November 20, 2020

AAE 564 Fall 2020

Homework Thirteen

Due: Friday December 4

Exercise 1 Determine whether or not each of the following systems are observable, detectable, or not detectable.

$$\begin{array}{rcl} \dot{x}_1 &=& -x_1 \\ \dot{x}_2 &=& x_2 + u \\ y &=& x_1 + x_2 \end{array} \qquad \begin{array}{rcl} \dot{x}_1 &=& -x_1 \\ \dot{x}_2 &=& x_2 + u \\ y &=& x_2 \end{array} \qquad \begin{array}{rcl} \dot{x}_1 &=& x_1 \\ \dot{x}_2 &=& x_2 + u \\ y &=& x_1 + x_2 \end{array}$$

Exercise 2 Consider the system described by

$$\begin{aligned} \dot{x}_1 &= -x_2 + u \\ \dot{x}_2 &= -x_1 - u \\ y &= x_1 - x_2 \end{aligned}$$

where all quantities are scalars.

(a) Is this system observable?

(b) Is this system detectable?

(c) Does there exist an asymptotic state estimator for this system? If an estimator does not exist, explain why; if one does exist, give an example of one.

(b) If the answer to part (c) is yes, illustrate the effectiveness of your observer with a simulation

Exercise 3 Consider the system,

$$\begin{aligned} \dot{x}_1 &= x_2 + u_1 \\ \dot{x}_2 &= u_2 \\ y &= x_1 \end{aligned}$$

where all quantities are scalar. Obtain (by hand) an output feedback controller which results in an asymptotically stable closed loop system.

Exercise 4 Consider the system

$$\begin{aligned} \dot{x}_1 &= -x_1 + x_3 \\ \dot{x}_2 &= u \\ \dot{x}_3 &= x_2 \\ y &= x_3 \end{aligned}$$

with scalar control input u and scalar measured output y.

(a) Obtain (by hand) an observer-based output feedback controller which results in an asymptotically stable closed loop system.

(b) Can all the eigenvalues of the closed loop system be arbitrarily placed?

Exercise 5 Consider the system,

$$
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_1 + u \\
y &= x_1
\end{aligned}
$$

where all quantities are scalar. Obtain (by hand) an output feedback controller which results in an asymptotically stable closed loop system.

Exercise 6 (Stabilization of cart pendulum system via output feedback.) Consider the cart pendulum system with the displacement y as the measured output. Carry out the following for parameter set P4 and equilibriums $E1$ and $E2$. Illustrate the effectiveness of your controllers with numerical simulations.

Using eigenvalue placement techniques, obtain a output feedback controller which stabilizes the nonlinear system about the equilibrium.

What is the largest value of δ (in degrees) for which your controller guarantees convergence of the closed loop system to the equilibrium for initial condition

$$(y,\, \theta_1,\, \theta_2,\, \dot{y},\, \dot{\theta}_1,\, \dot{\theta}_2)(0) = (0,\, \theta_1^e{-}\delta,\, \theta_2^e{+}\delta,\, 0,\, 0,\, 0)$$

where θ_1^e and θ_2^e are the equilibrium values of θ_1 and θ_2.

Exercise 7 Using the Lyapunov equation determine (by hand) whether or not the system $\dot{x} = Ax$ is asymptotically stable for each one of the following A matrices.

$$
\begin{pmatrix} -1 & 2 \\ 0 & -1 \end{pmatrix} \qquad \begin{pmatrix} -1 & 2 \\ 0 & 1 \end{pmatrix} \qquad \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}
$$

Check your answers using the MATLAB command lyap.

Exercise 8 Consider the system with disturbance input w and performance output z described by

$$
\begin{aligned}
\dot{x}_1 &= -x_1 + x_2 + w \\
\dot{x}_2 &= -x_1 - 4x_2 + 2w \\
z &= x_1 \,.
\end{aligned}
$$

Using an appropriate Lyapunov equation, determine (by hand)

$$\int_0^\infty \|z(t)\|^2 \, dt$$

for each of the following situations.

(a)

$$w = 0 \qquad \text{and} \qquad x(0) = (1, 0)\,.$$

(b)

$$w(t) = \delta(t) \qquad \text{and} \qquad x(0) = 0\,.$$