A picture containing fireworks, dark, water, flying

Description automatically generated

College of Engineering

School of Aeronautics and Astronautics

AAE 564

System Analysis and Synthesis

Homework 10

Controllability of Control Systems

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**Exercise 1**

Determine (by hand) whether or not each of the following systems are controllable.

(a)

(b)

(c)

(a)

The *A* matrix and *B* matrix are

Then the controllability matrix becomes

The reduced echelon form of this matrix is

Thus, this system is controllable.

(b)

The *A* matrix and *B* matrix are

Then the controllability matrix becomes

The reduced echelon form of this matrix is

Thus, this system is uncontrollable.

(c)

The *A* matrix and *B* matrix are

Then the controllability matrix becomes

The reduced echelon form of this matrix is

Thus, this system is uncontrollable.

MATLAB Verification

function res = checkControllability(A, B)

dim = size(A); n = dim(1);

Qc = ctrb(A, B);

res.check = rank(Qc) == n;

res.Qc = Qc;

end

% Ex1

% (a)

A = [-1, 0; 0, 1];

B = [1; 1];

res = checkControllability(A, B);

res.check

res.Qc

% (b)

A = [-1, 0; 0, 1];

B = [0; 1];

res = checkControllability(A, B);

res.check

res.Qc

% (c)

A = [1, 0; 1, 0];

B = [1; 1];

res = checkControllability(A, B);

res.check

res.Qc

**Exercise 2**

(By hand) Determine whether or not the following system is controllable.

If the system is uncontrollable, compute the uncontrollable eigenvalues.

The *A* and *B* matrices of this system are

Then,

Thus, the controllability matrix becomes

The reduced echelon form of this matrix is

This system is uncontrollable.

Then we conduct the PBH test to find the uncontrollable eigenvalues.

For ,

This eigenvalue is observable.

For ,

This eigenvalue of 4 is uncontrollable.

For ,

This eigenvalue is observable.

**Exercise 3**

Carry out the following for linearizations L1, L3, L7 of the two pendulum cart system.

1. Determine which linearizations are controllable?
2. Compute the singular values of the controllability matrix.
3. Determine the uncontrollable eigenvalues for the uncontrollable linearizations.

You may want to use MATLAB.

The system equation for the double pendulum cart system is

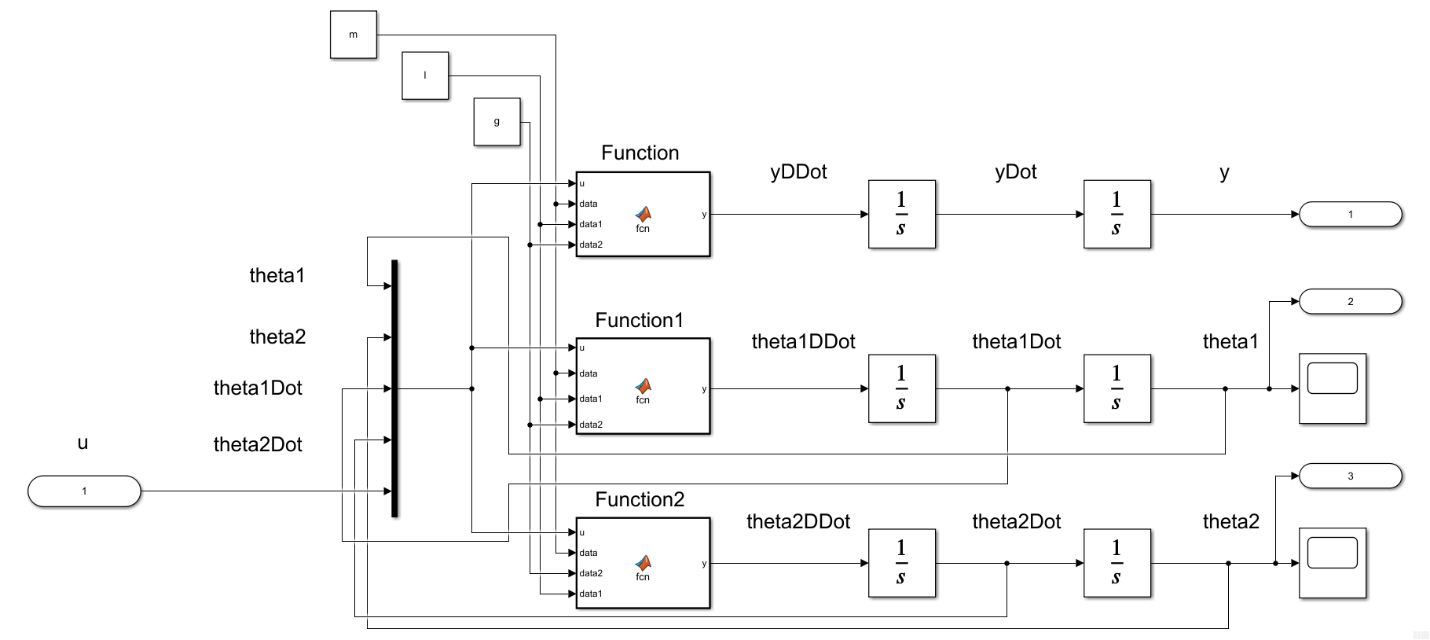
Have the system be a single output of the displacement y.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |
| P1 | 2 | 1 | 1 | 1 | 1 | 1 | 0 |
| P2 | 2 | 1 | 1 | 1 | 0.99 | 1 | 0 |
| P3 | 2 | 1 | 0.5 | 1 | 1 | 1 | 0 |
| P4 | 2 | 1 | 1 | 1 | 0.5 | 1 | 0 |

|  |  |  |
| --- | --- | --- |
| L1 | P1 | E1 |
| L2 | P1 | E2 |
| L3 | P2 | E1 |
| L4 | P2 | E2 |
| L5 | P3 | E1 |
| L6 | P3 | E2 |
| L7 | P4 | E1 |
| L8 | P4 | E2 |

(a)

The Simulink model used for this is shown below,



Embedded MATLAB Block – Function (code)

function y = fcn(u, data, data1, data2)

%{

EMBEDDED MATLAB BLOCK FUNCTION

%}

m0 = data(1); m1 = data(2); m2 = data(3); l1 = data1(1); l2 = data1(2);

g = data2;

num = -m1\*l1\*sin(u(1))\*u(3)\*u(3) - m2\*l2\*sin(u(2))\*u(4)\*u(4)...

- m1\*g\*sin(u(1))\*cos(u(1)) - m2\*g\*sin(u(2))\*cos(u(2))...

+ u(5);

den = m0 + m1 + m2 - m1\*cos(u(1))^2 - m2\*cos(u(2))^2;

y = num / den;

end

Embedded MATLAB Block – Function1 (code)

function y = fcn(u, data, data1, data2)

%{

EMBEDDED MATLAB BLOCK FUNCTION1

%}

m0 = data(1); m1 = data(2); m2 = data(3); l1 = data1(1); l2 = data1(2);

g = data2;

num = -(m1\*l1\*cos(u(1))\*sin(u(1))\*u(3)\*u(3) + m2\*l2\*cos(u(1))\*sin(u(2))\*u(4)\*u(4))...

+ m2\*g\*(sin(u(1))\*cos(u(2))^2 - cos(u(1))\*sin(u(2))\*cos(u(2)))...

- (m0 + m1 + m2)\*g\*sin(u(1)) + u(5)\*cos(u(1));

den = l1\*(m0 + m1 + m2 - m1\*cos(u(1))^2 - m2\*cos(u(2))^2);

y = num / den;

end

Embedded MATLAB Block – Function2 (code)

function y = fcn(u, data, data2, data1)

%{

EMBEDDED MATLAB BLOCK FUNCTION2

%}

m0 = data(1); m1 = data(2); m2 = data(3); l1 = data1(1); l2 = data1(2);

g = data2;

num = -(m1\*l1\*cos(u(2,1))\*sin(u(1))\*u(3)\*u(3) + m2\*l2\*cos(u(2))\*sin(u(2))\*u(4)\*u(4))...

+ m1\*g\*(sin(u(2))\*cos(u(1))^2 - cos(u(2))\*sin(u(1))\*cos(u(1)))...

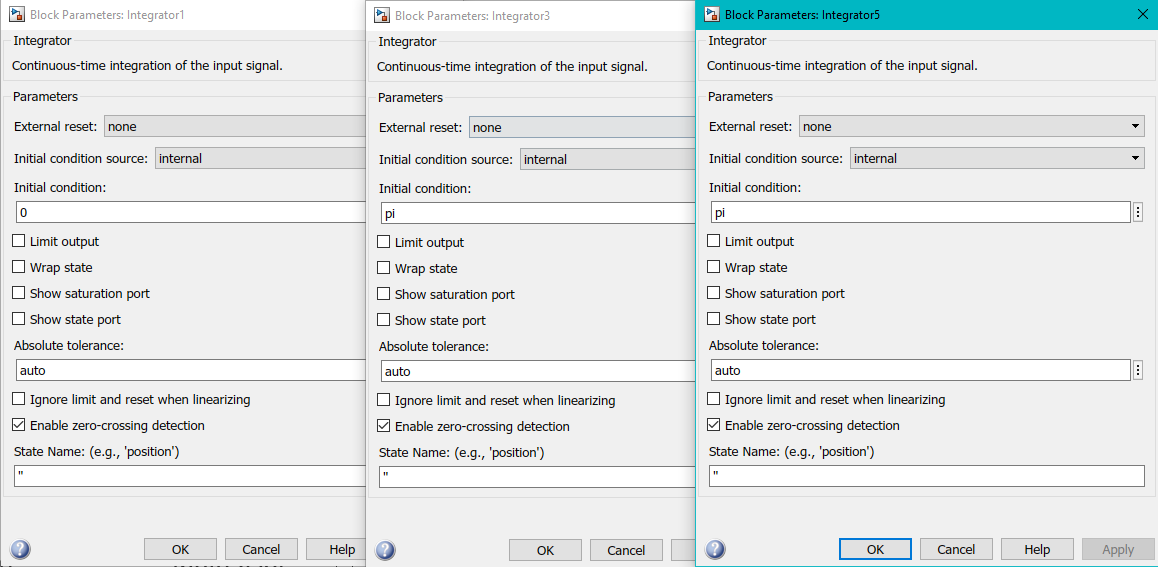
- (m0 + m1 + m2)\*g\*sin(u(2)) + u(5)\*cos(u(2));

den = l2\*(m0 + m1 + m2 - m1\*cos(u(1))^2 - m2\*cos(u(2))^2);

y = num / den;

end

For the conditions E1 and E2, we set the initial conditions of the integrator block of y, , and correspondingly to ; like in the following windows,



L1:

|  |  |
| --- | --- |
| A = 6×6  0 0 0 1.0000 0 0  0 0 0 0 1.0000 0  0 0 0 0 0 1.0000  0 -0.5000 -0.5000 0 0 0  0 -1.5000 -0.5000 0 0 0  0 -0.5000 -1.5000 0 0 0 | B = 6×1  0  0  0  0.5000  0.5000  0.5000 |
| C = 1×6  1 0 0 0 0 0 | **D = 0** |

The controllability matrix for this system is

|  |
| --- |
| Qc\_L1 = 6×6  0 0.5000 0 -0.5000 0 1.0000  0 0.5000 0 -1.0000 0 2.0000  0 0.5000 0 -1.0000 0 2.0000  0.5000 0 -0.5000 0 1.0000 0  0.5000 0 -1.0000 0 2.0000 0  0.5000 0 -1.0000 0 2.0000 0 |

The reduced echelon form of this matrix is

|  |
| --- |
| Qc\_L1\_rref = 6×6  1.0000 0 0 0 0 0  0 1.0000 0 0 0 0  0 0 1.0000 0 -2.0000 0  0 0 0 1.0000 0 -2.0000  0 0 0 0 0 0  0 0 0 0 0 0 |

Thus,

This system linearized by L1 is uncontrollable.

L3:

|  |  |
| --- | --- |
| A = 6×6  0 0 0 1.0000 0 0  0 0 0 0 1.0000 0  0 0 0 0 0 1.0000  0 -0.5000 -0.5000 0 0 0  0 -1.5000 -0.5000 0 0 0  0 -0.5051 -1.5152 0 0 0 | B = 6×1  0  0  0  0.5000  0.5000  0.5051 |
| C = 1×6  1 0 0 0 0 0 | **D = 0** |

The controllability matrix for this system is

|  |
| --- |
| Qc\_L3 = 6×6  0 0.5000 0 -0.5025 0 1.0101  0 0.5000 0 -1.0025 0 2.0127  0 0.5051 0 -1.0178 0 2.0484  0.5000 0 -0.5025 0 1.0101 0  0.5000 0 -1.0025 0 2.0127 0  0.5051 0 -1.0178 0 2.0484 0 |

The reduced echelon form of this matrix is

|  |
| --- |
| Qc\_L3\_rref = 6×6  1 0 0 0 0 0  0 1 0 0 0 0  0 0 1 0 0 0  0 0 0 1 0 0  0 0 0 0 1 0  0 0 0 0 0 1 |

Thus,

This system linearized by L3 is controllable.

L7:

|  |  |
| --- | --- |
| A = 6×6  0 0 0 1.0000 0 0  0 0 0 0 1.0000 0  0 0 0 0 0 1.0000  0 -0.5000 -0.5000 0 0 0  0 -1.5000 -0.5000 0 0 0  0 -1.0000 -3.0000 0 0 0 | B = 6×1  0  0  0  0.5000  0.5000  1.0000 |
| C = 1×6  1 0 0 0 0 0 | **D = 0** |

The controllability matrix for this system is

|  |
| --- |
| Qc\_L7 = 6×6  0 0.5000 0 -0.7500 0 2.3750  0 0.5000 0 -1.2500 0 3.6250  0 1.0000 0 -3.5000 0 11.7500  0.5000 0 -0.7500 0 2.3750 0  0.5000 0 -1.2500 0 3.6250 0  1.0000 0 -3.5000 0 11.7500 0 |

The reduced echelon form of this matrix is

|  |
| --- |
| Qc\_L7\_rref = 6×6  1 0 0 0 0 0  0 1 0 0 0 0  0 0 1 0 0 0  0 0 0 1 0 0  0 0 0 0 1 0  0 0 0 0 0 1 |

Thus,

This system linearized by L7 is controllable.

% (a)

global m l g ye theta1e theta2e

param\_combo = ["L1","L3","L7"];

for i = 1:numel(param\_combo)

define\_params(param\_combo(i));

[A, B, C, D] = linmod('db\_pend\_cart\_lin');

lin\_sys(i).Amat = A;

lin\_sys(i).Bmat = B;

lin\_sys(i).Cmat = C;

lin\_sys(i).Dmat = D;

sys\_ss = ss(A, B, C, D); % get the state space system

CTR(i) = checkControllability(A, B); % check the observability of the system

eigCTR{i} = find\_unctrb\_eigVal(A, B); % check the observability of the eigenvalues

end

Qc\_L1 = CTR(1).Qc

Qc\_L1\_rref = rref(Qc\_L1)

Qc\_L3 = CTR(2).Qc

Qc\_L3\_rref = rref(Qc\_L3)

Qc\_L7 = CTR(3).Qc

Qc\_L7\_rref = rref(Qc\_L7)

function define\_params(L)

% Function to define parameters

global m l g ye theta1e theta2e

if L == "L1"

m = [2,1,1]; l = [1,1]; g = 1; % P1

ye = 0; theta1e = 0; theta2e = 0; % E1

elseif L == "L2"

m = [2,1,1]; l = [1,1]; g = 1; % P1

ye = 0; theta1e = pi; theta2e = pi; % E2

elseif L == "L3"

m = [2,1,1]; l = [1,0.99]; g = 1; % P2

ye = 0; theta1e = 0; theta2e = 0; % E1

elseif L == "L4"

m = [2,1,1]; l = [1,0.99]; g = 1; % P2

ye = 0; theta1e = pi; theta2e = pi; % E2

elseif L == "L5"

m = [2,1,0.5]; l = [1,1]; g = 1; % P3

ye = 0; theta1e = 0; theta2e = 0; % E1

elseif L == "L6"

m = [2,1,0.5]; l = [1,1]; g = 1; % P3

ye = 0; theta1e = pi; theta2e = pi; % E2

elseif L == "L7"

m = [2,1,1]; l = [1,0.5]; g = 1; % P4

ye = 0; theta1e = 0; theta2e = 0; % E1

elseif L == "L8"

m = [2,1,1]; l = [1,0.5]; g = 1; % P4

ye = 0; theta1e = pi; theta2e = pi; % E2

else

print('error: did not match any')

end

end

(b)

For this problem, we conduct single value decomposition using MATLAB.

% (b)

[U\_L1, S\_L1, V\_L1] = svd(Qc\_L1);

[U\_L3, S\_L3, V\_L3] = svd(Qc\_L3);

[U\_L7, S\_L7, V\_L7] = svd(Qc\_L7);

S\_L1

S\_L3

S\_L7

L1:

|  |
| --- |
| S\_L1 = 6×6  3.4565 0 0 0 0 0  0 3.4565 0 0 0 0  0 0 0.2287 0 0 0  0 0 0 0.2287 0 0  0 0 0 0 0.0000 0  0 0 0 0 0 0.0000 |

L3:

|  |
| --- |
| S\_L3 = 6×6  3.5019 0 0 0 0 0  0 3.5019 0 0 0 0  0 0 0.2290 0 0 0  0 0 0 0.2290 0 0  0 0 0 0 0.0016 0  0 0 0 0 0 0.0016 |

L7:

|  |
| --- |
| S\_L7 = 6×6  13.1371 0 0 0 0 0  0 13.1371 0 0 0 0  0 0 0.3513 0 0 0  0 0 0 0.3513 0 0  0 0 0 0 0.1083 0  0 0 0 0 0 0.1083 |

(c)

The uncontrollable system is only L1.

The eigenvalues for L1 are

|  |
| --- |
| eigVal = 6×1 complex  0.0000 + 0.0000i  0.0000 + 0.0000i  0.0000 + 1.4142i  0.0000 - 1.4142i  -0.0000 + 1.0000i  -0.0000 - 1.0000i |

For ,

|  |
| --- |
| Z = 6×7  0 0 0 1.0000 0 0 0  0 0 0 0 1.0000 0 0  0 0 0 0 0 1.0000 0  0 -0.5000 -0.5000 0 0 0 0.5000  0 -1.5000 -0.5000 0 0 0 0.5000  0 -0.5000 -1.5000 0 0 0 0.5000 |

The reduced echelon form of *Z* is

|  |
| --- |
| Zrref = 6×7  0 1 0 0 0 0 0  0 0 1 0 0 0 0  0 0 0 1 0 0 0  0 0 0 0 1 0 0  0 0 0 0 0 1 0  0 0 0 0 0 0 1 |

The eigenvalue 0 is controllable.

For ,

|  |
| --- |
| Z = 6×7 complex  -0.0000 - 1.4142i 0.0000 + 0.0000i 0.0000 + 0.0000i 1.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i  0.0000 + 0.0000i -0.0000 - 1.4142i 0.0000 + 0.0000i 0.0000 + 0.0000i 1.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i  0.0000 + 0.0000i 0.0000 + 0.0000i -0.0000 - 1.4142i 0.0000 + 0.0000i 0.0000 + 0.0000i 1.0000 + 0.0000i 0.0000 + 0.0000i  0.0000 + 0.0000i -0.5000 + 0.0000i -0.5000 + 0.0000i -0.0000 - 1.4142i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.5000 + 0.0000i  0.0000 + 0.0000i -1.5000 + 0.0000i -0.5000 + 0.0000i 0.0000 + 0.0000i -0.0000 - 1.4142i 0.0000 + 0.0000i 0.5000 + 0.0000i  0.0000 + 0.0000i -0.5000 + 0.0000i -1.5000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i -0.0000 - 1.4142i 0.5000 + 0.0000i |

The reduced echelon form of *Z* is

|  |
| --- |
| Zrref = 6×7 complex  1.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.3536i 0.0000 + 0.0000i  0.0000 + 0.0000i 1.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.7071i 0.0000 + 0.0000i  0.0000 + 0.0000i 0.0000 + 0.0000i 1.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i -0.0000 + 0.7071i 0.0000 + 0.0000i  0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 1.0000 + 0.0000i 0.0000 + 0.0000i -0.5000 + 0.0000i 0.0000 + 0.0000i  0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 1.0000 + 0.0000i -1.0000 + 0.0000i 0.0000 + 0.0000i  0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 1.0000 + 0.0000i |

The eigenvalue 1.4142j is controllable.

For ,

|  |
| --- |
| Z = 6×7 complex  -0.0000 + 1.4142i 0.0000 + 0.0000i 0.0000 + 0.0000i 1.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i  0.0000 + 0.0000i -0.0000 + 1.4142i 0.0000 + 0.0000i 0.0000 + 0.0000i 1.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i  0.0000 + 0.0000i 0.0000 + 0.0000i -0.0000 + 1.4142i 0.0000 + 0.0000i 0.0000 + 0.0000i 1.0000 + 0.0000i 0.0000 + 0.0000i  0.0000 + 0.0000i -0.5000 + 0.0000i -0.5000 + 0.0000i -0.0000 + 1.4142i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.5000 + 0.0000i  0.0000 + 0.0000i -1.5000 + 0.0000i -0.5000 + 0.0000i 0.0000 + 0.0000i -0.0000 + 1.4142i 0.0000 + 0.0000i 0.5000 + 0.0000i  0.0000 + 0.0000i -0.5000 + 0.0000i -1.5000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i -0.0000 + 1.4142i 0.5000 + 0.0000i |

The reduced echelon form of *Z* is

|  |
| --- |
| Zrref = 6×7 complex  1.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 - 0.3536i 0.0000 + 0.0000i  0.0000 + 0.0000i 1.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 - 0.7071i 0.0000 + 0.0000i  0.0000 + 0.0000i 0.0000 + 0.0000i 1.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i -0.0000 - 0.7071i 0.0000 + 0.0000i  0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 1.0000 + 0.0000i 0.0000 + 0.0000i -0.5000 - 0.0000i 0.0000 + 0.0000i  0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 1.0000 + 0.0000i -1.0000 - 0.0000i 0.0000 + 0.0000i  0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 1.0000 + 0.0000i |

The eigenvalue -1.4142j is controllable.

For ,

|  |
| --- |
| Z = 6×7 complex  0.0000 - 1.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 1.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i  0.0000 + 0.0000i 0.0000 - 1.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 1.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i  0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 - 1.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 1.0000 + 0.0000i 0.0000 + 0.0000i  0.0000 + 0.0000i -0.5000 + 0.0000i -0.5000 + 0.0000i 0.0000 - 1.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.5000 + 0.0000i  0.0000 + 0.0000i -1.5000 + 0.0000i -0.5000 + 0.0000i 0.0000 + 0.0000i 0.0000 - 1.0000i 0.0000 + 0.0000i 0.5000 + 0.0000i  0.0000 + 0.0000i -0.5000 + 0.0000i -1.5000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 - 1.0000i 0.5000 + 0.0000i |

The reduced echelon form of *Z* is

|  |
| --- |
| Zrref = 6×7 complex  1.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 - 0.0000i  0.0000 + 0.0000i 1.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 - 1.0000i -1.0000 - 0.0000i  0.0000 + 0.0000i 0.0000 + 0.0000i 1.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i -0.0000 + 1.0000i -0.0000 + 0.0000i  0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 1.0000 + 0.0000i 0.0000 + 0.0000i -0.0000 + 0.0000i 0.0000 + 0.0000i  0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 1.0000 + 0.0000i 1.0000 + 0.0000i 0.0000 - 1.0000i  0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i |

The eigenvalue j is uncontrollable.

For ,

|  |
| --- |
| Z = 6×7 complex  0.0000 + 1.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 1.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i  0.0000 + 0.0000i 0.0000 + 1.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 1.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i  0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 1.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 1.0000 + 0.0000i 0.0000 + 0.0000i  0.0000 + 0.0000i -0.5000 + 0.0000i -0.5000 + 0.0000i 0.0000 + 1.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.5000 + 0.0000i  0.0000 + 0.0000i -1.5000 + 0.0000i -0.5000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 1.0000i 0.0000 + 0.0000i 0.5000 + 0.0000i  0.0000 + 0.0000i -0.5000 + 0.0000i -1.5000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 1.0000i 0.5000 + 0.0000i |

The reduced echelon form of *Z* is

|  |
| --- |
| Zrref = 6×7 complex  1.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 - 0.0000i 0.0000 + 0.0000i  0.0000 + 0.0000i 1.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 1.0000i -1.0000 + 0.0000i  0.0000 + 0.0000i 0.0000 + 0.0000i 1.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i -0.0000 - 1.0000i -0.0000 - 0.0000i  0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 1.0000 + 0.0000i 0.0000 + 0.0000i -0.0000 - 0.0000i 0.0000 - 0.0000i  0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 1.0000 + 0.0000i 1.0000 - 0.0000i 0.0000 + 1.0000i  0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i |

The eigenvalue -j is uncontrollable.

**Exercise 4**

(BB in laundromat: external excitation) Obtain a state representation of the following system.

Determine whether or not your state space representation system is controllable.

Manipulating the system, we obtain

Let , then the state representation of this system becomes

Thus, the *A* and *B* matrices become

Then the controllability matrix becomes

Thus,

The reduced echelon form of this matrix is

This system is controllable.

**Exercise 5**

(BB in Laundromat: self excited.) (By hand.) Obtain a state space representation of the following system.

1. Determine the uncontrollable eigenvalues. Consider .
2. Obtain a basis for its controllable subspace
3. Obtain a reduced order controllable system which has the same input-output behavior as the original system when initial conditions are zero.

(a)

Manipulating the system, we obtain

Let , then the state representation of this system becomes

Then the *A* and *B* matrices become

Then the controllability matrix become

Thus,

The reduced echelon form of this matrix is

This system in uncontrollable.

Now we find the uncontrollable eigenvalues.

The eigenvalues of this system are

When ,

The reduced echelon form is

The eigenvalue of is uncontrollable.

When ,

The reduced echelon form is

The eigenvalue of is uncontrollable.

When ,

The reduced echelon form is

The eigenvalue of is controllable.

When ,

The reduced echelon form is

The eigenvalue of is controllable.

(b)

From part (a) we know that the reduced echelon form of the controllability matrix is

Then the basis of the controllable subspace becomes the column space of

(c)

The reduced order is

From the basis

Then

Thus,

**Exercise 6**

(By hand.) Consider a system described by

where all quantities are scalar. Obtain conditions on the numbers and which are necessary and sufficient for the controllability of this system. (Hint: PBH time.)

The *A* matrix of this system is

The *C* matrix is

Then the PBH test for controllability is

The leading columns are surrounded by rounded rectangle, and these leading columns cannot be zero in order for the matrix to have a full rank. Also, the system must have an input that is non-zero. Thus, the condition for controllability becomes

and

**Exercise 7**

Consider the system described by

Find (by hand) a non-zero such for every input , every solution of this system satisfies

The *A* and *B* matrix of this system is as follows.

The controllability matrix of this system is

This system is uncontrollable.

The eigenvalues of this system is

Choosing we check if this eigenvalue is uncontrollable or not.

Thus, we verified that this eigenvalue is uncontrollable.

Since,

**Exercise 8**

Suppose that is an uncontrollable complex eigenvalue of the system

where and are real. Show that there are real vectors and such that for every initial condition and every ,

where .

Let . Then if the eigenvalue of is uncontrollable for the system we know that there exists a that satisfies

Where is a non-zero vector such that for every input , every solution of this system suffices the equation above.

This will then become

Since

Which can be separated into the real and imaginary part

Which proves to be