How do trusted setups work?

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Necessary background: elliptic curves and elliptic curve pairings. See also: Dankrad Feist's article on KZG polynomial commitments.

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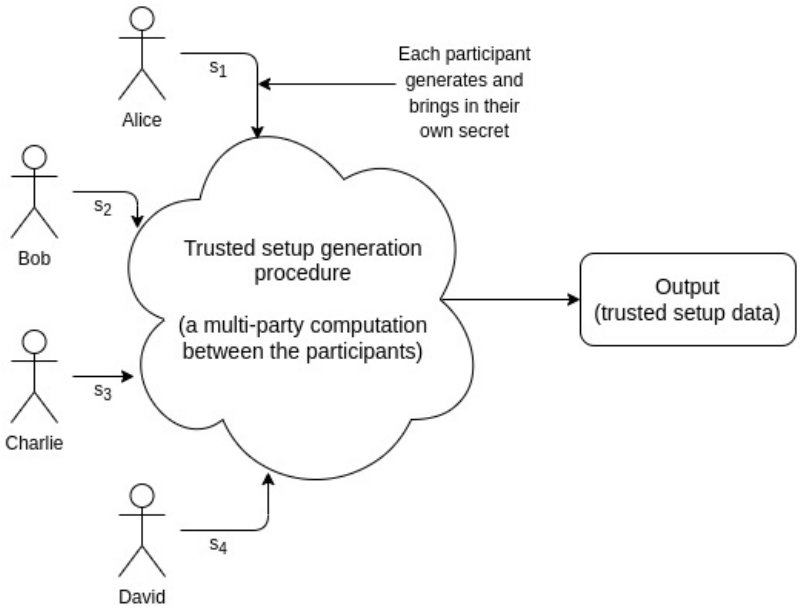
Many cryptographic protocols, especially in the areas of data availability sampling and ZK-SNARKs depend on trusted setups. **A trusted setup ceremony is a procedure that is done once to generate a piece of data that must then be used every time some cryptographic protocol is run**. Generating this data requires some secret information; the "trust" comes from the fact that some person or some group of people has to generate these secrets, use them to generate the data, and then publish the data and forget the secrets. But once the data is generated, and the secrets are forgotten, no further participation from the creators of the ceremony is required.



There are many types of trusted setups. The earliest instance of a trusted setup being used in a major protocol is the original Zcash ceremony in 2016. This ceremony was very complex, and required many rounds of communication, so it could only have six participants. Everyone using Zcash at that point was effectively trusting that at least one of the six participants was honest. More modern protocols usually use the **powers-of-tau** setup, which has a 1-of-N trust model with \(N\) typically in the hundreds. That is to say, **hundreds of people participate in generating the data together, and only one of them needs to be honest and not publish their secret for the final output to be secure. Well-executed setups like this are often considered "close enough to trustless" in practice**.

This article will explain how the KZG setup works, why it works, and the future of trusted setup protocols. Anyone proficient in code should also feel free to follow along this code implementation: https://github.com/ethereum/research/blob/master/trusted_setup/trusted_setup.py.

What does a powers-of-tau setup look like?

A powers-of-tau setup is made up of two series of elliptic curve points that look as follows:

\([G_1, G_1 * s, G_1 * s^2 ... G_1 * s^{n_1-1}]\)

\([G_2, G_2 * s, G_2 * s^2 ... G_2 * s^{n_2-1}]\)

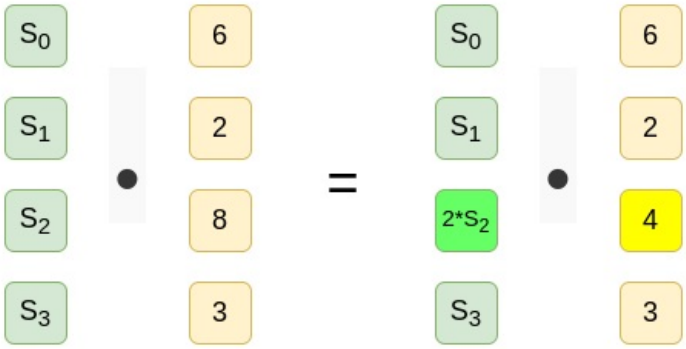
\(G_1\) and \(G_2\) are the standardized generator points of the two elliptic curve groups; in BLS12-381, \(G_1\) points are (in compressed form) 48 bytes long and \(G_2\) points are 96 bytes long. \(n_1\) and \(n_2\) are the lengths of the \(G_1\) and \(G_2\) sides of the setup. Some protocols require \(n_2 = 2\), others require \(n_1\) and \(n_2\) to both be large, and some are in the middle (eg. Ethereum's data availability sampling in its current form requires \(n_1 = 4096\) and \(n_2 = 16\)). \(s\) is the secret that is used to generate the points, and needs to be forgotten.

To make a KZG commitment to a polynomial \(P(x) = \sum_i c_i x^i\), we simply take a linear combination \(\sum_i c_i S_i\), where \(S_i = G_1 * s^i\) (the elliptic curve points in the trusted setup). The \(G_2\) points in the setup are used to verify evaluations of polynomials that we make commitments to; I won't go into verification here in more detail, though Dankrad does in his post.

Intuitively, what value is the trusted setup providing?

It's worth understanding what is philosophically going on here, and why the trusted setup is providing value. A polynomial commitment is committing to a piece of size-\(N\) data with a size \(O(1)\) object (a single elliptic curve point). We *could* do this with a plain Pedersen commitment: just set the \(S_i\) values to be \(N\) random elliptic curve points that have no known relationship with each other, and commit to polynomials with \(\sum_i c_i S_i\) as before. And in fact, this is exactly what IPA evaluation proofs do.

However, any IPA-based proofs take \(O(N)\) time to verify, and there's an unavoidable reason why: a commitment to a polynomial \(P(x)\) using the base points \([S_0, S_1 ... S_i ... S_{n-1}]\) would commit to a different polynomial if we use the base points \([S_0, S_1 ... (S_i * 2) ... S_{n-1}]\).



A valid commitment to the polynomial \(3x^3 + 8x^2 + 2x + 6\) under one set of base points is a valid commitment to \(3x^3 + 4x^2 + 2x + 6\) under a different set of base points.

If we want to make an IPA-based *proof* for some statement (say, that this polynomial evaluated at \(x = 10\) equals \(3826\)), the proof should pass with the first set of base points and fail with the second. Hence, whatever the proof verification procedure is cannot avoid somehow taking into account each and every one of the \(S_i\) values, and so it unavoidably takes \(O(N)\) time.

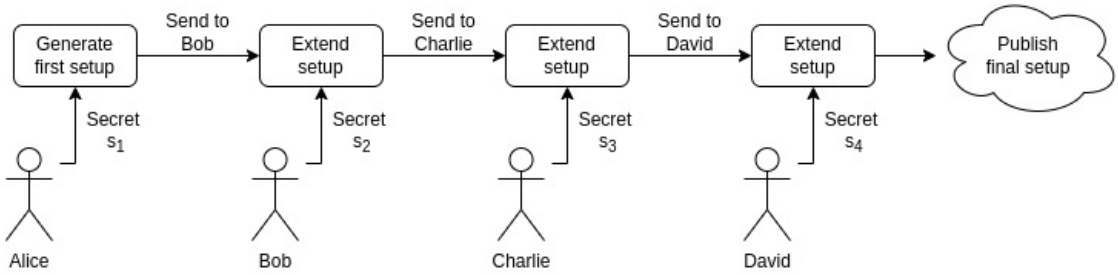
But with a trusted setup, there is a hidden mathematical relationship between the points. It's guaranteed that \(S_{i+1} = s * S_i\) with the same factor \(s\) between any two adjacent points. If \([S_0, S_1 ... S_i ... S_{n-1}]\) is a valid setup, the "edited setup" \([S_0, S_1 ... (S_i * 2) ... S_{n-1}]\) *cannot also be a valid setup*. **Hence, we don't need \(O(n)\) computation; instead, we take advantage of this mathematical relationship to verify anything we need to verify in constant time.**

However, the mathematical relationship has to remain secret: if \(s\) is known, then anyone could come up with a commitment that stands for many different polynomials: if \(C\) commits to \(P(x)\), it also commits to \(\frac{P(x) * x}{s}\), or \(P(x) - x + s\), or many other things. This would completely break all applications of polynomial commitments. **Hence, while some secret \(s\) must have existed at one point to make possible the mathematical link between the \(S_i\) values that enables efficient verification, the \(s\) must also have been forgotten.**

How do multi-participant setups work?

It's easy to see how one participant can generate a setup: just pick a random \(s\), and generate the elliptic curve points using that \(s\). But a single-participant trusted setup is insecure: you have to trust one specific person!

The solution to this is multi-participant trusted setups, where by "multi" we mean *a lot* of participants: over 100 is normal, and for smaller setups it's possible to get over 1000. Here is how a multi-participant powers-of-tau setup works.



Take an existing setup (note that you don't know \(s\), you just know the points):

\([G_1, G_1 * s, G_1 * s^2 ... G_1 * s^{n_1-1}]\)

\([G_2, G_2 * s, G_2 * s^2 ... G_2 * s^{n_2-1}]\)

Now, choose your own random secret \(t\). Compute:

\([G_1, (G_1 * s) * t, (G_1 * s^2) * t^2 ... (G_1 * s^{n_1-1}) * t^{n_2-1}]\)

\([G_2, (G_2 * s) * t, (G_2 * s^2) * t^2 ... (G_2 * s^{n_2-1}) * t^{n_2-1}]\)

Notice that this is equivalent to:

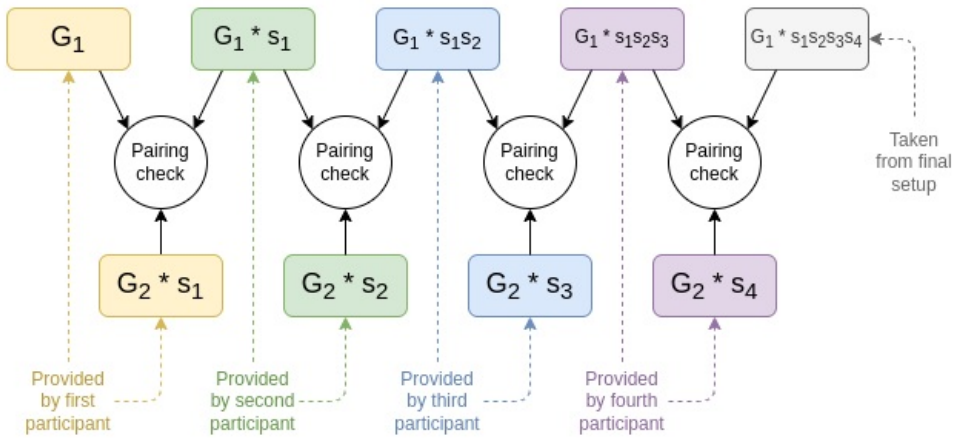
\([G_1, G_1 * (st), G_1 * (st)^2 ... G_1 * (st)^{n_1-1}]\)

\([G_2, G_2 * (st), G_2 * (st)^2 ... G_2 * (st)^{n_2-1}]\)

That is to say, you've created a valid setup with the secret \(s * t\)! You never give your \(t\) to the previous participants, and the previous participants never give you their secrets that went into \(s\). And as long as any one of the participants is honest and does not reveal their part of the secret, the combined secret does not get revealed. In particular, finite fields have the property that if you know know \(s\) but not \(t\), and \(t\) is securely randomly generated, then you know *nothing* about \(s*t\)!

Verifying the setup

To verify that each participant actually participated, each participant can provide a proof that consists of (i) the \(G_1 * s\) point that they received and (ii) \(G_2 * t\), where \(t\) is the secret that they introduce. The list of these proofs can be used to verify that the final setup combines together all the secrets (as opposed to, say, the last participant just forgetting the previous values and outputting a setup with just their own secret, which they keep so they can cheat in any protocols that use the setup).



\(s_1\) is the first participant's secret, \(s_2\) is the second participant's secret, etc. The pairing check at each step proves that the setup at each step actually came from a combination of the setup at the previous step and a new secret known by the participant at that step.

Each participant should reveal their proof on some publicly verifiable medium (eg. personal website, transaction from their .eth address, Twitter). Note that this mechanism does *not* prevent someone from claiming to have participated at some index where someone else has (assuming that other person has revealed their proof), but it's generally considered that this does not matter: if someone is willing to lie about having participated, they would also be willing to lie about having deleted their secret. As long as at least one of the people who publicly claim to have participated is honest, the setup is secure.

In addition to the above check, we also want to verify that all the powers in the setup are correctly constructed (ie. they're powers of the same secret). To do this, we *could* do a series of pairing checks, verifying that $e(S_{i+1}, G_2) = e(S_i, T_1)$ (where T_1 is the $G_2 * s$ value in the setup) for every i. This verifies that the factor between each S_i and S_{i+1} is the same as the factor between T_1 and G_2. We can then do the same on the G_2 side.

But that's a lot of pairings and is expensive. Instead, we take a random linear combination $L_1 = \sum_{i=0}^{n_1-2} r_iS_i$, and the same linear combination shifted by one: $L_2 = \sum_{i=0}^{n_1-2} r_iS_{i+1}$. We use a single pairing check to verify that they match up: $e(L_2, G_2) = e(L_1, T_1)$.

We can even combine the process for the G_1 side and the G_2 side together: in addition to computing L_1 and L_2 as above, we also compute $L_3 = \sum_{i=0}^{n_2-2} q_iT_i$ (q_i is another set of random coefficients) and $L_4 = \sum_{i=0}^{n_2-2} q_iT_{i+1}$, and check $e(L_2, L_3) = e(L_1, L_4)$.

Setups in Lagrange form

In many use cases, you don't want to work with polynomials *in coefficient form* (eg. $P(x) = 3x^3 + 8x^2 + 2x + 6$), you want to work with polynomials in *evaluation form* (eg. $P(x)$ is the polynomial that evaluates to $[19, 146, 9, 187]$ on the domain $[1, 189, 336, 148]$ modulo 337). Evaluation form has many advantages (eg. you can multiply and sometimes divide polynomials in $O(N)$ time) and you can even use it to evaluate in $O(N)$ time. In particular, data availability sampling expects the blobs to be in evaluation form.

To work with these cases, it's often convenient to convert the trusted setup to evaluation form. This would allow you to take the evaluations ($[19, 146, 9, 187]$ in the above example) and use them to compute the commitment directly.

This is done most easily with a Fast Fourier transform (FFT), but passing the curve points as input instead of numbers. I'll avoid repeating a full detailed explanation of FFTs here, but here is an implementation; it is actually not that difficult.

The future of trusted setups

Powers-of-tau is not the only kind of trusted setup out there. Some other notable (actual or potential) trusted setups include:

- The more complicated setups in older ZK-SNARK protocols (eg. see here), which are sometimes still used (particularly Groth16) because verification is cheaper than PLONK.
- Some cryptographic protocols (eg. **DARK**) depend on **hidden-order groups**, groups where it is not known what number an element can be multiplied by to get the zero element. Fully trustless versions of this exist (see: class groups), but by far the most efficient version uses RSA groups (powers of x mod $n = pq$ where p and q are not known). Trusted setup ceremonies for this with 1-of-n trust assumptions are possible, but are very complicated to implement.
- If/when **indistinguishability obfuscation** becomes viable, many protocols that depend on it will involve someone creating and publishing an obfuscated program that does something with a hidden internal secret. This is a trusted setup: the creator(s) would need to possess the secret to create the program, and would need to delete it afterwards.

Cryptography continues to be a rapidly evolving field, and how important trusted setups are could easily change. It's possible that techniques for working with IPAs and Halo-style ideas will improve to the point where KZG becomes outdated and unnecessary, or that quantum computers will make anything based on elliptic curves non-viable ten years from now and we'll be stuck working with trusted-setup-free hash-based protocols. It's also possible that what we can do with KZG will improve even faster, or that a new area of cryptography will emerge that depends on a different kind of trusted setup.

To the extent that trusted setup ceremonies are necessary, it is important to remember that **not all trusted setups are created equal**. 176 participants is better than 6, and 2000 would be even better. A ceremony small enough that it can be run inside a browser or phone application (eg. the ZKopru setup is web-based) could attract far more participants than one that requires running a complicated software package. Every ceremony should ideally have participants running multiple independently built software implementations and running different operating systems and environments, to reduce common mode failure risks. Ceremonies that require only one round of interaction per participant (like powers-of-tau) are far better than multi-round ceremonies, both due to the ability to support far more participants and due to the greater ease of writing multiple implementations. Ceremonies should ideally be *universal* (the output of one ceremony being able to support a wide range of protocols). These are all things that we can and should keep working on, to ensure that trusted setups can be as secure and as trusted as possible.