AE 6230 – HW1: Free Vibrations and Harmonic Excitation of SDOF Systems

Out: September 6, 2022; **Due:** September 13, 2022 by 11:59 PM ET in Canvas

Guidelines

- Read each question carefully before doing any work;
- If you find yourself doing pages of math, pause and consider if there is an easier approach;
- You can consult any relevant materials;
- You can discuss solution approaches with others, but your submission must be your own work;
- If you have doubts, please ask questions in class, during office hours, and/or Piazza (no questions via email);
- The solution to each question should concisely and clearly show the steps;
- Simplify your results as much as possible;
- Box the final answer for each question;
- If you use any code, please submit it with the solution.

Problem 1 – 40 points

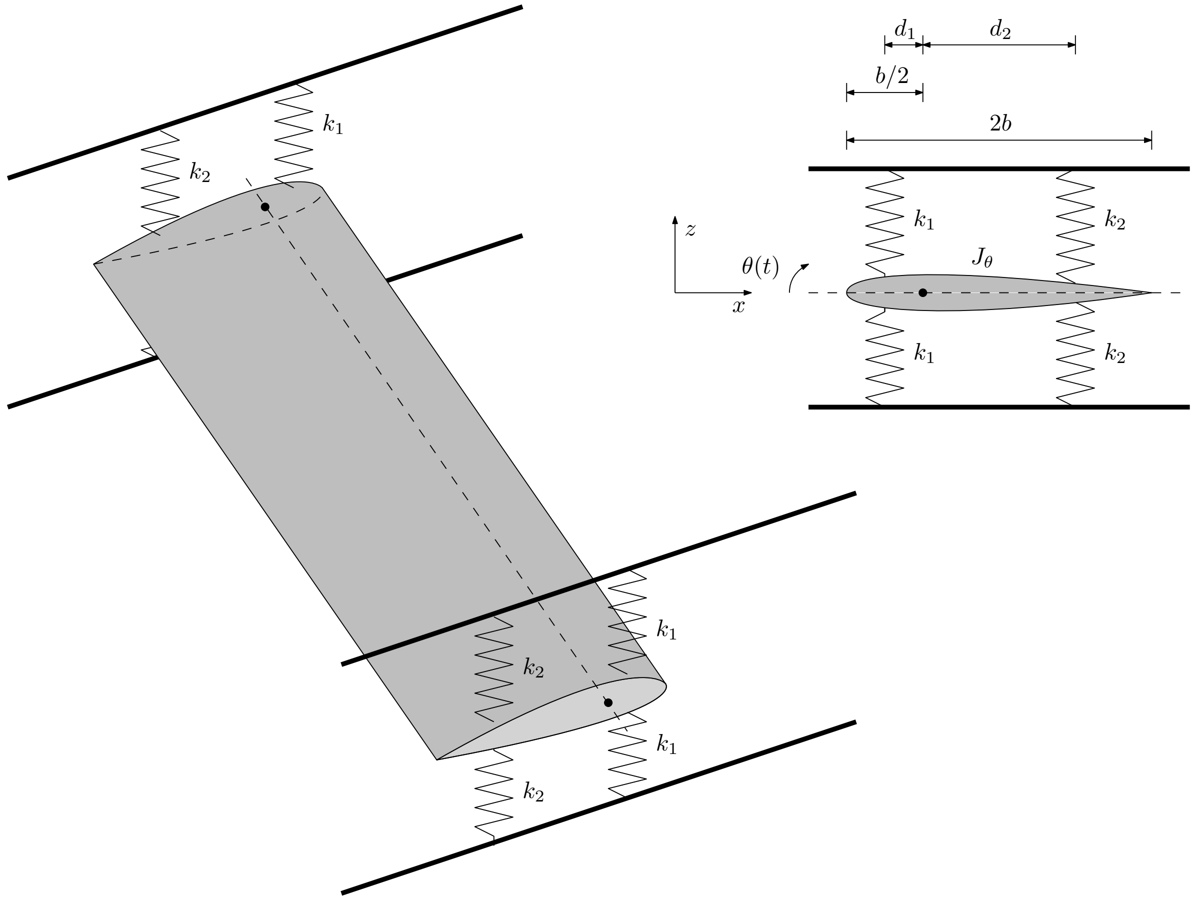

Figure 1: Schematic of wind-tunnel wing model.

Consider a uniform rigid wing mounted in a wind-tunnel test section (Fig. 1). The wing can pitch about the quarter-chord axis and the pitch motion is restrained by four springs on each end (near the wind-tunnel walls). The front springs have spring constant k_1 and are attached to the wing upper and lower surfaces at a distance d_1 ahead of the quarter chord (toward the leading edge); the rear springs have spring constant k_2 and connect to the wing at a distance d_2 downstream of the quarter chord (toward the trailing edge). The wing moment of inertia about the pitch axis is denoted by J_θ. Assuming the pitch angle θ as the degree of freedom (see the convention in Fig. 1) and neglecting the wing self-weight, answer the following questions:

1. After drawing the free-body diagram for the system
   (a) Derive the equation of motion for studying its free vibrations;
   (b) Determine the natural frequency ω_n and evaluate it for the parameters in Table 1;
2. Modify the attachment point of either the front or the rear springs to increase ω_n by 15%;
3. Assuming that four linear viscous dampers c_1 are added to the initial system (one for each front spring)
   (a) Find the minimum value of c_1 such that any free response satisfies

$$\delta = \ln \frac{x(t_1)}{x(t_2)} \geq 0.2 \tag{1}$$

   where δ is the logarithmic decrement and t_1 and $t_2 = t_1 + T$ are two consecutive oscillation peaks;
   (b) Evaluate the frequency of the damped motion ω_d and compare it with ω_n;
4. Considering the system with the viscous dampers
   (a) Determine the free response for the initial conditions $\theta_0 = 5$ deg and $\dot{\theta}_0 = 0$;
   (b) Plot the free response for $t \in [0, 5]$ s.

Table 1: Parameter values for Problem 1.

Parameter	Symbol	Value
Spring constant of the front springs	k_1	25 N/m
Spring constant of the rear springs	k_2	$0.75k_1$
Half chord	b	10 cm
Distance of the front springs from the pitch axis	d_1	$b/4$
Distance of the rear springs from the pitch axis	d_2	b
Moment of inertia about the pitch axis	J_θ	0.0004 kg·m^2

Recommendations and clarifications:

- **Question 1b:** only substitute the values in Table 1 at the end of the question to evaluate ω_n;
- **Question 2:** assume that the attachment points for all the front or all the rear springs change by the same amount and that you can only move one set of springs (either all the front or all the rear springs). The front springs must remain ahead of the pitch axis and the rear springs must remain downstream of the pitch axis;
- **Question 3:** answer this question for the initial system, that is, before the changes in Question 2. The viscous dampers connect to the upper and lower surfaces of the wing at the same distance d_1 from the pitch axis as the front springs;
- **Question 4a:** simplify your solution as appropriate;
- **Question 4b:** use a reasonable time step when plotting the free response and do not forget the units. Note that you can use the results from this question to check whether the logarithmic decrement is correct.

Problem 2 – 40 points

Consider the system in Problem 1 but now with the parameters in Table 2. The system is excited by a moment

$$M(t) = M_0 \sin \omega t \tag{2}$$

about the pitch axis, with zero initial conditions. Answer the following questions:

1. Plot the magnitude $|H(i\omega)|$ and phase lag $\phi(\omega)$ of the frequency response for $\omega/\omega_n \in [0, 4]$;
2. Using the complex response method
   (a) Determine the steady-state forced response;
   (b) Plot the steady-state forced response for $t \in [10T, 10T + 0.5]$ s where T is the period of the excitation;
3. Using the time-domain method
   (a) Determine the steady-state forced response;
   (b) Plot the steady-state forced response for $t \in [10T, 10T + 0.5]$ s;
   (c) Determine the complete forced response including the transient phase;
   (d) Plot the complete forced response and the transient terms for $t \in [0, 4]$ s.

Table 2: Parameters for Problem 2.

Parameter	Symbol	Value
Moment of inertia about pitch axis	J_θ	0.0004 kg·m^2
Natural frequency	ω_n	50 rad/s
Viscous damping factor	ζ	0.04
Excitation amplitude	M_0	0.1 Nm
Excitation frequency	ω	$0.5\omega_n$

Recommendations and clarifications:

- **Question 1:** use a reasonable frequency step when plotting the results and do not forget the units. Note that the phase lag is denoted by $\phi(\omega)$ instead of $\theta(\omega)$ as done in class to avoid confusion with $\theta(t)$;
- **Question 2 and 3:** use a reasonable time step when plotting the results and do not forget the units.

Problem 3 – 20 points

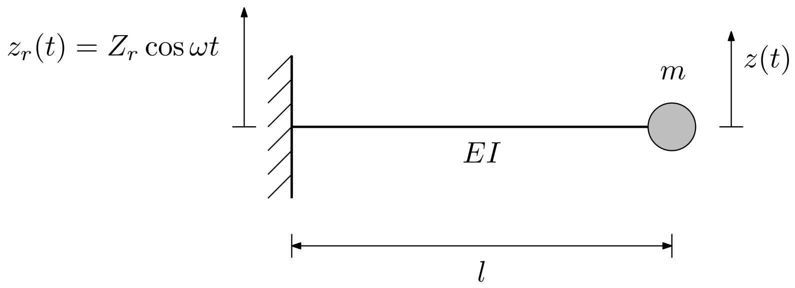

Figure 2: Schematic of a cantilevered beam in bending with a tip mass subject to harmonic motion of its root.

Consider the massless uniform isotropic cantilevered beam in bending in Fig. 2 with a tip mass. The beam root undergoes harmonic motion

$$z_r(t) = Z_r \cos \omega t \tag{3}$$

Very slight damping is present in the system such that, after a transient phase, the tip mass motion $z(t)$ contains only the excitation frequency ω. The impact of such slight damping on the amplitude and phase of $z(t)$ is assumed to be negligible. Answer the following questions:

1. After showing the free-body diagram, derive the equation of motion for the tip mass;
2. Considering the parameters in Table 3, determine
   (a) The natural frequency ω_n;
   (b) The maximum excitation frequency $\omega < \omega_n$ such that $|z(t)| \leq 1.1|Z_r|$.

Table 3: Parameter values for Problem 3.

Parameter	Symbol	Value
Beam length	l	0.5 m
Beam bending stiffness	EI	5 N·m^2
Tip mass	m	0.5 kg

Recommendations and clarifications:

- **Question 2a:** only substitute the values in Table 3 at the end of the question to evaluate ω_n.