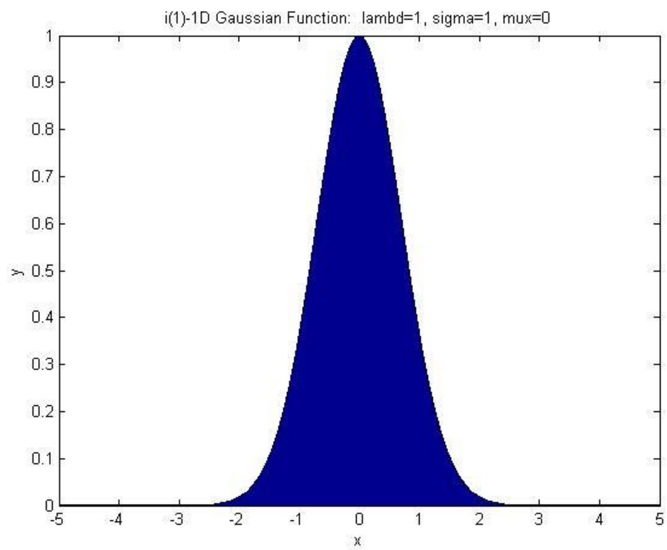
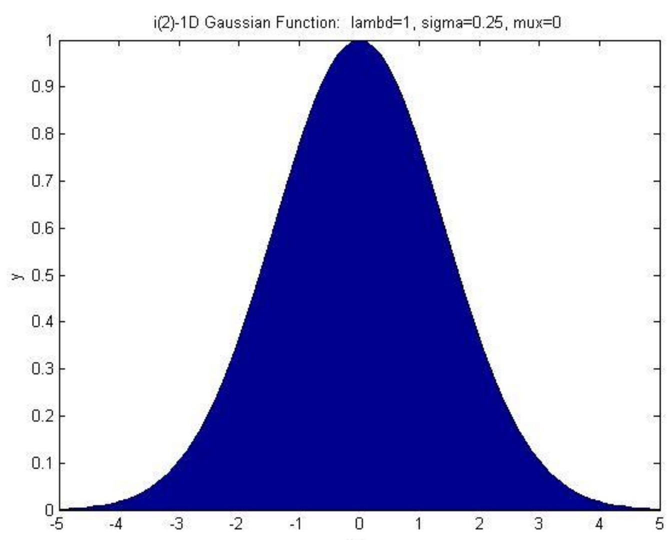
Lab #2 Report

Introduction:

The Gaussian Function is useful in many fields such as math, engineering, science and statistics. In this Lab work, I use Matlab to explore the Gaussian Function. 1D Gaussian Function has the form: $f(x) = \lambda e^{-\sigma(x-\mu x)^{\wedge}2}$, where λ, σ, μx are real values. And 2D Gaussian Function has the form: $f(x,y) = \lambda e^{-\sigma((x-\mu x)^{\wedge}2+(y-\mu y)^{\wedge}2)}$, where λ, σ, μx, μy are real values. I use Matlab to make Gaussian Function's plots according to different parameters' value (λ, σ, μx, μy) and to explain the plots.

Results:

i(1)	$f(x) = \lambda e^{-\sigma(x-\mu x)^{\wedge}2}$ λ=1 σ=1 μx=0	
i(2)	$f(x) = \lambda e^{-\sigma(x-\mu x)^{\wedge}2}$ λ=1 σ=0.25 μx=0	

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<tr>
<td>i(3)</td>
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$$f(x) = \lambda e^{-\sigma(x-\mu x)^{\wedge}2}$$

λ=0.5
σ=1
μx=0
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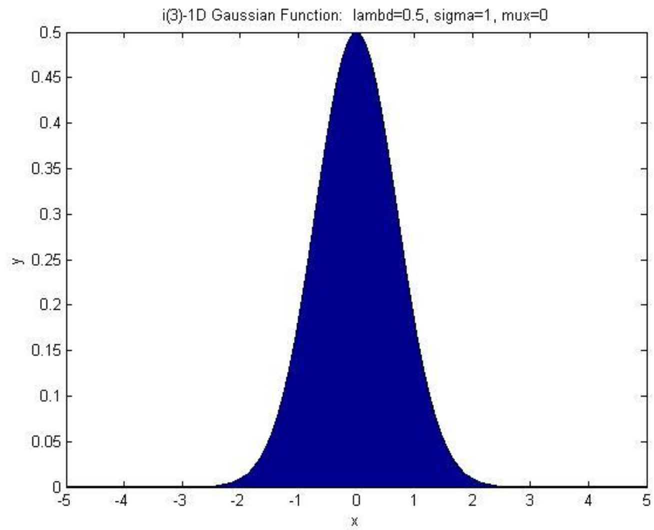

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<td>i(4)</td>
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$$f(x) = \lambda e^{-\sigma(x-\mu x)^{\wedge}2}$$

λ=1
σ=1
μx=1
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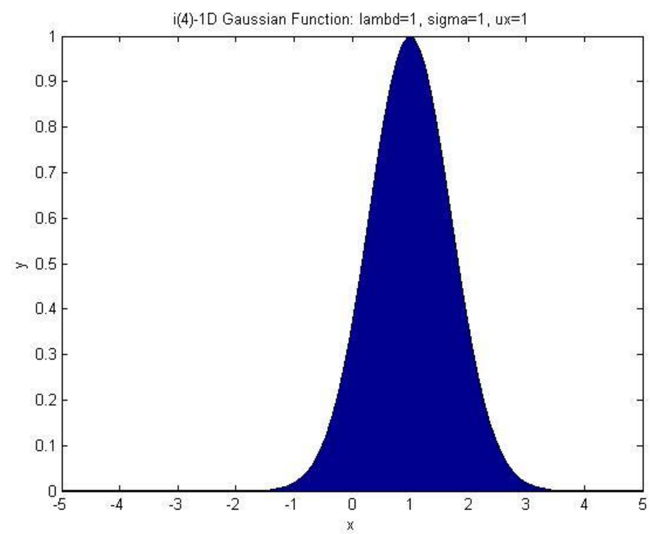

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<tr>
<td>i(5)</td>
<td>

$$f(x) = \lambda e^{-\sigma(x-\mu x)^{\wedge}2}$$

λ=-1
σ=1
μx=0
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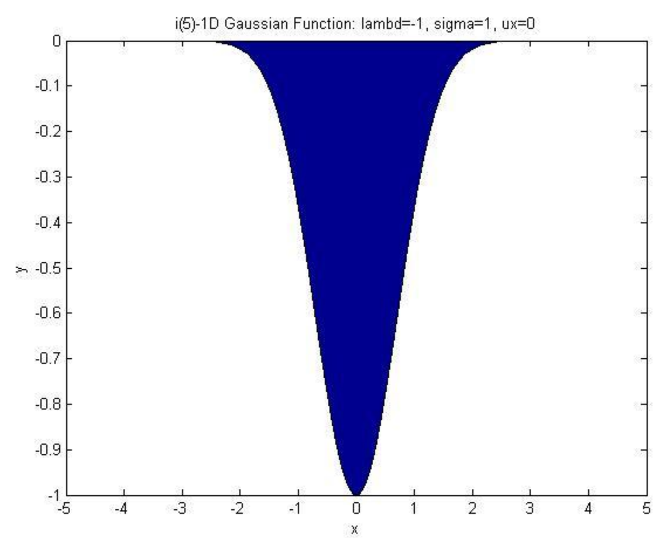

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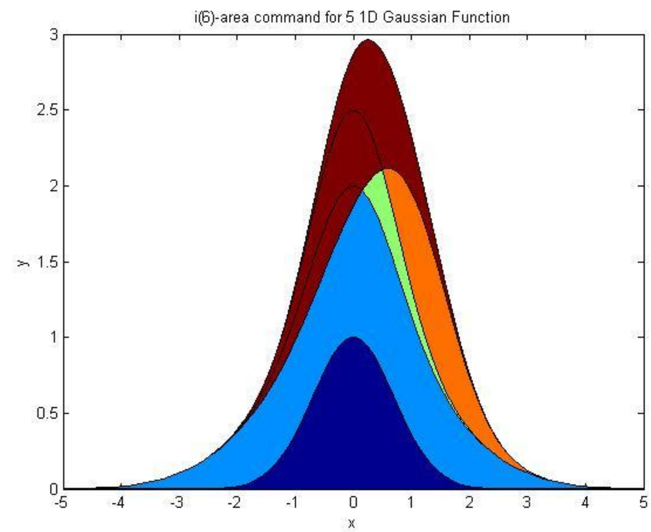
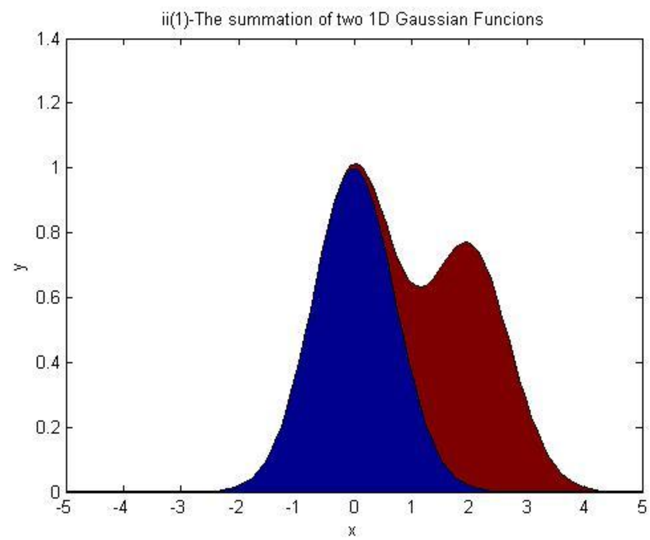
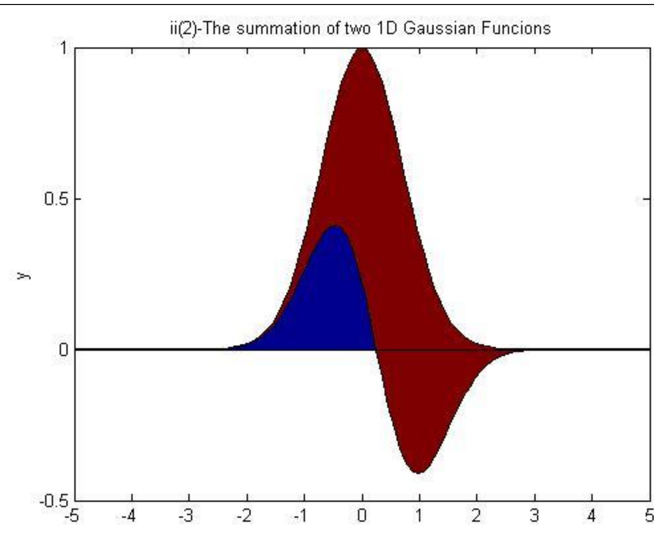
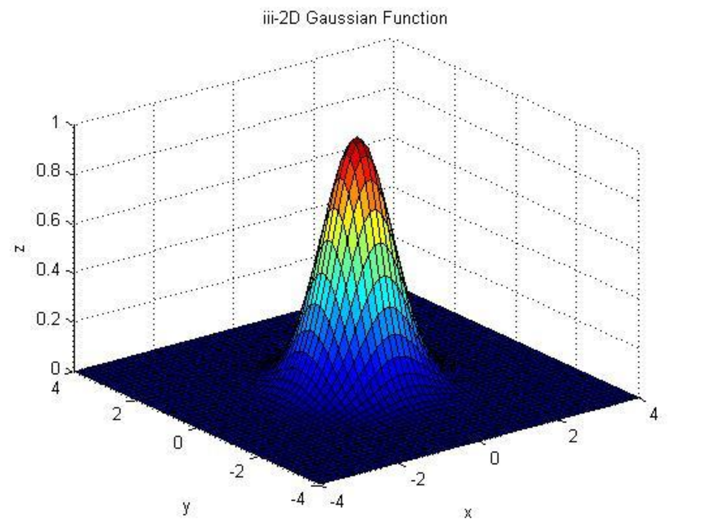
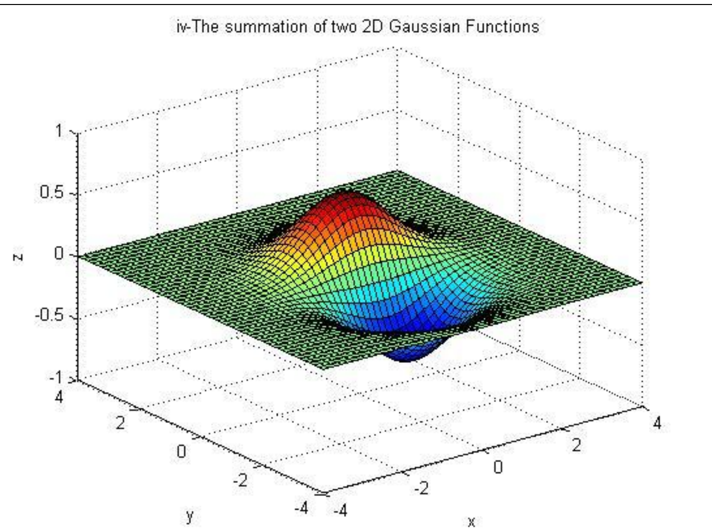
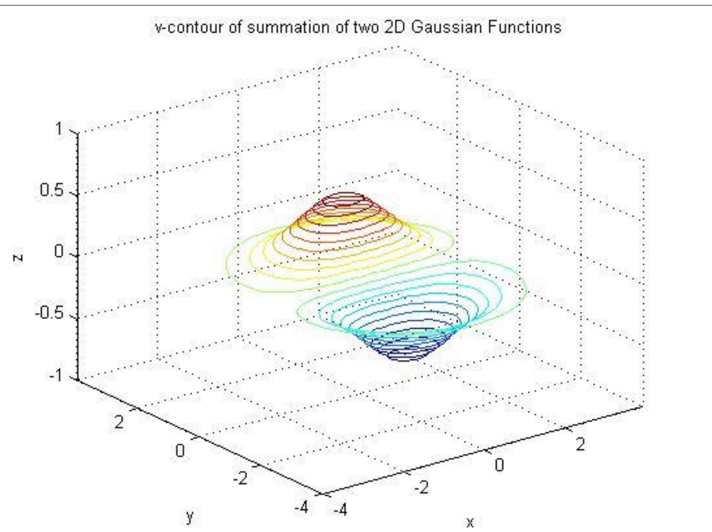
i(6)	Area command for first 5 plots	
ii(1)	$f(x) =$ $\lambda 1 e^{-\sigma 1(x-\mu x1)^{\wedge}2} +$ $\lambda 2 e^{-\sigma 2(x-\mu x2)^{\wedge}2}$ λ1=1 σ1=1 μx=0 λ1=0.75 σ1=1 μx=2	
ii(2)	$f(x) =$ $\lambda 1 e^{-\sigma 1(x-\mu x1)^{\wedge}2} +$ $\lambda 2 e^{-\sigma 2(x-\mu x2)^{\wedge}2}$ λ1=1 σ1=1 μx=0 λ1=-1 σ1=1 μx=0.5	

iii	$f(x,y) = \lambda e^{-\sigma((x-\mu x)^2+(y-\mu y)^2)}$ λ=1 σ=1 µx=0 µy=0	
iv	$f(x,y) = \lambda 1 e^{-\sigma 1\left(\begin{smallmatrix}(x-\mu x1)^2\\+(y-\mu y1)^2\end{smallmatrix}\right)}$ + $\lambda 2 e2^{-\sigma 2\left(\begin{smallmatrix}(x-\mu x2)^2\\+(y-\mu y2)^2\end{smallmatrix}\right)}$ Surf λ1=1 σ1=0.5 µx1=0 µy1=0 λ2=-1 σ2=0.5 µx2=0 µy2=-1	
v	Contour	

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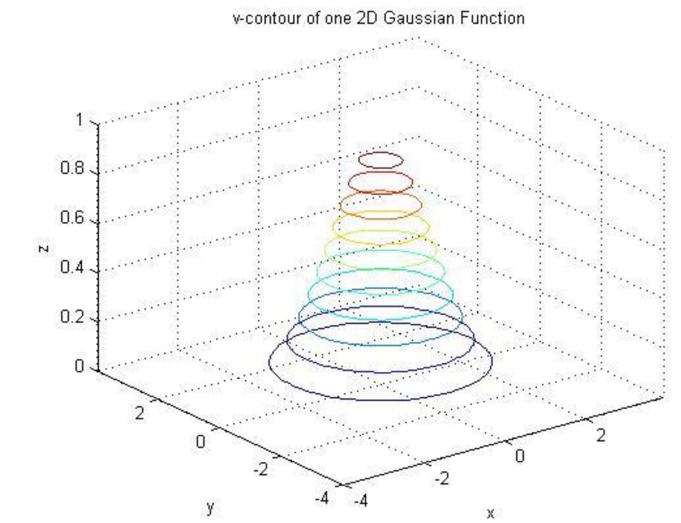
<td>v</td>
<td>Contour

$f(x,y) = \lambda 1 e^{-\sigma 1\left((x-\mu x1)^{\wedge}2 + (y-\mu y1)^{\wedge}2\right)}$

λ1=1
σ1=0.5
µx1=0
µy1=0</td>
<td>
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Analysis:

For part i, first 5 plots stand for 1D Gaussian Function. λ is the amplitude of the function. (1)(2)(4) has λ=1 and (3) has λ=0.5 so (1)(2)(4)'s maximal value is two times of (3). (5) has λ=-1, so its minimal value is -1. σ is related with the shape of the plot. For example, (2) has σ=0.25 and (1)(3)(4)(5) have σ =1. So the shape of (2) is much wider than others (the value of function (2) goes down more slowly than value of others when |x| becomes large). µx is related to the position of the plots. For (4), µx=1 and for others, µx=0. So the symmetry axis of (4) is at x=1 and for others, symmetry axis is at x=0. For plot of (6), the net result is the sum of corresponding values from each y function.

For part ii, there are 2 plots of the summation of two 1D Gaussian functions. I use area command, so for each plot, there are two contour lines. One just stands for the first 1D Gaussian function, and the other stands for the summation of the two functions.

For part iii. I use surf to draw a 2D Gaussian function. As same as 1D Gaussian function, . λ is the amplitude of the function and σ is related with the shape of the plot. But there are two µ. One is µx and one is µy. µx stands for the position in x axis and µy stands for the position in y axis.

For part iv. I draw the summation of two 2D Gaussian functions

$$f(x,y) = \lambda 1 e^{-\sigma 1\left((x-\mu x1)^{\wedge}2+(y-\mu y1)^{\wedge}2\right)} + \lambda 2 e 2^{-\sigma 2\left((x-\mu x2)^{\wedge}2+(y-\mu y2)^{\wedge}2\right)}$$

λ1 is positive and λ2 is negative. So the shape of function is that one part has positive z value and one part has negative z value. σ1=σ2=0.5, so the positive and negative part is symmetric. µx1= µx2=0 and µy1= 0, µy2=-1. So the figure is symmetric about line y=-0.5 and z=0.

For part v. I draw two contour plots. One is the summation of two 2D Gaussian functions as described in part iv. For the summation I use command " contour3(X, Y, Z, 20) so that there 20 loops for the plot. Another is one 2D Gaussian function: $f(x,y) = \lambda 1 e^{-\sigma 1((x-\mu x1)^2+(y-\mu y1)^2)}$ with λ1=1, σ1=0.5, µx1=0, µy1=0. For the one 2D Gaussian Function I use command " contour3 (X, Y, Z1,10) so that there are 10 loops for the plot. Becourse that µx1=0, µy1=0, center axis of plot is (x=0, y=0).

Conclusion:

Using Matlab can easily draw the plots of functions. By exploring the Gaussian Function, I know that each parameters of Gaussian function affects the shape of the plots. For example, λ is the amplitude of the function. The value of function with larger σ goes down more quickly than value of function with smaller σ when |x| becomes large. µx, µy stand for the position of plot in x axis and y axis. The shape of summation of Gaussian function is decided by all the parameters of sub functions.