A picture containing fireworks, dark, water, flying

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College of Engineering

School of Aeronautics and Astronautics

AAE 36401 Lab

Control Systems Lab

Lab 5 Report

The Control of Inverted Pendulum to Balance with Sine Wave Input

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## Introduction

### Objective

The object of this lab is to control an inverted pendulum on a cart which is place on a track. Alike the previous lab, the pendulum will be controlled in its upright orientation; however, in this experiment the pendulum will maintain the orientation while having the cart move sideways based on a sine wave input. This is almost identical to the last experiment where the input was a square wave. The objective is the input gains into the feedback system to maintain a stable upright orientation with a more complicated sine wave input.

### Method

To accomplish the objective, we have to come up with the gains of the feedback control system. For this experiment, the gains will be computed by the LQR method and the pole placement method. Then the gains will be input to the Simulink model that is synced to the experimental setup. The cart will move based on the square wave input and the data is obtain from the response.

## Results

### Part (i)

The first gains used were the gains obtained from the prelab

The gains using LQR:

Table : gains of LQR from prelab

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  |  |  |
| -49.9747 | 156.3945 | -45.5208 | 36.5653 | 31.6228 |

The diagonal matrix, Q used for the linear quadratic regulator is the following

The weight, R is

The poles for LQR:

Table : poles of LQR from prelab

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  |  |  |
| -64.3972 | -2.0915 + 1.3698i | -2.0915 - 1.3698i | -1.2125 + 1.1168i | -1.2125 - 1.1168i |

The gains using Pole Placement:

Table : gains of pole placement from prelab

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  |  |  |
| -76.5544 | 91.0760 | -37.9289 | 18.5217 | 94.4953 |

The corresponding poles were

Table : poles of pole placement from prelab

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  |  |  |
| -5+3.5i | -5-3.5i | -3+4.5i | -3-4.5i | -3 |

The gains were tuned because of excessive oscillation. The tuned gains are tabulated below.

The tuned gains for LQR:

Table : the tuned gains of LQR

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  |  |  |
| -57.6834 | 136.7788 | -47.5748 | 27.8695 | 31.6228 |

The diagonal matrix, Q used for the linear quadratic regulator is the following

The weight, R is

The poles for this was,

Table : the tuned gains of LQR

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  |  |  |
| -24.3929 | -6.9848 | -3.0906 | -1.3639+0.4658i | -1.3639-0.4658i |

The tuned gains for Pole Placement:

Table : the tuned gains of pole placement

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  |  |  |
| -88.2269 | 114.1343 | -46.3868 | 23.6105 | 88.4623 |

The corresponding poles were

Table : the tuned poles of pole placement

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  |  |  |
| -5+3i | -5-3i | -5 | -6 | -3 |

## Analysis & Discussions

### Part (i)

The time histories for LQR and Pole Placement are graphed as the following.

Chart, line chart

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Figure : time history for tuned gains of LQR system

Chart, line chart

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Figure : time history for tuned gains of pole placement system

Because the gains from the prelab had 2 complex pole pairs the response resulted in a highly shaky response. That is because when there are 2 complex pole pairs there is going to oscillation in the response. By fixing this, we were able to obtain a response that is not oscillating and follows the sine wave fairy well. As we can observe from the angle plot, the angle after the pendulum is lifted up slowly to an upright orientation is able to maintain the angle with a very small error with minimal perturbations.

However, looking closely we can see that when the amplitude is at the point of absolute magnitude the cart tends to shake to stabilize itself. We can observe more of this shake in the pole placement method than the LQR method.

## Conclusion & Recommendation

### Main Points

From the results, we can observe that the experiment is successful with a system that maintains a pendulum at an upright position while the cart has a dynamic motion. The angle of the pendulum is maintaining 0 degrees with a significantly small error while the cart follows a somewhat complicated sine wave input.

### Theoretical/Experimental Limitations

The motion of the cart is one limitation that we can see from the plots. The motion that causes a subtle shake of the cart at turning points.

Another limitation common to all the other labs performed is the voltage limit that prevents us to have a gain larger than 200.

### Lessons Learned & Suggestions for Improvement

The utmost lesson from this lab is the implementation of linear quadratic regulators to control complex tasks. It is rather better to use LQR for a more stable response compared to the pole placement method. We can consider evaluating the poles by conducting a Root Locus or Nyquist Analysis on the poles of the control system.

## Appendix

### Experiment Setup

A desktop computer sitting on top of a desk

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A picture containing indoor, cabinet, table, desk

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### Notations for Variables

|  |  |  |  |
| --- | --- | --- | --- |
| **Symbol** | **Description** | **Value** | **Unit** |
| *Rm* | motor armature resistance | 2.6 | Ω |
| *Lm* | motor armature inductance | 0.18 | *mH* |
| *Kt* | motor torque constant | 0.00767 | *N.m/A* |
| *ηm* | motor eﬃciency | 100% | % |
| *Km* | back-electromotive-force(EMF) constant | 0.00767 | *V.s/rad* |
| *Jm* | rotor moment of inertia | 3*.*9 *×* 10*−*7 | *kg.m*2 |
| *Kg* | planetary gearbox ratio | 3*.*71 |  |
| *ηg* | planetary gearbox eﬃciency | 100% | % |
| *Mc*2 | cart mass | 0*.*57 | *kg* |
| *Mw* | cart weight mass | 0*.*37 | *kg* |
| *Mc* | total cart weight mass including motor inertia | 1*.*0731 | *kg* |
| *Beq* | viscous damping at motor pinion | 5*.*4000 | *N.s/m* |
| *Lt* | track length | 0*.*990 | *m* |
| *Tc* | cart travel | 0*.*814 | *m* |
| *Pr* | rack pitch | 1*.*664 *×* 10*−*3 | *m/tooth* |
| *rmp* | motor pinion radius | 6*.*35 *×* 10*−*3 | *m* |
| *Nmp* | motor pinion number of teeth | 24 |  |
| *rpp* | position pinion radius | 0*.*01482975 | *m* |
| *Npp* | position pinion number of teeth | 56 |  |
| *KEP* | cart encoder resolution | 2*.*275 *×* 10*−*5 | *m/count* |
| *Mp* | long pendulum mass with T-fitting | 0*.*230 | *kg* |
| *Mpm* | medium pendulum mass with T-fitting | 0*.*127 | *kg* |
| *Lp* | long pendulum length from pivot to tip | 0*.*6413 | *m* |
| *Lpm* | medium pendulum length from pivot to tip | 0*.*3365 | *m* |
| *lp* | long pendulum length: pivot to center of mass | 0*.*3302 | *m* |
| *lpm* | medium pendulum length: pivot to center of mass | 0*.*1778 | *m* |
| *Jp* | long pendulum moment of inertia ⟳ center of mass | 7*.*88 *×* 10*−*3 | *kg.m*2 |
| *Jpm* | medium pendulum moment of inertia ⟳ center of mass | 1*.*20 *×* 10*−*3 | *kg.m*2 |
| *Bp* | viscous damping at pendulum axis | 0*.*0024 | *N.m.s/rad* |
| *g* | gravitational constant | 9*.*81 | *m/s*2 |
| *v* | voltage of servo motor | variable | *V* |

### MATLAB Code

% AAE 364L Lab5 MATLAB Code

% TOMOKI KOIKE

clear all; close all; clc;

set(groot, 'defaulttextinterpreter','latex');

set(groot, 'defaultAxesTickLabelInterpreter','latex');

set(groot, 'defaultLegendInterpreter','latex');

lqr\_xc\_data = load("koike\_lab5\_lqr\_xc.mat");

lqr\_ang\_data = load("koike\_lab5\_lqr\_ang.mat");

pp\_xc\_data = load("koike\_lab5\_pp\_xc.mat");

pp\_ang\_data = load("koike\_lab5\_pp\_ang.mat");

% LQR

t = lqr\_xc\_data.x\_and\_command\_part3.time;

xc = lqr\_xc\_data.x\_and\_command\_part3.signals.values(:,1);

command = lqr\_xc\_data.x\_and\_command\_part3.signals.values(:,2);

theta = lqr\_ang\_data.theta\_part3.signals.values;

% Plotting

fig = figure('Renderer',"painters", 'Position', [10 10 900 800]);

subplot(2,1,1)

plot(t,xc)

grid on; grid minor; box on; hold on;

plot(t, command, '--')

hold off

ylabel('$x\_c$ [m]')

subplot(2,1,2)

plot(t,theta, '-g')

grid on; grid minor; box on;

ylabel('$\alpha$ [rad]')

xlabel('t, [sec]')

sgtitle('Time Histories for $x\_c$ and $\alpha$ for Sine Wave Response with LQR - T. Koike')

saveas(fig, 'response\_lqr.png')

% PP

t = pp\_xc\_data.x\_and\_command\_part3.time;

xc = pp\_xc\_data.x\_and\_command\_part3.signals.values(:,1);

command = pp\_xc\_data.x\_and\_command\_part3.signals.values(:,2);

theta = pp\_ang\_data.theta\_part3.signals.values;

% Plotting

fig = figure('Renderer',"painters", 'Position', [10 10 900 800]);

subplot(2,1,1)

plot(t,xc)

grid on; grid minor; box on; hold on;

plot(t, command, '--')

hold off

ylabel('$x\_c$ [m]')

subplot(2,1,2)

plot(t,theta, '-g')

grid on; grid minor; box on;

ylabel('$\alpha$ [rad]')

xlabel('t, [sec]')

sgtitle('Time Histories for $x\_c$ and $\alpha$ for Sine Wave Response with Pole Placement - T. Koike')

saveas(fig, 'response\_pp.png')

### Simulink Models











