Paper Review: Consensus and Disagreement of Heterogeneous Belief Systems in Influence Networks

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***Abstract*—We will analyze and dissect a paper by Mengbin Ye, Ji Liu, Lili Wang, Brian D. O. Anderson, and Ming Cao concerning how consensus theory systematically explains agreements and disagreements in a variety of discussions[1]. The belief system of each individual can be captured as a logic matrix, and this research postulates how the logic matrix and its analysis determines the convergence or divergence of people's opinions. This then gives forth an explanation to how strong diversities arise for some certain topics and individual's belief systems which is a highlighted theme in opinion dynamics.**

***Index Terms*—Agent-based models; influence networks; mutliagent systems; opinion dynamics; social networks; logic matrix; heterogeneity; cascade interdependence relationships**

I. Introduction and Motivation

In the field of opinion dynamics, there have been many models that aim to capture a more realistic network on how people's opinions evolve over time. Many of the proposed models differ from each other in that they successfully model a new trait that can be observed in human interactions such as homophily, antagonistic reaction, stubbornness, and diversity. The extension of classical models have led to show that logical interdependencies can be expressed as logic matrices and provide an explanation to the mechanism of our belief system.

This paper investigates the heterogeneity of logic matrices and attempts to generalize existing models to explain how opinions reach consensus or not. This all accumulates to the goal of modelling *strong diversity* within reformation of opinions through interactions.

II. Problem Formulation

To formulate the problem the paper first establishes the ground for the theory using graph theory and consensus theory. The investigated model, the multidimensional DeGroot model is theorized as

$$x_i(t+1) = \sum_{j=1}^{n} w_{ij} C_i x_j(t) \quad .$$

The index i indicates each individual and w_{ij} represents the influence weight from one indivudal to another. The x is the individuals opinion with the range of $[-1, 1]$, where the value is closer to 1 it means that they are more supportive and if otherwise they disagree. Most importantly, the C matrix is the logic matrix which represents the logical interdependence between m topics perceived by individual i. Thus, this matrix is a *m*-dimensional square matrix showing how interdependent multiple topics are to one another, or in other words a mathematical representation of an individual's belief system for several topics. Moving forward with this model, the paper posits its objective as investigating whether or not a certain structure of the logic matrix always guarantees consensus depending on a set of assumptions for a limiting opinion configuration.

III. Main Results

With the help of the assumptions and consensus theory the paper is able to prove that for some *n* individuals and the vector $y(t)$ representing their opinions evolve according to a discrete-time system with a positive, irreducible, and row stochastic matrix. This implies that there exists a consensus value y^* for the convergence of $\lim_{t\to\infty} y(t) = y^*$. This satisfies the objective of investigating the consensus of the model for the limiting opinion configuration.

Furthermore, the paper proves that for a given set of logic matrices C and a strongly connected social network $\mathcal{G}$ a consensus can be reached for all topics but the derivation also concluded necessary conditions for a disagreement to be formed for a topic or all topics. Thus, despite the assumptions, sufficient amount of theoretical analysis has done to prove that it is possible to model an influence network based on heterogeneous belief systems

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while predicting whether or not the network will reach a consensus or not.

IV. Your Ideas of Further Improvements

Building up on my previous reviews, this paper has given me a interesting insight on how it may be possible to adopt consensus algorithms for more abstract tasks like opinion dynamics. Where it may be for path planning or object avoidance, the consensus of a heterogeneous system may be something applicable to some kind of decision making for robots. Some tasks in which the environment is uncontrolled and dynamic the irregularities cause the decision making for robots to leave convoluted due to the complexity of the problem or excessive computation that it can not handle. To cope with that, confidence bounds could be one technique, but applying the theory postulated in this paper would be worthwhile to explore.

References

[1] Mengbin Ye, Ji Liu, Lili Wang, Brian D. O. Anderson, and Ming Cao. Consensus and disagreement of heterogeneous belief systems in influence networks. *IEEE Transactions on Automatic Control*, 65(11):4679–4694, 2020.