JT BP

Monday, October 19, 2020 02:51 PM

Transfers

Goal: Shift to an orbit that does NOT intersect the original orbit

→ To accomplish: use multi impulses transfer

Usually propellant is the limiting factor so use the transfer that requires the minimum total Δv

<Note: min ΔV ≠ min # of impulses>

Approach transfer problems:

(1) **Define transfer geometry**

Given transfer orbit type : what are departure & arrival points initial/final orbits ; departure & arrival conditions

(2) **Define departure/arrival points** ← **much more difficult**

solve for transfer path that meets the specification

Since (2) more difficult, begin by considering some types from (1)

simplest possible example: circle-to-circle planar transfer

sketches ⟷ consistent with numerical results

$\gamma \rightarrow \theta^* \rightarrow E \rightarrow H$

transformation matrix } angles ⇒ quadrant check

local horizon — add to sketch/plot

Simplest two-impulse transfer (also the minimum Δv two-impulse solution)

→ Hohmann Transfer

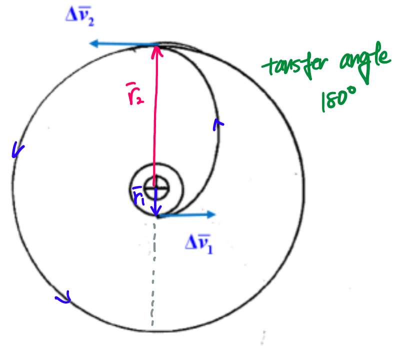
Walter Hohmann – first to draw attention to problem and compute mission times

1925 (Munich) "The Accessibility of the Heavenly Bodies" simplest version of Hohmann transfer : circle-to-circle planar transfer

Example $r_1 = 2R_\oplus$ $r_2 = 4R_\oplus$

transfer geometry known

Solution:



(a) Establish current orbit

$$a = r_1 = 2R_\oplus$$
$$e = 0$$

(b) Conditions at thrust point before maneuver

insert into process

$$r_1 = 2R_\oplus$$
$$v_1 = 5.59\,\text{km/s}$$
$$\gamma_1 = 0^\circ$$

To calculate Δv requires conditions on the transfer ellipse so transfer ellipse must be defined

(c) Determine transfer ellipse

$$a_T = \frac{1}{2}(r_p + r_a) = 3R_\oplus$$

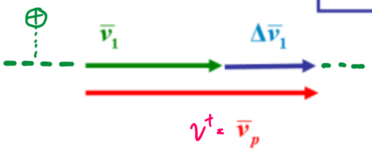
Transfer angle : TAng

$$r_p = a(1-e) \longrightarrow e_T = \frac{1}{3}$$

(d) Conditions at thrust point (on transfer) after maneuver

$$r = r_1$$
$$\frac{v_p^2}{2} = \frac{\mu}{r_1} - \frac{\mu}{2a_T} \longrightarrow v^+ = v_p = 6.45\ \text{km/s}$$
$$\gamma_1 = 0^\circ$$

(e) Vector Diagram for $\Delta\bar{v}_1$ $\boxed{\textbf{ALWAYS sketch the vector diagram}}$



$$\begin{cases} \bar{v}^+ = \bar{v}^- + \Delta\bar{v} \\ \Delta\bar{v} = \bar{v}^+ - \bar{v}^- \end{cases}$$

$\rightarrow$ $\Delta v_1 = 0.861$ km/s

$\alpha = 0°$

(f) move to the next maneuver point

Conditions at thrust point before 2[nd] maneuver
(now in transfer orbit)

$$r_a = r_2 = 4R_\oplus$$

$$\frac{v_a^2}{2} = \frac{\mu}{r_2} - \frac{\mu}{2a_T} \longrightarrow v^- = v_a = 3.22 \text{ km/s}$$

$$\gamma_2 = 0^\circ \quad \text{(apogee)}$$

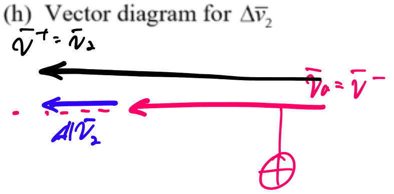
(g) Conditions required after maneuver in final orbit

$$r_2 = 4R_\oplus$$

$$v_2 = \sqrt{\frac{\mu}{r_2}} = 3.95\,\text{km/s}$$

$$\gamma = 0^\circ$$

(h) Vector diagram for $\Delta\bar{v}_2$



Increase vel

$\bar{v}^+ = \bar{v}^- + \Delta\bar{v}$

$\Delta v_2 = 0.73$ km/s

$\alpha_2 = 0°$

(i) Total $\Delta v = |\Delta\bar{v}_1| + |\Delta\bar{v}_2|$ $\boxed{\Delta v_{Total} = 1.59 \text{ km/s}}$

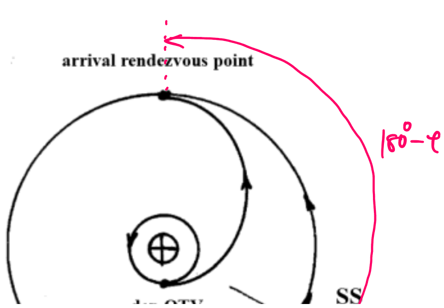
Hohmann → inexpensive → Δv is tangential
efficient easy to compute
takes time to transfer TOF (time of flight)

Conditions for Rendezvous

Transfers shift vehicles from one orbit to another

Additional complexity if rendezvous:
Just reaching target orbit is not sufficient
Timing becomes a critical factor

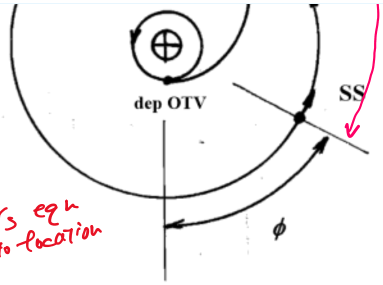
Example: $\oplus$ orbiting OTV departing low $\oplus$ orbit to rendezvous with a space station



TOF ↳ transfer time

$\frac{1}{2}P$ of transfer orbit

transfer orbit



If destination object in elliptical orbit ⇒ use Kepler's eq^n to relate time to location in orbit.

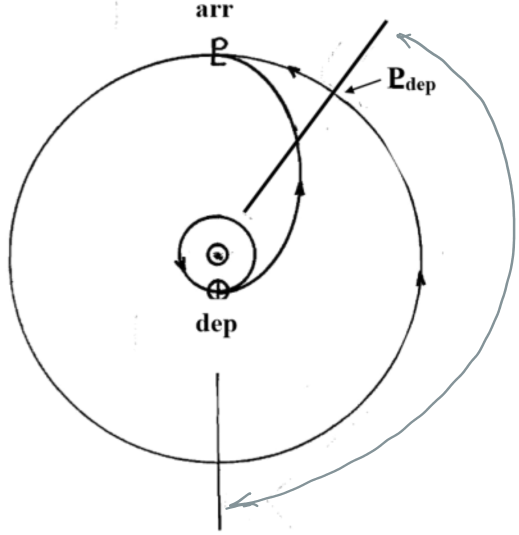
φ : phase angle at departure

circular orbit for SS

$(n_2)(TOF) = 180° - \varphi \longrightarrow \varphi = 63.1°$

ang vel = mean motion (circular)

Example: Hohmann Earth-to-Pluto



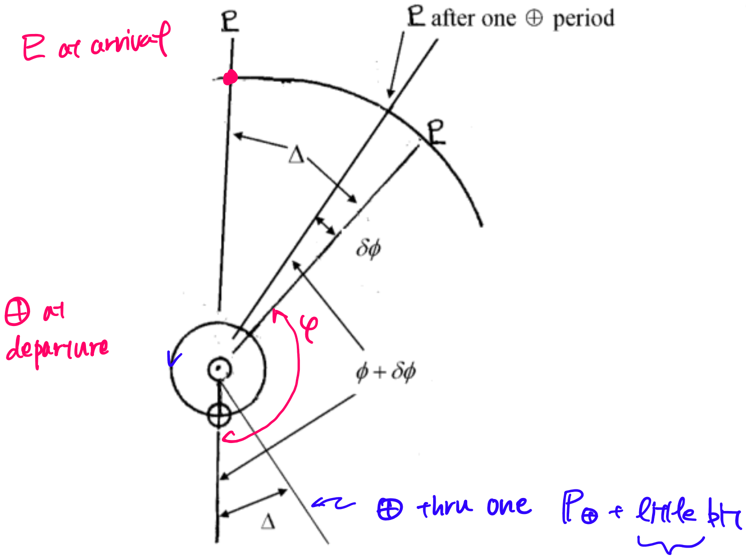
Requirement for rendezvous/interception determines initial geometry

If this "launch" opportunity is missed, how long until proper alignment again available?

→ synodic period?

note: just need φ to occur again – do NOT require that planets be in same position in space

♇ ♇ after one ⊕ period



⟵ ⊕ thru one $P_\oplus$ + $\underbrace{\text{little bit}}_{\delta\varphi}$

1. $IP_{Pluto} = 247$ yrs; P does not move far in one Earth IP

2. After one IP_{Earth} , angle between Earth and Pluto = $\phi + \delta\phi$

3. Earth moves faster than Pluto, so if we let both move a little, Earth will "catch up"

$IP = \dfrac{2\pi}{n}$ $nP = 2\pi\ (360°)$ nt = angle thru time

Earth time to go one period plus a little = $\underbrace{IP + \Delta}$ $= t_s$

$$\left.\begin{aligned} n_{Earth}\, t_s &= 2\pi + \Delta \\ n_{Pluto}\, t_s &= \Delta \end{aligned}\right\}$$

if both ⊕ and P move thru same Δ → recreate geometry

subtract

$(n_{Earth} - n_{pluto})\, t_s = 2\pi$

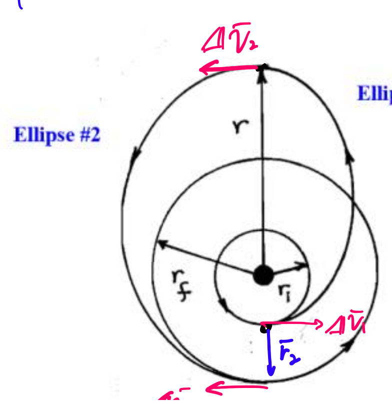
$\boxed{t_s = \dfrac{2\pi}{n_{earth} - n_{pluto}}}$ synodic period $\boxed{S = \dfrac{2\pi}{n_1 - n_2}}$

Bi-Elliptical Transfers   **Hoelker-Silber**

all tangential

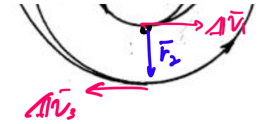
Extension to Hohmann transfer that uses three impulses

predetermined geometry → 360° transfer



r: intermediate apoapsis

best bi-elliptic save ~10% than Hohmann



save ~10% than Hohmann

Characteristics:

1. Initial orbit circular (?)
2. 1^{st} impulse applies <u>tangentially</u>; shift to periapsis of transfer Ellipse #1 (E1)
3. Apogee on E1 $= r > r_f$

   2^{nd} impulse applied <u>tangentially</u>; shifts from apoapsis of E1 tp apoapsis of transfer Ellipse #2 (E2)
4. Periapsis on E2 $= r_f$

   3^{rd} impulse applied <u>tangentially</u>; shifts into final circular (?) orbit
5. Total cost $= |\Delta\bar{v}_1| + |\Delta\bar{v}_2| + |\Delta\bar{v}_3|$

   Tot time $= \frac{1}{2} P_1 + \frac{1}{2} P_2$

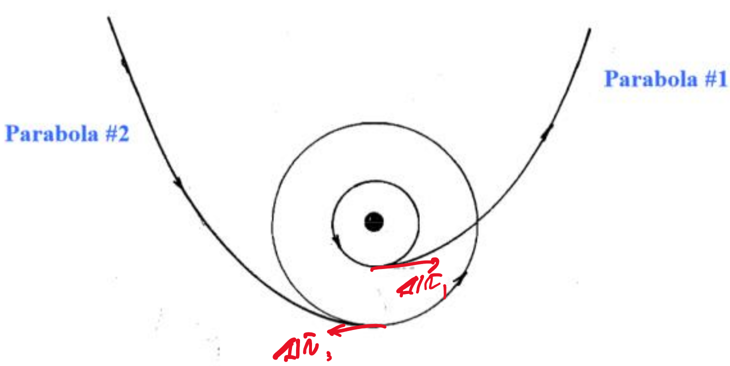
Bi-Parabolic Transfers

Move the intermediate radius out to infinity ($r \to \infty$)

→ Transfer paths become parabolic

2^{nd} impulse becomes infinitesimally small ($\Delta v_2 \approx 0$)

$\Delta\bar{v}_2 \approx 0$



No practical value because duration infinite

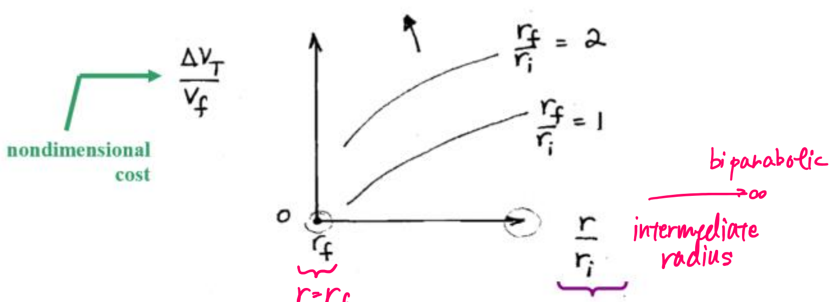
Gain achieved by use of bi-parabolic (in-plane) small (max ≈ 10%) so Hohmann preferred in practice

Return to Bi-elliptic

$$\Delta v_{Total} = |\Delta \bar{v}_1| + |\Delta \bar{v}_2| + |\Delta \bar{v}_3|$$

This total cost a fcn of intermediate radius

To clarify the relationship between Δv_{Total} and r, consider a plot for circle-to-circle bi-elliptic transfers



Next page: Find conditions for minimum cost

Check limits $r = r_f$ (two-impulse Hohmann)
$r \to \infty$ (bi-parabolic)

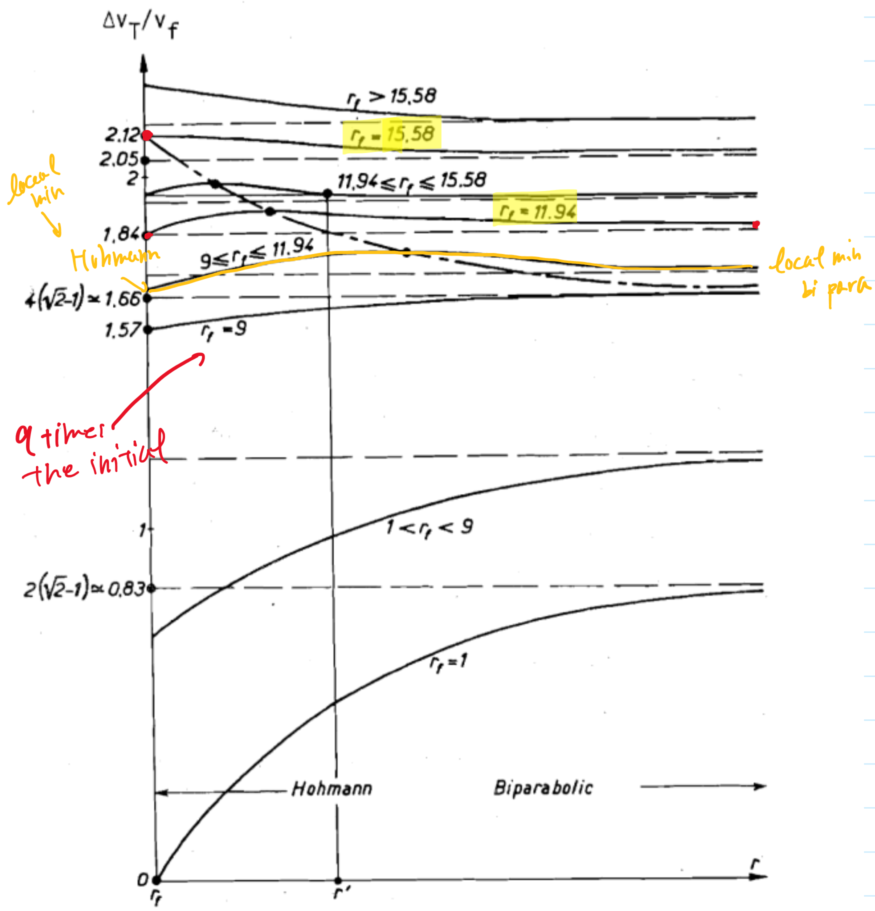
(a) $1 \le r_f \le 9$ ⟹ ΔV_{total} increases; absolute min Hohmann

(b) $9 \le r_f \le 15.58$ ⟹ ΔV_{total} local mins for Hohmann & bi-parabolic

(i) $9 \le r_f \le 11.94$ absolute min Hohmann

(ii) $11.94 \le r_f \le 15.58$ absolute min bi-parabolic

(c) $r_f \ge 15.58$ ⟹ Hohmann not even local min
all bi-elliptic transfers require less ΔV than Hohmann


$\Delta v_T / v_f$
$r_f > 15.58$
2.12
$r_f = 15.58$
2.05
2
$11.94 \leq r_f \leq 15.58$
$r_f = 11.94$
1.84
local min
Hohmann
$9 \leq r_f \leq 11.94$
local min
bi para
$4(\sqrt{2}-1) \simeq 1.66$
1.57
$r_f = 9$
9 times the initial
1
$1 < r_f < 9$
$2(\sqrt{2}-1) \simeq 0.83$
$r_f = 1$
Hohmann
Biparabolic
0
r_f
r'
r