PARUL UNIVERSITY
FACULTY OF ENGINEERING AND TECHNOLOGY
B. TECH. PROGRAMME (3rd SEM) (CSE/ IT)
DISCRETE MATHEMATICS (303191202)
ACADEMIC YEAR 2023-24

Assignment for detain students

Q.1. List the ordered pairs in the relation R from A = {0, 1, 2, 3, 4} to B = {0, 1, 2, 3}, where (a, b)$\in$ R if and only if 1) $a \leq b$ 2) $a + b \geq 3$.

Q.2. Given A = {1, 2, 3, 4} and B = {x, y, z}. Let $R = \{(1, x), (1, z), (3, y), (4, x), (4, z)\}$

(a) Determine the matrix of the relation.
(b) Draw Digraph of R
(c) Find the inverse of R.
(d) Determine the domain and range of R.

Q.3 Let A = {1,2,3,4} and B = {2, 4, 6, 7}. R be on relation on A defined by 'x divides y'.

(a) Write R as a set of ordered pairs.
(b) Find the inverse of R.
(c) Matrix Representation of inverse R.

Q.4 Each of following defines a relation on the positive integers

1) xy is the square of an integer.
2) x+y=10

determine which relations are reflexive, symmetric, antisymmetric and/or transitive.

Q.5 Prove that if R is equivalence relation on a Set A, then R^{-1} is also an equivalence relation.

Q.6 Check if $\mathbb{N}$ with the '$divides$' relation is a POSet.

Q.7 Let R and S be the following relations on A = {a, b, c, d}.
$R = \{(a.a), (a, c), (c, b), (c, d), (d, b)\}$ and $S = \{(b, a), (c, c), (c, d), (d, a)\}$
Find the composition relation a) $R0S, SOR$ b) Matrix representation of $R0S, SOR$

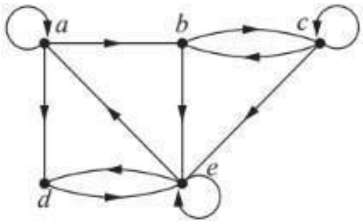
Q.8 Let R and S be relations on a set A represented by the matrices

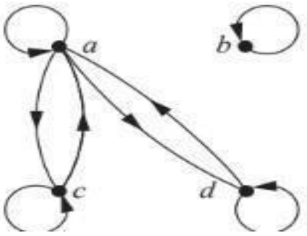
$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \text{ and } M_s \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

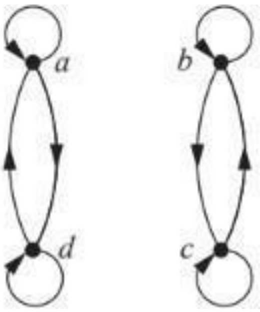
Find the matrices representing the following relations.
(a) $R \cup S$ (b) $R \cap S$ (c) $S \circ R$ (d) $R \circ$ S (e) $R \oplus S$

Q.10 Let $A = \{1,2,3 \ldots\ldots\ldots\ldots, 9\}$. Let R be relation on $A \times A$ defined by $(a, b)\ R\ (c, d)$ $if\ a + d = b + c$ Check that relation is equivalence or partial ordering relation.

Q.10 Write the relation represented by the following digraph and also write the matrix representing this relation.







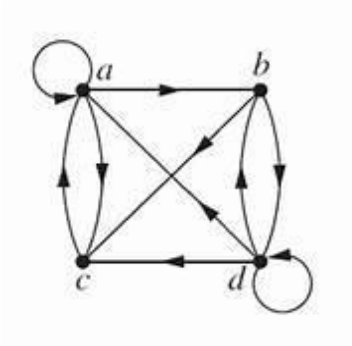
Q.11 Write the relation represented by the following matrices and also draw the corresponding digraph.
Also check that relation is equivalence or partial ordering relation.

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

Q.12 Consider the following. Find reflexive closure, symmetric closure and transitive closure

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

Q.13 Find reflexive closure, symmetric closure and transitive closure



Q.14 Prove that using principle of mathematical induction $1 + 2 + 2^2 + \cdots + 2^n = 2^{n+1} - 1$

Q.15 Using Euclidean algorithm find a)gcd(252,105) b) gcd(1220,516) c)gcd(1701,3768)

Q.16 How many different bit strings of length seven are there?

Q.17 How many different licenses plate can be made if each plate contains a sequence of three upper case English letters followed by three digits?

Q.18 There are 18 mathematics majors and 325 computer sciences majors at a college.

(a) In how many can two representatives be picked so that one is a mathematics major and the other is a computer sciences major?

(b) In how many ways can one representative be picked who is either a mathematics or acomputer science?

Q.19 How many bits of strings of length seven either begin with two 0s or end with three 1s?

Q.20 In how many ways can we select three students from a group of five students to stand in line picture?

Q.21 How many ways are there to select a first prize, second prize and third prize winner from 100 different who have entered a contest?

Q.22 How many permutations of the letters ABCDEFGH contain string ABC?

Q.23 Suppose that a department contains 10 men and 15 women. How many ways are there to a committee with six members if it must have the same number of men and women?

Q.24 Define the following terms:

(a) Proposition
(b) Truth value of proposition
(c) Tautology and Contradiction
(d) Converse, contrapositive and Inverse of propositions.
(e) Universal quantifier.
(f) Logically equivalent propositions.
(g) Propositional satisfiability

Q.25 Construct a truth table for each of these compound propositions

a) $(p \rightarrow q) \rightarrow (\neg q \rightarrow p)$
b) $(p \leftrightarrow q) \oplus (\neg p \leftrightarrow r)$
c) $(p \rightarrow q) \vee (\neg p \rightarrow \neg r)$

Q.26 Show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent by using the laws of logical equivalences.

Q.27 State the converse, contrapositive and Inverse of propositions of each of the following statements:

a) If you eat too much junk food, then you might gain weight.
b) If the power goes out, then the computer will shut down.
c) A positive integer is a prime only if it has no divisors other than 1 and itself.

Q.28 Let p and q be the propositions

p : The program is readable.
q : The program is well structured.

Express each of these propositions as an English sentence.

a) ¬p b) $p \vee q$ c) $p \rightarrow q$ d) $p \wedge q$
e) $p \leftrightarrow q$ f) $\neg p \rightarrow \neg q$ g) $\neg p \wedge \neg q$ h) $\neg p \vee (p \wedge q)$

Q.29 Construct a truth table for each of following compound proposition:

(a) $p \Longrightarrow \sim p$, (b) $p \oplus (p \vee q)$,(c) $(p \vee q) \Longrightarrow (p \wedge q)$

Q.30 Find the bitwise OR, bitwise And & bitwise XOR of each of the following pairs of bit strings:

(a) 1011110100, 1110001110
(b) 110010000, 111110001

Q.31 State the following Laws for mathematical logic and prove them using truth table:

(a) Associative laws.
(b) Distributive laws.
(c) De Morgan’s laws.

Q.32 Show that each of these conditional statements is a tautology by using truth tables.
a) $[(p \wedge q) \rightarrow (p \vee q)]$
b) $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$

Q.33 Determine whether the given compound propositions is satisfiable.
$(p \leftrightarrow q) \wedge (p \leftrightarrow \neg q)$

Q.34 Prove that if *n* is an integer and n^2 is odd, then *n* is odd.

Q.35 Use the method of contradiction, prove that $\sqrt{p}$, p is prime is irrational number

Q.36 Use a direct proof to show that the sum of two odd integers is even.

Q.37 Give a direct proof that if *m* and *n* are both perfect squares, then *nm* is also a perfect square.

Q.38 Show that if *n* is an integer and n^3+5 is odd, then *n* is even using
a) a proof by contraposition.
b) a proof by contradiction.

Q.39 Prove that For all integers a, b and c , if a divides b and b divides c, then a divides c.

Q.40 Identify the identity element in $\mathbb{Z}$ under the operation $*$ given as $a * b = a + b - 3$, for any $a, b \in \mathbb{Z}$. Also identify the inverse element of any member $a \in \mathbb{Z}$

Q.41 Check if the set of all non-negative integers is an abelian group under usual addition of integers.

Q.42 Show that the binary operation $*$ defined on the set of positive rational numbers Q^+, by

$a * b = \frac{ab}{4}$ forms an abelian group.