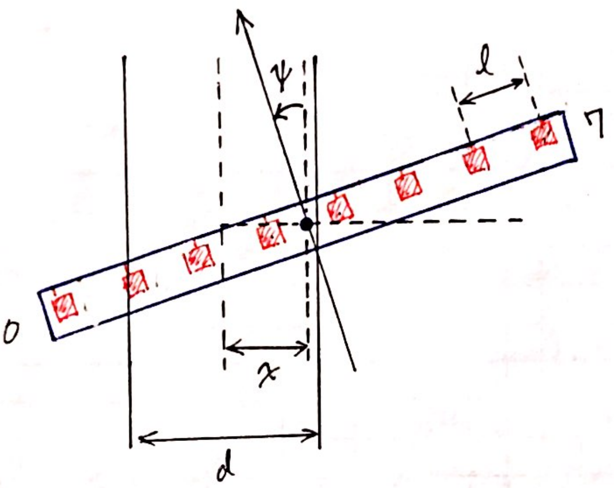
Simpler scheme

$0 \sim 7 \begin{cases} \text{High} & \text{black} \\ \text{low} & \text{white} \end{cases}$



2 , 3 High

0,1,4,5,6,7 low

how to compute x-displacement ?

In the above example if sensor #2 & #3 are high the position becomes

position = 1000 × mean(2,3) = 2500

if error > 0 or position < 3500 then turn left → $\omega_l - \Delta\omega$, $\omega_r + \Delta\omega$

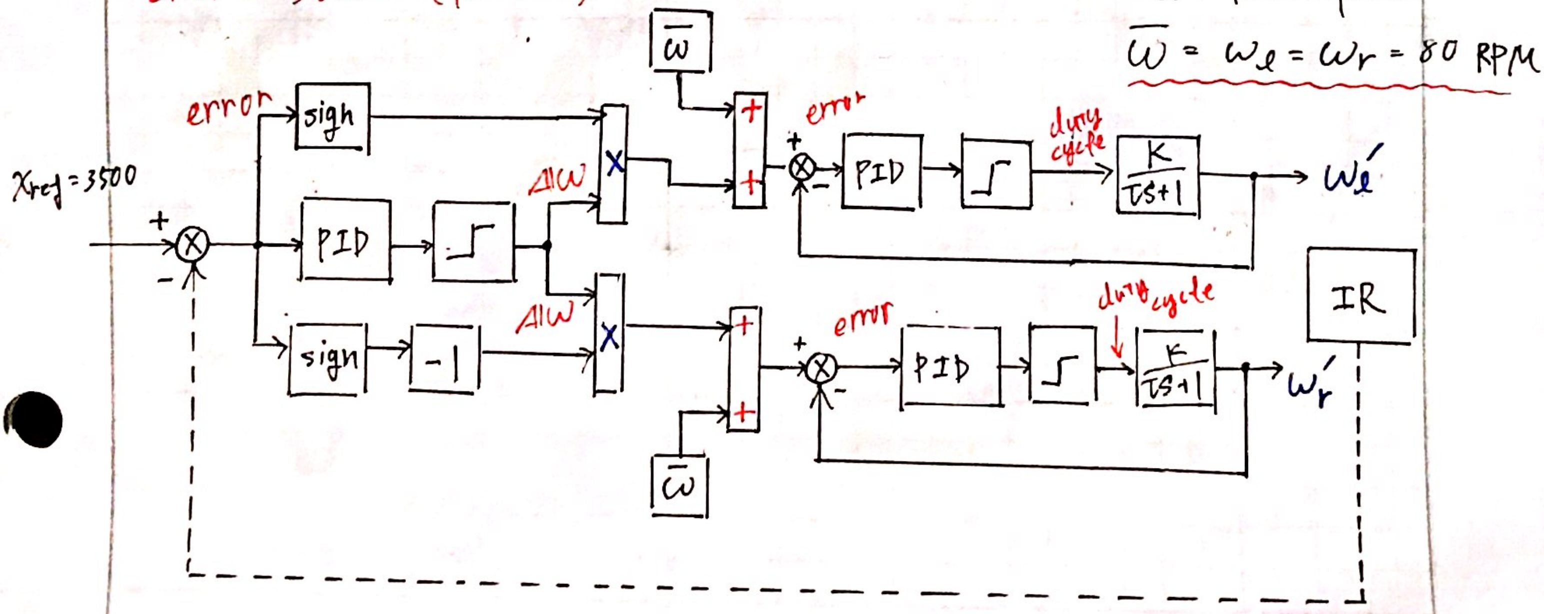
if error = 0 or position = 3500 then go straight → $\omega_l = \omega_r$

if error < 0 or position > 3500 then turn right → $\omega_l + \Delta\omega$, $\omega_r - \Delta\omega$

error = 3500 − (position)

let base speed

$\bar{\omega} = \omega_l = \omega_r = 80$ RPM



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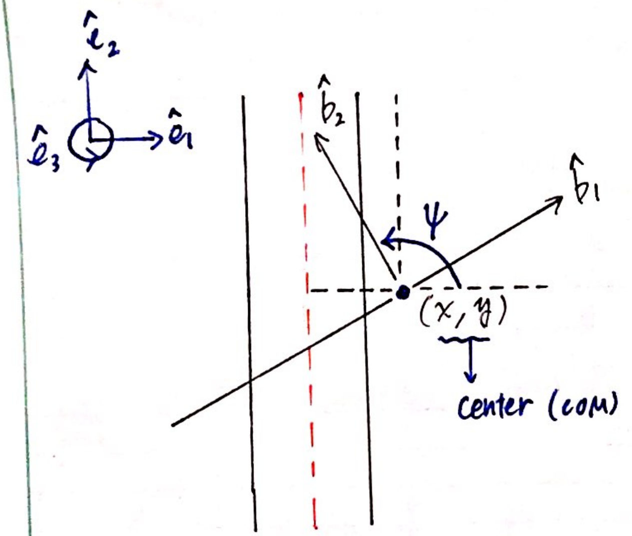

Dubins path

$$\begin{cases} \dot{x} = v\cos\psi \\ \dot{y} = v\sin\psi \\ \dot{\psi} = u \end{cases}$$

from linear momentum conservation

$$m_{robot} V_{CM} = m_{tire} V_l + m_{tire} V_r$$

$$\begin{cases} m_{robot} : \text{total mass of robot} \\ m_{tire} : \text{mass of tire} \\ V_{CM} : \text{velocity of robot @ COM} \\ V_l : \text{velocity of left tire} \\ V_r : \quad '' \quad \text{right tire} \end{cases}$$

since $v_l = \omega_l R$

$v_r = \omega_r R$

$$\begin{cases} R : \text{radius of tire} \\ \omega_l, \omega_r : \text{RPM of left \& right tire} \end{cases}$$

$$\boxed{v = V_{CM} = \frac{m_{tire}}{m_{robot}}(\omega_l + \omega_r)R}$$

$$\begin{cases} \omega_r \rightarrow +\hat{e}_3 \\ \omega_l \rightarrow -\hat{e}_3 \end{cases}$$

from angular momentum conservation

$$I_{robot}\dot{\psi} = -I_{tire}\omega_l + I_{tire}\omega_r \longrightarrow \boxed{\dot{\psi} = \frac{I_{tire}}{I_{robot}}(\omega_r - \omega_l)}$$

$$\begin{cases} I_{robot} : \text{moment of inertia of robot} \\ I_{tire} : \quad '' \quad \text{of tire} \end{cases}$$

Thus,

$$\begin{cases} \dot{x} = \frac{m_{tire}}{m_{robot}} R(\omega_l + \omega_r)\cos\psi \\ \dot{y} = \frac{m_{tire}}{m_{robot}} R(\omega_l + \omega_r)\sin\psi \\ \dot{\psi} = \frac{I_{tire}}{I_{robot}}(\omega_r - \omega_l) \end{cases}$$

let,

$$\begin{cases} \alpha = \frac{m_{tire}}{m_{robot}} R \\ \beta = \frac{I_{tire}}{I_{robot}} \end{cases}$$

control

$$u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \omega_r \\ \omega_l \end{bmatrix}$$

states

$$X = \begin{bmatrix} x \\ y \\ \psi \end{bmatrix}$$

$$\rightarrow \begin{cases} \dot{x} = \alpha(\omega_l + \omega_r)\cos\psi \\ \dot{y} = \alpha(\omega_l + \omega_r)\sin\psi \\ \dot{\psi} = \beta(\omega_r - \omega_l) \end{cases}$$

$$\boxed{\begin{aligned} \dot{X}_1 &= \alpha(u_1 + u_2)\cos X_3 \\ \dot{X}_2 &= \alpha(u_1 + u_2)\sin X_3 \\ \dot{X}_3 &= \beta(u_1 - u_2) \end{aligned}}$$

The system is nonlinear but we can linearize the system w.r.t the equilibrium points

$$x_e = 0, \quad y_e = 0, \quad \psi_e = \frac{\pi}{2}, \quad \omega_{re} = \omega_{le} = \omega_e = \text{const.}$$

which are the desired values.

(1) $\dot{x} = \alpha\omega_r\cos\psi + \alpha\omega_l\cos\psi$

(2) $\dot{y} = \alpha\omega_r\sin\psi + \alpha\omega_l\sin\psi$

(3) $\dot{\psi} = \beta\omega_r - \beta\omega_l$

(1) $\delta\dot{x} = \alpha\delta\omega_r\cancel{\cos\psi_e} - \alpha\omega_{re}\sin\psi_e\delta\psi + \alpha\delta\omega_l\cancel{\cos\psi_e} - \alpha\omega_{le}\sin\psi_e\delta\psi$

$\delta\dot{x} = -2\alpha\omega_e\delta\psi$

(2) $\delta\dot{y} = \alpha\delta\omega_r\sin\psi_e + \alpha\omega_{re}\cancel{\cos\psi_e}\delta\psi + \alpha\delta\omega_l\sin\psi_e + \alpha\omega_{le}\cancel{\cos\psi_e}\delta\psi$

$\delta\dot{y} = \alpha\delta\omega_r + \alpha\delta\omega_l$

(3) $\delta\dot{\psi} = \beta\delta\omega_r - \beta\delta\omega_l$

$$\boxed{\begin{aligned} \dot{X}_1 &= -2\alpha\omega_e X_3 \\ \dot{X}_2 &= \alpha u_1 + \alpha u_2 \\ \dot{X}_3 &= \beta u_1 - \beta u_2 \end{aligned}}$$

Hence, the linearized system is

$$A = \begin{bmatrix} 0 & 0 & -2\alpha\omega_e \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ \alpha & \alpha \\ \beta & -\beta \end{bmatrix}$$

controllability Matrix Q_c

$$Q_c = [B \;\; AB \;\; A^2B]$$

$$Q_c = \begin{bmatrix} 0 & 0 & -2\alpha\beta\omega_e & 2\alpha\beta\omega_e & 0 & 0 \\ \alpha & \alpha & 0 & 0 & 0 & 0 \\ \beta & -\beta & 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \end{bmatrix}$$

This system Is "controllable"