A picture containing fireworks, dark, water, flying

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College of Engineering

School of Aeronautics and Astronautics

AAE 564

System Analysis and Synthesis

Homework 11

Hermitian Matrices and Single Value Decomposition

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Exercise 1

Determine (by hand) whether each one of the following matrices is pd, psd, nd, nsd, or none of the above.

Check you answers using MATLAB command eig().

For the Hermitian matrix

the eigenvalues become

All of the eigenvalues are positive, so this matrix is positive definite.

For the Hermitian matrix

the eigenvalues become

All the eigenvalues are non-negative, so this matrix is positive semi-definite.

For the Hermitian matrix

The eigenvalues of this matrix are

Since there are positive and negative eigenvalues this matrix is indefinite.

The Hermitian matrix

has the eigenvalues

All the eigenvalues are negative values, so this matrix is negative definite.

MATLAB Verification

% (1)

P = [1, 1j, 0; -1j, 2, 1; 0, 1, 4];

[v, d] = eig(P)

% (2)

P = [1, 1j; -1j, 1];

[v, d] = eig(P)

% (3)

P = [0, 2; 2, 0];

[v, d] = eig(P)

% (4)

P = [-1, 1; 1, -2];

[v, d] = eig(P)

The outputs are

|  |  |
| --- | --- |
| (1) | (2) |
| d = 3×3  0.3004 0 0  0 2.2391 0  0 0 4.4605 | d = 2×2  0 0  0 2 |
| (3) | (4) |
| d = 2×2  -2 0  0 2 | d = 2×2  -2.6180 0  0 -0.3820 |

**Exercise 2**

Determine (by hand) the maximum singular value of the following matrices.

(1)

The eigenvalues of become

Then the of the singular value decomposition becomes

Hence, the maximum singular value is 5.

(2)

The eigenvalues of become

Then the of the singular value decomposition becomes

Hence, the maximum singular value is 5.

(3)

The eigenvalues of become

Then the of the singular value decomposition becomes

Hence, the maximum singular value is 1.

**Exercise 3**

Determine (by hand) the singular value decomposition of

Check your answer in MATLAB.

We use the equations

We then start with,

Then,

Next, we find the eigenvectors for *M*.

If

If

If

Then normalize

Then,

Thus,

MATLAB Verification

A = [3,0,1; 1,0,3];

[U, S, V] = svd(A)

|  |  |
| --- | --- |
| U = 2×2  -0.7071 0.7071  -0.7071 -0.7071 | S = 2×3  4.0000 0 0  0 2.0000 0 |
| V = 3×3  -0.7071 0.7071 0  0 0 -1.0000  -0.7071 -0.7071 0 |  |