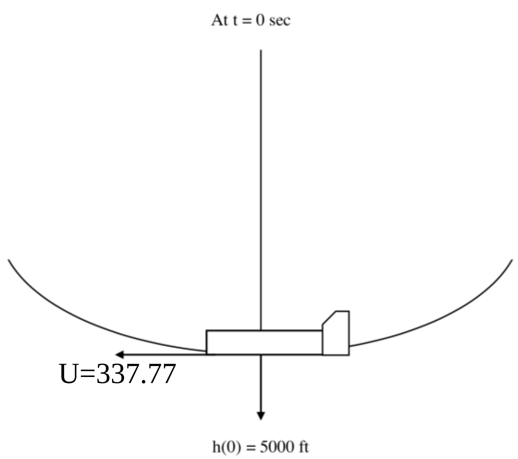
Homework 2 AAE 421 Due 09/29 5:00 pm **Name___________________**

Problem 1. (15pts)

An aircraft is flying straight and level at a constant velocity of 337. 77 ft/sec, and then performs a symmetric pull up such that $\dot{\Theta} = 0.05$ rad/s=constant. Assume the aircraft's x-axis is aligned with the flight path throughout the motion and that at $t = 0$, the aircraft's location in North-East-Altitude coordinate is $p_N = 0$, $p_E = 0$, and $h = 5000$ ft. Find the position coordinates (p_N, p_E, h) at $t = 5$ sec. Assume $\Psi = 0$.



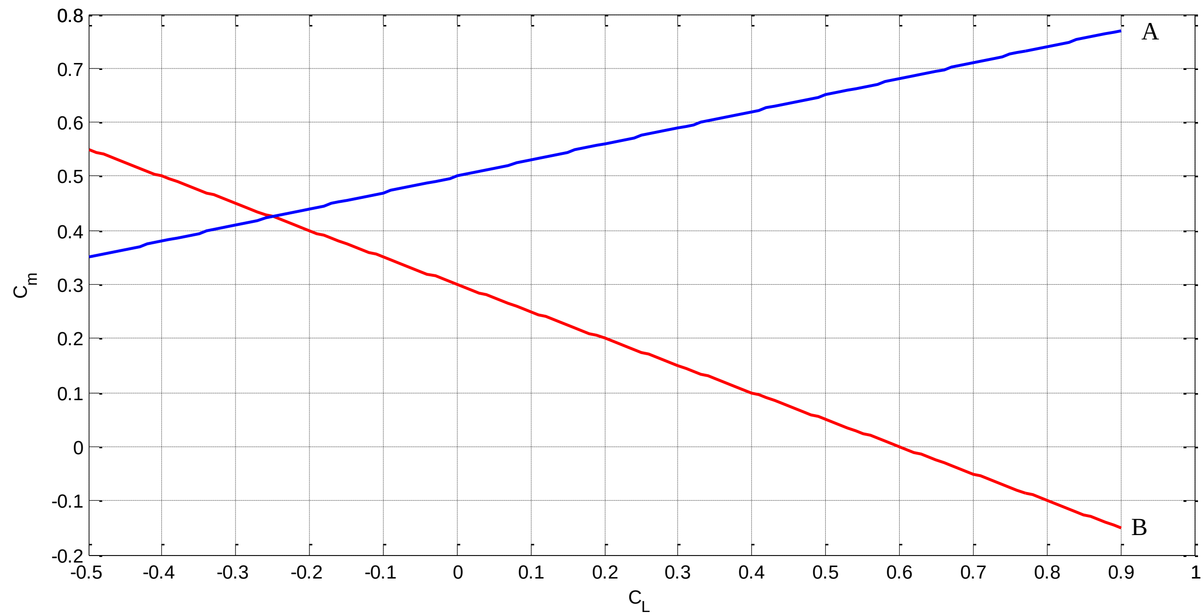
Problem 2. (10pts)

The aircraft velocity vector expressed in the Earth-fixed reference frame is
$\bar{V}_I = U\boldsymbol{I} + V\boldsymbol{J} + W\boldsymbol{K} = 6.6637\boldsymbol{I} + 289.1164\boldsymbol{J} - 407.8815\boldsymbol{K}$ (ft/sec)
and in the aircraft fixed body reference frame it is given by
$\bar{V}_b = u\boldsymbol{i} + v\boldsymbol{j} + w\boldsymbol{k} = 497.7939\boldsymbol{i} + 17.4497\boldsymbol{j} + 43.5513\boldsymbol{k}$ (ft/sec)
Find the attitude of the aircraft in terms of its Euler angles (Ψ, Θ, Φ). Is your answer unique?

Problem 3 (10pts)

For the C_L and C_m relationship shown in the following plots

(1) Find the linear expressions of C_m in term of C_L for line A and B, respectively.
(2) To obtain a trim condition, which line should be selected?
(3) Assuming $\frac{x_{cg}}{\bar{c}}$=0.6, how to relocate the a.c center ($\frac{x_{ac}}{\bar{c}}$) to obtain a new $C_{L,trim}$=0.8?



Problem 4. (15pts)

Wind tunnel test on a full-scale flying wing yielded the following database

Angle of Attack, deg	C_L	$C_{m_{cg}}$
8.0	0.64	-0.014
5.0	0.40	0.010
2.0	0.16	0.034
-3.0	-0.24	0.074

The configuration c.g is located 0.58 ft from the leading edge of the chord and the chord length is 2.6 ft.

(a) Estimate the configuration lift curve slope
(b) Is the configuration, as tested, statically stable? Explain your answer.
(c) Estimate values for C_m at the aerodynamic center and aerodynamic center location.
(d) Can this configuration, as tested, be trimmed in a steady glide condition? Explain your answer.
(e) If the answer to part (d) is yes, then estimate values for trim angle of attack and trim lift coefficient

Problem 5. (10pts)

Consider the following nonlinear 2nd-order system

$$\ddot{y} + a_1\dot{y}^2 + \frac{a_0}{y^2} = f$$

where a_0 and a_1 are constant, and $a_0 > 0$

(1) For a constant input $f = f_0 > 0$, determine the equilibrium points of the system
(2) Obtain the linearized equations of the system at the equilibrium points
(3) Express the linearized model in state equations, choosing $x_1 = \Delta y, x_2 = \Delta\dot{y}, u = \Delta f$

Problem 6. (10pts)

Consider an airplane in constant-altitude, straight-line flight. The velocity equation is

$$\dot{V} = T - \frac{1}{2}kV^2$$

where the second term represents aerodynamic drag, and assume k = constant, and T is the engine thrust acceleration. Treat T as the control (input). Let V^* be a given constant cruise speed. Obtain the linearized differential equation for the velocity around V^*.

Problem 7. (15pts)

From the nonlinear flight dynamics model, derive the following linear perturbation equations for Y force

$$m(\dot{v} + u_0 r) = \Delta Y + mgcos(\theta_0)\phi$$

and moments:

$$\Delta L = I_{xx}\dot{p} - I_{xz}\dot{r}$$
$$\Delta M = I_{yy}\dot{q}$$
$$\Delta N = -I_{xz}\dot{p} + I_{zz}\dot{r}$$

Show all the steps!

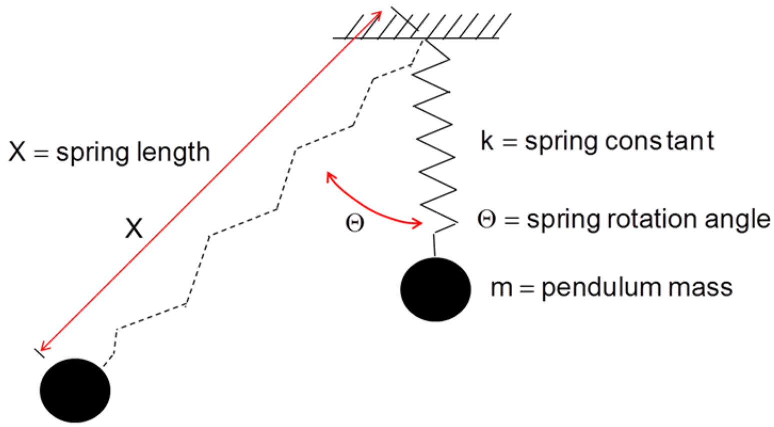
Problem 8. (15pts)

Consider the 2-degree-of-freedom spring mass pendulum shown below (All motion is in the plane of the picture shown). The nonlinear equations of motion are given by

(1) $\ddot{X} + \frac{k}{m}(X - L) - gcos\Theta - X\dot{\Theta}^2 = 0$
(2) $X^2\ddot{\Theta} + gXsin\Theta + 2\dot{\Theta}X\dot{X} = 0$
where L is the original spring length.

Linearize the equations of motion for this system. Let the reference condition be the equilibrium (no motion) state for the pendulum mass. In particular

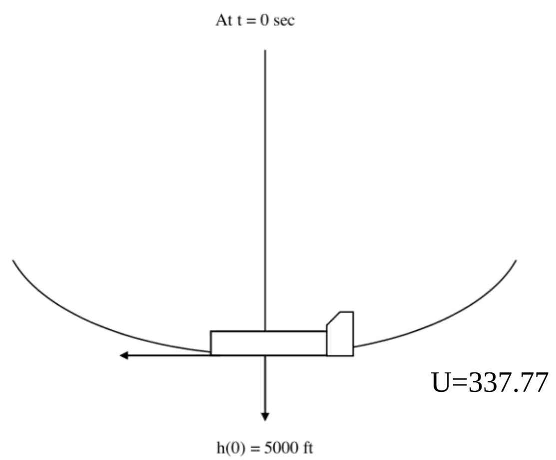
(a) Define a set of perturbation variables
(b) Substitute the results of Part (a) into the equations of motion
(c) Expand the equations and discard appropriate terms (show the terms that are to be discarded)


X = spring length
X
k = spring constant
Θ
Θ = spring rotation angle
m = pendulum mass

Homework 2 AAE 421 Due: 09/29/2020 5:00 PM Name: Rashi Jain

PROBLEM 1 (15 pts)

An aircraft is flying straight and level at a constant velocity of 337.77 ft/sec, and then performs a symmetric pull up such that $\dot{\Theta} = 0.05$ rad/s=constant. Assume the aircraft's x-axis is aligned with the flight path throughout the motion and that at $t = 0$, the aircraft's location in North-East-Altitude coordinate is $p_N = 0$, $p_E = 0$, and $h = 5000$ ft. Find the position coordinates (p_N, p_E, h) at $t = 5$ sec. Assume $\Psi = 0$.



This is a very special case of the general motion discussed in the equations of motion. Given that: $337.77 = \frac{ft}{s}, V = 0$ (motion only in the vertical plane).

Since the body x-axis aligned with the freestream velocity vector throughout the motion, we see that $W = 0$.

Also, as far as rotational motion is concerned $P = 0$ (no roll), $R = 0$ (no yaw), $\psi = 0, \phi = 0$ (because symmetrical pull up), and $\dot{\theta} = Q = constant = 0.05\frac{rad}{s}$. $\dot{\theta} = Q$ because in this motion throughout, the body y-axis and the inertial Y – axis are parallel to each other.

Now,

$$\begin{bmatrix} \dot{p_N} \\ \dot{p_E} \\ -\dot{h} \end{bmatrix} = \begin{bmatrix} \cos\theta\cos\psi & * & * \\ \cos\theta\sin\psi & * & * \\ -\sin\theta & * & * \end{bmatrix} \begin{bmatrix} U \\ V \\ W \end{bmatrix}$$

where * denotes entries we do not care or worry about. For the above given situation, the Navigation Equations are:

$$\begin{bmatrix} \dot{p_N} \\ \dot{p_E} \\ -\dot{h} \end{bmatrix} = \begin{bmatrix} \cos\theta & * & * \\ 0 & * & * \\ \sin\theta & * & * \end{bmatrix} \begin{bmatrix} U \\ V \\ W \end{bmatrix}$$

Thus,
$\dot{p}_N = U\cos\theta\,, \dot{p}_E = U\sin\theta$

Given that,
$\dot{\theta} = 0.05\; rads^{-1}, \theta(t) = 0.05t$

$$\dot{p}_N = 337.7\cos(0.05t)\,, p_N = \int_0^5 337.77\cos(0.05t)\;dt = 1671.3ft$$

$$p_E = \int_0^5 0\;dt = 0ft$$

$$\dot{h} = 337.7\sin(0.05t)\,, where\; h(0) = 5000ft$$

$$h(t) = \int_0^5 337.77\sin(0.05t)\;dt = 5210ft$$

PROBLEM 2 (10 pts)

The aircraft velocity vector expressed in the Earth-fixed reference frame is $\bar{V}_I = UI + VJ + WK = 6.6637I + 289.1164J - 407.8815K$ (ft/sec) and in the aircraft fixed body reference frame it is given by $\bar{V}_b = ui + vj + wk = 497.7939i + 17.4497j + 43.5513k$ (ft/sec) Find the attitude of the aircraft in terms of its Euler angles (Ψ, Θ, Φ). Is your answer unique?

We have $V_I = L_{IB}V_B$

$$\begin{bmatrix} U \\ V \\ E \end{bmatrix} = \begin{bmatrix} \cos\theta\cos\psi & \sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi & \cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi \\ \cos\theta\sin\psi & \sin\phi\sin\theta\sin\psi + \cos\phi\cos\psi & \cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi \\ -\sin\theta & \sin\phi\cos\theta & \cos\phi\cos\theta \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

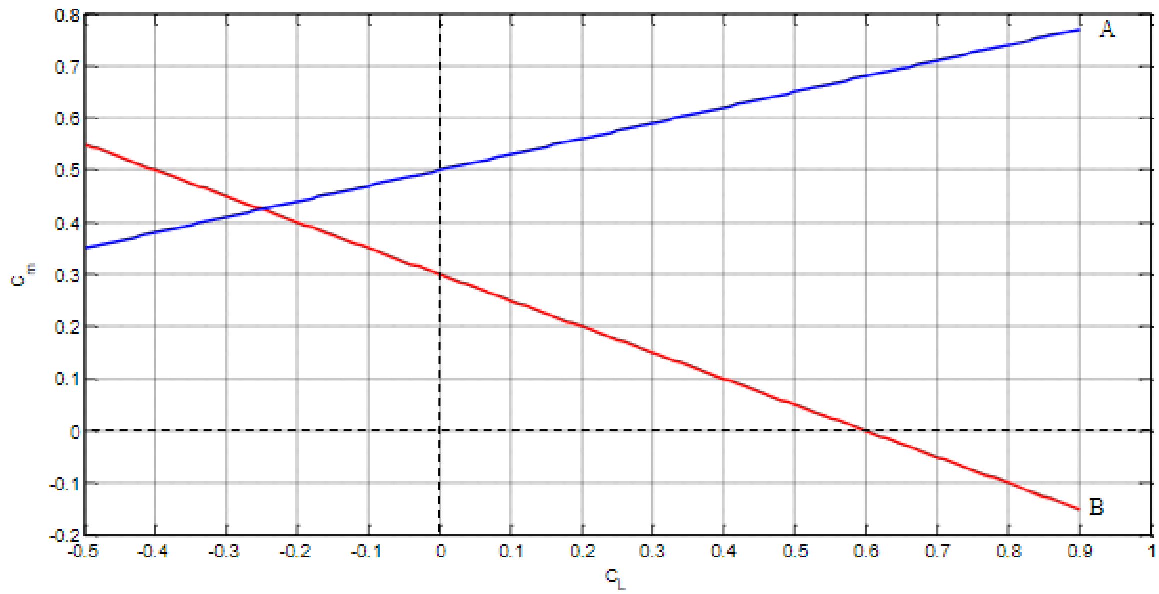
Substitute values, you may either construct a MATLAB for loop, or fsolve to solve for the three angles. The answers are not unique.

$$\psi = \frac{\pi}{2}, \theta = \frac{\pi}{3}, and\ \phi = \frac{\pi}{6}$$

PROBLEM 3 (10 pts)

For the C_L and C_m relationship shown in the following plots

1. **Find the linear expressions of C_m in term of C_L for line A and B, respectively.**
2. **To obtain a trim condition, which line should be selected?**
3. **Assuming $\frac{x_{cg}}{\bar{c}}$=0.6, how to relocate the a.c center ($\frac{x_{ac}}{\bar{c}}$) to obtain a new $C_{L,trim}$=0.8?**



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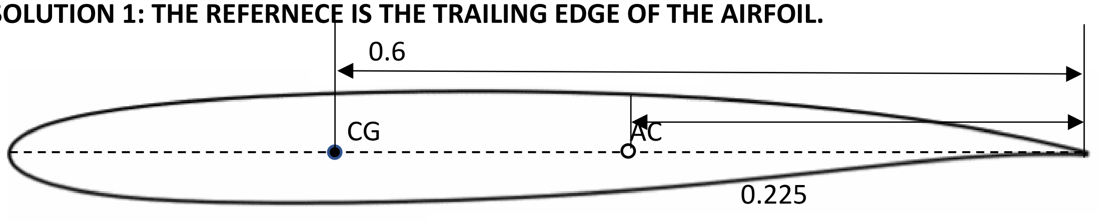
(1) Line A: $C_M = 0.5 + 0.3C_L$

Line B: $C_M = 0.3 - 0.5C_L$

(2) Line B should be selected. ($C_{M_\alpha} < 0$ implies a stable configuration in B)

(3) **SOLUTION 1: THE REFERNECE IS THE TRAILING EDGE OF THE AIRFOIL.**



$$C_{M,cg} = C_{M,R} - \frac{x_{cg}}{c} C_L \qquad C_{M,ac} = C_{M,R} - \frac{x_{ac}}{c} C_L$$

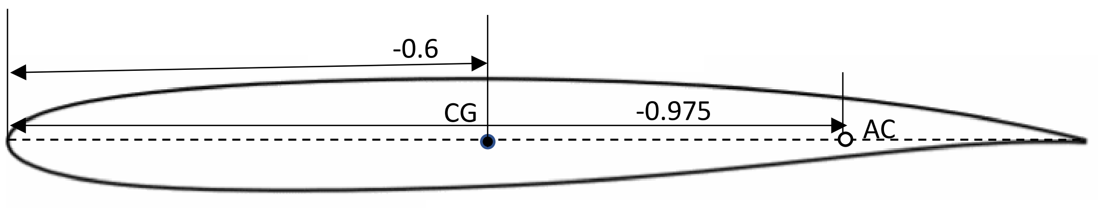
$$C_{M,cg} = C_{M,ac} - \left(\frac{x_{cg} - x_{ac}}{c}\right) C_L$$

Note: $C_{M,cg}$ = 0 at trim. $C_{L,trim} = 0.8$ (given). $C_{M,ac}$ is y-intercept on Line B. ($C_{M,ac} = 0.3$)

$$0 = 0.3 - \left(0.6 - \frac{x_{ac}}{c}\right) 0.8$$

$$\frac{x_{ac}}{c} = 0.225$$

SOLUTION 2: THE REFERNECE IS THE LEADING EDGE OF THE AIRFOIL.



$C_{M,cg} = C_{M,ac} - \left(\frac{x_{cg} - x_{ac}}{c}\right) C_L, \boldsymbol{\frac{x_{cg}}{c} = -0.6}$

Note: $C_{M,cg}$ = 0 at trim. $C_{L,trim} = 0.8$ (given). $C_{M,ac}$ is y-intercept on Line B. ($C_{M,ac} = 0.3$)

$$0 = 0.3 - \left(-0.6 - \frac{x_{ac}}{c}\right) 0.8$$

$$\boldsymbol{\frac{x_{ac}}{c} = -0.975}$$

Both solutions are correct.

PROBLEM 4

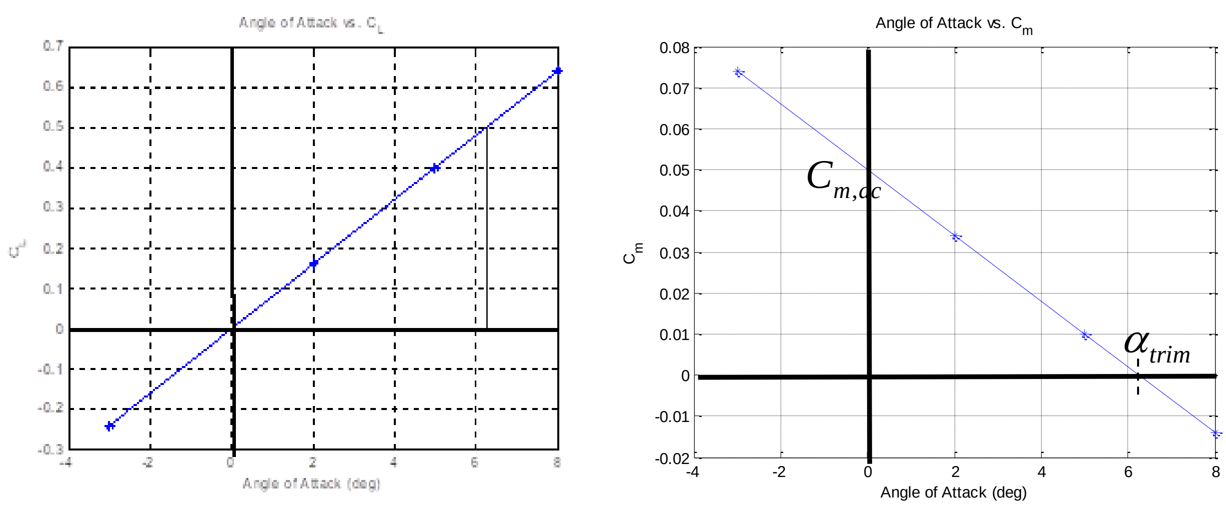
Wind tunnel test on a full-scale flying wing yielded the following database

Angle of Attack, deg	C_L	$C_{m_{cg}}$
8.0	0.64	-0.014
5.0	0.40	0.010
2.0	0.16	0.034
-3.0	-0.24	0.074

The configuration c.g is located 0.58 ft from the leading edge of the chord and the chord length is 2.6 ft.

- **(a) Estimate the configuration lift curve slope**
- **(b) Is the configuration, as tested, statically stable? Explain your answer.**
- **(c) Estimate values for C_m at the aerodynamic center and aerodynamic center location.**
- **(d) Can this configuration, as tested, be trimmed in a steady glide condition? Explain your answer.**
- **(e) If the answer to part (d) is yes, then estimate values for trim angle of attack and trim lift coefficient**

(a) The C_L, α information is translated into the plot. See Figure below:



It is a straight line. $C_{L,\alpha}$ is found by simply the slope of the straight line. $C_{L_\alpha} = \frac{\Delta C_L}{\Delta \alpha} = 0.08/^o$

(b) Yes, the configuration is statistically stable. A plot of $C_M vs\ \alpha$, shows that $C_{M,\alpha}(= -0.008/^o) < 0$

(c) $C_{M,ac}\ (y - intercept) = 0.05.$

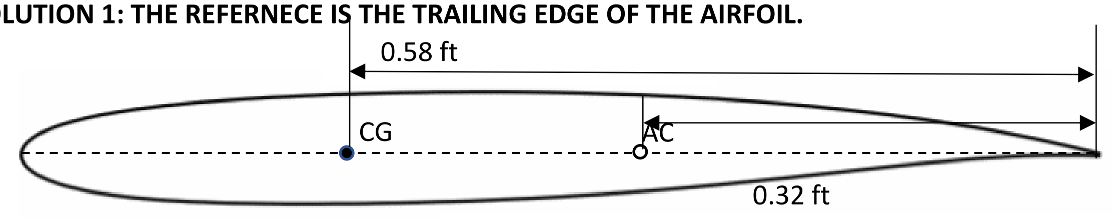
Using equation in P3, at trim condition:

$$C_{M,cg} = C_{M,ac} - \left(\frac{x_{cg} - x_{ac}}{c}\right)\ C_L$$

$$0 = 0.05 - \left(\frac{0.58}{2.6} - \frac{x_{ac}}{c}\right) 0.5$$

$$\frac{x_{ac}}{c} = 0.123, x_{ac} = 0.32ft$$

SOLUTION 1: THE REFERNECE IS THE TRAILING EDGE OF THE AIRFOIL.

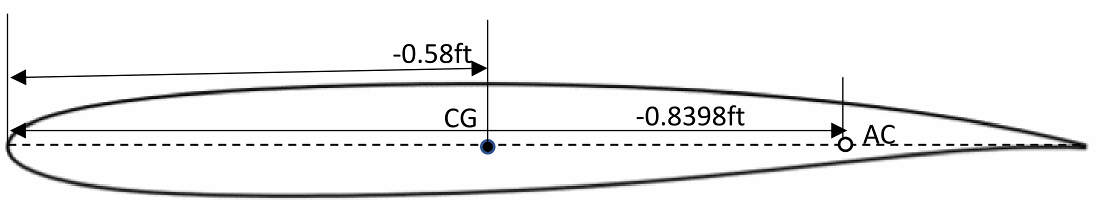


Note: $C_{M,cg}$ = 0 at trim. $C_{L,trim} = 0.5$ (given). $C_{M,ac}$ is y-intercept on Line B. ($C_{M,ac} = 0.05$)

$$0 = 0.05 - \left(\frac{0.58}{2.6} - \frac{x_{ac}}{c}\right) 0.5$$

$$\frac{x_{ac}}{c} = 0.123, x_{ac} = 0.32ft$$

SOLUTION 2: THE REFERNECE IS THE LEADING EDGE OF THE AIRFOIL.



$$0 = 0.05 - \left(-\frac{0.58}{2.6} - \frac{x_{ac}}{c}\right) 0.5$$

$$\frac{x_{ac}}{c} = -0.323, x_{ac} = -0.8398ft$$

Both solutions are correct.

(d) Yes, $C_{m_\alpha} < 0$

(e) Using the plots, we obtain $\alpha_{trim} = 6.25^o$. Then from (a) we have $C_{L_{trim}} \cong 0.50$.

Problem 5

(1)

Given system:

$$\ddot{y} + a_1\dot{y}^2 + \frac{a_0}{y^2} = f$$

Finding equilibrium pint for constant input f_0.
At equilibrium, $\ddot{y} = 0$ and $\dot{y} = 0$:

$$y_0 = \pm\sqrt{\frac{a_0}{f}}$$

(2)

Finding coefficients of linearized equation:

$$g = \ddot{y} + a_1\dot{y}^2 + \frac{a_0}{y^2} - f$$

$$\left.\frac{\partial g}{\partial \ddot{y}}\right|_0 = 1$$

$$\left.\frac{\partial g}{\partial \dot{y}}\right|_0 = 2a_1\dot{y}_0 = 0$$

$$\left.\frac{\partial g}{\partial y}\right|_0 = \frac{-2a_0}{y_0^3}$$

$$\left.\frac{\partial g}{\partial f}\right|_0 = -1$$

Therefore, the linearized system:

$$\left.\frac{\partial g}{\partial \ddot{y}}\right|_0 \Delta\ddot{y} + \left.\frac{\partial g}{\partial \dot{y}}\right|_0 \Delta\dot{y} + \left.\frac{\partial g}{\partial y}\right|_0 \Delta y + \left.\frac{\partial g}{\partial f}\right|_0 \Delta f = 0$$

$$\Delta\ddot{y} = \frac{2a_0}{y_0^3}\Delta y + \Delta f$$

Substituting for y_0:

$$\Delta\ddot{y} = 2a_0 \times \pm\sqrt{\frac{a_0}{f}}^{-3} \times \Delta y + \Delta f$$

$$\Delta\ddot{y} = \pm 2\frac{f^{3/2}}{\sqrt{a_0}}\Delta y + \Delta f$$

The two systems:

$$\Delta\ddot{y} = 2\frac{f^{3/2}}{\sqrt{a_0}}\Delta y + \Delta f$$

$$\Delta\ddot{y} = -2\frac{f^{3/2}}{\sqrt{a_0}}\Delta y + \Delta f$$

(3)

Choosing,

$$x_1 = \Delta y, \qquad x_2 = \Delta \dot{y}, \qquad u = \Delta f$$

We have the linear system:

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = \frac{2a_0}{y_0^3} x_1 + u$$

In matrix form:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{2a_0}{y_0^3} & 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

Substituting for y_0, we have the 2 linear systems:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{2a_0}{y_0^3} & 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{2a_0}{y_0^3} & 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

Substituting for y_0, the two systems:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 2\frac{f^{3/2}}{\sqrt{a_0}} & 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2\frac{f^{3/2}}{\sqrt{a_0}} & 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

Problem 6

Given Velocity equation:

$$\dot{V} = T - \frac{1}{2} k V^2$$

Thrust at given constant cruise speed:

$$\dot{V}^* = T^* - \frac{1}{2} k V^{*2}$$
$$\implies T^* = \frac{1}{2} k V^{*2}$$

Since, $\dot{V}^* = 0$.
Finding the coefficients of the linearized system:

$$f = \dot{V} + \frac{1}{2}kV^2 - T$$
$$\left.\frac{\partial f}{\partial \dot{V}}\right|_* = 1$$
$$\left.\frac{\partial f}{\partial V}\right|_* = kV^*$$
$$\left.\frac{\partial f}{\partial T}\right|_* = -1$$

The linearized system:

$$\frac{\partial f}{\partial \dot{V}}\Delta\dot{V} + \frac{\partial f}{\partial V}\Delta V + \frac{\partial f}{\partial T}\Delta T = 0$$
$$\Delta\dot{V} = -kV^*\Delta V + \Delta T$$

Where,

$$\Delta V = V - V^* \qquad \Delta T = T - T^*$$

Problem 7

We have the non-linear equations:

$$\dot{V} = \frac{1}{m}Y + g\cos\Theta\sin\Phi - RU + PW$$
$$L = I_{xx}\dot{P} - I_{xz}(\dot{R} + PQ) - (I_{yy} - I_{zz})QR$$
$$M = I_{yy}\dot{Q} - I_{zx}(R^2 - P^2) - (I_{zz} - I_{xx})PR$$
$$N = I_{zz}\dot{R} - I_{zx}(\dot{P} - QR) - (I_{xx} - I_{yy})PQ$$

Perturbation variables:

$V = V_0 + v, \quad \Theta = \Theta_0 + \theta, \quad \Phi = \Phi_0 + \phi, \quad R = R_0 + r, \quad U = U_0 + u, \quad P = P_0 + p, \quad W = W_0 + w$
$L = L_0 + \Delta L, \quad M = M_0 + \Delta M, \quad N = N_0 + \Delta N$

At equilibrium,

$$R_0 = P_0 = W_0 = 0$$
$$Y_0 = 0$$
$$\Phi_0 = 0$$
$$\dot{V}_0 = 0$$
$$L_0 = M_0 = N_0 = 0$$

Assuming, all the higher order infinitesimals (product and powers of infinitesimals) are zero and,

$$\cos(\theta + \delta\theta) = \cos\theta\cos\delta\theta - \sin\theta\sin\delta\theta = \cos\theta - \delta\theta\sin\theta$$
$$\sin(\theta + \delta\theta) = \sin\theta\cos\delta\theta + \cos\theta\sin\delta\theta = \sin\theta + \delta\theta\cos\theta$$

Therefore,

$$\begin{aligned} \sin(\Phi_0 + \phi) &= \phi \\ \cos(\Theta_0 + \theta) &= \cos\Theta - \theta \sin\theta \\ \implies \sin(\Phi_0 + \phi)\cos(\Theta_0 + \theta) &= \phi \cos\Theta \end{aligned}$$

Using above results we get the following linear equations:

1.

$$\begin{aligned} \dot{V} &= \frac{1}{m} Y + g \cos\Theta \sin\Phi - RU + PW \\ \dot{v} &= \frac{1}{m}(Y_0 + \Delta Y) + g\phi\cos\Theta - (R_0 + r)(U_0 + u) + (P_0 + p)(W_0 + w) \\ \dot{v} &= \frac{1}{m}(\Delta Y) + g\phi\cos\Theta - U_0 r \end{aligned}$$

2.

$$\begin{aligned} L &= I_{xx}\dot{P} - I_{xz}(\dot{R} + PQ) - (I_{yy} - I_{zz})QR \\ L_0 + \Delta L &= I_{xx}(\dot{P}_0 + \dot{p}) - I_{xz}((\dot{R}_0 + \dot{r}) + (P_0 + p)(Q_0 + q)) - (I_{yy} - I_{zz})(Q_0 + q)(R_0 + r) \\ \Delta L &= I_{xx}\dot{p} - I_{xz}\dot{r} \end{aligned}$$

3.

$$\begin{aligned} M &= I_{yy}\dot{Q} - I_{zx}(R^2 - P^2) - (I_{zz} - I_{xx})PR \\ M_0 + \Delta M &= I_{yy}(\dot{Q}_0 + \dot{q}_0) - I_{zx}((R_0 + r)^2 - (P_0 + p)^2) - (I_{zz} - I_{xx})(P_0 + p)(R_0 + r) \\ \Delta M &= I_{yy}\dot{q} \end{aligned}$$

4.

$$\begin{aligned} N &= I_{zz}\dot{R} - I_{zx}(\dot{P} - QR) - (I_{xx} - I_{yy})PQ \\ N_0 + \Delta N &= I_{zz}(\dot{R}_0 + \dot{r}) - I_{zx}((\dot{P}_0 + \dot{p}) - (Q_0 + q)(R_0 + r)) - (I_{xx} - I_{yy})(P_0 + p)(Q_0 + q) \\ \Delta N &= -I_{xz}\dot{p} + I_{zz}\dot{r} \end{aligned}$$

Problem 8

(a)

Assuming the subscript "e" represents the equilibrium points. We have the perturbation variables:

$$\begin{aligned} x &= x_e + \Delta x, \quad \dot{x} = \dot{x}_e + \Delta\dot{x}, \quad \ddot{x} = \ddot{x}_e + \Delta\ddot{x} \\ \theta &= \theta_e + \Delta\theta, \quad \dot{\theta} = \dot{\theta}_e + \Delta\dot{\theta}, \quad \ddot{\theta} = \ddot{\theta}_e + \Delta\ddot{\theta} \end{aligned}$$

(b)

Given state equations:

$$\ddot{x} + \frac{k}{m}(x - L) - g\cos\theta - x\dot{\theta}^2 = 0$$
$$x^2\ddot{\theta} + gx\sin\theta + 2\dot{\theta}x\dot{x} = 0$$

Substituting the perturbation variables:

$$(\ddot{x}_e + \Delta\ddot{x}) + \frac{k}{m}((x_e + \Delta x) - L) - g\cos(\theta_e + \Delta\theta) - (x_e + \Delta x)(\dot{\theta}_e + \Delta\dot{\theta})^2 = 0$$
$$(x_e + \Delta x)^2(\ddot{\theta}_e + \Delta\ddot{\theta}) + g(x_e + \Delta x)\sin(\theta_e + \Delta\theta) + 2(\dot{\theta}_e + \Delta\dot{\theta})(x_e + \Delta x)(\ddot{x}_e + \Delta\ddot{x}) = 0$$

(c)

At equilibrium, we have the following:

$$\ddot{x}_e = \dot{x}_e = \theta_e = \dot{\theta}_e = \ddot{\theta}_e = 0$$
$$\frac{k}{m}(x_e - L) = g$$

Also, all the higher order infinitesimals (product and powers of infinitesimals) are zero and,

$$\cos(\theta_e + \Delta\theta) = \cos\theta_e\cos\Delta\theta - \sin\theta_e\sin\Delta\theta = \cos\theta_e - \sin\theta_e\Delta\theta = 1$$
$$\sin(\theta_e + \Delta\theta) = \sin\theta_e\cos\Delta\theta + \cos\theta_e\sin\Delta\theta = \sin\theta_e + \cos\theta_e\Delta\theta = \Delta\theta$$

Simplifying the state equations using above results:

$$\Delta\ddot{x} + g + \frac{k}{m}\Delta x - g = 0$$
$$x_e^2\Delta\ddot{\theta} + gx_e\Delta\theta = 0$$

Hence, we have the linearized state equations:

$$\Delta\ddot{x} + \frac{k}{m}\Delta x = 0$$
$$\Delta\ddot{\theta} + \frac{g}{x_e}\Delta\theta = 0$$