A picture containing fireworks, dark, water, flying

Description automatically generated

College of Engineering

School of Aeronautics and Astronautics

AAE 564

System Analysis and Synthesis

Homework 9

Observability of Control Systems

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**Exercise 1**

Determine (by hand) whether or not each of the following systems are observable.

(a)

(b)

(c)

(d)

(a)

The *A* and *C* matrix for this system is

The observability matrix becomes

Thus, this system is observable.

(b)

The *A* and *C* matrix for this system is

The observability matrix becomes

Thus, this system is unobservable.

(c)

The *A* and *C* matrix for this system is

The observability matrix becomes

Thus, this system is unobservable.

(d)

The *A* and *C* matrix for this system is

The observability matrix becomes

Thus, this system is unobservable.

MATLAB code for verification

function res = checkObservability(A, C)

dim = size(A); n = dim(1);

Qo = obsv(A, C);

res.check = rank(Qo) == n;

res.Qo = Qo;

end

% Ex1

% (a)

A = [-1, 0; 0, 1];

C = [1, 1];

res = checkObservability(A, C);

res.check

res.Qo

% (b)

A = [-1, 0; 0, 1];

C = [0, 1];

res = checkObservability(A, C);

res.check

res.Qo

% (c)

A = [1, 0; 0, 1];

C = [1, 1];

res = checkObservability(A, C);

res.check

res.Qo

% (d)

A = [0, 1; 4, 0];

C = [-2, 1];

res = checkObservability(A, C);

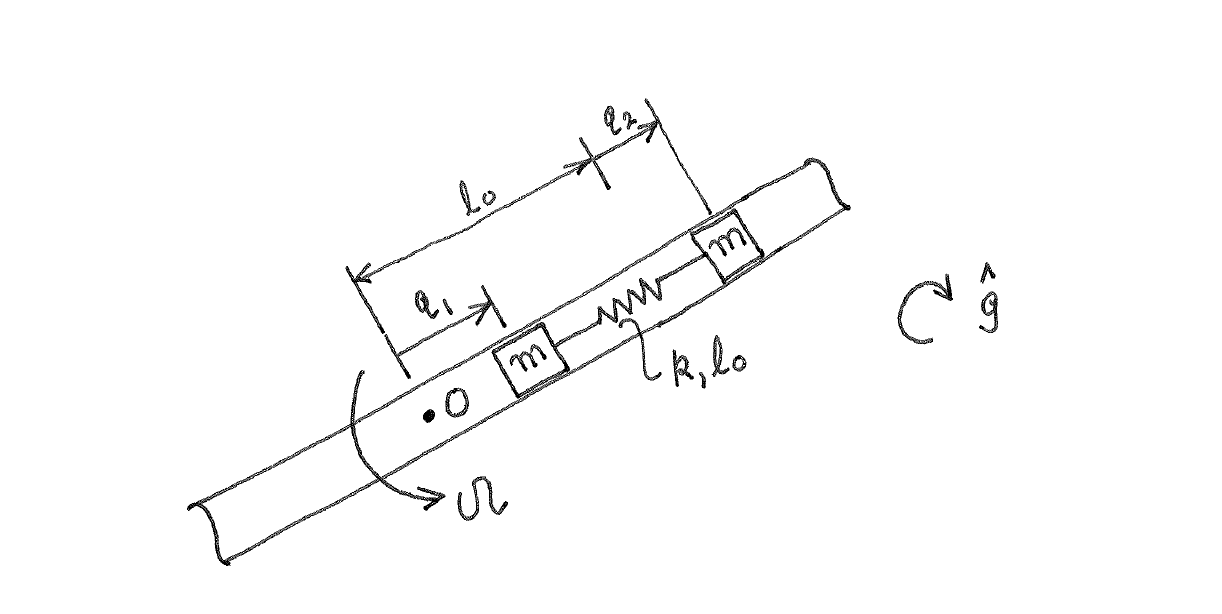
res.check

res.Qo

**Exercise 2**

(BB in laundromat) Obtain a state space representation of the following system.

Determine whether or not your state space representation is observable.



Manipulating the system, we obtain

If , the *A* and *C* matrices become

Then the observability matrix becomes

It is very easy to tell that the reduced echelon form of this observability matrix is

And the rank of this is 4.

Thus, this system is observable.

**Exercise 3**

For each system in Exercise 1 which is not observable, obtain a basis for the unobservable subspace.

The unobservable systems were (b), (c), and (d).

(b)

Since we know that

The basis of the null space is

Then the basis of the unobservable subspace is

(c)

Since we know that

The basis of the null space is

Then the basis of the unobservable subspace is

(d)

Since we know that

The basis of the null space is

Then the basis of the unobservable subspace is

**Exercise 4**

Determine the unobservable eigenvalues for each of the systems of Exercise 1.

(a)

Since the system is observable there are no unobservable eigenvalues.

(b)

The eigenvalues are .

For ,

This eigenvalue is observable.

For ,

The unobservable eigenvalue is -1.

(c)

The eigenvalue is .

For ,

The unobservable eigenvalue is 1.

(d)

The eigenvalues are .

For ,

The eigenvalue 2 is unobservable.

For ,

This eigenvalue is observable.

MATLAB Code for verification

function res = find\_unobsv\_eigVal(A, C)

[v, d] = eig(A);

sz = size(d);

n = sz(1);

for i = 1:n

lambda = d(i,i);

Z = [A-lambda\*eye(n); C];

res(i).observability = rank(Z) == n;

res(i).Z = Z;

res(i).rrefZ = rref(Z);

res(i).lambda = lambda;

end

end

% Ex4

% (b)

A = [-1, 0; 0, 1];

C = [0, 1];

res = find\_unobsv\_eigVal(A, C)

% (c)

A = [1, 0; 0, 1];

C = [1, 1];

res = find\_unobsv\_eigVal(A, C)

% (d)

A = [0, 1; 4, 0];

C = [-2, 1];

res = find\_unobsv\_eigVal(A, C)

**Exercise 5**

Determine (by hand) whether or not the following system is observable.

If the system is unobservable, compute the unobservable eigenvalues.

The *A* and *C* matrix of this system is

The corresponding observability matrix

The rank of this observability matrix is

The system is unobservable.

To find the unobservable eigenvalues we first find the eigenvalues of this system

For ,

This eigenvalue is observable.

For ,

This eigenvalue of 4 is unobservable.

For ,

This eigenvalue is observable.

**Exercise 6**

Consider a system described by

where all quantities are scalar. Obtain conditions on the numbers and which are necessary and sufficient for the observability of this system. (Hint: PBH time.)

The *A* matrix of this system is

The *C* matrix is

Since *A* is a diagonal matrix the diagonal values are the eigenvalues. Thus, for the observability of the system to hold true the PBH test for all eigenvalues must be true. This means that

For this to be true,

cannot have linearly dependent rows, which means that

and

**Exercise 7**

Using MATLAB, carry out the following for linearizations L1, L3, L7 of the two pendulum cart system.

1. Determine which linearizations are observable.
2. Determine the unobservable eigenvalues for the unobservable linearizations.

The system equation for the double pendulum cart system is

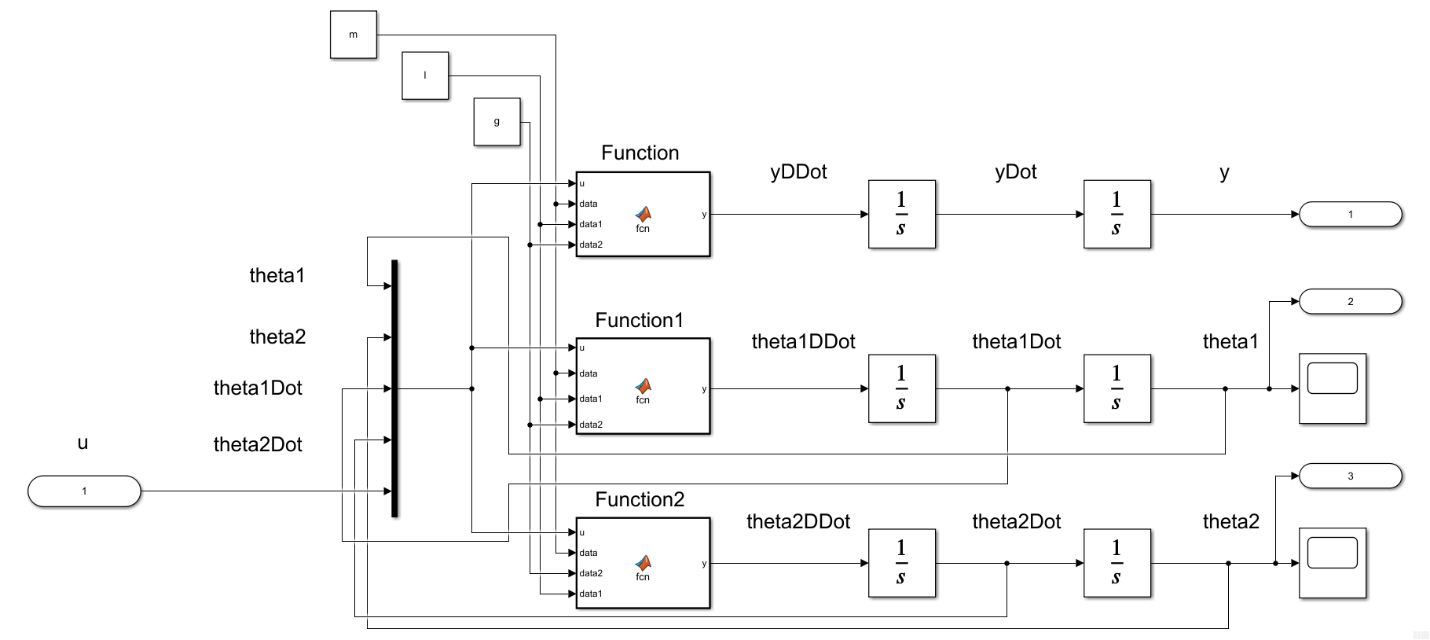
Have the system be a single output of the displacement y.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |
| P1 | 2 | 1 | 1 | 1 | 1 | 1 | 0 |
| P2 | 2 | 1 | 1 | 1 | 0.99 | 1 | 0 |
| P3 | 2 | 1 | 0.5 | 1 | 1 | 1 | 0 |
| P4 | 2 | 1 | 1 | 1 | 0.5 | 1 | 0 |

|  |  |  |
| --- | --- | --- |
| L1 | P1 | E1 |
| L2 | P1 | E2 |
| L3 | P2 | E1 |
| L4 | P2 | E2 |
| L5 | P3 | E1 |
| L6 | P3 | E2 |
| L7 | P4 | E1 |
| L8 | P4 | E2 |

(a)

The Simulink model used for this is shown below,



Embedded MATLAB Block – Function (code)

function y = fcn(u, data, data1, data2)

%{

EMBEDDED MATLAB BLOCK FUNCTION

%}

m0 = data(1); m1 = data(2); m2 = data(3); l1 = data1(1); l2 = data1(2);

g = data2;

num = -m1\*l1\*sin(u(1))\*u(3)\*u(3) - m2\*l2\*sin(u(2))\*u(4)\*u(4)...

- m1\*g\*sin(u(1))\*cos(u(1)) - m2\*g\*sin(u(2))\*cos(u(2))...

+ u(5);

den = m0 + m1 + m2 - m1\*cos(u(1))^2 - m2\*cos(u(2))^2;

y = num / den;

end

Embedded MATLAB Block – Function1 (code)

function y = fcn(u, data, data1, data2)

%{

EMBEDDED MATLAB BLOCK FUNCTION1

%}

m0 = data(1); m1 = data(2); m2 = data(3); l1 = data1(1); l2 = data1(2);

g = data2;

num = -(m1\*l1\*cos(u(1))\*sin(u(1))\*u(3)\*u(3) + m2\*l2\*cos(u(1))\*sin(u(2))\*u(4)\*u(4))...

+ m2\*g\*(sin(u(1))\*cos(u(2))^2 - cos(u(1))\*sin(u(2))\*cos(u(2)))...

- (m0 + m1 + m2)\*g\*sin(u(1)) + u(5)\*cos(u(1));

den = l1\*(m0 + m1 + m2 - m1\*cos(u(1))^2 - m2\*cos(u(2))^2);

y = num / den;

end

Embedded MATLAB Block – Function2 (code)

function y = fcn(u, data, data2, data1)

%{

EMBEDDED MATLAB BLOCK FUNCTION2

%}

m0 = data(1); m1 = data(2); m2 = data(3); l1 = data1(1); l2 = data1(2);

g = data2;

num = -(m1\*l1\*cos(u(2,1))\*sin(u(1))\*u(3)\*u(3) + m2\*l2\*cos(u(2))\*sin(u(2))\*u(4)\*u(4))...

+ m1\*g\*(sin(u(2))\*cos(u(1))^2 - cos(u(2))\*sin(u(1))\*cos(u(1)))...

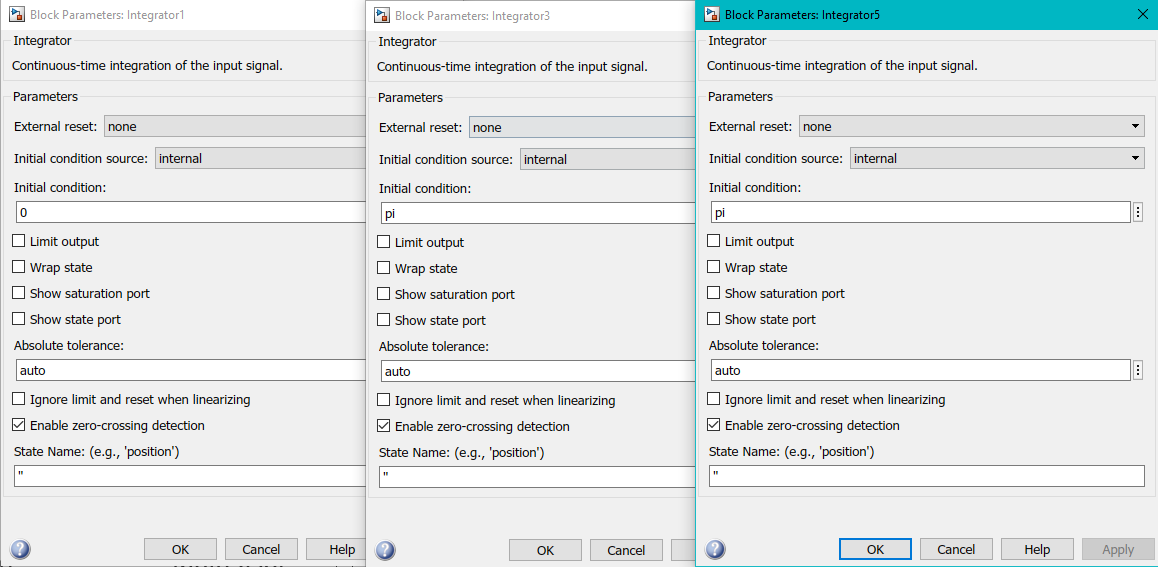
- (m0 + m1 + m2)\*g\*sin(u(2)) + u(5)\*cos(u(2));

den = l2\*(m0 + m1 + m2 - m1\*cos(u(1))^2 - m2\*cos(u(2))^2);

y = num / den;

end

For the conditions E1 and E2, we set the initial conditions of the integrator block of y, , and correspondingly to ; like in the following windows,



L1:

|  |  |
| --- | --- |
| A = 6×6  0 0 0 1.0000 0 0  0 0 0 0 1.0000 0  0 0 0 0 0 1.0000  0 -0.5000 -0.5000 0 0 0  0 -1.5000 -0.5000 0 0 0  0 -0.5000 -1.5000 0 0 0 | B = 6×1  0  0  0  0.5000  0.5000  0.5000 |
| C = 1×6  1 0 0 0 0 0 | **D = 0** |

The observability matrix for this system is

|  |
| --- |
| Qo\_L1 = 6×6  1.0000 0 0 0 0 0  0 0 0 1.0000 0 0  0 -0.5000 -0.5000 0 0 0  0 0 0 0 -0.5000 -0.5000  0 1.0000 1.0000 0 0 0  0 0 0 0 1.0000 1.0000 |

The reduced echelon form of this matrix is

|  |
| --- |
| Qo\_L1\_rref = 6×6  1 0 0 0 0 0  0 1 1 0 0 0  0 0 0 1 0 0  0 0 0 0 1 1  0 0 0 0 0 0  0 0 0 0 0 0 |

Thus,

This system linearized by L1 is unobservable.

L3:

|  |  |
| --- | --- |
| A = 6×6  0 0 0 1.0000 0 0  0 0 0 0 1.0000 0  0 0 0 0 0 1.0000  0 -0.5000 -0.5000 0 0 0  0 -1.5000 -0.5000 0 0 0  0 -0.5051 -1.5152 0 0 0 | B = 6×1  0  0  0  0.5000  0.5000  0.5051 |
| C = 1×6  1 0 0 0 0 0 | **D = 0** |

The observability matrix for this system is

|  |
| --- |
| Qo\_L3 = 6×6  1.0000 0 0 0 0 0  0 0 0 1.0000 0 0  0 -0.5000 -0.5000 0 0 0  0 0 0 0 -0.5000 -0.5000  0 1.0025 1.0076 0 0 0  0 0 0 0 1.0025 1.0076 |

The reduced echelon form of this matrix is

|  |
| --- |
| Qo\_L3\_rref = 6×6  1 0 0 0 0 0  0 1 0 0 0 0  0 0 1 0 0 0  0 0 0 1 0 0  0 0 0 0 1 0  0 0 0 0 0 1 |

Thus,

This system linearized by L3 is observable.

L7:

|  |  |
| --- | --- |
| A = 6×6  0 0 0 1.0000 0 0  0 0 0 0 1.0000 0  0 0 0 0 0 1.0000  0 -0.5000 -0.5000 0 0 0  0 -1.5000 -0.5000 0 0 0  0 -1.0000 -3.0000 0 0 0 | B = 6×1  0  0  0  0.5000  0.5000  1.0000 |
| C = 1×6  1 0 0 0 0 0 | **D = 0** |

The observability matrix for this system is

|  |
| --- |
| Qo\_L7 = 6×6  1.0000 0 0 0 0 0  0 0 0 1.0000 0 0  0 -0.5000 -0.5000 0 0 0  0 0 0 0 -0.5000 -0.5000  0 1.2500 1.7500 0 0 0  0 0 0 0 1.2500 1.7500 |

The reduced echelon form of this matrix is

|  |
| --- |
| Qo\_L7\_rref = 6×6  1 0 0 0 0 0  0 1 0 0 0 0  0 0 1 0 0 0  0 0 0 1 0 0  0 0 0 0 1 0  0 0 0 0 0 1 |

Thus,

This system linearized by L3 is observable.

(b)

The unobservable system is only L1.

The eigenvalues for L1 are

|  |
| --- |
| eigVal = 6×1 complex  0.0000 + 0.0000i  0.0000 + 0.0000i  0.0000 + 1.4142i  0.0000 - 1.4142i  -0.0000 + 1.0000i  -0.0000 - 1.0000i |

For ,

|  |
| --- |
| Z = 7×6  0 0 0 1.0000 0 0  0 0 0 0 1.0000 0  0 0 0 0 0 1.0000  0 -0.5000 -0.5000 0 0 0  0 -1.5000 -0.5000 0 0 0  0 -0.5000 -1.5000 0 0 0  1.0000 0 0 0 0 0 |

The reduced echelon form of *Z* is

|  |
| --- |
| Z\_rref = 7×6  1 0 0 0 0 0  0 1 0 0 0 0  0 0 1 0 0 0  0 0 0 1 0 0  0 0 0 0 1 0  0 0 0 0 0 1  0 0 0 0 0 0 |

The eigenvalue 0 is observable.

For ,

|  |
| --- |
| Z = 7×6 complex  -0.0000 - 1.4142i 0.0000 + 0.0000i 0.0000 + 0.0000i 1.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i  0.0000 + 0.0000i -0.0000 - 1.4142i 0.0000 + 0.0000i 0.0000 + 0.0000i 1.0000 + 0.0000i 0.0000 + 0.0000i  0.0000 + 0.0000i 0.0000 + 0.0000i -0.0000 - 1.4142i 0.0000 + 0.0000i 0.0000 + 0.0000i 1.0000 + 0.0000i  0.0000 + 0.0000i -0.5000 + 0.0000i -0.5000 + 0.0000i -0.0000 - 1.4142i 0.0000 + 0.0000i 0.0000 + 0.0000i  0.0000 + 0.0000i -1.5000 + 0.0000i -0.5000 + 0.0000i 0.0000 + 0.0000i -0.0000 - 1.4142i 0.0000 + 0.0000i  0.0000 + 0.0000i -0.5000 + 0.0000i -1.5000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i -0.0000 - 1.4142i  1.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i |

The reduced echelon form of *Z* is

|  |
| --- |
| Z\_rref = 7×6  1 0 0 0 0 0  0 1 0 0 0 0  0 0 1 0 0 0  0 0 0 1 0 0  0 0 0 0 1 0  0 0 0 0 0 1  0 0 0 0 0 0 |

The eigenvalue 1.4142j is observable.

For ,

|  |
| --- |
| Z = 7×6 complex  -0.0000 + 1.4142i 0.0000 + 0.0000i 0.0000 + 0.0000i 1.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i  0.0000 + 0.0000i -0.0000 + 1.4142i 0.0000 + 0.0000i 0.0000 + 0.0000i 1.0000 + 0.0000i 0.0000 + 0.0000i  0.0000 + 0.0000i 0.0000 + 0.0000i -0.0000 + 1.4142i 0.0000 + 0.0000i 0.0000 + 0.0000i 1.0000 + 0.0000i  0.0000 + 0.0000i -0.5000 + 0.0000i -0.5000 + 0.0000i -0.0000 + 1.4142i 0.0000 + 0.0000i 0.0000 + 0.0000i  0.0000 + 0.0000i -1.5000 + 0.0000i -0.5000 + 0.0000i 0.0000 + 0.0000i -0.0000 + 1.4142i 0.0000 + 0.0000i  0.0000 + 0.0000i -0.5000 + 0.0000i -1.5000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i -0.0000 + 1.4142i  1.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i |

The reduced echelon form of *Z* is

|  |
| --- |
| Z\_rref = 7×6  1 0 0 0 0 0  0 1 0 0 0 0  0 0 1 0 0 0  0 0 0 1 0 0  0 0 0 0 1 0  0 0 0 0 0 1  0 0 0 0 0 0 |

The eigenvalue -1.4142j is observable.

For ,

|  |
| --- |
| Z = 7×6 complex  0.0000 - 1.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 1.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i  0.0000 + 0.0000i 0.0000 - 1.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 1.0000 + 0.0000i 0.0000 + 0.0000i  0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 - 1.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 1.0000 + 0.0000i  0.0000 + 0.0000i -0.5000 + 0.0000i -0.5000 + 0.0000i 0.0000 - 1.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i  0.0000 + 0.0000i -1.5000 + 0.0000i -0.5000 + 0.0000i 0.0000 + 0.0000i 0.0000 - 1.0000i 0.0000 + 0.0000i  0.0000 + 0.0000i -0.5000 + 0.0000i -1.5000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 - 1.0000i  1.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i |

The reduced echelon form of *Z* is

|  |
| --- |
| Z\_rref = 7×6 complex  1.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i  0.0000 + 0.0000i 1.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 - 1.0000i  0.0000 + 0.0000i 0.0000 + 0.0000i 1.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i -0.0000 + 1.0000i  0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 1.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i  0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 1.0000 + 0.0000i 1.0000 + 0.0000i  0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i  0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i |

The eigenvalue j is unobservable.

For ,

|  |
| --- |
| Z = 7×6 complex  0.0000 + 1.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 1.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i  0.0000 + 0.0000i 0.0000 + 1.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 1.0000 + 0.0000i 0.0000 + 0.0000i  0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 1.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 1.0000 + 0.0000i  0.0000 + 0.0000i -0.5000 + 0.0000i -0.5000 + 0.0000i 0.0000 + 1.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i  0.0000 + 0.0000i -1.5000 + 0.0000i -0.5000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 1.0000i 0.0000 + 0.0000i  0.0000 + 0.0000i -0.5000 + 0.0000i -1.5000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 1.0000i  1.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i |

The reduced echelon form of *Z* is

|  |
| --- |
| Z\_rref = 7×6 complex  1.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i  0.0000 + 0.0000i 1.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 1.0000i  0.0000 + 0.0000i 0.0000 + 0.0000i 1.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i -0.0000 - 1.0000i  0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 1.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i  0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 1.0000 + 0.0000i 1.0000 - 0.0000i  0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i  0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i |

The eigenvalue -j is unobservable.

MATLAB code

% AAE 564 HW9 Ex7

% Tomoki Koike

close all; clear all; clc;

set(groot, 'defaulttextinterpreter','latex');

set(groot, 'defaultAxesTickLabelInterpreter','latex');

set(groot, 'defaultLegendInterpreter','latex');

% (a)

global m l g ye theta1e theta2e

param\_combo = ["L1","L3","L7"];

for i = 1:numel(param\_combo)

define\_params(param\_combo(i));

[A, B, C, D] = linmod('db\_pend\_cart\_lin');

lin\_sys(i).Amat = A;

lin\_sys(i).Bmat = B;

lin\_sys(i).Cmat = C;

lin\_sys(i).Dmat = D;

sys\_ss = ss(A, B, C, D); % get the state space system

OB(i) = checkObservability(A, C); % check the observability of the system

eigOB{i} = find\_unobsv\_eigVal(A, C); % check the observability of the eigenvalues

end

function define\_params(L)

% Function to define parameters

global m l g ye theta1e theta2e

if L == "L1"

m = [2,1,1]; l = [1,1]; g = 1; % P1

ye = 0; theta1e = 0; theta2e = 0; % E1

elseif L == "L2"

m = [2,1,1]; l = [1,1]; g = 1; % P1

ye = 0; theta1e = pi; theta2e = pi; % E2

elseif L == "L3"

m = [2,1,1]; l = [1,0.99]; g = 1; % P2

ye = 0; theta1e = 0; theta2e = 0; % E1

elseif L == "L4"

m = [2,1,1]; l = [1,0.99]; g = 1; % P2

ye = 0; theta1e = pi; theta2e = pi; % E2

elseif L == "L5"

m = [2,1,0.5]; l = [1,1]; g = 1; % P3

ye = 0; theta1e = 0; theta2e = 0; % E1

elseif L == "L6"

m = [2,1,0.5]; l = [1,1]; g = 1; % P3

ye = 0; theta1e = pi; theta2e = pi; % E2

elseif L == "L7"

m = [2,1,1]; l = [1,0.5]; g = 1; % P4

ye = 0; theta1e = 0; theta2e = 0; % E1

elseif L == "L8"

m = [2,1,1]; l = [1,0.5]; g = 1; % P4

ye = 0; theta1e = pi; theta2e = pi; % E2

else

print('error: did not match any')

end

end

function res = checkObservability(A, C)

dim = size(A); n = dim(1);

Qo = obsv(A, C);

res.check = rank(Qo) == n;

res.Qo = Qo;

end

function res = find\_unobsv\_eigVal(A, C)

[v, d] = eig(A);

sz = size(d);

n = sz(1);

for i = 1:n

lambda = d(i,i);

Z = [A-lambda\*eye(n); C];

res(i).observability = rank(Z) == n;

res(i).Z = Z;

res(i).rrefZ = rref(Z);

res(i).lambda = lambda;

end

end

**Exercise 8**

(BB in laundromat: mass center observations.) Obtain a state space representation of the following system.

(a) Obtain a basis for its unobservable subspace.

(b) Determine the unobservable eigenvalues. Consider .

(a)

Manipulating the system, we obtain

If , the *A* and *C* matrices become

Then the observability matrix becomes

The reduced echelon form of this becomes

Thus,

This system is unobservable.

From the reduced echelon form of the observability matrix we can get the null space bases

(b)

The eigenvalues of this system is

When ,

The reduced echelon form is

The eigenvalue of is observable.

When ,

The reduced echelon form is

The eigenvalue of is observable.

When ,

The reduced echelon form is

The eigenvalue of is unobservable.

When ,

The reduced echelon form is

The eigenvalue of is unobservable.