AAE 339: Aerospace Propulsion

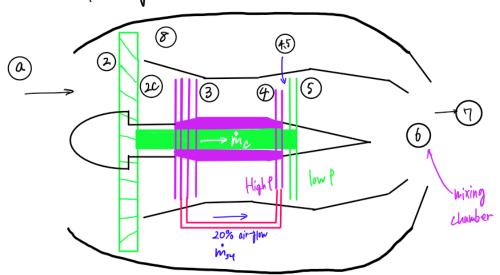
HW6: Realistic Turbofan Cycle Analysis

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- (a) Draw a neat schematic of the engine that clearly includes the changes noted above.
 - 2 spoul engine turbo-fan schematic



Gliven Properties

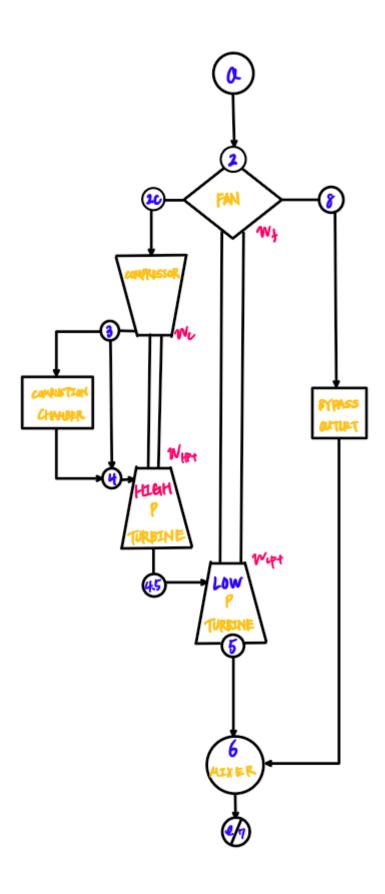
@ freestream

$$R = 287$$
 $Teg-k$
 $Q_{r} = 45,000 + Teg$ $\eta_{b} = 0.98$ $\eta_{f} = \frac{h_{00}s - h_{02}}{h_{00} - h_{02}} = \frac{T_{00}s - T_{02}}{T_{00} - T_{02}} = 0.85$

$$\eta_d = \frac{h_{02}s - h_a}{h_{02} - h_a} = \frac{T_{02}s - T_a}{T_{02} - T_a} = 0.97 , \eta_c = \frac{h_{03}s - h_{02}}{h_{03} - h_{02}} = \frac{T_{03}s - T_{02}}{T_{03} - T_{03}} = 0.85$$

$$7_{t} = \frac{h_{04} - h_{05}}{h_{04} - h_{05}} = \frac{T_{04} - T_{05}}{T_{04} - T_{05}} = 0.90 , \quad 7_{h} = \frac{h_{06} - h_{7}}{h_{06} - h_{75}} = \frac{T_{06} - T_{7}}{T_{06} - T_{05}} = 0.98$$

Burner Energy Equation



To solve (b) we use answers from part (d)

(b) Calculate the specific work required by the fan and the compressor.

Use stagnation properties at the stations

$$T_{02} = 248.92 \text{K}$$
 $T_{03} = 741.44 \text{K}$
 $T_{02c} = T_{08} = 281.39 \text{K}$

fan

$$w_{f} = (1+\beta) C_{p2} (70s - 702)$$

$$w_{f} = 3.3990 \times 10^{5} \frac{J}{+9}$$

Compressor

$$W_{c} = C_{p2} (T_{03} - T_{02}c)$$

$$W_{c} = 5.0409 \times 10^{5} \frac{J}{+9}$$

(c) Determine the fuel-air ratio in the combustor.

since, there is air bleeding from the compressor to cool the turbine.

$$\dot{m}_{0} = \dot{m}_{34} + \dot{m}_{c}$$

Since $\int = \frac{\dot{m}_{34}}{\dot{m}_{0}} = 0.2$, $\xi = \frac{\dot{m}_{c}}{\dot{m}_{0}} = 0.8$

now, the Burner Energy Equation can be rouritten as

$$\int_{cs} (h_0) \rho(\overline{u} \cdot \overline{h}) dA = \overline{Q} - \frac{1}{164}$$

$$(\dot{m}_c + \dot{m}_f) h_{04} - (\dot{m}_c) h_{03} = \dot{m}_f \eta_b Q_r$$

$$\Leftrightarrow (\xi + f) h_{04} - \xi h_{03} = f \eta_b Q_r$$

$$(\xi + f) C_{p2} T_{04} - \xi C_{p2} T_{03} = f \eta_b Q_r$$

$$\xi C_{p2} (T_{04} - T_{03}) = f (\eta_b Q_r - C_{p2} T_{04})$$

$$f = \frac{\xi C_{p2} (T_{04} - T_{03})}{\eta_b Q_r - C_{p2} T_{04}}$$

$$f = \frac{0.8 (\frac{1}{16} (\frac{1}{16} (\frac{1}{16} \cos k - \frac{1}{16} (\frac{1}{16} \cos k)))}{0.98 (\frac{1}{16} \cos k)(\frac{1}{16} \frac{1}{16} (\frac{1}{16} \cos k))}$$

$$f = 0.022 | 16$$

(d) Calculate stagnation conditions in the free stream, at the engine face (2), at the entry of the bypass duct and compressor, at the combustor inlet (3), at the inlets to the turbines (4 and 4.5), exit of the low-pressure turbine (5), at the exit of the mixing section (call it 6), and engine exit (7).

MATLAB was utilized for calculations; code is in Appendix

Some as pl the properties are

from textbook appendix

$$T_a = 389.97 R = 216.65 K$$
 $P_a = 3.9312 \times 10^2 \text{ lb/ft}^2 = 18823 Pa$

then $P_{0a} = P_a \left[1 + \frac{k-1}{2} M_a^2 \right]^{\frac{k-1}{k-1}}$
 $P_{0a} = \left(18823P_0 \right) \left[1 + \frac{(4-1)}{2} (0.85)^2 \right]^{\frac{4}{k-1}} \approx 30189 Pa$
 $T_{0a} = T_a \left[1 + \frac{k-1}{2} M_a^2 \right]$
 $T_{0a} = \frac{247.96 K}{2}$

$$P_{02} = 30|89$$
 Pa
 $T_{02} = 247.96$ K

now, from 2 , we know that

When from Isentropic velations
$$\left(\frac{\rho_{02}}{\rho_{0a}}\right) = \left(\frac{\sigma_{02}}{\sigma_{0a}}\right)^{\sigma_{02}/\sigma_{01}} \Rightarrow \rho_{02} = 29790 \rho_{02}$$

Bypass duct inlet (8)

Then
$$\frac{102c5}{102} = \frac{\binom{P02c}{P02}}{\binom{P02c}{P02}} = \frac{\binom{P0$$

lii) Compustor inlet (3)

from the relations
$$\frac{P_{03}}{P_{02}} = 42$$
, $\frac{P_{02c}}{P_{02}} = \frac{3}{2}$
 $\frac{P_{03}}{P_{02c}} = \frac{P_{03}}{P_{02c}} = \frac{P_{02}}{P_{02c}} = 42 \cdot \frac{2}{3} = 28$

then,

 $\frac{P_{03}}{P_{03}} = 28 P_{02c} = 28 \left(446 \% 6 P_{a} \right) = 1.2512 \times 10^6 P_{a}$

next, from isentropic relations

$$\frac{\sqrt{1035}}{\sqrt{1026}} = \left(\frac{\frac{1}{103}}{\frac{1}{1036}}\right)^{\frac{1}{1035}}$$

$$\sqrt{1035} = \sqrt{1036} = \sqrt{1036} = \left(\frac{\frac{1}{103}}{\frac{1}{1035}}\right)^{\frac{1}{1035}}$$

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$$\sqrt{1035} = \frac{1}{1036} =$$

now using 7c,

$$T_{03} = T_{02} + \frac{T_{035} - T_{02}}{7c}$$

$$T_{03} = 738,55 \text{ K}$$

<iv-a> High p turbine inlet (4)

it is given that the turbine inlet is

and it is also given that

<iv-b> Intermediate of High P and Low & turbine (4.5)

$$\therefore \mathcal{E} + \delta = 1 \implies \overline{C_{045}} = 2\overline{C_{04}} + \delta\overline{C_{03}} - \frac{w_c}{C_{p_1}}$$

2V> exit of Low P turbine (5)

Since

Work of low P turbine = -Wf

$$C_{P-}(Tos-Tors) = -W_{f}$$

$$Tos = Tors - \frac{W_{f}}{C_{P2}} - ... 2$$

$$T_{OS} = \frac{822}{Tor} - \frac{W_{f}}{C_{P2}} - ... 2$$

$$T_{HP,f} = \frac{Tor - Tors}{Tor - Tors}$$

$$T_{HP,f} = \frac{Tors - Tos}{Tors}$$

$$T_{HP,f} = \frac{Tors}{Tors} - \frac{Tors}{Tors}$$

$$T_{HP,f} = \frac{Tors}{Tors} - \frac{Tors}{Tors}$$

$$T_{HP,f} = \frac{Tors}{Tors} - \frac{Tors}{Tors$$

Po45 = 1.5933 × 10 Pa

$$T_{055} = T_{045} - \frac{T_{045} - T_{05}}{\eta_{LP,t}} = \eta_{88,17} k$$

$$\frac{P_{05}}{P_{145}} = \left(\frac{T_{055}}{T_{045}}\right)^{\frac{p_L}{p_L-1}}$$

$$P_{05} = P_{145} \left(\frac{T_{075}}{T_{045}}\right)^{\frac{p_L}{p_L-1}}$$

$$P_{05} = 3.9771 \times 10^4 P_a$$

2 Mixing chamber (6)

Assuming no energy/heart transfer

Using the
$$l^{st}$$
 law of thermodynamics

2 Mix $(e + \frac{u^{2}}{2} + 92)edV + \int_{cs} (h + \frac{u^{2}}{2} + 93)e(\overline{u} \cdot \overline{n})dA = \frac{\partial^{2} - iv^{2}}{\partial x^{2}} + \frac{\partial^{2}$

(e) Calculate temperature, pressure, and velocity at engine exit $7(T_7, p_7, u_7)$.

$$\frac{p_{07}}{p_{7}} = \left(1 + \frac{y-1}{2}M_{e}^{2}\right)^{\frac{32}{32-1}}$$

$$\left(\frac{p_{09}}{p_{9}}\right)^{\frac{\gamma_{5}-1}{\beta_{5}}} = \left(+\frac{\gamma_{5}-1}{2}M_{e}\right)^{\frac{\gamma_{5}-1}{2}}$$

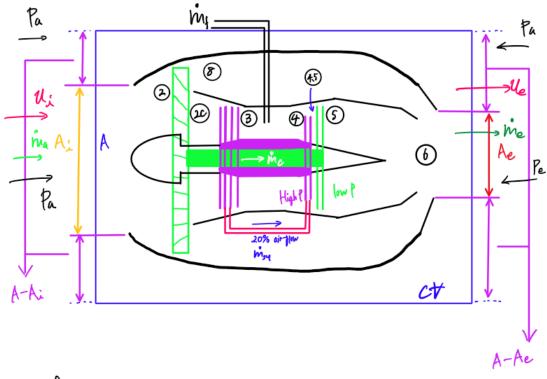
$$\frac{r-1}{2l}e^{2} = \left(\frac{p_{0\eta}}{p_{\eta}}\right)^{\frac{r}{r}} - 1$$

$$M_e = \sqrt{\frac{2}{\tilde{l}_2 - 1} \left[\left(\frac{\tilde{l}_0 \eta}{\tilde{l}_0} \right)^{\frac{\tilde{l}_2 - 1}{\tilde{l}_2}} - 1 \right]}$$

now

then using In

(f) Not everyone was able to correctly develop an equation for thrust in Exam 1, so let's do it again. Draw a control volume around the engine, clearly showing control surfaces and external forces. Starting with the conservation equations, and using your CV, derive an equation for thrust.



$$(\dot{m}_{a}+\dot{m}_{b}+\dot{m}_{f}) v_{e} - (\dot{m}_{a}+\dot{m}_{b}) v_{i}$$

$$= \dot{m}_{a} [(I+\beta+f) v_{e} - v_{i} - \beta v_{i}]$$

$$= \dot{m}_{a} [(I+f) v_{e} - v_{i} + \beta (v_{e} - v_{i})]$$

RHS

Thus

$$ST = \frac{T}{m_a} = (+f)ue - Ui + p(Ve - Ui)$$
 $Ui = Ma\sqrt{F}Ta$
 $Ui = 0.85 - \sqrt{1.4(287\%-k)(216.65k)}$
 $Ui = 250.79 \text{ m/s}$

plug in all the numbers

 $Ue = 436.68 \text{ m/s}$
 $f = 0.0222$
 $G = 8.5$

we have a

$$\frac{ST-ST}{ST} \times 100 = 23.79\%$$
improvement from $HW5$ (2)

Appendix

```
close all; clear all; clc
% Given properties
Ma = 0.85;
gamma1 = 1.4;
Cp1 = 1.0*10^3; % [J/kg/K]
gamma2 = 1.35;
Cp2 = 1.10*10^3; \% [J/kg/K]
R = 287; % [J/kg/K]
Qr = 45000*10^3; % [J/kg]
eta d = 0.97;
eta_c = 0.85;
eta_f = eta_c;
eta_t = 0.90;
eta_n = 0.98;
eta_b = 0.98;
beta = 8.5;
Po3 Po2 ratio = 42;
Po2c_Po2_ratio = 1.5;
To4 = 1800; % [K]
% freestream (a)
Ta = 216.65; % [K]
Pa = 18823; % [Pa]
Poa = p_from_M_and_gamma(Pa, Ma, gamma1, "stagnation")
Toa = T from M and gamma(Ta, Ma, gamma1, "stagnation")
% engine entry (2)
To2 = Toa
To2s = T_from_adiabatic_eff_stagnation(Ta, To2, eta_d, "diffuser")
Po2 = P_from_isentropic_relation(Poa, To2s, Toa, gamma1)
% compressor inlet (2c)
Po2c = Po2c Po2 ratio*Po2
To2cs = T_from_isentropic_relation(To2, Po2c, Po2, gamma2)
To2c = T_from_adiabatic_eff_static(To2, To2cs, eta_f, "fan")
% combustor inlet (3)
Po3_Po2c_ratio = Po3_Po2_ratio/Po2c_Po2_ratio
Po3 = Po3 Po2c ratio*Po2c
To3s = T from isentropic relation(To2c, Po3, Po2c, gamma2)
To3 = T_from_adiabatic_eff_static(To2, To3s, eta_c, "compressor")
% combustor exit/High pressure turbine inlet (4)
To4 = 1800 % [K]
Po4 = Po3
% bypass duct inlet (8)
Po8 = Po2c
```

AAE 339 HW6 MATLAB CODE

```
To8 = To2c
% calculate the work done by fan and compressor
Wf = (1 + beta) * Cp2 * (To8 - To2)
Wc = Cp2 * (To3 - To2c)
% calculate the fuel-air-ratio
delta = 0.2;
epsilon = 0.8;
f = epsilon*Cp2*(To4 - To3)/(eta_b*Qr - Cp2*To4)
disp(vpa(f, 7))
% intermediate of high pressure and low pressure turbines (4.5)
To45 = epsilon*To4 + delta*To3 - Wc/Cp2
eta_HP_t = eta_t;
To45s = To4 - (To4 - To45)/eta HP t
Po45 = P_from_isentropic_relation(Po4, To45s, To4, gamma2)
% exit of the low pressure turbine (5)
To5 = To45 - Wf/Cp2
eta_LP_t = eta_t;
To5s = To45 - (To45 - To5)/eta_LP_t
Po5 = P_from_isentropic_relation(Po45, To5s, To45, gamma2)
% mixing chamber (6)
Cp2_alt = Cp2/1000;
R = R/1000;
To6 = (To5 + beta*To8)/(1 + beta)
a1 = Po5^R alt;
a2 = Po8^(beta*R_alt);
a3 = To5^(Cp2_alt);
a4 = To8^(beta*Cp2 alt);
b1 = To6^(Cp2 alt/R alt);
A1 = (a1*a2/a3/a4)^(1/(1 + beta)/R_alt);
Po6 = A1*b1
% Calculating properties at exit (7)
P7 = Pa
Po7 = Po6
To7 = To6
a1 = (Po7/Pa)^{(gamma2 - 1)/gamma2) - 1;
Me = sqrt(2/(gamma2 - 1)*a1)
T7s = To7/(1 + (gamma2 - 1)/2*Me^2)
T7 = T_from_adiabatic_eff_static(To6, T7s, eta_n, "nozzle")
u7 = Me * sqrt(gamma2 * R * T7)
% calculate velocity at engine entry
ua = Ma*sqrt(gamma1*R*Ta)
% Specfic thrust
ST = (1 + f)*u7 - ua + beta*(u7 - ua)
imp = (ST - 1434.5)/1434.5*100
```

```
function T2 = T from M and gamma(T1, M, gamma, type)
    if type == "stagnation"
        T2 = T1 * (1 + (gamma - 1) / 2 * M^2);
    elseif type == "static"
       T2 = T1 / (1 + (gamma - 1) / 2 * M^2);
    else
        disp("Error. Incorrect type. Type can only be 'stagnation' or 'static'.")
    end
end
function T2 = T from isentropic relation(T1, P2, P1, gamma)
    T2 = T1 * (P2 / P1)^{((gamma - 1)/ gamma)};
end
function To2 = T_from_adiabatic_eff_static(To1, To2s, eta, type)
    if type == "diffuser" || type == "compressor" || type == "fan"
        To2 = To1 + (To2s - To1) / eta;
    elseif type == "nozzle" || type == "turbine"
        To2 = To1 - eta * (To1 - To2s);
    else
        disp("Error, incorrect type. Accepted types are 'diffuser,' " + ...
            "'compressor,' 'nozzle,' 'turbine,' or 'fan.'");
    end
end
function To2s = T_from_adiabatic_eff_stagnation(To1, To2, eta, type)
    if type == "diffuser" || type == "compressor" || type == "fan"
        To2s = To1 + (To2 - To1) * eta;
    elseif type == "nozzle" || type == "turbine"
        To2s = To1 - (To1 - To2) / eta;
    else
        disp("Error, incorrect type. Accepted types are 'diffuser,' " + ...
            "'compressor,' 'nozzle,' 'turbine,' or 'fan.'");
    end
end
function p2 = p_from_M_and_gamma(p1, M, gamma, type)
    if type == "stagnation"
        p2 = p1 * (1 + (gamma - 1) / 2 * M^2)^(gamma/(gamma - 1));
    elseif type == "static"
       p2 = p1 / (1 + (gamma - 1) / 2 * M^2)^(gamma/(gamma - 1));
    else
        disp("Error. Incorrect type. Type can only be 'stagnation' or 'static'.")
    end
end
function P2 = P from isentropic relation(P1, T2, T1, gamma)
    P2 = P1 * (T2 / T1)^{gamma} / (gamma - 1));
end
```