A picture containing fireworks, dark, water, flying

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College of Engineering

School of Aeronautics and Astronautics

AAE 564

System Analysis and Synthesis

Homework 4

Linear Algebra

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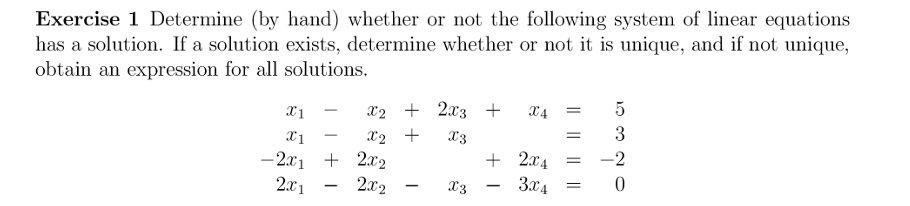
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September 25th, 2020 Friday

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Do some Gaussian Elimination

Swap rows:

~

Cancel leading column in row 2:

~

Cancel leading column in row 3:

~

Cancel leading column in row 4:

~

Cancel leading column in row 4:

~

Cancel :

~

Divide common factors for columns in :

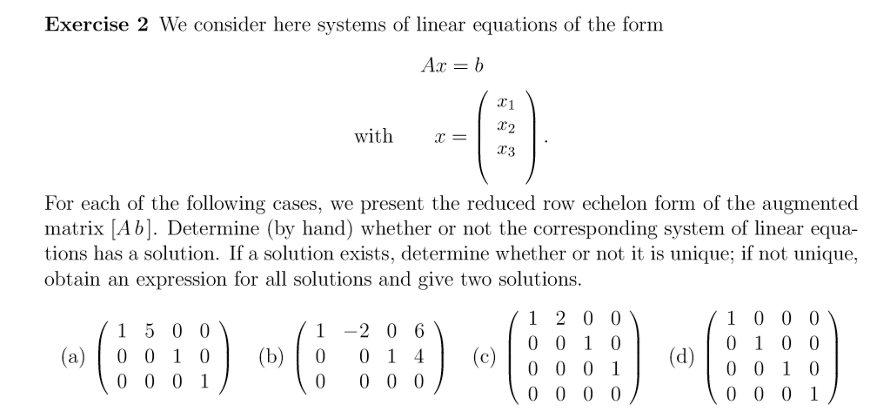
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For this reduced echelon form, we can tell that the linear system equations have an infinite number of solutions and not unique.

Hence, the expression of all solutions become the following

Now, let and

Thus,



1. This matrix does not have a solution because the last row has a leading column in the last column of the matrix and zero cannot be equal to one.
2. The second matrix has a solution but has an infinite number of solutions and not unique.

Let, . Then,

Here, and may be anything as long as they satisfy the first equation above.

1. For this matrix, there is no solution because the third row has a leading column at the last column and 0 = 1 is impossible.
2. The fourth matrix also has no solution since it the last row has a leading column at the last column. And 0 = 1 is impossible.

**Exercise 3** (By hand) Consider the three vectors

1. Express the vector

as a linear combination of , , and .

1. Can every vector of the form

be expressed as a linear combination of , , and ?

1. Are the vectors , , and linearly independent?
2. .

This is the same as

Thus, conduct Gaussian Elimination on the following augmented matrix

Swap rows:

~

Cancel leading column in row 2:

~

Cancel leading column in row 3:

Cancel row 3:

Divide row 1 with 3 and multiply row 2 by 3:

Cancel the second column of row 1:

This indicates that there is an infinite number of solutions for x, y, and z.

So, we will choose,

Hence,

1. .

To figure out what the span of these three vectors are we conduct a Gaussian Elimination on the following matrix

We have already done this in part (a). We simply end up with the same kind of matrix but disregarding the 4th column in our calculations of part (a). Thus, the after Gaussian Elimination, we end up with the following

We see that there are 2 non-zero rows in this final matrix (or 2 pivots). Thus, we can say that the three vectors , , and span a plane in . Thus, any arbitrary vector, in cannot be expressed in terms of , , and .

(c).

Take the determinant of the matrix

Also, from the calculations in part (b), we know that

Since the determinant is zero and one of the three vectors can be expressed by the other two vectors we know that the 3 vectors are NOT linearly independent.

**Exercise 4** (By hand)

1. Find a basis for the **null space** of the matrix,
2. What is the nullity of A?

Check your answers using MATLAB.

(a).

First, conduct Gaussian Elimination on the following matrix

Swap rows:

~

Cancel leading column in row 2:

~

Cancel leading column in row 3:

Cancel row 3:

Divide row 1 with 3 and multiply row 2 by 3:

Cancel the second column of row 1:

Consider the solutions for .

Since, the leading columns do not show up in the 3rd and 4th column we know that and are free variables. Thus, let and . Then can be expressed as

Thus, the basis for the null space of A is

(b).

The nullity, or in other words,

MATLAB verification

A = [1,2,3,4;2,3,4,5;3,4,5,6]

E\_A = rref(A)

N\_A = null(A,'r')

|  |
| --- |
| A = 3×4  1 2 3 4  2 3 4 5  3 4 5 6 |
| E\_A = 3×4  1 0 -1 -2  0 1 2 3  0 0 0 0 |
| N\_A = 4×2  1 2  -2 -3  1 0  0 1 |

**Exercise 5** (By hand)

(a) Find a basis for the **range** of the matrix,

(b) What is the rank of A?

Check your answers using MATLAB.

(a).

First, conduct Gaussian Elimination on the following matrix

Swap rows:

~

Cancel leading column in row 2:

~

Cancel leading column in row 3:

Cancel row 3:

Divide row 1 with 3 and multiply row 2 by 3:

Cancel the second column of row 1:

The first and second column has a leading column of 1. Thus, the basis of range of A are the first and second column of matrix A,

(b).

The rank of the matrix A is

MATLAB verification

R\_A = rank(A)

R\_A = 2

**Exercise 6** Consider a 5 x 5 matrix

where and are 5 x 1 matrices.

1. What is the rank of A?
2. What is the nullity of A?

(a).

Let matrices and be

Then, becomes

|  |
| --- |
| A = |

Now, performing the Gaussian Elimination, we obtain the following outcome

|  |
| --- |
| E\_A = |

We know that there is only one column with a leading column of 1 in the row reduced echelon form of A. Hence,

Therefore,

(b).

Since,

MATLAB verification

vv = sym('v',[5,1]); ww = sym('w',[5,1]);

assume(ww,'real')

A = vv\*ww'

E\_A = rref(A)

R\_A = rank(A)

N\_A = null(A)

Exercise 7 Suppose and are subspaces of a vector space . Prove or disprove (by counterexample) the following statements.

1. is a subspace of .
2. is a subspace of .

(a).

Theorem. To say that *B* is a subspace of *A*.

1. *B* is a subset of *A.*
2. *0 B,*
3. For all it holds that , and
4. For all and it holds that .

We follow the theorem above.

1. We know that and are subspaces of a vector space
2. Since and are subspaces of we know that and and therefore
3. Let, . Then we have that and individually. This shows that and . Therefore, we have
4. Let and be a scalar value. If we have and it indicates that and , and therefore, . This is true because and are subspaces of a vector space .

Since, all 4 of the criteria in the theorem above is satisfied, the statement “ is a subspace of ” is true.

(b).

Say and . Also, there is a vector that suffices but . There is another vector that suffices but .

Seeking for a contradiction, let us assume that the union, is a subspace of . The vectors , lie in the vector space . Thus, there sum is also in . This implies that we have either

If , then there exists a vector such that

.

Since and , then the difference is also in . Thus,

However, this contradicts with our assumption .

Thus, we must have .

In this case, there exists a such that

Since both , are vectors of it should follow

which happens to contradict with .

Hence, we can say that “ is a subspace of ” is false when and .