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1.4 Formal proof with mapping reduction

Suppose we have D_{HALT} , a decider for HALT. We define the mapping reduction as:

$$f(\langle M, w \rangle) = \langle M', w \rangle$$

where the TM M' is constructed as computing the function:

$M'(x)$	=	if (eval($\langle M, x \rangle$))
		then <i>accept</i>
		else <i>loop</i>

This means exactly (but concisely) what the above informal description says:

Construct the following machine M' .

“On input x :

1. Run M on x .
2. If M accepts, *accept*.
3. If M rejects, enter a loop.”

Notice although M' takes input named “ x ”, the actual input is w when it is simulated. This is because when $\langle M', w \rangle$ is passed as input to a decider of HALT, it uses the w part as the *actual argument* for M' . This is like the difference between a formal parameter and the actual argument in a function call when using a programming language such as Java or Python.

We can see the behavior of M , M' and D_{HALT} in the following table:

M accepts w	M' accepts w	D_{HALT} accepts $\langle M', w \rangle$
M does not accept w	M' loops	D_{HALT} rejects $\langle M', w \rangle$

Table 1. Behavior table for M , M' and D_{HALT}

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3. Prove by mathematical induction that

$$\forall n \geq 2 : 1 + 2^n < 3^n$$

Proof. Base case. For $n=2$, we have $1 + 2^2 = 5 < 9 = 3^2$. So the base case holds.

Inductive step. Suppose the proposition holds for $n-1$, that is, $1 + 2^{n-1} < 3^{n-1}$ (IH), we prove that it holds for n . We have

$1 + 2^n$	$= 1 + 2(1 + 2^{n-1} - 1)$
	$\leq 1 + 2(3^{n-1} - 1)$
	$\leq 1 + 3(3^{n-1} - 1)$
	$= 3^n - 2$
	$\leq 3^n$.

The inductive step holds, thus the proposition holds for all $n \geq 2$. □

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