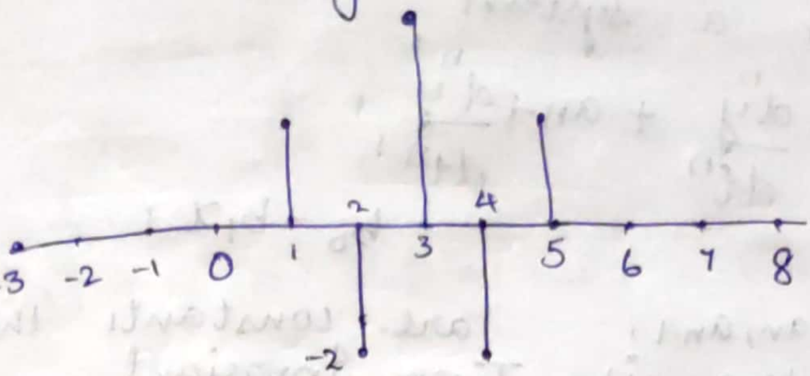
Assignment

1) a)

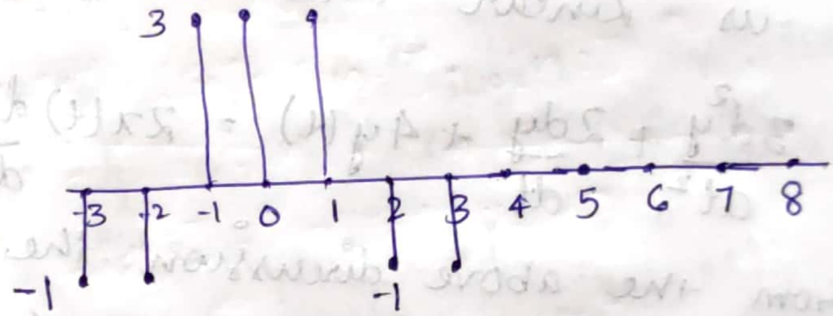


$$x[n] = \sum_{k=-\infty}^{\infty} a_k \, \delta[n-k]$$

$$= a_1 \delta[n-1] + a_2 \delta[n-2] + a_3 \delta[n-3] + a_4 \delta[n-4] + a_5 \delta[n-5]$$

$$\therefore x[n] = \delta[n-1] - 2\delta[n-2] + 3\delta[n-3] - 2\delta[n-4] + \delta[n-5]$$

b)



$$s[n] = a_3 \delta[n+3] + a_{-2} \delta[n+2] + a_{-1} \delta[n+1] + a_0 \delta[n] + a_1 \delta[n-1] + a_2 \delta[n-2]$$

$$= -1\left(u[n+3] - u[n+2]\right) + (-1)\left(u[n+2] - u[n+1]\right)$$
$$3\left(u[n+1] - u[n]\right) + 3\left(u[n] - u[n-1]\right)$$
$$+ 3\left(u[n-1] - u[n-2]\right) + (-1)\left(u[n-2] - u[n-3]\right)$$
$$+ (-1)\left(u[n-3] - u[n-4]\right)$$

$$\therefore s[n] = -u[n+3] + 4u[n+1] - 4u[n-2] + u[n-4]$$

2)

a) For a system

$$a_n \frac{d^n y}{dt^n} + a_{n-1} \frac{d^{n-1} y}{dt^{n-1}} + \cdots$$

$$= b_0 + b_1 x + \cdots$$

If $a_n, a_{n-1}, \cdots$ are constants then the system is Time Invariant.

If degree of the differential equation is 1 then system is linear

So the system

$$(t^2-3)\frac{dy}{dt} + 2ty = x$$

is Linear and Time Variant

b) $$3\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 4y(t) = 2x(t)\frac{dx}{dt}$$

From the above discussion the System is Linear, Time Invariant

c) $$\left(\frac{dy}{dt}\right)^2 + 2y(t) = x(t)$$

The system is Non-Linear, Time Invariant

d) $$y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau$$

by differentiating on both sides

$$y'(t) = x(t)$$

③ The system is Linear Time Invariant

e) $y[n] = \cos(wn)\, x[n]$

The system is Linear Time Variant

f) $y[n+1] + y^2[n] = 2x[n+1] - x[n]$

The system is Non Linear,

Time Invariant

Non Linear because of $y^2[n]$ part

3) a) $x(t) = \sin\left(10\pi t + \frac{\pi}{6}\right)$

$$= \frac{e^{i\left(10\pi t + \frac{\pi}{6}\right)} - e^{-i\left(10\pi t + \frac{\pi}{6}\right)}}{2}$$

$$C_n = \begin{cases} 1/2 \\ -1/2 \end{cases} = \frac{e^{i\pi/6} \cdot e^{i(10\pi t)} - e^{-i\pi/6} \cdot e^{-i(10\pi t)}}{2i}$$

$$= \frac{\sqrt{3}+i}{4i} e^{i(10\pi t)} - \frac{\sqrt{3}-i}{4i} e^{-i(10\pi t)}$$

Now period of $\sin(10\pi t + \pi/6)$ is $\frac{2\pi}{10\pi} = 1/5$

$$x(t) = \sum_{n=-\infty}^{\infty} f(n)\, e^{2\pi i n \frac{n}{T} t} = \sum_{n=-\infty}^{\infty} f(n)\, e^{10\pi i t n}$$

As fourier series is unique we can compare both sides to find the co-efficients

$$f(n) = \begin{cases} \frac{\sqrt{3}+i}{4i} \text{ or } \frac{1-i\sqrt{3}}{4} & \text{for } n=1 \\ -\left(\frac{\sqrt{3}-i}{4i}\right) \text{ or } +\frac{1+i\sqrt{3}}{4} & \text{for } n=-1 \\ 0 & \text{otherwise} \end{cases}$$

b) $x(t) = 1 + \cos 2\pi t$

$$= 1 + \frac{e^{i2\pi t} + e^{-i2\pi t}}{2}$$

Now the period of $x(t)$ is $\frac{2\pi}{2\pi} = 1$

$$x(t) = e^{i2\pi i(0)t} + \frac{1}{2}e^{i2\pi t} + \frac{1}{2}e^{-i2\pi t}$$

$$x(t) = \sum_{n=-\infty}^{\infty} f(n)\, e^{2\pi i t n}$$

By comparision we get

$$f(n) = \begin{cases} 1 & \text{for } n=0 \\ 1/2 & \text{for } n = 1, -1 \\ 0 & \text{otherwise} \end{cases}$$

c) $x(t) = [1 + \cos 2\pi t]\left[\sin\left(10\pi t + \frac{\pi}{6}\right)\right]$

$$= \left[1 + \frac{e^{i2\pi t} + e^{-i2\pi t}}{2}\right]\left[\frac{1-i\sqrt{3}}{4}e^{i10\pi t} + \frac{1+i\sqrt{3}}{4}e^{-i10\pi t}\right]$$

$$= \frac{1-i\sqrt{3}}{4}e^{i10\pi t} + \frac{1+i\sqrt{3}}{4}e^{-i10\pi t} + \frac{1-i\sqrt{3}}{8}e^{i12\pi t} + \frac{1-i\sqrt{3}}{8}e^{i8\pi t}$$

$$+ \frac{1+i\sqrt{3}}{8}e^{-i8\pi t} + \frac{1+i\sqrt{3}}{8}e^{-12i\pi t}$$

Scanned by CamScanner

(5)

$$x(t) = \sin\left(10\pi t + \frac{\pi}{6}\right) + \sin\left(10\pi t + \frac{\pi}{6}\right)\cos 2\pi t$$

$$= \sin\left(10\pi t + \frac{\pi}{6}\right) + \frac{2\sin(10\pi t + \pi/6)\cos 2\pi t}{2}$$

$$= \sin\left(10\pi t + \frac{\pi}{6}\right) + \frac{\sin\left(12\pi t + \frac{\pi}{6}\right) + \sin\left(8\pi t + \frac{\pi}{6}\right)}{2}$$

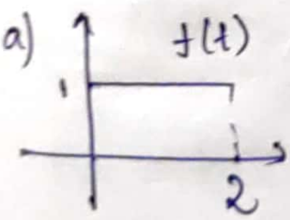
$[\because 2\sin A\cos B = \sin(A+B) + \sin(A-B)]$

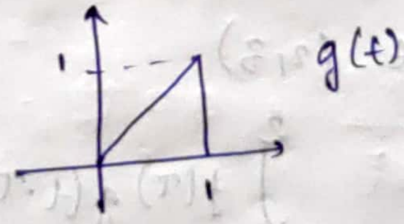
period of $x(t) = 1$

$$x(t) = \sum_{n=-\infty}^{\infty} f(n)\, e^{2\pi i t n}$$

$$f(n) = \begin{cases} \frac{1-i\sqrt{3}}{4} & \text{for } n=5 \\ \frac{1+i\sqrt{3}}{4} & \text{for } n=-5 \\ \frac{1-i\sqrt{3}}{8} & \text{for } n=6,4 \\ \frac{1+i\sqrt{3}}{8} & \text{for } n=-6,-4 \\ 0 & \text{otherwise} \end{cases}$$

4) a)

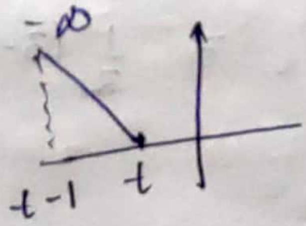




output $h(t) = f(t) * g(t)$

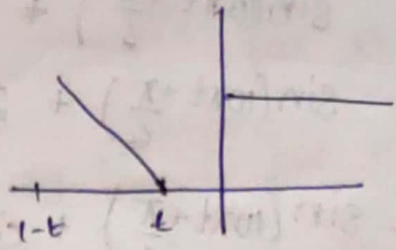
$$= \int_{-\infty}^{\infty} f(t)\, g(t-\tau)\, d\tau$$

$g(t-\tau)$ graph is



* For $t < 0$

the graph is



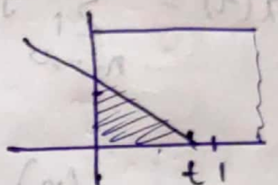
There is no common area

So $h(t) = 0$

* For $t \in (0,1)$

$$h(t) = \int_0^t f(\tau)\, g(t-\tau)$$

[∵ $f(t) = 1$]

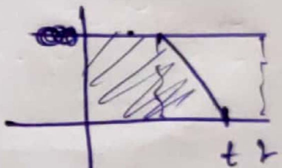


$$= \int_0^t 1\cdot(t-\tau)\,d\tau$$

$$= t^2 - \frac{t^2}{2} = \frac{t^2}{2}$$

* For $t \in (1,2)$

$$h(t) = \int_{t-1}^{t} 1\cdot(t-\tau)\,d\tau$$



$$= t(t-(t-1)) - \frac{t^2-(t-1)^2}{2}$$

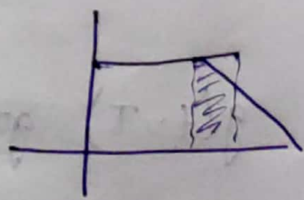
$$= 1/2$$

* For $t \in (2,3)$

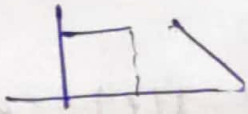
$$h(t) = \int_{t-1}^{2} f(\tau)\, g(t-\tau)\,d\tau$$

$$= \int_{t-1}^{2} (t-\tau)\,d\tau = t(3-t) - \frac{4-(t-1)^2}{2}$$

$$= \frac{1}{2}(3-t)(t-1)$$

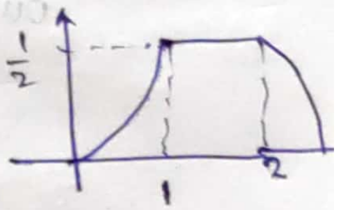


* For $t > 3$



$h(t) = 0$

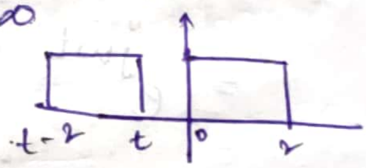
$$\therefore h(t) = \begin{cases} 0 & \text{for } t<0 \text{ and } t>3 \\ \frac{t^2}{2} & \text{" } t \in (0,1) \\ \frac{1}{2} & \text{" } t \in (1,2) \\ \frac{(3-t)(t-1)}{2} & \text{" } t \in (2,3) \end{cases}$$



b) Now $k(t) = f(t) * f(t)$

$$= \int_{-\infty}^{\infty} f(t)\, f(t-\tau)\, d\tau$$

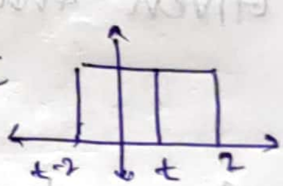
* For $t < 0$



$k(t) = 0$

* For $t \in (0,2)$

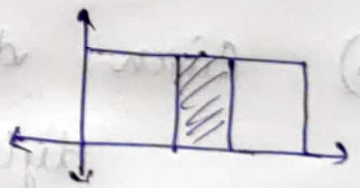
$$k(t) = \int_0^t f(\tau)\, f(t-\tau)\, d\tau$$



$$= t$$

* For $t \in (2,4)$

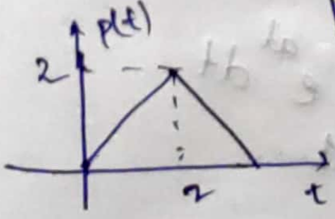
$$k(t) = \int_{t-2}^{2} 1 \cdot d\tau = 4-t$$



* For $t > 4$

$k(t) = 0$

$$\therefore k(t) = \begin{cases} 0 & \text{for } t<0,\ t>4 \\ t & \text{" } t \in (0,2) \\ 4-t & \text{" } t \in (2,4) \end{cases}$$



6)

a) Given $f(t) = e^{j\omega t}$

$$\text{output} = \int_{-\infty}^{\infty} f(\tau)\, h(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} f(t-\tau)\, h(\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} e^{j\omega(t-\tau)}\, h(\tau)\, d\tau$$

$$= \cancel{e^{j\omega t} \int_{-\infty}^{\infty} h(\tau)\, d\tau}.$$

$$= e^{j\omega t} \int_{-\infty}^{\infty} e^{-j\omega\tau}\, h(\tau)\, d\tau$$

Given that $H(\omega) = \int_{-\infty}^{\infty} h(\tau)\, e^{-j\omega\tau}\, d\tau$

So output $= e^{j\omega t} H(\omega)$

b) Given differential equation is

$$\frac{dy(t)}{dt} + a\,y(t) = x(t).$$

$$e^{\int a\,dt}\, y(t) = \int x(t)\, e^{\int a\,dt}\, dt$$

$$e^{\int a\,dt}\, y(t) = x(t) \int e^{\int a\,dt}\, dt$$

$$e^{at}\, y(t) = x(t) \int e^{at}\, dt.$$

$$\cancel{e^{at}}\, y(t) = \frac{x(t)\, \cancel{e^{at}}}{a}$$

$$\Rightarrow \quad y(t) = \frac{x(t)}{a} = \frac{e^{j\omega t}}{a}$$

Given $y(t) = H(\omega)\, e^{j\omega t}$

$$\therefore \quad H(\omega) = \frac{1}{a}$$

8) a) Given $x[n] = \delta[n-n_0]$

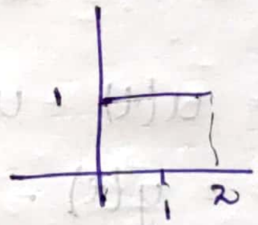
and $h[n] = \left(\frac{1}{2}\right)^n$

we know that output

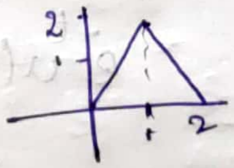
$$= \sum_{m=-\infty}^{\infty} x[m]\, h[n-m]$$

$$= \sum_{m=-\infty}^{\infty} x[n-m]\, h[m]$$

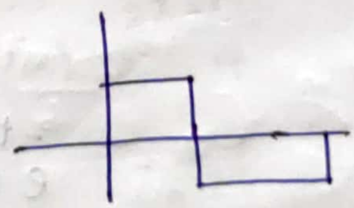
7) $x_1(t)$ is



$y_1(t)$ is



a) Now $x_2(t)$ is

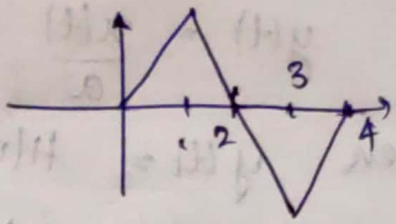


we can write $x_2(t) = x_1(t) - x_1(4-t)$

As the given system is LTI

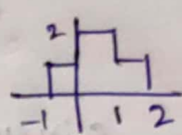
$$y_2(t) = y_1(t) - y_1(4-t)$$

So $y_2(t)$ is



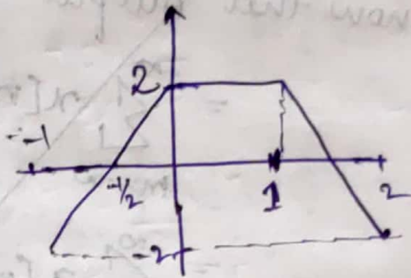
b) $x_3(t)$ is

$$x_3(t) = x(t+1) + x(t)$$



So $\quad y_3(t) = y(t+1) + y(t)$

So the sketch is



c) Given $\quad y(t) = e^{-t}u(t) + u(-1-t)$

$$x_1(t) = u(t) - u(t-1)$$

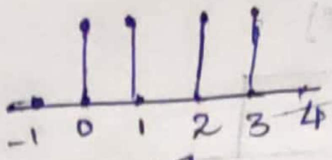
$$y_1(t) = y(t) - y(t-1)$$

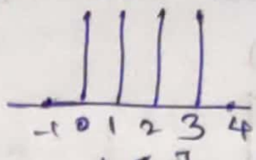
$$= e^{-t}u(t) + u(-1-t) - e^{-(t-1)}u(t-1) - u(-1-(t-1))$$

$$= e^{-t}u(t) + u(-1-t) - e^{-(t-1)}u(t-1) - u(-t)$$

9)

a)





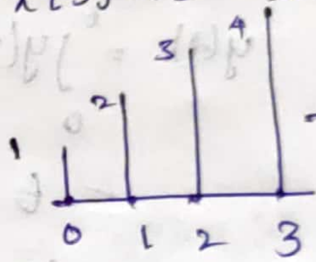
$$x[n] * h[n] = \sum_{m=-\infty}^{\infty} x[m]\, h[n-m]$$

$$y[n] = x[0]h[n] + x[1]h[n-1] + x[2]h[n-2] + x[3]h[n-3]$$

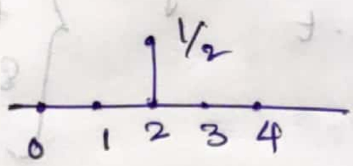
$y[0] = h[0] = 1$

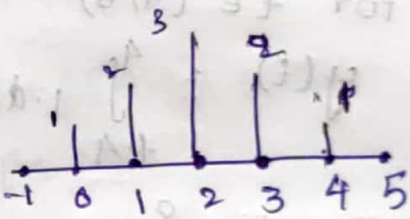
$y[1] = h[1] + h[0] = 2$

$y[2] = 3$, $y[3] = 4$



b)





$x[n] * h[n]$

$$y[n] = \sum_{m=-\infty}^{\infty} x[m]\, h[n-m]$$

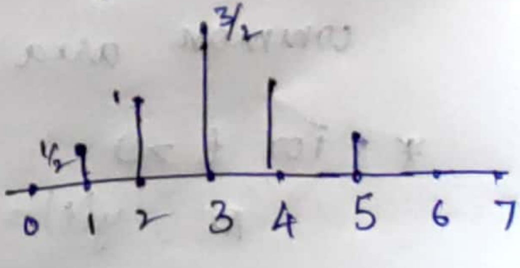
$$= \cancel{x[0]h[n]} + x[2]h[n-2]$$

$y[0], y[1] = 0$

$y[2] = \frac{1}{2} h[0] = 1/2$, $y[3] = \frac{1}{2} \times 2 = 1$

$y[4] = \frac{1}{2} \times 3 = 3/2$ $y[5] = \frac{1}{2} \times 2 = 1$

$y[6] = \frac{1}{2} \times 1 = 1/2$

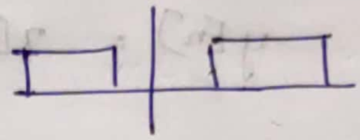


Continuous time convolution (12)

a)

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

* For $t < 0$

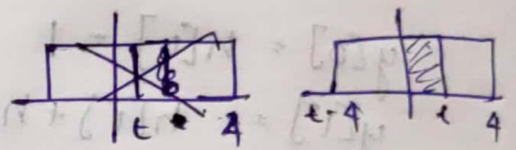


$y(t) = 0$

* For $t \in (0, 4)$

$y(t) = \int_0^t y(\tau) f(t-\tau)\, d\tau$

$= t$



* For $t \in (4, 8)$

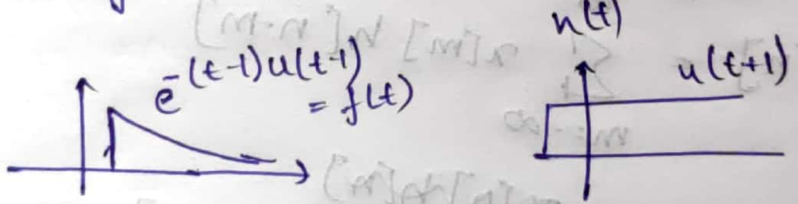
$y(t) = \int_{t-4}^{4} 1 \cdot d\tau = 8 - t$

$\therefore y(t) = \begin{cases} 0 & \text{for } t<0, t>8 \\ t & t \in (0,4) \\ 8-t & t \in (4,8) \end{cases}$

* For $t > 8$

$y(t) = 0$

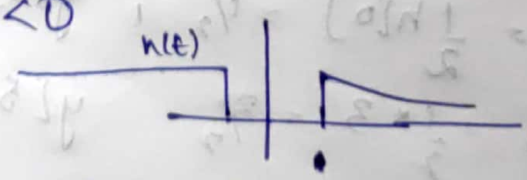
b)



let $f(t+1) = x(t)$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

* For $t < 0$

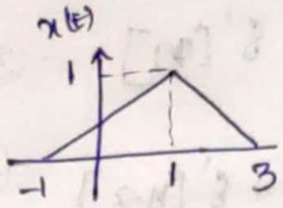


common area $= 0$

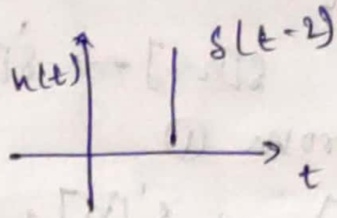
* For $t > 0$

$y(t) = \int_0^t e^{-t}\, dx = -(e^{-t})_0^t = e^{-t}$

$\therefore\; y(t) = \begin{cases} 0 & t<0 \\ e^{-t} & t>0 \end{cases}$

c) 

$x(t)$



$y(t) = x(t) * \delta(t-2)$

$= x(t-2)$

$y(t) = x(t-2)$ $\quad y(t) = \begin{cases} 0 & t<1 \\ t/3 & 1<t<3 \\ \frac{1}{2}(5-t) & 3<t<5 \\ 0 & t>5 \end{cases}$

8)

a) we can write

$x_1[n] = \delta[n]$

$x_2[n] = \delta[n] + \delta[n-1] \Rightarrow x_2[n] - x_1[n] = \delta[n-1]$

$x_3[n] = \delta[n] + \delta[n-1] + \delta[n-2]$

$= x_2[n] + \delta[n-2]$

$\Rightarrow \delta[n-2] = x_3[n] - x_2[n]$

$x_4[n] = \delta[n] - \delta[n-1] + \delta[n-2]$

$= x_1[n] - (x_2[n] - x_1[n]) + x_3[n] - x_2[n]$

$= 2x_1[n] - 2x_2[n] + x_3[n]$

b) As the system is linear we have

$y_4[n] = 2y_1[n] - 2y_2[n] + y_3[n]$

For y_1

$\delta[n] \rightarrow \delta'[n+1] + \delta'[n] + \delta'[n-1]$ — (1)

$\delta[n] + \delta[n-1] \rightarrow \delta'[n] + \delta'[n-1] + \delta'[n-2]$ — (2)

Now by comparing ① ② we get

$$\delta[n-1] \Leftrightarrow \delta'[n-2] - \delta'[n] \quad -③$$

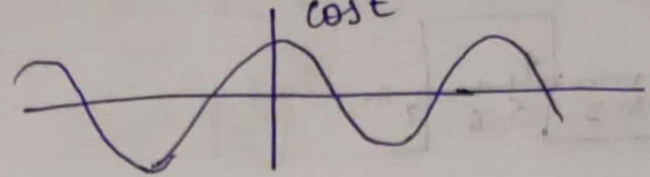
But from ①

$$\delta[n-1] \rightarrow \delta'[n] + \delta'[n-1] + \delta'[n-2] \quad -④$$

As ③, ④ are not equal the system is time variant

10)

a)



$f(t) = \cos t$

$$f(t) = \sum_{n=-\infty}^{\infty} x(n)\, e^{2\pi i \frac{nt}{T}}$$

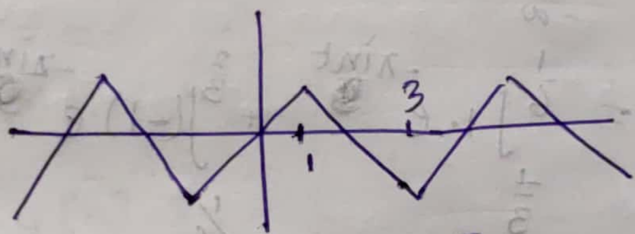
where $x(n) = \begin{cases} \frac{1}{2} & n = 1, -1 \\ 0 & \text{otherwise} \end{cases}$

here $T = 2\pi$

$$\cos t = \frac{e^{it} + e^{-it}}{2}$$

$$\cos t = \sum_{n=-\infty}^{\infty} x(n)\, e^{2\pi i \frac{n}{2\pi} t} = \sum_{n=-\infty}^{\infty} x(n)\, e^{int}$$

b)



the given function

$$f(t) = \sin^{-1}\left(\sin \frac{\pi t}{2}\right)$$

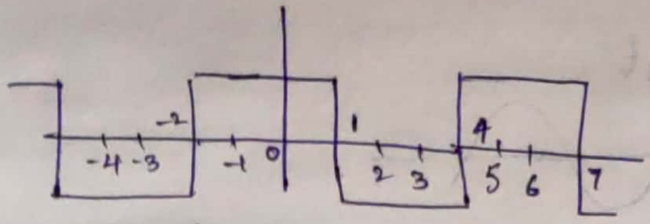
period $= 4$

$$f(t) = \sum_{n=-\infty}^{\infty} x(n)\, e^{2\pi i \frac{nt}{4}}$$

$$= \sum_{n=-\infty}^{\infty} x(n)\, e^{\frac{\pi i n t}{2}}$$

$$x_n = \int_0^4 \sin^{-1} \sin \frac{\pi t}{2}\, e^{-\frac{\pi i n t}{2}}\, dt$$

c)



period of this function is 6

$$f(t) = \begin{cases} 1 & t \in [-2, 1] \\ -1 & t \in [1, 4] \end{cases}$$

$$C_n = \int_{-\infty}^{\infty} f(6t)\, e^{-\frac{2\pi i n t}{6}}\, dt$$

$$C_n = \int_{-\infty}^{\infty} f(6t)\, e^{-\frac{\pi i n t}{3}}\, dt$$

$$= \int_{-\frac{1}{3}}^{\frac{1}{6}} 1 \cdot e^{-\frac{\pi i n t}{3}} + \int_{\frac{1}{6}}^{\frac{2}{3}} (-1)\, e^{-\frac{\pi i n t}{3}}\, dt$$

$$= \frac{-1}{n i \pi} \left[e^{-\frac{\pi i n t}{3}} \right]_{-1/3}^{1/6} + \frac{1}{n i \pi} \left[e^{-\frac{\pi i n t}{3}} \right]_{1/6}^{2/3}$$

$$= \frac{-1}{n i \pi} \left[e^{-\frac{n i \pi}{6}} - e^{\frac{n i \pi}{3}} \right] + \frac{1}{n i \pi} \left[e^{-\frac{2 i \pi n}{3}} - e^{-\frac{n i \pi}{6}} \right]$$

$$f(t) = \sum_{n=-\infty}^{\infty} C_n\, e^{\frac{2\pi i n t}{6}} = \sum_{n=-\infty}^{\infty} C_n\, e^{\frac{\pi i n t}{3}}$$

5) a) Given $x[n] = \delta[n - n_0]$

$$h[n] = \left(\frac{1}{2}\right)^n u[n]$$

$$y[n] = x[n] * h[n]$$

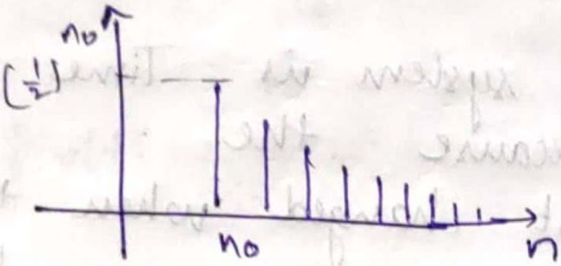
17

$$= \delta[n-n_0] * \left(\tfrac{1}{2}\right)^n u[n]$$

$$= \left(\tfrac{1}{2}\right)^n \left(\delta[n-n_0] * u[n]\right)$$

$$y[n] = \left(\tfrac{1}{2}\right)^n u[n-n_0]$$

$$y[n] = \begin{cases} \left(\tfrac{1}{2}\right)^n & \text{for } n \geq n_0 \\ 0 & \text{for } n < n_0 \end{cases}$$



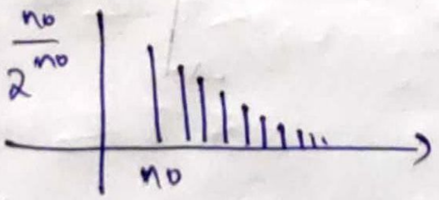
(b) $h[n] = \left(\tfrac{1}{2}\right)^n u[n] \qquad x[n] = u[n]$

$$y[n] = x[n] * h[n]$$

$$= u[n] * \left(\tfrac{1}{2}\right)^n u[n]$$

$$= \left(\tfrac{1}{2}\right)^n \cdot n$$

AS $u[n] * u[n] = n$



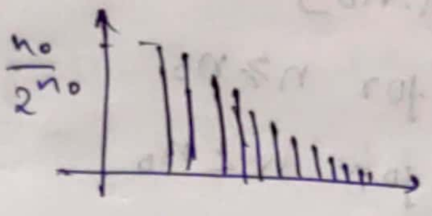
(c) $h[n] = u[n]$

$$x[n] = \left(\tfrac{1}{2}\right)^n u[n]$$

$$y[n] = x[n] * h[n]$$

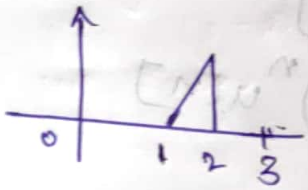
Given $y[n]$ is a linear and time invariant. So convolution follows commutative law, By interchanging the function we get same output

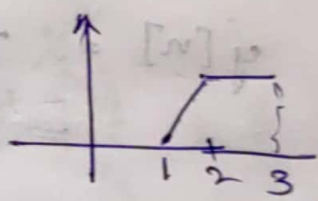
$$y[n] = \frac{n}{2^n}$$



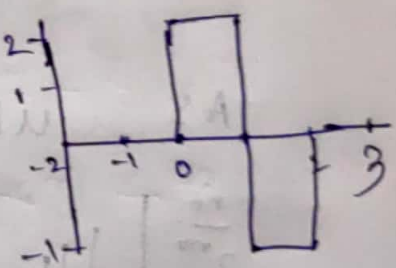
11) a) The given system is Time variant because the output got changed when the input got delayed by 1 second

Expected output Actual output is

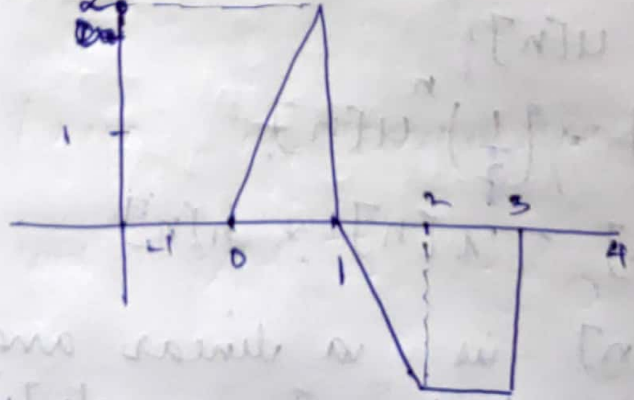




b) If the ~~out~~ input is



the output is



12) a) Given $L\,v(t) = \int_{-\infty}^{t} v(\tau)\, d\tau.$

Take $\cancel{v(t) = \int_{-\infty}^{P} v(\tau - n)}\, d\tau$

put $\tau+n$ instead of τ

where the upper limit changes to p

Now replace τ with $\tau - a$ to have

$$L\,v(t) = \int_{-\infty}^{t} v(\tau - a)\, d\tau)$$

put $\tau - a = t$

so

$$L\,v(t) = \int_{-\infty}^{\boxed{t+a}} v(\tau)\, d\tau = L\,v(t+a)$$

as $v(t) = v(t+a)$

The system is Time Invariant

13) a) $\Pi(x) * \Pi(x) = \int_{-\infty}^{\infty} \Pi(y)\, \pi(x-y)\, dy$

* For $x < -1/2$

$\Pi(x) * \Pi(x) = 0$

* For $-1/2 < x < 1/2$

$\Pi(x) * \Pi(x) = \int_{-1/2}^{x} dy = x + 1/2$

* For $\frac{1}{2} < x < 3/2$

$\Pi(x) * \Pi(x) = \int_{x-1}^{\cancel{x}} = \frac{1}{2} - x + 1 = \frac{3}{2} - x$

* For $x > 3/2$

$\Pi(x) * \Pi(x) = 0$

$$\therefore \quad \Pi(x) * \Pi(x) = \begin{cases} 0 & \text{for } x < -1/2, x > 3/2 \\ x + 1/2 & x \in (-1/2, 1/2) \\ \frac{3}{2} - x & x \in (1/2, 3/2 \end{cases}$$

b) $$\Pi_a * \Pi_a = \begin{cases} 0 & x < -1/2, x > 3/2 \\ a^2(x + 1/2) & x \in (-1/2, 1/2) \\ a^2\left(\frac{3}{2} - x\right) & x \in [1/2, 3/2) \end{cases}$$

14)

a) i) $y(t) = \int_{-\infty}^{t-1} x(\tau)\,d\tau$

we know $y(t) = \int_{-\infty}^{\infty} h(t-\tau)\,d\tau$

by comparision

$$h(t-\tau) = \begin{cases} 1 & -\infty < \tau < t-1 \\ 0 & t-1 < \tau < \infty \end{cases}$$

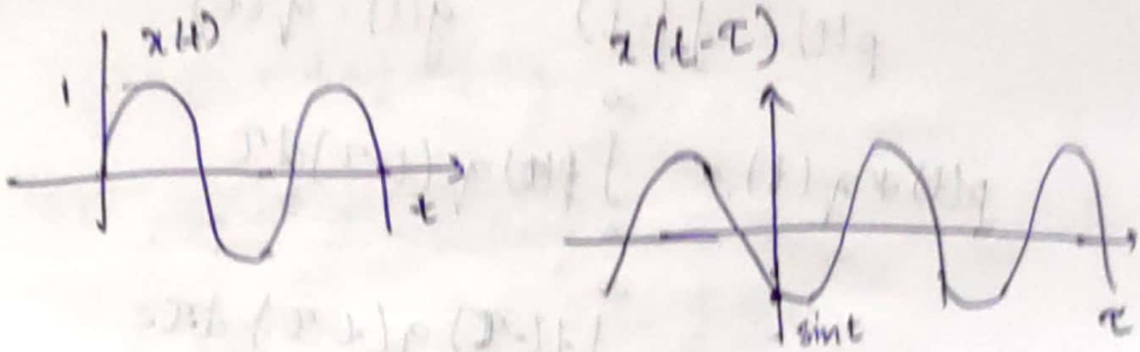
$$\Rightarrow h(\tau) = \begin{cases} 1 & -\infty < \tau \leq 1 \\ 0 & 1 < \tau < \infty \end{cases}$$

ii) Again by comparision

$$h(t-\tau) = \begin{cases} 1 & t - T/2 < t < t + T/2 \\ 0 & \text{otherwise} \end{cases}$$

$$h(\tau) = \begin{cases} 1 & -T/2 < \tau < T/2 \\ 0 & \text{otherwise} \end{cases}$$

b) $h(t) = \Pi(t - 1/2)$ $x(t) = \sin 2\pi t\, u(t)$



$y(t) = x * h(t)$

$= \int_0^{\infty} h(t-\tau)\, x(\tau)\, d\tau$

$= \begin{cases} \int_0^t \sin 2\pi\tau\, d\tau & 0 < t < 1 \\ \int_{t-1}^{t} \sin 2\pi\tau\, d\tau & 1 < t \le 2 \end{cases}$

$= \dfrac{1 - \cos 2\pi t}{2\pi} = \dfrac{\sin^2 2\pi t}{\pi}$ $0 \le t \le 1$

15) a) $y(t) = f(t) * g(t)$

$p(t) = f(-t)$ $q(t) = g(-t)$

$p(t) * q(t)$

$= \int_{-\infty}^{\infty} p(t)\, q(t-\tau)\, d\tau$

$y(t) = \int_{-\infty}^{\infty} f(-t)\, g(\tau - t)\, d\tau$

$y(-t) = \int_{-\infty}^{\infty} f(t)\, g(t-\tau)\, d\tau$

$y(-t) = y(t) = \int_{-\infty}^{\infty} f(t)\, g(t-\tau)\, d\tau$

22

ii) ~~f(t)~~ = ~~p(t)~~ q(

$$p(t) = f(-t) \qquad g(t) = q(t)$$

$$p(t) * q(t) = \int_{-\infty}^{\infty} p(\tau)\, q(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} f(-\tau)\, g(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} f(\tau)\, g(t+\tau)\, d\tau$$

b) $$\underbrace{(T_b f) * g}_{P} = \underbrace{T_b (f*g)}_{Q} = \underbrace{f * (T_b g)}_{R}$$

$$P = f(t-a) * g(t)$$

$$= \int_{-\infty}^{\infty} f(\tau-a)\, g(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} f(\tau)\, g(t-a-\tau)\, d\tau$$

$$= f(t) * g(t-a)$$

$$P = R$$

$$y(t) = \int_{-\infty}^{\infty} f(\tau)\, g(t-\tau)\, d\tau$$

$$y(t-a) = \int_{-\infty}^{\infty} f(\tau)\, g(t-a-\tau)\, d\tau$$

$$Q = f(t) * g(t-a) = P = R$$

$$\therefore \boxed{P = Q = R}$$

17) $\mathcal{F} f'(s) = 2\pi i s \mathcal{F} f(s)$

$$(f*g)' = f'*g = f*g'$$

For $y(t) = f(t)\, g(t)$

$$\mathcal{F}\{y(t)\} = \mathcal{F}(f(t)) \cdot \mathcal{F} g(t)$$

$$\mathcal{F}(f*g)'(t) = 2\pi i s\, \mathcal{F}(f*g)(t)$$

$$= 2\pi i s\, \mathcal{F} f(t) \cdot \mathcal{F} g(t) \quad -①$$

$$\mathcal{F}(f'*g)t = \mathcal{F} f'(t) \cdot \mathcal{F} g(t)$$

$$= 2\pi i s\, \mathcal{F} f(t) \cdot \mathcal{F} g(t) \quad -②$$

$$\mathcal{F}(f*g')(t) = \mathcal{F} f(t) \cdot \mathcal{F} g'(t)$$

$$= 2\pi i s\, \mathcal{F} f(t) \cdot \mathcal{F} g(t) \quad -③$$

As ①, ②, ③ are equal

$$(f*g)' = f'*g = f*g'$$

19) a) $f(x) = \sin 2\pi m x + \sin 2\pi n x$

period $= \frac{1}{m}$ period $= \frac{1}{n}$

As m, n both are integers $\frac{1}{m}, 1/n$ are rational numbers.

common period is LCM of $\frac{1}{m}, \frac{1}{n}$

$$\text{Period} = \frac{1}{HCF(m,n)}$$

b) $g(x) = \sin 2\pi p x + \sin 2\pi q x$

period $= \frac{1}{p}$ period $= \frac{1}{q}$

take $p = \frac{a}{b}$, $q = \frac{c}{d}$

So period for $g(x) = \frac{\text{LCM of numerators}}{\text{HCF of denominators}}$

$$= \frac{\text{LCM}(a, c)}{\text{HCF}(b, d)}$$

Here period exists because $\frac{\frac{1}{p}}{\frac{1}{q}} = \frac{q}{p}$ is rational again

c) $f(t) = \cos t + \cos \sqrt{2} t$

Let us consider T to be the period

$f(t) = f(t + nT)$ where n is integer.

$\cos t + \cos(\sqrt{2} t) = \cos(t + nT) + \cos(t + n\sqrt{2} T)$

Now for $\cos(\sqrt{2} t) = \cos(t + n\sqrt{2} T)$

we must have $n\sqrt{2}$ as an integer. But here $n\sqrt{2}$ is an irrational

$\therefore$ By contradiction it is proved that $f(t)$ is not periodic

20) $f(t) = \sin 3t + \cos 5t$

a) period of $f(t)$ is LCM of $\frac{2\pi}{3}, \frac{2\pi}{5}$

$$= 2\pi$$

$$f(t) = \frac{e^{3it} - e^{-3it}}{2i} + \frac{e^{5it} + e^{-5it}}{2}$$

$$f(t) = \sum_{n=-\infty}^{\infty} F(n)\, e^{\frac{2\pi i n}{2\pi} t}$$

$$= \sum_{n=-\infty}^{\infty} F(n)\, e^{int}$$

$$F(n) = \begin{cases} \frac{1}{2i} & \text{for } n = 3 \\ \frac{-1}{2i} & \text{for } n = -3 \\ \frac{1}{2} & \text{for } n = 5, -5 \\ 0 & \text{otherwise} \end{cases}$$

b) $g(t) = \sin 3t \cos 5t$

$$= \frac{2 \sin 3t \cos 5t}{2}$$

$$= \frac{\sin(8t) - \sin(2t)}{2}$$

period is LCM of $\frac{2\pi}{8}, \frac{2\pi}{2} = \pi$

$$g(t) = \frac{e^{i8t} - e^{-i8t}}{4i} - \frac{e^{2it} - e^{-2it}}{4i}$$

$$= \frac{1}{4i}\left(e^{i8t} - e^{-i8t} - e^{2it} + e^{-2it}\right)$$

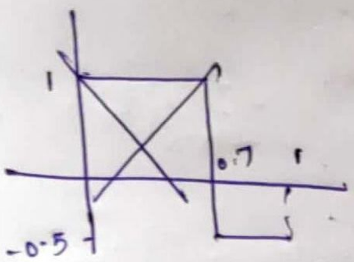
Scanned by CamScanner

$$g(t) = \sum_{n=-\infty}^{\infty} f(n)\, e^{2\pi i \frac{n}{\pi} t}$$

$$= \sum_{n=-\infty}^{\infty} f(n)\, e^{2int}$$

$$f(n) = \begin{cases} \frac{1}{4i} & n = 4, -1 \\ -\frac{1}{4i} & n = -4, 1 \\ 0 & \text{otherwise} \end{cases}$$

20)

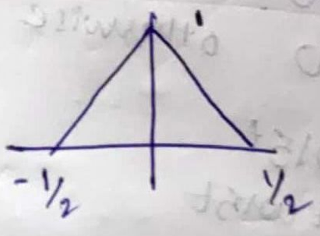


21)

$$\Lambda_p(t) = \begin{cases} 1 - \frac{|t|}{p} & |t| \le p \\ 0 & |t| \ge p \end{cases}$$

given $p = 1/2$

$\Lambda_{1/2} \Rightarrow$



$$g(t) = \sum_{n=-\infty}^{\infty} \Lambda_p(t - nT)$$

For $T = 1/2$

$$\Lambda_p(t - m) = \sum_{n=-\infty}^{\infty} \Lambda_p\left(t - \frac{n}{2}\right)$$

(27)

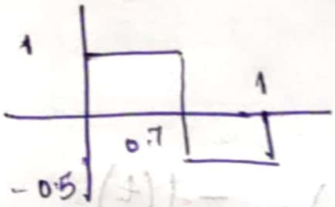
22) b) $d_n = (-1)^n c_n = e^{2\pi i n} \cdot c_n$

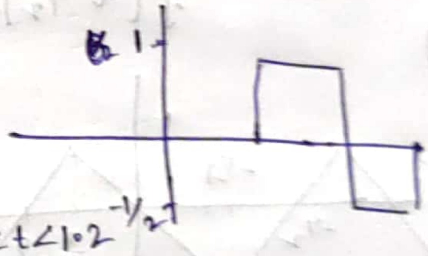
$d_n = e^{2\pi i n (1/2)} c_n$

$f(t) \to f(t - 1/2)$

$c_n \to d_n$

$s(t) = f(t - 1/2)$

$f(t) =$ 

$g(t) = f(t - 0.5)$ 

$$g(t) = \begin{cases} 1 & \frac{1}{2} < t < 1.2 \\ -1/2 & 1.2 < t < 1.5 \\ 0 & \text{otherwise} \end{cases}$$

c) $d_n = \begin{cases} c_n & n \text{ is even} \\ 0 & n \text{ is odd} \end{cases}$

$= \frac{1}{2} (c_n + \bar{c}_n)$

$$\int_{-1/2}^{1/2} g(t) e^{-2\pi i n t} dt = \frac{1}{2} \left[\int_{-1/2}^{1/2} f(t) e^{-2\pi i n t} dt + \int_{-1/2}^{1/2} f(t) e^{2\pi i n t} dt \right]$$

Scanned by CamScanner

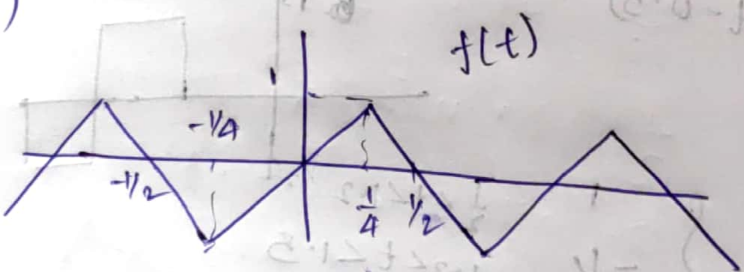
$$g(t) = \frac{f(t) + f(-t)}{2}$$

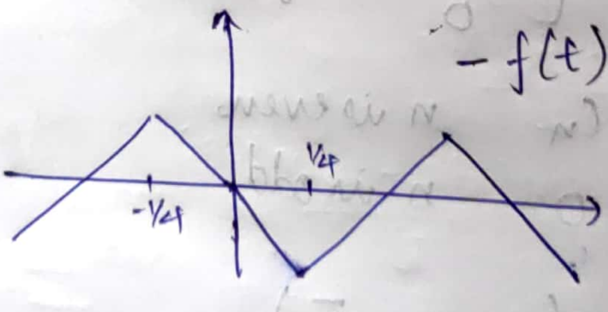
$$f(t) = \begin{cases} 1 & 0 < t < 0.7 \\ -0.5 & 0.7 < t < 1 \end{cases}$$

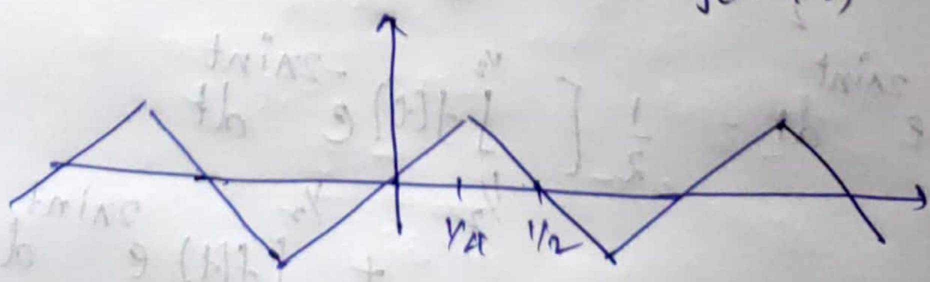
$$g(t) = \begin{cases} -1/2 & -1 < t < -0.7 \\ 1 & -0.7 < t < 0 \\ 1 & 0 < t < 0.7 \\ -0.5 & 0.7 < t < 1 \\ 0 & \text{otherwise} \end{cases}$$

23) Given

a) $f(t - 1/2) = -f(t)$







Clearly $-f(t - 1/2) = f(t)$

b) $f(t) = \sum_{n=-\infty}^{\infty} c_n e^{2\pi i n t}$

$- f(t-1/2) = -\sum_{n=-\infty}^{\infty} c_n e^{\pi i n (t-1/2)}$

$= f(t)$

$c_n e^{\pi i n}$

$= -c_n$

if n is odd.

$-c_n = -\int_0^1 e^{-2\pi i n t} f(t)\, dt$

$d_n = \int_0^1 f(t-1/2) e^{-2\pi i n t} dt$

$d_n = c_n e^{\pi i n}$ $f(t-1/2) = \sum_{n=-\infty}^{\infty} d_n e^{2\pi i n t}$

$c_n = -d_n$

$= f(t) = \sum_{n=-\infty}^{\infty} c_n e^{2\pi i n t}$

By solving both we get

n is even