College of Engineering
School of Aeronautics and Astronautics

AAE 36401 Lab
Control Systems Lab

Lab 3 Report
The Control of Inverted Pendulum

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Introduction

Objective

The object of this lab is to control an inverted pendulum on a cart which is place on a track. The cart will be able to have the pendulum be in an upright position even if there is a slight perturbation such as a slight push applied on the it.

Method

To accomplish the objective, we have to come up with the gains of the feedback control system. Two methods to compute the gains are implemented in this lab. One is the pole placement and the second is the linear quadratic regulator. We compute two different gains from these methods and compare the response of the system.

After we initialize the pendulum at the downward position, we slowly lift the pendulum up to the upward position. Then we push the pendulum with a slight force and see its response. An image of the actual setup of the experiment is in the appendix.

Results

Part (i)

The gains used for each method are the following

The gains using Pole Placement:

Table 1: gains of pole placement

K_1	K_2	K_3	K_4
-47.9001	67.5785	-29.6405	9.5819

The gains using LQR:

Table 2: gains for linear quadratic regulator

K_1	K_2	K_3	K_4
-18.7083	74.5253	-26.4867	9.5819

The diagonal matrix, Q used for the linear quadratic regulator is the following

$$Q = \begin{pmatrix} 35 & 0 & 0 & 0 \\ 0 & 35 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}$$

The weight, R is

$$R = 0.01$$

The corresponding poles for each method are

The poles for Pole Placement:

Table 3: poles for pole placement

λ_1	λ_2	λ_3	λ_4
-3+3.5i	-3-3.5i	-10	-15

The poles for LQR:

Table 4: poles for linear quadratic regulator

λ_1	λ_2	λ_3	λ_4
-46.4366	-3.1942+1.7633i	-3.1942-1.7633i	-2.0139

The plots of the poles on the complex plane are the following

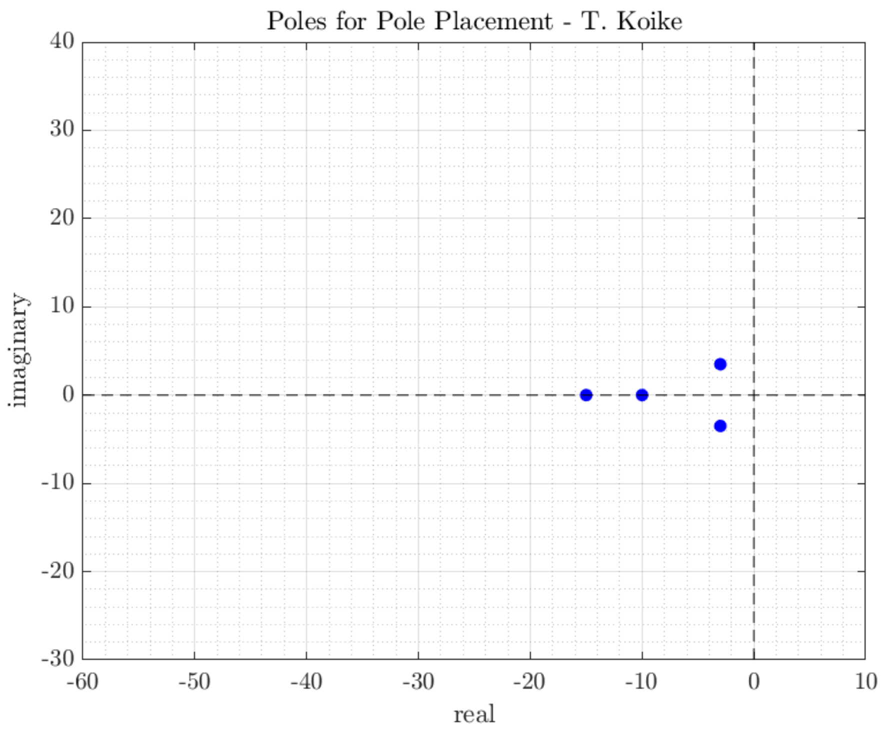


Figure 1: poles on complex plane for pole placement

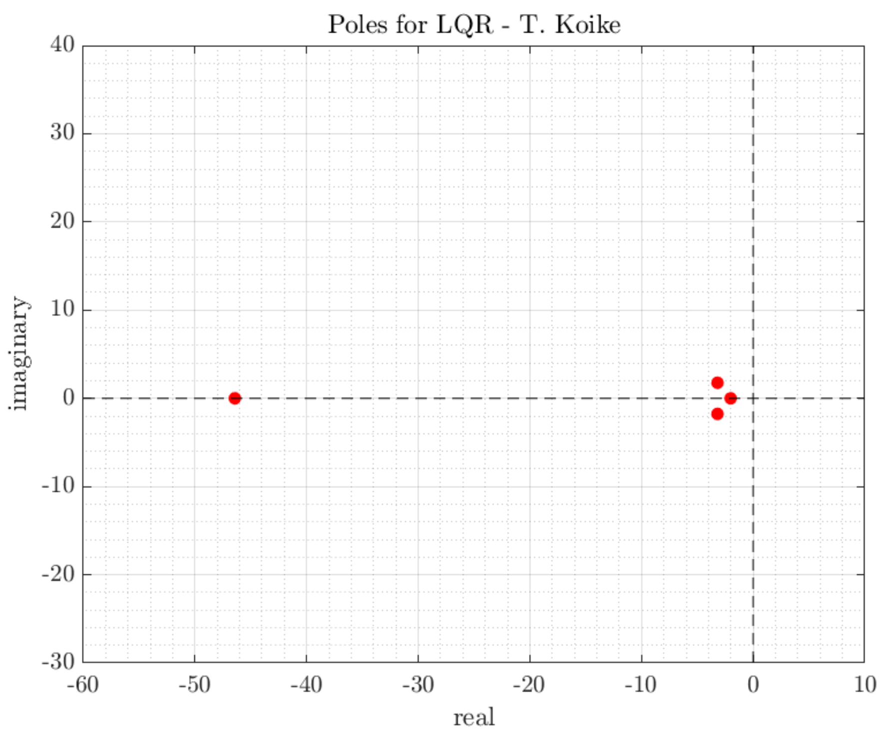


Figure 2: poles on complex plane for linear quadratic regulator

Analysis & Discussions

Part (i)

The time history of each the position of the cart and the angle of the pendulum are graphed from the data that we have achieved for each pole placement and LQR.

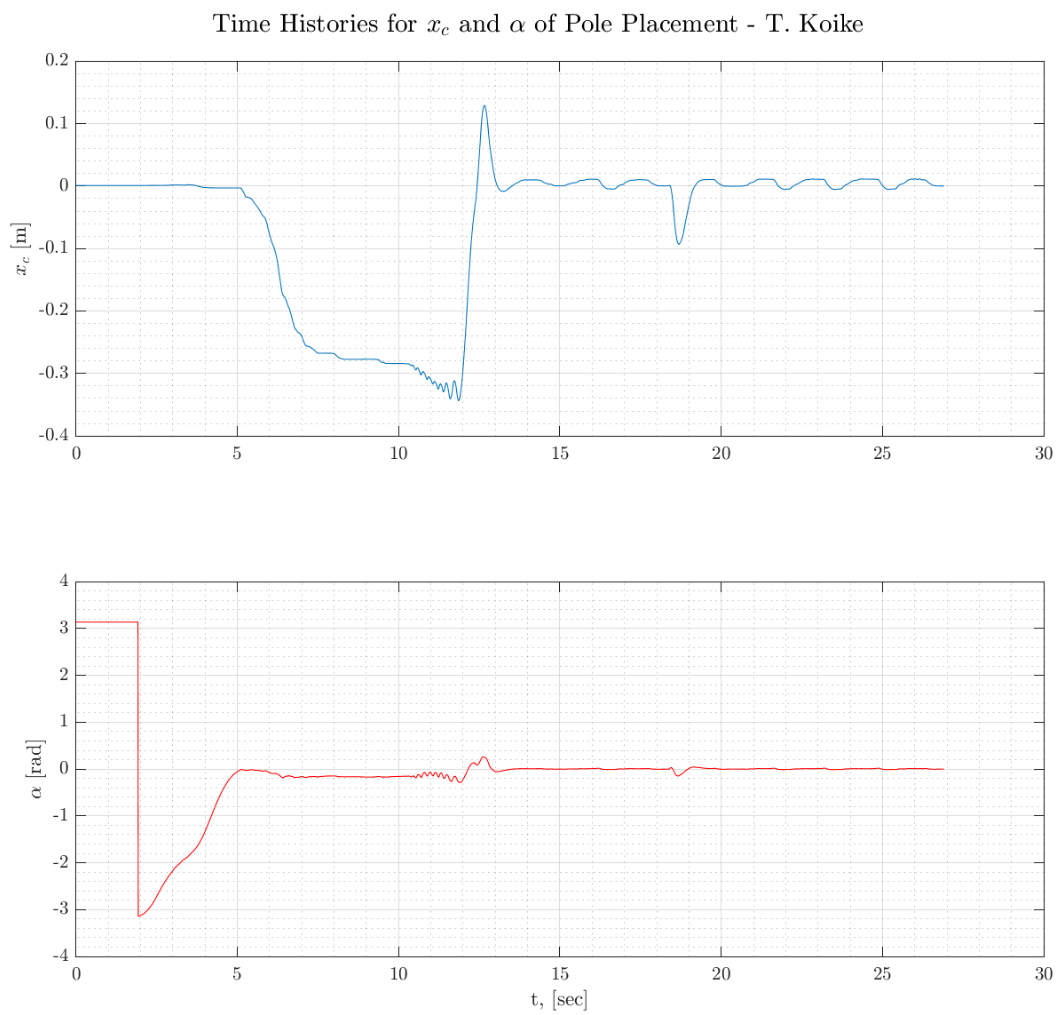


Figure 3: time history for pole placement

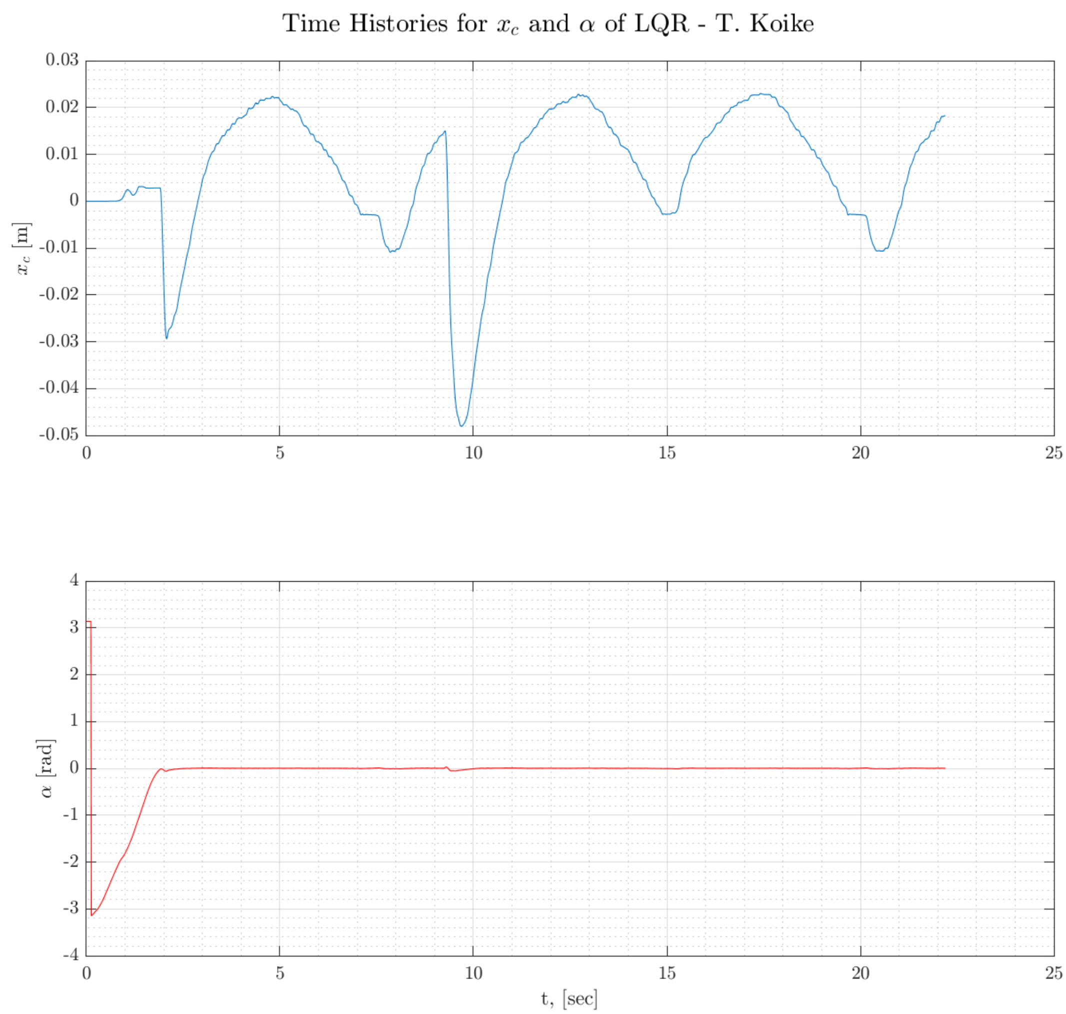


Figure 4: time history of linear quadratic regulator

Because of the system settings initially the pendulum at the downward position is perceived as $\alpha = 180°$. Then we slowly move the pendulum to be upward. This makes $\alpha = -180°$.

We can see that for the pole placement the rod takes approximately 1 second longer to stabilize the angle to 0 degrees than the LQR method. Also, for the pole placement method the largest magnitude of the cart's displacement is about 0.3. Whereas for the LQR method, the largest magnitude of displacement is about 0.05.

Conclusion & Recommendation

Main Points

From the results, we can observe that for the pole placement and LQR have stable but different poles on the complex plane. Due to this difference in the poles we can tell from the figures 3 and 4 that the LQR outputs a better response in terms of stabilizing both the pendulum and the cart for this experiment.

Theoretical/Experimental Limitations

One big factor that have affected this experiment is the strength we applied when we pushed the rod at the upward position. Though we were conscious of keeping the force applied on the pendulum to be very small, it might have varied for when we were experimenting for the pole placement and the LQR.

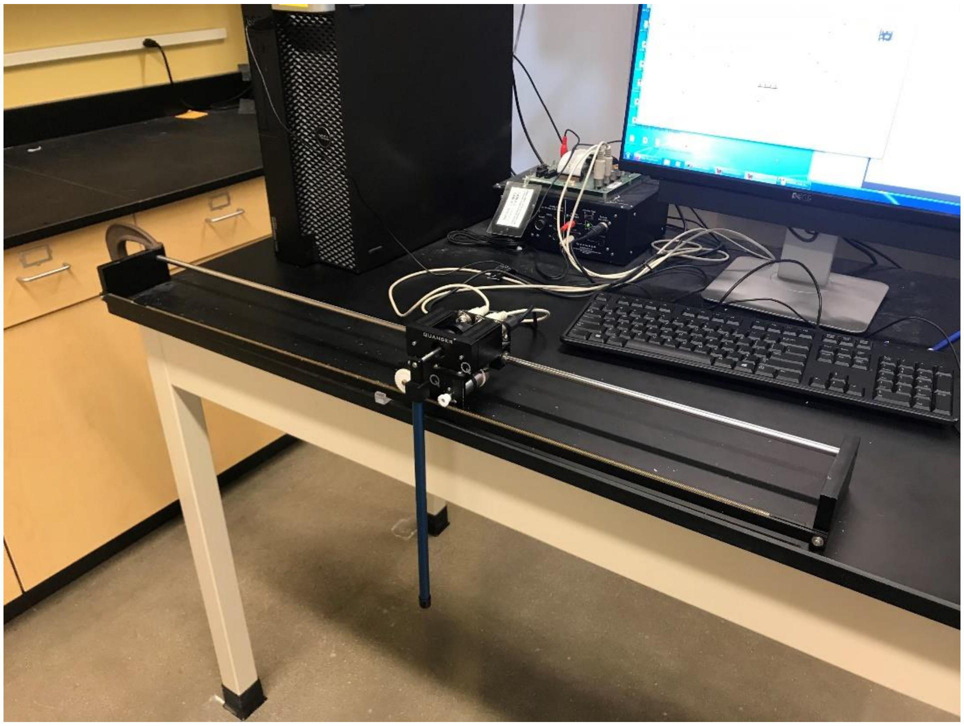
Another limitation common to all the other labs performed is the voltage limit that prevents us to have a gain larger than 200.

Lessons Learned & Suggestions for Improvement

As discussed in the previous section, it would be better to have a robot arm of some sort to apply force on the pendulum to prevent human errors for the responses.

Appendix

Experiment Setup



Notations for Variables

Symbol	Description	Value	Unit
R_m	motor armature resistance	2.6	Ω
L_m	motor armature inductance	0.18	mH
K_t	motor torque constant	0.00767	$N.m/A$
η_m	motor efficiency	100%	%
K_m	back-electromotive-force(EMF) constant	0.00767	$V.s/rad$
J_m	rotor moment of inertia	3.9×10^{-7}	$kg.m^2$
K_g	planetary gearbox ratio	3.71	
η_g	planetary gearbox efficiency	100%	%
M_{c2}	cart mass	0.57	kg
M_w	cart weight mass	0.37	kg
M_c	total cart weight mass including motor inertia	1.0731	kg
B_{eq}	viscous damping at motor pinion	5.4000	$N.s/m$
L_t	track length	0.990	m
T_c	cart travel	0.814	m
P_r	rack pitch	1.664×10^{-3}	$m/tooth$
r_{mp}	motor pinion radius	6.35×10^{-3}	m
N_{mp}	motor pinion number of teeth	24	
r_{pp}	position pinion radius	0.01482975	m
N_{pp}	position pinion number of teeth	56	
K_{EP}	cart encoder resolution	2.275×10^{-5}	$m/count$
M_p	long pendulum mass with T-fitting	0.230	kg
M_{pm}	medium pendulum mass with T-fitting	0.127	kg
L_p	long pendulum length from pivot to tip	0.6413	m
L_{pm}	medium pendulum length from pivot to tip	0.3365	m
l_p	long pendulum length: pivot to center of mass	0.3302	m
l_{pm}	medium pendulum length: pivot to center of mass	0.1778	m
J_p	long pendulum moment of inertia ⟳ center of mass	7.88×10^{-3}	$kg.m^2$
J_{pm}	medium pendulum moment of inertia ⟳ center of mass	1.20×10^{-3}	$kg.m^2$
B_p	viscous damping at pendulum axis	0.0024	$N.m.s/rad$
g	gravitational constant	9.81	m/s^2
v	voltage of servo motor	variable	V

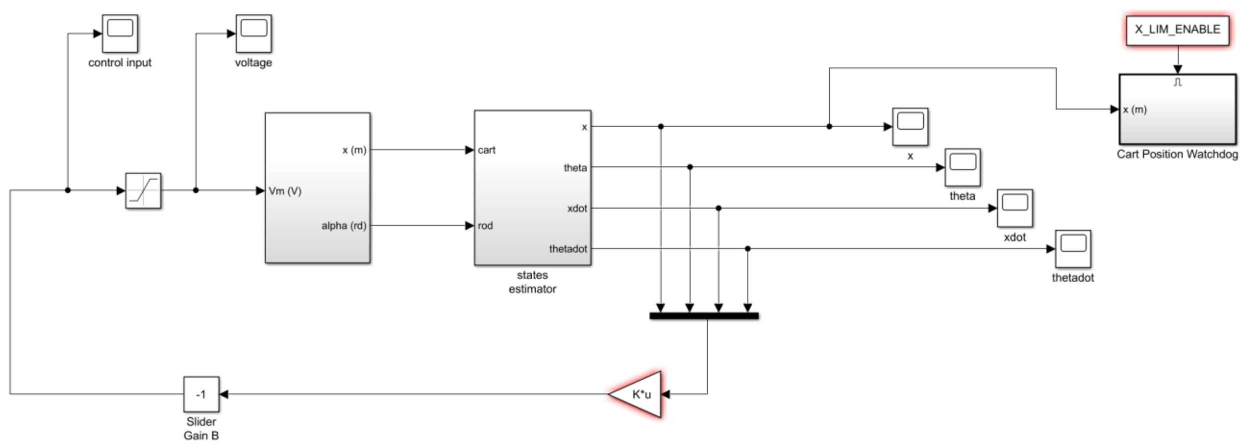
MATLAB Code

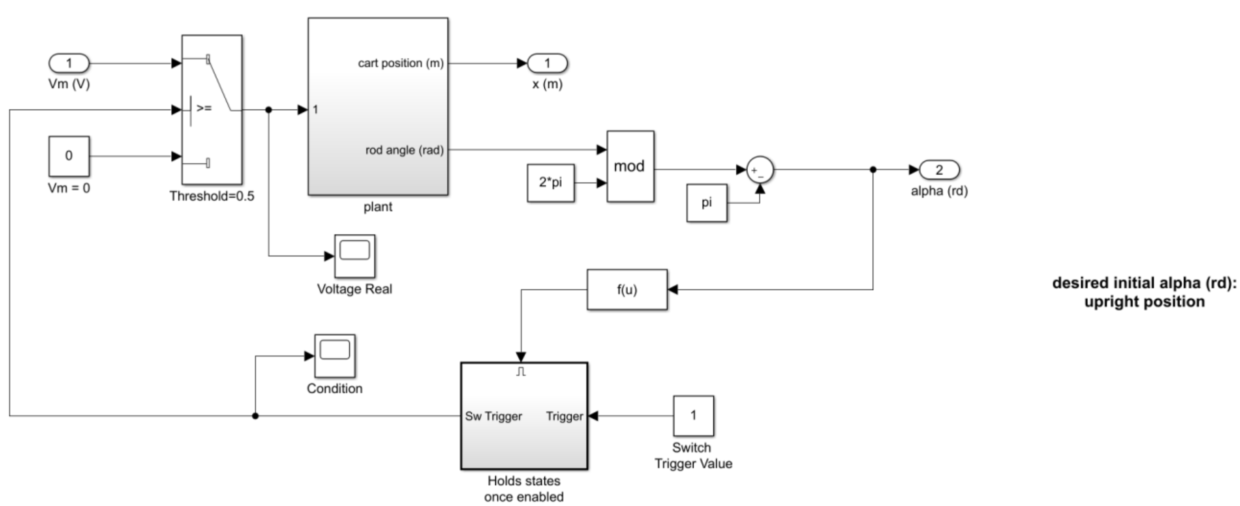
```
% AAE 364L MATLAB CODE
% TOMOKI KOIKE
clear all; close all; clc;
set(groot, 'defaulttextinterpreter','latex');
set(groot, 'defaultAxesTickLabelInterpreter','latex');
set(groot, 'defaultLegendInterpreter','latex');

% Load result data

pp_xc = load("koike_lab3_results\koike_position_PP_lab3.mat");
pp_theta = load("koike_lab3_results\koike_theta_PP_lab3.mat");
lqr_xc = load("koike_lab3_results\koike_position_LQR_lab3.mat");
lqr_theta = load("koike_lab3_results\koike_theta_LQR_lab3.mat");

% PP

t = pp_xc.position_part2.time;
xc = pp_xc.position_part2.signals.values;
theta = pp_theta.theta_part2.signals.values;

% Plotting
fig = figure('Renderer',"painters", 'Position', [10 10 900 800]);
subplot(2,1,1)
plot(t,xc)
grid on; grid minor; box on;
ylabel('$x_c$ [m]')
subplot(2,1,2)
plot(t,theta, '-r')
grid on; grid minor; box on;
ylabel('$\alpha$ [rad]')
xlabel('t, [sec]')
sgtitle('Time Histories for $x_c$ and $\alpha$ of Pole Placement - T. Koike')
saveas(fig, 'pp_history.png')

% PP

t = lqr_xc.position_part2.time;
xc = lqr_xc.position_part2.signals.values;
theta = lqr_theta.theta_part2.signals.values;

% Plotting
fig = figure('Renderer',"painters", 'Position', [10 10 900 800]);
subplot(2,1,1)
plot(t,xc)
grid on; grid minor; box on;
ylabel('$x_c$ [m]')
subplot(2,1,2)
plot(t,theta, '-r')
grid on; grid minor; box on;
```

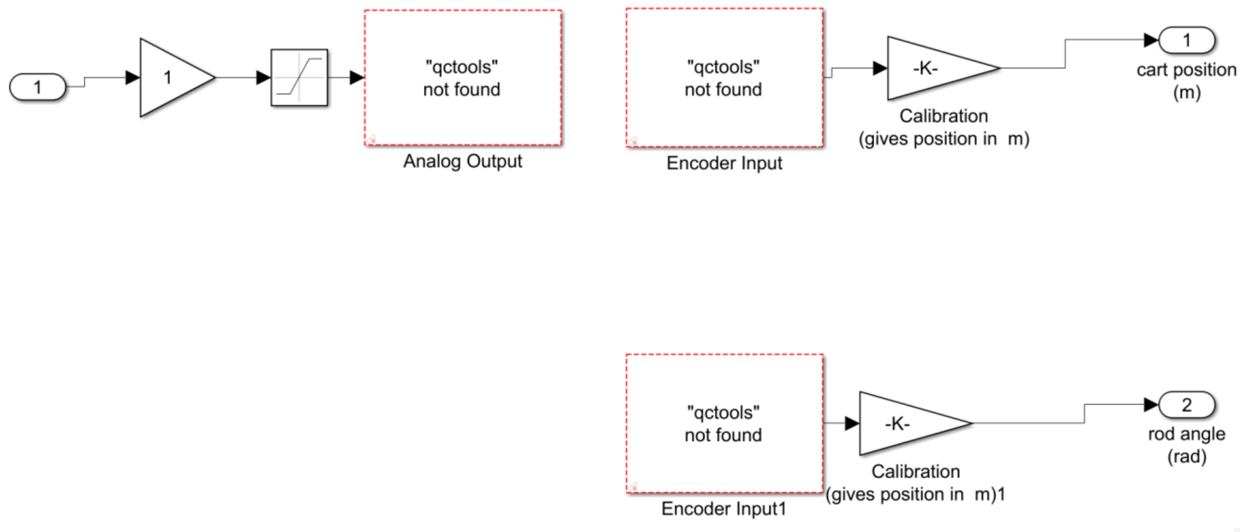
```
ylabel('$\alpha$ [rad]')
xlabel('t, [sec]')
sgtitle('Time Histories for $x_c$ and $\alpha$ of LQR - T. Koike')
saveas(fig, 'lqr_history.png')
```

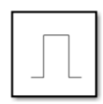
Simulink Models

Prelab Models

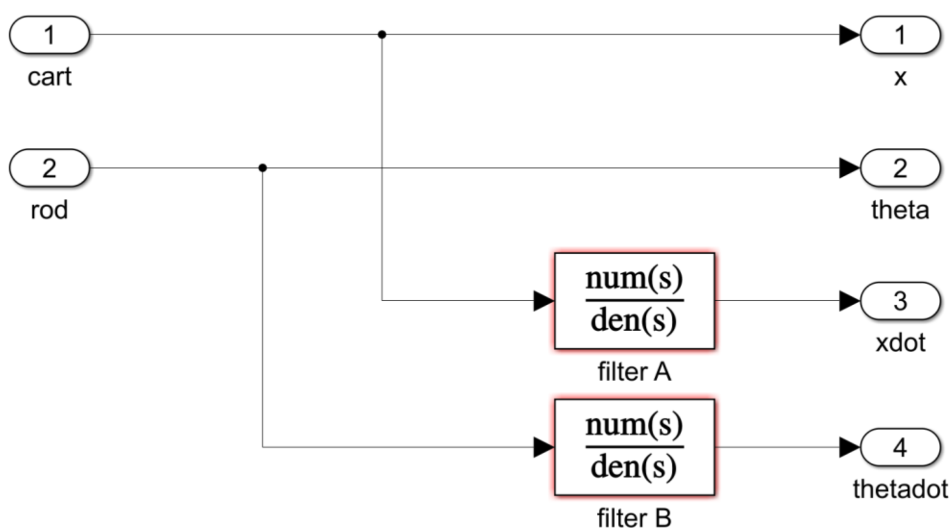


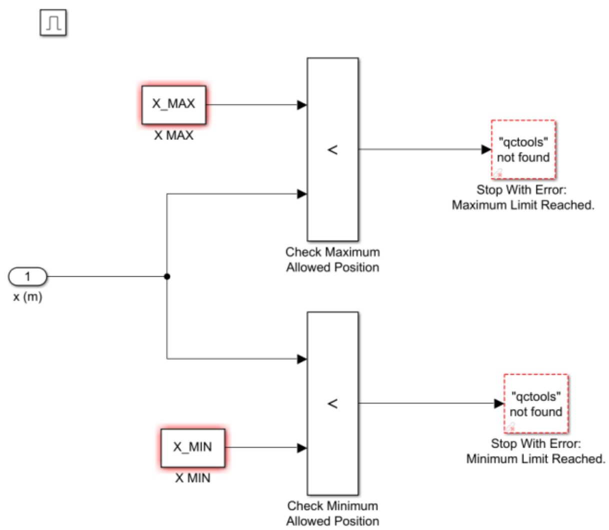



1
1
"qctools" not found
Analog Output
"qctools" not found
Encoder Input
-K-
Calibration (gives position in m)
1
cart position (m)
"qctools" not found
Encoder Input1
-K-
Calibration (gives position in m)1
2
rod angle (rad)




1
Trigger
1
Sw Trigger


1
cart
2
rod
num(s)
den(s)
filter A
num(s)
den(s)
filter B
1
x
2
theta
3
xdot
4
thetadot


X_MAX
X MAX
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Stop With Error:
Maximum Limit Reached.
Check Maximum
Allowed Position
1
x (m)
<
"qctools" not found
Stop With Error:
Minimum Limit Reached.
X_MIN
X MIN
Check Minimum
Allowed Position