion describing the longitudinal motion of the aircraft are given below. A detailed derivation of these equations is beyond the scope of the class and will be covered in AAE 421.

$$\dot{\alpha} = \mu\Omega\sigma[-(C_L + C_D)\alpha + \frac{1}{(\mu - C_L)}q - (C_W sin\gamma)\theta + C_L] \tag{1}$$

$$\dot{q} = \frac{\mu\Omega}{2i_{yy}}[[C_M - \eta(C_L + C_D)]\alpha + [C_M + \sigma C_M(1 - \mu C_L)]q + (\eta C_W sin\gamma)\delta] \tag{2}$$

$$\dot{\theta} = \Omega q \tag{3}$$

Nomenclature

α	Angle of attack	C_D	Coefficient of drag
q	Pitch rate	C_L	Coefficient of lift
θ	Pitch angle	C_W	Coefficient of weight
δ	Elevator deflection angle	C_M	Coefficient of pitching moment
ρ	Density of air	γ	Flight path angle
S	Wing planform area	i_{yy}	Normalized moment of inertia
c	Average chord length	μ	$\frac{\rho S c}{4m}$ (constant)
m	Aircraft mass	η	$\mu \sigma C_M$ (constant)
U	Equilibrium flight speed	Ω	$= \frac{2U}{c}$
C_T	Coefficient of thrust		

Once we have the above equations of motion, we can substitute values for the various aircraft characteristics and aerodynamic coefficients for a particular aircraft to obtain the longitudinal equations of motion for that aircraft. Taking these values for a representative commercial aircraft, we obtain the simplified form of the equations of motion shown below:

$$\dot{\alpha} = -0.312\alpha + 53.4q + 0.232\delta \tag{4}$$

$$\dot{q} = -0.0125\alpha - 0.426q + 0.0207\delta \tag{5}$$

$$\dot{\theta} = 53.4q \tag{6}$$

Using the equations of motion in (4)-(6), derive the transfer function describing the aircraft pitch angle response output θ to the elevator deflection input δ.