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MASTER THESIS

Enhanced Real Options Valuation with Machine Learning: Applied Case to Energy Finance

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Abstract
This thesis explores real option valuation in the energy industry using deep learning methodologies. Despite the theoretical foundation of real options in financial analysis, their practical application in the volatile energy sector remains under-explored. This study bridges this gap by integrating advanced data science techniques with traditional financial models. Utilizing machine learning architectures, particularly deep learning, the study evaluates these models' efficacy in capturing the uncertainties and dynamic investment opportunities in energy projects, comparing their performance against traditional financial approaches and integrating predictions within the Black-Scholes-Merton model. The empirical case focuses on the European energy generation industry. This research validates deep learning's utility in enhancing cash flow prediction and optimizing investment decisions under uncertainty. The thesis contributes to finance, energy economics, and AI, providing valuable tools and techniques for industry practitioners and researchers.

Keywords: Predictive Modeling, Machine Learning, Energy Markets.

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1. Introduction

1.1 Background and Importance of Real Option Valuation

Real option valuation has emerged as a powerful tool for enhancing investment decision-making under uncertainty. Building upon the foundational work of Black and Scholes (1973) and Merton (1973) in options pricing, real option valuation extends these concepts to the evaluation of real investment opportunities. This approach recognizes the value of flexibility and strategic decision-making in the face of uncertain future conditions. Traditional valuation methods, such as discounted cash flow analysis (**DCF**), often fall short in capturing the value of managerial flexibility and the dynamic nature of uncertainty. These methods typically rely on static assumptions and fail to account for the ability of managers to adapt their strategies in response to changing market conditions, technological advancements, and regulatory shifts. As a result, companies may undervalue or overvalue their investment opportunities, leading to suboptimal capital allocation decisions. Real options analysis provides a framework for assessing the value of flexibility in investment decisions. It acknowledges that investment opportunities often come with embedded options, such as the option to delay, expand, contract, or abandon a project. By explicitly valuing these options, real option valuation helps managers make more informed and strategic decisions, particularly in industries characterized by high uncertainty and irreversible investments.

The importance of real option valuation lies in its ability to bridge the gap between financial theory and strategic decision-making. By incorporating the value of flexibility and the dynamic nature of uncertainty into the valuation process, real option valuation provides a more comprehensive and realistic assessment of investment opportunities. This approach enables managers to identify and capitalize on strategic opportunities that may be overlooked by traditional valuation methods.

Despite the theoretical appeal of real options, their practical application has been hindered by the complexity of the underlying mathematical models and the difficulty in estimating key input parameters. The successful implementation of real option valuation requires a deep understanding of the industry dynamics, market uncertainties, and the specific characteristics of the investment opportunity.

This thesis aims to explore the frontier of real option valuation by integrating advanced data science techniques, particularly deep learning, with traditional financial models. By leveraging the power of machine learning to capture complex patterns and relationships in data, this research seeks to enhance the accuracy and applicability of real option valuation in real-world investment decisions. The ultimate goal is to provide practitioners and researchers with valuable insights and tools to make more informed and strategic decisions in the face of uncertainty.

1.2 The Case for the Energy Industry

The energy sector presents a compelling case for the application of real option valuation. Investment decisions in this industry are characterized by high levels of uncertainty, long investment horizons, and significant capital expenditures. The complex interplay of technological advancements, shifting market dynamics, and evolving regulatory landscapes makes the energy sector particularly vulnerable to the limitations of traditional valuation methods. In recent years, the energy industry has witnessed a rapid transformation driven by the increasing emphasis on sustainability, the transition to low-carbon economies, and the emergence of disruptive technologies. These developments have introduced new sources of uncertainty and have challenged the conventional wisdom of investment decision-making. The ability to adapt to changing market conditions and to make strategic investment decisions has become a critical factor for success in the energy sector.

Real option valuation offers a powerful framework for addressing the unique challenges of the energy industry. By explicitly accounting for the value of flexibility and the dynamic nature of uncertainty, real option valuation enables energy companies to make more informed and strategic investment decisions. This approach helps managers identify and evaluate the strategic value of investment opportunities, such as the option to delay, expand, or abandon a project based on evolving market conditions.

The application of real option valuation in the energy sector has gained increasing attention in recent years. Researchers and practitioners have recognized the potential of this approach to enhance the decision-making process and to optimize capital allocation in the face of uncertainty. However, the practical implementation of real option valuation in the energy industry remains a challenge, owing to the complexity of the sector and the limitations of traditional valuation models.

1.3 Scope of the Research

This thesis aims to explore the frontier of real option valuation within the energy industry by integrating advanced data science techniques, particularly deep learning, with traditional financial models. The research seeks to address the limitations of existing valuation methods and to provide a comprehensive framework for the application of real option valuation in the energy sector.

This thesis aims to advance the field of real option valuation within the energy industry by integrating cutting-edge deep learning techniques with traditional financial models. The research has three main objectives:

1. **Literature Review**: To conduct an exhaustive review of existing literature on real option valuation, specifically focusing on its application in the energy sector. This review will identify key challenges and opportunities, laying the foundation for subsequent research.
2. **Methodology Development**: To investigate and develop novel valuation models using deep learning techniques, particularly focusing on capturing complex patterns and relationships within data to account for the dynamic nature of uncertainty in the energy sector. These models will be rigorously tested and validated using empirical data from the energy industry.
3. **Framework for Application**: To provide a comprehensive framework for the practical application of real option valuation in the energy sector. This framework will incorporate insights from the literature review and empirical analysis, offering practical guidelines for implementation in investment decision-making. It will address critical challenges such as the estimation of input parameters and the interpretation of results, aiming to enhance the accuracy and strategic value of investment decisions under uncertainty.

1.4 Machine Learning in Finance

The advent of machine learning in finance has opened up new possibilities for the application of advanced data science techniques in investment decision-making. Machine learning, and particularly deep learning, has emerged as a powerful tool for capturing complex patterns and relationships in financial data that traditional statistical methods may struggle to identify.

Deep learning techniques, such as artificial neural networks, have demonstrated remarkable success in a wide range of applications, from image recognition to natural language processing. In the context of finance, deep learning has been applied to various tasks, including stock price prediction, credit risk assessment, and fraud detection (Dixon, Halperin, and Bilokon 2020).

The application of deep learning to real option valuation represents a novel and promising approach. By leveraging the ability of deep learning models to learn from vast amounts of historical data and to capture non-linear relationships, this approach aims to provide more accurate and reliable valuations of real investment opportunities. Deep learning techniques, such as Recurrent Neural Networks (**RNN**s) and Long Short-Term Memory (**LSTM**) networks, are particularly well-suited for modeling the temporal dependencies and sequential nature of financial data.

The integration of deep learning with traditional financial models, such as the Black-Scholes model, offers the potential to enhance the robustness and adaptability of these models. By incorporating machine learning techniques, the limitations of traditional models in capturing the complexities and uncertainties of real-world investment scenarios can be addressed, leading to more accurate and reliable valuations.

However, the application of deep learning in finance also presents significant challenges. These challenges include the availability and quality of data, the interpretability of complex models, and the potential for overfitting. Addressing these challenges requires a deep understanding of both the technical aspects of machine learning and the domain-specific knowledge of finance.

2. Literature Review

2.1 Foundations of Real Option Theory

The seminal works of Black and Scholes (1973) and Merton (1973) laid the groundwork for the development of option pricing theory, revolutionizing the field of corporate liability pricing. Their pioneering approach, albeit mathematically advanced, provided a complete equilibrium option pricing model under certain restrictive assumptions.[1] Subsequent research has sought to relax these assumptions and generalize the model to more realistic and complex scenarios.

Merton (1973) demonstrated that the basic mode of analysis remains valid even with the introduction of stochastic interest rates, early exercise, and dividend payments. The critical assumption, as pointed out by Merton, is the continuity of trading and the underlying asset price dynamics. Merton (1976) further extended the model to incorporate discontinuous "jumps" in stock prices, capturing the empirically observed negative skewness and excess kurtosis implied by option prices.

The practical applicability of option pricing models has been enhanced through the development of numerical and approximation methods. Cox, Ross, and Rubinstein (1979) presented a simple discrete-time model that yields the Black-Scholes formula as a special limiting case, illustrating the economic principles of option pricing in a more intuitive setting. Their approach gives rise to efficient numerical procedures for valuing options with early exercise.

Empirical investigations have played a crucial role in assessing the performance of alternative option pricing models. Bakshi, Cao, and Chen (1997) conducted a comprehensive study, developing a model that admits stochastic volatility, stochastic interest rates, and random jumps. They found that incorporating stochastic volatility and jumps is important for pricing and internal consistency, while modeling stochastic volatility alone yields the best hedging performance.

Heston (1993) proposed a closed-form solution for options with stochastic volatility, allowing for arbitrary correlation between volatility and spot asset returns. This model has been widely adopted due to its analytical tractability and ability to capture the "smile" pattern observed in implied volatilities across strike prices.

The scope of option pricing theory has expanded beyond its initial focus on equity options, with applications to various financial instruments and economic problems. Merton (1973) demonstrated how corporate securities can be viewed as contingent claims on the firm's assets, opening up new avenues for the application of option pricing techniques.

2.2 Real Options in the Energy Sector

Real options analysis (ROA) is a robust framework for managing the inherent uncertainties and capturing the strategic value of flexibility in investment decisions, particularly within the volatile

[1] The Black-Scholes model is based on the fundamental insight that, under certain assumptions, the price of an option can be determined by constructing a riskless hedge using the underlying asset and a risk-free bond. The model is derived using stochastic calculus, specifically the Itô's lemma, which leads to a partial differential equation (PDE) that the option price must satisfy. The solution to this PDE, subject to appropriate boundary conditions, yields the famous Black-Scholes formula for pricing European call and put options.

energy sector. This section delves into the traditional approaches to real options valuation in the energy sector, drawing insights from key studies.

Kozlova (2017) examines the application of real options analysis to renewable energy projects, with a focus on support schemes like feed-in tariffs and renewable energy certificates. Kozlova highlights the importance of policy-driven incentives in shaping investment decisions in renewable energy. By applying a real options framework, the study demonstrates how these support schemes can enhance the value of renewable energy investments by providing the flexibility to delay, expand, or abandon projects based on changing market conditions and policy landscapes. This flexibility is crucial for mitigating the risks associated with the high capital costs and long payback periods typical of renewable energy projects.

The work by Santos et al. (2014) explores the valuation of energy efficiency projects using real options analysis. The study emphasizes the limitations of traditional valuation methods, such as discounted cash flow, which often fail to capture the value of managerial flexibility in responding to technological advancements and market uncertainties. By applying a real options approach, the authors illustrate how incorporating the ability to delay or stage investments can lead to more accurate valuations and better decision-making in energy efficiency projects. This approach is particularly relevant for projects with significant initial costs and uncertain future benefits, as it allows for a more strategic allocation of resources.

Siddiqui, Marnay, and Wiser (2007) investigate the application of real options analysis to distributed generation investments in the energy sector. Their research highlights the strategic value of flexibility in the deployment of distributed generation technologies, such as microgrids and combined heat and power (CHP) systems. By using a real options framework, the study captures the value of options to expand, contract, or defer investments in response to changing market conditions, regulatory environments, and technological developments. This flexibility is essential for managing the risks and uncertainties inherent in the deployment of distributed generation technologies, which are often influenced by fluctuating energy prices and evolving policy landscapes.

Venetsanos, Angelopoulou, and Tsoutsos (2002) apply the Black-Scholes option pricing model to renewable energy projects, demonstrating the inadequacies of traditional net present value (NPV) approaches in capturing the full value of investments under uncertainty. Their study emphasizes the positive option value derived from managerial flexibility, such as the ability to delay or abandon projects. By incorporating real options analysis, the authors highlight the importance of accounting for uncertainty and the strategic value of flexibility in investment decisions.

Boomsma, Meade, and Fleten (2012) extend the application of real options to renewable energy investments, focusing on the impact of different support schemes like subsidies and feed-in tariffs. Their findings reveal that the option to delay investment until more favorable conditions arise significantly enhances project valuation. This is crucial in the renewable energy sector, where policy changes and technological advancements can drastically affect project viability and attractiveness.

Fernandes, Cunha, and Ferreira (2011) focus on the traditional energy investments such as oil and gas. They critique traditional discounted cash flow (DCF) methods for not adequately capturing the value of flexibility in investment decisions. By applying real options analysis, they demonstrate how considering the stochastic behavior of oil prices and the ability to adjust investment timing can lead to more accurate and strategic valuations.

2.3 Machine Learning in Finance and Valuation

In the context of applying machine learning techniques to financial valuation and option pricing, several recent studies have made significant contributions to the literature. Fischer and Krauss (2018) present a comprehensive study on the application of Long Short-Term Memory networks for large-scale financial market prediction tasks using S&P 500 data. Their research provides a detailed methodology for implementing LSTM networks in financial time series prediction, demonstrating superior performance compared to other classification methods. The study challenges the semi-strong form of market efficiency, showing statistically and economically significant returns. It also analyzes the risk exposure of LSTM strategies and develops a profitable trading strategy based on LSTM predictions. The authors note, however, that market efficiency has increased post-2010, impacting the predictive power of LSTM models.

Sezer, Gudelek, and Ozbayoglu (2020) focus on the application of Deep Learning (DL) models, such as Convolutional Neural Networks (CNNs), Long-Short Term Memory, and Deep Reinforcement Learning (DRL), in financial time series forecasting. Their literature review addresses the gap in existing surveys by providing a comprehensive analysis of DL models in the context of financial predictions. The study highlights the increasing interest in DL models over traditional machine learning techniques and emphasizes the need for better forecasting models in the finance industry. By categorizing studies based on forecasting implementation areas and DL model choices, the paper aims to guide researchers and model developers in effectively implementing DL models.

Liu, Oosterlee, and Bohte (2019) explore a data-driven approach using Artificial Neural Networks (ANNs) to price financial options and compute implied volatilities. Their work aims to enhance the efficiency of numerical methods in computational finance by training ANNs to approximate option prices and implied volatilities based on different financial models and parameters. The study focuses on the Black-Scholes model, the Heston stochastic volatility model, and the iterative root-finding Brent method for implied volatility. The researchers demonstrate the ANN's ability to accurately approximate option prices and implied volatilities without requiring the characteristic function of the financial model during testing, significantly reducing computing time for pricing financial options, particularly in high-dimensional financial models.

In the domain of electricity price forecasting, Lago, De Ridder, and De Schutter (2018) introduce an innovative modeling framework utilizing deep learning methodologies. The authors propose four distinct deep learning architectures and compare their predictive accuracy against 27 prevalent methods used in electricity price forecasting. This investigation addresses a significant gap in the field by thoroughly examining the application of deep learning algorithms in this domain, which has been relatively unexplored. The results show that these methods surpass state-of-the-art techniques, delivering statistically significant enhancements in predictive accuracy.

2.4 Machine Learning Approaches to Cash Flow Prediction

While the application of machine learning techniques to cash flow prediction in the energy sector remains relatively unexplored, several studies have demonstrated the potential of these methods in related financial forecasting tasks. This emerging body of research provides valuable insights that can be adapted to our specific context of real options valuation in the energy industry.

Zhu, Yan, and Bai (2022) proposed a Backpropagation Neural Network model based on an improved genetic algorithm for predicting enterprise free cash flow. Their approach, which incorporated a sliding window technique, demonstrated superior prediction accuracy compared to traditional methods. The study underscored the potential of hybrid models that combine evolutionary algorithms with neural networks to enhance the robustness of cash flow forecasts.

In a comparative study, Hongjiu, Rieg, and Yanrong (2012) evaluated the performance of various artificial intelligence methods for cash flow prediction. Their research examined Response Surface Models, Backpropagation Neural Networks, Radial Basis Function Neural Networks, and Support Vector Machines. The results indicated that Support Vector Machines exhibited the most robust performance for cash flow forecasting, particularly in scenarios with limited historical data. This finding is particularly relevant to our research, given the often constrained availability of historical cash flow data in energy sector projects.

These studies collectively highlight the potential of machine learning approaches to enhance the accuracy and reliability of cash flow predictions. By leveraging techniques such as neural networks, support vector machines, and hybrid models, we can potentially overcome some of the limitations associated with traditional valuation methods in capturing the complex dynamics of energy sector investments.

2.5 Integrating Deep Learning with Real Option Valuation within the Energy Sector

The integration of deep learning techniques, valuation methods, and energy sector projects remains a relatively unexplored area in the current literature. While there has been a growing interest in applying machine learning and deep learning models to various financial and economic problems, the specific application of these techniques to the valuation of energy sector projects, particularly in the context of real options analysis, is still limited.

The study of Bachouch et al. (2021) is one of the few studies that bridge the gap between deep learning and stochastic control problems, with some applications in the energy sector. The authors propose deep learning-based algorithms, such as NNcontPI, Hybrid-Now, and Hybrid-LaterQ, which extend actor-critic methods from reinforcement learning to solve discrete-time stochastic control problems with a finite time horizon. They demonstrate the efficiency and effectiveness of these algorithms through numerical tests on various examples, including high-dimensional nonlinear PDEs, quadratic Backward Stochastic Differential Equations, and linear quadratic stochastic control problems.

While the paper touches upon the application of these algorithms to energy storage problems, such as gas storage and microgrid management, it does not specifically focus on the valuation of energy sector projects using real options analysis. The study primarily aims to introduce deep learning algorithms for finite-horizon control problems and showcase their applicability across different domains, rather than delving into the specific challenges and considerations of energy project valuation.

The scarcity of research on the intersection of machine learning, valuation, cash flow forecasting, and energy sector projects highlights a significant gap in the literature. The energy sector is characterized by complex decision-making processes, uncertainties, and long-term investments, making it an ideal candidate for the application of advanced valuation techniques like real options analysis. Incorporating deep learning models into these valuation frameworks could potentially enhance the accuracy and efficiency of project assessments, risk management, and strategic decision-making.

This thesis aims to fill this gap by exploring the integration of deep learning techniques with real options valuation methods, specifically focusing on energy sector projects. By leveraging the power of deep learning to capture complex patterns, dependencies, and uncertainties in energy markets and project dynamics, this research seeks to develop novel approaches for forecasting and subsequently valuing energy investments. The thesis will build upon the foundation laid by studies like Bachouch et al. (2021), while addressing the unique challenges and opportunities at the intersection of deep learning, valuation, and the energy sector.

In conclusion, the limited research on the integration of deep learning, cash flow forecasting, valuation, and energy sector projects presents a significant opportunity for this thesis to contribute to the advancement of knowledge and practice in this field. By bridging the gap between these domains, the thesis aims to unlock new insights, methodologies, and tools that can support more informed and effective decision-making in the energy sector, ultimately contributing to the sustainable development and management of energy resources.

3. Theoretical Framework and Methodologies

This section elucidates the theoretical underpinnings and methodological approaches employed in our investigation of real options valuation within the energy sector, harnessing the power of deep learning techniques. We commence by introducing Recurrent Neural Networks, a class of advanced machine learning models particularly adept at processing sequential data and capturing intricate temporal dependencies. We then juxtapose these cutting-edge approaches with traditional valuation methods, such as the Black-Scholes model, highlighting the potential of RNNs—specifically Long Short-Term Memory and Gated Recurrent Units (**GRUs**)—to surmount the limitations of conventional techniques. Furthermore, we unveil the innovative Time2Vec (**T2V**) layer, a groundbreaking approach to encoding temporal information that amplifies the predictive prowess of our models. In subsequent subsections, we delve into the application of these sophisticated techniques within the energy industry and delineate the dataset underpinning our study.

3.1 Recurrent Neural Networks

Recurrent Neural Networks have emerged as a formidable tool in the deep learning arsenal, demonstrating unparalleled capability in deciphering non-linear relationships and complex temporal dependencies inherent in time series data (Goodfellow, Bengio, and Courville 2016). In stark contrast to conventional statistical models, which are constrained by a predetermined number of lagged values, RNNs possess an inherent faculty to retain and utilize information from previous time steps. This mnemonic mechanism empowers RNNs to model intricate patterns and dynamics in sequential data, rendering them exceptionally well-suited for financial time series forecasting tasks (Preeti, Bala, and Singh 2019; Kim et al. 2004).

The application of RNNs in real options valuation holds immense promise, stemming from their capacity to assimilate non-linear and non-stationary characteristics of financial time series without necessitating explicit differencing or logarithmic transformations for stationarity. By harnessing the computational might of RNNs, particularly variants such as Long Short-Term Memory and Gated Recurrent Units, we aspire to uncover latent patterns in estimating the theoretical value of options, thereby elevating the accuracy and reliability of our valuation framework.

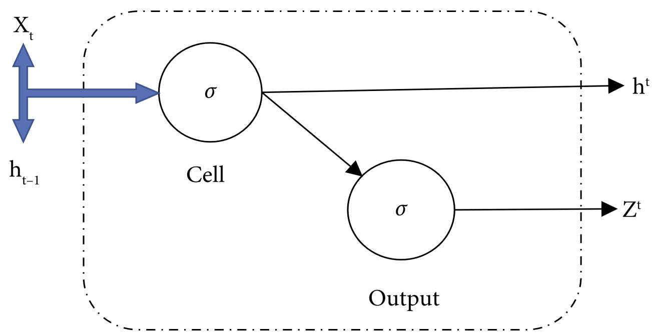

Figure 1. RNN Cell Unit Representation (Tomar et al. 2022).

3.1.1 Gated Recurrent Units

Gated Recurrent Units are a variant of RNNs designed to effectively capture and retain long-term dependencies in sequence data (Cho et al. 2014). GRUs address the vanishing gradient problem commonly encountered in traditional RNNs by introducing a gating mechanism consisting of an update gate and a reset gate. These gates control the flow of information, allowing the model to strike a balance between preserving relevant past information and incorporating new input data.

In our study, the GRU layer is parameterized with a variable number of units ranging from 64 to 128, with a step size of 32. The number of units determines the dimensionality of the output space and the capacity of the layer. The activation function used in the GRU layer is the hyperbolic tangent (`tanh`), which helps to center the data and return outputs between -1 and 1.

The mathematical formulation of GRUs is as follows:

$$
\begin{aligned}
z(t) &= \sigma(W_z \cdot [h(t-1), x(t)]) \\
r(t) &= \sigma(W_r \cdot [h(t-1), x(t)]) \\
\tilde{h}(t) &= \tanh(W \cdot [r(t) \cdot h(t-1), x(t)]) \\
h(t) &= (1 - z(t)) \cdot h(t-1) + z(t) \cdot \tilde{h}(t)
\end{aligned}
$$

where $z(t)$ is the update gate, $r(t)$ is the reset gate, $\tilde{h}(t)$ is the candidate hidden state, $h(t)$ is the current hidden state, and σ denotes the sigmoid function.

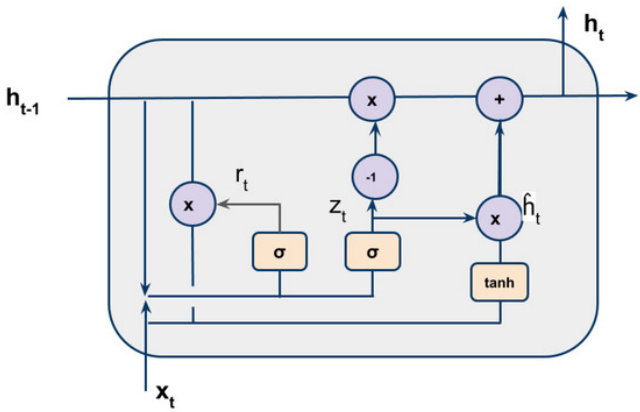

Figure 2. GRU Cell Unit Representation (Hosseini et al. 2020).

3.1.2 Long Short-Term Memory

Long Short-Term Memory units are another specialized type of RNN architecture designed to handle long-term dependencies in sequence data (Goodfellow, Bengio, and Courville 2016). LSTMs incorporate a gating mechanism similar to GRUs, but with an additional cell state and output gate. The forget gate, input gate, and output gate work together to control the flow of information, allowing the model to selectively remember or forget relevant information over varying time periods.

In our study, the LSTM layer is parameterized with a variable number of units, determining the dimensionality of the output space and the capacity of the layer. The activation function used in the LSTM cell state is the hyperbolic tangent (`tanh`), consistent with the GRU layer.

The mathematical formulation of LSTMs is as follows:

$$
\begin{aligned}
f(t) &= \sigma(W_f \cdot [h(t-1), x(t)] + b_f) \\
i(t) &= \sigma(W_i \cdot [h(t-1), x(t)] + b_i) \\
\tilde{C}(t) &= \tanh(W_C \cdot [h(t-1), x(t)] + b_C) \\
C(t) &= f(t) * C(t-1) + i(t) * \tilde{C}(t) \\
o(t) &= \sigma(W_o \cdot [h(t-1), x(t)] + b_o) \\
h(t) &= o(t) * \tanh(C(t))
\end{aligned}
$$

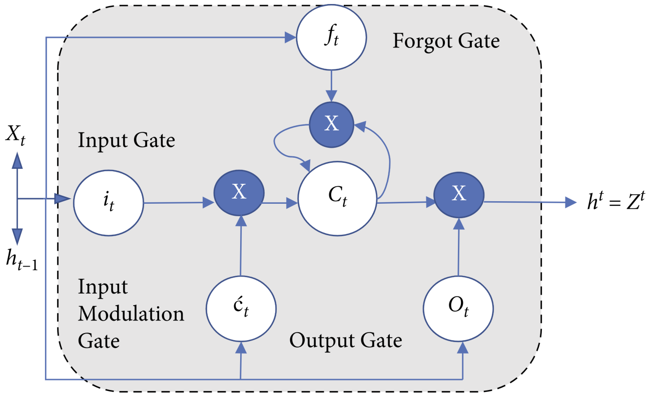

Figure 3. LSTM Cell Unit Representation (Tomar et al. 2022).

where $f(t)$, $i(t)$, and $o(t)$ are the forget, input, and output gates, respectively, $C(t)$ is the cell state, $\tilde{C}(t)$ is the candidate cell state, $h(t)$ is the hidden state, and σ denotes the sigmoid function.

By leveraging the power of RNNs, particularly GRUs and LSTMs; we aim to capture the complex dynamics and uncertainties inherent in real life valuation scenarios. This innovative approach has the potential to enhance the accuracy and reliability of real options valuation in the energy sector, providing valuable insights for investment decision-making and risk management.

3.1.3 Time2Vec

Time2Vec (**T2V**) represents an innovative algorithmic strategy that enhances the predictive capabilities of Recurrent Neural Networks by encoding temporal data into vector representations. Serving as a layer within RNNs, T2V transforms the temporal information of the input data into a higher-dimensional vector space, where each dimension can potentially encode different temporal aspects. This capability allows the model to retain both ordinal and cyclical characteristics of time, embedding additional temporal dynamics and significantly bolstering the predictive efficacy of Gated Recurrent Units and Long Short-Term Memory cells.

The T2V layer is composed of a linear term and a sum of weighted sine functions; the linear term models non-periodic components and aids in extrapolation, while the weighted sine functions capture the periodic behavior of time series. This is mathematically expressed as follows Kazemi et al. 2019:

$$a(\tau, k)[i] = \theta_{i,0}(\omega_0\tau + \phi_0) + \sum_{j=1}^{k} \theta_{i,j} \sin(\omega_i\tau + \phi_i)$$

Here, $(a(\tau, k)[i]$ symbolizes the $i - th$ component of the output vector, τ is the input time value, and k stands for the number of sine functions utilized. During the training phase, the T2V layer optimizes the learned parameters—such as the number of sine functions and the weights assigned to each function—by minimizing a loss function. These learned parameters are pivotal in determining the dimensionality of the output vector and the importance of each frequency component. Upon optimizing these parameters, they can be employed to convert time values into vector representations, of temporal patterns; hoping to enhance the predictive capability. In this case, applied for cash flow predictions.

3.2 Data Processing and Model Architecture

We apply our methodology to a case study within the European energy generation industry. This empirical analysis involves selecting a representative set of energy companies, collecting relevant data, and applying our deep learning and real options valuation framework. The case study aims to demonstrate the practical utility of our approach in real-world investment scenarios, highlighting its ability to enhance cash flow prediction accuracy and optimize investment decisions under uncertainty.

3.2.1 Data

The first step in applying our framework involves collecting comprehensive datasets relevant to the energy sector. This includes historical data on MNCs financial statements and relevant metrics. In our case, data points are obtained from *Orbis Europe* (2022) database; which contains detailed data on approximately 128 million companies, financial institutions, and insurers across Europe. This database provided us with a comprehensive range of essential data for our analysis, including measures on environmental, social, and climate risks through the ESG Score Predictor module. Utilizing this reliable source ensures the accuracy and relevance of the data used in our study. Data processing involves several critical steps:

- Data Cleaning: Addressing missing values, outliers, and inconsistencies to ensure data quality.
- Exploratory Data Analysis (**EDA**): Creating visualization and statistical summary tools to understand the used data.
- Normalization: Scaling data to a standard range to facilitate better model convergence.
- Feature Engineering: Creating new features that capture important patterns, such as lagged variables, moving averages, and volatility measures.
- Time Series Encoding: Utilizing the Time2Vec layer to encode temporal data, capturing both ordinal and cyclical characteristics of time.

3.2.2 Training and Validation

The neural network models are trained on the preprocessed datasets using a supervised learning (labeled target variable) approach. We divide the data into training, and test sets, typically using a 80-20 split. The models are trained to predict future cash flows and other relevant financial metrics, leveraging historical data. Key steps in the training process include:

- Loss Function: Using mean squared error (MSE) for regression tasks to minimize prediction errors.
- Optimization: Employing the Adam optimizer for efficient gradient-based optimization.
- Regularization: Implementing techniques such as dropout and L2 regularization to prevent overfitting.

The models' performance is evaluated on the validation set, tuning hyperparameters to achieve optimal predictive accuracy. Once validated, the models are tested on the test set to assess their generalization capability.

3.2.3 Real Options Valuation with Black-Scholes

To integrate real options analysis, we use the predicted cash flows from the neural network models as inputs into the Black-Scholes-Merton framework. This involves calculating the present value of expected cash flows and incorporating volatility estimates to value the flexibility inherent in investment decisions. The Black-Scholes model for real options valuation is expressed as:

$$C = S_0 N(d_1) - X e^{-rT} N(d_2)$$

where d_1 and d_2 are calculated as:

$$d_1 = \frac{\ln(S_0/X) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

Here, S_0 is the present value of the predicted cash flows, X is the initial investment cost, r is the risk-free interest rate, T is the time to expiration, and σ is the volatility of the project value.

4. Data

To test our methodology for predicting cash flows in the energy sector, we utilize a comprehensive dataset of financial variables for European oil, gas, and power companies. The selection of variables is informed by recent literature on cash flow prediction, particularly the work of Ball and Nikolaev (2022), who demonstrate the importance of considering both accrual-based earnings measures and operating cash flows when forecasting future cash flows.

Our dataset comprises key financial metrics for the years 2019 through 2023, focusing on the upper quartile of companies within the sector in regard to total revenue (as we find these companies to be the most consistent in financial reporting). All financial data is reported in millions of USD, providing a consistent basis for comparison across companies and time periods. The inclusion of data from 2019 is particularly valuable as it allows us to account for pre-pandemic financial conditions and subsequent adjustments, offering insights into the sector's resilience and adaptability in the face of significant global disruptions.

This approach allows us to analyze the most significant players in the European energy market. The variables include operating revenue, net income, cash flow (defined as net income before depreciation and amortization), total assets, profit margins, solvency ratios, cost of goods sold, EBITDA, and cash and cash equivalents. These metrics were chosen to capture various aspects of financial performance and position that are theoretically and empirically linked to future cash flows.

The inclusion of both accrual-based measures (such as net income and EBITDA) and cash-based measures (such as operating cash flow) aligns with the findings of Ball and Nikolaev (2022), who show that accrual-based earnings often dominate operating cash flows in predicting future cash flows, particularly when firm heterogeneity is accounted for. By incorporating multiple years of data, we can capture trends and potential cyclicality in the energy sector, which is known for its volatility. The solvency and profitability ratios provide insights into the financial health and efficiency of the companies, which can be indicative of future cash flow generating ability. Additionally, the inclusion of cost of goods sold and cash and cash equivalents allows us to consider the impact of operational efficiency and liquidity on future cash flows.

This rich dataset, sourced from the *Orbis Europe* database, provides a solid foundation for applying our deep learning models to predict cash flows in the energy sector. By focusing on the upper quartile of companies, we can examine how well our models perform for the industry leaders, which often have more stable and predictable financial patterns. However, it is important to note that this focus may limit the generalizability of our findings to smaller or more volatile companies within the sector.

4.1 Data Preparation

To ensure the robustness and reliability of our analysis, we implemented a comprehensive data processing strategy. Our dataset initially encompassed a wide range of financial metrics for European energy sector companies over the period 2019-2023. The processing steps were as follows:

1. **Data Truncation:** We confined our analysis to companies with full reported information. Although ideally we would try to fetch this information or find imputation methods, we find

that confining the dataset still yields a relevant amount of companies (118); whilst eliminating any potentially illogical values that might arise from imputation. This threshold ensures focus on significant market players, mitigating potential noise from smaller entities and incomplete rows.

2. **Data Validation:** Post-deletion, we rigorously verified the completeness of our dataset to ensure no missing values remained.

This data processing approach yields a robust dataset that preserves the temporal characteristics of financial data while addressing the challenges posed by missing information. The resulting dataset forms a solid foundation for our subsequent analysis and modeling of cash flows in the energy sector, aligning with the methodological rigor required for real options valuation in this complex and dynamic industry.

4.2 Exploratory Data Analysis

After processing our dataset, we conducted an in-depth exploratory analysis to uncover key insights into the financial characteristics of leading energy companies.

4.2.1 Descriptive Statistics

Our sample comprises 118 firms spanning the crude oil, natural gas, and electricity sectors. The summary statistics highlight considerable variation in financial metrics:

- 2023 Operating Revenue spans from 0.46 million to 316,620 million USD, averaging 13,106 million USD.
- Net Income ranges widely, from -1,274 million to 21,384 million USD, with a mean of 866 million USD.
- Total Assets vary substantially, from 1.39 million to 406,270 million USD, averaging 21,718 million USD.

This broad spectrum of values reflects the diverse landscape of company sizes and financial performances within our dataset.

4.2.2 Variable Distributions

The histograms of financial metrics (Figure 4) generally exhibit right-skewed distributions, indicating that a small number of large firms significantly influence industry averages. This skew is particularly evident in metrics such as Operating Revenue, Total Assets, and EBITDA. In contrast, Profit Margin and Solvency Ratio display more balanced distributions, suggesting a more uniform spread of financial health across companies.

4.2.3 Correlation Analysis

The correlation heatmap (Figure 5) unveils strong relationships between several financial indicators:

- Operating Revenue shows robust positive correlations with Net Income (0.89), Cash Flow (0.96), Total Assets (0.95), and EBITDA (0.98).
- COGS exhibits a strong negative correlation with Operating Revenue (-0.99), reflecting the inverse relationship between revenue and cost of goods sold.
- Interestingly, Profit Margin and Solvency Ratio demonstrate weak correlations with other metrics, suggesting these ratios capture unique aspects of financial performance not directly mirrored in absolute financial figures.

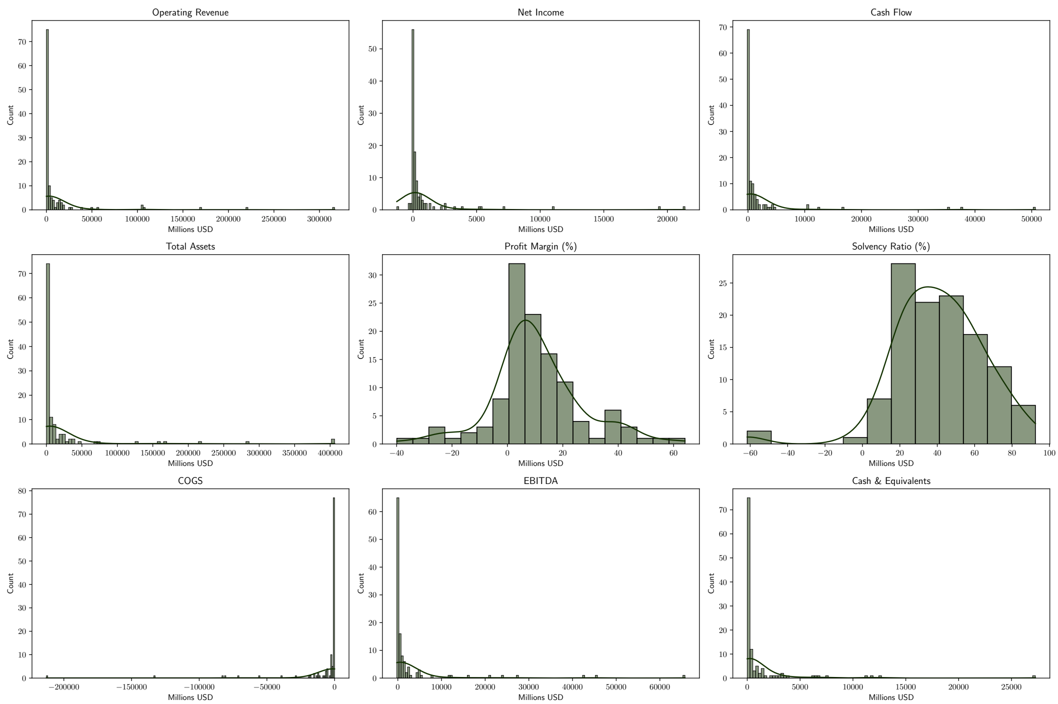

Figure 4. Distribution of Key Financial Metrics for 2023.

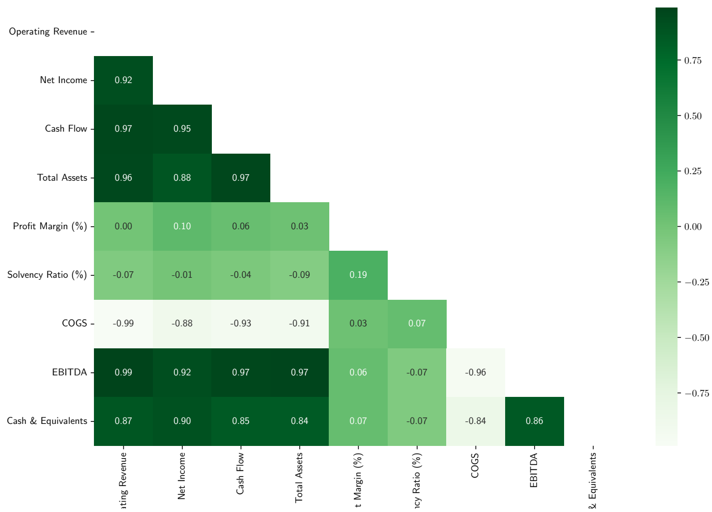

Figure 5. Correlation Heatmap of Financial Metrics for 2023.

4.2.4 Financial Trends by Industry

Examination of financial patterns from 2019 to 2023 (Figure 6) reveals distinct industry-specific trends:

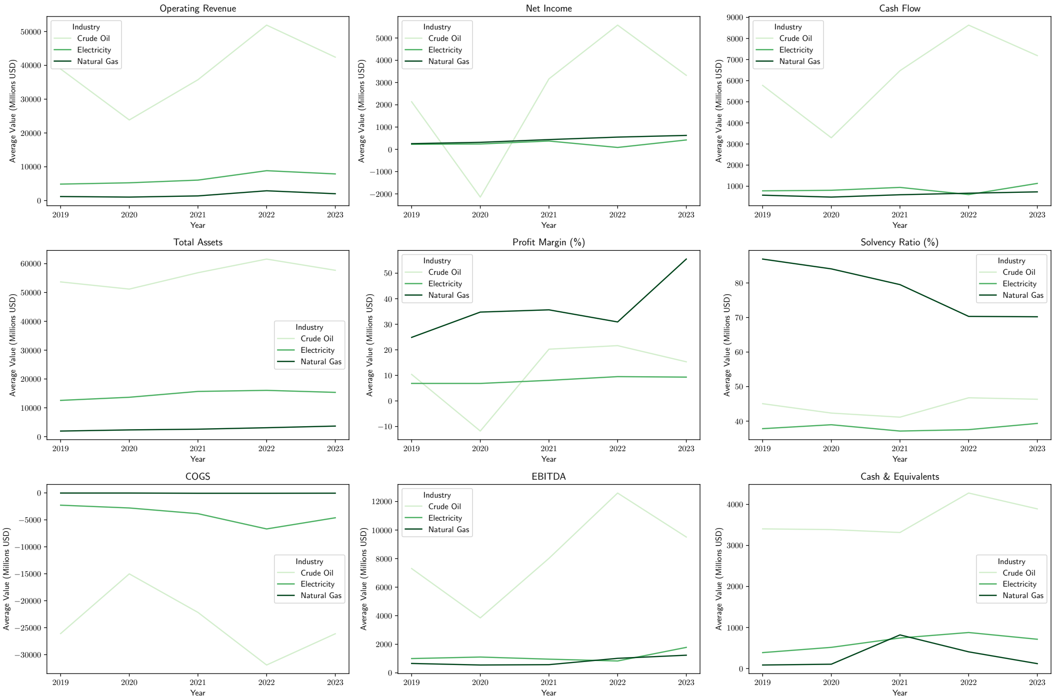

Figure 6. Time Trends of Financial Metrics by Industry (2019-2023).

- Crude Oil firms consistently display higher absolute values across most financial metrics, particularly in Operating Revenue, Total Assets, and EBITDA. This aligns with the capital-intensive nature and large-scale operations typical of the oil industry.
- Natural Gas companies stand out with notably lower COGS compared to other sectors. This could stem from the nature of gas extraction and processing, which often entails lower variable costs once infrastructure is established. Moreover, these firms boast higher Profit Margins and Solvency Ratios, hinting at potentially more stable and lucrative operations in this sector.
- Electricity firms exhibit more consistent trends over the years, especially in metrics like Total Assets and Solvency Ratio. This stability might reflect the regulated nature of many electricity markets and the long-term infrastructure investments characteristic of this sector.
- All sectors show a noticeable dip in financial performance in 2020, likely mirroring the impact of the COVID-19 pandemic, followed by a recovery in subsequent years.

This exploratory analysis lays the groundwork for understanding the financial dynamics of top-performing energy companies across different sectors, paving the way for our subsequent cash flow prediction and real options valuation models.

5. Model Evaluation

Having explored the financial landscape of leading energy companies, we now turn our attention to the core of our study: predicting cash flows and evaluating the effectiveness of various modeling approaches. This section outlines our methodology for developing and comparing different predictive models, ranging from traditional statistical methods to advanced machine learning techniques.

Our evaluation process unfolds in three key stages:

5.1 Baseline Regression

We begin by establishing a benchmark using a straightforward auto-regressive (**AR**) model. This time-tested approach serves as our baseline, providing a point of comparison for more sophisticated techniques. The AR model's performance will help us gauge the added value of more complex methodologies in capturing the nuances of cash flow dynamics in the energy sector. The most basic form of this model can be expressed as:

$$y_t = \phi \cdot y_{t-1} + \epsilon_t$$

However, our initial analysis of the cash flow data revealed important characteristics that necessitate a more nuanced approach. We conducted Augmented Dickey-Fuller (**ADF**) and Kwiatkowski-Phillips-Schmidt-Shin (**KPSS**) tests on our 5-year cash flow series for each company. These tests are complementary and help us determine the stationarity of the time series. Table 1 presents the results for a sample of companies from each industry in our dataset.

Table 1. Stationarity Test Results for Sample Companies.

Company	Industry	ADF Statistic	KPSS Statistic
Shell plc	Crude Oil	-1.136	0.285
Électricité de France	Electricity	-4.170***	0.500**
Romgaz S.A.	Natural Gas	-0.251	0.309

Note: *p<0.1; **p<0.05; ***p<0.01.

The ADF test has a null hypothesis (H_0) of non-stationarity (unit root present), while the KPSS test has a null hypothesis of stationarity. Ideally, we aim to reject the null hypothesis of the ADF test and fail to reject the null hypothesis of the KPSS test to conclude that a series is stationary. However, in practice, we often encounter conflicting results, as seen in our sample.

For Shell plc and Romgaz S.A., we fail to reject the null hypothesis of both tests, indicating non-stationarity. Électricité de France presents an interesting case where we reject both null hypotheses, suggesting a more complex underlying process that may require further investigation.

Given these results, particularly the predominant indication of non-stationarity, we adopt an ARIMA(1,1,0) model as our refined baseline. This model incorporates first-order differencing to address the non-stationarity issue, while maintaining the autoregressive component. The ARIMA(1,1,0) can be expressed as:

$$\Delta y_t = \phi \cdot \Delta y_{t-1} + \epsilon_t$$

where $\Delta y_t = y_t - y_{t-1}$ represents the first difference of the series. This approach offers several advantages:

1. It addresses the non-stationarity issue revealed by our statistical tests, ensuring more reliable predictions.
2. It captures both the trend and the autoregressive nature of the cash flow series.
3. It maintains model simplicity, serving as an interpretable yet effective benchmark.

By employing this ARIMA(1,1,0) model, we establish a robust baseline that accounts for the time series' characteristics while remaining straightforward. This will provide a meaningful comparison point for our subsequent, more complex models, allowing us to quantify the potential improvements offered by advanced machine learning techniques in capturing the intricacies of cash flow dynamics in the energy sector.

While the ARIMA(1,1,0) model provides a baseline for our analysis, it's important to note its limitations in this context. As illustrated in Figures 7-9, the model's fit varies considerably across our

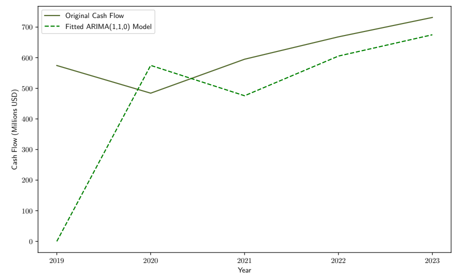

Figure 7. ARIMA(1,1,0) Model Fit for Romgaz S.A.

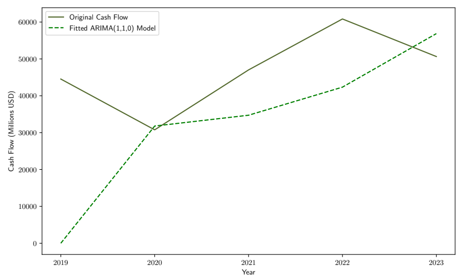

Figure 8. ARIMA(1,1,0) Model Fit for Shell plc.

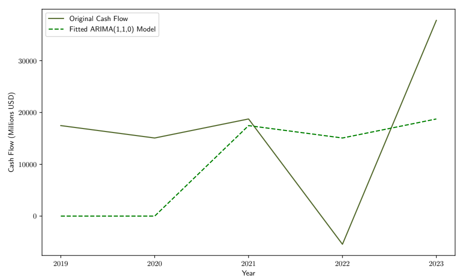

Figure 9. ARIMA(1,1,0) Model Fit for Électricité de France.

sample companies. The small sample size (only five annual data points) poses significant challenges. It limits our ability to conduct robust regression diagnostics or perform meaningful heteroskedasticity checks, which are crucial for ensuring the reliability of time series models. Moreover, with such limited data, the model's predictive power is inherently constrained. The graphs demonstrate that while the ARIMA model captures the general trend in some cases (e.g., Romgaz S.A.), it struggles to adequately represent the more volatile patterns observed in others (e.g., Shell plc and Électricité de France). This showcases a critical point: relying solely on traditional time series models like ARIMA may not be sufficient for accurate cash flow predictions in the energy sector, especially given the limited historical data available. This limitation provides a strong rationale for exploring more sophisticated techniques, such as the deep learning approaches we will discuss in subsequent sections, which may be better equipped to capture complex patterns even with limited data points, given more inputs that can be taken into account; such as the other variables we have discussed.

5.2 Deep Learning Architectures

Building on our baseline, we explore the potential of deep learning in enhancing cash flow predictions. Specifically, we implement recurrent neural network (RNN) architectures, known for their ability to capture temporal dependencies in sequential data. To optimize our RNN models, we employ a Bayesian grid search for hyperparameter tuning, aiming to strike a balance between model complexity and predictive accuracy.

Before feeding our financial data into the model, we perform two important preparatory steps. First, we scale all our numerical variables. This is in nature converting different units of measurement (say, meters and feet) into a common scale, ensuring that all financial metrics are treated equally by the model, regardless of their original magnitude. Second, we organize our data into company-specific *"windows"* of time (5-year period). For grasp, it is easy to imagine each company's financial history as a story - we're giving our model several chapters of this story at once, rather than isolated sentences. This approach allows the model to understand the context and progression of each company's financial situation over time, rather than looking at disconnected snapshots. By structuring our data this way, we enable the model to detect company-specific patterns and trends, leading to more accurate and personalized predictions.

Our deep learning approach centers on a sophisticated RNN model that incorporates several key components. At its core, we utilize a Time2Vec layer (Kazemi et al. 2019), which enhances the model's ability to capture temporal patterns by encoding time information into a higher-dimensional space. This is followed by bidirectional LSTM or GRU layers, allowing the network to process information both forwards and backwards in time, thereby capturing a more comprehensive view of the temporal dependencies in our financial data.

The model architecture is further augmented with an attention mechanism, which enables the network to focus on the most relevant parts of the input sequence when making predictions. This is particularly valuable in our context, as certain financial indicators may carry more weight in predicting future cash flows at different points in time.

To determine the optimal configuration of our model, we employ a Bayesian optimization approach for hyperparameter tuning (Snoek, Larochelle, and Adams 2012). This method efficiently explores the hyperparameter space, including the choice between LSTM and GRU cells, the number of recurrent layers, the number of units in each layer, and the learning rate. The Bayesian optimization process iteratively refines the model configuration, guided by the validation mean absolute error (MAE) as the objective function. Table 2 presents the hyperparameter search space explored by our Bayesian optimization process, along with the best configuration found for our RNN model.

Table 2. Hyperparameter Search Space and Best Configuration.

Hyperparameter	Search Space	Best Value
Time2Vec kernel size	1 to 5	2
Number of RNN layers	1 to 3	2
RNN type	LSTM or GRU	LSTM
Units per layer	32, 64, 96, 128	96
Dropout	True or False	False
Learning rate	1e-2, 1e-3, 1e-4	1e-3

Figure 10 illustrates the structure of our best-performing model, as determined by the hyperparameter search. This architecture consists of a Time2Vec layer followed by two bidirectional LSTM layers, an attention layer, and a final dense layer for output.

To put this in simpler terms, our model can be thought of as a sophisticated pattern recognition system. The Time2Vec layer acts like a translator, converting raw time data into a form the model can better understand, much like translating dates into day-of-week patterns. The bidirectional LSTM layers are akin to two analysts reading a financial report - one from start to finish, the other from end to beginning - each picking up on different trends and patterns. The attention layer then works like a highlighter, emphasizing the most important pieces of information for making predictions. Finally, the dense layer combines all this processed information to produce our cash flow forecast, much like a financial advisor would synthesize various insights to make a recommendation. This layered approach allows our model to capture complex, time-dependent patterns in financial data that might be missed by simpler forecasting methods.

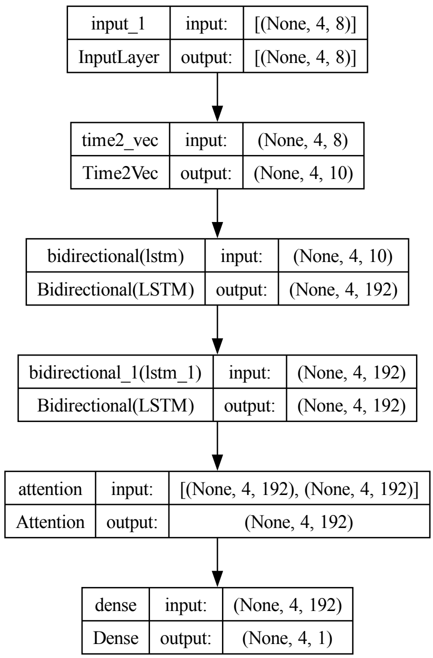

Figure 10. Architecture of the Best-Performing RNN Model.

The training process of our model is visualized in Figure 11, which shows the evolution of both training and validation loss over 100 epochs. This graph provides valuable insights into the model's learning dynamics and helps us assess potential overfitting or underfitting issues.

As observed in Figure 11, both the training and validation loss decrease rapidly in the initial epochs, indicating that the model is effectively learning from the data. The convergence of the training and validation loss curves suggests that our model achieves a good balance between fitting the training data and generalizing to unseen data. The slight oscillations in the validation loss towards the later epochs are typical in deep learning models and may be attributed to the complexity of the financial data and the relatively small dataset size.

This approach represents a significant advancement over traditional time series forecasting methods, leveraging the power of deep learning to capture complex, non-linear relationships in

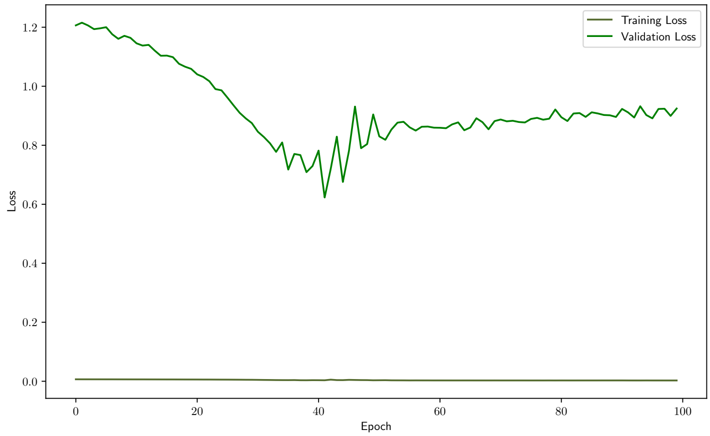

Figure 11. Training and Validation Loss Over Epochs.

financial data. By combining sophisticated neural network architectures with careful hyperparameter tuning, we aim to achieve superior predictive performance in cash flow forecasting for energy sector companies.

5.3 Performance Comparison

To evaluate the effectiveness of our models, we compare their performance using several key metrics. Table 3 presents a comprehensive comparison of the ARIMA(1,1,0) model and our RNN model, including both training and test performance for the latter.

Table 3. Performance Comparison of ARIMA and RNN Models.

Metric	ARIMA(1,1,0)	RNN (Train)	RNN (Test)
RMSE	3181.89	0.229	0.223
MAE	759.53	0.067	0.096
sMAPE (%)	80.51	18.33	37.48
R^2	0.706	0.809	0.987

These metrics provide different insights into model performance. The Root Mean Square Error (RMSE) and Mean Absolute Error (MAE) quantify the average magnitude of prediction errors, with RMSE giving more weight to larger errors. The Symmetric Mean Absolute Percentage Error (sMAPE) offers a percentage-based measure of prediction accuracy, ranging from 0 to 2, as defined by Chen and Yang (2004). This metric is particularly useful for comparing performance across different scales. The R-squared value indicates the proportion of variance in the dependent variable that is predictable from the independent variable(s), providing a measure of how well the model fits the data.

The results clearly demonstrate the superior performance of the RNN model over the ARIMA model. Even on the test set, which the RNN had not seen during training, it significantly outperforms the ARIMA model across all metrics. The RNN achieves lower RMSE and MAE values, indicating more accurate predictions on average. The substantially lower sMAPE for the RNN model suggests it produces predictions that are closer to the actual values in percentage terms. Furthermore, the high R-squared values for the RNN model, particularly on the test set, indicate an excellent fit to the data.

It's worth noting that the RNN model's performance is particularly impressive given the relatively short 5-year span of our dataset. This suggests that the RNN's ability to capture complex, non-linear relationships in the data allows it to extract more information from the available features compared to the simpler ARIMA model. The RNN's capacity to process multiple input features simultaneously, as opposed to the univariate nature of the ARIMA model, likely contributes significantly to its superior performance. These results exemplify the potential of deep learning approaches in enhancing cash flow predictions in the energy sector, even with limited historical data.

6. Real Option Valuation Application

7. Conclusions and Future Directions

7.1 Summary of Findings

7.2 Recommendations for Practitioners

7.3 Suggestions for Future Research

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Appendix 1.
Supplementary Data and Model Code

All code related to the scraping of data, processing, model setup, and data itself can be found in this public GitHub repository.