A picture containing fireworks, dark, water, flying

Description automatically generated

College of Engineering

School of Aeronautics and Astronautics

AAE 421

Flight Dynamics and Controls

HW 5

Modern Control Theory

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**Problem 1 (20pts)**

For the differential equations that follow, rewrite the equations in the state-space formulation.

(1)

Let,

Then,

Thus, the state space representation becomes

(2)

Reorganize the equation as

Plug of the second equation into the first equation and we obtain

Let,

Then,

Thus, the state space representation becomes

**Problem 2 (20 pts)**

The transfer functions for a feedback control system follow. Determine the states space equations for the closed-loop system.

(1)

The transfer function of the closed loop feedback system is

Thus, the state space realization becomes

Thus, the state space equations are

(2)

The transfer function of the closed loop feedback system is

Thus, the state space realization becomes

Thus, the state space equations are

**Problem 3 (20 pts)**

The state space equations are given as follows:

1. Determine if the system is controllable.

The *A* and *B* matrices of this system are

Then the controllability matrix becomes

Since,

This system is uncontrollable.

1. Determine if the system is observable.

The *A* and *C* matrices of this system are

Then the observability matrix becomes

Since

Thus, the system is observable.

**Problem 4 (20 pts)**

The longitudinal equations for an airplane having poor handling qualities follow. Use state feedback to provide stability augmentation so that the augmented aircraft has the following short- and long-period (phugoid) characteristics:

Use the Ackermann’s formula to design the feedback gain to locate the closed-loop eigenvalues.

We know that …

The longitudinal dynamics of a plane are described by

The short period mode has the approximate 2-D dynamics described by:

and the phugoid mode has the approximate 2-D dynamics described by:

Short Period Mode:

The desired poles are

Now we use a MATLAB function that conducts Ackermann’s formula step-by-step, which is the one as follows.

function res = ackermannMethod(A, B, dp, tol)

% Function that computes the eigVal placement with Ackermann's method

% A: system A matrix

% B: system B matrix

% dp: array of desired poles

% tol: tolerance

%Checking for user inputed tolerance

if nargin == 3

%using default value

tol = 2;

elseif nargin > 4

error('Too many inputs.')

elseif nargin < 3

error('Too few inputs.')

end

sz = size(A); n = sz(1);

% Step-1

ad\_s = poly(dp);

% Step-2

ad\_A = 0; idx = 1;

for i = n:-1:0

ad\_A = ad\_A + A^i \* ad\_s(idx);

idx = idx + 1;

end

% Step-3

Qc = ctrb(A,B);

% Step-4

e = zeros(1, n); e(end) = 1;

K = e\*inv(Qc)\*ad\_A;

% Step-5

Ad = A - B \* K;

for p = eig(Ad)'

ct = 0;

for pp = dp

if round(p,tol) == round(pp,tol)

ct = ct + 1;

end

end

if ct == 0

error('The gains do not produce the desired poles.');

end

end

% Results

res.check = 1;

res.K = K;

res.Ad = Ad;

res.Qc = Qc;

res.DA = ad\_A;

res.Ds = ad\_s;

end

From the desired poles we know that

Then plug in the *A* matrix

The controllability matrix becomes

where

Then,

REMINDER: I use the convention .

We verify the results by using

and the eigenvalues of this new matrix is the same as the desired poles.

Phugoid Mode:

The desired poles are

From the desired poles we know that

Then plug in the *A* matrix

The controllability matrix becomes

where

Then,

REMINDER: I use the convention .

We verify the results by using

and the eigenvalues of this new matrix is the same as the desired poles.

**Problem 5 (20 pts)**

The rolling motion of an aerospace vehicle is given by these state equations:

Where and are the aileron deflection angle, roll rate, and voltage input to the aileron actuator motor. Note that in this problem the aileron angle is considered a state and the control voltage, is the input. Determine the optimal control law that minimizes the performance index, J, as follows:

Where

For the problem, assuming the following:

For this problem we will use MATLAB. First we setup the provided parameters and matrices.

% Given parameters

tau = 0.1;

L\_da = 30;

L\_p = -1.0;

d\_amax = 0.436;

phi\_max = 0.787;

d\_vmax = 10;

% System A and B matrices

A = [-1/tau, 0, 0; L\_da, L\_p, 0; 0, 1, 0];

B = [1/tau; 0; 0];

% Weighting matrices Q and R

Q = diag([1/d\_amax, 0, 1/phi\_max^2]);

R = [1/d\_vmax^2];

Then we run “lqr()” to get the LQR gains

% Obtain the LQR gains

K = lqr(A, B, Q, R);

Then the optimal control law of this system is to use a state variable feedback with the feedbacks generated with the LQR method

where