Name	Team Number
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AAE 251: Introduction to Aerospace Design

Assignment 3—Orbits and Launch to LEO

Due Tuesday February 5, 10:00 am on Blackboard

NO 24 hr extension

Instructions

This assignment has six problems—four for the lecture Intro to Orbits, and two for lecture, Launch to LEO. The questions are a mix of derivations and numerical problems. Start the HW early, or you will run out of time!

Carefully read the lectures notes as they will be helpful to answer the questions. If you have questions, always ask for help from the TA.

Write or type your answers into the appropriate boxes. ***Make sure you submit a single PDF on Blackboard.*** *For the Matlab Code, you can either use Matlab's publishing feature and attach that to your homework or simply copy paste the code in Word and then make a PDF.*

There is no 24hr extension on this homework. Any submission after February 5, 10:00 am will not be accepted.

Problem Number	Points Possible	Points Earned
Problem 1	13	
Problem 2	4	
Problem 3	13	
Problem 4	15	
Problem 5	30	
Problem 6	5	
Total	80	

Introduction to Orbits

Question 1

In class, we saw that the equation of a conic section is given by:

$$r = \frac{p}{1+e\cos(\theta^*)} \quad ......(1)$$

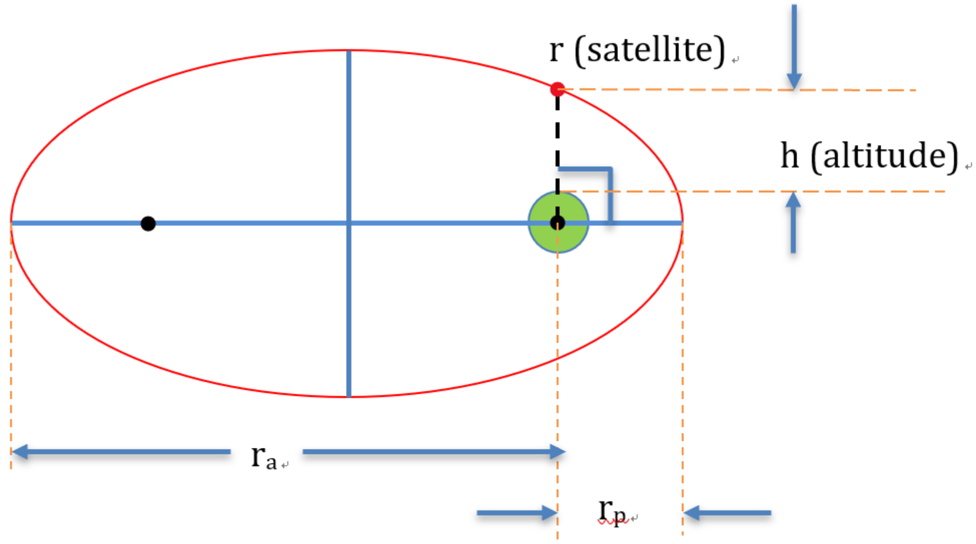
where $p = a(1 - e^2)$ is a geometrical constant of the conic called the "parameter" or "semi-latus rectum", e is the eccentricity, and θ^* is the true anomaly.

Your team launches a satellite into an orbit with a perigee radius of 7,000 km and an apogee radius of 10,000 km measured from the center of the Earth. You wish to calculate the altitude above the Earth's surface your satellite will attain when it has reached a point $90°$ past the perigee. To do so, follow these steps:

a) Draw a labeled sketch of the problem showing the orbit, perigee and apogee radius, and the position of the satellite at which we want to calculate the altitude.
b) Develop an expression for eccentricity in terms of perigee and apogee radius and calculate the value of eccentricity.
c) Calculate the semimajor axis for the orbit.
d) Use the polar equation of a conic section, Eq. (1), to calculate the desired altitude. Note that you need to report the altitude above the Earth's surface.

Answer 1:

(a)



r_a ≡ apogee radius = 10000 km

r_p ≡ perigee radius = 7000 km

(b)

$$r(\theta^*) = \frac{p}{1 + e\cos(\theta^*)} \quad \cdots (1) \quad \& \quad p = a(1-e^2) \quad \cdots (2)$$

$$\text{apogee} \equiv r_a = \frac{p}{1 + e\cos(180^\circ)} = \frac{a(1-e^2)}{1-e} = \boxed{a(1+e)}$$

$$\text{perigee} \equiv r_p = \frac{p}{1 + e\cos(0^\circ)} = \frac{a(1-e^2)}{1+e} = \boxed{a(1-e)}$$

Answer 1:

(c)

$$\text{semimajor axis} \equiv a = \frac{a(1+e)+a(1-e)}{2}$$

$$= \frac{r_a + r_p}{2}$$

$$= \frac{(10000 + 7000)\ \text{km}}{2} = \boxed{8500\ \text{km}}$$

(d)

$$\text{eccentricity} \equiv e = \frac{a-r_p}{a} = \frac{8500-7000}{8500} \cong 0.1765 \quad \cdots (3)$$

$$\text{altitude} \equiv h = r(90°) - R_e = \frac{P}{1+e\cdot 0} - R_e = a(1-e^2) - R_e$$

$$\because (2), (3)$$

$$h = (8500\ \text{km})(1 - 0.1765^2) - 6380\ \text{km}$$

$$\cong 8235.2\ \text{km} - 6380\ \text{km}$$

$$= 1855.2$$

$$\boxed{\text{altitude} = 1855\ \text{km}}$$

Question 2

Consider the satellite and the orbit you launched the satellite into in Question 1. What is the velocity magnitude of this satellite when the true anomaly is $90°$? How long before the spacecraft returns to this point in the orbit?

Answer 2:

From Question #1 and Vi Viva Equation

$$v = \sqrt{\frac{2GM_e}{r(90^\circ)} - \frac{GM_e}{a}} \qquad \because GM_e = \mu = 3.986 \times 10^{14} \frac{m^3}{s^2}$$

$$= \sqrt{\frac{2(3.986 \times 10^{14} \frac{m^3}{s^2})}{(8.235 \times 10^6 m)} - \frac{3.986 \times 10^{14} \frac{m^3}{s^2}}{8.5 \times 10^6 m}}$$

$$\cong 7.064 \ km/s$$

$$\boxed{7.064 \ km/s}$$

When the satellite returns to this point is when the satellite undergoes one period of orbit.

$$\text{period} \equiv T = 2\pi \sqrt{\frac{a^3}{\mu}} = 2\pi \sqrt{\frac{(8.5 \times 10^6 m)^3}{3.986 \times 10^{14} \frac{m^3}{s^2}}}$$

$$\cong 7799.01 \ s$$

$$\cong 2.17 \ hrs$$

$$\boxed{7799 \ s \ ; \ 2.17 \ hrs}$$

Question 3

At two points on a geocentric orbit, the altitude and true anomaly are:

Point 1: $R_1 = 1545\ km$ and $\theta_1{}^* = 126^O$

Point 2: $R_2 = 852\ km$ and $\theta_2{}^* = 58^O$

Find:

a. the eccentricity of the orbit defined by these two points
Hint: equate the absolute value of the specific angular momentum, $h = \sqrt{\mu p}$ at these two points
b. the altitude of perigee
c. the semimajor axis
d. the orbital period

Answer 3:

(a)

Given that: $R_1 + R_e = \dfrac{p}{1+e\cos\theta_1^*}$ ...(1) , $R_2 + R_e = \dfrac{p}{1+e\cos\theta_2^*}$ ... (2)

$$\frac{(1)}{(2)} \Rightarrow \frac{1+e\cos\theta_2^*}{1+e\cos\theta_1^*} = \frac{R_1'}{R_2'} \quad \langle \because R_1' = R_1 + R_e ,\ R_2' = R_2 + R_e \rangle$$

$$R_2' + (R_2'\cos\theta_2^*)e = R_1' + (R_1'\cos\theta_1^*)e$$

$$(R_2'\cos\theta_2^* - R_1'\cos\theta_1^*)e = R_1' - R_2'$$

$$\therefore e = \frac{R_1' - R_2'}{R_2'\cos\theta_2^* - R_1'\cos\theta_1^*}$$

$\langle$ since $R_1' = 7925$ km, $R_2' = 7232$ km $\rangle$ $= \dfrac{(1545 - 852)\text{ km}}{(7232\cos(58°) - 7925\cos(126°))\text{ km}}$

$$\cong 0.08162$$

$$\boxed{e = 0.08162}$$

(b)

because semilatus rectum $\equiv p = a(1-e^2)$ ... (3)

where $a \equiv$ semimajor axis

then $\because$ (1) & (3)

$$a = \frac{(R_1+R_e)(1+e\cos\theta_1^*)}{1-e^2} \cong 7595.4 \text{ km} \quad \cdots (4)$$

now since perigee radius $\equiv r_p = a(1-e)$

perigee altitude $\equiv R_p = r_p - R_e = a(1-e) - R_e$

$$= (7595.4 \text{ km})(1-0.08162) - (6380 \text{ km})$$

$$\cong 544.03 \text{ km}$$

$$\boxed{R_p = 544 \text{ km}}$$

(c)

$\because (4)$ $$\boxed{a = 7595 \text{ km}}$$

(d)

orbital period $\equiv T = 2\pi\sqrt{\frac{a^3}{\mu}}$

$\because \mu = \mu_{earth} = 3.986 \times 10^{14} \frac{m^3}{s^2}$

$\therefore T = 2\pi\sqrt{\frac{(7595 \times 10^3 \, m)^3}{3.986 \times 10^{14} \frac{m^3}{s^2}}}$

$\cong 6587.23 \text{ s}$

$\cong 1.83 \text{ hrs}$

$$\boxed{T = 6587 \text{ s} \; ; \; T = 1.83 \text{ hrs}}$$

Question 4

The date is 02 September 1986 and you are an aspiring astrodynamics expert working at JPL. Today, Halley's comet is passing through perihelion. Using your state-of-the-art algorithm, you found that its orbit has eccentricity $e = 0.9671429$, and semi-major axis $a = 17.834144 \ AU$.

A colleague runs in the room telling you that they have finally managed to figure out all the decimals of the Sun's gravitational parameter and it is equal to:

$$\mu_{Sun} = 1.327124400189 \ \times 10^{20} \ m^3/s^2$$

Then, they ask you to calculate the perihelion and aphelion distances, the current true anomaly, the specific total energy, the specific angular momentum, the period, and the current velocity magnitude of Halley's comet. (Use AU for distance units)

After completing your calculations, plot Halley's orbit in MATLAB. You need to use the conic equation as a function of the true anomaly, $0° \le \theta^* \le 360°$. Place the Sun at the center of your plot and add the Earth's orbit as well ($e_E = 0.0167, a_E = 1AU$). Is there a chance that the comet passes close to Earth? Do NOT forget to attach your MATLAB code at the end.

Answer 4:

- calculations -

$$\begin{cases} e = 0.9671429 \\ a = 17.834144 \text{ AU} = 2.667949968 \times 10^{12} \text{ m} \\ \mu_{sun} = 1.32712440018 \times 10^{20} \frac{m^3}{s^2} \\ e_E = 0.0167 \\ a_E = 1.495978707 \times 10^{11} \text{ m} = 1 \text{ AU} \end{cases}$$

assumption : same plane of reference (sun, Halley's Comet, and earth)

(i) perihelion & aphelion

$$\text{perihelion} \equiv r_p = a(1-e)$$
$$= (2.667949968 \times 10^{12} \text{ m})(1 - 0.9671429)$$
$$= \boxed{8.766109889 \times 10^{10} \text{ m}}$$
$$= \boxed{0.5859782528 \text{ AU}}$$

$$\text{aphelion} \equiv r_a = a(1+e)$$
$$= \boxed{5.248238837 \times 10^{12} \text{ m}}$$
$$= \boxed{35.08230975 \text{ AU}}$$

(ii) The comet is passing through the perihelion thus,

$$\text{current true anomaly} \equiv \boxed{\theta^* = 0}$$

(iii) Specific Energy $E_t = -\frac{GM}{2a} = -\frac{\mu_{sun}}{2a}$

$$\therefore E_t = \frac{-1.32712440018 \times 10^{20} \frac{m^3}{s^2}}{2 \times 2.667949968 \times 10^{12} \text{ m}}$$
$$= \boxed{-24871613.37 \text{ J/kg}}$$
$$= \boxed{-24.87161337 \text{ MJ/kg}}$$

Answer 4:

(iv) specific angular momentum $\equiv h_{sun}$

if we equate $h_{sun} = \sqrt{\mu_{sun} p}$

where $p \equiv$ semilatus rectum $= a(1-e^2)$

$$\therefore h_{sun} = \sqrt{\mu_{sun} a(1-e^2)}$$

$$= \boxed{4.783846403 \times 10^{15} \ \frac{m^2}{s}}$$

(v) period T

$$T = 2\pi \sqrt{\frac{a^3}{\mu_{sun}}}$$

$$= 2376788693 \text{ s}$$

$$= 27509.1284 \text{ days}$$

$$= 75.36747506 \text{ years}$$

(vi) current velocity Vi Viva Eqn.

$$v = \sqrt{\frac{2\mu_{sun}}{r_p} - \frac{\mu_{sun}}{a}}$$

$$= 54572.05606 \ \frac{m}{s}$$

$$= 54.57205606 \ km/s$$

Problem #4 Graphing Orbit

Here, the orbit of the Halley's comet will be graphed with the sun in the center, along with the orbit of the earth around the sun.

Set-up

```
e_halley = 0.9671429; % The eccentricity of Halley's comet-sun orbit
a_halley = 2.667949968 * 10^(12); % Semimajor axis of the Halley's comet-sun orbit [m]
myu_sun = 1.327124400189 * 10^(20); %The standard gravitaional parameter of the
                                    % sun [m^3/s^2]
e_E = 0.0167; % The eccentricity of the earth-sun orbit
a_E = 1.49597870700 * 10^(11); % The standard gravitational parameter of the
                               % earth [m^3/s^2]
```

Calculations

In order to plot the elliptical orbit we must figure out two properties: the semimajor axis, "a" and the semiminor axis, "b" of the orbit. Since the semimajor axis is already given and perihelion and aphelion can be expressed using the eccentricity and semimajor axis as

$$r_p \ = a(1-e) \ \cdots (1) \quad \text{and} \quad r_a \ = \ a(1+e) \ \cdots (2)$$

Now from the mathematic property of the ellipse the realtion between the semimajor axis, the semiminor axis, and the eccentricity is

$$e \ = \frac{\sqrt{a^2 - b^2}}{a}$$

maniuplate this and you can obtain

$$b \ = \sqrt{a^2(1 \ - e^2)}$$

and by plugging in (1) and (2) from above

$$b \ = \ \sqrt{r_p \cdot r_a}$$

Then we shall compute the periapsis, apoapsis, and the semiminor axis

```
% Halley's comet
r_p_halley = a_halley * (1 - e_halley); % Perihelion [m]
r_a_halley = a_halley * (1 + e_halley); % Aphelion [m]
b_halley = sqrt(r_p_halley * r_a_halley); % Semiminor axis [m]

% Earth
r_p_E = a_E * (1 - e_E); % Perigee [m]
r_a_E = a_E * (1 + e_E); % Apogee [m]
b_E = sqrt(r_p_E * r_a_E); % Semiminor axis of earth-sun orbit [m]
```

Plotting

Now that we have all the parameters for plotting the orbit for the Hally's comet and the earth; for both of the orbits the sun is the center of the orbit.

The plot will be done using the parameterization of ellipse equation. In which a general form of a ellipse equation is expressed as

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

and when fixed into a parametic equation, in our case using the true anomaly as the parameter. The equations become

$$x = a \cdot \cos(\theta) \quad \text{and} \quad y = b \cdot \sin(\theta)$$

For our plot the center (* x = 0, y = 0) is the sun; and therefore the parameterization becomes

$$x = a \cdot \cos(\theta^*) - \sqrt{a^2 - b^2} \quad \text{and} \quad y = b \cdot \sin(\theta^*)$$

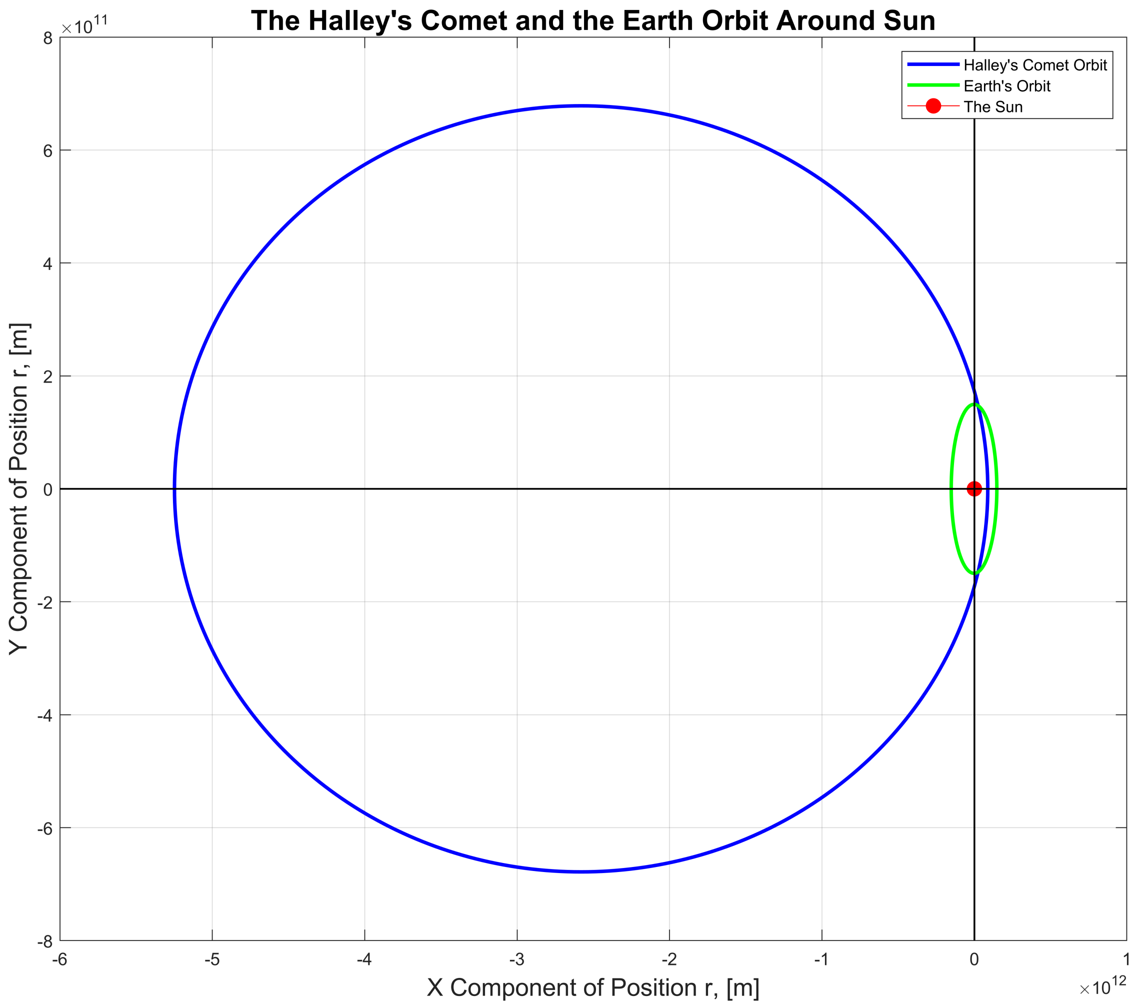
which is in other words

$$x = a \cdot \cos(\theta^*) - \sqrt{a^2 - (r_p \cdot r_a)} \quad \text{and} \quad y = \sqrt{r_p \cdot r_a} \cdot \sin(\theta^*)$$

```
% Adjustments for the plot
linewidth = 2;
fontsize = 14;

% Set-up to plot Halley's comet orbit
theta = 0:360; % The true anomaly ranging from 0 to 360 degrees
x_halley = a_halley * cosd(theta) - sqrt(a_halley^2 - b_halley^2); % X-value parameter
y_halley = b_halley * sind(theta); % Y-value parameter
% Set-up to plot Earth's orbit
x_E = a_E * cosd(theta) - sqrt(a_E^2 - b_E^2); % X-value parameter
y_E = b_E * sind(theta); % Y-value parameter

% Plotting
figure(1)
plot(x_halley, y_halley,'Color','b', 'LineStyle','-', 'LineWidth',linewidth);
title('The Halley''s Comet and the Earth Orbit Around Sun', 'FontSize',16)
xlabel('X Component of Position r, [m]', 'FontSize',fontsize)
ylabel('Y Component of Position r, [m]', 'Fontsize', fontsize)
box on
grid on
hold on
plot(x_E, y_E, 'Color', 'g', 'LineStyle','-','LineWidth',linewidth)
plot(0,0,'marker', '.', 'MarkerSize',30, 'color','r') % Plotting the Sun as the center
line([0 0], ylim, 'color', 'k', 'linewidth', 1)
line(xlim, [0 0], 'color', 'k', 'linewidth', 1)
hold off
legend('Halley''s Comet Orbit', 'Earth''s Orbit', 'The Sun', 'Location','northeast')
% Control position of figure
set(gcf,'PaperPositionMode','auto','Position',[0 0 1050 875])
```



Analysis:

Is there a chance of Halley's comet passing by the earth?

Yes there is most likely a chance of such event occurring. The figure plotted by matlab does not consider the inclination between the orbital planes so is not precisely accurate; however, even having the inclination from the graph we can tell that the two orbits have an instance where the comet and earth approach each other.

The last time that Halley's comet entered the inner solar system was 1986, and is estimated to return in mid-2061.

Launch to LEO

Question 5:

a. Fill out the Table with launch site information. Include the latitude, longitude, and location information of some important launch sites in the world.

b. At each launch site, compute the velocity due to Earth's rotation and include the value in the Table. Assume that the Earth's axis of rotation is perpendicular to the equatorial plane.

c. Assume that a multinational scientific organization wants to launch a spacecraft to LEO and is exploring various theoretical scenarios.

   i. Launch from Cape Canaveral to achieve a 90° inclination and 200 km altitude circular orbit.
   ii. Launch from Kourou due East to achieve a 300 km altitude circular orbit.
   iii. Launch from Svalbard Rocket Range to achieve 90° inclination and 400 km altitude circular orbit.

Draw the launch velocity triangles and evaluate the "launch ΔV" for each of these scenarios and make your suggestion. Why did you pick that option?

Launch Site	**Location**	**Latitude, deg**	**Longitude, deg**	$v_{Earth\_rotation}$**, m/s**
Cape Canaveral	USA	28.396837	-80.605659	407.44
Sriharikota	India	13.733000	80.235000	449.93
Kourou	French Guiana	5.155180	-52.647790	461.30
Svalbard Rocket Range	Norway	78.925496	11.850163	88.97
Baikonur Cosmodrome	Kazakhstan	45.616669	63.316666	323.97

Answer 5:

(b)

$$\Delta V_{EH} = \omega_E r_E \cos(L_a)\sin(A_z)$$

where $\omega_E = 7.27\times10^{-5}\ \frac{rad}{s}$

$r_{E,avg} = 6371\ km = 6371\times10^3\ m$

and assume $A_z \equiv$ azimuth is all $90°$ (east)

where ΔV_{EH} is at MAX.

(Sample calculation)

- Cape Canaveral

$$\Delta V_{EH,1} = \left(7.27\times10^{-5}\ \frac{rad}{s}\right)\left(6371\times10^3\ m\right)\cos(28.396837°)\sin(90°)$$

$$\cong \boxed{407.44\ m/s}$$

likewise

- Sriharikota $\Delta V_{EH,2} \cong \boxed{449.93\ m/s}$
- Kourou $\Delta V_{EH,3} \cong \boxed{461.30\ m/s}$
- Svalbard Rocket Range $\Delta V_{EH,4} \cong \boxed{88.97\ m/s}$
- Baikonur Cosmodrome $\Delta V_{EH,5} \cong \boxed{323.97\ m/s}$

Answer 5:

(c)

(c)

(i) $v_{circ} = \sqrt{\dfrac{\mu_E}{R_E + h_0}} \simeq \boxed{7.783 \times 10^3\ m/s}$

since $\mu_E = 3.986 \times 10^{14}\ \frac{m^3}{s^2}$

$R_E = 6.38 \times 10^6\ m$

$h_0 = 200 \times 10^3\ m$

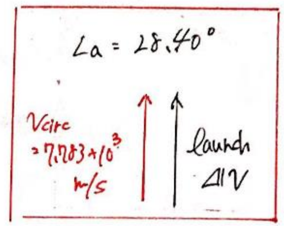
for inclination $i = 90°$ & $L_a = 28.396837°$ @ Cape Canaveral

$\cos i = \cos(L_a)\sin(A_z)$

$A_z = \arcsin\left(\dfrac{\cos 90°}{\cos 28.396837}\right) = 0 \quad (180°)$

$\therefore v_{EH} = \omega_E r_E \cos(L_a)\sin(A_z) = \boxed{0}$

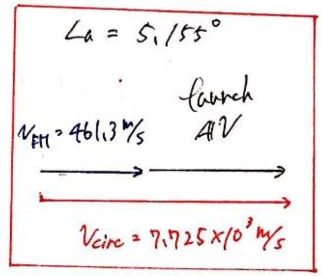
$\Delta V = V_{circ}$



(ii) $A_z = 90°$ $\quad V_{EH} = \boxed{461.30\ m/s}$

$V_{circ} = \sqrt{\dfrac{\left(3.986 \times 10^{14} \frac{m^3}{s^2}\right)}{(6.38 \times 10^6 + 300 \times 10^3)m}} \simeq \boxed{7.725 \times 10^3\ m/s}$

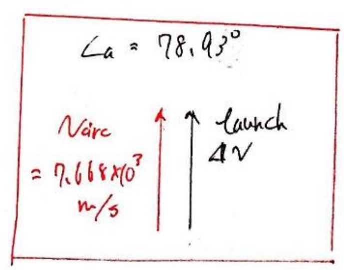
$\Delta V = V_{circ} - V_{EH}$



(iii) $i = 90°$ $\quad A_z = 0°$ $\quad V_{EH} = 0$

$V_{circ} = \sqrt{\dfrac{\left(3.986 \times 10^{14} \frac{m^3}{s^2}\right)}{(6.38 \times 10^6 + 400 \times 10^3)m}} \simeq 7.668 \times 10^3\ m/s$

$\Delta V = V_{circ}$



Question 6:

A spacecraft is to be launched from Cape Canaveral to the International Space Station (ISS). The ISS orbits at 51.6° inclination. Find the two possible launch azimuths for the launch vehicle. Which launch azimuth is more preferred and why?

Answer 6:

@ Cape Canaveral

inclination $\equiv i = 51.6^\circ$

latitude $\equiv La = 28.397^\circ$

from

$$\cos i = \cos(La)\sin(Az)$$

$$Az = \arcsin\left(\frac{\cos(51.6^\circ)}{\cos(28.397^\circ)}\right)$$

$$\simeq \underset{\theta_1}{\underset{\parallel}{44.919^\circ}} \text{ or } (180^\circ - 44.919^\circ) = 135.081^\circ = \theta_2$$

The safety azimuth at Cape Canaveral is approximately $30^\circ < \theta < 120^\circ$ to prevent the rocket flying over land. And if the azimuth were to be $\theta_2 = 135.081^\circ$ this will make the rocket fly over the archipelagoes in the vicinity of The Panamas. Therefore, in terms of safety measures it will be better to launch the rocket in the azimuth of $\theta_1 = 44.919^\circ$.