Name	Team Number
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AAE 251: Introduction to Aerospace Design

Assignment 6—The Rocket Equation, Rocket Thrust and Staging

Due Tuesday March 5, 10:00 am on Blackboard

No 24hr extensions

Instructions

Write or type your answers into the appropriate boxes. ***Make sure you submit a single PDF on Blackboard.***

Make sure you keep a record of submission receipts or the confirmation emails after each submission as a proof that your submission was accepted.

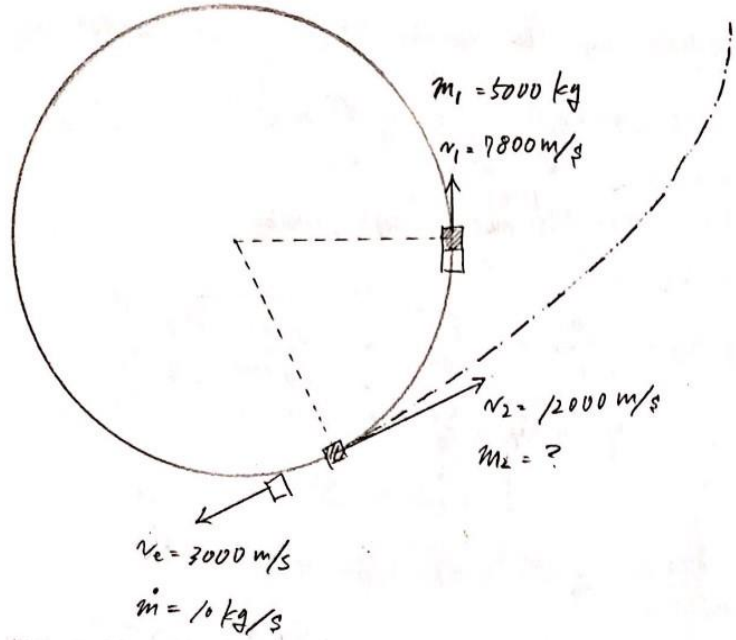
There is no 24hr extension on this homework. Any submission after March 5, 10:00 am will not be accepted.

	Score	Max
Question 1		**8**
Question 2		**9**
Question 3		**10**
Question 4		**10**
Question 5		**13**
Question 6		**25**
TOTAL		**75**

Question 1

A spacecraft weighing 5,000 kg is travelling in an orbit around Earth at a velocity of 7,800 m/s. Its engine places it on an escape trajectory by accelerating it to a velocity of 12,000 m/s. The engine expels mass at a rate of 10 kg/s at an effective exhaust velocity of 3,000 m/s. What is the duration of the burn?

Answer 1:



Total change in momentum $\equiv \Delta p = m_1 v_1 - m_2 v_2$

$$F_{thrust} = \frac{dm\, v_e}{dt} = \dot{m} v_e$$

and $(F_{thrust})\Delta t = \Delta p \iff \Delta t = \dfrac{\Delta p}{F_{thrust}} = \dfrac{m_1 v_1 - m_2 v_2}{F_{thrust}} \quad \cdots ①$

since we still do not know m_2

$$\because \quad dv = -v_e \frac{dm}{m}$$

$$\int_{v_1}^{v_2} dv = -v_e \int_{m_1}^{m_2} \frac{dm}{m} \rightarrow v_2 - v_1 = -v_e \ln \frac{m_2}{m_1}$$

$$m_2 = m_1 \exp\left(-\frac{v_2 - v_1}{v_e}\right) \quad \cdots ②$$

$$\because ② \Rightarrow m_2 = (5000\text{ kg}) \exp\left(-\frac{12000\text{ m/s} - 7800\text{ m/s}}{3000\text{ m/s}}\right) \cong 944.378\text{ kg}$$

$\because$ ① & ②

$$\Delta t = \frac{(5000\text{ kg})(7800\text{ m/s}) - (944.378\text{ kg})(12000\text{ m/s})}{(10\text{ kg/s})(3000\text{ m/s})} \cong \boxed{922.25\text{ s}}$$

Question 2

A single-stage rocket is used to launch a satellite weighing 1000 kg into a circular orbit at an altitude of 200 km. The specific impulse of the rocket is 20,000 m/s. The structural mass of the rocket is 20% of the initial mass. Calculate the mass of propellant needed for this mission.

$GM = 3.986 \times 10^5\ km^3/s^2; \quad R_e = 6378\ km$

Answer 2:

this is mass · based so v_e has unit m/s $v_e = 20000$ m/s

$GM = \mu = 3.986 \times 10^5\ km^3/s^2 = 3.986 \times 10^{14}\ m^3/s^2$

$R_e = 6378\ km = 6.378 \times 10^6\ m$

$R = 200\ km$, satellite mass $= m = 1000\ kg$

initial velocity $v_i = 0$

final velocity $v_f = \sqrt{\frac{\mu}{r}} = \sqrt{\frac{\mu}{R+R_e}} = \sqrt{\frac{3.986 \times 10^{14}\ m^3/s^2}{2.00 \times 10^5 + 6.378 \times 10^6}}$

$\cong 7784.34\ m/s$

now using

$$dv = -v_e \frac{dm}{m}$$

$$v_f - v_i = v_e \ln \frac{m_i}{m_f}$$

$$f_{inert} = \frac{0.2 m_i}{m_i - m_{pay}}$$

thus

$m_{pay} = 1000\ kg$

$m_{inert} = 0.2 m_i$

$m_i = m_{prop} + m_{pay} + m_{inert} = m_{prop} + m_{pay} + 0.2 m_i$

$m_{prop} = 0.8 m_i - m_{pay}$ ... ①

$m_f = m_{pay} + m_{inert} = m_{pay} + 0.2 m_i$

$v_f - \cancel{v_i}^{0} = v_e \ln \frac{m_i}{m_f}$

$m_f = m_i \exp(-\frac{v_f}{v_e})$

$m_f \cong 0.67759\ m_i$

$0.67759 m_i = m_{pay} + 0.2 m_i$

$m_i = 2093.85\ kg$

$m_{prop} = ① = 675.08$

$\boxed{675.08\ kg}$

Question 3

The ARIANE 5 launch vehicle has two P230 solid propellant boosters and a main Vulcain engine that are ignited at lift-off. The two components of the launch vehicle have the following characteristics:

Effective exhaust velocities: $c_{Vulcain} = 3285$ m/s; $c_{P230} = 2355$ m/s

Mass flows: $\dot{m}_{Vulcain} = 255$ kg/s; $\dot{m}_{P230} = 1835$ kg/s

Calculate the average effective exhaust velocity and average specific impulse for the launch vehicle.

Answer 3:

$$\text{mean}(C_{eff}) = \frac{C_{vulcain}\dot{m}_{vulcain} + 2C_{p230}\dot{m}_{p230}}{\dot{m}_{vulcain} + 2\dot{m}_{p230}}$$

$$= \frac{(3285\ m/s)(255\ kg/s) + 2(2355\ m/s)(1835\ kg/s)}{255\ kg/s + 2(1835\ kg/s)}$$

$$\cong \boxed{2415.42\ m/s}$$

$$\text{mean}(I_{sp}) = \frac{\text{mean}(C_{eff})}{g_0} = \frac{2415.42\ m/s}{8.31\ m/s^2} \cong \boxed{290.66\ s}$$

Question 4

The mass of the payload of a spacecraft is 1000 kg, and the inert mass fraction is 0.10. If the velocity change, ΔV and the exit velocity of the propellant are 4860 m/s and 4267 m/s respectively,

a) What is the mass of the propellant on board the spacecraft?

b) What is the inert mass of the spacecraft?

c) What is the initial mass of the spacecraft?

Answer 4:

$m_{pay} = 1000\ kg$

$f_{inert} = \beta = 0.10$

$\Delta V = 4860\ m/s$

$v_e = 4267\ m/s$

(a) using the formula

$$m_{prop} = \frac{m_{pay}\left(e^{\frac{\Delta V}{v_e}} - 1\right)(1-\beta)}{1-\beta e^{\frac{\Delta V}{v_e}}} = \frac{(1000\ kg)\left(e^{\frac{4860}{4267}} - 1\right)(1-0.10)}{1-(0.10)e^{\frac{4860}{4267}}}$$

$$\approx \boxed{2779.35}\ kg$$

(b)

$$\beta = \frac{m_{inert}}{m_{inert} + m_{prop}} \Leftrightarrow m_{inert} = \frac{\beta}{1-\beta} m_{prop} = \frac{0.10}{0.90}(2779.35\ kg)$$

$$\approx \boxed{308.82\ kg}$$

(c)

$$m_i = m_{pay} + m_{inert} + m_{prop}$$

$$= 1000\ kg + 308.82\ kg + 2779.35\ kg$$

$$= \boxed{4088.17\ kg}$$

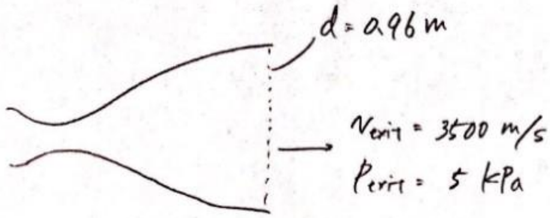
<u>Question 5</u>

A rocket's engine has an exit diameter of $0.96\, m$. It is ejecting mass at a rate of $30\ kg/s$ with an exit velocity of $3{,}500\ m/s$. The pressure at the exit of the nozzle is $5\ kPa$. Calculate:

a) The thrust of the engine in vacuum
b) Final mass of the rocket if the initial mass was $1000\ kg$ and the burn duration was $25s$.
c) Specific impulse
d) Effective exhaust velocity

Answer 5:

Q5.



(a) at vacuum $P_{atm} = 0$

so

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.96\,m)^2 \simeq 0.7238\ m^2$$

and

$$F_{THRUST} = \dot{m}[v_{exit} + (P_{exit} - \overset{0}{\cancel{P_{atm}}})A]$$

$$= (30\ kg/s)[3500\ m/s] + (5\times10^3\ Pa)(0.7238\ m^2)$$

$$= \boxed{108619\ N}$$

(b)

$$\frac{dm}{dt} = \dot{m}$$

$$\int_i^f dm = \int_0^{25} -\dot{m}\,dt \quad\rightarrow\quad m_f - m_i = -\dot{m}\Delta t$$

$$m_f = -\dot{m}\Delta t + m_i$$

$$= -(30\ kg/s)(25\ s) + 1000\ kg = \boxed{250\ kg}$$

(c) specific impulse $\equiv I_{sp} = \dfrac{F_{THRUST}}{\dot{m}g_0}$

$$= \frac{108619\,N}{(30\ kg/s)(9.81\ m/s^2)} \simeq \boxed{369.08\ s}$$

(d)

$$c = v_e = \frac{F_{THRUST}}{\dot{m}} = \boxed{3620.13\ m/s}$$

Question 6

The shape of NACA airfoils is described through their numerical identifier. We can input the parameters from the identifier into equations to precisely generate the cross-section of the airfoil. The four-digit series, in particular, defines the profile through three parameters. The first digit describes maximum camber as a percentage of the chord. The second digit provides the distance of the maximum camber from the airfoil leading edges in tens of percentages of the chord. The last two digits give us the maximum thickness of the airfoil as a percentage of the chord.

For example, the NACA 2215 airfoil has a maximum camber of 2%, located at 0.2 chords (20%) from the leading edge of the airfoil, with a maximum thickness of 15% of the chord. Since we can use equations to describe the shape of the airfoil, we can automate the process of drawing different airfoils!

Write a MATLAB script that takes a NACA 4-digit series code and the chord length as inputs and plots the shape of the airfoil. On your plot, include the chord and mean camber line. Plot three different airfoils to show that your algorithm works with various inputs. Include a symmetric airfoil, a cambered airfoil, and a third airfoil of your choice.

If you need inspiration when choosing NACA airfoils to plot, you can find airfoil designations for a variety of aircraft on "*The Incomplete Guide to Airfoil Usage*" at https://m-selig.ae.illinois.edu/ads/aircraft.html. On this website, you will notice that the Supermarine Spitfire has a different airfoil for the root and the tip of the wing. The Air Tractor, on the other hand, maintains the same airfoil from root to tip. Why do you think that is?

For this question, you will be graded on your MATLAB code, plot, and discussion on the last prompt. Make sure your code is well commented and describes the variables and equations you have used to generate the plots. Make is easy for a third person to read and understand.