AAE 564 Fall 2020

Homework Twelve

Due: Friday, November 20

Exercise 1 (By hand.) Determine whether or not each of the following systems are controllable, stabilizable, or not stabilizable.

$$\begin{aligned}\dot{x}_1 &= -x_1 + u \\ \dot{x}_2 &= x_2 + u\end{aligned} \qquad \begin{aligned}\dot{x}_1 &= -x_1 \\ \dot{x}_2 &= x_2 + u\end{aligned} \qquad \begin{aligned}\dot{x}_1 &= x_1 + u \\ \dot{x}_2 &= x_2 + u\end{aligned}$$

Exercise 2 Obtain an open loop control which drives the following system from $x(0) = -1$ to $x(1) = 1$

$$\dot{x} = x + u\,.$$

Exercise 3 (By hand.) Consider the system described by

$$\begin{aligned}\dot{x}_1 &= x_1 + x_2 + u \\ \dot{x}_2 &= u\end{aligned}$$

Obtain (by hand) a state feedback controller which results in a closed loop system which is asymptotically stable about the zero state.

Exercise 4 (By hand.) Consider the system described by

$$\begin{aligned}\dot{x}_1 &= -x_2 + u \\ \dot{x}_2 &= -x_1 - u\end{aligned}$$

where all quantities are scalars.

(a) Is this system stabilizable via state feedback?

(b) Does there exist a linear state feedback controller which results in closed loop eigenvalues $-1, -4$?

(c) Does there exist a linear state feedback controller which results in closed loop eigenvalues $-2, -4$?

In parts (b) and (c): If no controller exists, explain why; if one does exist, give an example of one.

Exercise 5 (Stabilization of cart pendulum system via state feedback.) (MATLAB) Carry out the following for parameter sets P2 and P4 and equilibriums $E1$ and $E2$. Illustrate the effectiveness of your controllers with numerical simulations.

Using eigenvalue placement techniques, obtain a state feedback controller which stabilizes the nonlinear system about the equilibrium.

What is the largest value of δ (in degrees) for which your controller guarantees convergence of the closed loop system to the equilibrium for initial condition

$$(y,\, \theta_1,\, \theta_2,\, \dot{y},\, \dot{\theta}_1,\, \dot{\theta}_2)(0) = (0,\, \theta_1^e - \delta,\, \theta_2^e + \delta,\, 0,\, 0,\, 0)$$

where θ_1^e and θ_2^e are the equilibrium values of θ_1 and θ_2.

Exercise 6 (By hand.) Consider the discrete-time system

$$\begin{aligned} x_1(k+1) &= x_1(k) + x_2(k) \\ x_2(k+1) &= x_2(k) + u(k) \end{aligned}$$

Obtain a state feedback controller which always drives the state of this system to zero in at most two steps.

Exercise 7 For the system described by

$$\begin{aligned} 2\ddot{q}_1 + \ddot{q}_2 - q_2 &= u_1 \\ \ddot{q}_1 + 2\ddot{q}_2 - q_1 &= u_2 \end{aligned}$$

obtain a feedback controller generating u_1 and u_2 which stabilizes this system. Assume that $q_1, q_2, \dot{q}_1$ and $\dot{q}_2$ can be measured. You can use MATLAB for some of this.

Exercise 8 Consider the system described by

$$\dot{x}_1 = x_1 + u \tag{1}$$
$$\dot{x}_2 = -x_2 + 2u \tag{2}$$

Obtain (by hand) a state-feedback controller (it will not be a static controller) which always results in the state of the closed loop going to zero in at most 2 secs. Illustrate your results with a simulation.