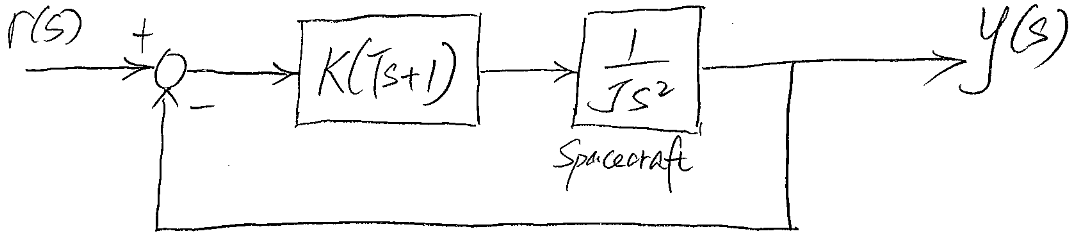
Problem 1.

Given:



$T = 3\,sec \qquad \frac{K}{J} = \frac{2}{9}$ and $K, J > 0$

Required:
1. Is the spacecraft stable?
2. Is the close-loop system stable?
3. Find the damping ratio of the close-loop system.

Solution:

1. poles $= \{0, 0\}$

The open-loop spacecraft is $\boxed{\text{not stable}}$

(or say <u>marginally stable</u>)

2. CE: $1 + K(Ts+1)\frac{1}{Js^2} = 0$

or

$$Js^2 + KTs + K = 0$$

or

$$s^2 + T\frac{K}{J}s + \frac{K}{J} = 0$$

or

$$s^2 + \frac{2}{3}s + \frac{2}{9} = 0$$

RSC:

$$\begin{array}{lll} s^2 & 1 & \frac{2}{9} \\ s^1 & \frac{2}{3} & 0 \\ s^0 & \frac{2}{9} & \end{array}$$

$\Rightarrow$ All poles of the CL system are in LHP.

Therefore the CL system is stable.

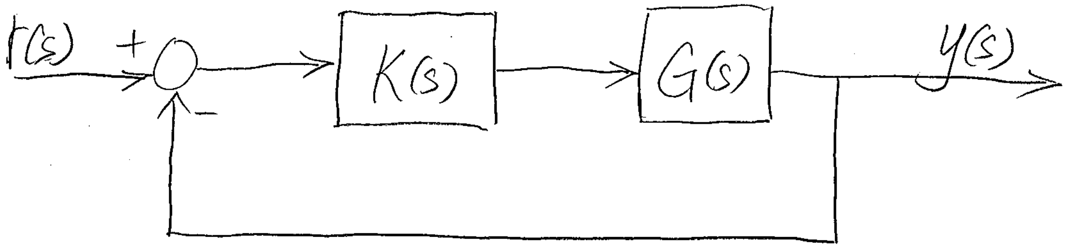
3.

$$\begin{cases} \omega_n^2 = \frac{2}{9} \\ 2\zeta\omega_n = \frac{2}{3} \end{cases} \Rightarrow \begin{cases} \omega_n = \frac{\sqrt{2}}{3} \\ \zeta_{CL} = \frac{\sqrt{2}}{2} \end{cases}$$

$$\boxed{\zeta_{CL} = \frac{\sqrt{2}}{2}}$$

Problem 2.

Given:



(1). $K(s) = k$, $G = \frac{s+2}{s^2}$

(2). $K(s) = k$, $G = \frac{1}{s(s+2)(s^2+4s+5)}$

(3). $K(s) = k$, $G = \frac{1}{s(s+0.5)(s^2+0.6s+10)}$

(4). $K(s) = k$, $G = \frac{s+0.2}{s^2(s+3.6)}$

Required: Draw RL by hand;
Check it using MATLAB.

※ We assume $k > 0$ for this course.

Solution:

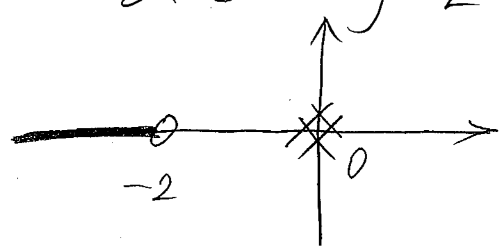
(1) CE: $1 + kL(s) = 0$

where $L(s) = G(s) = \frac{s+2}{s^2}$

1. $\boxed{\begin{cases} \text{poles: } 0, 0 \\ \text{zeros: } -2 \end{cases}}$ $\quad n = 2$, $m = 1$

2. Symmetric about the real axis

3. {Left of an odd number of real poles & zeros of $L(s)$} $\subset RL$



4. Asymptotes of RL: there are $(n-m)$ asymptotes.

$$\theta_a = \frac{180° + 360° l}{n-m} \quad (l = 0)$$

$$\boxed{\theta_a = 180°}$$

5. Break-in/away points: $\frac{d}{ds}\left(-\frac{1}{L(s)}\right) = 0$

$$\frac{d}{ds}\left(-\frac{1}{L(s)}\right) = -\frac{d}{ds}\,\frac{s^2}{s+2}$$

$$= -\frac{(s+2)(2s) - s^2}{(s+2)^2} = 0$$

$$\Downarrow$$

$$2s^2 + 4s - s^2 = 0$$

$$\boxed{S_b = \underset{\text{break-away}}{0}, \quad \underset{\text{break-in}}{-4}}$$

6. Angle of departure for complex poles : N/A.

7. Intersections with Im-axis :

$$1 + kL(j\omega) = 0$$

$$\Downarrow$$

$$1 + k\frac{j\omega + 2}{-\omega^2} = 0 \quad \text{or} \quad \omega^2 - k(j\omega + 2) = 0$$

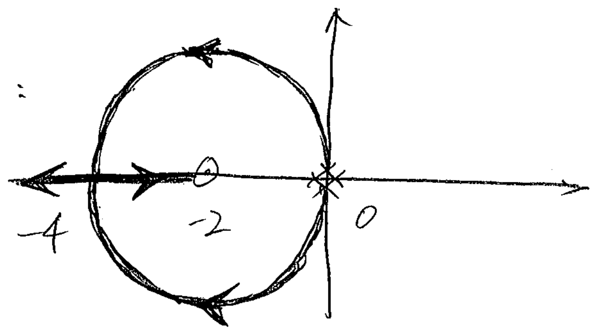
$$\text{so} \begin{cases} \omega^2 - 2k = 0 & ① \\ -kj\omega = 0 & ② \end{cases}$$

$$② \Rightarrow k = 0 \quad \text{or} \quad \omega = 0$$

If $k = 0$, $\omega = 0$;
If $\omega = 0$, $k = 0$. $\Rightarrow \begin{cases} k = 0 \\ \omega = 0 \end{cases}$ NOT appliable

Therefore there is no intersection

R.L:



(2). CE: $1 + kL(s) = 0$

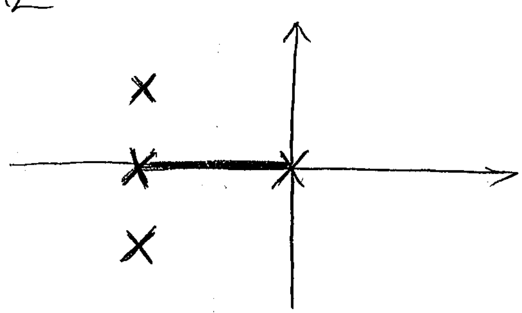
$$L(s) = \frac{1}{s(s+2)(s^2+4s+5)}$$

$$= \frac{1}{s(s+2)(s+2+j)(s+2-j)}$$

1. $\boxed{\begin{cases} \text{poles: } 0,\ -2,\ -2+j,\ -2-j & n = 4 \\ \text{zeros: } \phi & m = 0 \end{cases}}$

2. Symmetry

3. RL on real-axis :

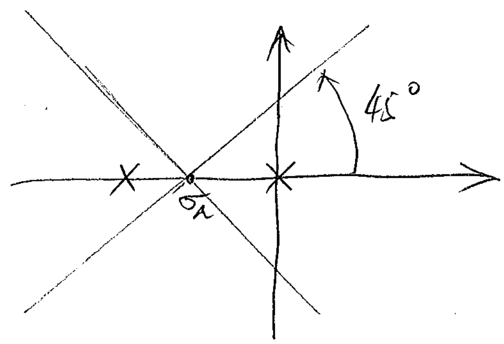


4. Asymptotes: # Asymptotes $= n - m = 4$.

$$\theta_a = \frac{180^\circ + 360^\circ l}{n-m} \quad ; \quad l = 0, 1, 2, 3$$

$$\boxed{\theta_a = 45^\circ, 135^\circ, 225^\circ, 315^\circ}$$

$$\sigma_a = \frac{\sum p_i - \sum z_i}{n-m} = \frac{0-2-2-2}{4} = \underline{-\frac{3}{2}}$$



5. Break-in/away points

$$\frac{d}{ds}\left(-\frac{1}{L(s)}\right) = 0$$

$$\Downarrow$$

$$\frac{d}{ds}\left[s(s+2)(s^2+4s+5)\right] = 0$$

$$LHS = \frac{d}{ds}\left[(s^2+2s)(s^2+4s+5)\right]$$

$$= (2s+2)(s^2+4s+5) + (s^2+2s)(2s+4)$$

$$= 2s^3 + 8s^2 + 10s + 2s^2 + 8s + 10 + 2s^3$$
$$+ 4s^2 + 4s^2 + 8s$$

$$= 4s^3 + 18s^2 + 26s + 10$$

The equation is equivalent to

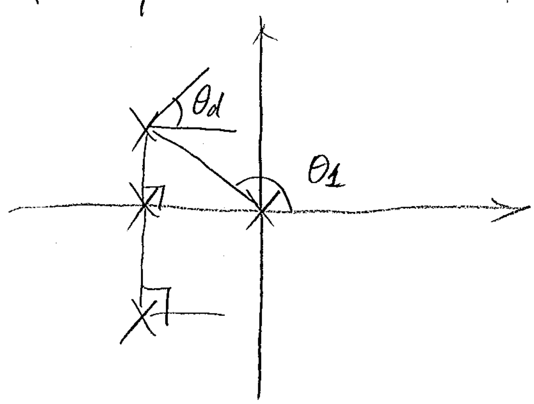
$$2s^3 + 9s^2 + 13s + 5 = 0$$

roots are -0.6018 , $-1.9491 \pm j\,0.5958$

$\uparrow$

break-away point

6. Angle of departure for complex poles



Consider a point, s^*, on RL that is very close to $s_p = -2 + j$.

The angles condition implies

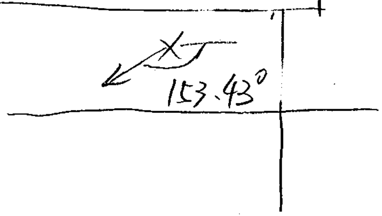
$$\angle L(s^*) = -180^\circ$$

$$\angle LHS = \underbrace{-\angle(s^* - 0)}_{\theta_1} \underbrace{- \angle(s^* - 2)}_{90^\circ} \underbrace{- \angle(s^* - 2 - j)}_{\theta_d} \underbrace{- \angle(s^* - 2 + j)}_{90^\circ}$$

$$-180^\circ - \theta_1 - \theta_d = -180$$

$$-\theta_d = \theta_1 = 180^\circ - \tan^{-1}\left(\frac{1}{2}\right) \approx 153.43^\circ$$

$$\boxed{\theta_d = -153.43^\circ}$$



7. Intersection with the Im-axis.

$$1 + kL(j\omega) = 0$$

$$1 + k\,\frac{1}{j\omega(j\omega+2)(-\omega^2+4j\omega+5)} = 0$$

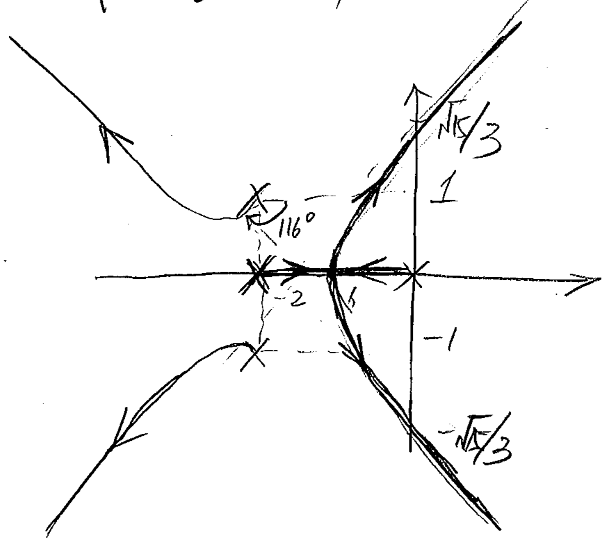
$$(+\omega^2 - 2j\omega)(+\omega^2 - 4j\omega - 5) + k = 0$$

$$(\cdots * \cdots) + j\left[-4\omega^3 - 2\omega^3 + 10\omega\right] = 0$$

$$\Downarrow$$

$$\begin{cases} \omega^4 - 13\omega^2 + k = 0 \\ -6\omega^3 - 10\omega = 0 \end{cases} \Rightarrow \quad \omega\left(\omega^2 + \tfrac{5}{3}\right) = 0$$

$$\boxed{\omega = \pm\frac{\sqrt{15}}{3},\ 0}$$



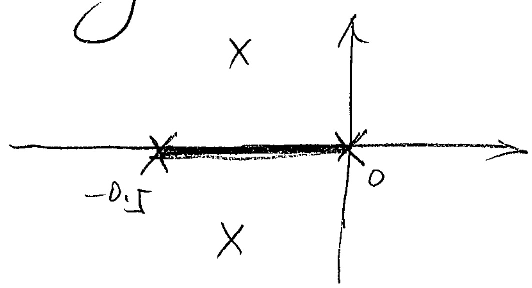
(3) CE: $1 + kL(s) = 0$, where

$$L(s) = \frac{1}{s(s+0.5)(s^2+0.6s+10)}$$

1. $\boxed{\begin{cases} \text{poles}: & 0,\ -0.5,\ -0.3 \pm j3.1480 \\ \text{zeros}: & \phi \end{cases}}$ $\quad n=4$, $m=0$

2. Symmetry

3.



4. $\theta_a = \dfrac{180° + 360° l}{n-m}$ $\quad l = 0, 1, \cdots n-m-1$

$= \boxed{45°,\ 135°,\ 225°,\ 315°}$

$\sigma_a = \dfrac{\sum_i p_i - \sum_i z_i}{n-m} = \dfrac{0-0.5-0.3-0.3}{4} = \dfrac{-1.1}{4}$

$\boxed{\sigma_a = 0.2750}$

5. Break-in/away points

$$\frac{d}{ds}\left(\frac{1}{L(s)}\right) = 0$$

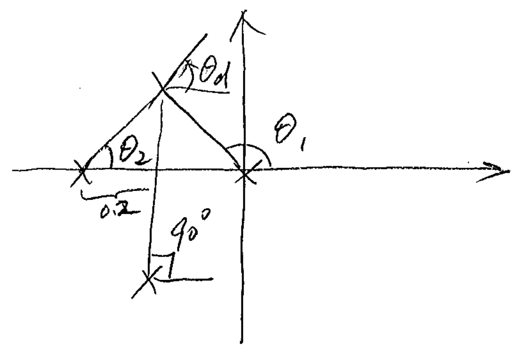
$$\Downarrow$$

$$4s^3 + \frac{33}{10}s^2 + \frac{103}{5}s + 5 = 0$$

roots: $\boxed{-0.2497}$, $-0.2877 \pm j2.2189$

$\uparrow$ break-away point.

6.



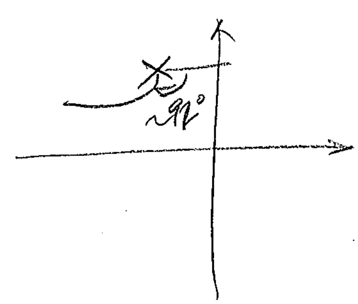
$$-\theta_d - \theta_1 - \theta_2 - 90° = -180°$$

$$\theta_d = 90° - \theta_1 - \theta_2$$

$$\theta_1 = 180° - \tan^{-1}\left(\frac{3.1480}{0.3}\right)$$

$$\theta_2 = \tan^{-1}\left(\frac{3.1480}{0.2}\right)$$

$$\boxed{\theta_d = -91.8085}$$



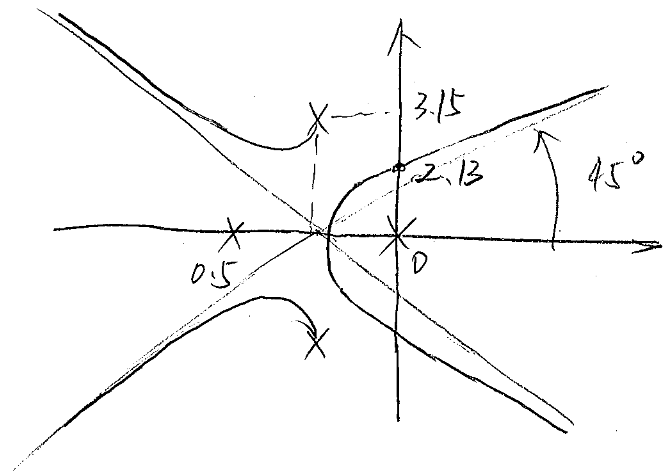
7. Intersection with Im-axis

$$1 + kL(j\omega) = 0$$

$$\Downarrow$$

$$j\omega(j\omega + 0.5)(-\omega^2 + 0.6j\omega + 10) + k = 0$$

$$\begin{cases} \omega^4 - \frac{103}{10}\omega^2 + k = 0 \\ -\frac{11}{10}\omega^3 + 5\omega = 0 \end{cases} \Rightarrow \begin{cases} \omega = 0 \\ k = 0 \end{cases} \text{ or } \begin{cases} \boxed{\omega = \sqrt{\frac{50}{11}}} \\ k = 26.16 \end{cases}$$



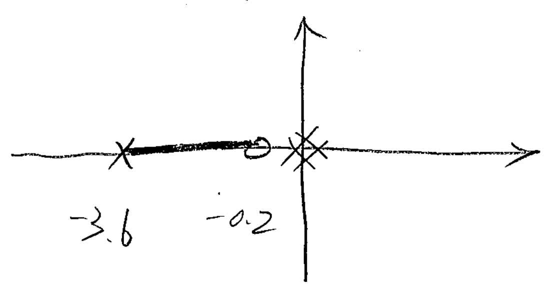
(4). CE: $1 + k\,L(s) = 0$

$$L(s) = \frac{s + 0.2}{s^2(s+3.6)}$$

1. $\begin{cases} \text{poles: } 0,\ 0,\ -3.6 & n = 3 \\ \text{zeros: } -0.2 & m = 1 \end{cases}$

2. Symmetry

3. R.L on Re-axis



4. Asymptotes:

$$\theta_a = \frac{180° + 360° l}{n-m}, \quad l = 0, 1, \cdots n-m-1$$

$$\boxed{= 90°, 270°}$$

$$\sigma_a = \frac{\sum \text{poles} - \sum \text{zeros}}{n-m} = \boxed{-1.7}$$

5. Break-in/away points.

$$\frac{d}{ds}\left(\frac{1}{L(s)}\right) = 0$$

$$\Downarrow$$

$$2s^3 + \frac{21}{5}s^2 + \frac{36}{25}s = 0$$

$$\boxed{\begin{array}{c} s = 0, \quad -1.6685, \quad -0.4315 \\ \uparrow \qquad\quad \uparrow \qquad\qquad \uparrow \\ \text{break-away points} \qquad \text{break-in point.} \end{array}}$$

6. N/A

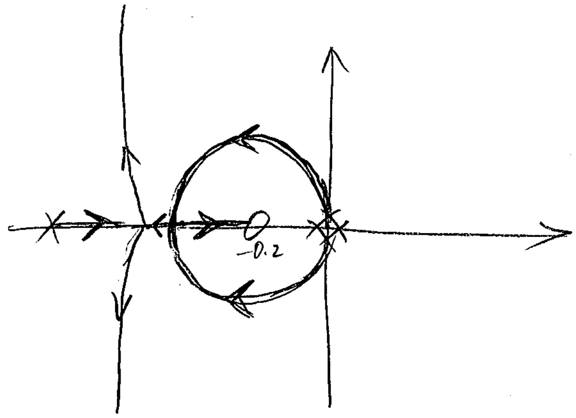
7. Intersection with Im-axis.

$$1 + kL(j\omega) = 0$$

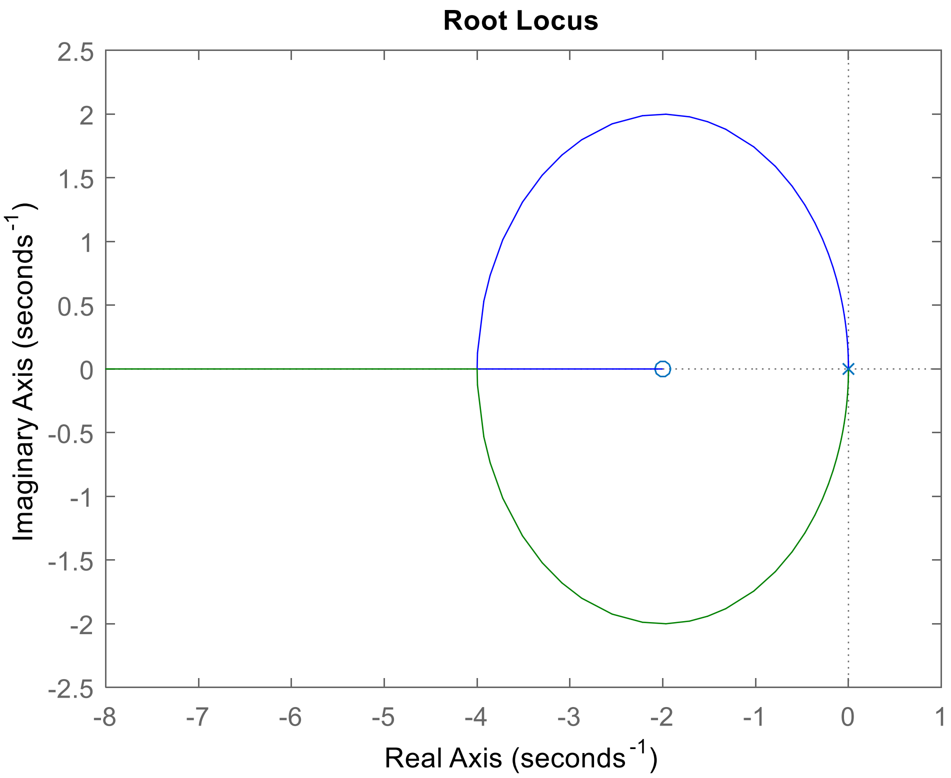
$$-\omega^2 (j\omega + 3.6) + k(j\omega + 0.2) = 0$$

$$\begin{cases} -3.6\,w^2 + 0.2k = 0 \\ -w^3 + kw = 0 \end{cases} \Rightarrow \begin{cases} w = 0 \\ k = 0 \end{cases}$$

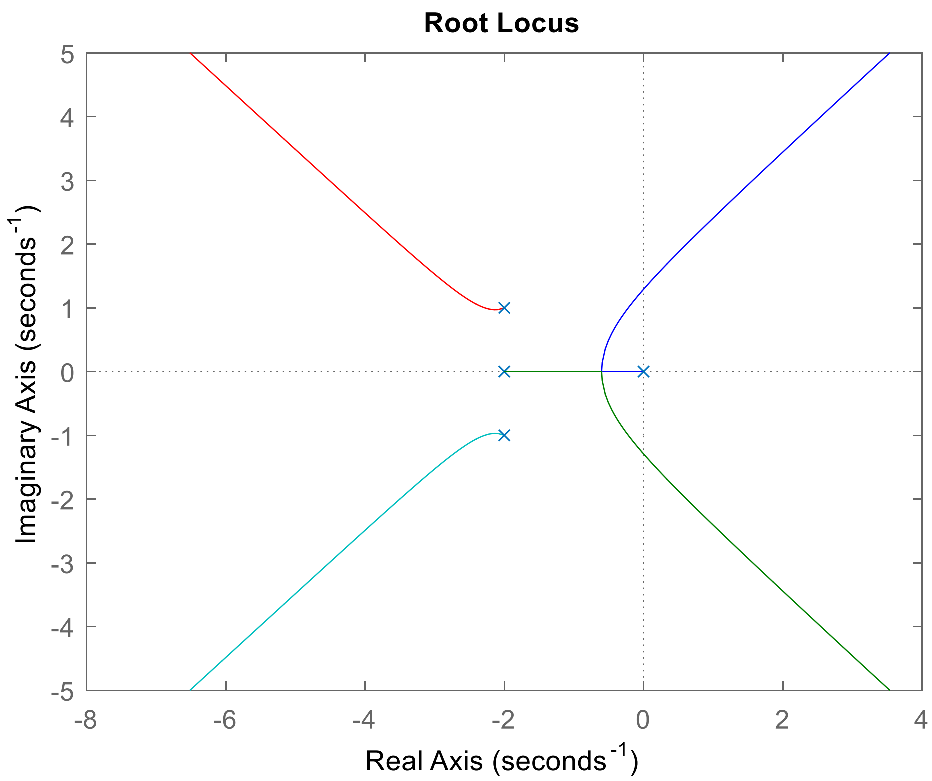
No intersection.



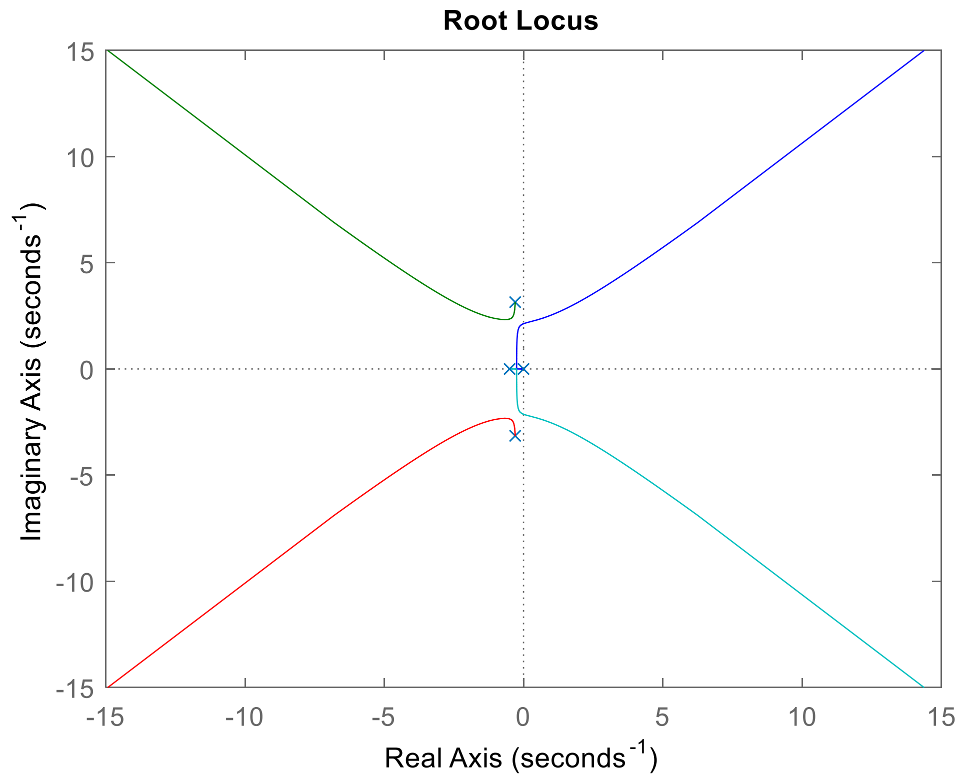
1.


Root Locus
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-8
-7
-6
-5
-4
-3
-2
-1
0
1

2.

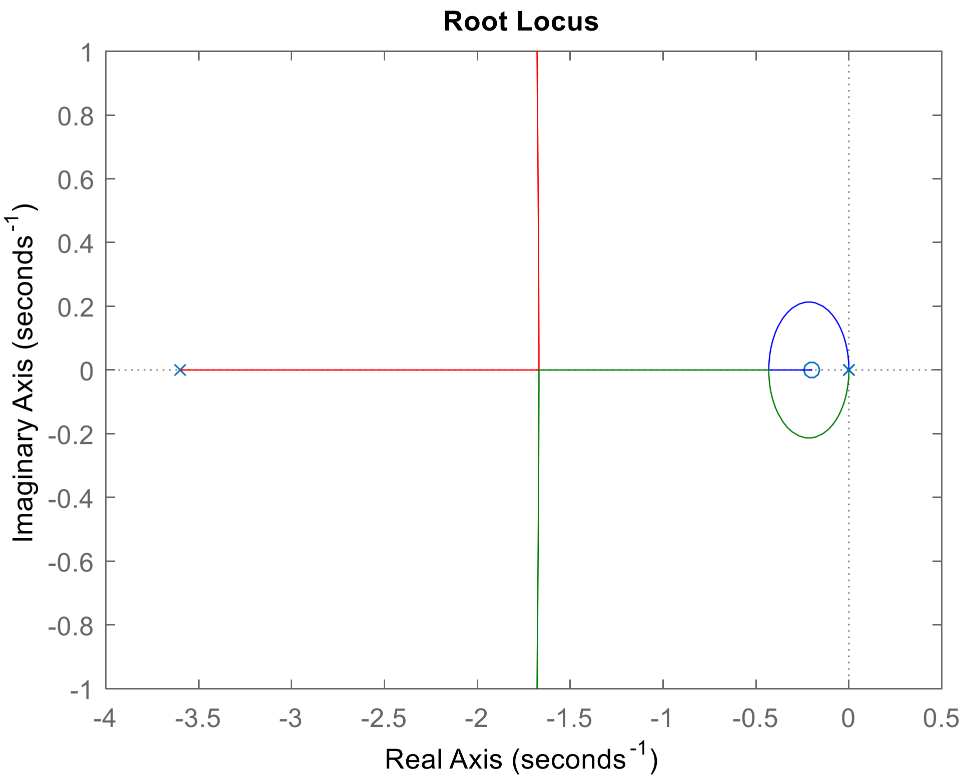

Root Locus
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})
5
4
3
2
1
0
-1
-2
-3
-4
-5
-8
-6
-4
-2
0
2
4

3.



Root Locus
15
10
5
0
-5
-10
-15
-15
-10
-5
0
5
10
15
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})

4.



Root Locus
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-4
-3.5
-3
-2.5
-2
-1.5
-1
-0.5
0
0.5
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})

```
%Part 1
G1 = zpk([-2],[0,0],1)
figure(1)
rlocus(G1)

%Part 2
G2 = zpk([],[0,-2,-2+sqrt(-1),-2-sqrt(-1)],1)
figure(2)
rlocus(G2)

%Part 3
G3 = tf([1],[1 11/10, 103/10, 5, 0])
figure(3)
rlocus(G3)

%Part 4
G4 = zpk([-0.2],[0,0,-3.6],1)
figure(4)
rlocus(G4)
```